Connect The Dot
or
Computing Feed Links with Minimum Dilation

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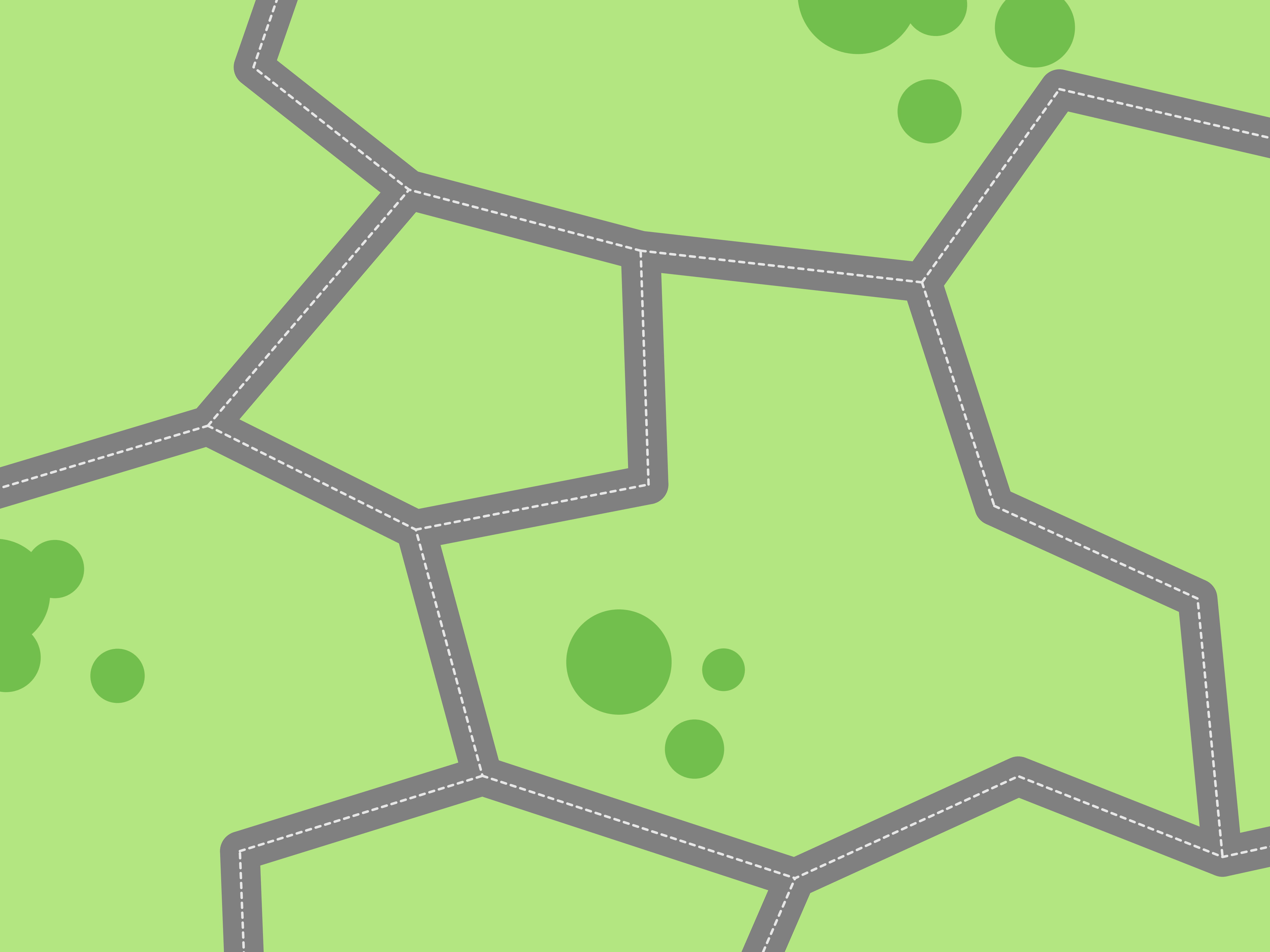
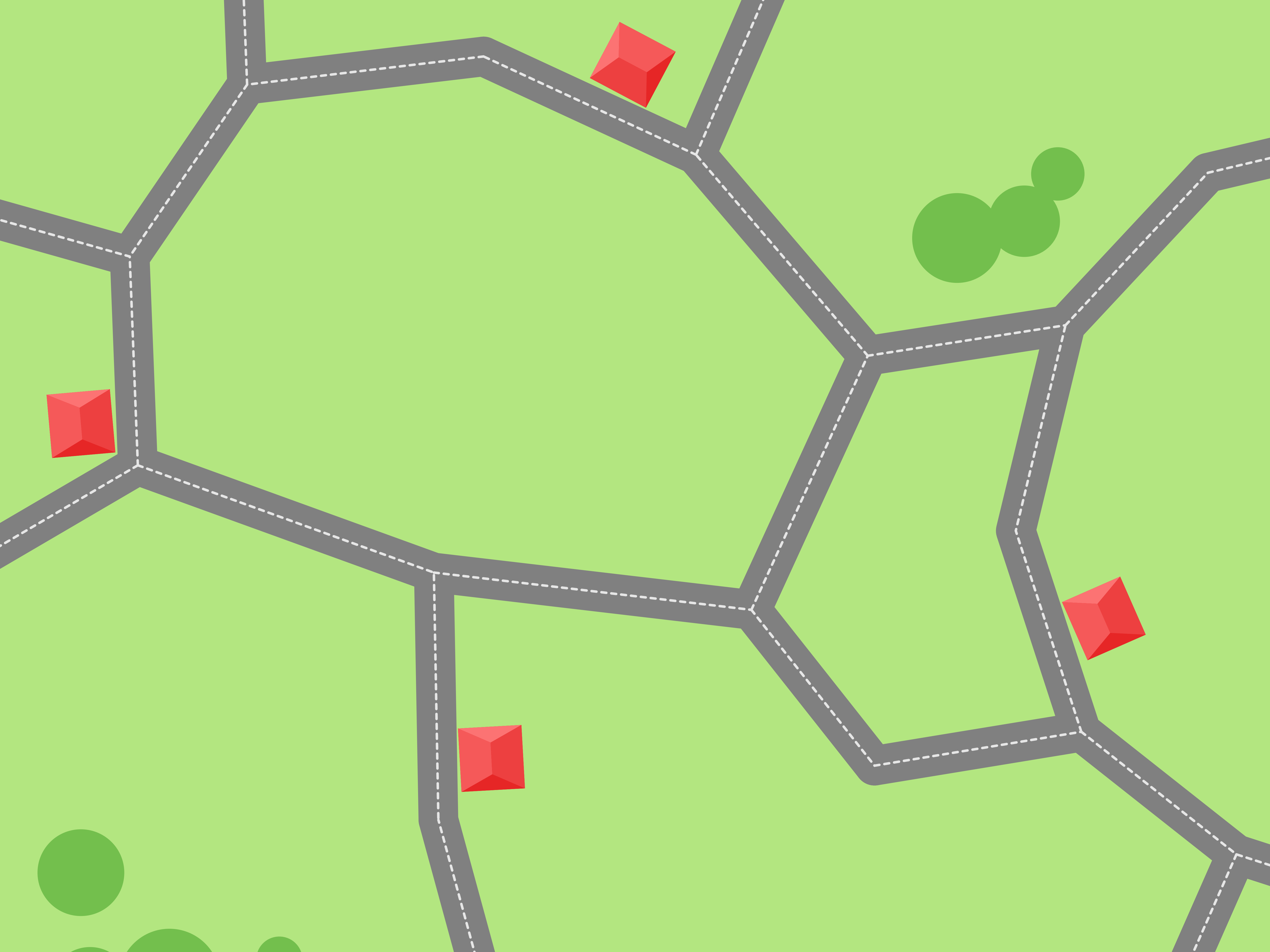
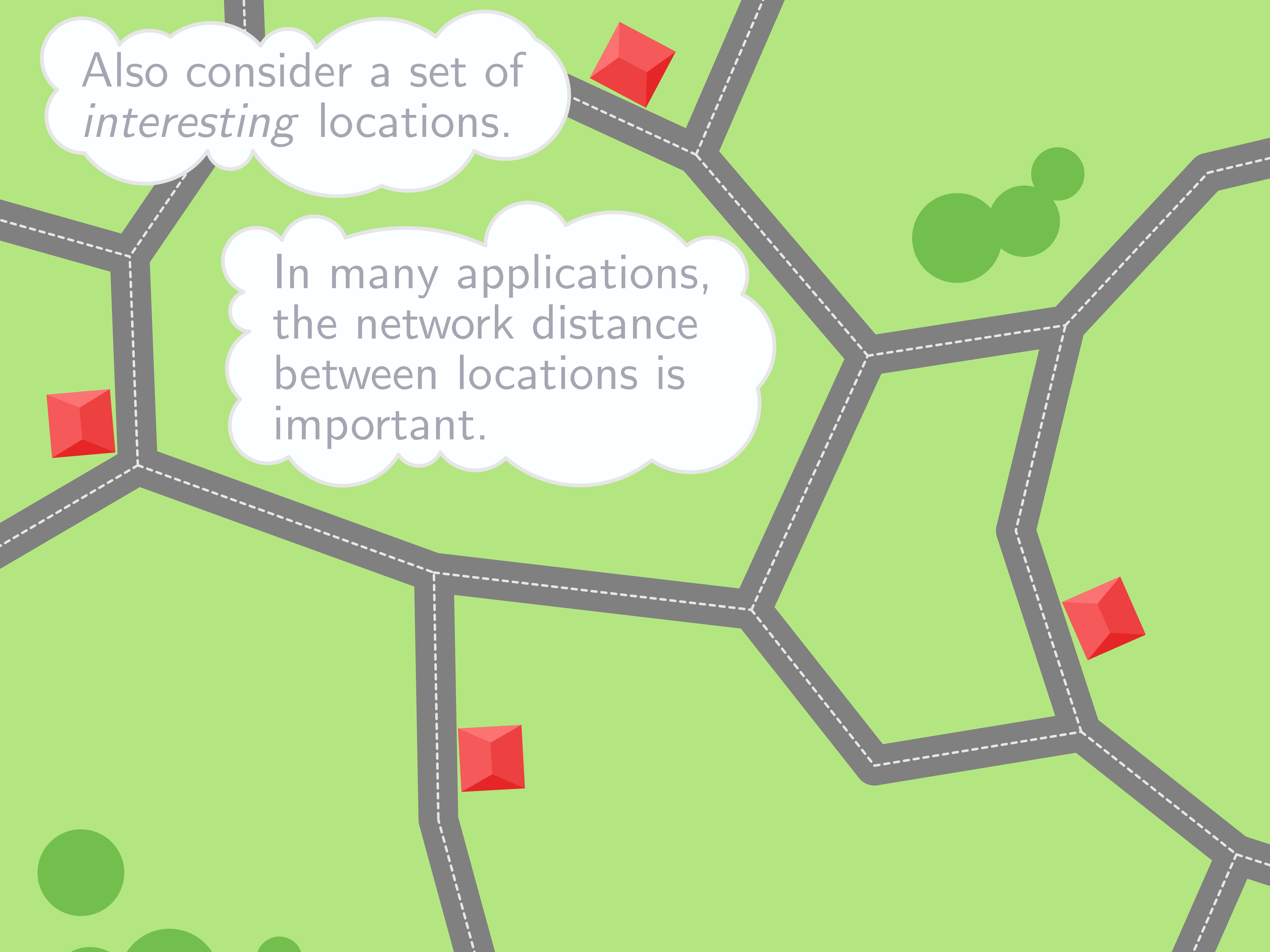
Maarten Löffler




Consider a network of roads.




Also consider a set of *interesting* locations.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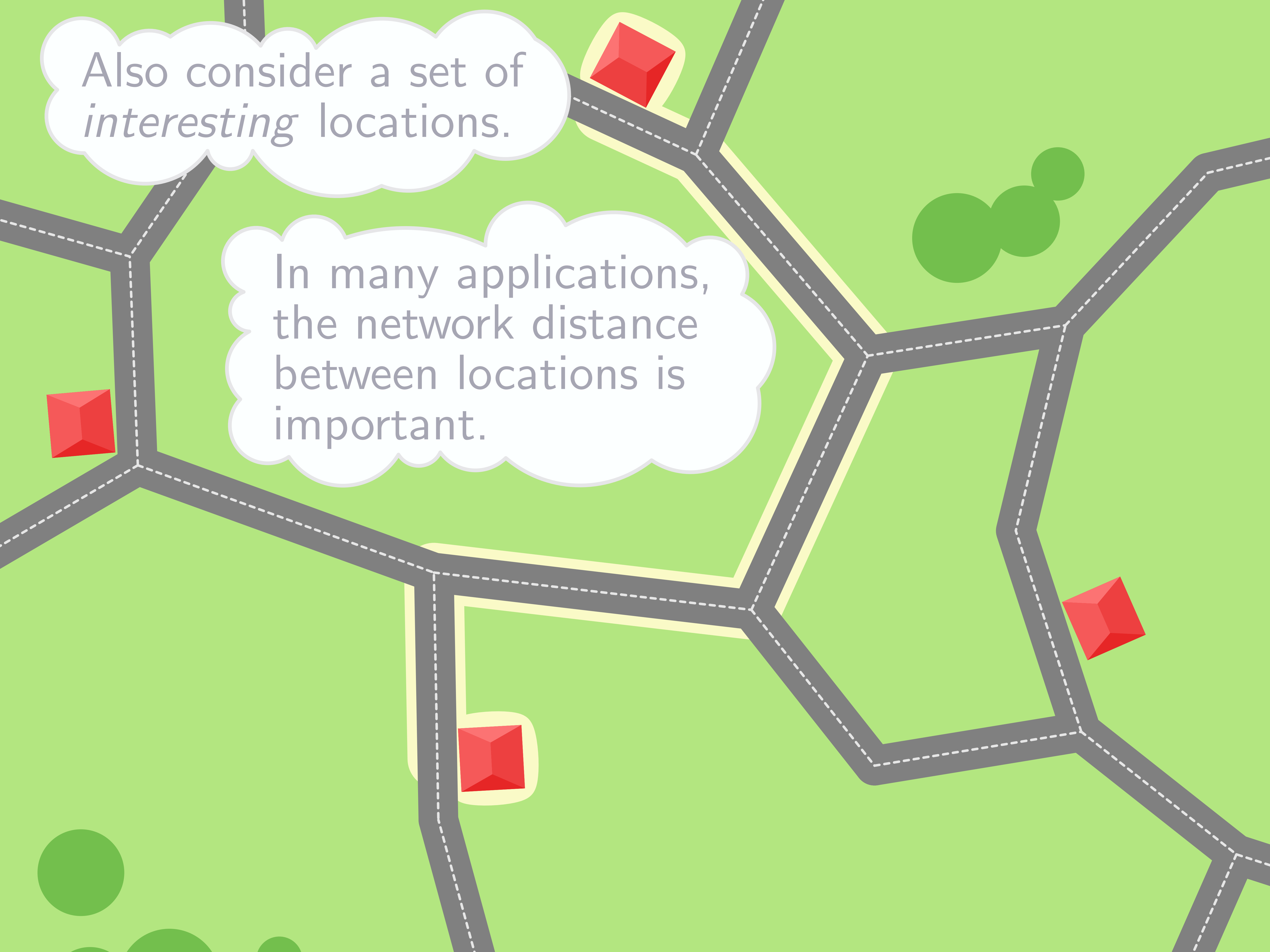
In many applications, the network distance between locations is important.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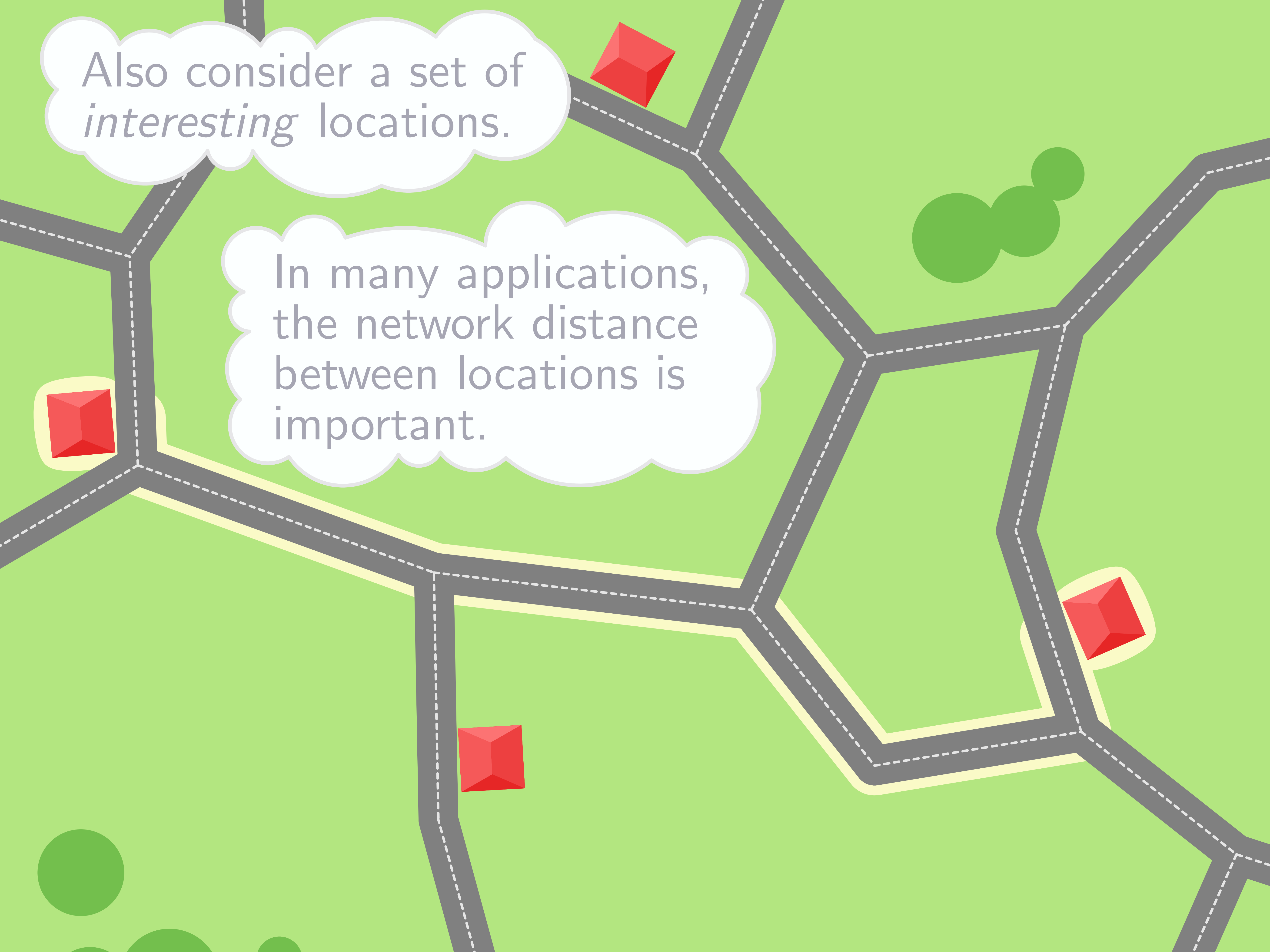
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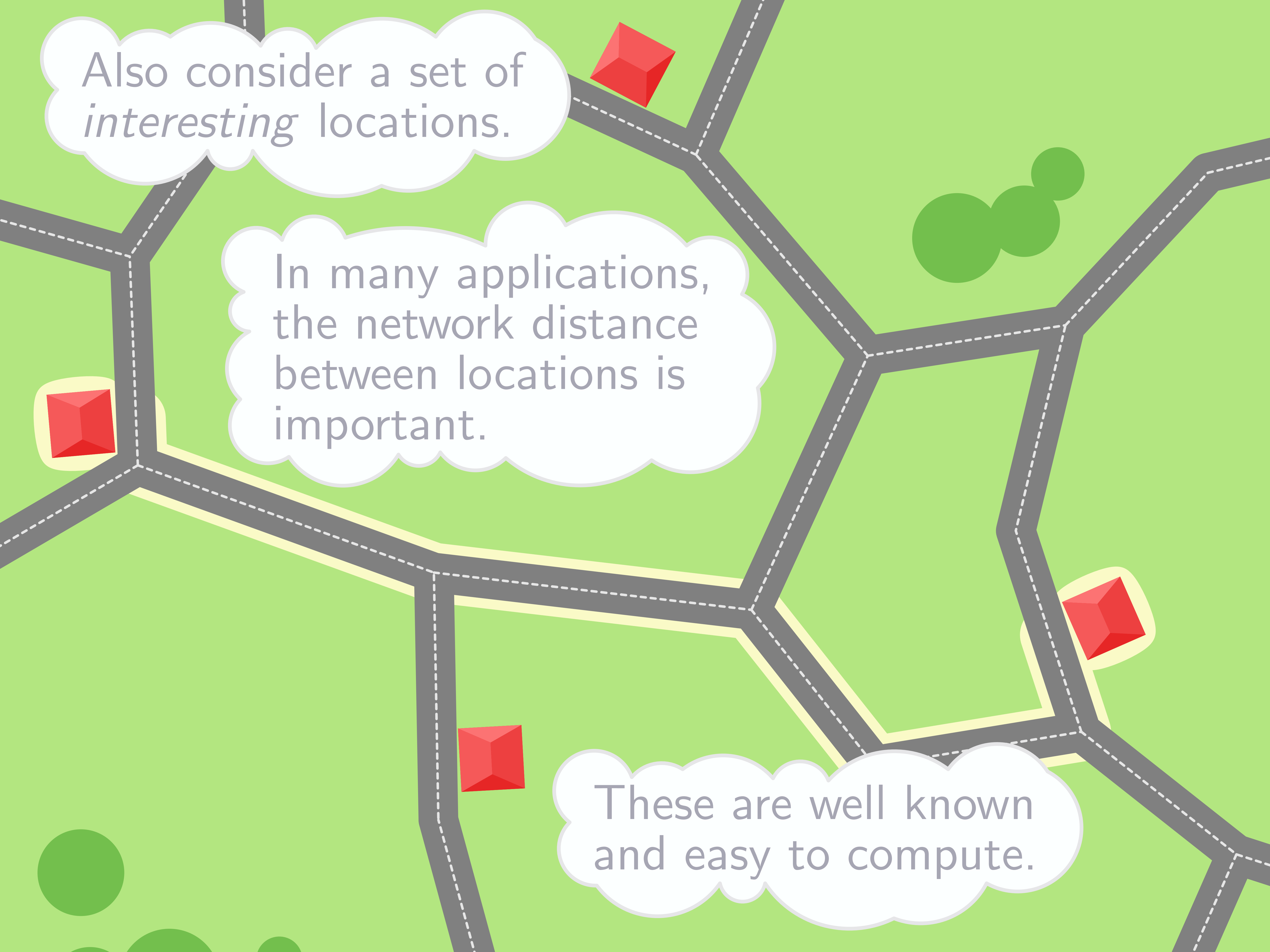
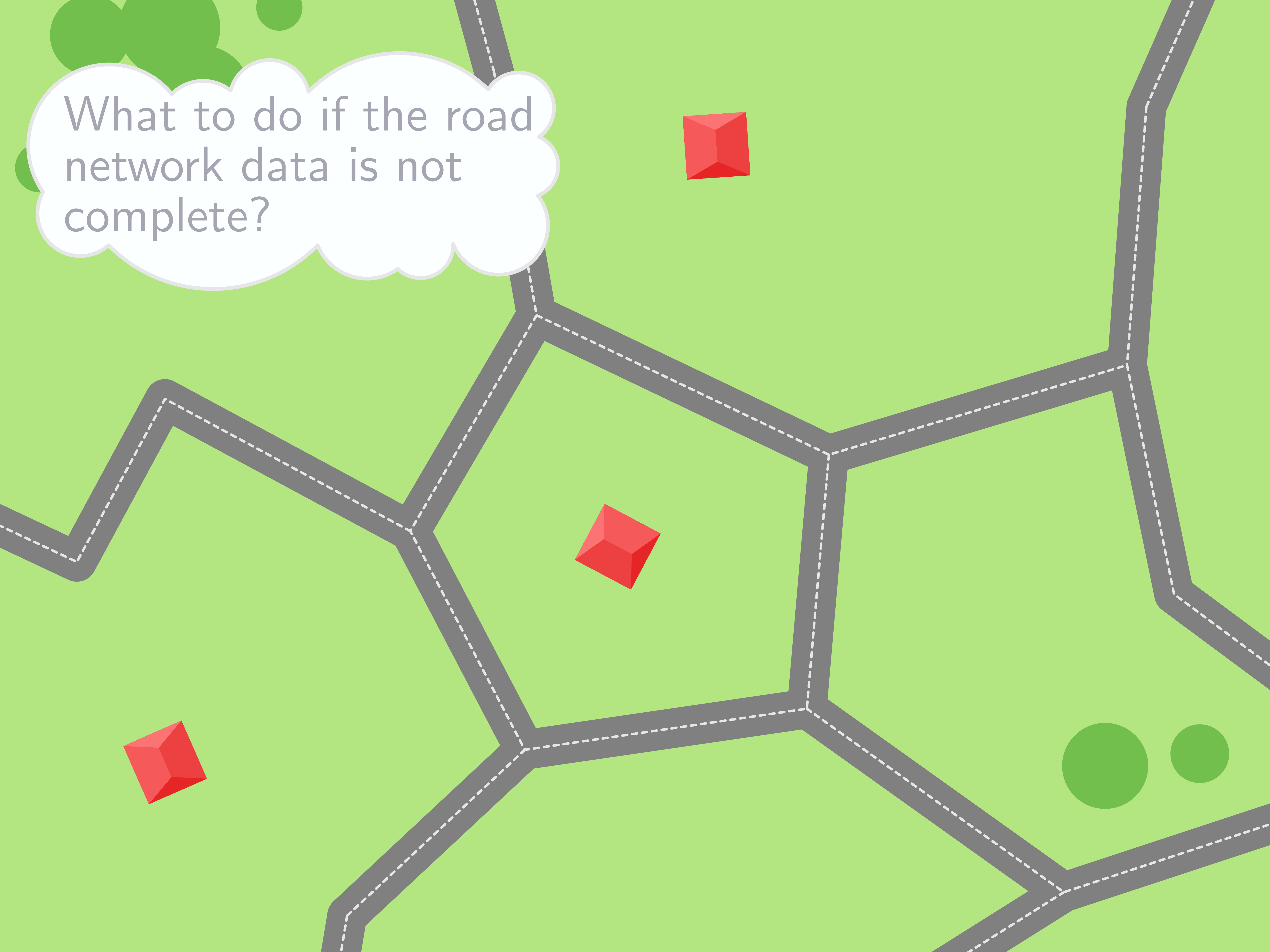
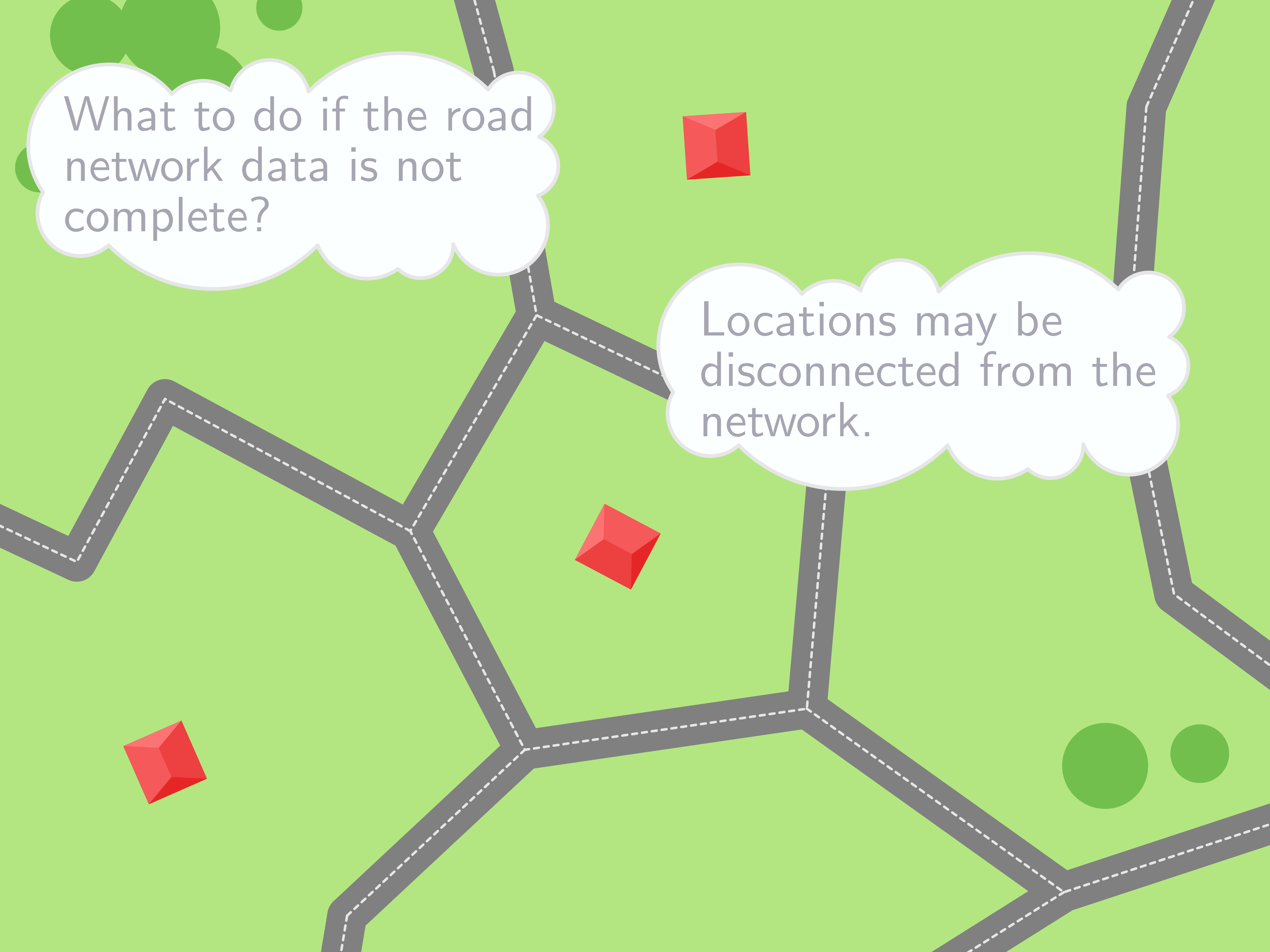

Also consider a set of *interesting* locations.
In many applications, the network distance between locations is important.
These are well known and easy to compute.


What to do if the road network data is not complete?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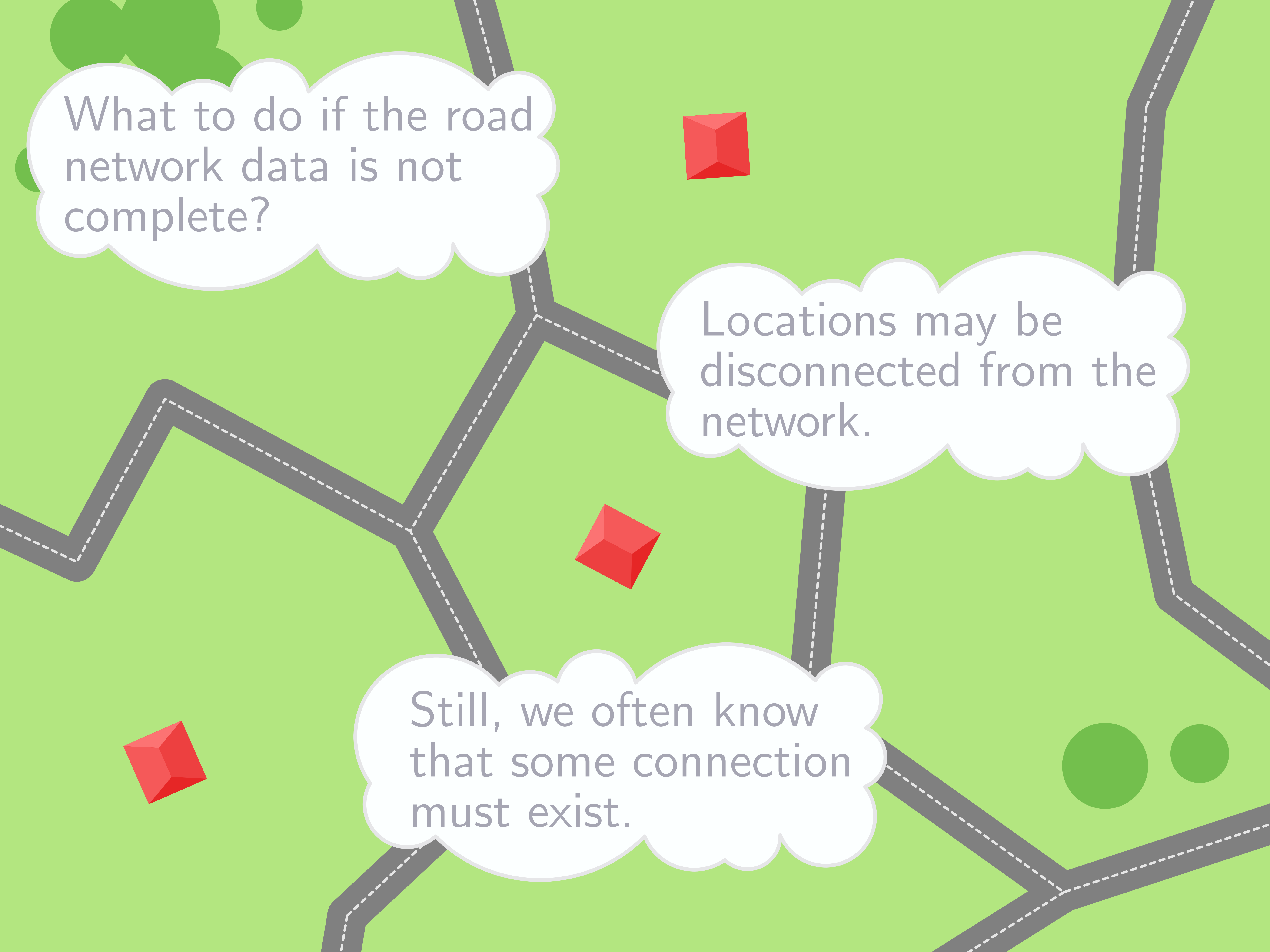
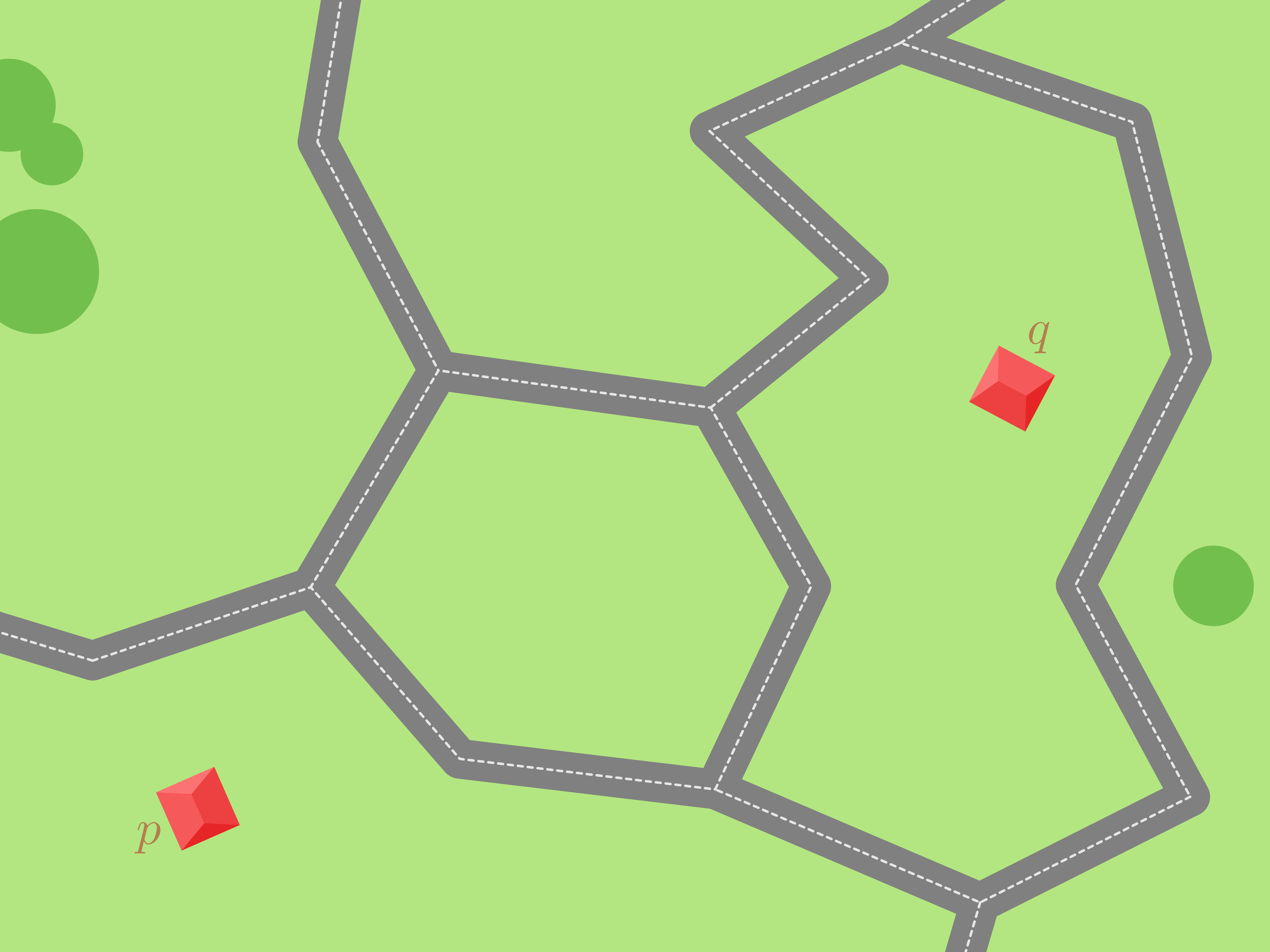
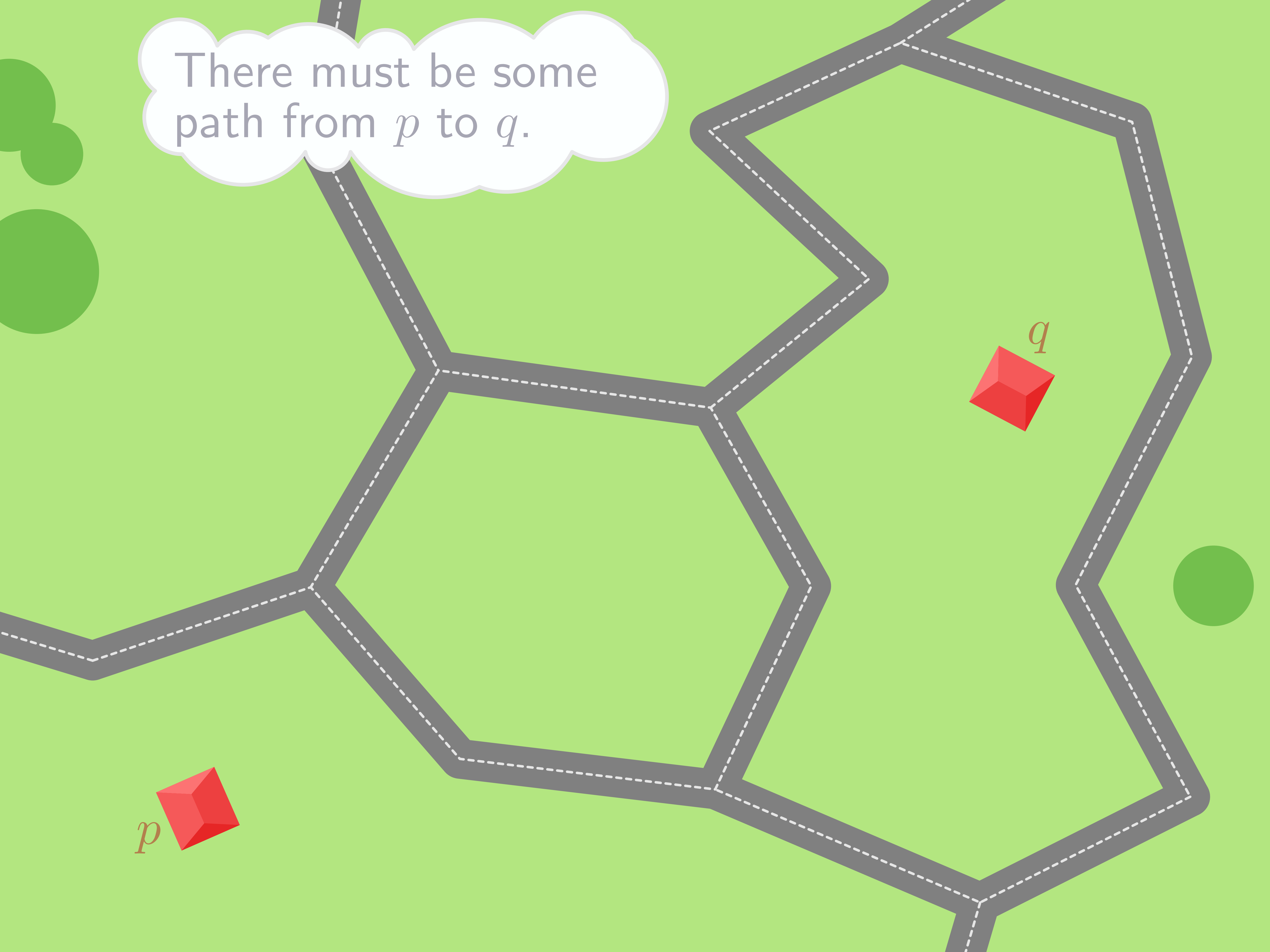
Locations may be disconnected from the network.


What to do if the road network data is not complete?
Locations may be disconnected from the network.
Still, we often know that some connection must exist.


q
p


There must be some path from p to q.
q
p


There must be some path from p to q.
q
?
p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q
p
?


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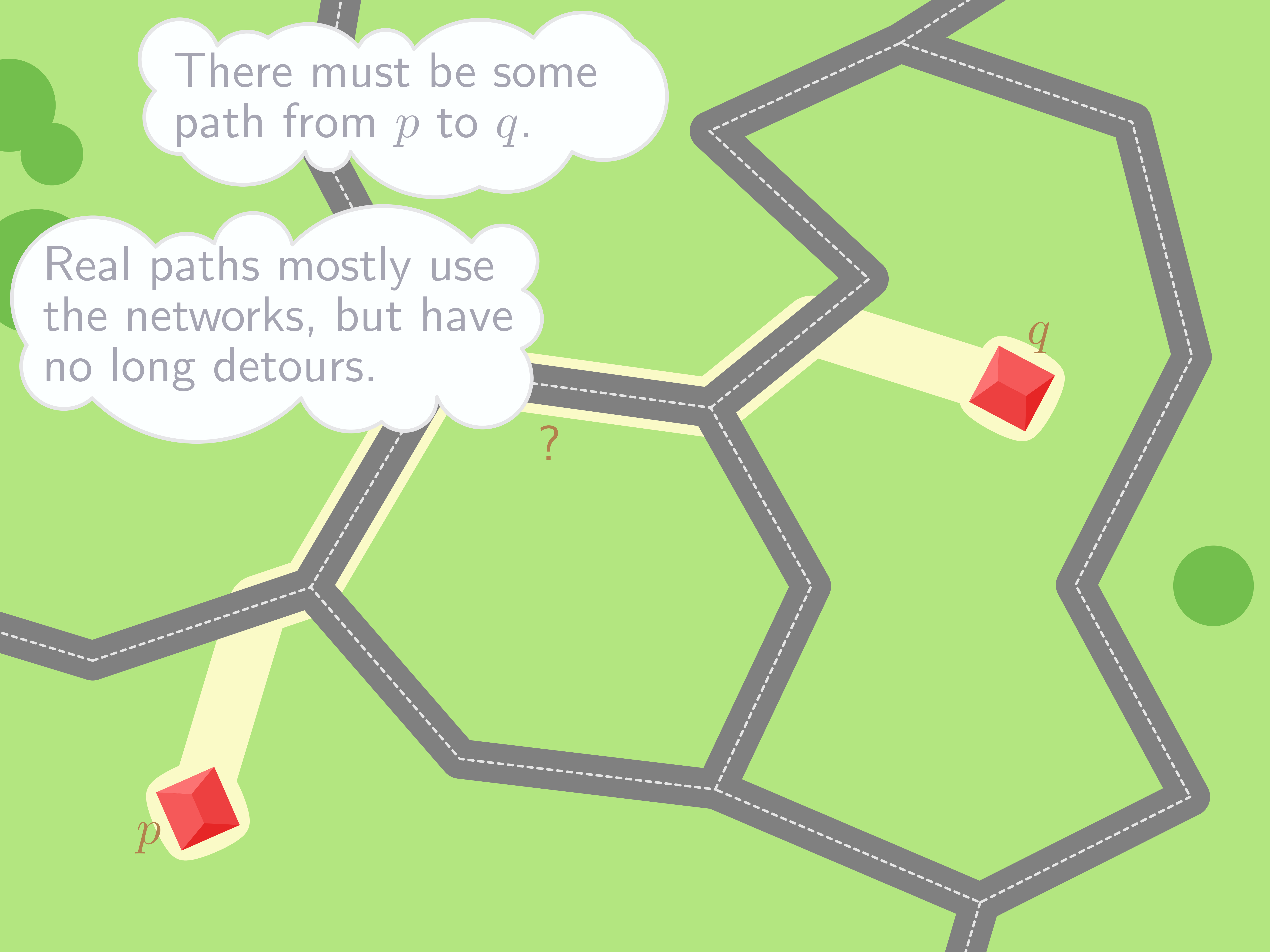
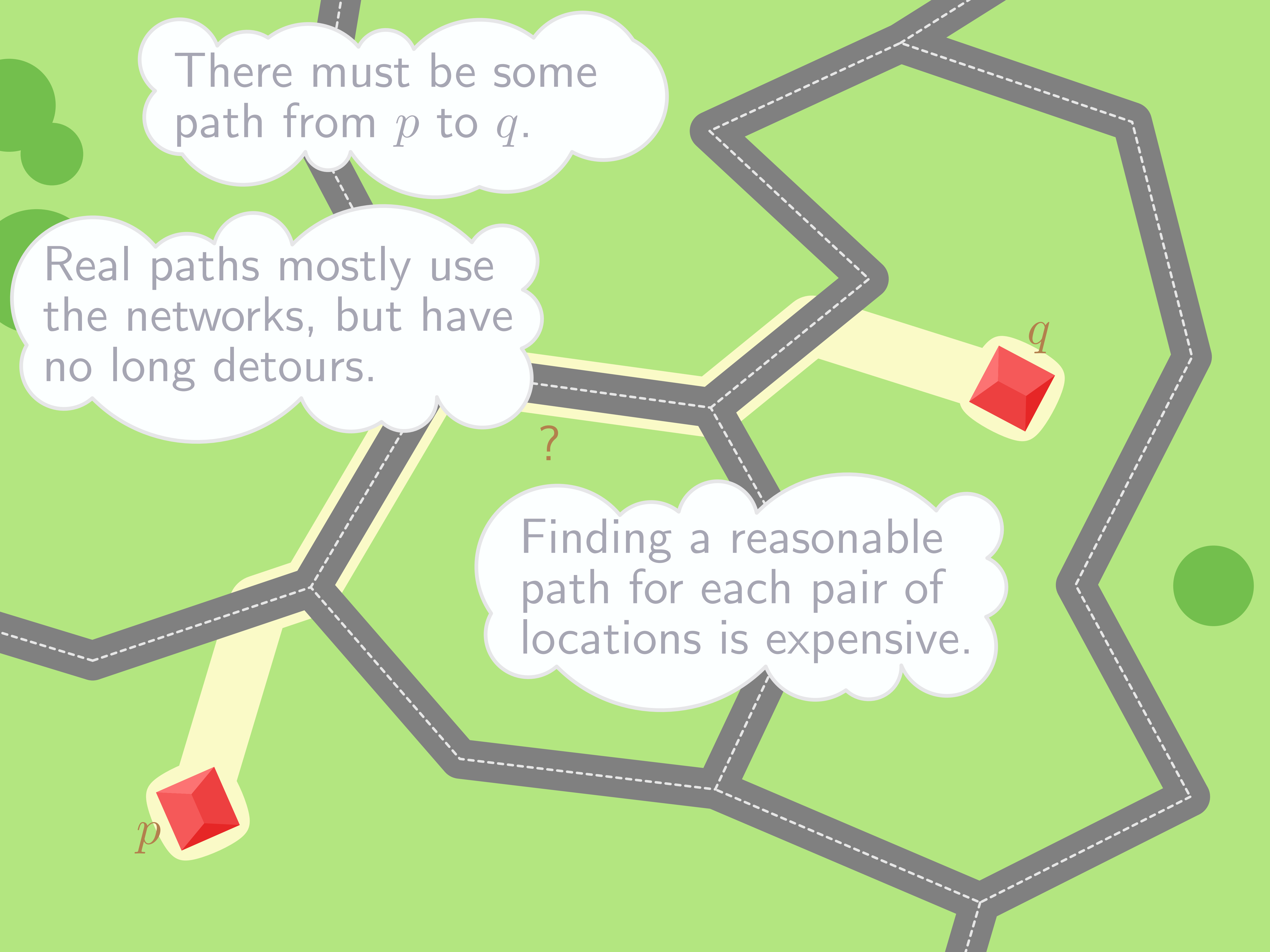
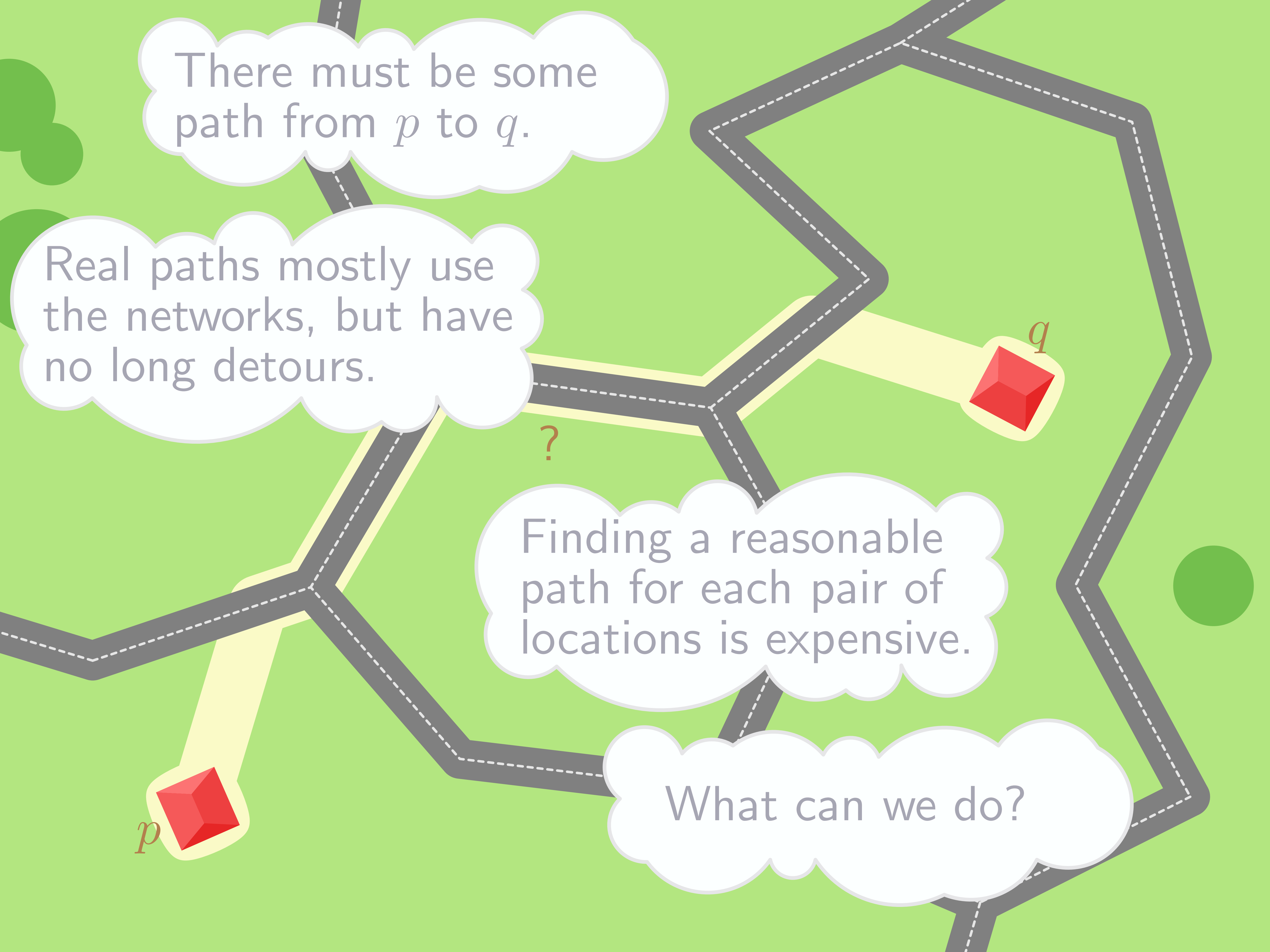
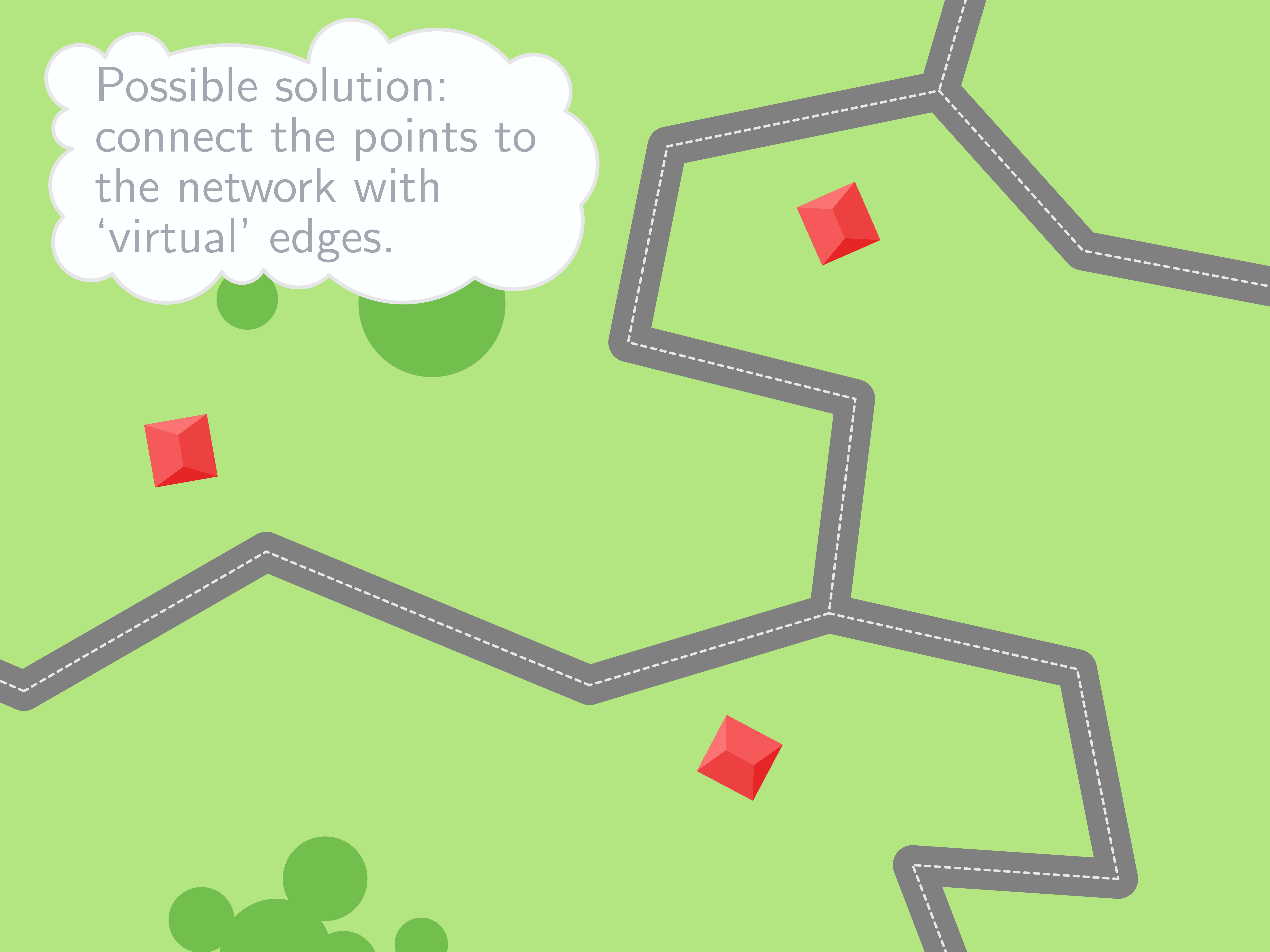
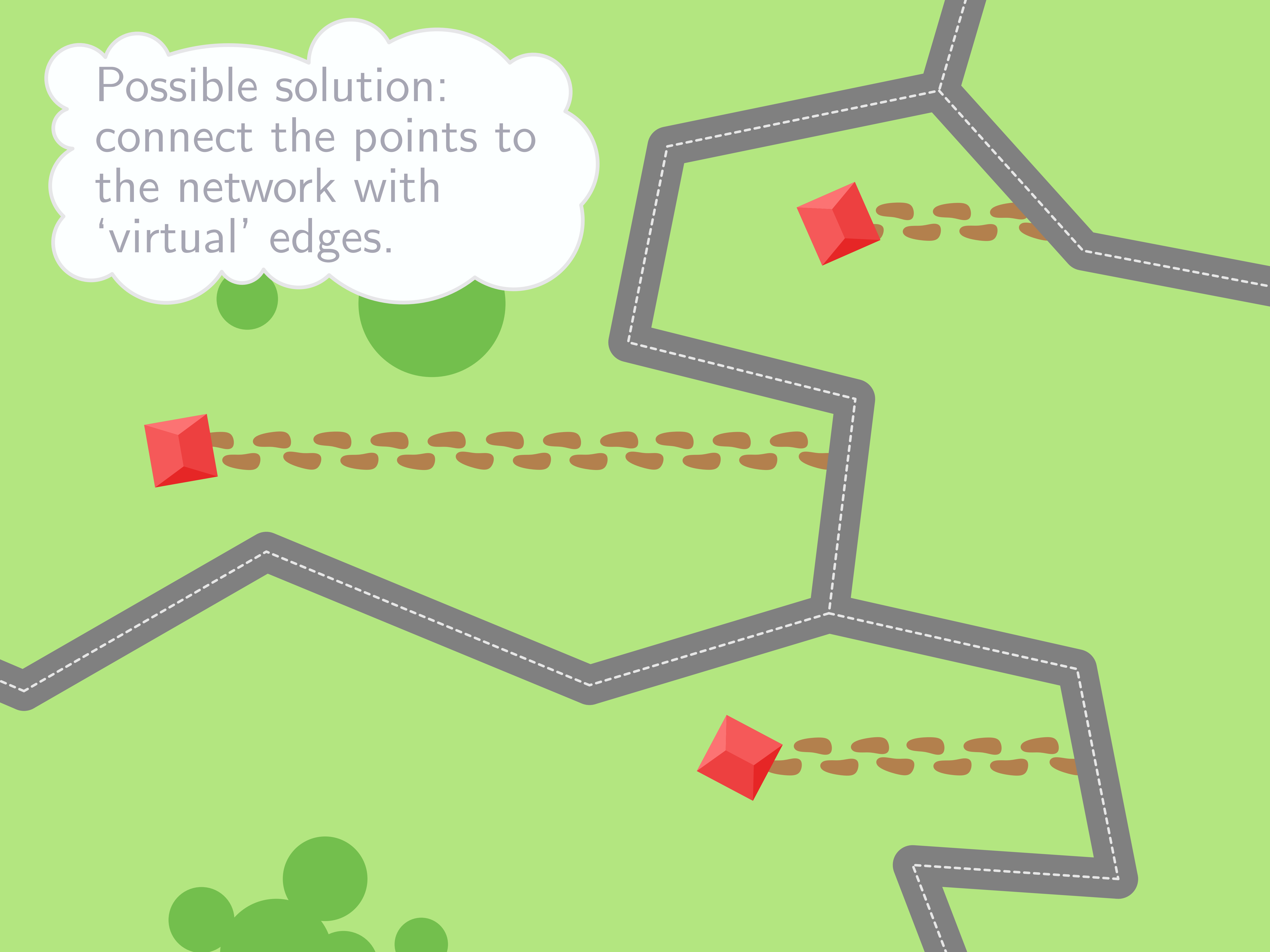

There must be some path from p to q.
Real paths mostly use the networks, but have no long detours.
q
?
p


There must be some path from p to q.
Real paths mostly use the networks, but have no long detours.
Finding a reasonable path for each pair of locations is expensive.
q
?
p


There must be some path from p to q.
Real paths mostly use the networks, but have no long detours.
Finding a reasonable path for each pair of locations is expensive.
What can we do?
q
?
p


Possible solution: connect the points to the network with 'virtual' edges.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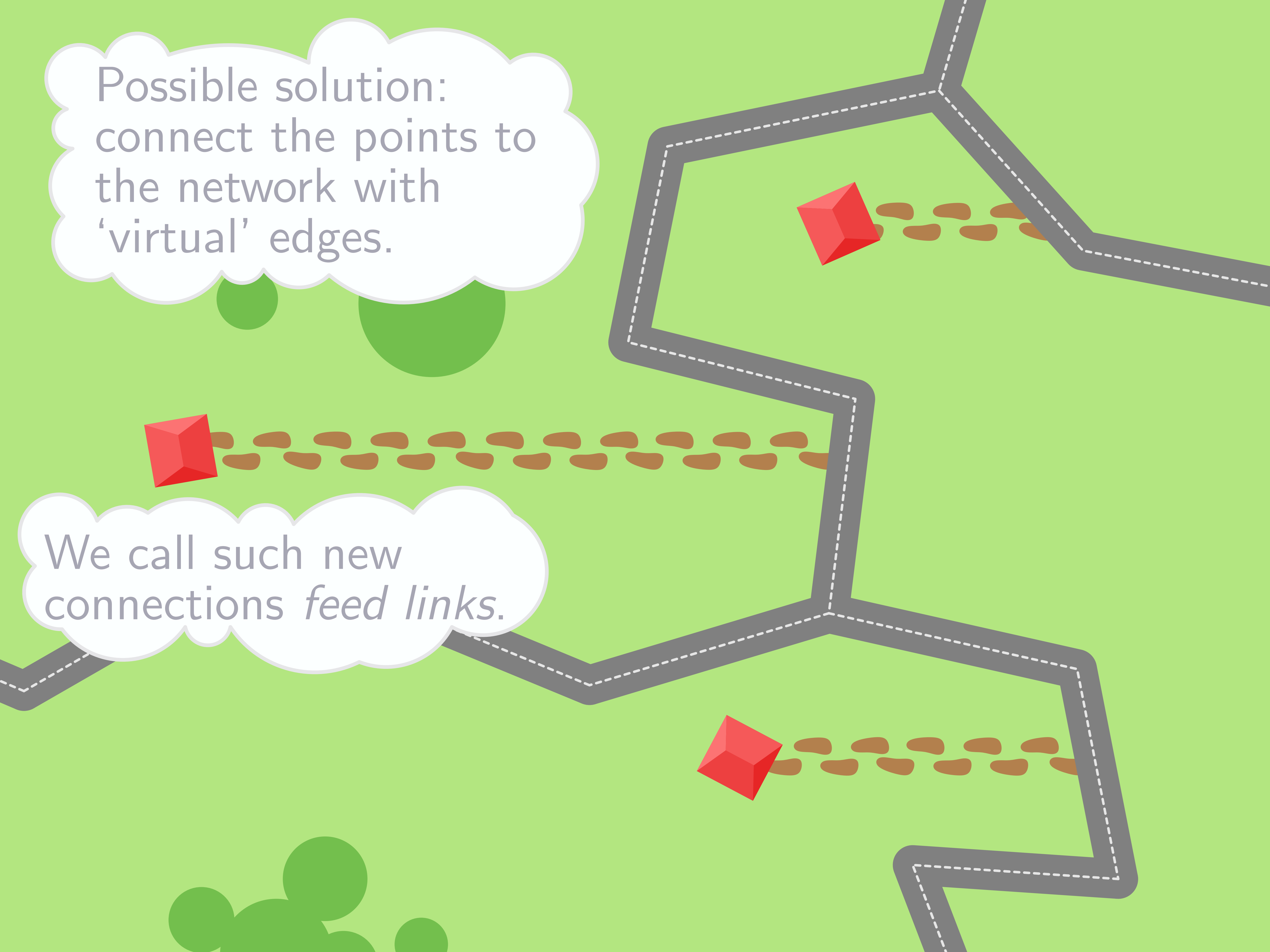
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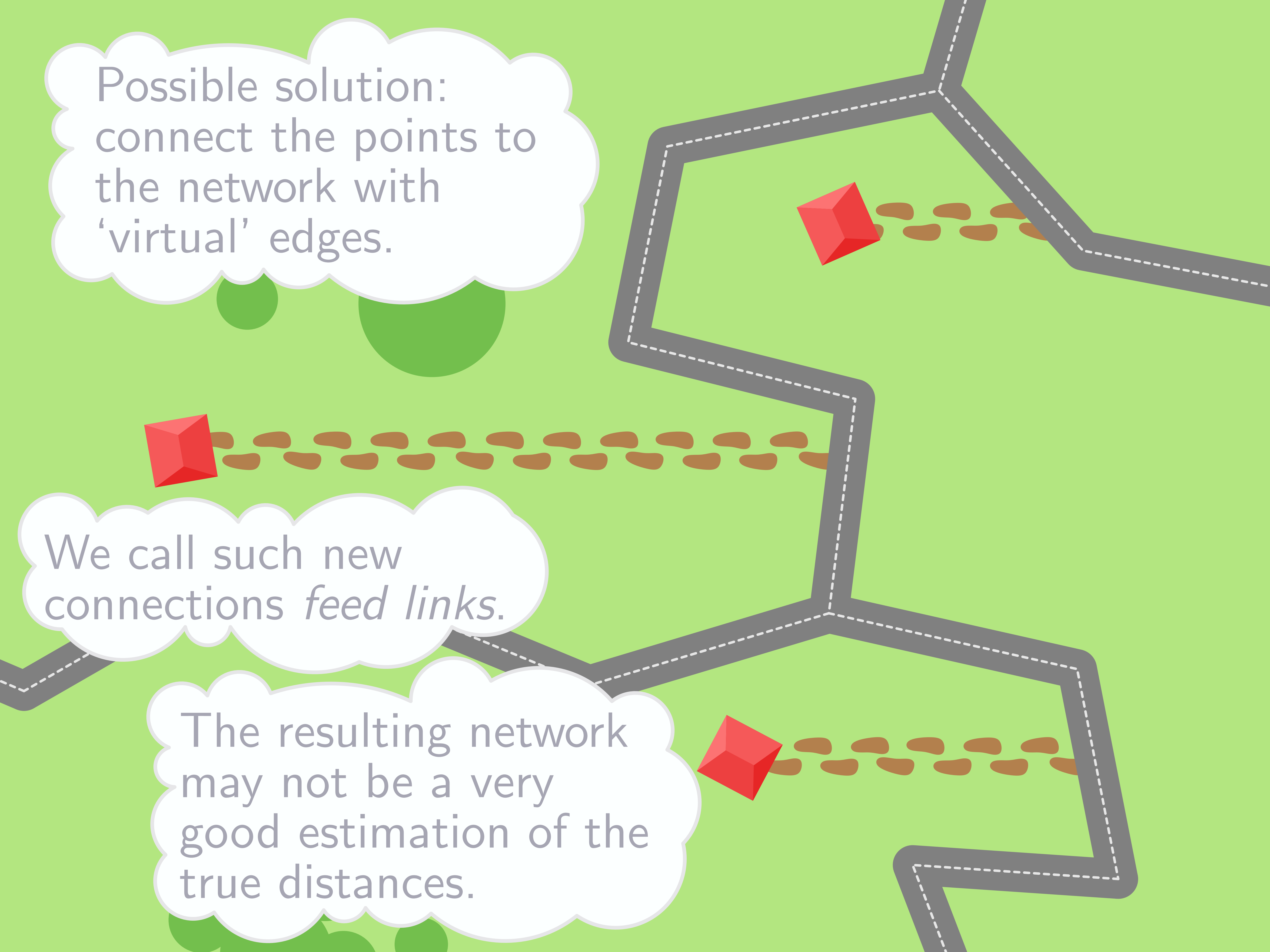
We call such new connections *feed links*.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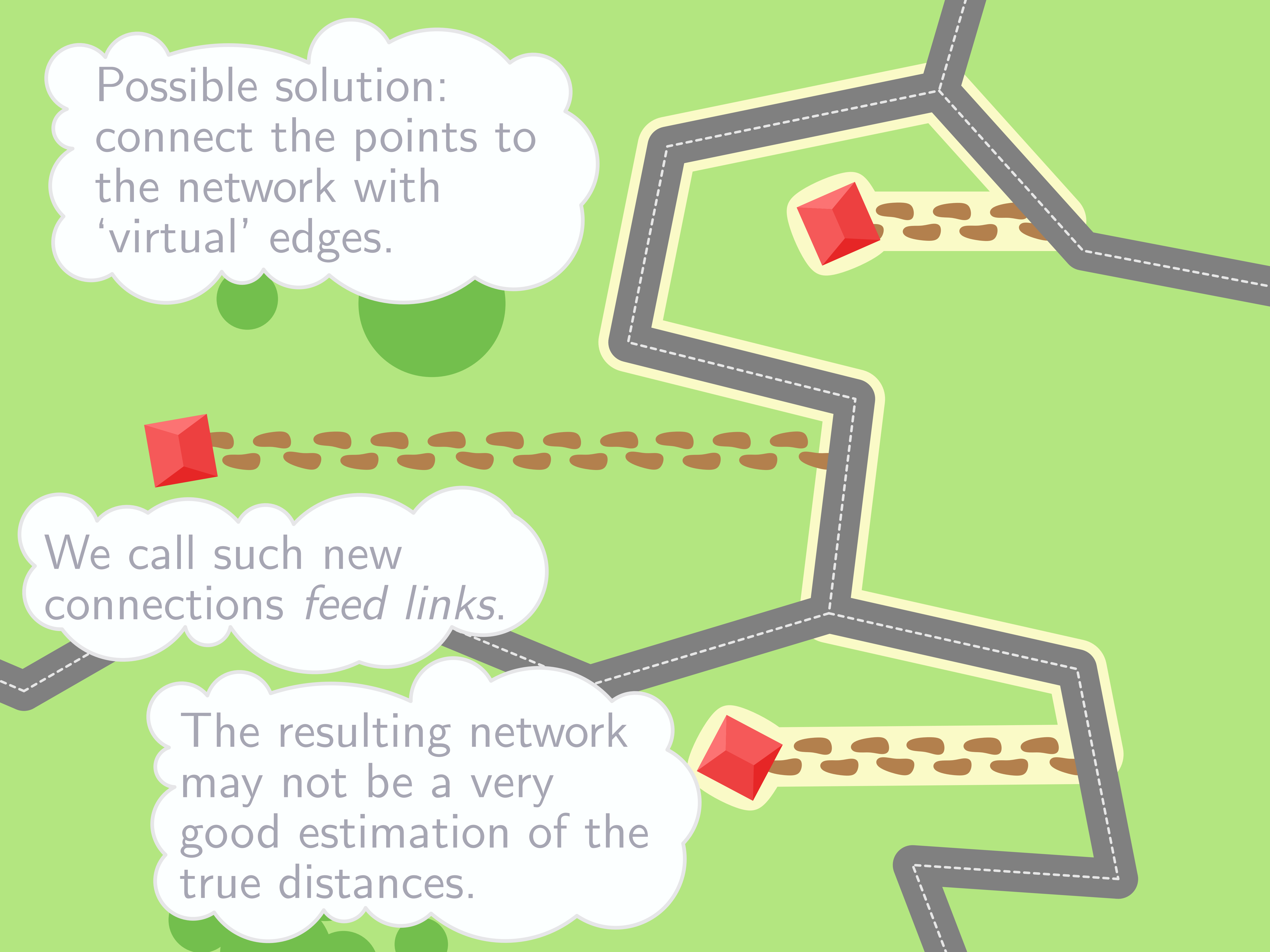
The resulting network may not be a very good estimation of the true distances.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We want to add feed links that do not induce long detours.


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p
r

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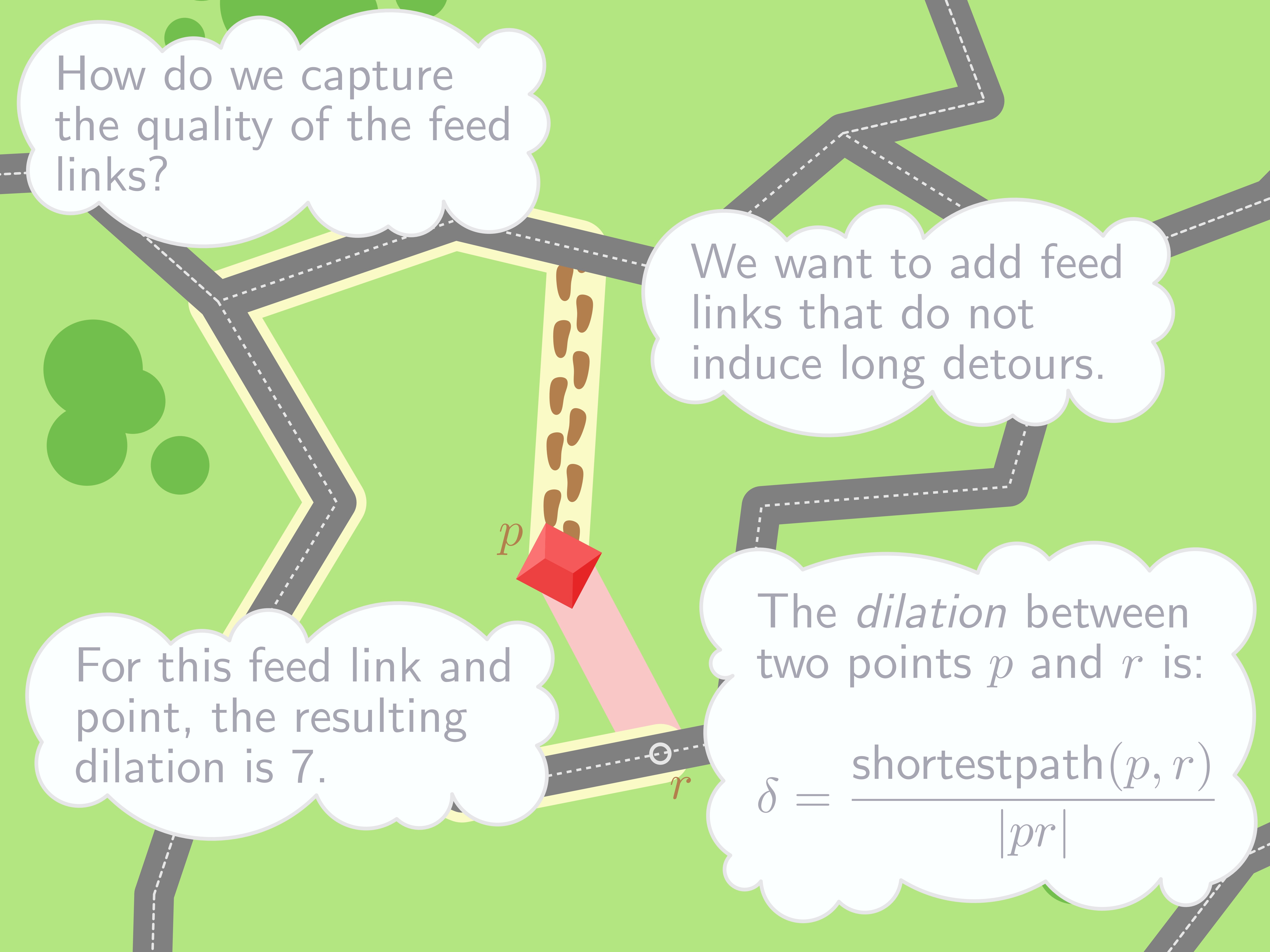
How do we capture the quality of the feed links?
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p
r
For this feed link and point, the resulting dilation is 7.
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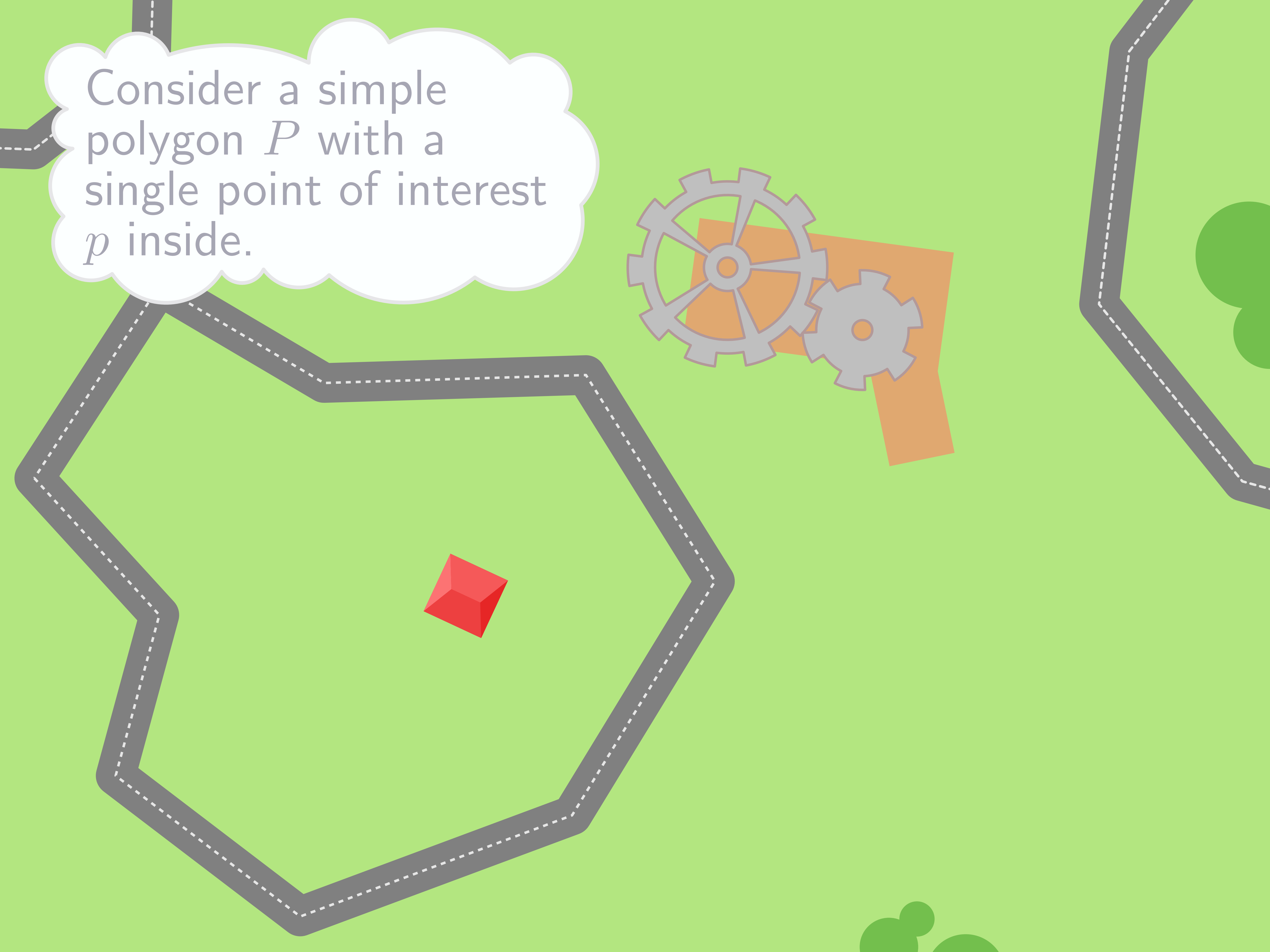
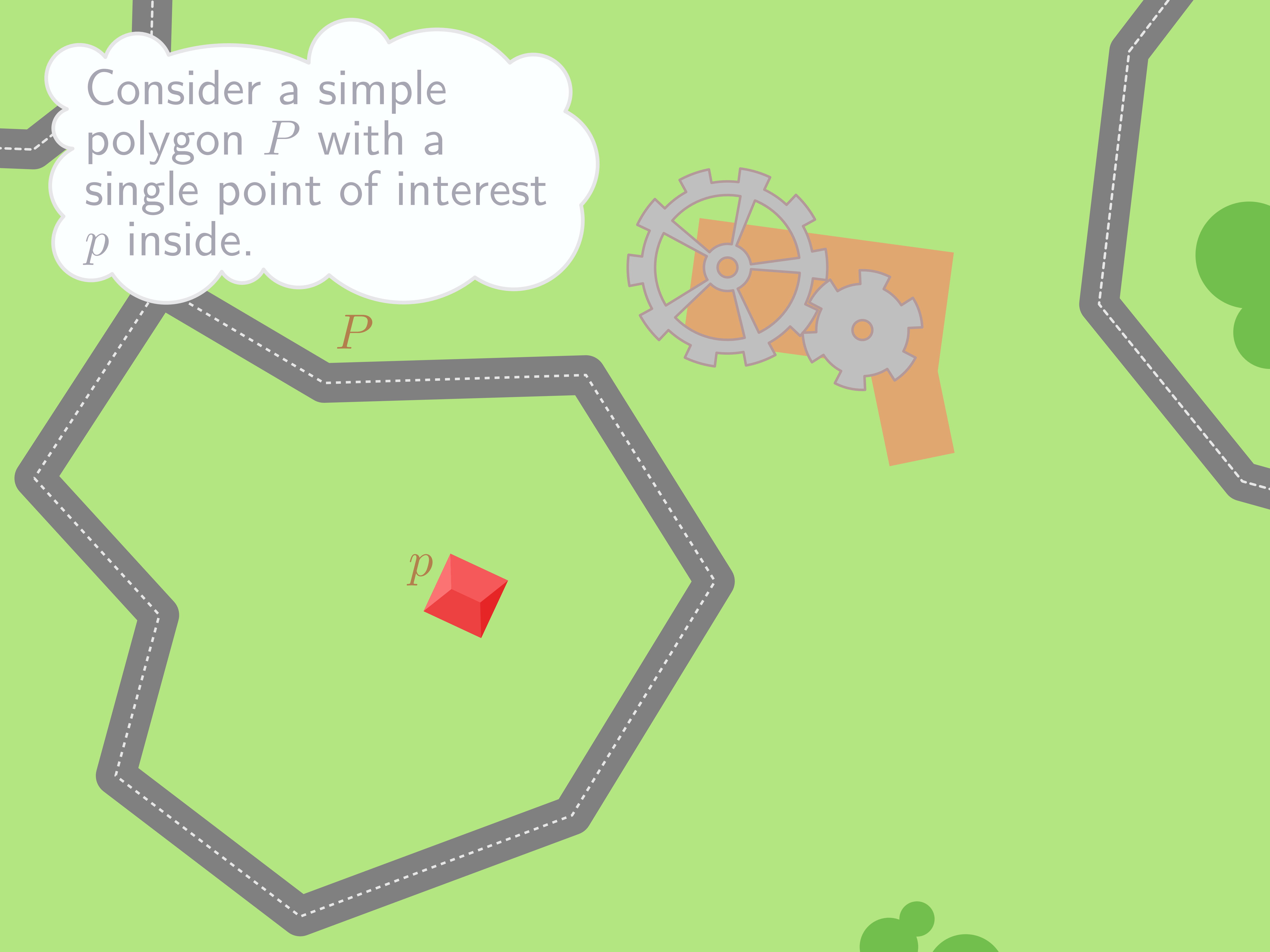

Consider a simple polygon P with a single point of interest p inside.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P
p

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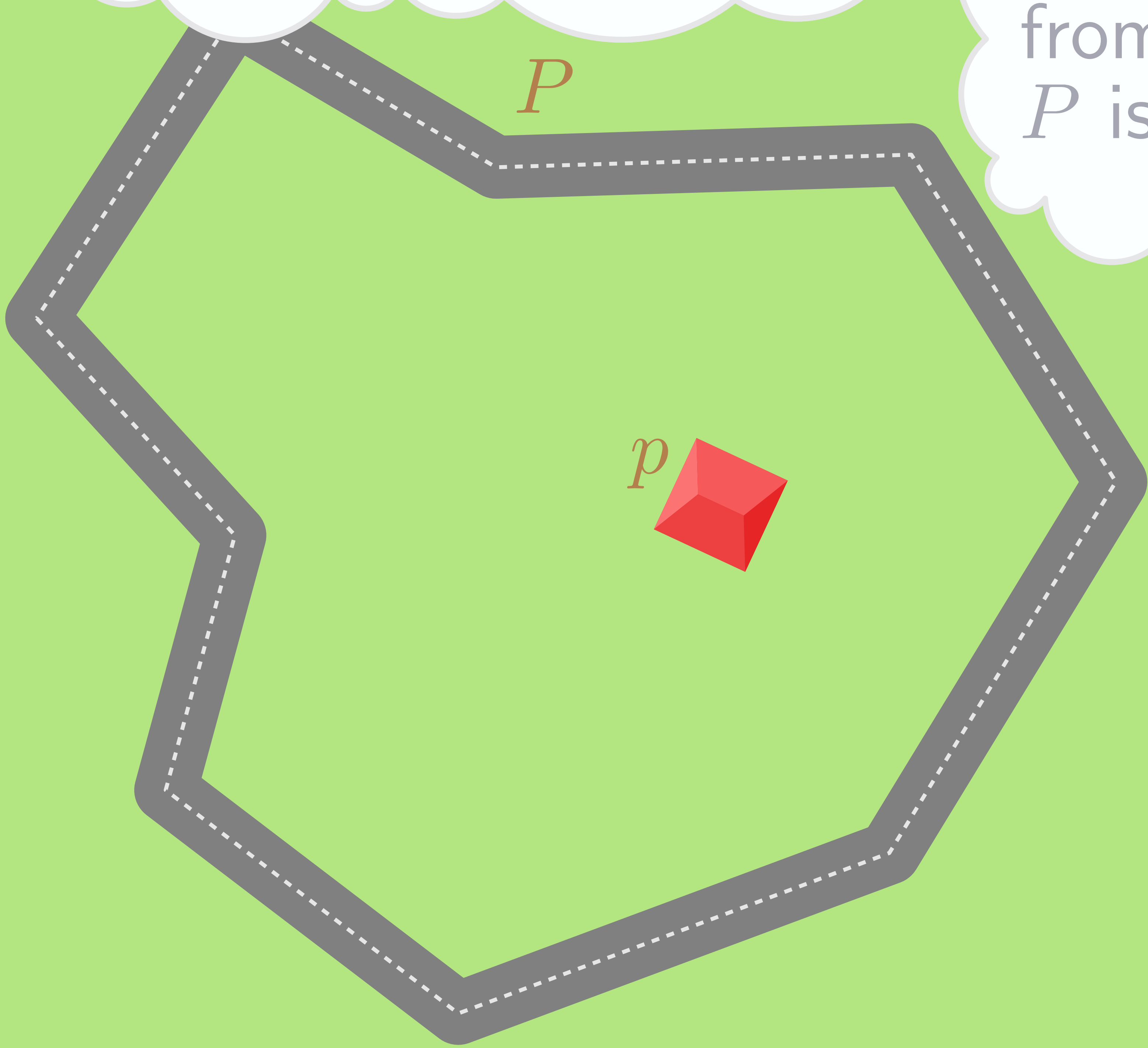
Question: Can we place one feed link so that the worst dilation from p to any point on P is minimised?



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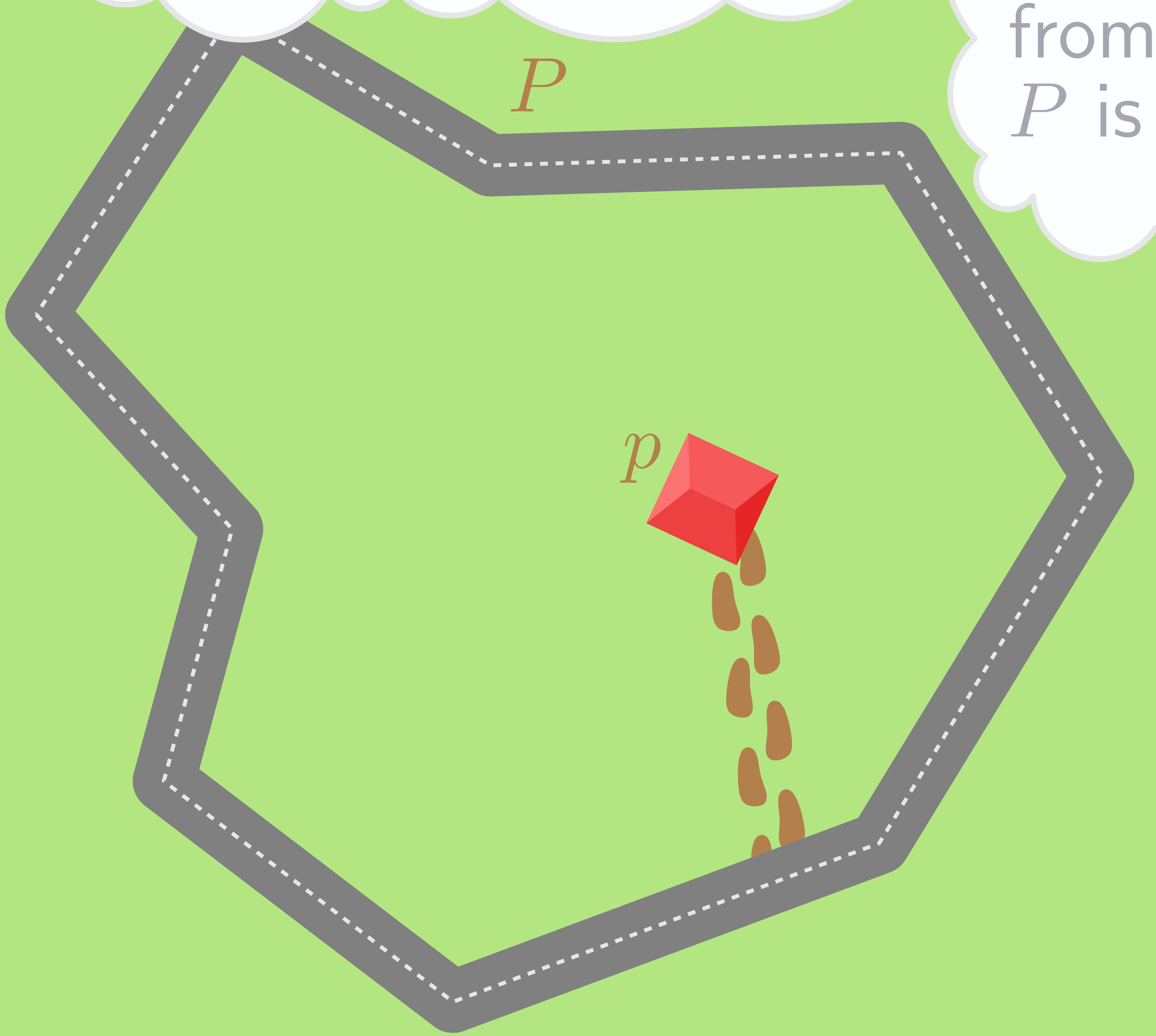

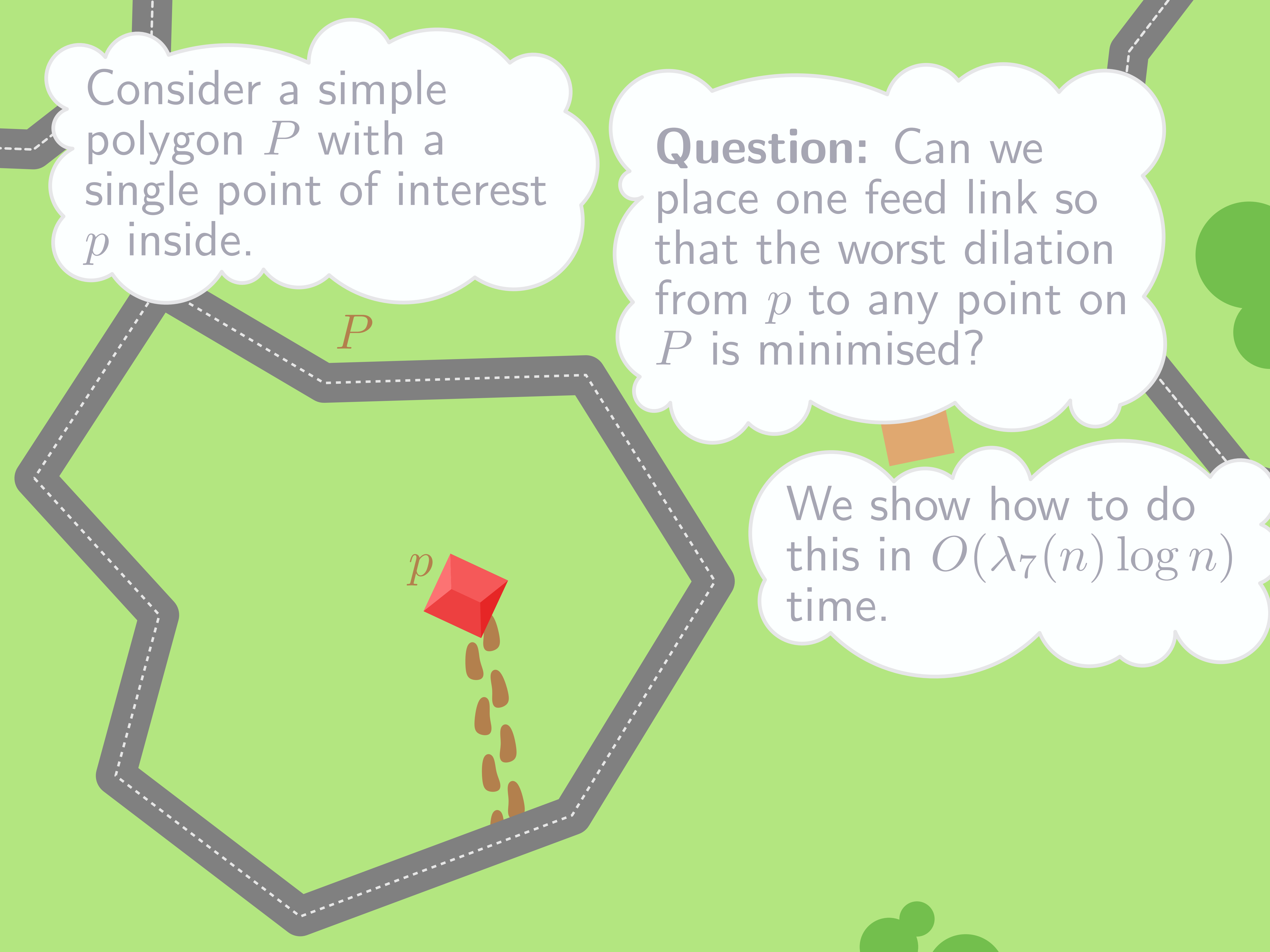
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We show how to do this in $O(\lambda_7(n)\log n)$ time.

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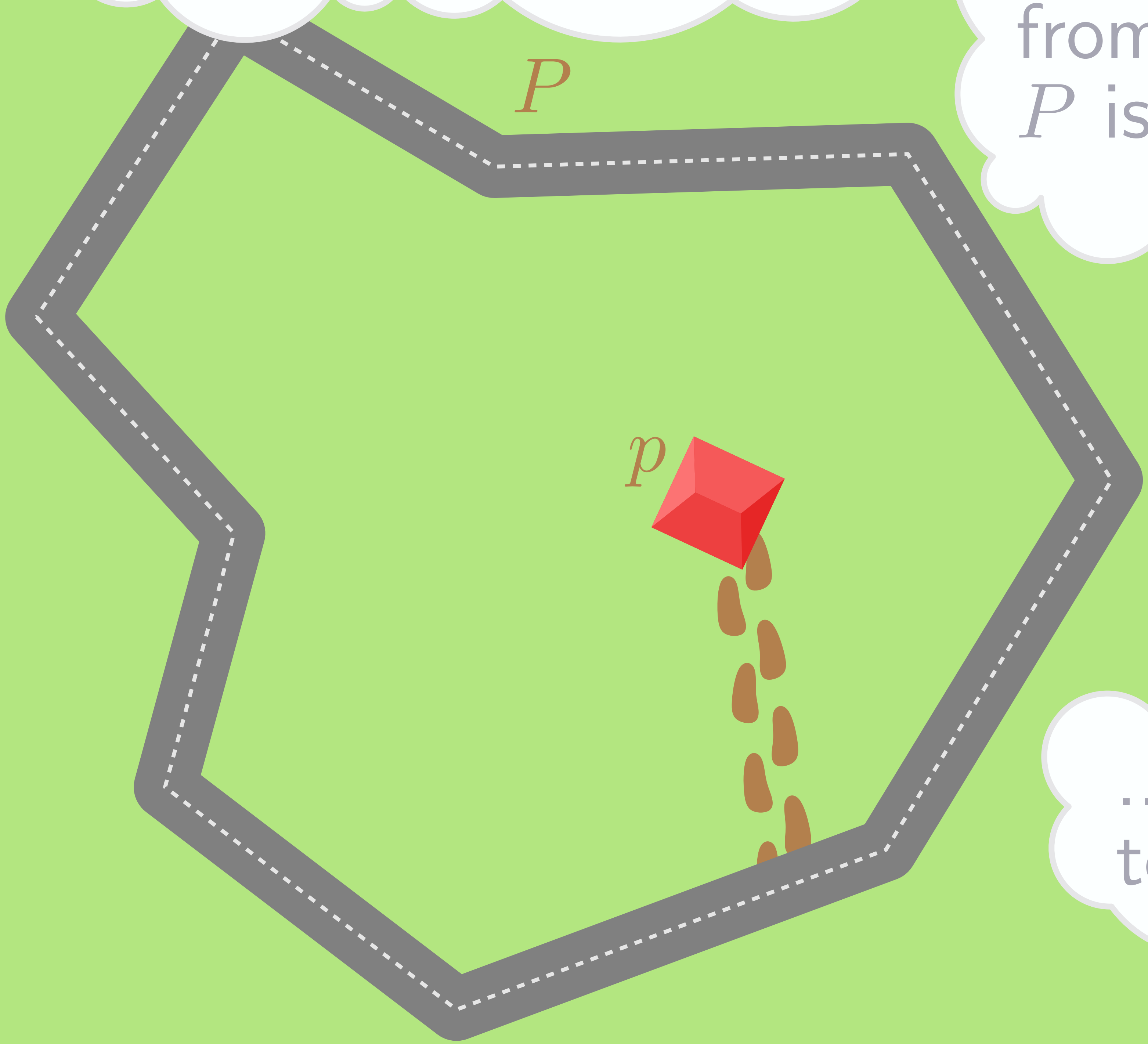

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... but, I'm not going to talk about this.


P
p


Instead, let's consider
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P
p


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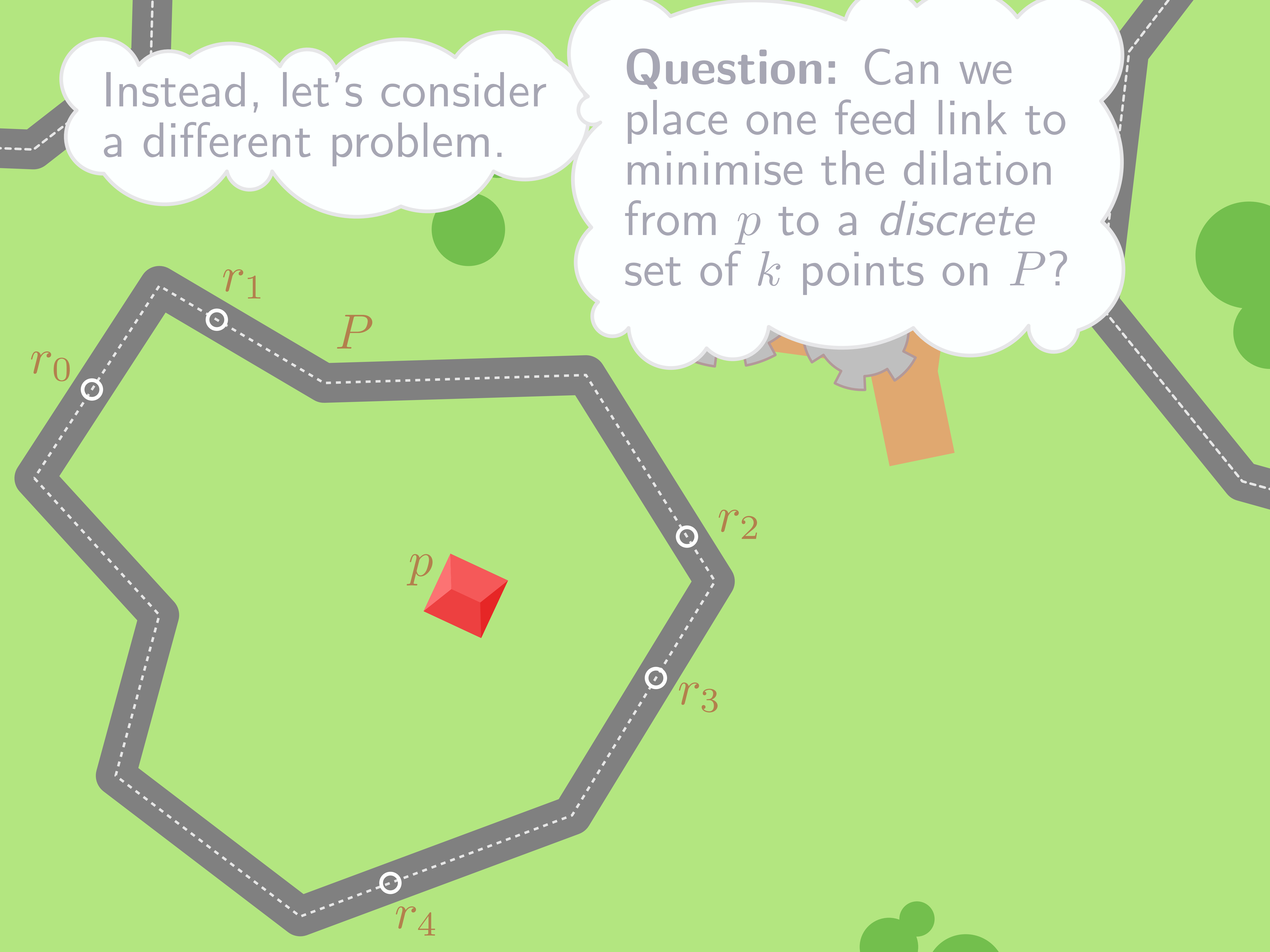
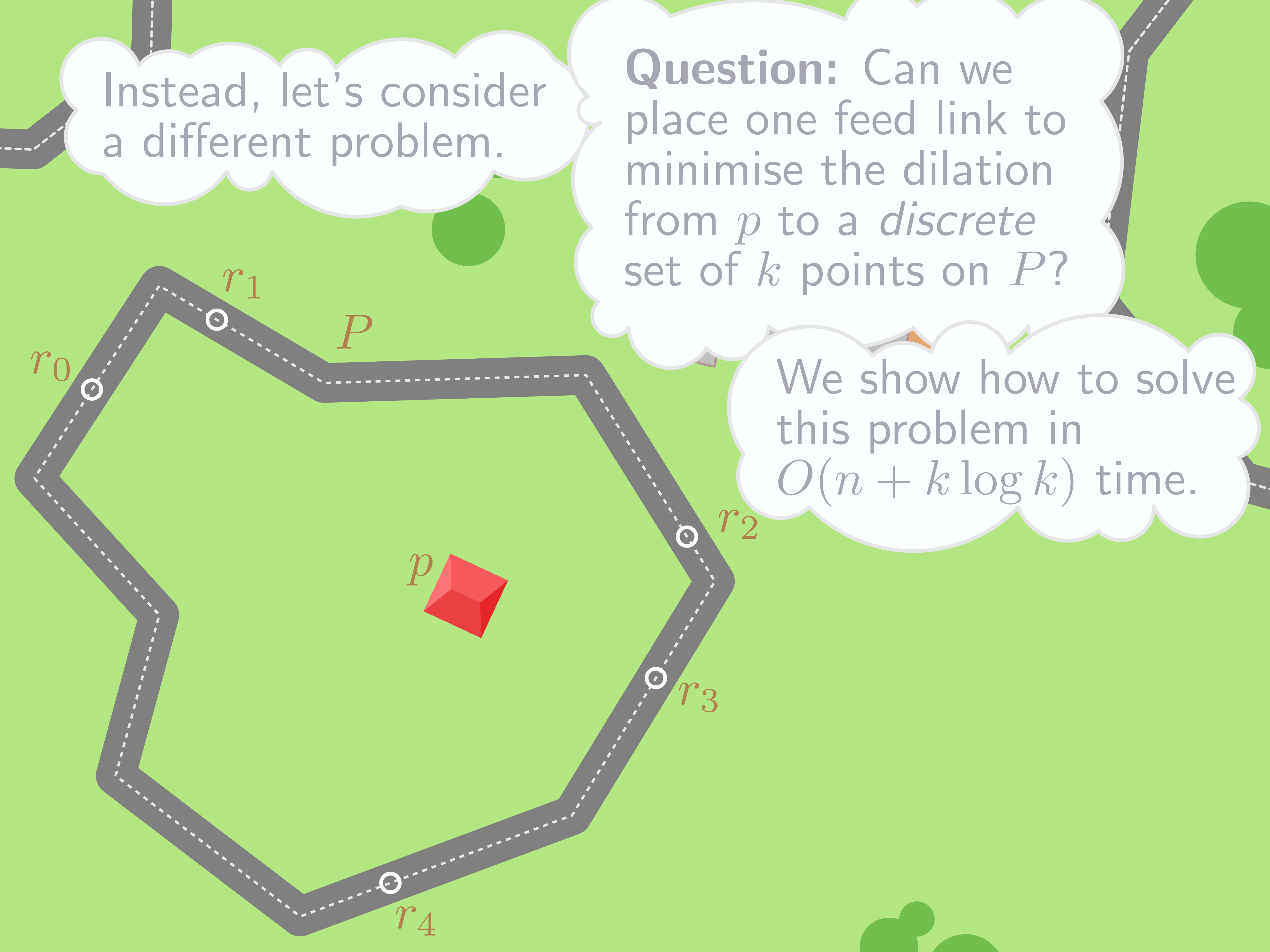
Question: Can we place one feed link to minimise the dilation from p to a *discrete* set of k points on P?
P
p


Instead, let's consider a different problem.
Question: Can we place one feed link to minimise the dilation from p to a *discrete* set of k points on P?
r_0
r_1
P
r_2
p
r_3
r_4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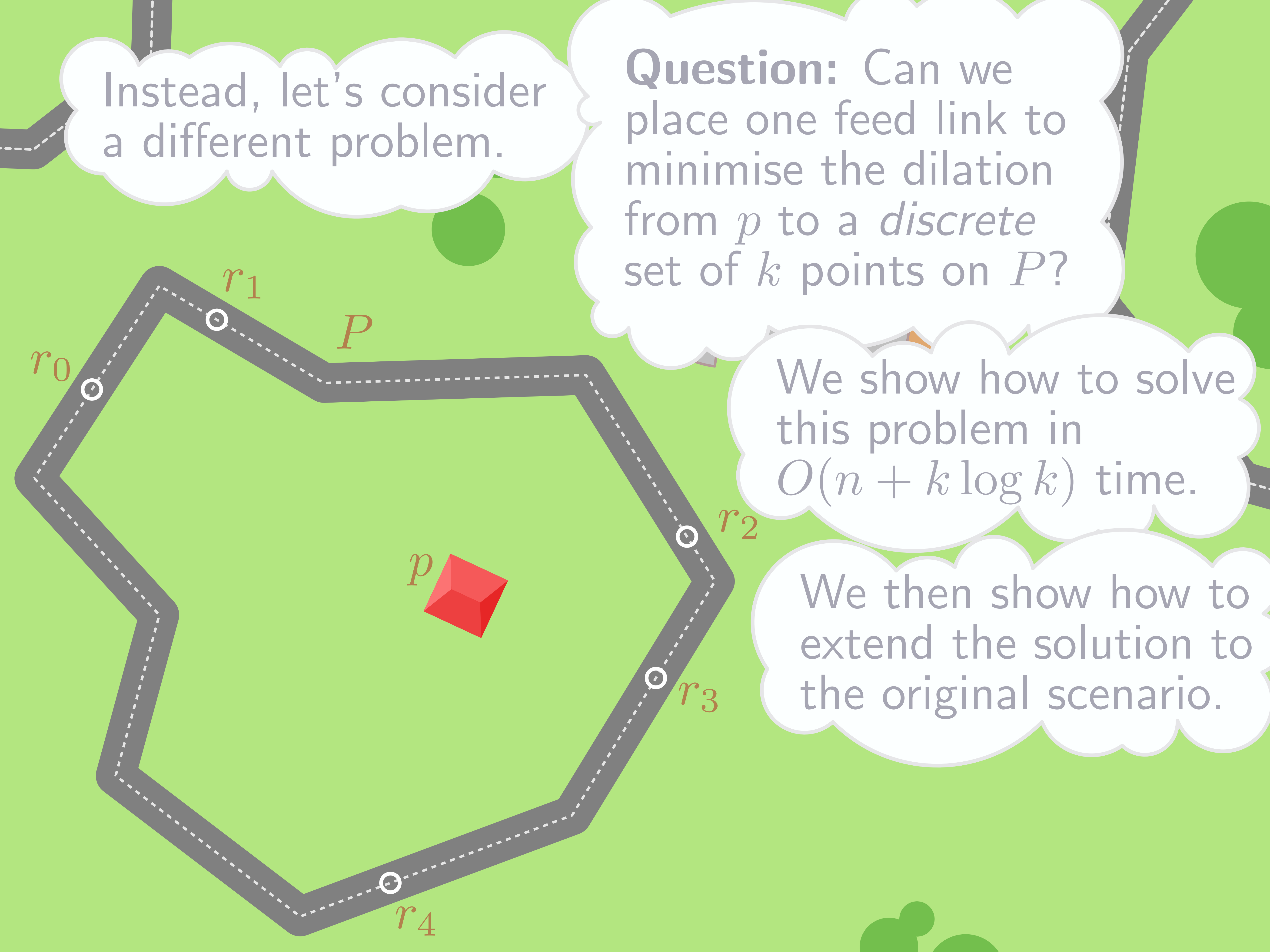
Question: Can we place one feed link to minimise the dilation from p to a *discrete* set of k points on P?
We show how to solve this problem in $O(n + k \log k)$ time.
r_0
r_1
P
r_2
p
r_3
r_4



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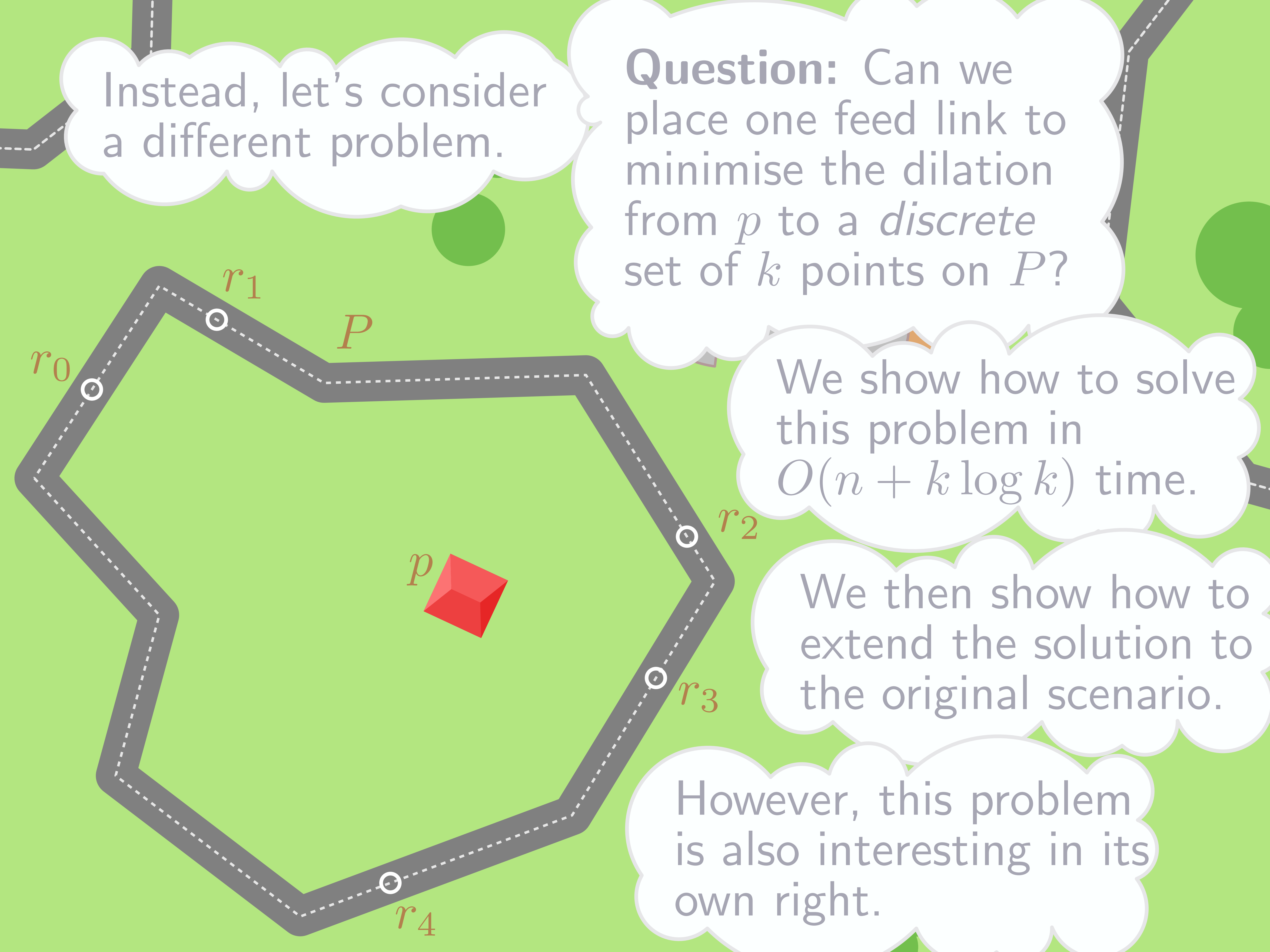
We then show how to extend the solution to the original scenario.
r_0
r_1
P
r_2
r_3
r_4
p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We then show how to extend the solution to the original scenario.
However, this problem is also interesting in its own right.
r_0
r_1
r_2
r_3
r_4
P
p


P
p


Idea: Consider a fixed point r_i on P.
P
p


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P
p
r_i


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Now, let q be the point the feed link attaches to.
P
p
r_i


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P
q
p
r_i


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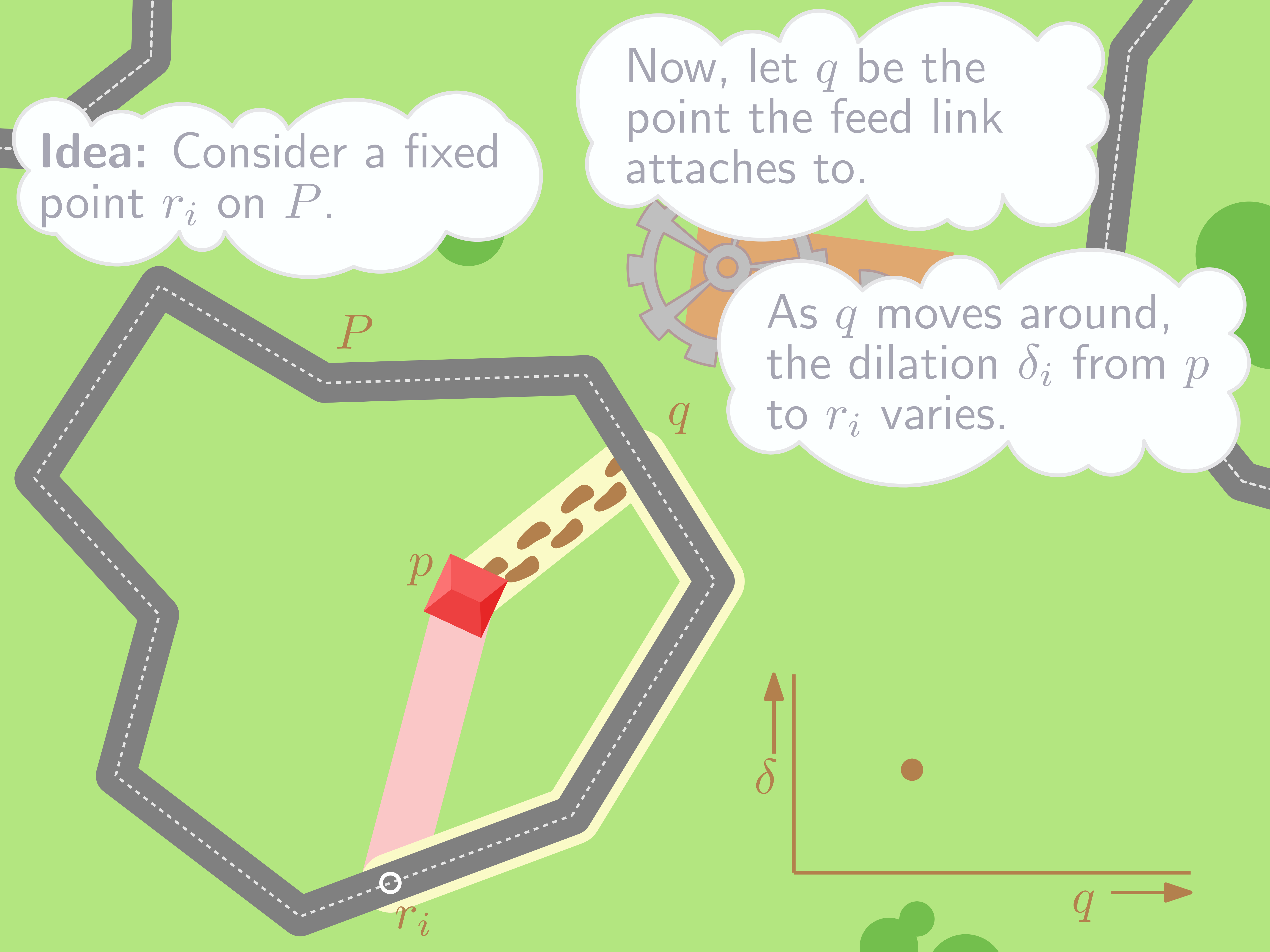
Now, let q be the point the feed link attaches to.
As q moves around, the dilation δ_i from p to r_i varies.
P
q
p
r_i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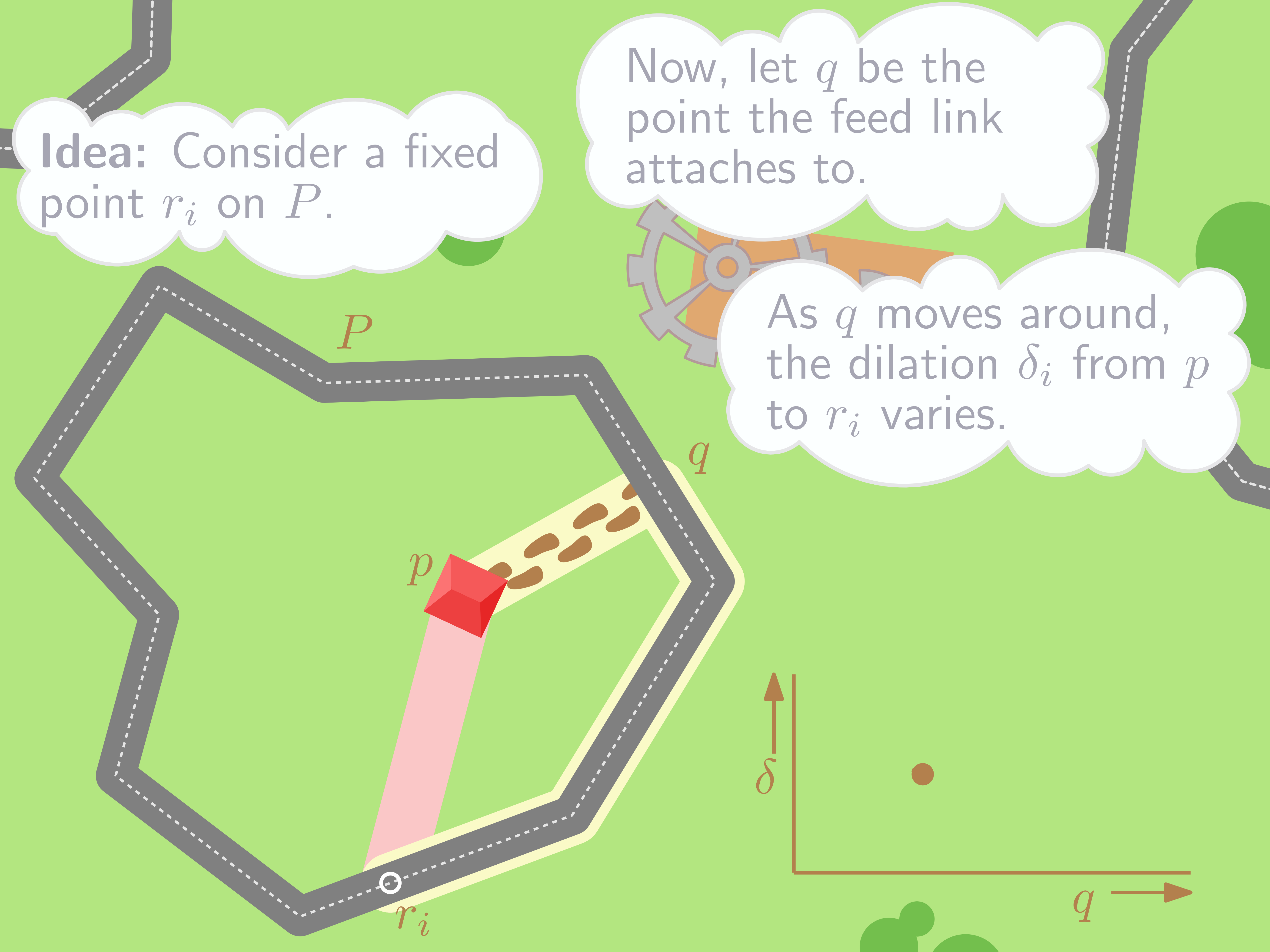
P
q
p
r_i
δ
q



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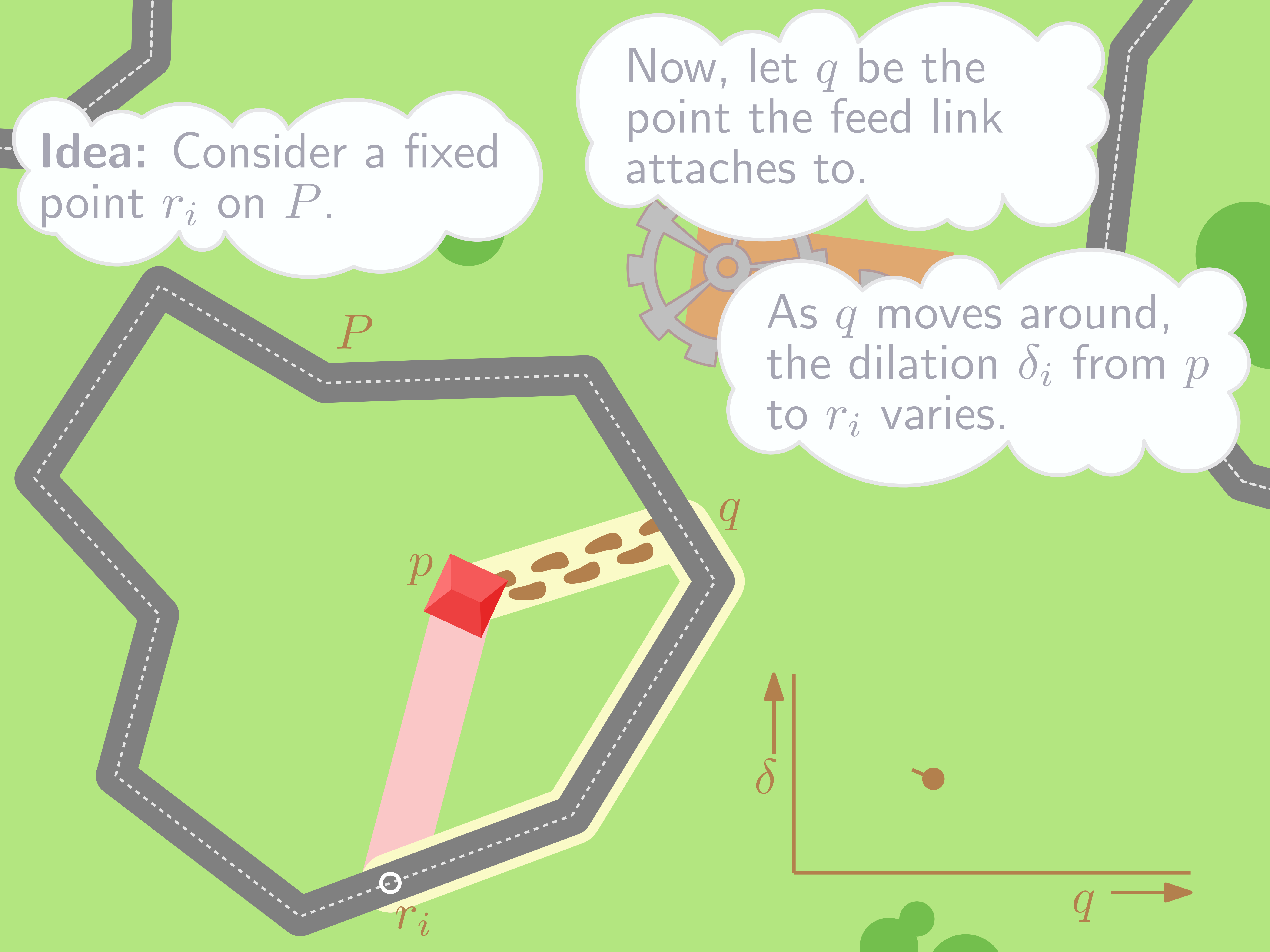
P
q
p
r_i
δ
q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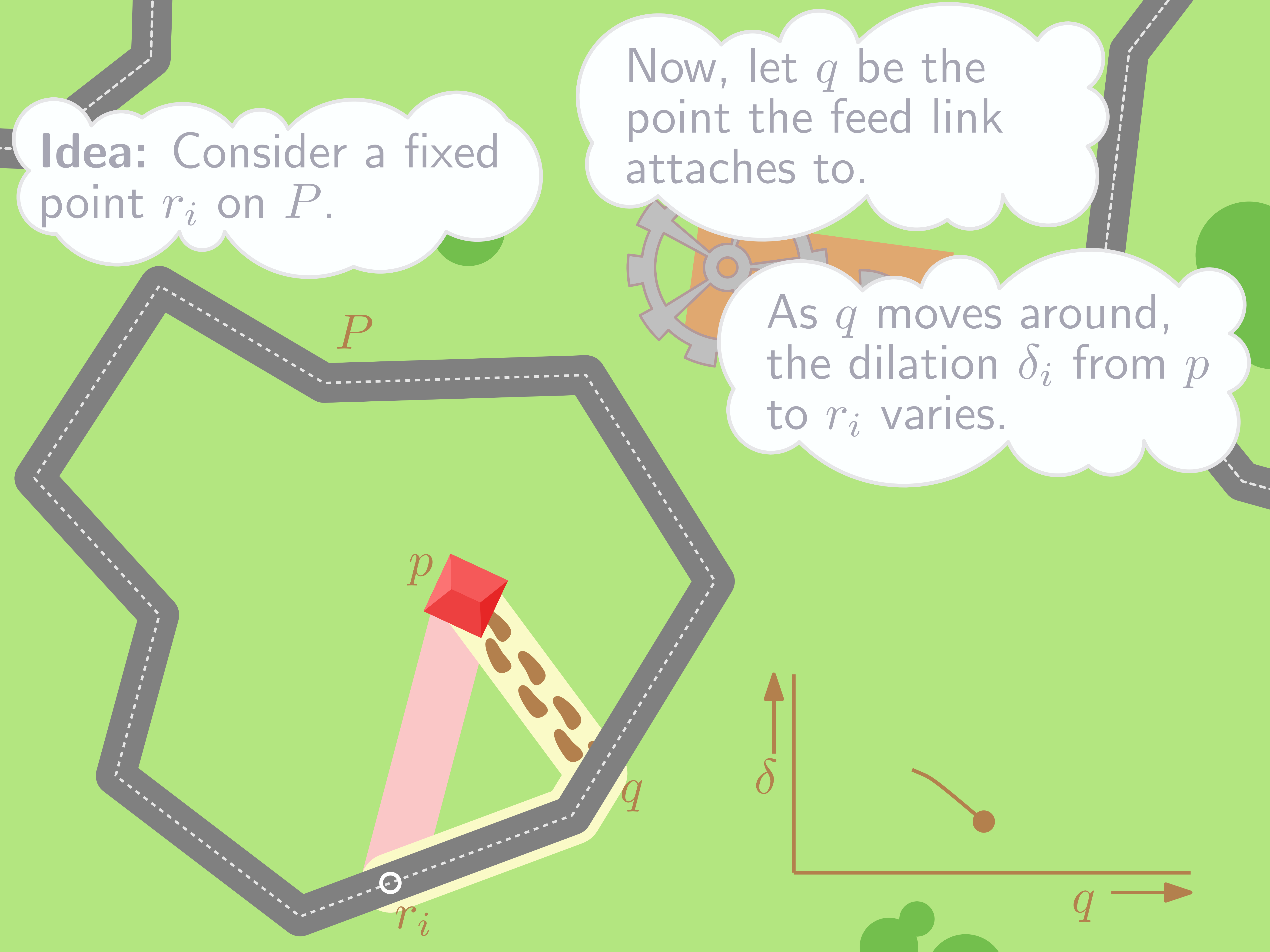
P
q
p
r_i
δ
q



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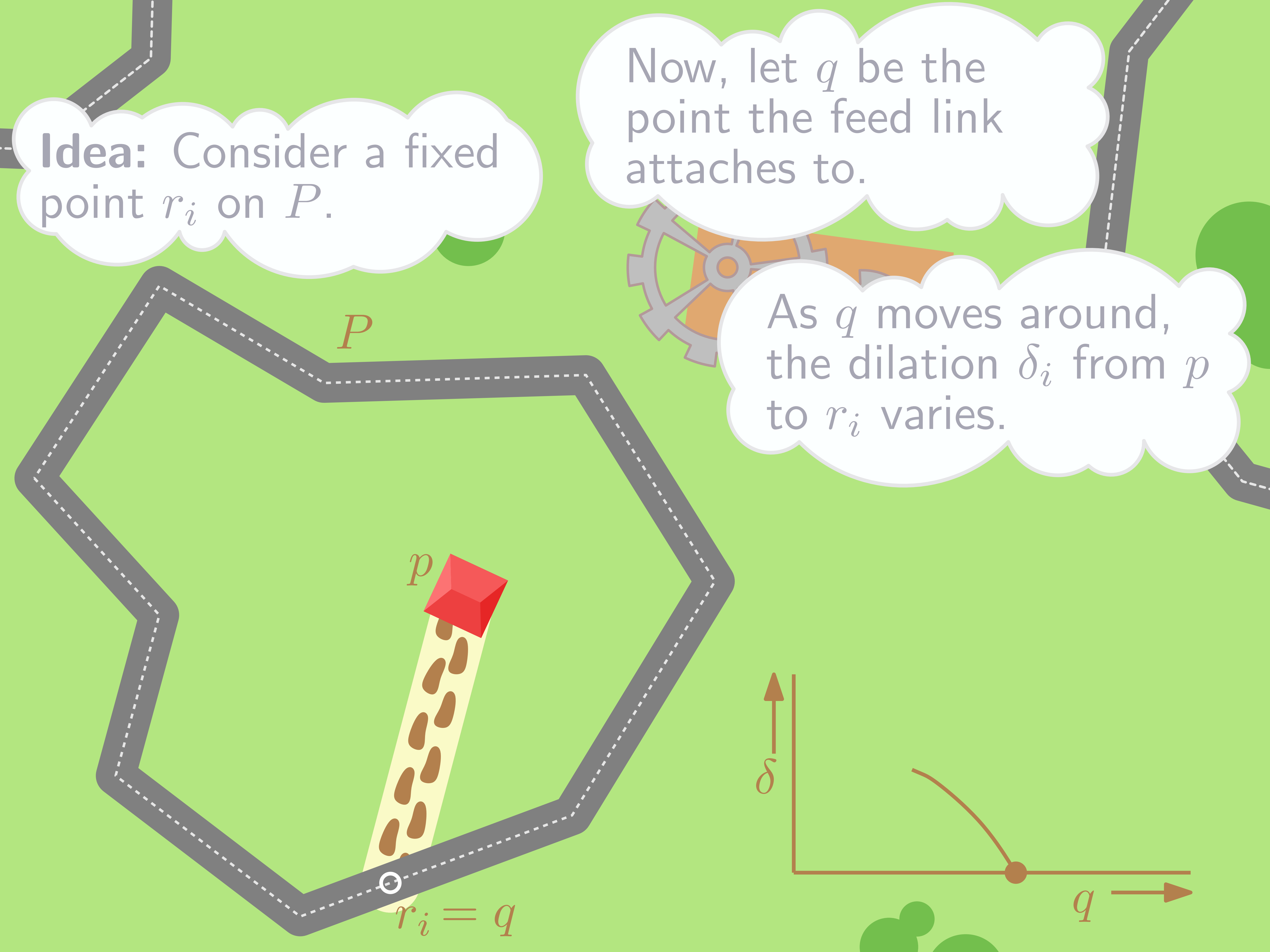
P
p
q
r_i
δ
q



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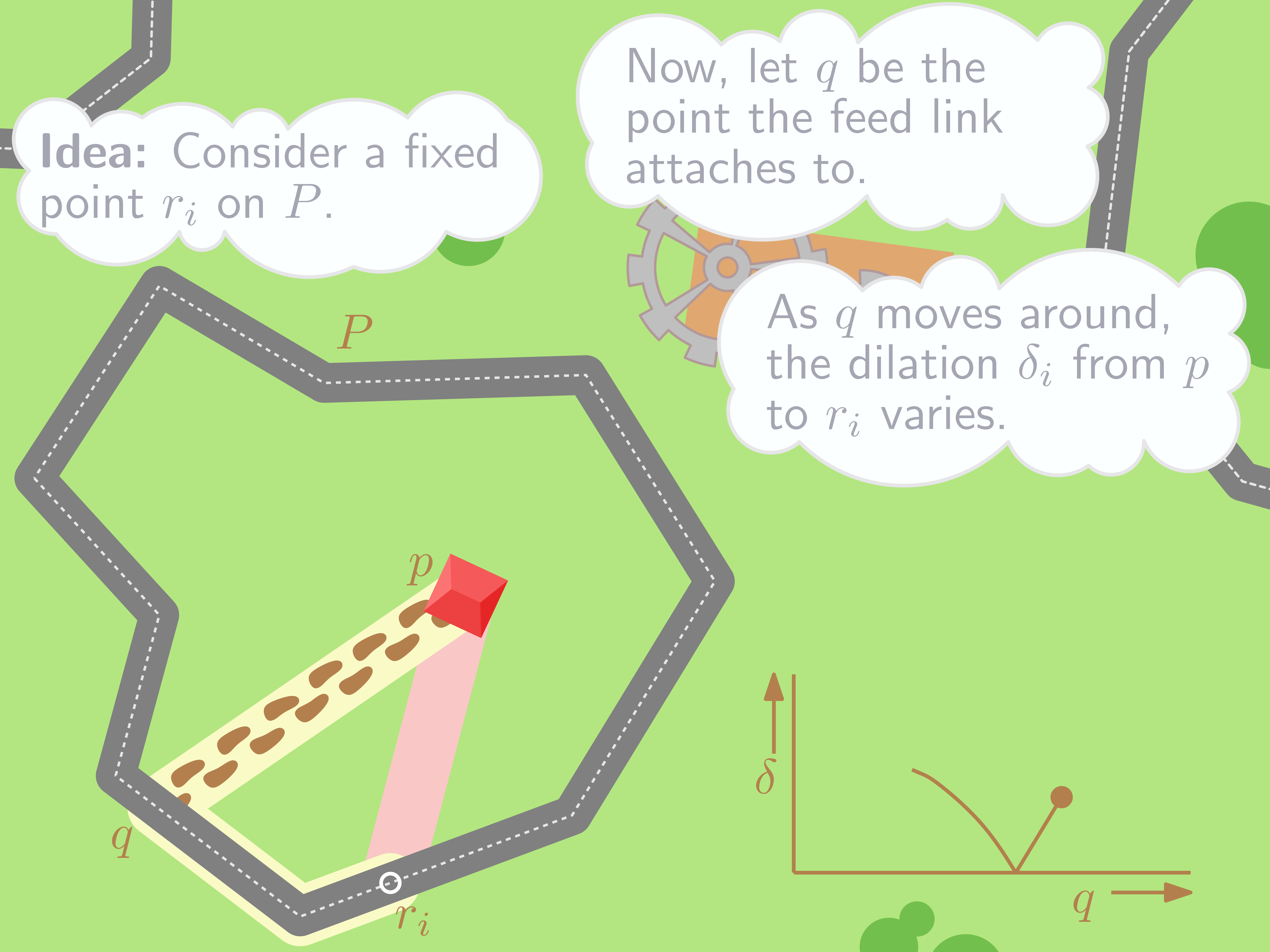
P
p
$r_i = q$
δ
q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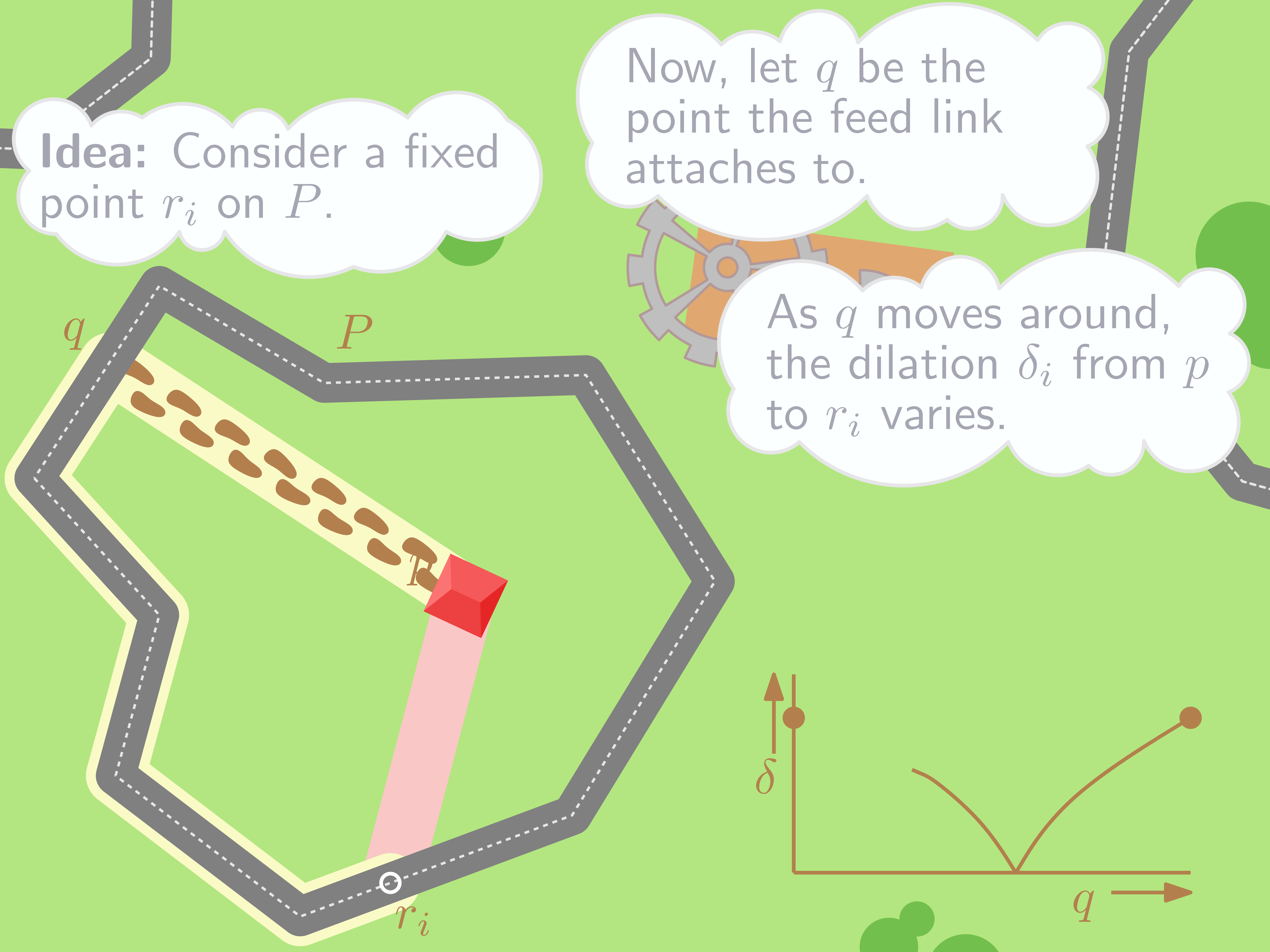
P
p
q
r_i
δ
q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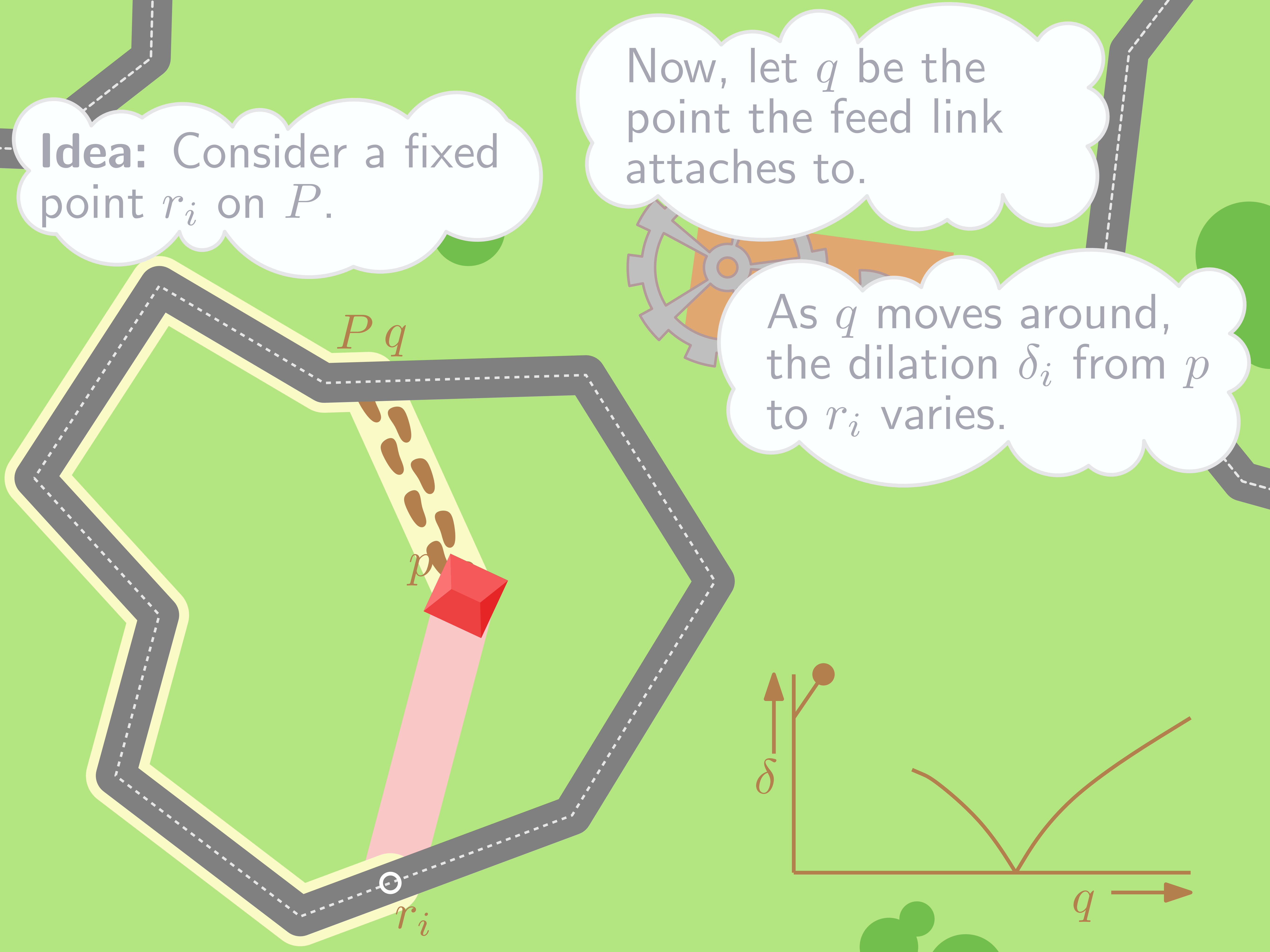
q
P
r_i
δ
q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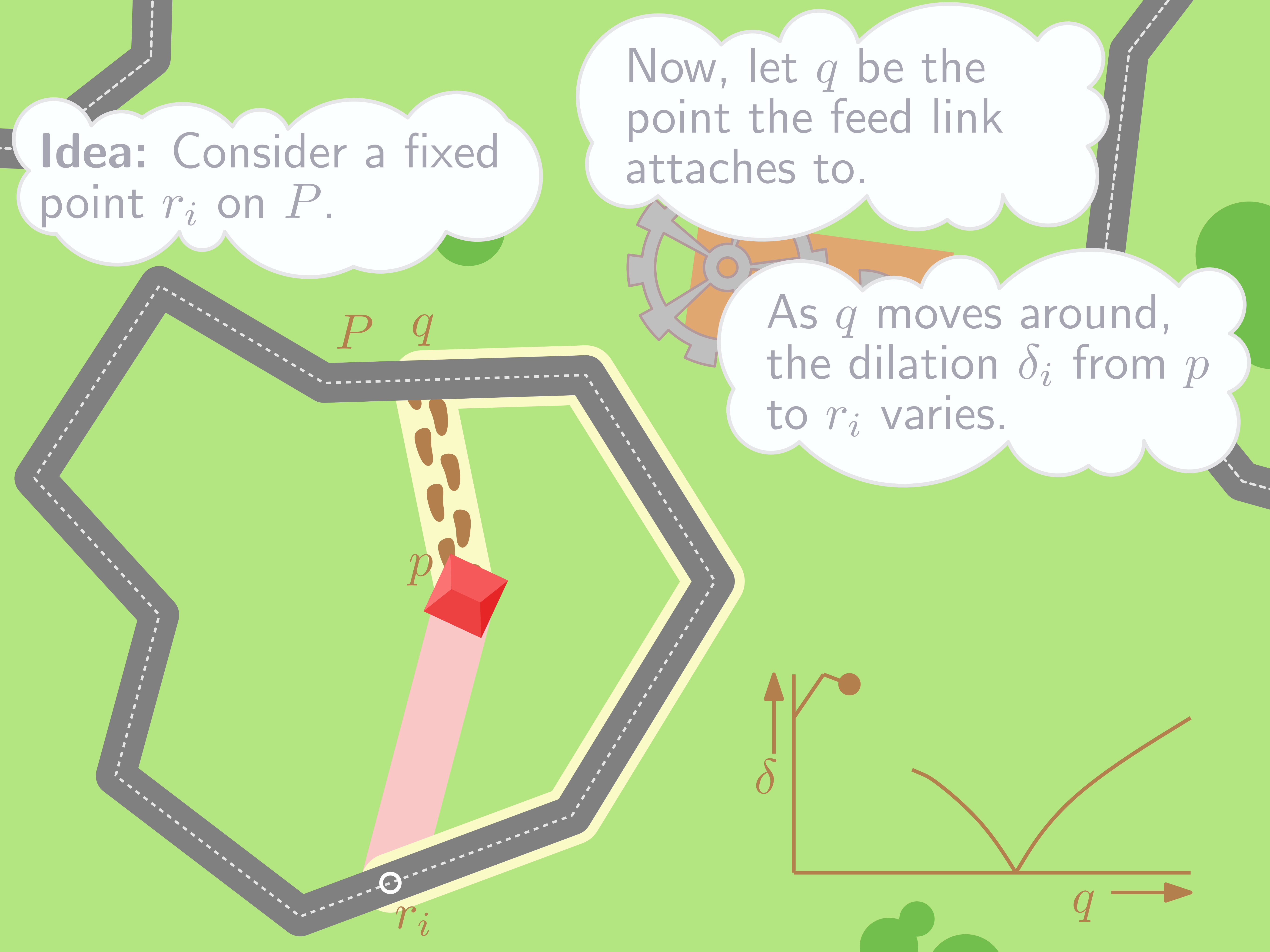
P q
p
r_i
δ
q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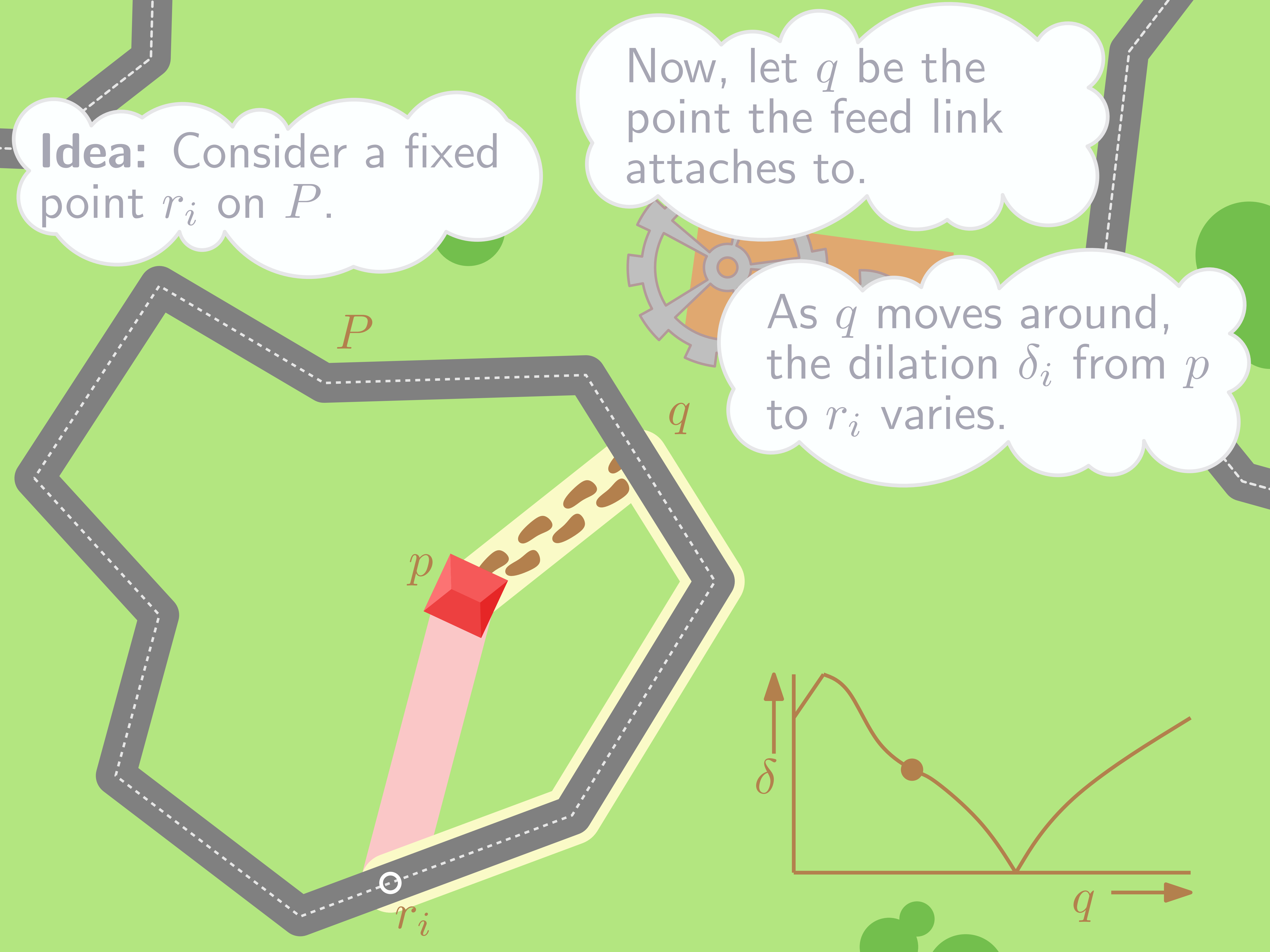
P
q
p
r_i
δ
q



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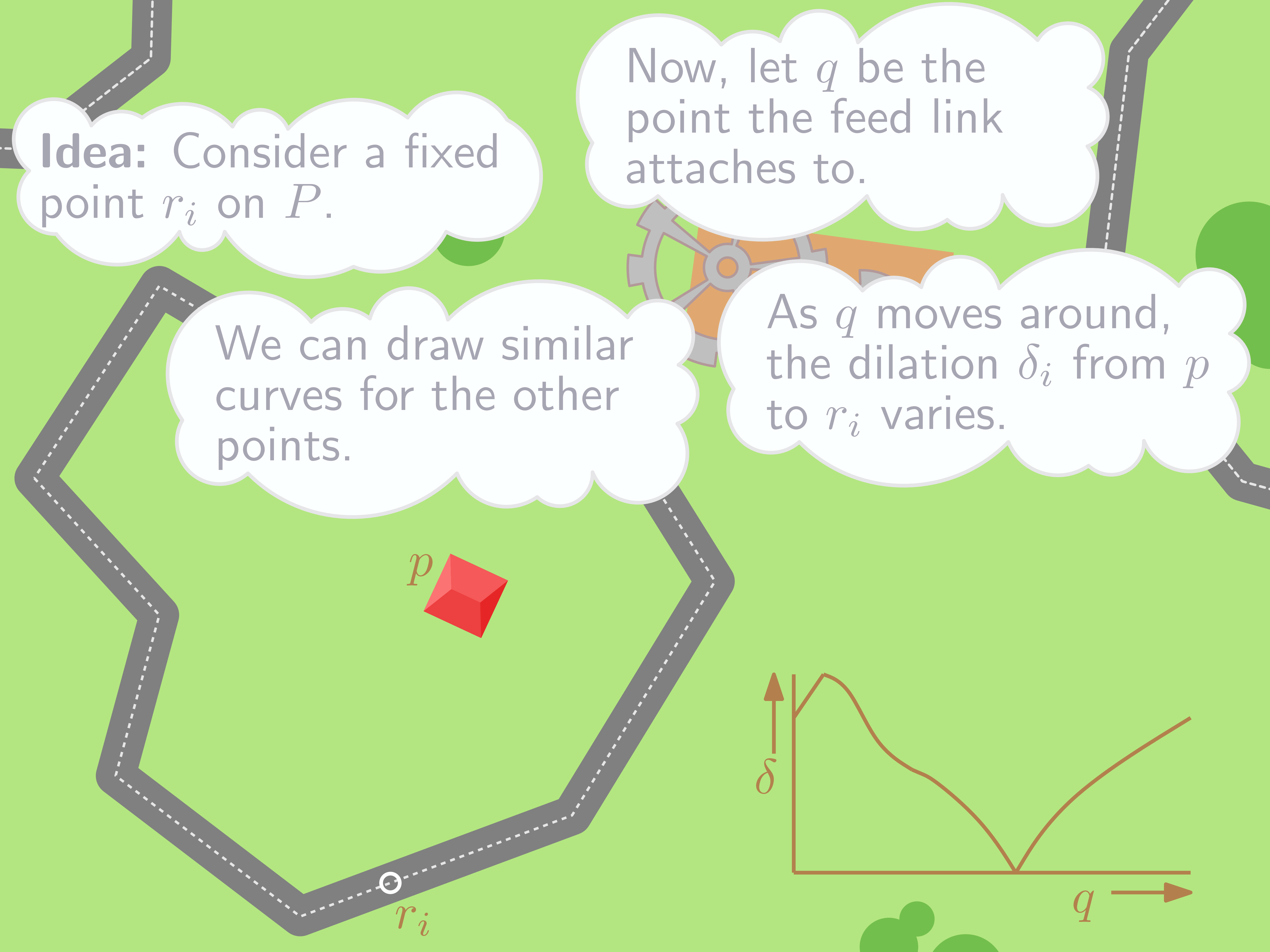
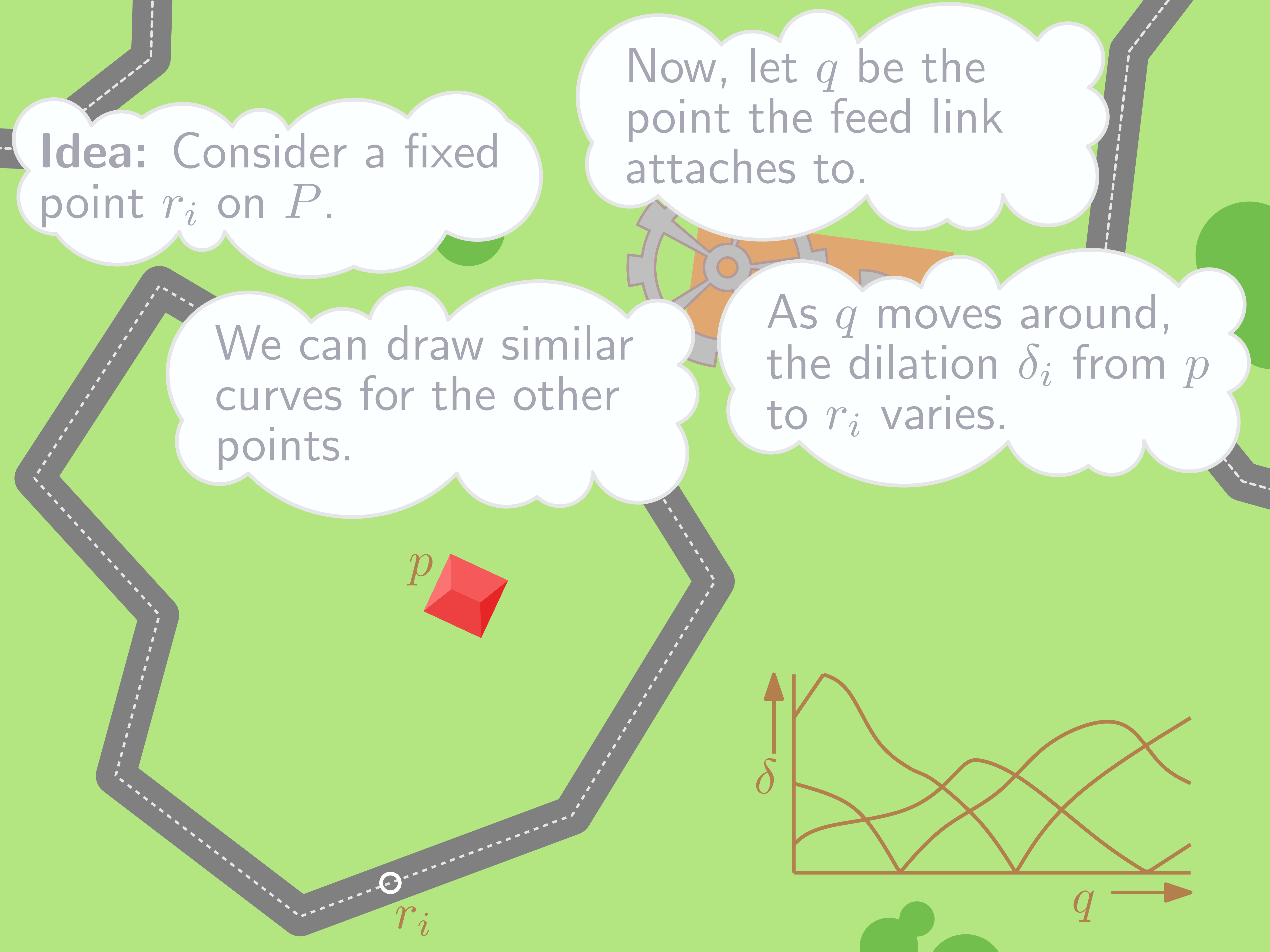
P
q
p
r_i
δ
q


Idea: Consider a fixed point r_i on P.
Now, let q be the point the feed link attaches to.
We can draw similar curves for the other points.
As q moves around, the dilation δ_i from p to r_i varies.
p
r_i
δ
q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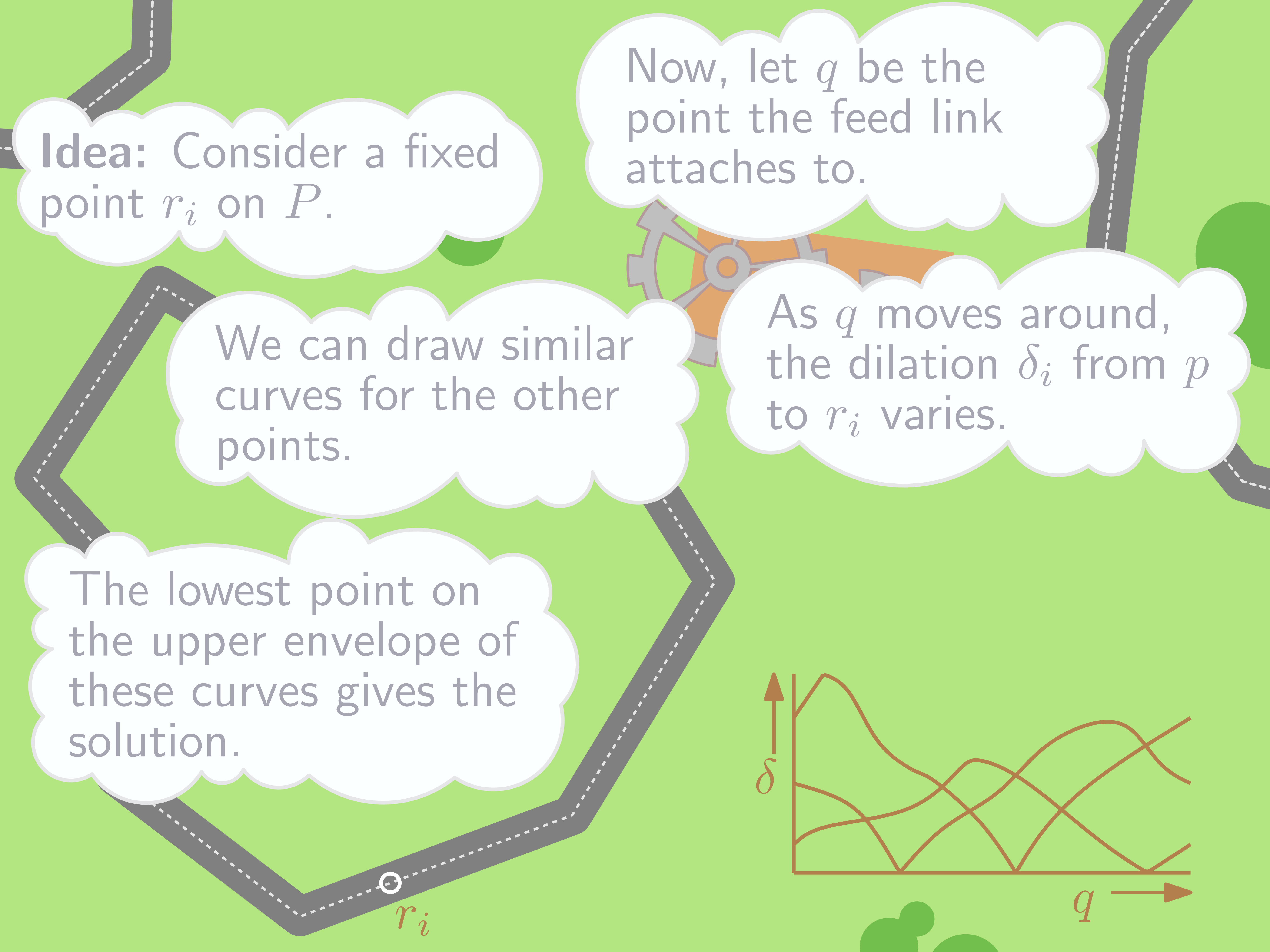
p
r_i
δ
q



Idea: Consider a fixed point r_i on P.
Now, let q be the point the feed link attaches to.
We can draw similar curves for the other points.
As q moves around, the dilation δ_i from p to r_i varies.
The lowest point on the upper envelope of these curves gives the solution.
δ
q
r_i

Idea: Consider a fixed point r_i on P.

Now, let q be the point the feed link attaches to.

As q moves around, the dilation δ_i from p to r_i varies.

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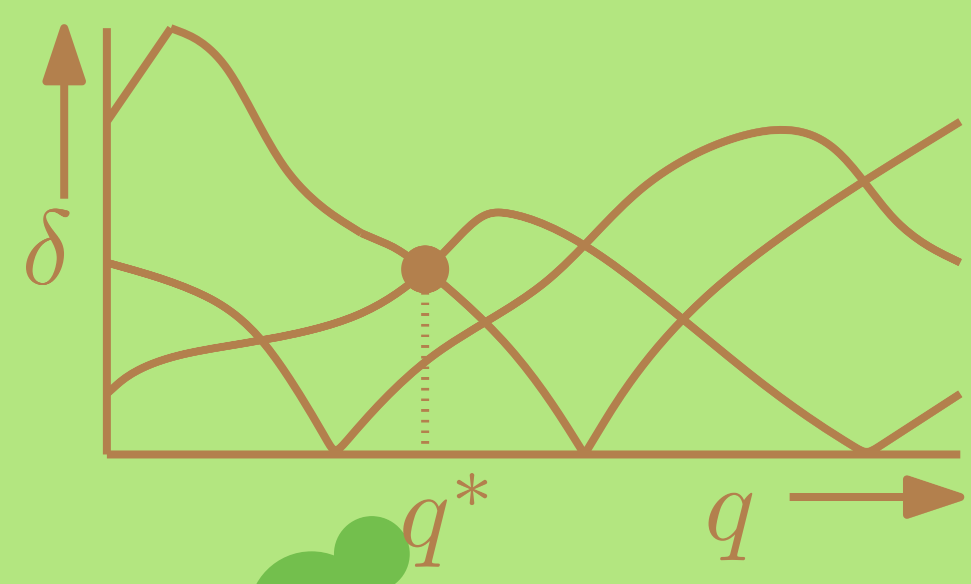



P
p

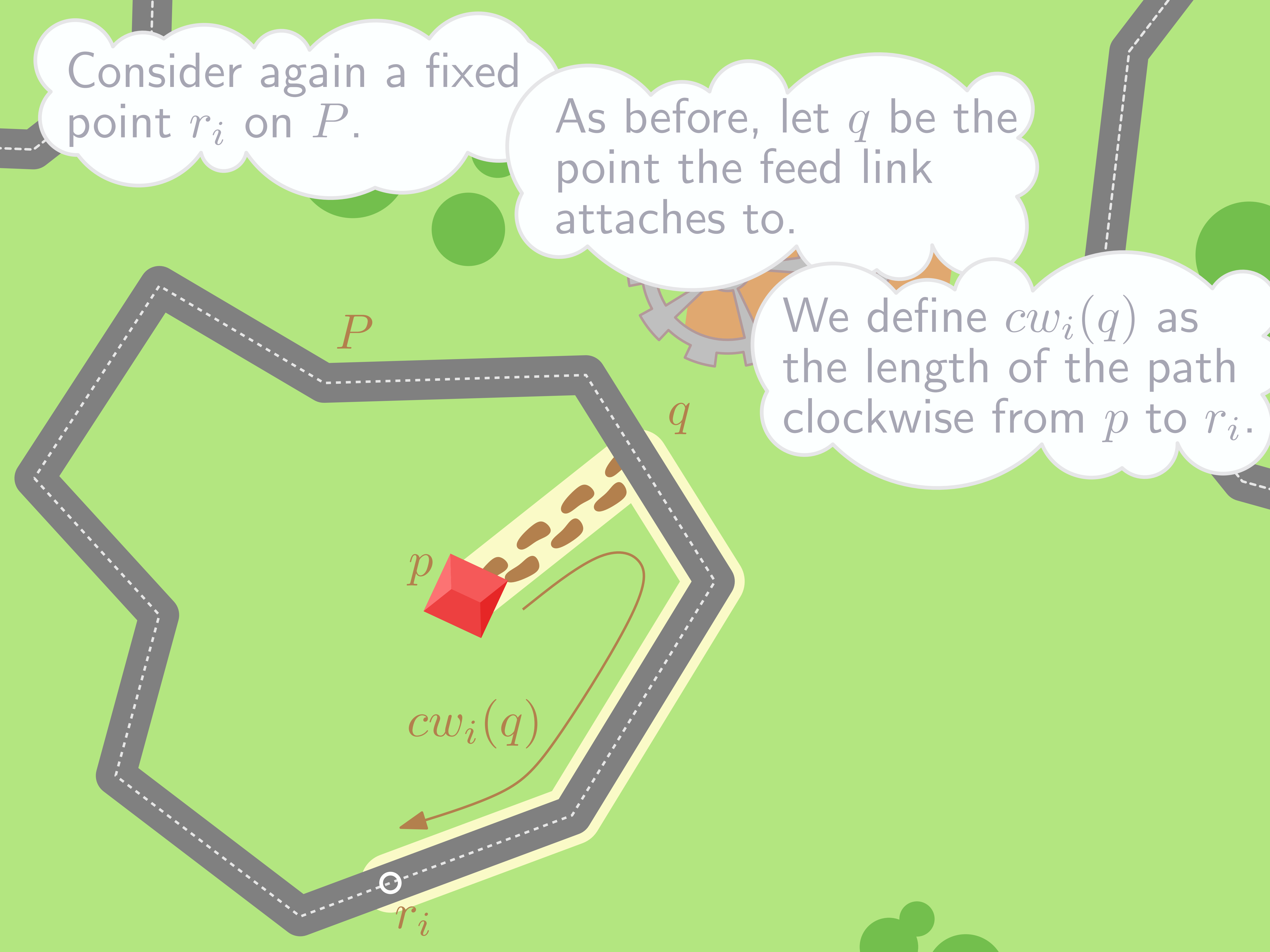

Consider again a fixed point r_i on P.
P
p


Consider again a fixed point r_i on P.
P
p
r_i


Consider again a fixed point r_i on P.
As before, let q be the point the feed link attaches to.
P
p
r_i


Consider again a fixed point r_i on P.
As before, let q be the point the feed link attaches to.
P
q
p
r_i


Consider again a fixed point r_i on P.
As before, let q be the point the feed link attaches to.
We define $cw_i(q)$ as the length of the path clockwise from p to r_i.
P
q
p
r_i



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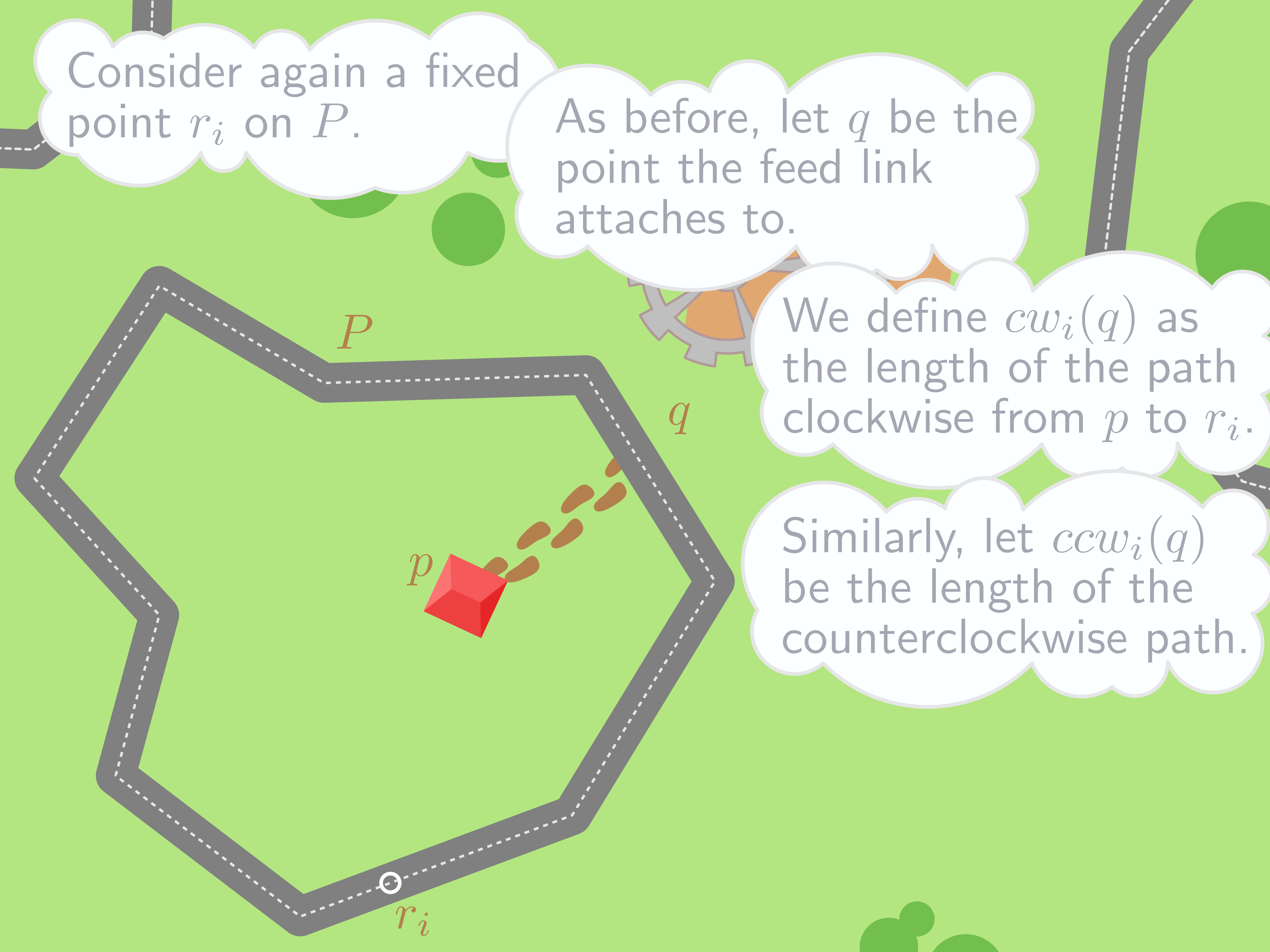
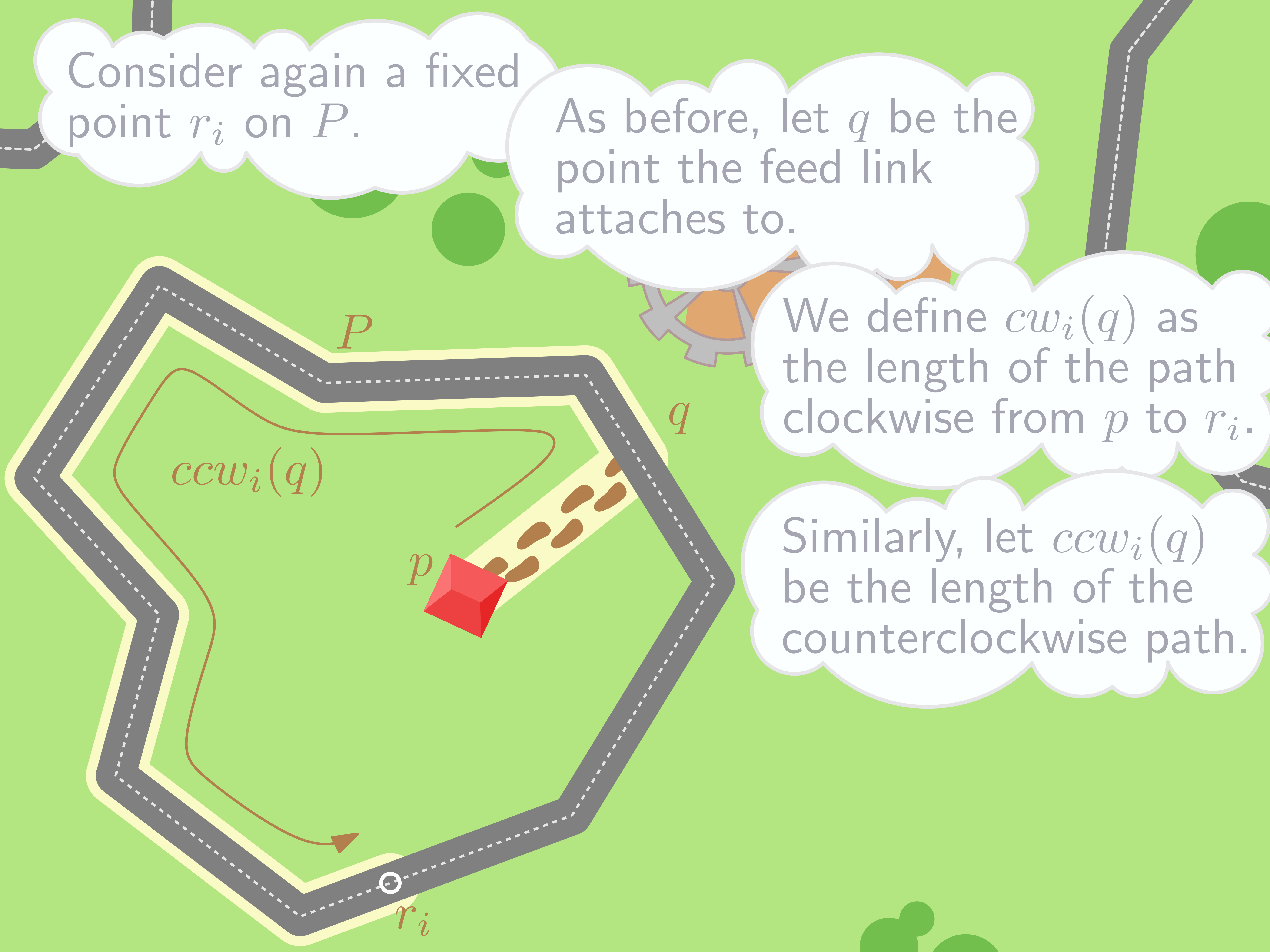
P
q
p
$cw_i(q)$
r_i



Consider again a fixed point r_i on P.
As before, let q be the point the feed link attaches to.
We define $cw_i(q)$ as the length of the path clockwise from p to r_i.
Similarly, let $ccw_i(q)$ be the length of the counterclockwise path.
P
q
p
r_i


Consider again a fixed point r_i on P.
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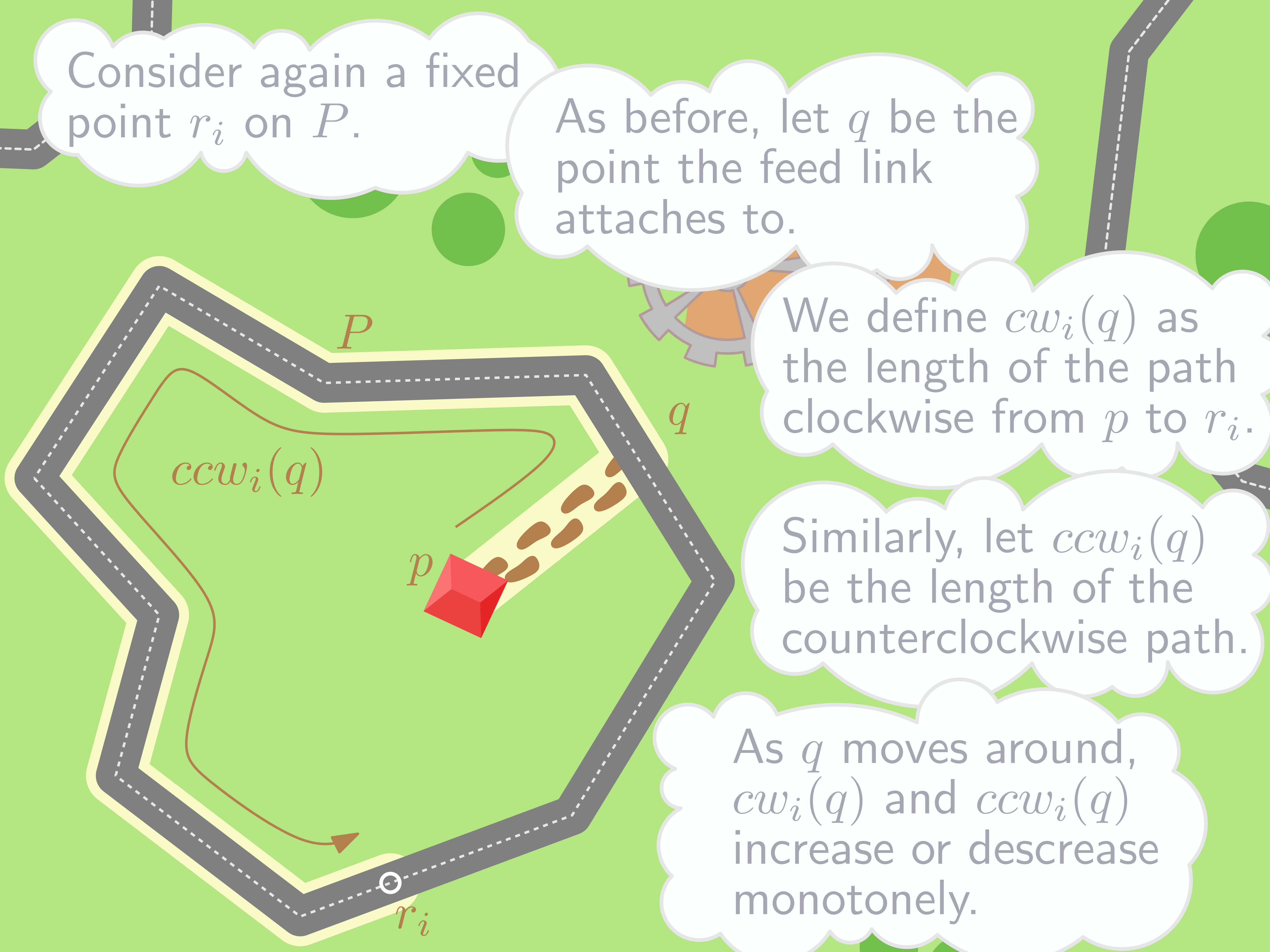
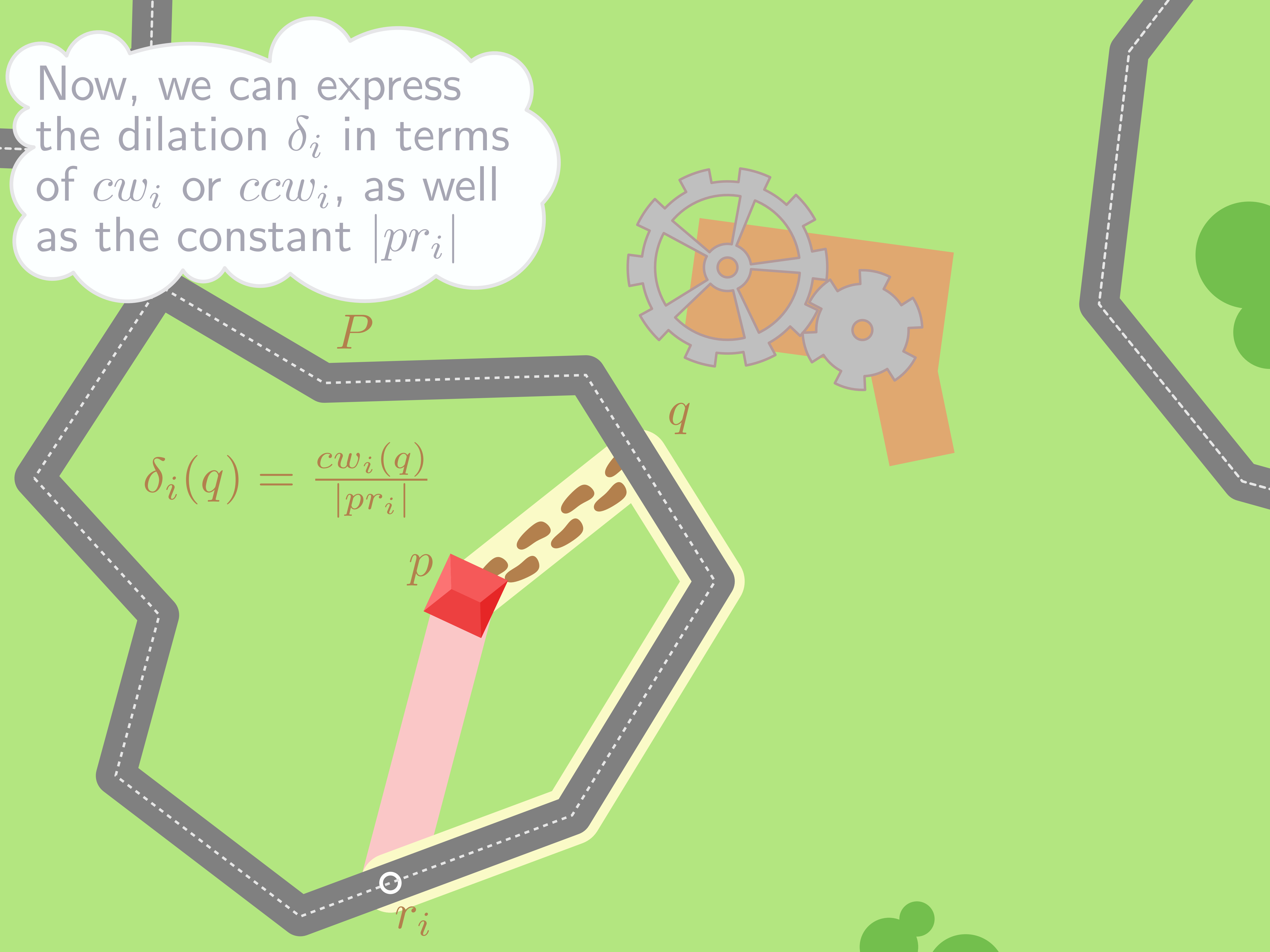
P
q
$ccw_i(q)$
p
r_i



Consider again a fixed point r_i on P.
As before, let q be the point the feed link attaches to.
We define $cw_i(q)$ as the length of the path clockwise from p to r_i.
Similarly, let $ccw_i(q)$ be the length of the counterclockwise path.
As q moves around, $cw_i(q)$ and $ccw_i(q)$ increase or descrease monotonely.
P
q
$ccw_i(q)$
p
r_i


P
p
r_i


Now, we can express the dilation δ_i in terms of cw_i or ccw_i, as well as the constant $|pr_i|$
P
p
r_i

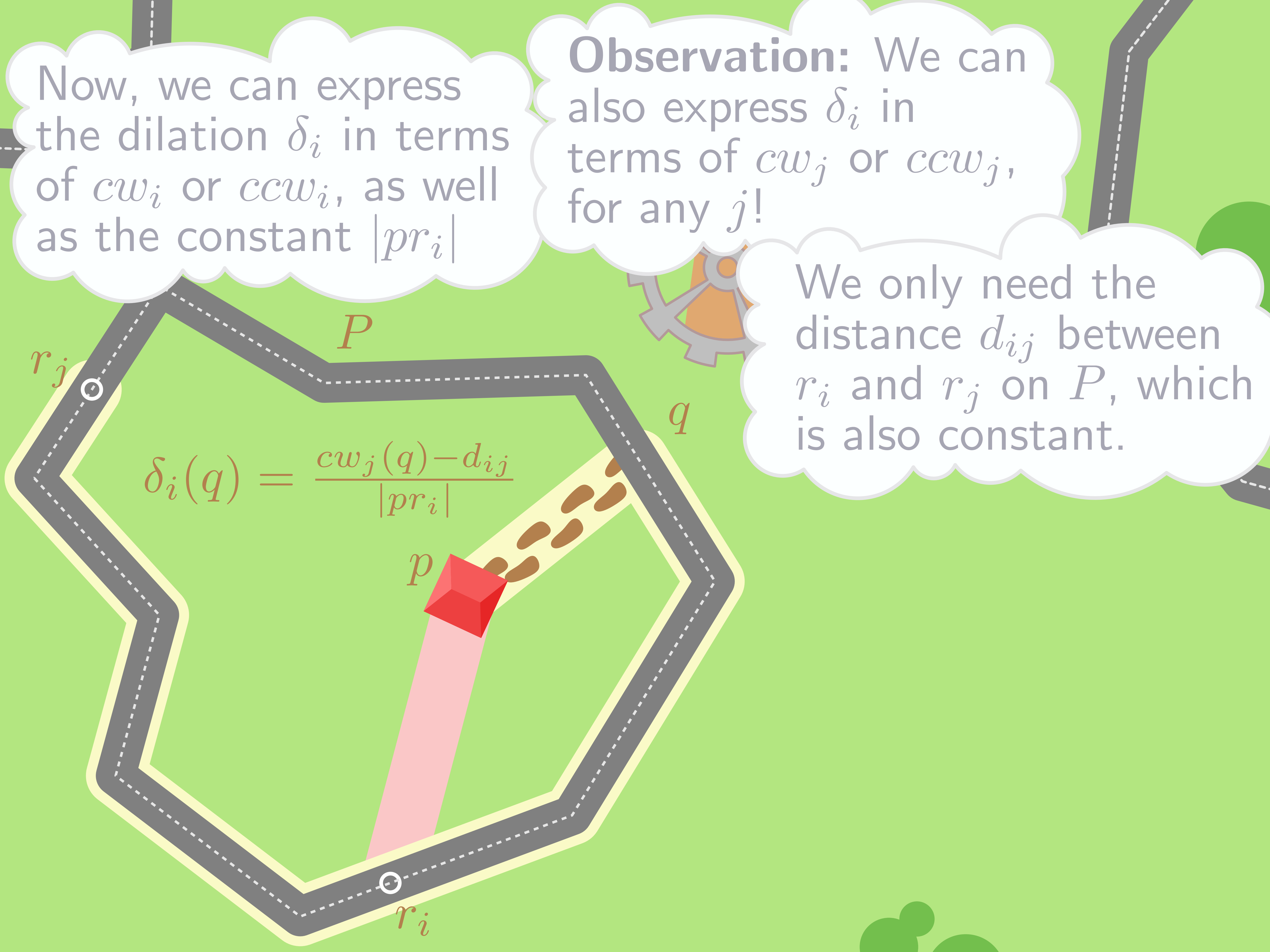


Now, we can express the dilation δ_i in terms of cw_i or ccw_i, as well as the constant $|pr_i|$
P
q
$\delta_i(q) = \frac{cw_i(q)}{|pr_i|}$
p
r_i


Now, we can express the dilation δ_i in terms of cw_i or ccw_i, as well as the constant $|pr_i|$
Observation: We can also express δ_i in terms of cw_j or ccw_j, for any j!
P
q
p
r_i

Now, we can express the dilation δ_i in terms of cw_i or ccw_i, as well as the constant $|pr_i|$
Observation: We can also express δ_i in terms of cw_j or ccw_j, for any j!
P
r_j
q
p
r_i


Now, we can express the dilation δ_i in terms of cw_i or ccw_i, as well as the constant $|pr_i|$
Observation: We can also express δ_i in terms of cw_j or ccw_j, for any j!
We only need the distance d_{ij} between r_i and r_j on P, which is also constant.
P
r_j
q
p
r_i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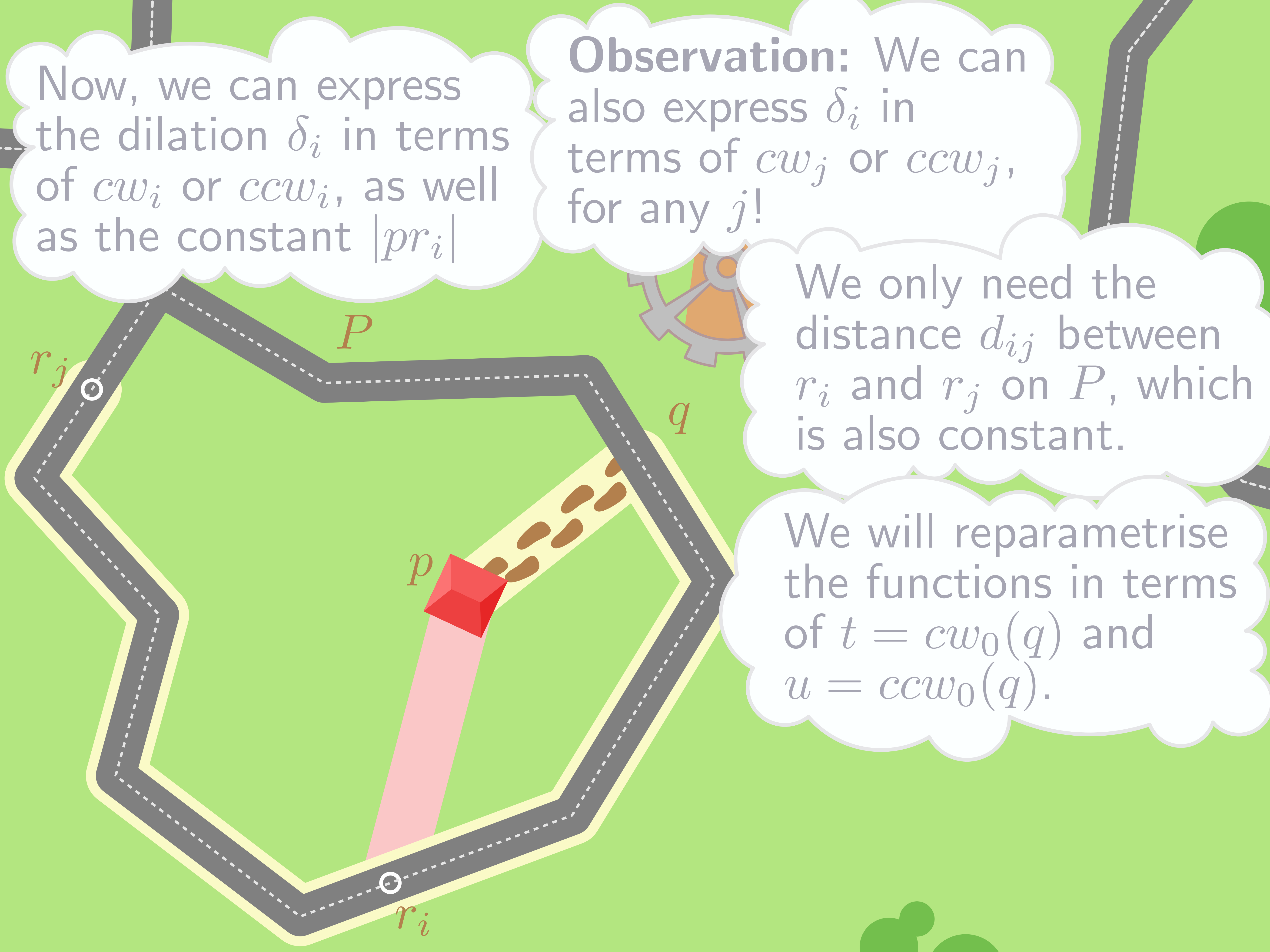
We only need the distance d_{ij} between r_i and r_j on P, which is also constant.
P
r_j
q
$\delta_i(q) = \frac{cw_j(q)-d_{ij}}{|pr_i|}$
p
r_i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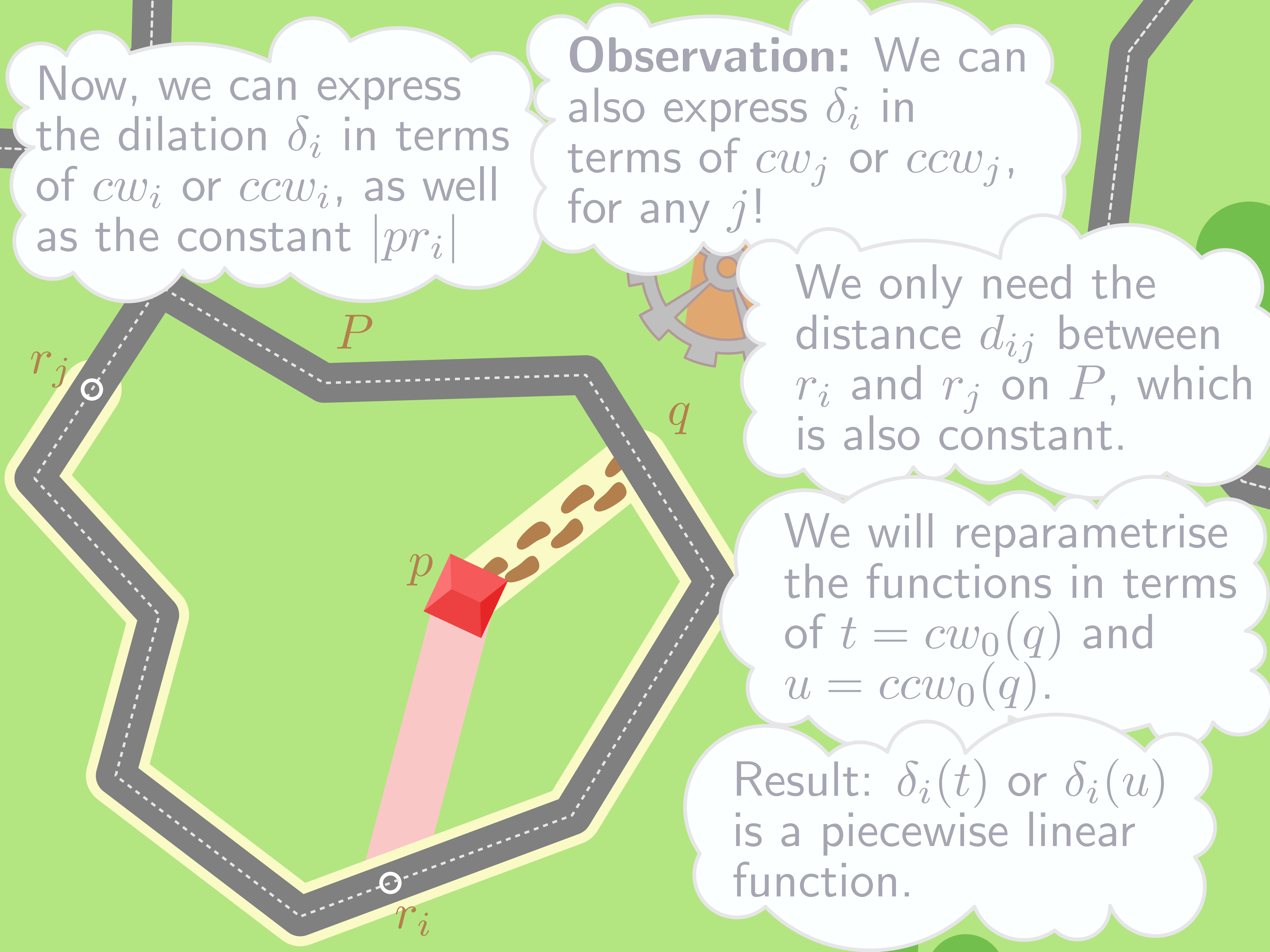
We only need the distance d_{ij} between r_i and r_j on P, which is also constant.
We will reparametrise the functions in terms of $t = cw_0(q)$ and $u = ccw_0(q)$.
P
r_j
q
p
r_i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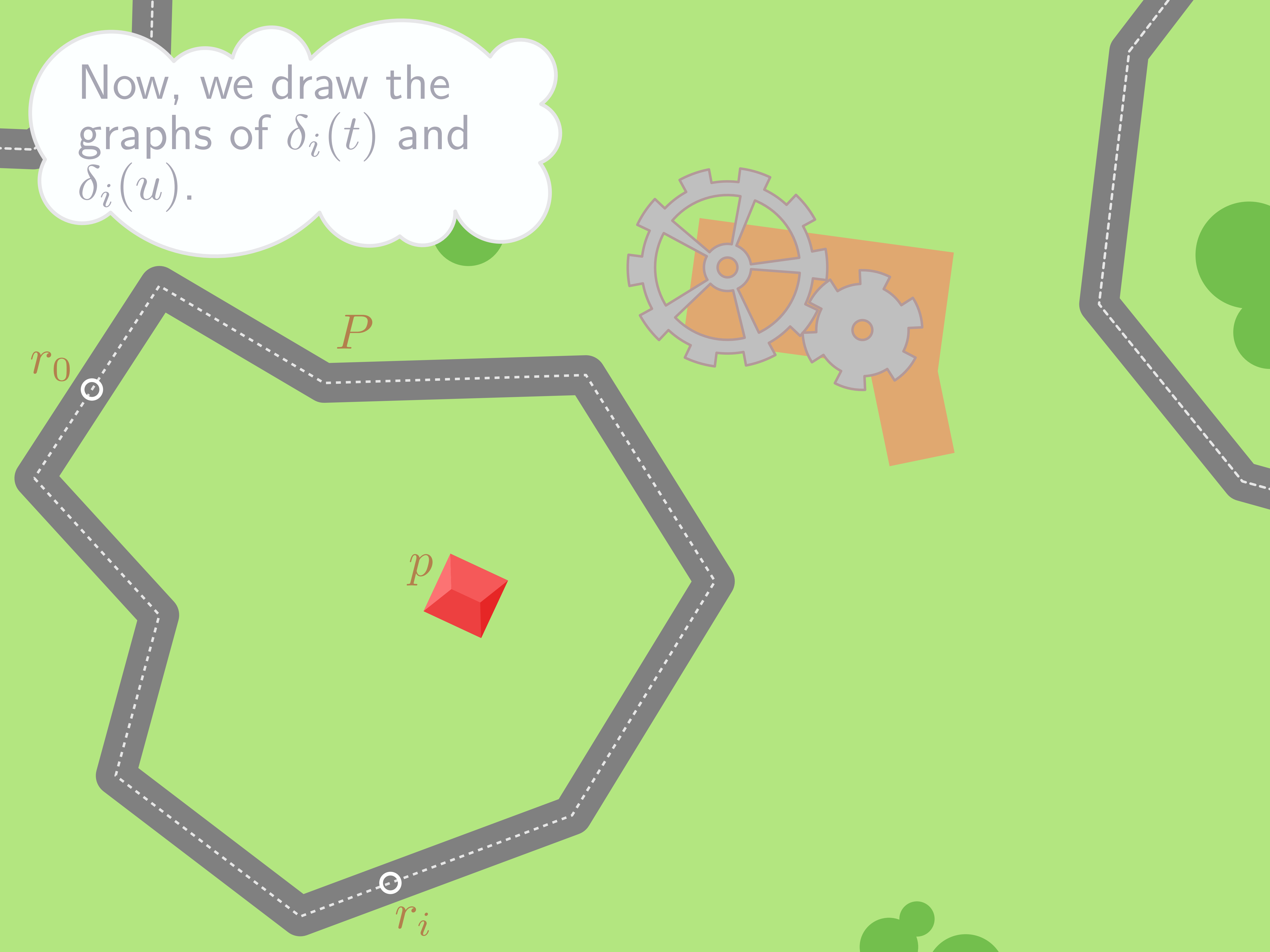
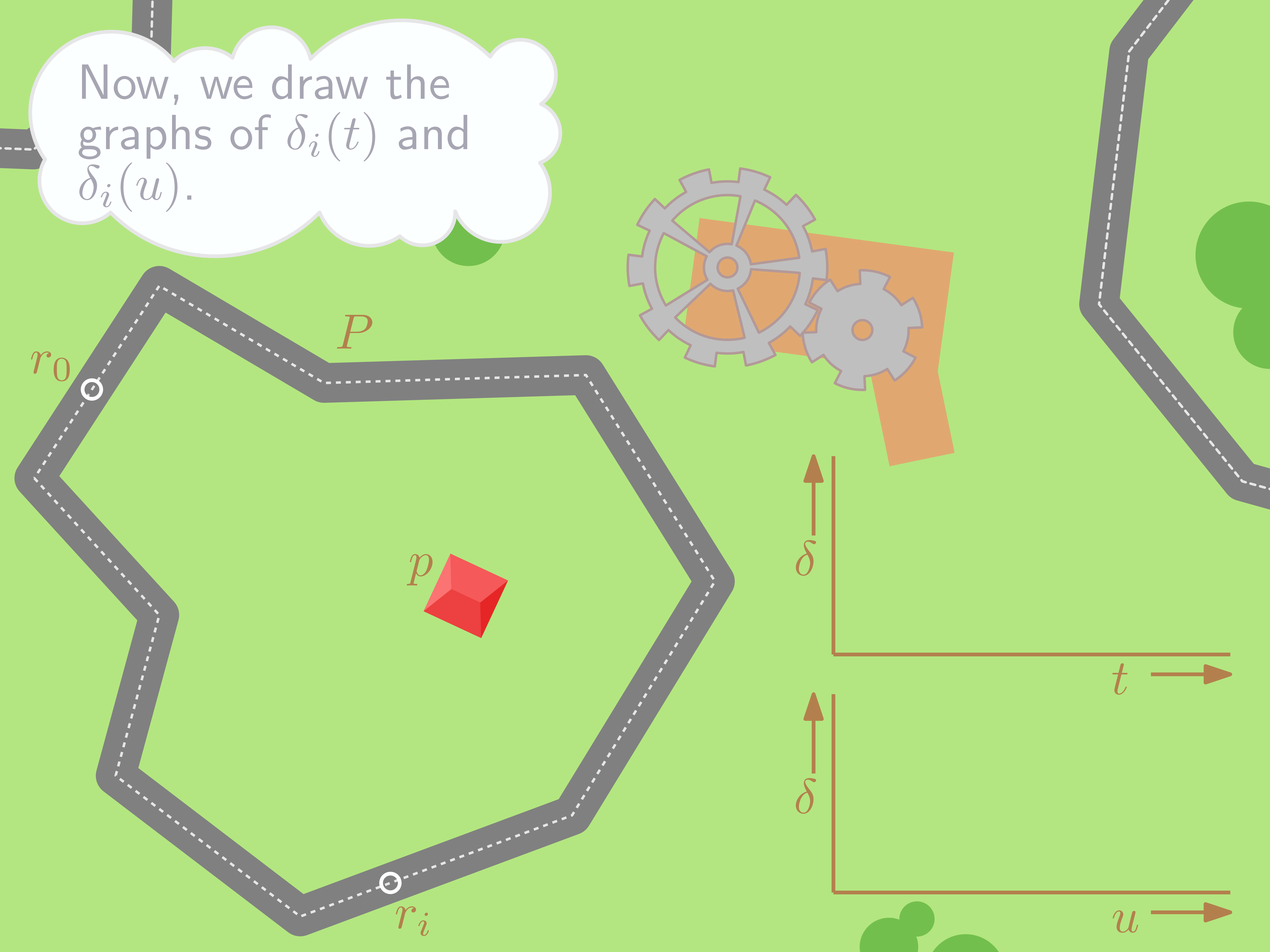
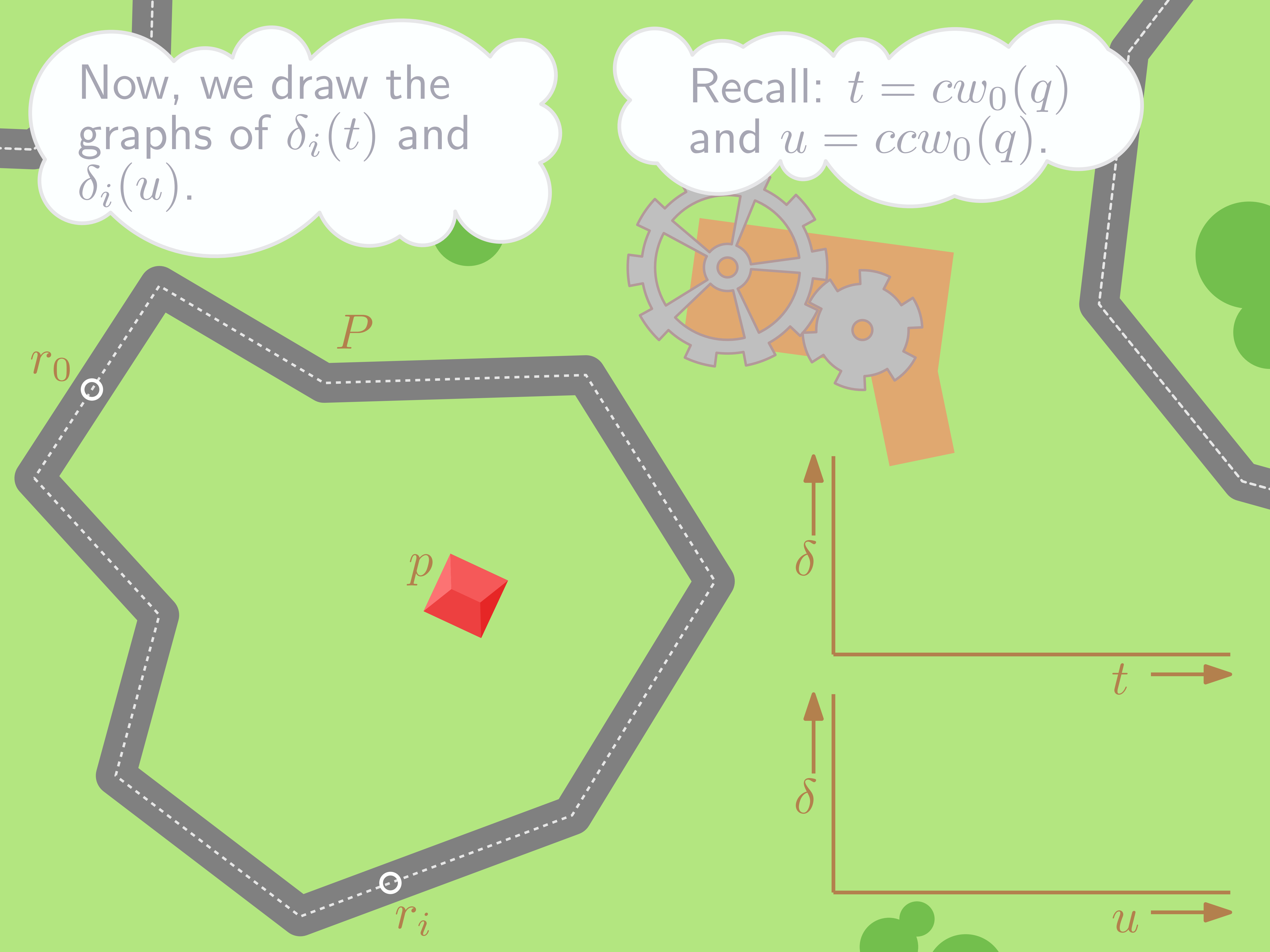
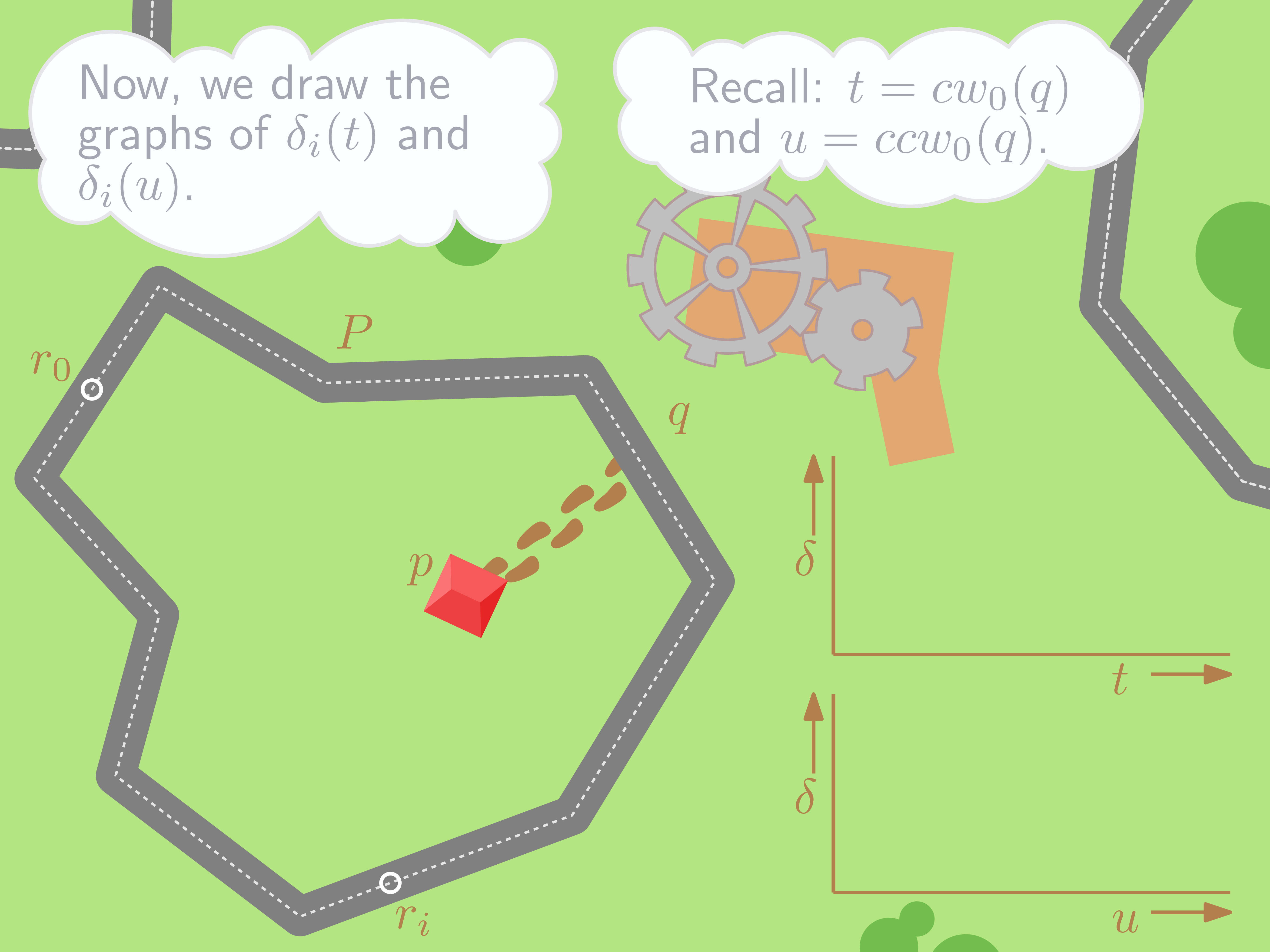
Observation: We can also express δ_i in terms of cw_j or ccw_j, for any j!
We only need the distance d_{ij} between r_i and r_j on P, which is also constant.
We will reparametrise the functions in terms of $t = cw_0(q)$ and $u = ccw_0(q)$.
Result: $\delta_i(t)$ or $\delta_i(u)$ is a piecewise linear function.
P
r_j
q
p
r_i

P
r_0
p
r_i


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
P
r_0
p
r_i


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
P
r_0
p
r_i
δ
t
δ
u


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
P
r_0
r_i
p
δ
t
δ
u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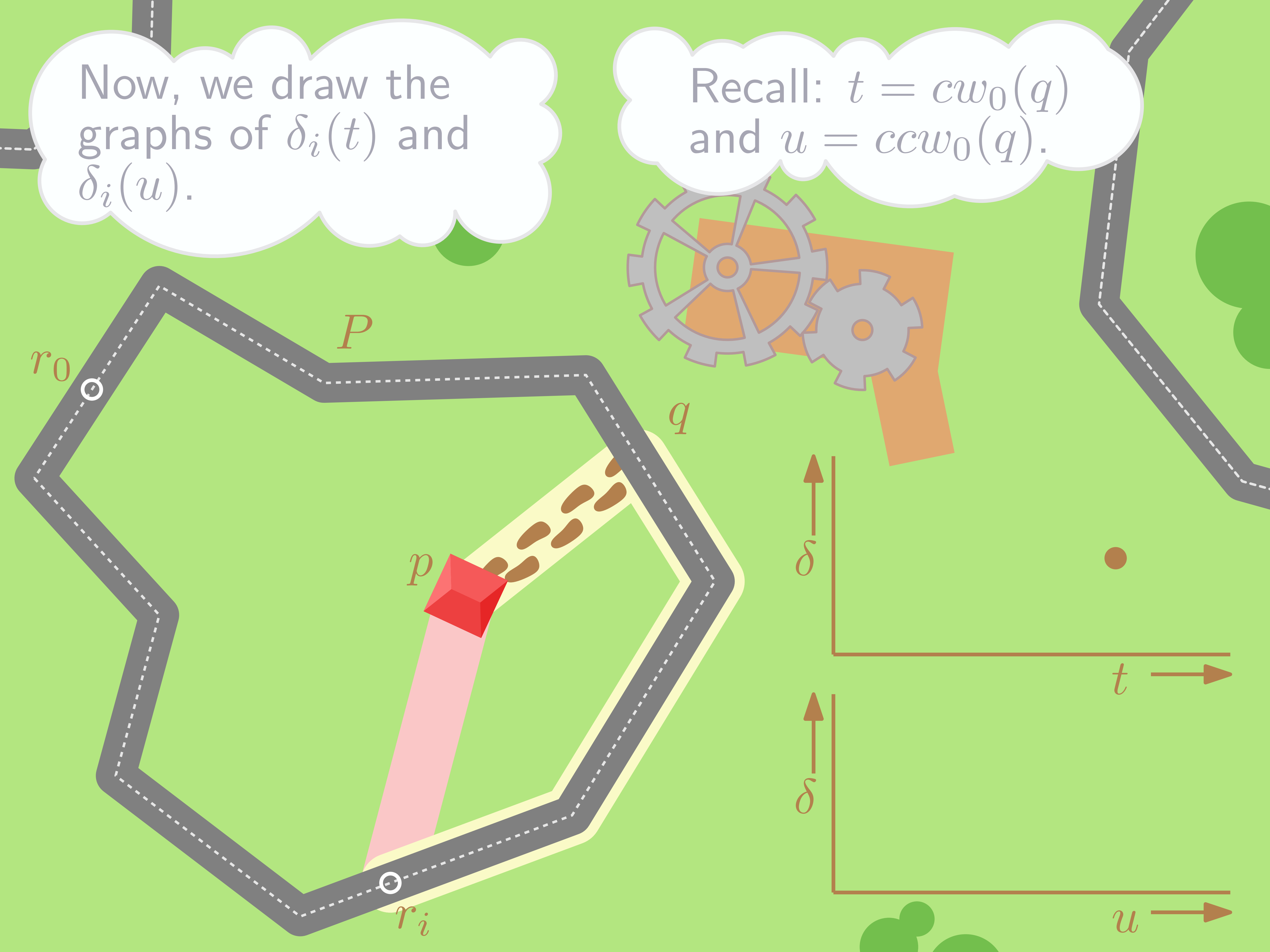
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
r_0
P
q
p
r_i
δ
t
δ
u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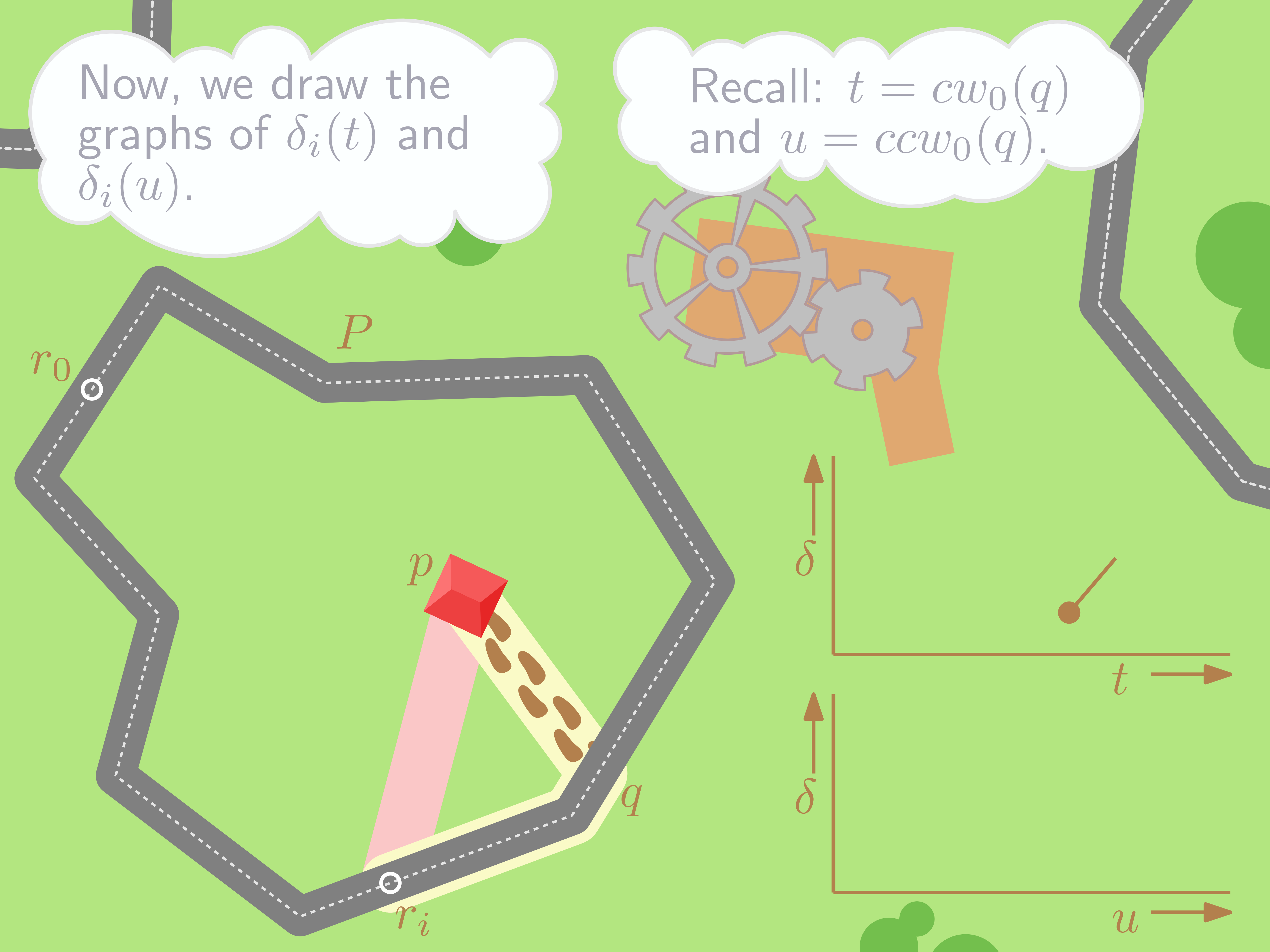
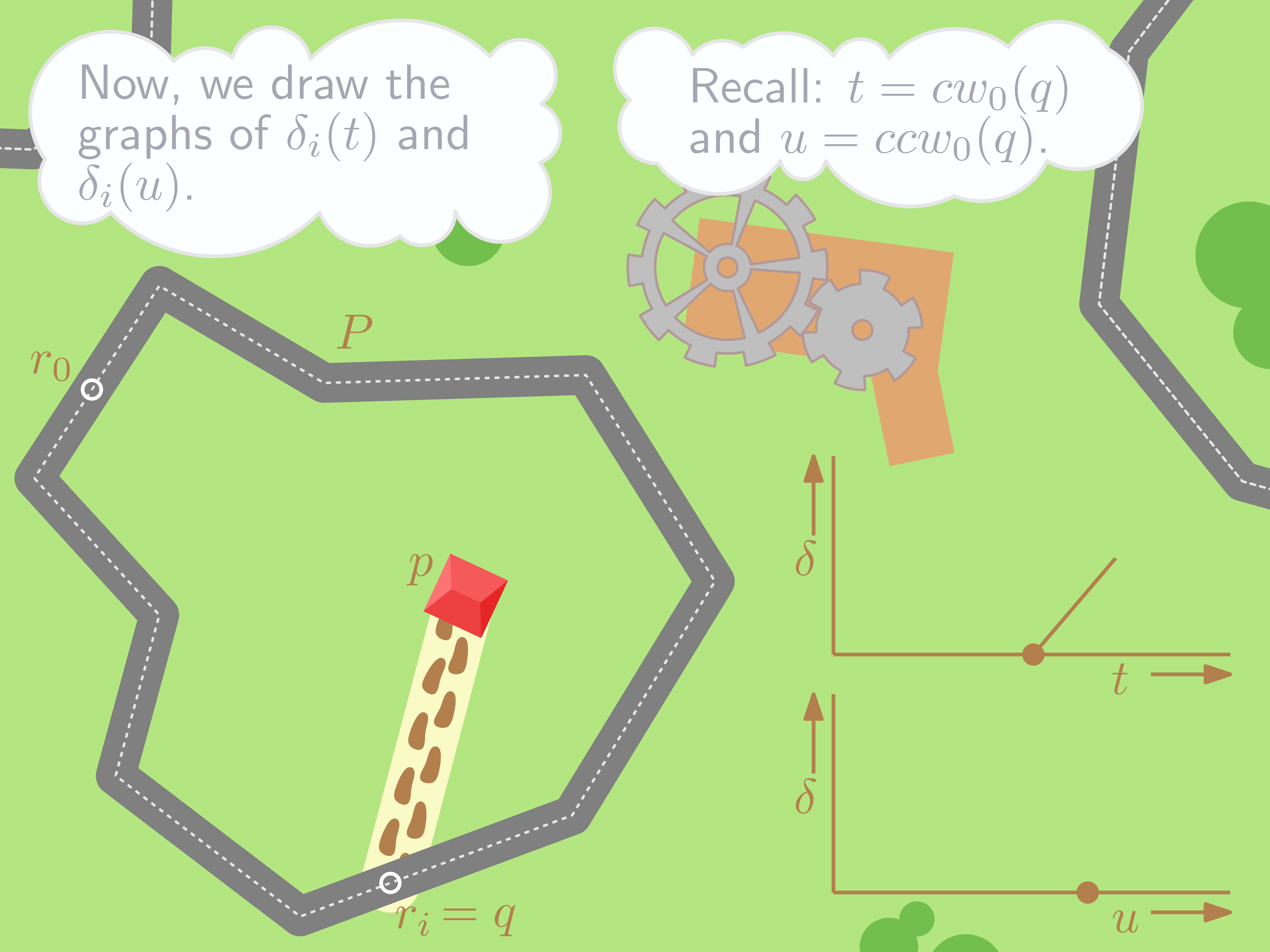
r_0
P
q
p
r_i
δ
t
δ
u



Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
P
r_0
p
q
r_i
δ
t
δ
u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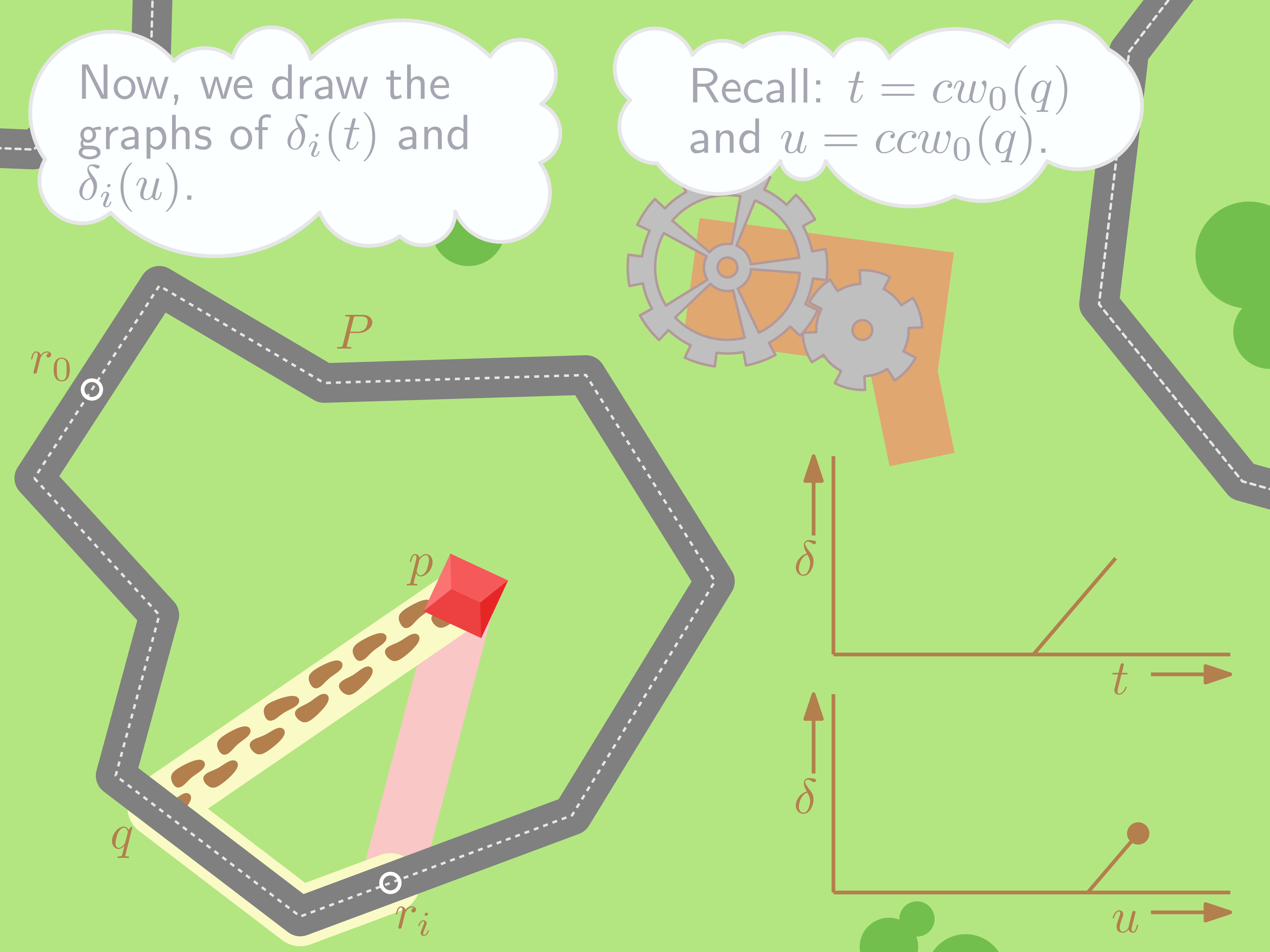
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
r_0
P
p
$r_i = q$
δ
t
δ
u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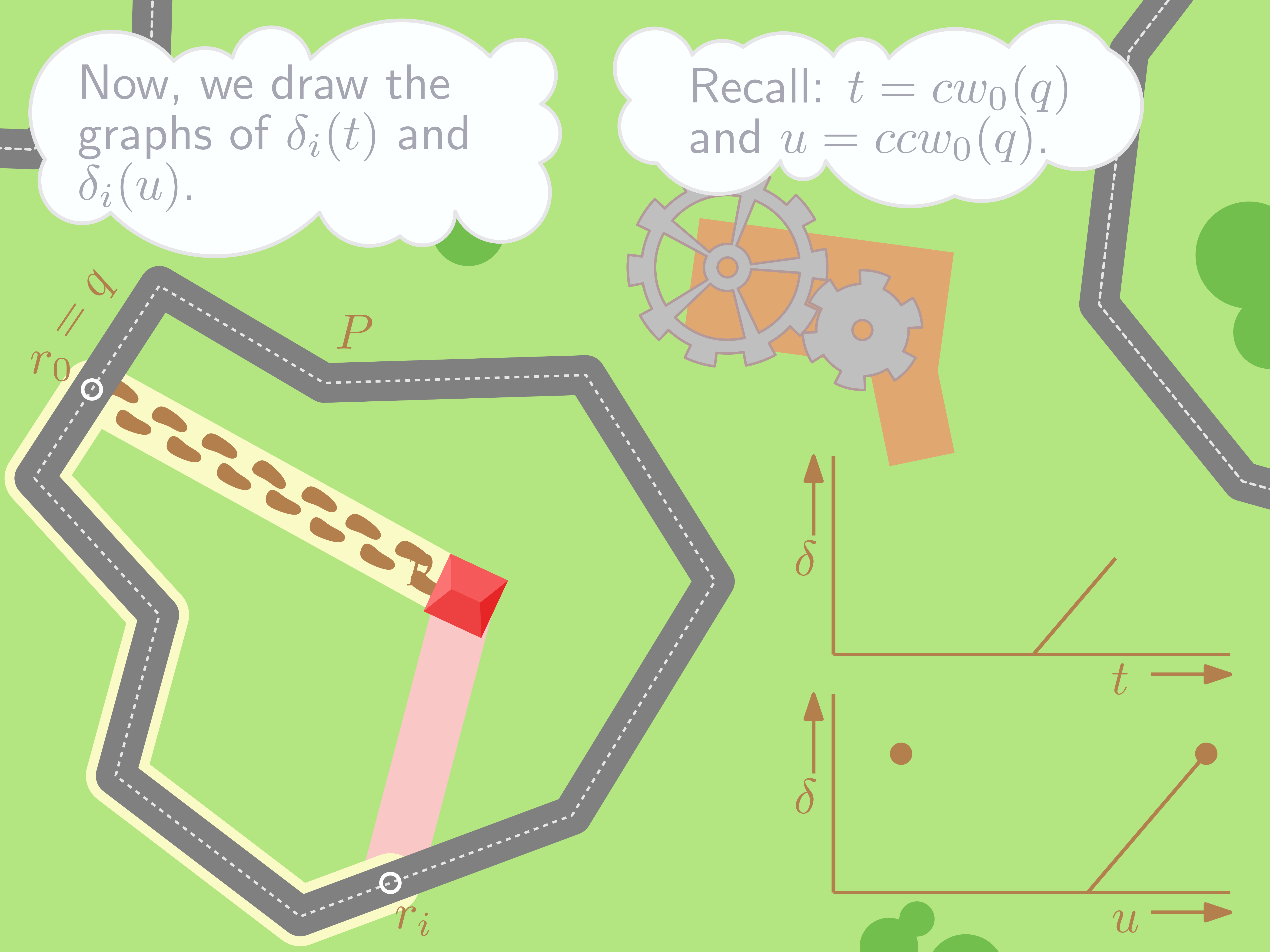
P
r_0
p
q
r_i
δ
t
δ
u



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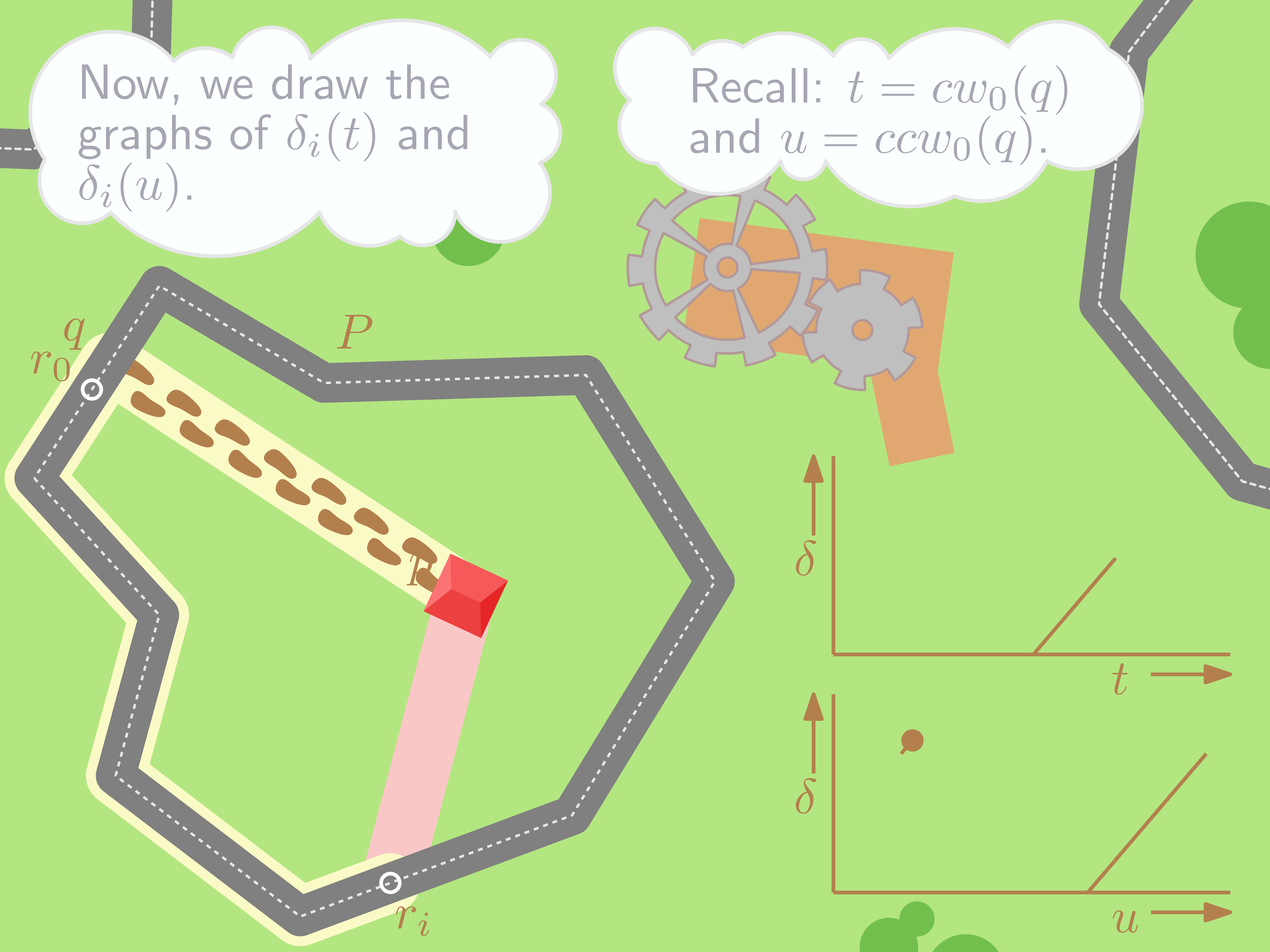
q
P
r_0
r_i
δ
t
δ
u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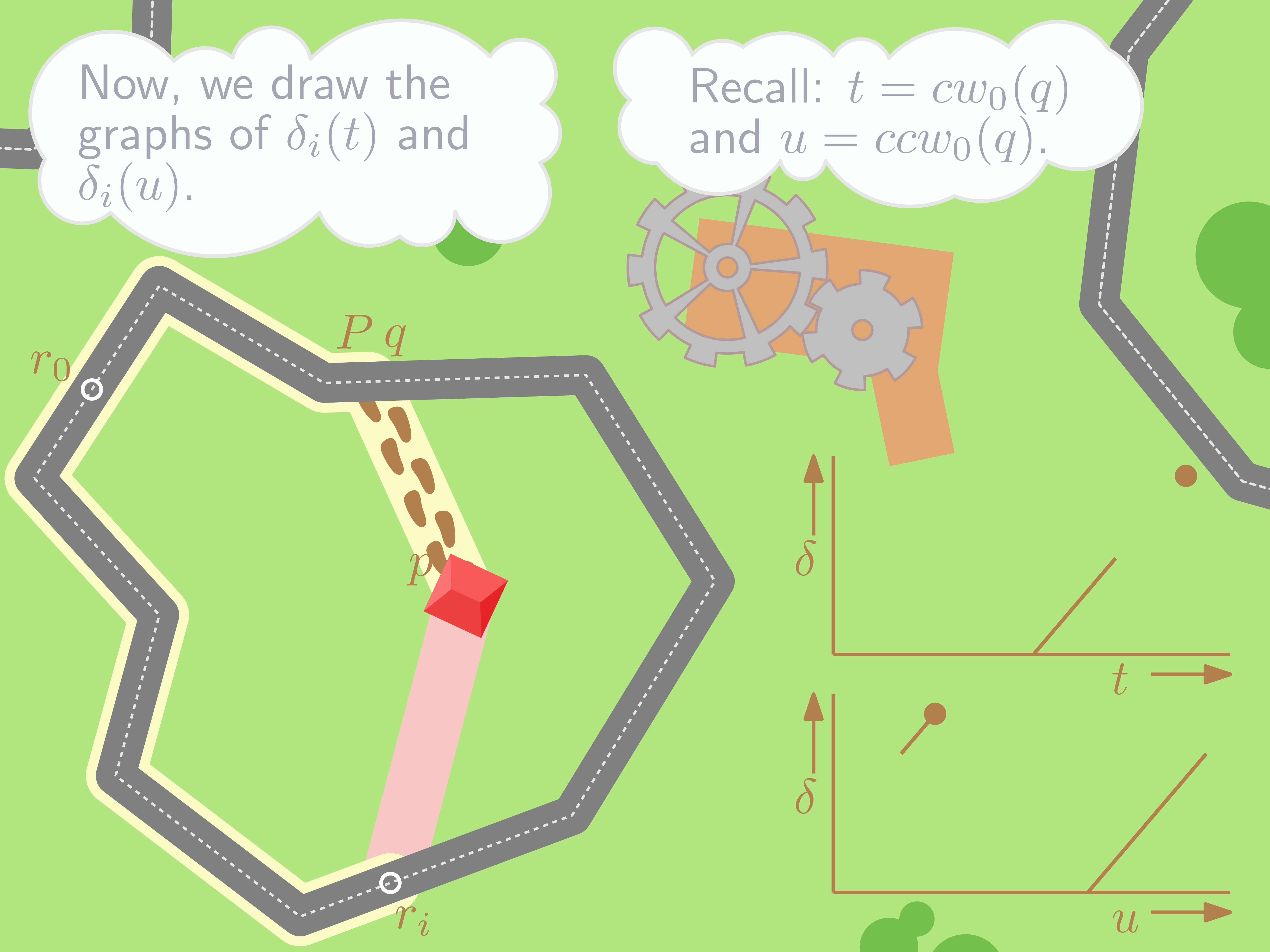
q
r_0
P
r_i
δ
t
δ
u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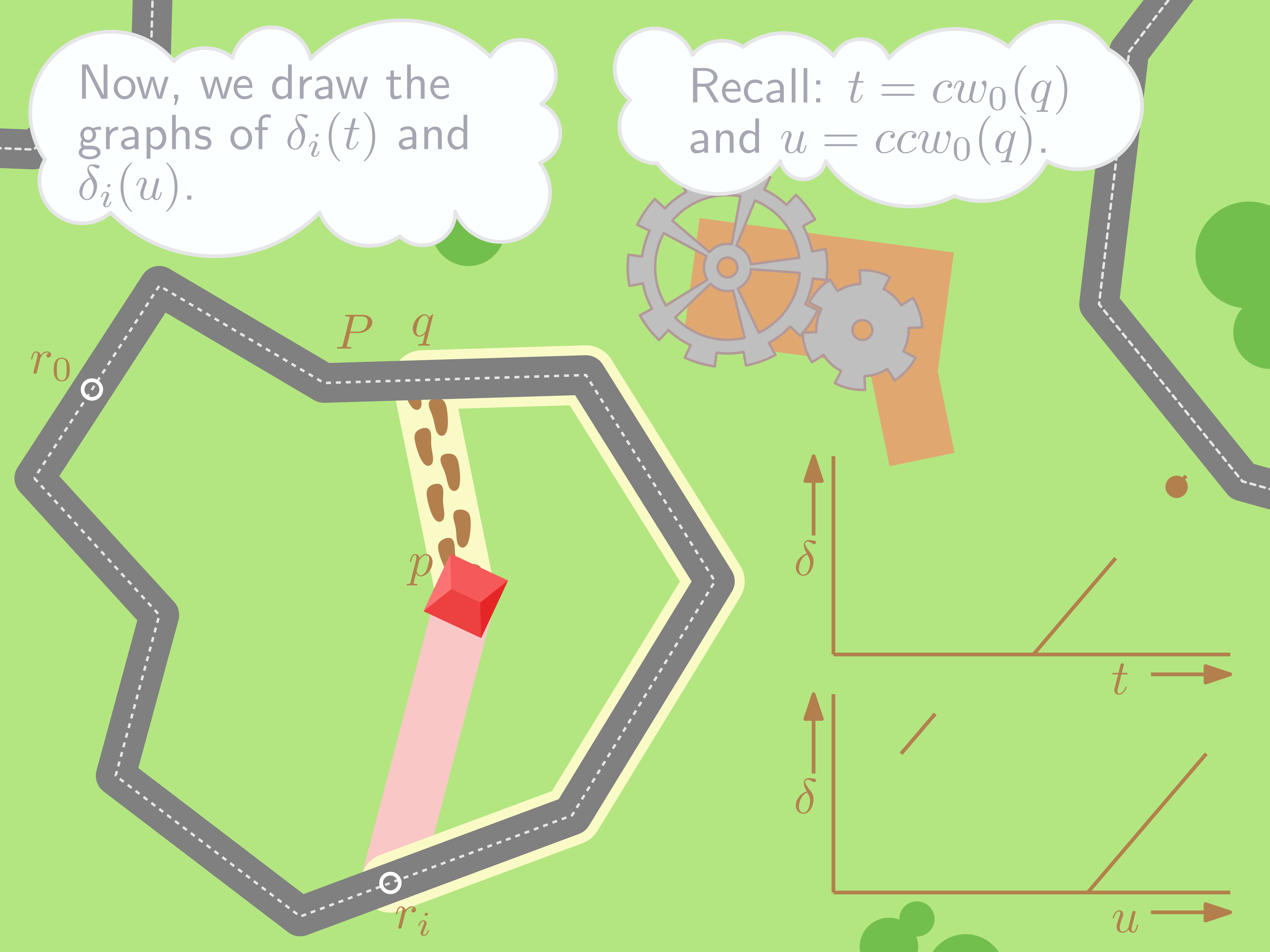
r_0
P q
p
r_i
δ
t
δ
u



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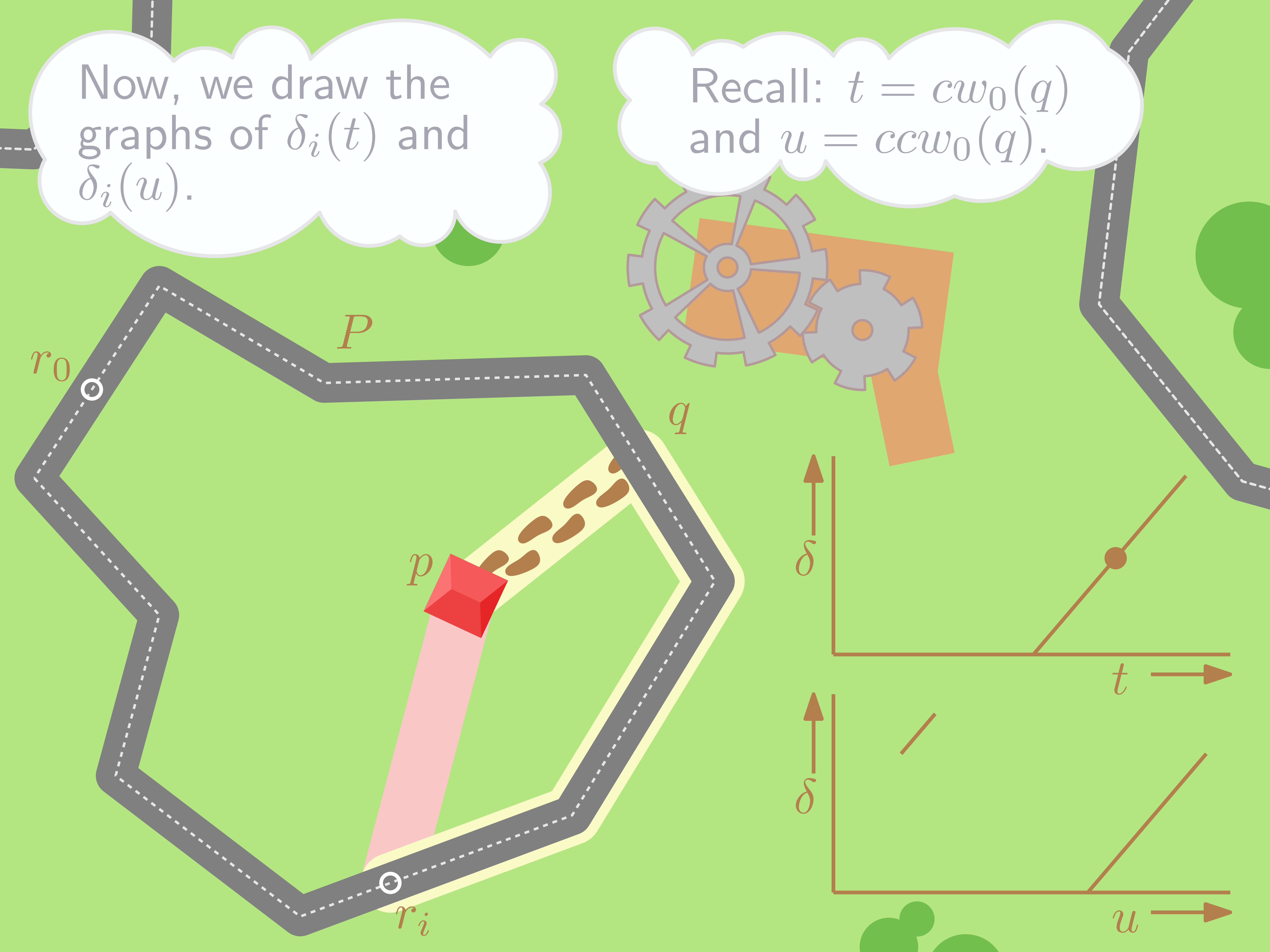
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
r_0
P
q
p
r_i
δ
t
δ
u



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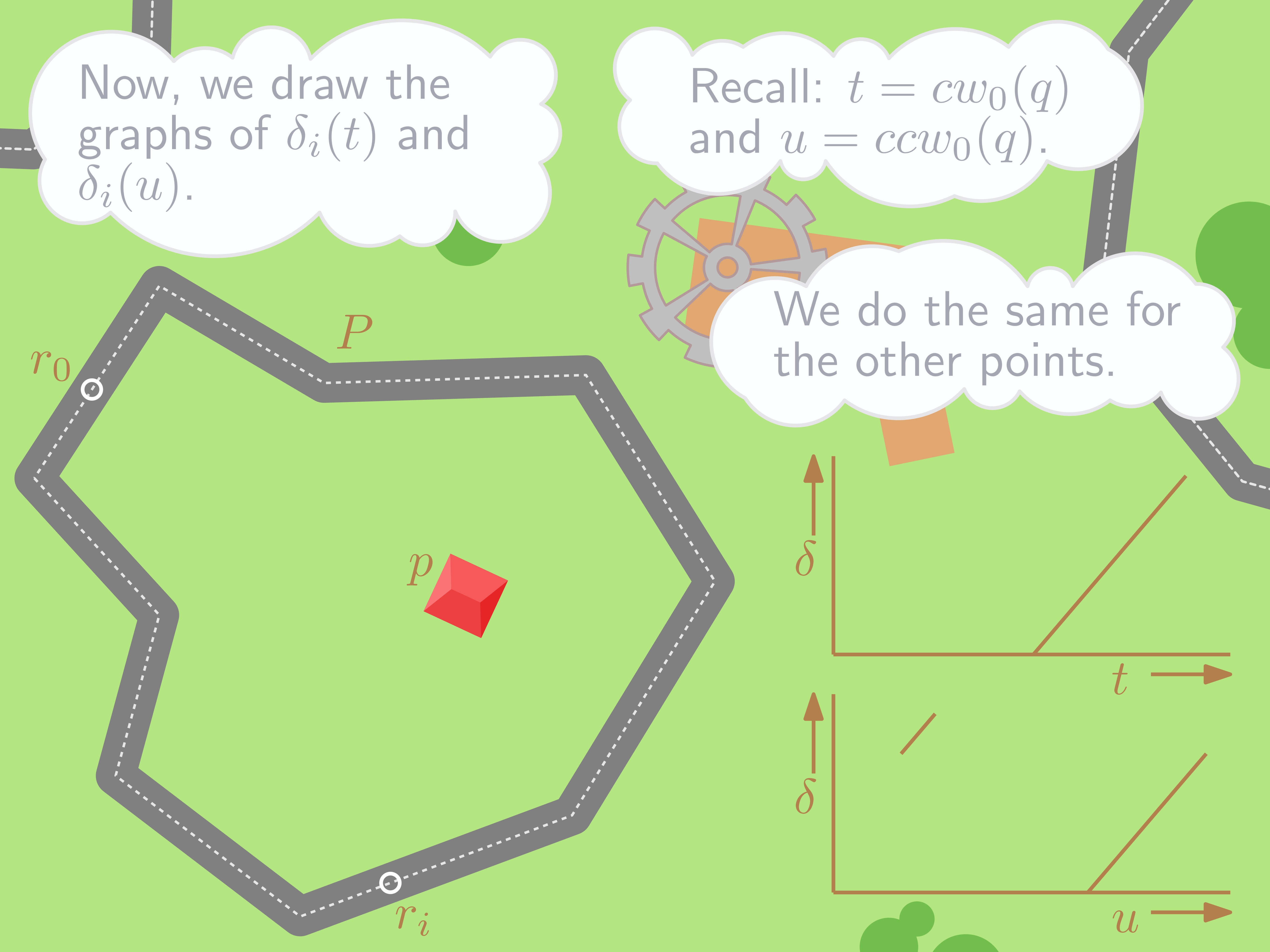
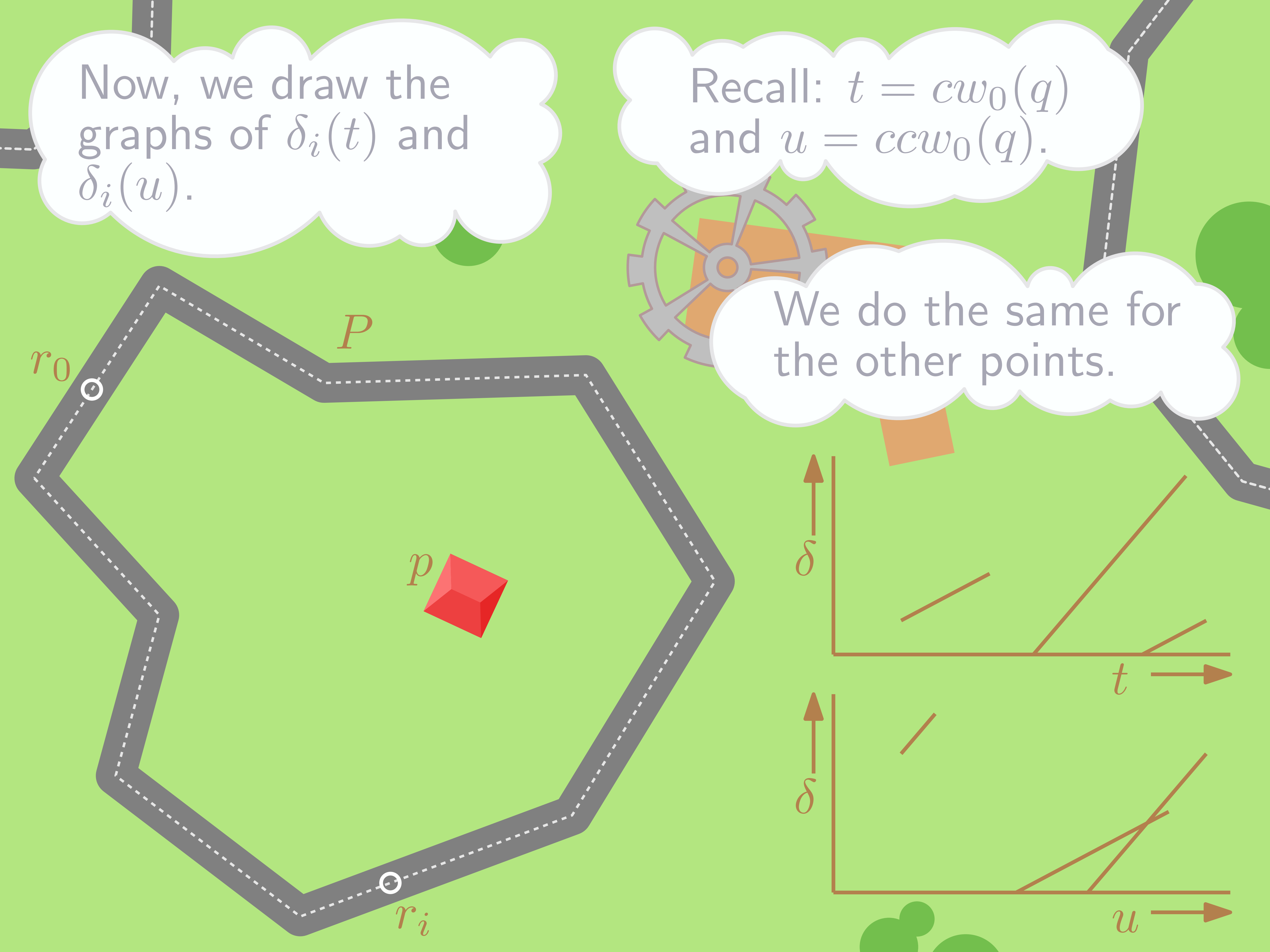
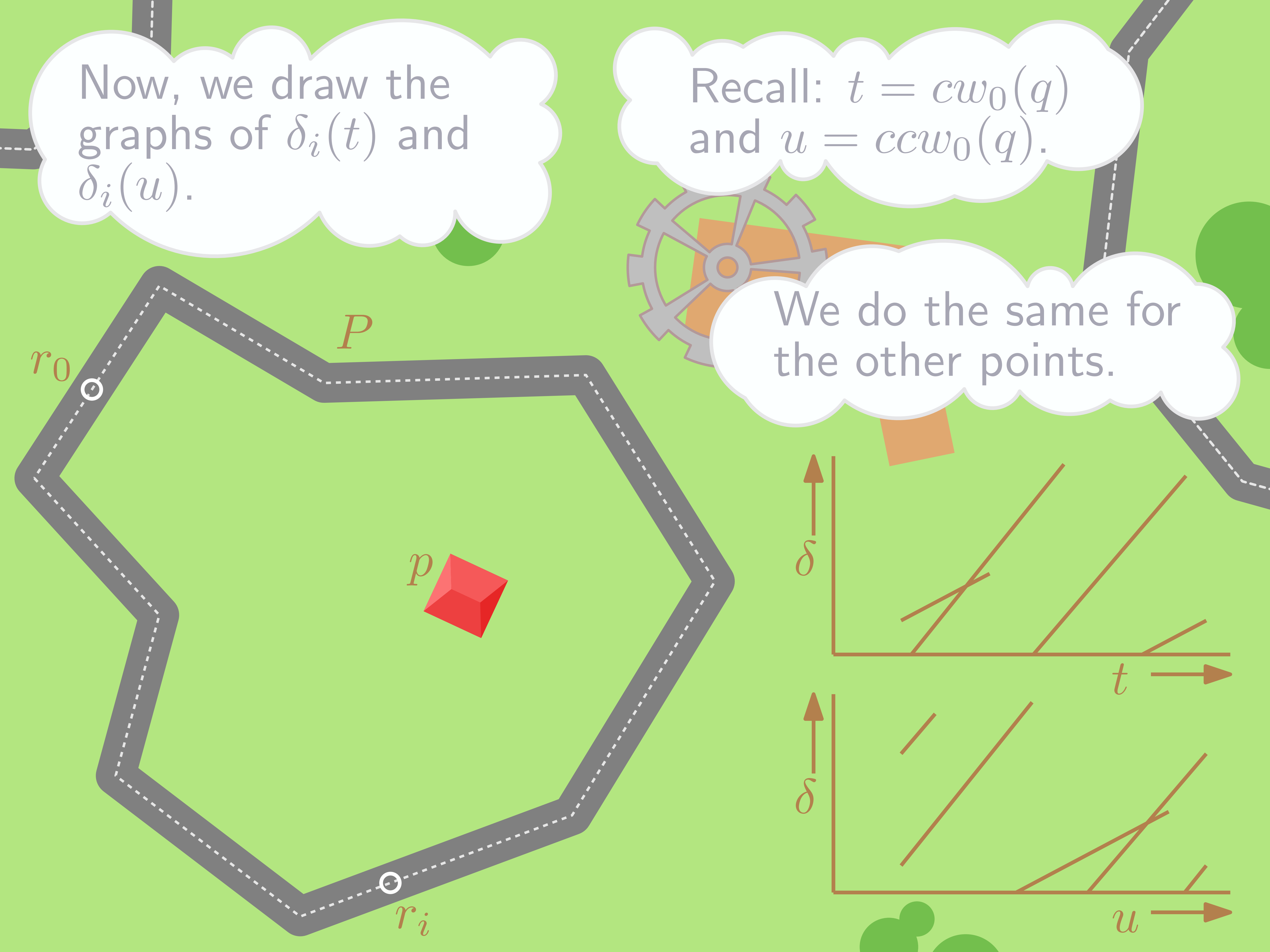
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
P
r_0
q
p
r_i
δ
t
δ
u


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
We do the same for the other points.
P
r_0
p
r_i
δ
t
δ
u



Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
We do the same for the other points.
P
r_0
p
r_i
δ
t
δ
u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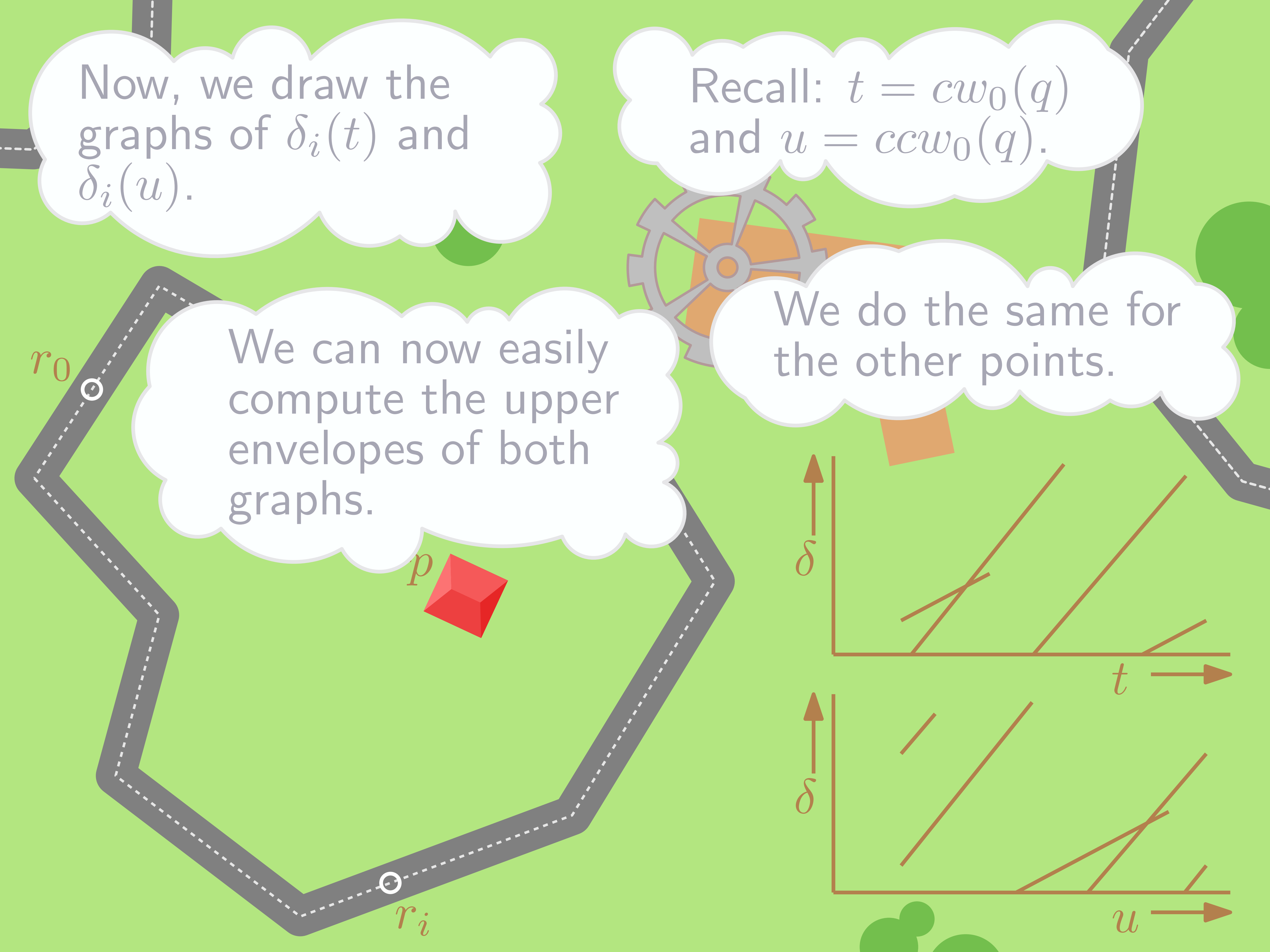
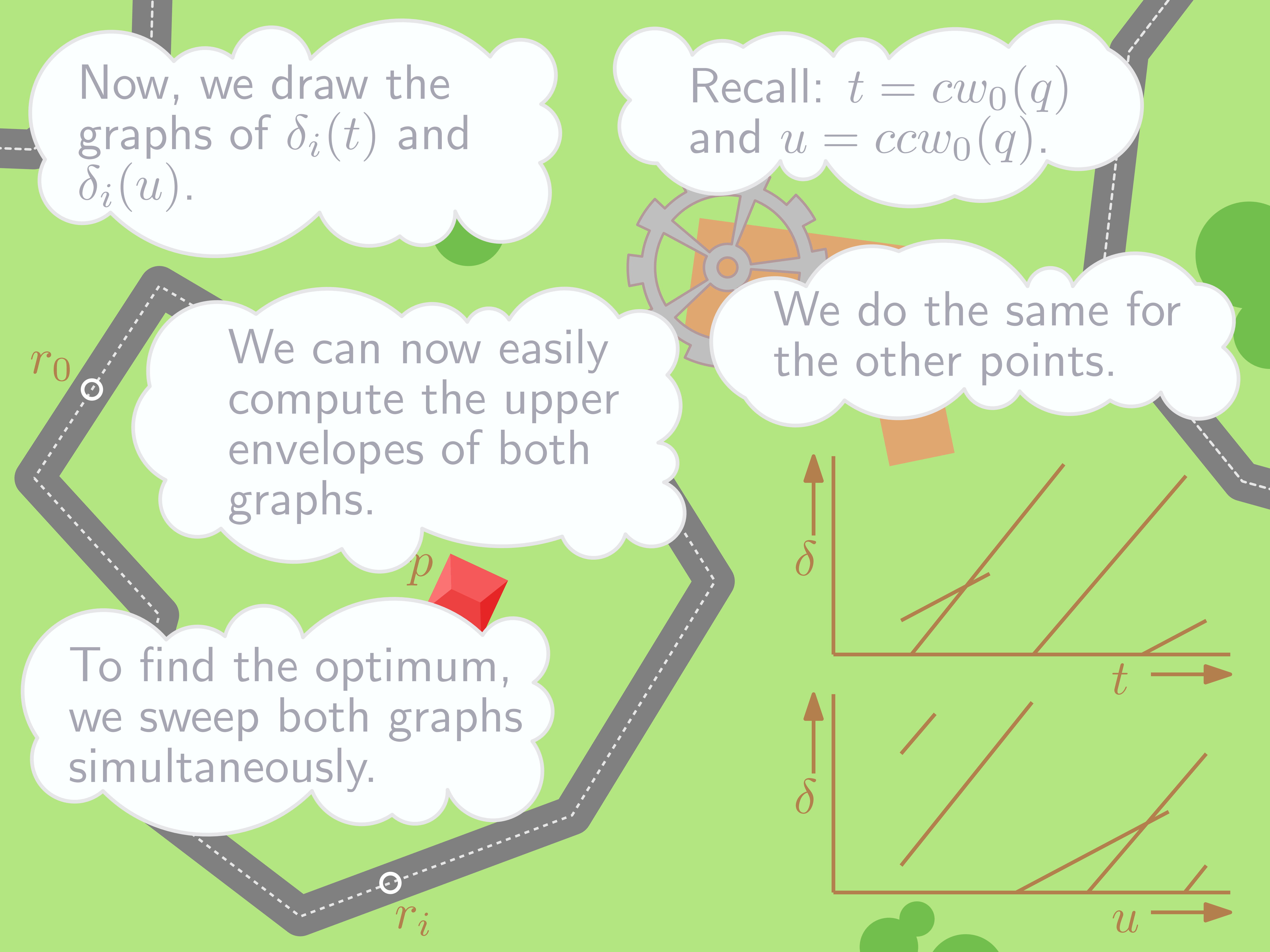
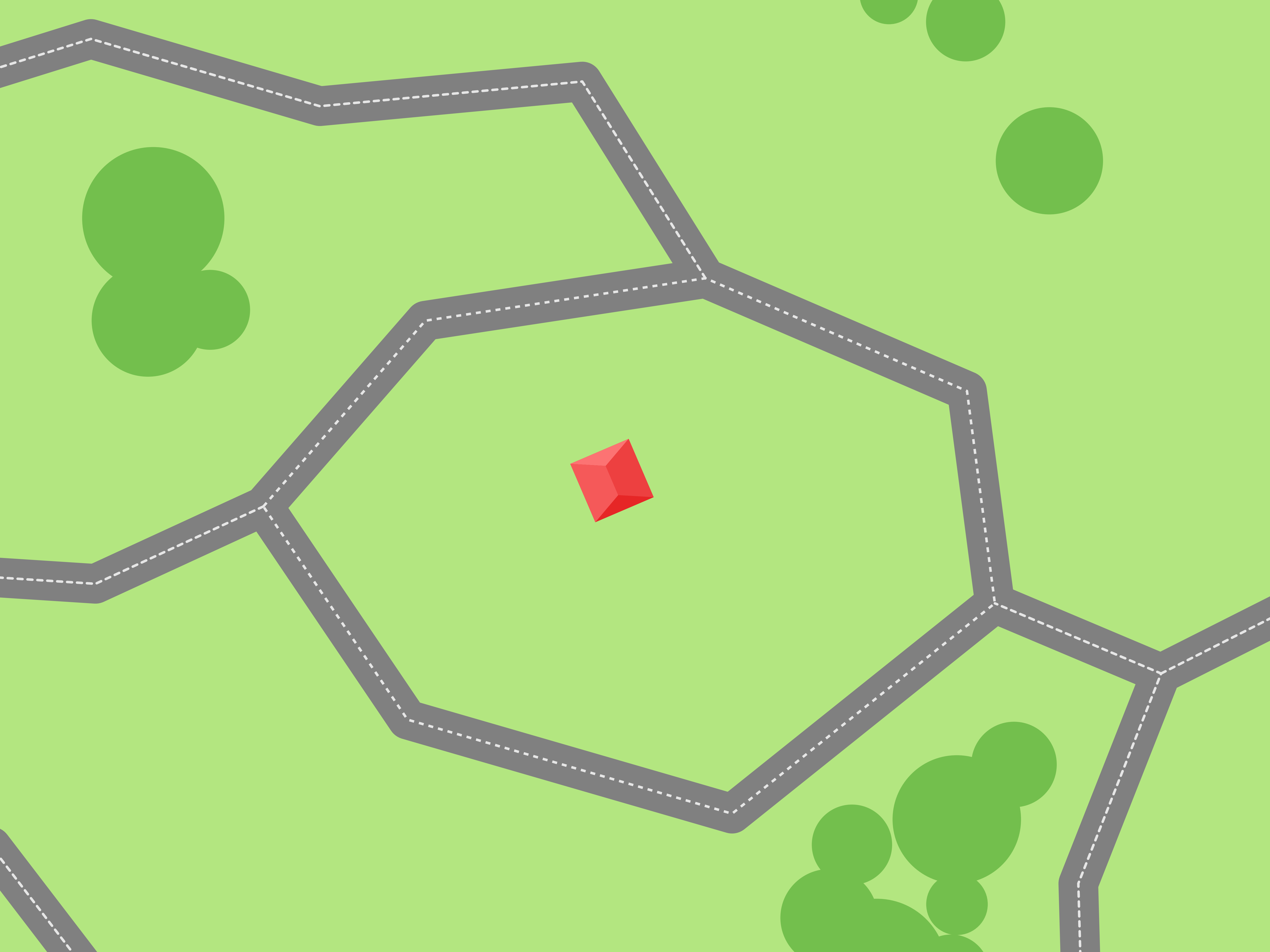
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
We do the same for the other points.
P
r_0
p
r_i
δ
t
δ
u


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
We can now easily compute the upper envelopes of both graphs.
We do the same for the other points.
r_0
p
r_i
δ
t
δ
u


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.
Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.
We can now easily compute the upper envelopes of both graphs.
We do the same for the other points.
To find the optimum, we sweep both graphs simultaneously.
r_0
p
r_i
δ
t
δ
u

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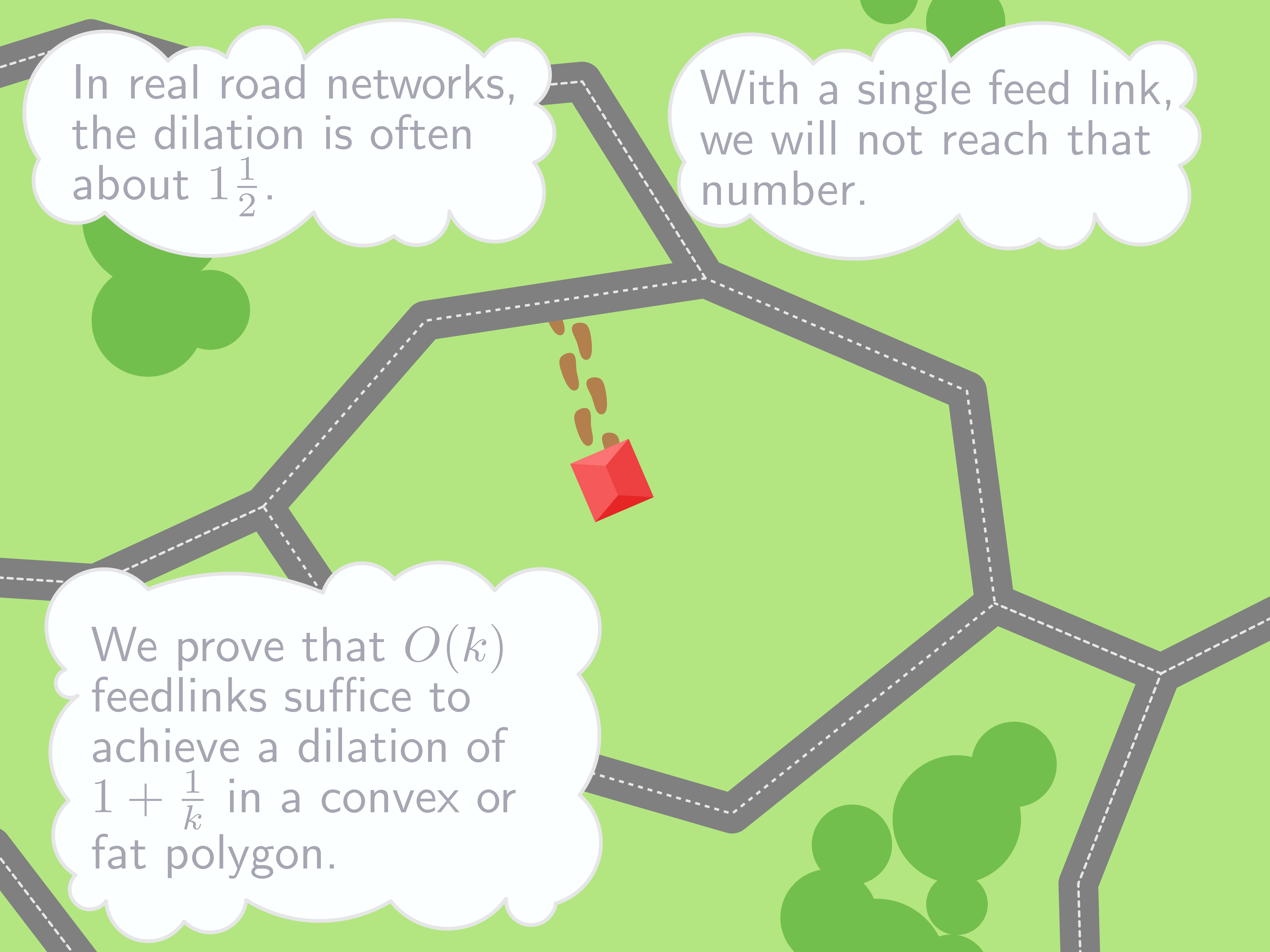
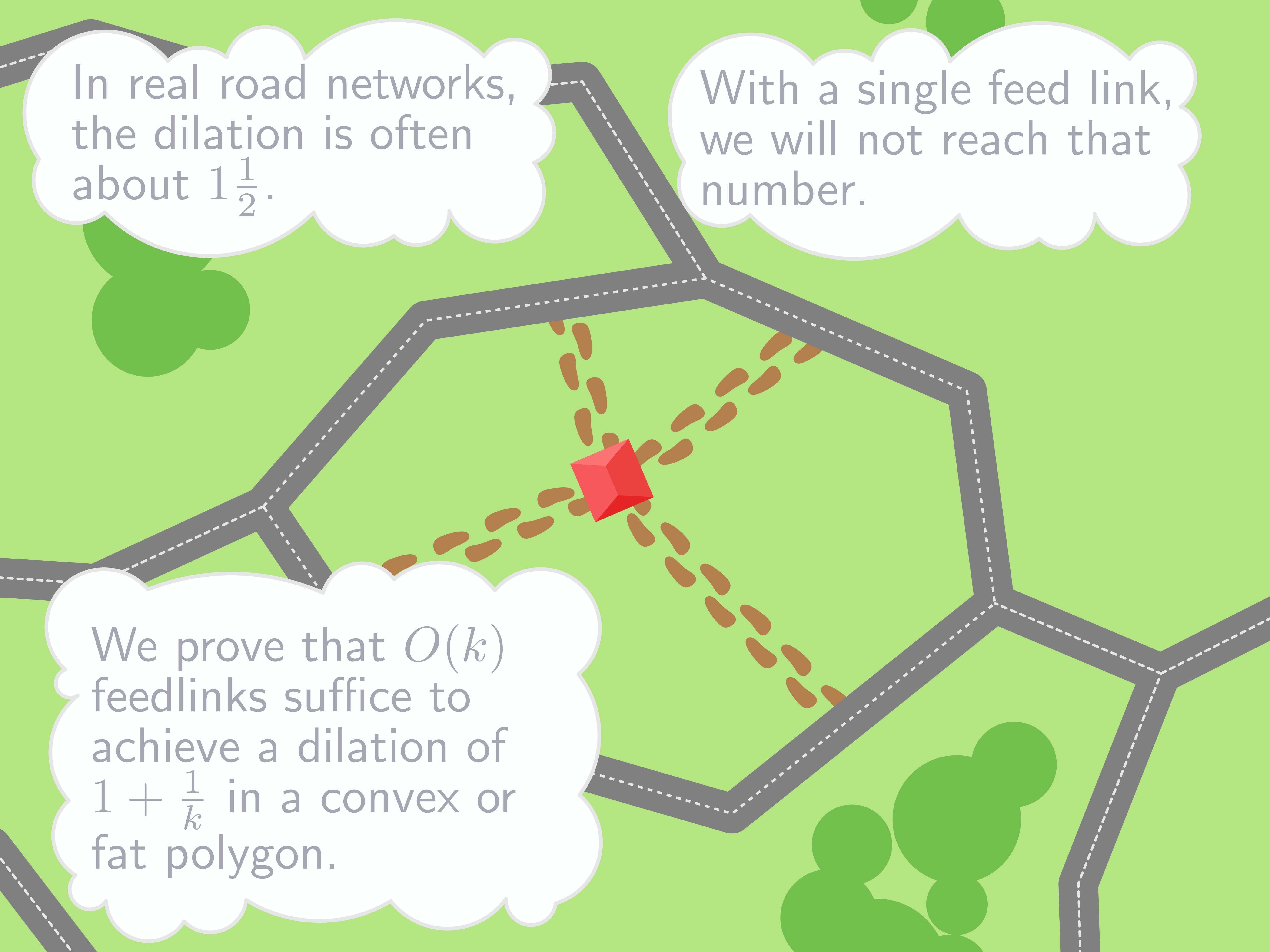

In real road networks, the dilation is often about $1\frac{1}{2}$.
With a single feed link, we will not reach that number.
We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1 + \frac{1}{k}$ in a convex or fat polygon.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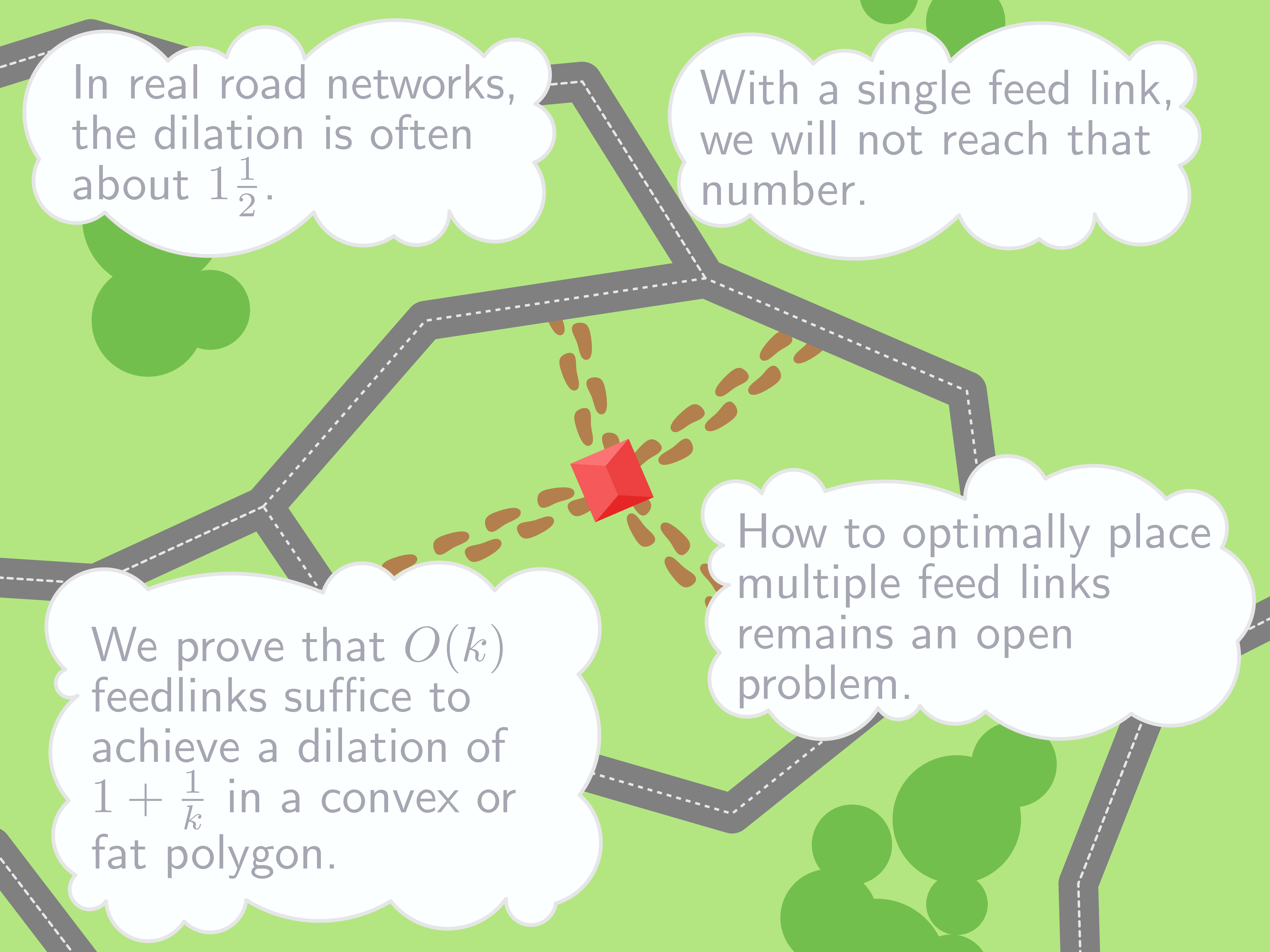
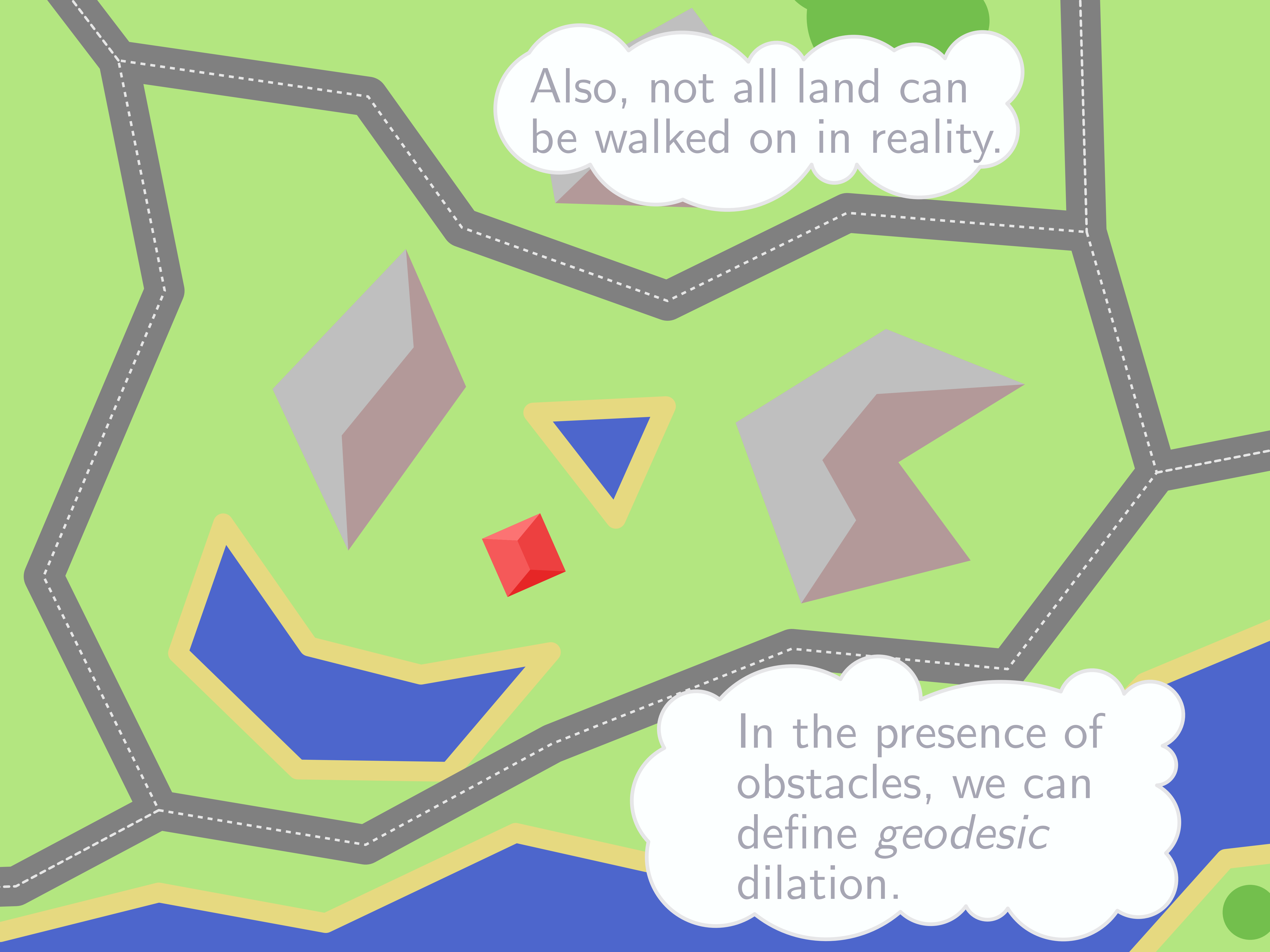
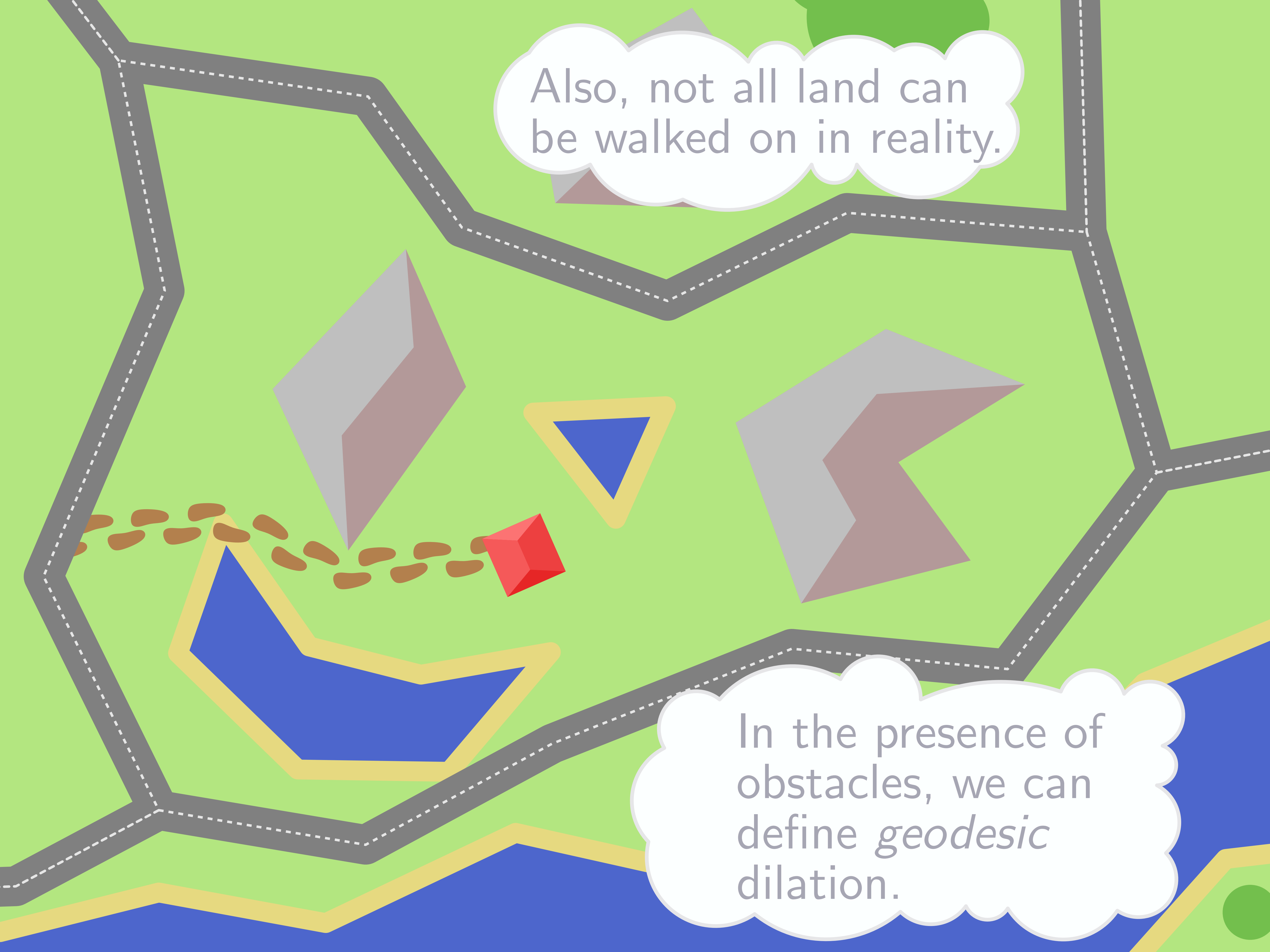

In real road networks, the dilation is often about $1\frac{1}{2}$.
With a single feed link, we will not reach that number.
We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1+\frac{1}{k}$ in a convex or fat polygon.
How to optimally place multiple feed links remains an open problem.


Also, not all land can be walked on in reality.


Also, not all land can be walked on in reality.
In the presence of obstacles, we can define *geodesic* dilation.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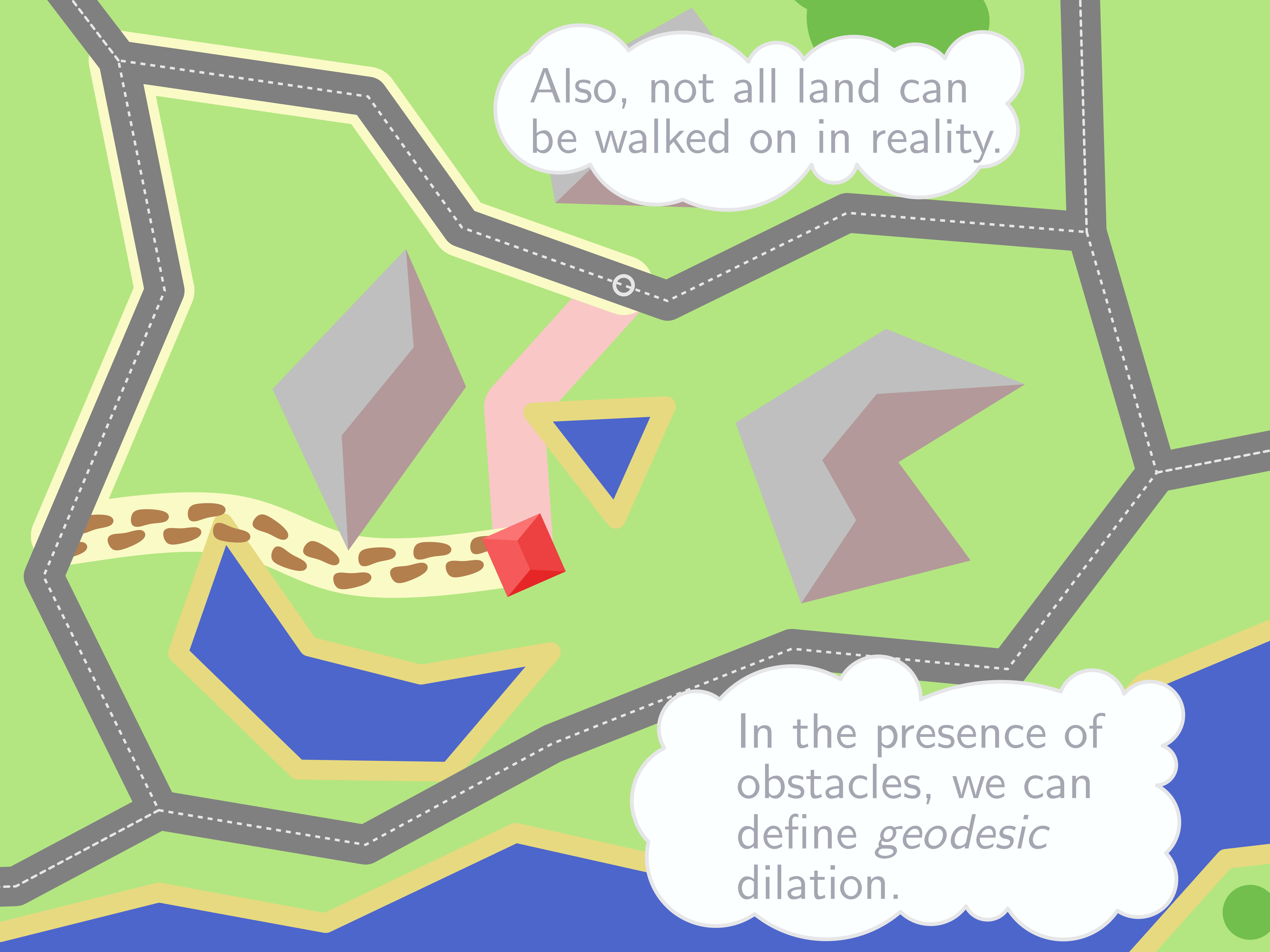
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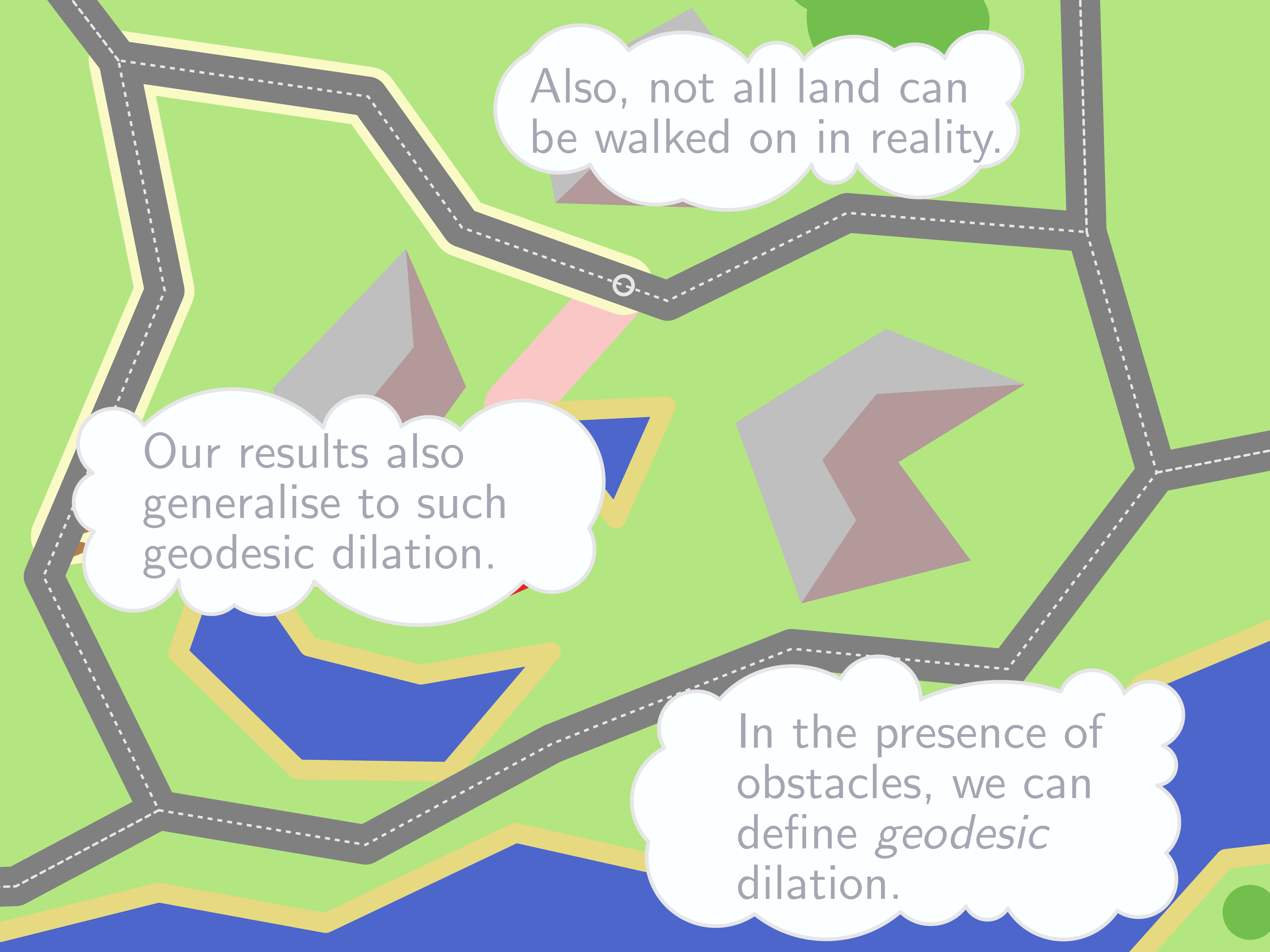
Also, not all land can be walked on in reality.
Our results also generalise to such geodesic dilation.
In the presence of obstacles, we can define *geodesic* dilation.


Thank you!


Boris Aronov
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Kevin Buchin
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Rodrigo Silveira
Marc van Kreveld
Maarten Löffler