Geometric Measures on Imprecise Points in Higher Dimensions

Hein Kruger Maarten Löffler
Utrecht University

What is this about?

Data is often measured from the *real world*

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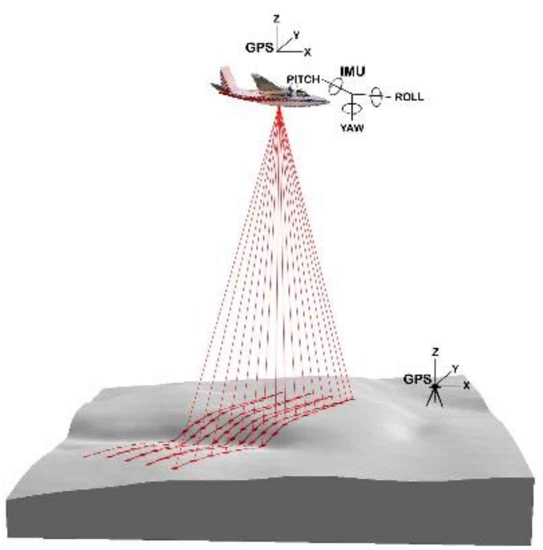
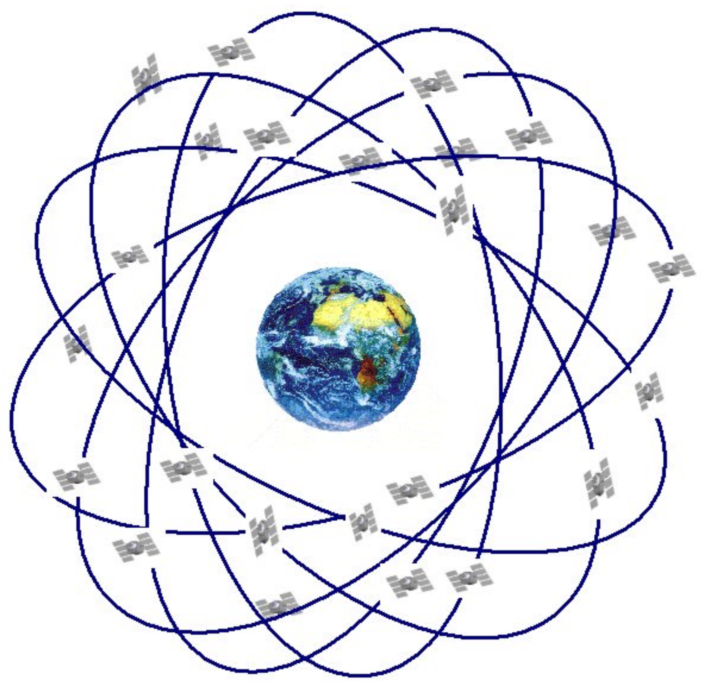


LIDAR



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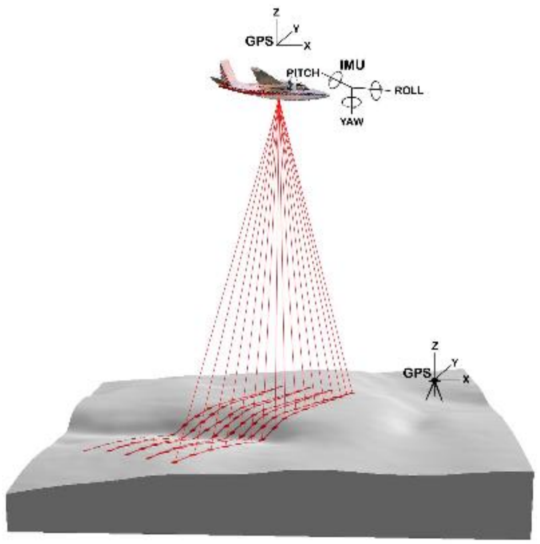

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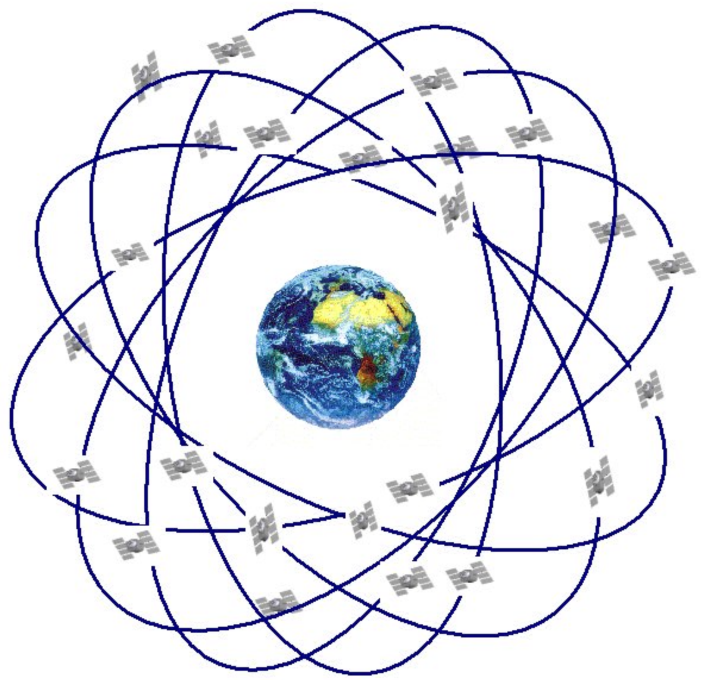

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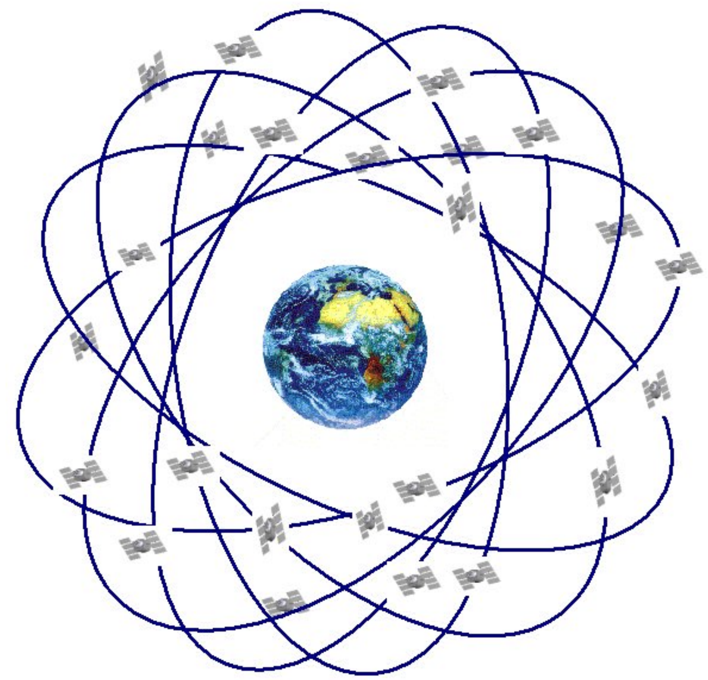

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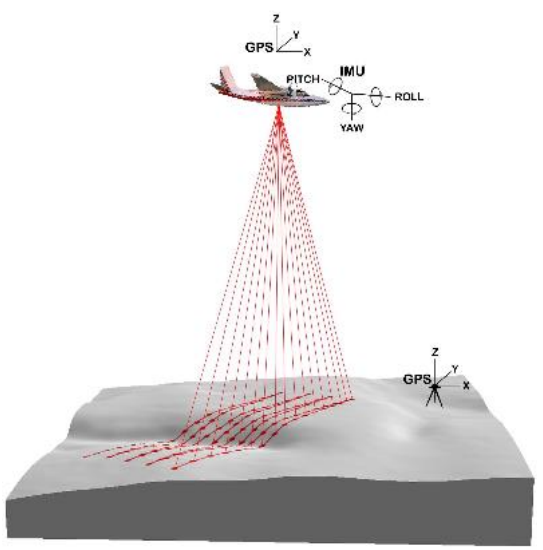

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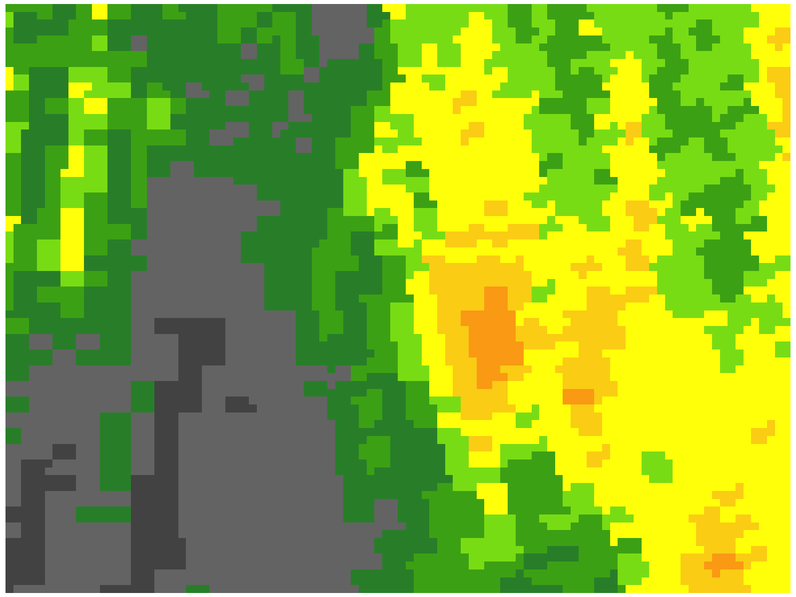

Grids

Let's define *imprecise points*.

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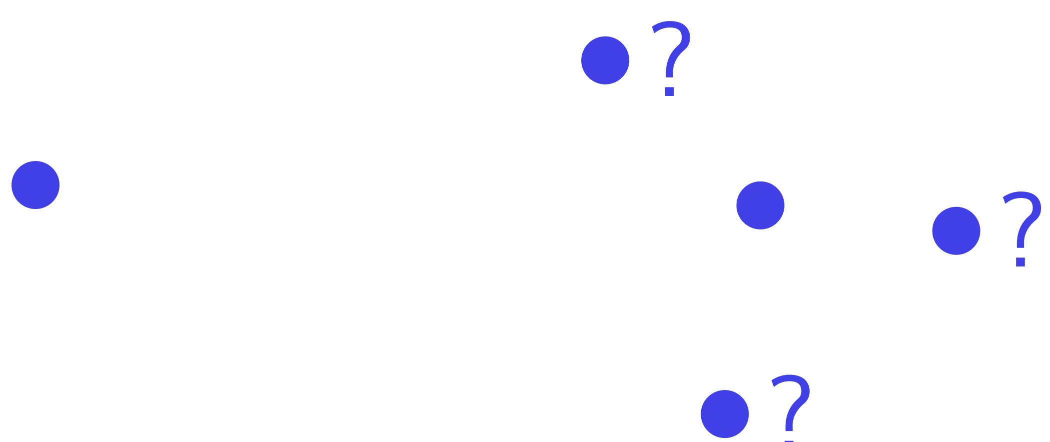

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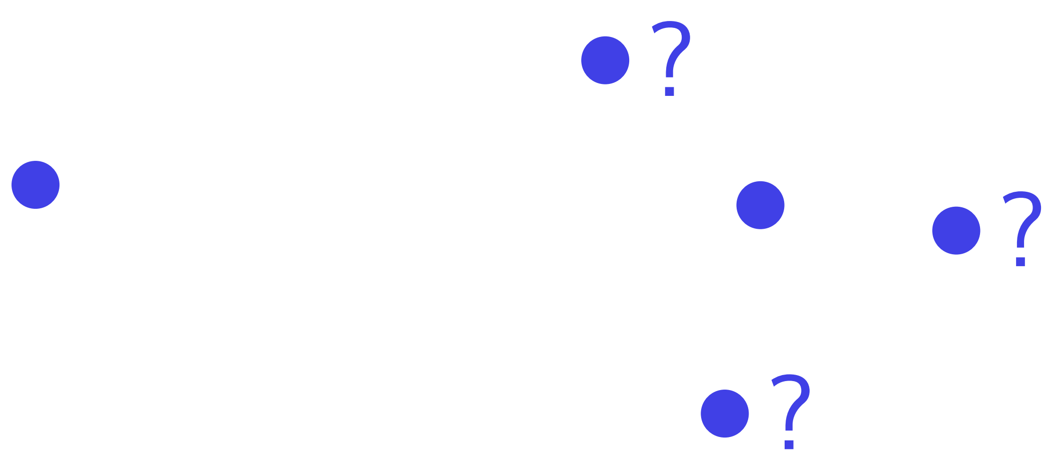

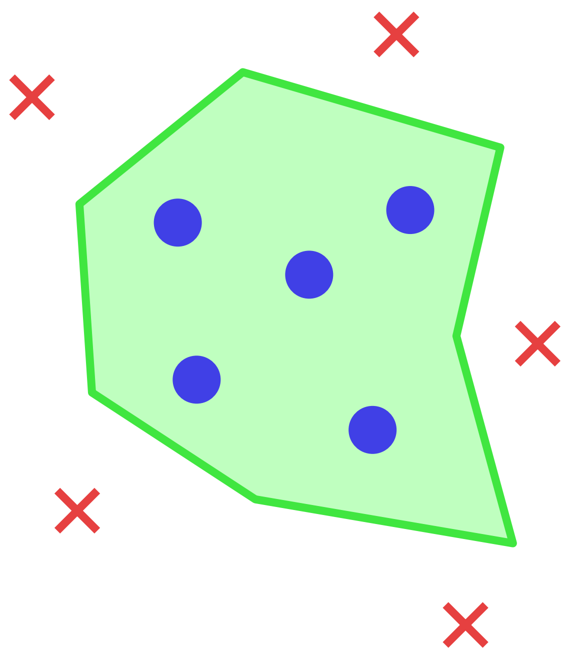

What can we do with imprecise data points?

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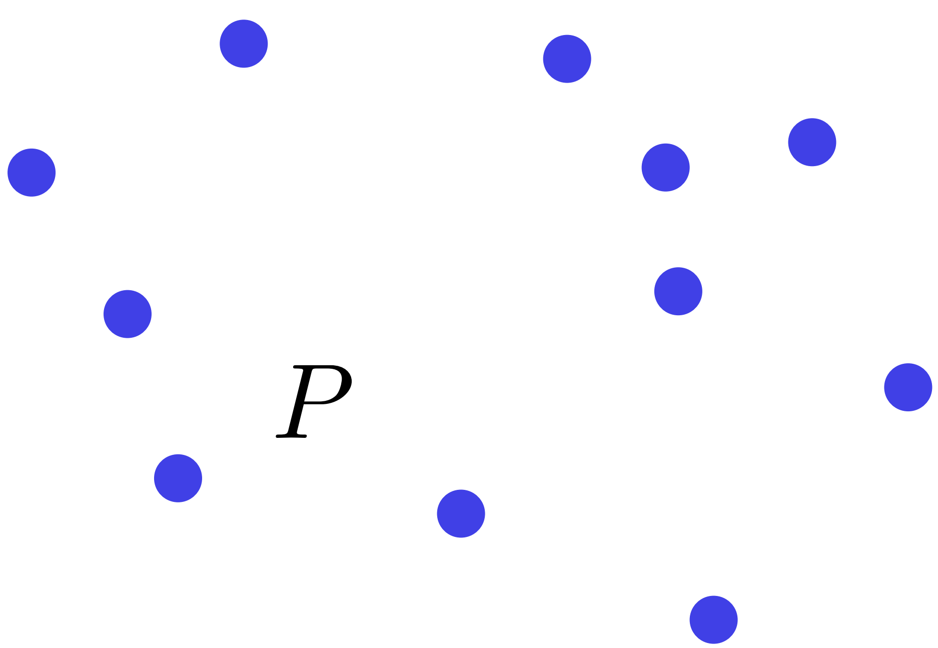

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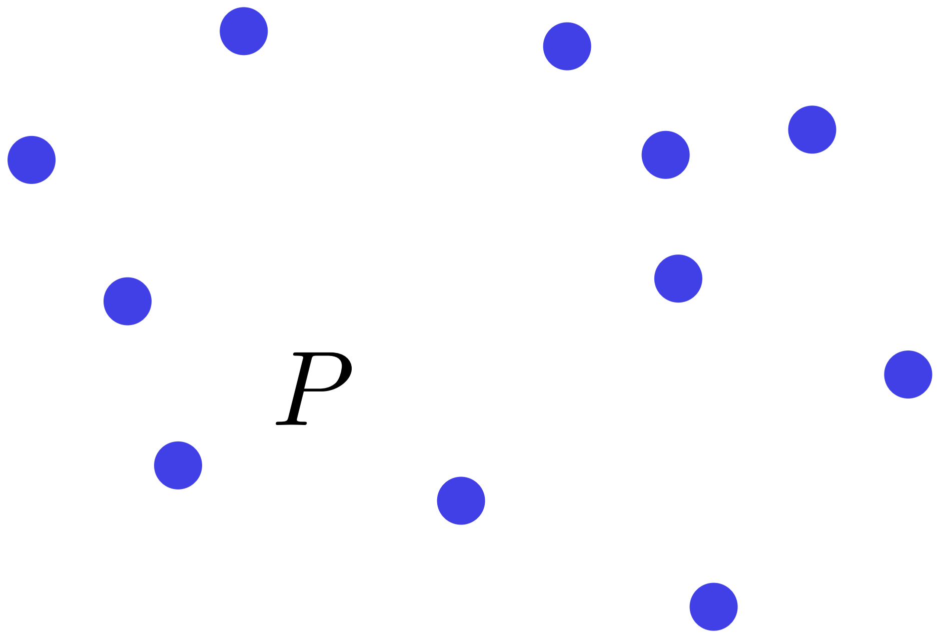

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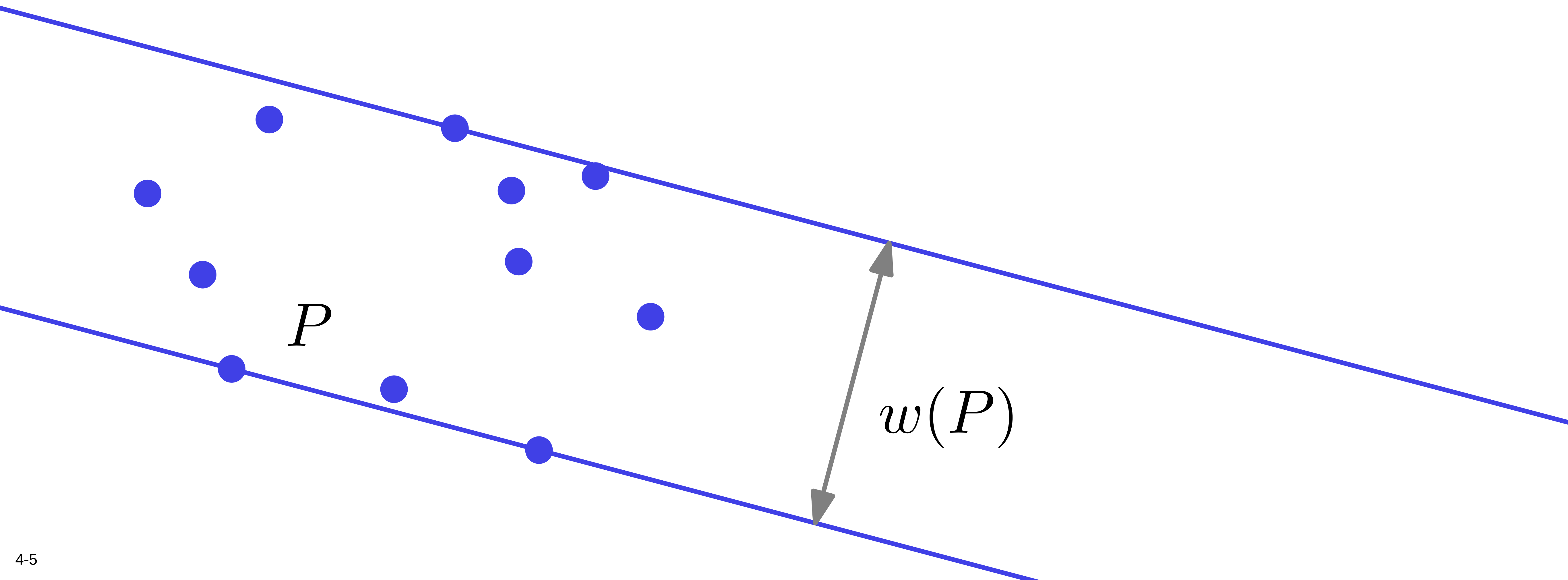

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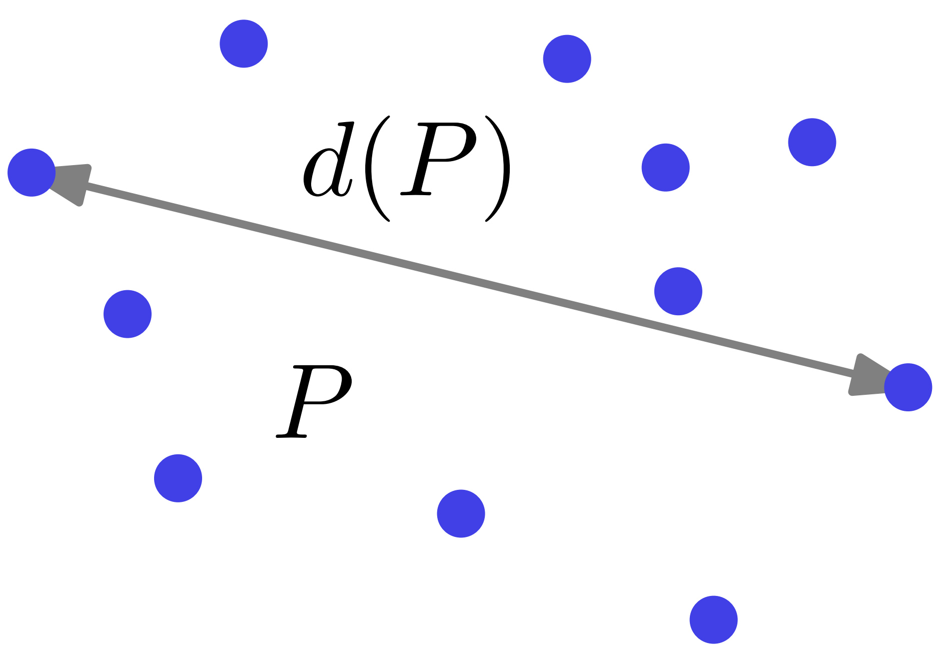

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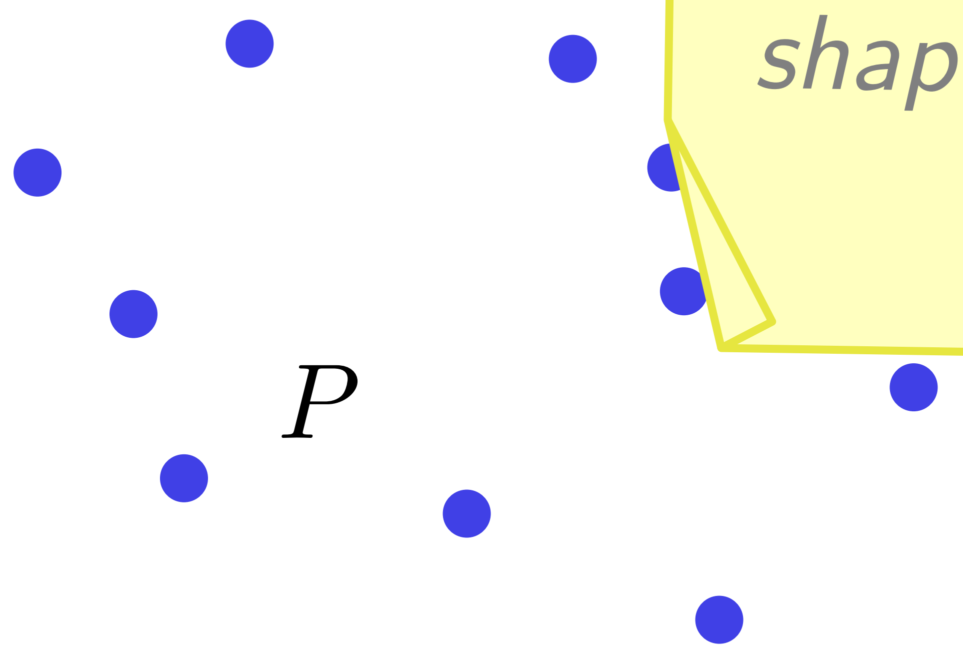

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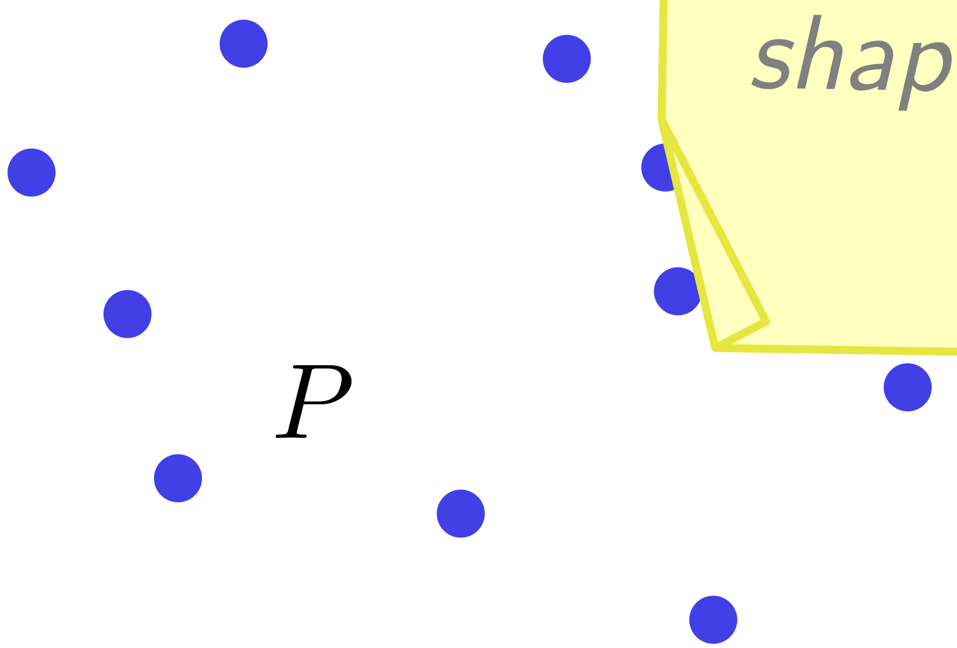

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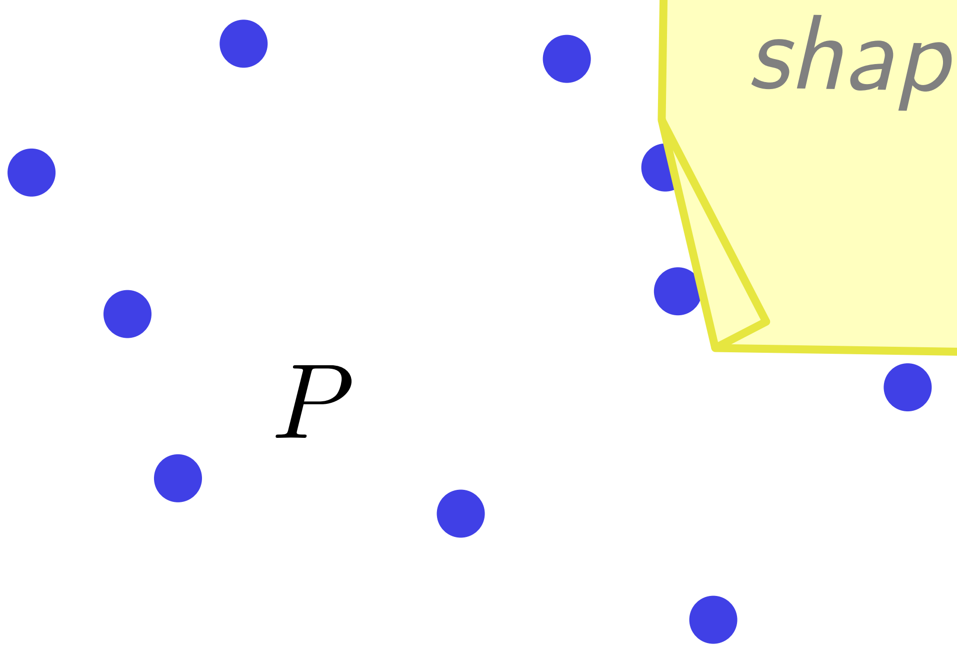

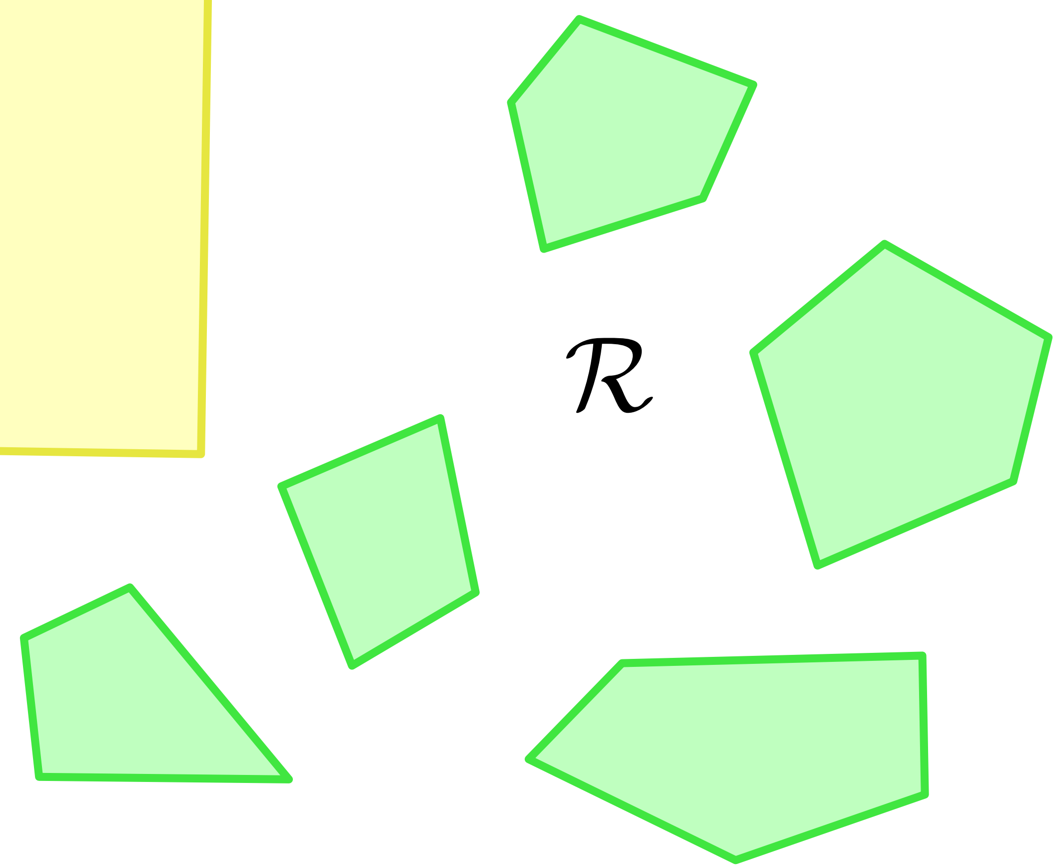

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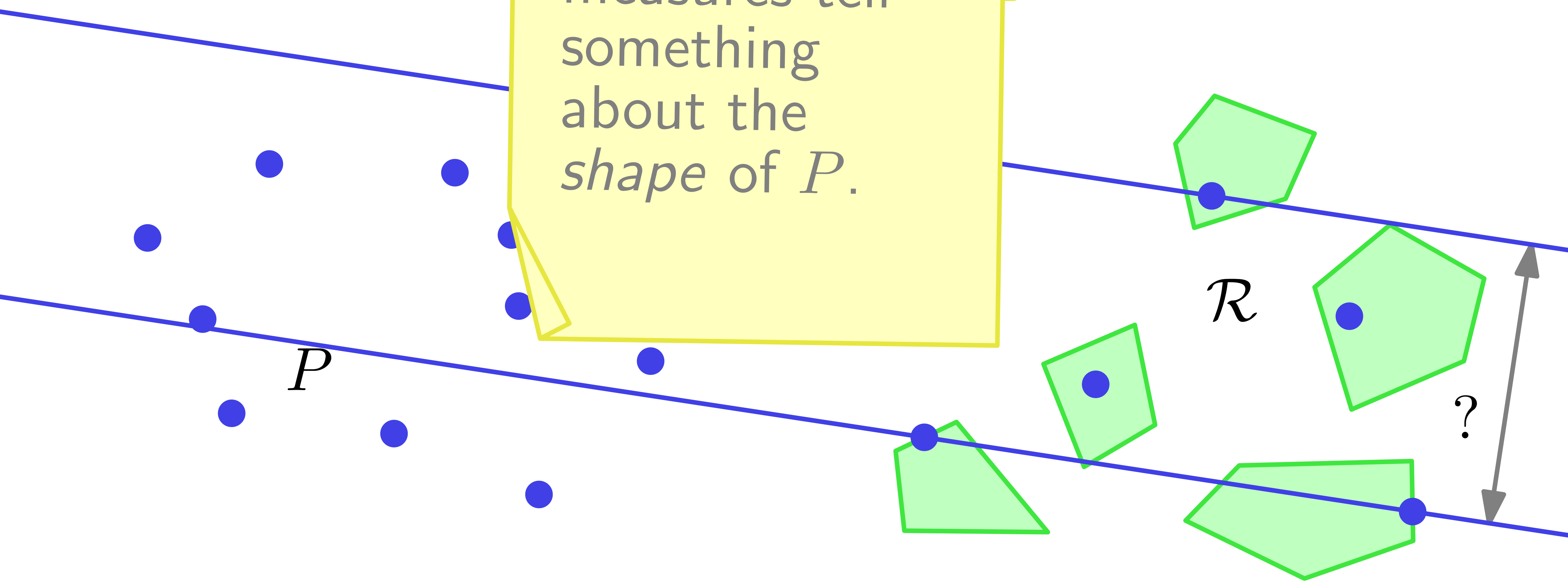

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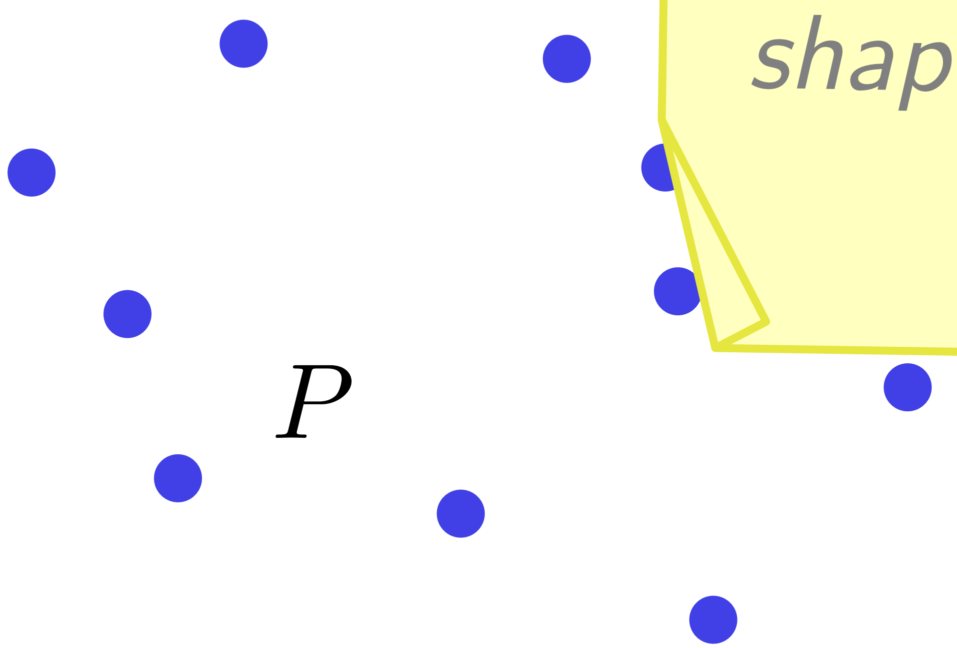

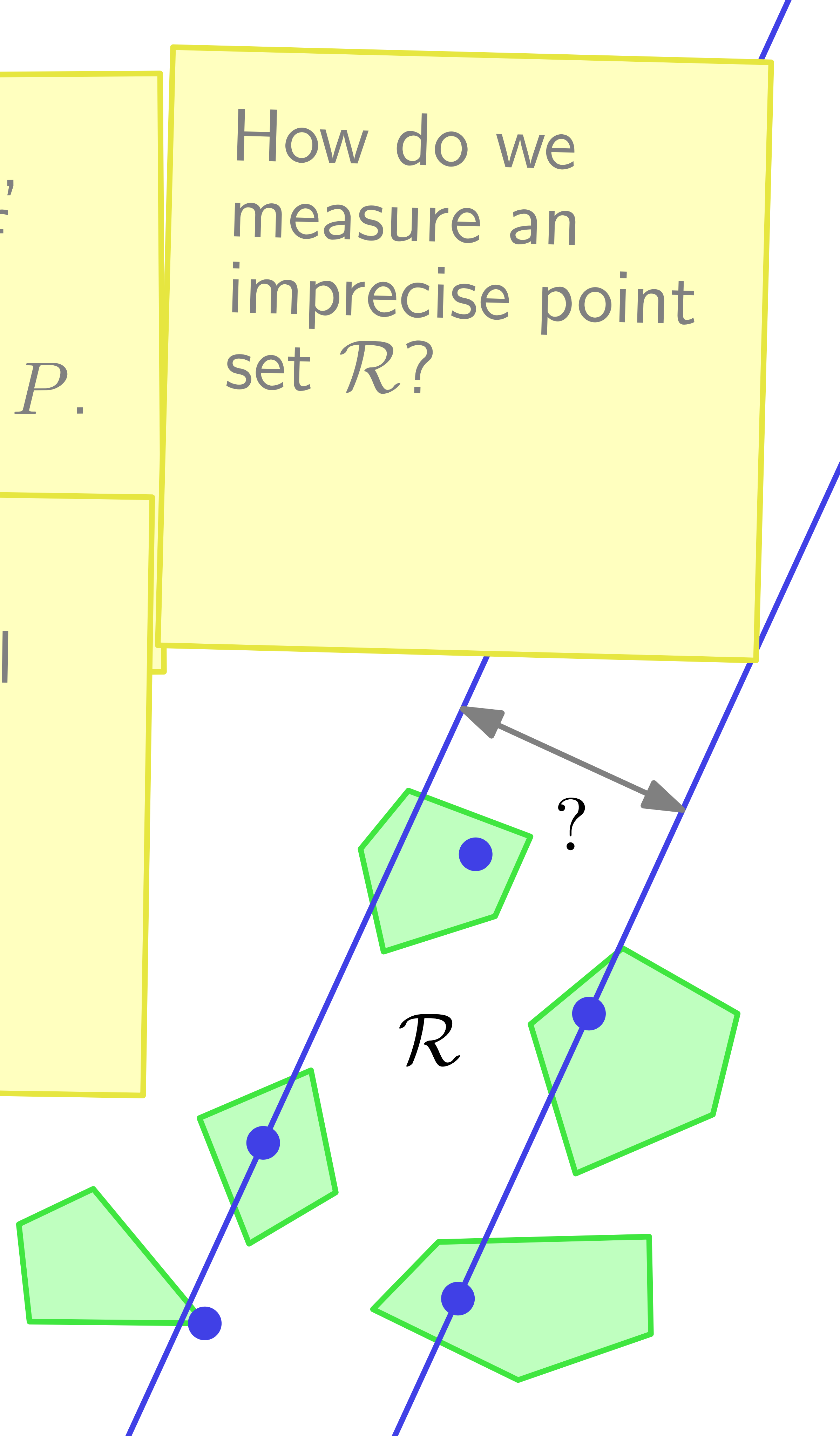

One possibility: compute *bounds* on the possible value.

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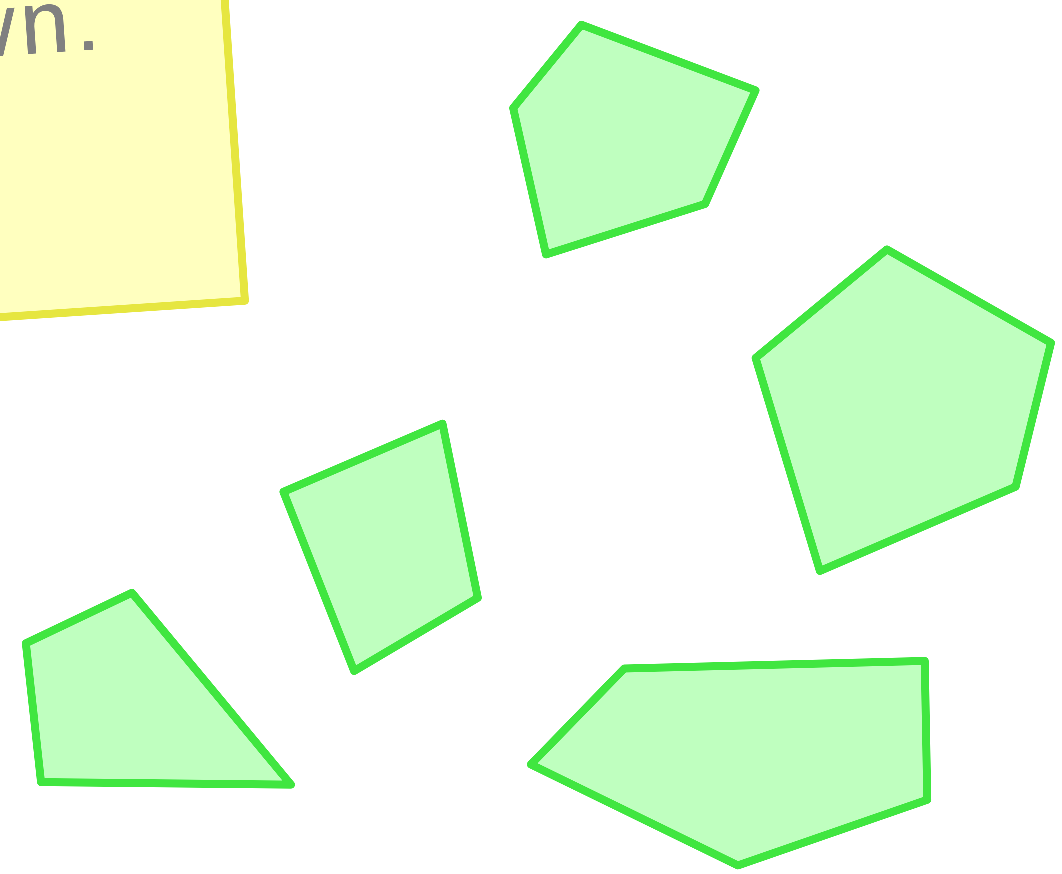

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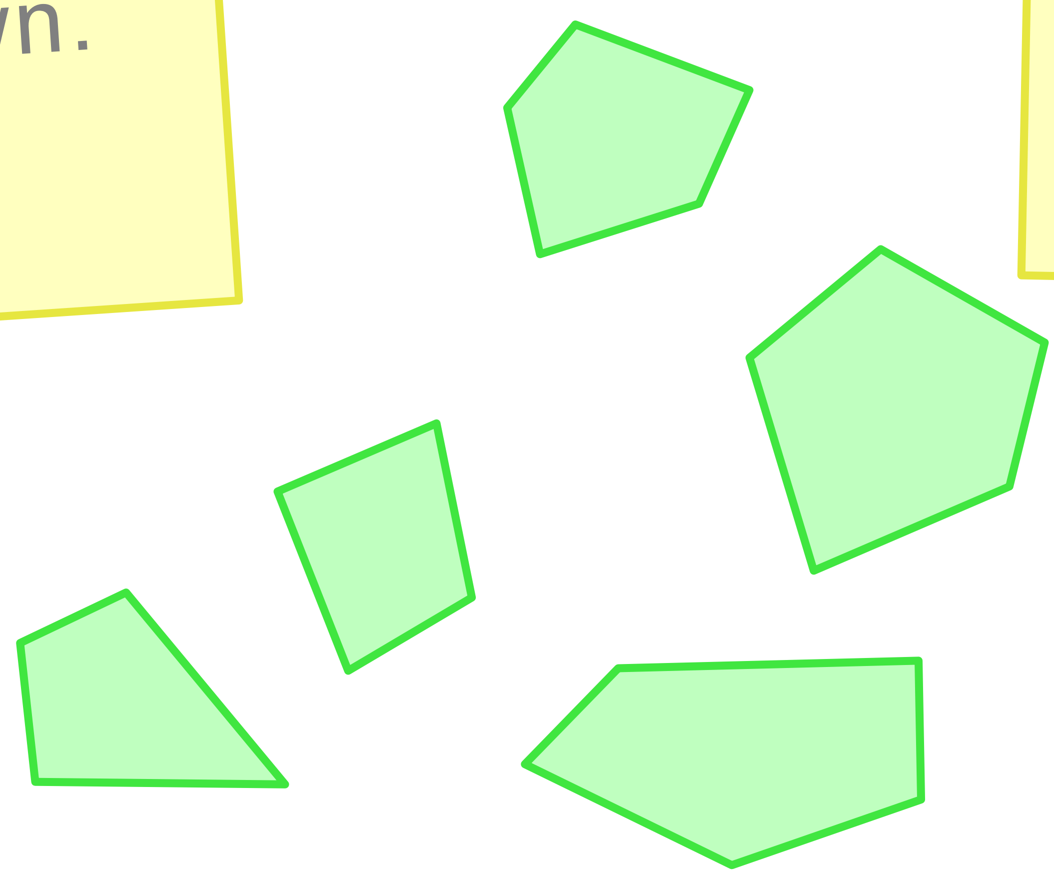
We want to compute the *smallest* possible value of a measure $\mu(P)$.



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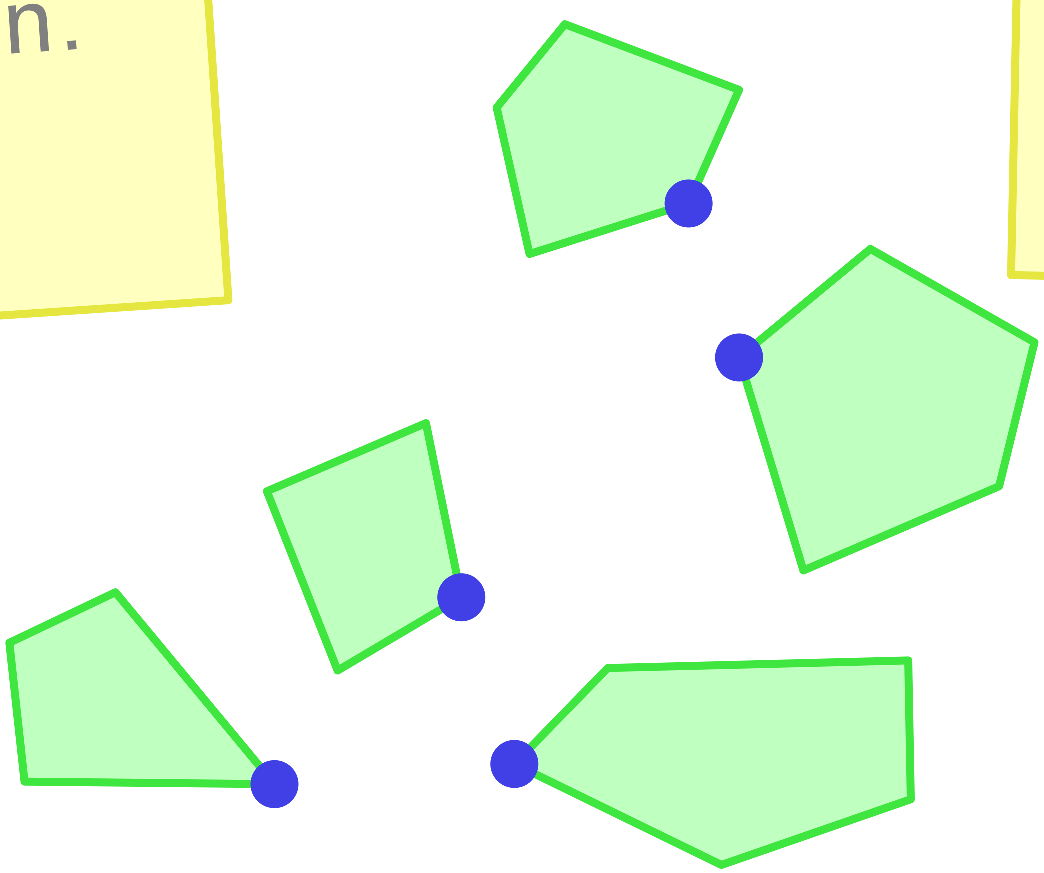

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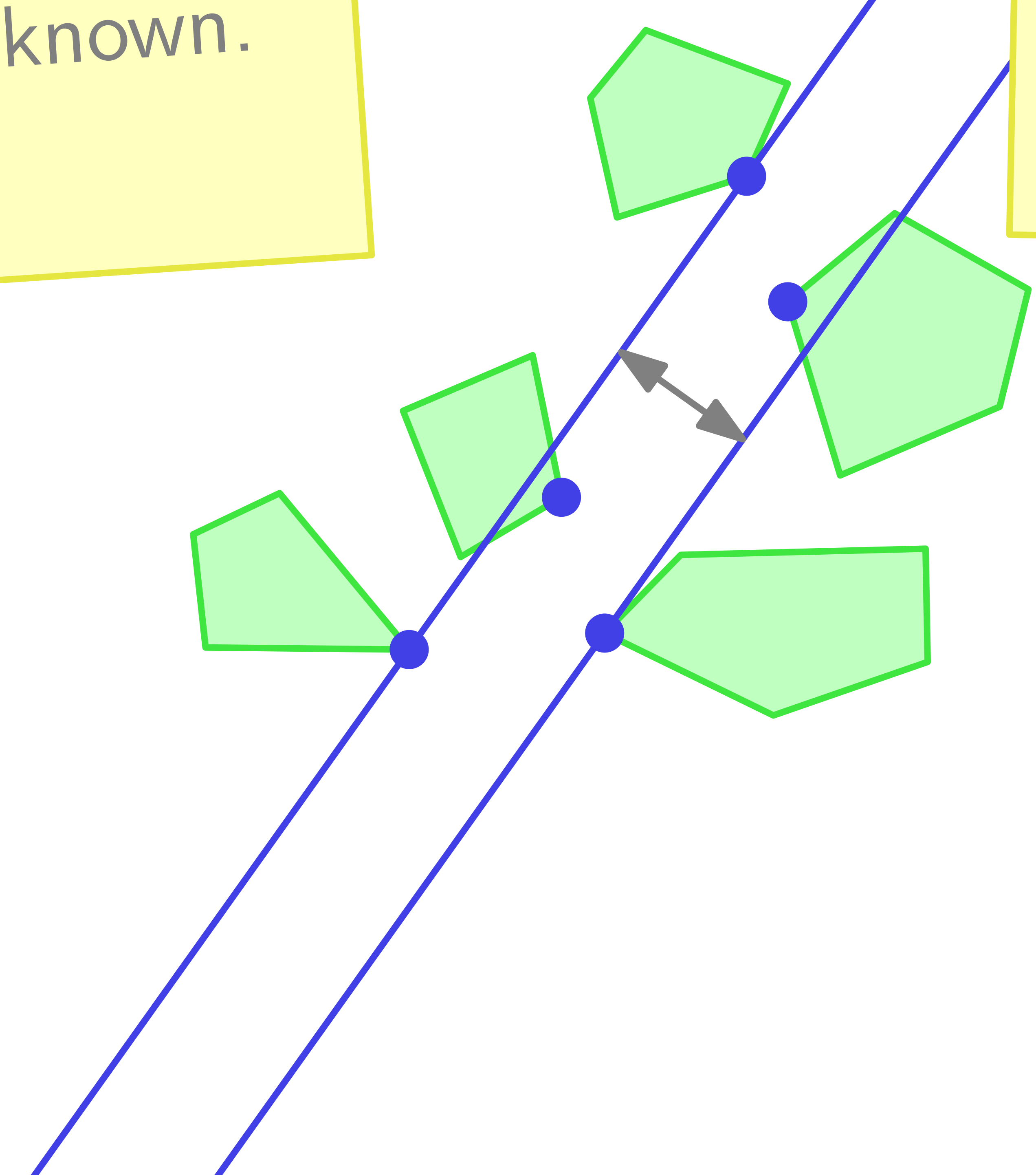

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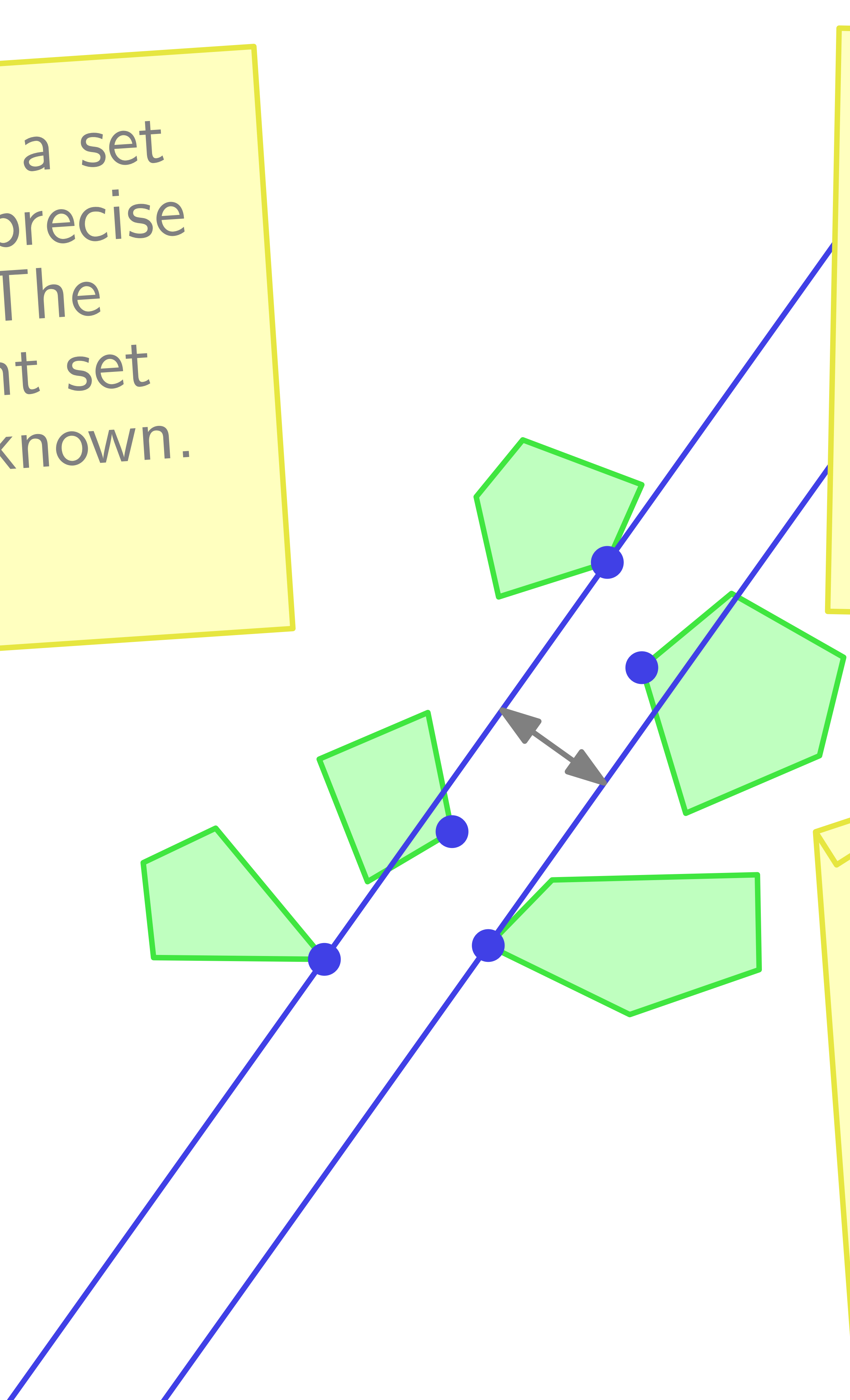

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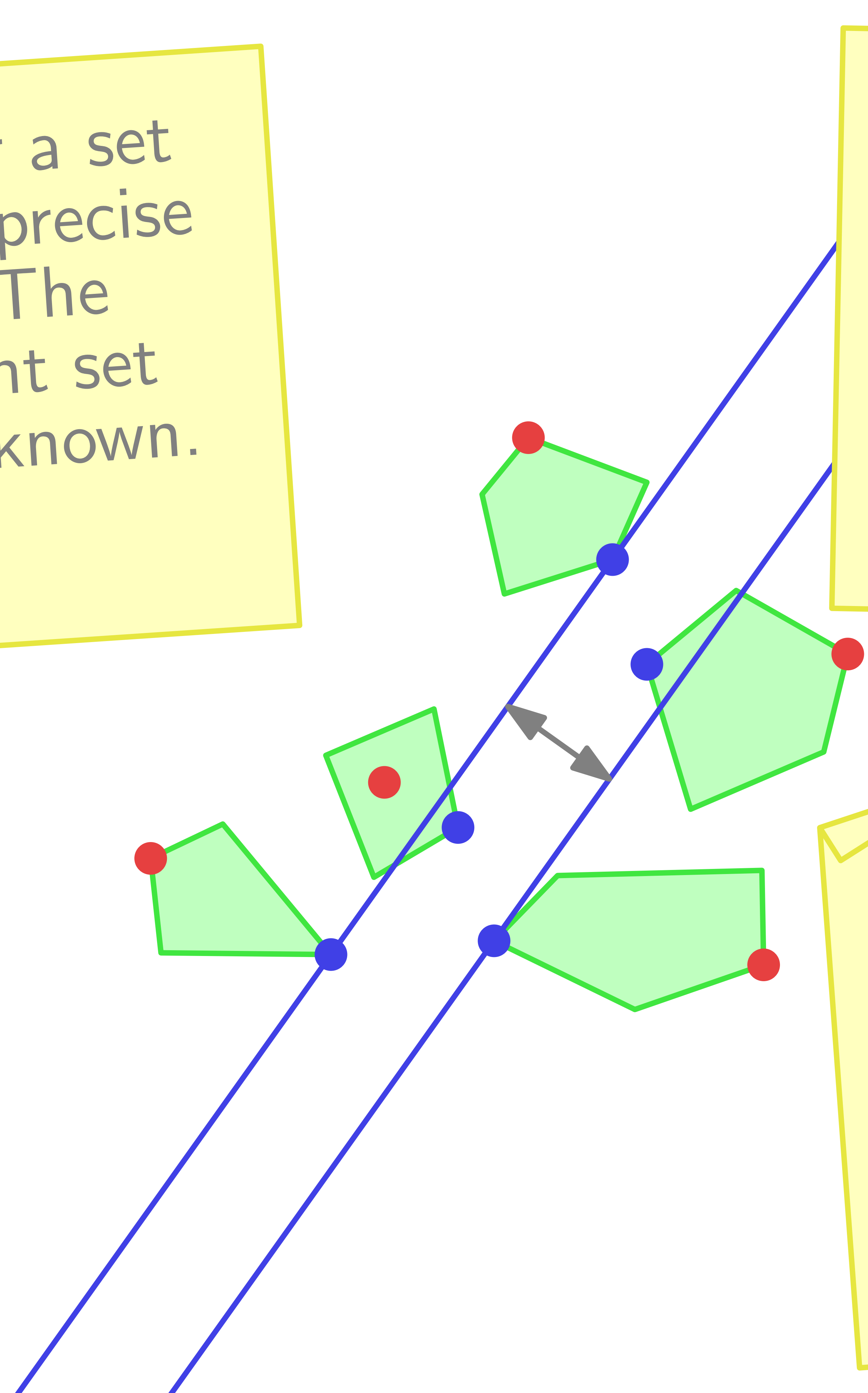

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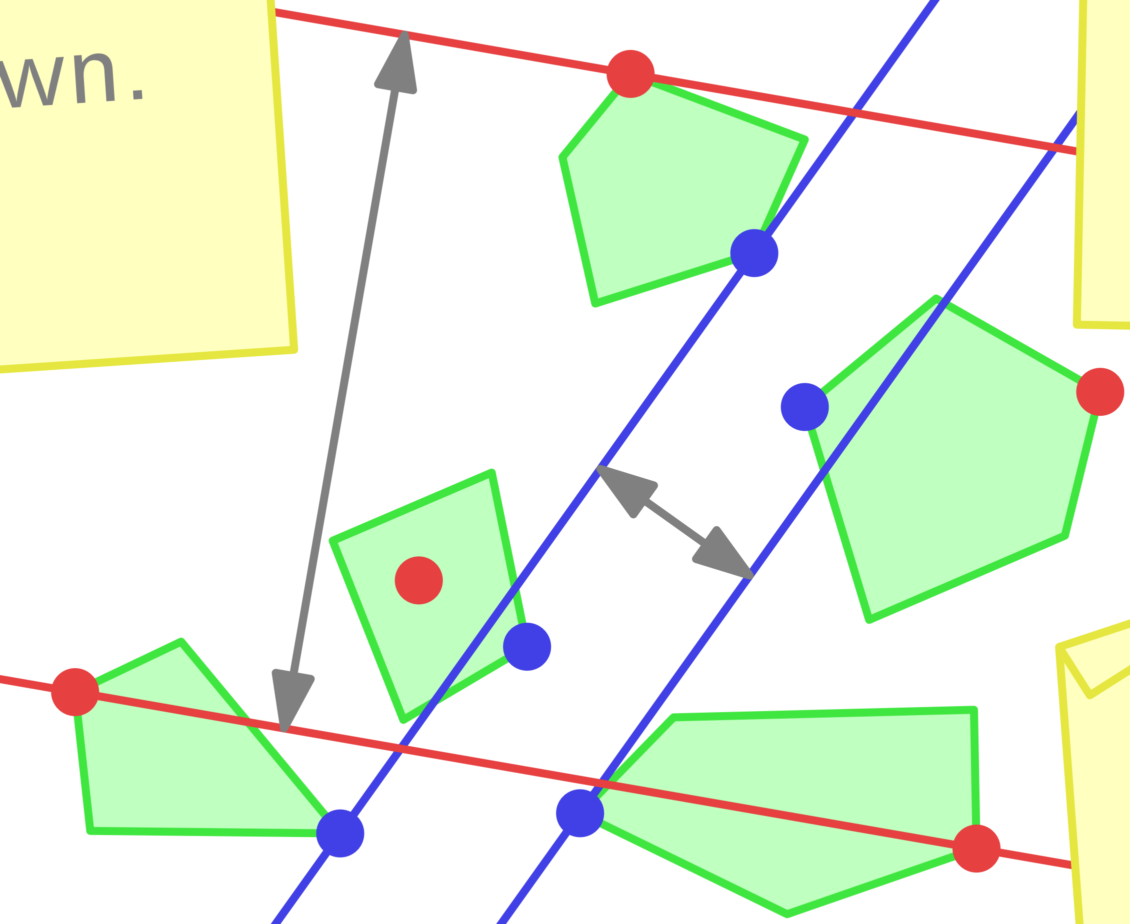

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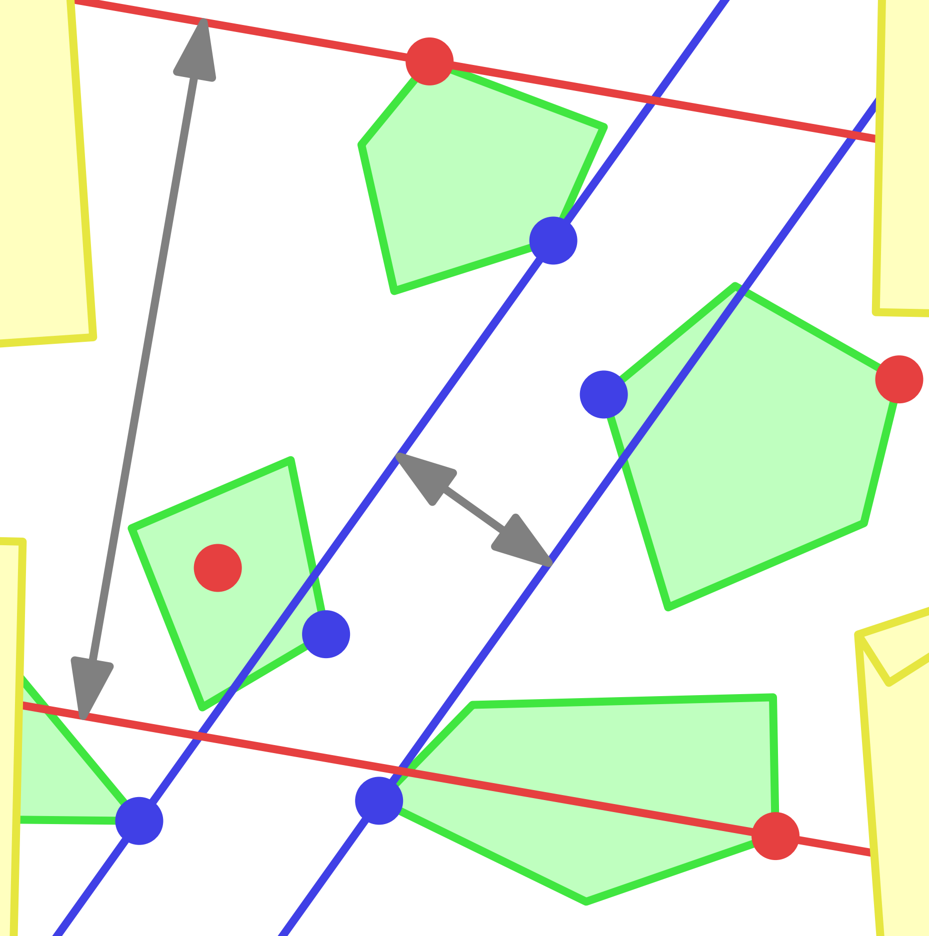

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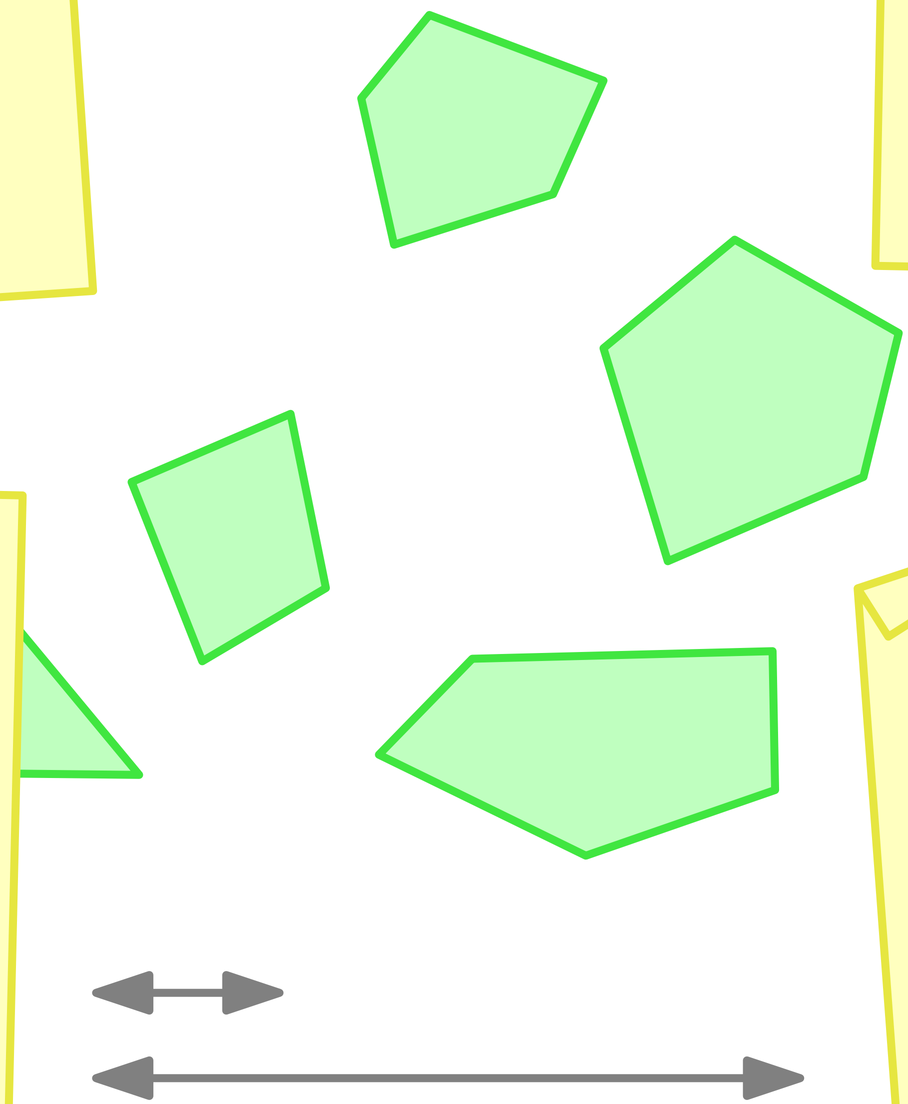

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What results of this kind are known?

Bounds for diameter and convex hull area in time $O(n \log n)$.

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However, these bounds are not *tight*: there may not be any point sets that achieve them.

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Times range rom $O(n \log n)$ to $O(n^{13})$ or even NP-hard.
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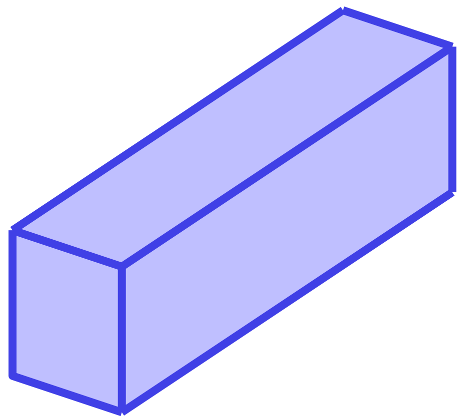
Can similar results be obtained in higher dimensions?

We show how to compute the smallest AABB in $\mathbb{R}^3$ in $O(n^7)$ time.



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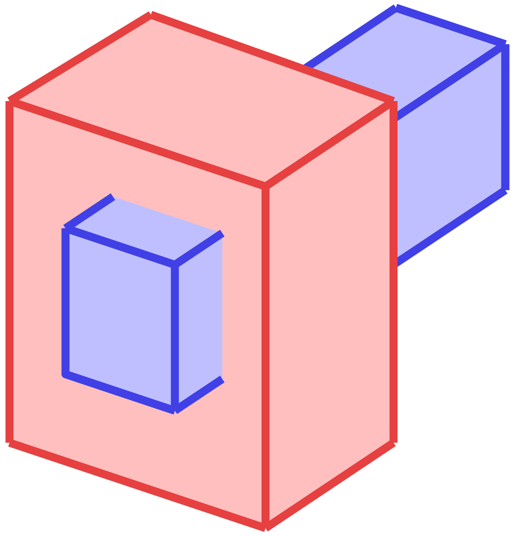
We can also compute the largest AABB in $\mathbb{R}^d$ in $O(d^2 n + f(d))$ time.



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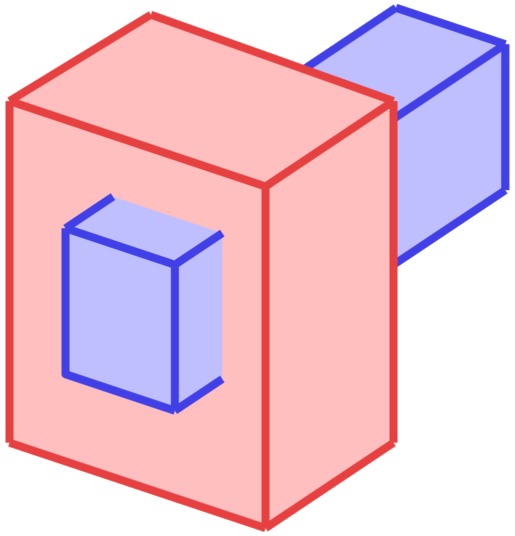

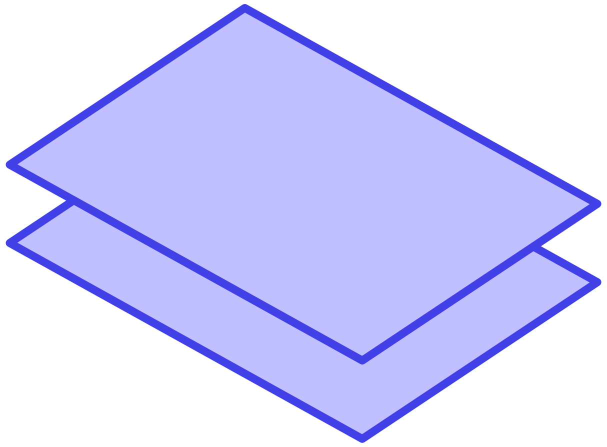

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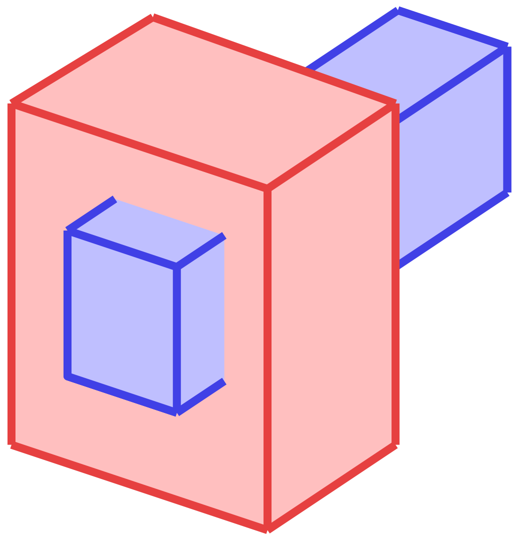

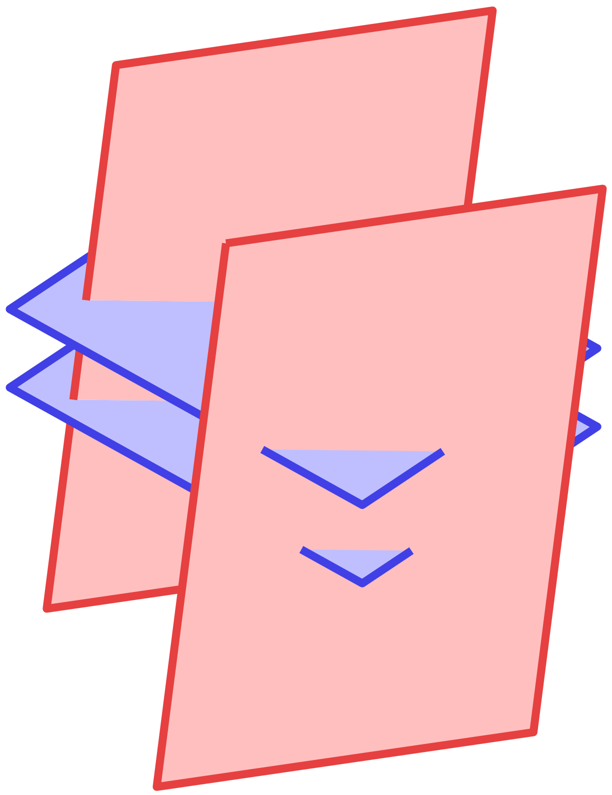

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Finally, we show that computing the largest width in $\mathbb{R}^d$ is NP-hard.

We will give some details on the *width* problem.

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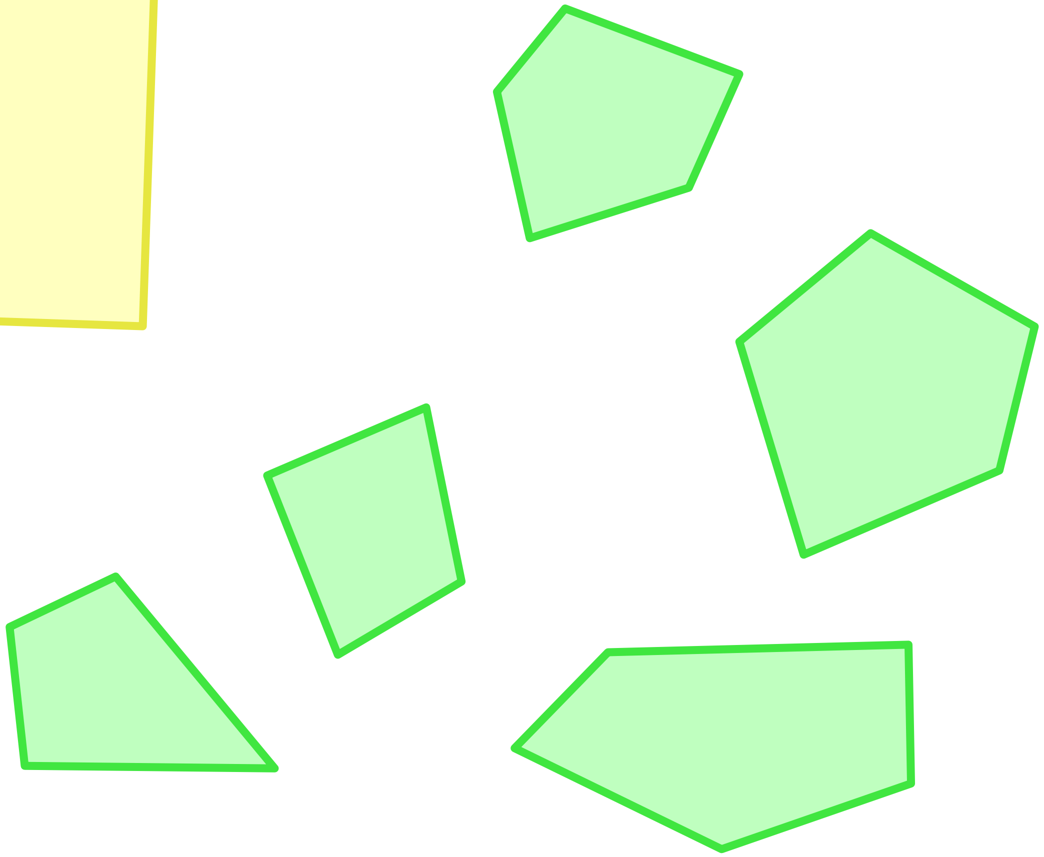

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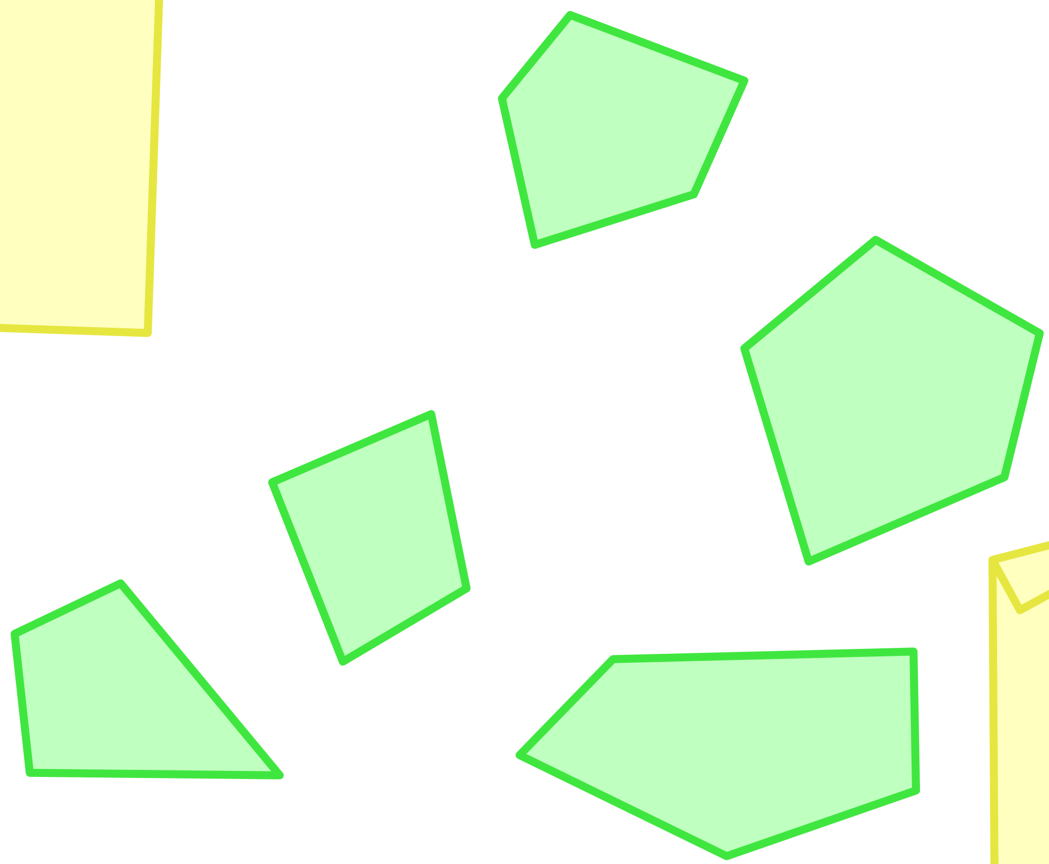

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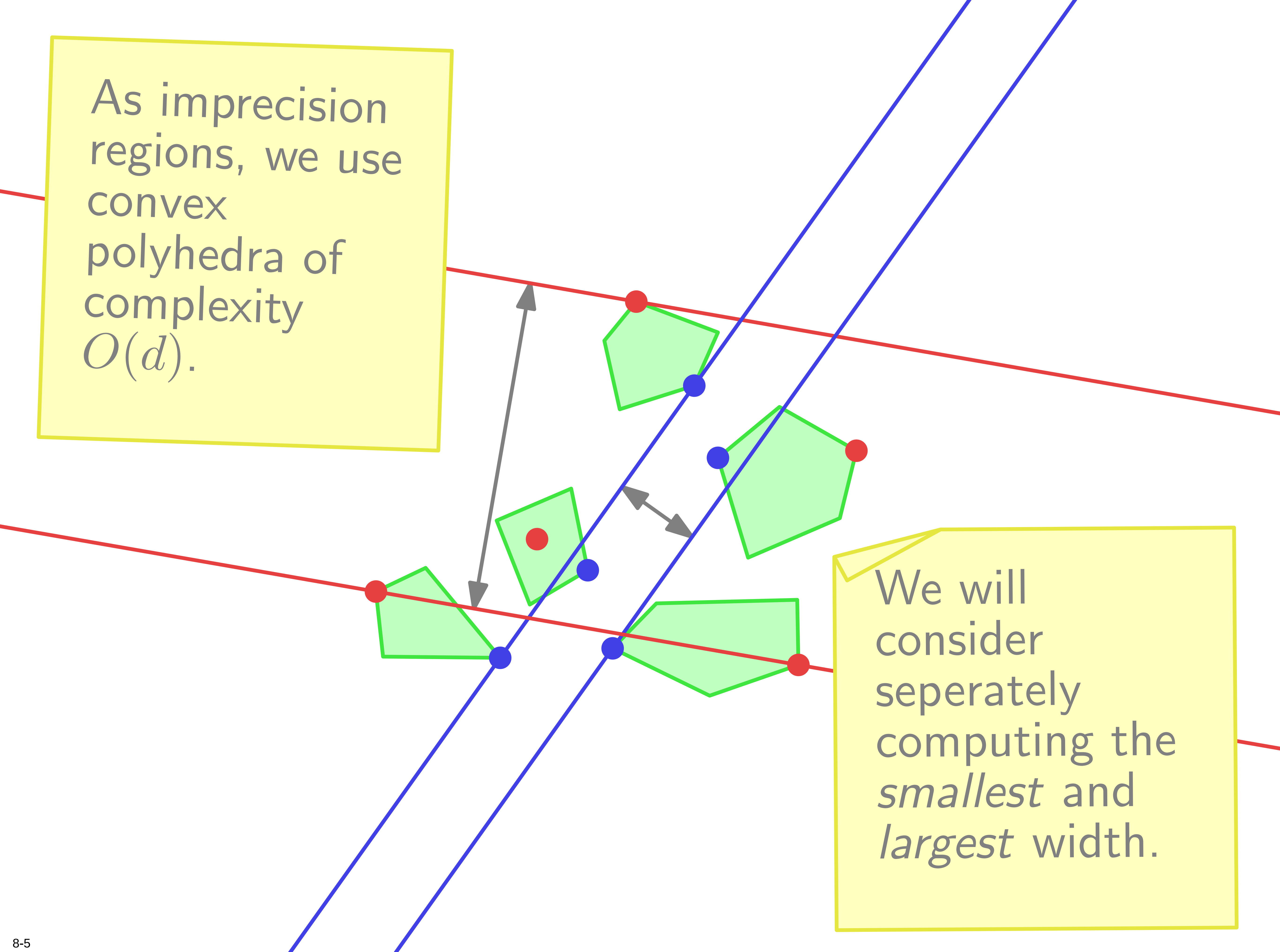
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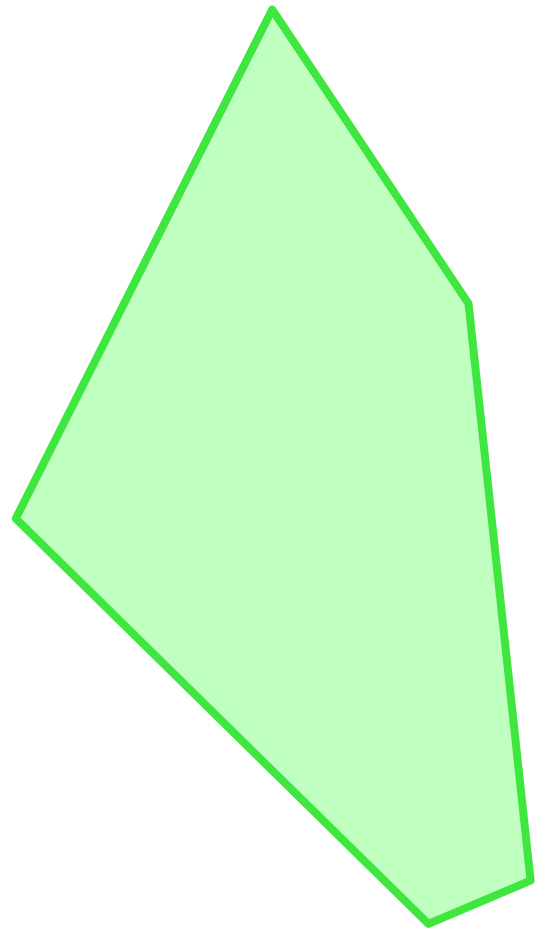
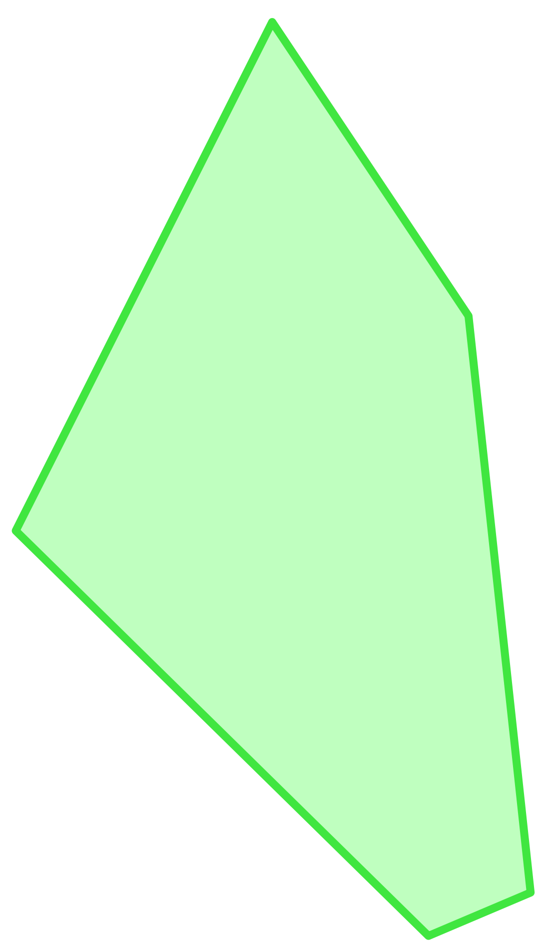


$\mathbb{R}^d$

The *directional* width of a region is defined by a pair of vertices.



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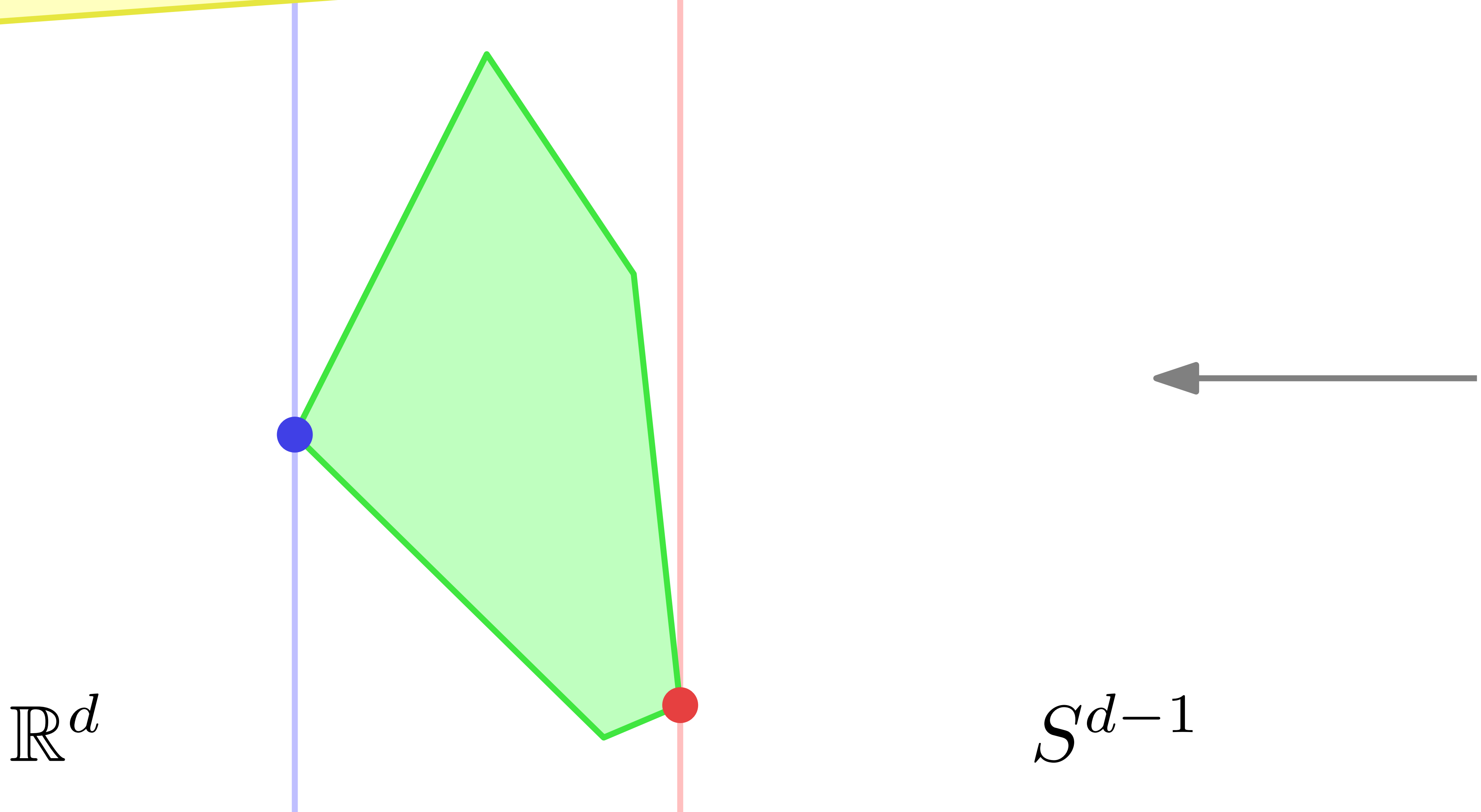

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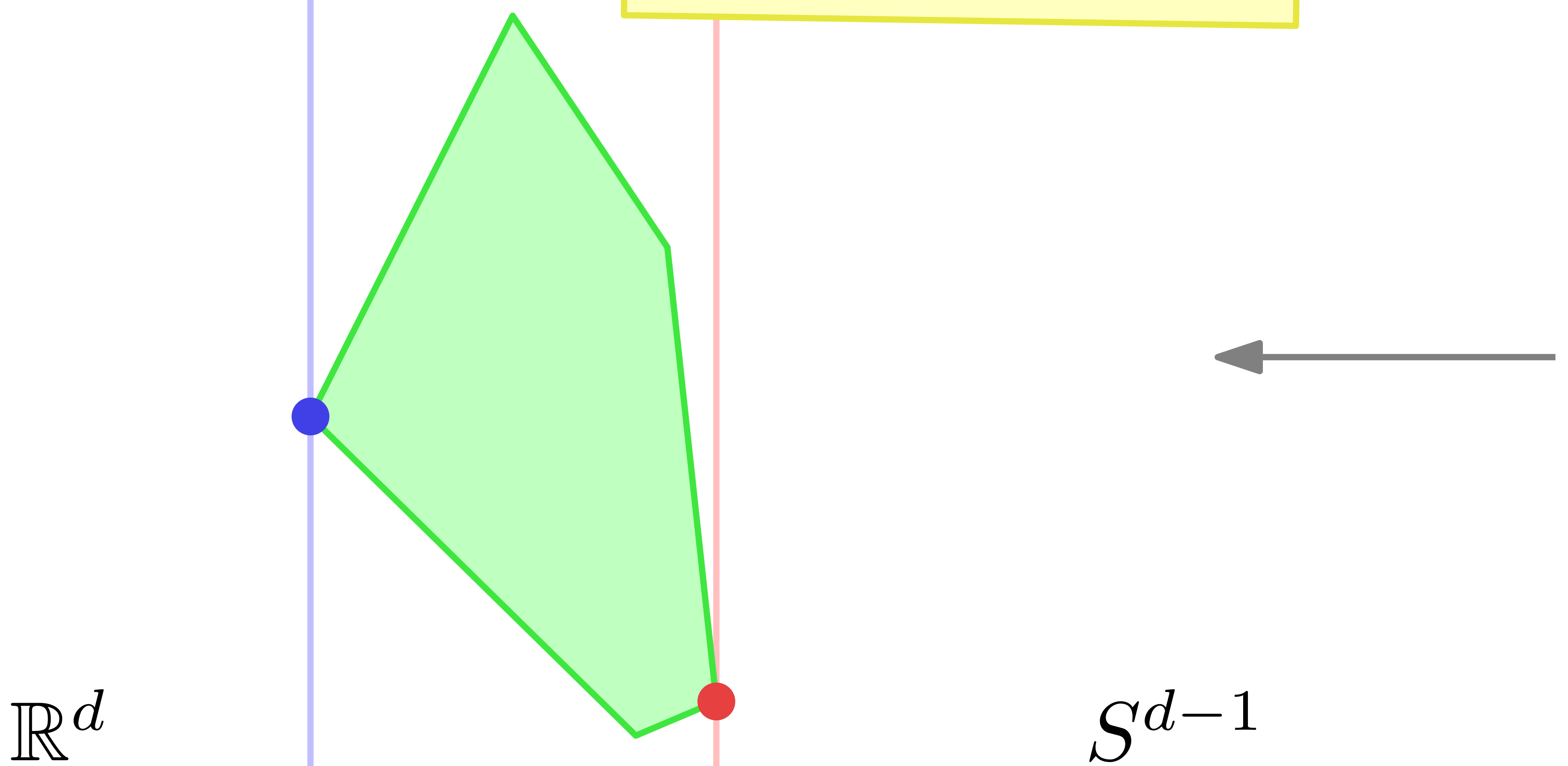

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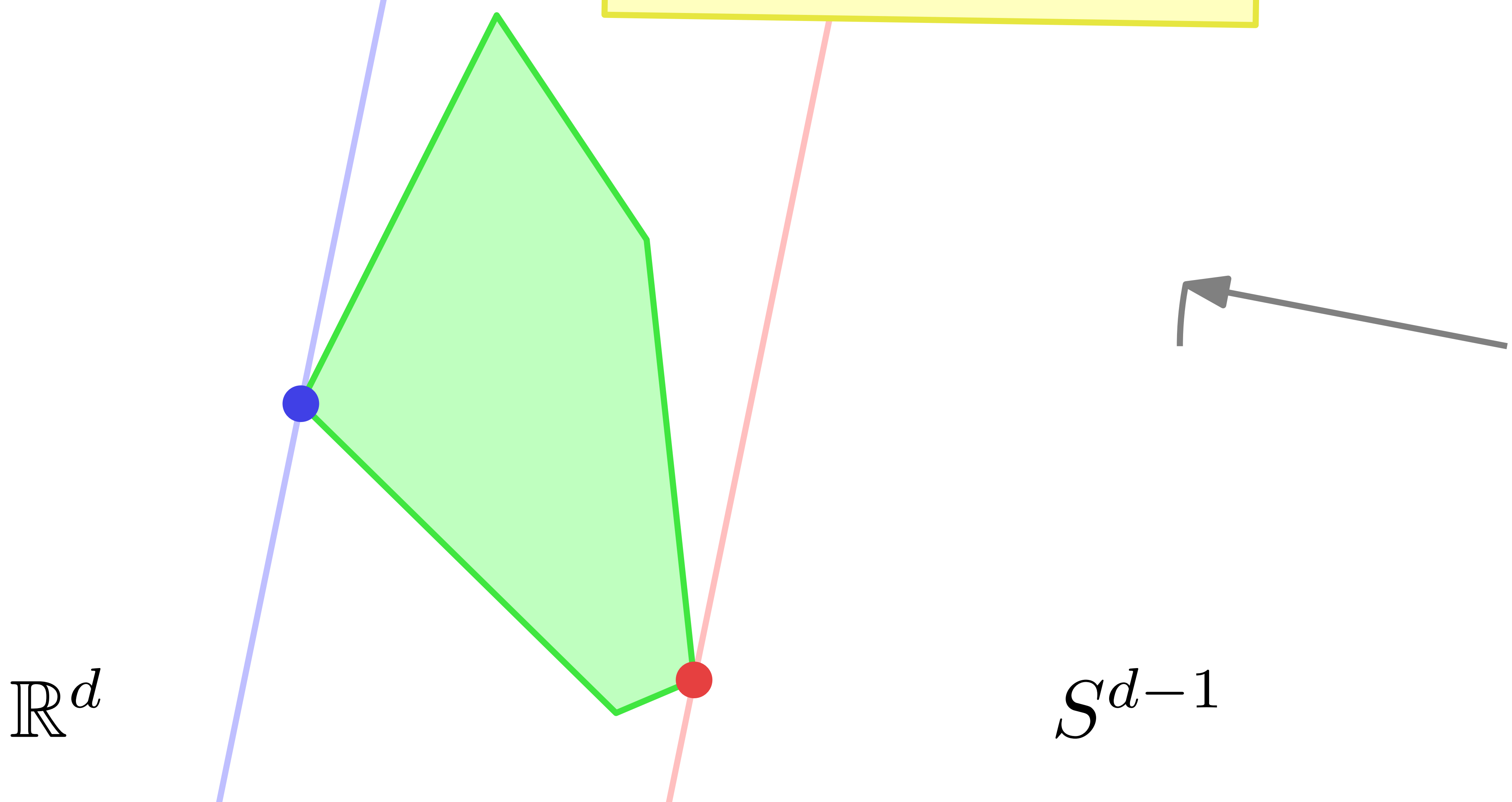

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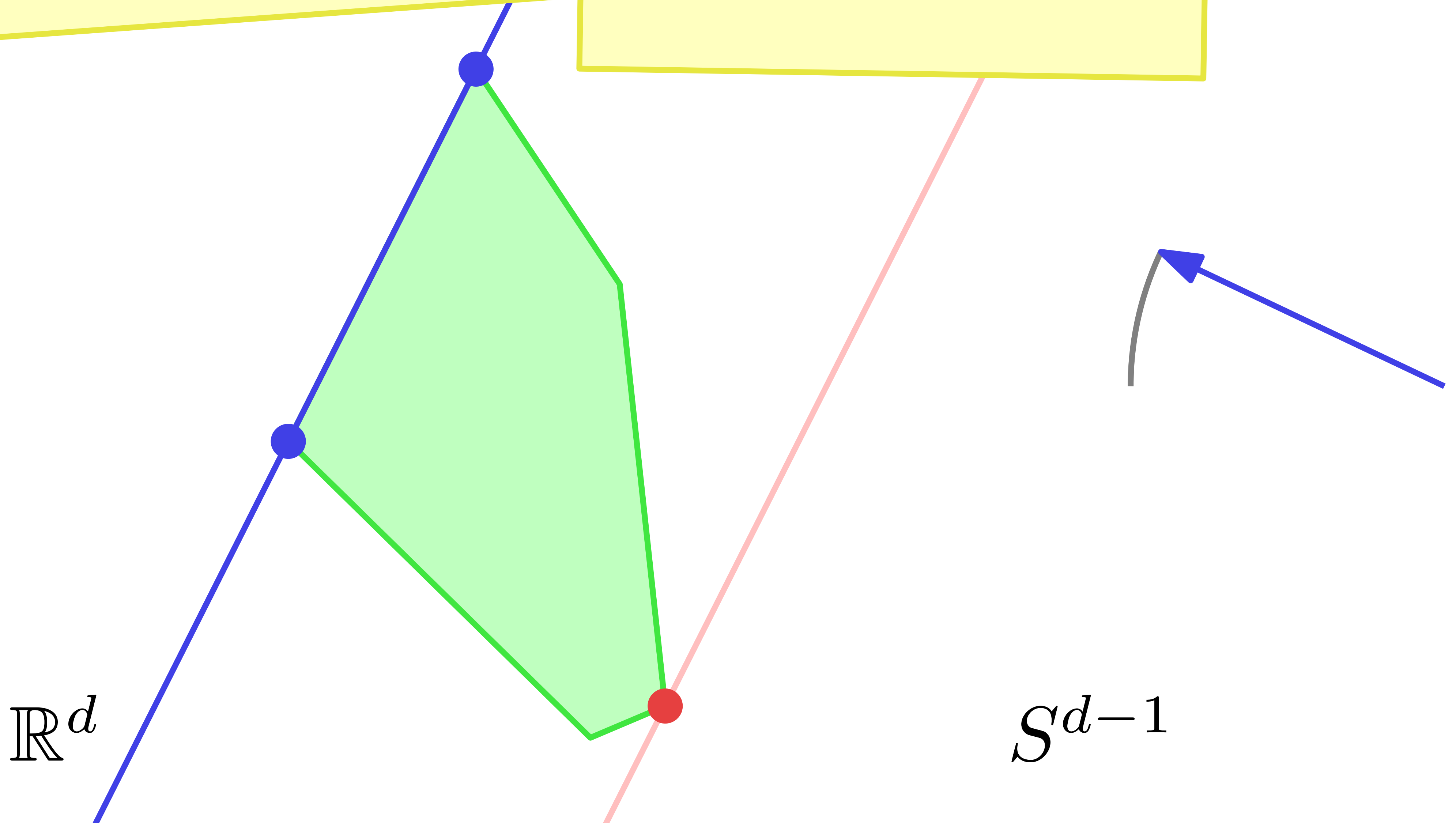

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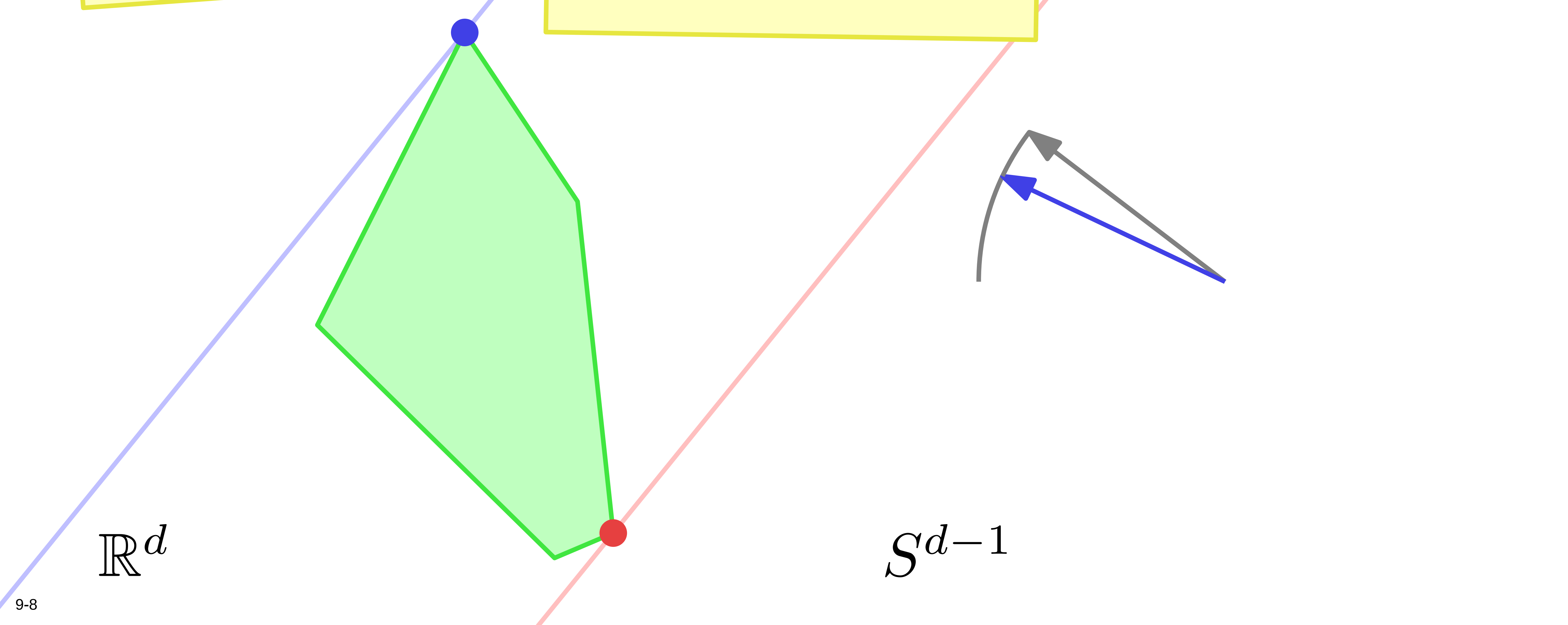

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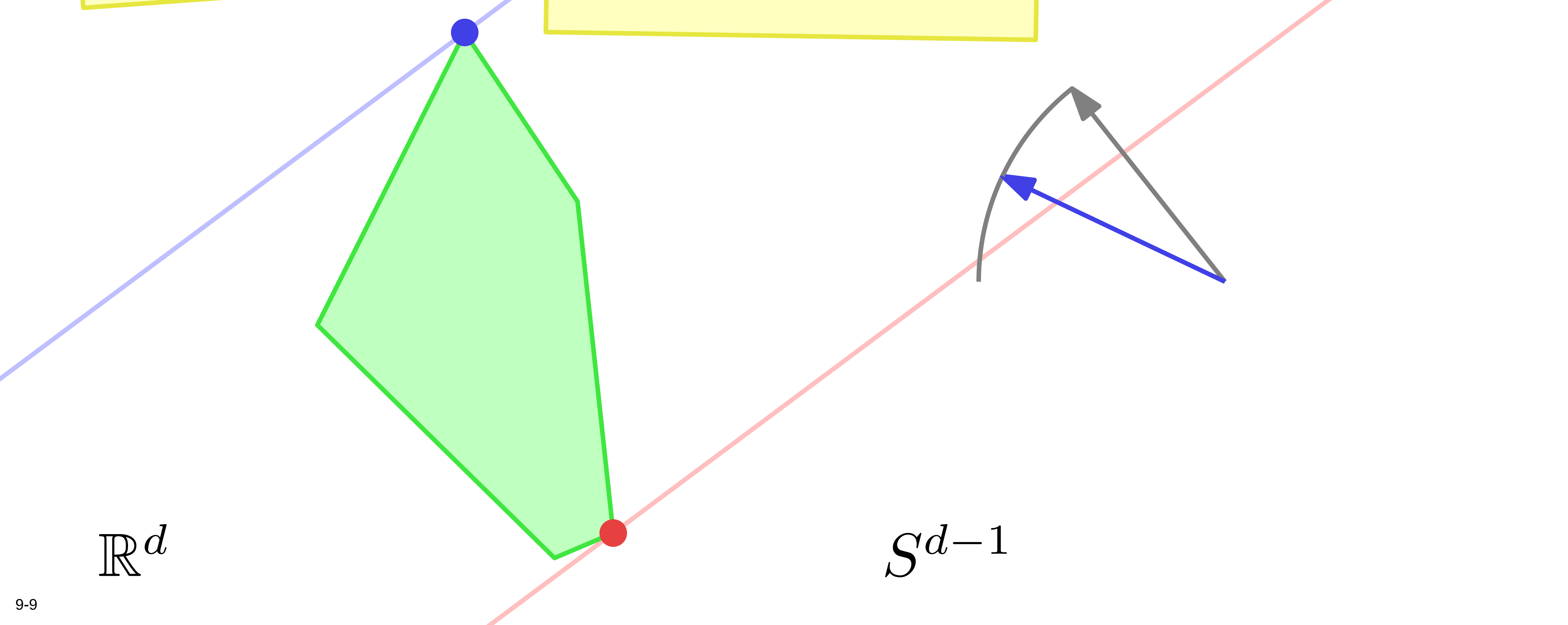

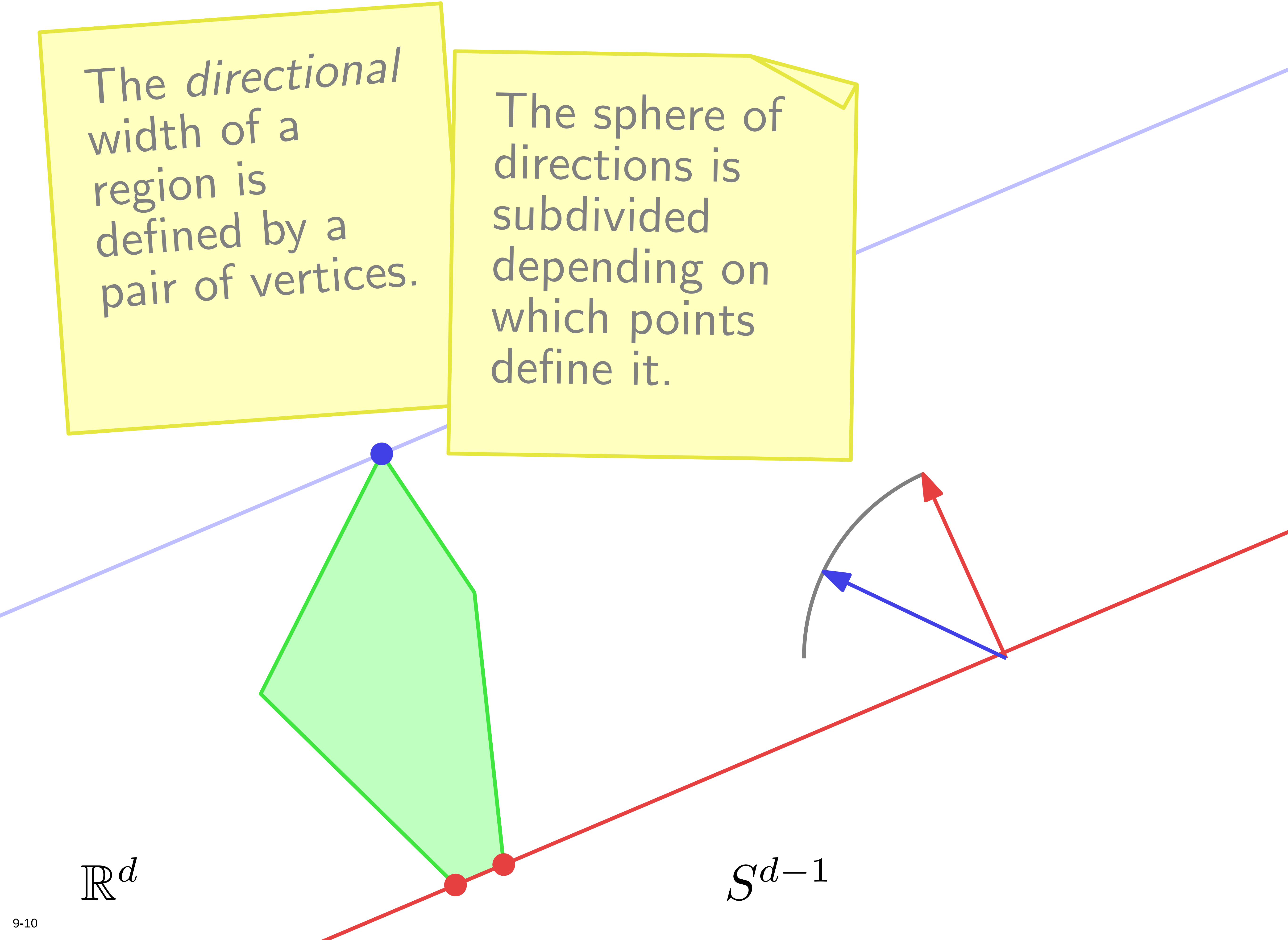
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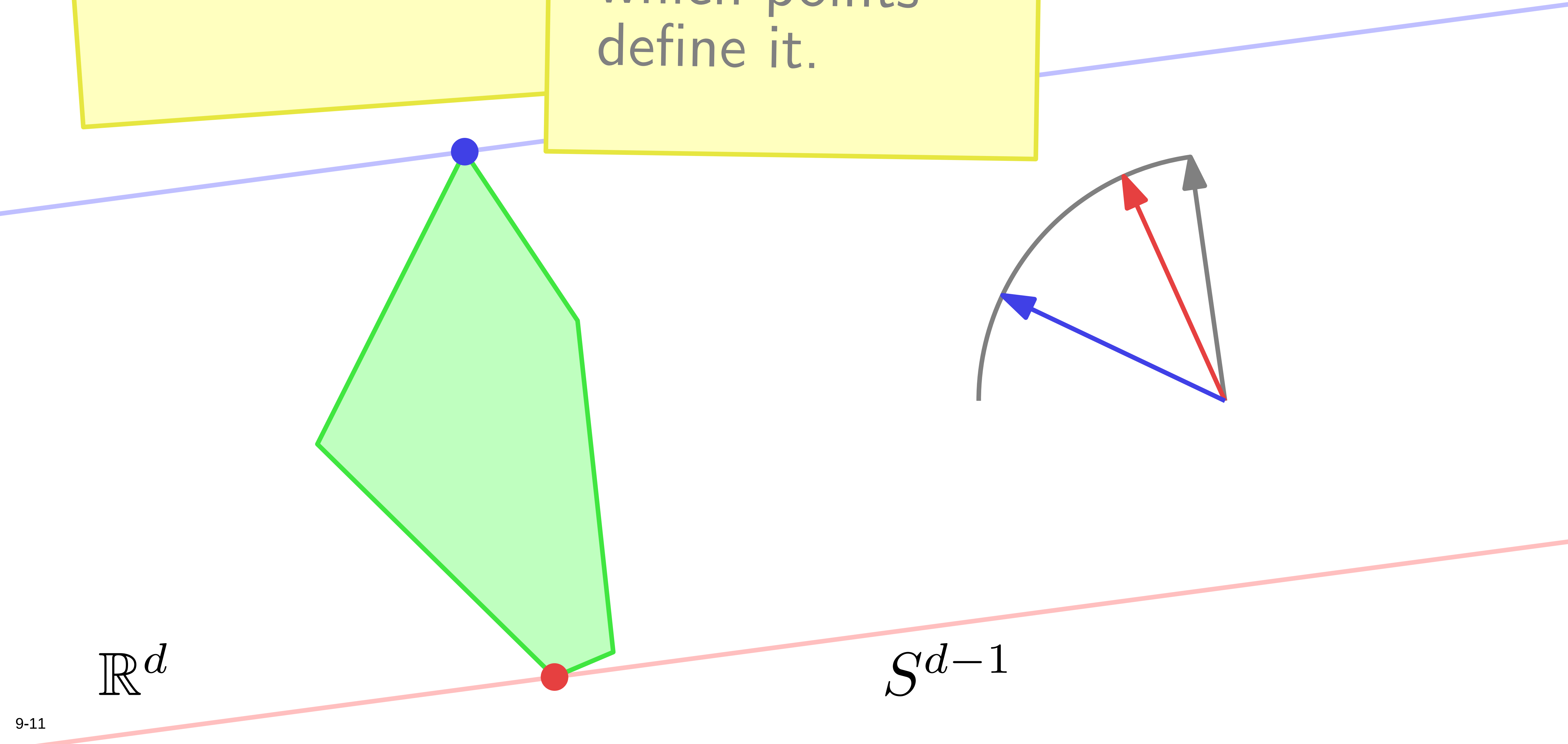

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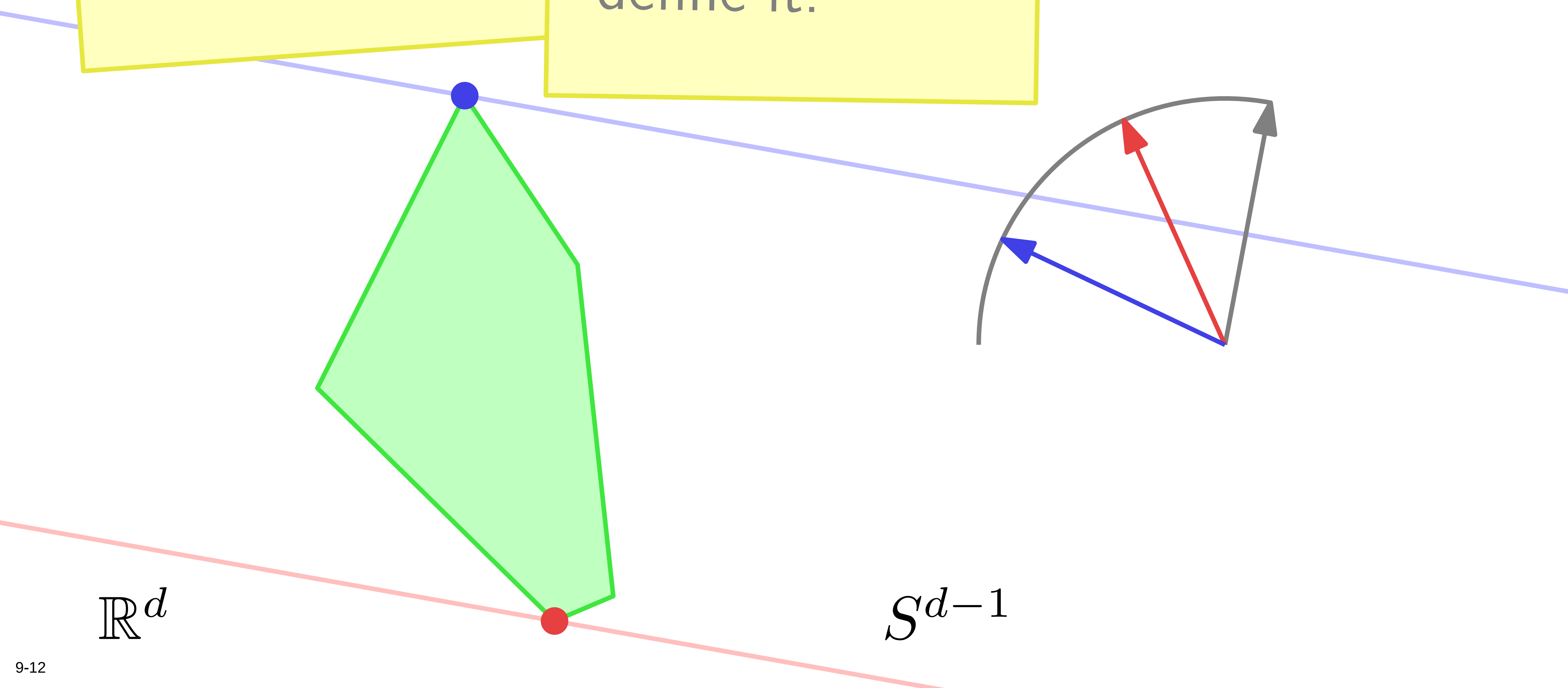

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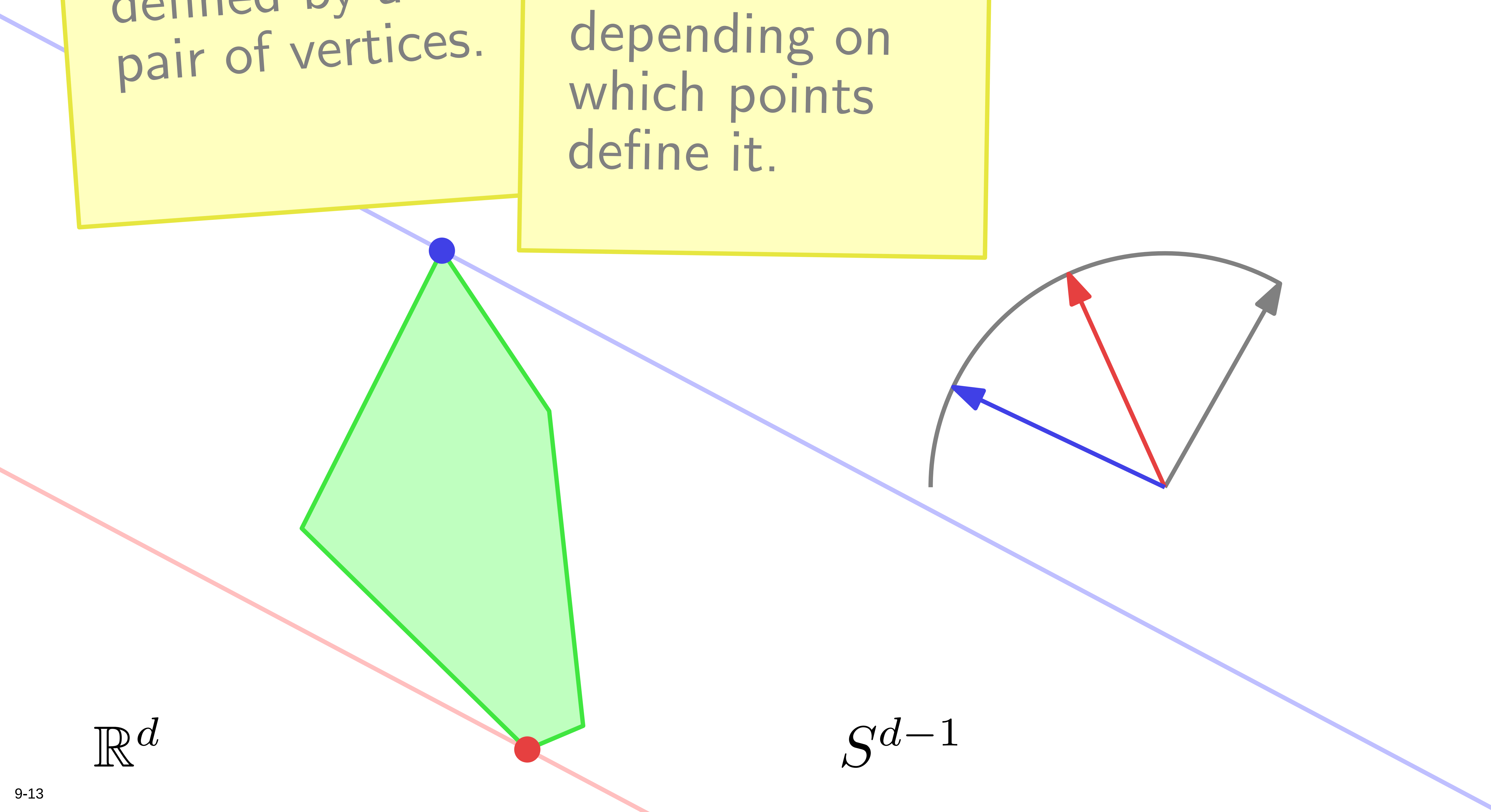

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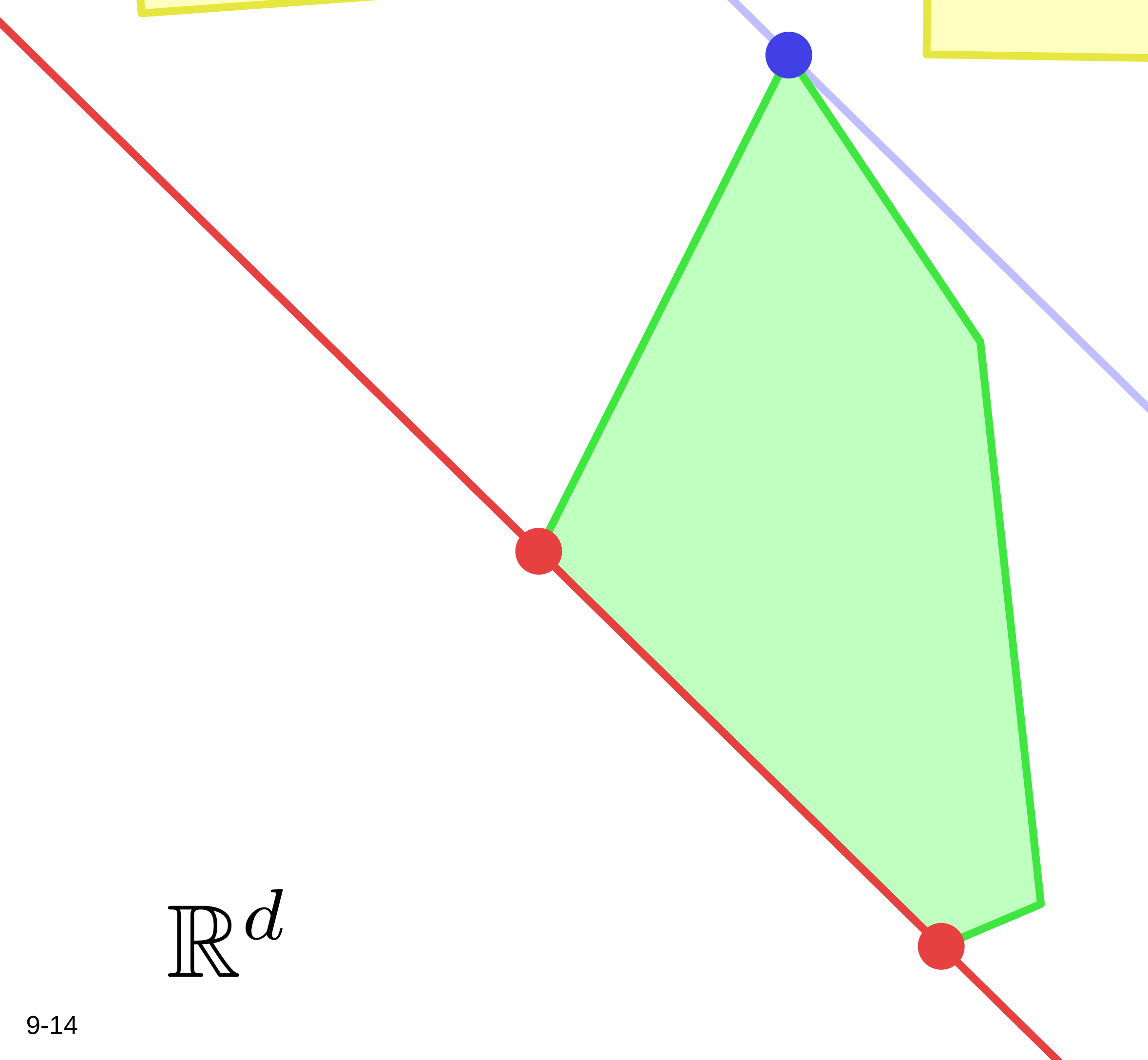

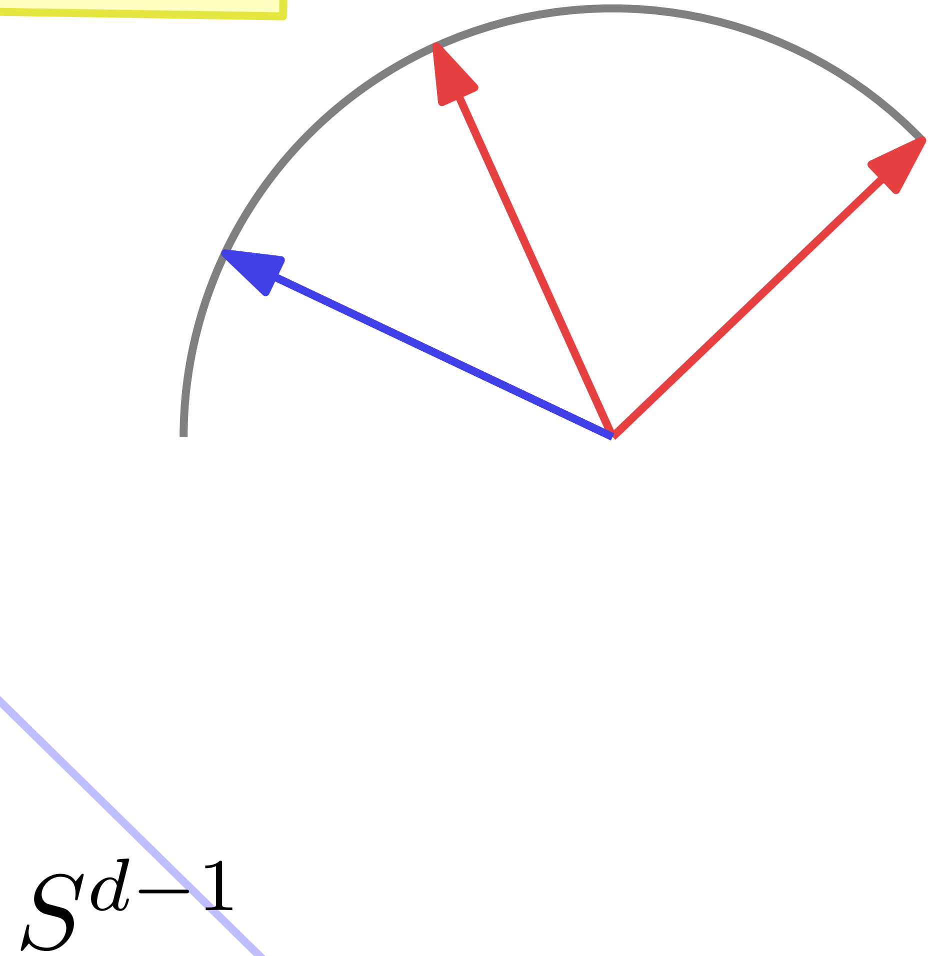

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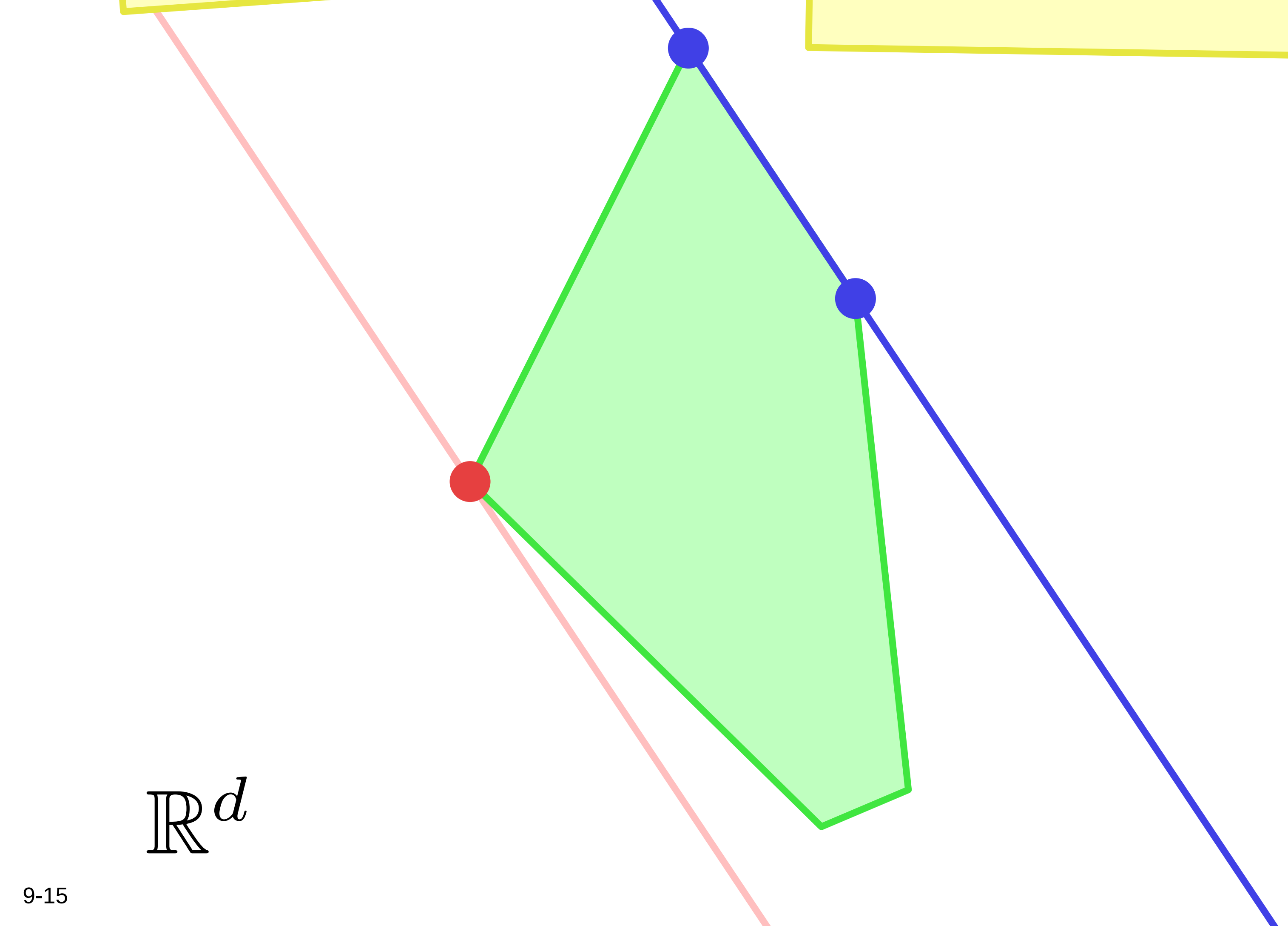

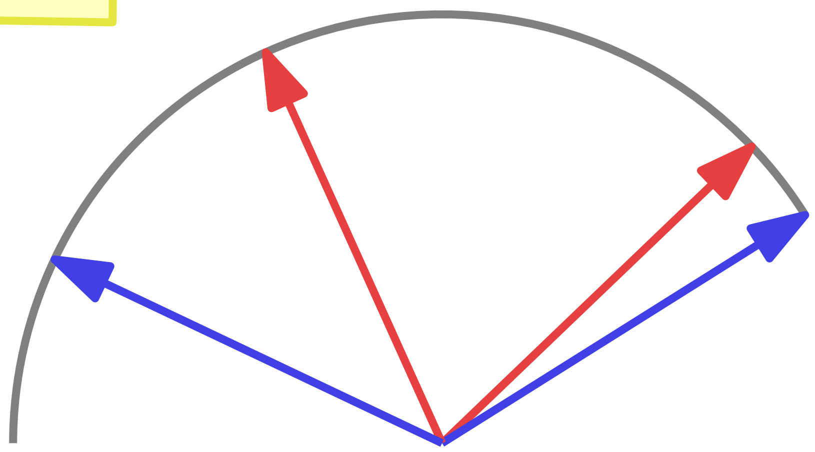

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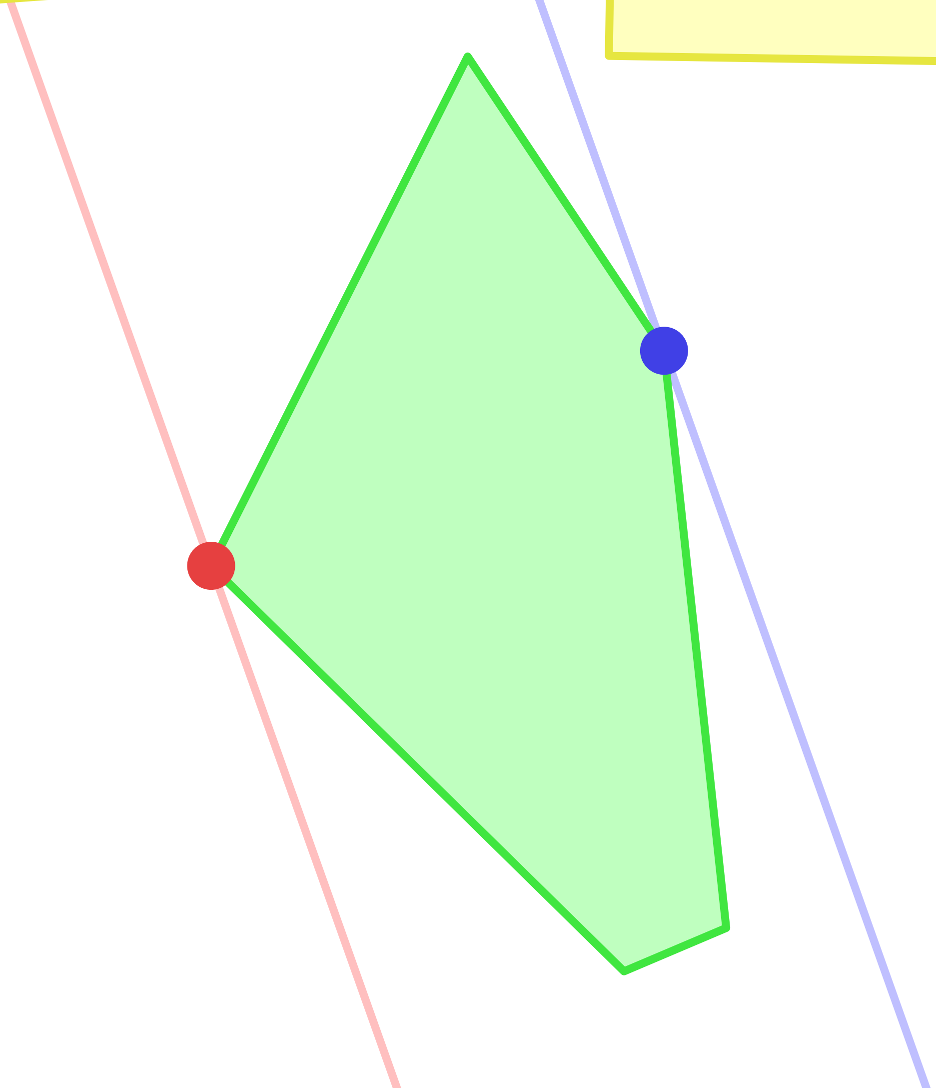
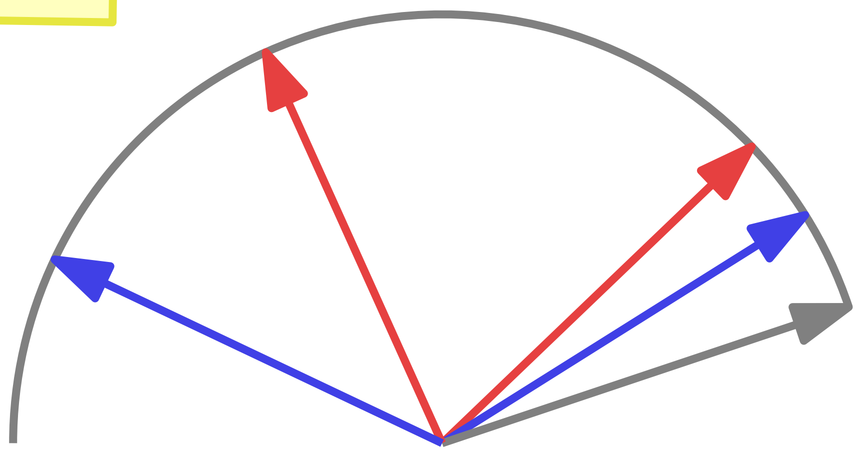


$\mathbb{R}^d$



S^{d-1}

The *directional* width of a region is defined by a pair of vertices.

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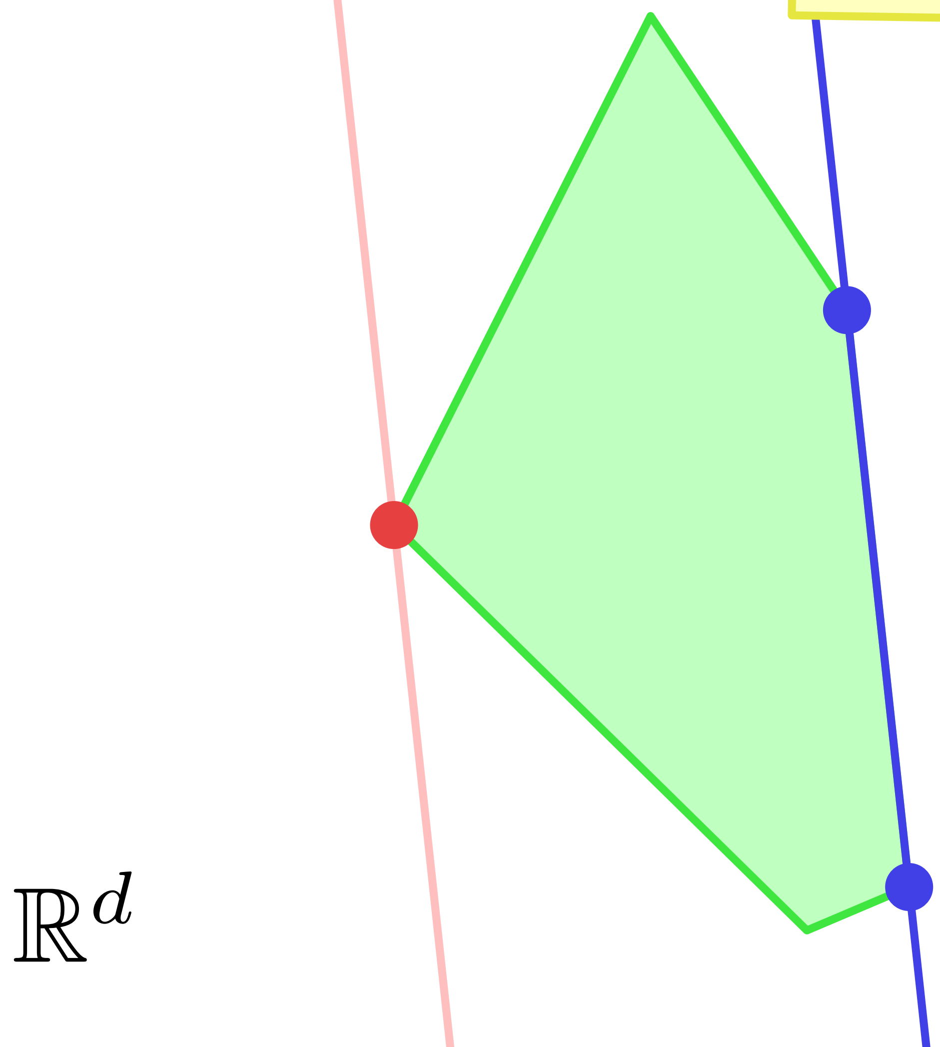

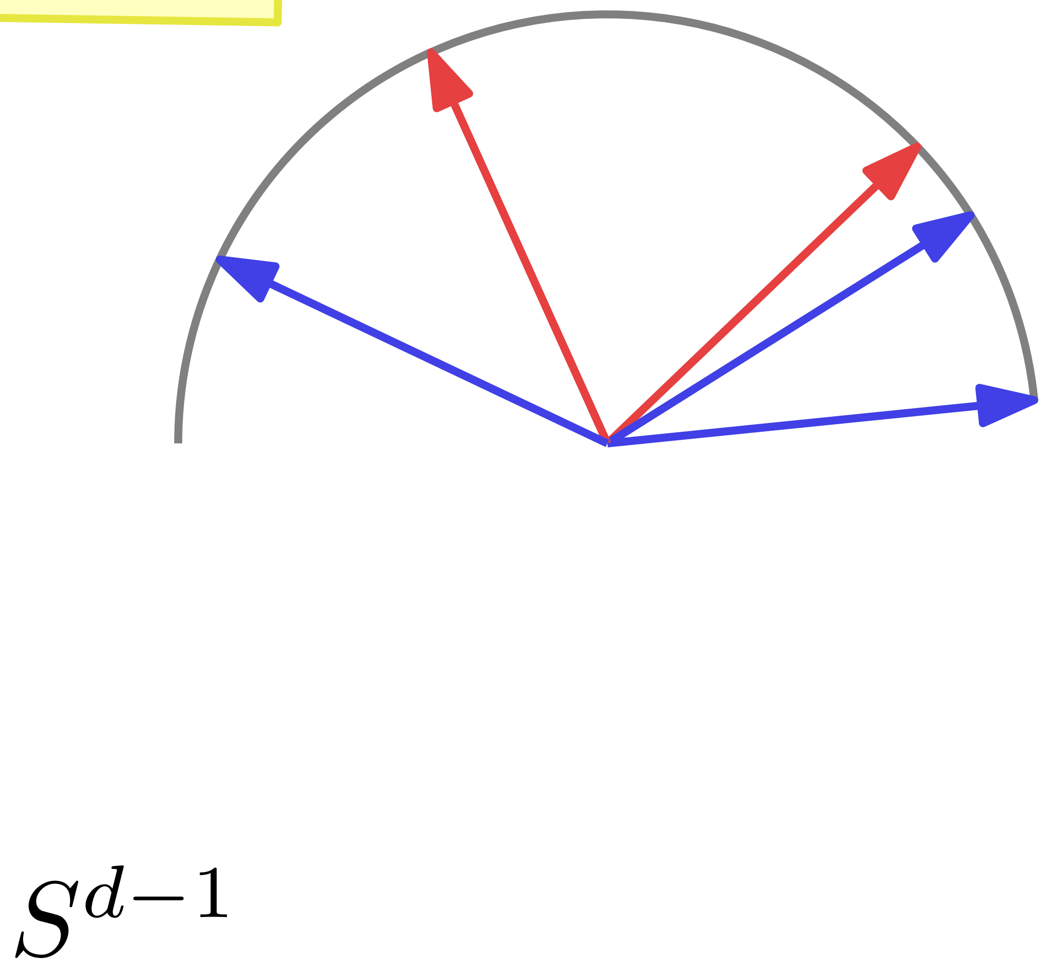

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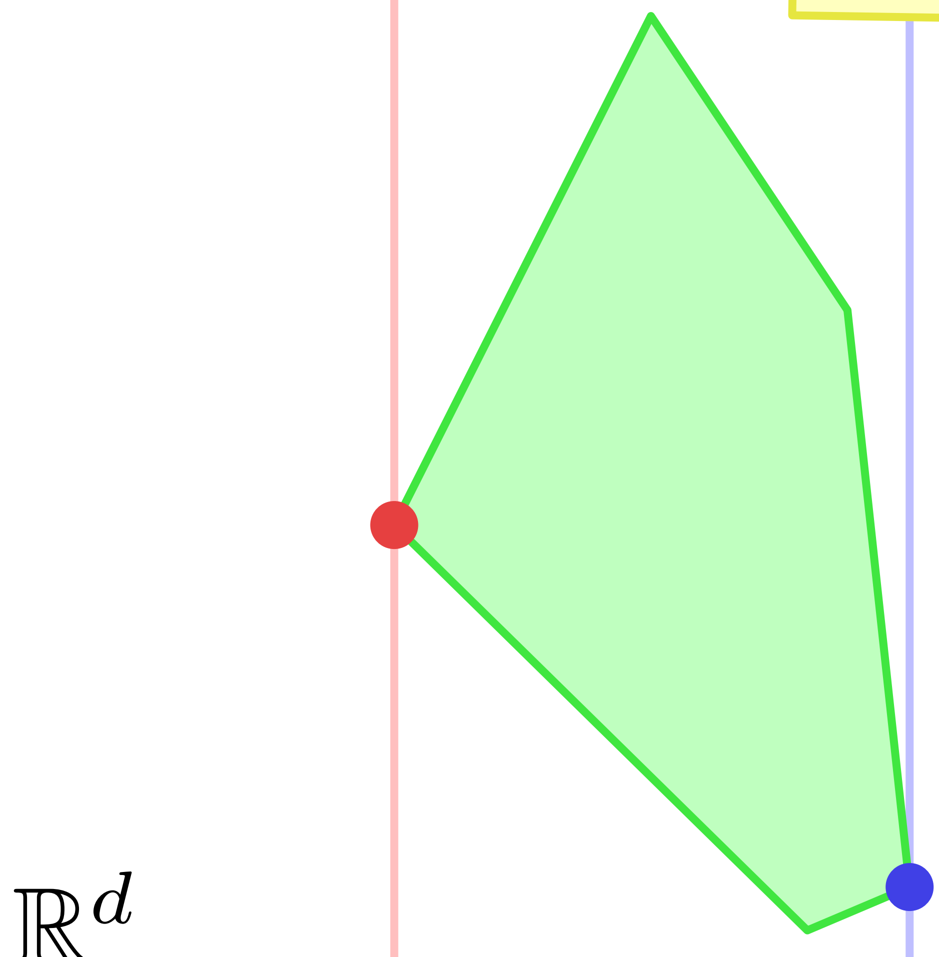

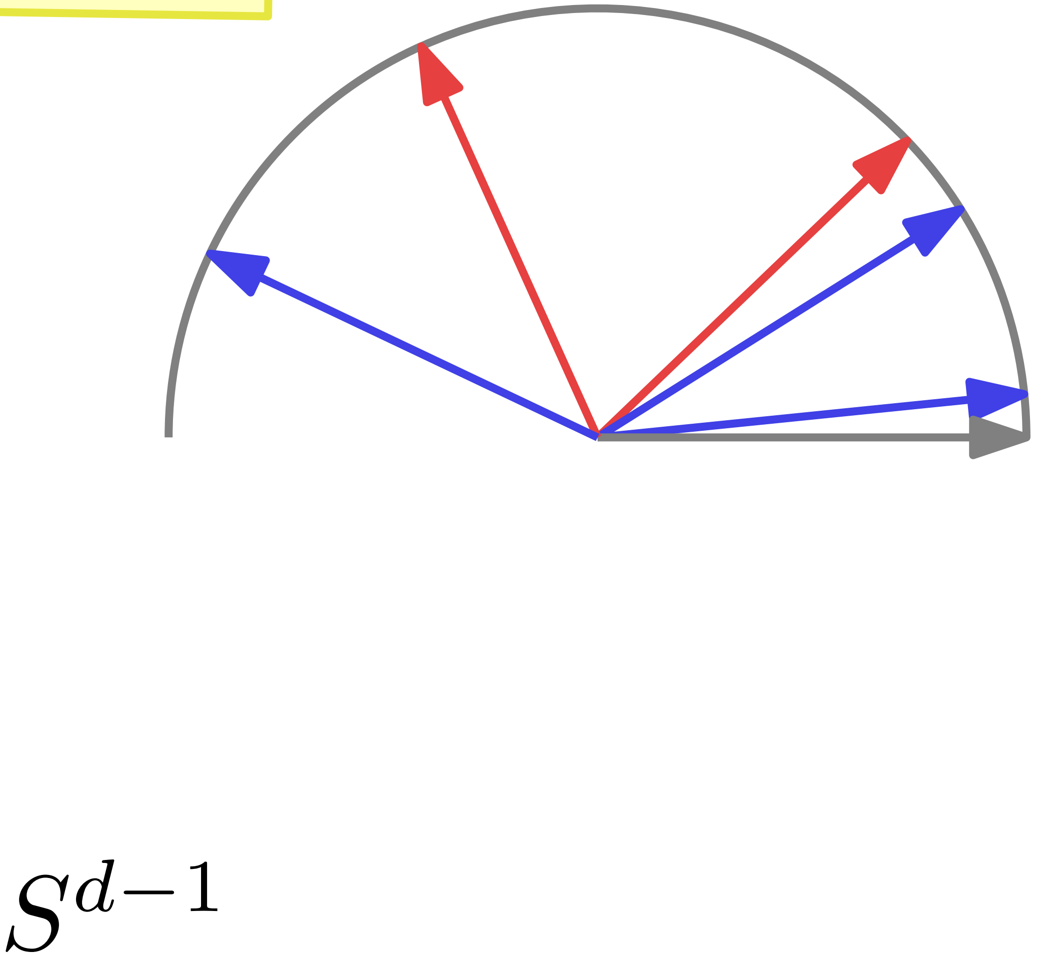

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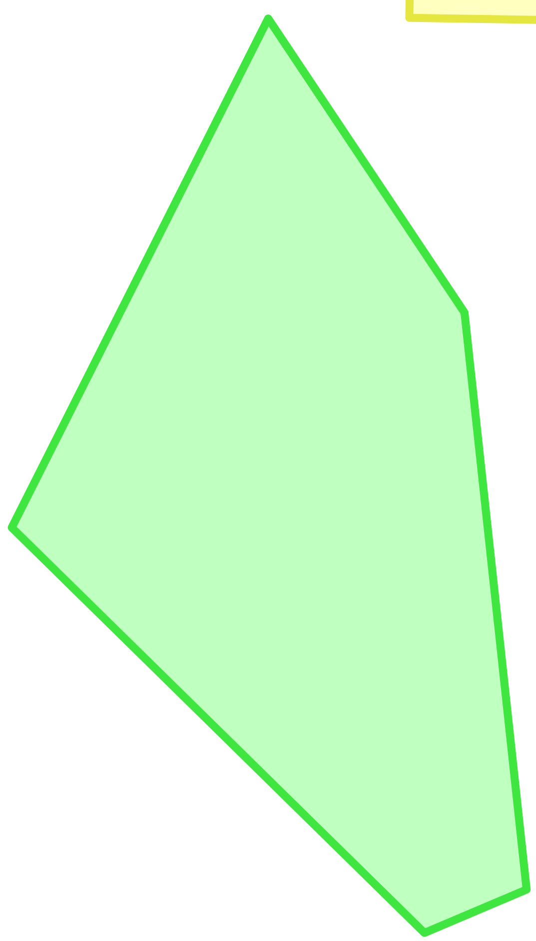
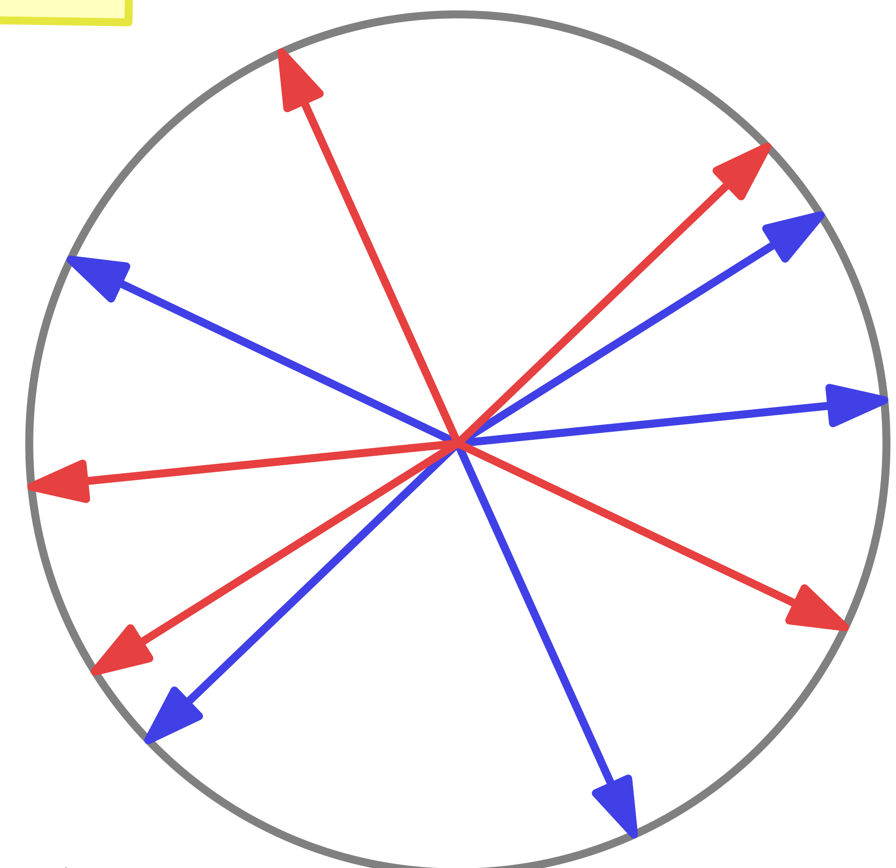


$\mathbb{R}^d$



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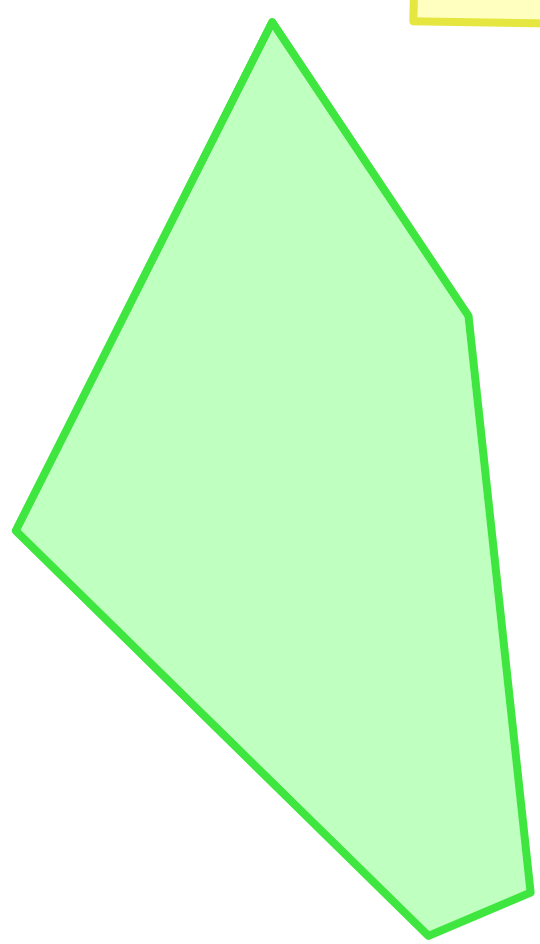

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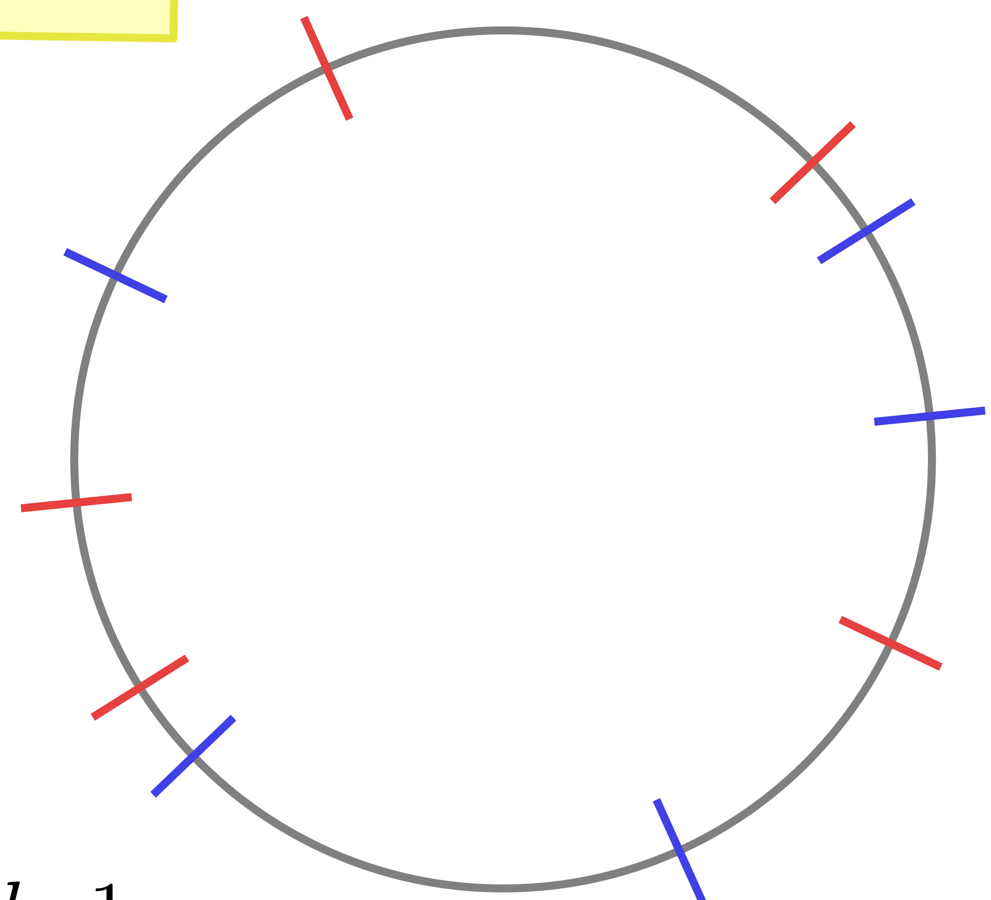

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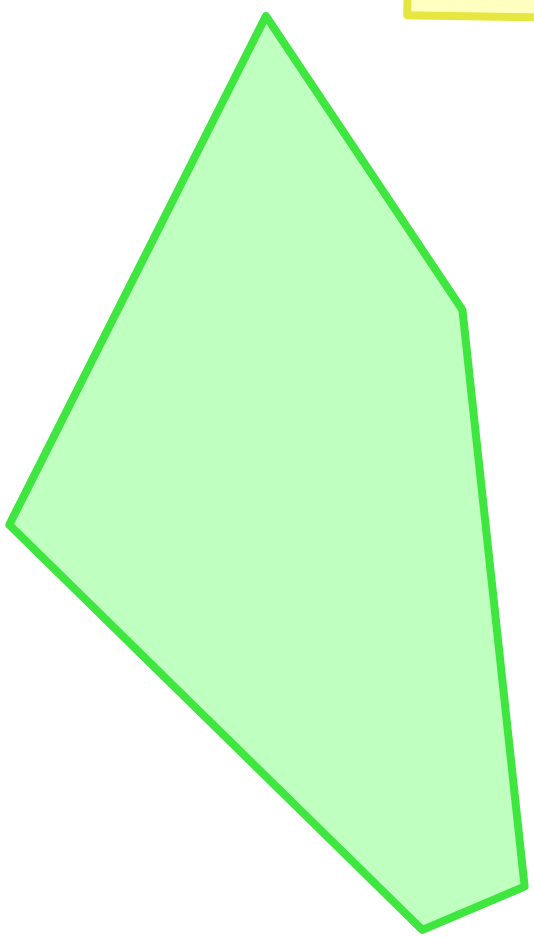
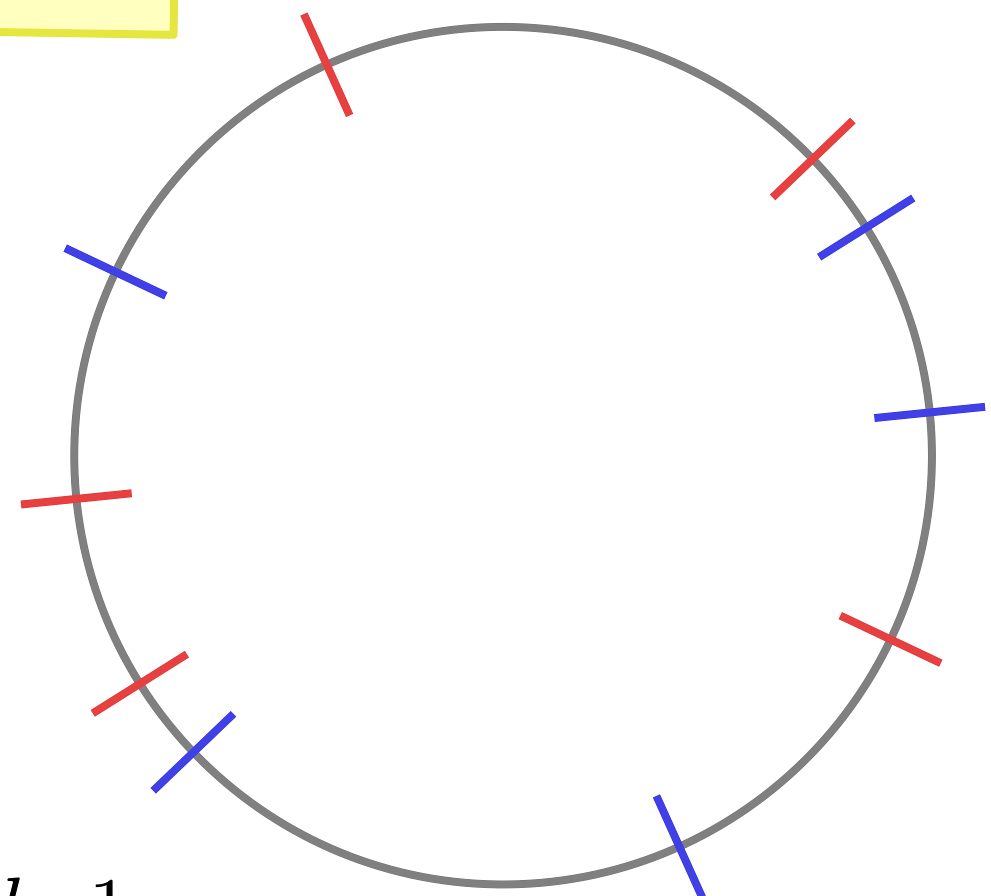
In d dimensions, this subdivision may have complexity $O(d^2)$.



$\mathbb{R}^d$



S^{d-1}

Now, how do we compute the *smallest* width?

Consider the set of regions $\mathcal{R}$.

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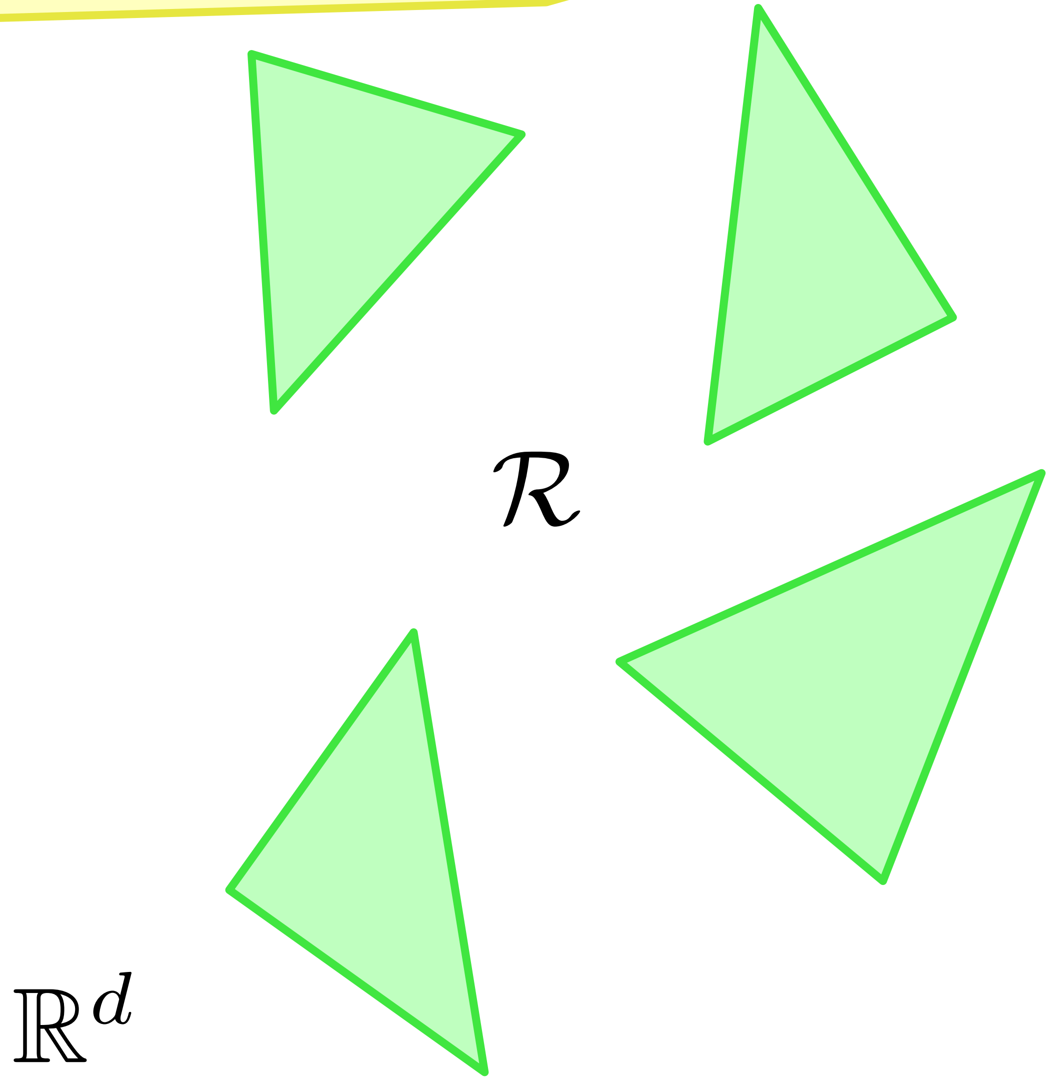
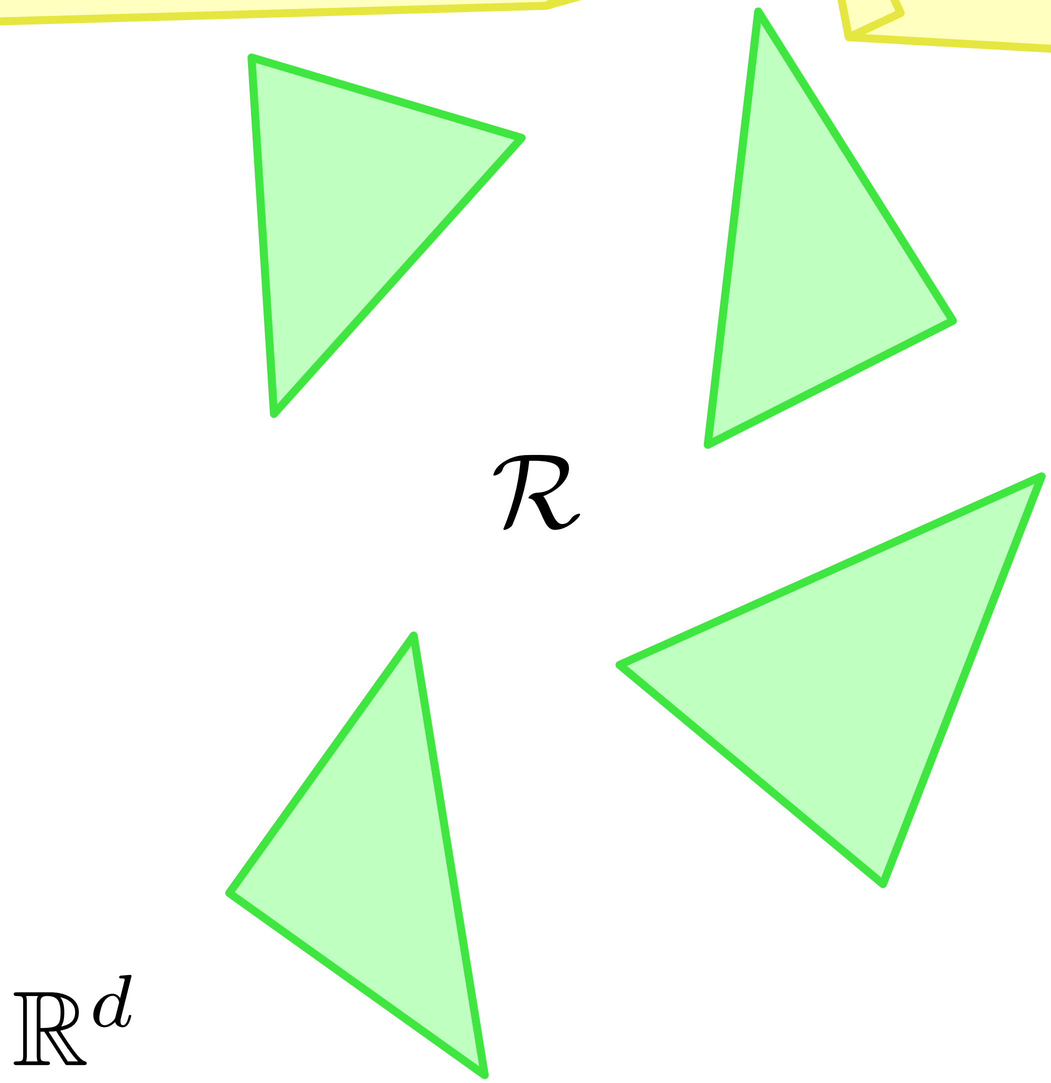


Consider the set of regions $\mathcal{R}$.

We can compute the overlay of the subdivisions of S^{d-1} of all regions.



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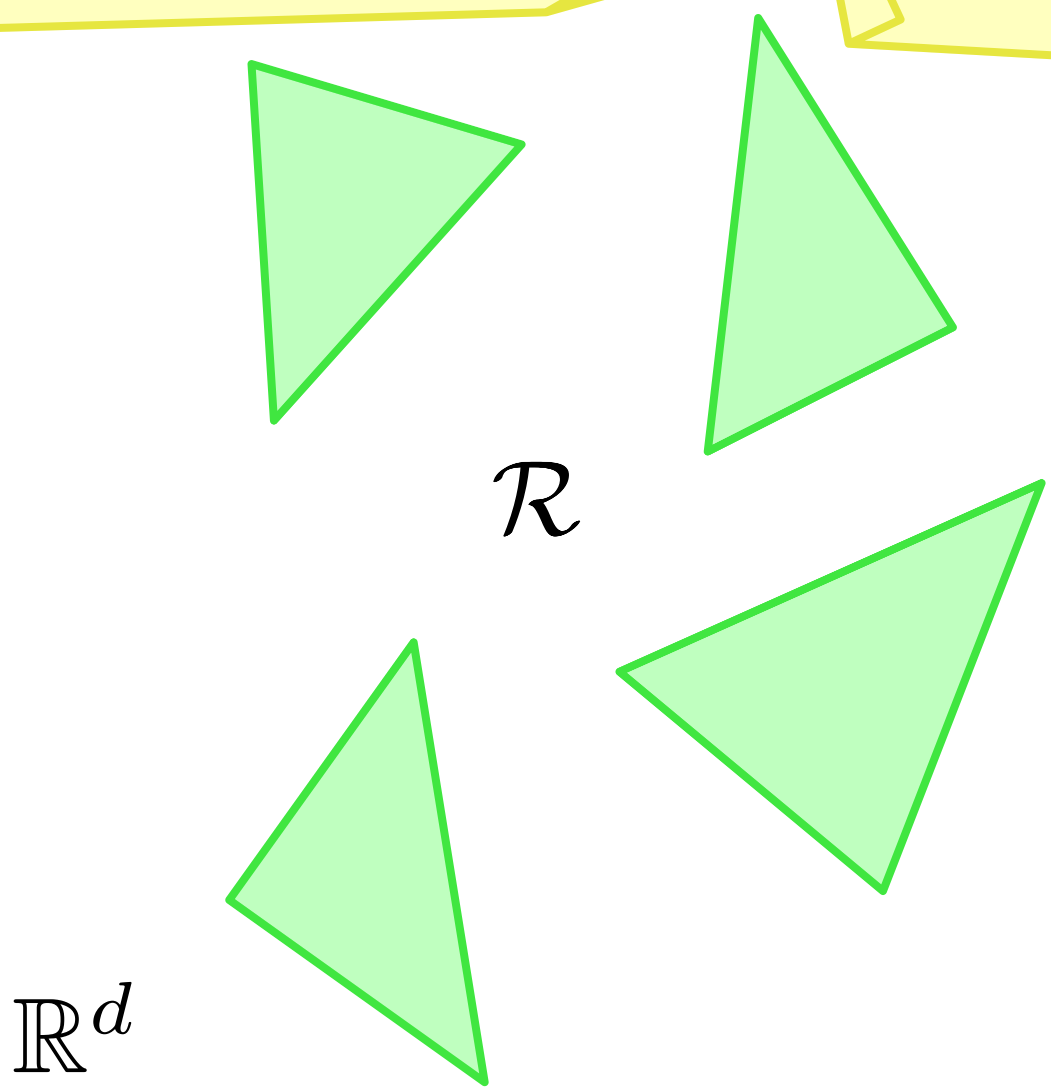
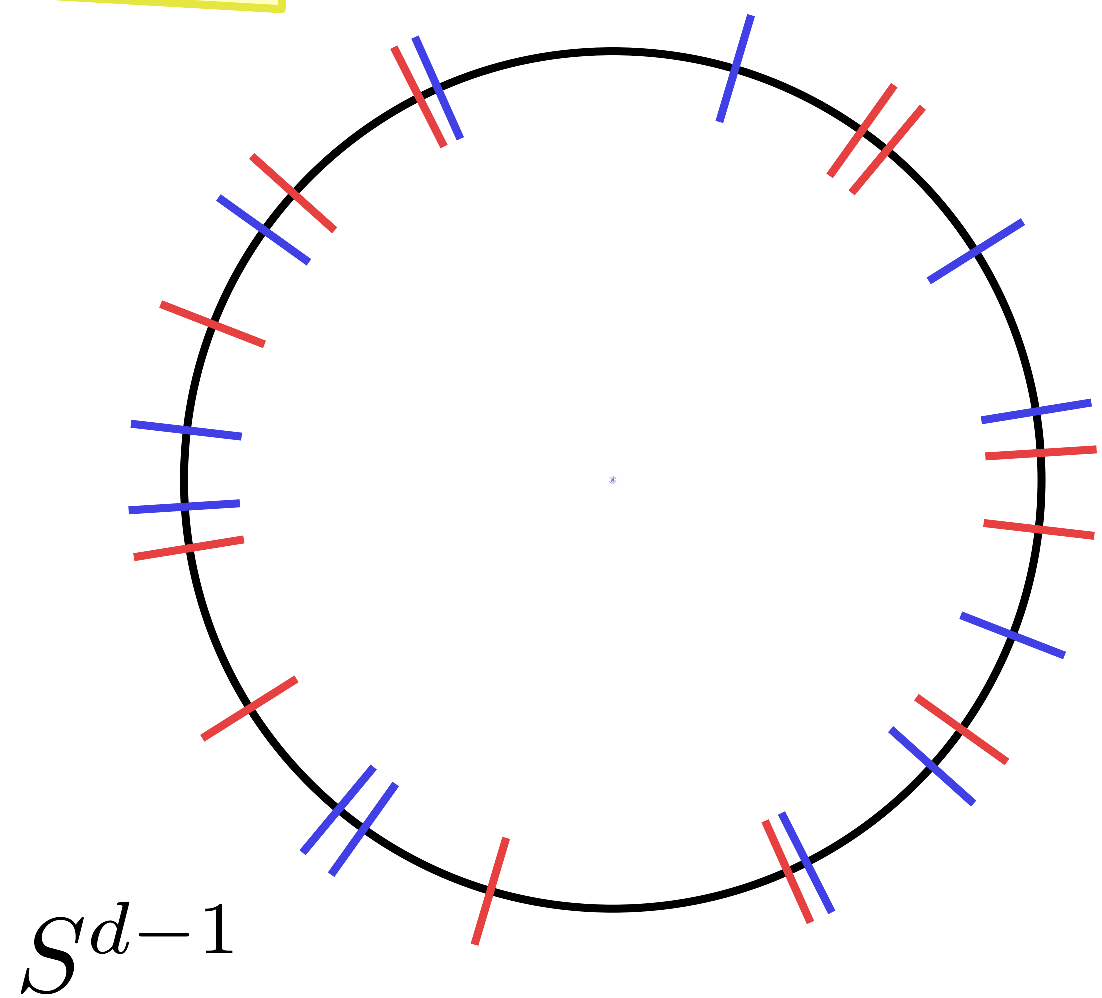
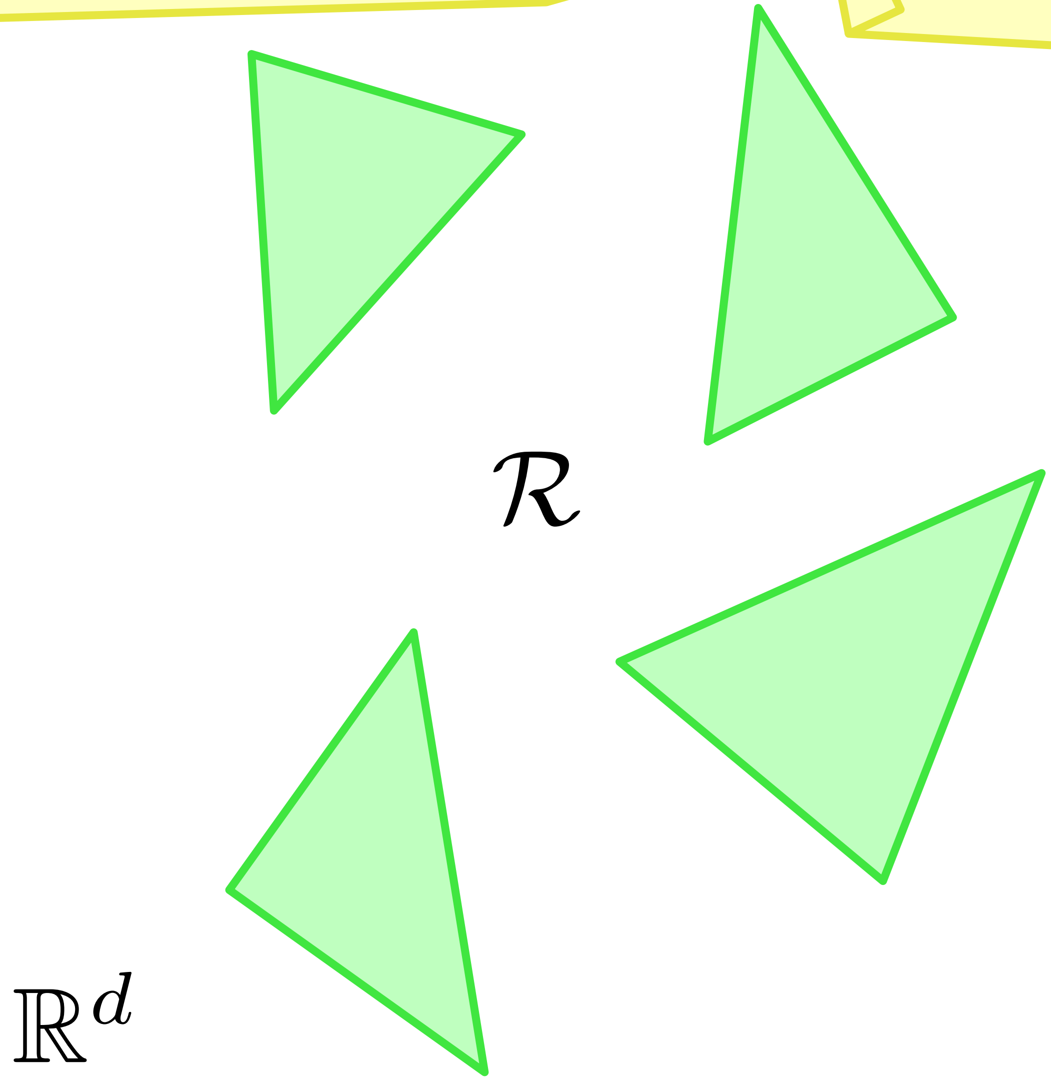


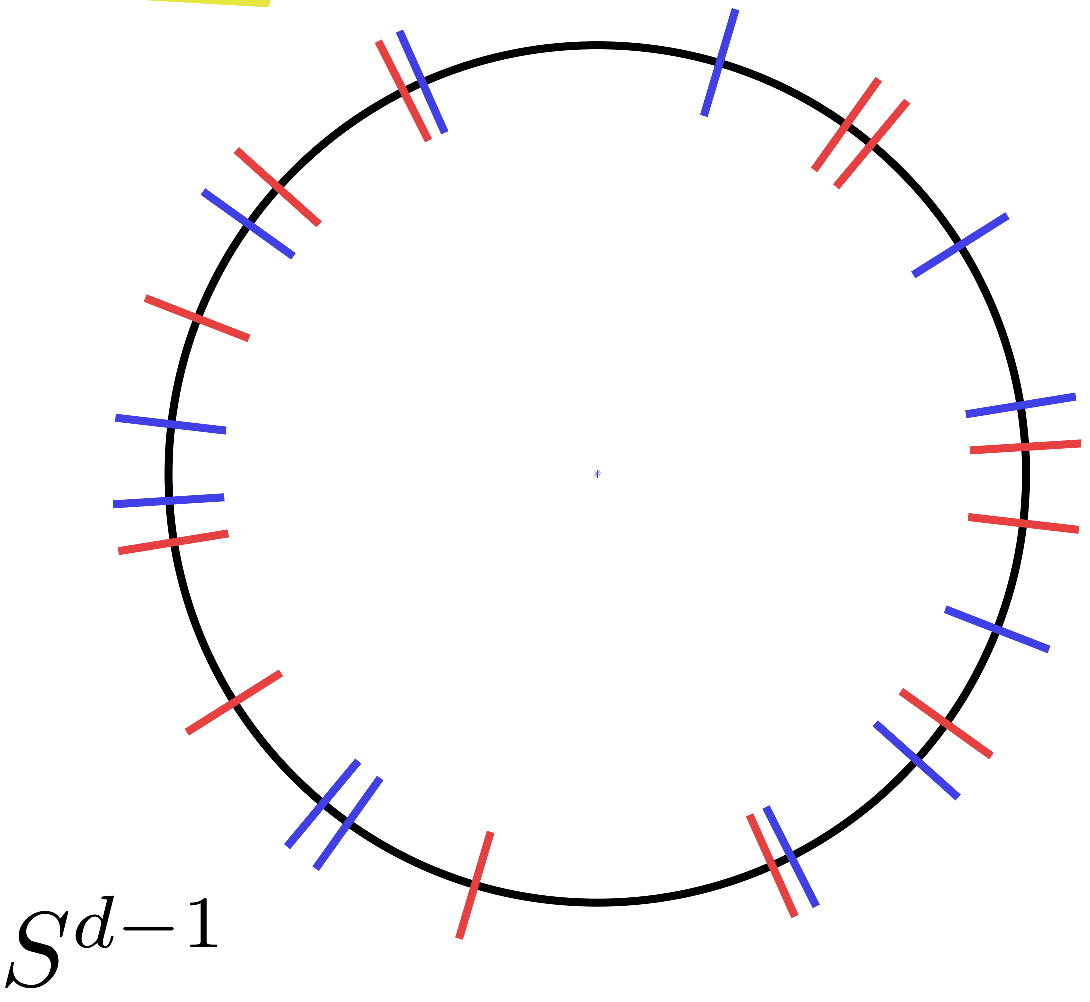


Consider the set of regions $\mathcal{R}$.

We can compute the overlay of the subdivisions of S^{d-1} of all regions.

Within each cell, we can now compute the smallest width.





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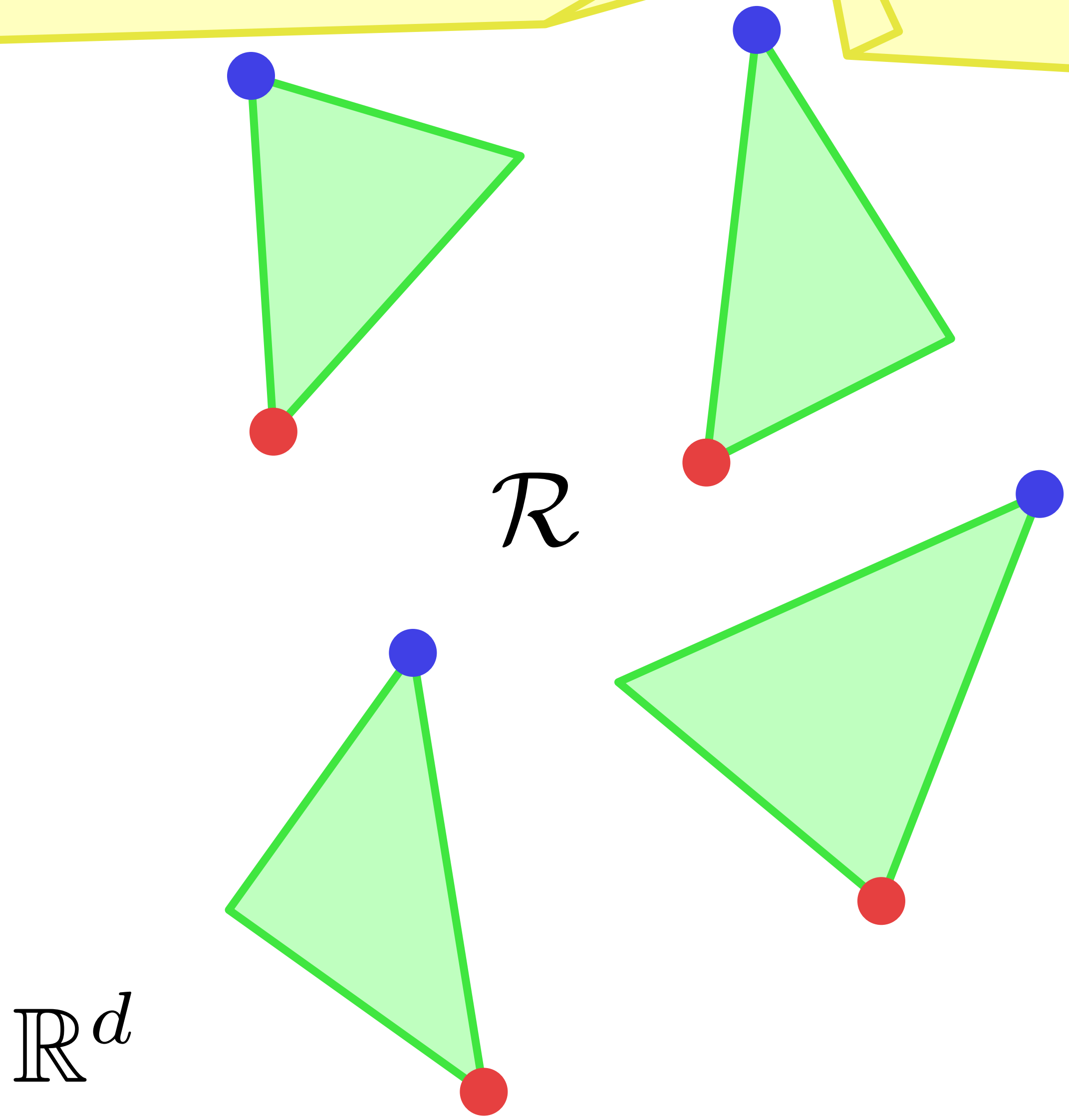

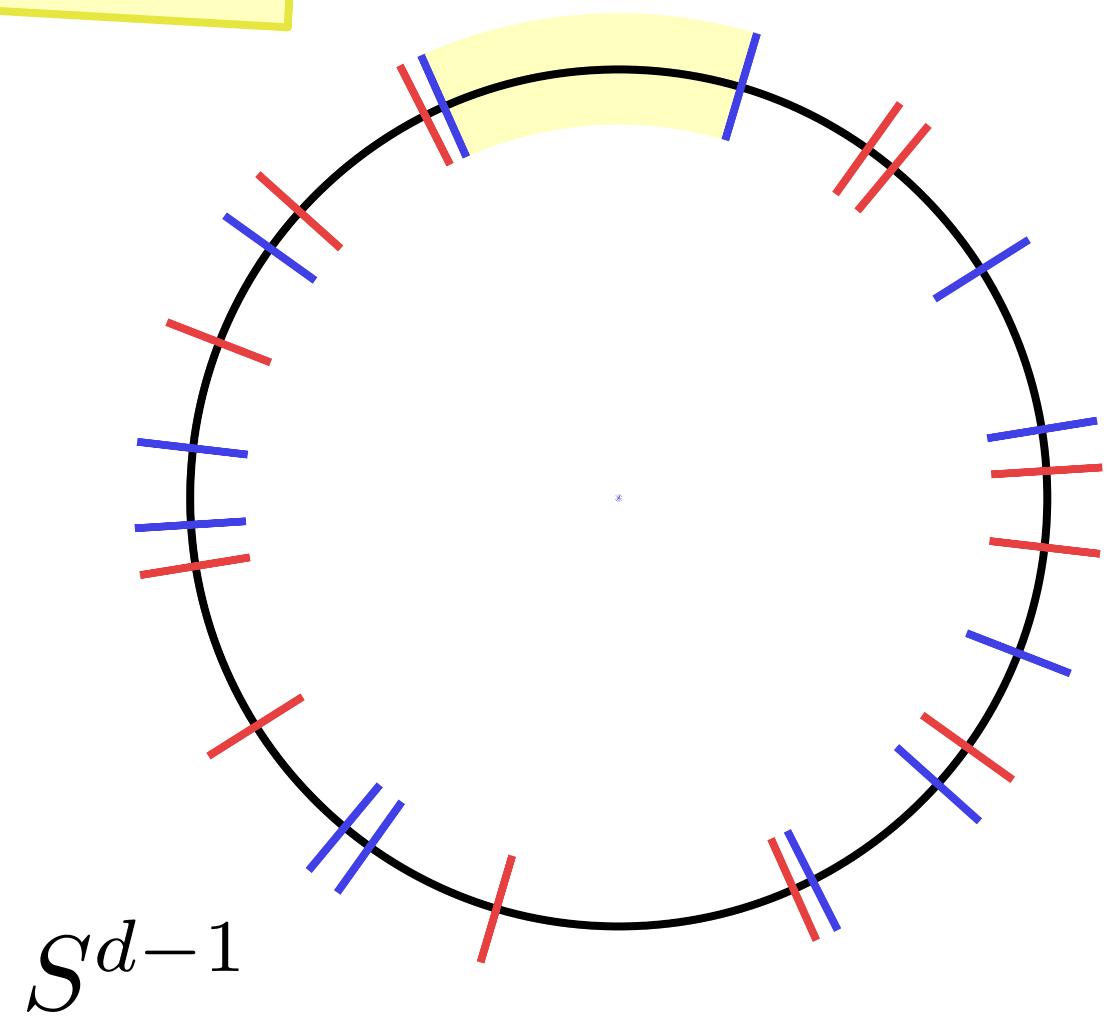

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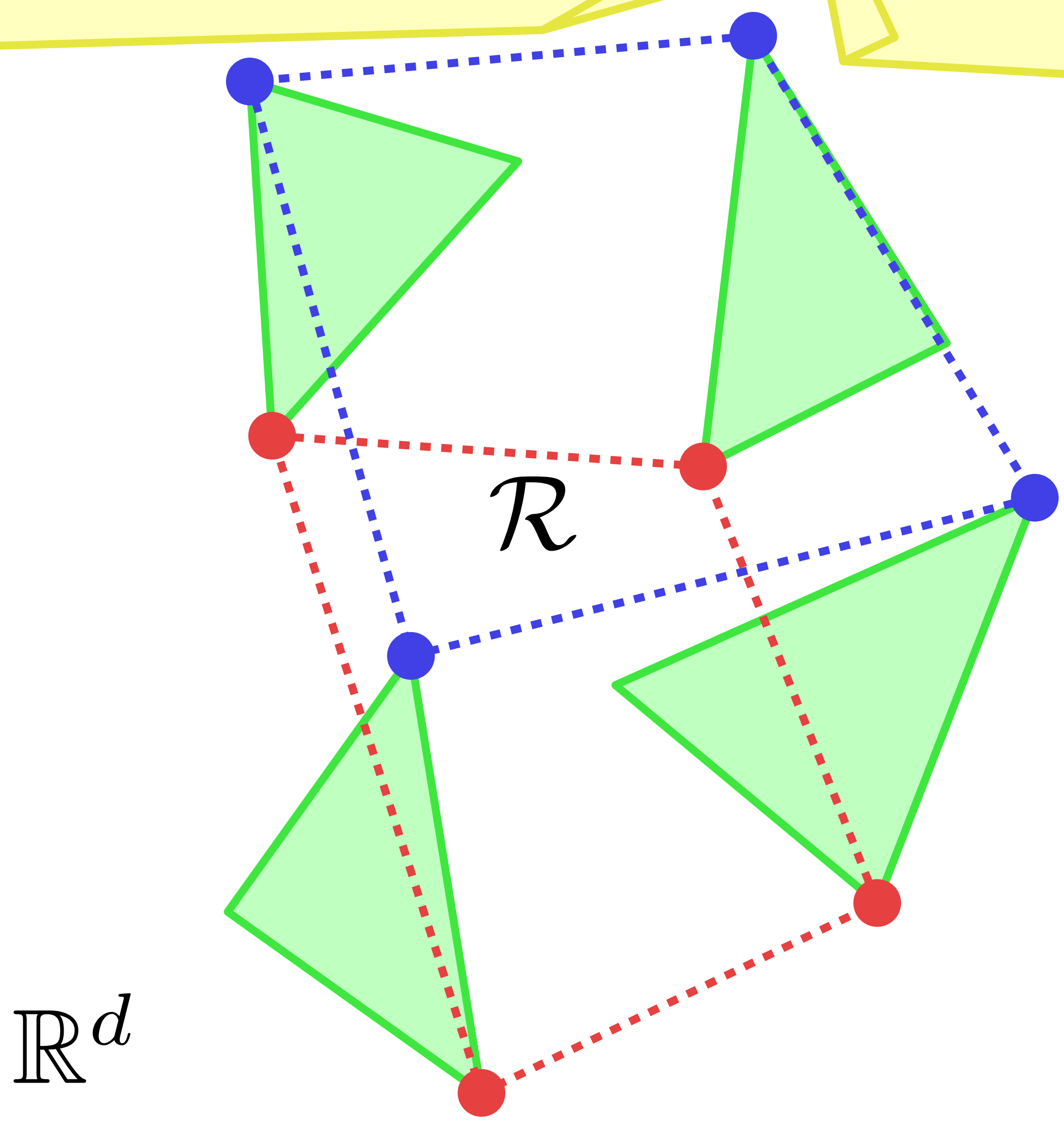

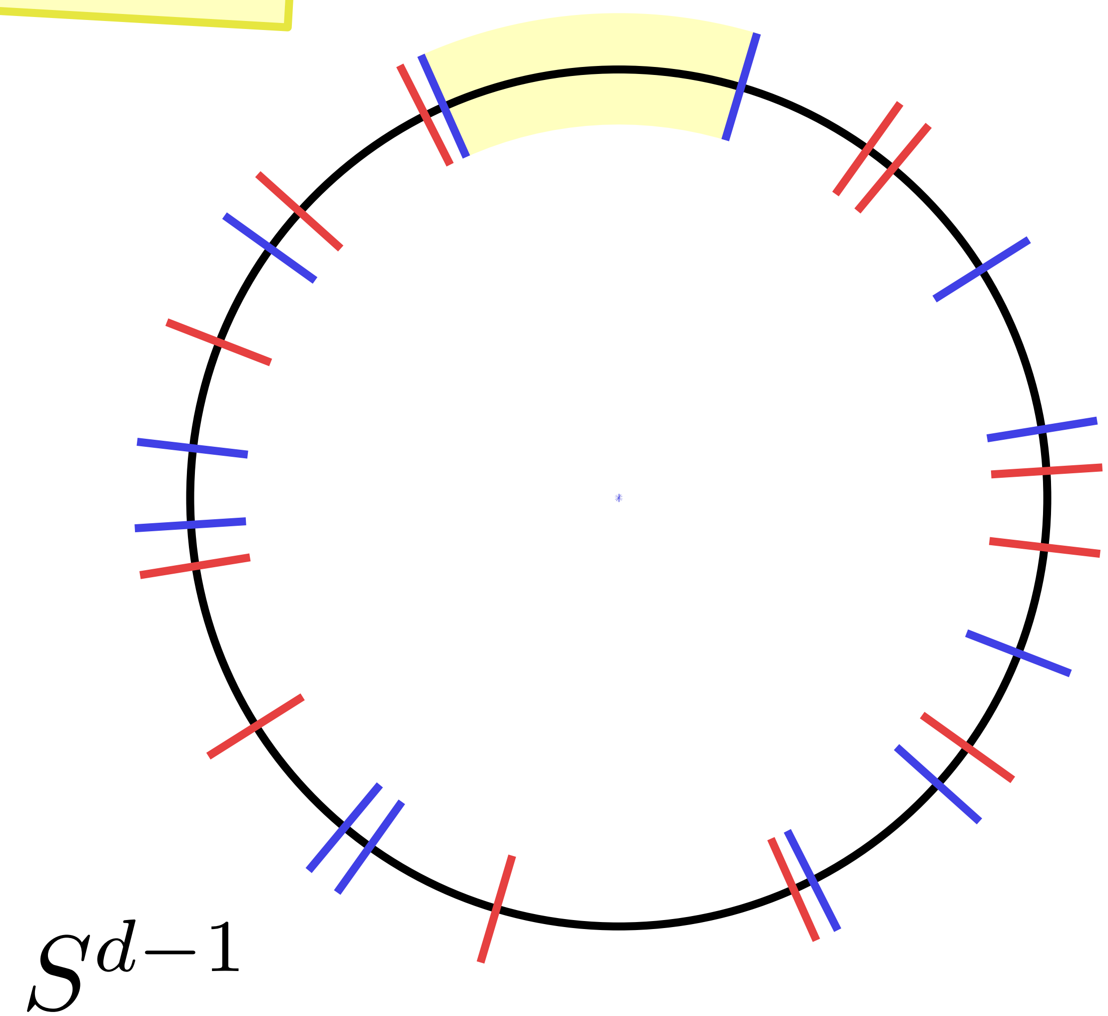

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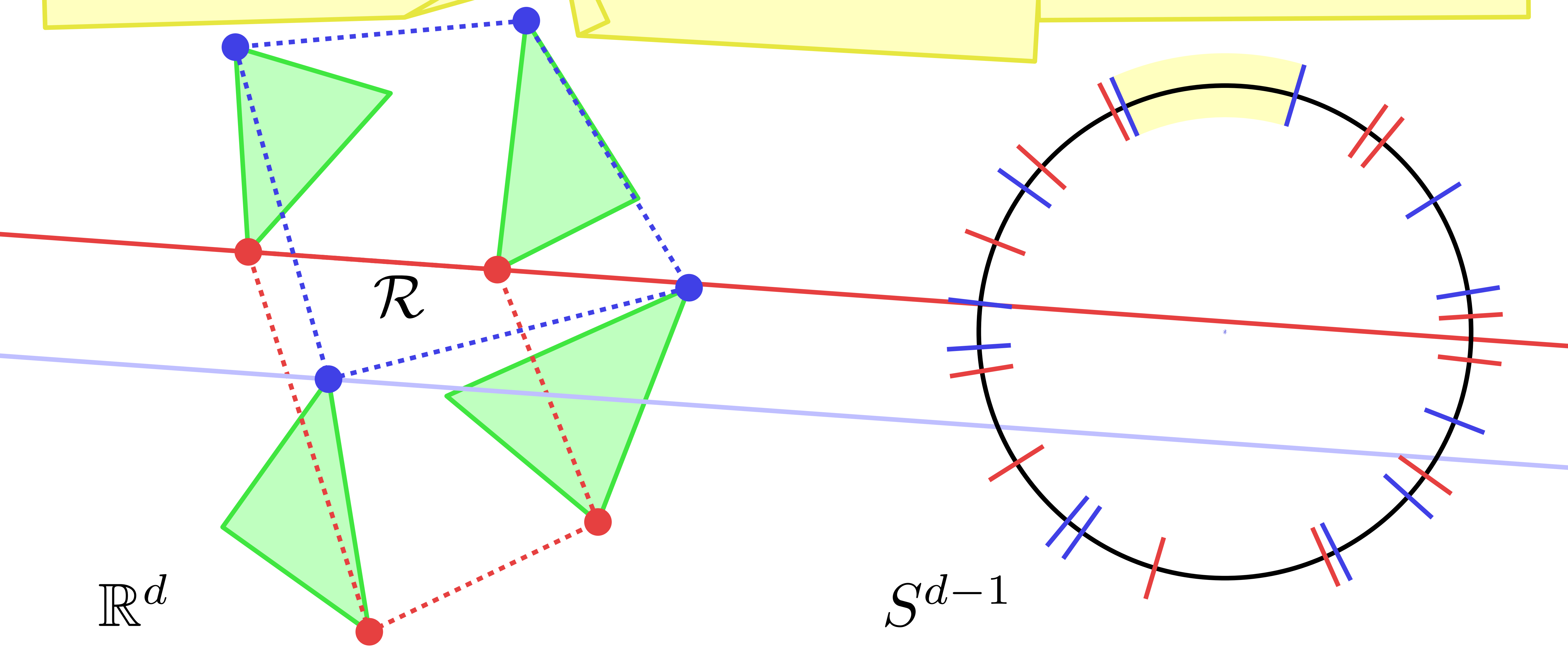

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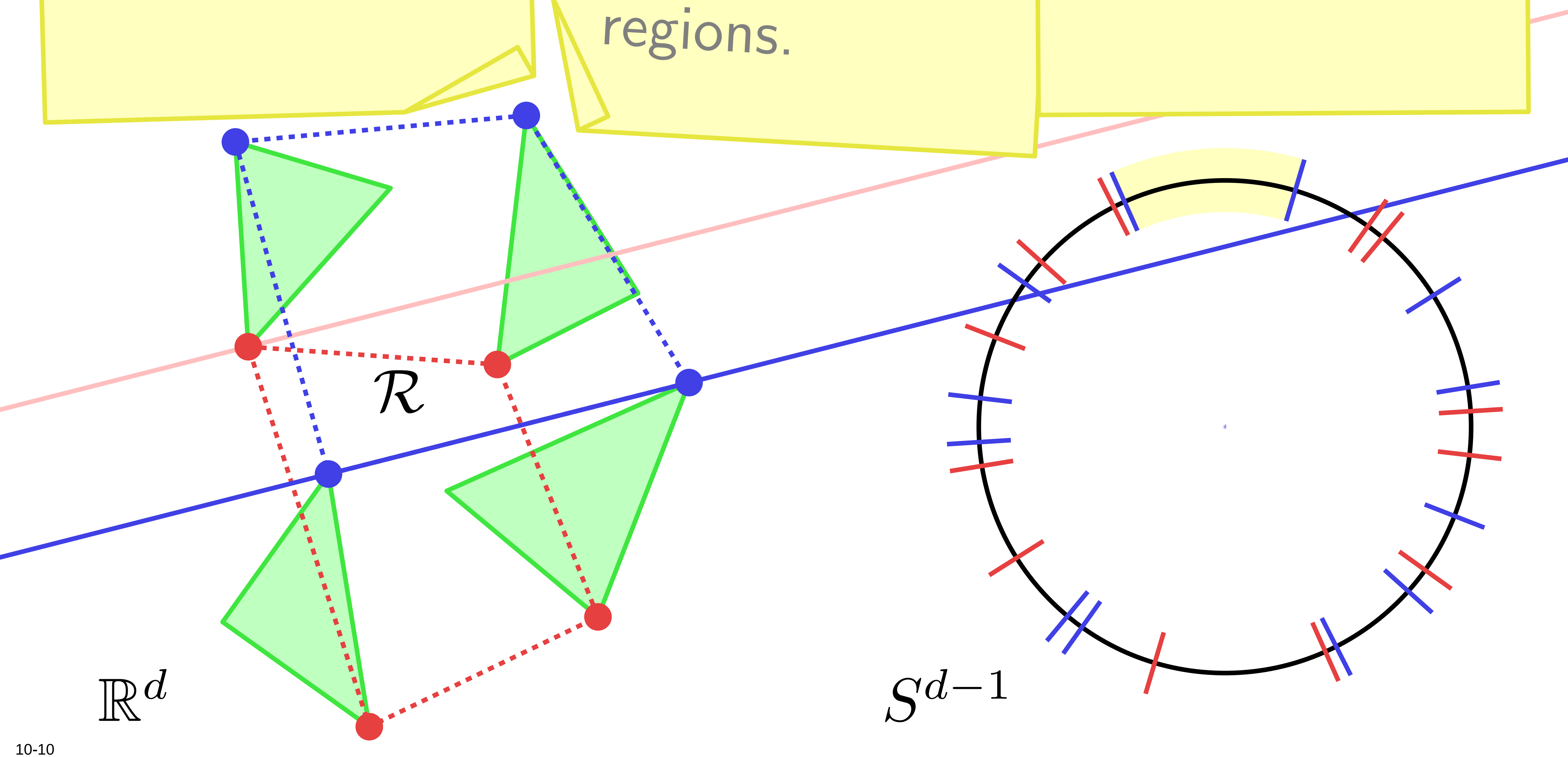
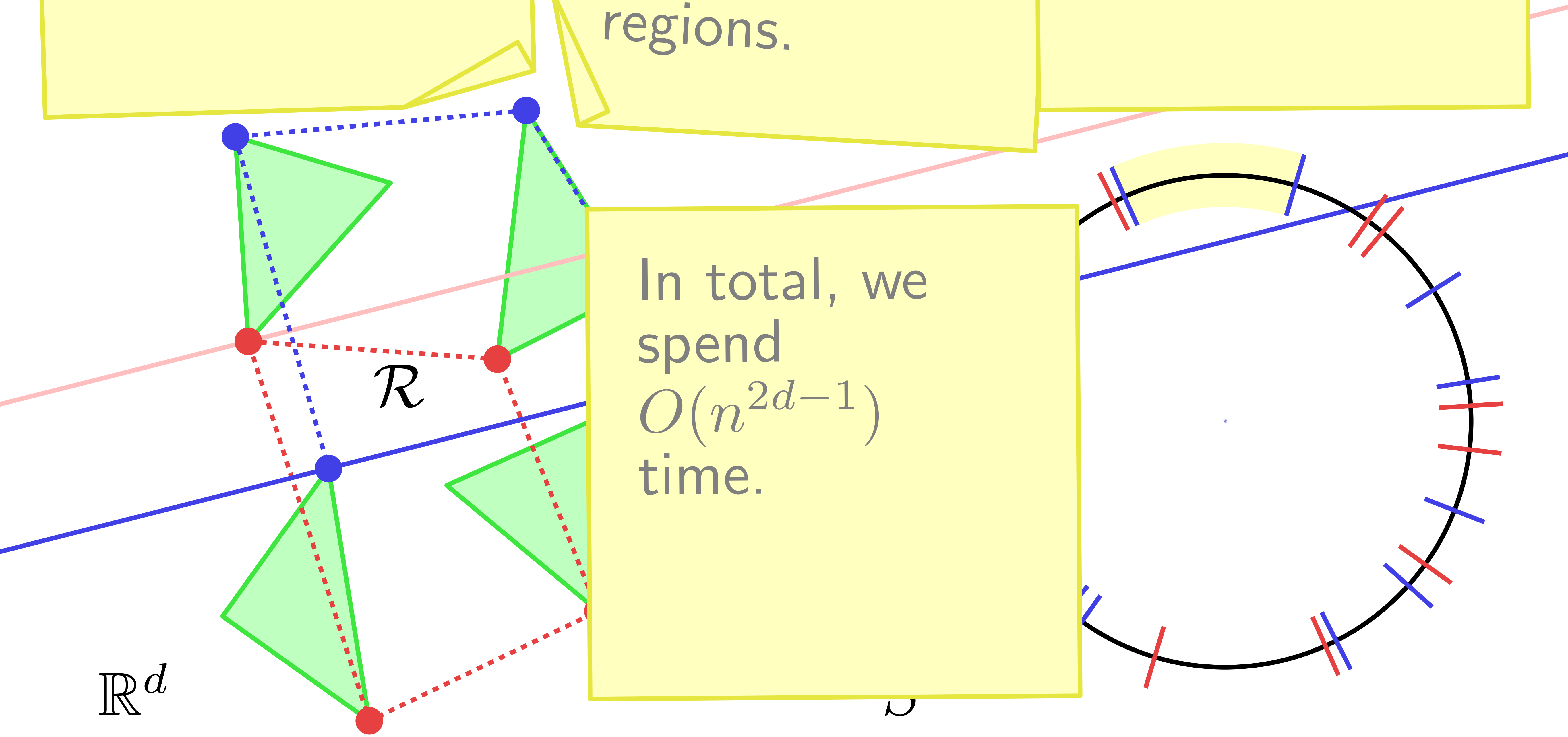


Consider the set of regions $\mathcal{R}$.

We can compute the overlay of the subdivisions of S^{d-1} of all regions.

Within each cell, we can now compute the smallest width.

In total, we spend $O(n^{2d-1})$ time.



What about the *largest* width?

The largest width could be determined by many triples of points, keeping each other in balance.

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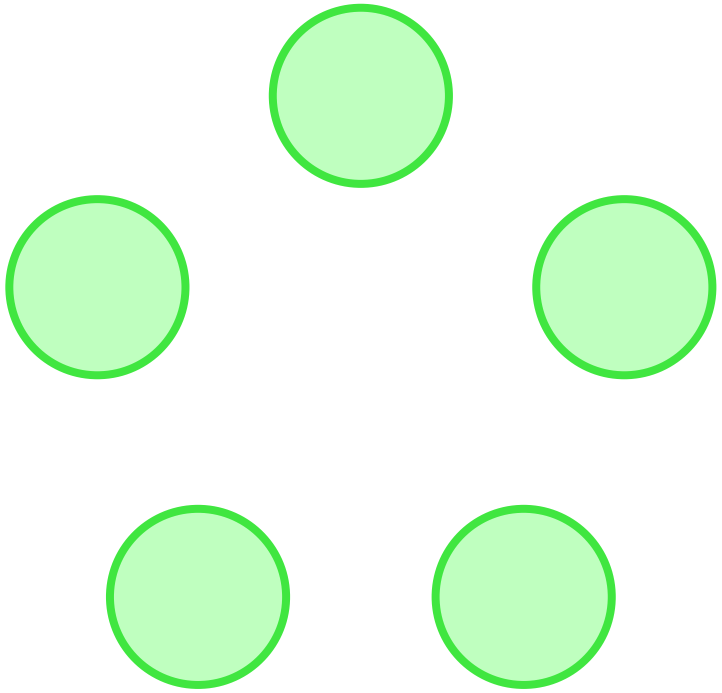

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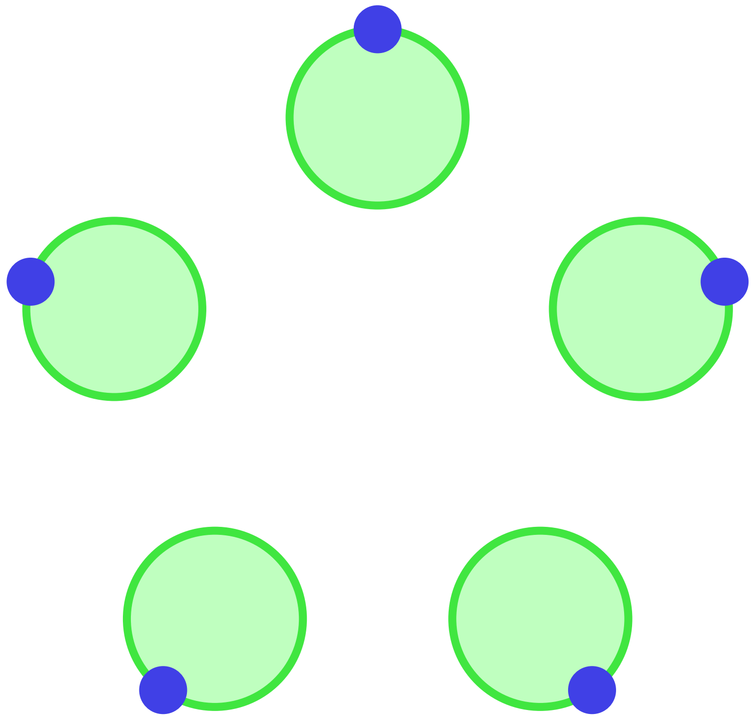

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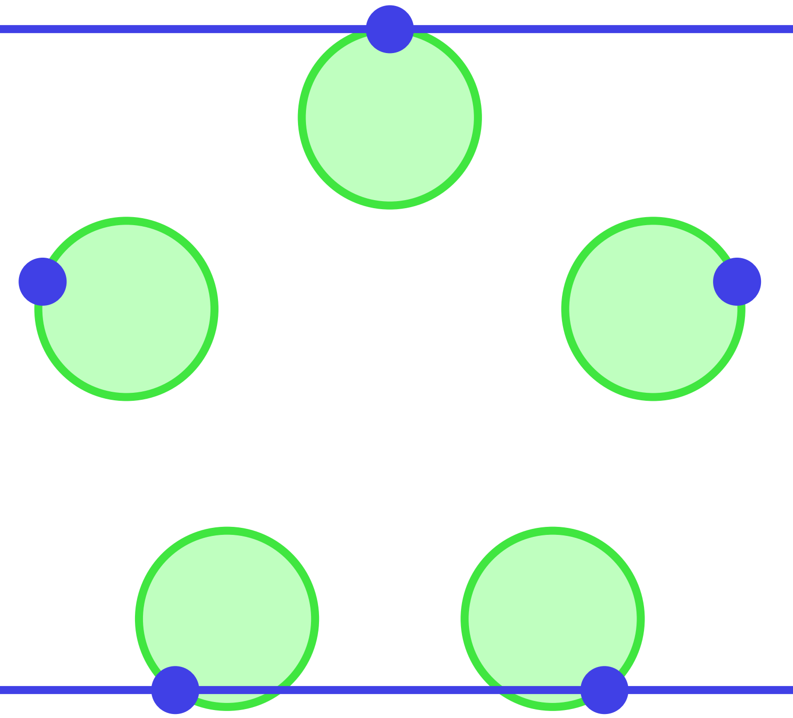

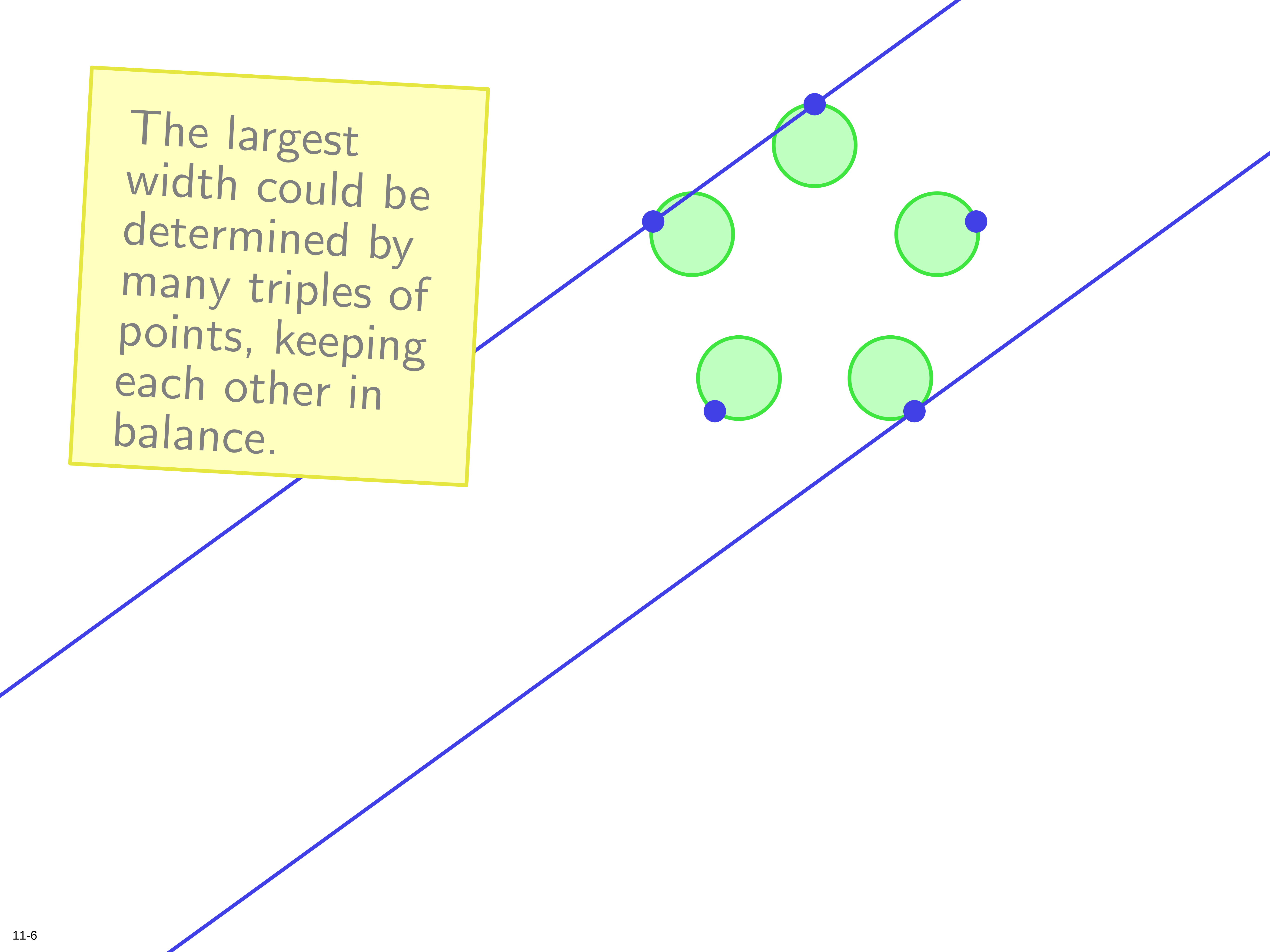
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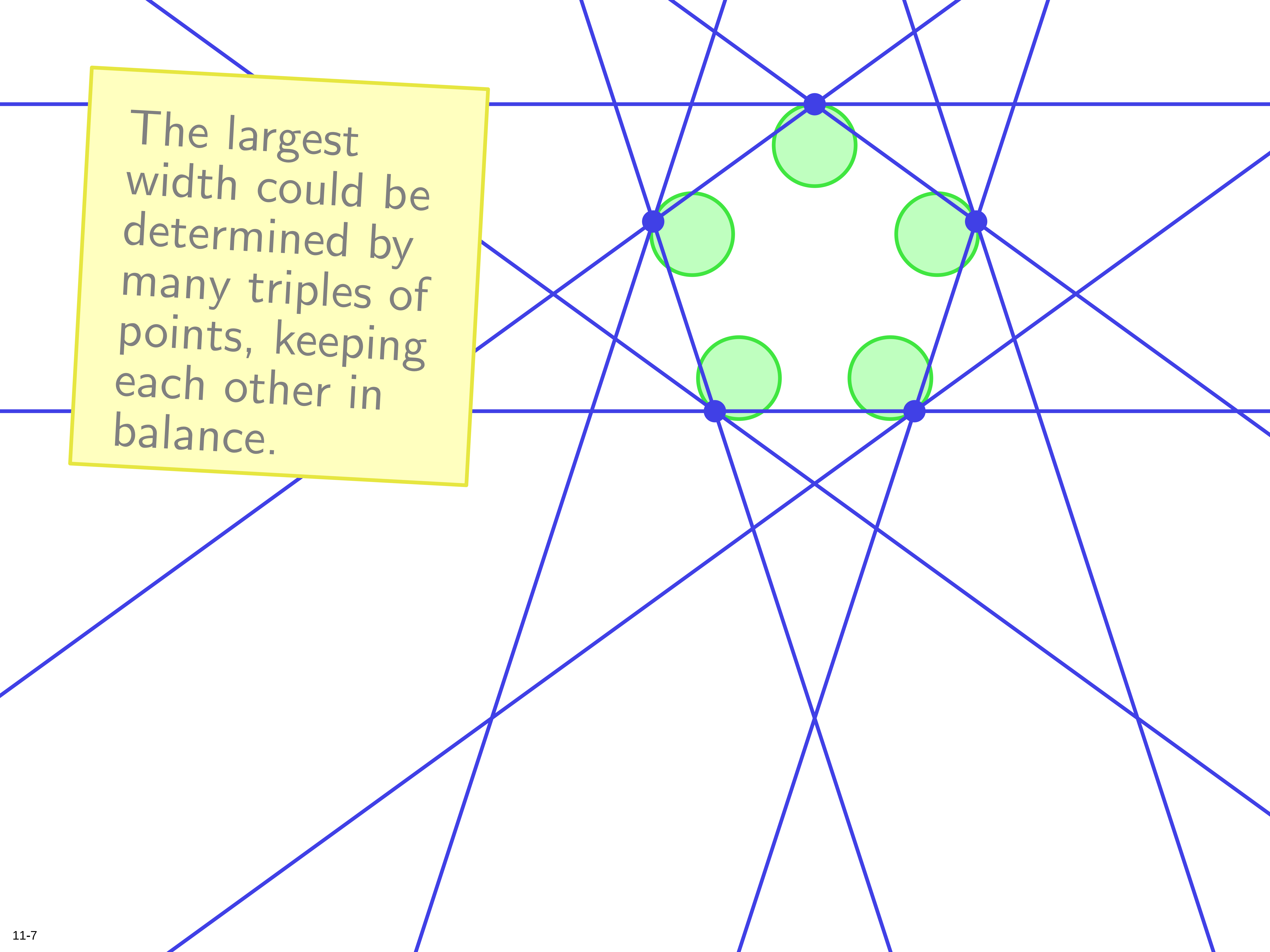
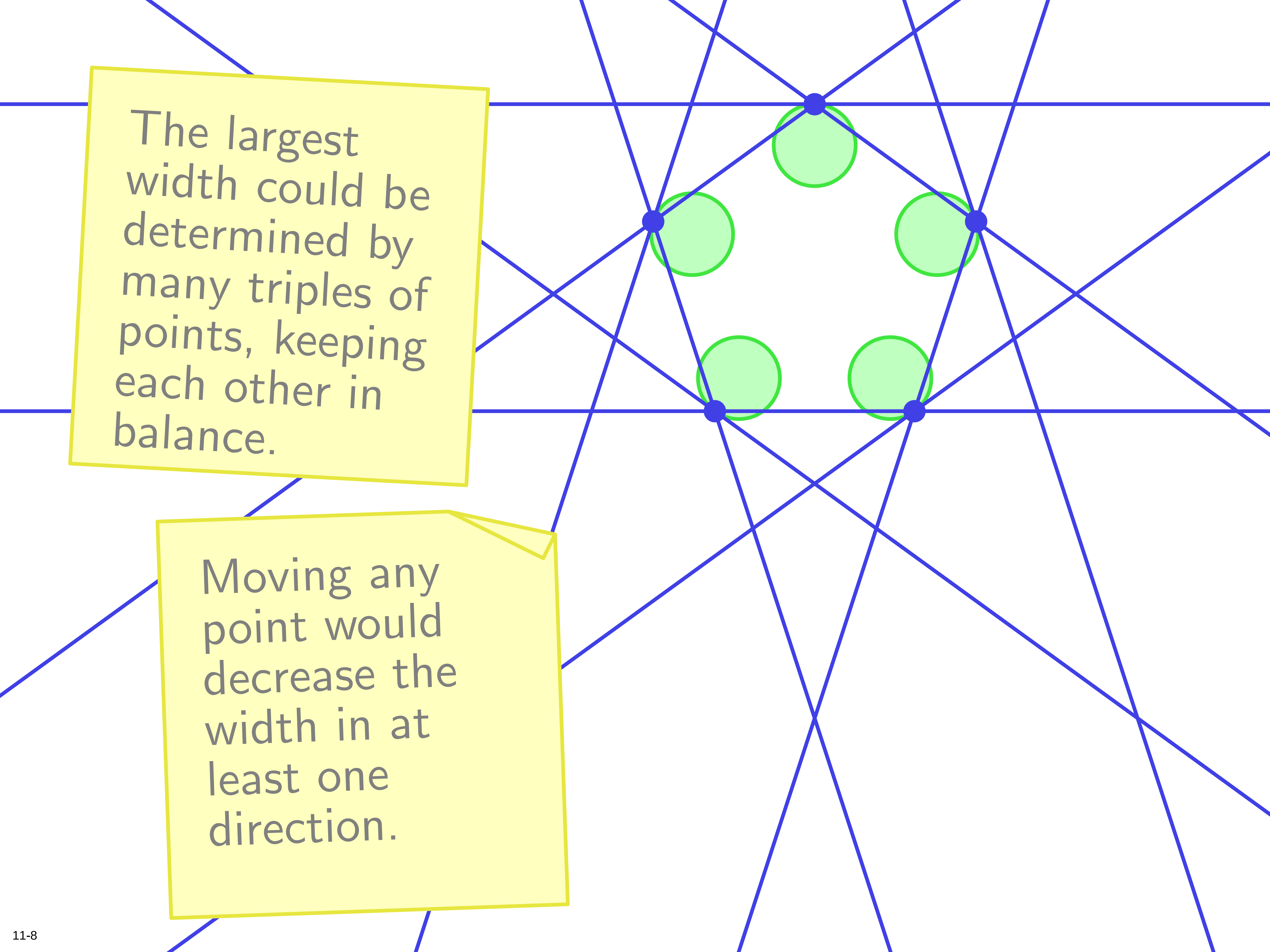
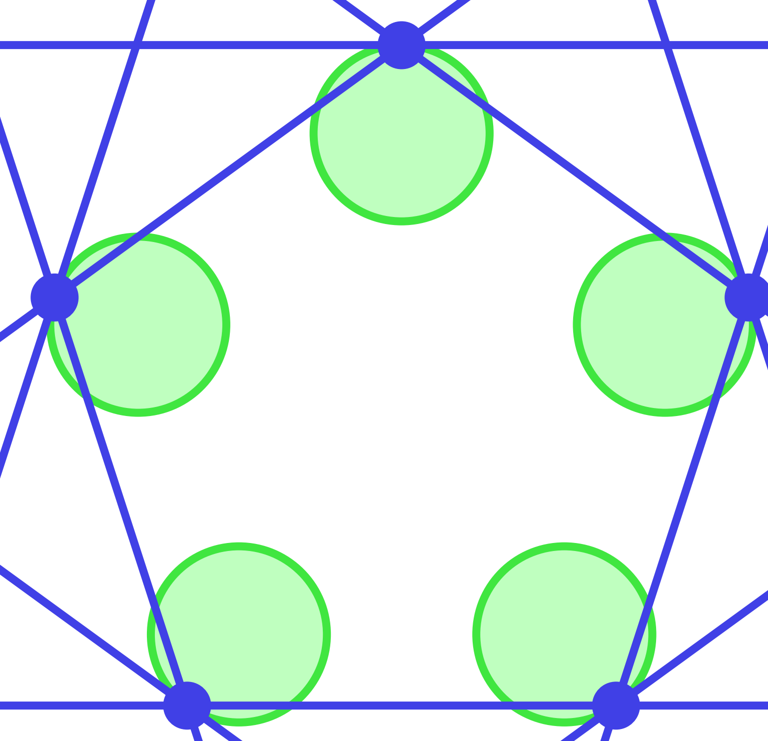




The largest width could be determined by many triples of points, keeping each other in balance.

Moving any point would decrease the width in at least one direction.



The largest width could be determined by many triples of points, keeping each other in balance.

Moving any point would decrease the width in at least one direction.

Indeed, computing the largest width in $\mathbb{R}^2$ is NP-hard. [van Kreveld & ML, 2006]

We can generalise that proof to $\mathbb{R}^d$.

Consider a set of regions $\mathcal{R}$ in the plane.

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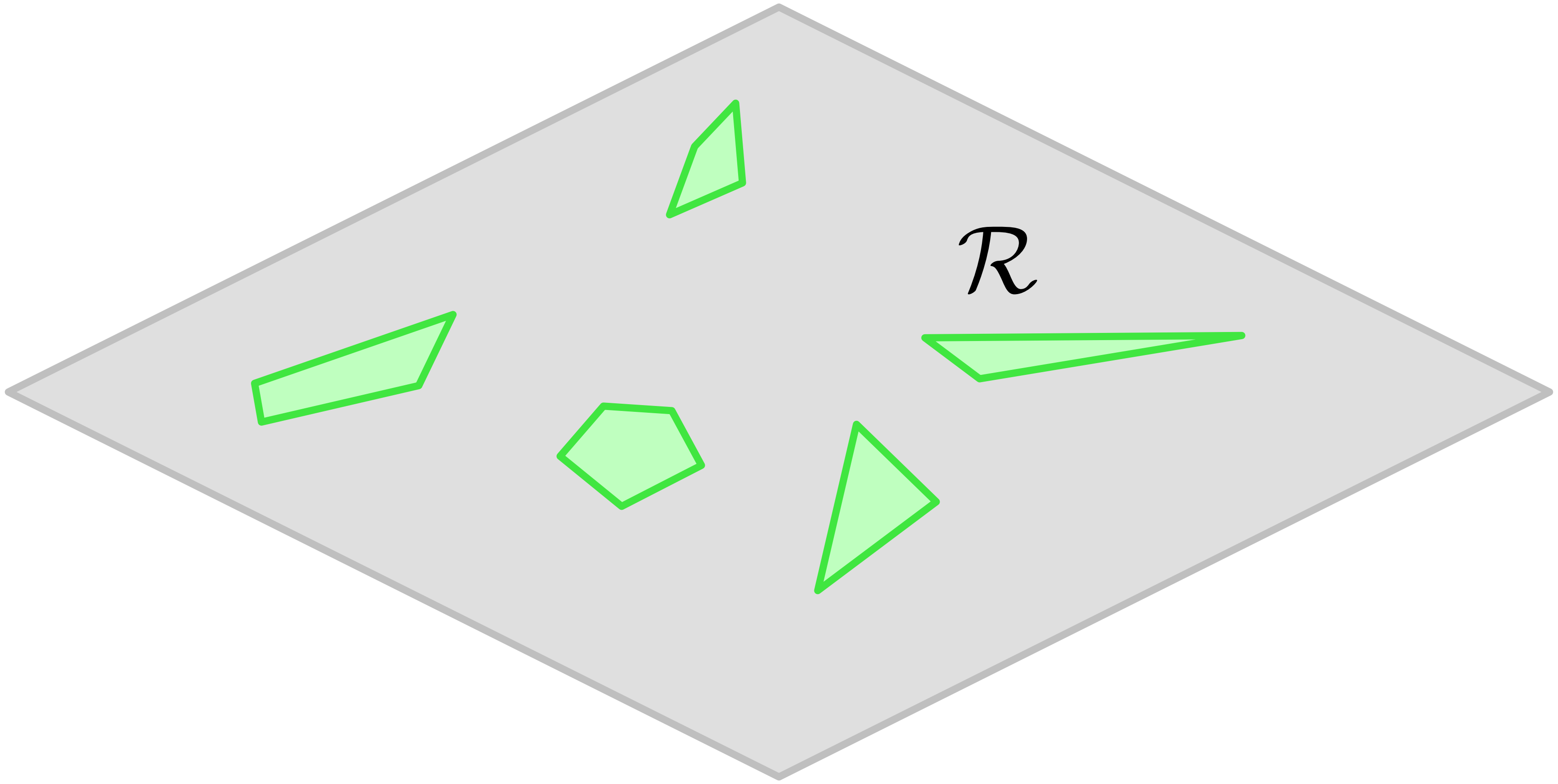
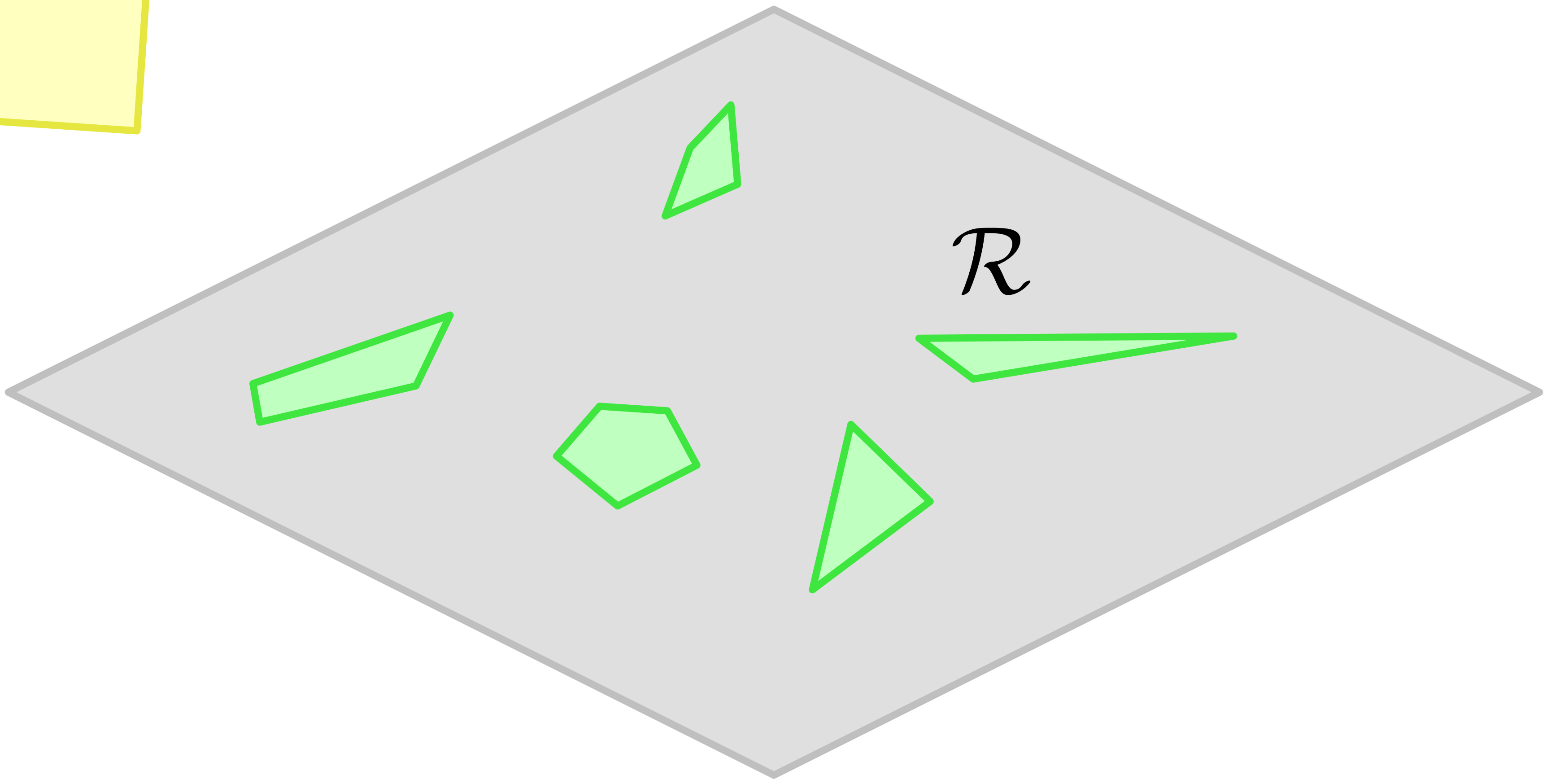


Consider a set of regions $\mathcal{R}$ in the plane.

We extend these regions very far in the $d-1$ other dimensions to a set $\mathcal{R}'$.



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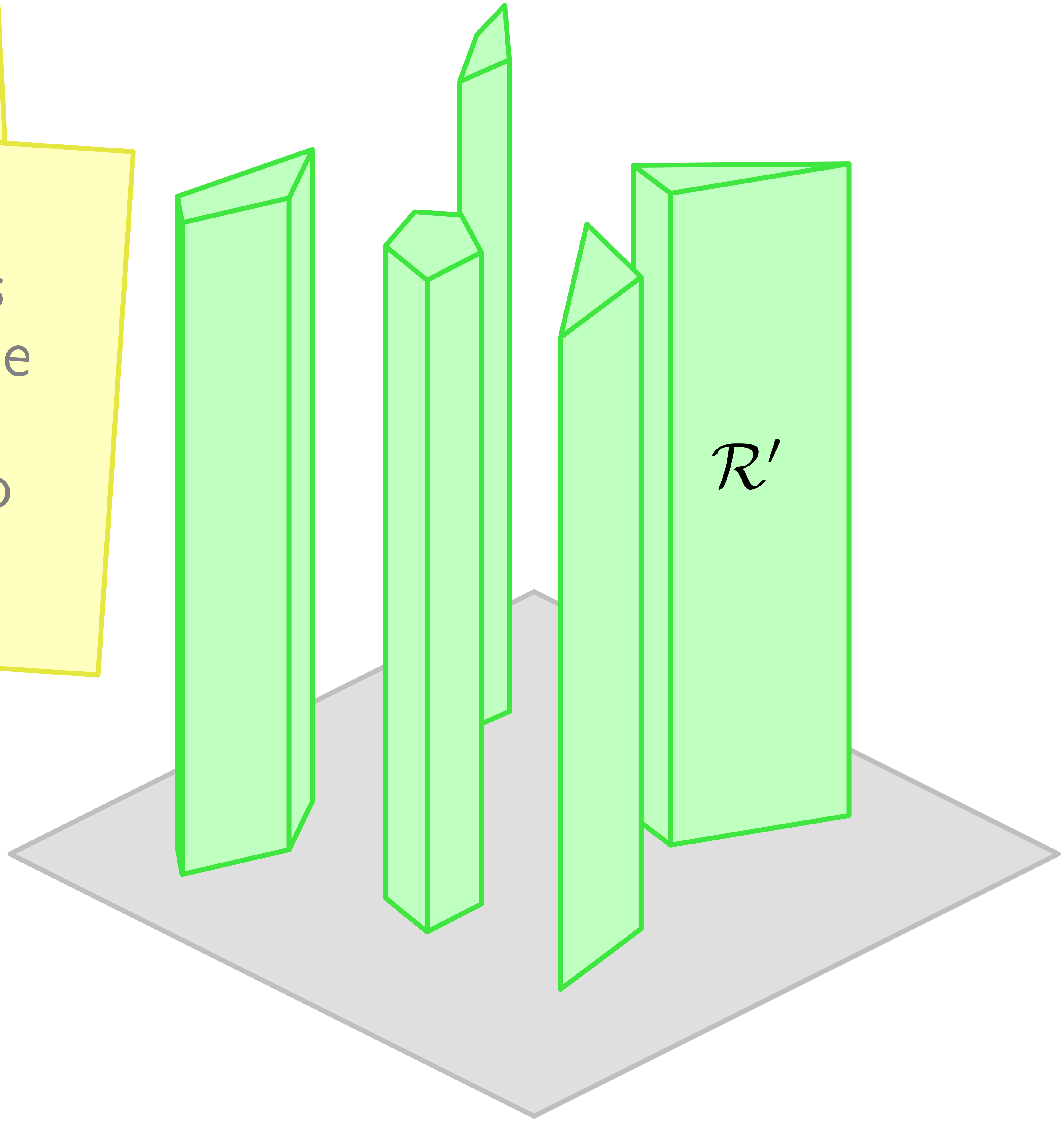
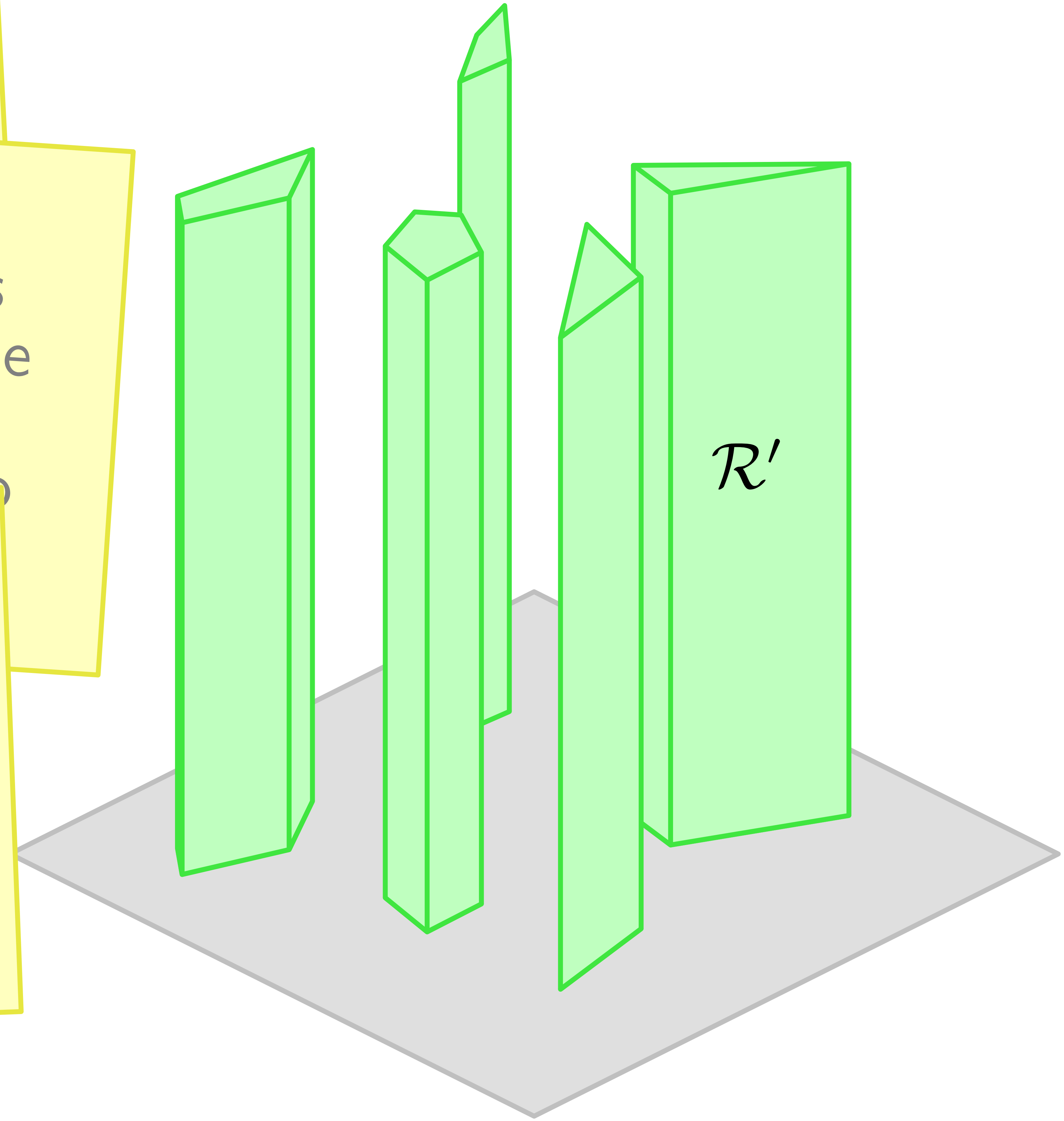


Consider a set of regions $\mathcal{R}$ in the pla

We extend these regions very far in the $d-1$ other dimensions to

Clearly, the width of $\mathcal{R}'$ cannot be larger than of $\mathcal{R}$.

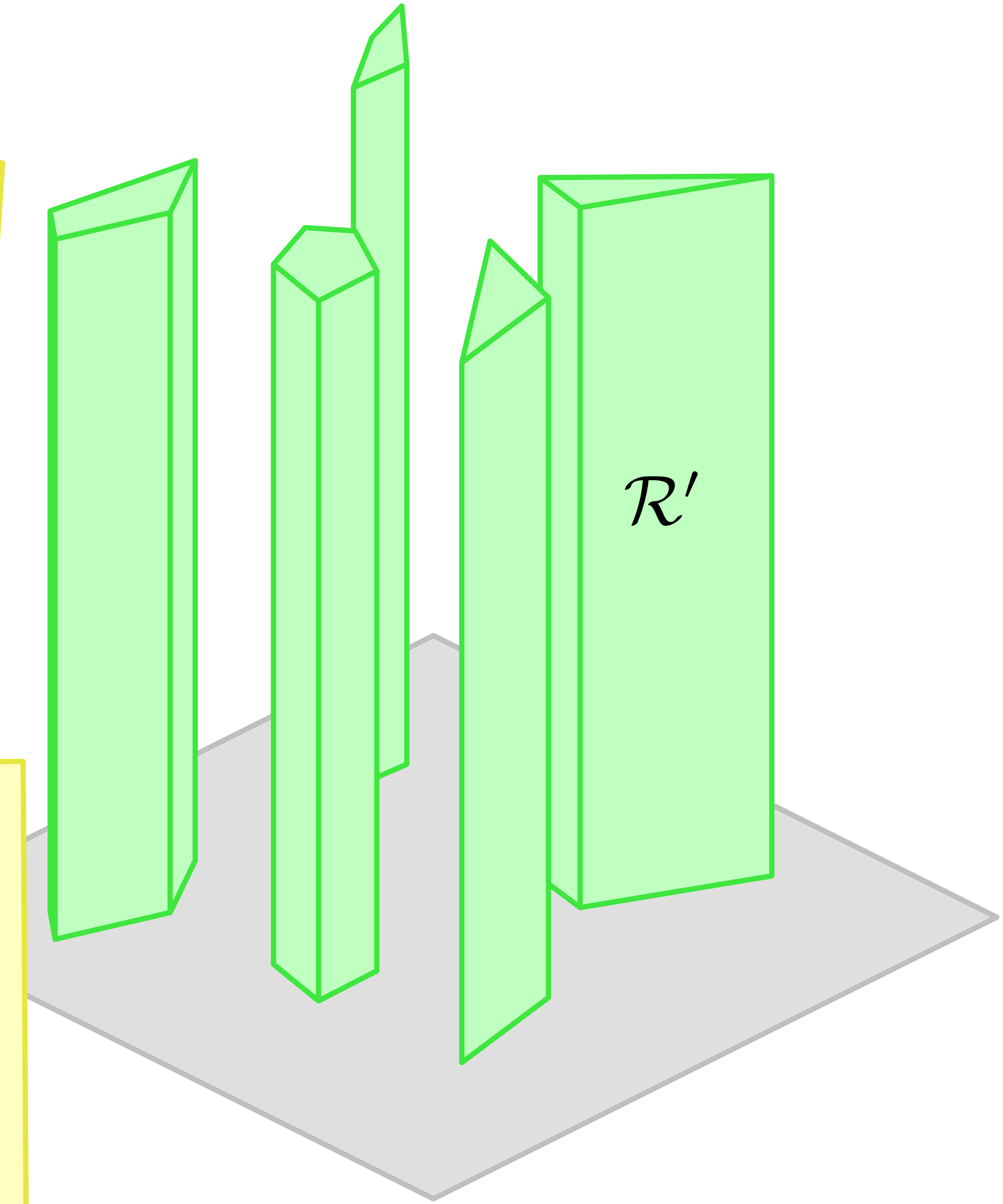


Consider a set of regions $\mathcal{R}$ in the pl

We extend these regions very far in the $d-1$ other dimensions to

Clearly, the width of $\mathcal{R}'$ cannot be larg $\mathcal{R}$.

For any P in $\mathcal{R}$ we can place points in $\mathcal{R}'$ such that the width comes arbitrary close.



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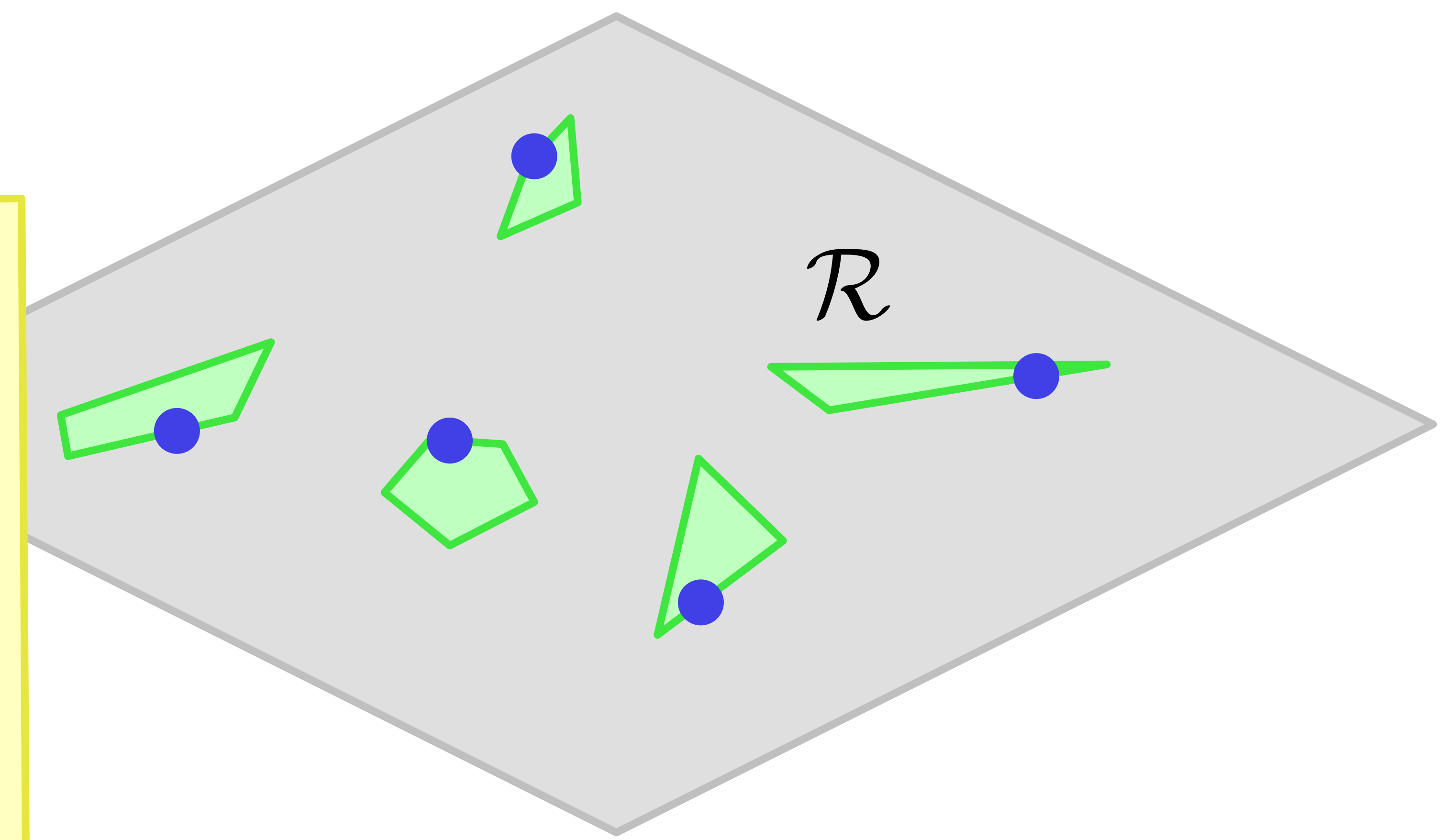

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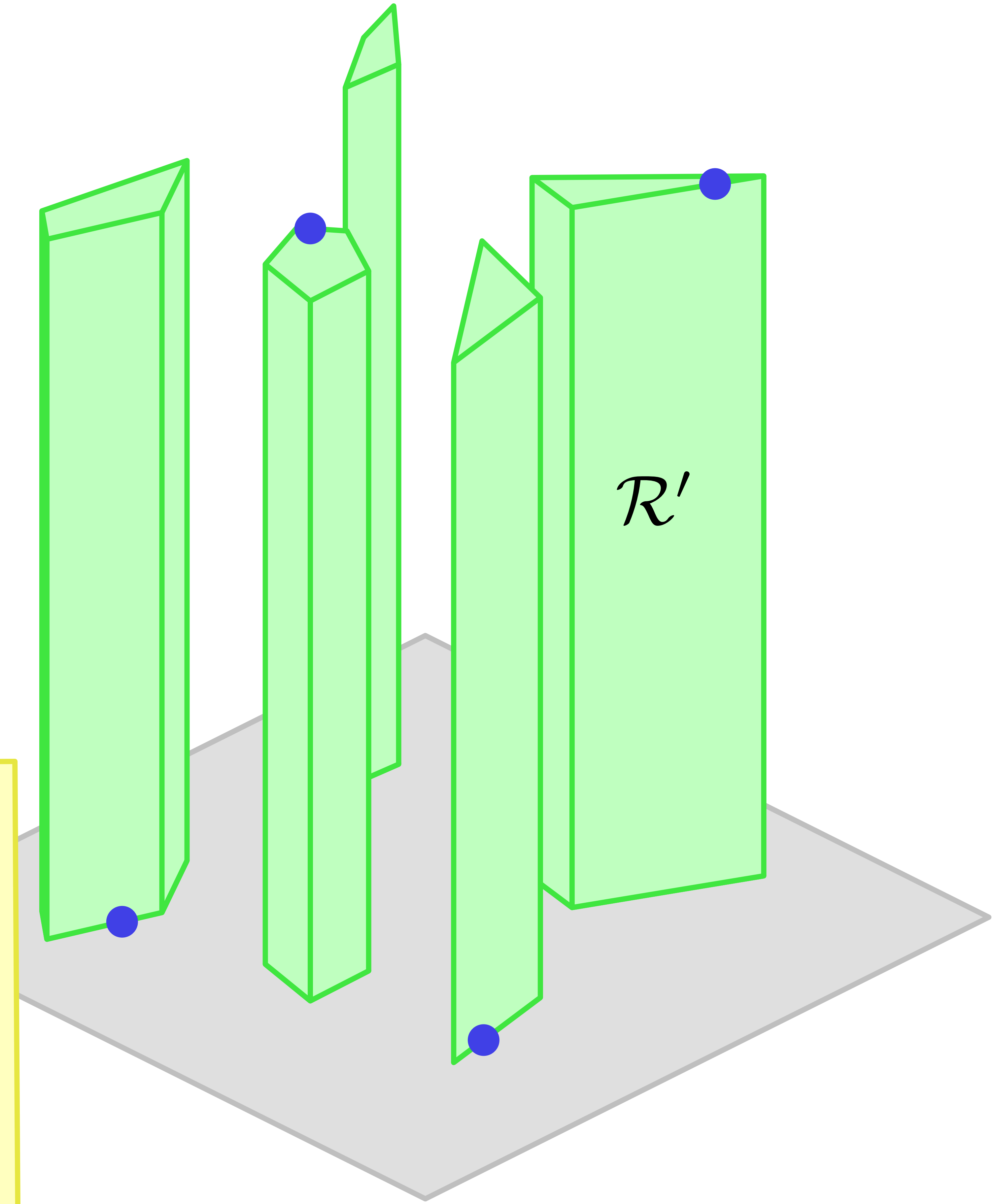

Thank you for your attention!

Thank you for your attention!

And see you all in 30 minutes in the other session! ☺