HOW MANY POTATOES ARE IN A MESH?

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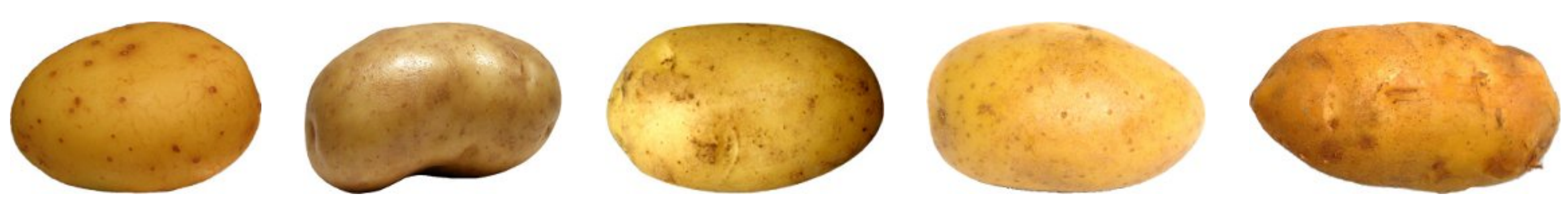
1 2 3 4 5



A LITTLE HISTORY

DEFINITION “POTATO”

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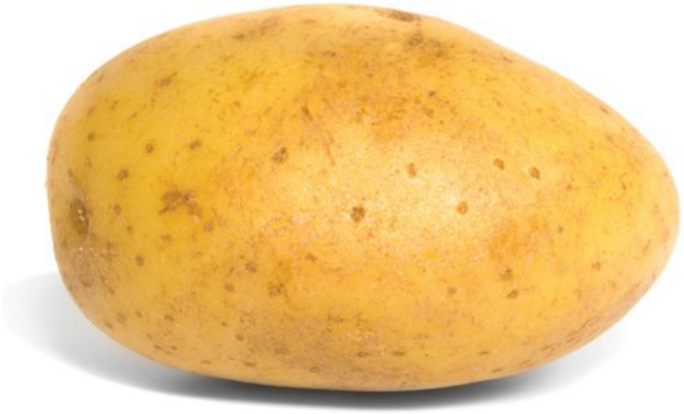

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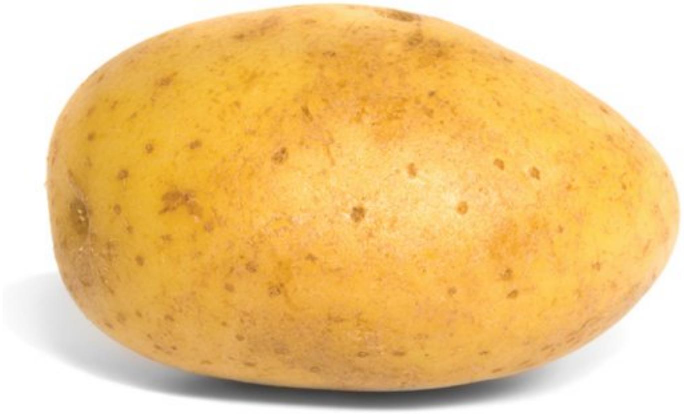

- Subset P of $\mathbb{R}^2$

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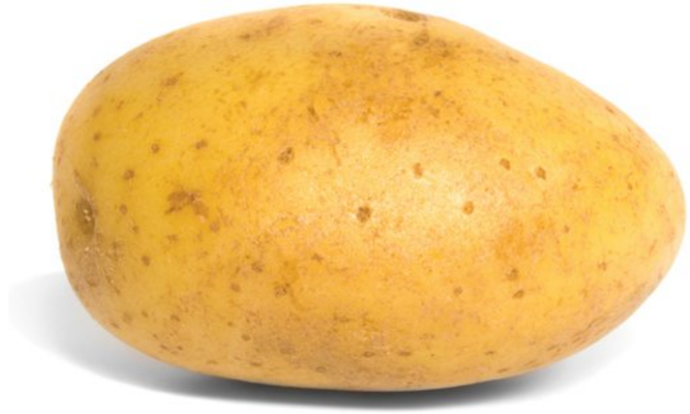

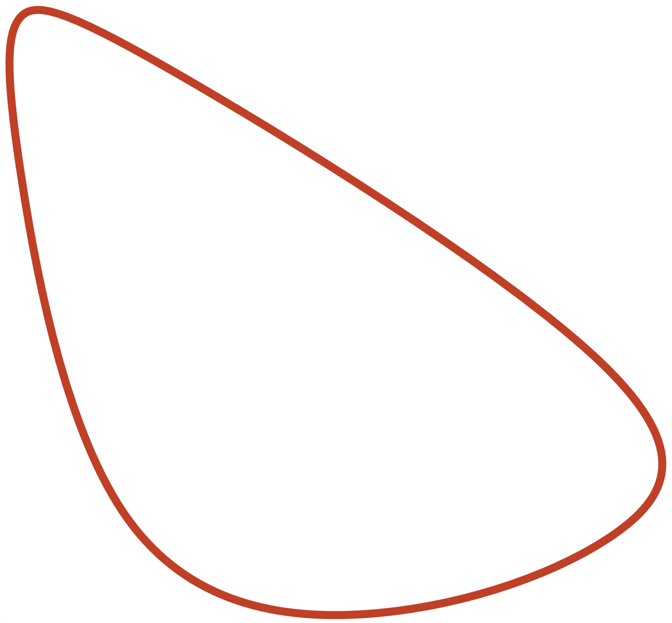

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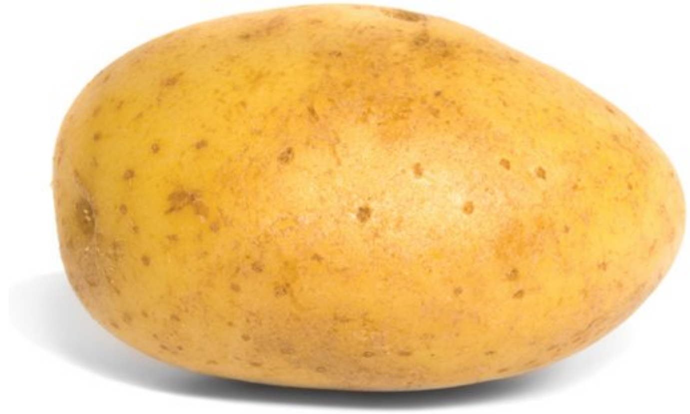

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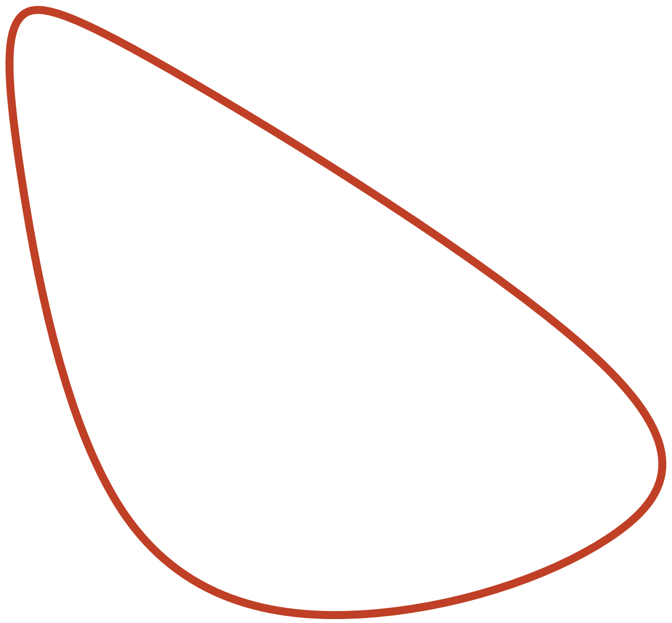

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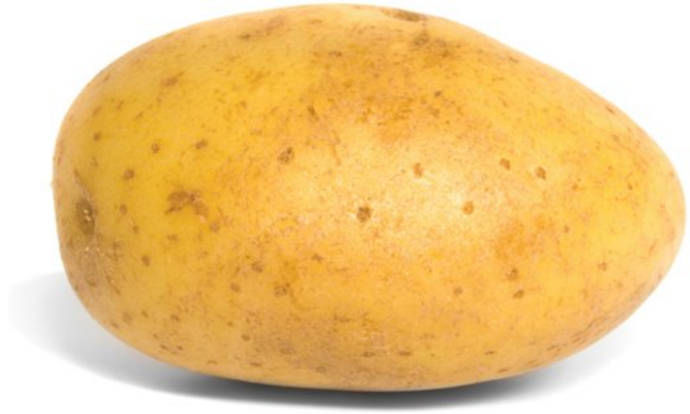

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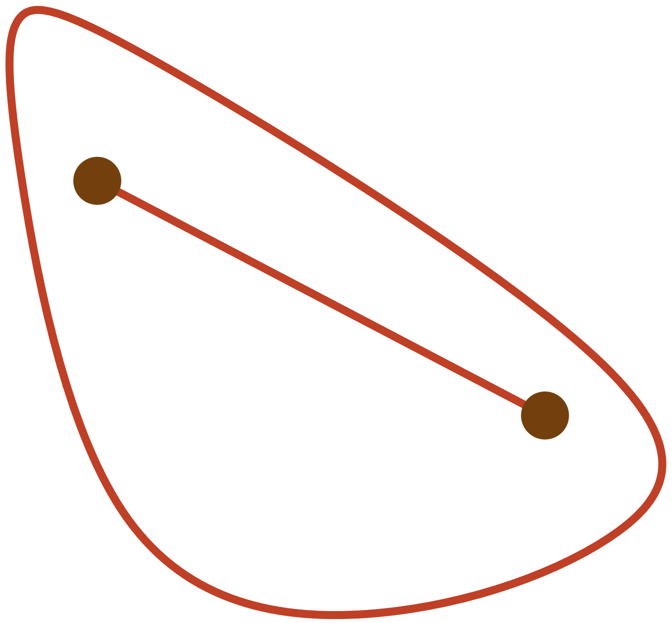

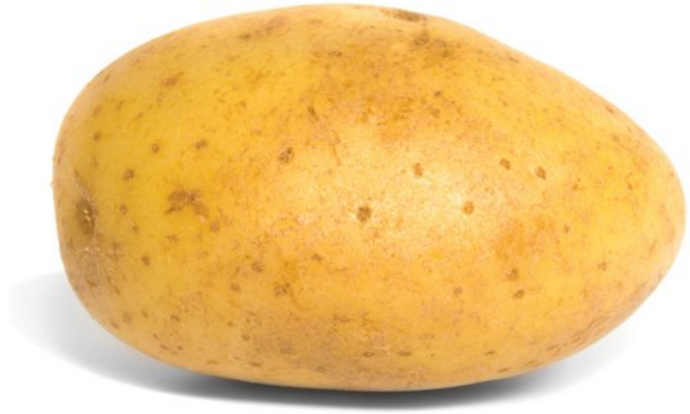

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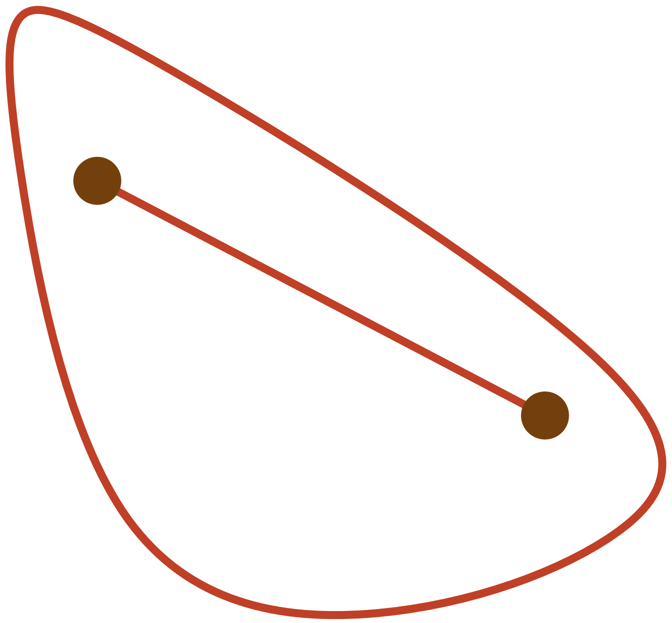

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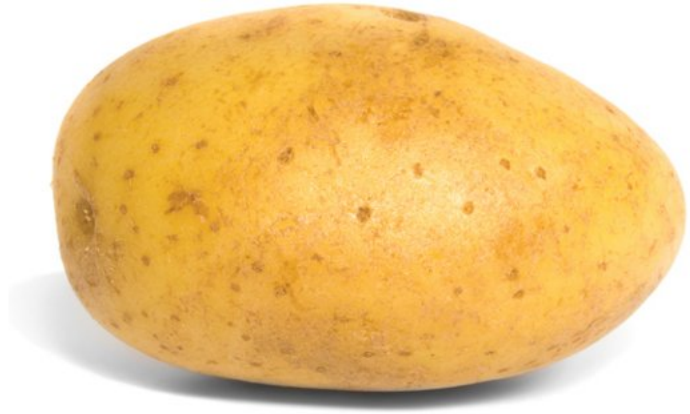

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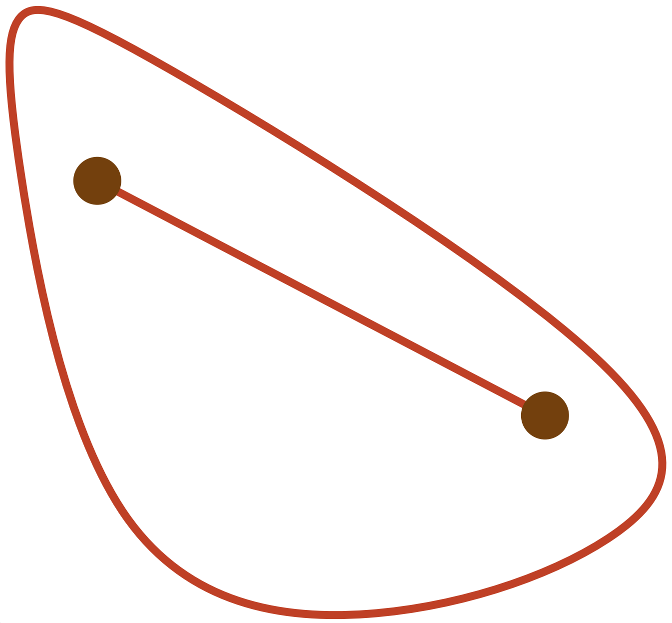

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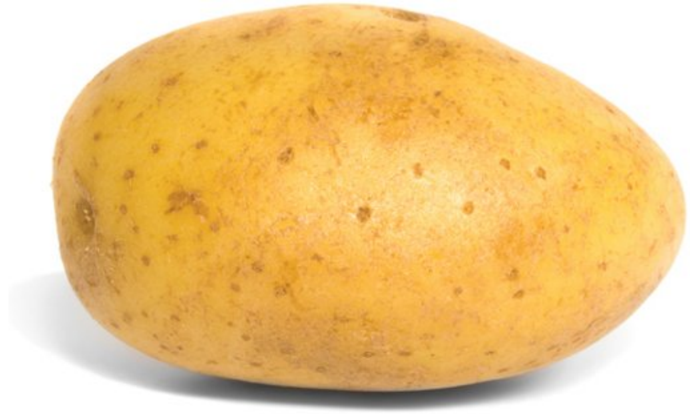

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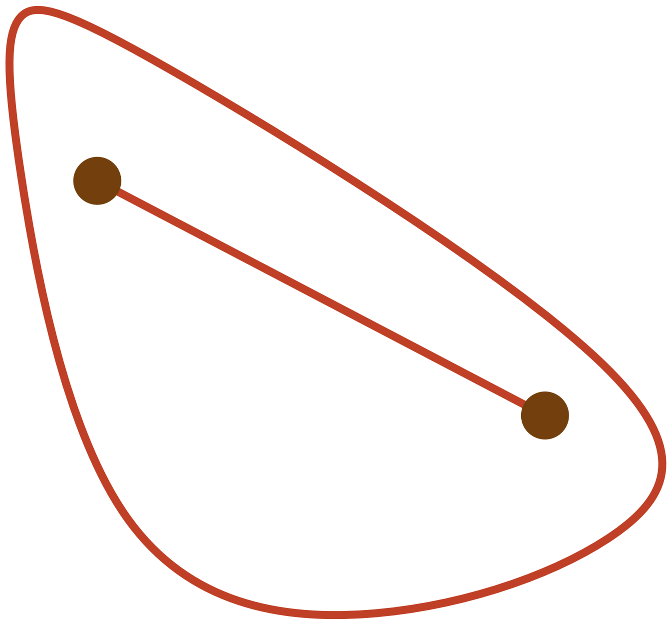

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Find the largest potato contained in a given polygon.

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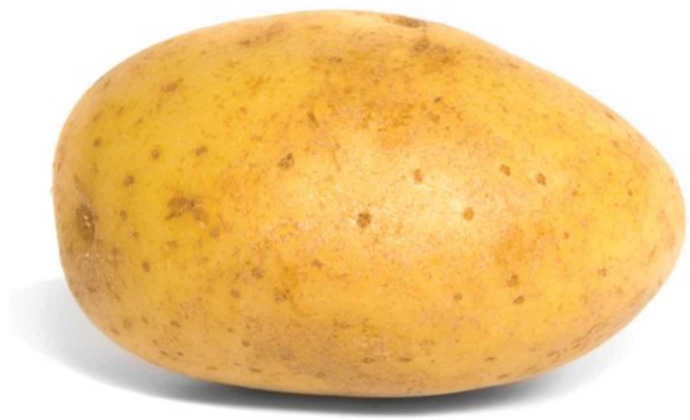

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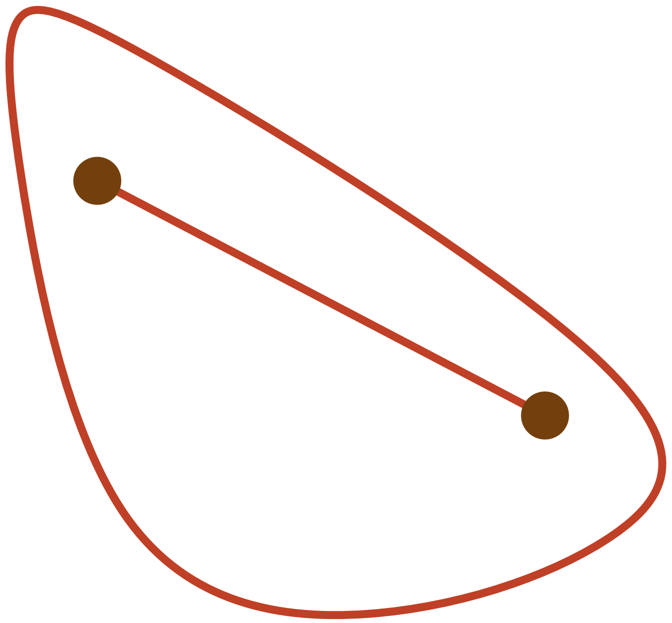

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(Goodman, 1981)

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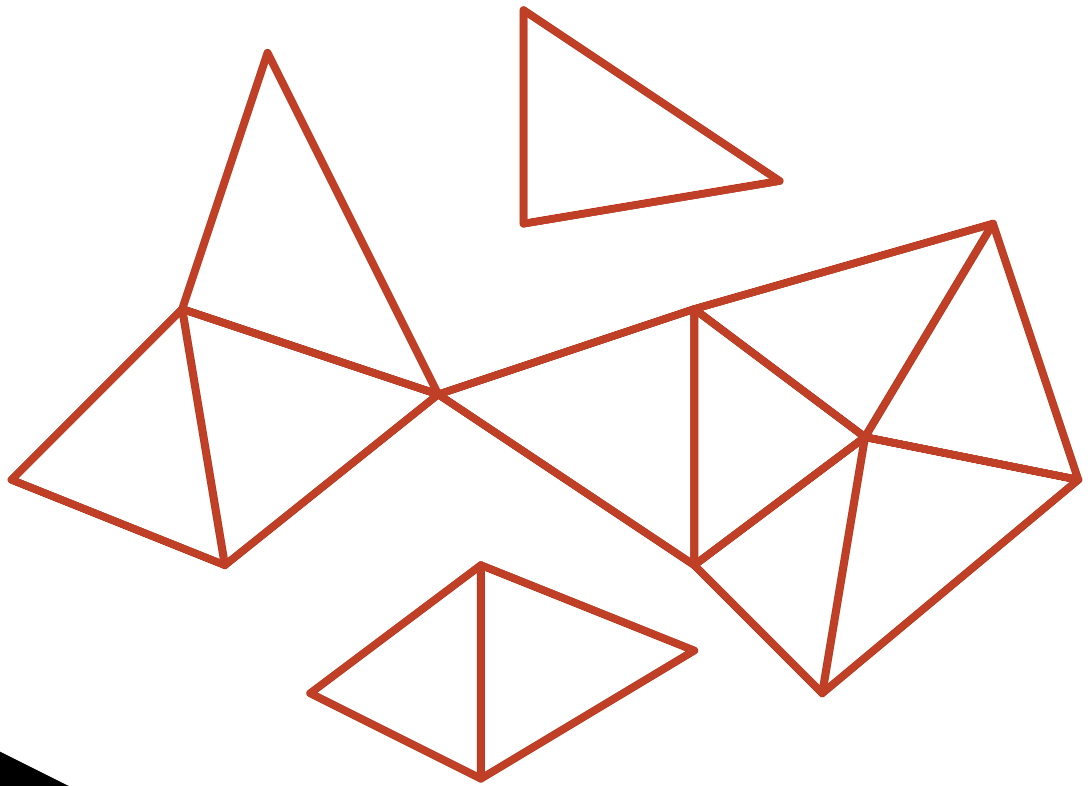

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- Any two triangles:
  - are disjoint,
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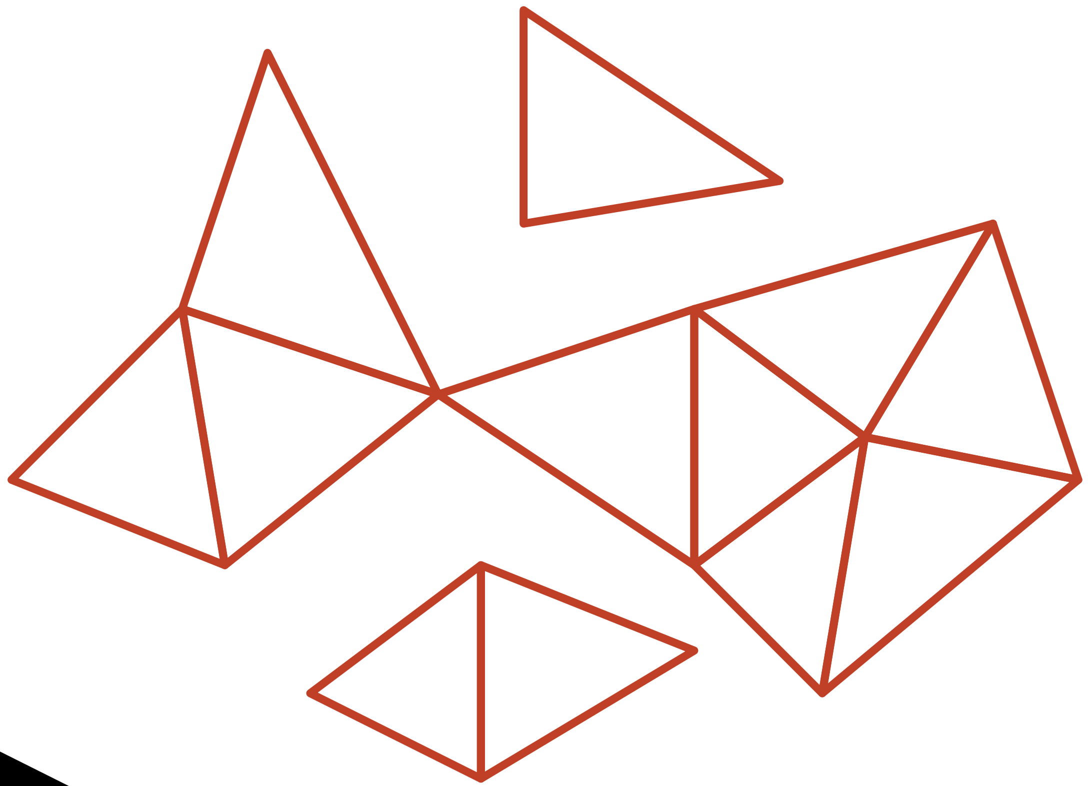

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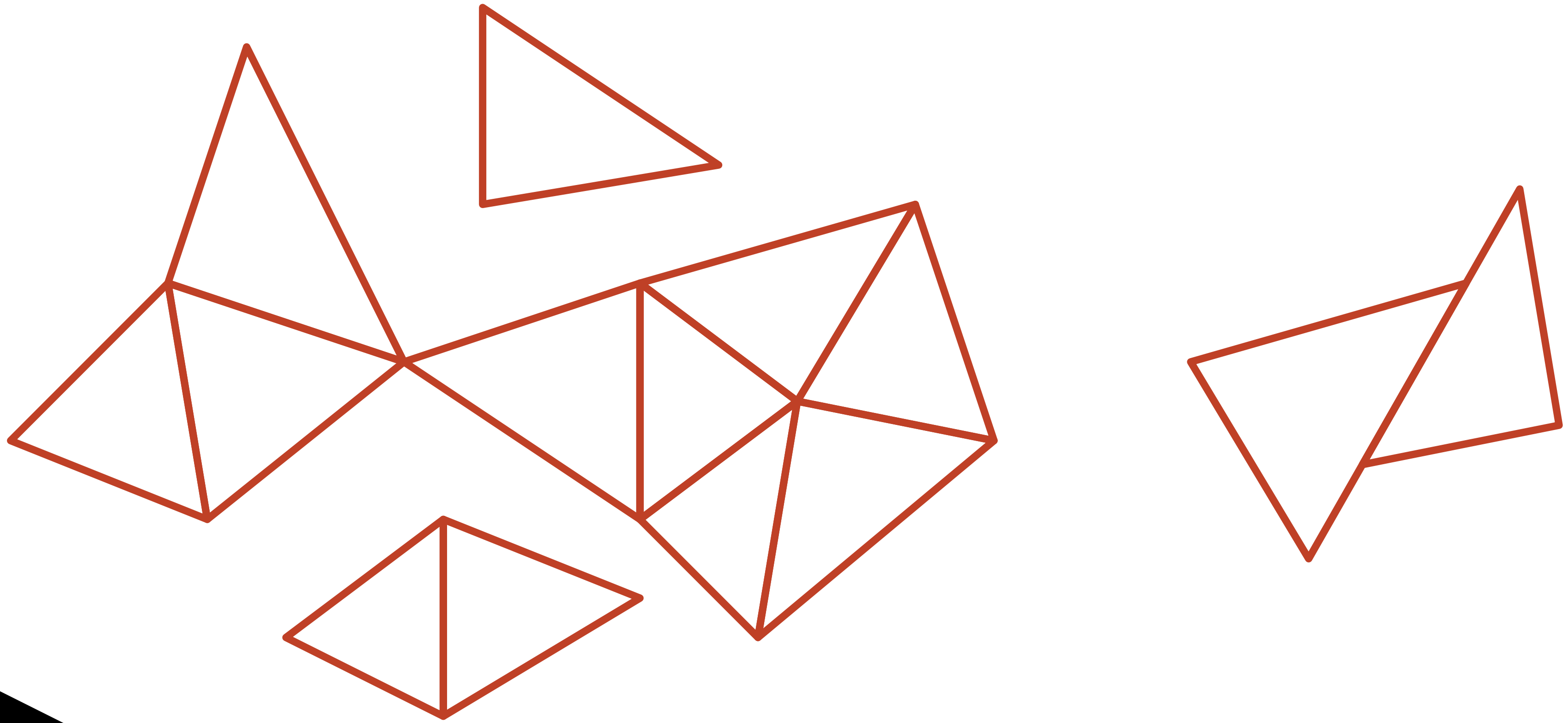

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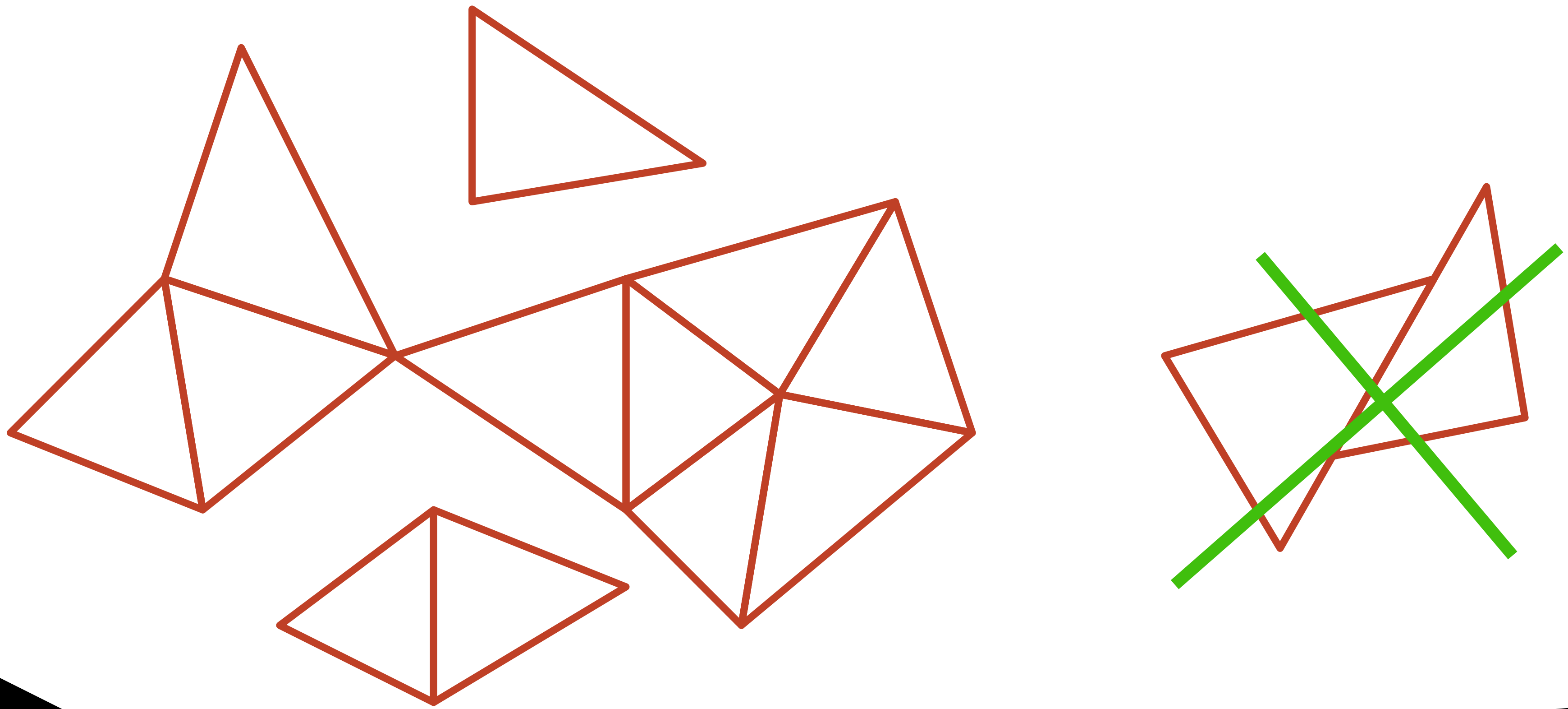

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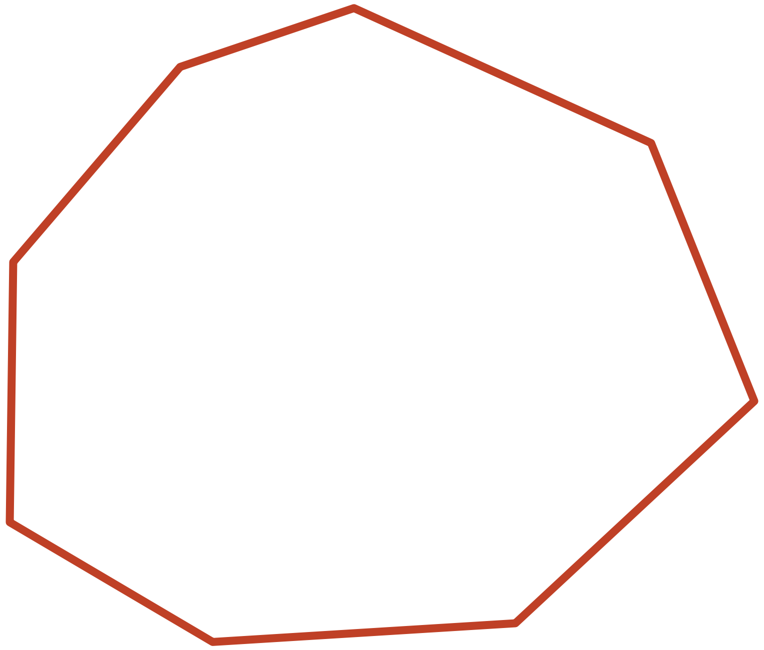

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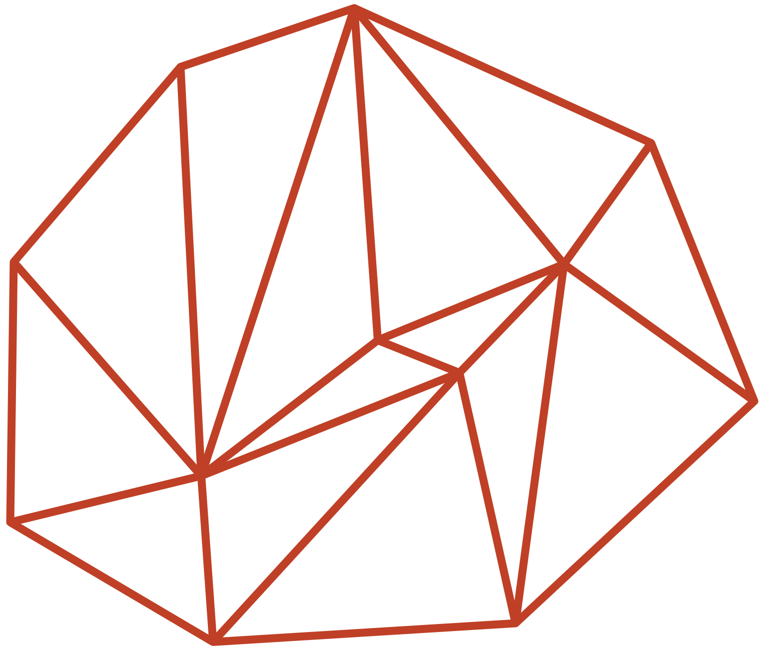

DEFINITION “MESHED POTATO”

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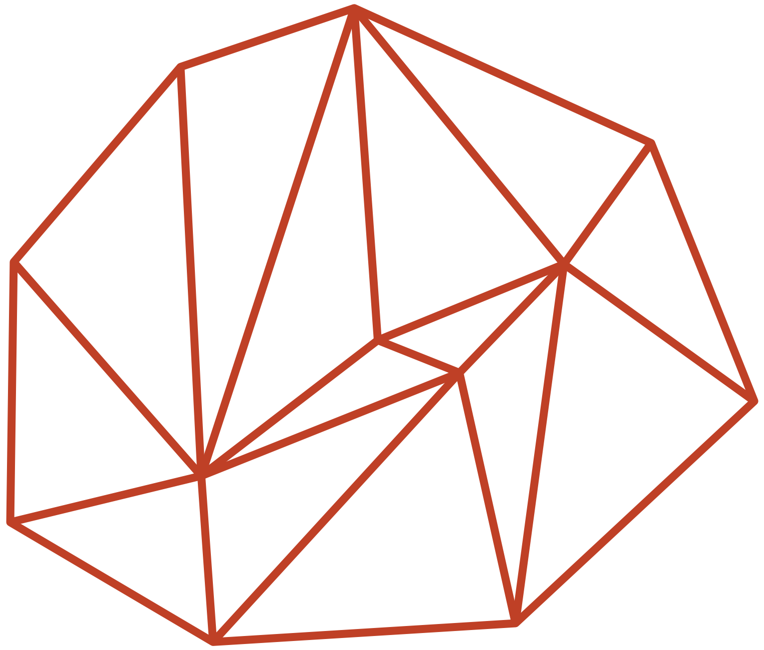
- (Also known as “triangulation”)



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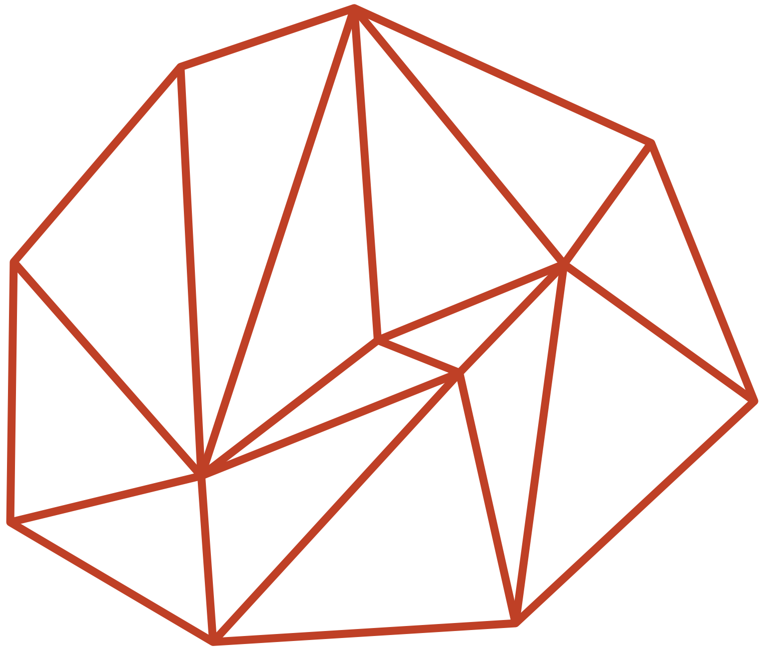

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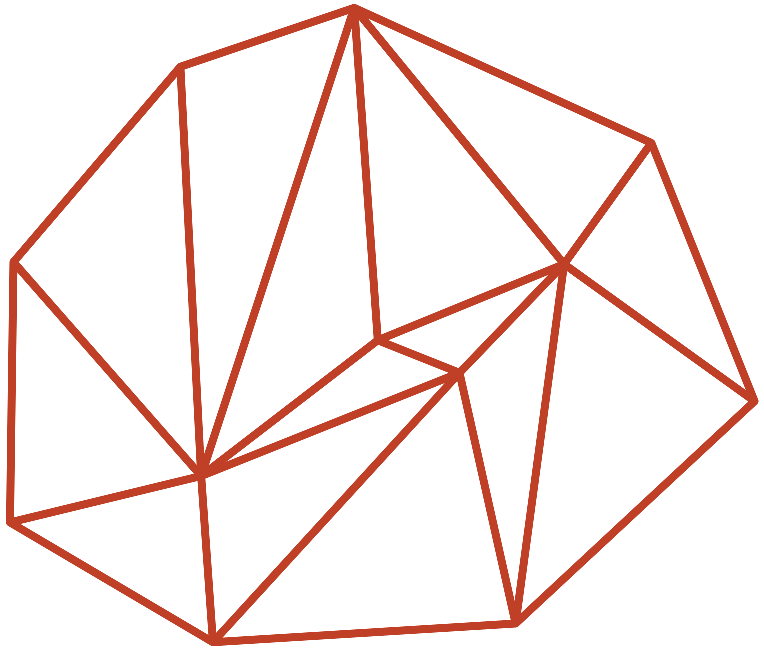

Find the largest meshed potato contained in a given mesh.

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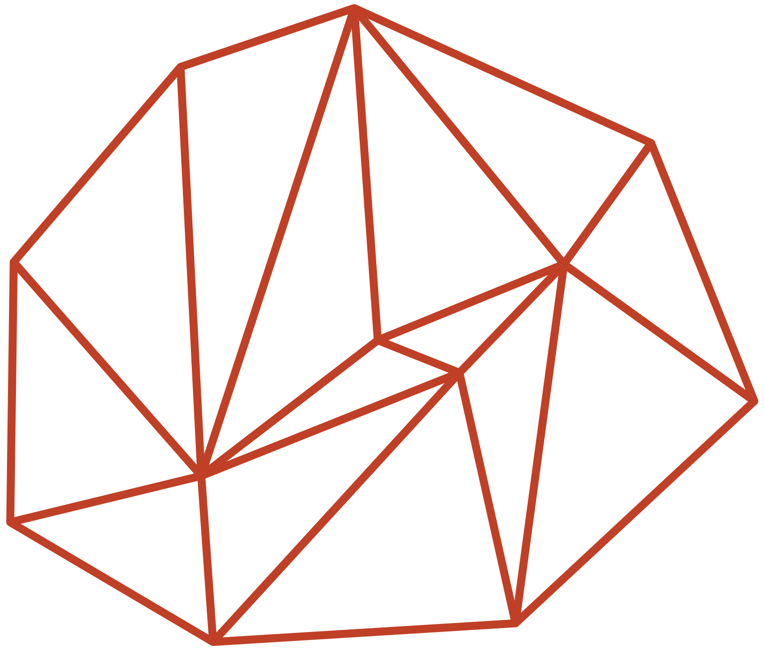

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(Aronov *et al.*, 2007)

NEW STUFF

QUESTION

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DEFINITION "POTATO NUMBER" $\mathcal{P}(\mathrm{M})$

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How many potatoes can there actually be?

DEFINITION "POTATO NUMBER" $\mathcal{P}(M)$

- Total number of distinct meshed potatoes in a given mesh M

EXAMPLE

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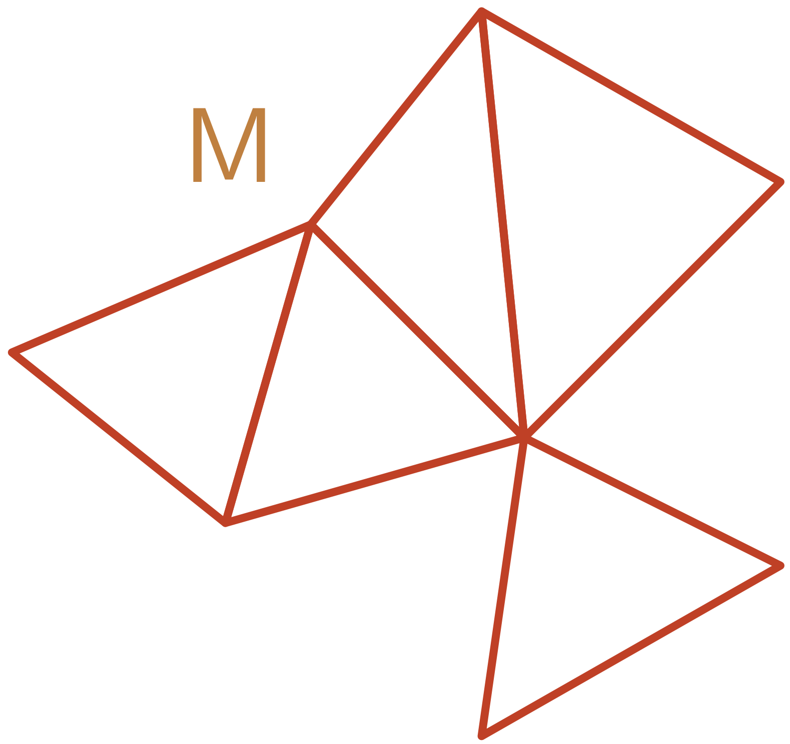

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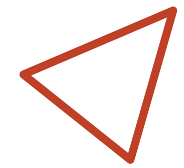

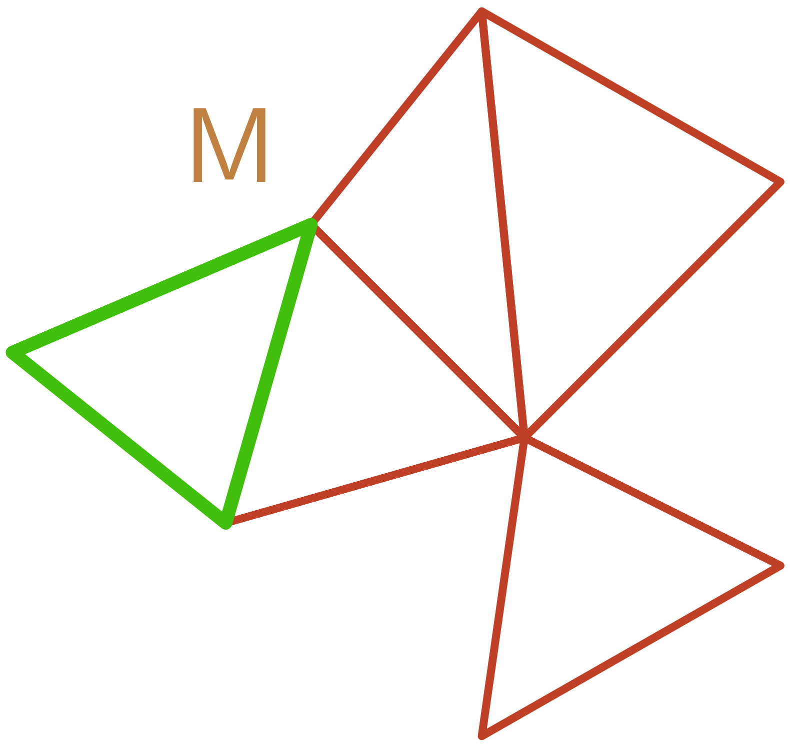

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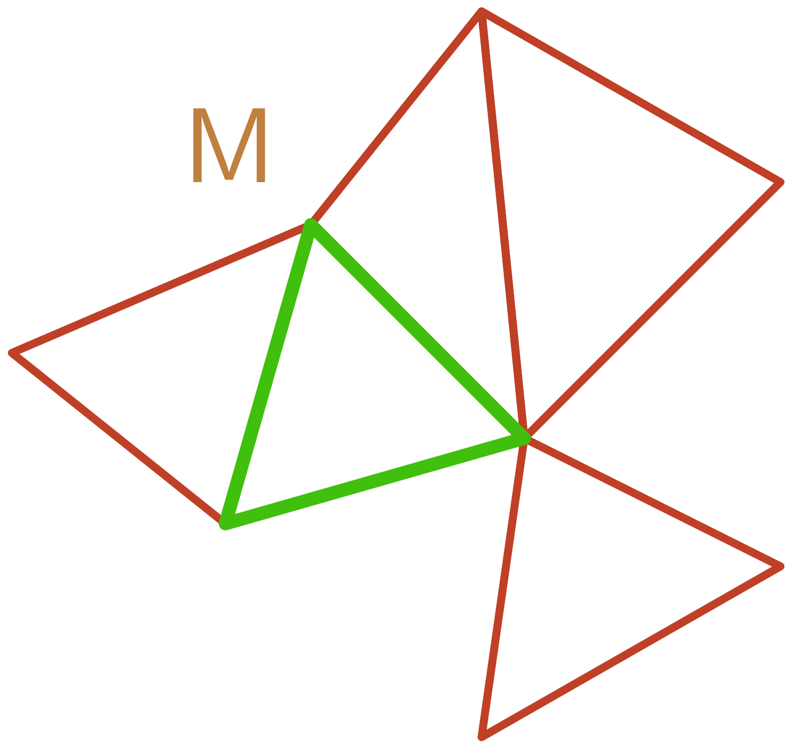

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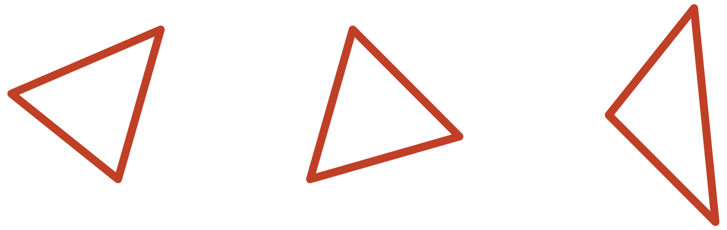

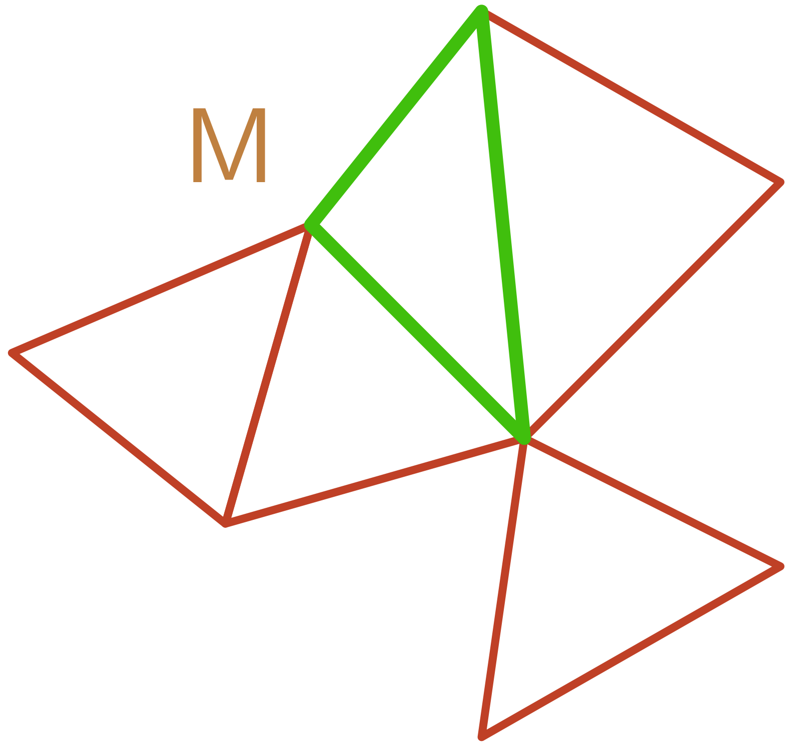

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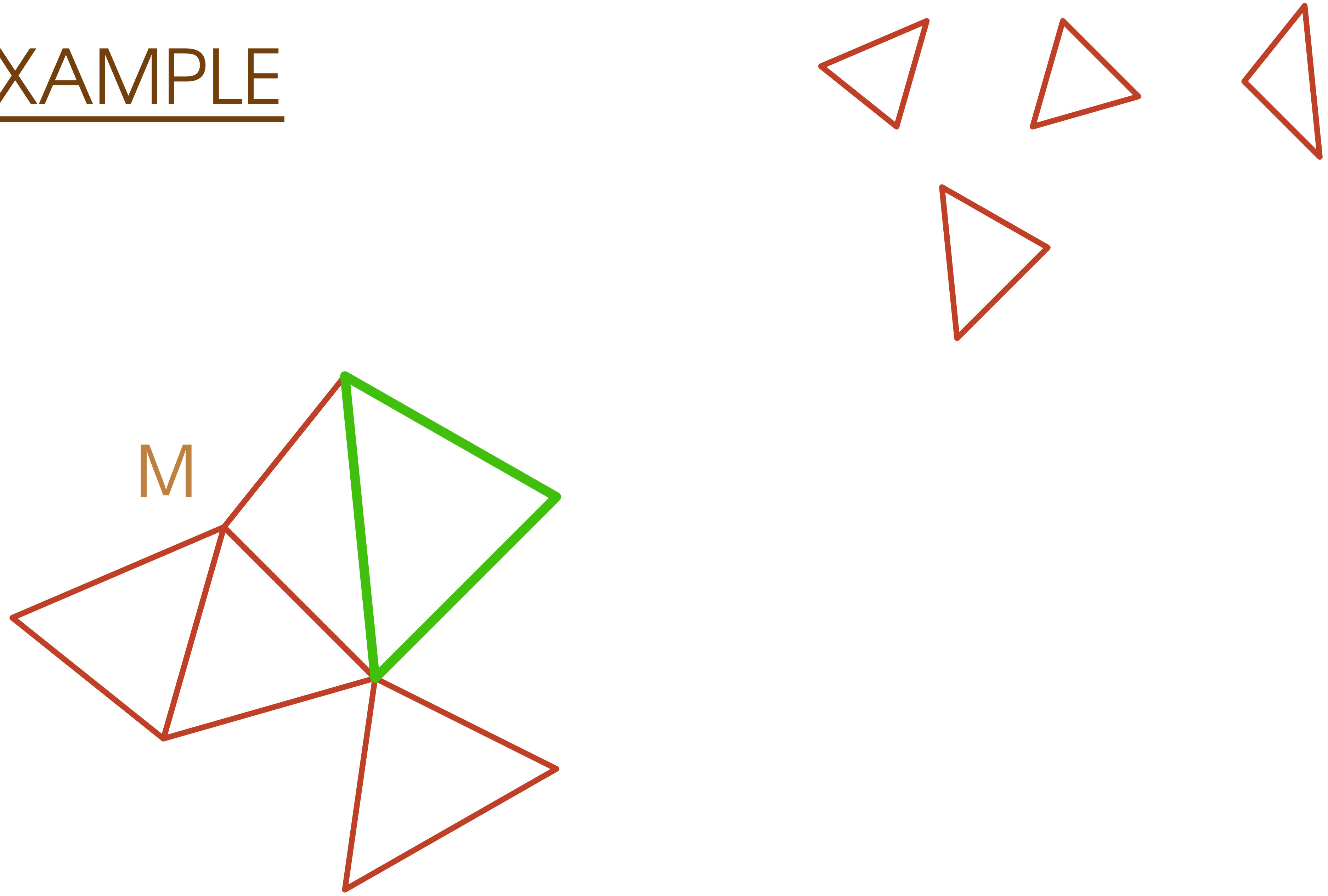

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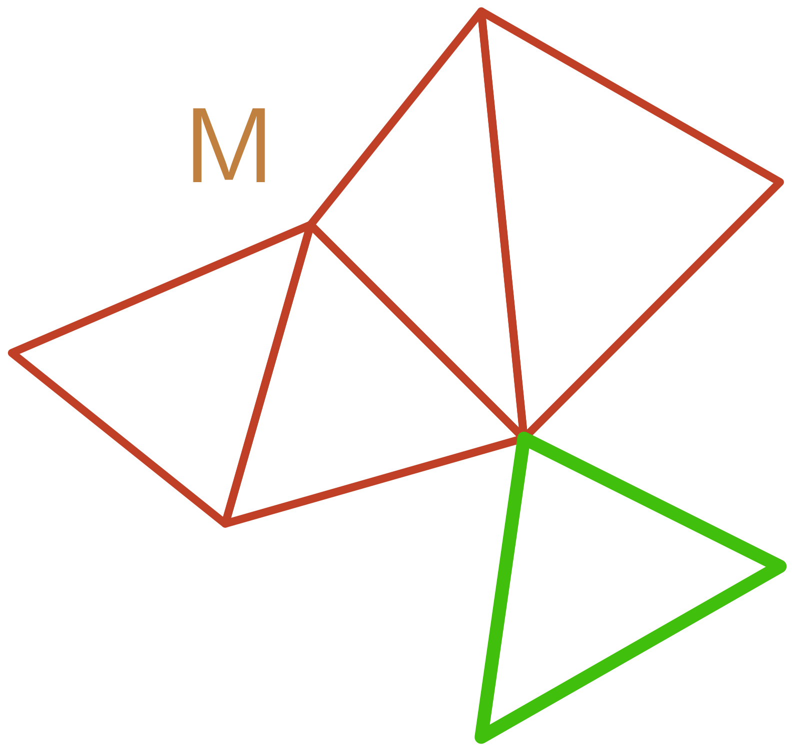

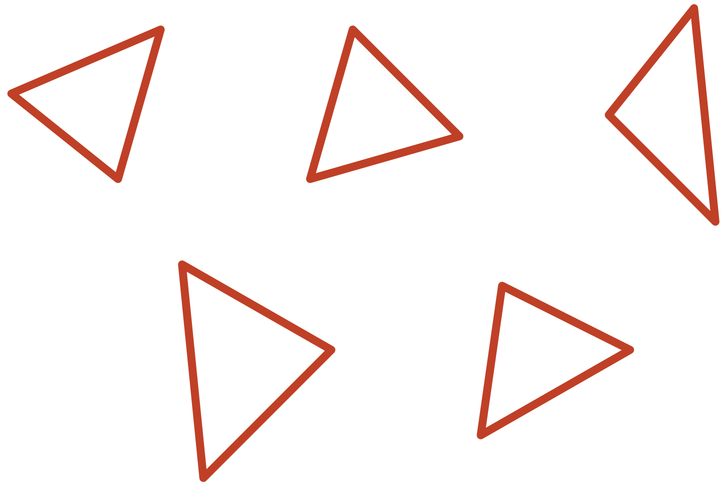

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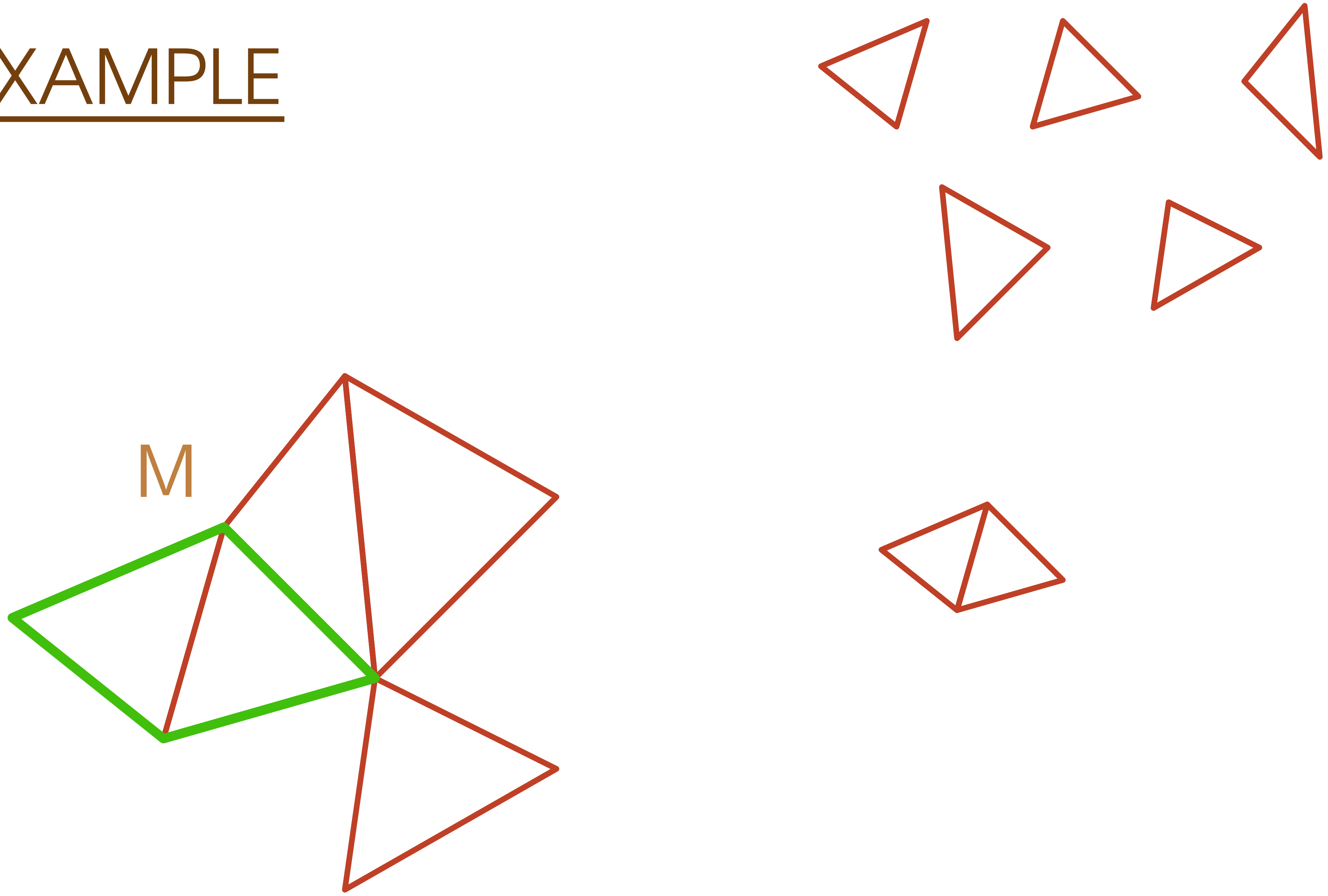

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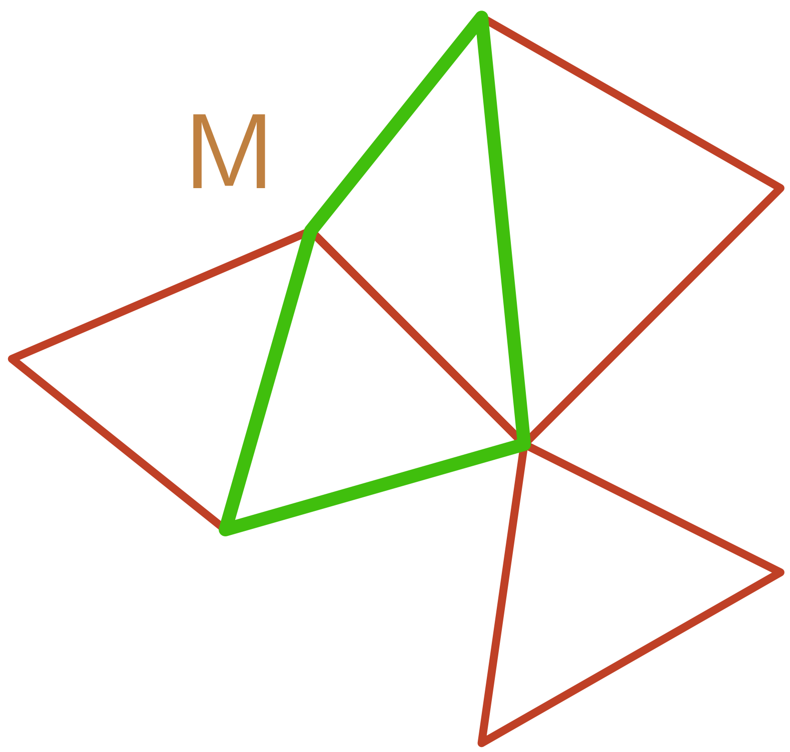

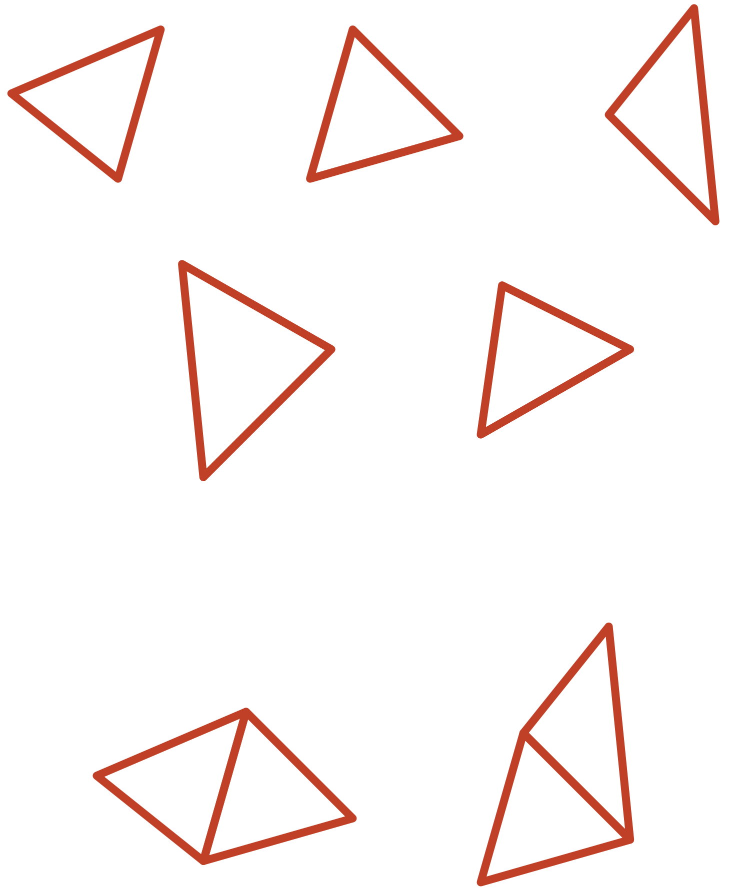

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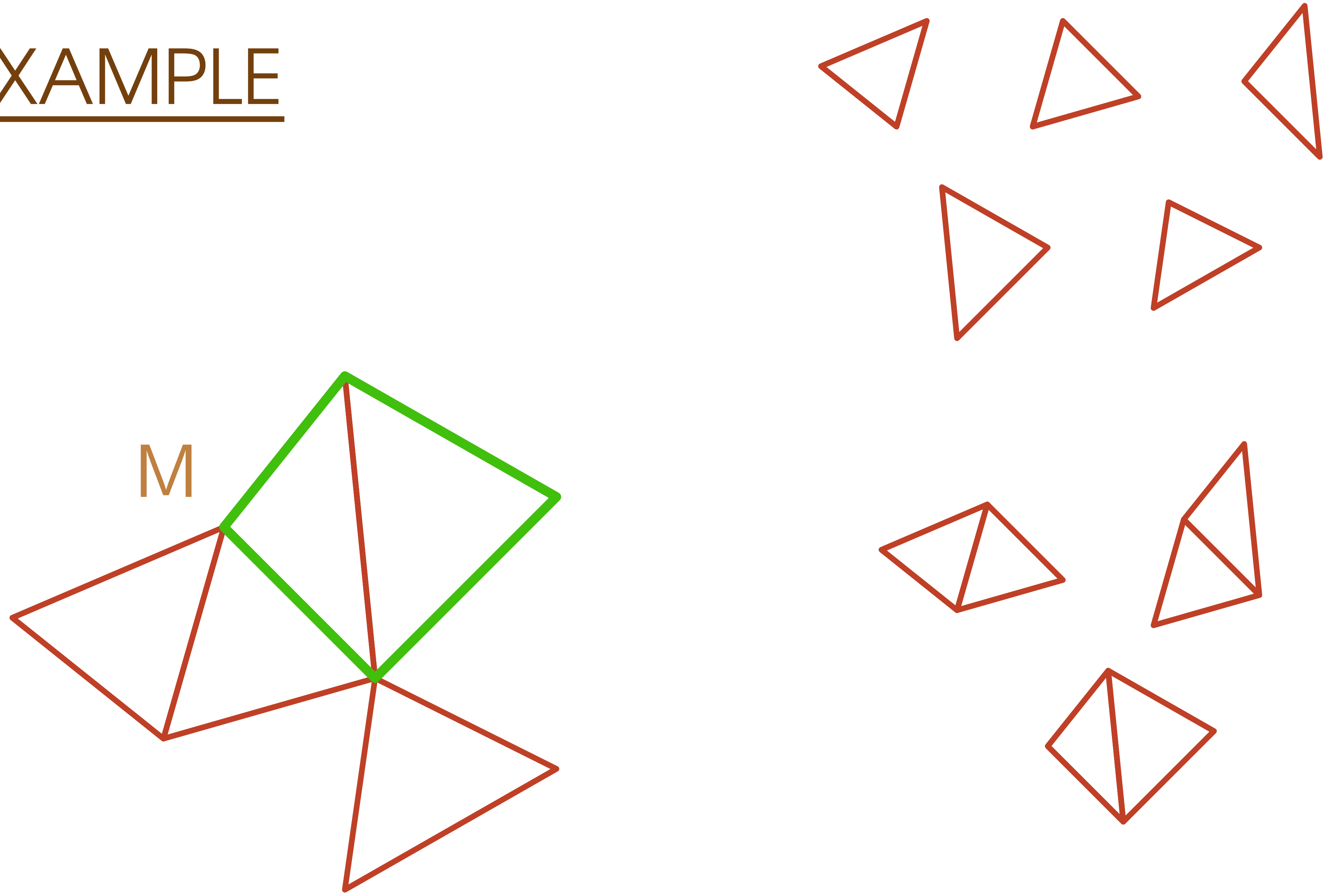

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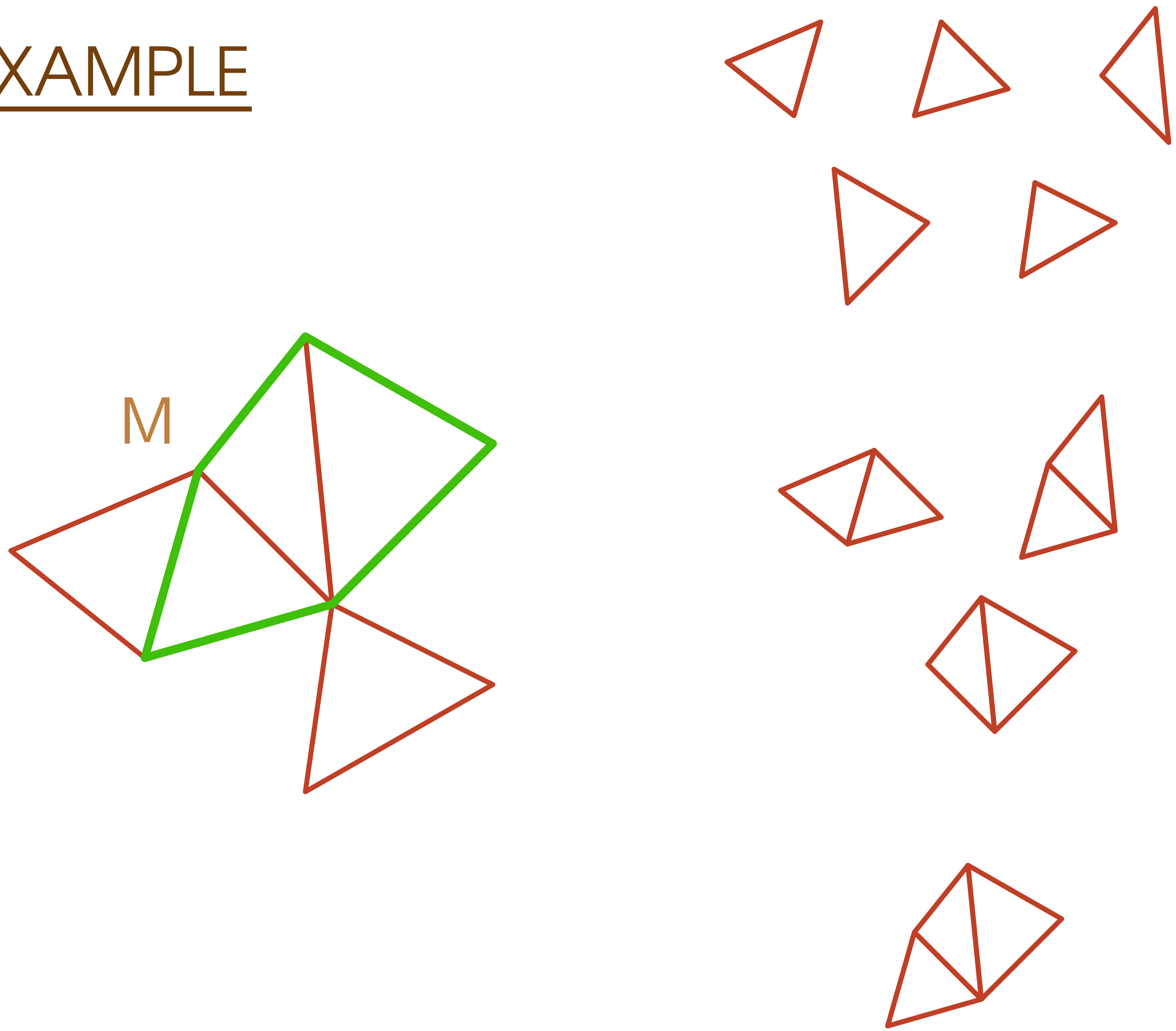

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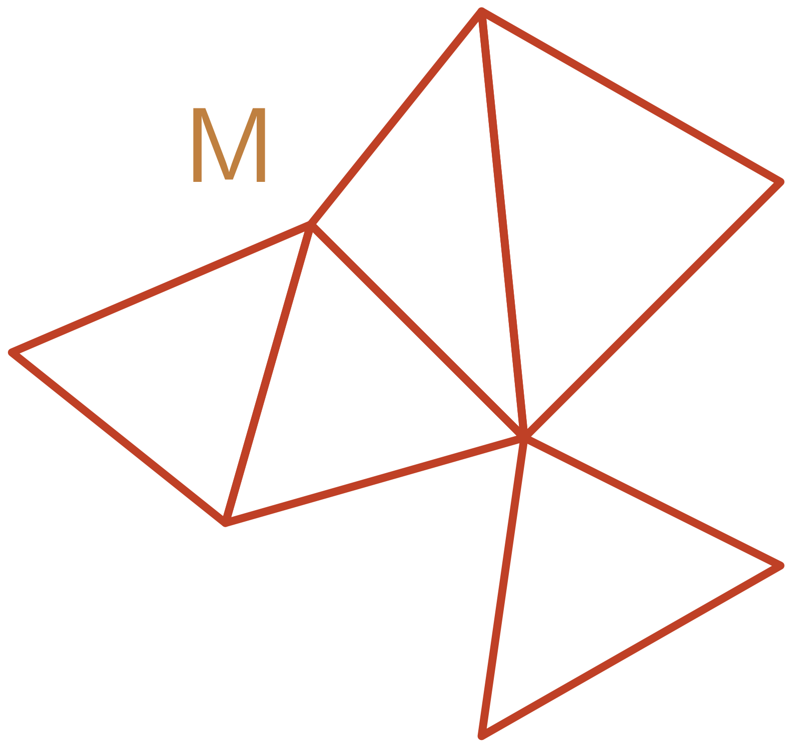
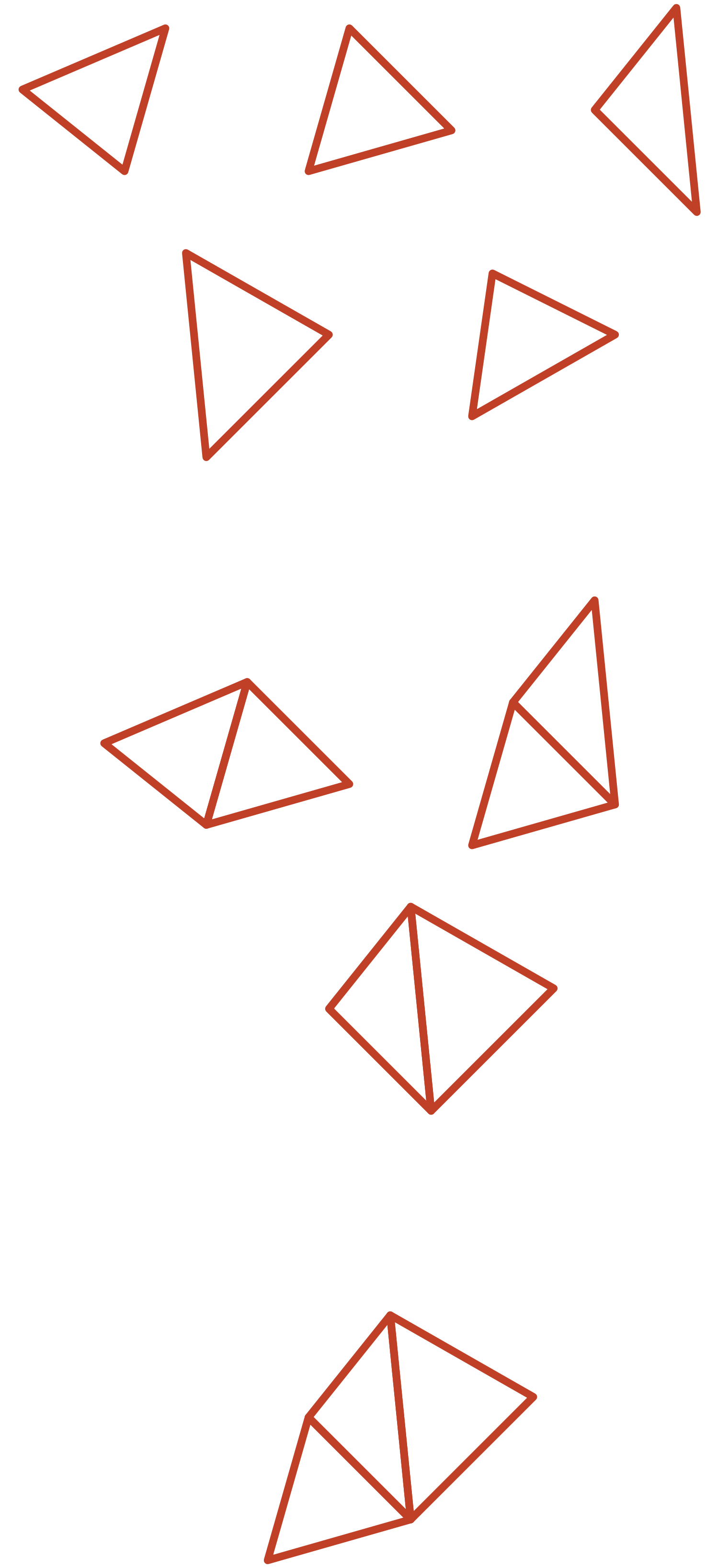


$$\mathcal{P}(\mathrm{M}) = 9$$



RESULTS

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- There exists a mesh with potato number $\Omega(1.5028^n)$.

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- There exists a mesh with potato number $\Omega(1.5028^n)$.
- For every mesh, the potato number is $O(1.6181^n)$.

WARNING: TECHNICAL DETAILS

LOWER BOUND

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- Place n points in convex position

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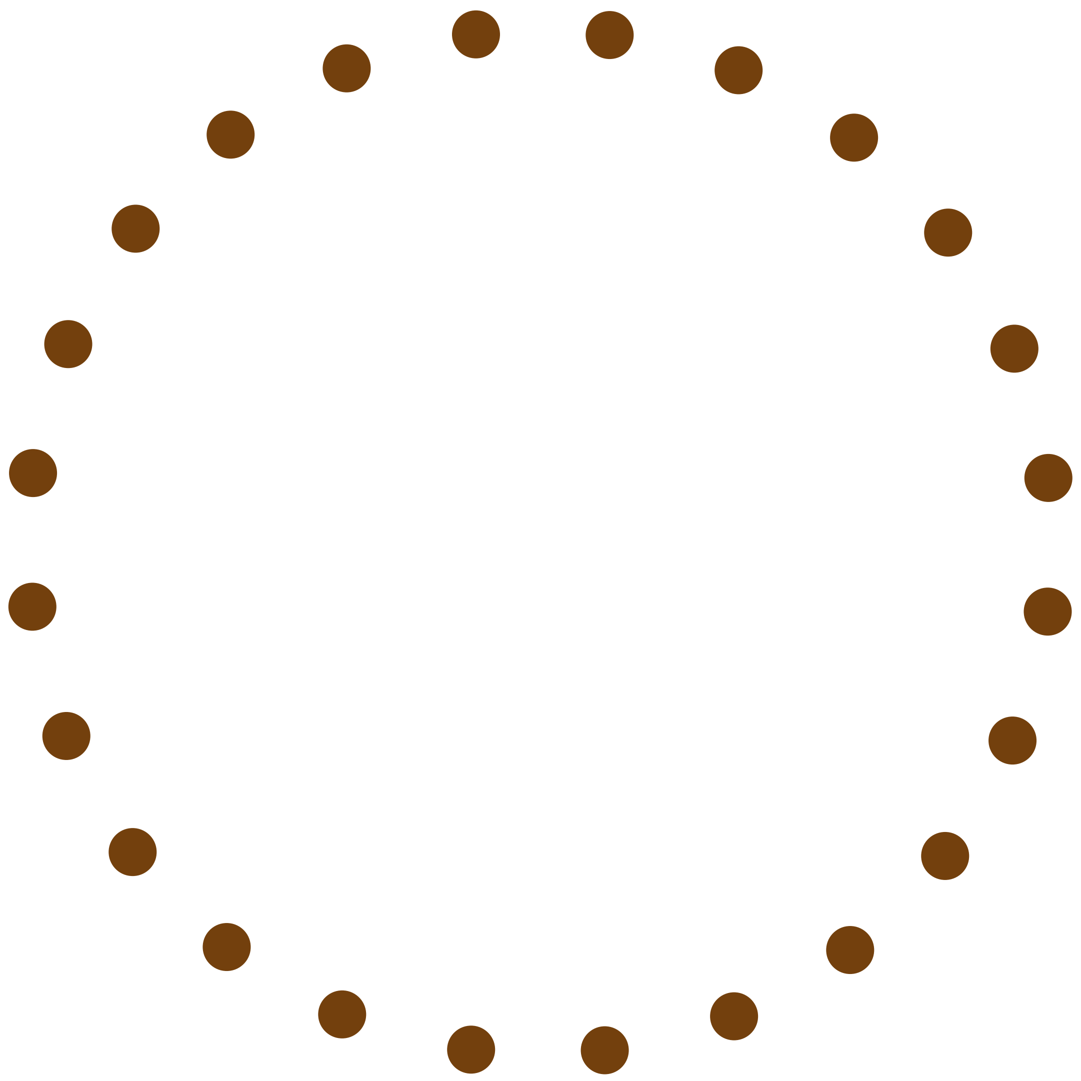

LOWER BOUND

- Place n points in convex position
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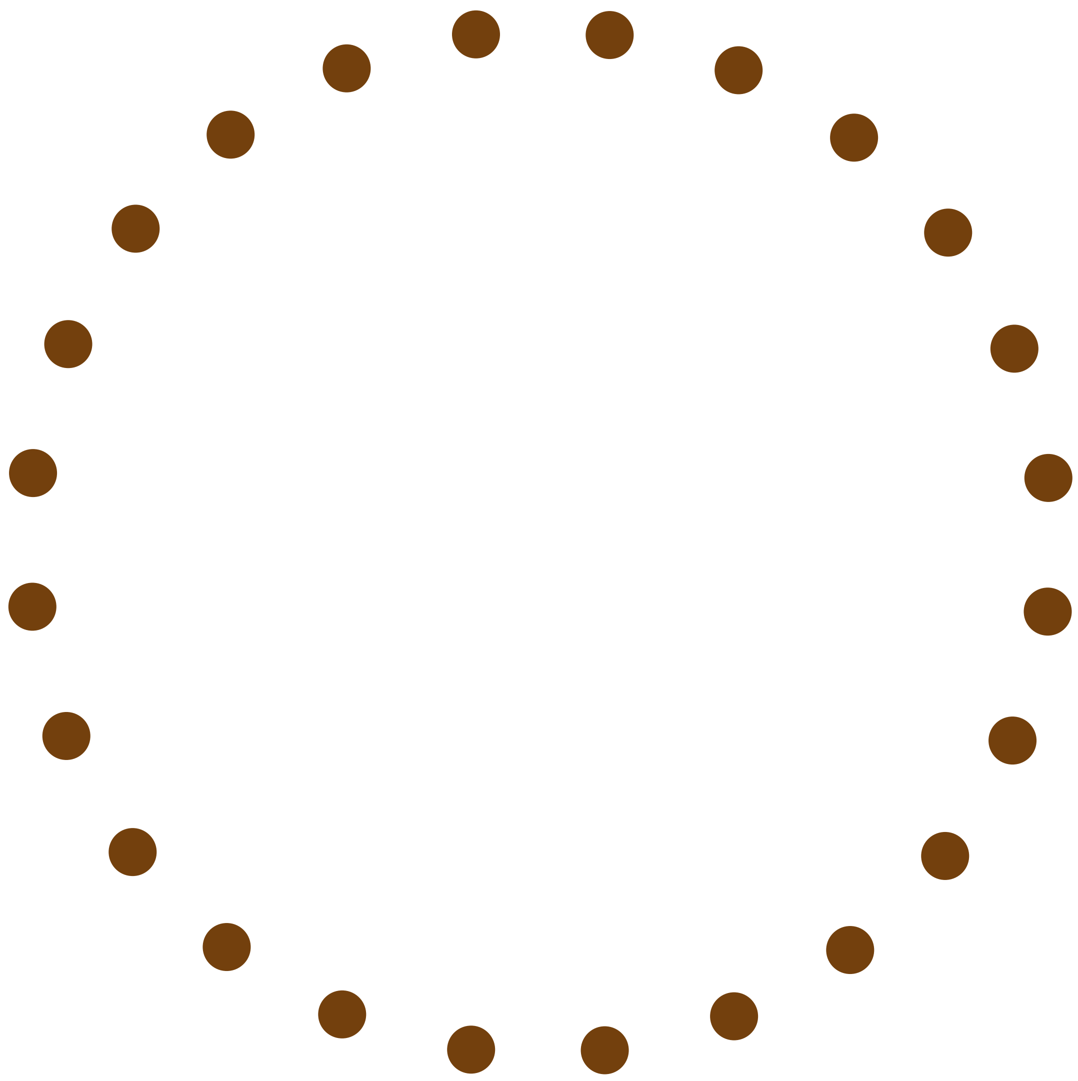

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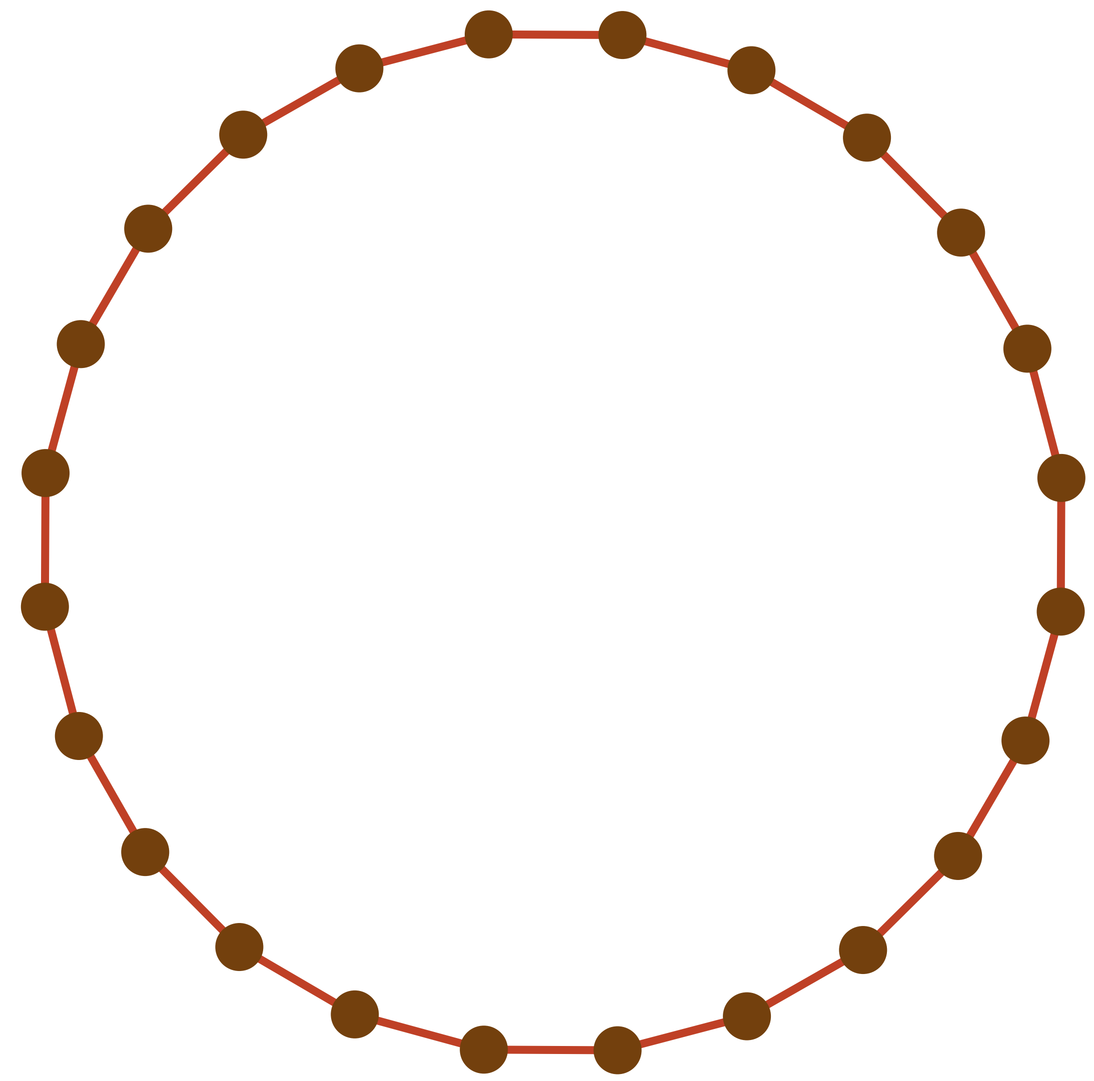

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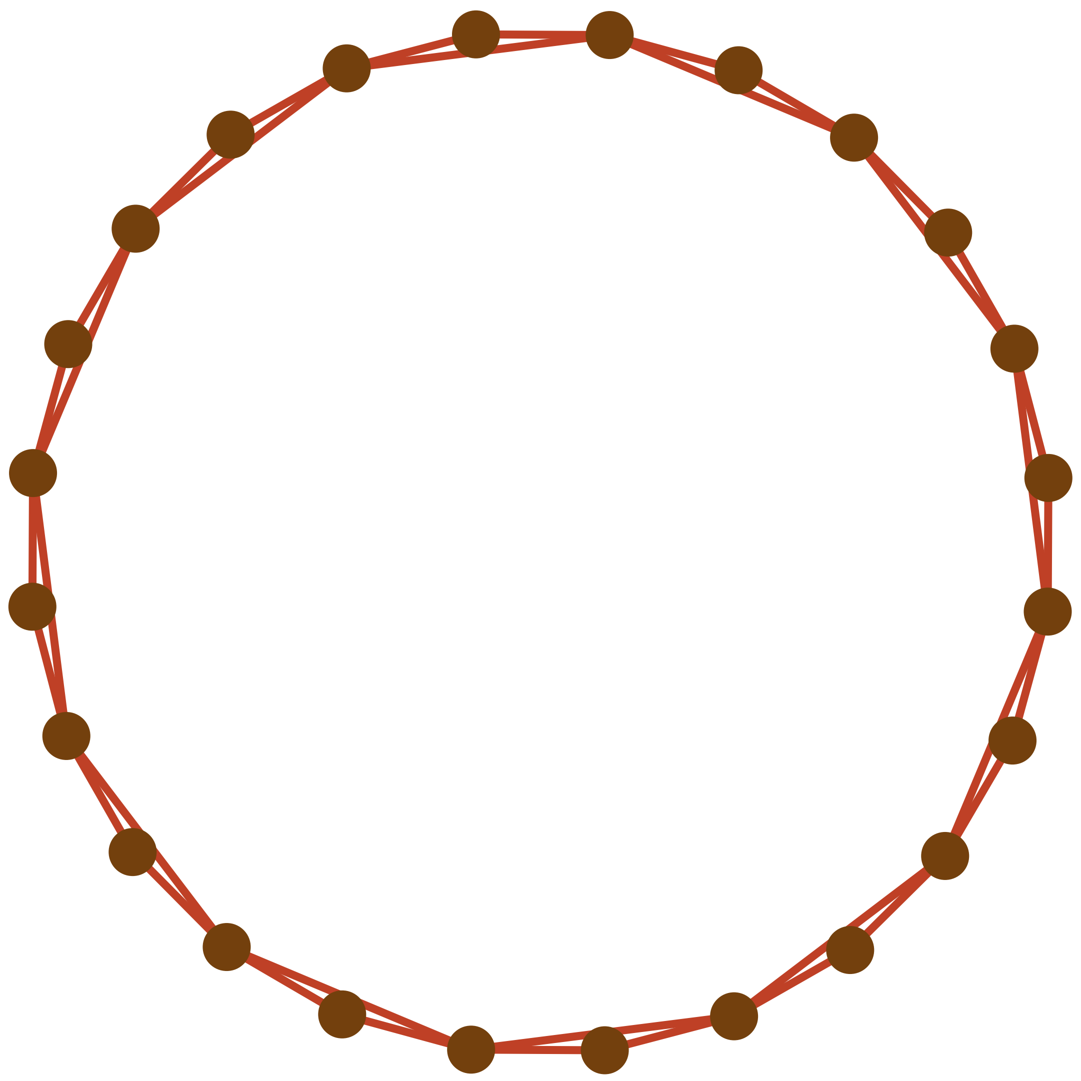

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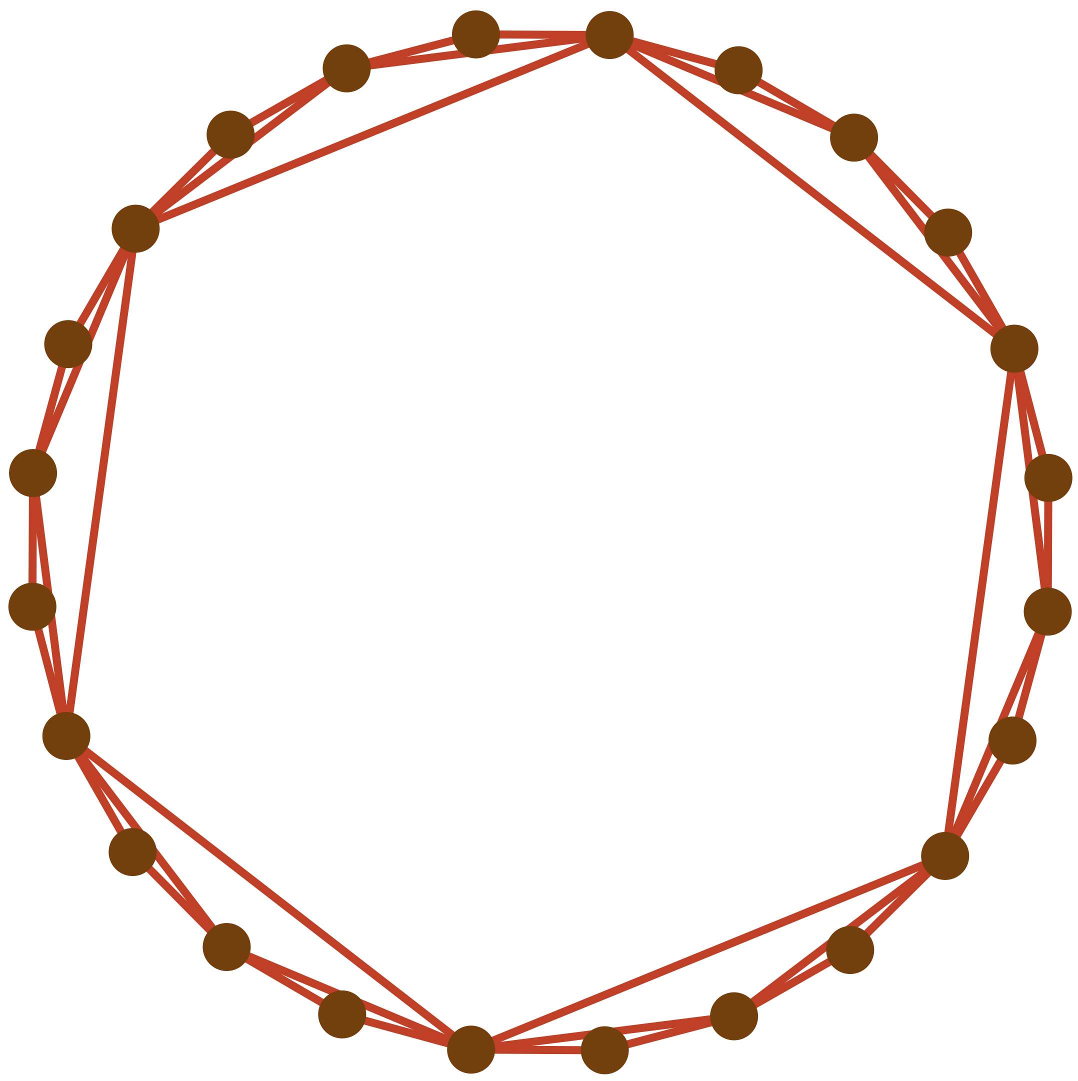

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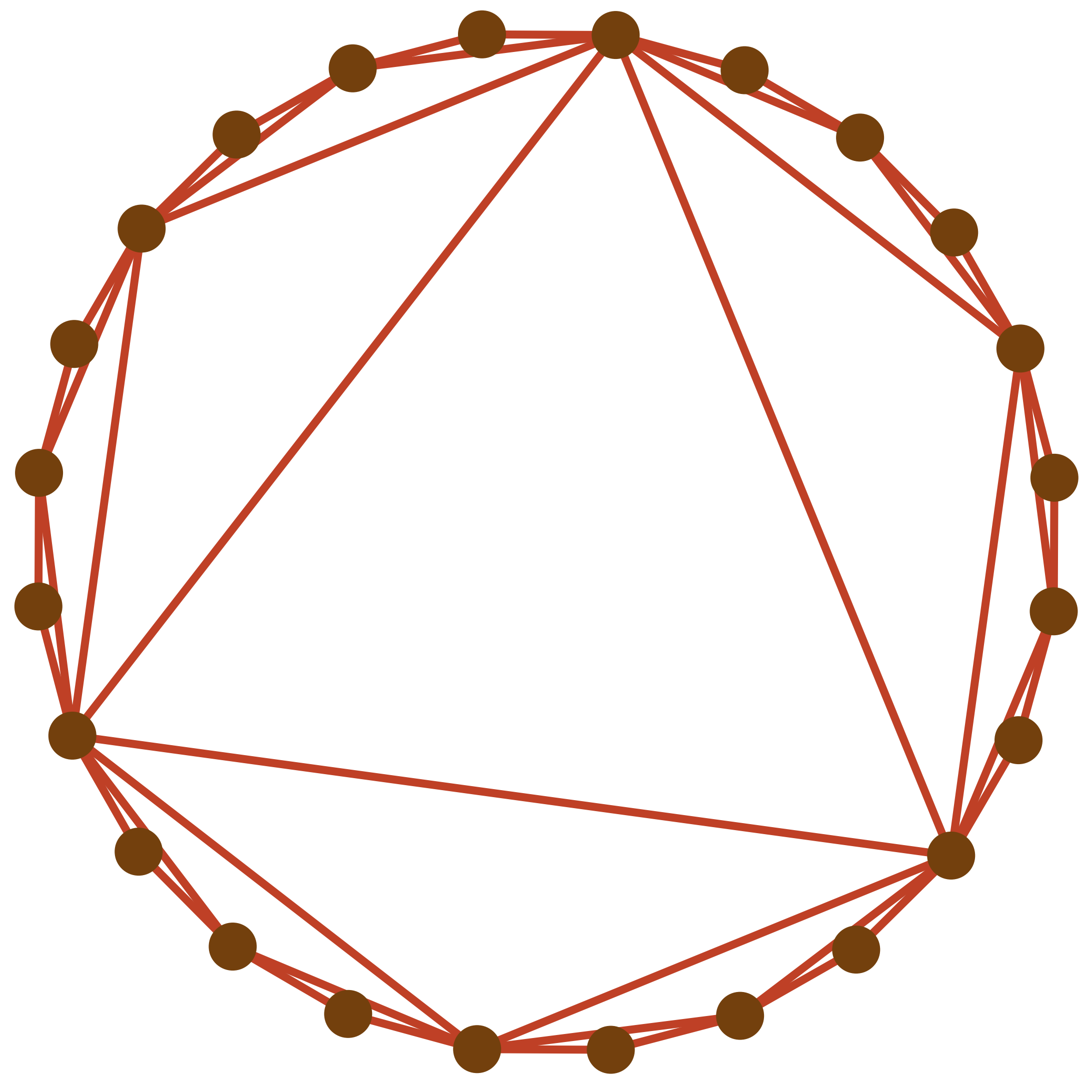

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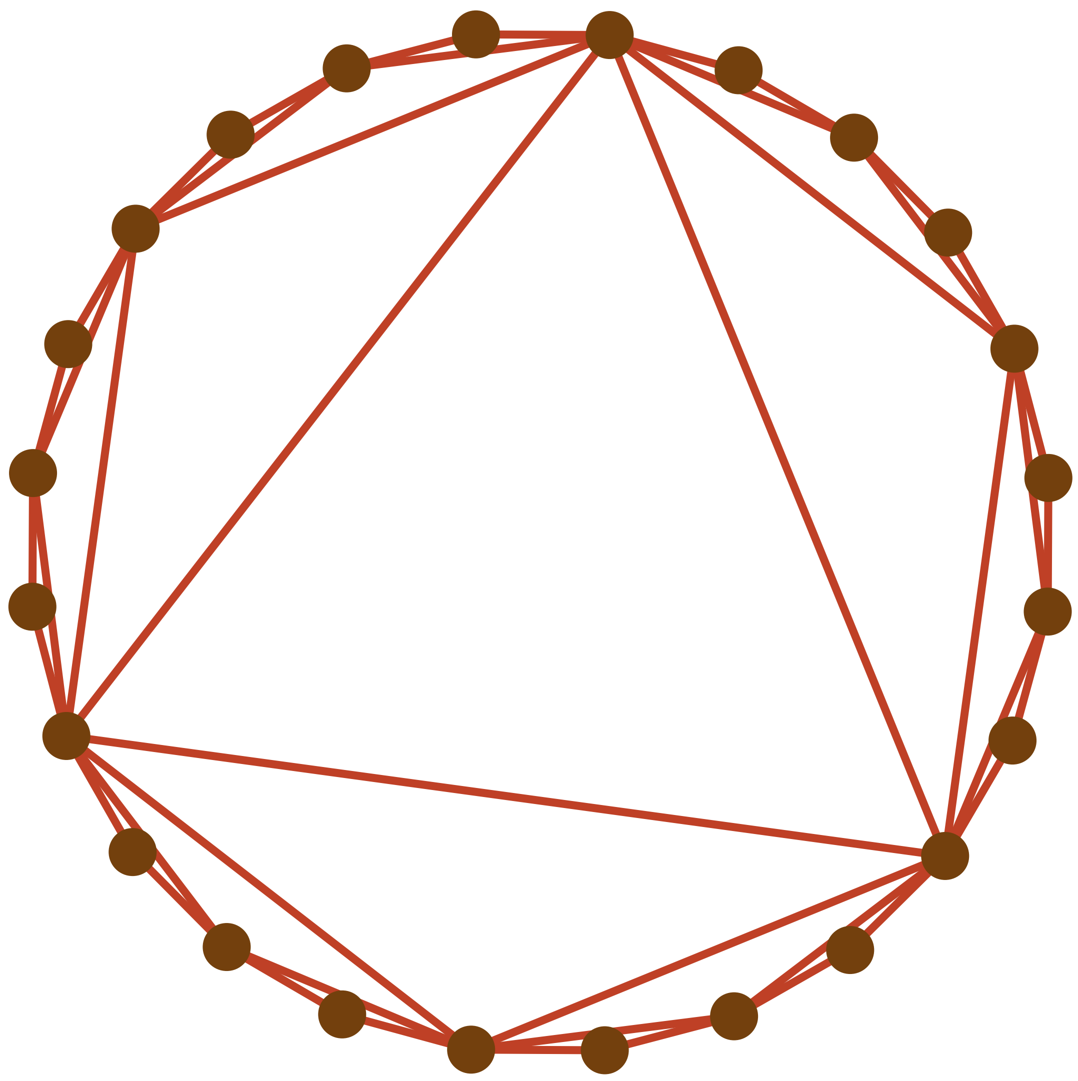

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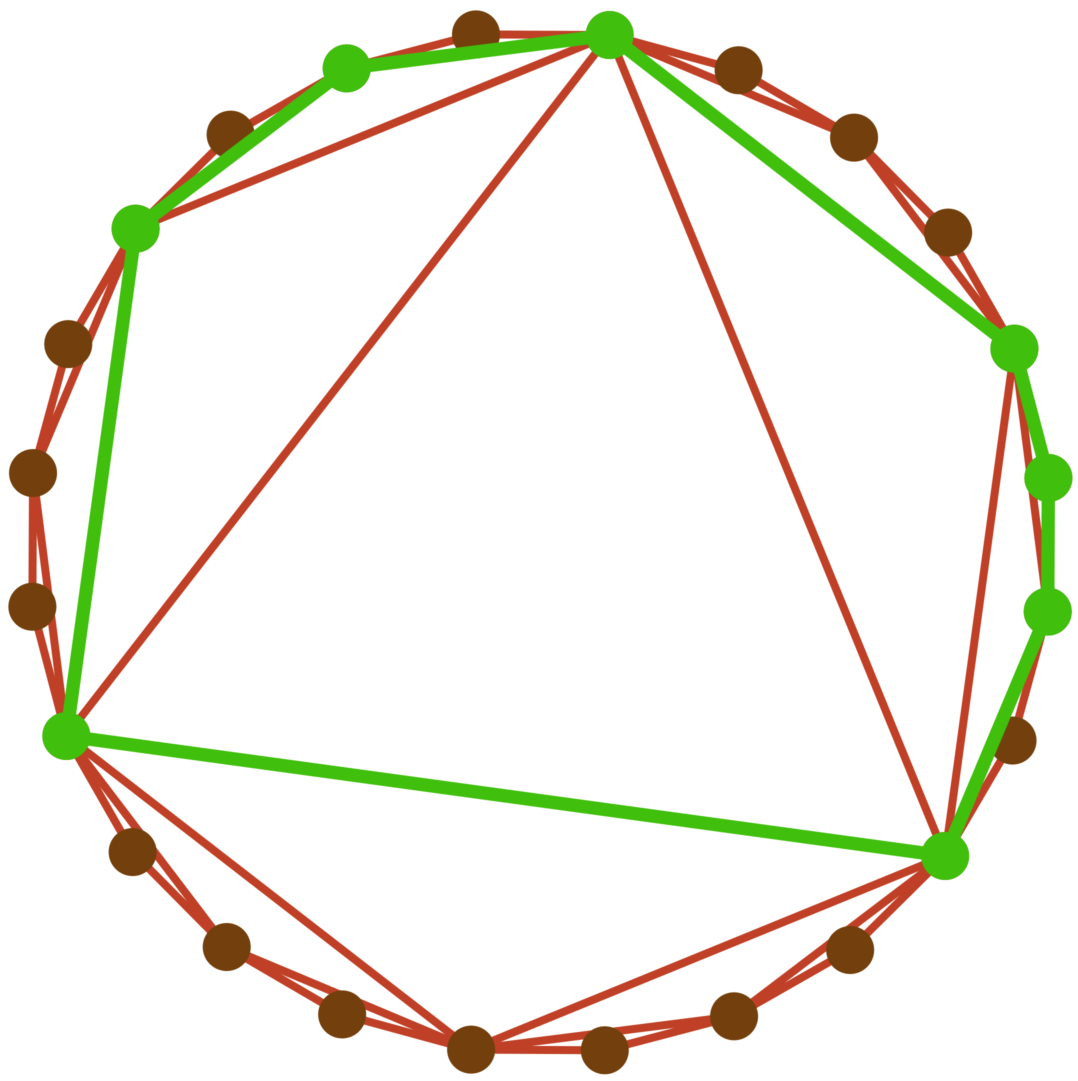

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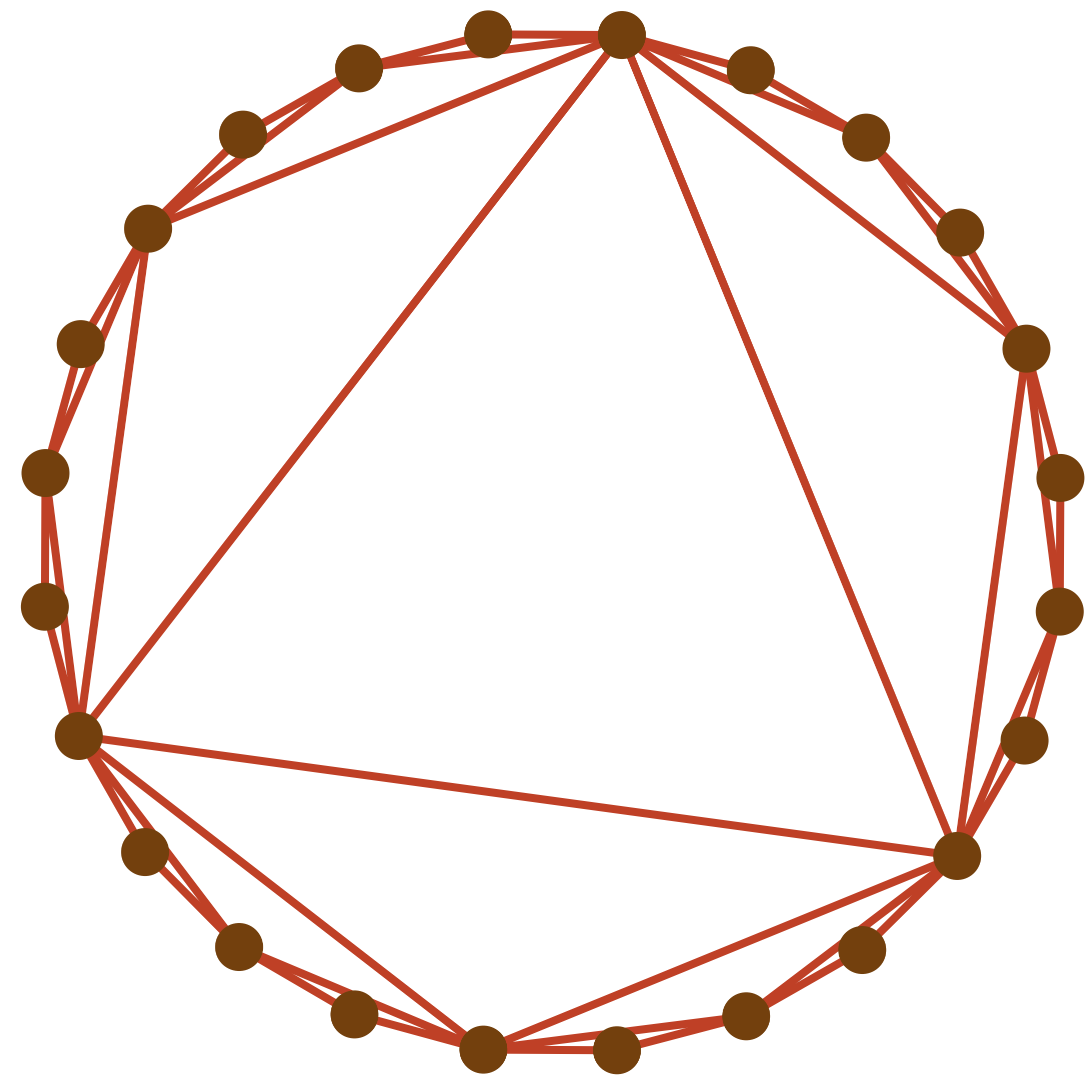

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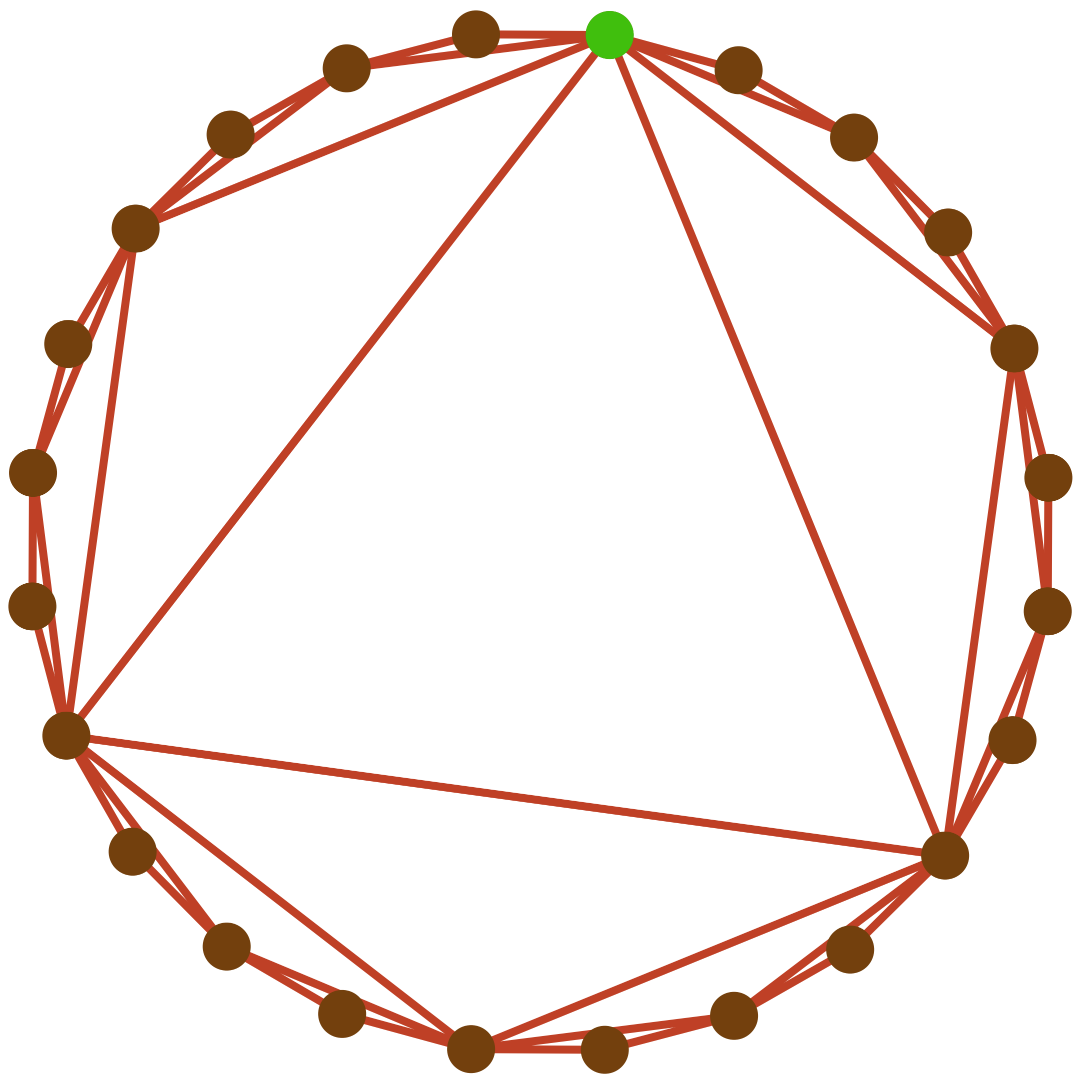

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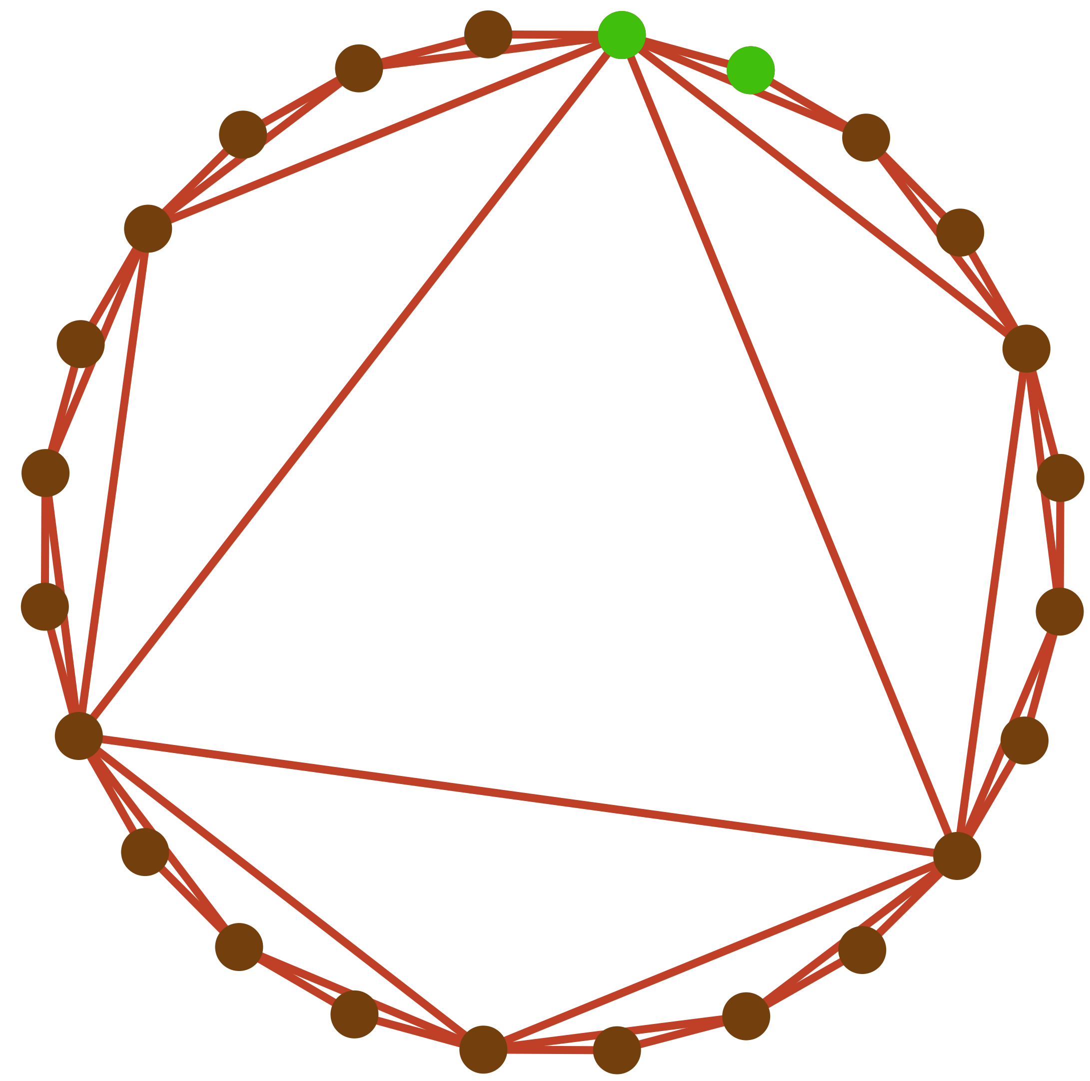

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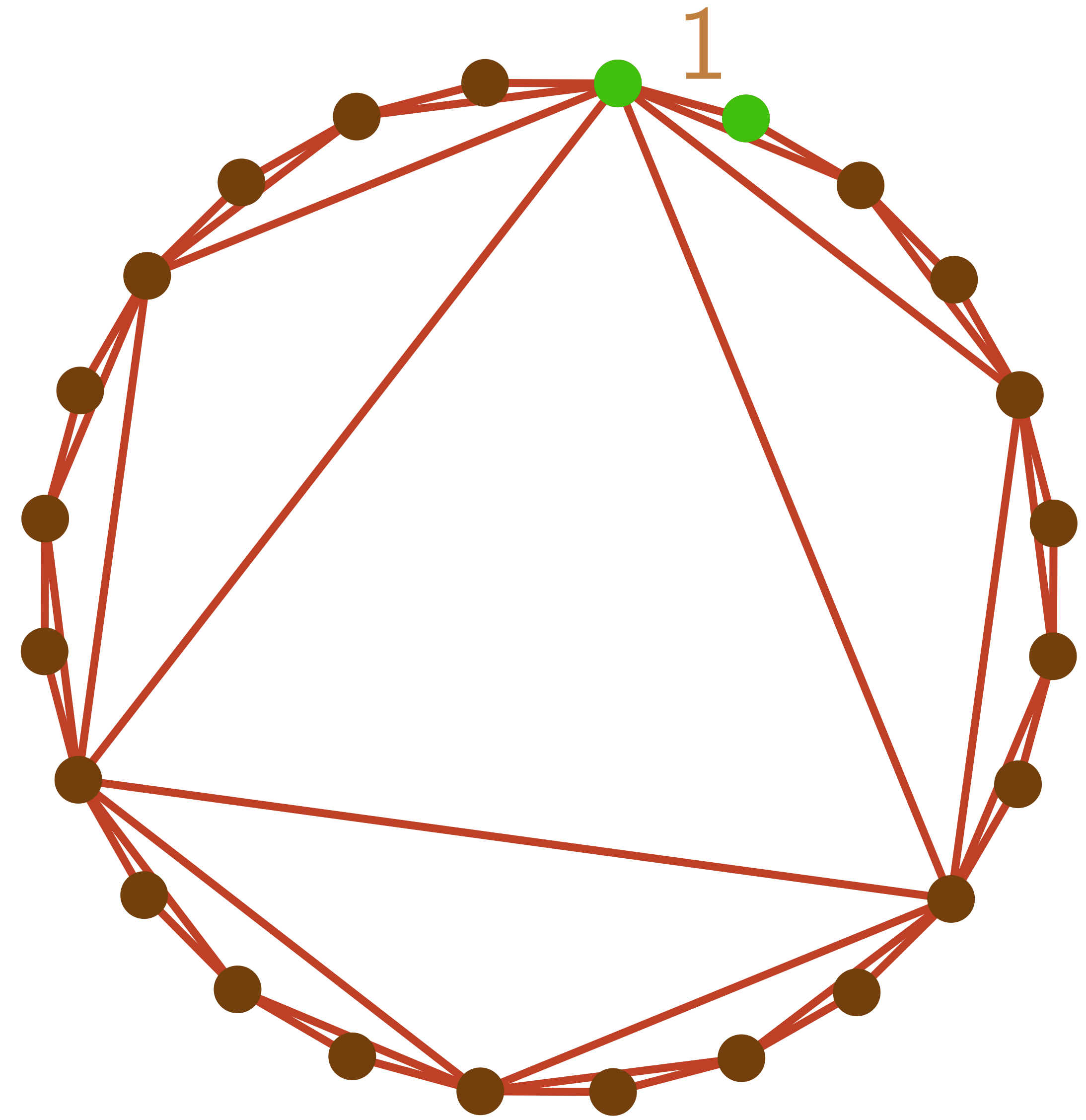

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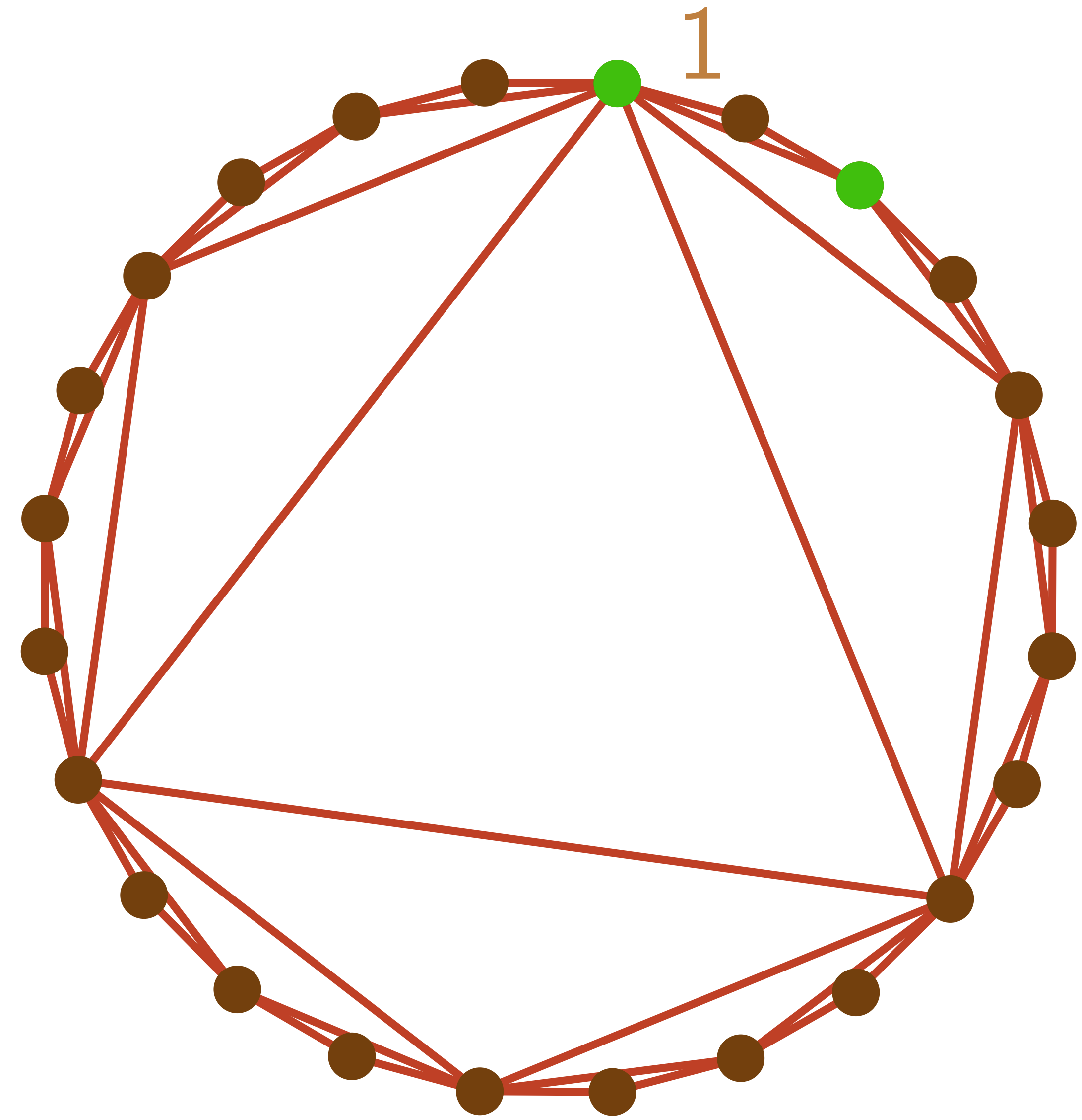

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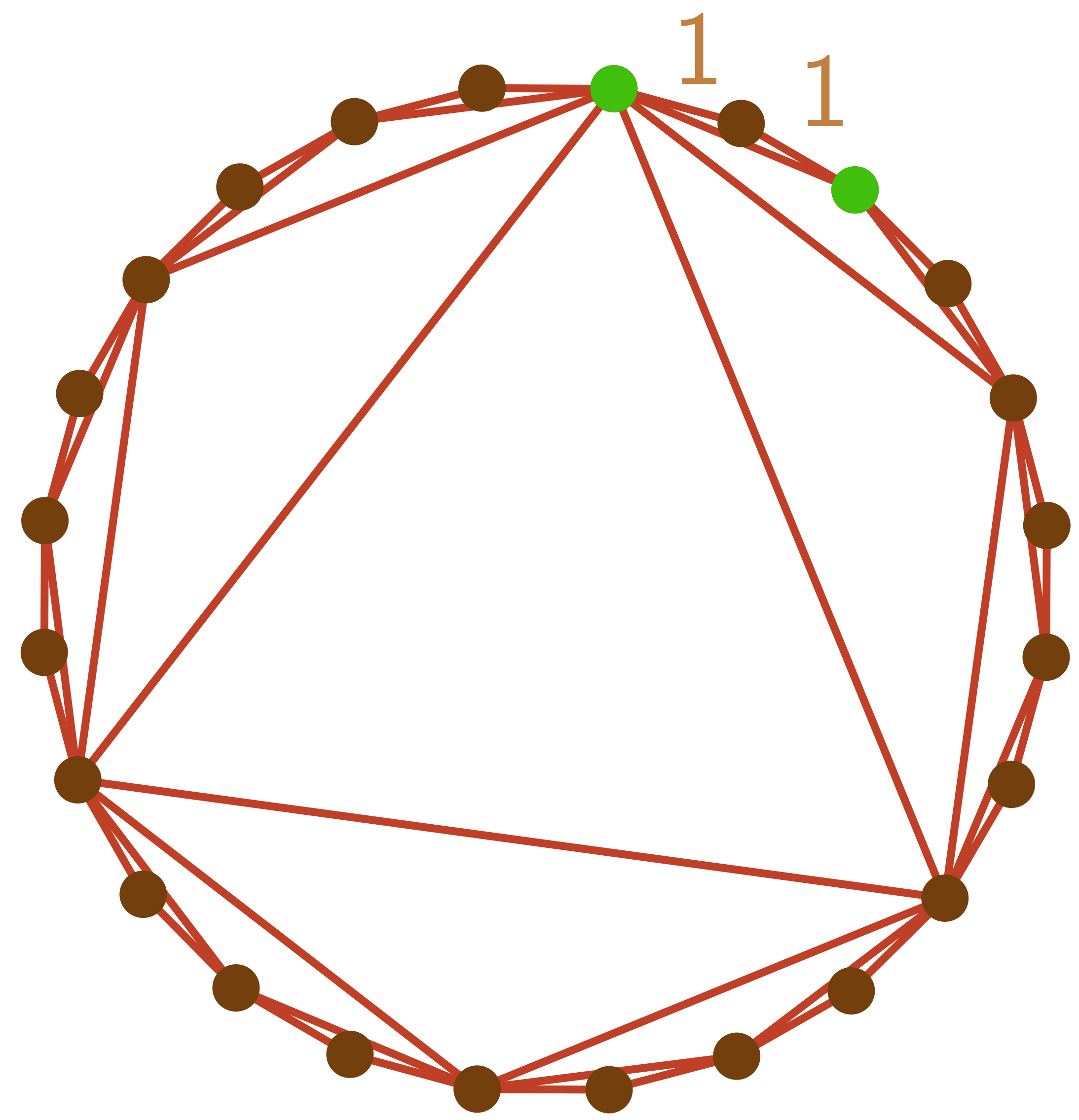

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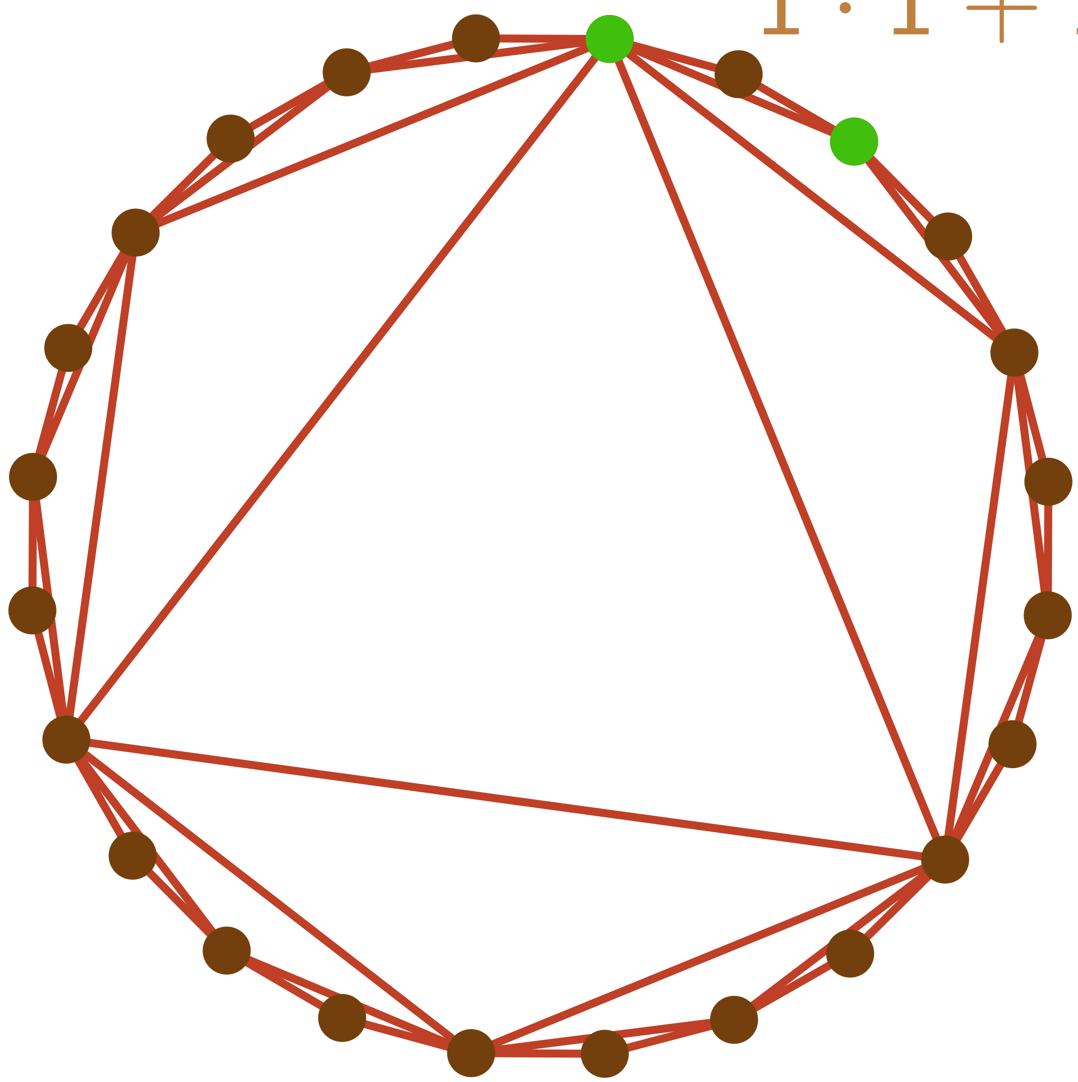

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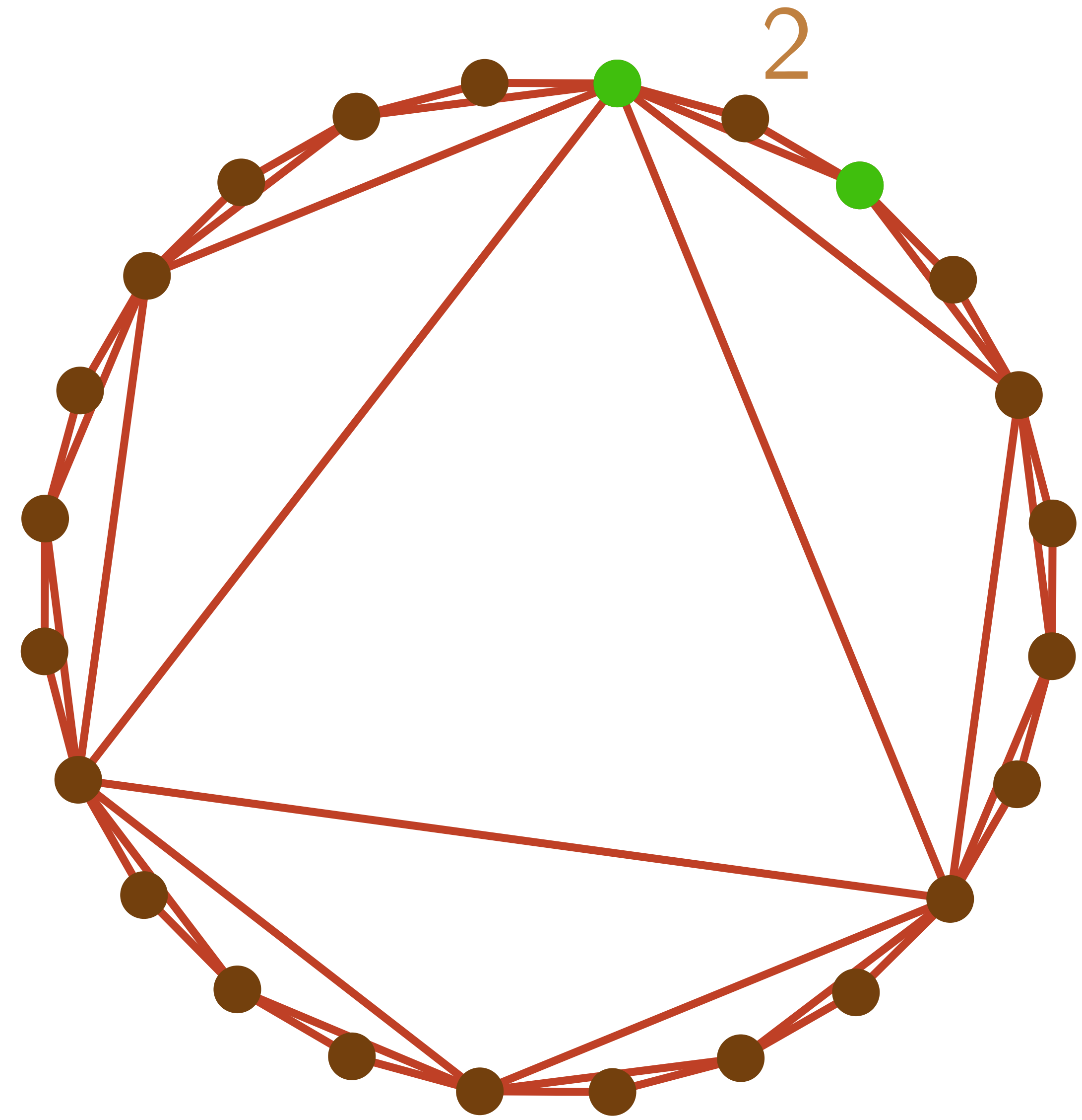

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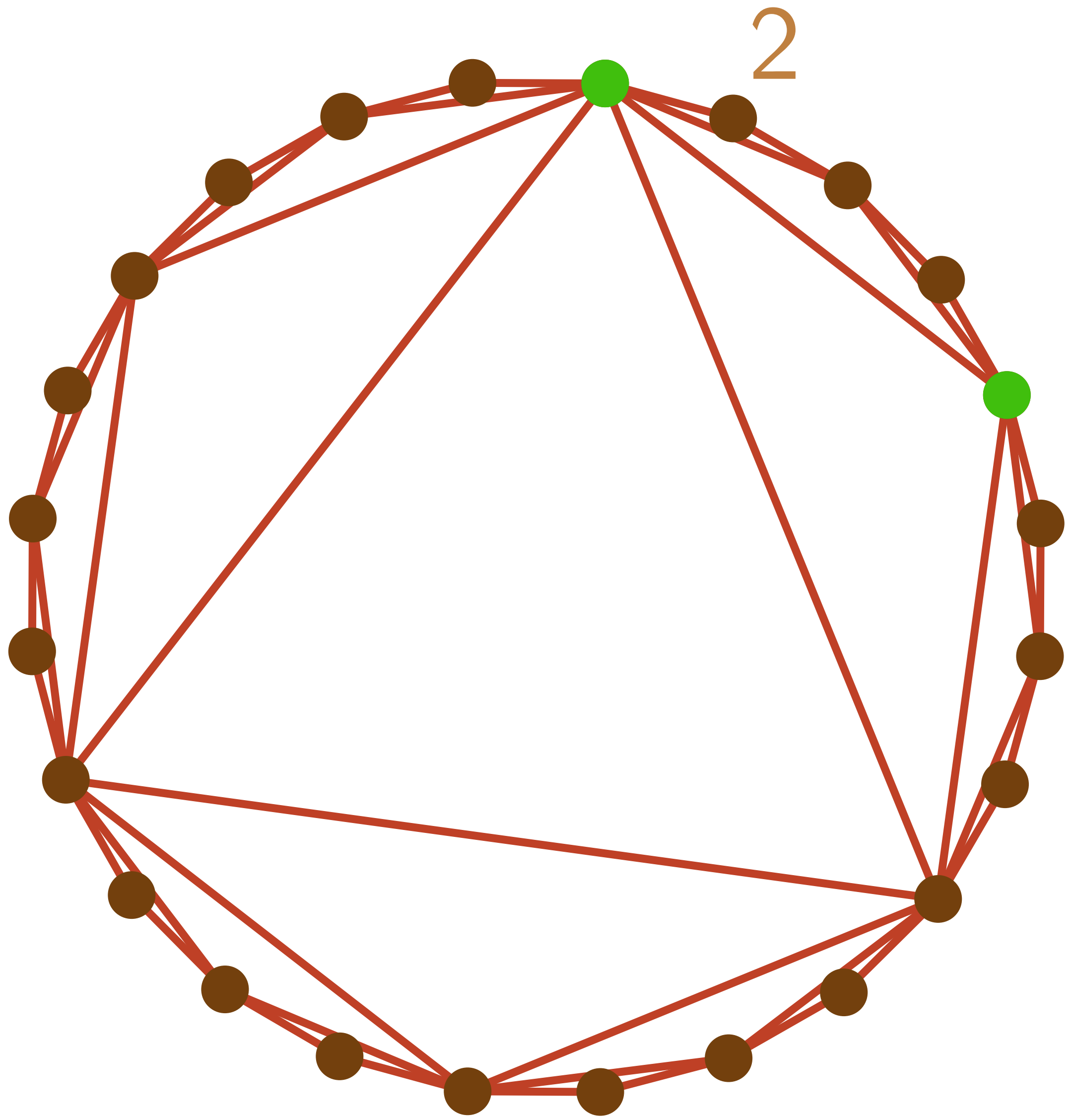

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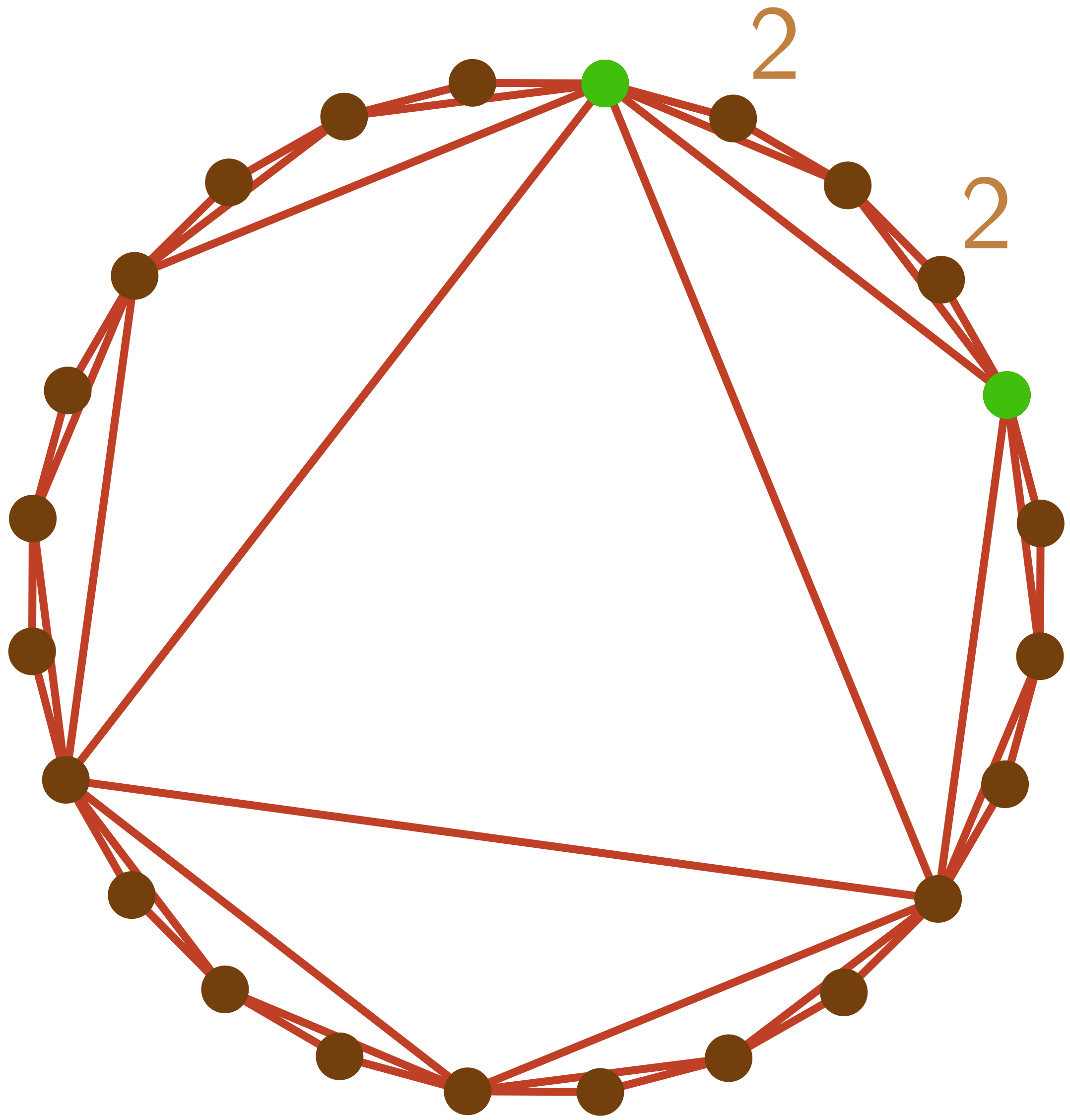
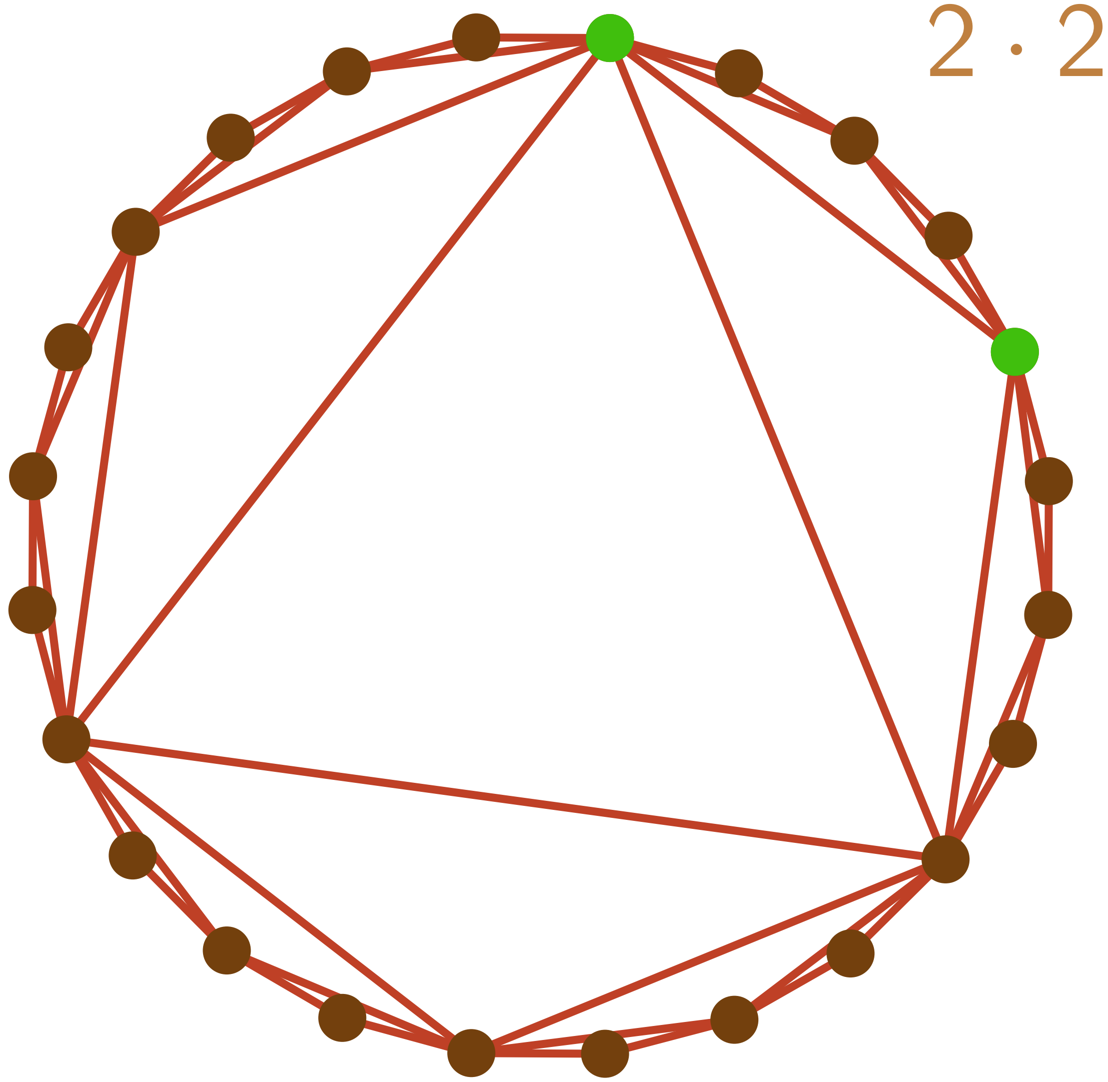


LOWER BOUND

- Place n points in convex position
- Triangulate iteratively
- Count potatoes



$2 \cdot 2 + 1$

LOWER BOUND

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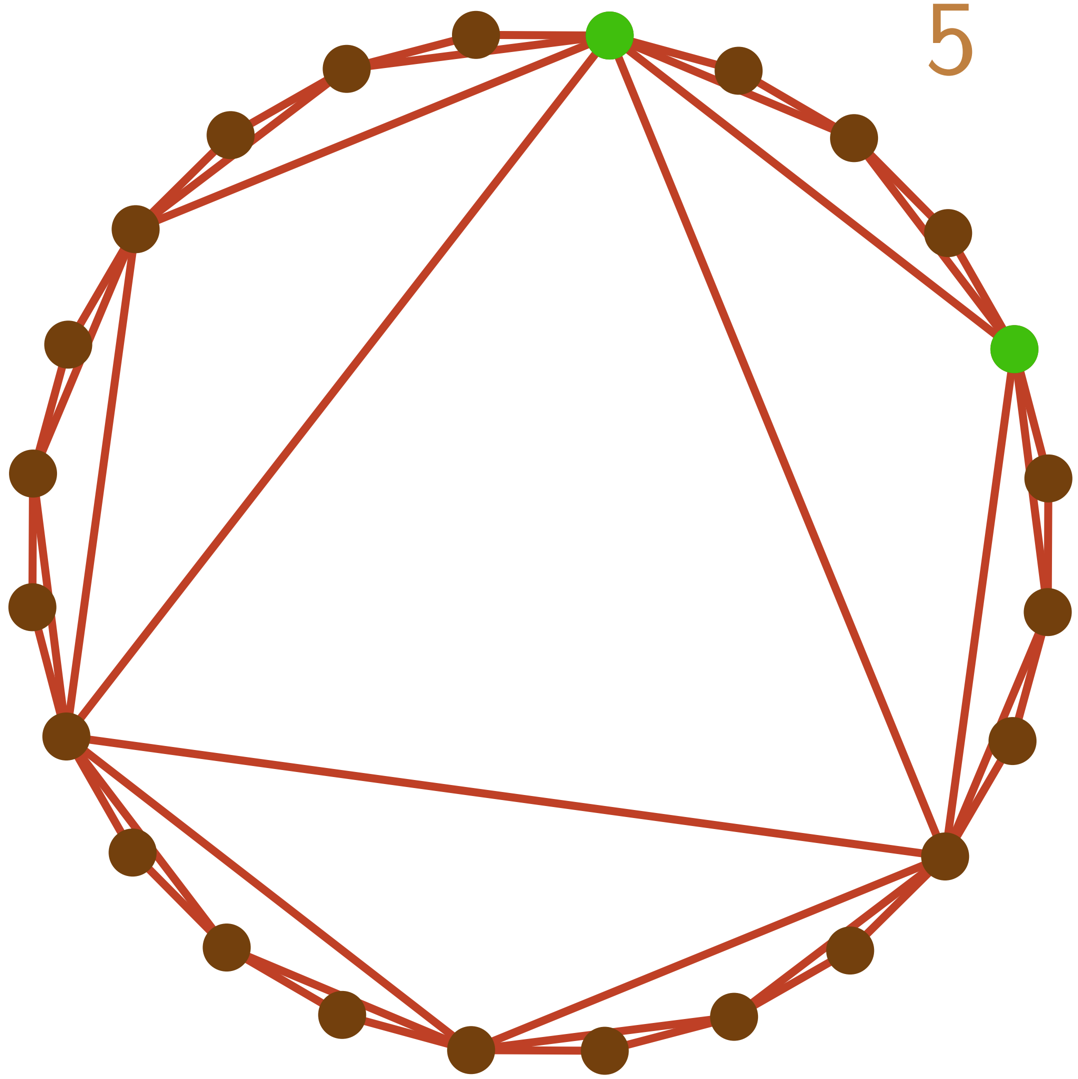

LOWER BOUND

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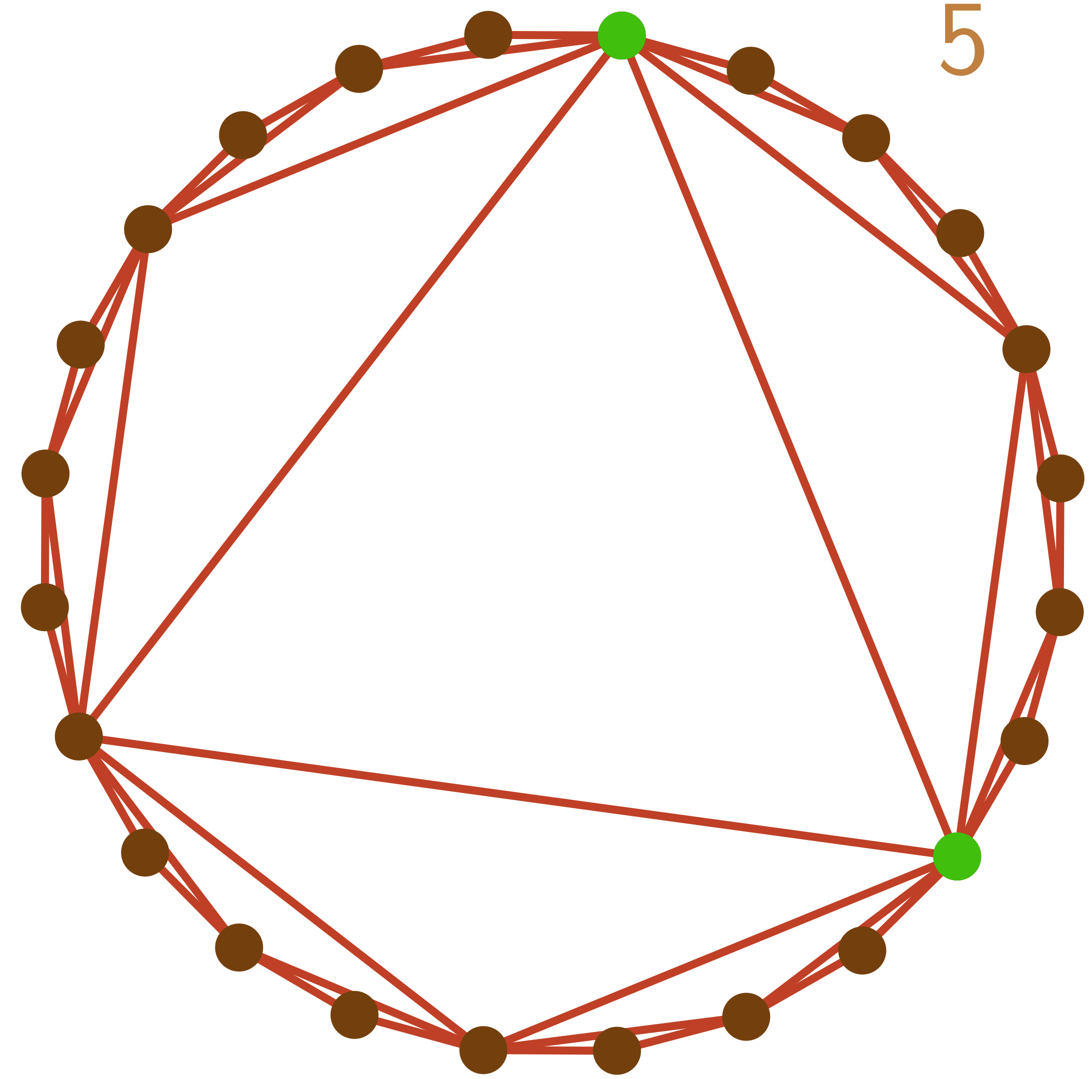

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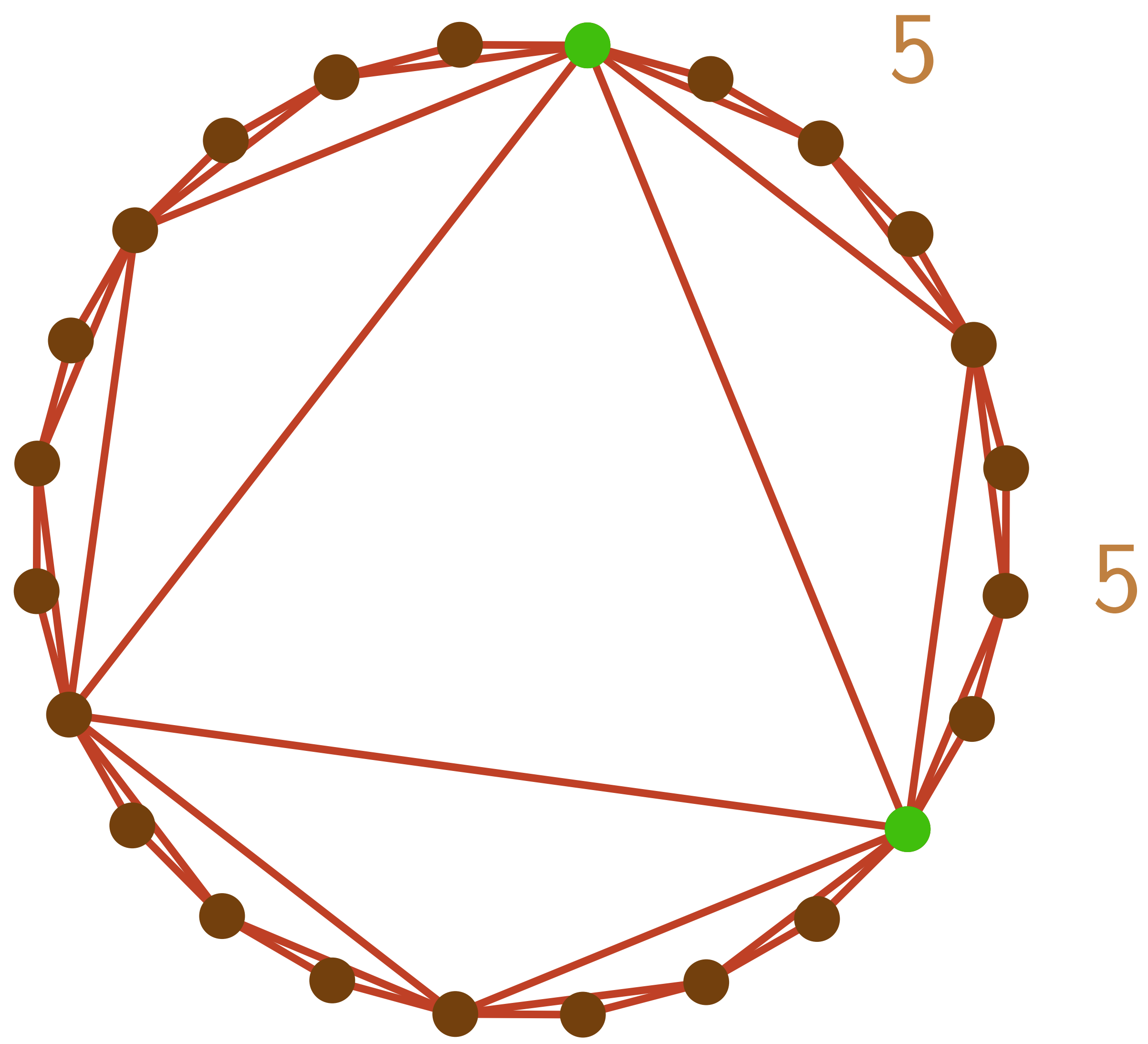

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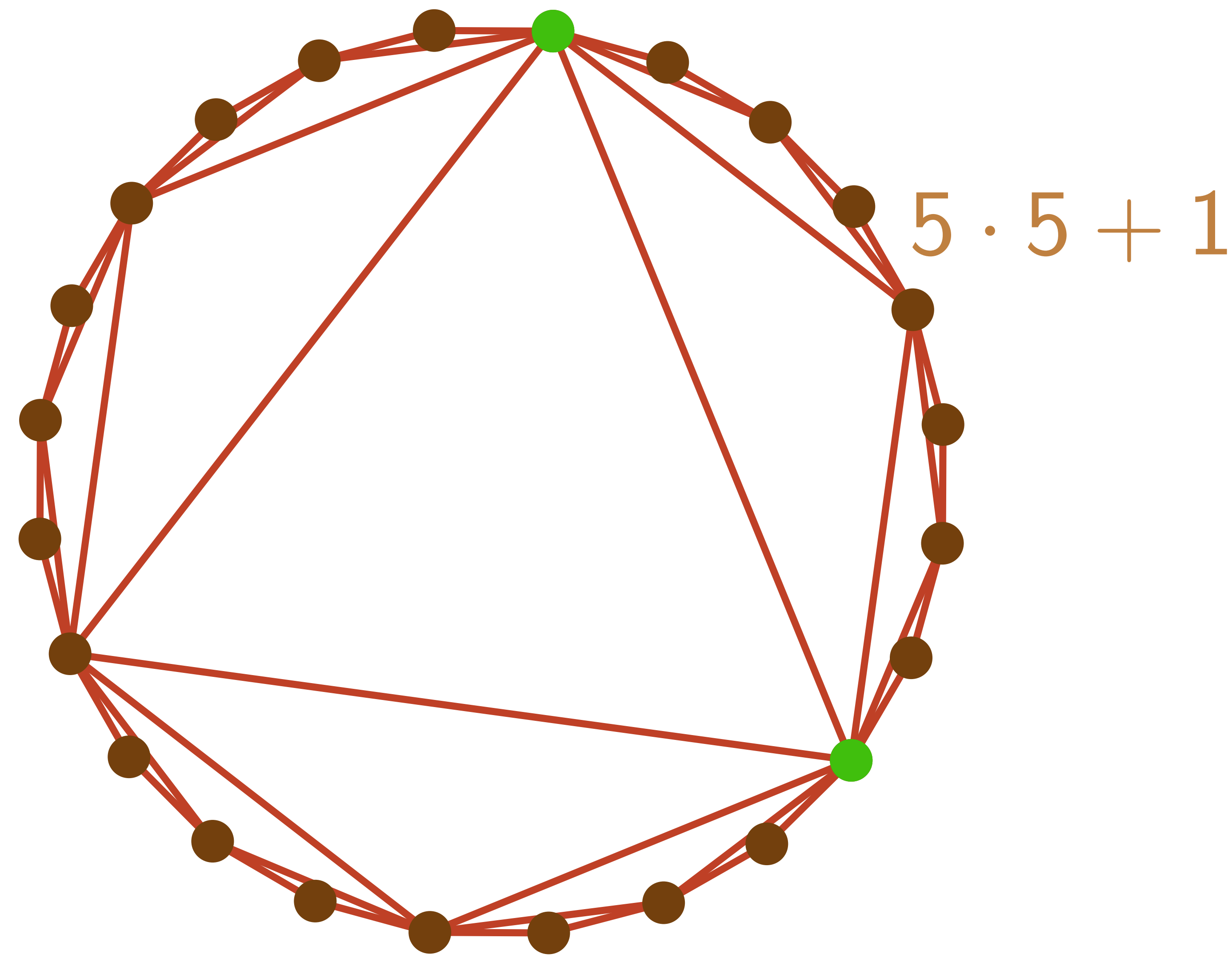

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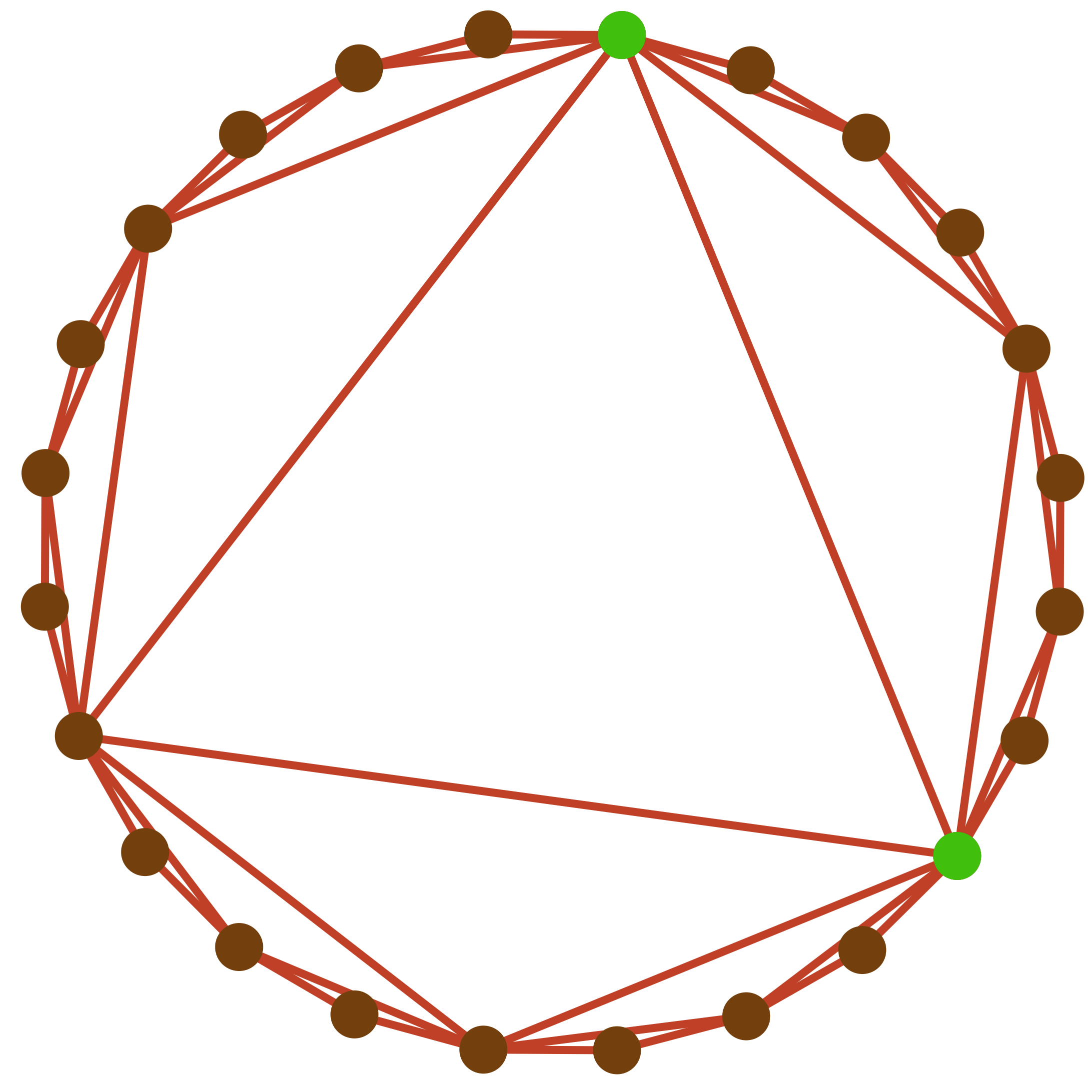

LOWER BOUND

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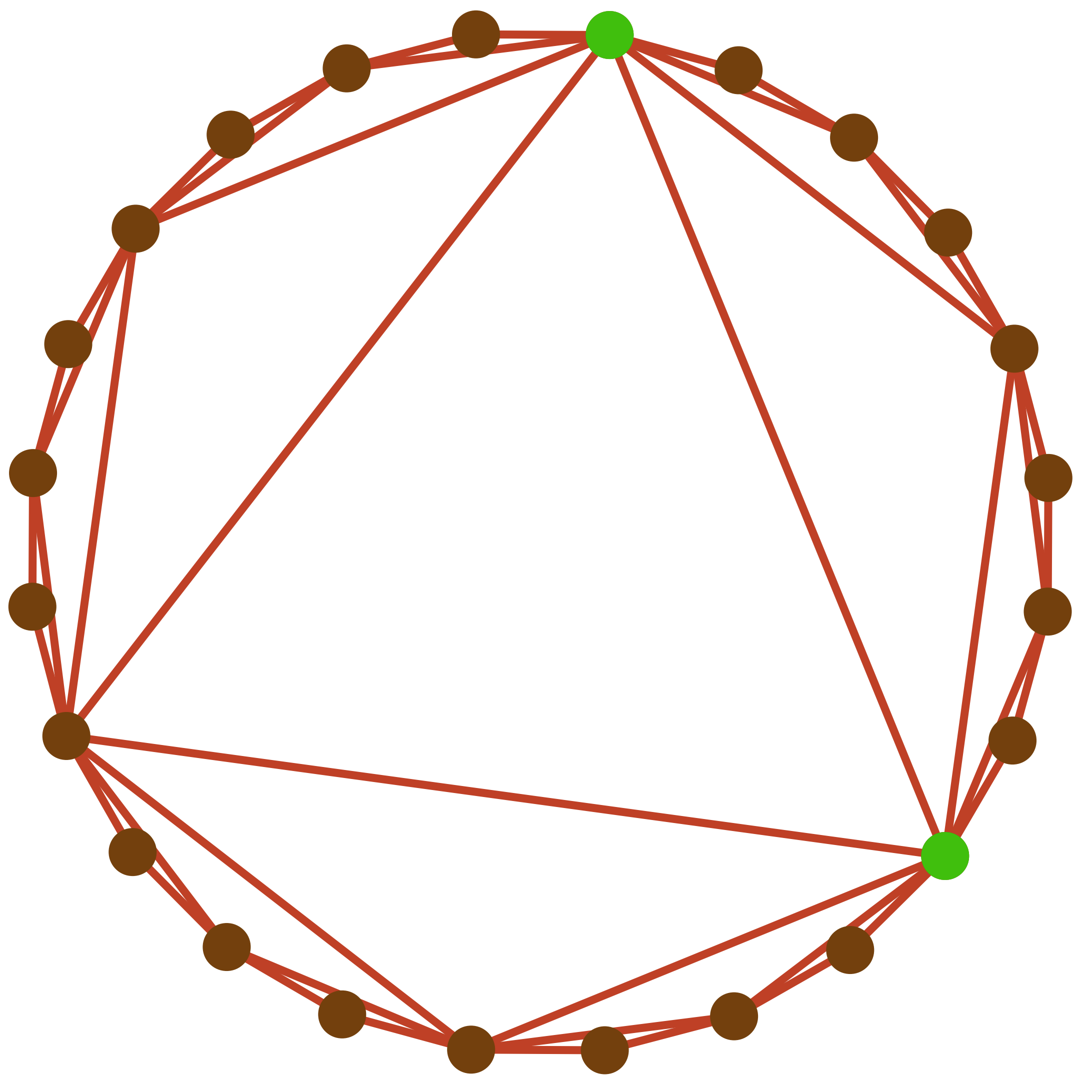
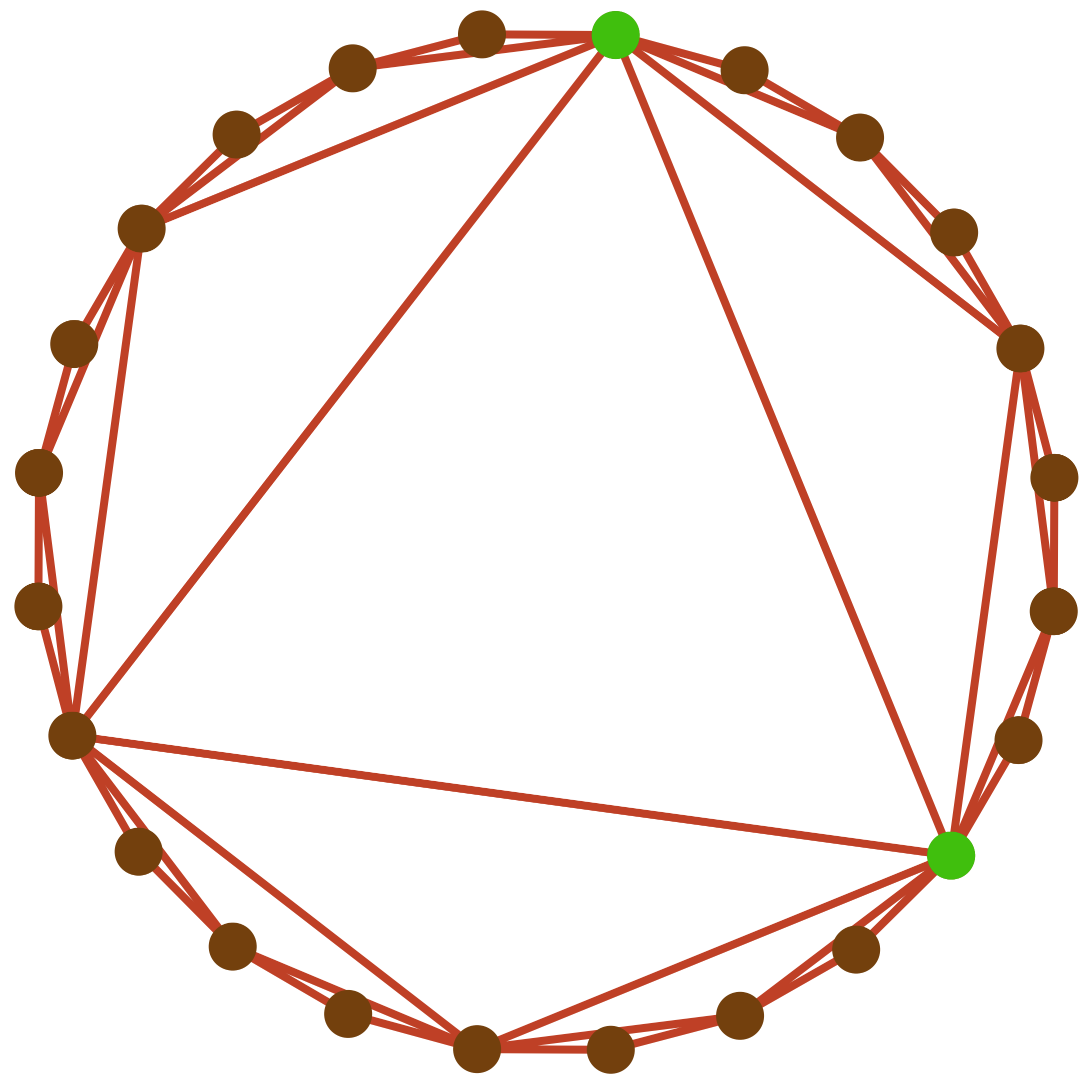
LEMMA



26

LOWER BOUND

- Place n points in convex position
- Triangulate iteratively
- Count potatoes



LEMMA

The number of potatoes in M *grows exponentially with base* > 1.5028

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- Place n points in convex position
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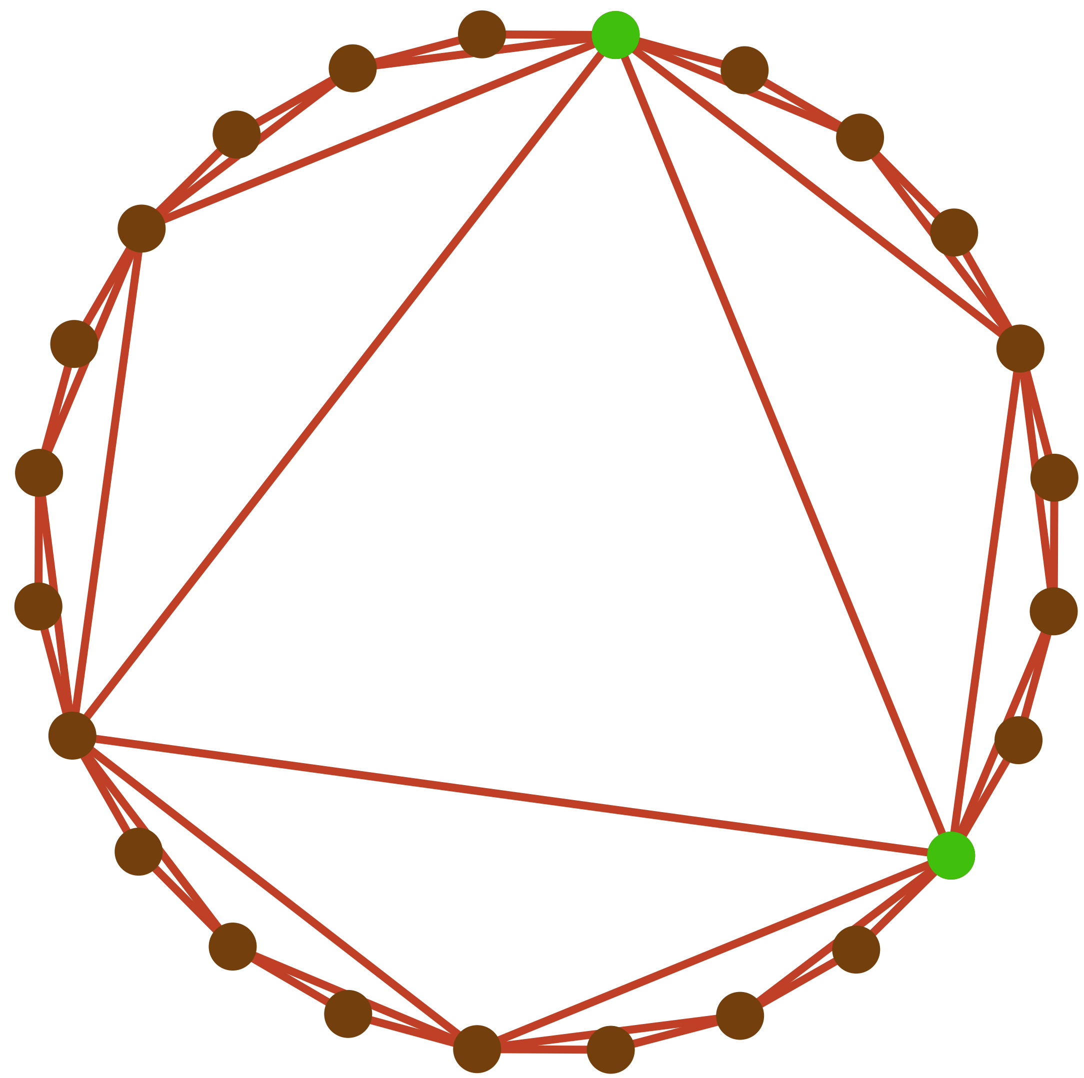

26

LEMMA

The number of potatoes in M *grows exponentially with base* > 1.5028

THEOREM

LOWER BOUND

- Place n points in convex position
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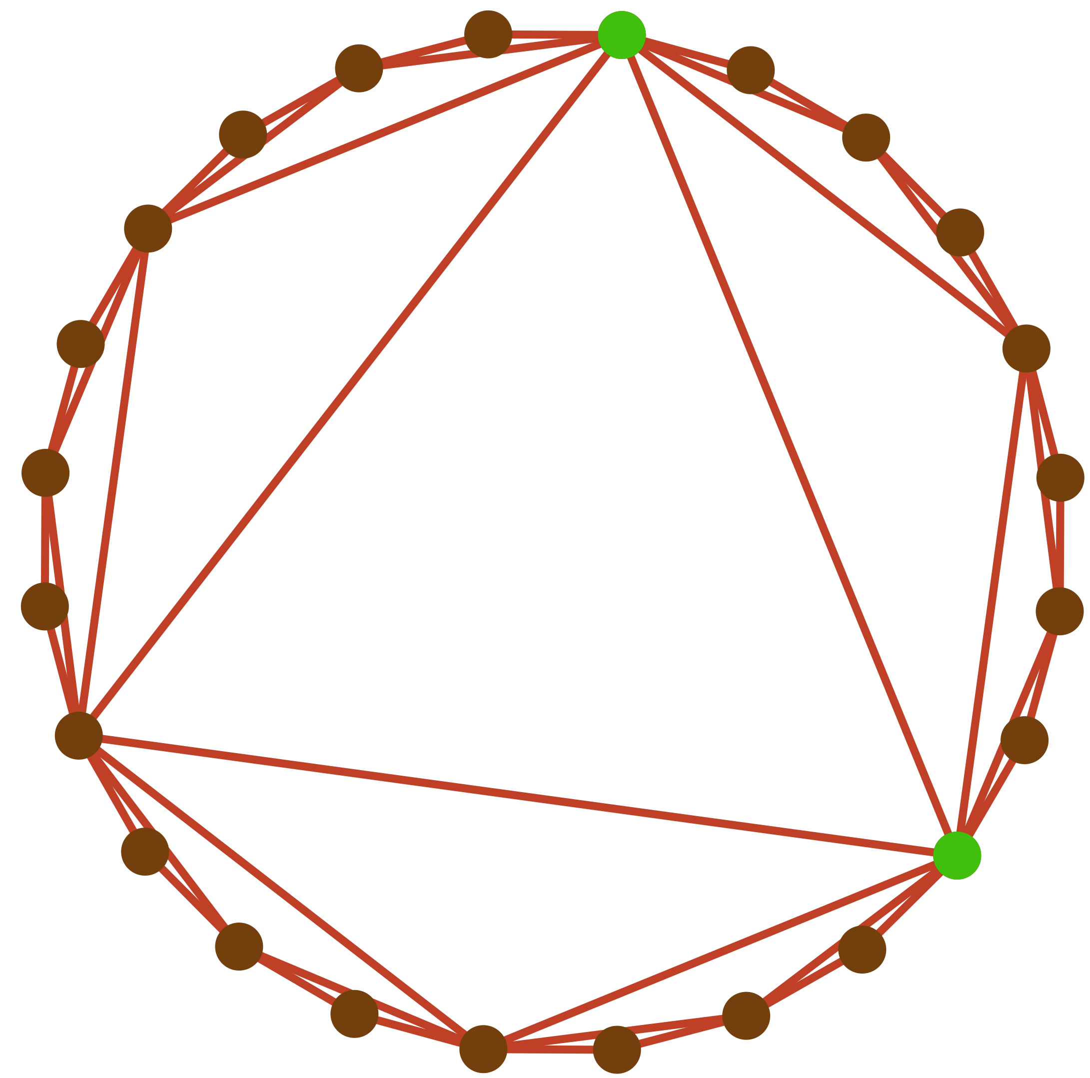


LEMMA

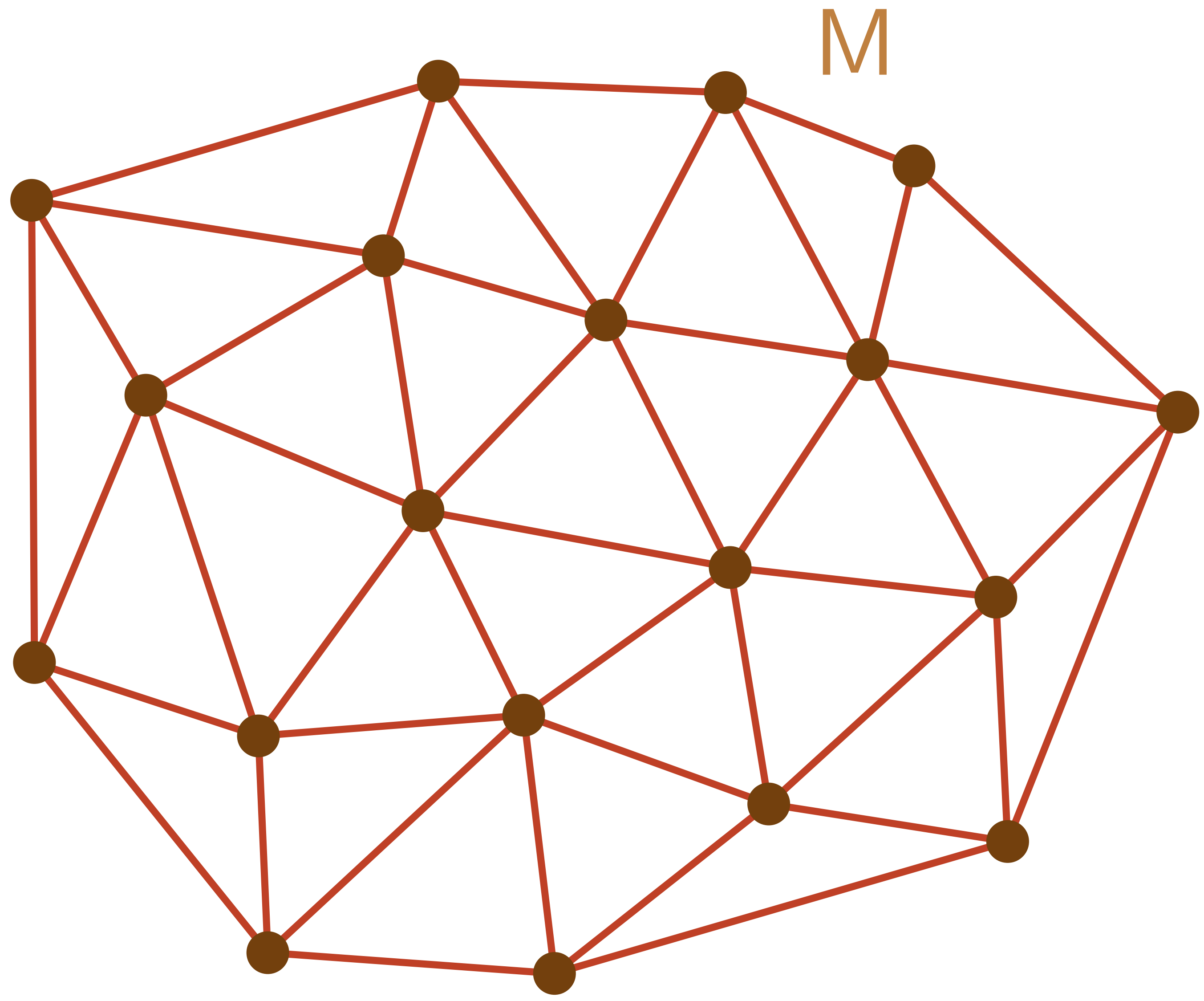
The number of potatoes in M *grows exponentially with base* > 1.5028

THEOREM

There is a mesh M *with* $\mathcal{P}(M)$ *in* $\Omega(1.5028^n)$.

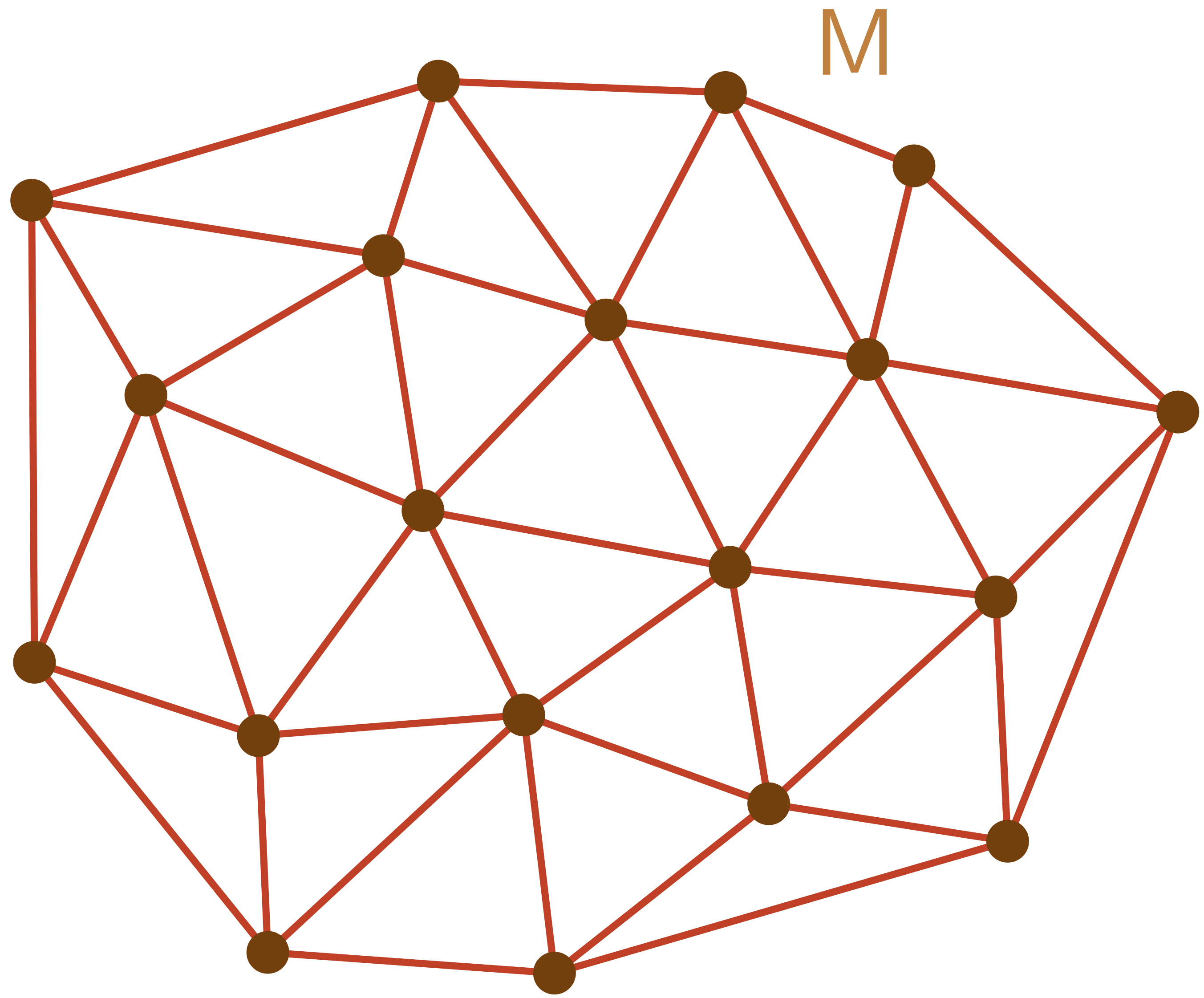
UPPER BOUND

UPPER BOUND



UPPER BOUND

- Only count all potatoes containing p



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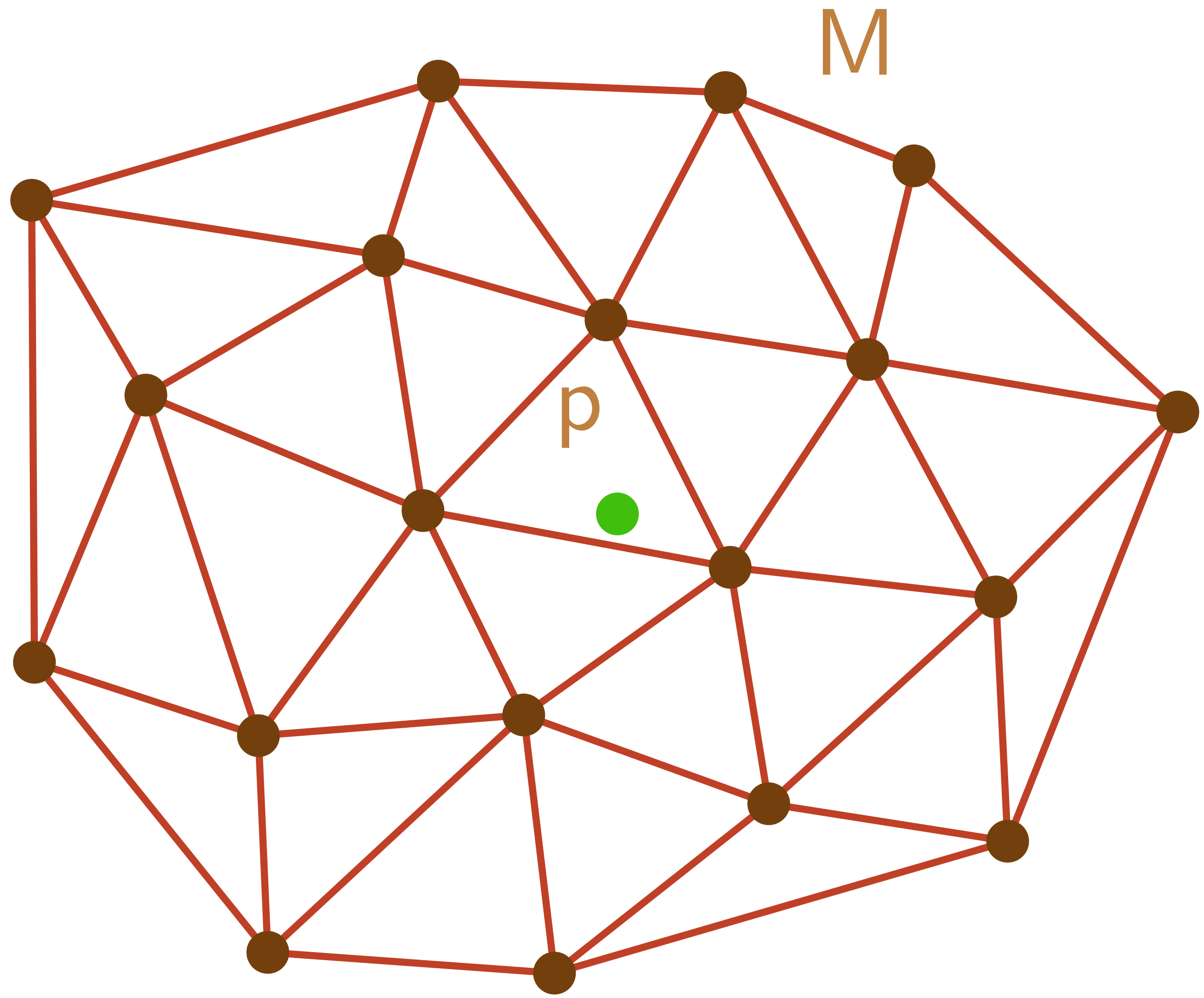
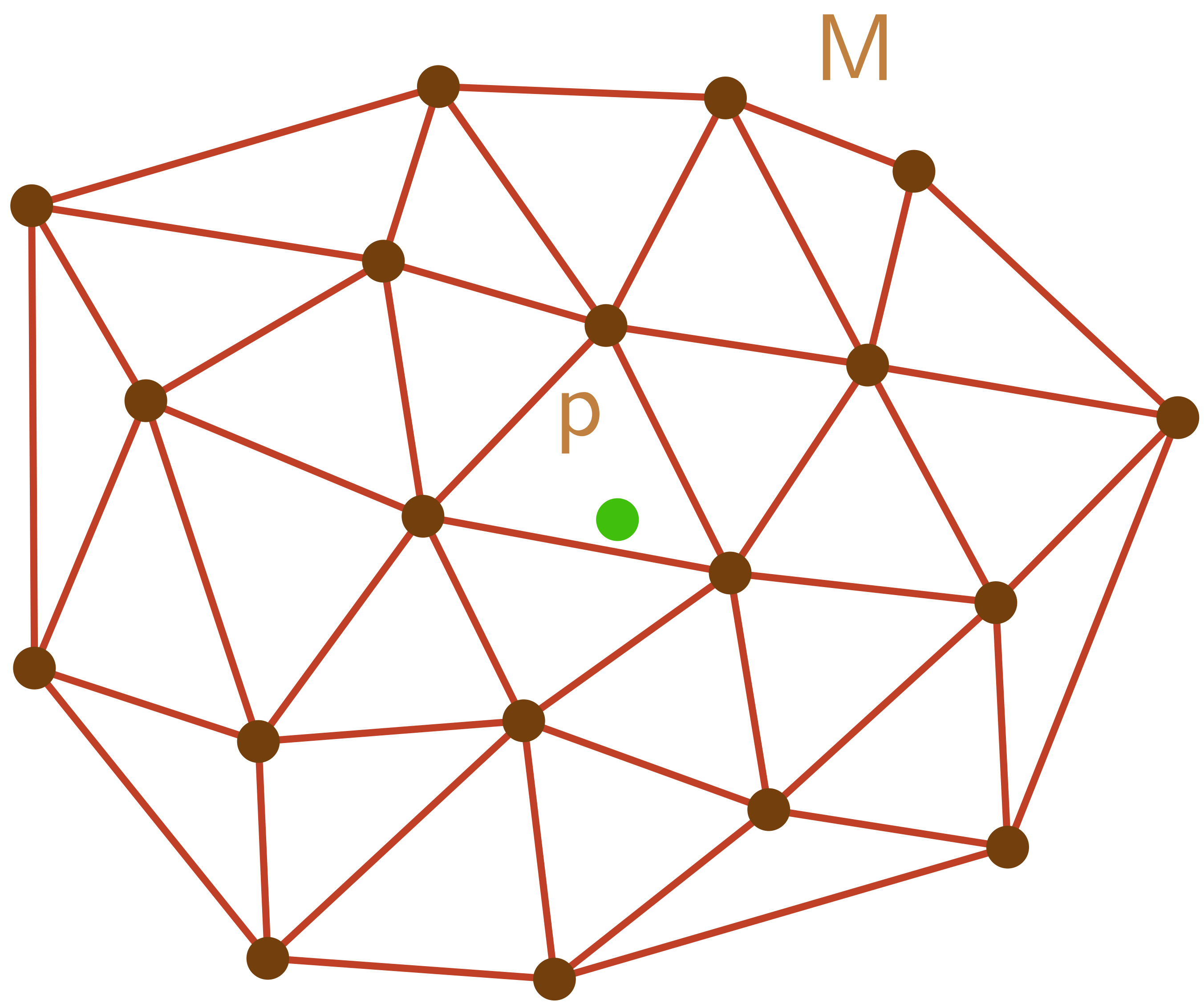


UPPER BOUND

- Only count all potatoes containing p
- Project all vertices from p and remove hit edges



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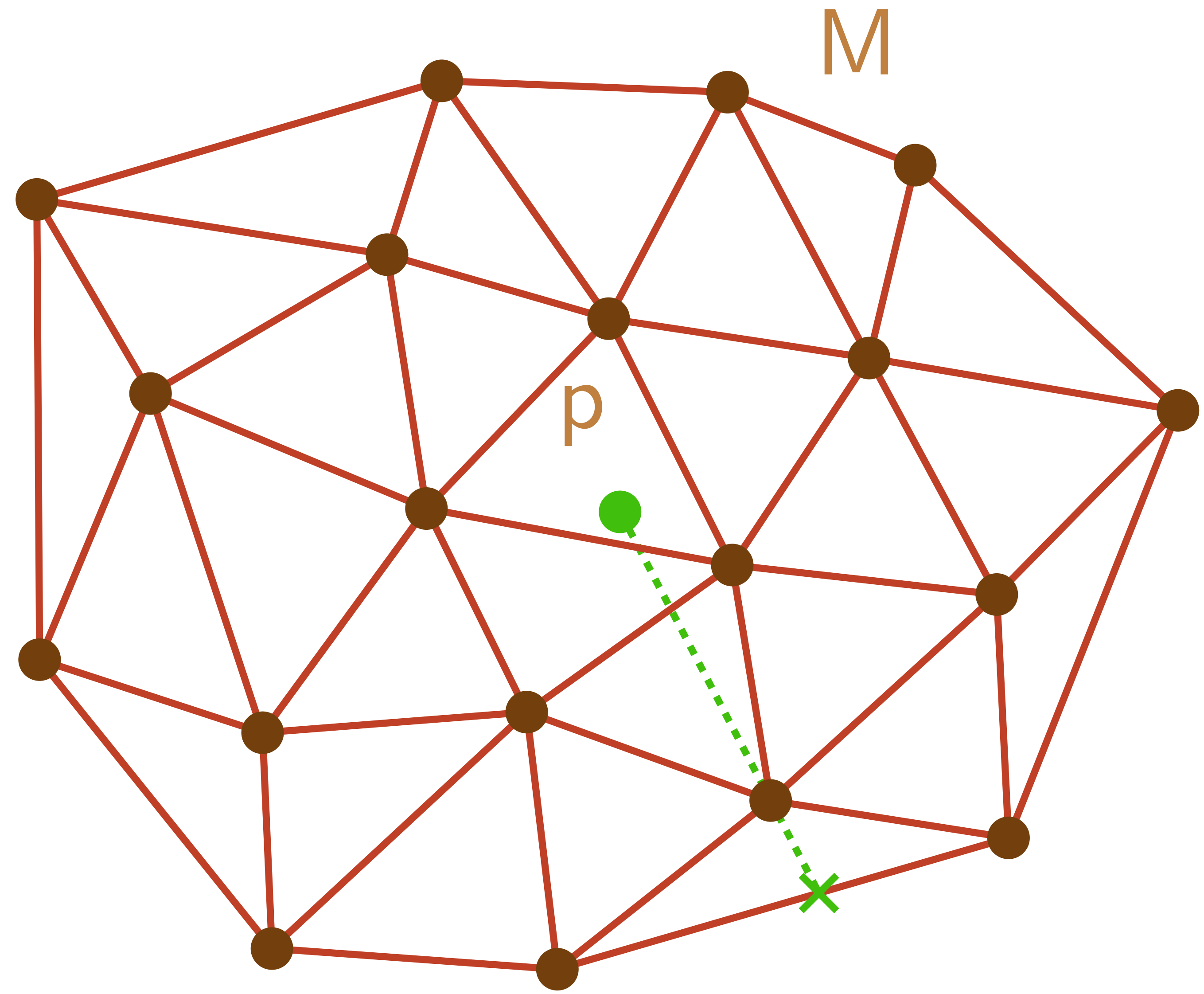

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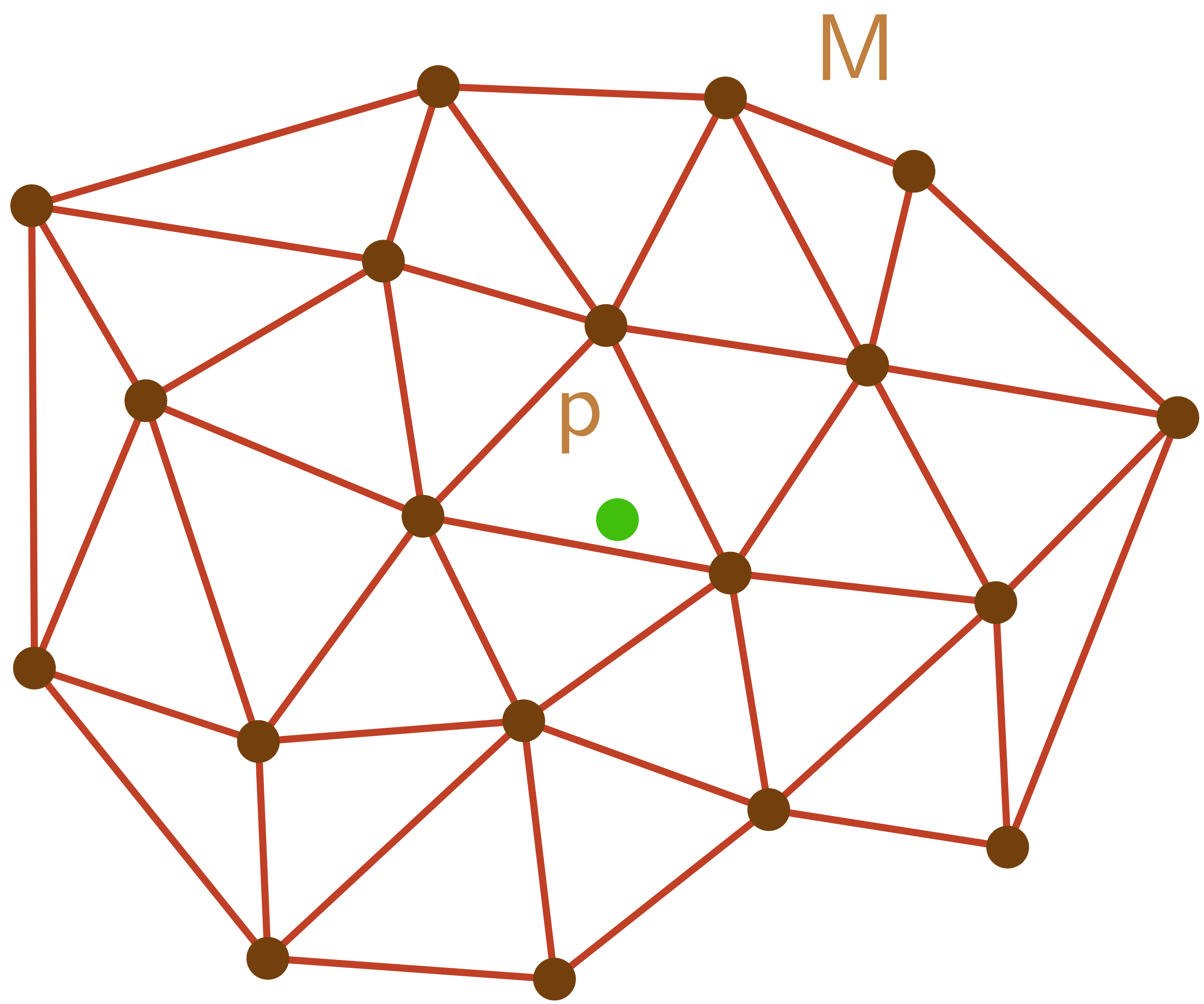

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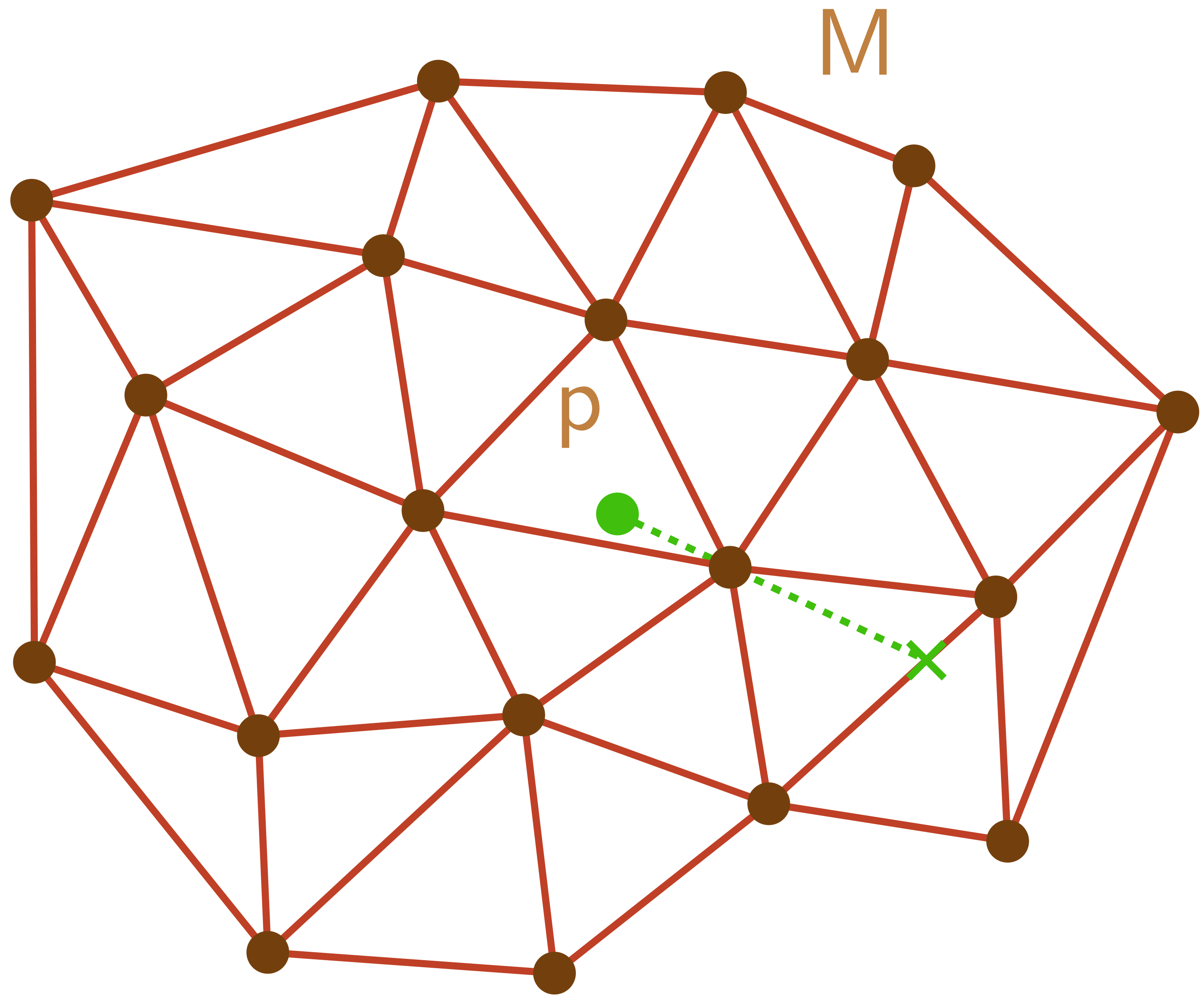

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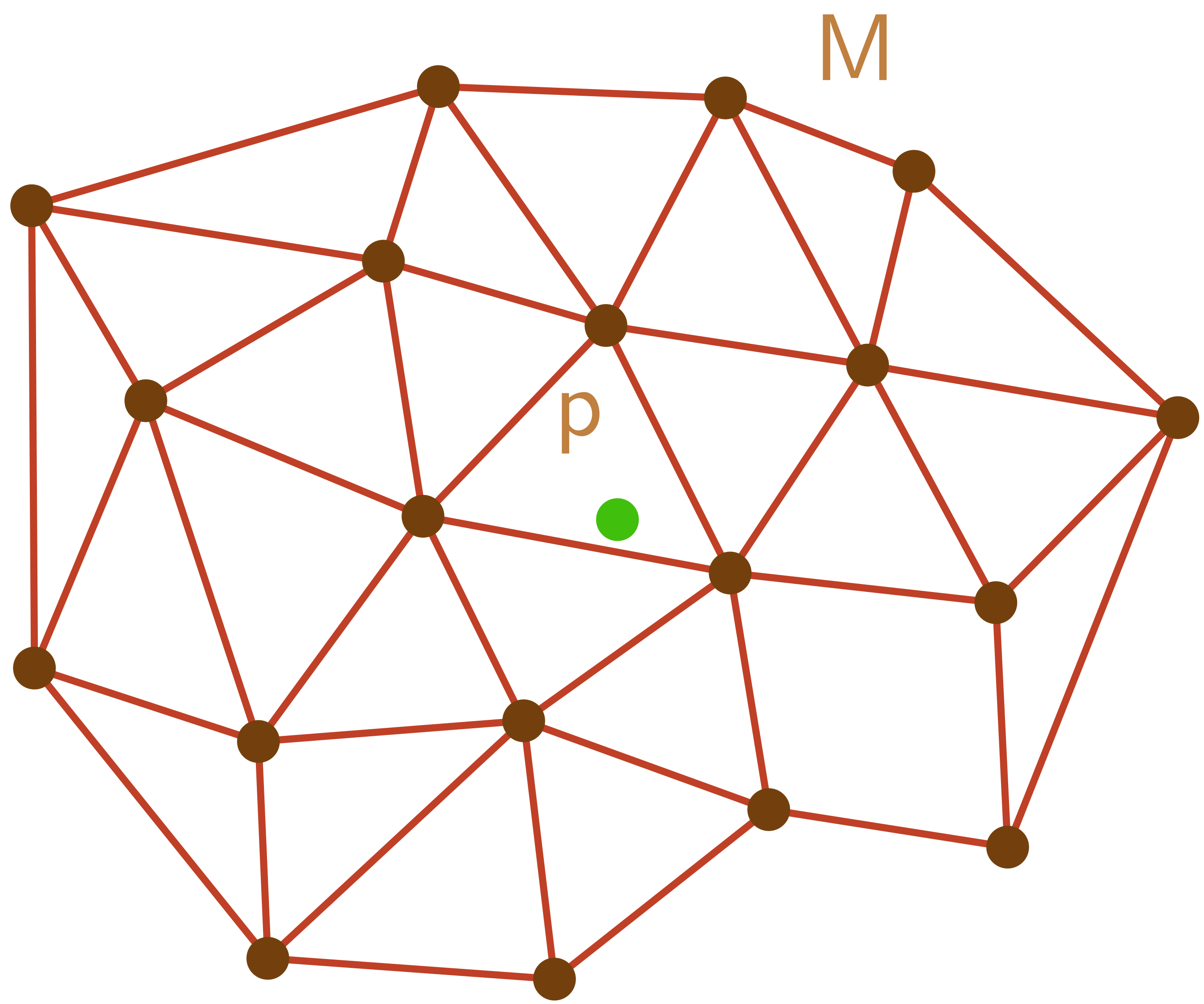

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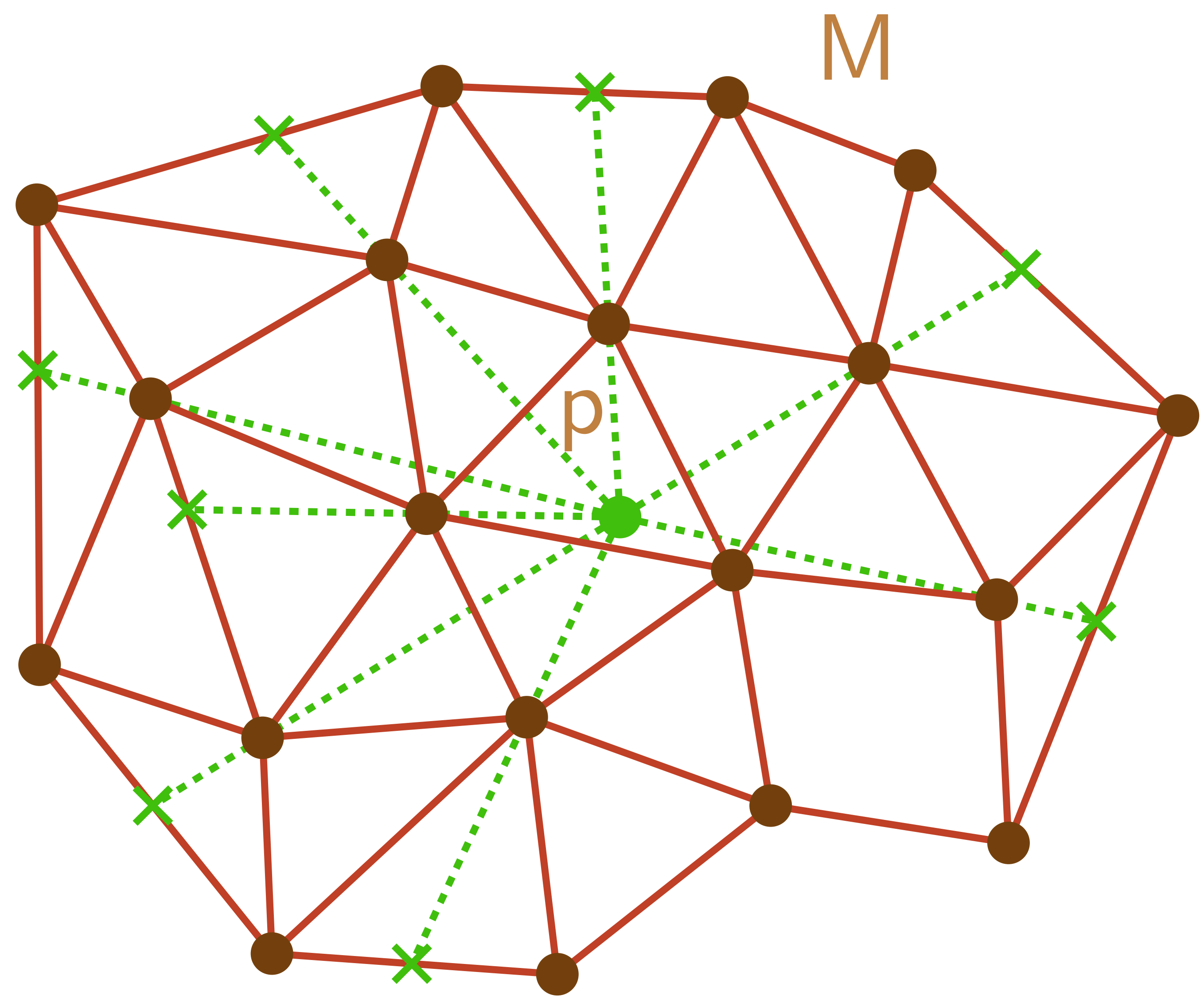

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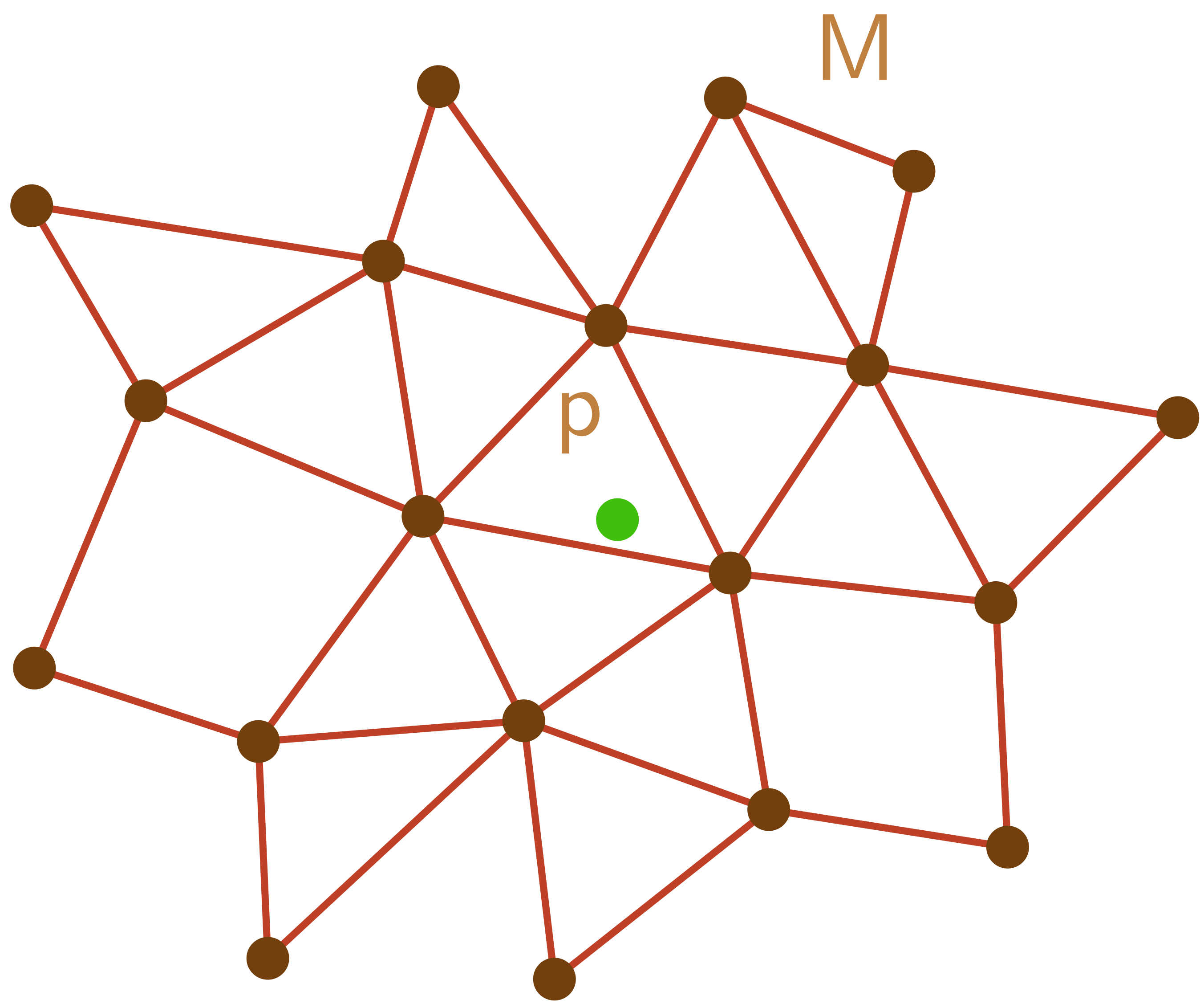
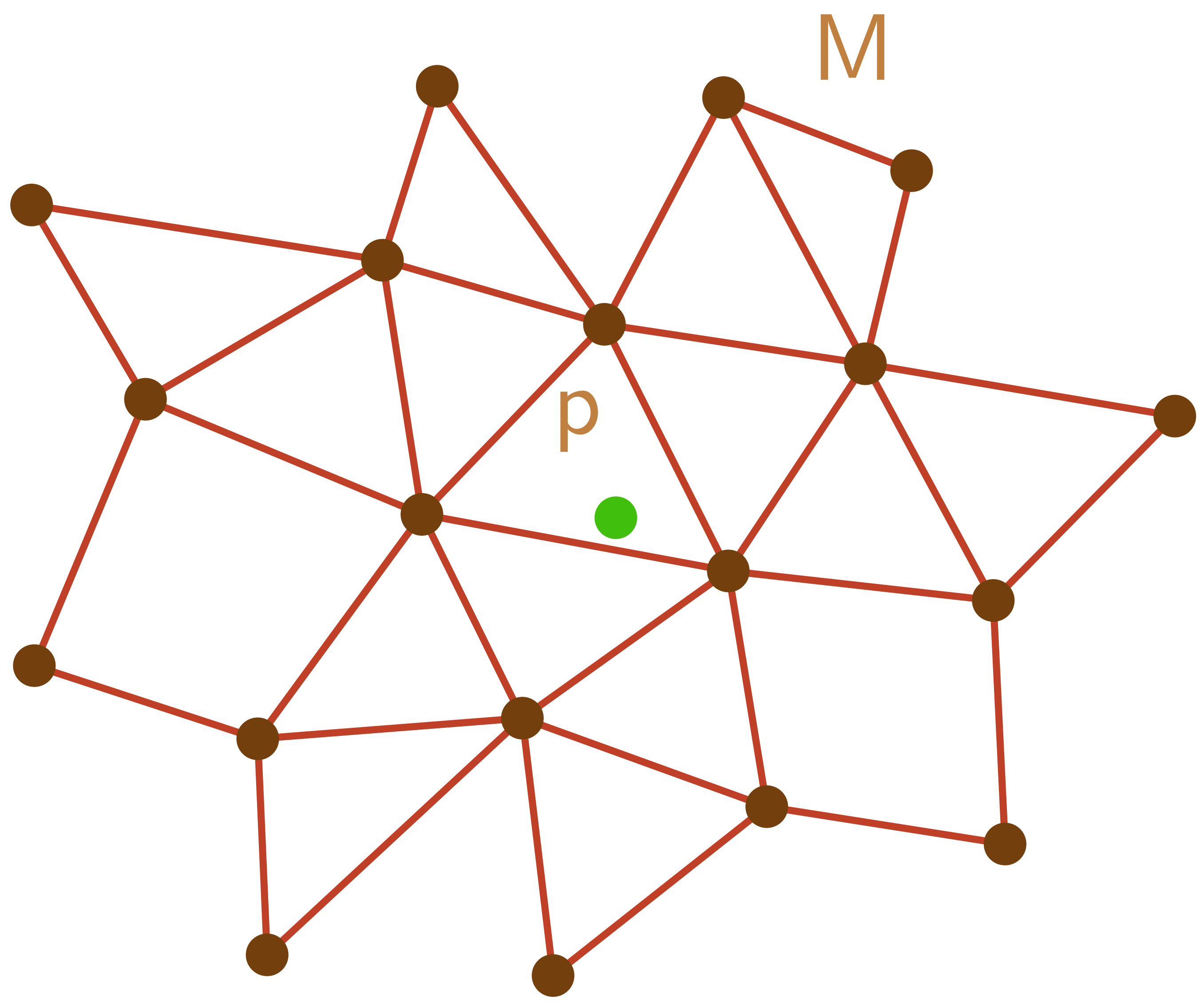


UPPER BOUND

- Only count all potatoes containing p
- Project all vertices from p and remove hit edges
- Orient edges around p



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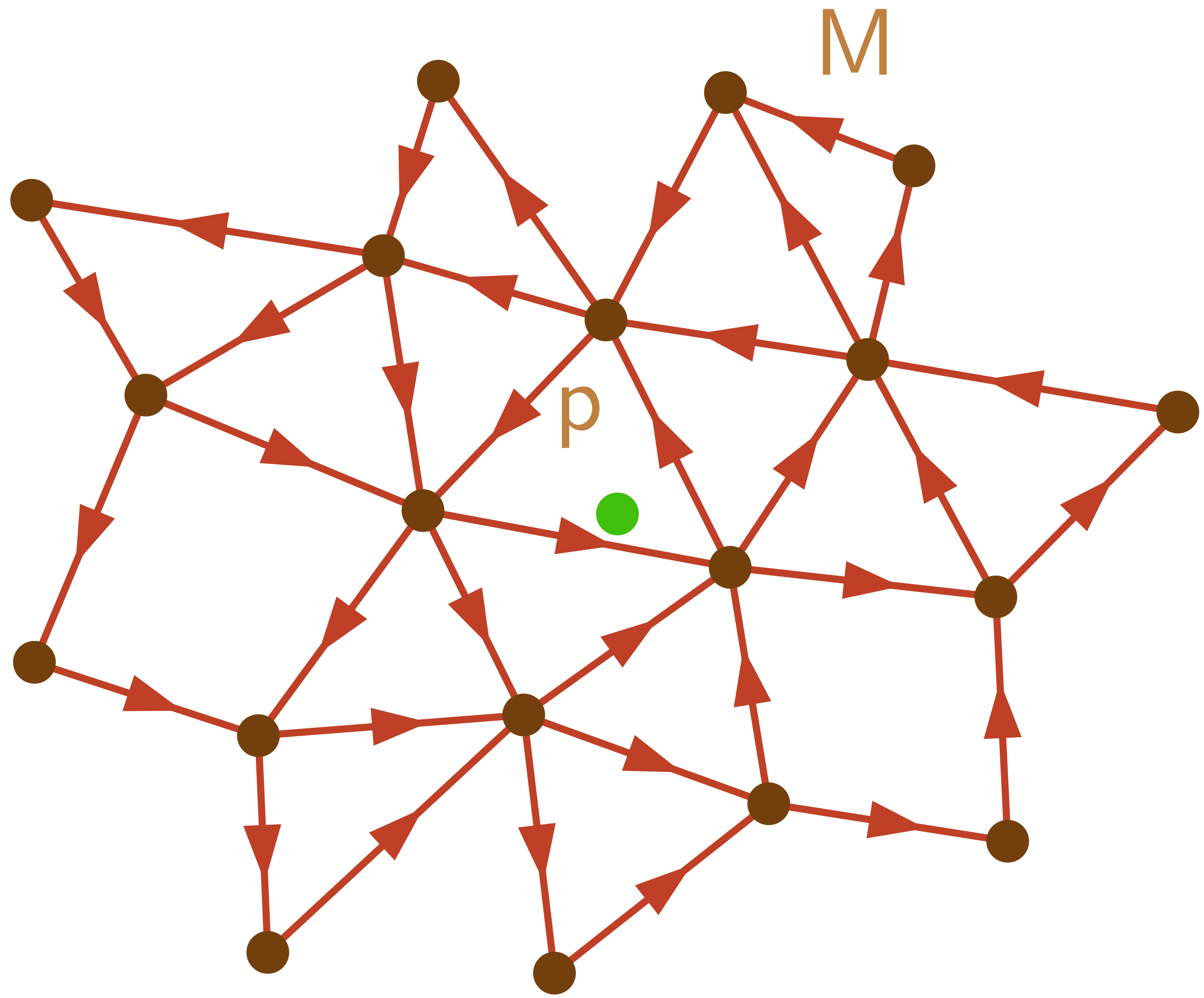
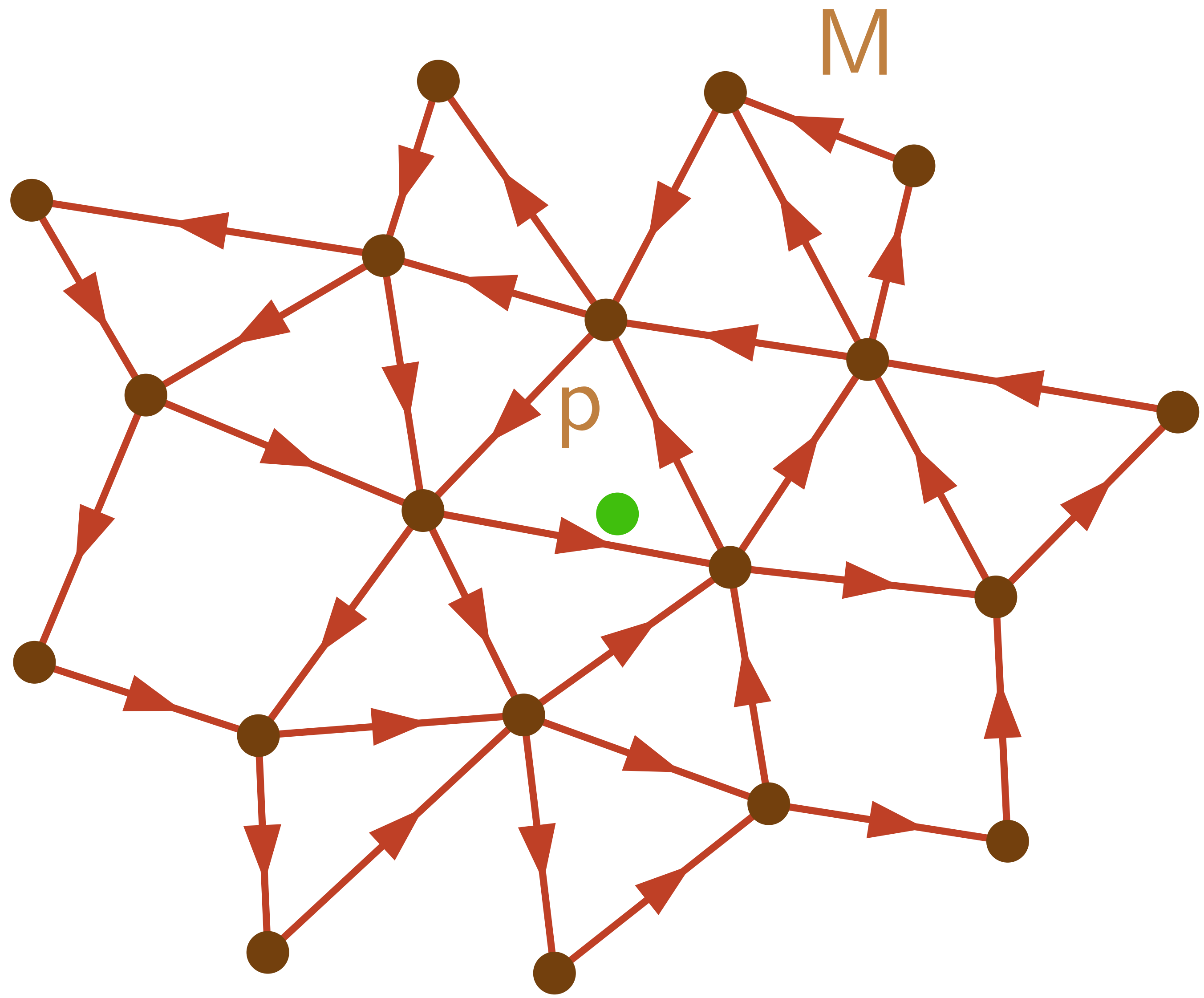


UPPER BOUND

- Only count all potatoes containing p
- Project all vertices from p and remove hit edges
- Orient edges around p
- Call resulting graph G



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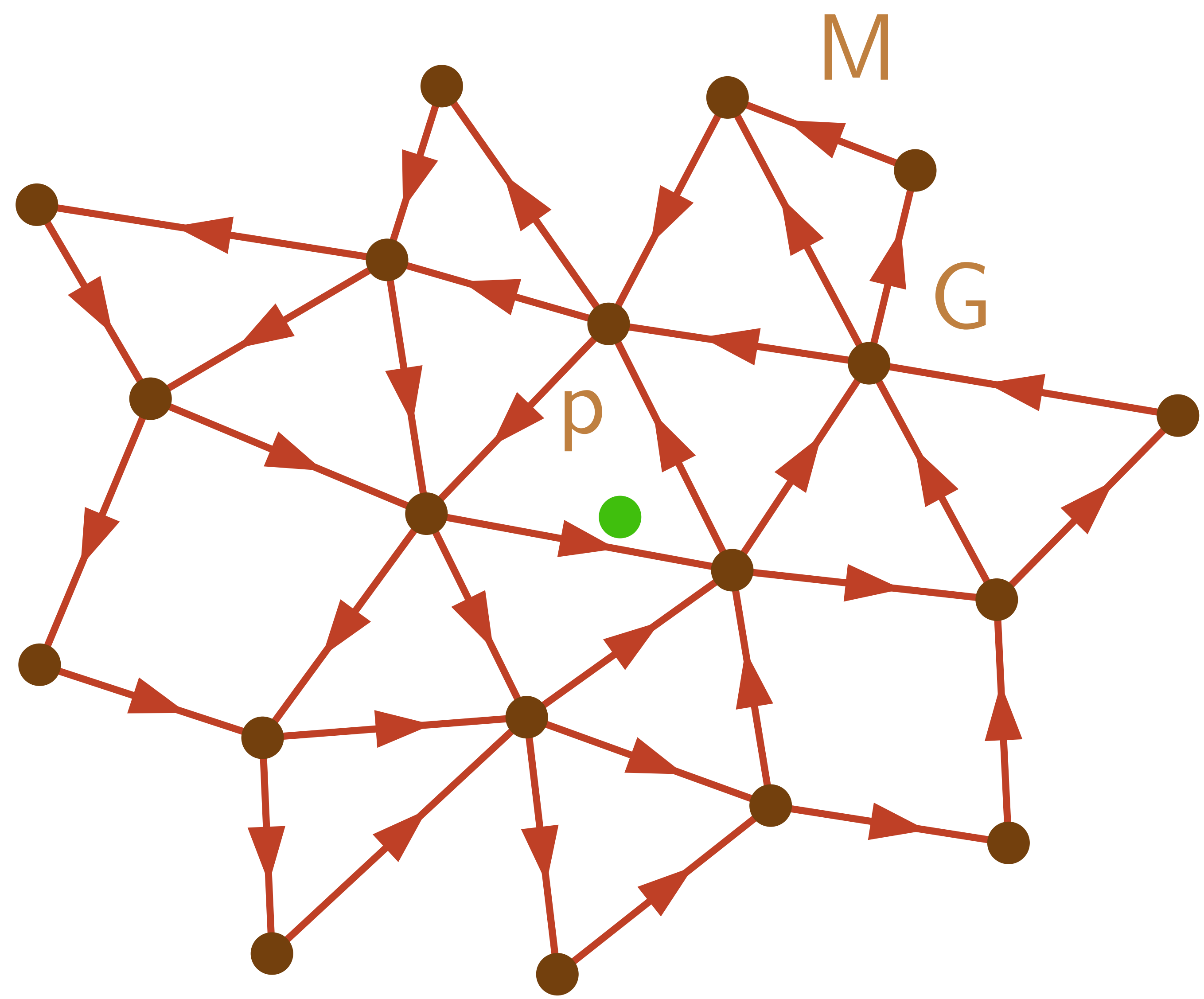
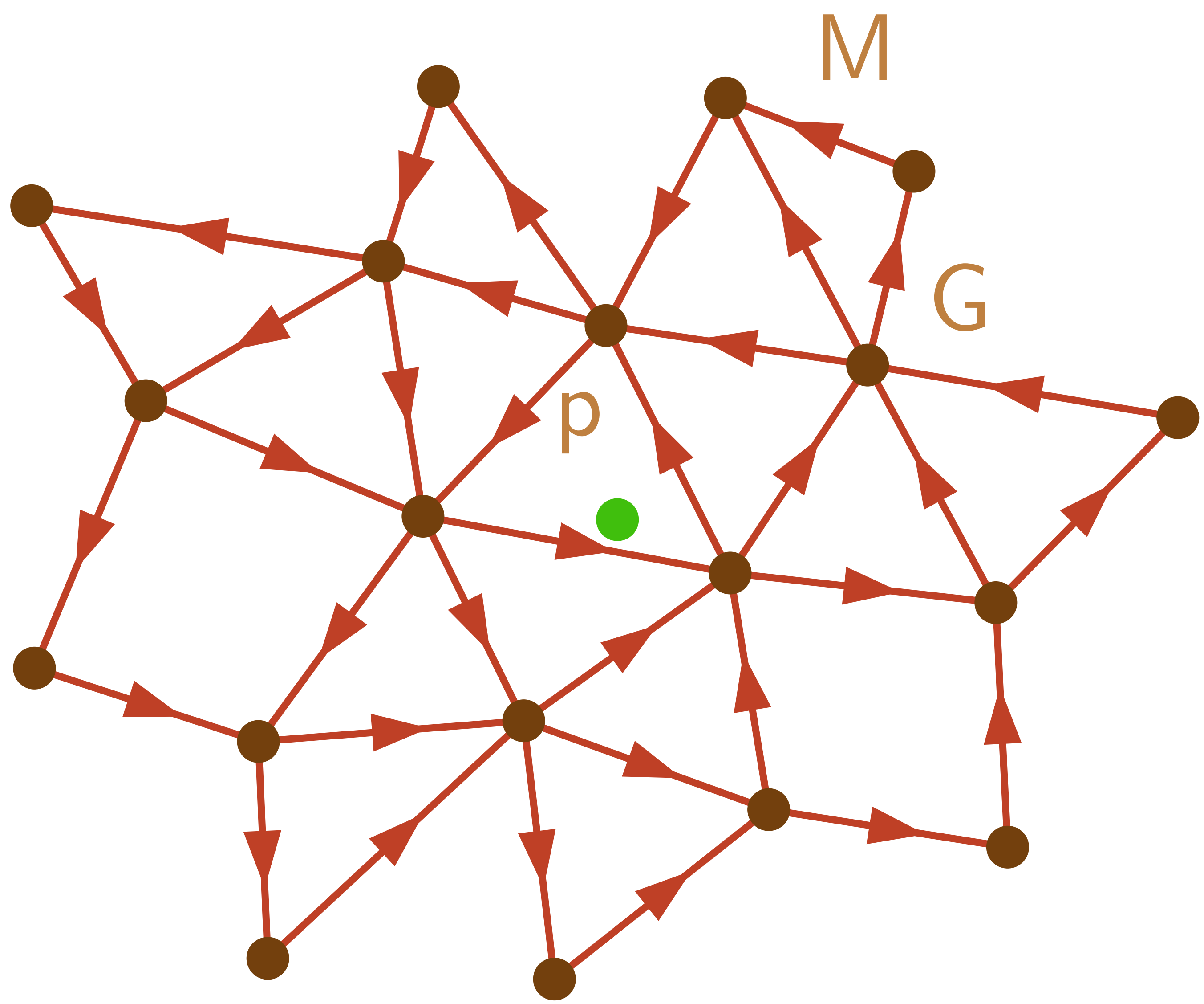
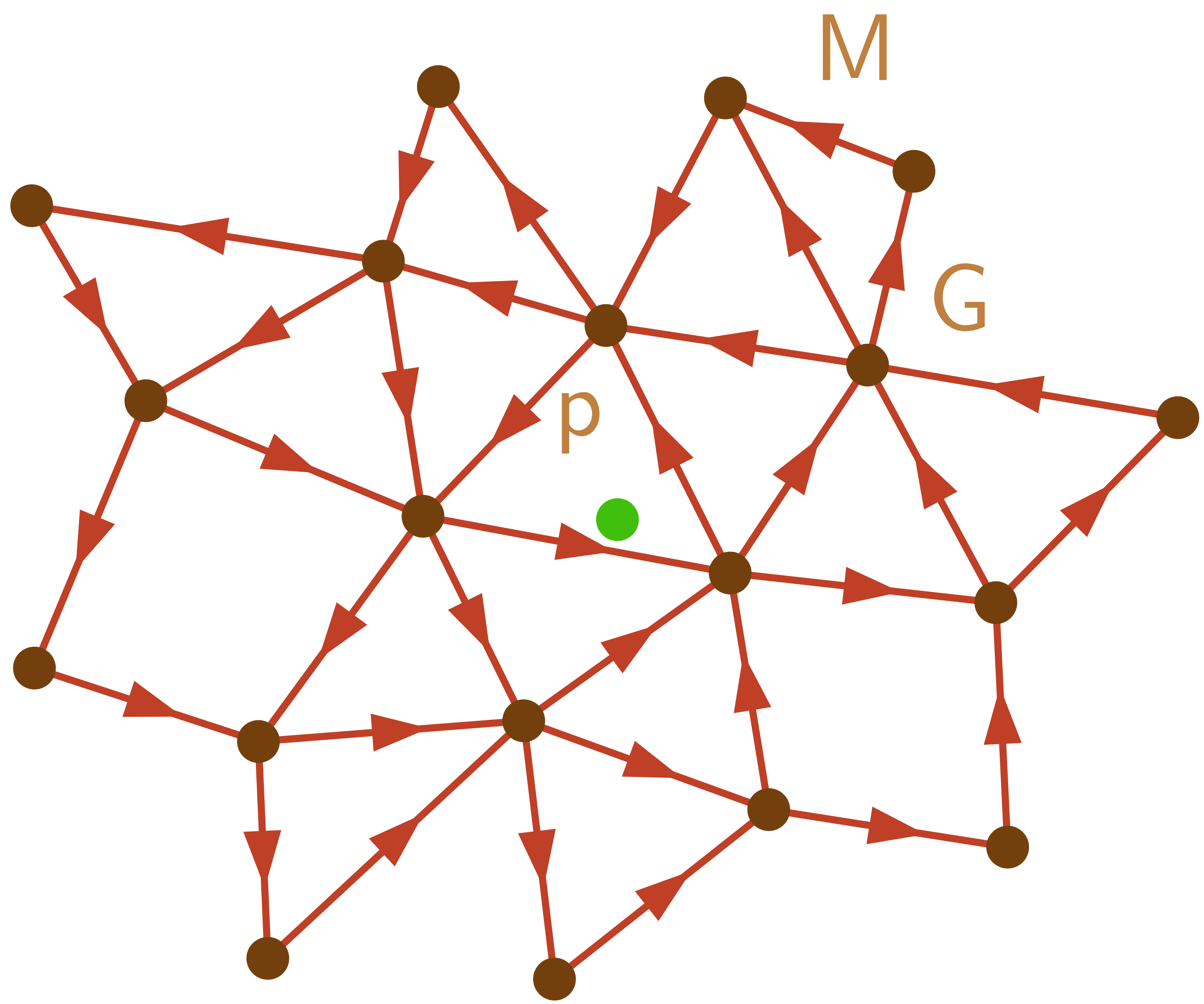


LEMMA



LEMMA

The number of potatoes in M *containing* p *is bounded by the number of cycles in* G



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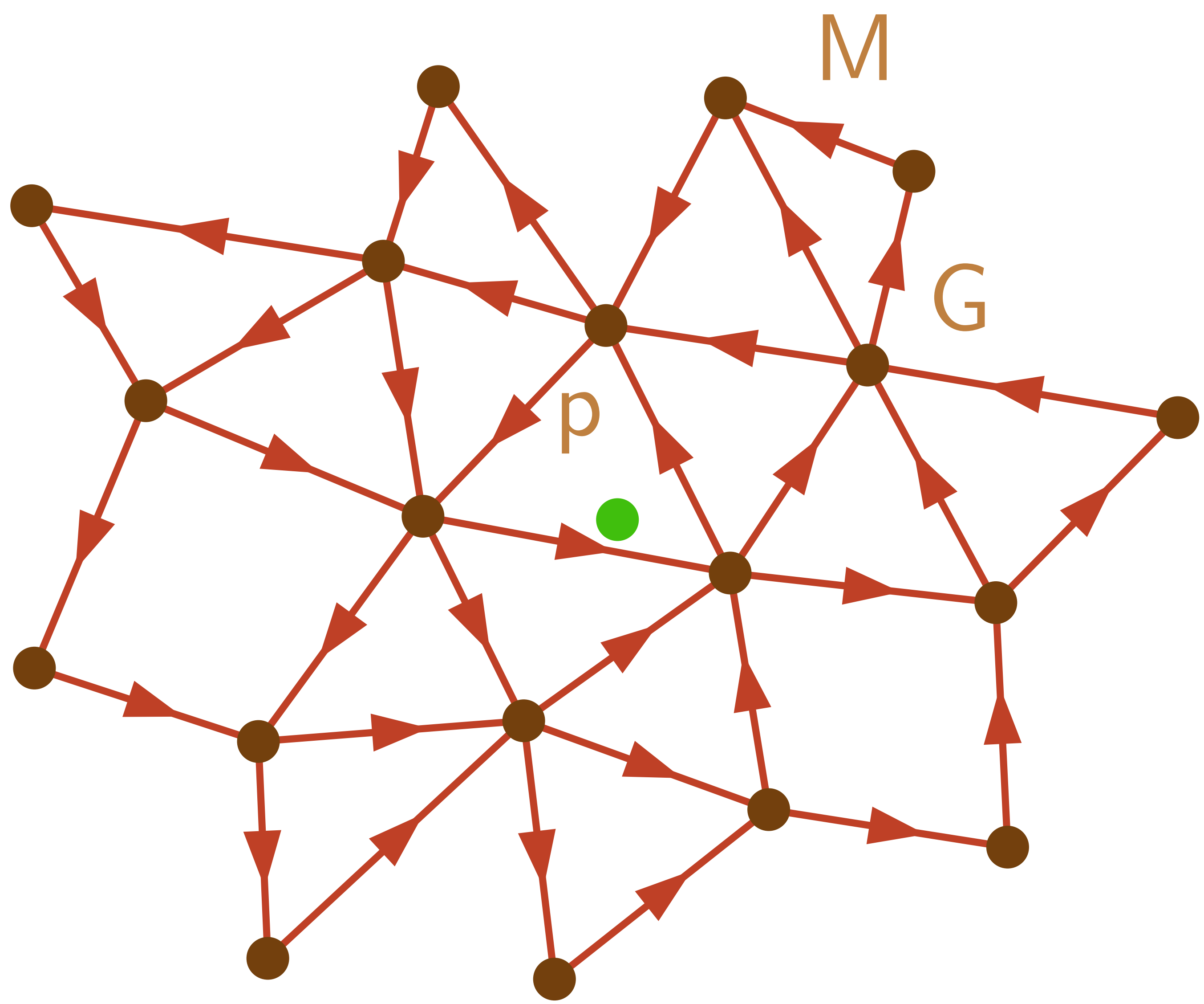

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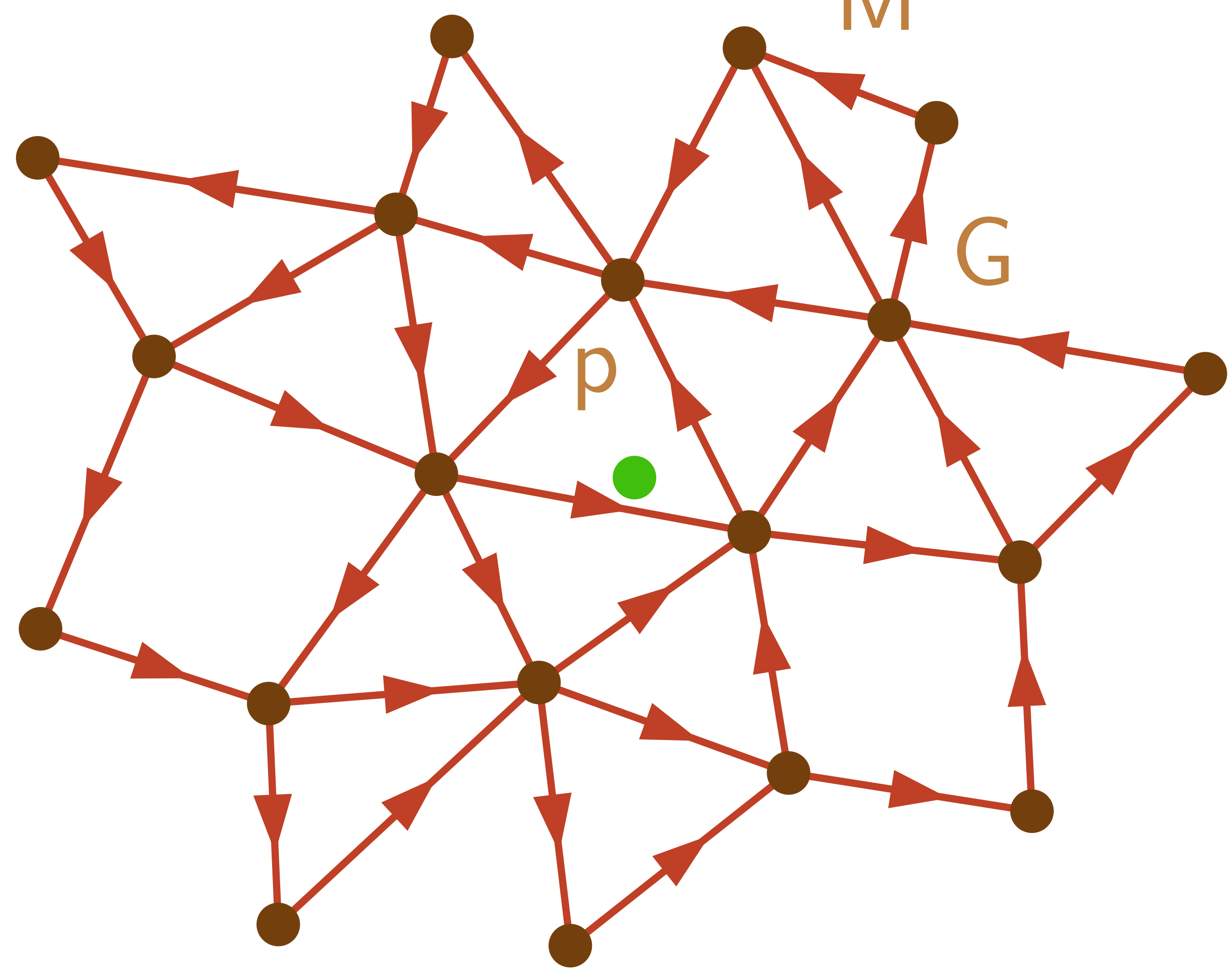
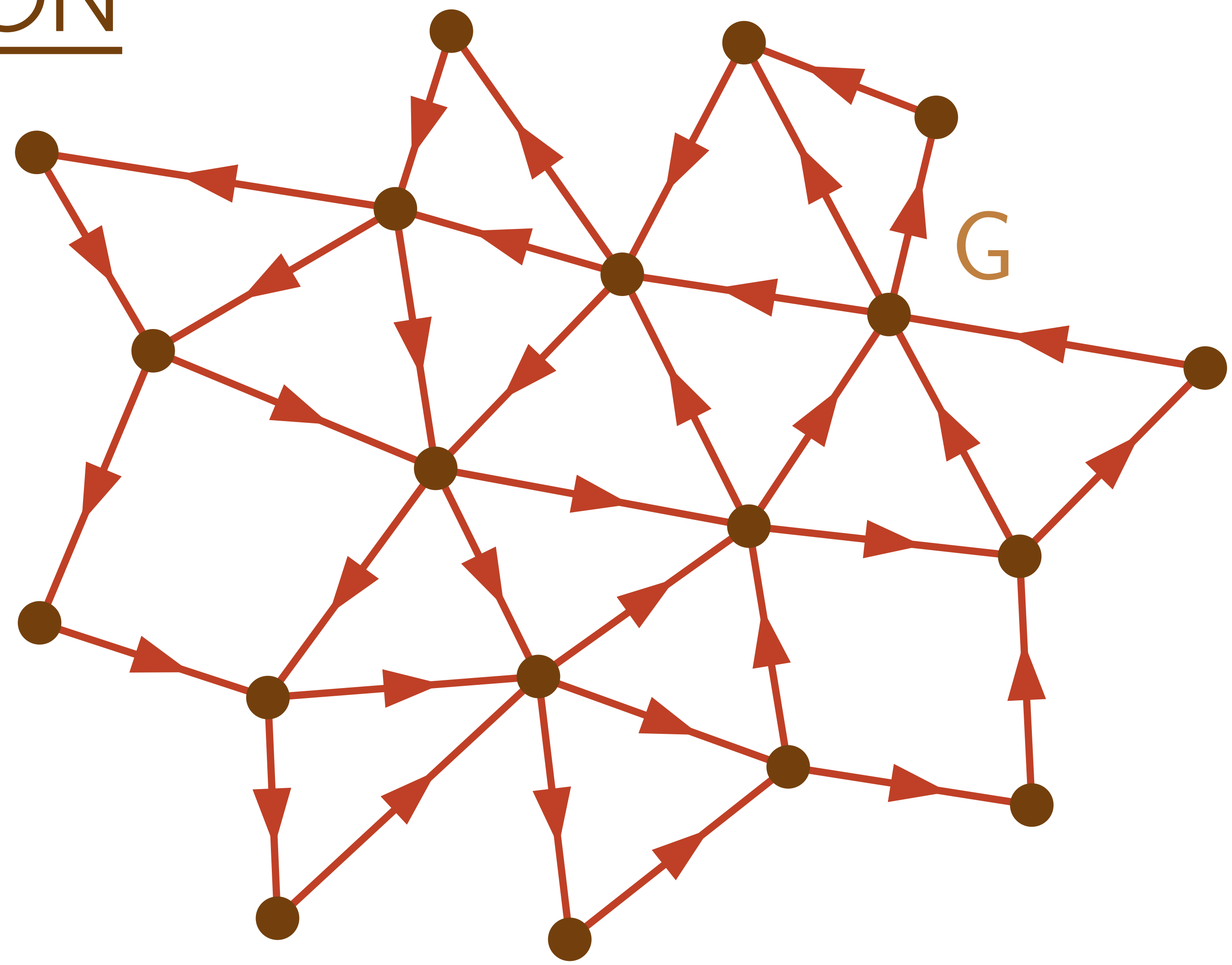


LEMMA

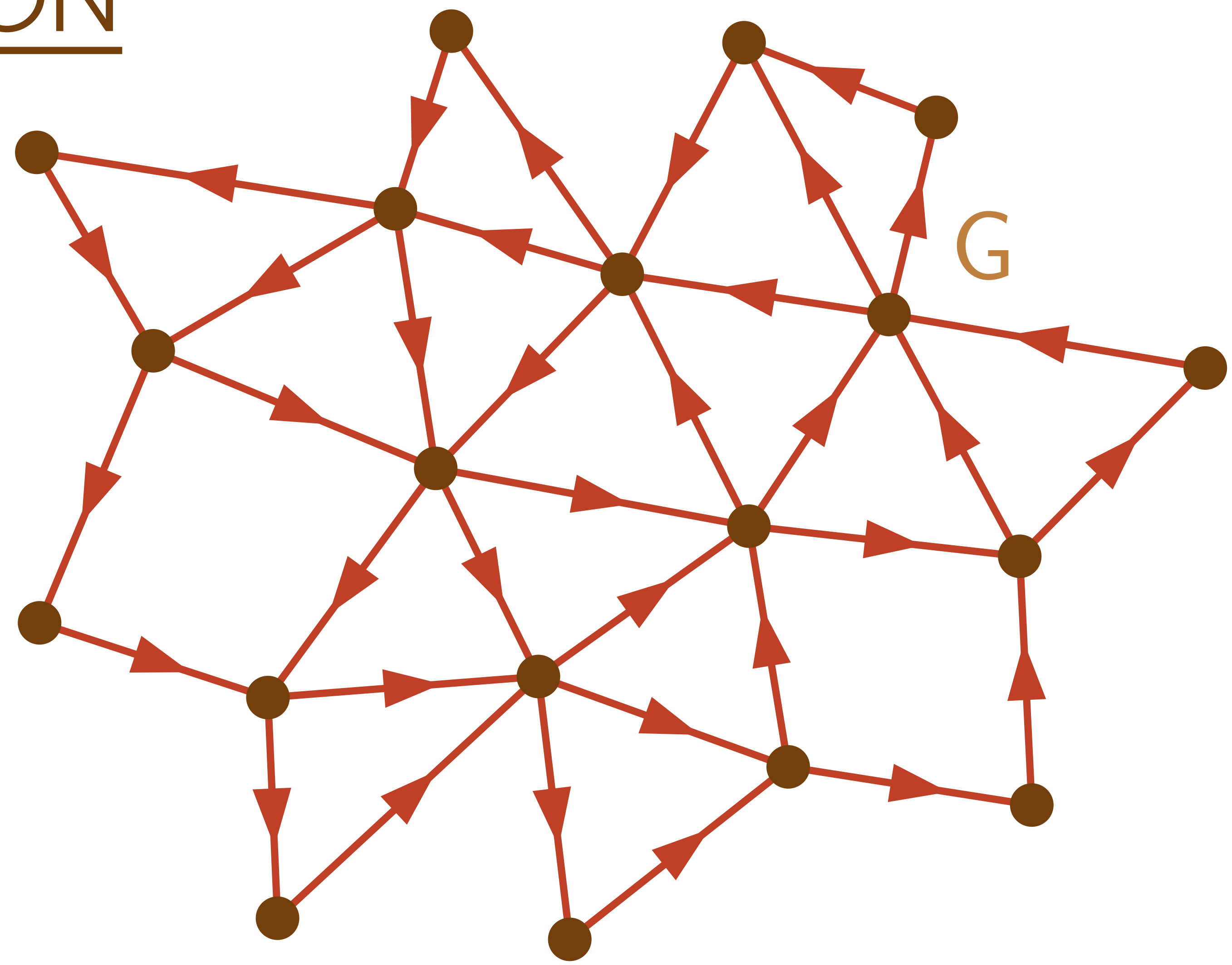
Every edge on the outer face of G *either comes from a vertex with outdegree* 1*, or goes to a vertex with indegree* 1*.*

POTENTIAL FUNCTION



POTENTIAL FUNCTION

- Let F be a subset of *fixed* edges of G



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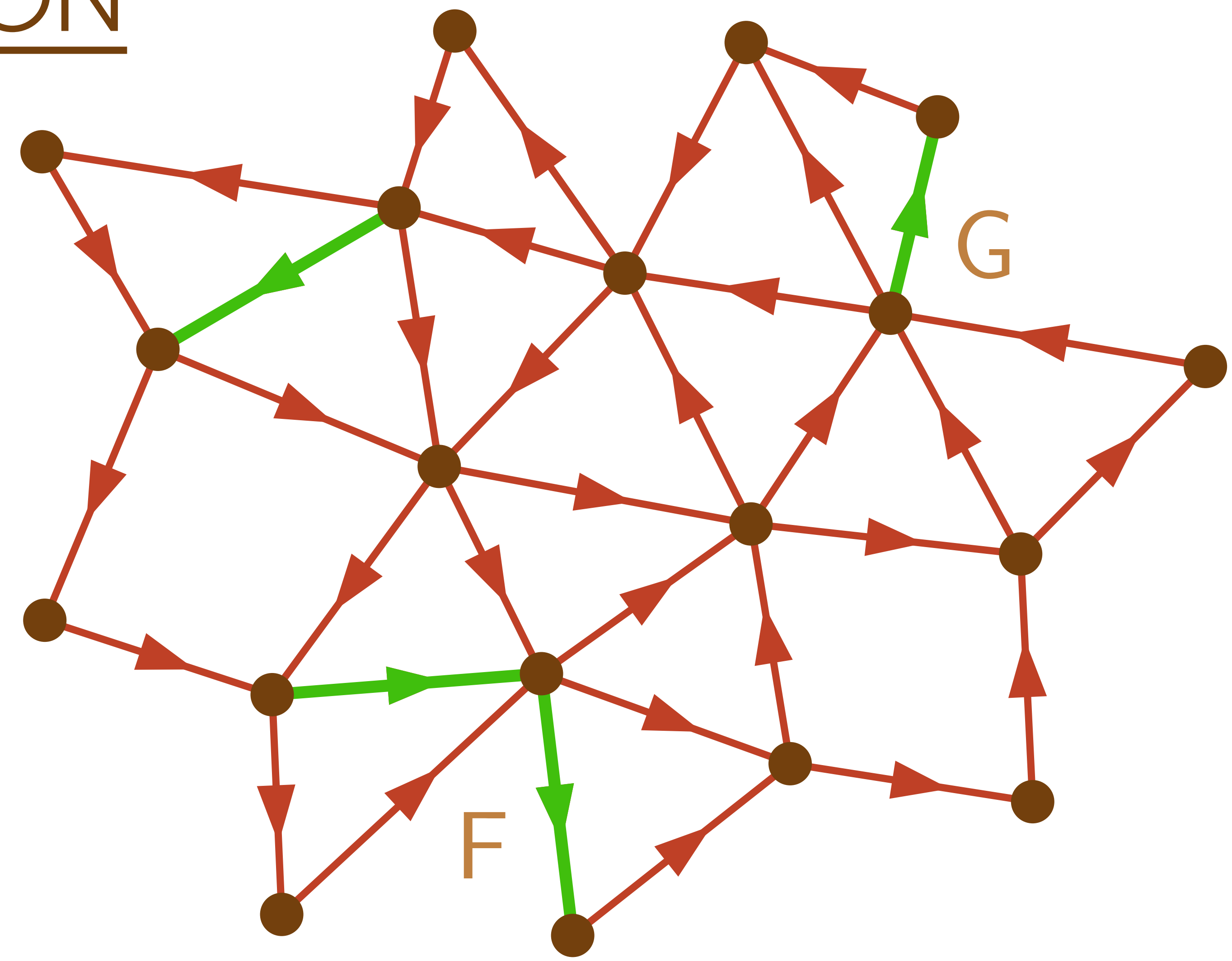
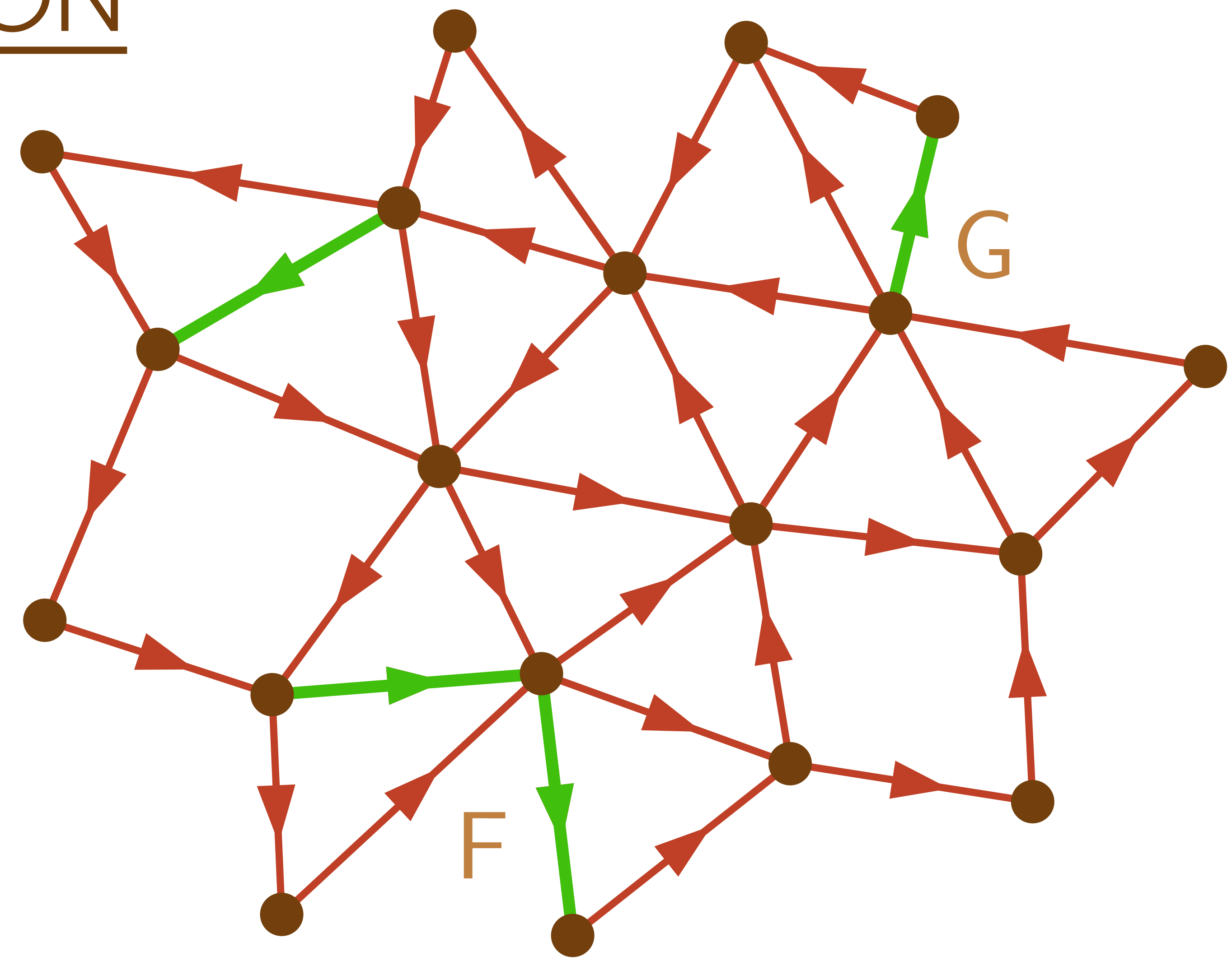


POTENTIAL FUNCTION

- Let F be a subset of *fixed* edges of G
- The *potential* k(G, F) is the #vertices – #fixed edges



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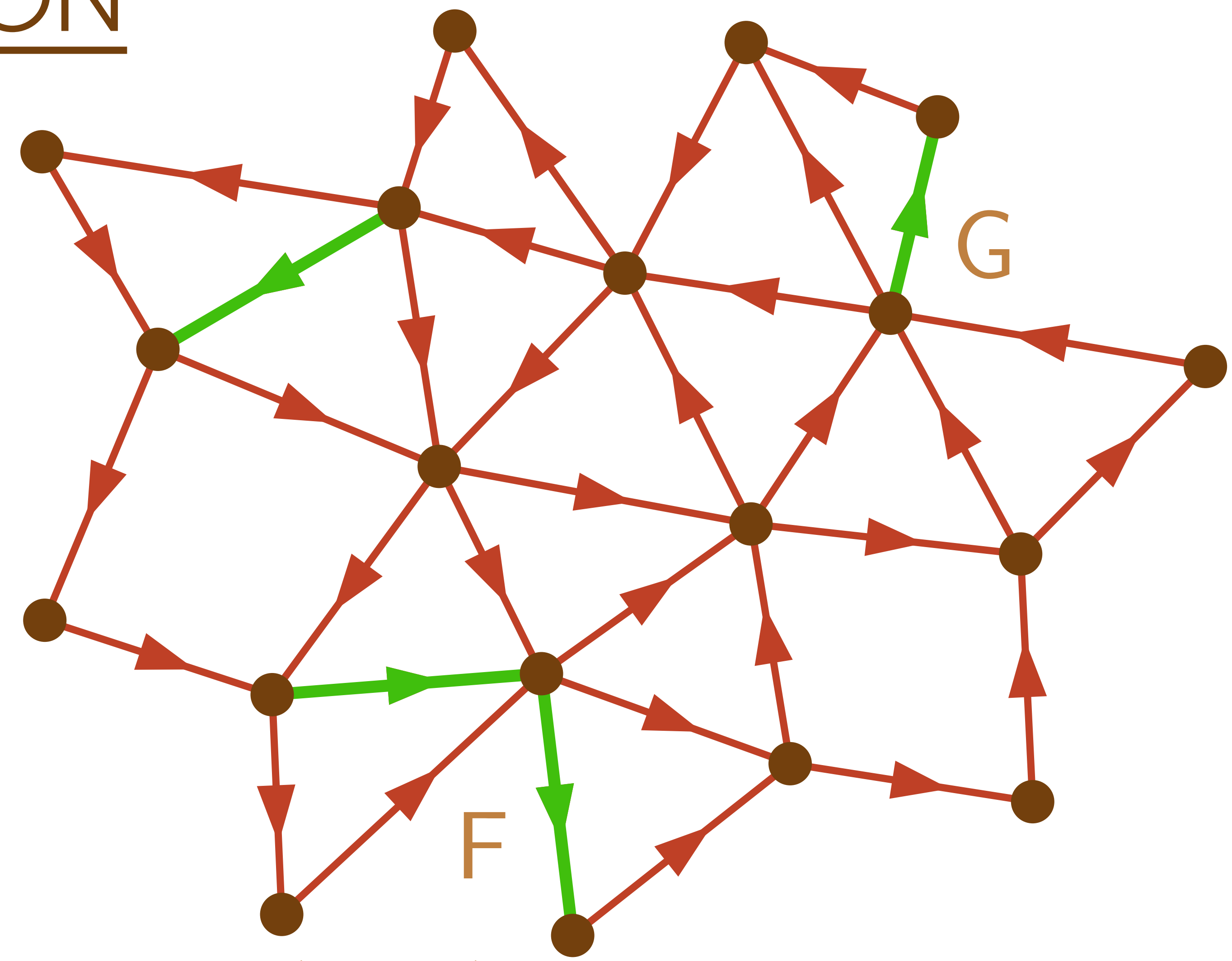
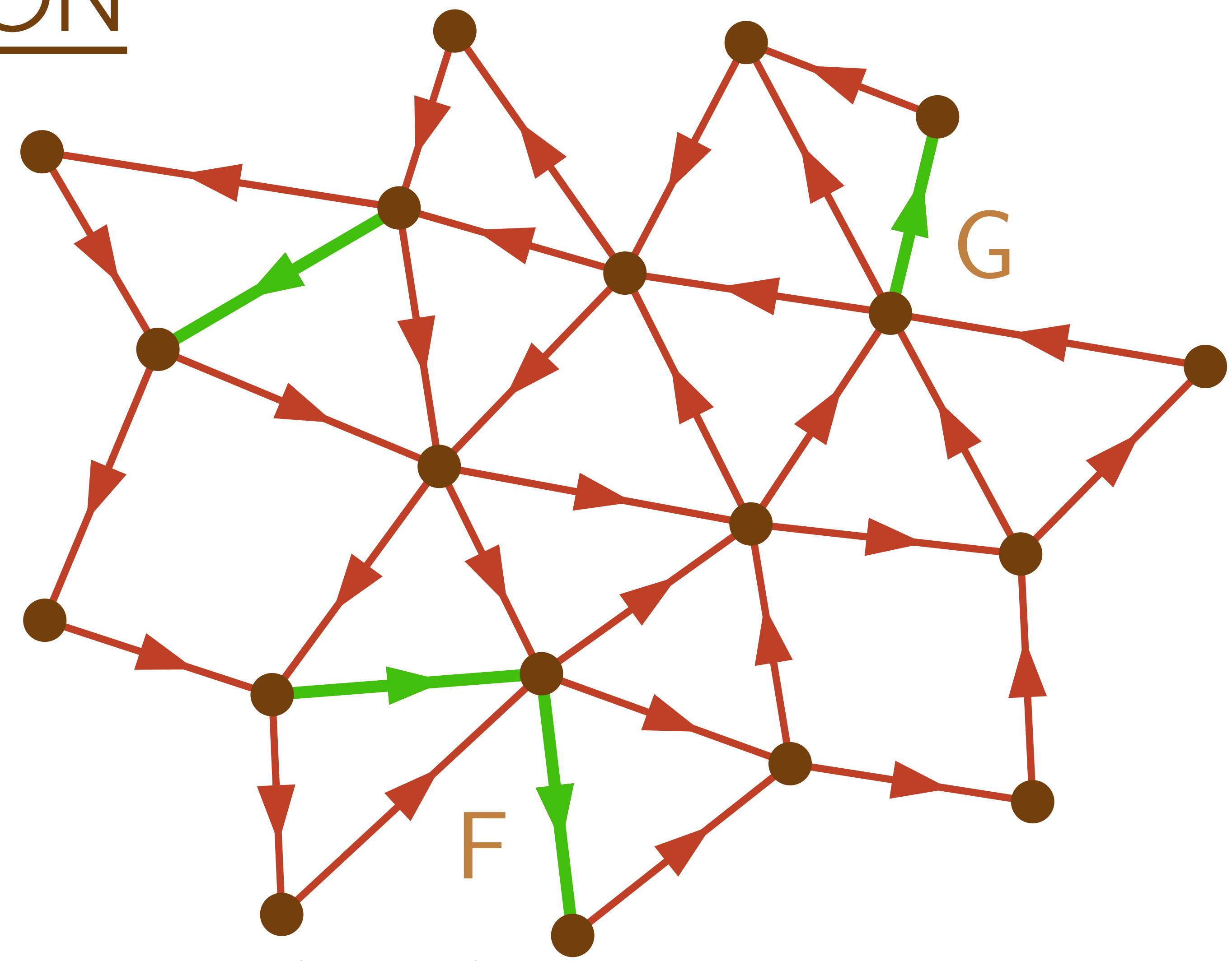


$k(F, G) = 19 - 4 = 15$

POTENTIAL FUNCTION

- Let F be a subset of *fixed* edges of G
- The *potential* $k(G, F)$ is the #vertices − #fixed edges
- Let $Q(k)$ be the max number of potatoes in a graph with potential k



$$k(F, G) = 19 - 4 = 15$$

LEMMA

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$$Q(k) \leq Q(k-1) + Q(k-2)$$

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- Consider an edge e on the outer face of G

LEMMA

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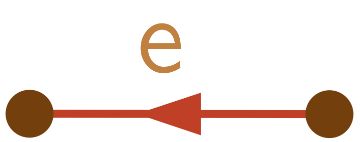

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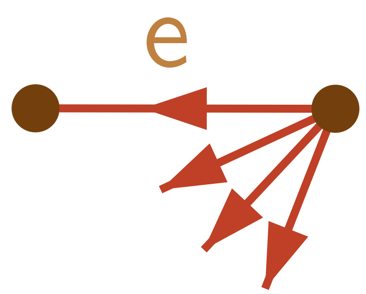

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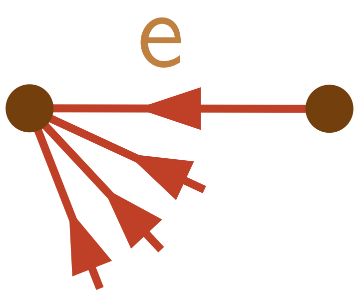

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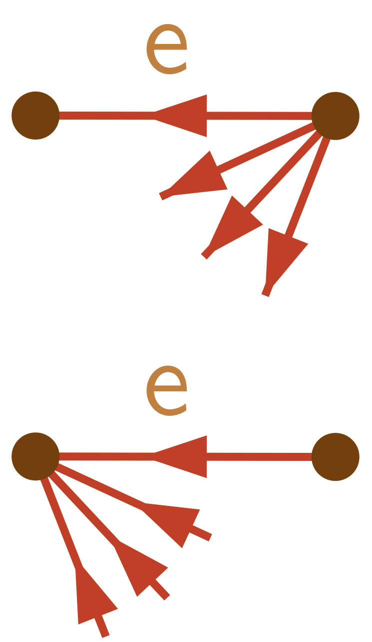

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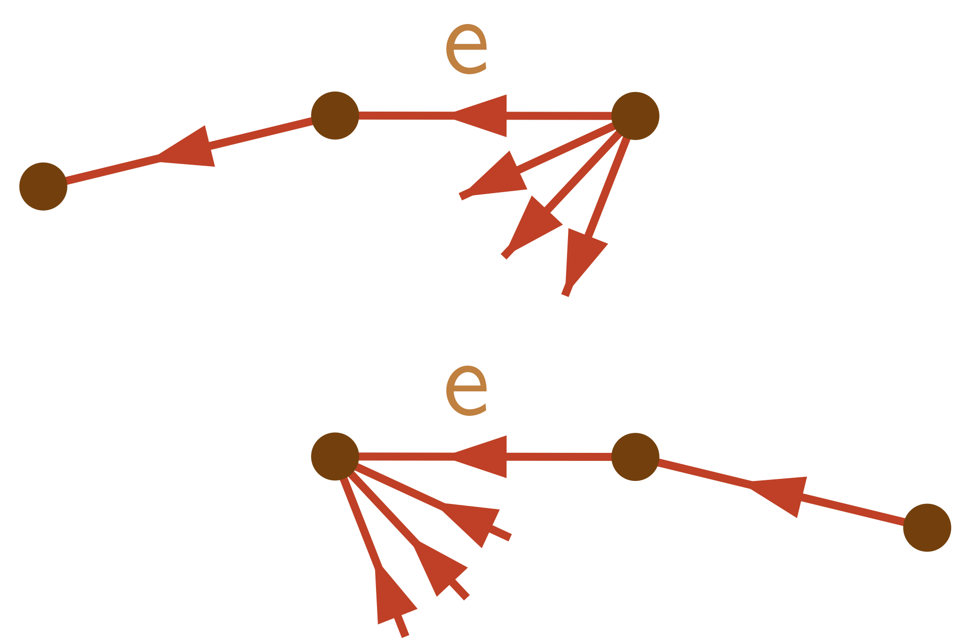

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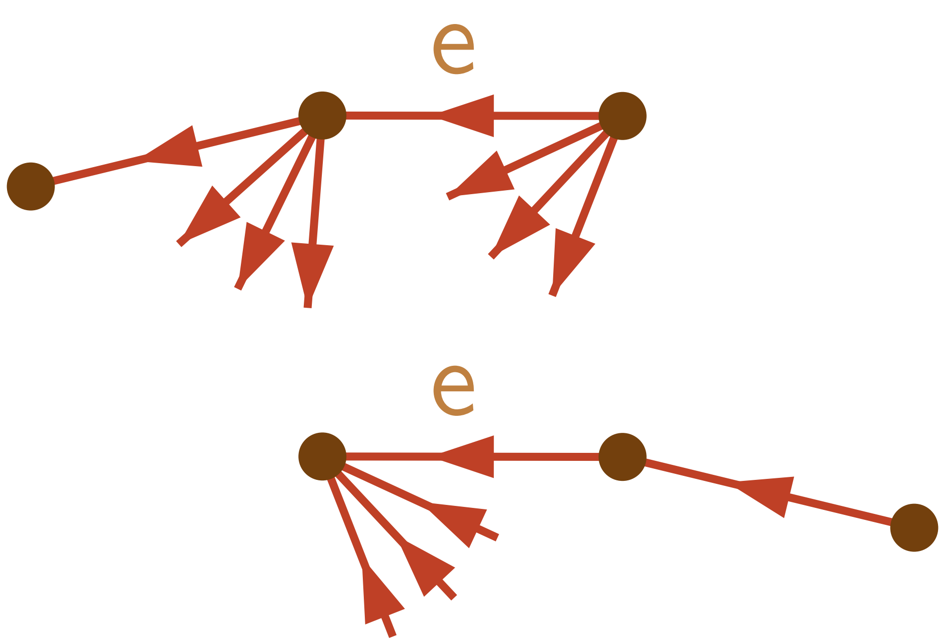

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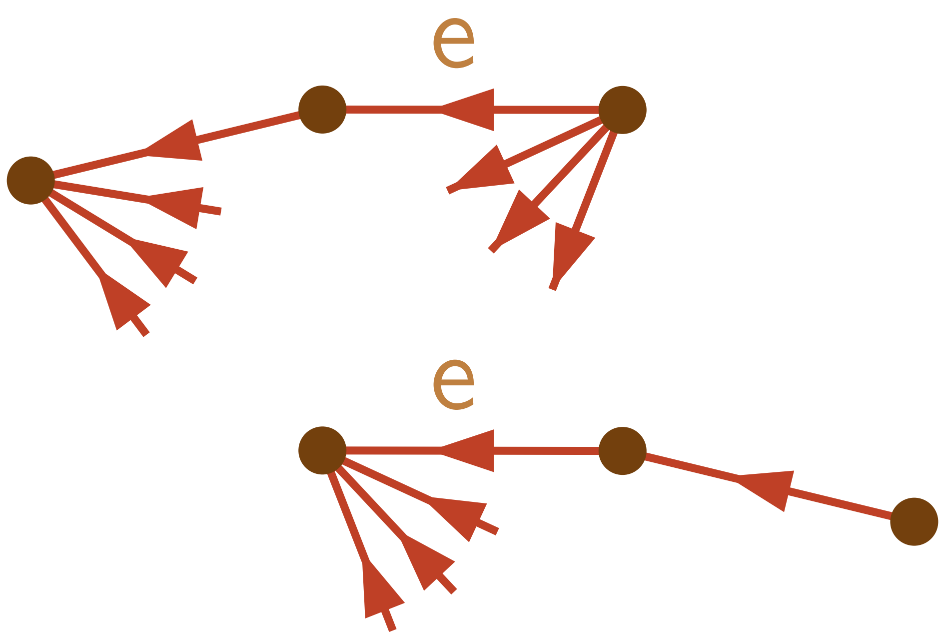

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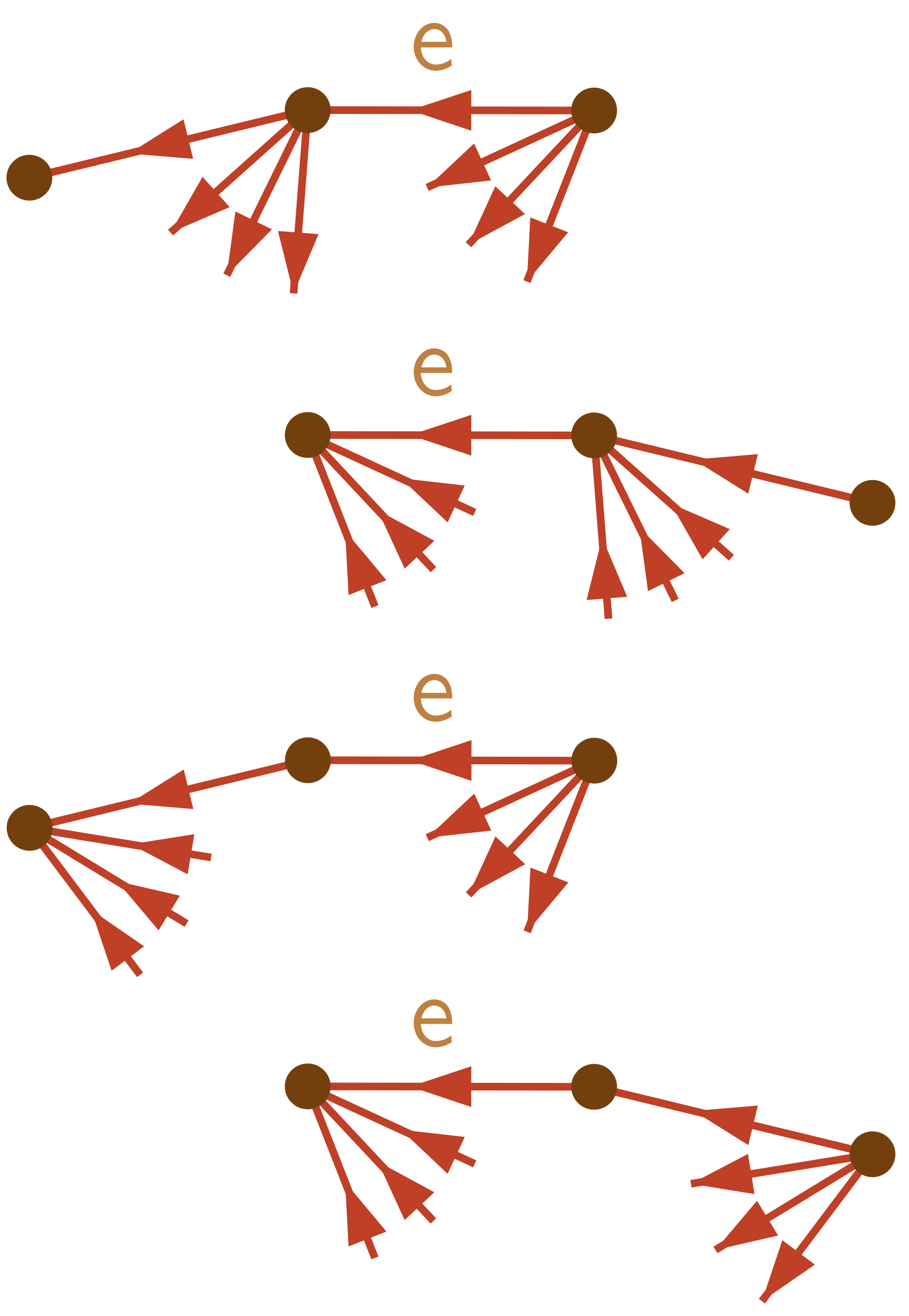

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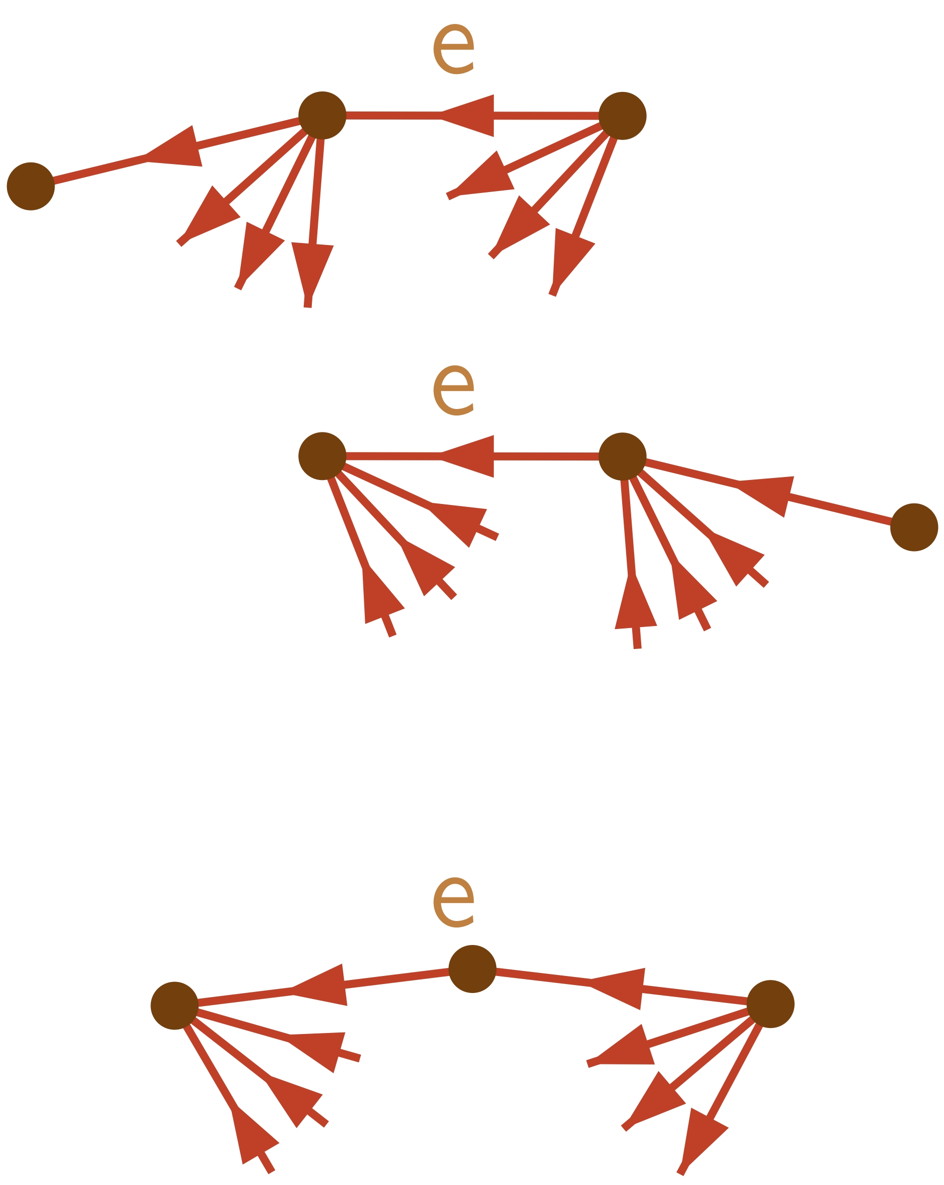
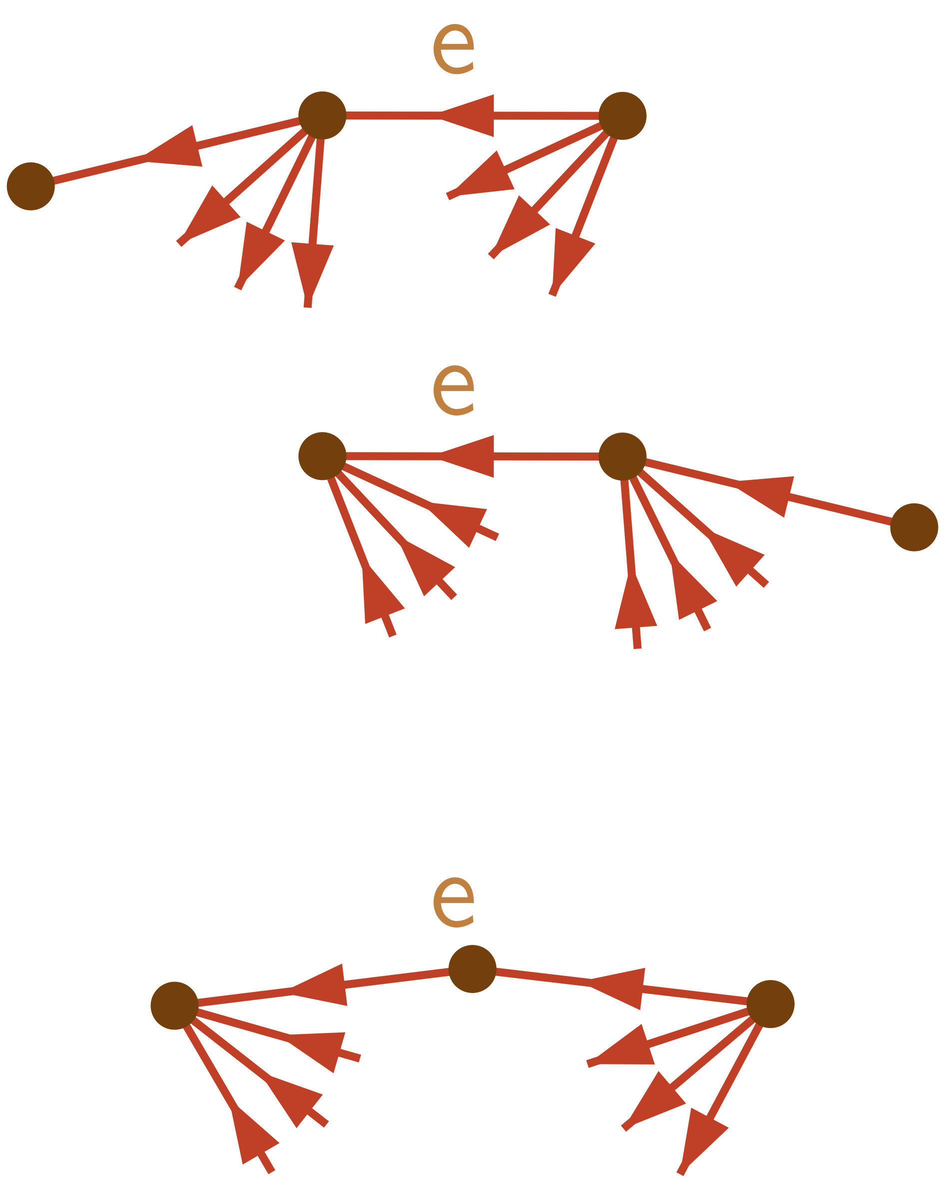


LEMMA

$$Q(k) \leq Q(k-1) + Q(k-2)$$

- Consider an edge e on the outer face of G
- Then either fix e, or remove its vertices.



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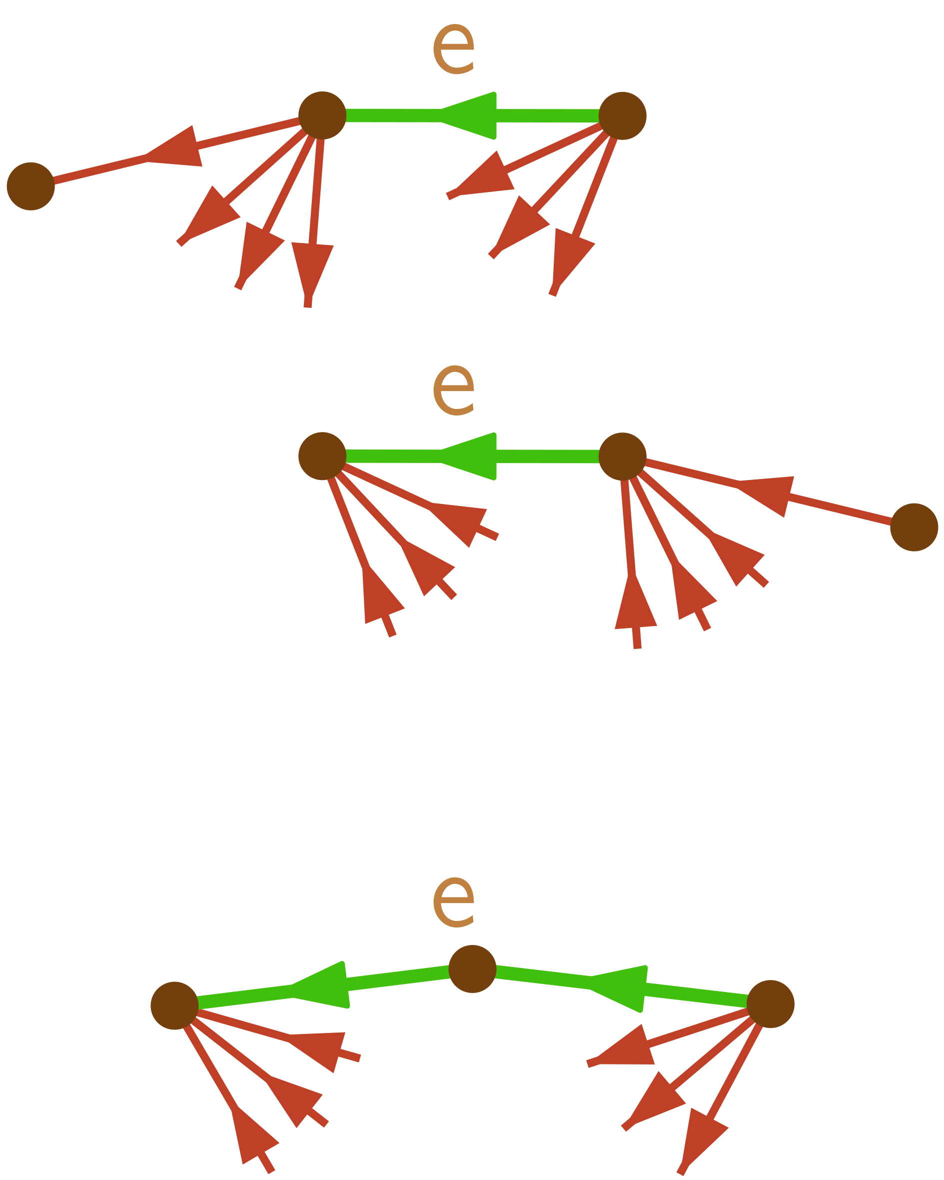

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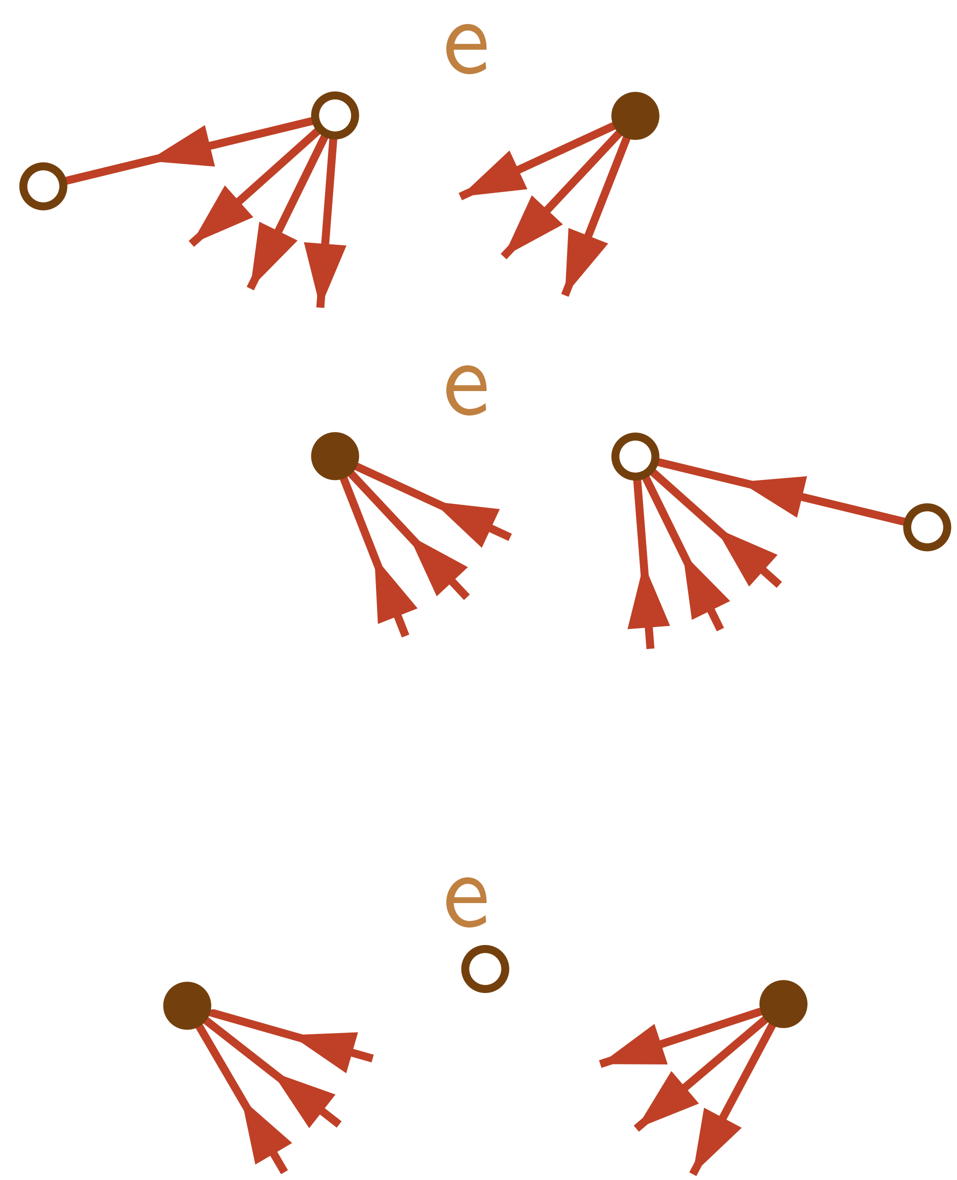

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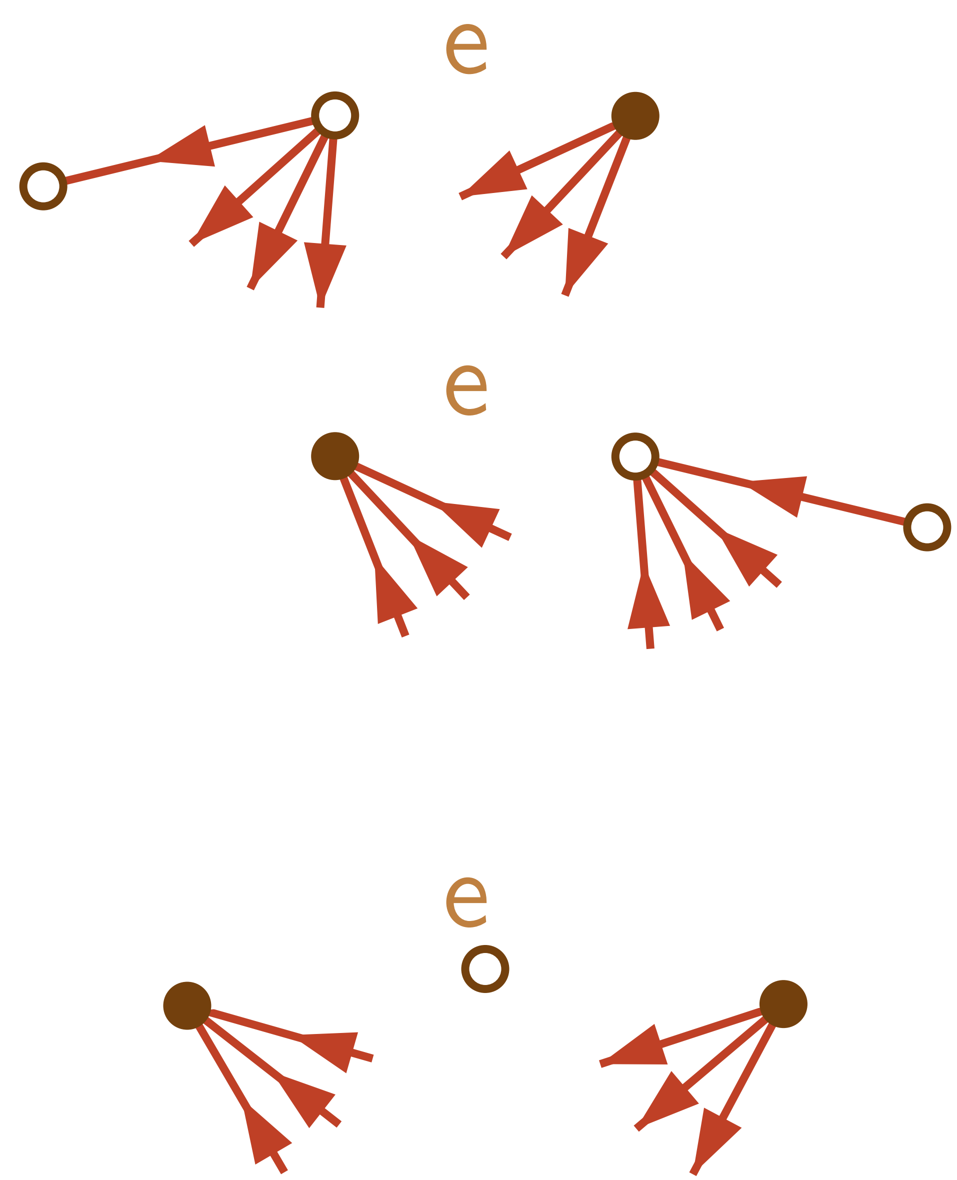

THEOREM

LEMMA

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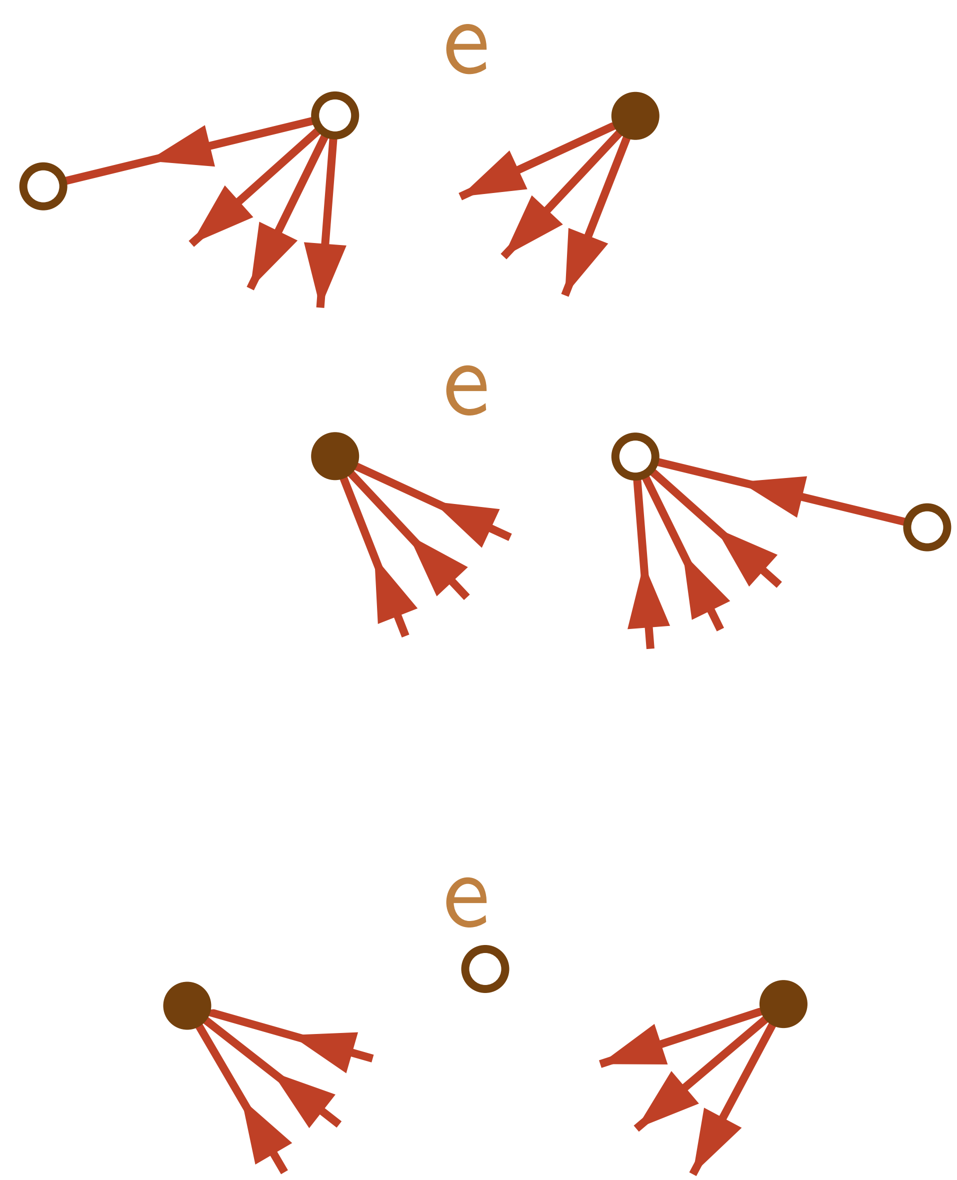

THEOREM

For every mesh M, $\mathcal{P}(M)$ *is in* $O(1.6181^n)$.

MORE NEW STUFF

DEFINITION "FAT TRIANGLE"

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- Triangle where all angles are $> \delta$

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- Triangle where all angles are $> \delta$

DEFINITION "FAT POTATO"

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DEFINITION “FAT POTATO”

- Potato that contains a large fat triangle

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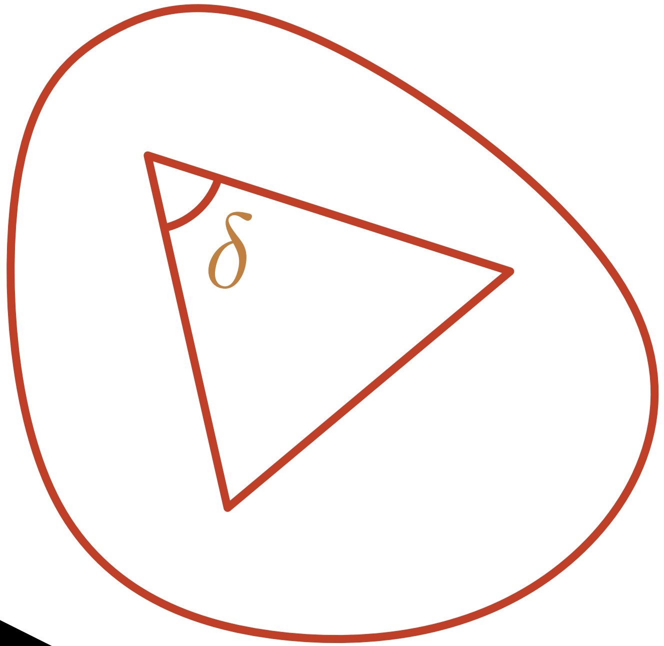

DEFINITION “FAT MESH“

DEFINITION "FAT MESH"

- Mesh where all triangles are fat

<u>DEFINITION</u> “FAT MESH“

- Mesh where all triangles are fat

<u>DEFINITION</u> “FAT MESHED POTATO“

DEFINITION “FAT MESH”

- Mesh where all triangles are fat

DEFINITION “FAT MESHED POTATO”

DEFINITION “FAT MESH”

- Mesh where all triangles are fat

DEFINITION “FAT MESHED POTATO”

- Fat potato that is also a fat mesh

DEFINITION "FAT MESH"

- Mesh where all triangles are fat

DEFINITION "FAT MESHED POTATO"

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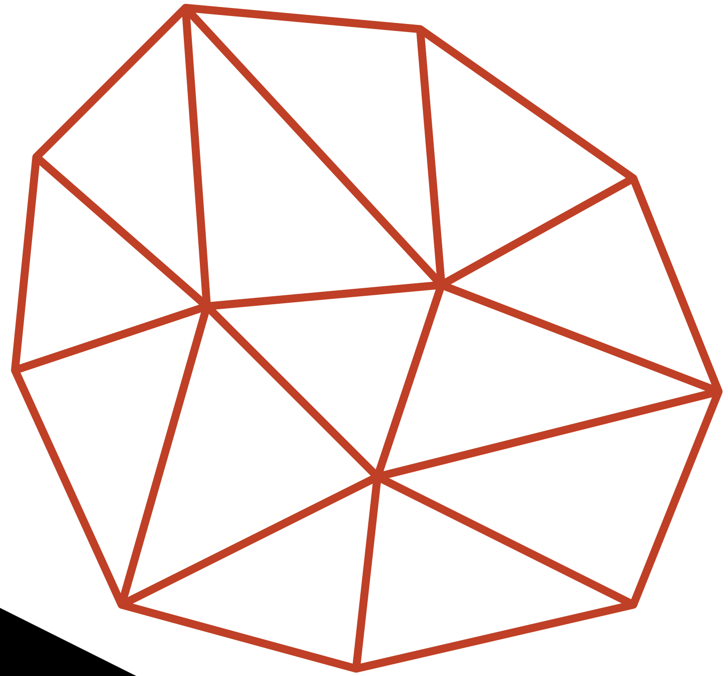

DEFINITION “CARROT”

<u>DEFINITION</u> "CARROT"

DEFINITION "CARROT"

- Potato without internal vertices

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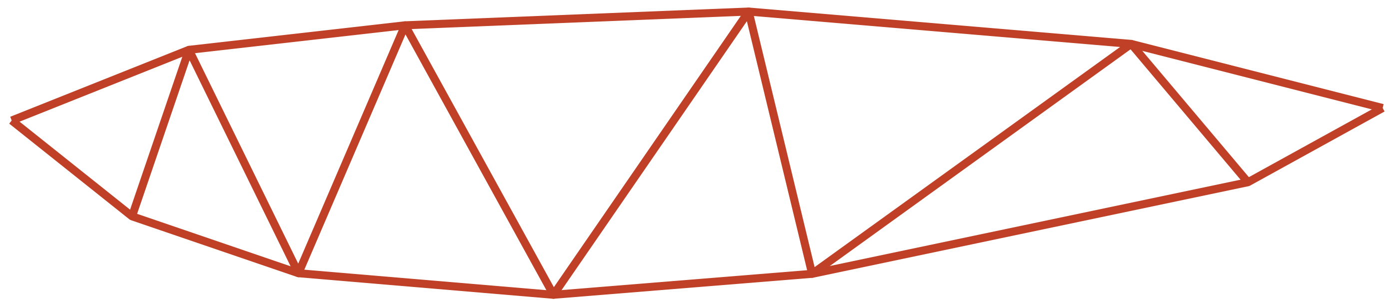
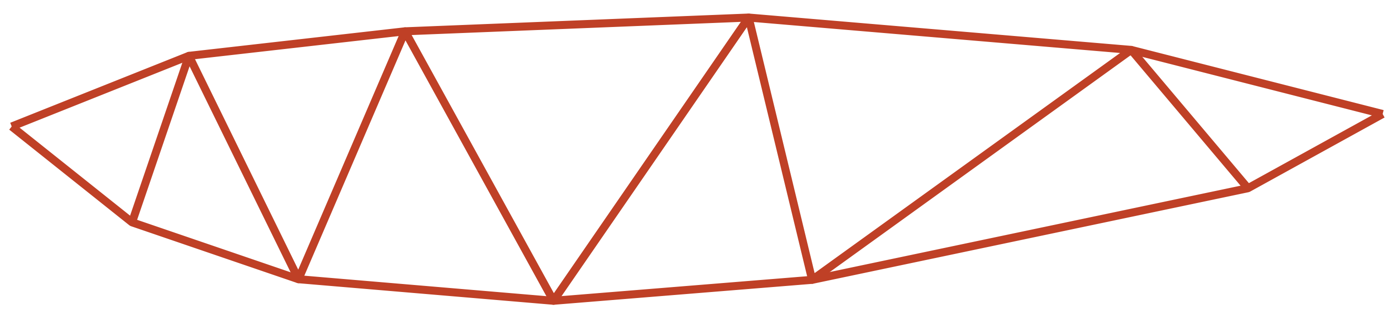


DEFINITION "CARROT"

- Potato without internal vertices
- (Also known as "outerplanar meshed potato")



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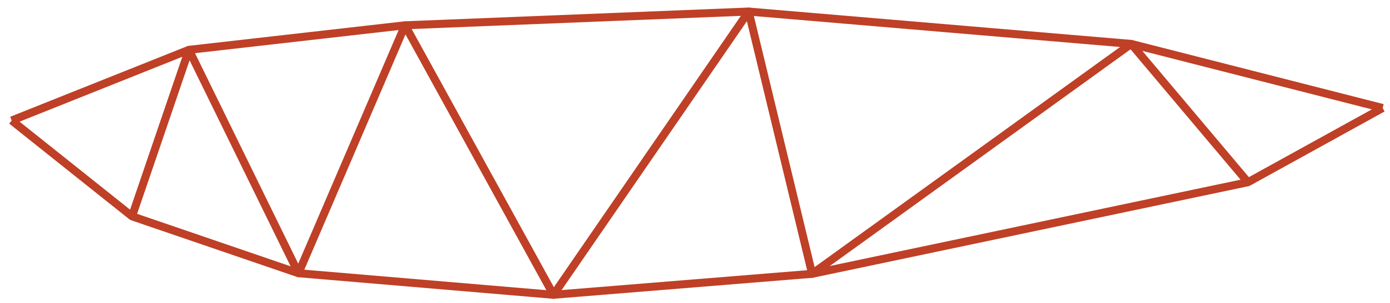
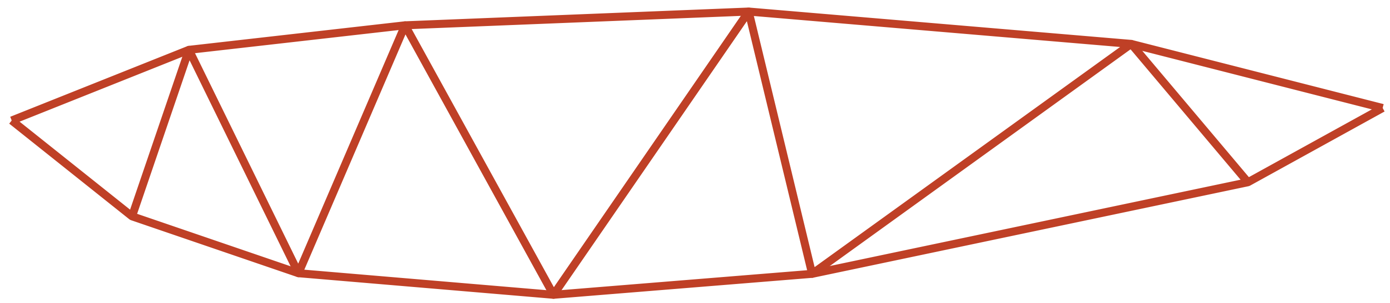


DEFINITION "CARROT NUMBER" $\mathcal{C}(M)$

DEFINITION "CARROT"

- Potato without internal vertices
- (Also known as "outerplanar meshed potato")



DEFINITION "CARROT NUMBER" $\mathcal{C}(M)$

- Total number of distinct carrots in a given mesh M

MORE RESULTS

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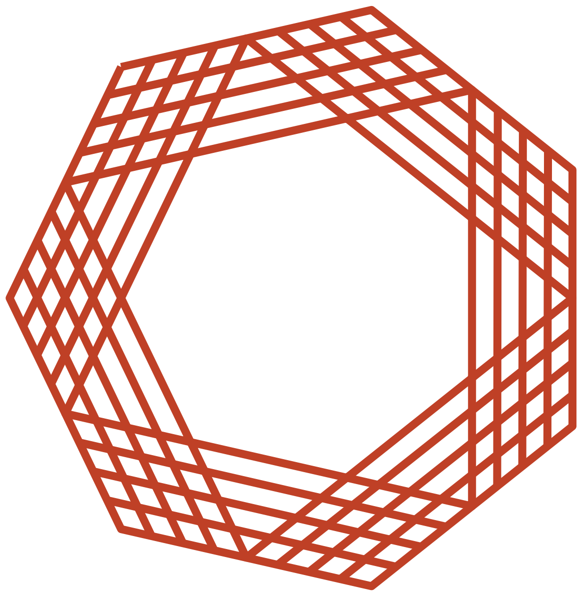
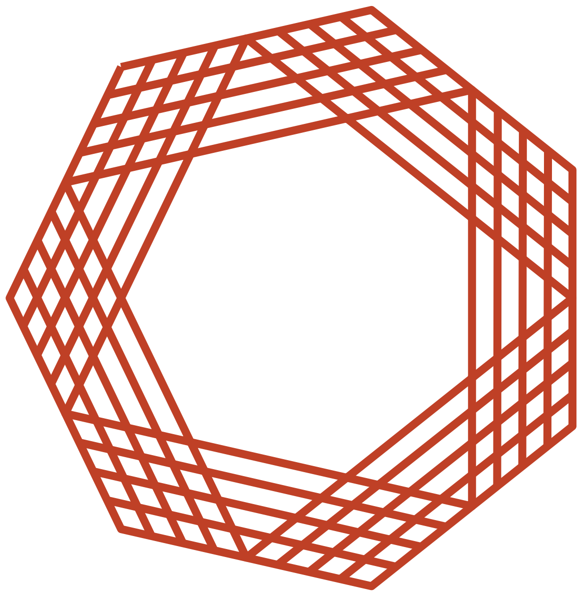


MORE RESULTS

- There exists a fat mesh with fat potato number $\Omega\left(n^{\frac{1}{2}\left\lfloor\frac{2\pi}{\delta}\right\rfloor}\right)$.
- For every fat mesh, the potato number is $O\left(n^{\left\lceil\frac{\pi}{\delta}\right\rceil}\right)$.



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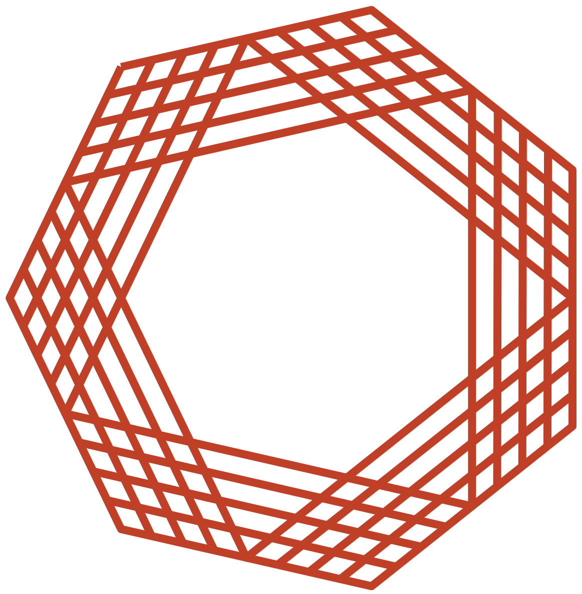

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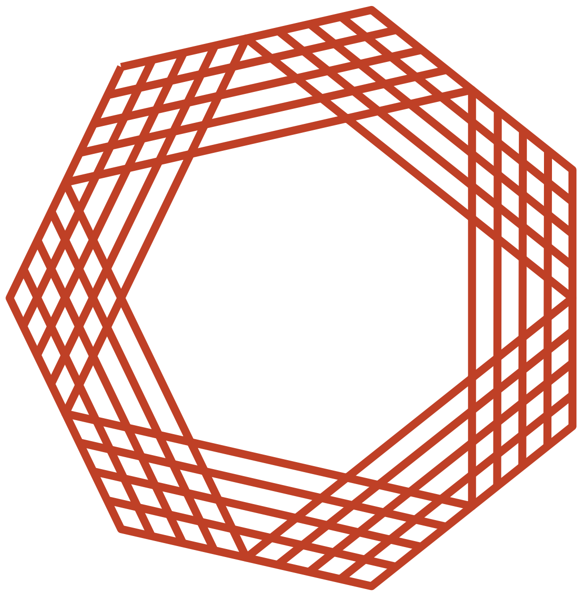

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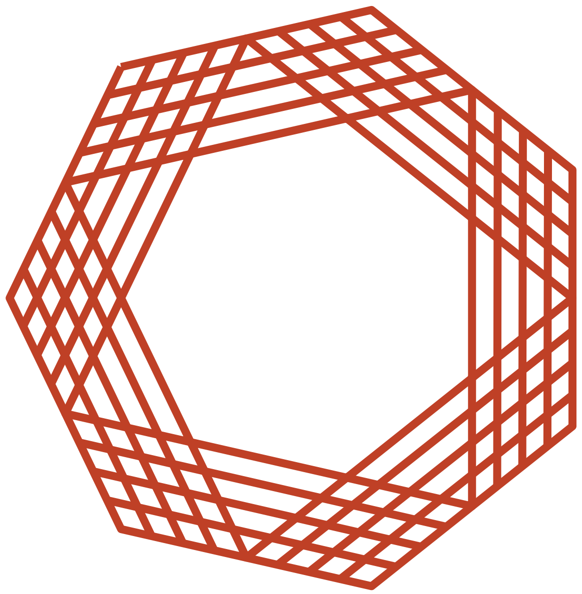

THANK YOU!

1 2 3 4 5