

Boris
Aronov

Bettina
Speckmann

Maike
Buchin

Kevin
Buchin

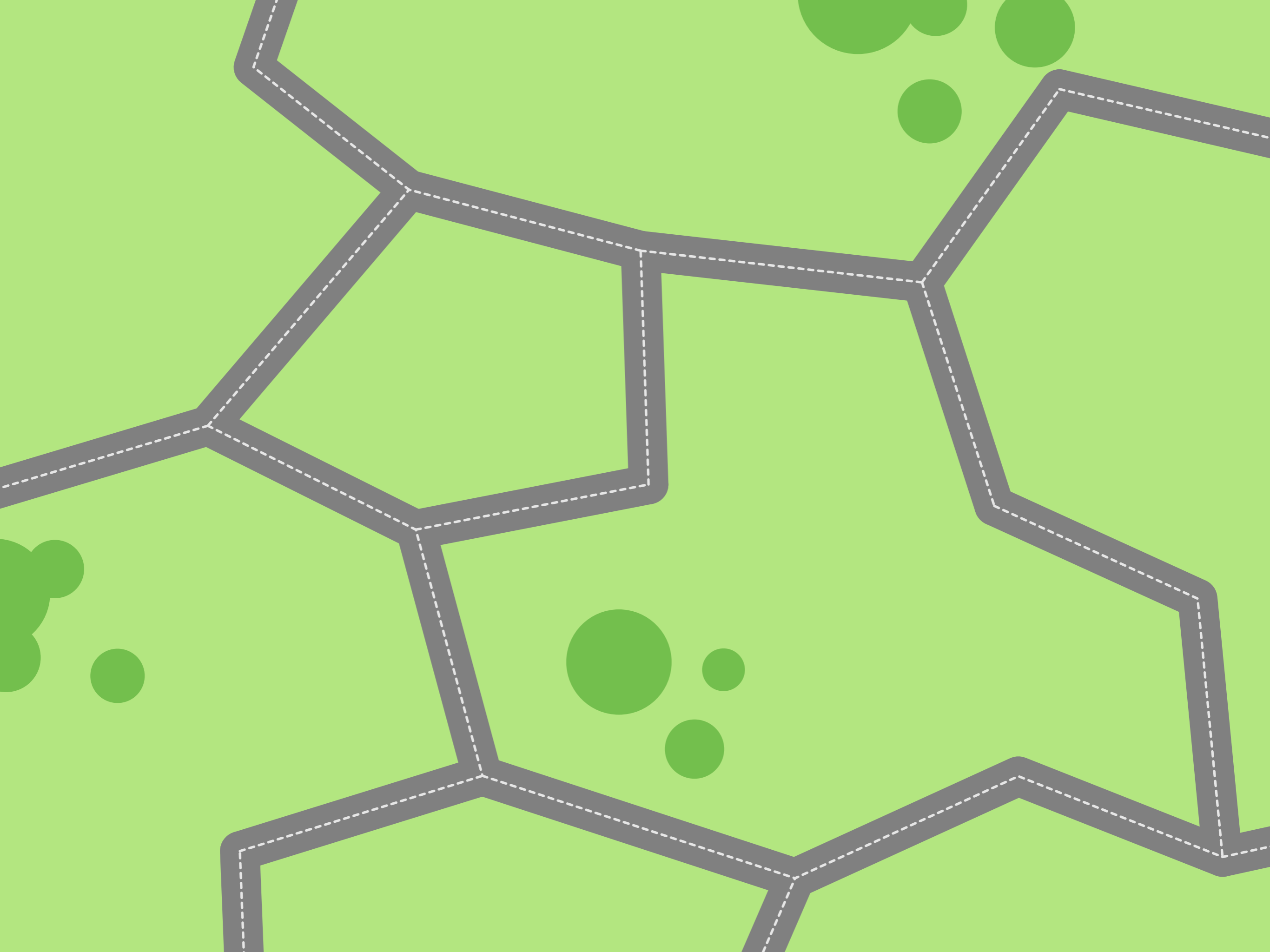
Connect The Dot
or
Computing Feed Links
with
Minimum Dilation

Jun
Luo

Rodrigo
Silveira

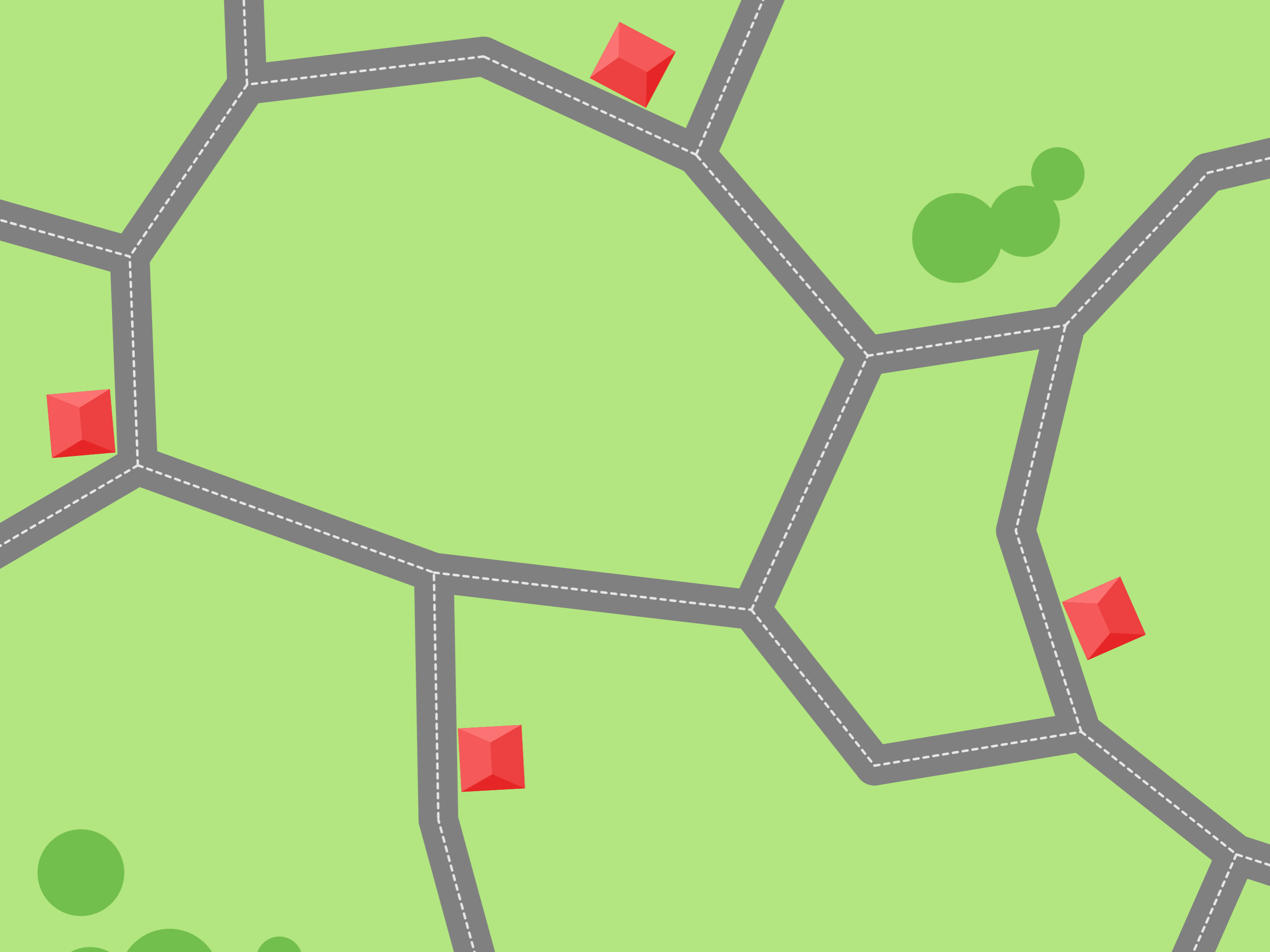
Marc
van Kreveld

Maarten
Löffler



The image features a light green background with a network of dark grey roads. The roads are represented by solid lines with a dashed white line running parallel to them, creating a sense of depth. The network consists of several interconnected paths that form irregular, roughly hexagonal shapes. In the center of the network, there is a white, cloud-like speech bubble with a thin grey outline. Inside this bubble, the text "Consider a network of roads." is written in a simple, grey, sans-serif font. Scattered throughout the green background are several solid green circles of varying sizes, some of which are partially obscured by the road lines.

Consider a network of roads.



Also consider a set of *interesting* locations.



A stylized map with a network of roads and interesting locations. The background is light green. A network of dark grey roads with dashed white lines runs across the map. There are four red 3D cube-like shapes representing interesting locations: one at the top, one on the left, one at the bottom, and one on the right. There are also several green circles of different sizes representing trees or bushes. Two white speech bubbles with grey outlines contain text.

Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.



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In many applications, the network distance between locations is important.

A network diagram on a green background. A grey road network is shown with dashed lines indicating connections. Four red cube markers are placed at various points on the network. A yellow path highlights a specific route through the network. Two white speech bubbles contain text.

Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.

A network diagram on a light green background. The network consists of several interconnected grey paths with dashed white lines. One path is highlighted in yellow. There are four red 3D cube markers placed at various points on the network. In the top right, there are three green circles of varying sizes. In the bottom left, there are several green circles of varying sizes.

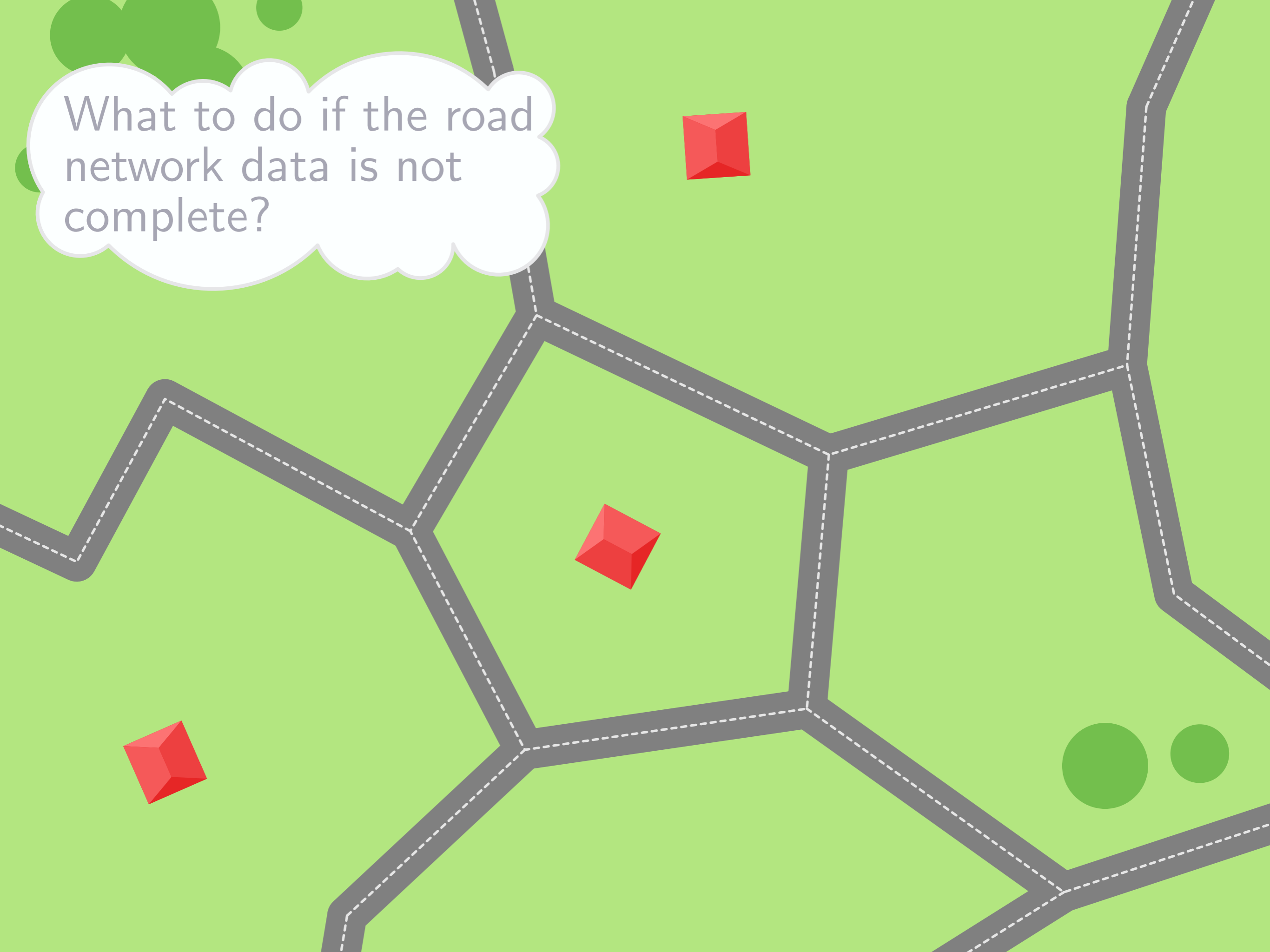
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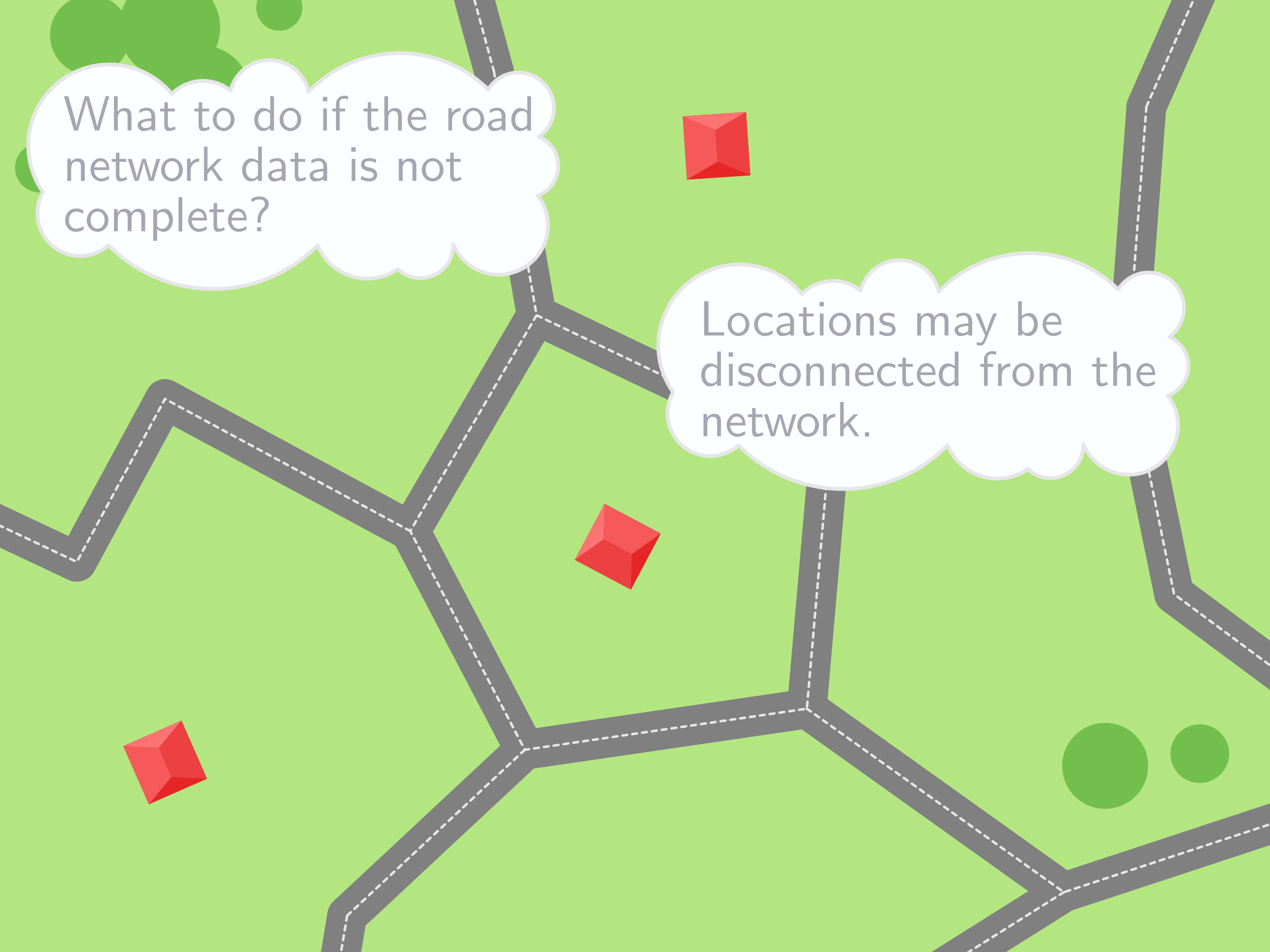
In many applications, the network distance between locations is important.

These are well known and easy to compute.



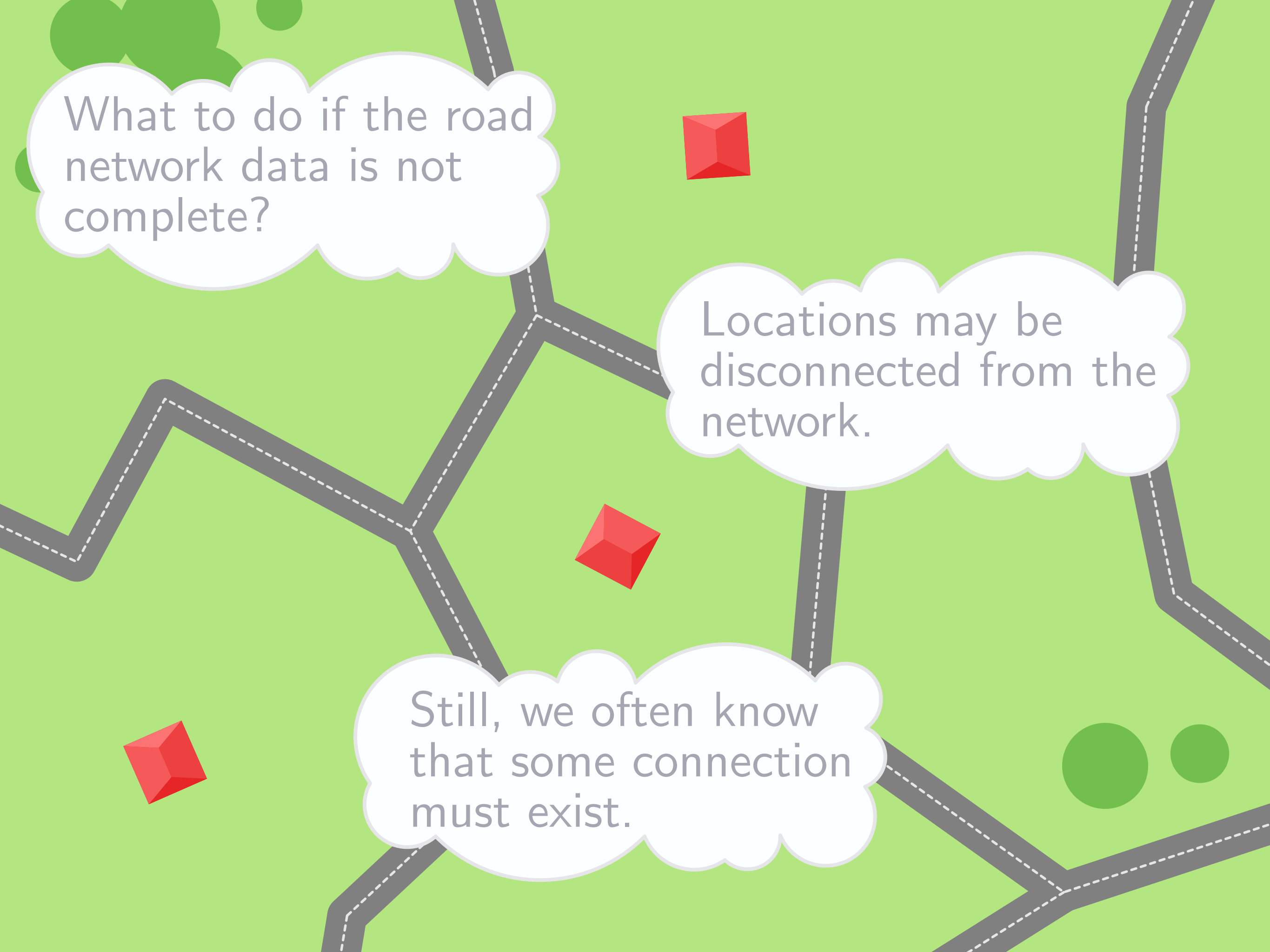
What to do if the road network data is not complete?





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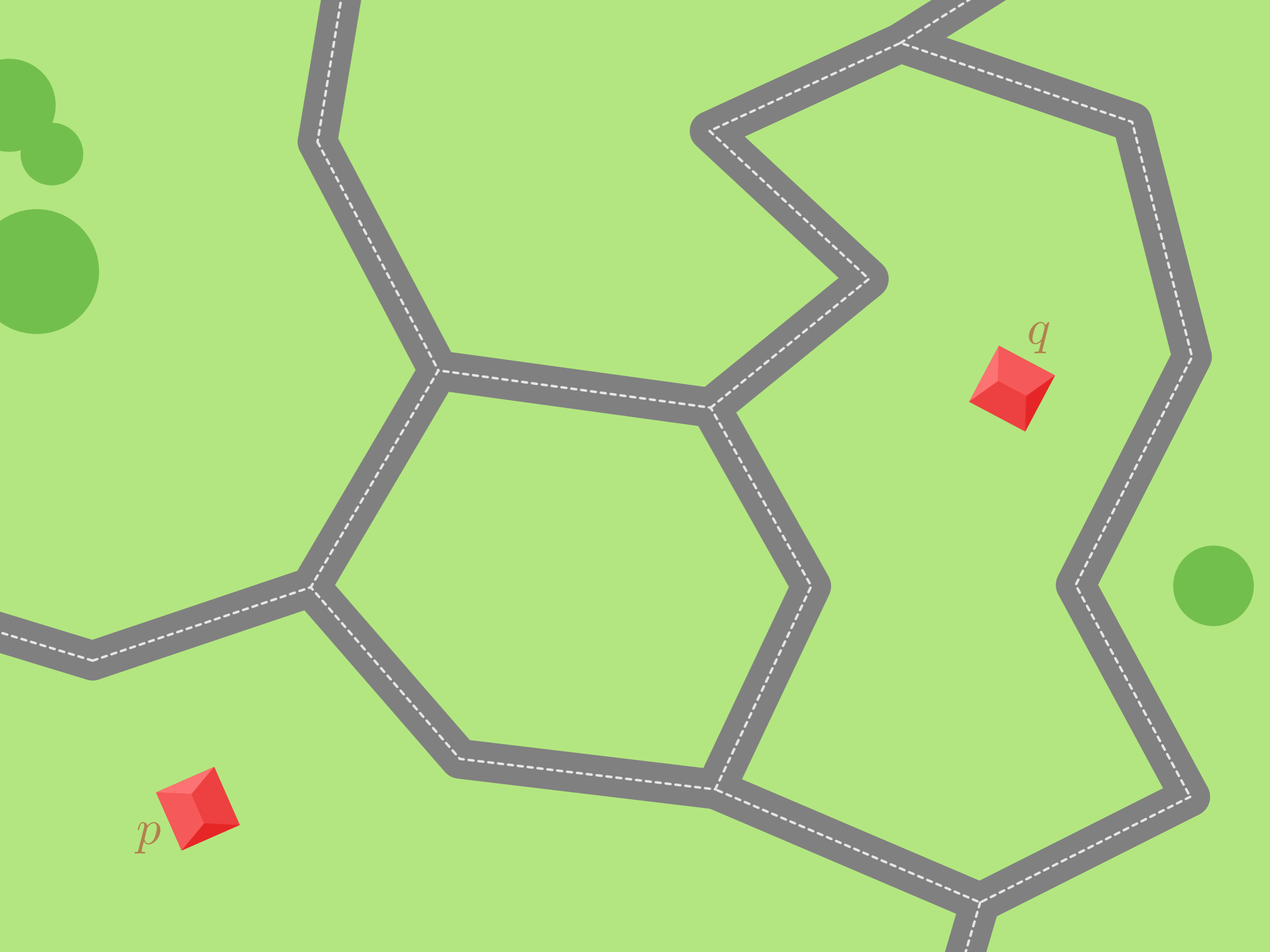
Locations may be disconnected from the network.



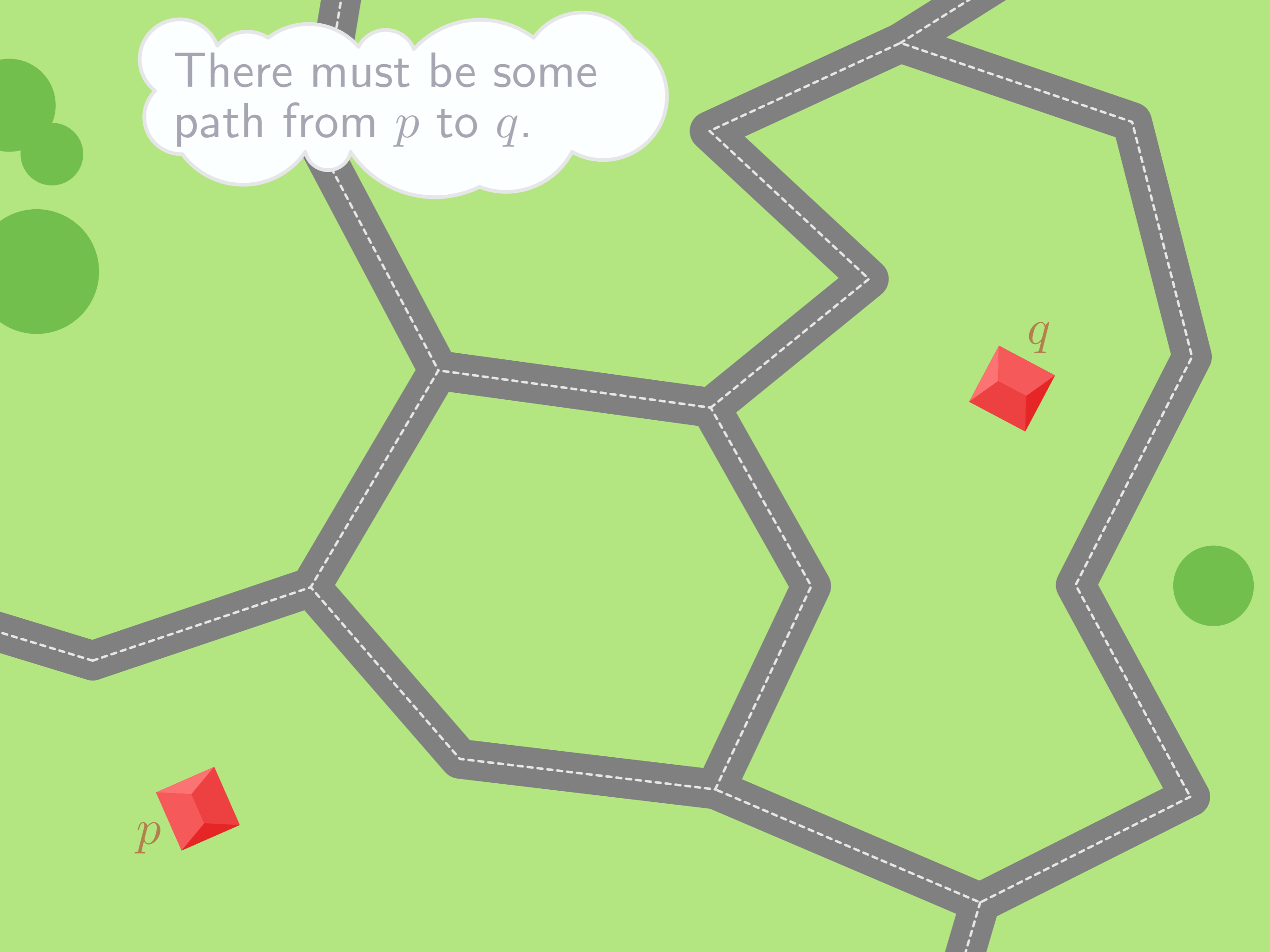
What to do if the road network data is not complete?

Locations may be disconnected from the network.

Still, we often know that some connection must exist.



There must be some path from p to q .



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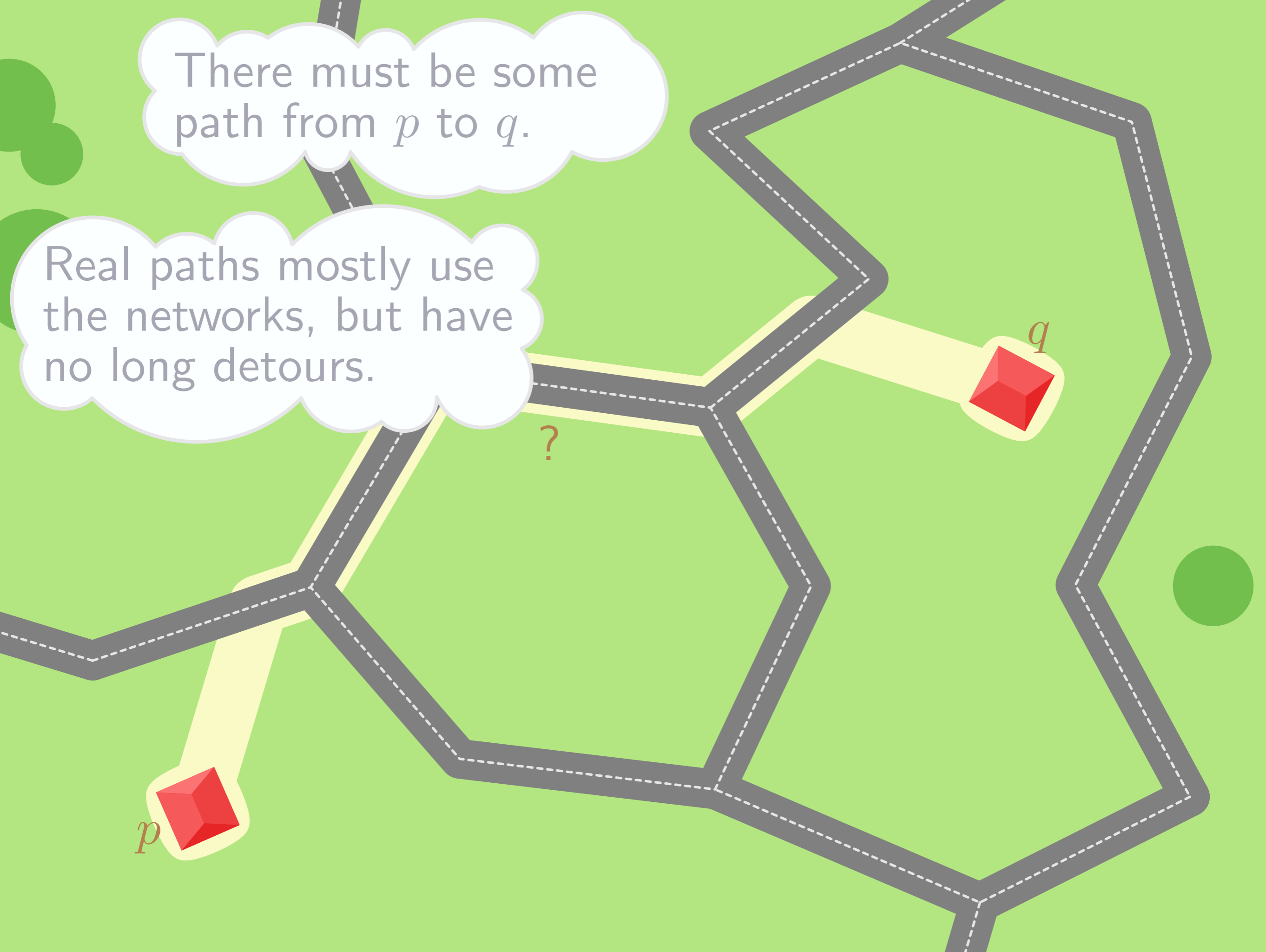


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Real paths mostly use the networks, but have no long detours.



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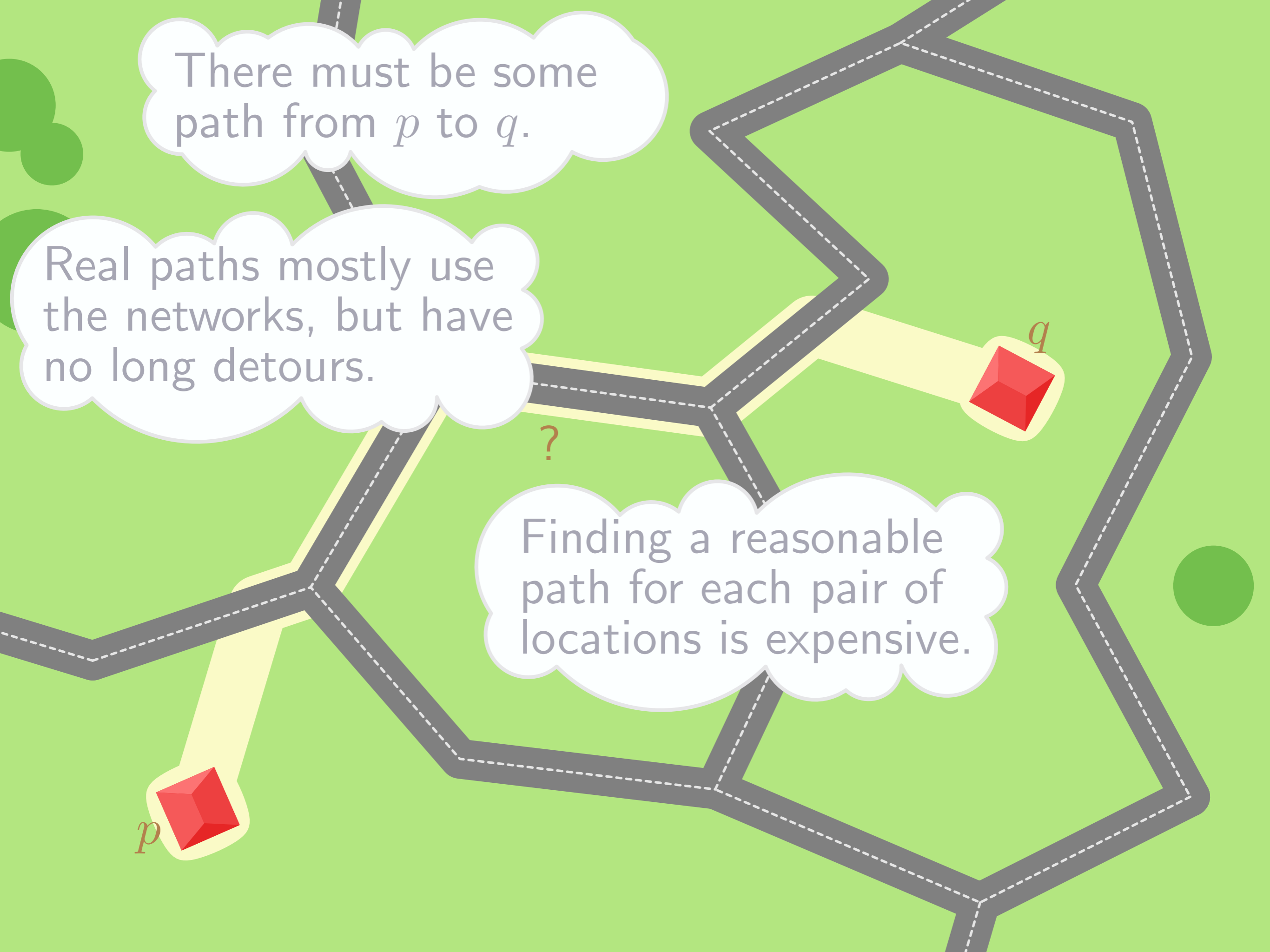
Real paths mostly use the networks, but have no long detours.

?

Finding a reasonable path for each pair of locations is expensive.

p

q



A network diagram on a green background. A dark grey path with a dashed white center line starts at a red gemstone labeled 'p' at the bottom left and ends at another red gemstone labeled 'q' at the top right. A yellow highlight follows the path from 'p' through several nodes and edges to 'q'. A red question mark is placed at a junction point on the path. Three white thought bubbles with grey text are overlaid on the diagram.

There must be some path from p to q .

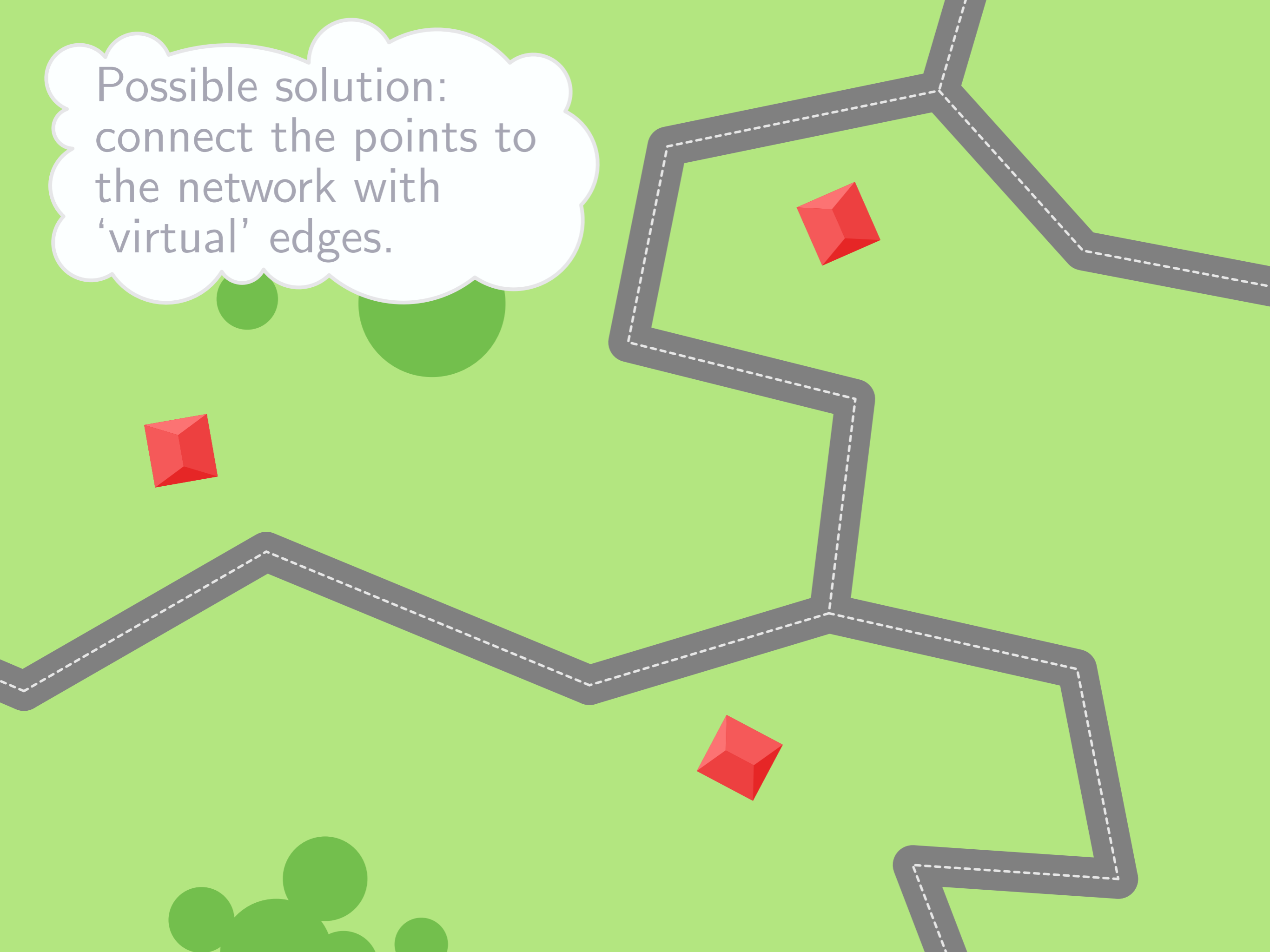
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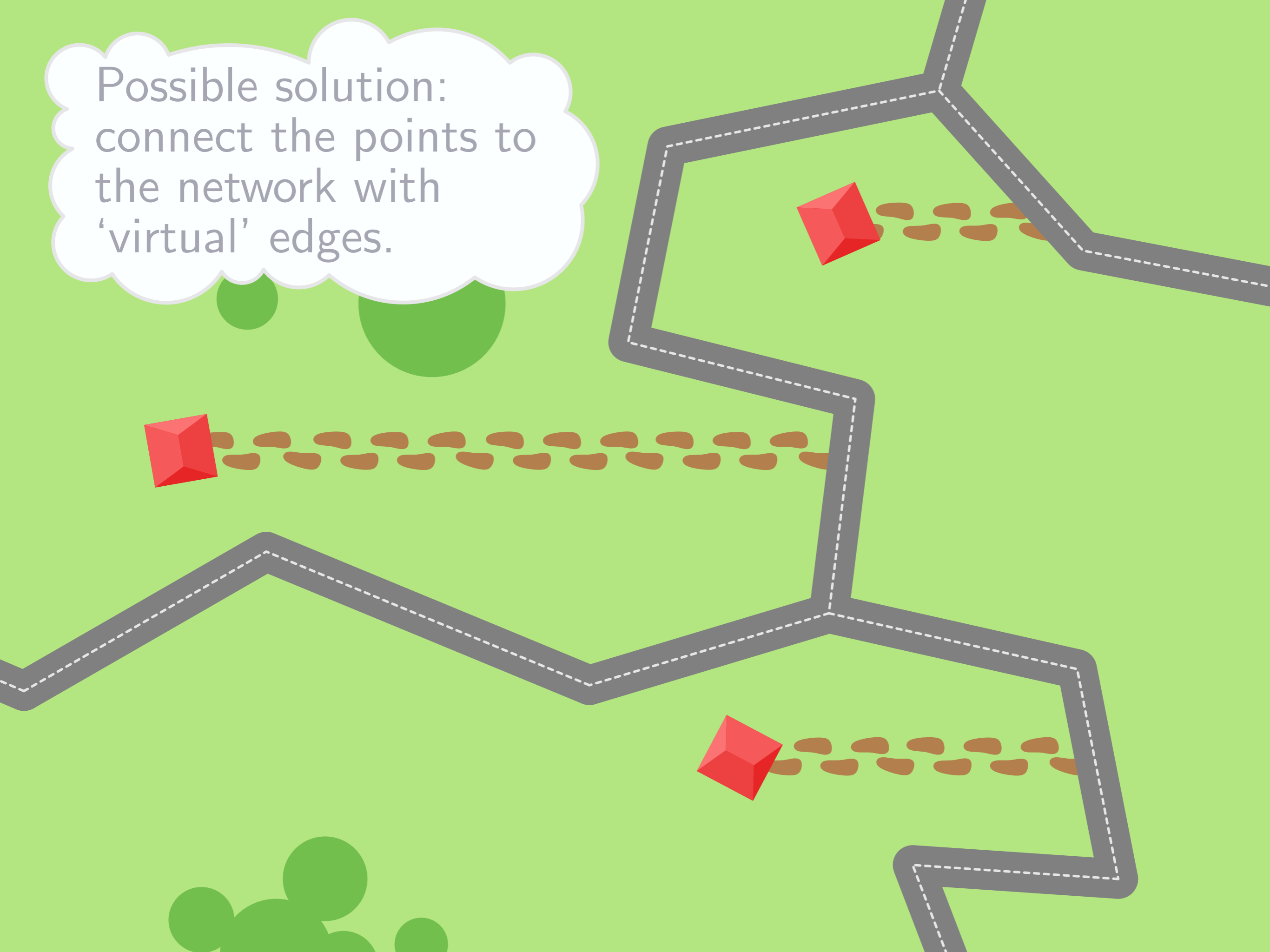
What can we do?



Possible solution:
connect the points to
the network with
'virtual' edges.

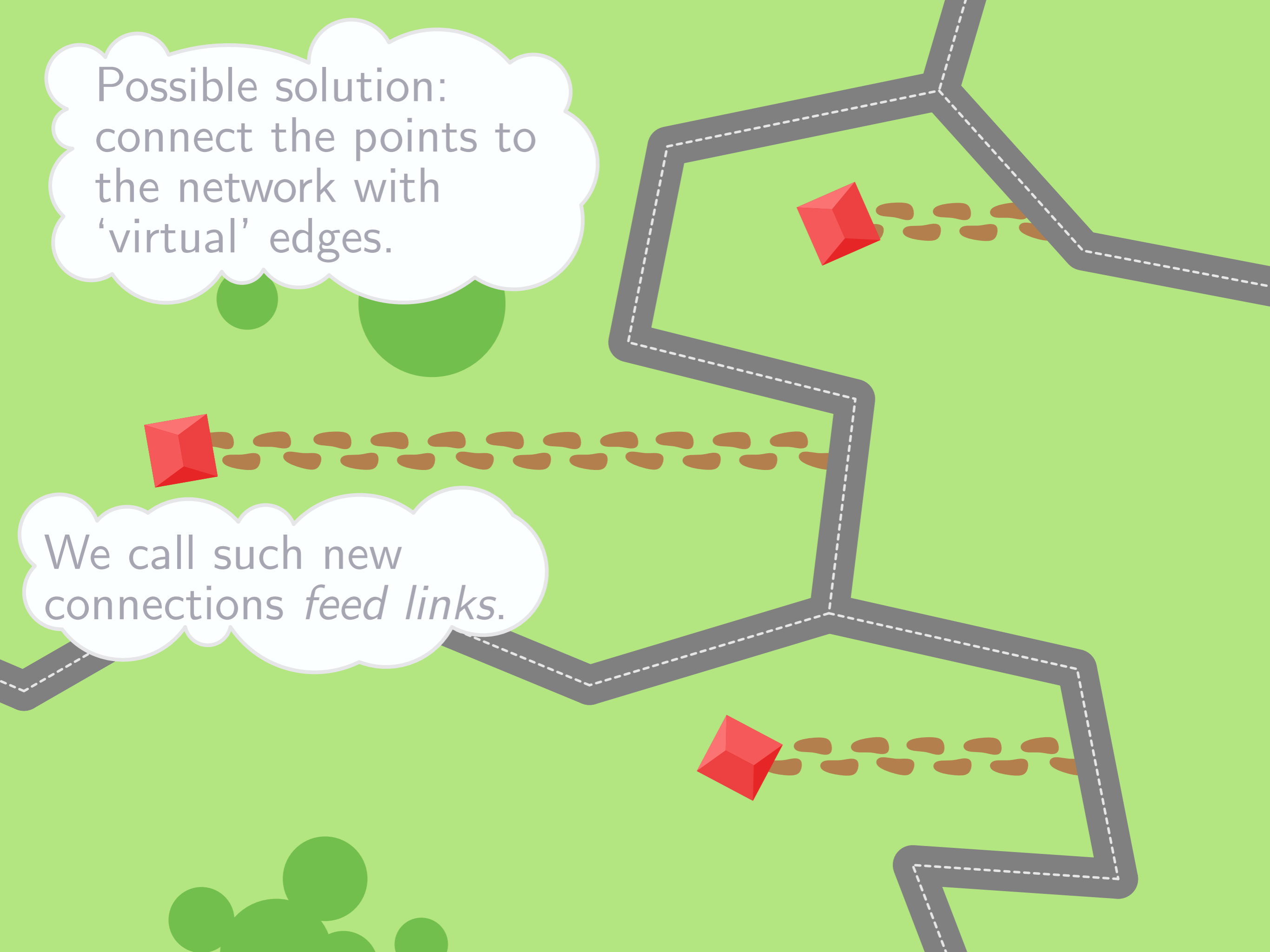


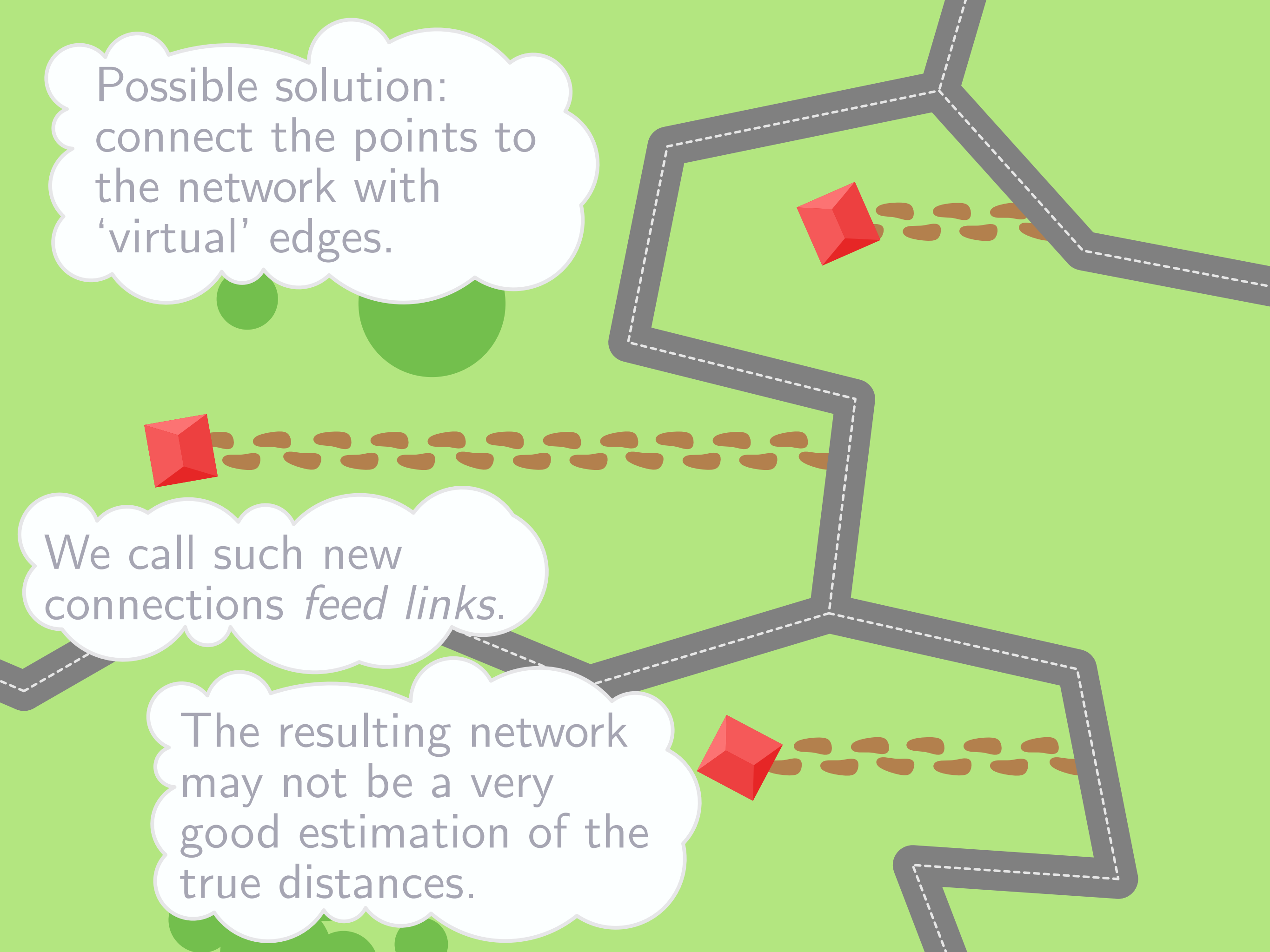
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We call such new
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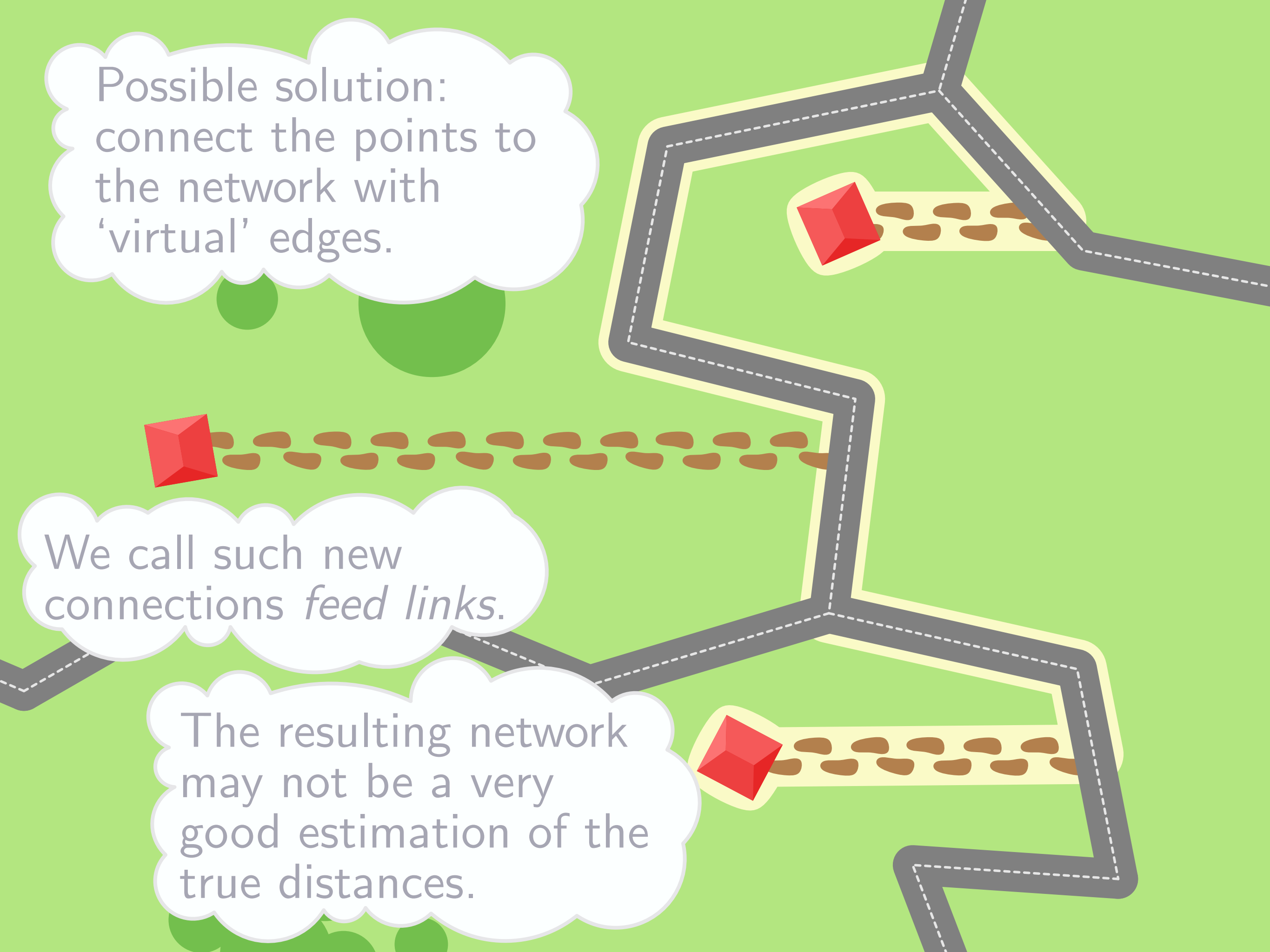




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How do we capture the quality of the feed links?



A stylized map on a light green background. A winding grey path with a dashed white line inside it starts from the left and moves towards the right. In the center of the map, there is a red 3D cube. There are several green circles of various sizes scattered around the map, representing trees or bushes. Two white thought bubbles with grey outlines are connected to the path.

How do we capture the quality of the feed links?

We want to add feed links that do not induce long detours.

A stylized map with a green background and grey road network. A red cube is located in the center, with a trail of brown footprints leading to it from a road junction. Two white thought bubbles are present: one on the left containing the question 'How do we capture the quality of the feed links?' and one on the right containing the statement 'We want to add feed links that do not induce long detours.' The road network consists of solid grey lines with dashed white lines inside, forming a complex path. There are several green circles of varying sizes scattered across the map, representing trees or bushes.

How do we capture the quality of the feed links?

We want to add feed links that do not induce long detours.

A network diagram on a green background. It features several grey paths with dashed white lines, representing a network. A red 3D cube is positioned in the lower-middle area. A brown, dotted path leads from the cube upwards towards the network. Three white thought bubbles with grey outlines contain text. The background has some green circular shapes representing trees or bushes.

How do we capture the quality of the feed links?

We want to add feed links that do not induce long detours.

The *dilation* between two points p and r is:

$$\delta = \frac{\text{shortestpath}(p, r)}{|pr|}$$

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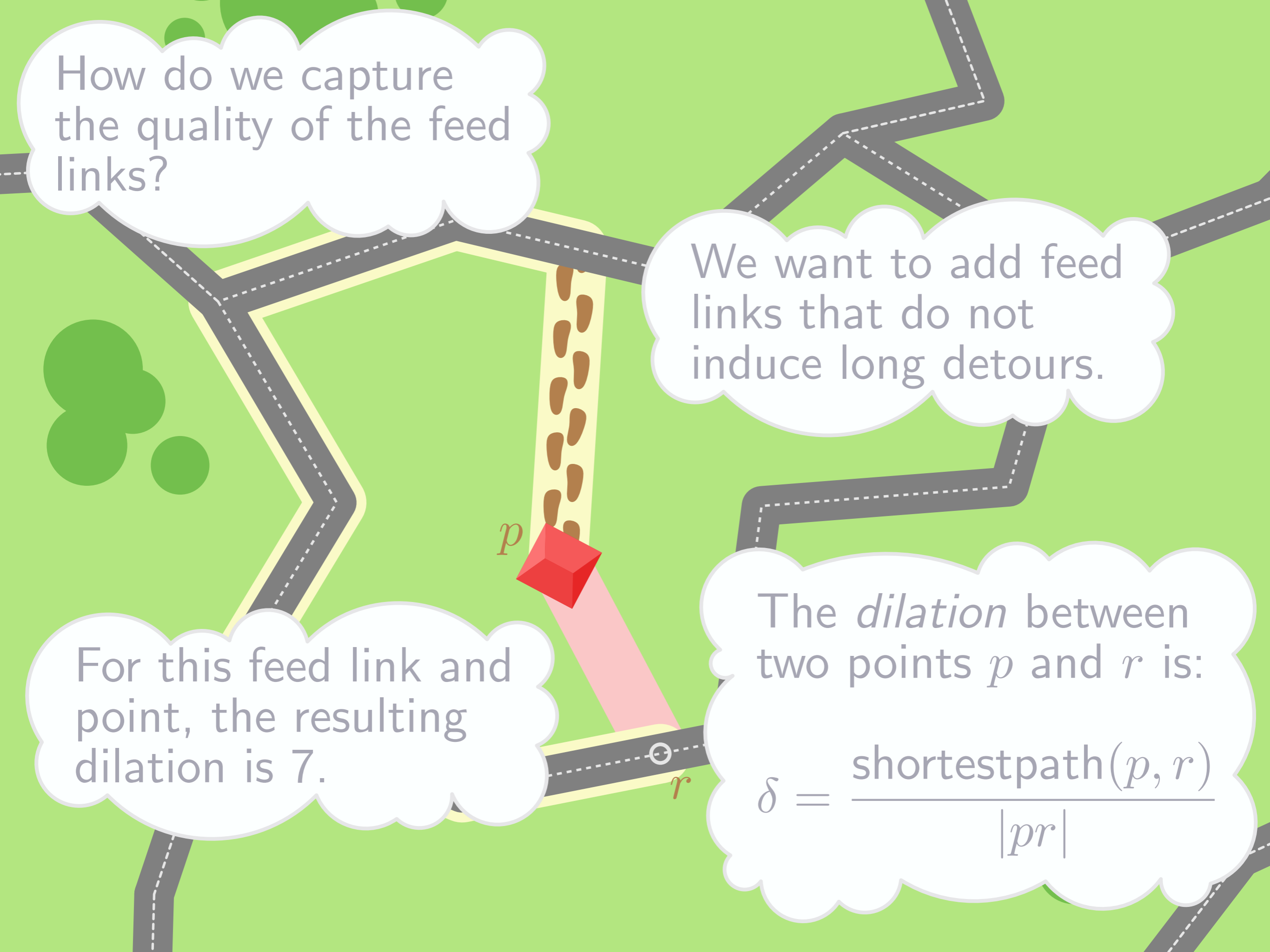
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We want to add feed links that do not induce long detours.

For this feed link and point, the resulting dilation is 7.

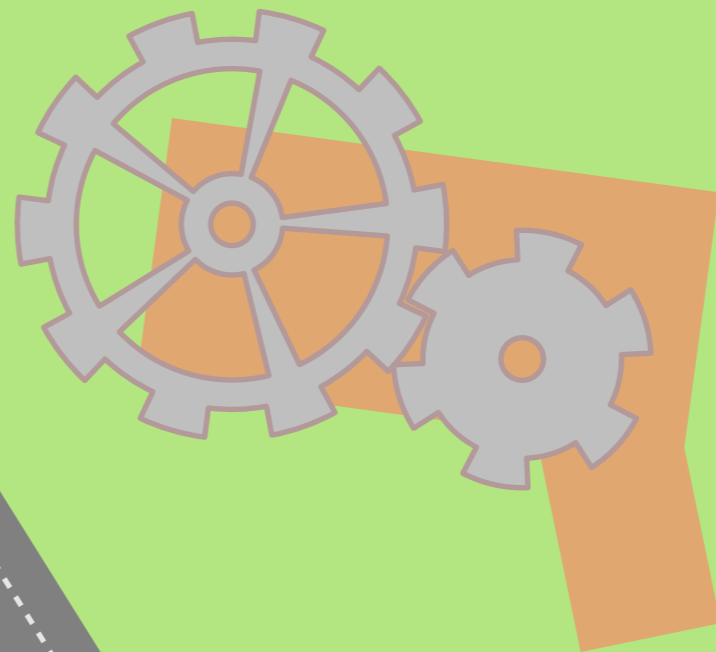
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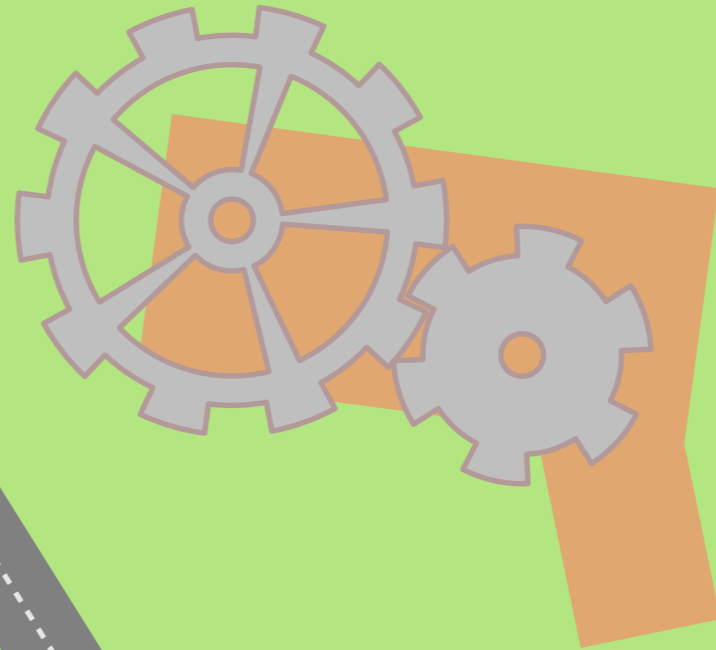
Consider a simple polygon P with a single point of interest p inside.



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P

p



Consider a simple polygon P with a single point of interest p inside.

Question: Can we place one feed link so that the worst dilation from p to any point on P is minimised?

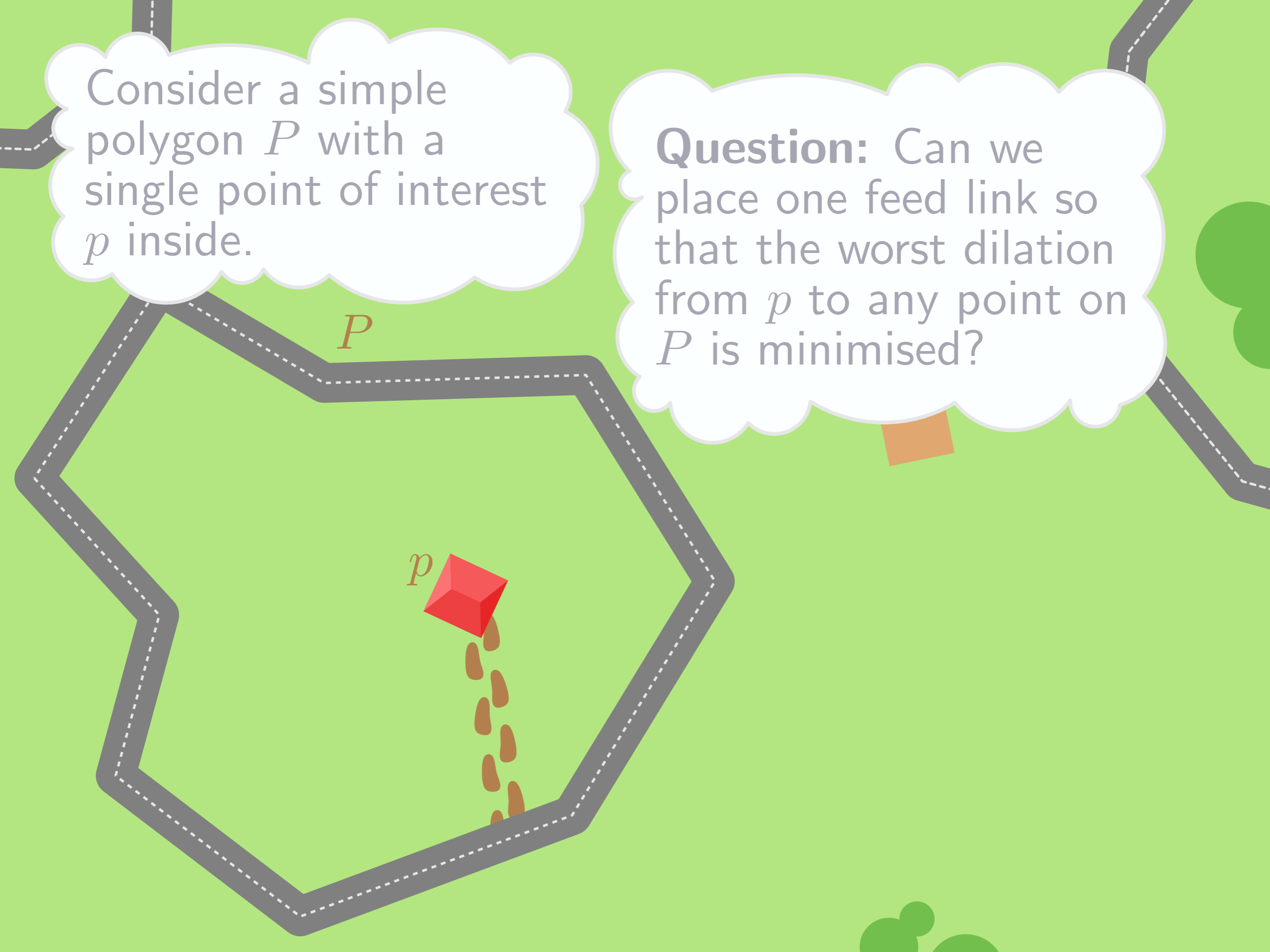
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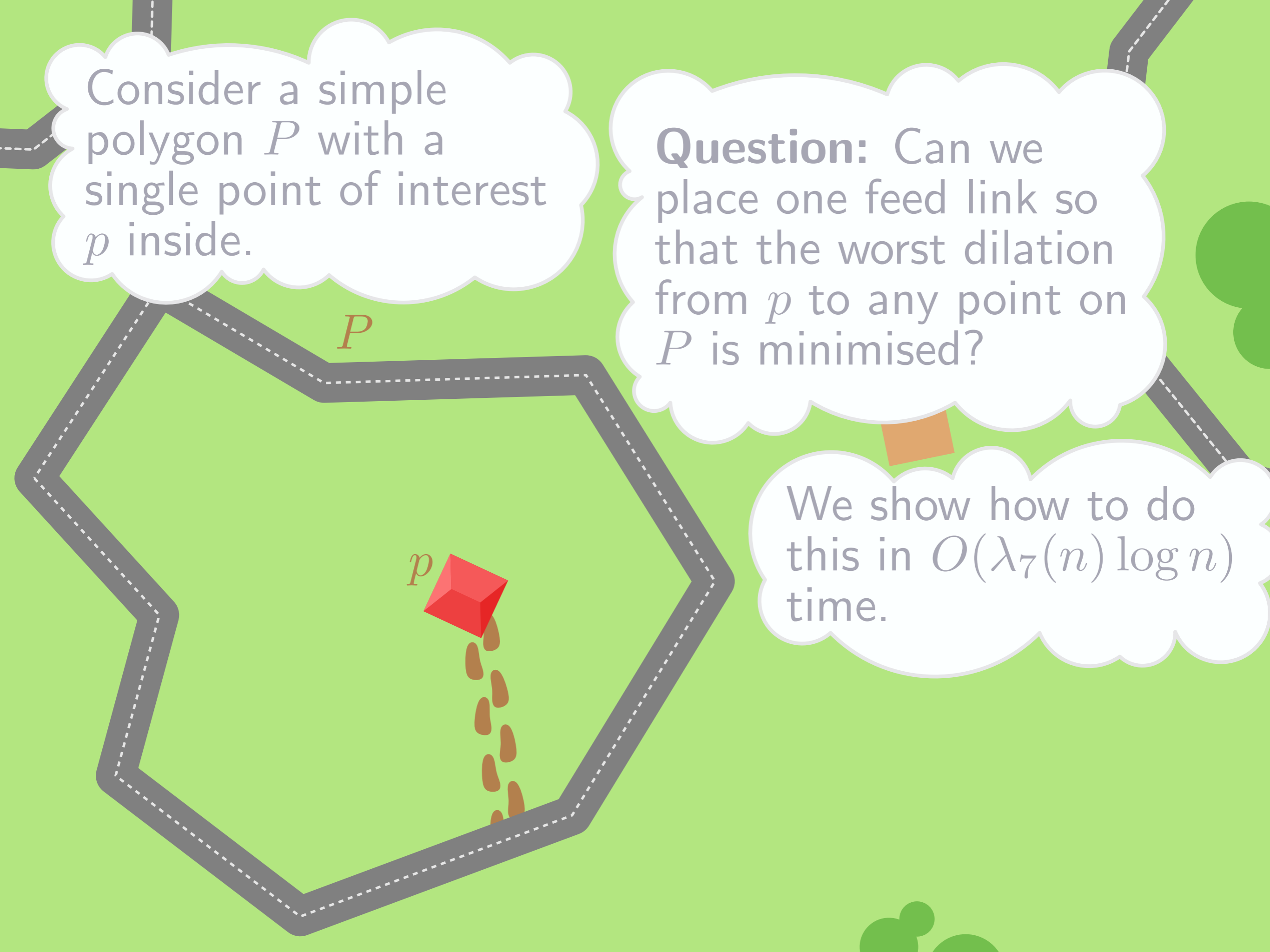
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We show how to do this in $O(\lambda_7(n) \log n)$ time.

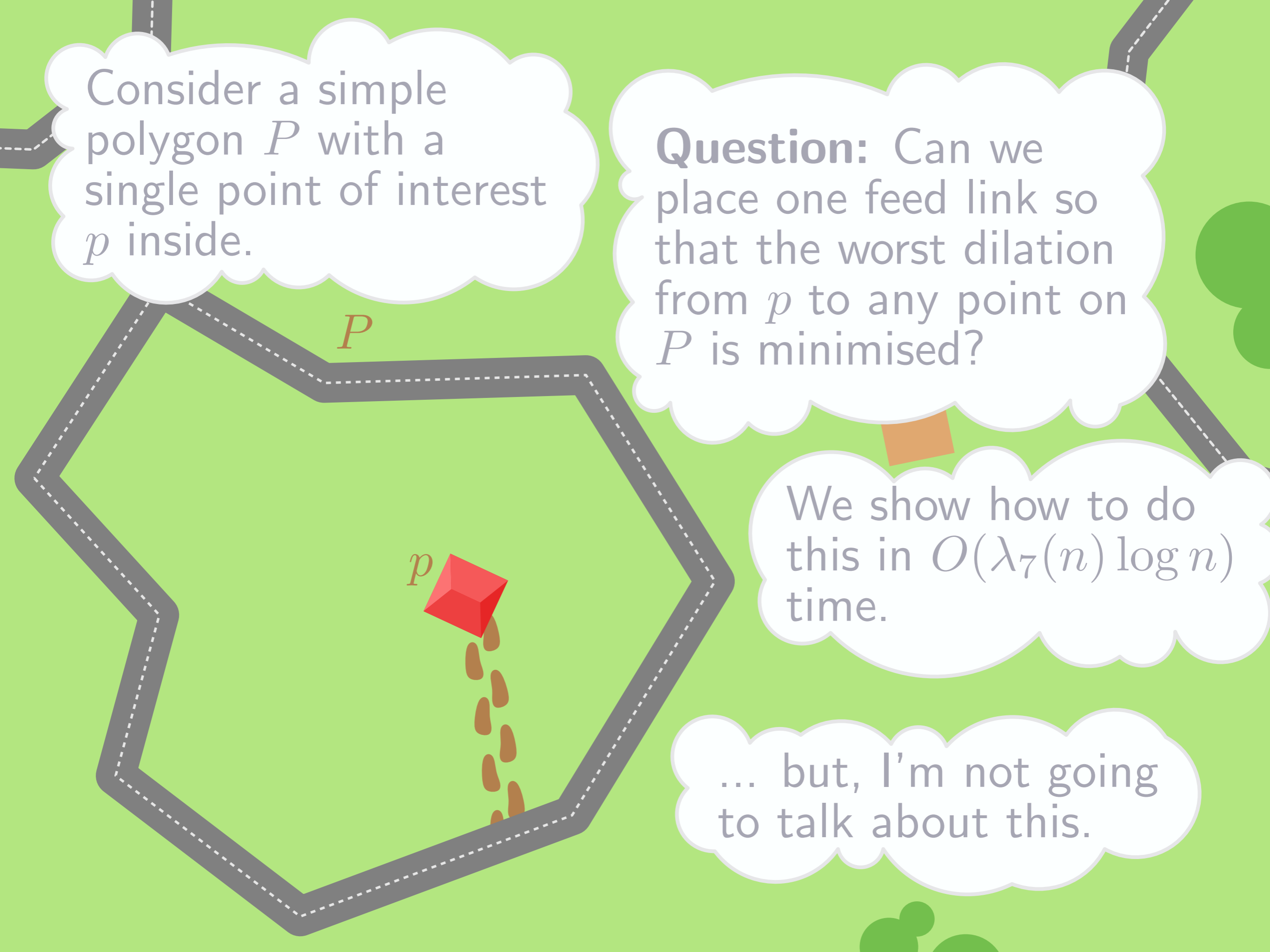


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... but, I'm not going to talk about this.





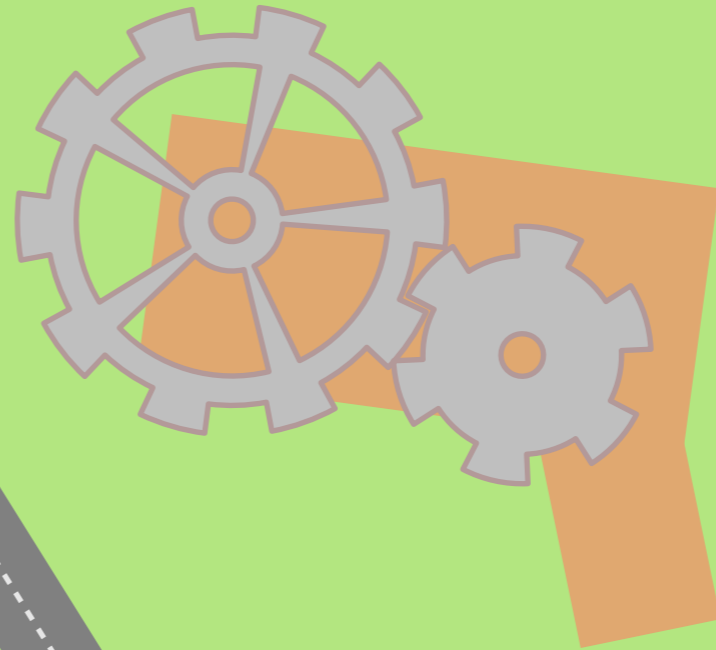
P

p

Instead, let's consider
a different problem.

P

p



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We show how to solve this problem in $O(n + k \log k)$ time.

We then show how to extend the solution to the original scenario.



Instead, let's consider a different problem.

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We show how to solve this problem in $O(n + k \log k)$ time.

We then show how to extend the solution to the original scenario.

However, this problem is also interesting in its own right.

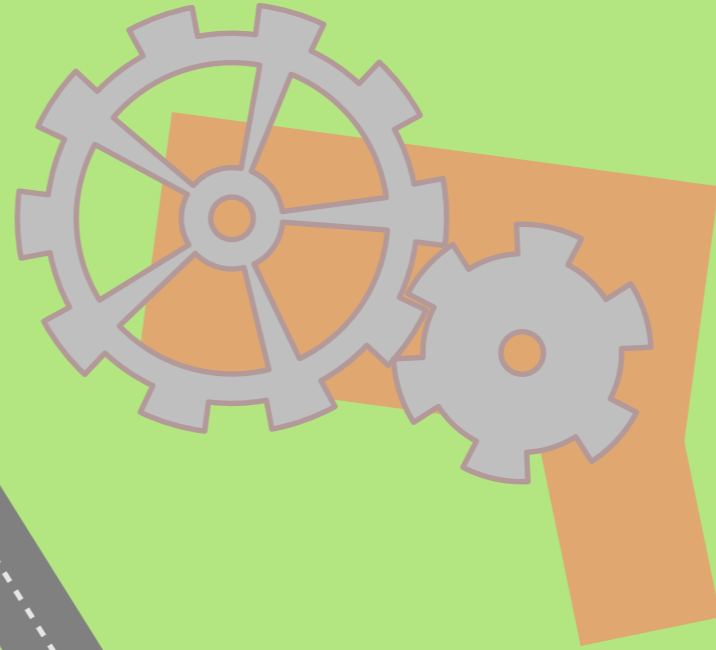




Idea: Consider a fixed point r_i on P .

P

p

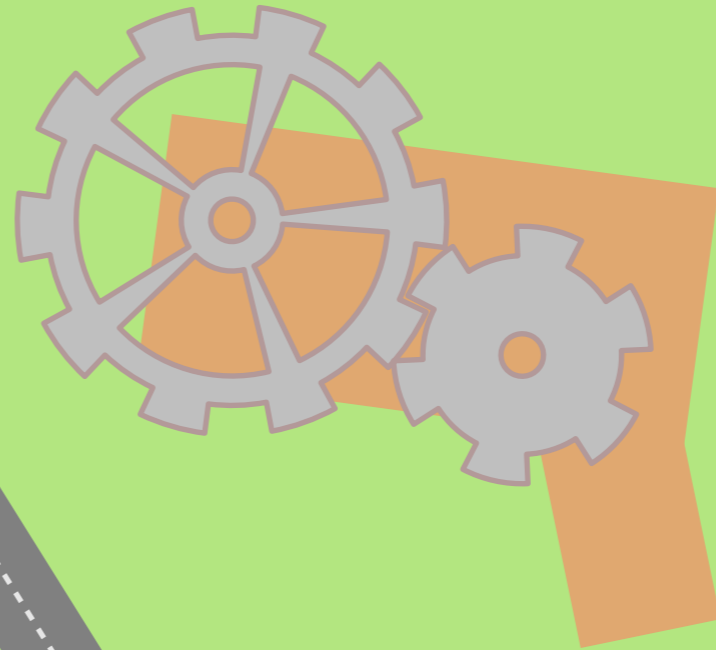


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P

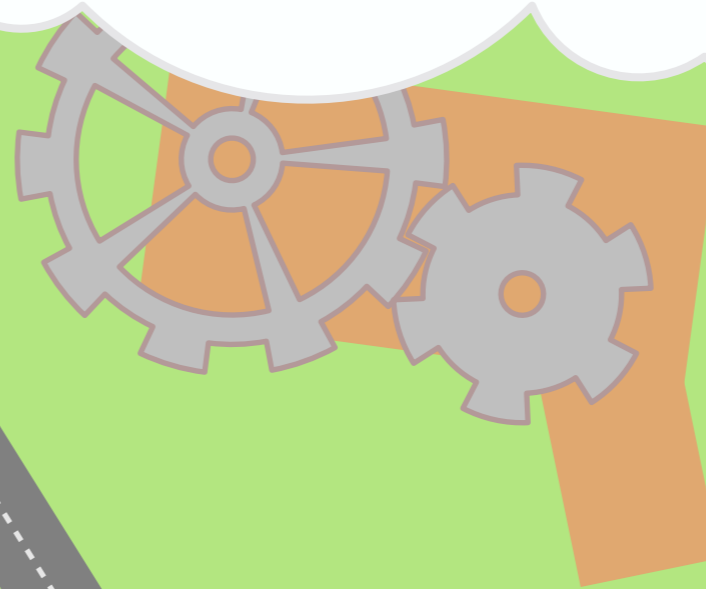
p

r_i



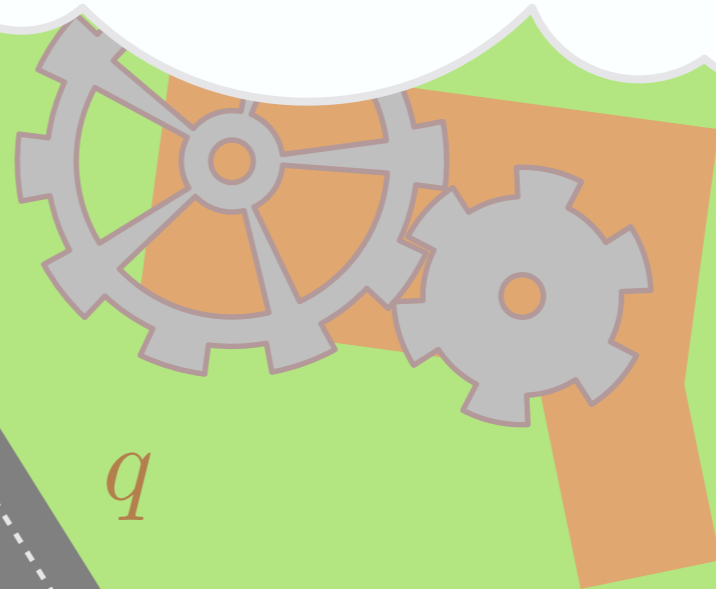
Idea: Consider a fixed point r_i on P .

Now, let q be the point the feed link attaches to.



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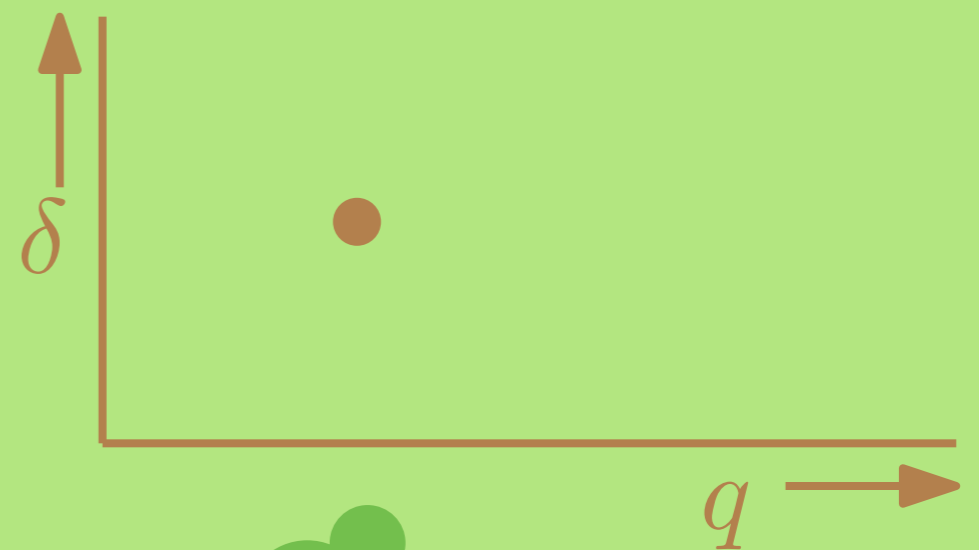
As q moves around, the dilation δ_i from p to r_i varies.



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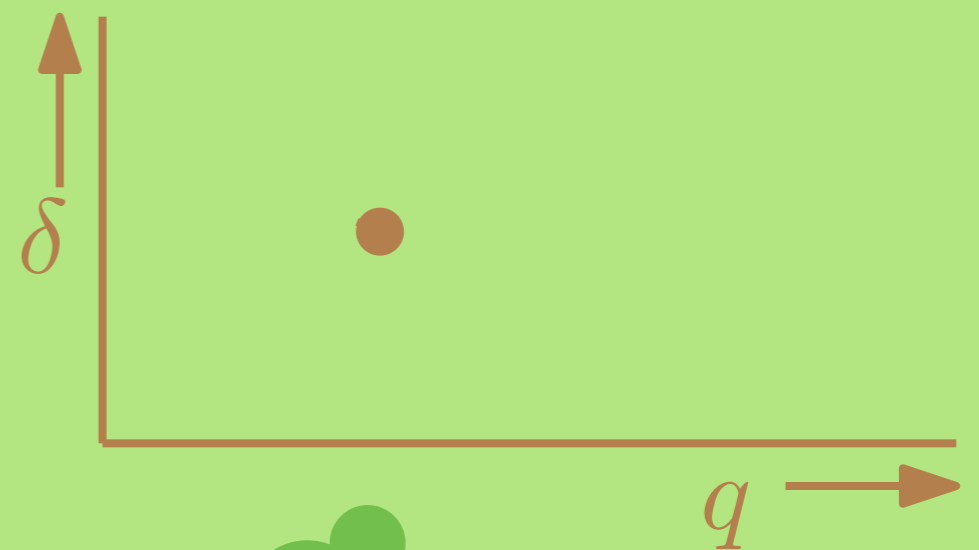
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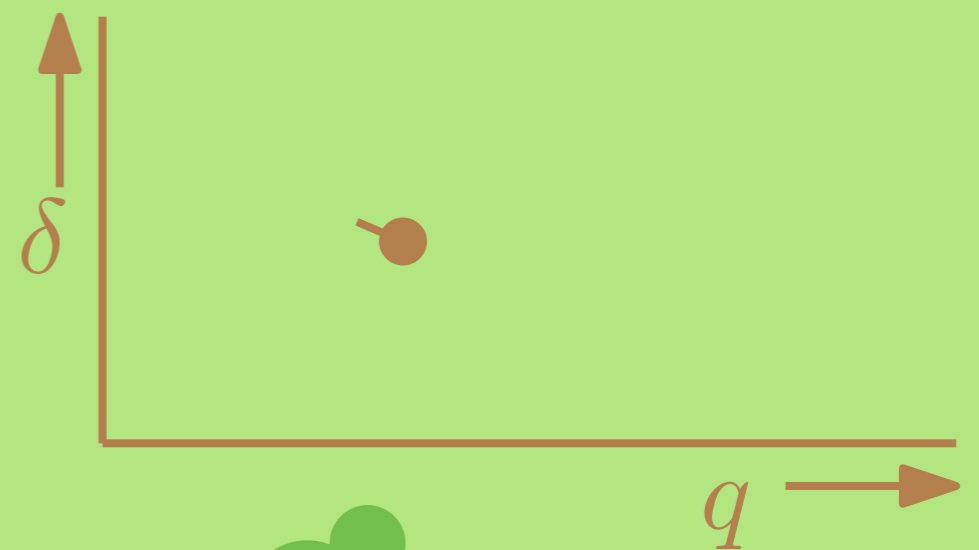
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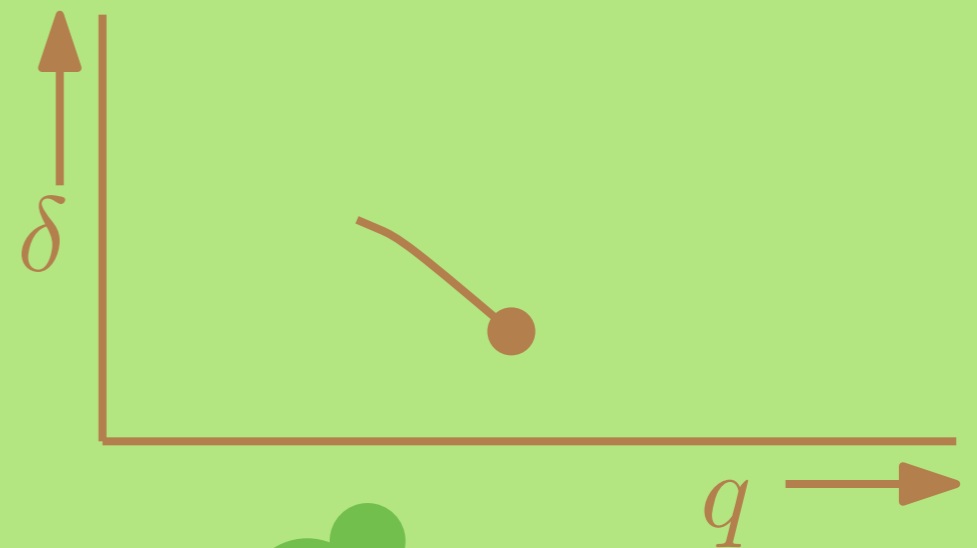
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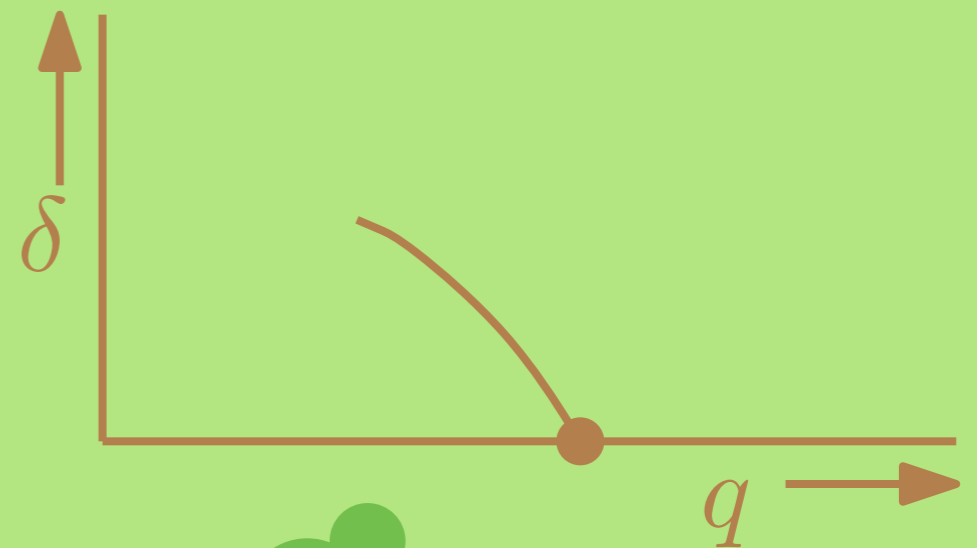
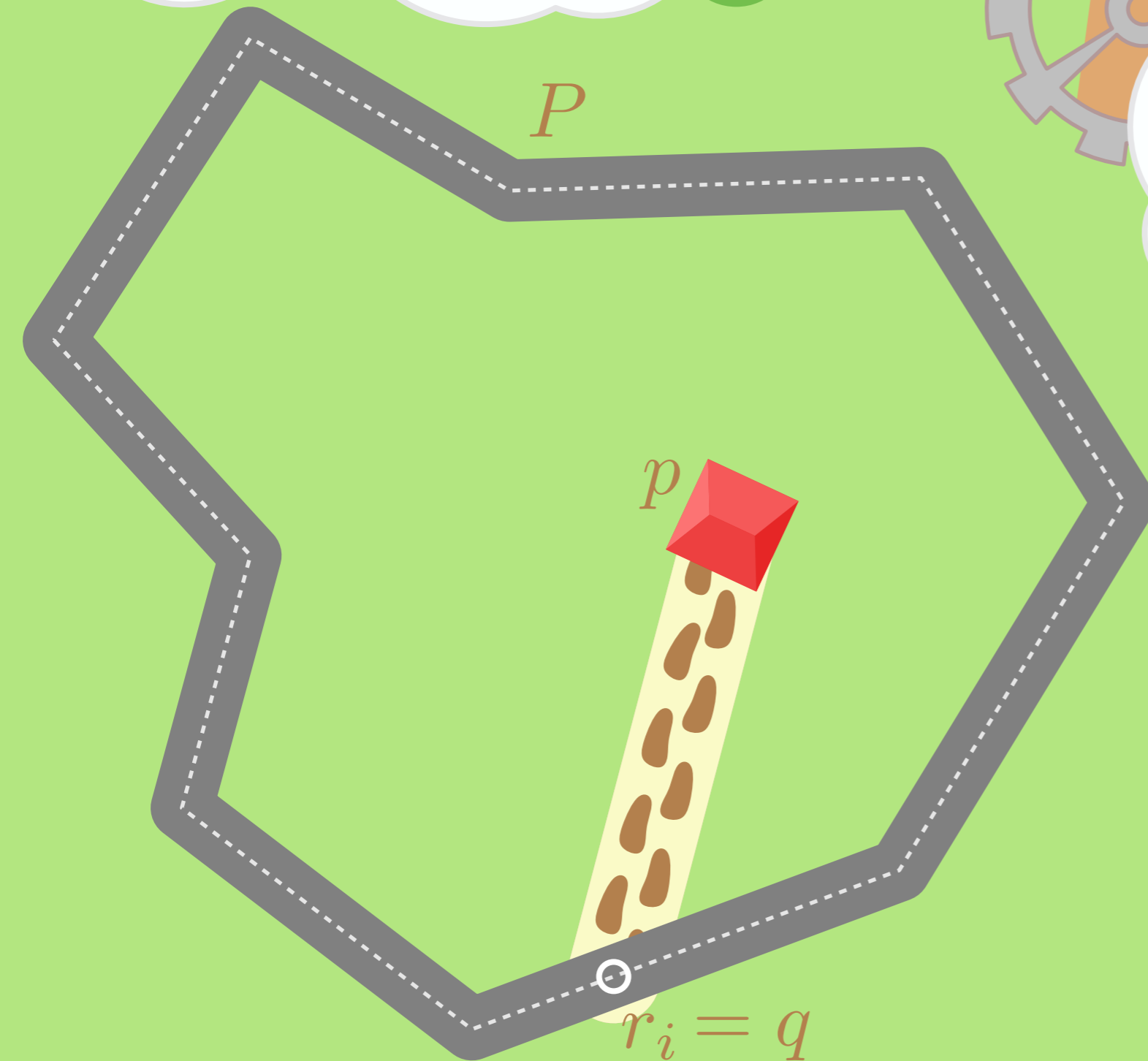
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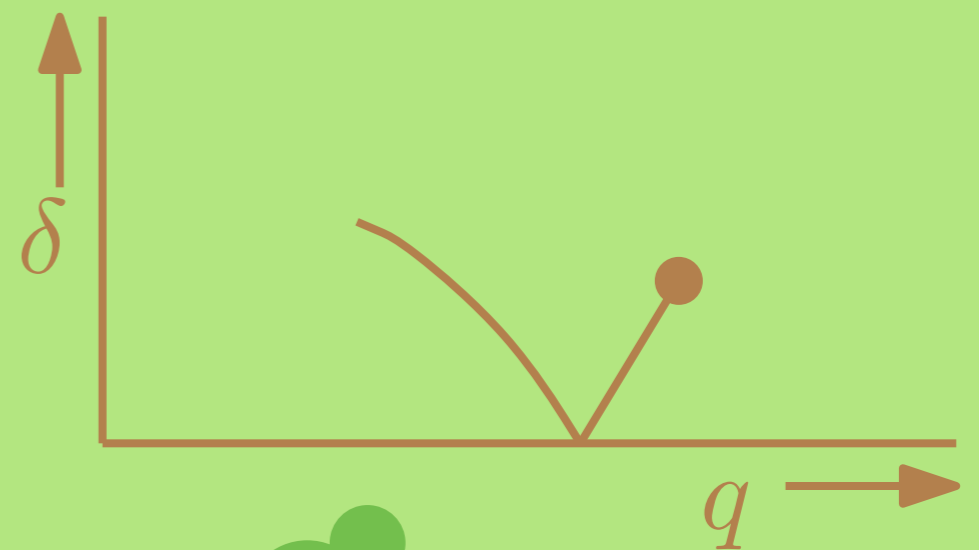
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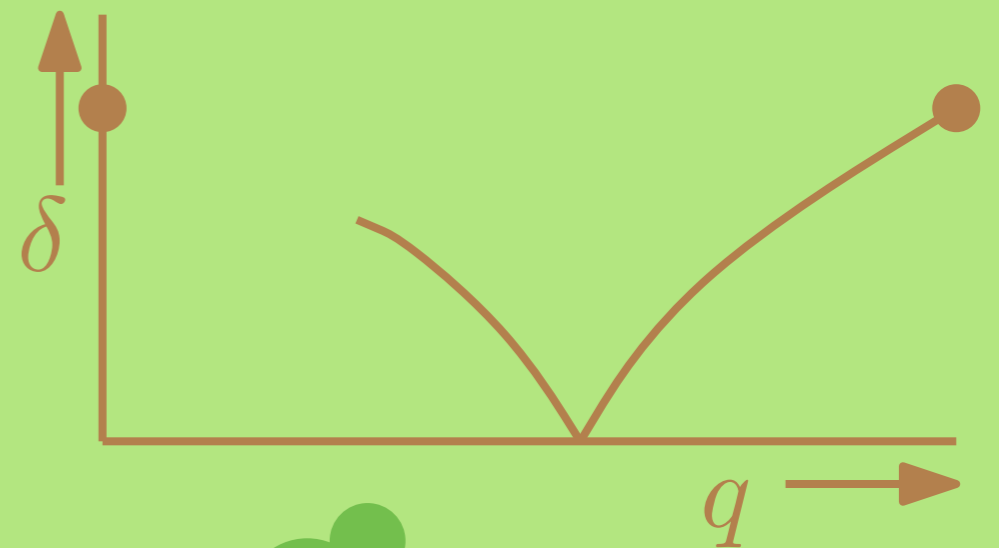
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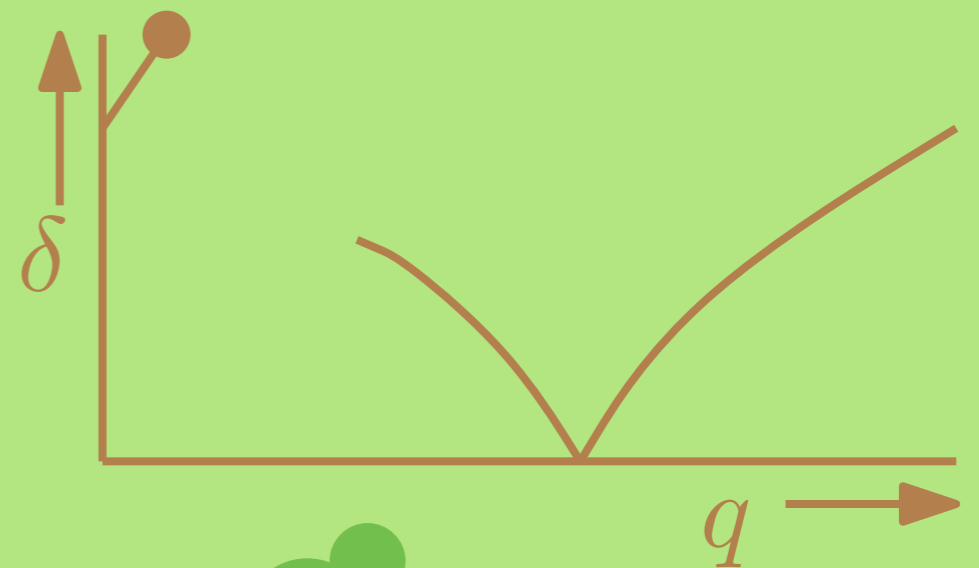
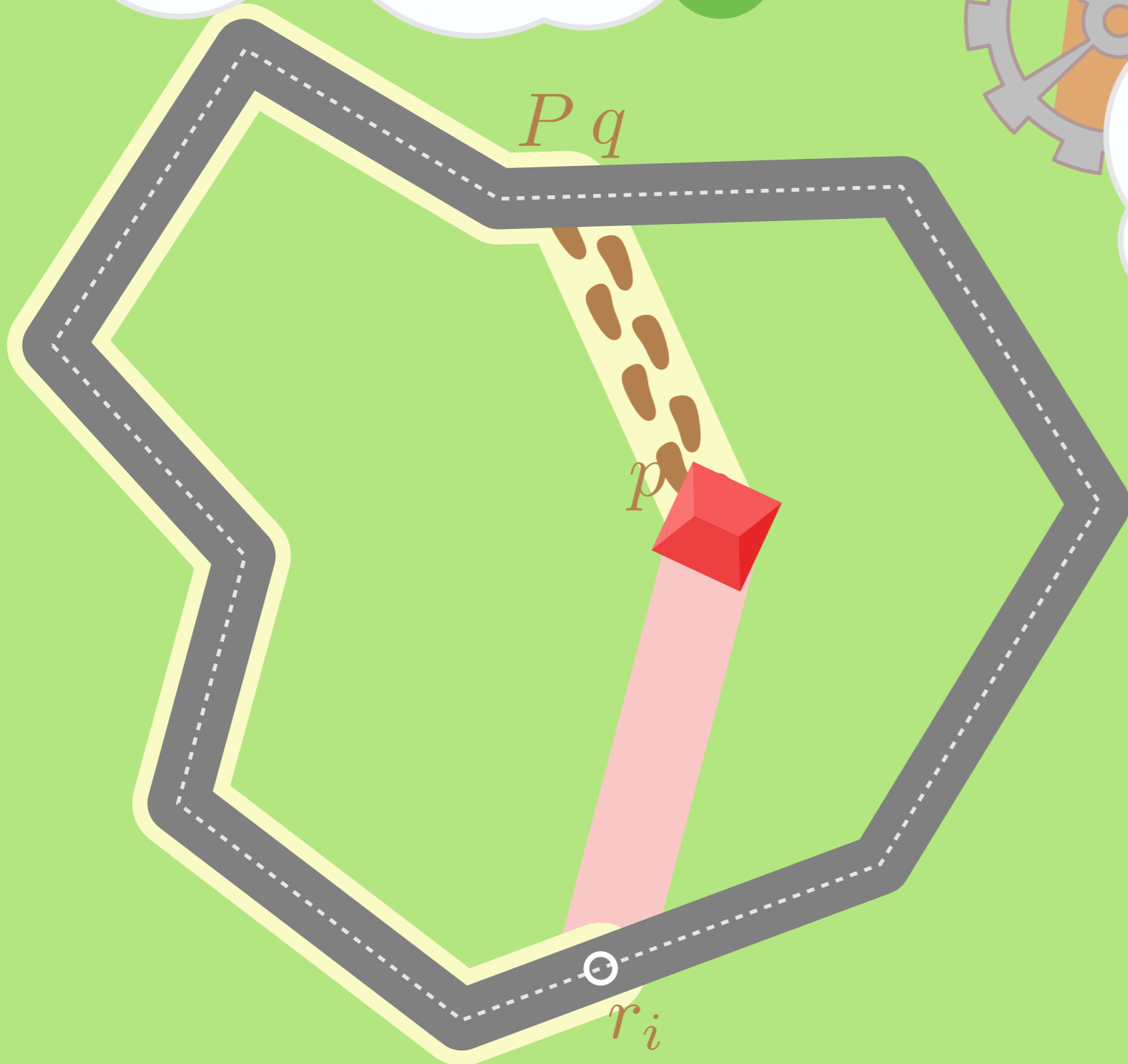
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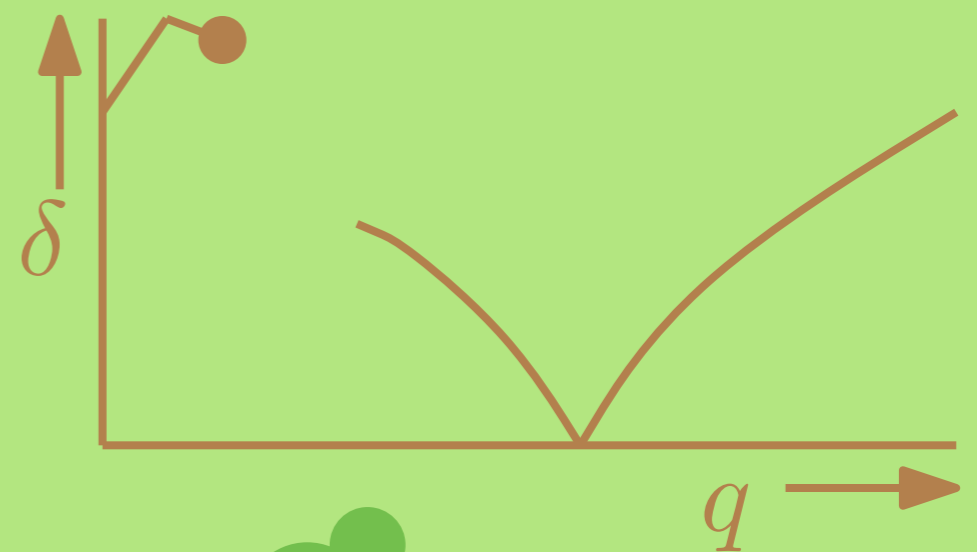
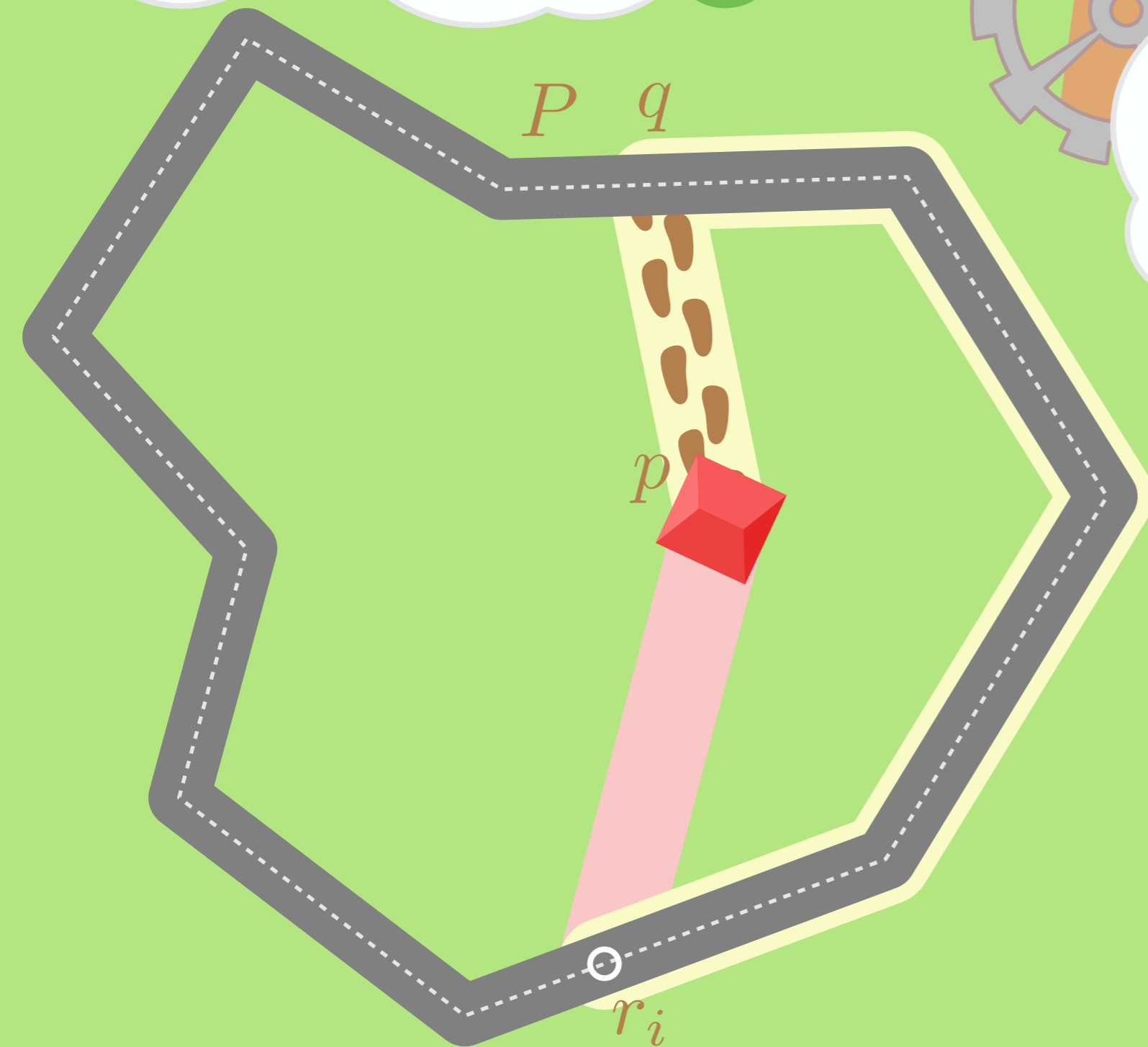
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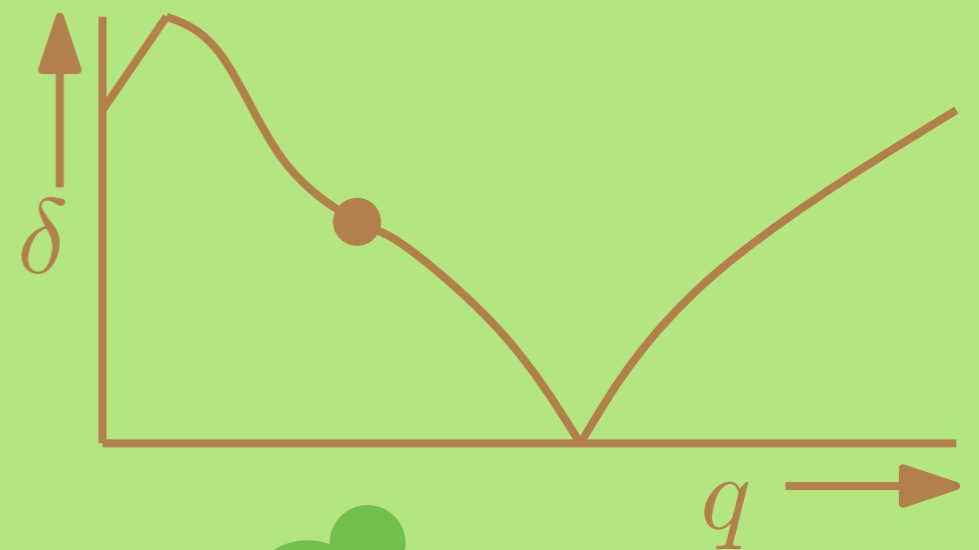
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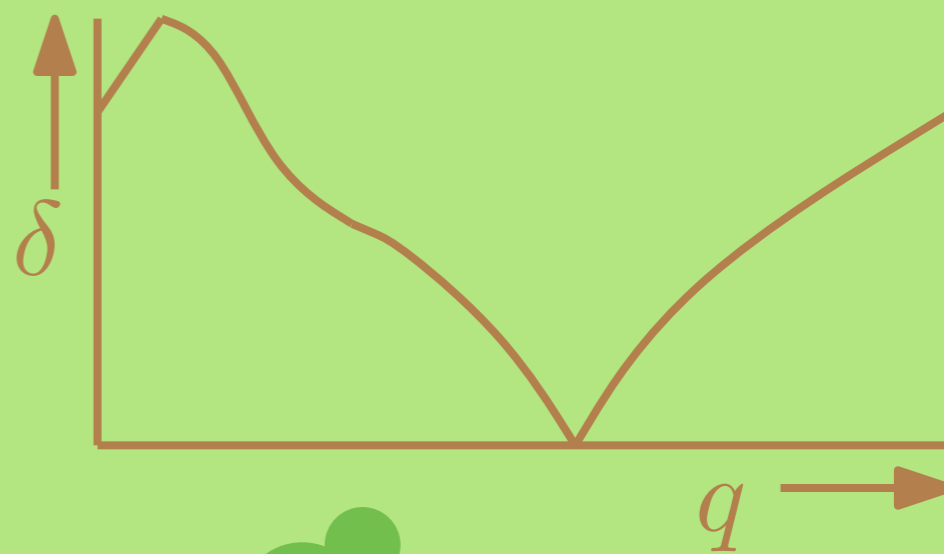


Idea: Consider a fixed point r_i on P .

We can draw similar curves for the other points.

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r_i



Idea: Consider a fixed point r_i on P .

We can draw similar curves for the other points.

The lowest point on the upper envelope of these curves gives the solution.

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As q moves around, the dilation δ_i from p to r_i varies.



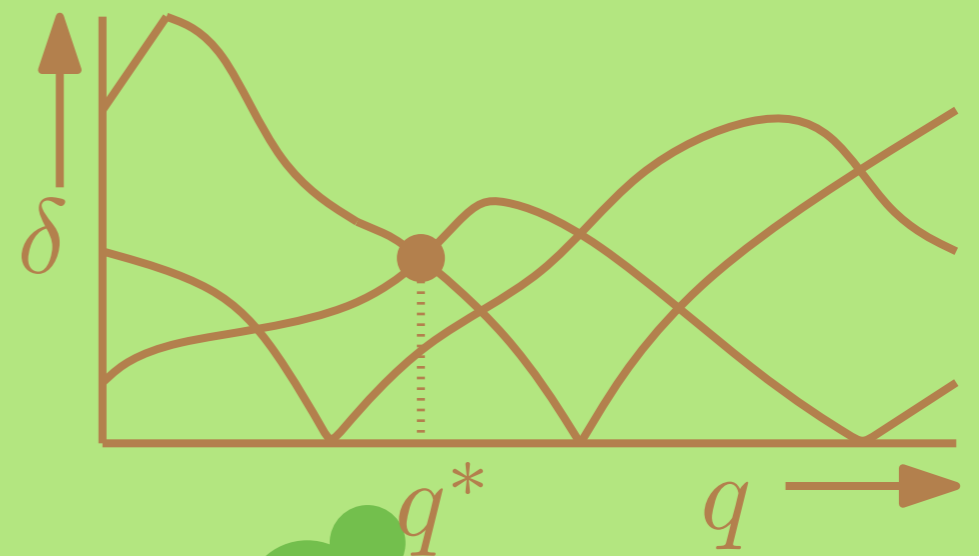
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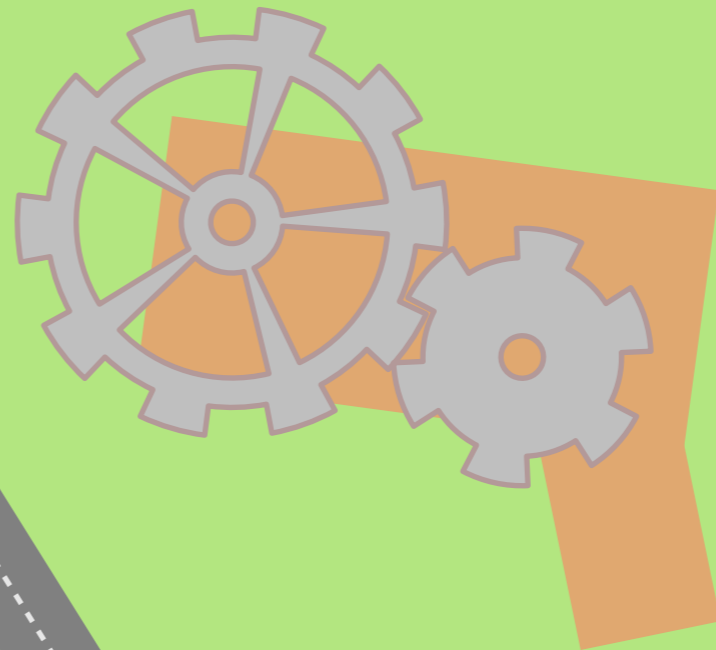
r_i



Consider again a fixed point r_i on P .

P

p



Consider again a fixed point r_i on P .



Consider again a fixed point r_i on P .

As before, let q be the point the feed link attaches to.



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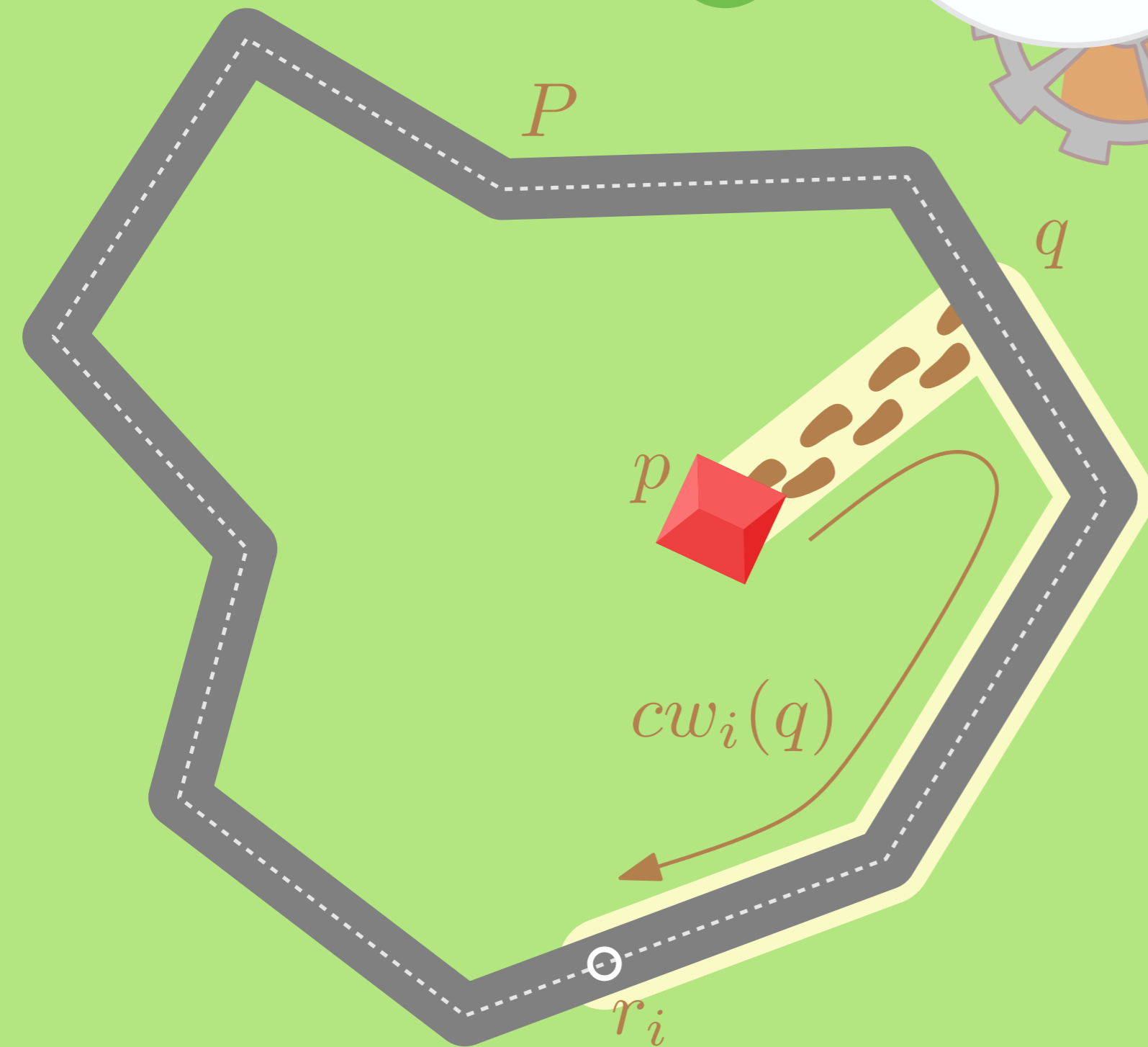
We define $cw_i(q)$ as the length of the path clockwise from p to r_i .



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We define $cw_i(q)$ as the length of the path clockwise from p to r_i .

Similarly, let $ccw_i(q)$ be the length of the counterclockwise path.



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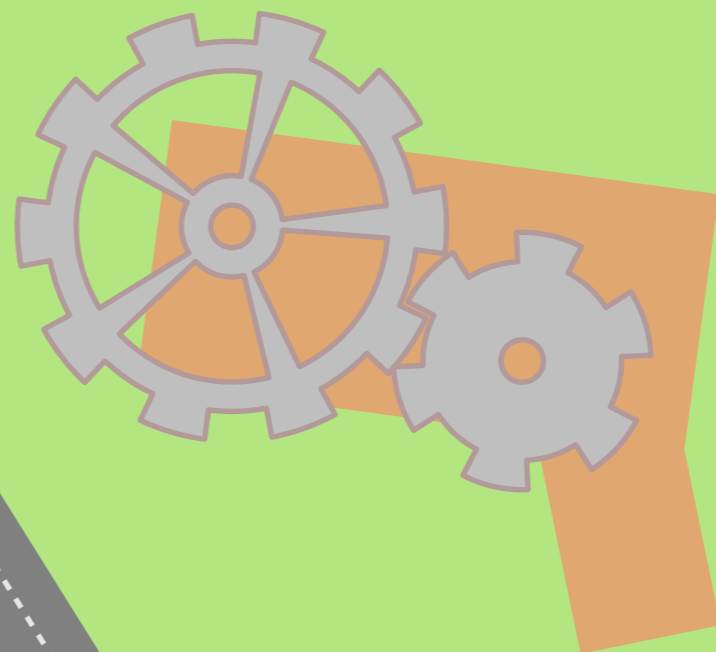
Similarly, let $ccw_i(q)$ be the length of the counterclockwise path.

As q moves around, $cw_i(q)$ and $ccw_i(q)$ increase or decrease monotonely.





Now, we can express the dilation δ_i in terms of cw_i or ccw_i , as well as the constant $|pr_i|$



P

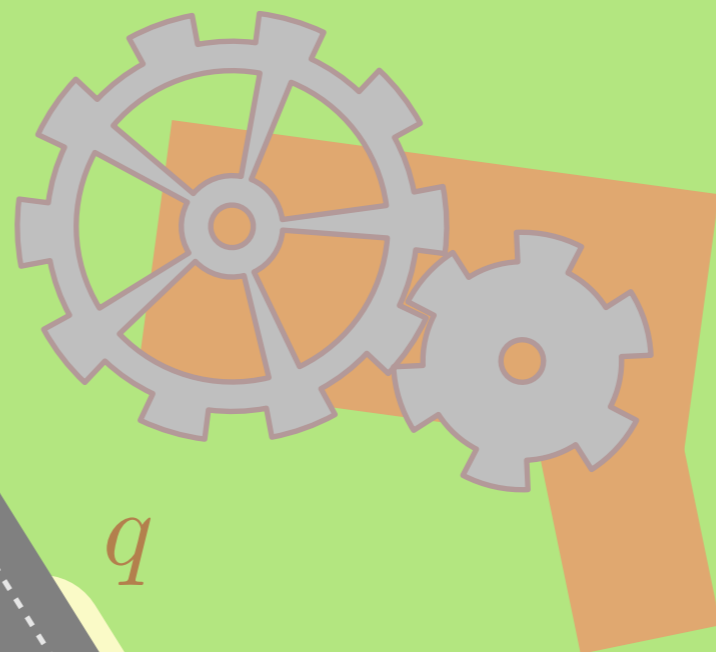
p



r_i

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$$\delta_i(q) = \frac{cw_i(q)}{|pr_i|}$$

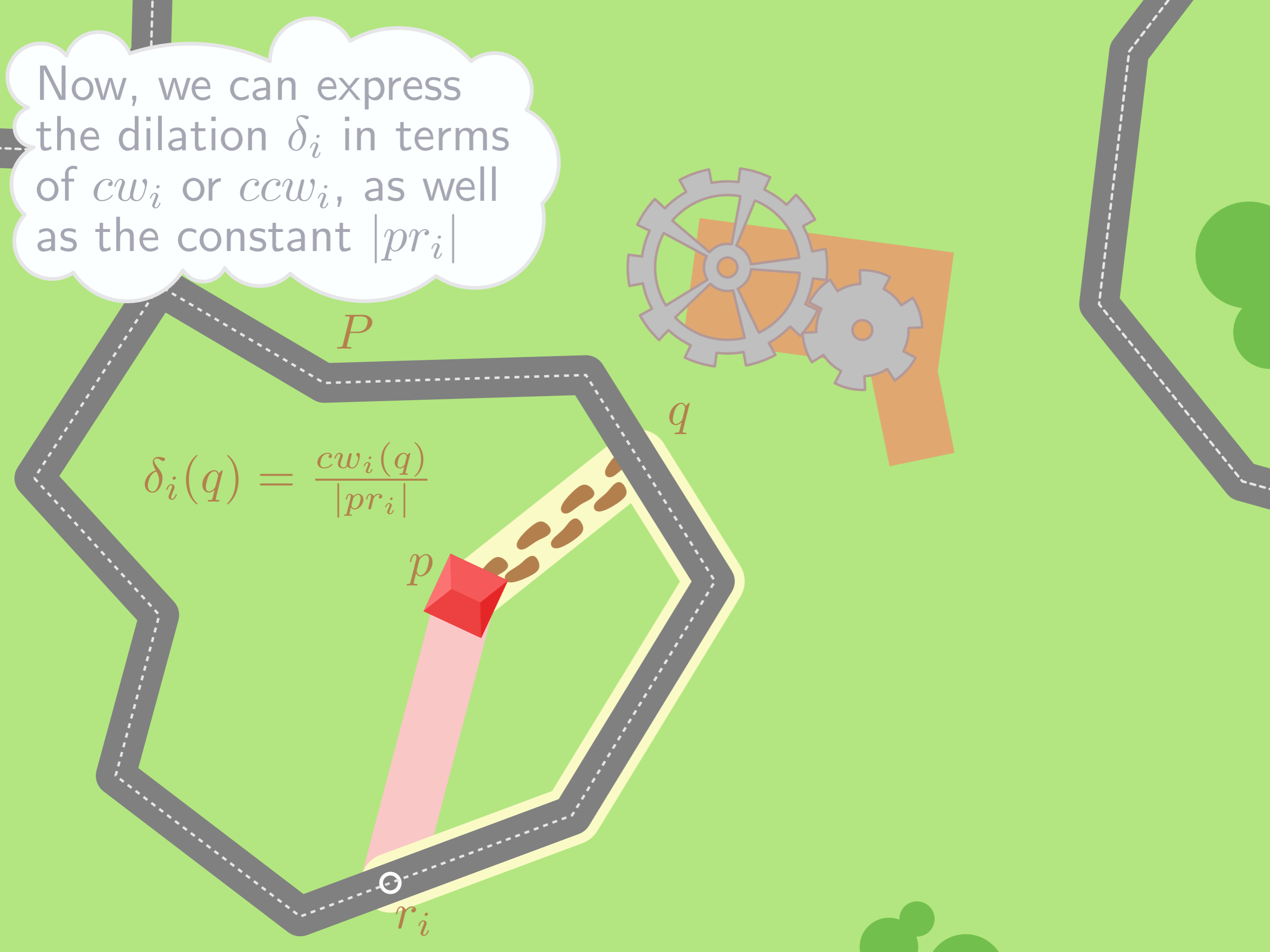


P

q

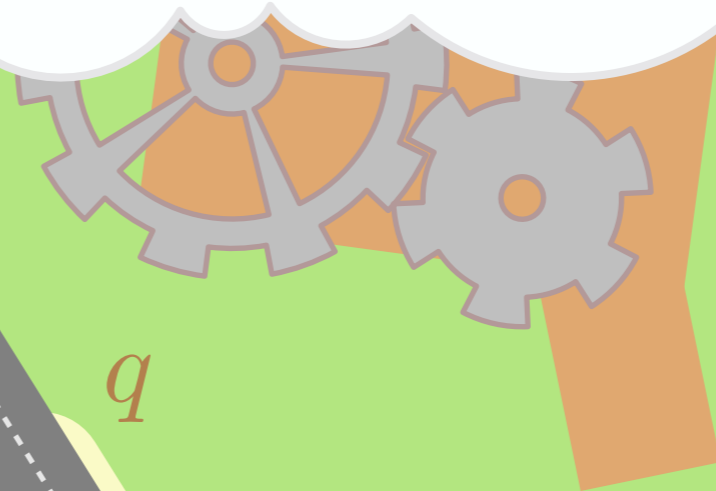
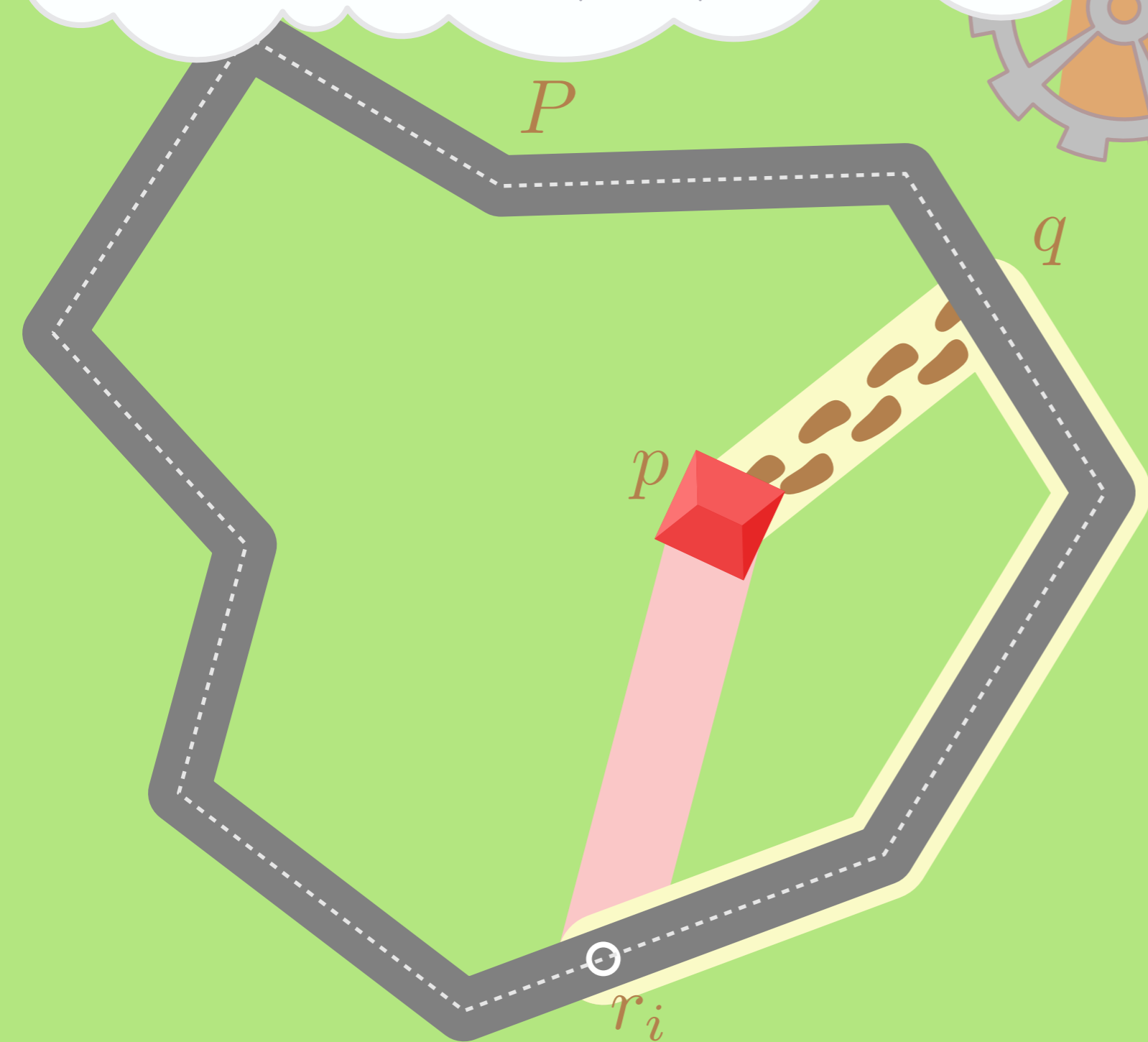
p

r_i



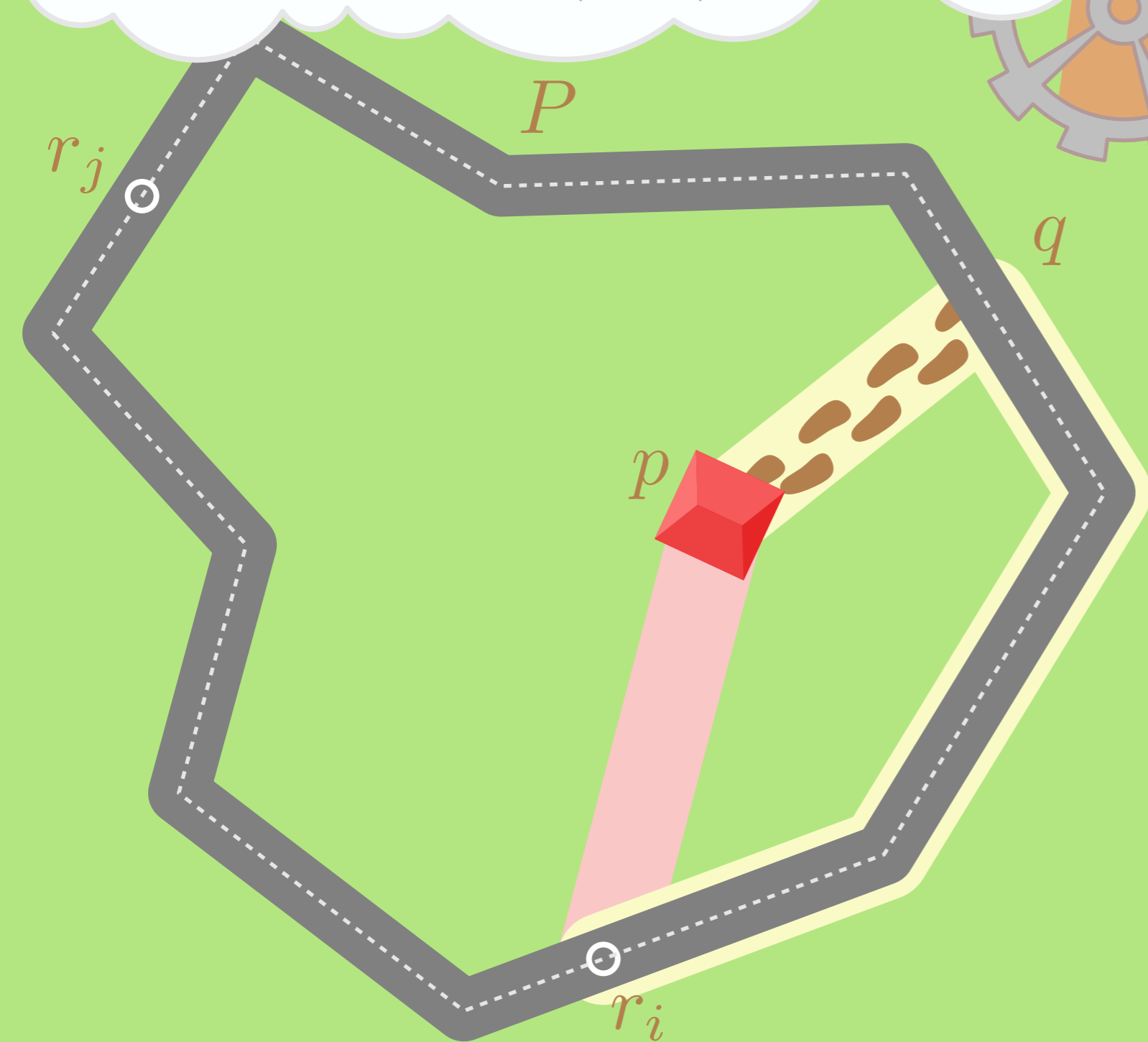
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Observation: We can also express δ_i in terms of cw_j or ccw_j , for any j !



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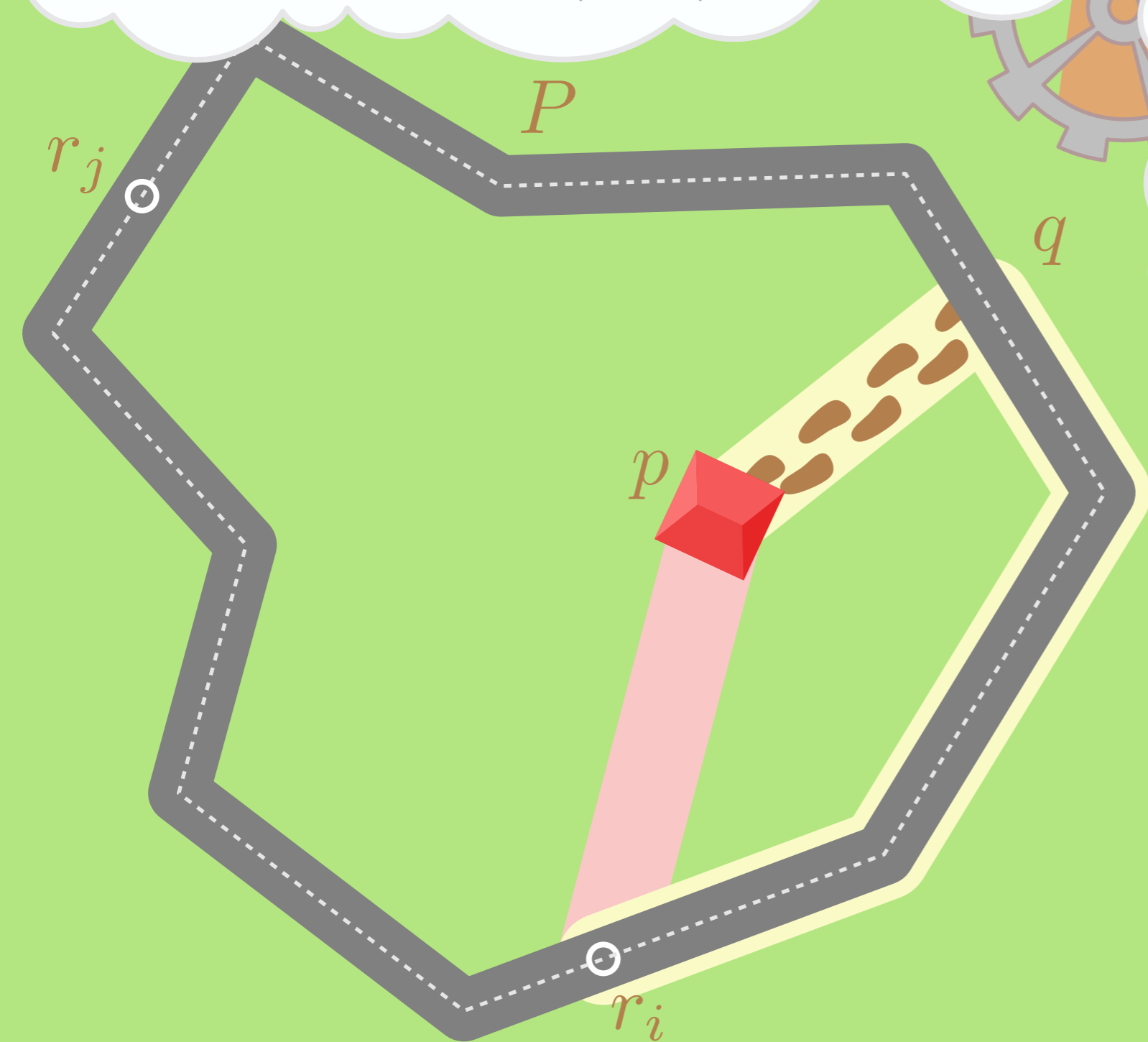
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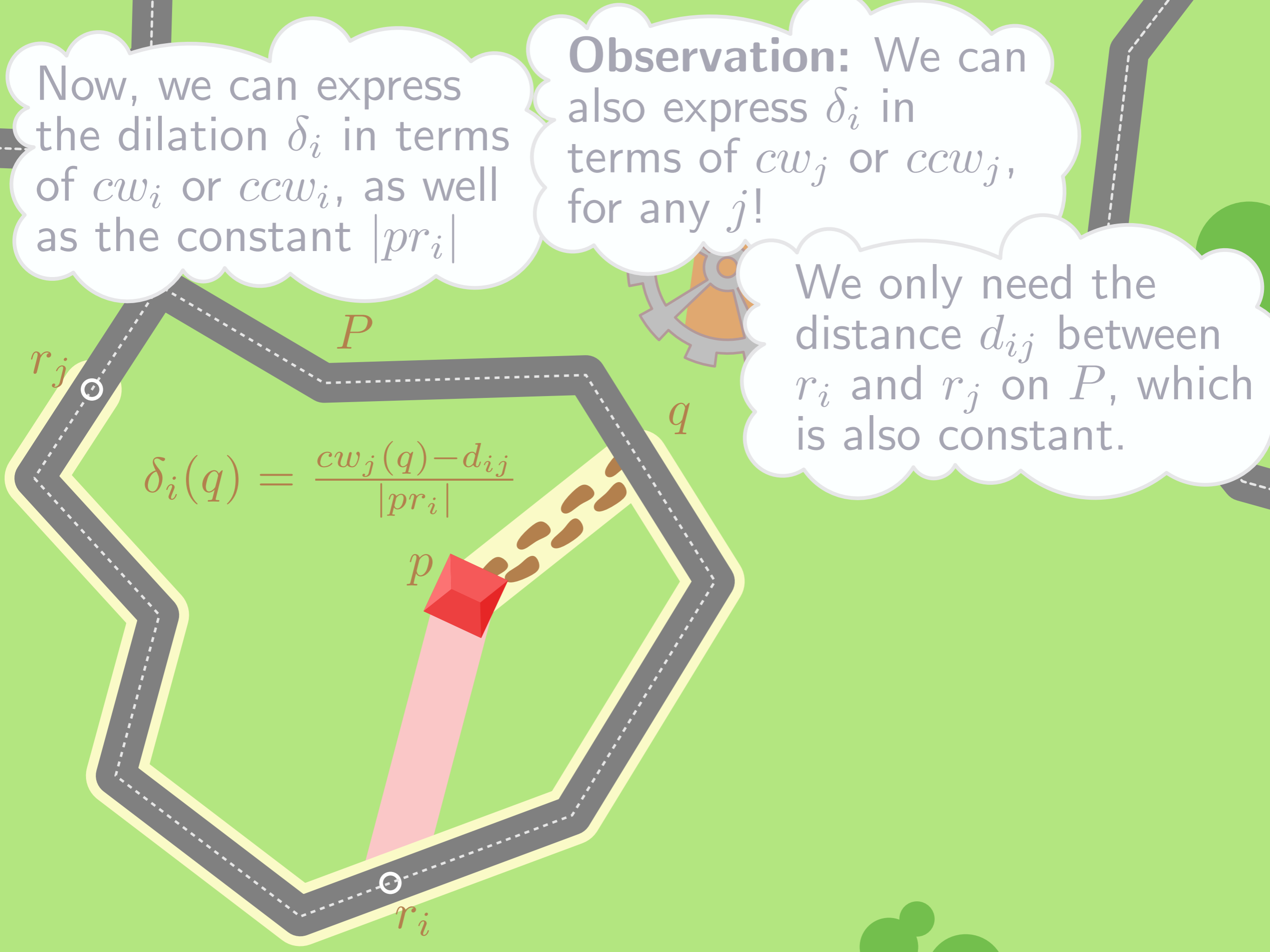


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$$\delta_i(q) = \frac{cw_j(q) - d_{ij}}{|pr_i|}$$

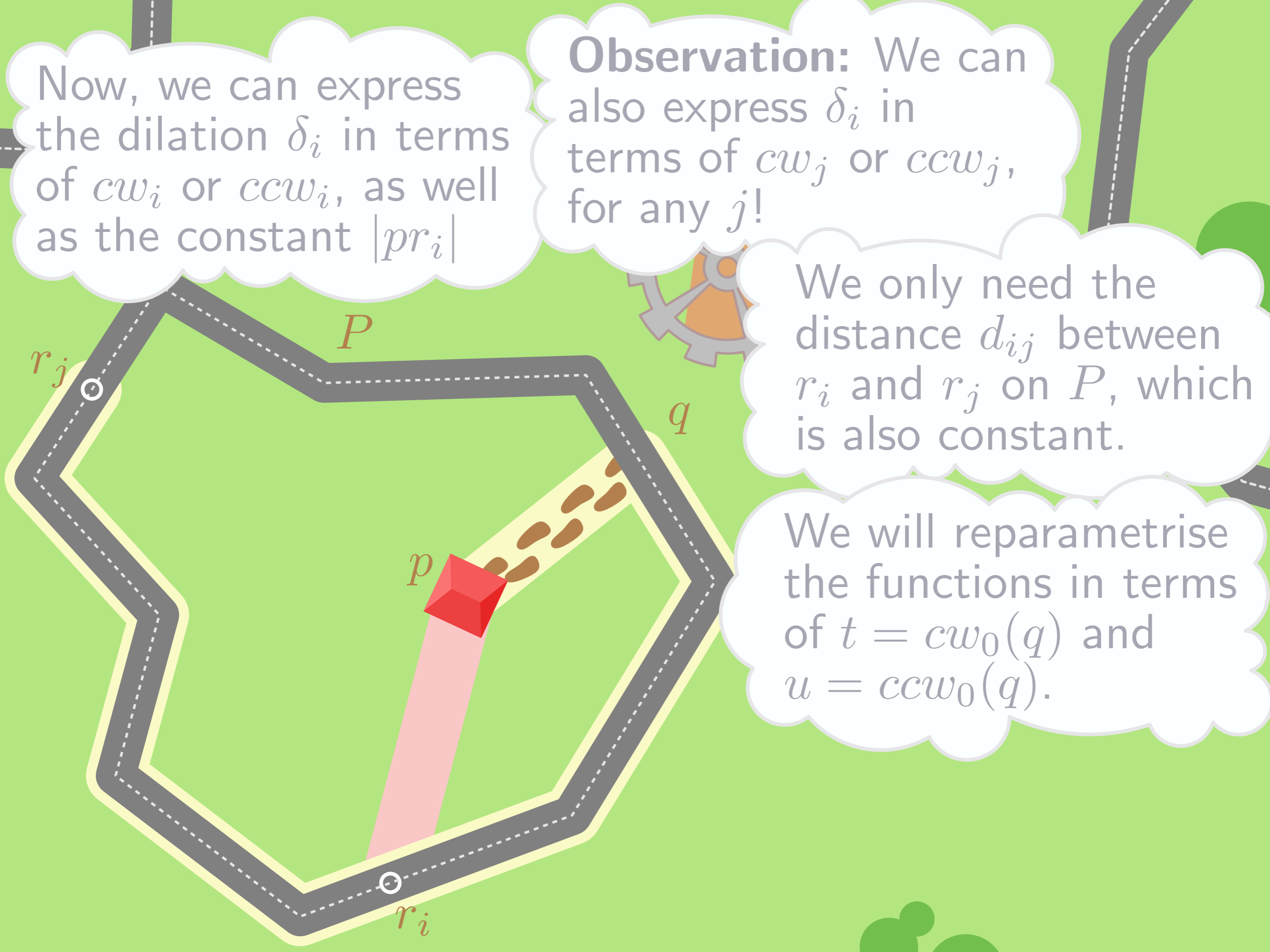


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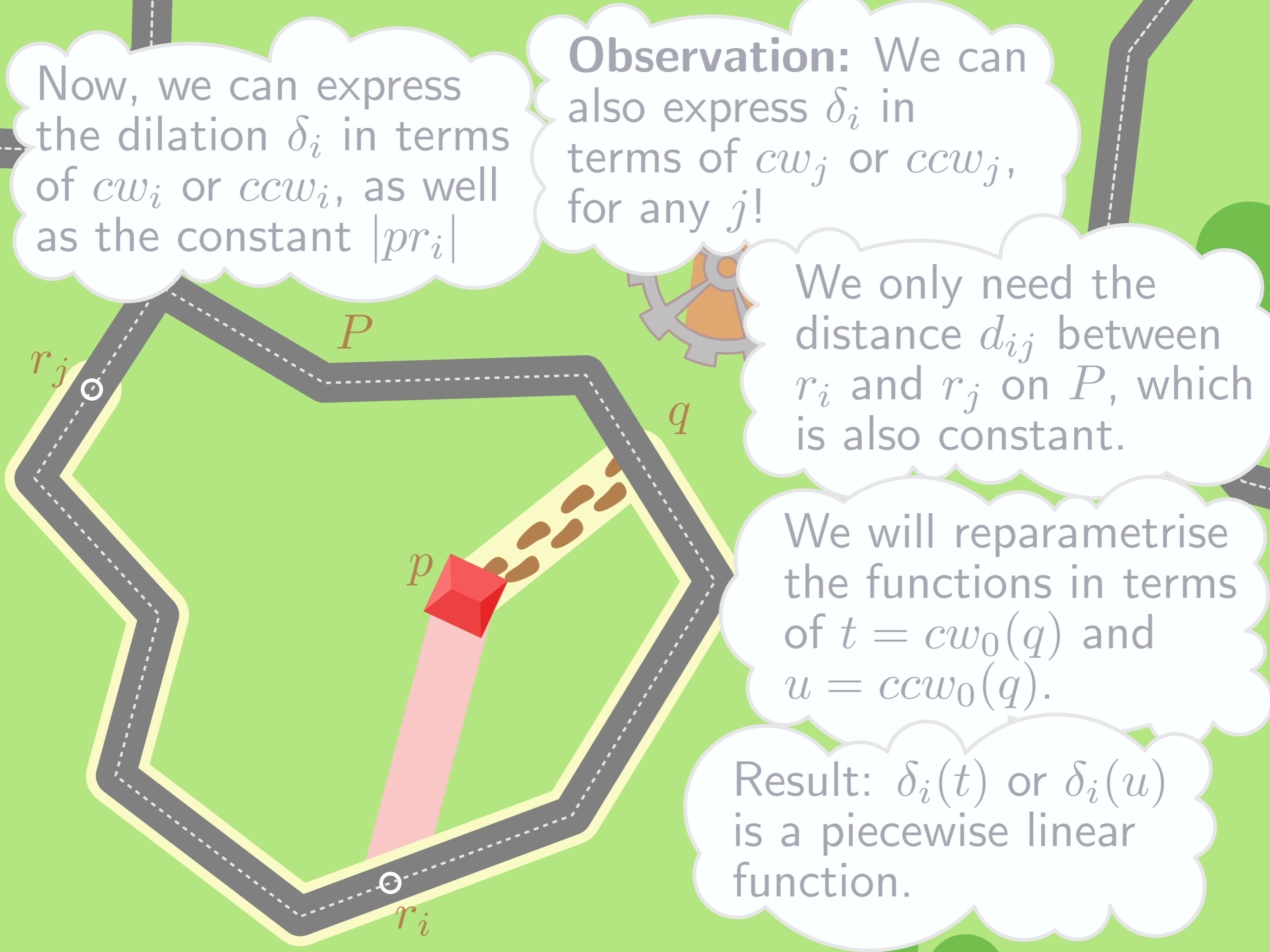
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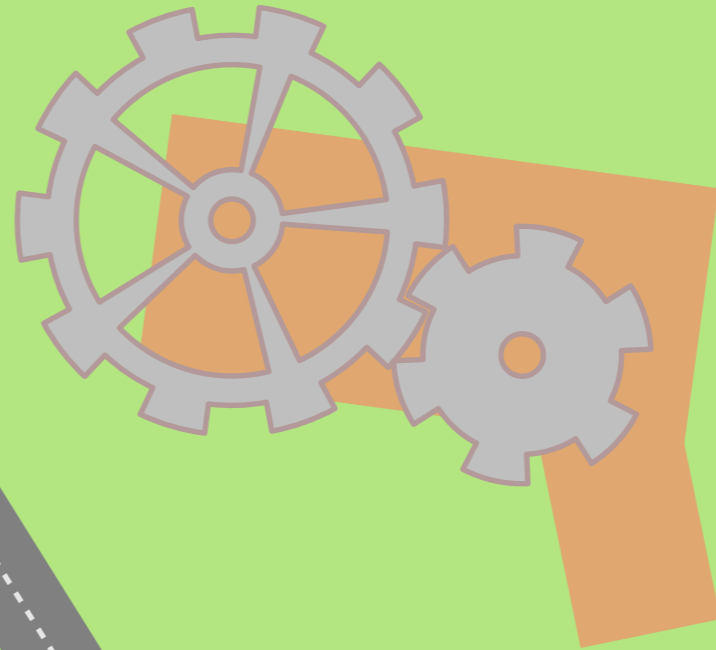
We will reparametrise the functions in terms of $t = cw_0(q)$ and $u = ccw_0(q)$.

Result: $\delta_i(t)$ or $\delta_i(u)$ is a piecewise linear function.





Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.

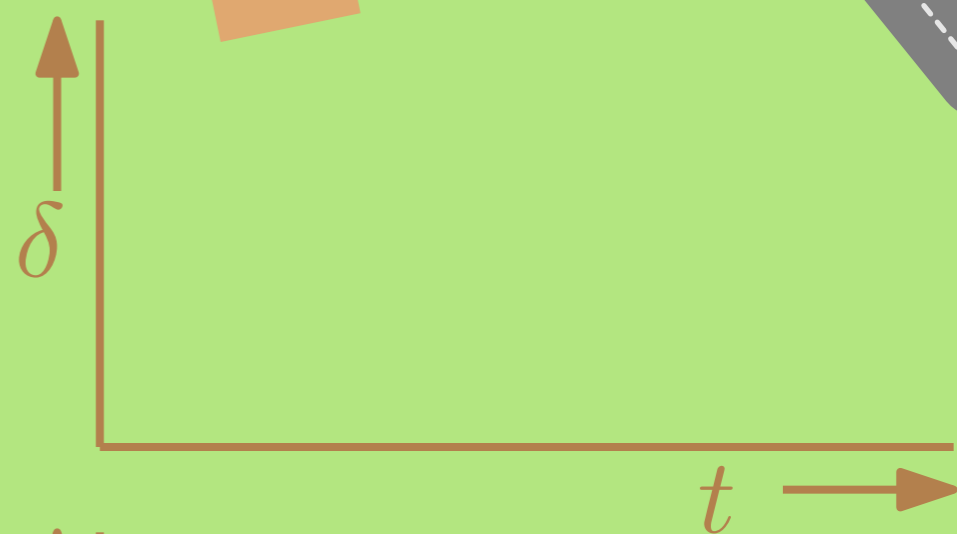
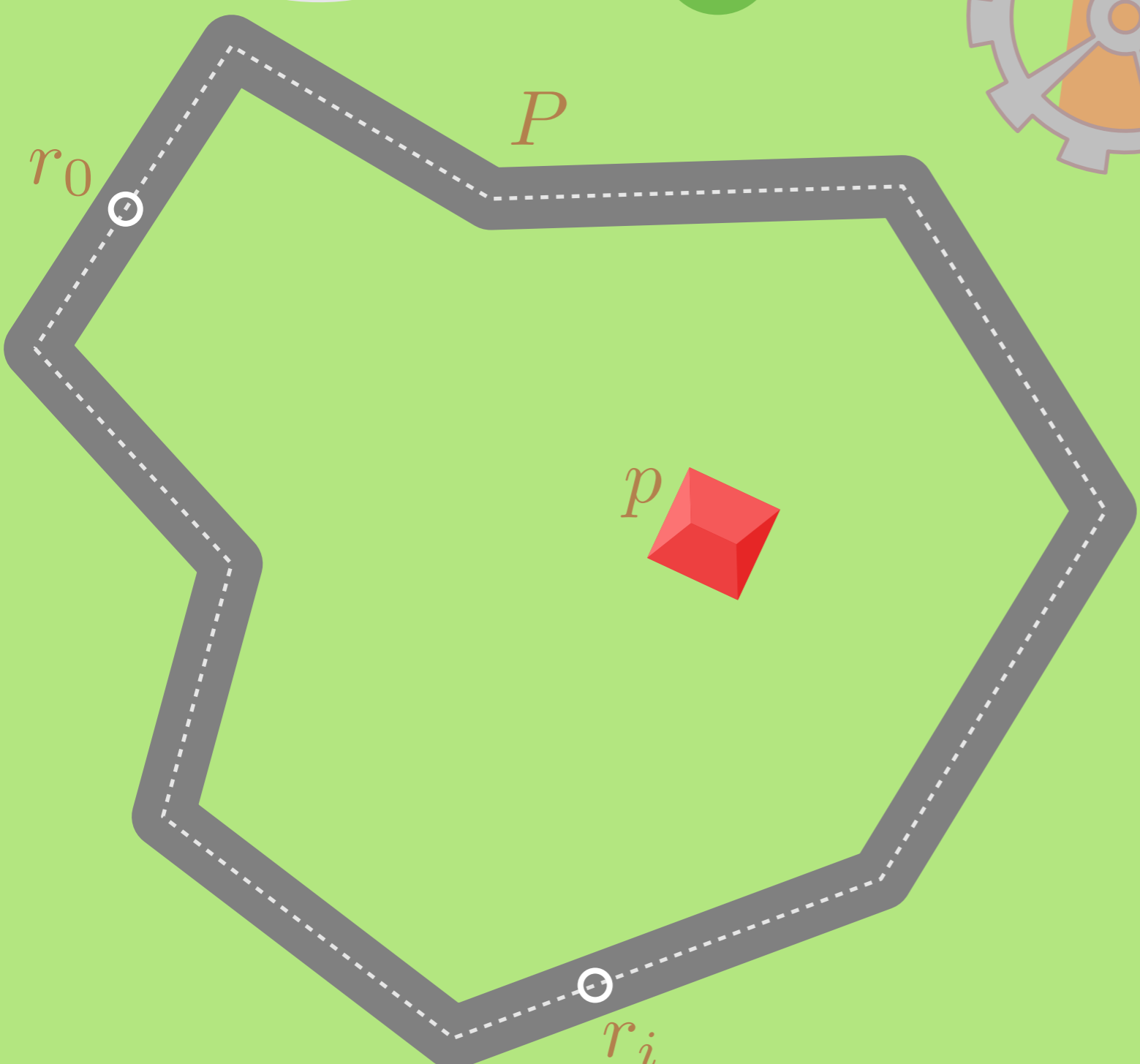


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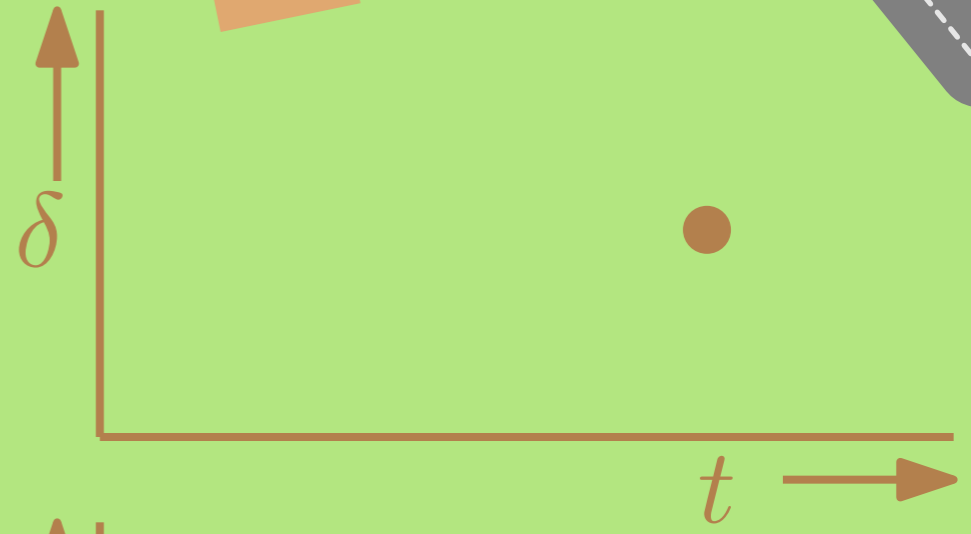
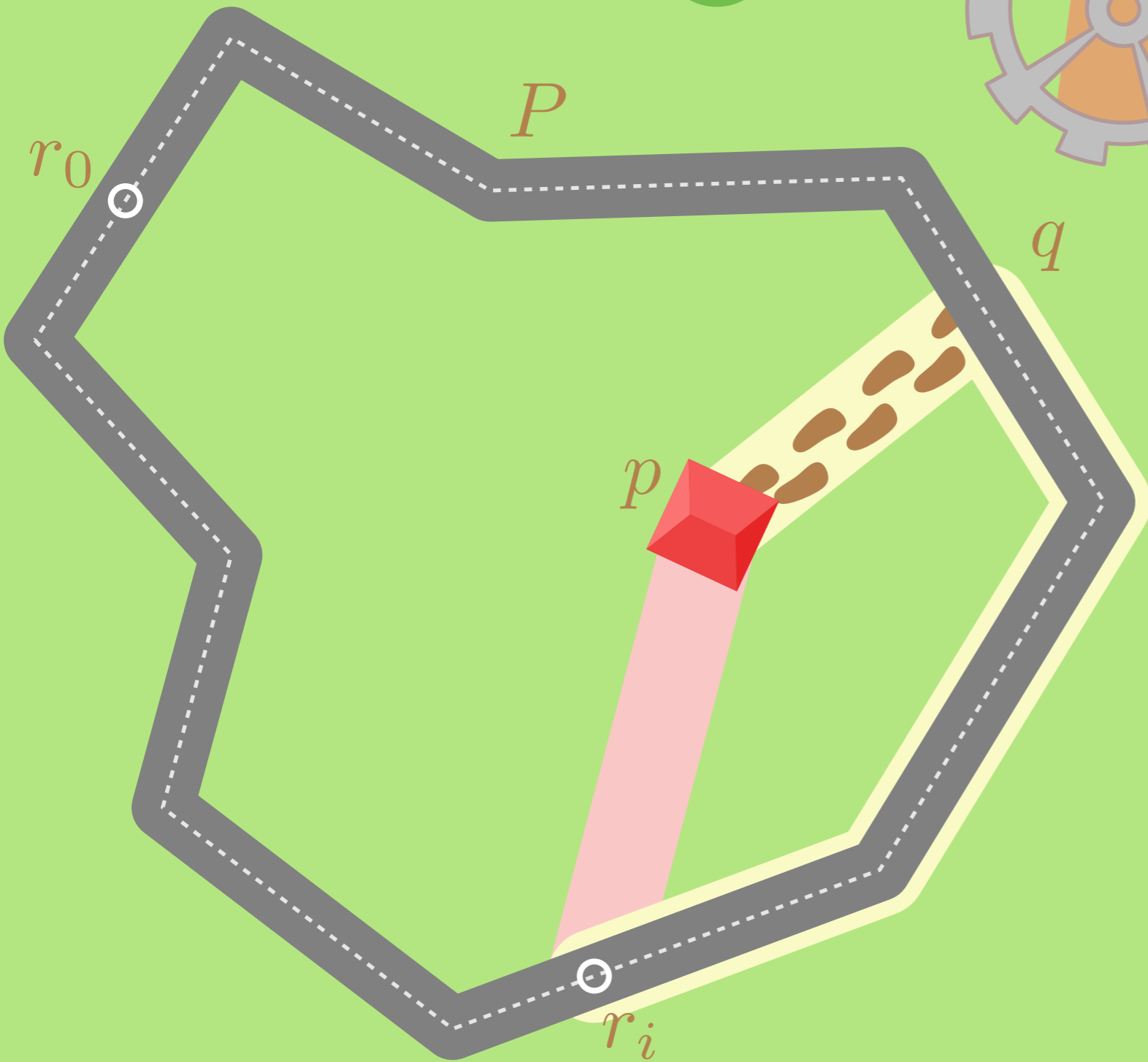
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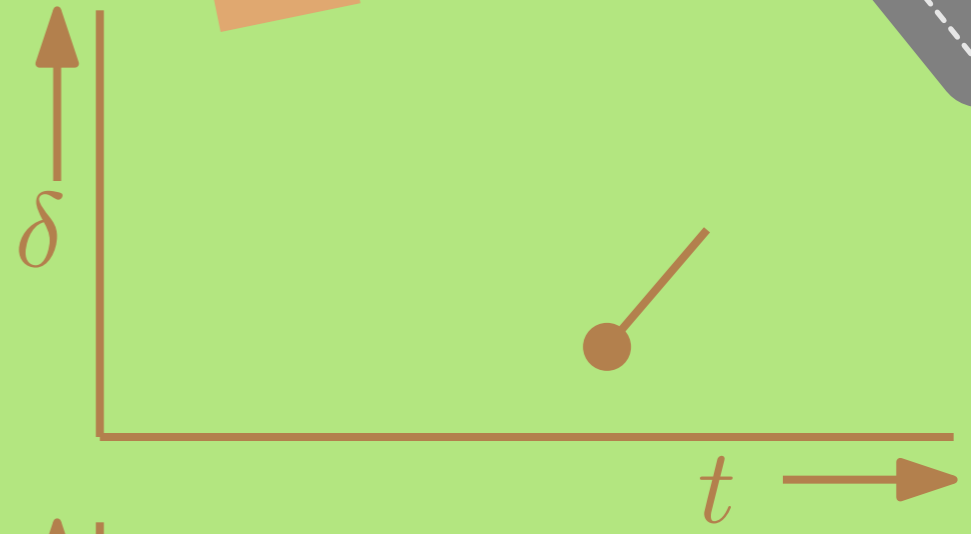
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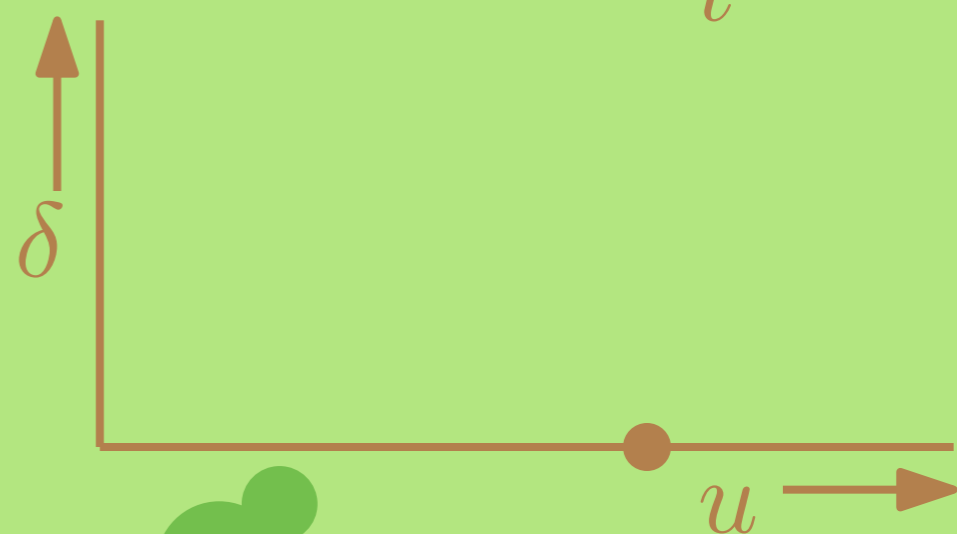
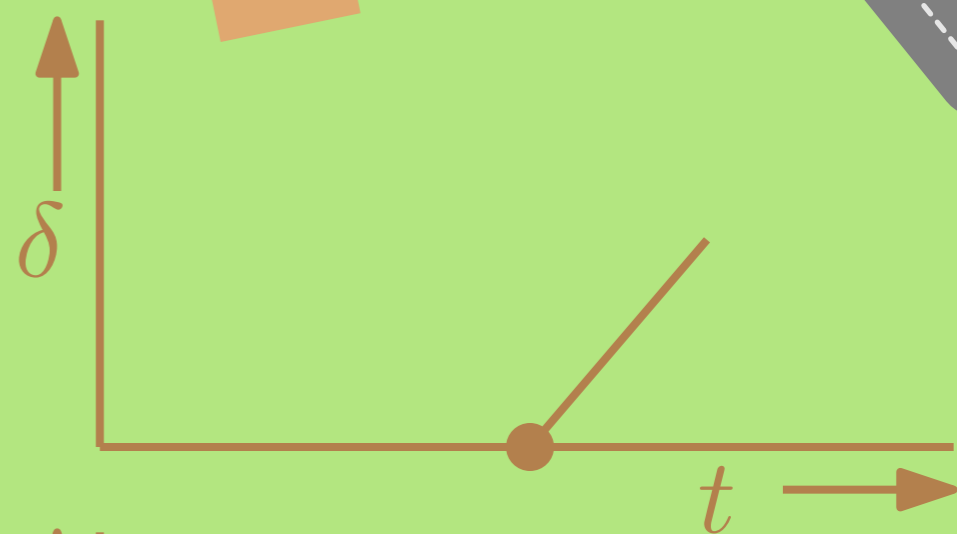
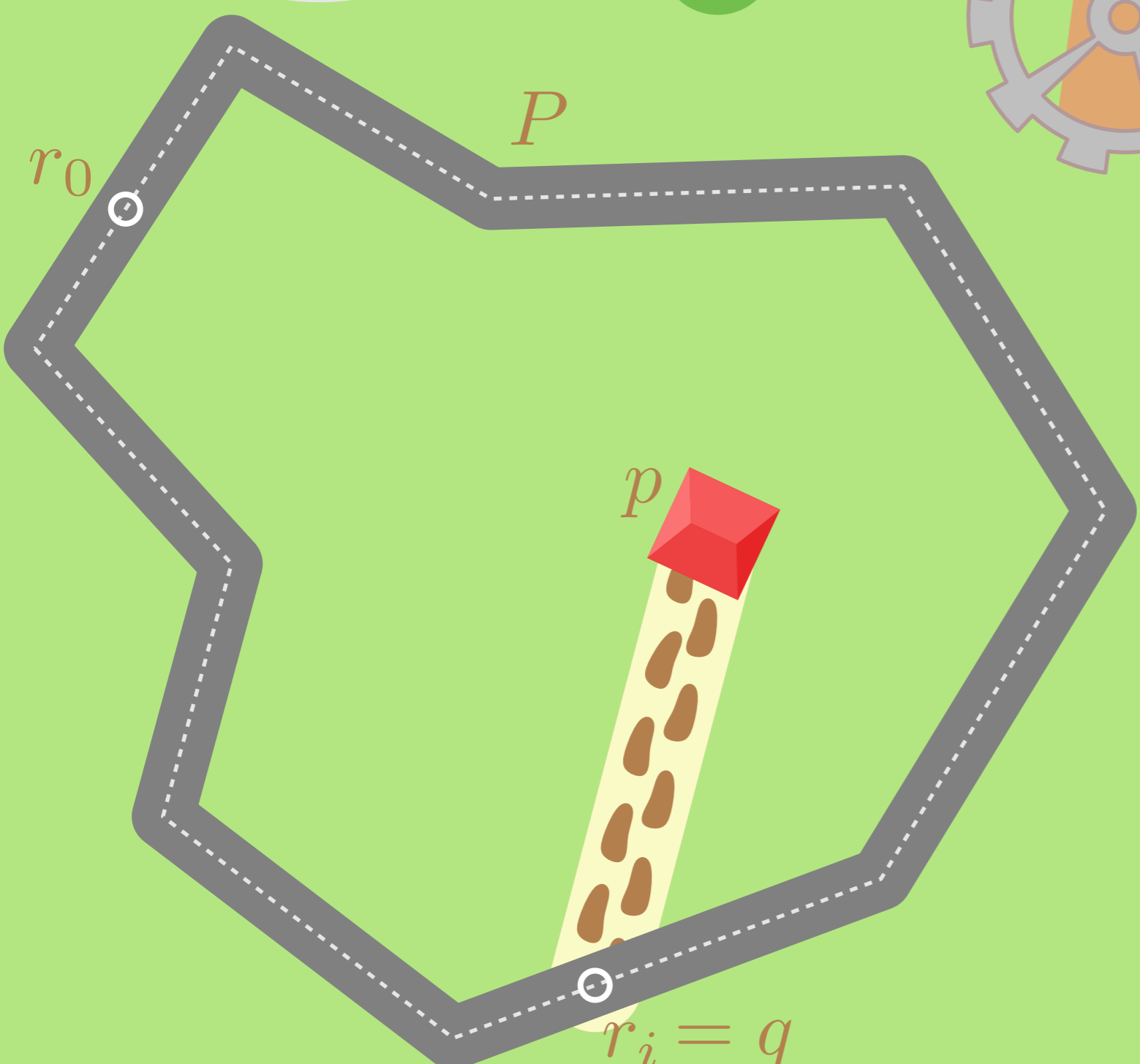
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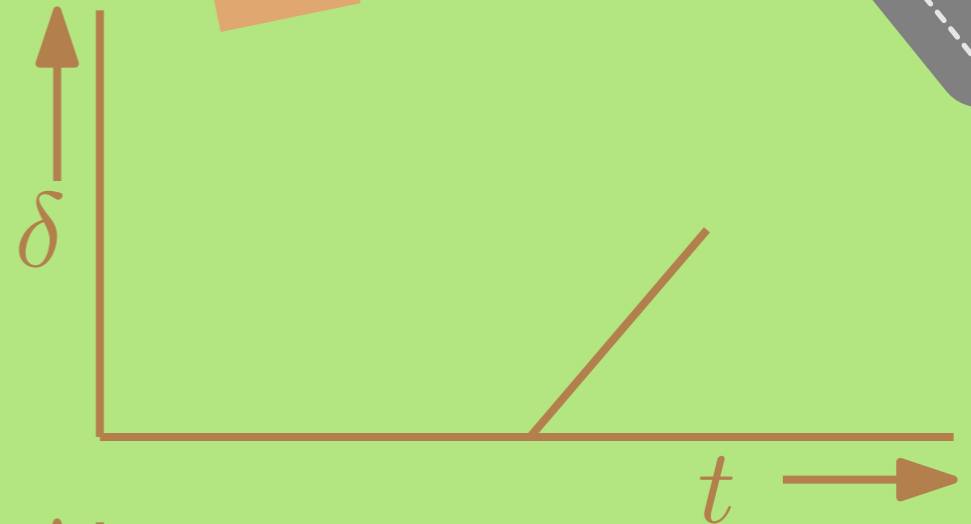
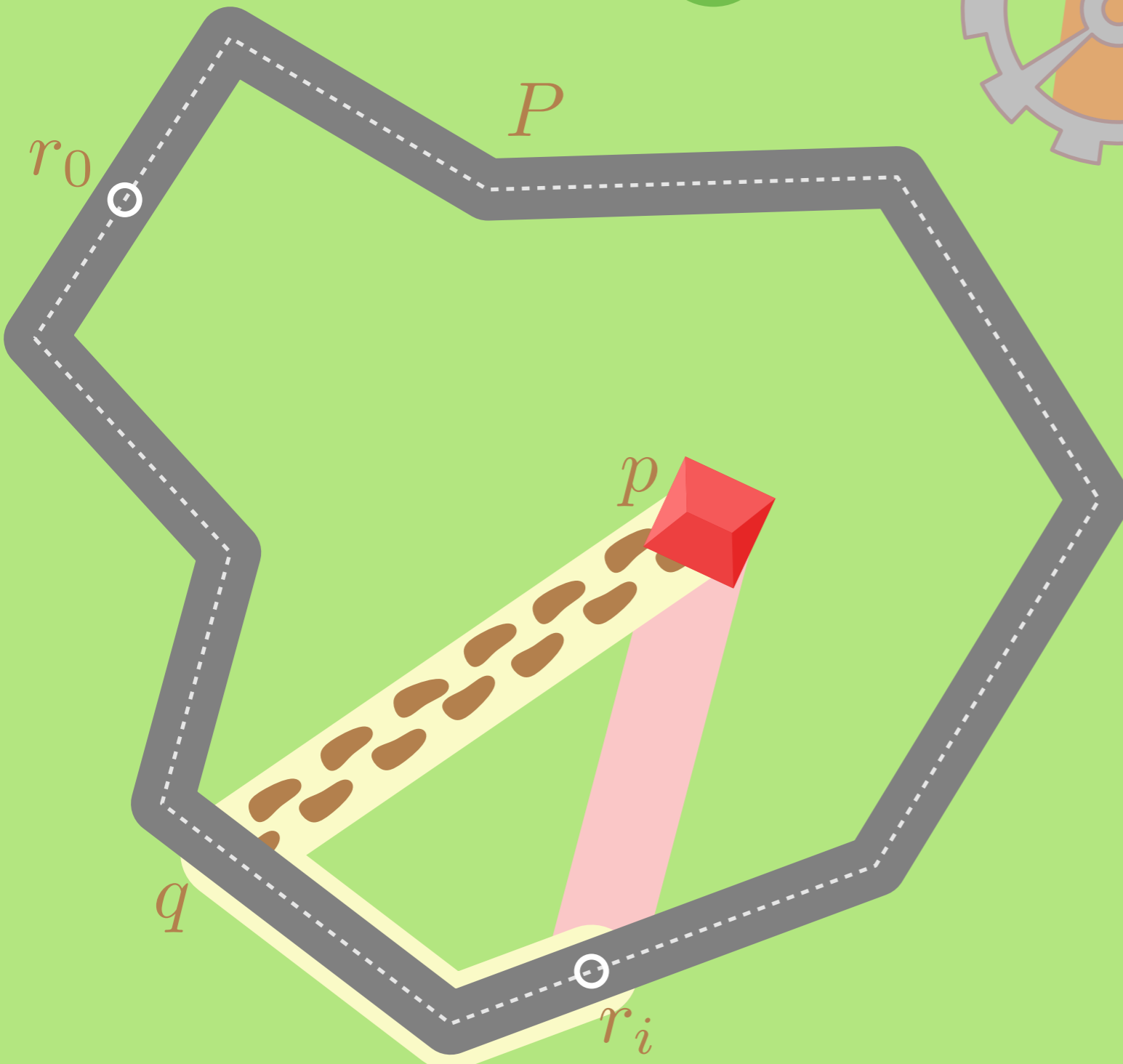
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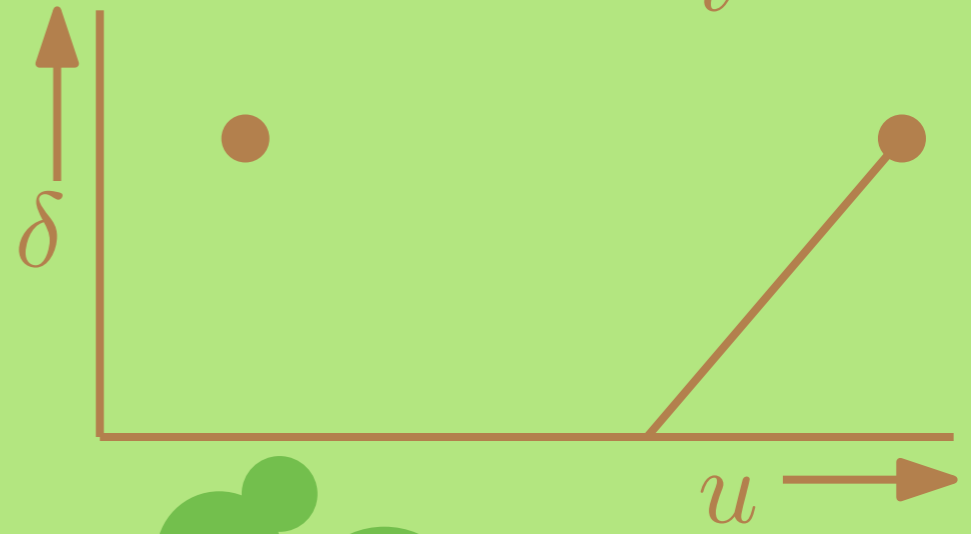
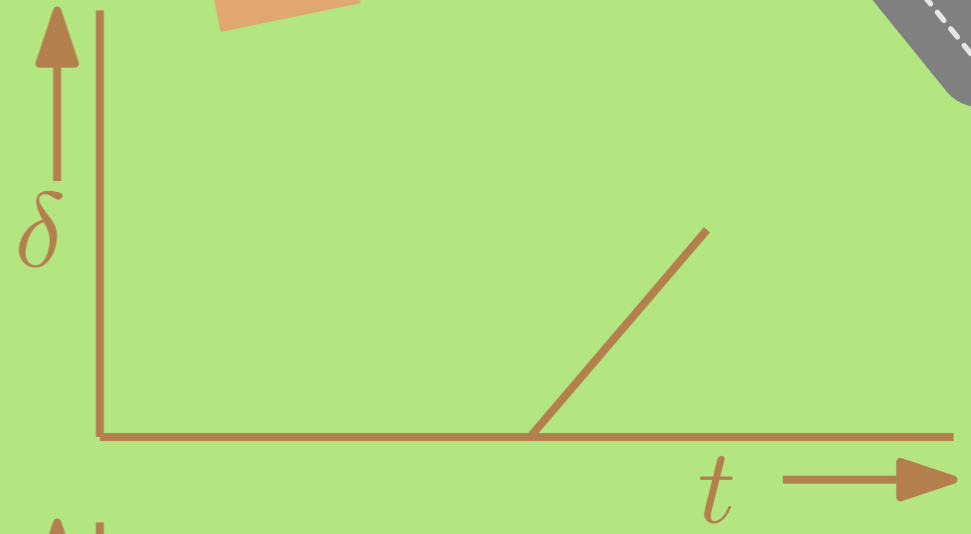
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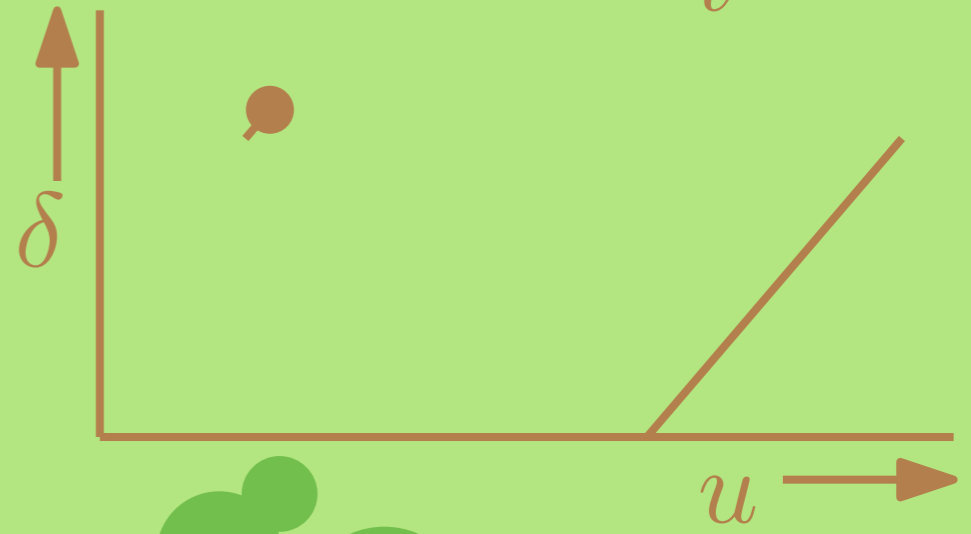
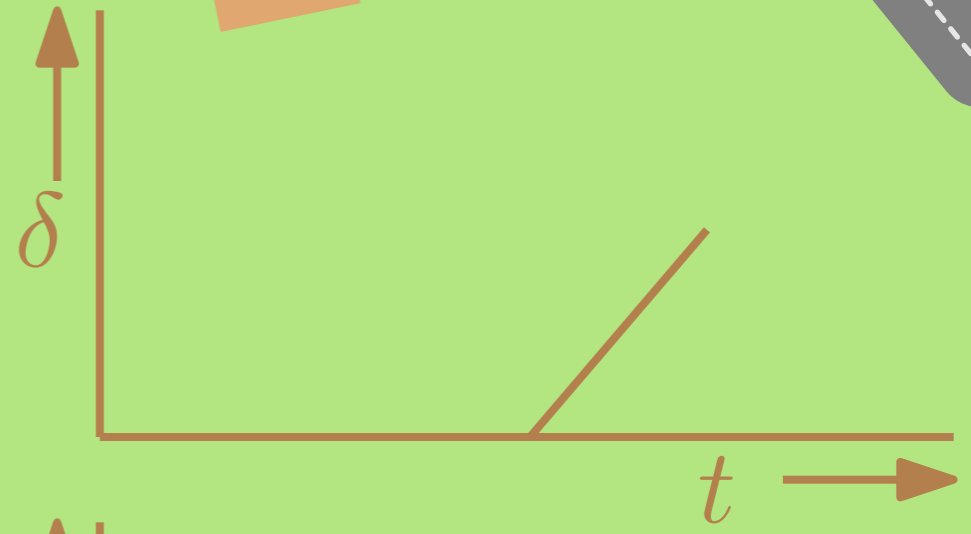
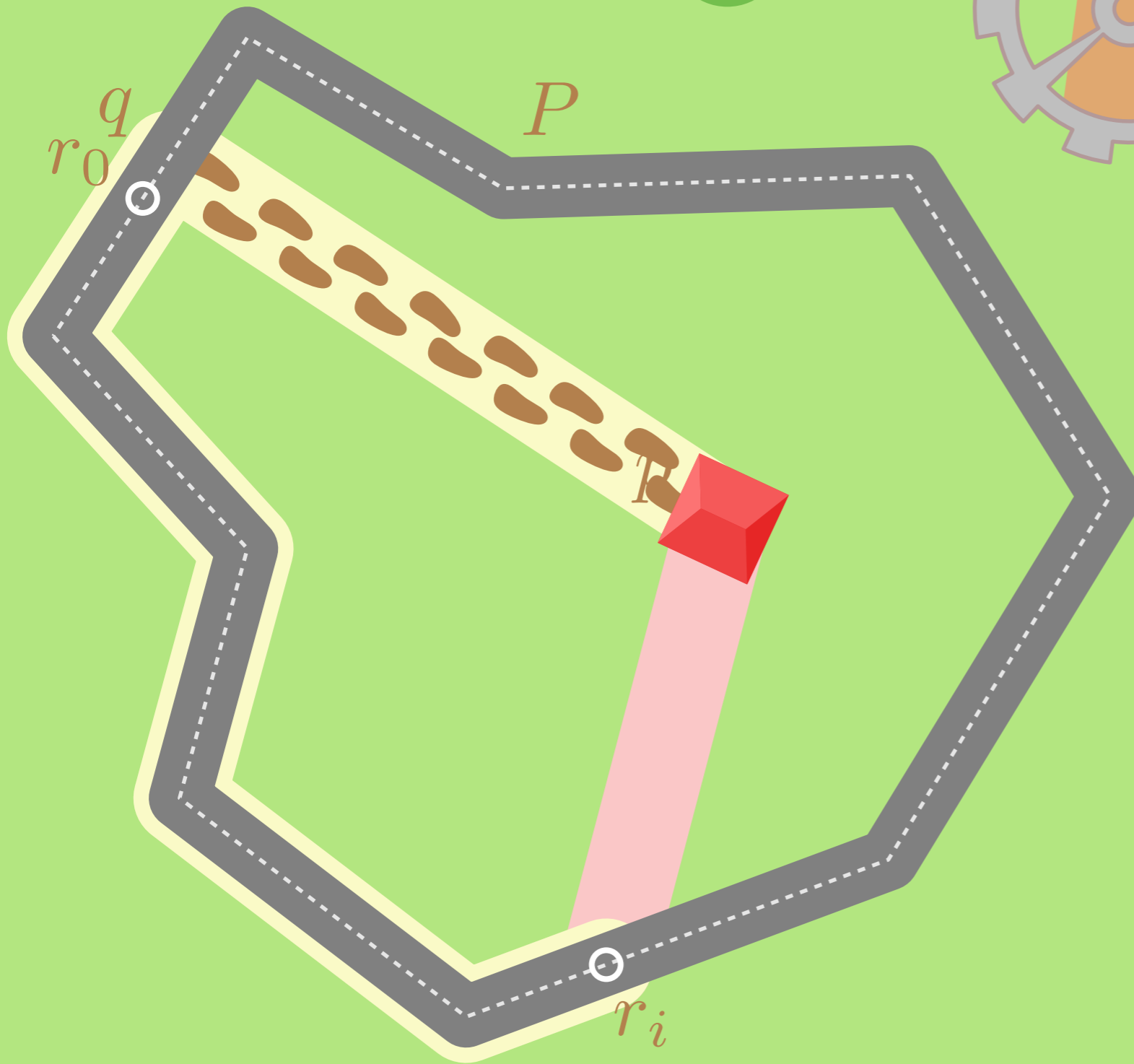
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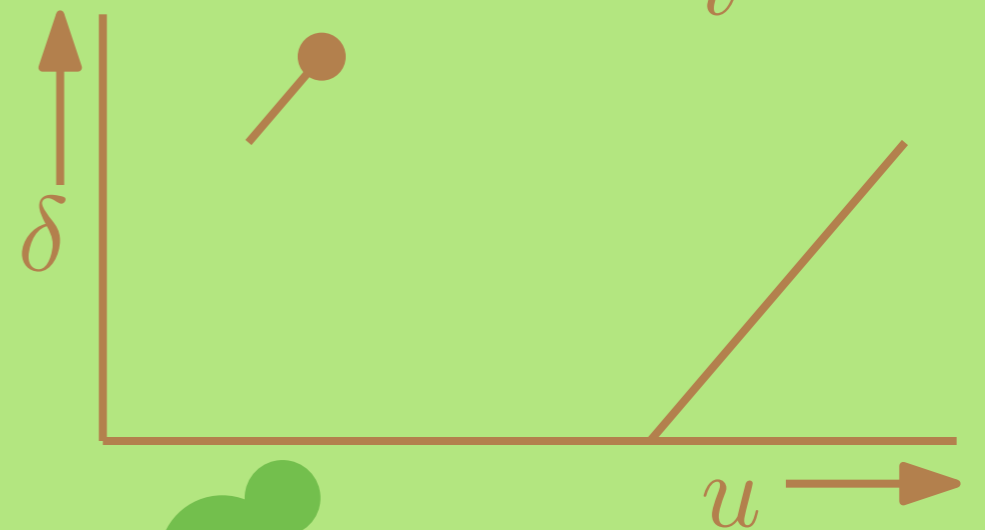
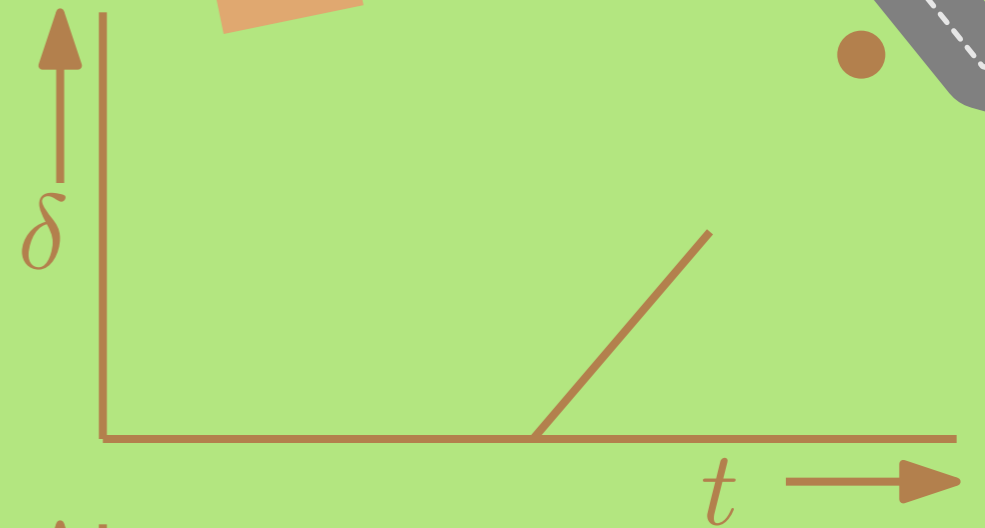
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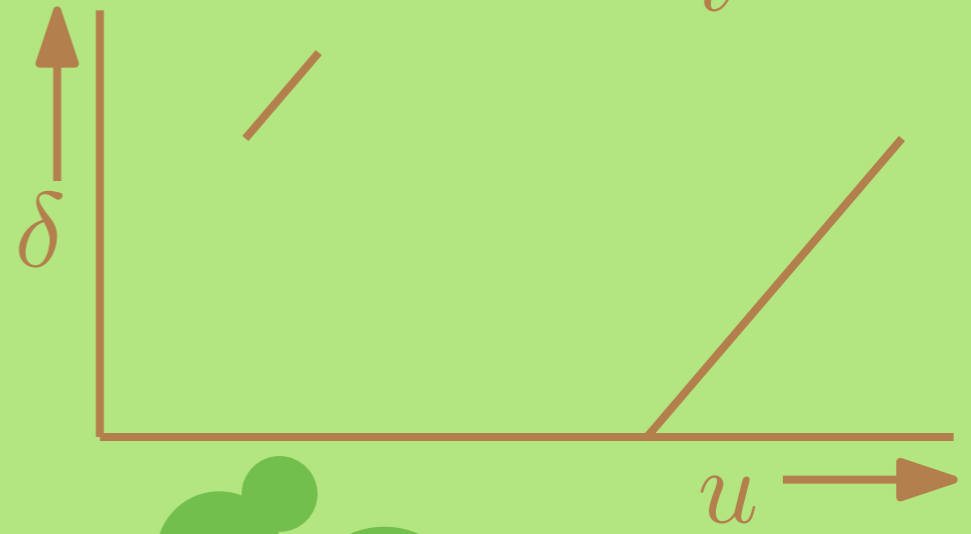
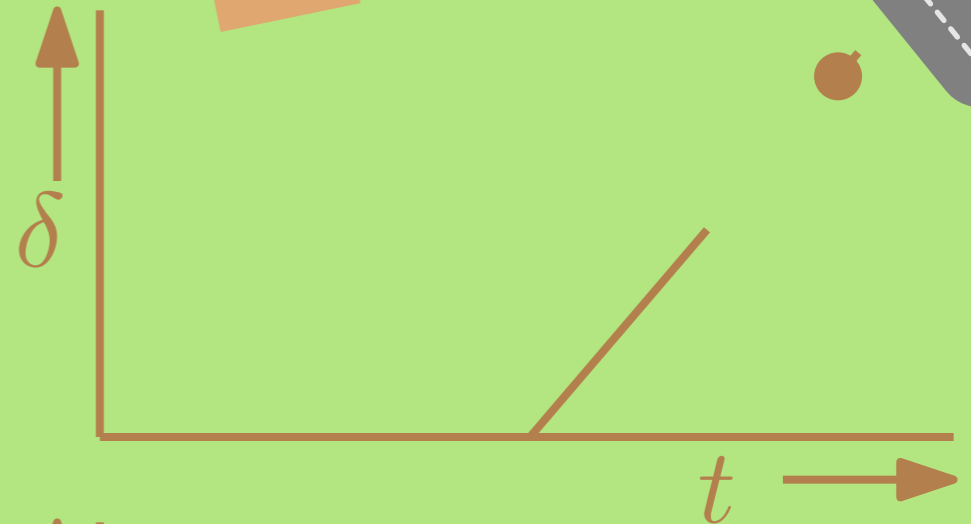
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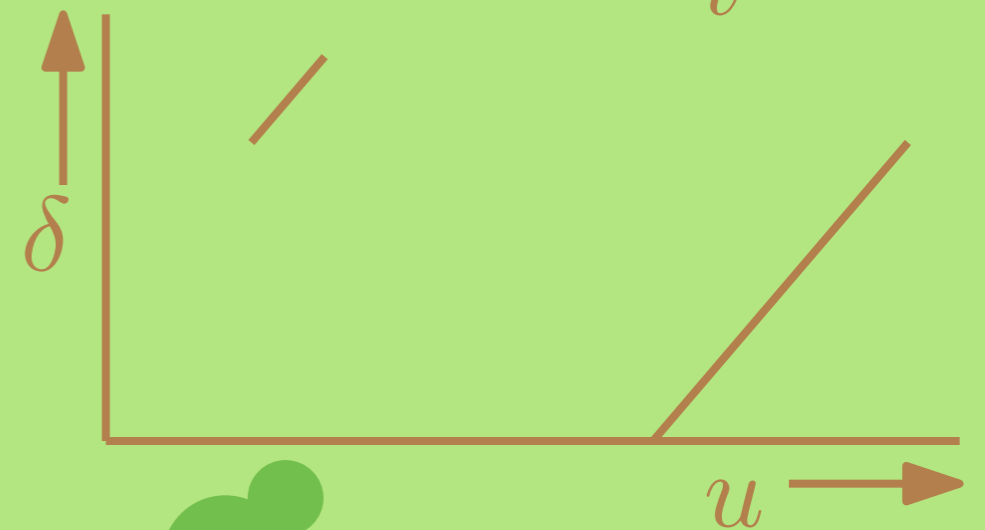
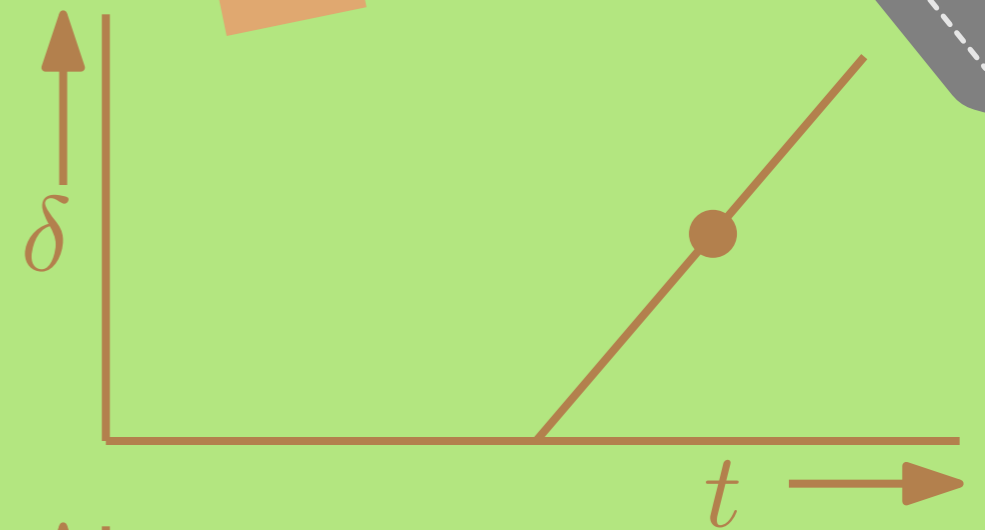
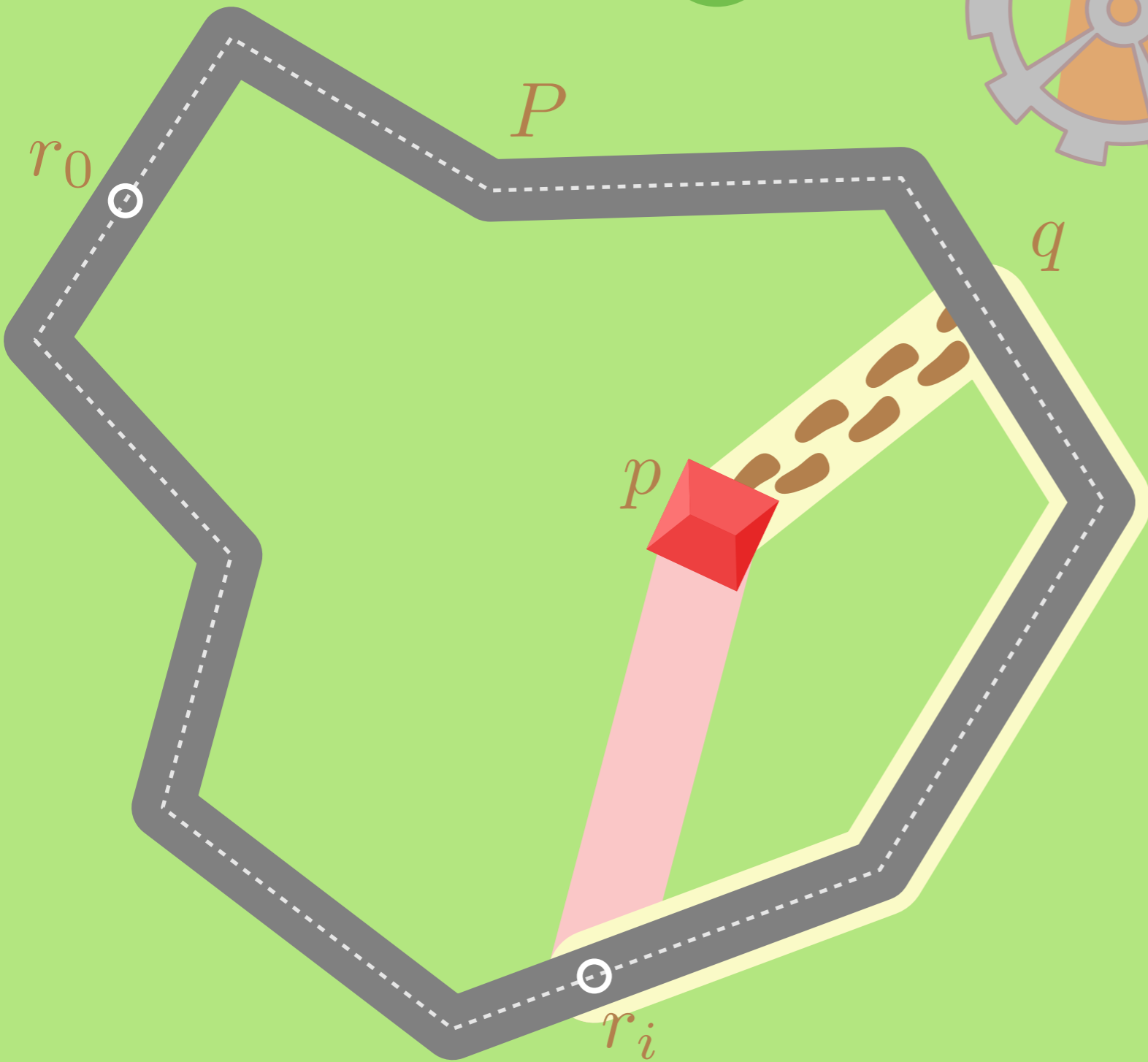
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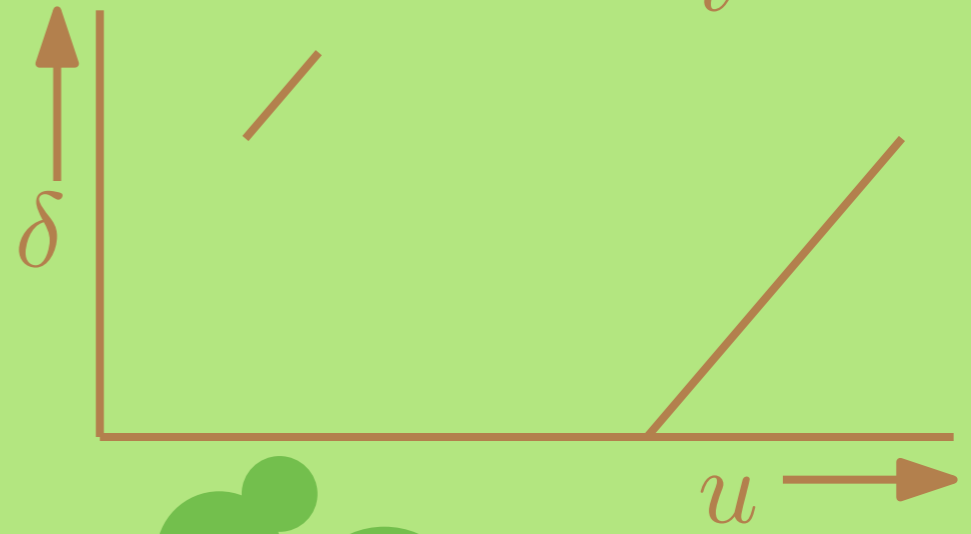
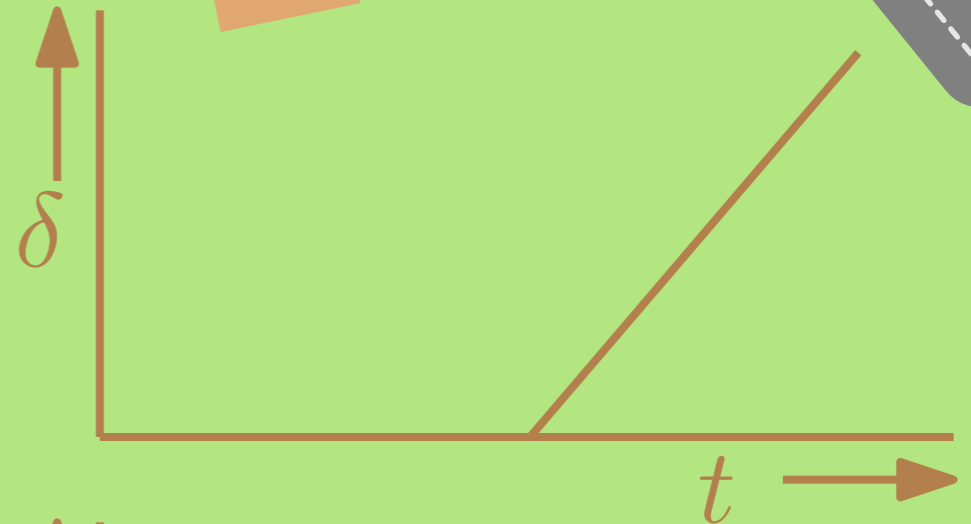
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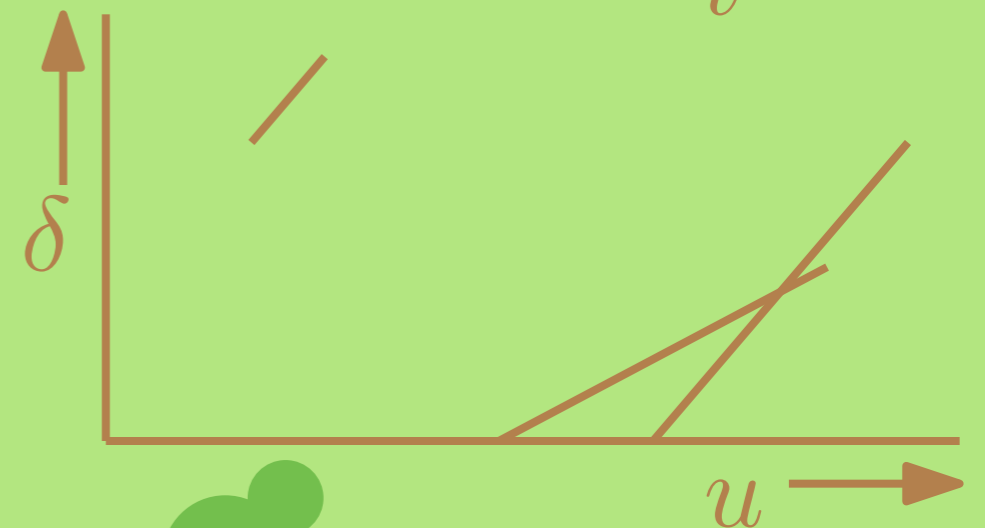
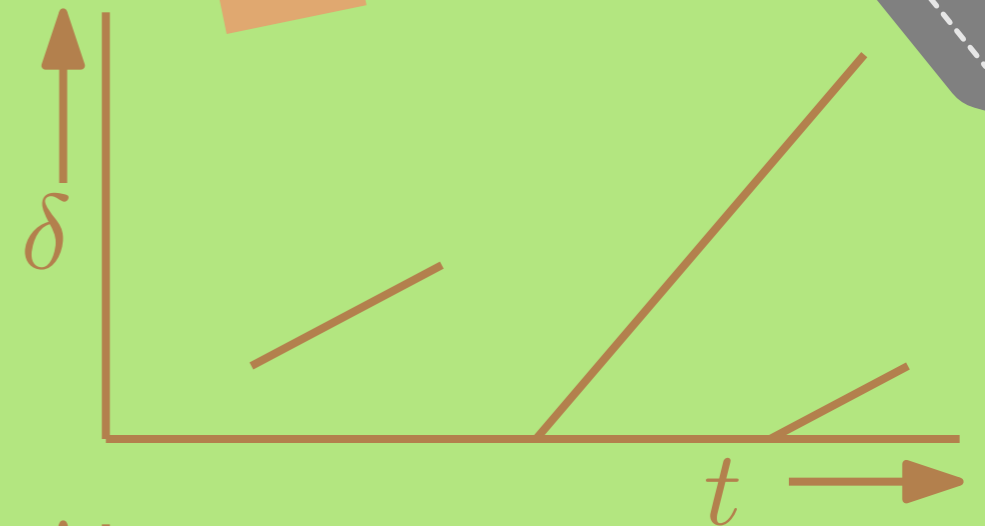
We do the same for the other points.



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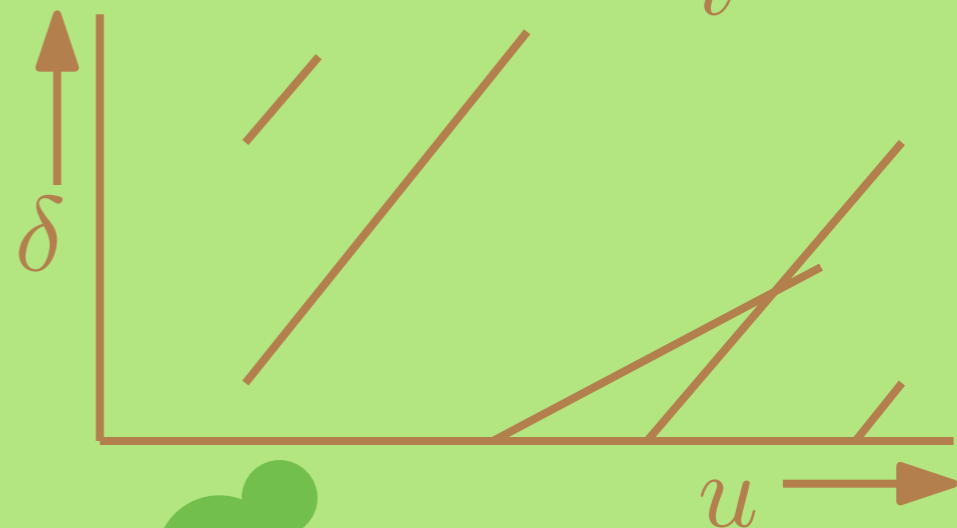
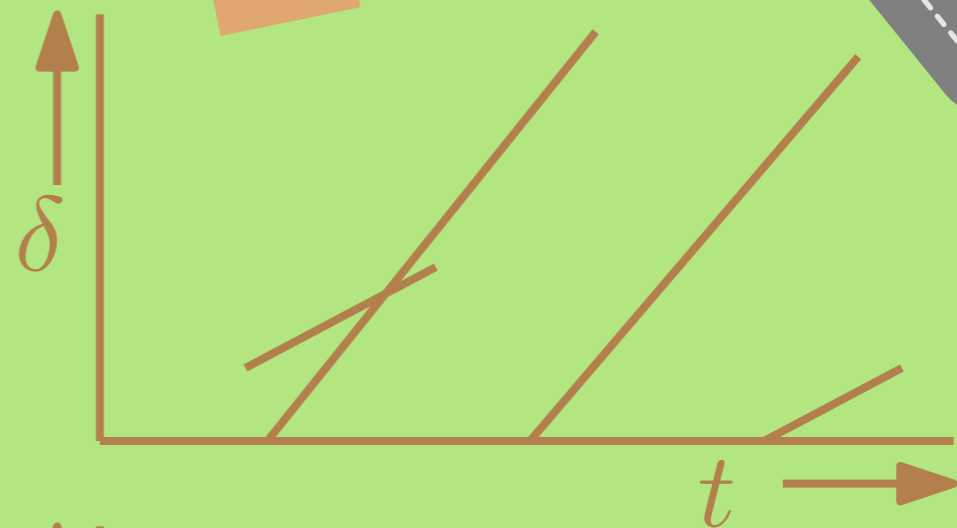
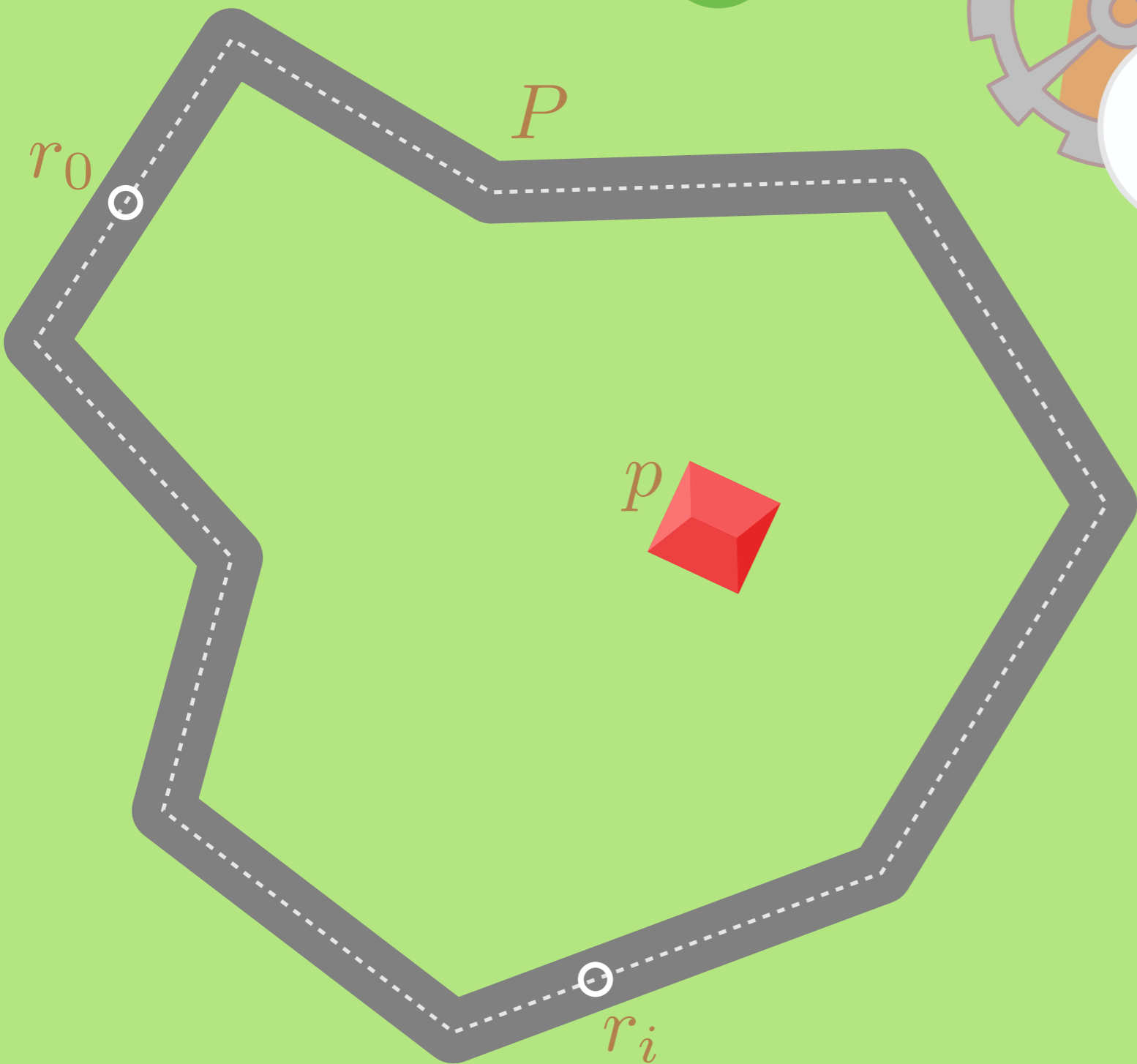
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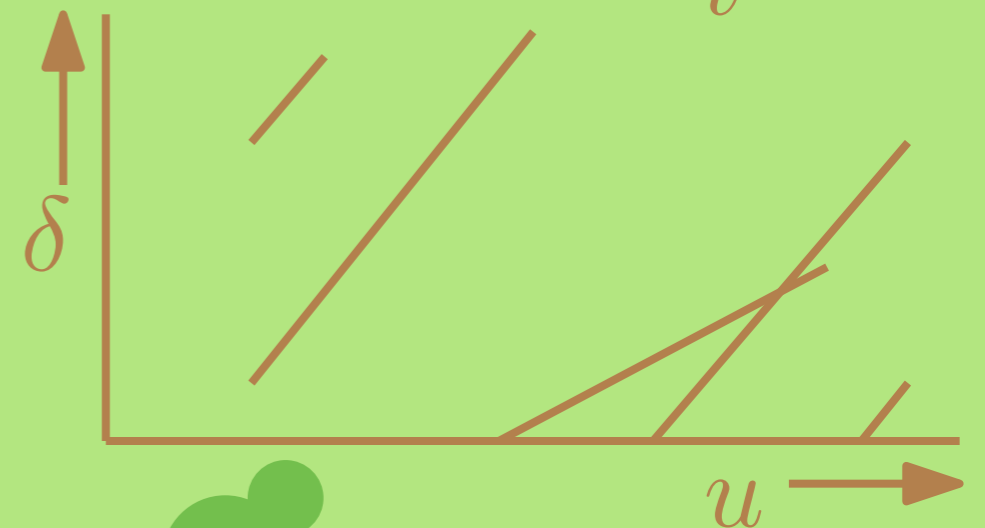
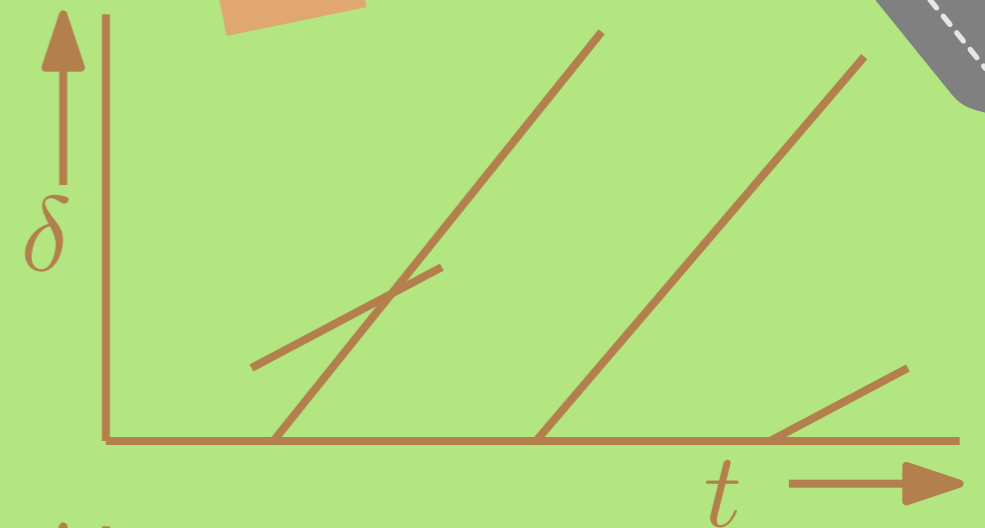
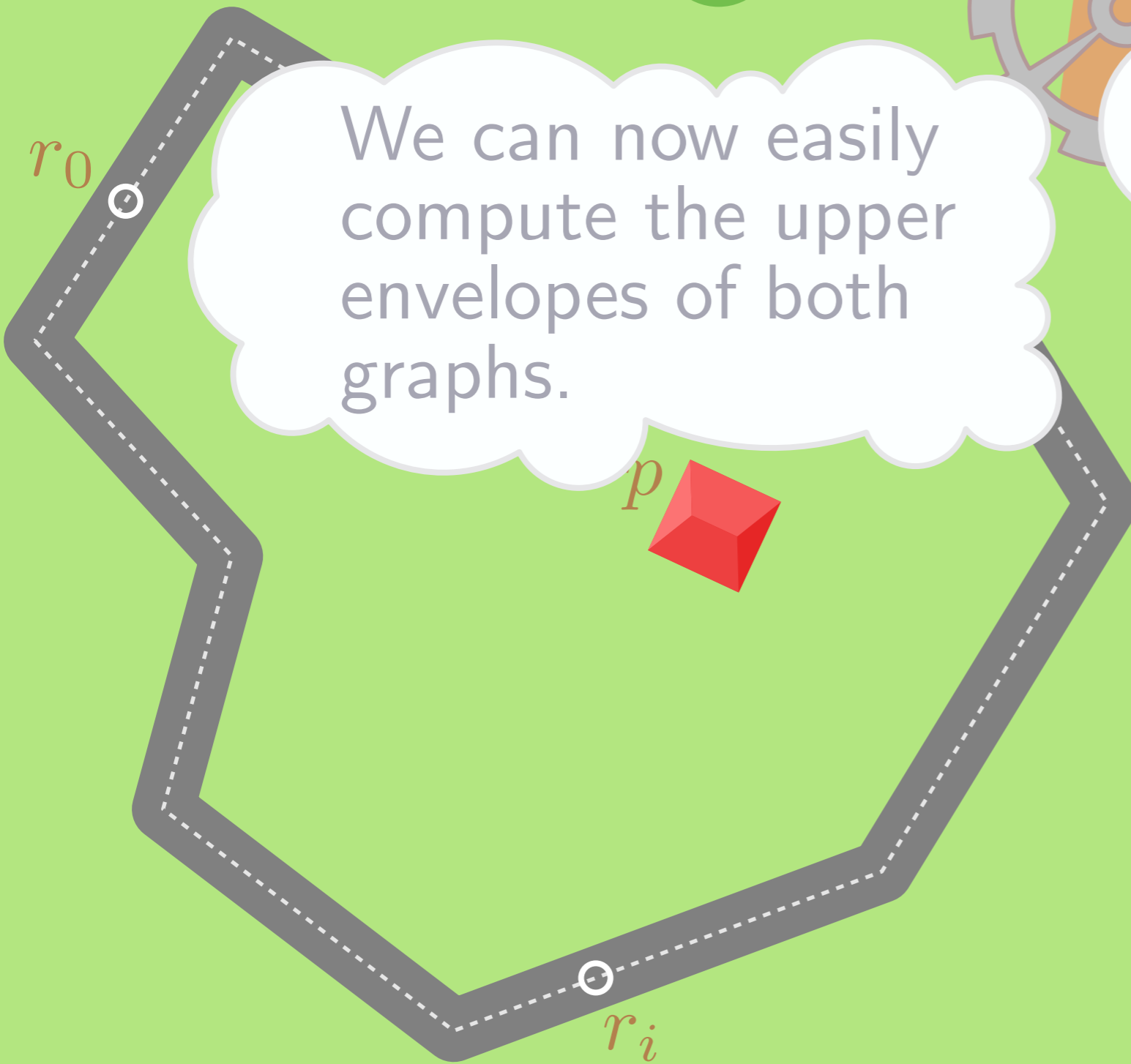


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.

Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.

We can now easily compute the upper envelopes of both graphs.

We do the same for the other points.



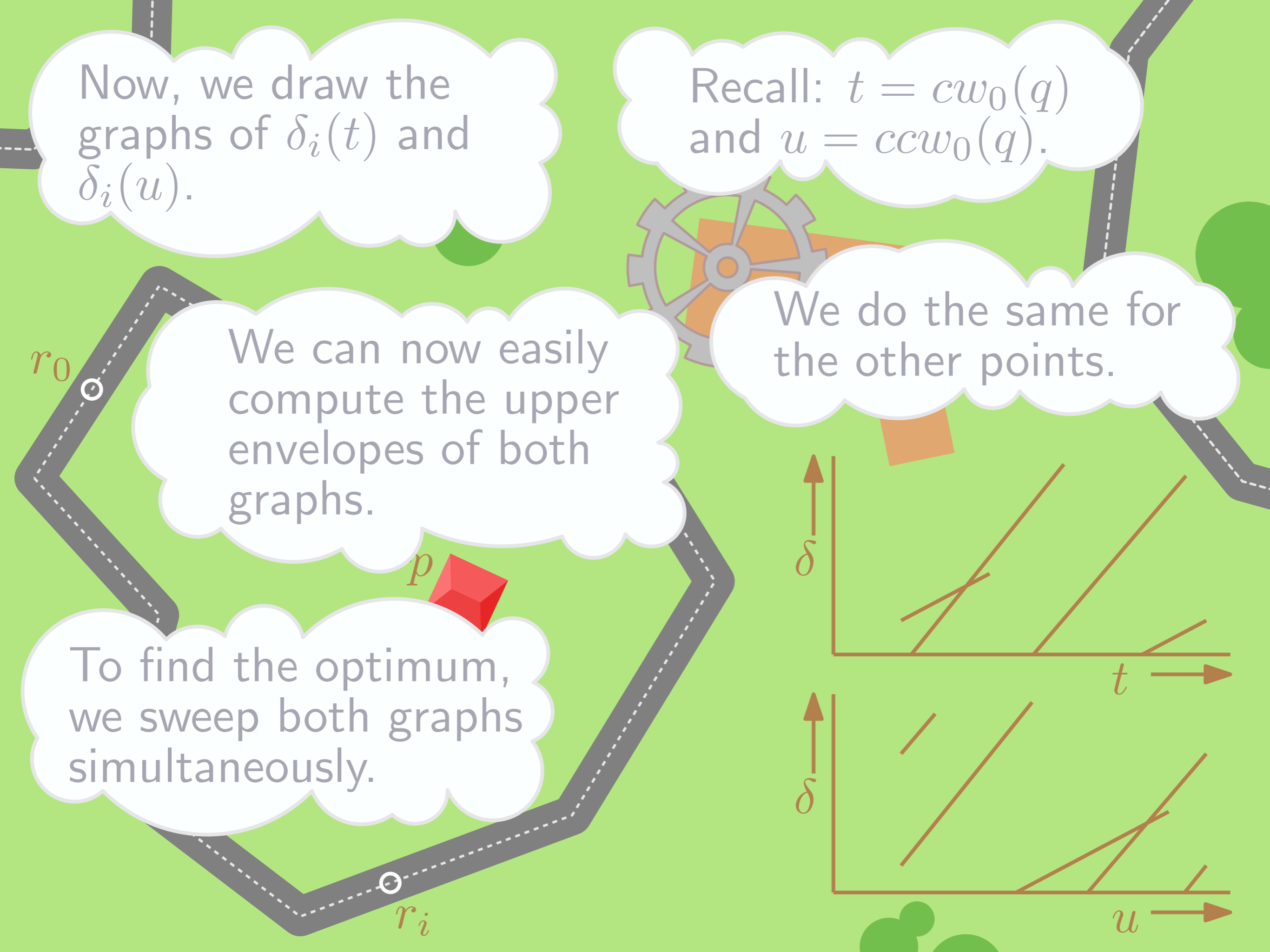
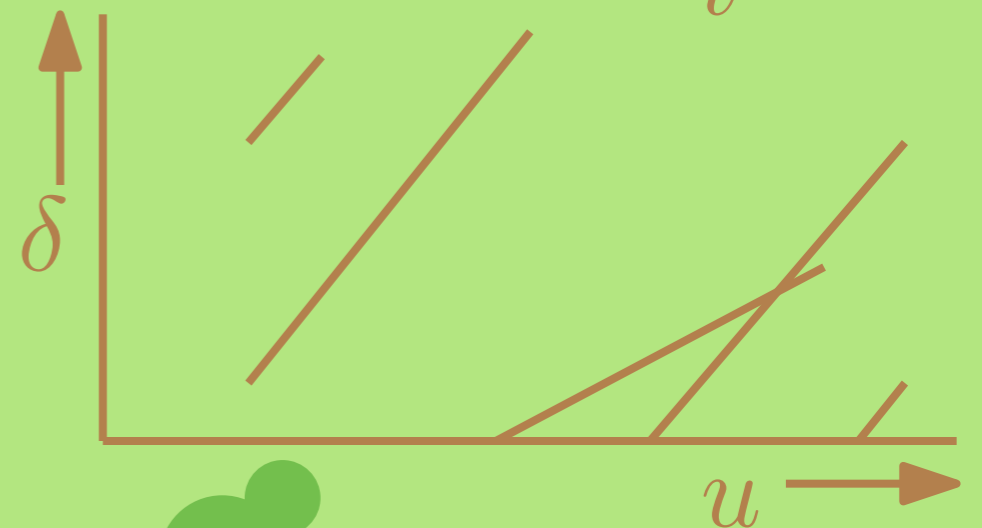
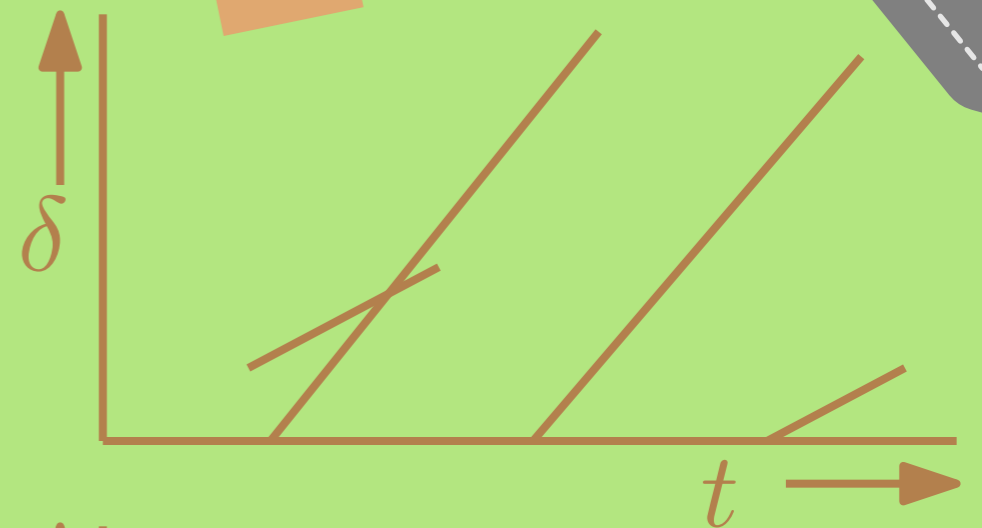
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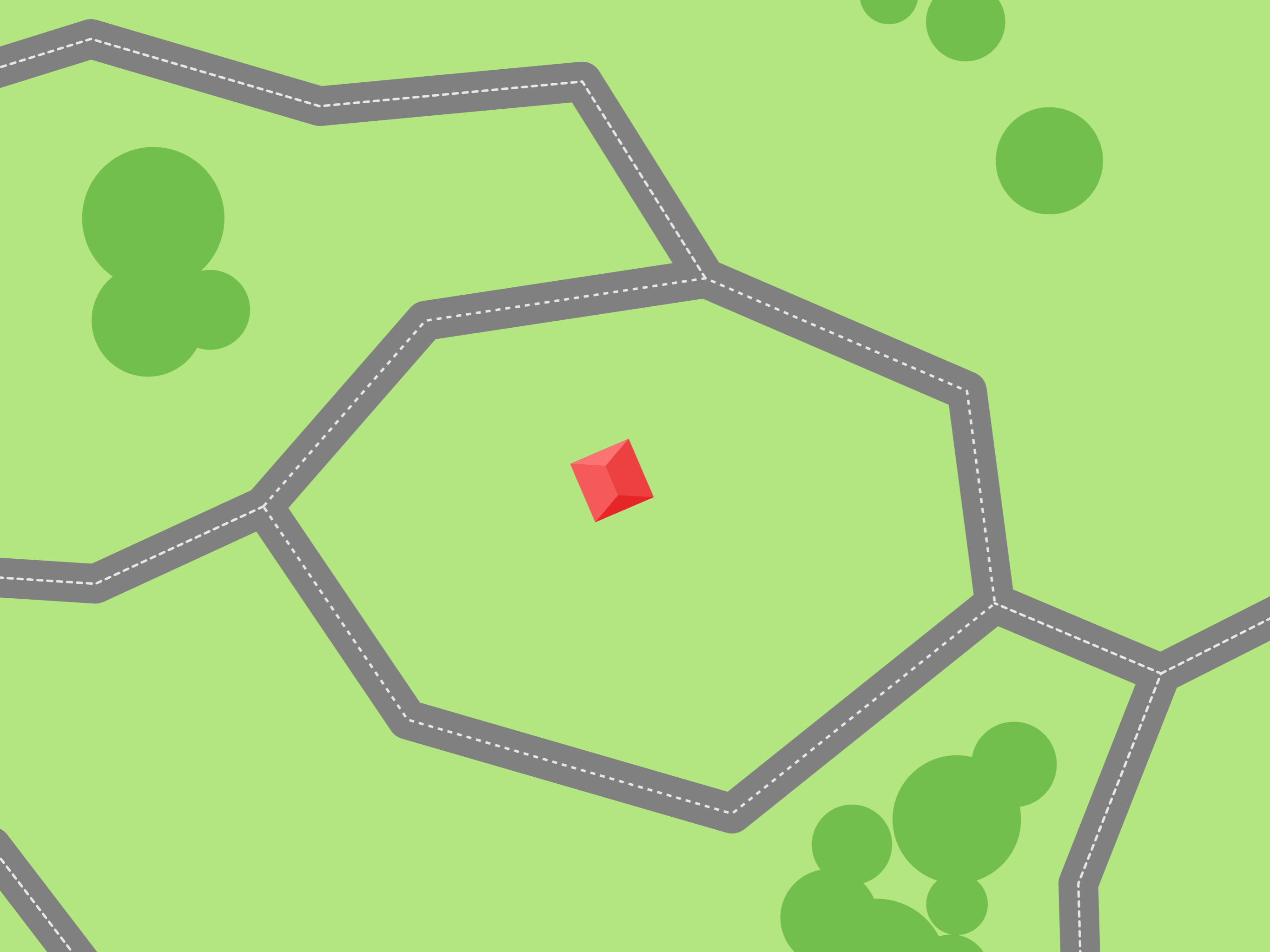
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.

We can now easily compute the upper envelopes of both graphs.

We do the same for the other points.

To find the optimum, we sweep both graphs simultaneously.





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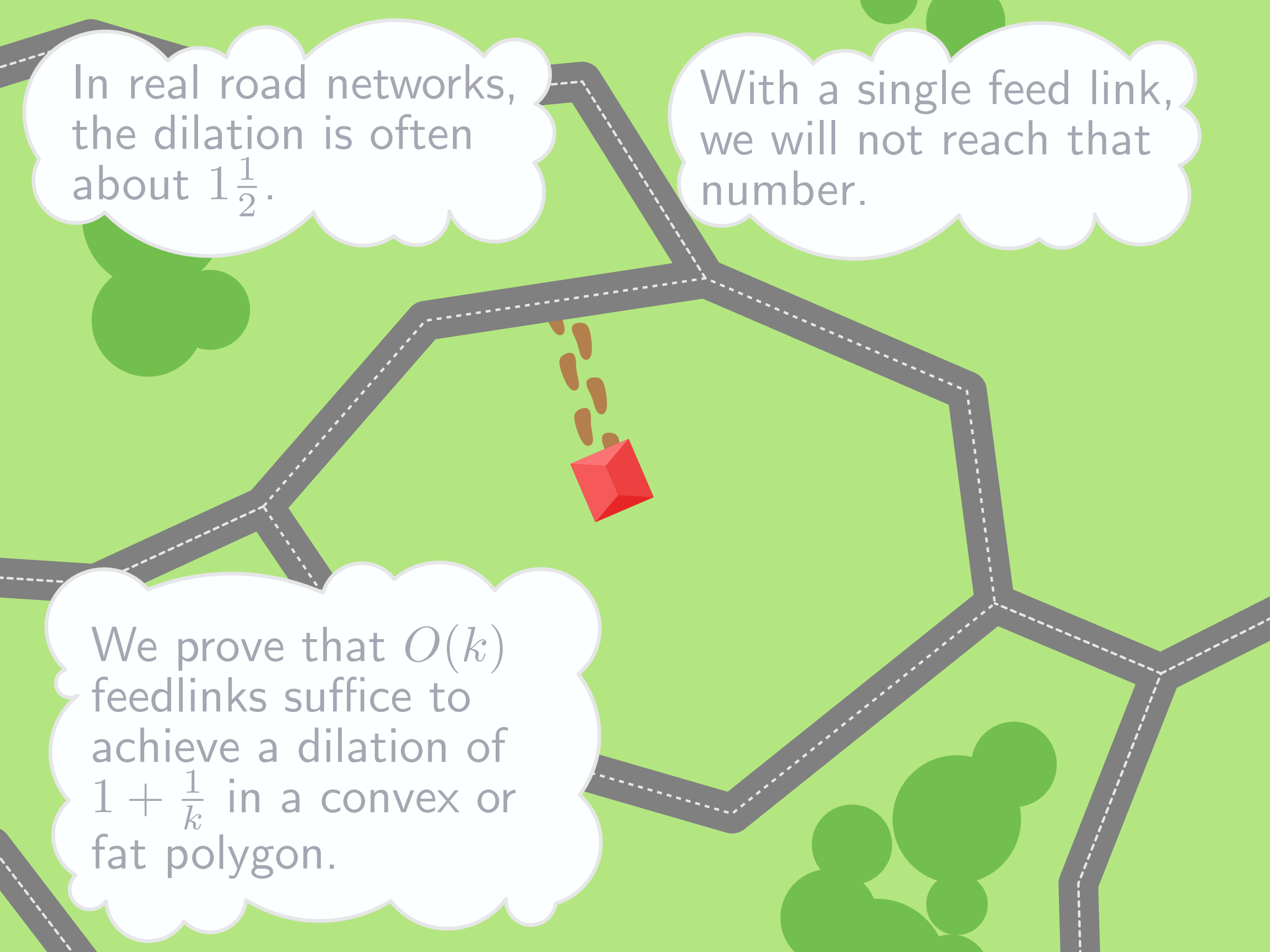
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we will not reach that
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number.



The diagram shows a network of dark grey roads with dashed white center lines. A red 3D cube is positioned in the center, with a dashed brown line representing a path leading from the cube towards the network. The background is light green with stylized green trees.

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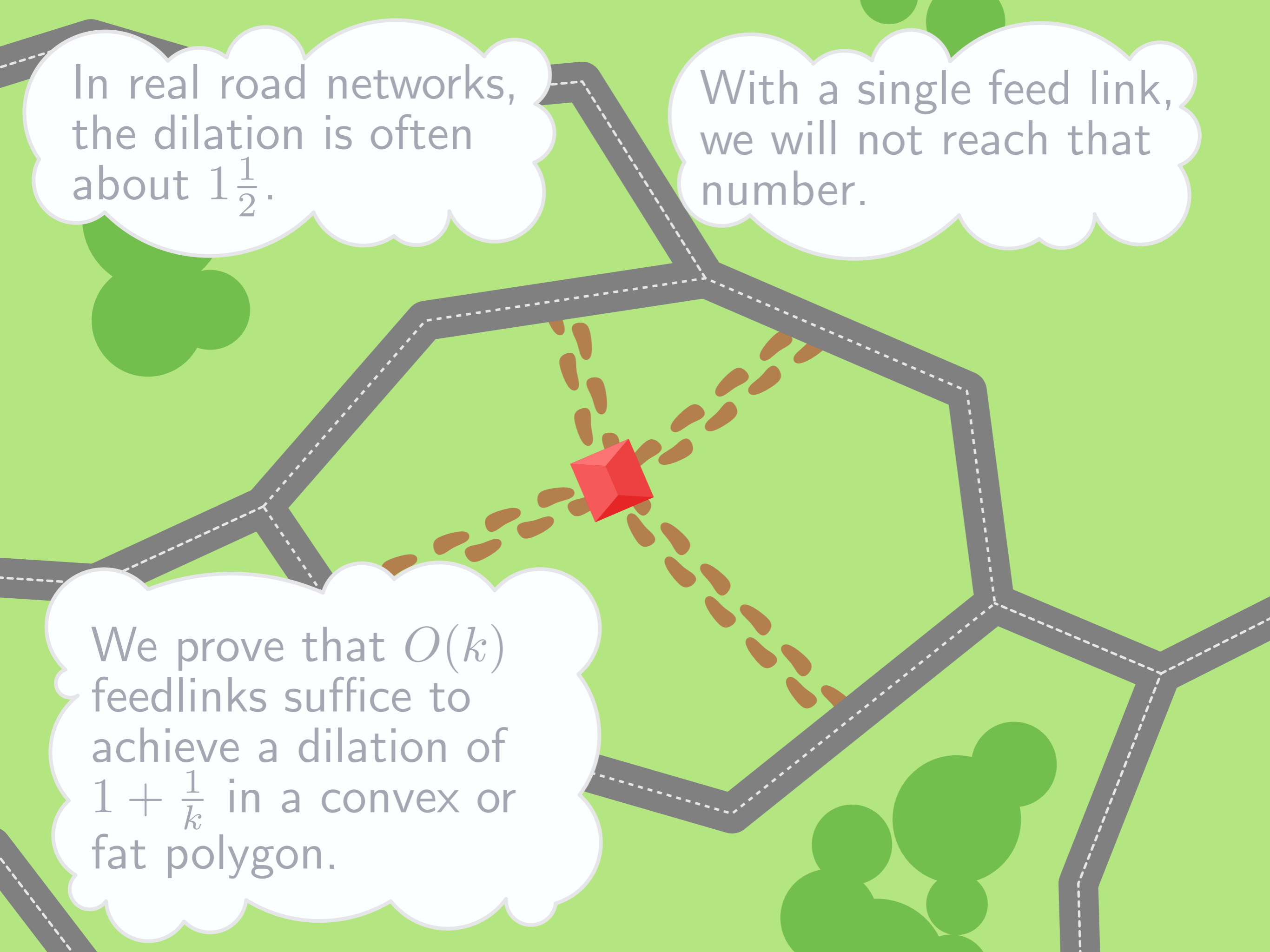
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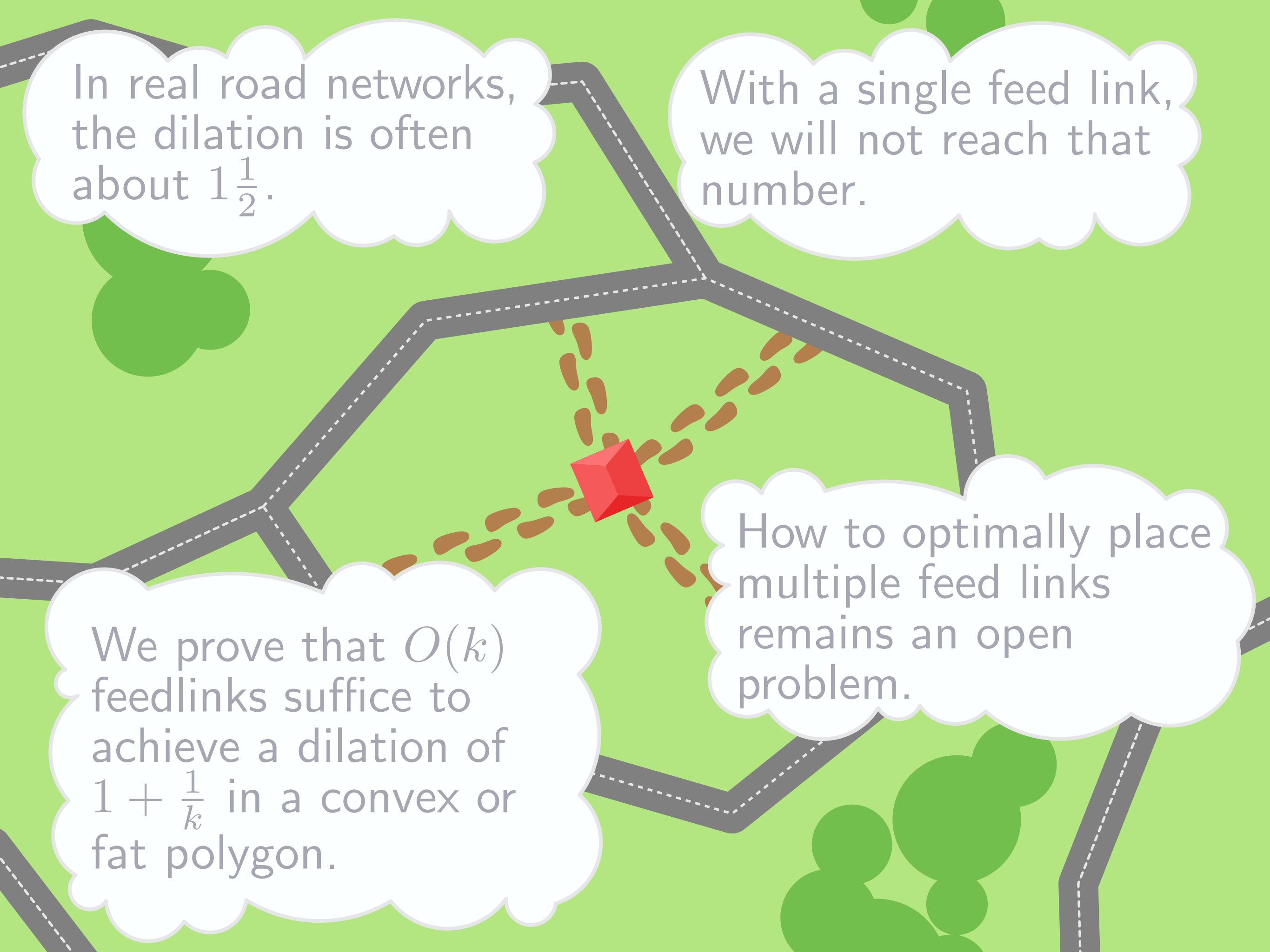
We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1 + \frac{1}{k}$ in a convex or fat polygon.

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We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1 + \frac{1}{k}$ in a convex or fat polygon.



The diagram shows a network of grey roads with dashed white center lines on a green background with stylized green trees. A red 3D cube is positioned in the center of the network. A dashed brown path leads from the cube towards the top-left of the network. Three white thought bubbles with grey outlines contain text. The top-left bubble says 'In real road networks, the dilation is often about 1 1/2.' The top-right bubble says 'With a single feed link, we will not reach that number.' The bottom-left bubble says 'We prove that O(k) feedlinks suffice to achieve a dilation of 1 + 1/k in a convex or fat polygon.' The bottom-right bubble says 'How to optimally place multiple feed links remains an open problem.'

In real road networks, the dilation is often about $1\frac{1}{2}$.

With a single feed link, we will not reach that number.

We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1 + \frac{1}{k}$ in a convex or fat polygon.

How to optimally place multiple feed links remains an open problem.



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In the presence of obstacles, we can define *geodesic* dilation.

A 2D environment with a green background. A grey path with a dashed white line runs through the scene. There are several obstacles: two grey 3D rectangular blocks, a blue triangle with a yellow border, and a red cube. A blue area with a yellow border is also present. A dashed brown line shows a path from the red cube towards the obstacles.

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Also, not all land can be walked on in reality.

Our results also generalise to such geodesic dilation.

In the presence of obstacles, we can define *geodesic* dilation.



A stylized landscape illustration. The background is a blue sky at the top. Below it are green hills and a grey road with a dashed white center line. There are several red cubes scattered across the scene. A white cloud-like shape in the center contains the text "Thank you!".

Thank
you!

Boris
Aronov

Bettina
Speckmann

Maike
Buchin

Kevin
Buchin

Thank
you!

Jun
Luo

Rodrigo
Silveira

Marc
van Kreveld

Maarten
Löffler