MEDIAN TRAJECTORIES

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OVERVIEW

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- I Introduction

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  - Motivation
  - Data representatives

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  - Implementation

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- Conclusion

I INTRODUCTION

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- Problem

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- Problem
  - Planar domain

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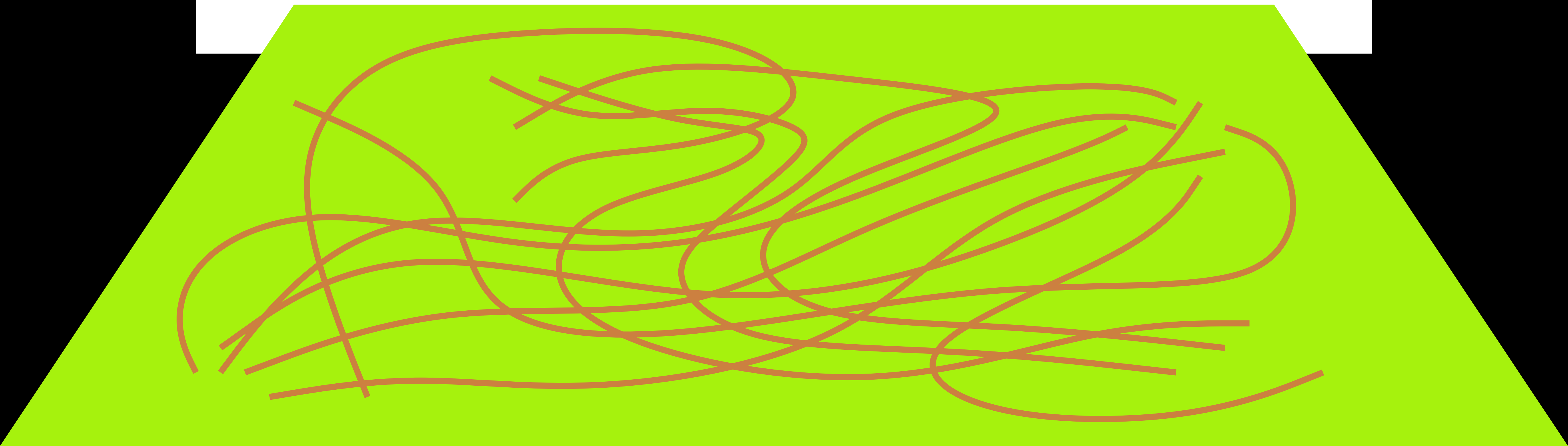
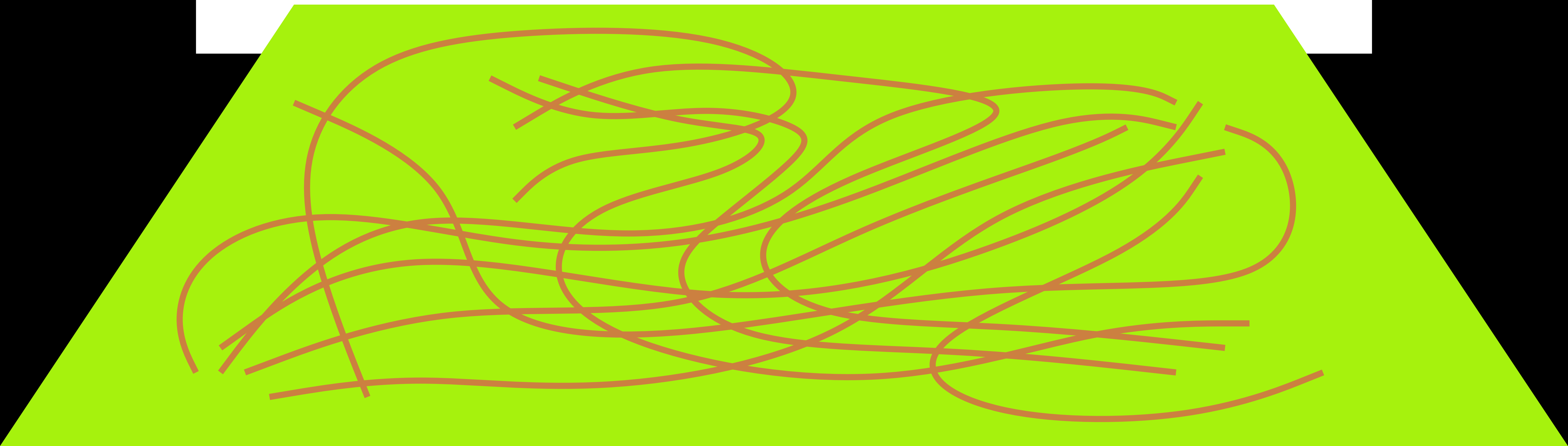
  - Big collection of trajectories



MOTIVATION

- Problem
  - Planar domain
  - Big collection of trajectories
  - Build catalogue of ‘common’ trajectories



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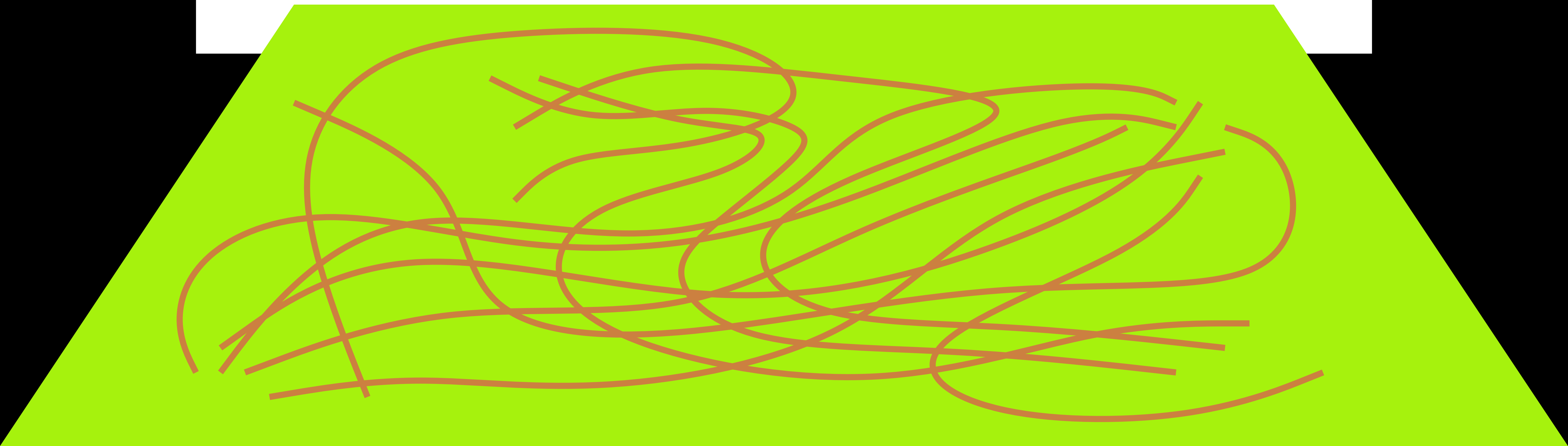
- Solution



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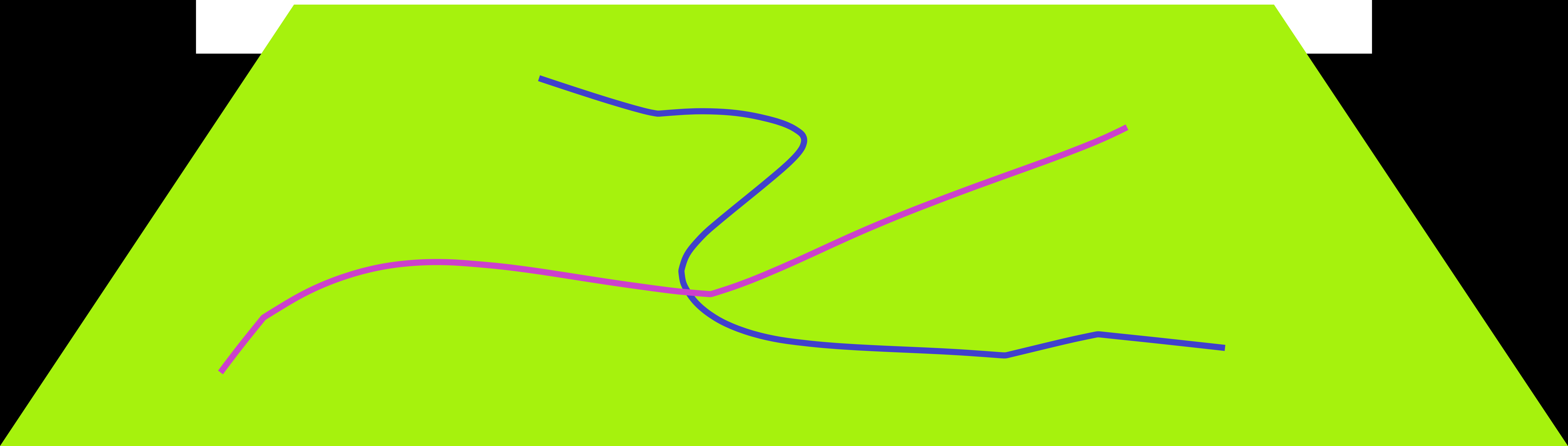

MOTIVATION

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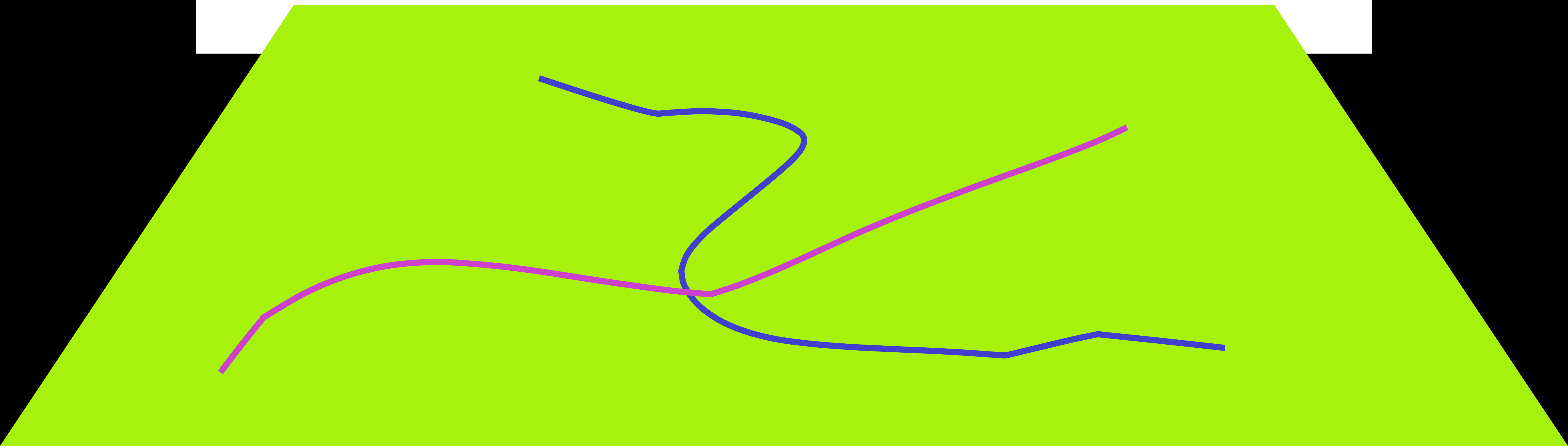
- Solution
  - Cluster the trajectories
  - Pick a good representative for each cluster
- ... how did we do that second step?



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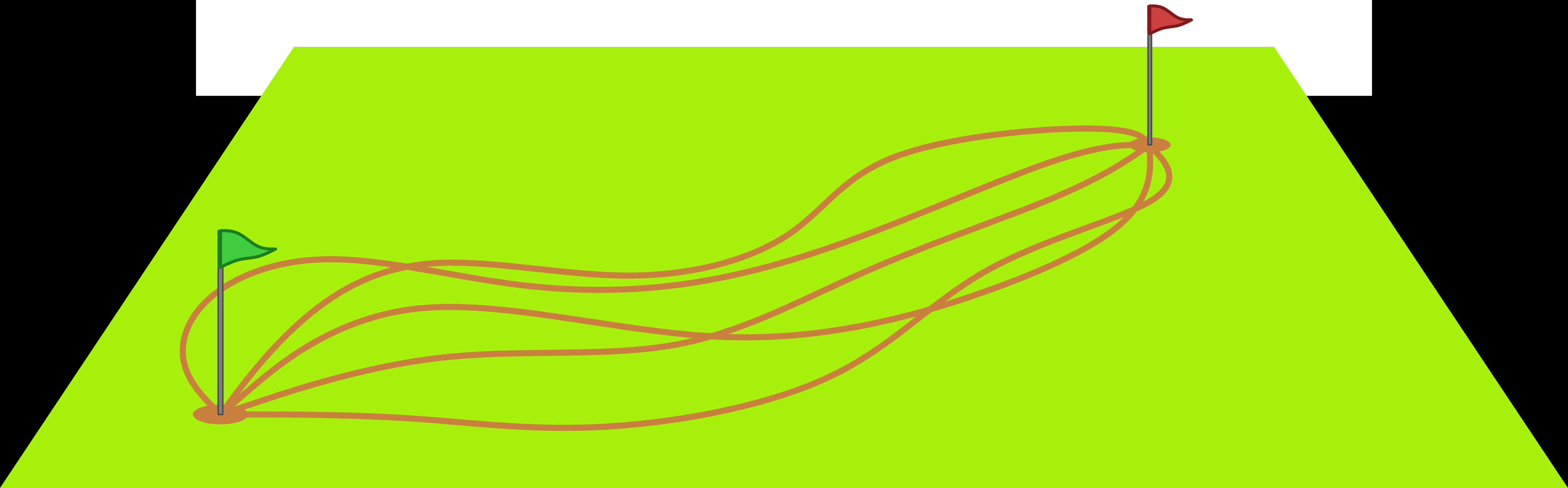
  - Sort of the same shape



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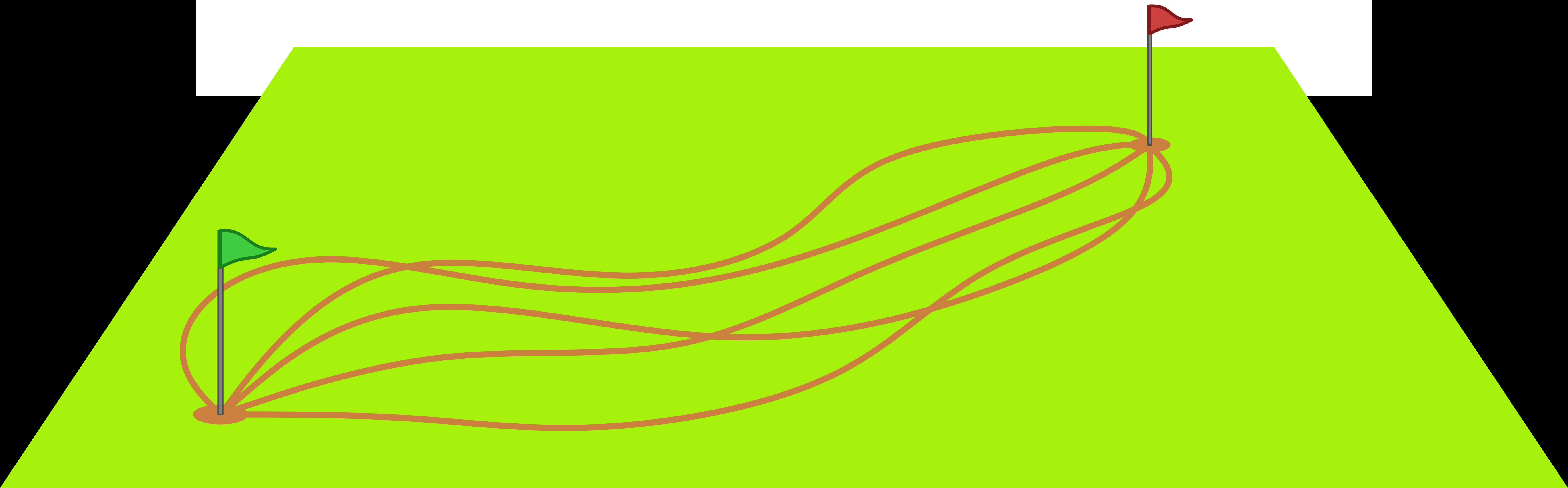

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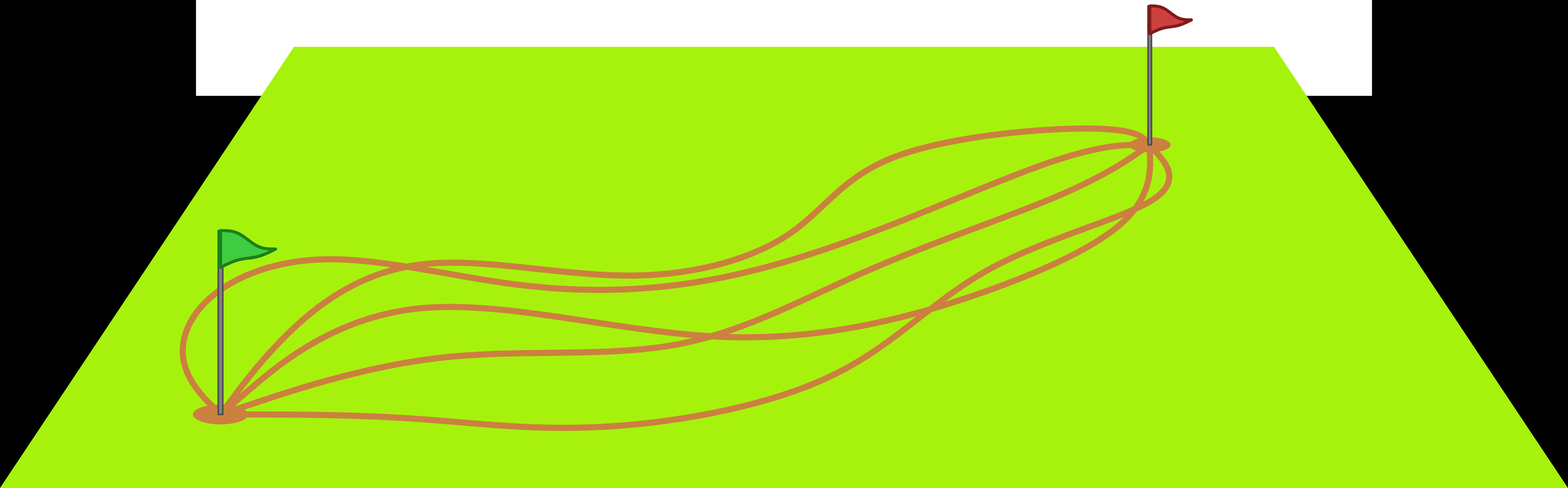

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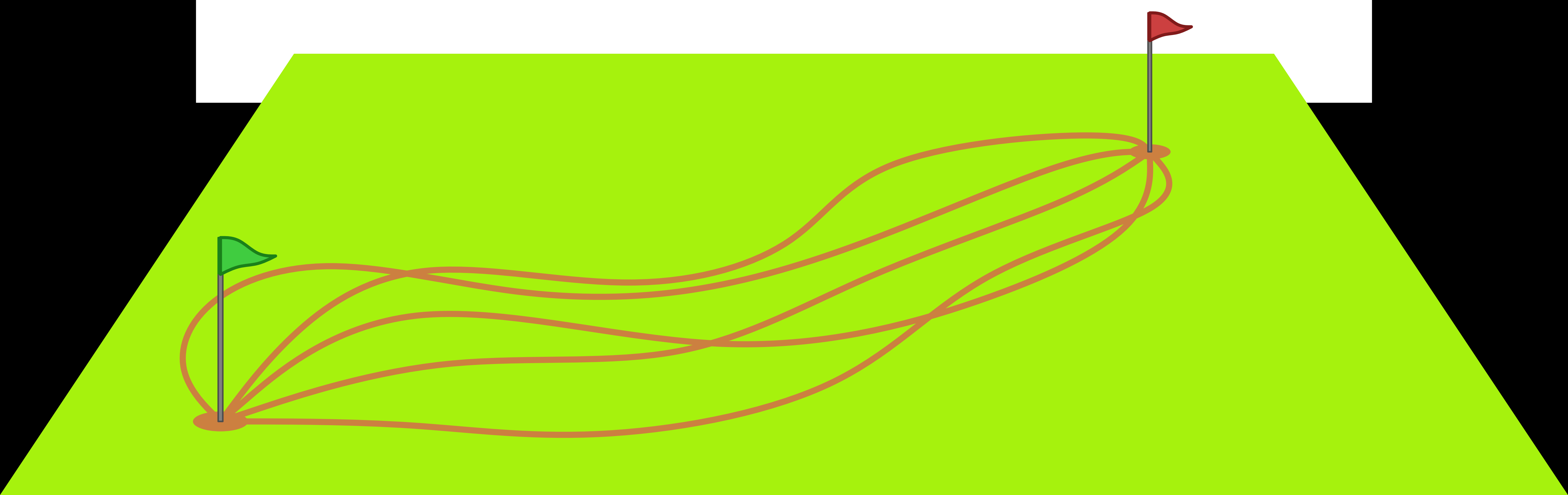

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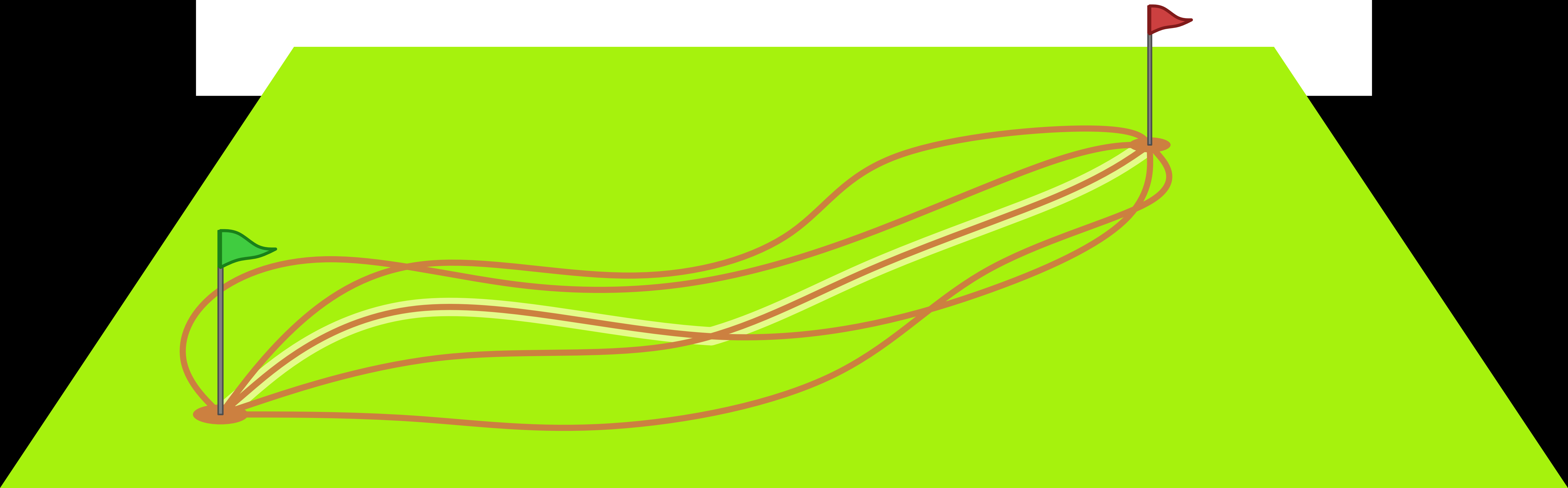

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- Set of numbers: $\{1, 1, 2, 5, 6\}$

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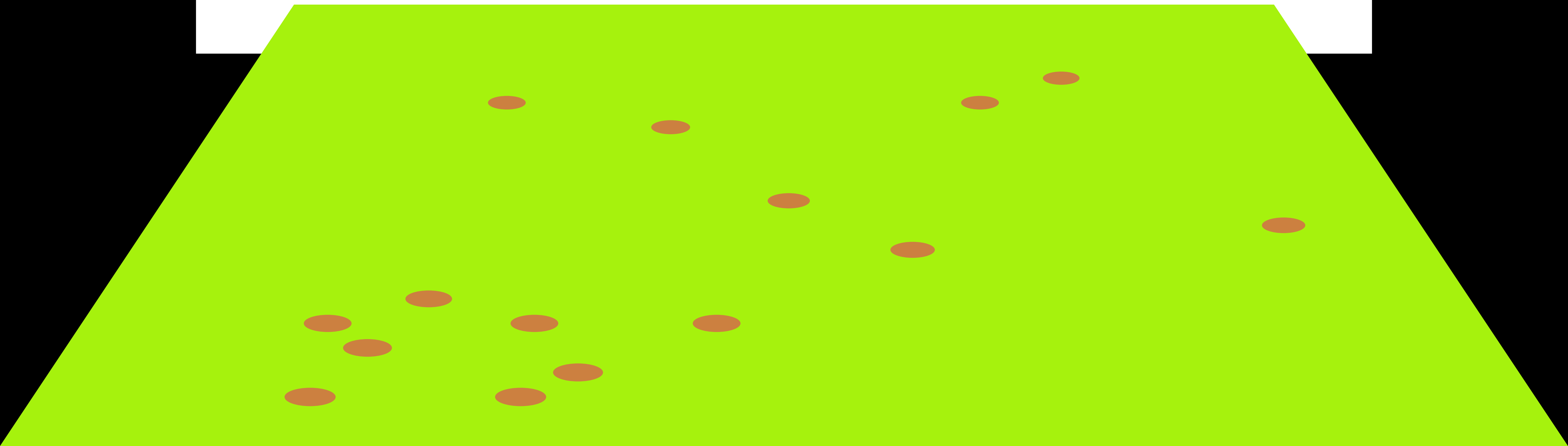

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- Set of numbers: $\{1, 1, 2, 5, 6\}$
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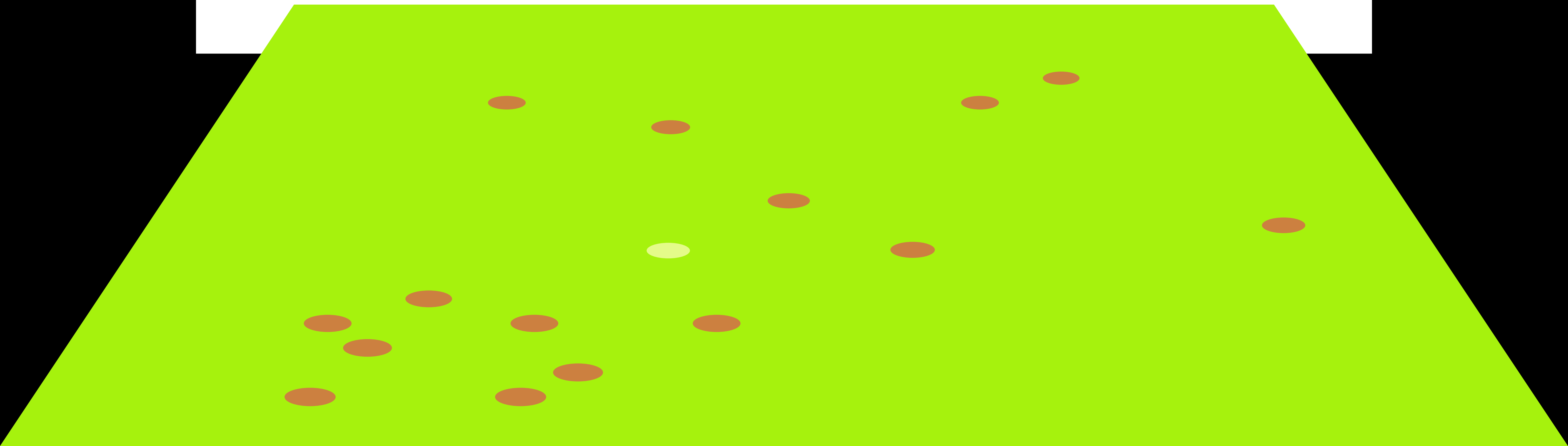

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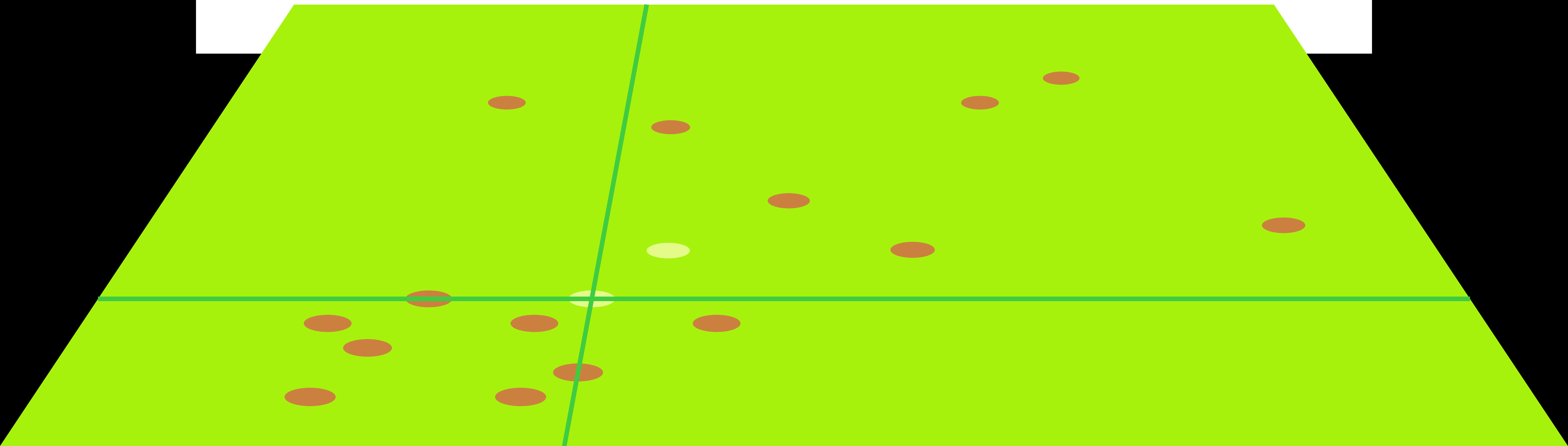

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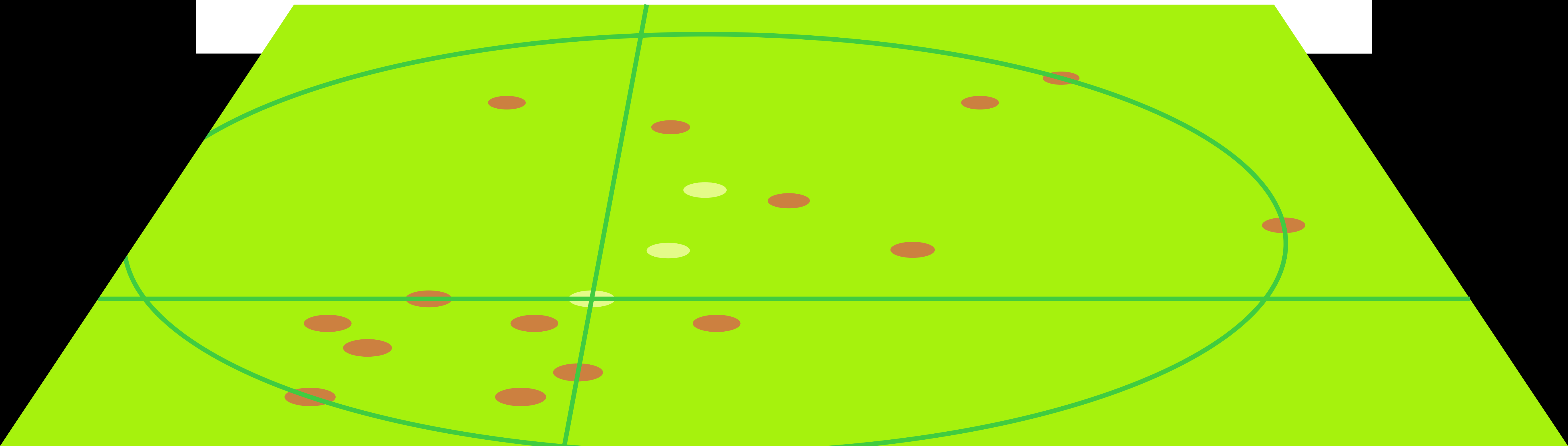

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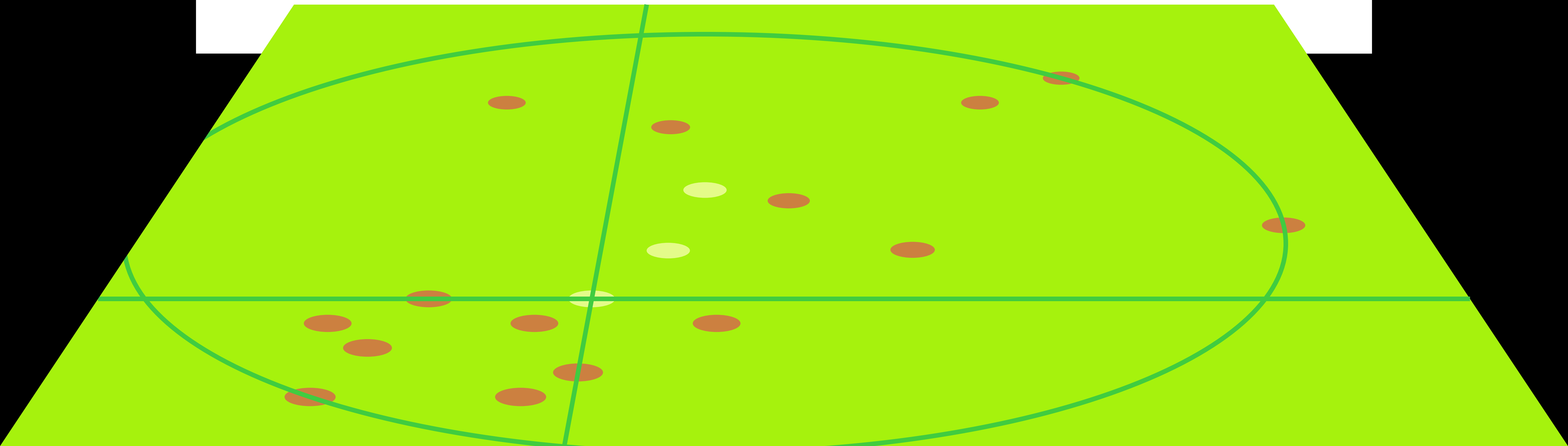
- Pick representative from input?



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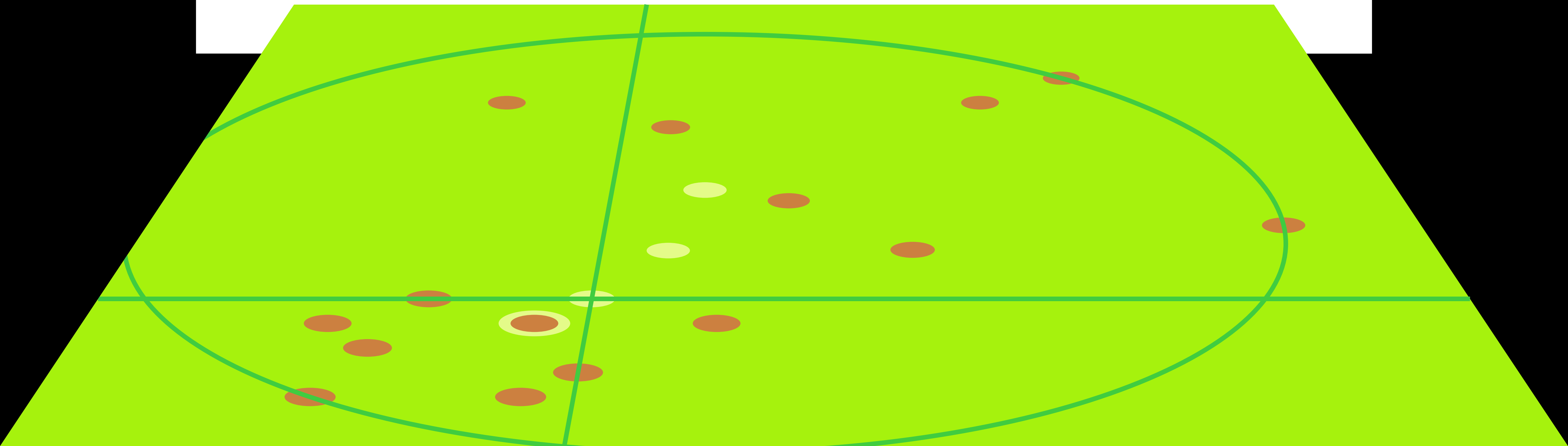

TRAJECTORIES: PICK REPRESENTATIVE FROM INPUT?

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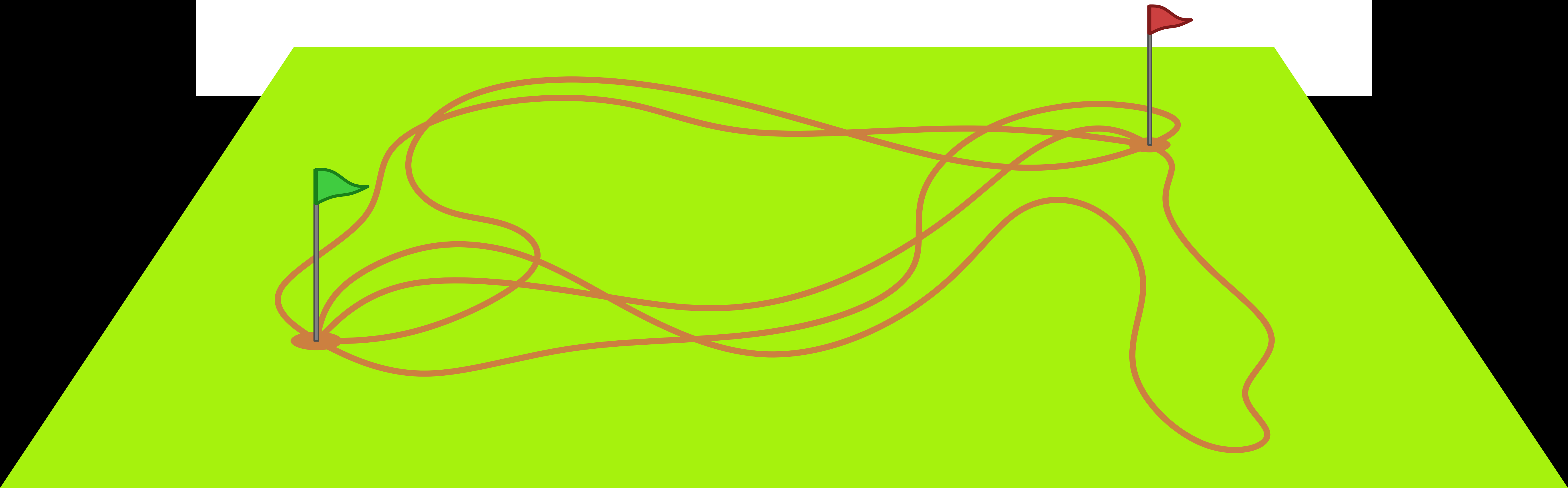
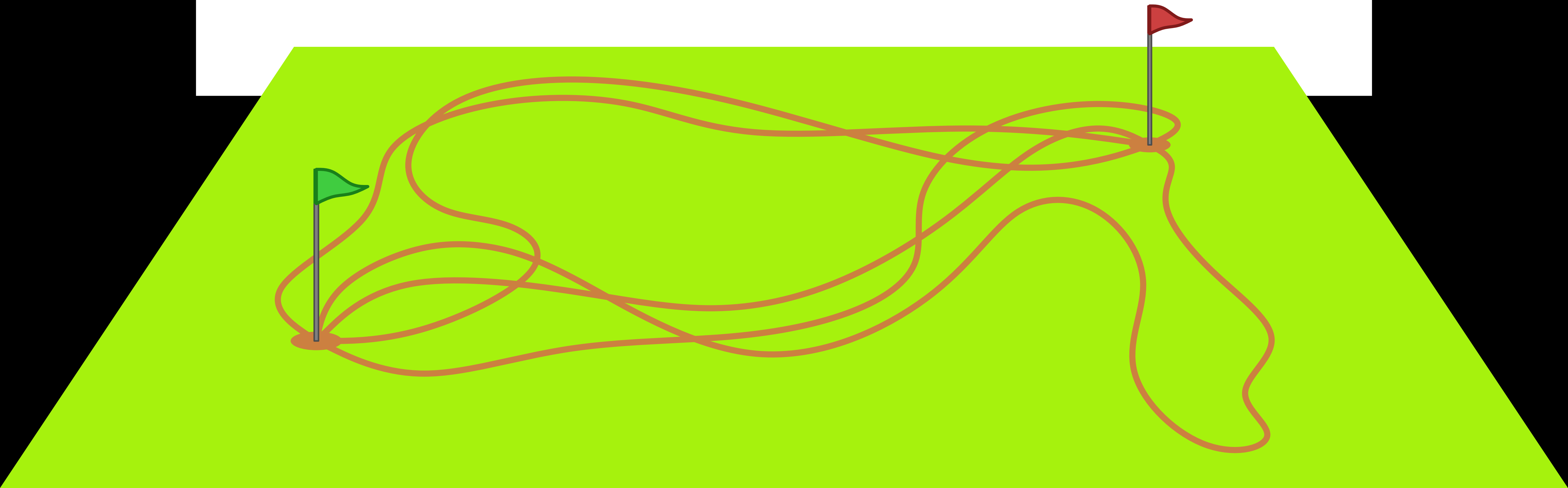


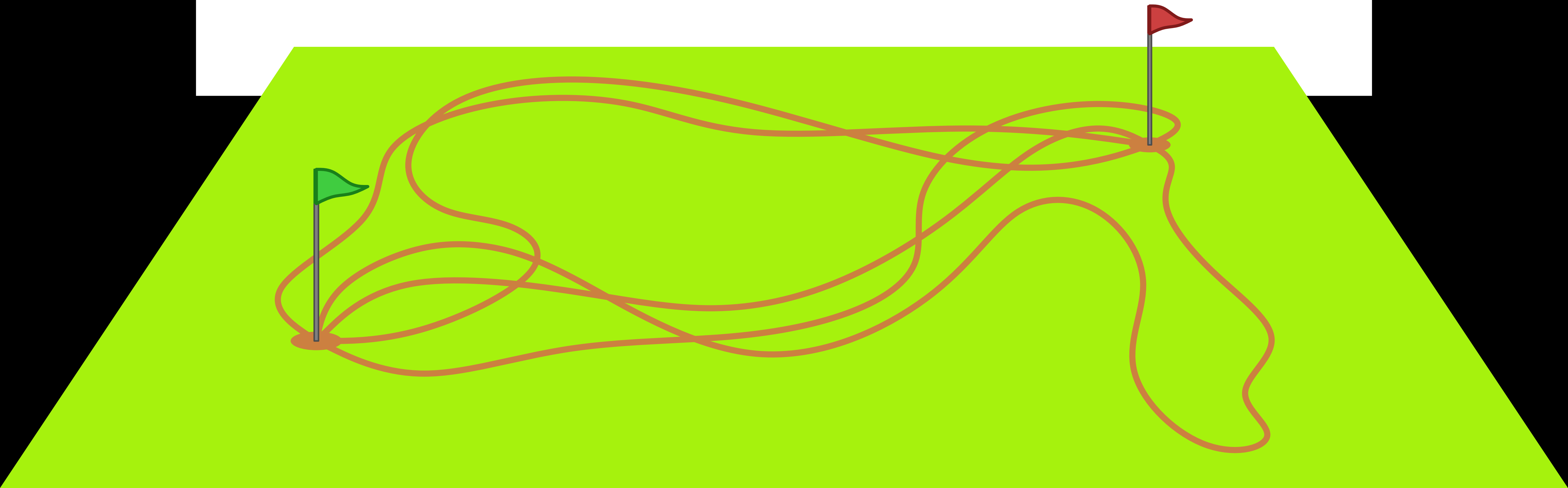
TRAJECTORIES: PICK REPRESENTATIVE FROM INPUT?

- No?



TRAJECTORIES: PICK REPRESENTATIVE FROM INPUT?

- No?
  - Parameterised mean trajectory



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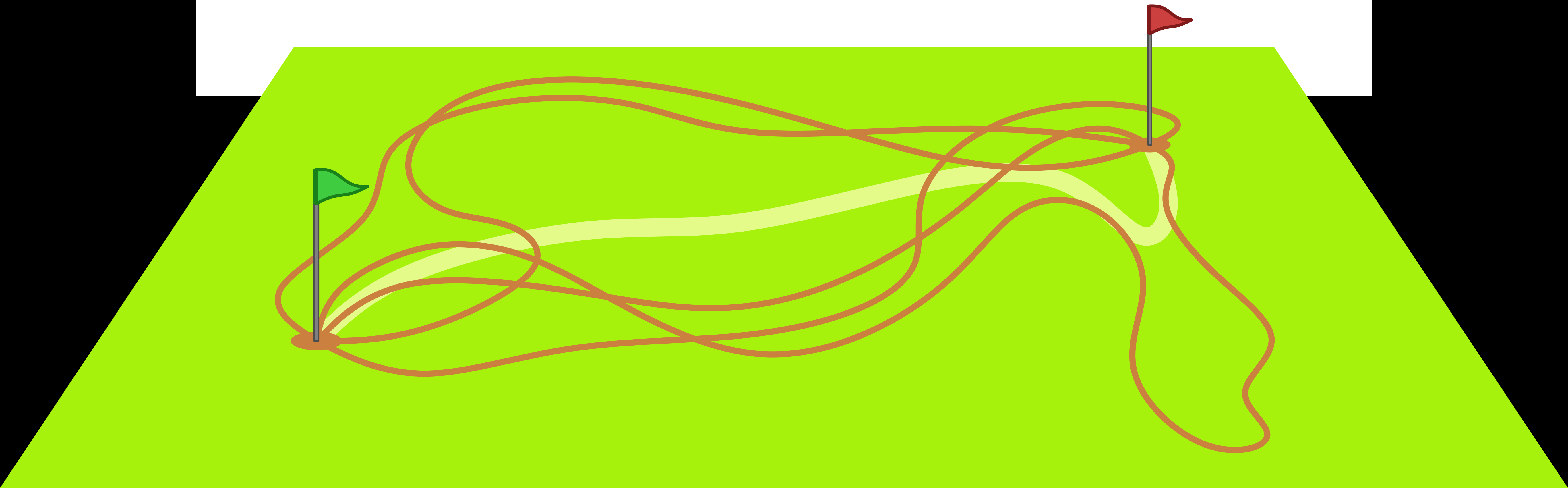

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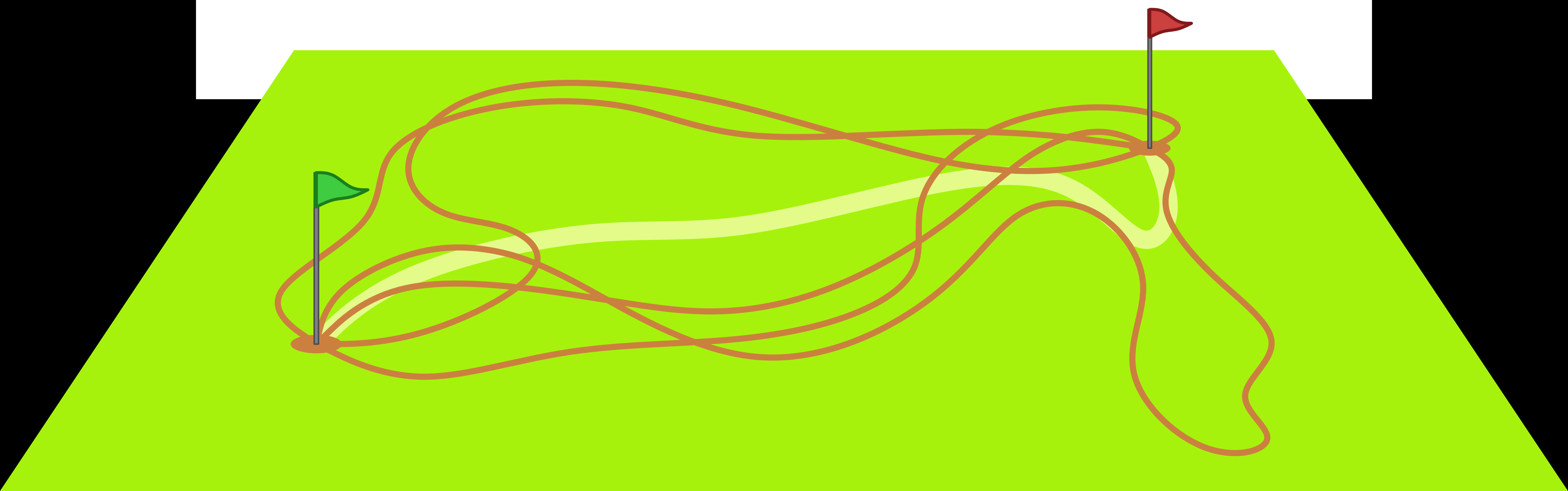

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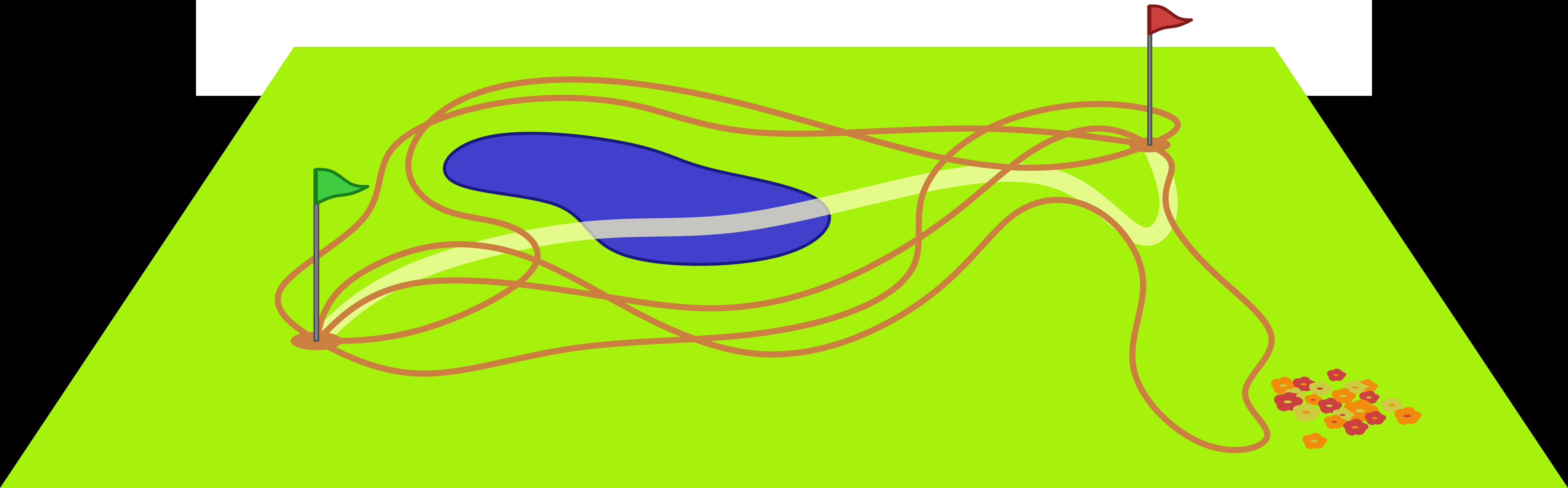

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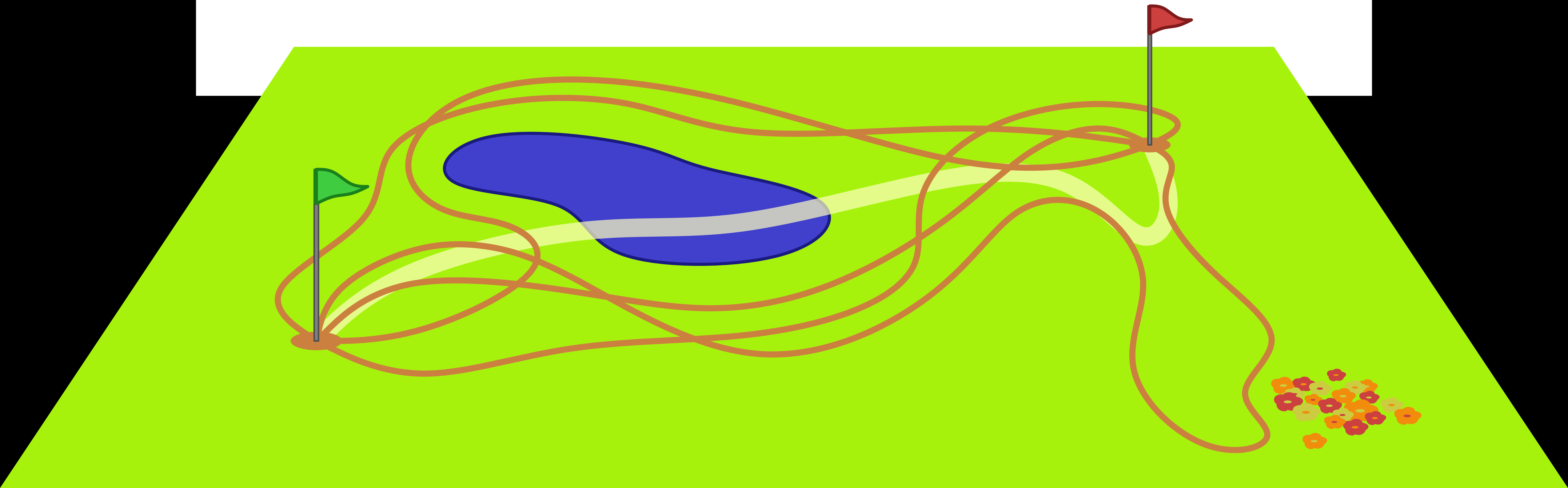

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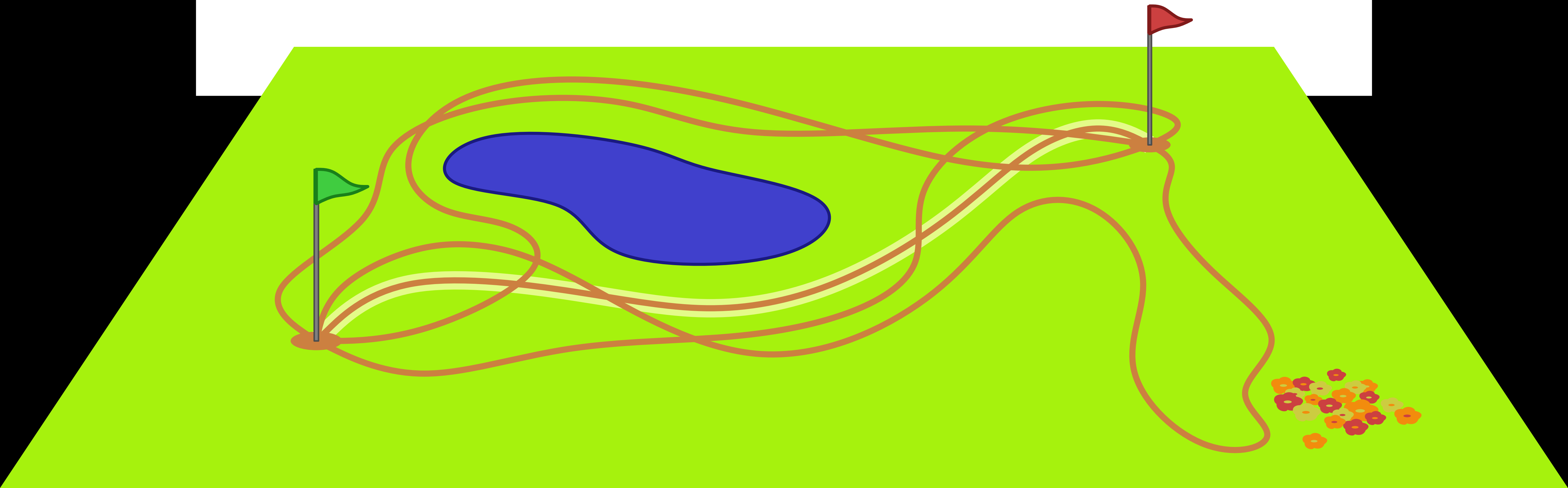

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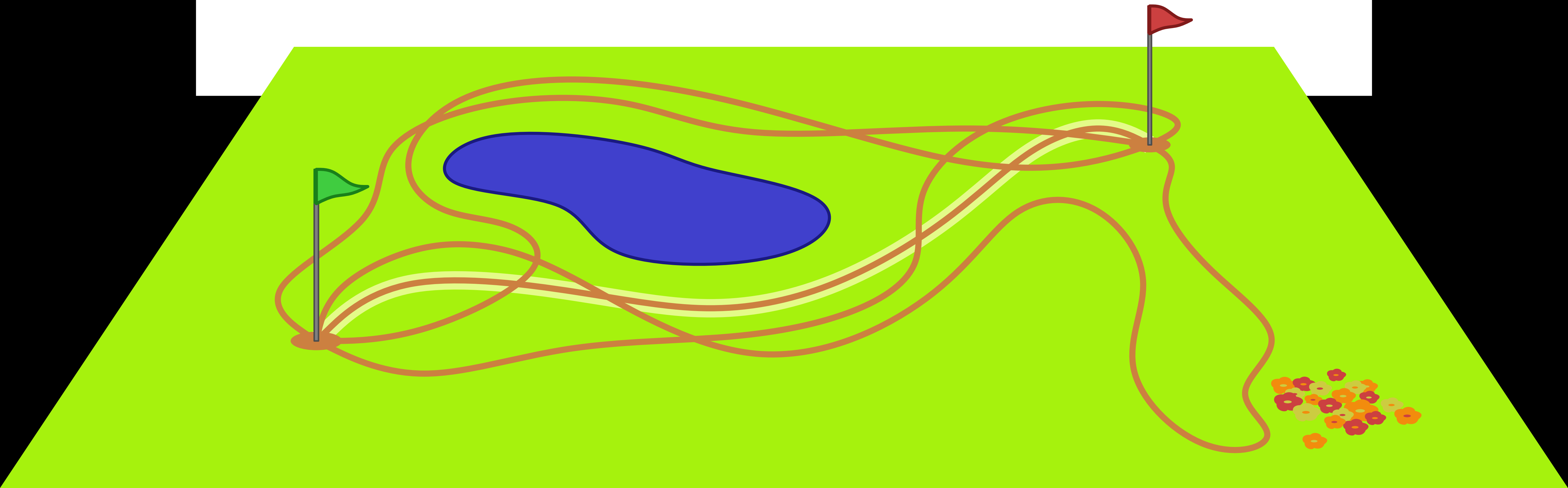

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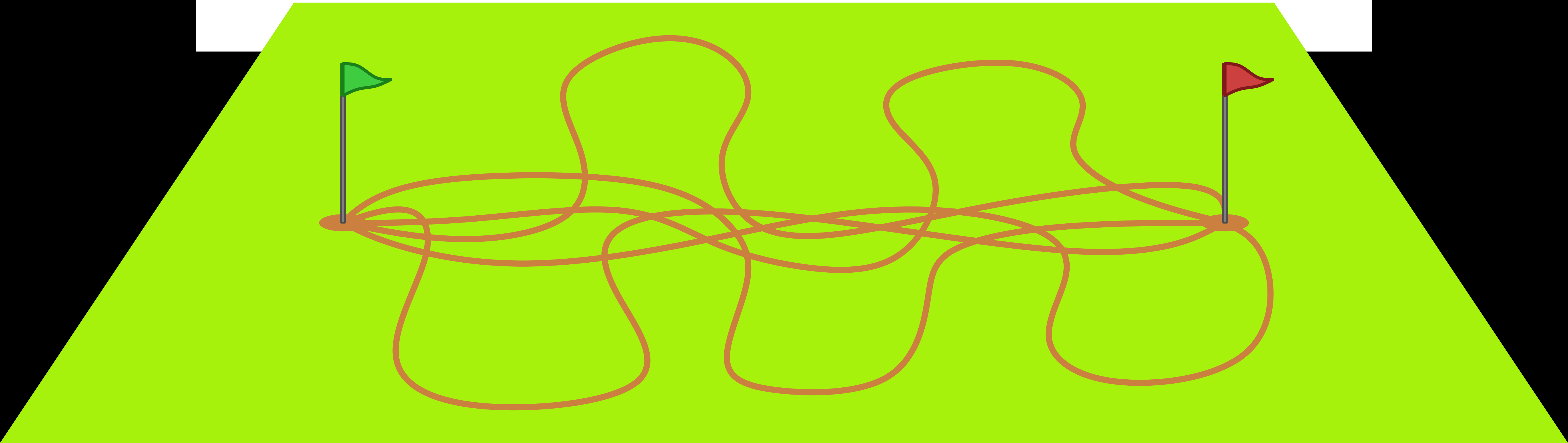

TRAJECTORIES: PICK REPRE–SENTATIVE FROM INPUT?

- No?
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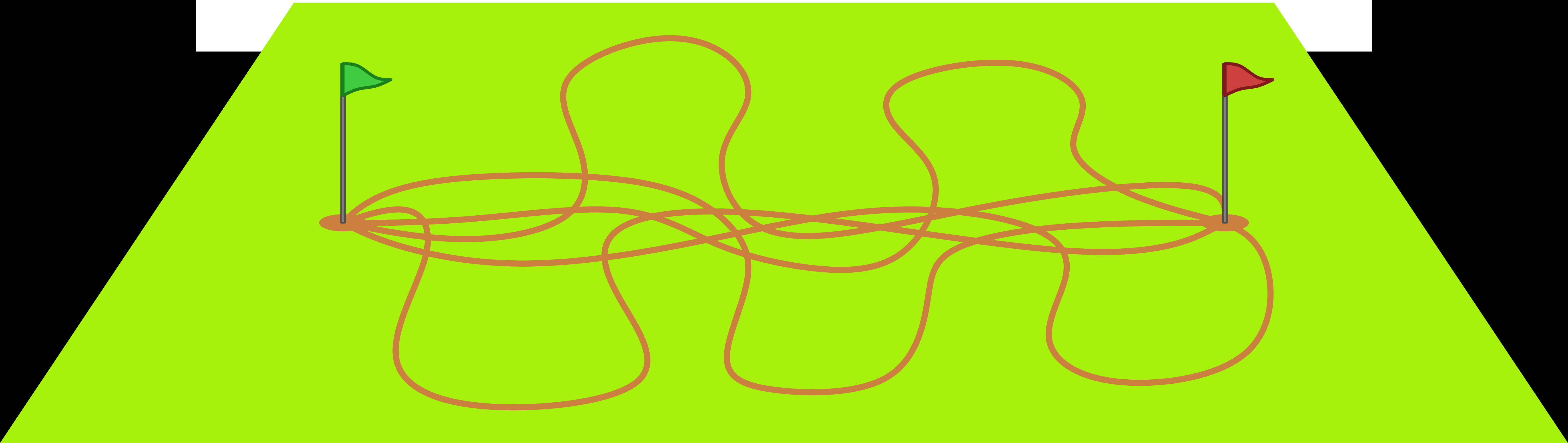
- Yes?
  - Which trajectory do we pick?
  - There may not be any good representative!



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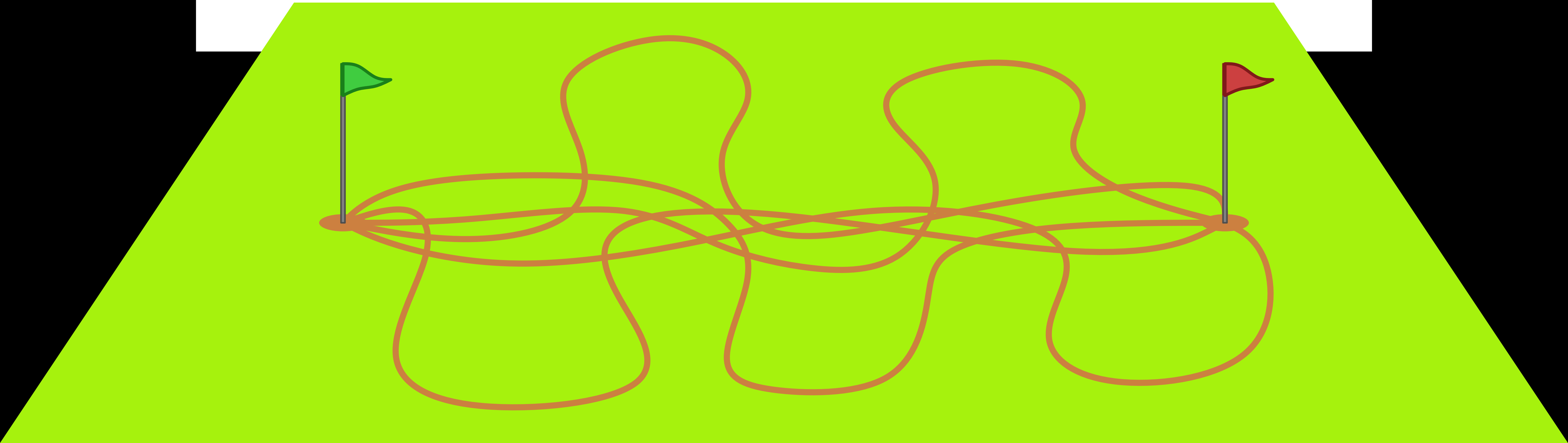
- Yes?
  - Which trajectory do we pick?
  - There may not be any good representative!
- Use pieces of different trajectories



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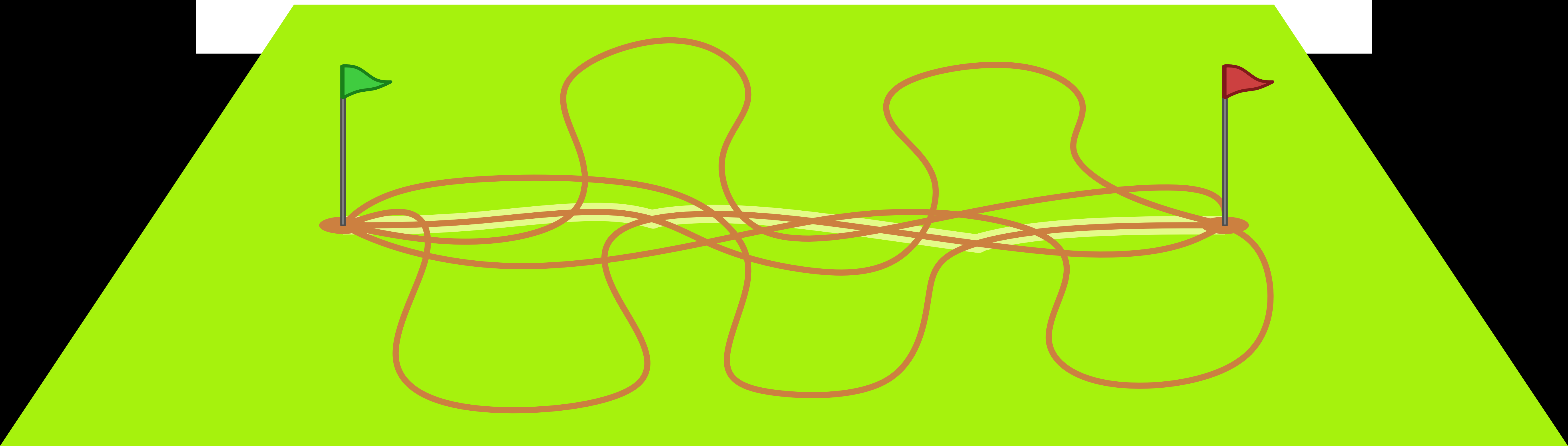
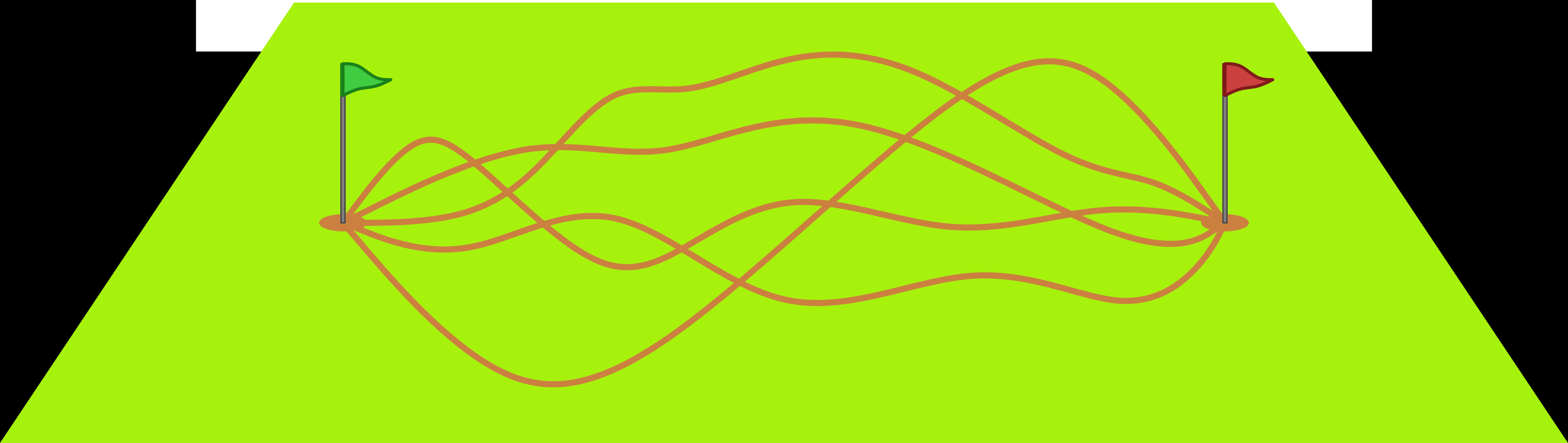


II DEFINING THE MEDIAN

SIMPLE MEDIAN

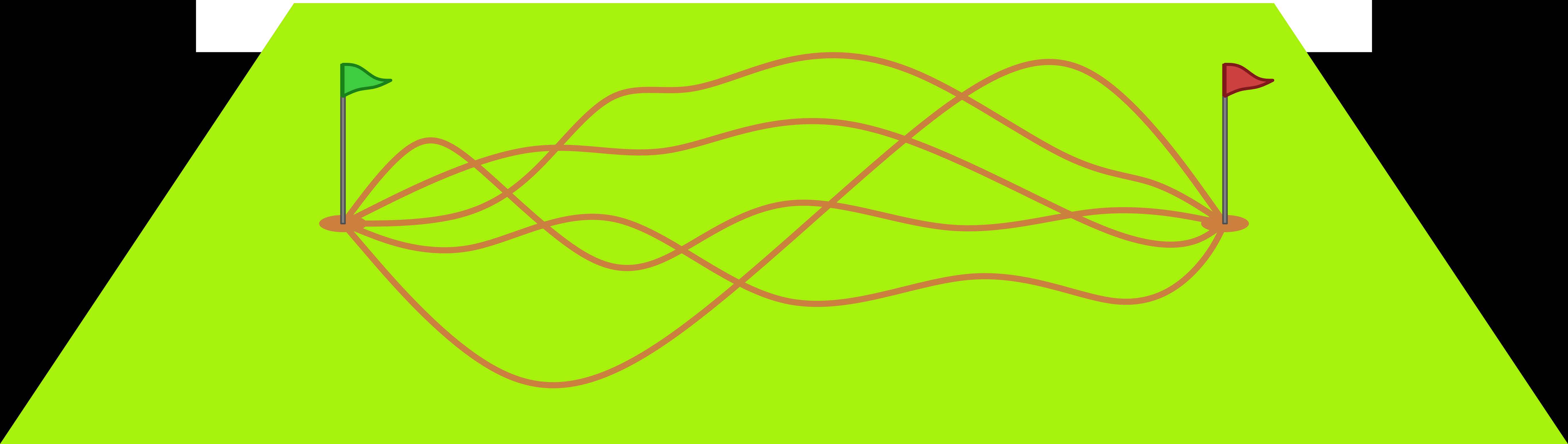
SIMPLE MEDIAN

- For x-monotone trajectories...



SIMPLE MEDIAN

- For x-monotone trajectories...
  - Take the median at each x-coordinate



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- For x-monotone trajectories...
  - Take the median at each x-coordinate
  - Result: the $n/2$-level
  - Start in the middle, switch at each crossing

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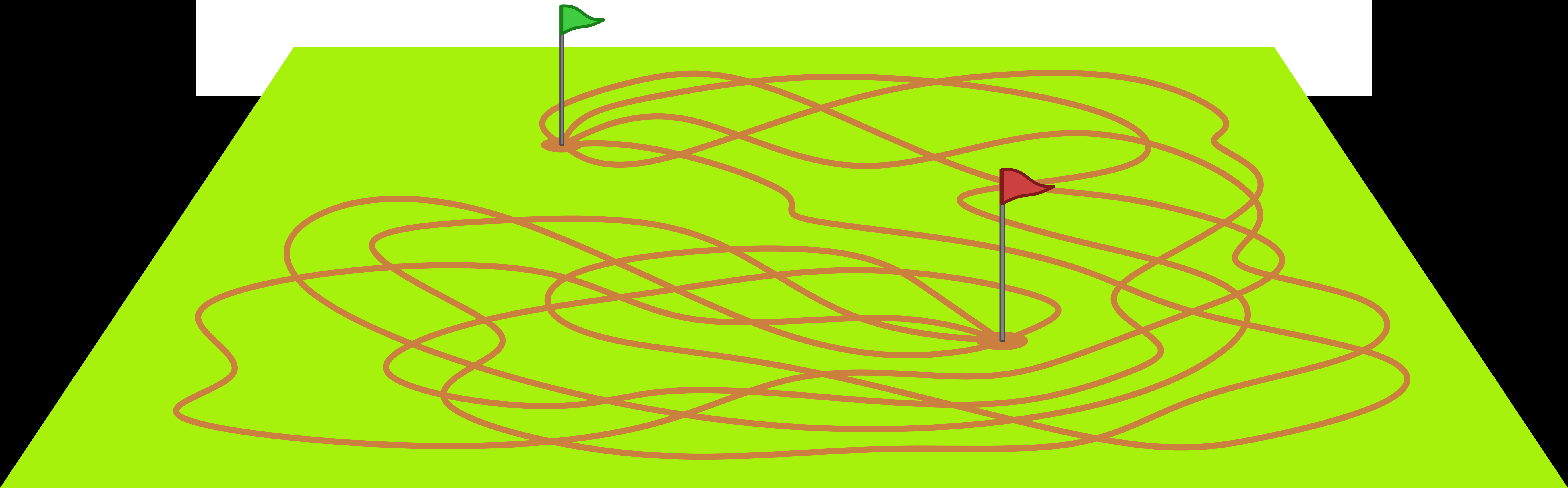
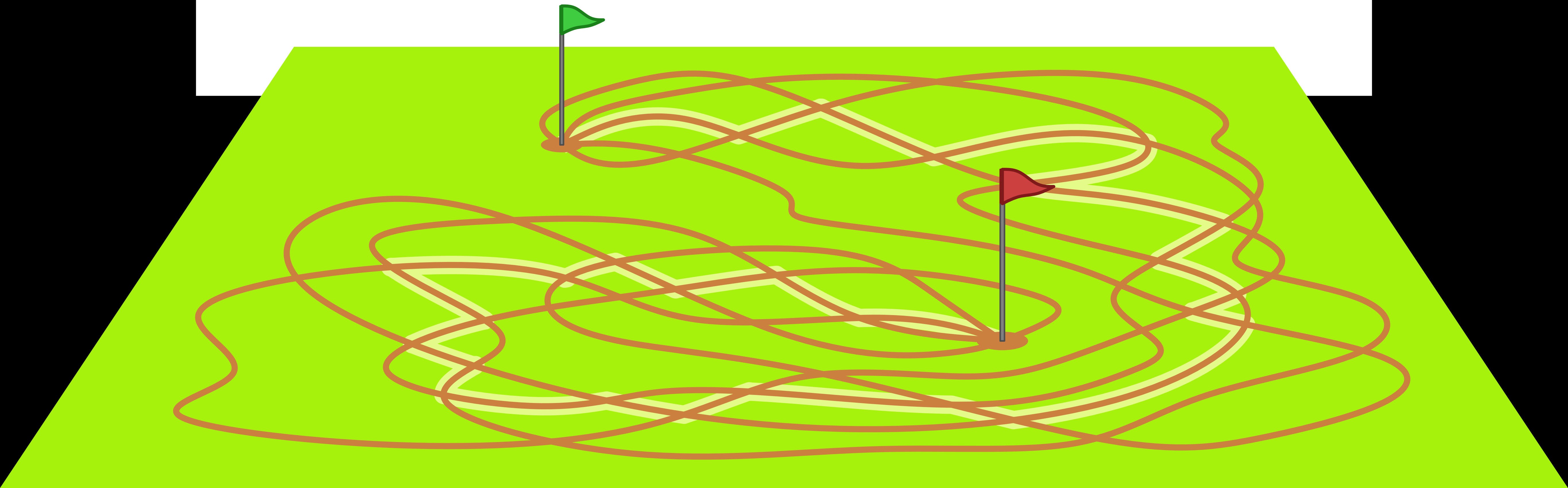
  - Take the median at each x-coordinate
  - Result: the $n/2$-level
  - Start in the middle, switch at each crossing
- For arbitrary trajectories...



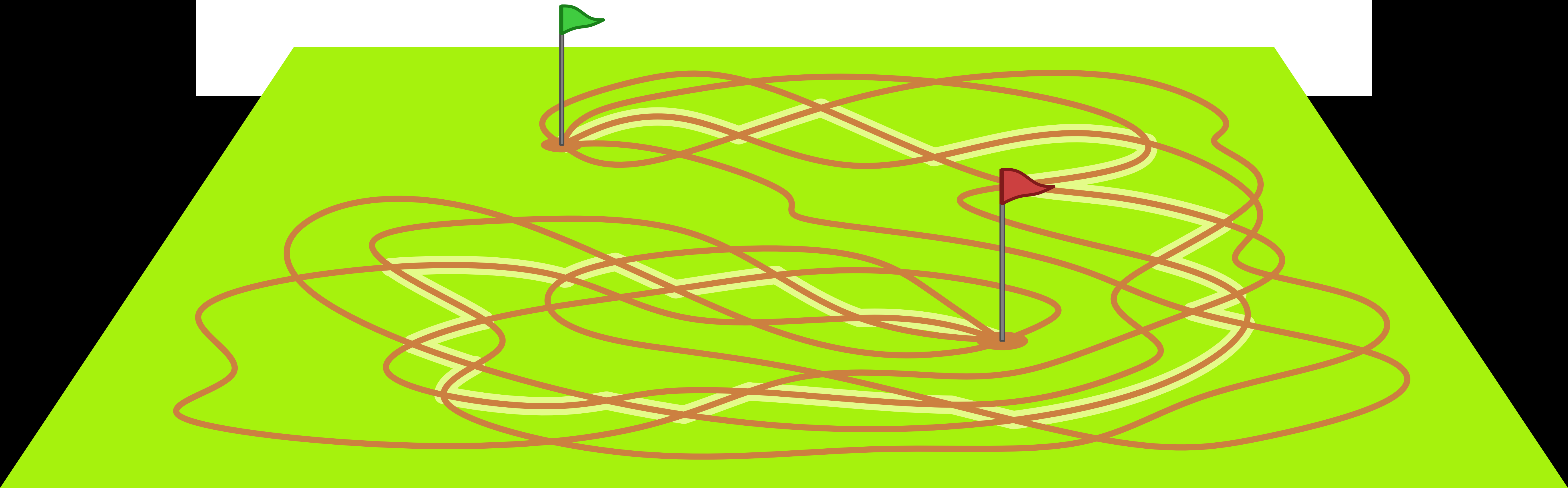
SIMPLE MEDIAN

- For x-monotone trajectories...
  - Take the median at each x-coordinate
  - Result: the $n/2$-level
  - Start in the middle, switch at each crossing
- For arbitrary trajectories...
  - Why not do the same thing?



SIMPLE MEDIAN

- For x-monotone trajectories...
  - Take the median at each x-coordinate
  - Result: the $n/2$-level
  - Start in the middle, switch at each crossing
- For arbitrary trajectories...
  - Why not do the same thing?
- We call this the *simple median* of a set of curves



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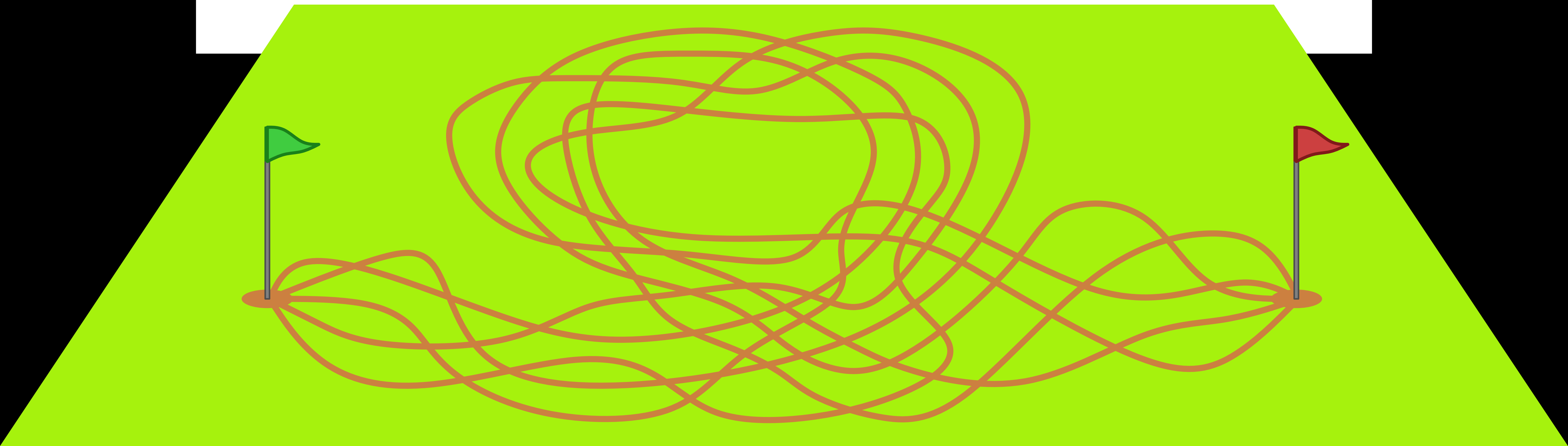

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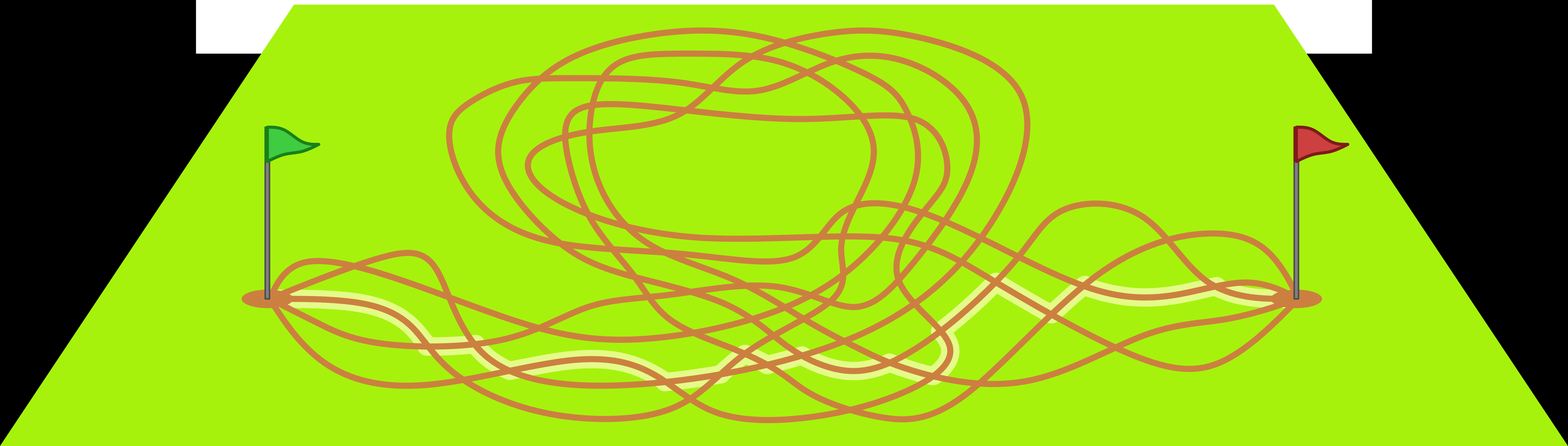

SHORTCUTS

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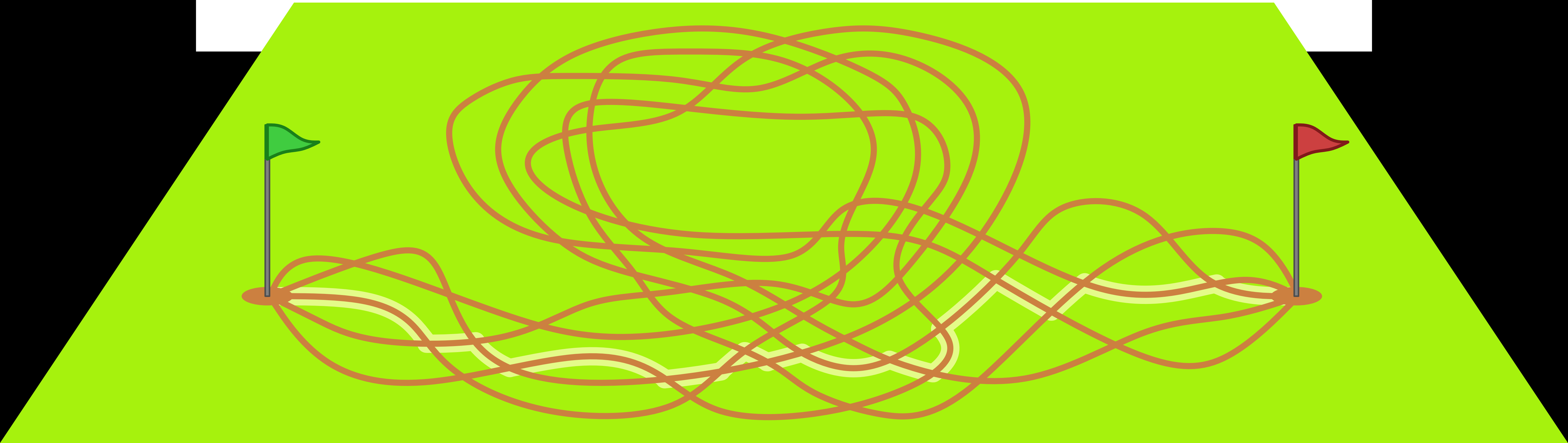

SHORTCUTS

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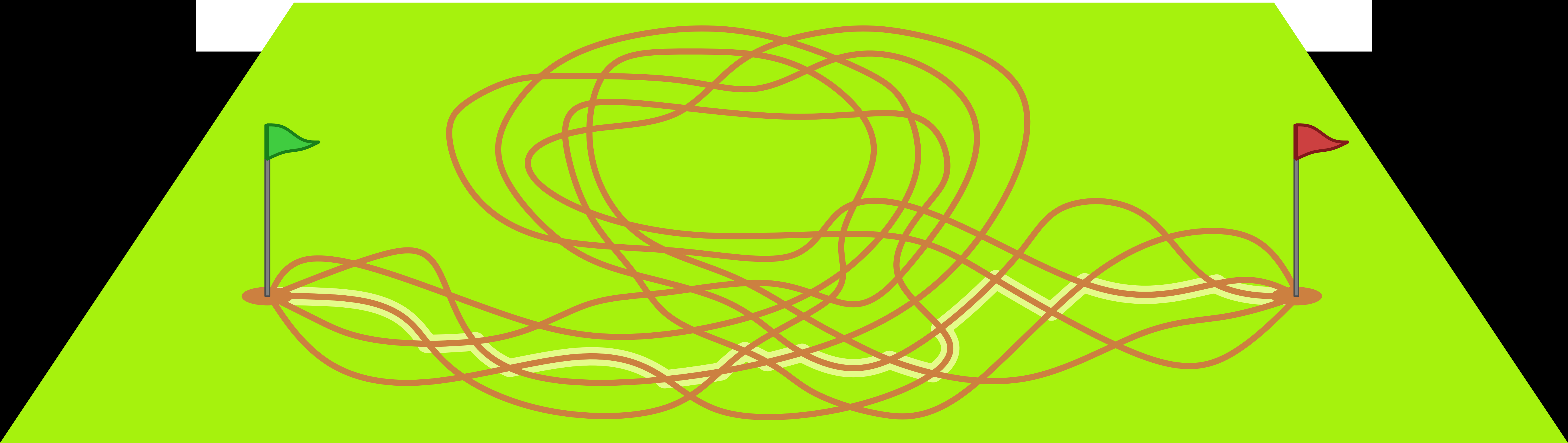

SHORTCUTS

- Problem
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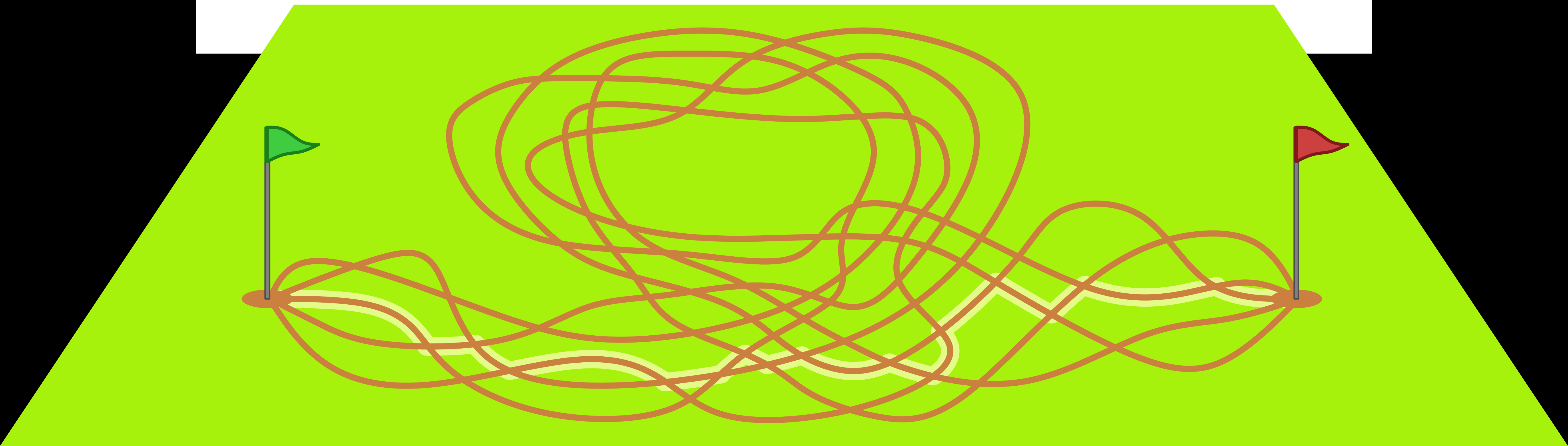
- Solution
  - Place a *pole*



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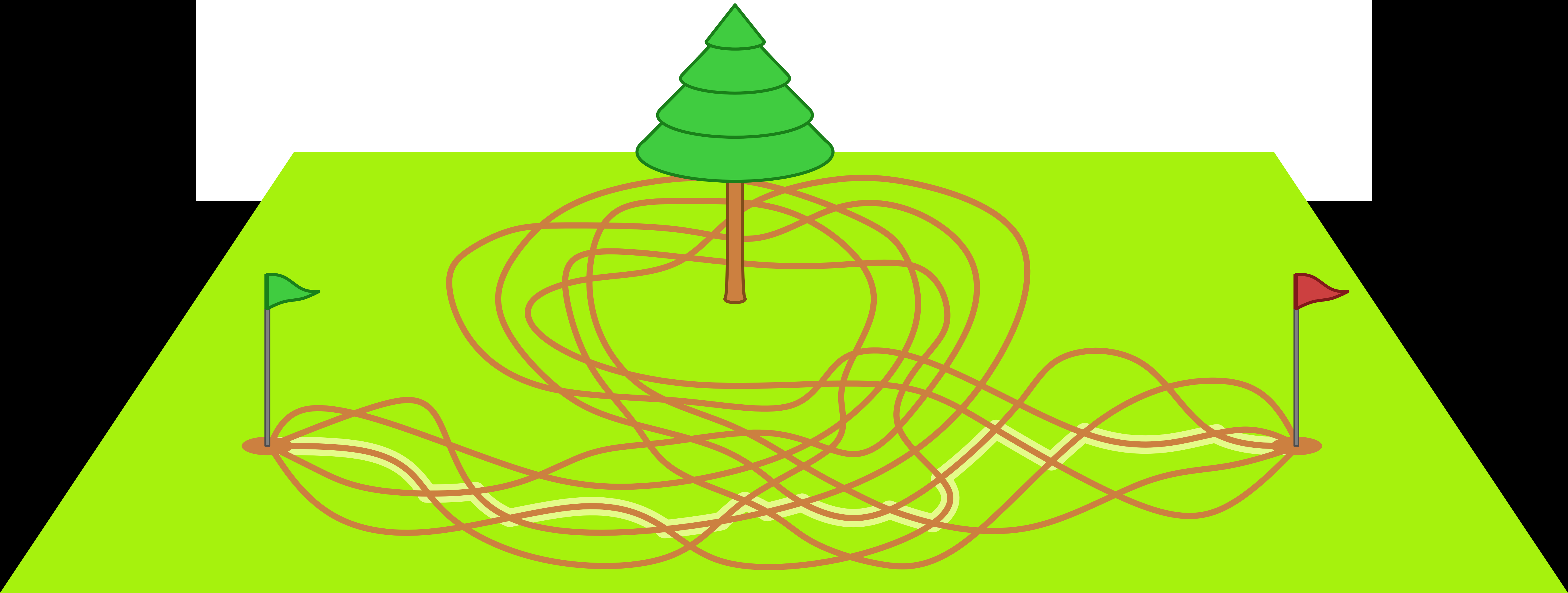
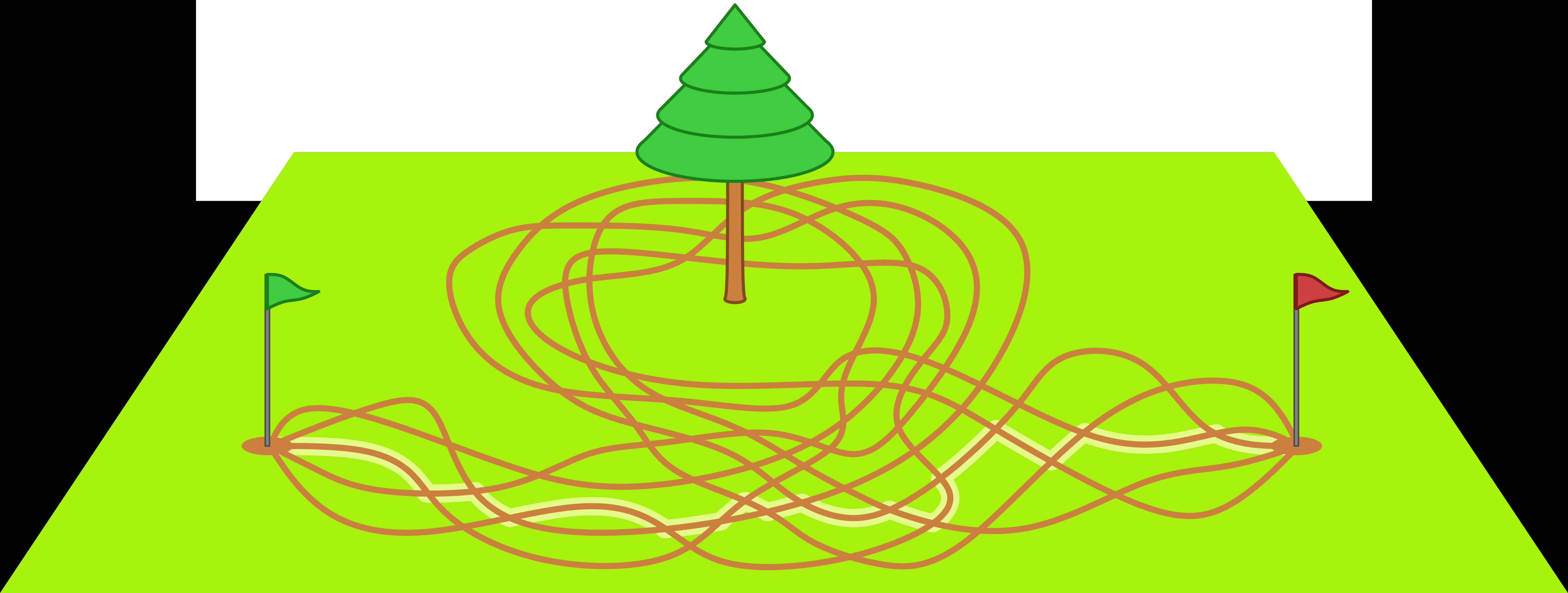


SHORTCUTS

- Problem
  - Simple median may miss large parts of the input trajectories
- Solution
  - Place a *pole*
  - Require the median to go around it



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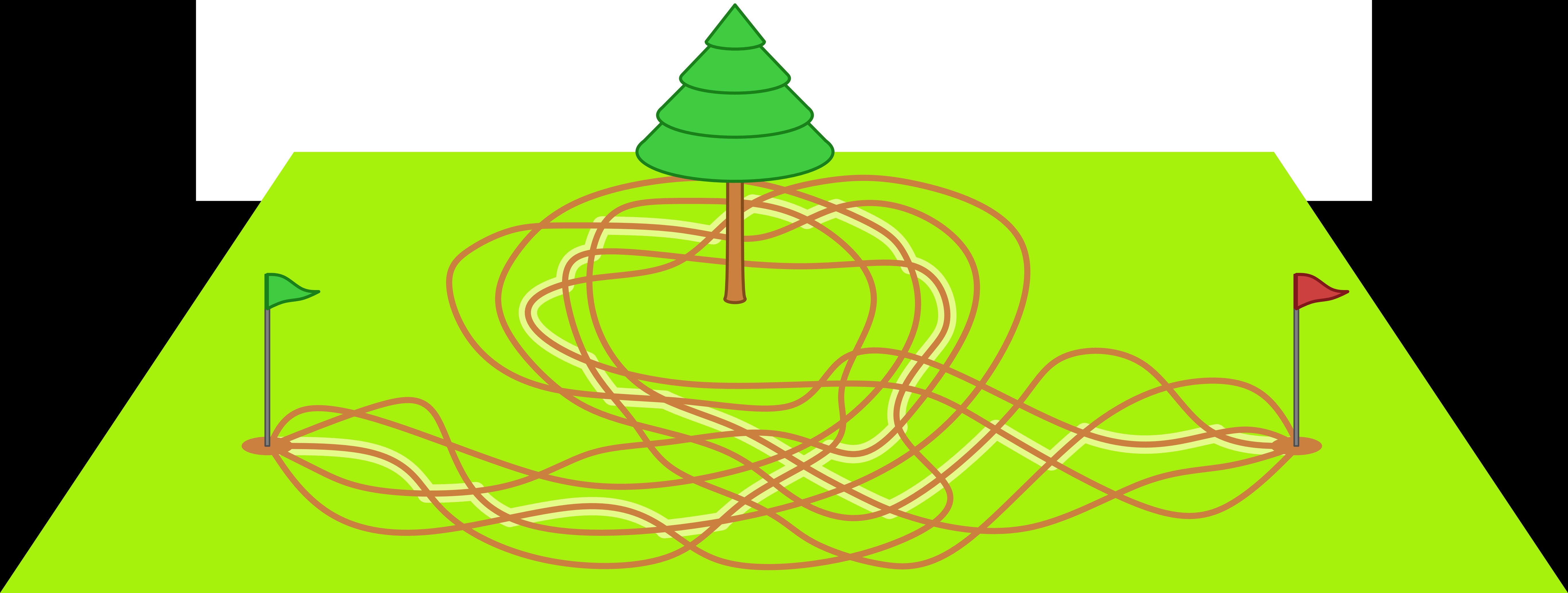

SHORTCUTS

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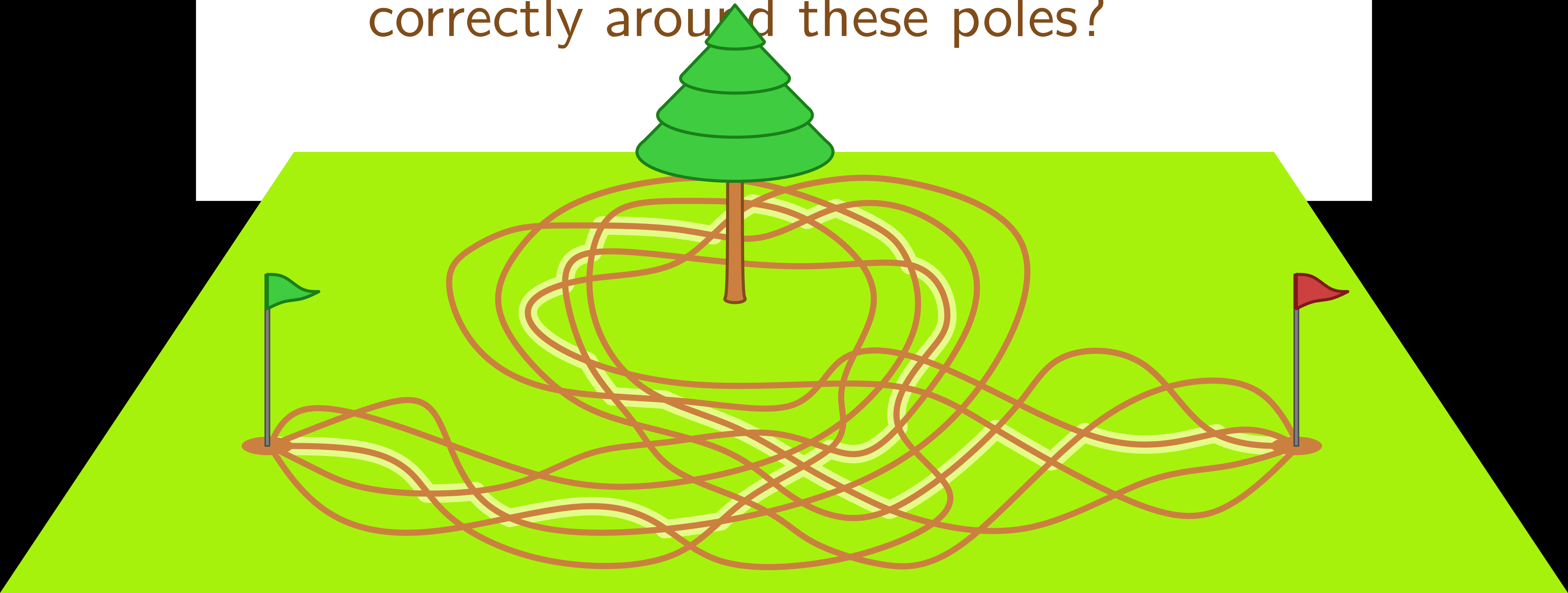
- Solution
  - Place a *pole*
  - Require the median to go around it
- ... how do we steer the median correctly around these poles?



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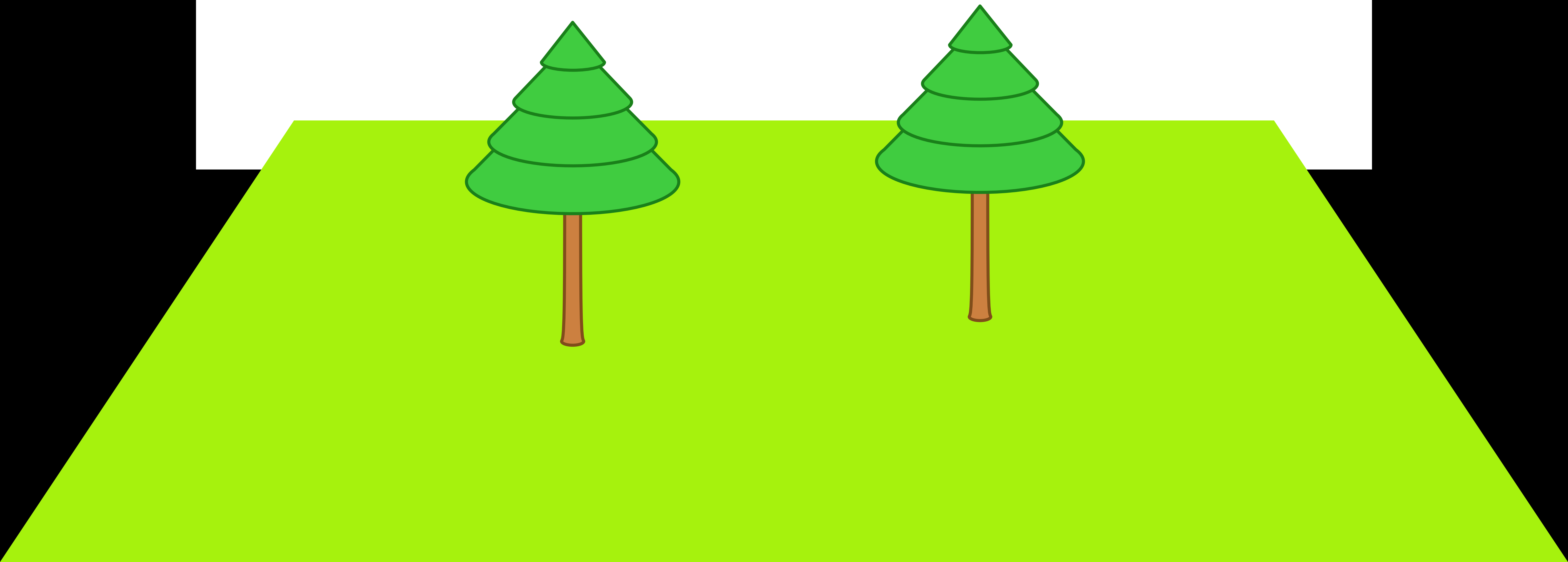
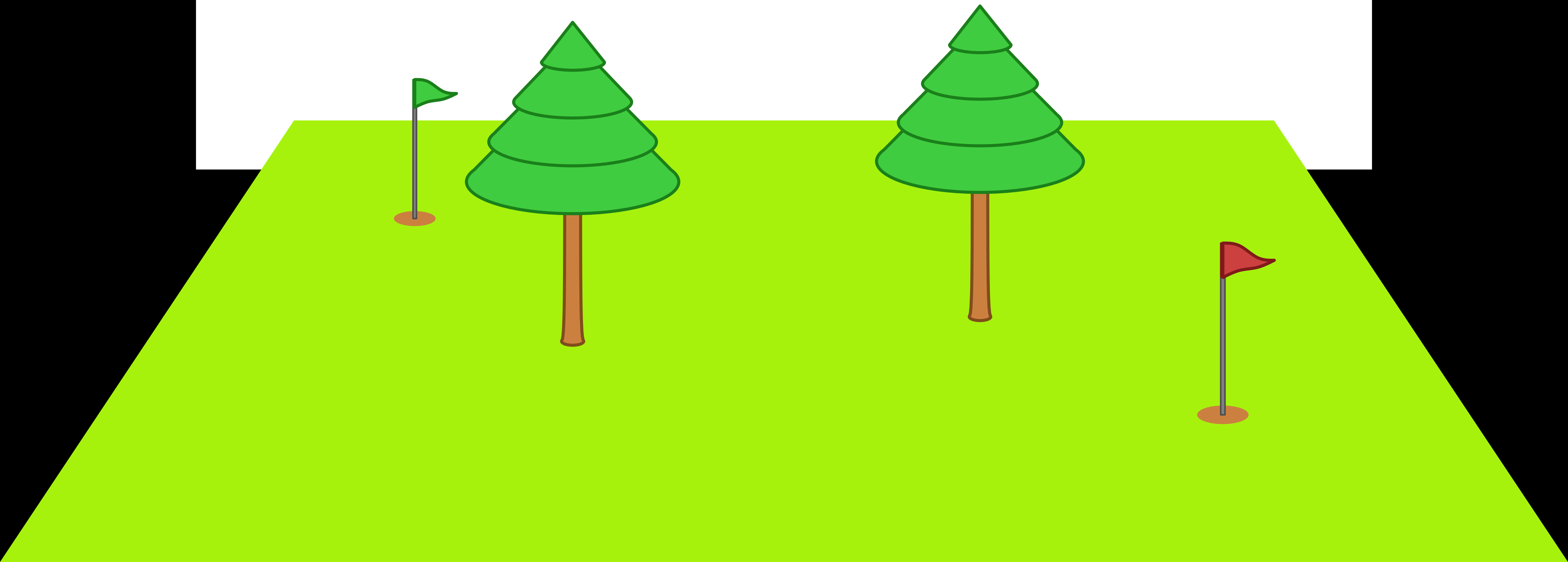


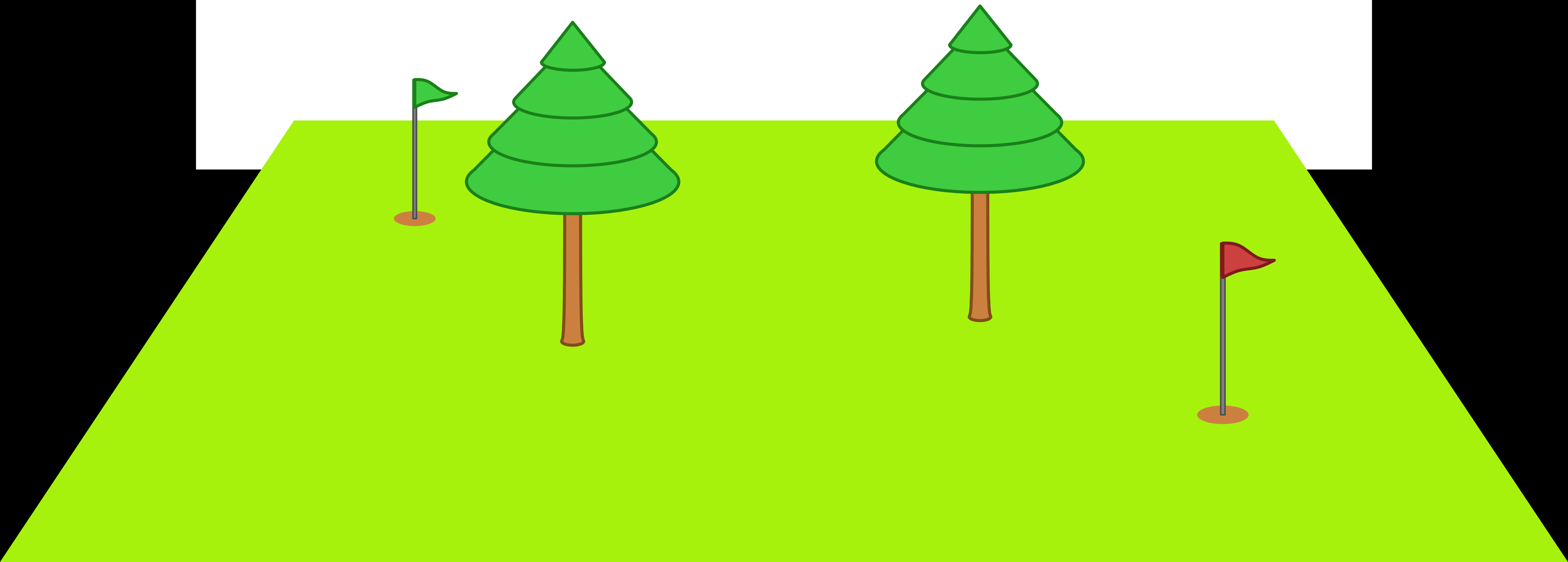
HOMOTOPY

- Ingredients
  - One punctured plane
  - Two points s and t in the plane



HOMOTOPY

- Ingredients
  - One punctured plane
  - Two points s and t in the plane
  - Two continuous curves from s to t



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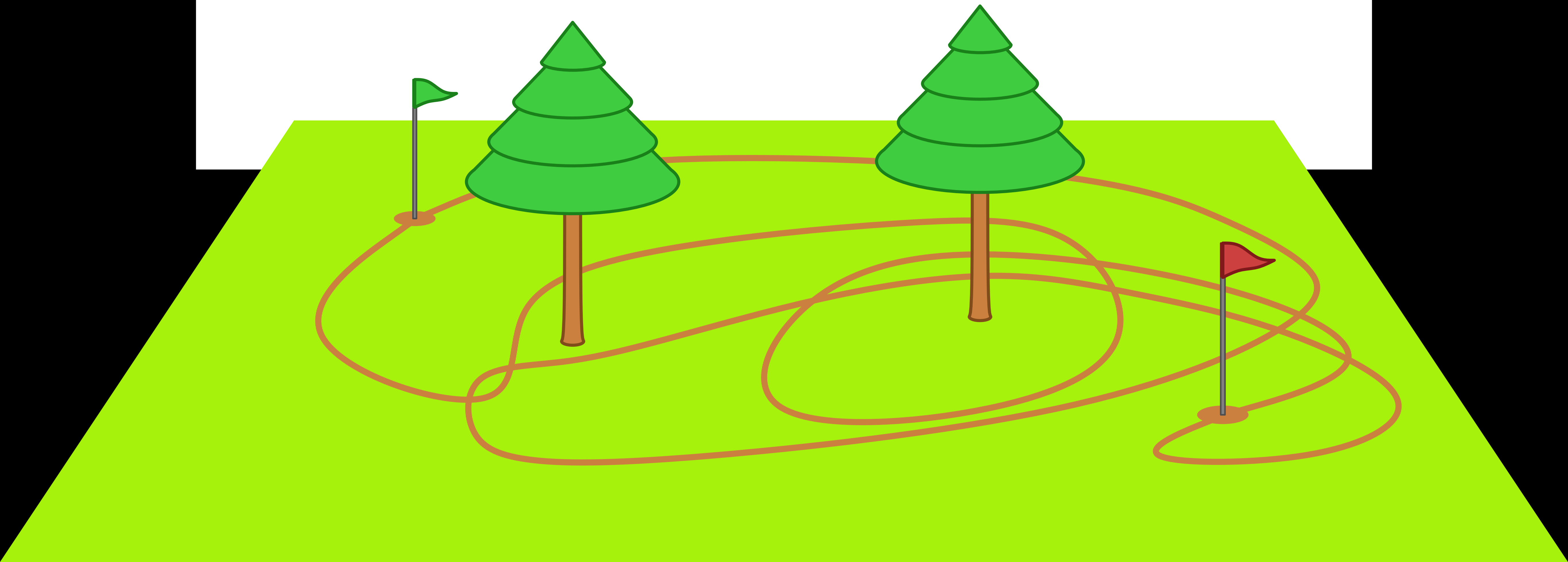

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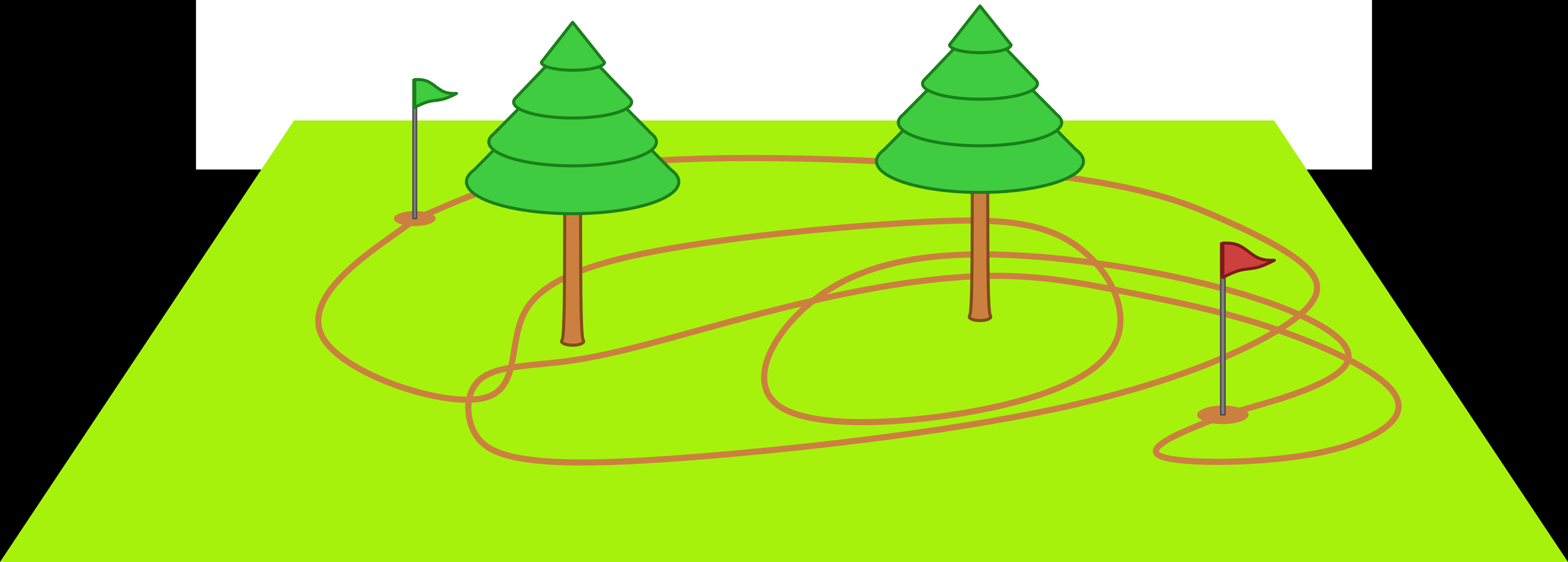

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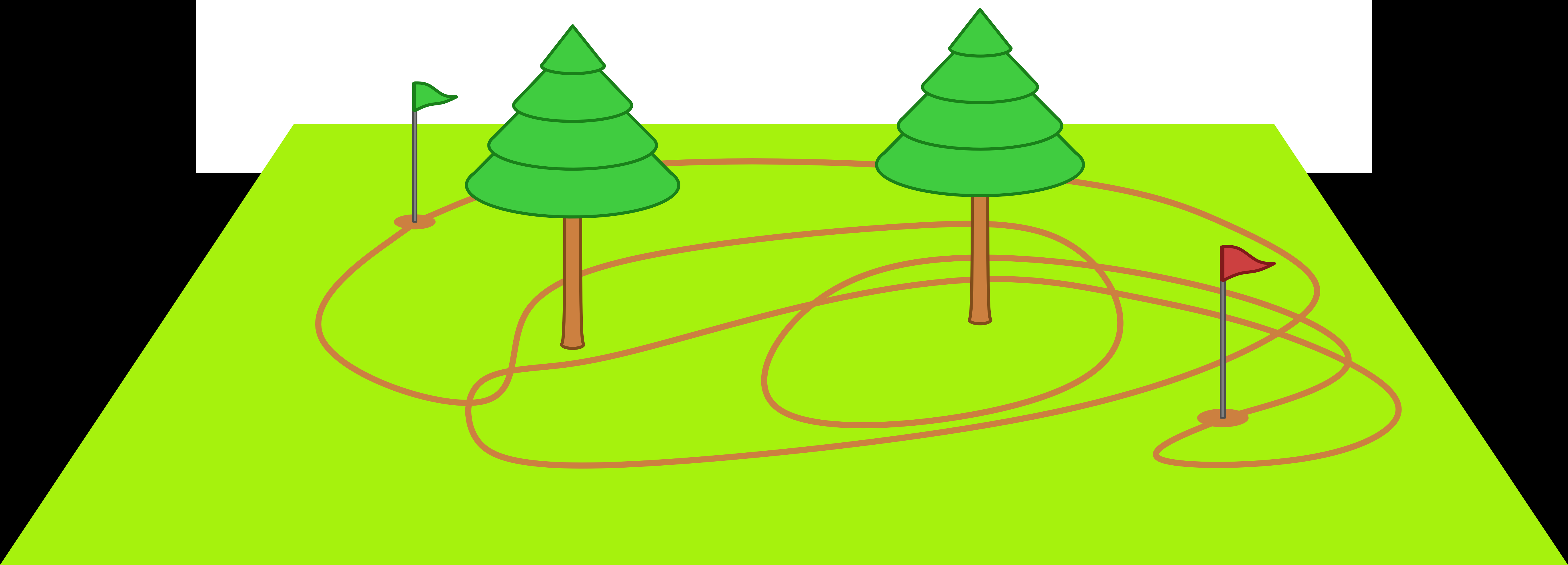

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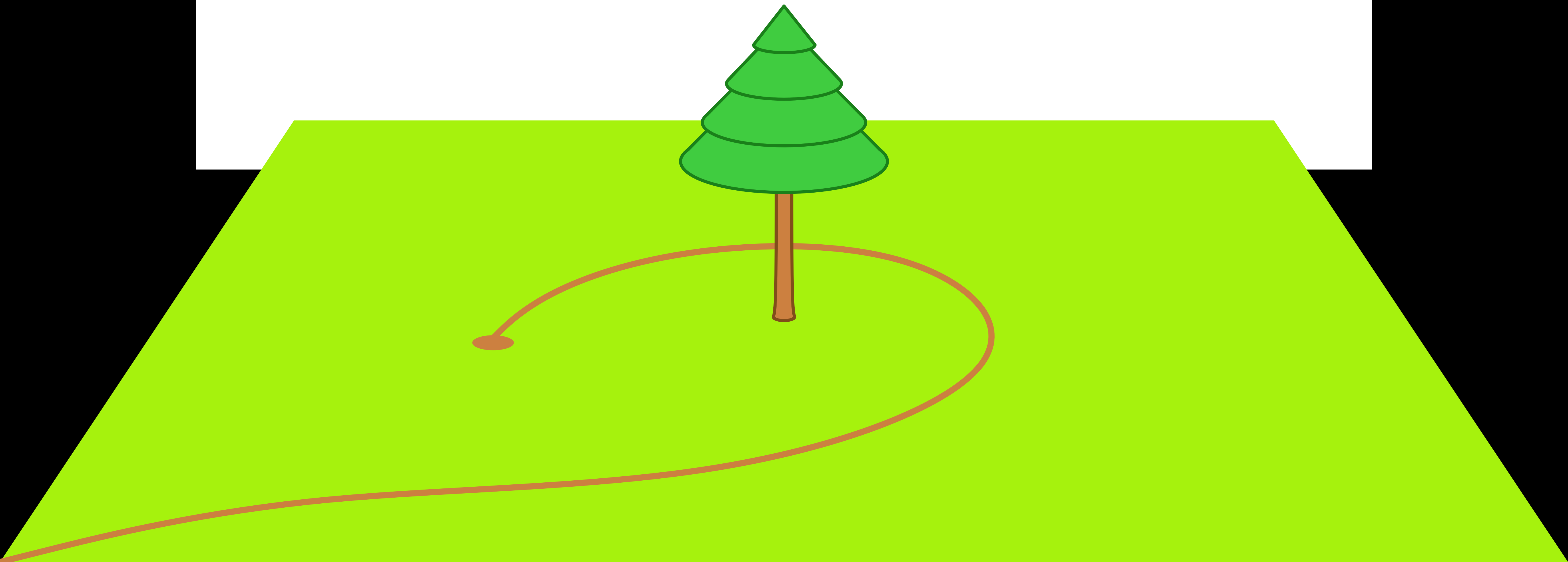

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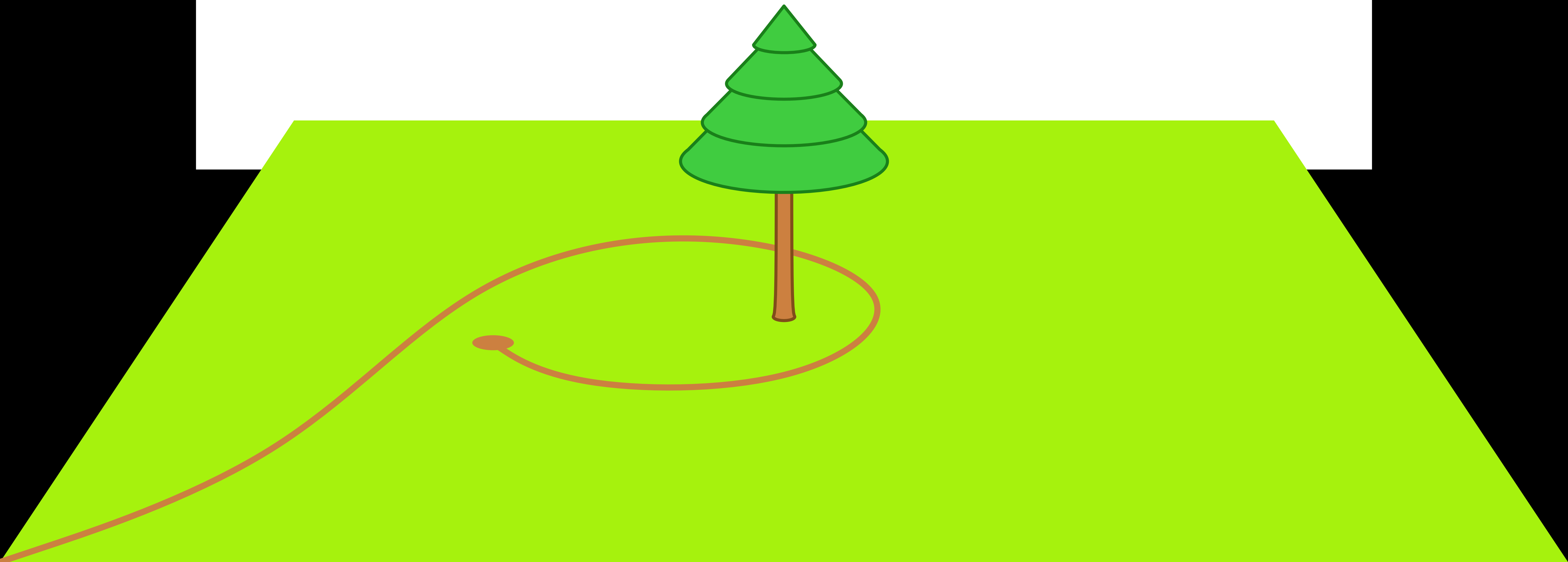

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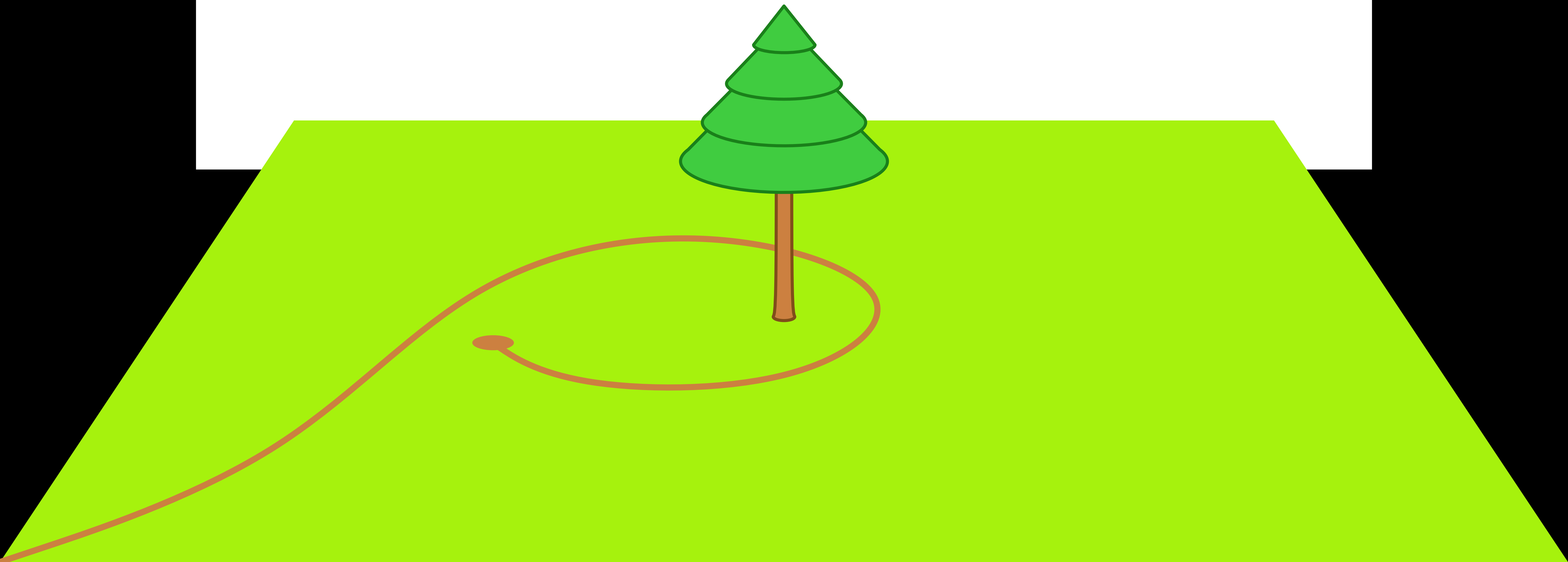

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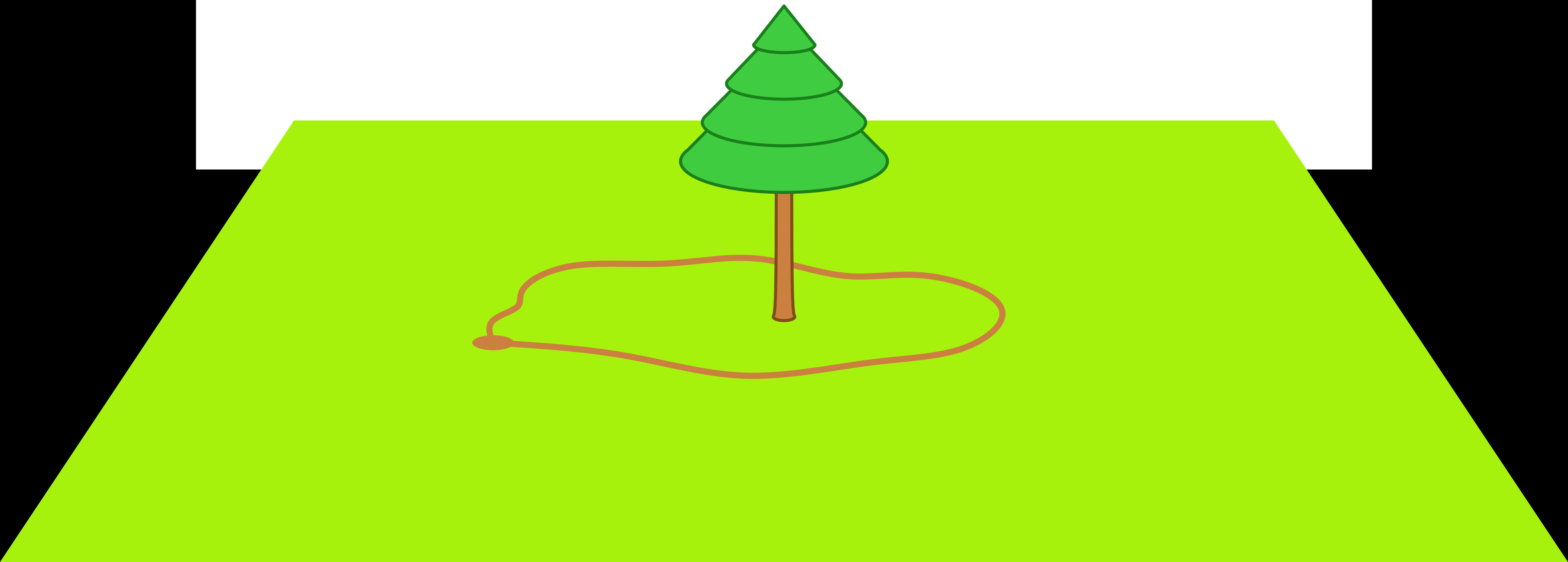

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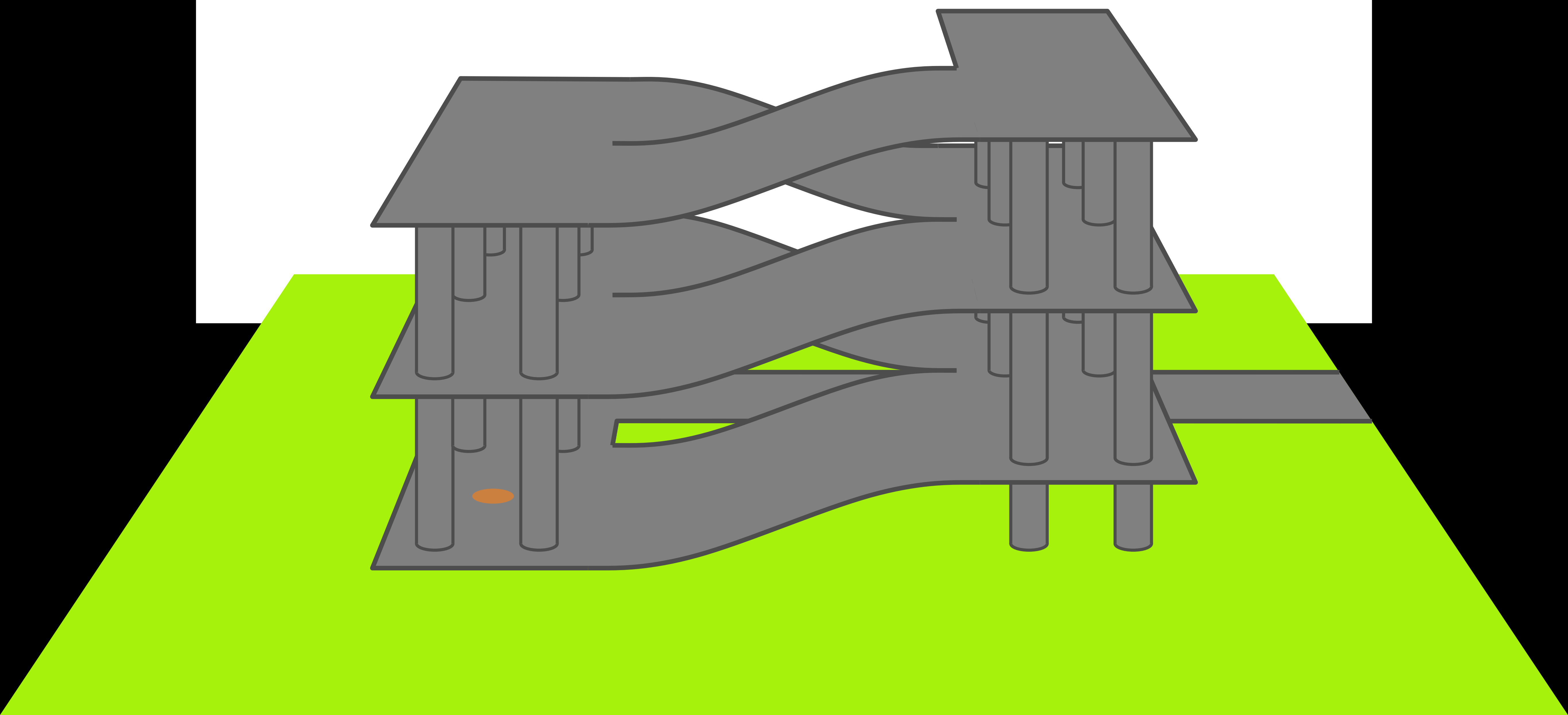

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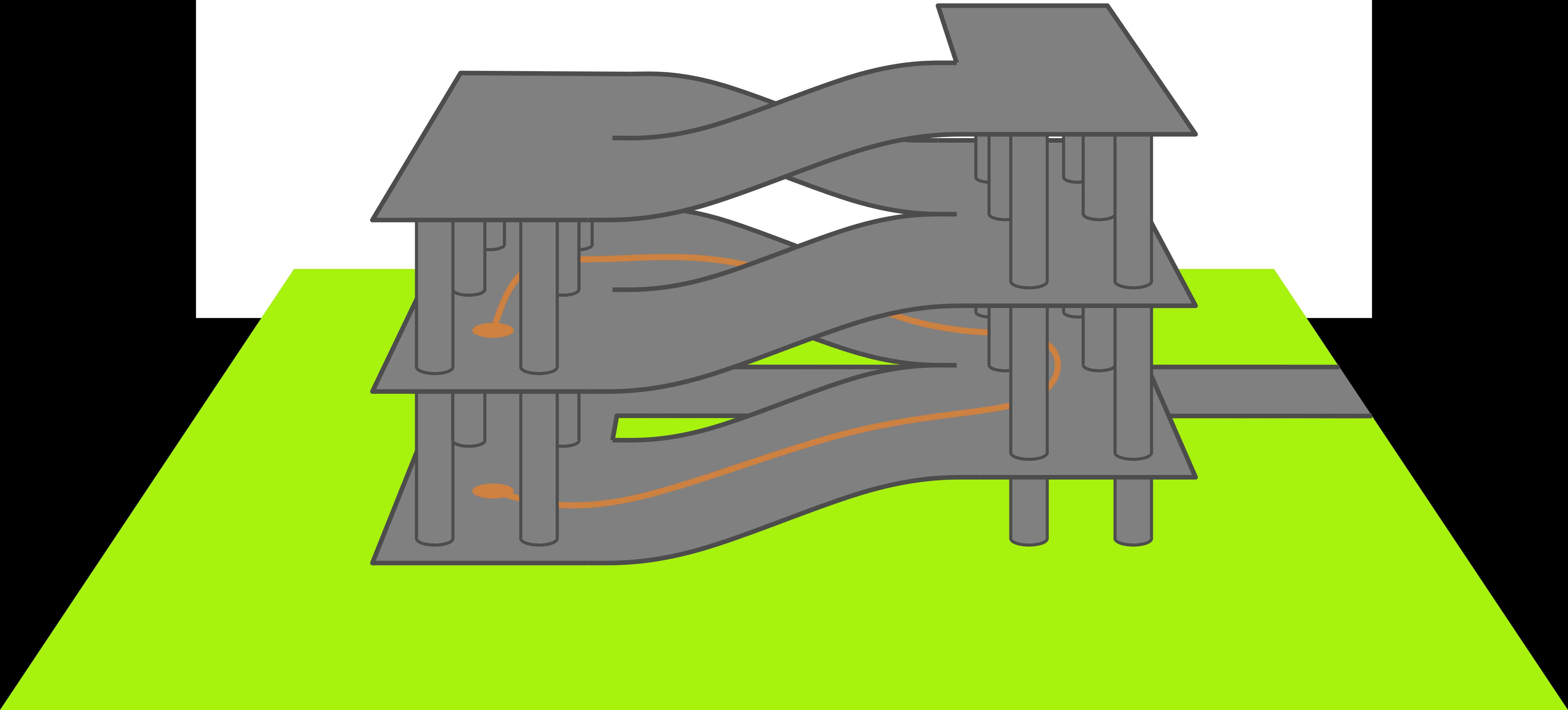

COVERING SPACE

- Consider a point in a space E
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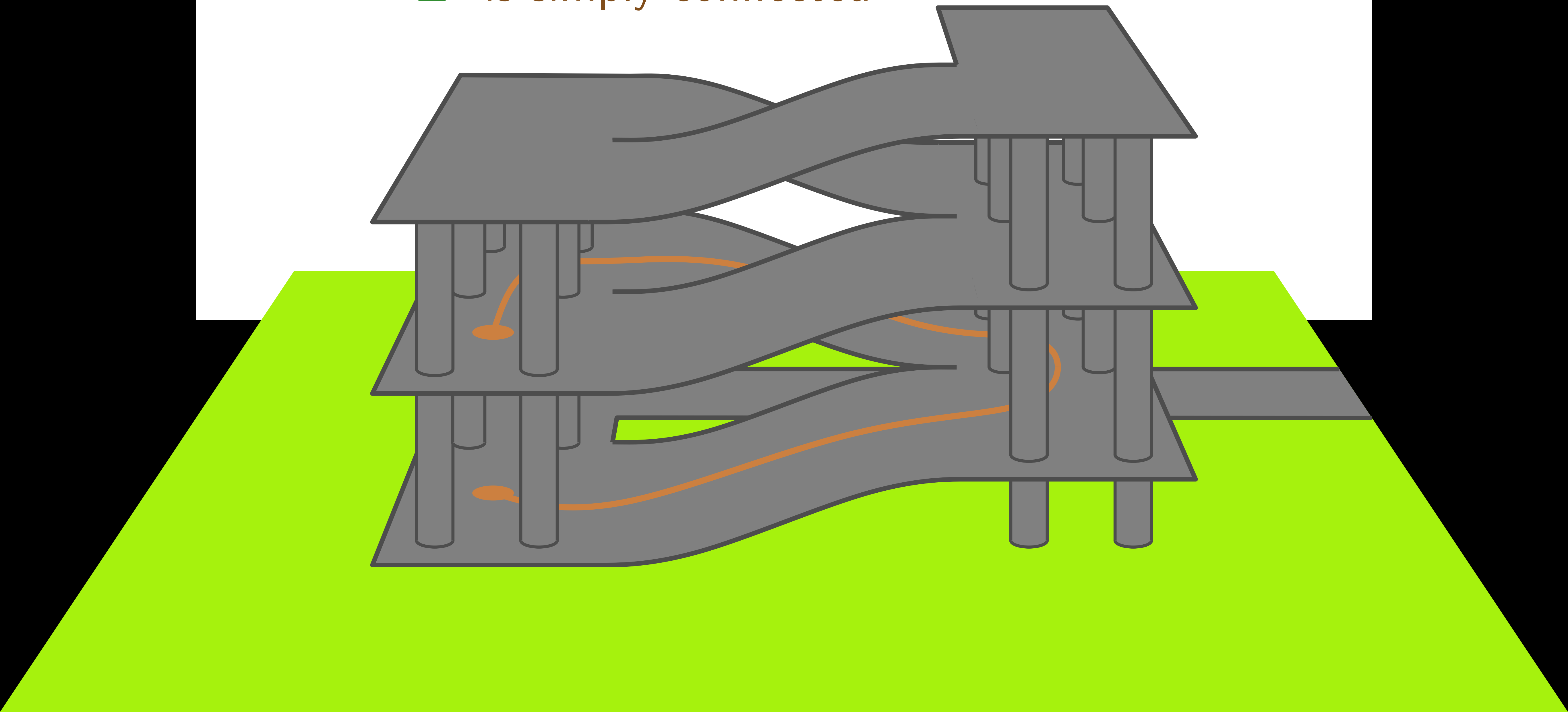
  - The resulting space E' is called the *covering space* of E
  - E' is simply connected



HOMOTOPIC MEDIAN

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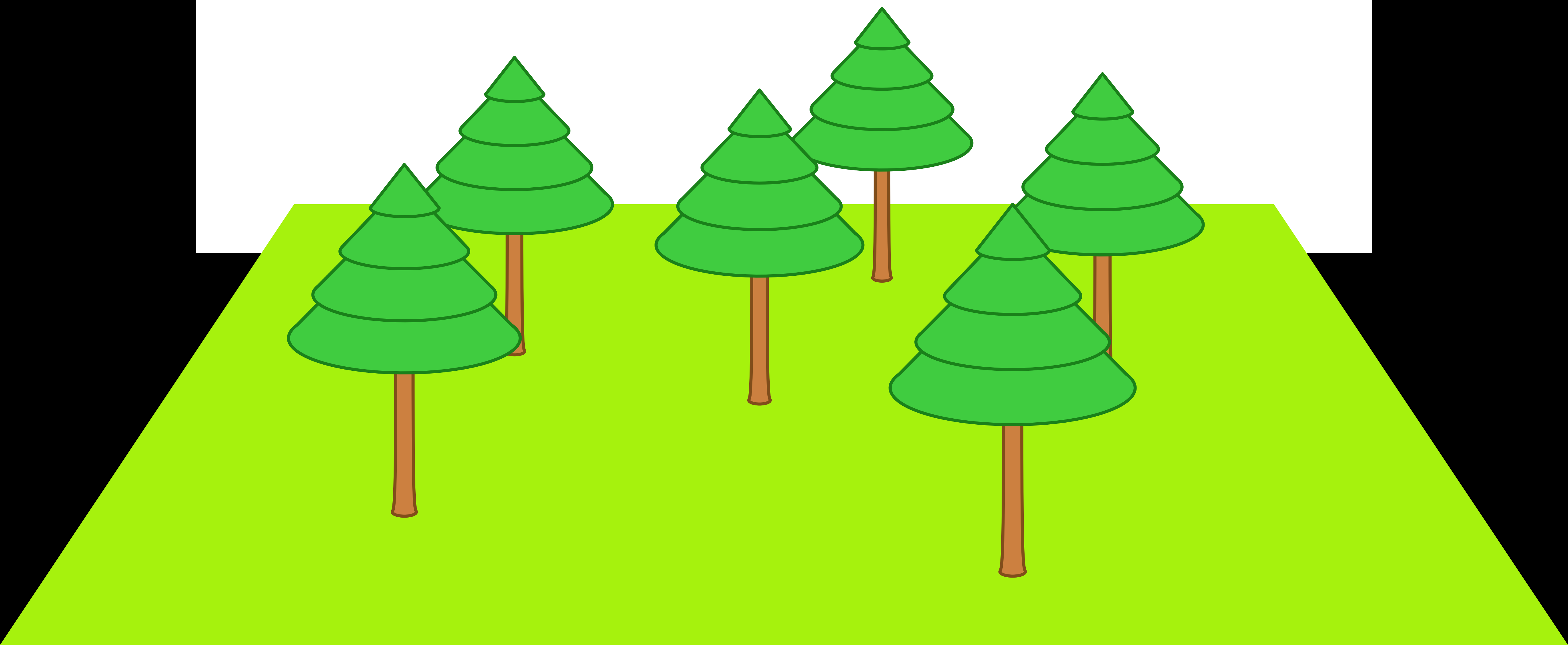

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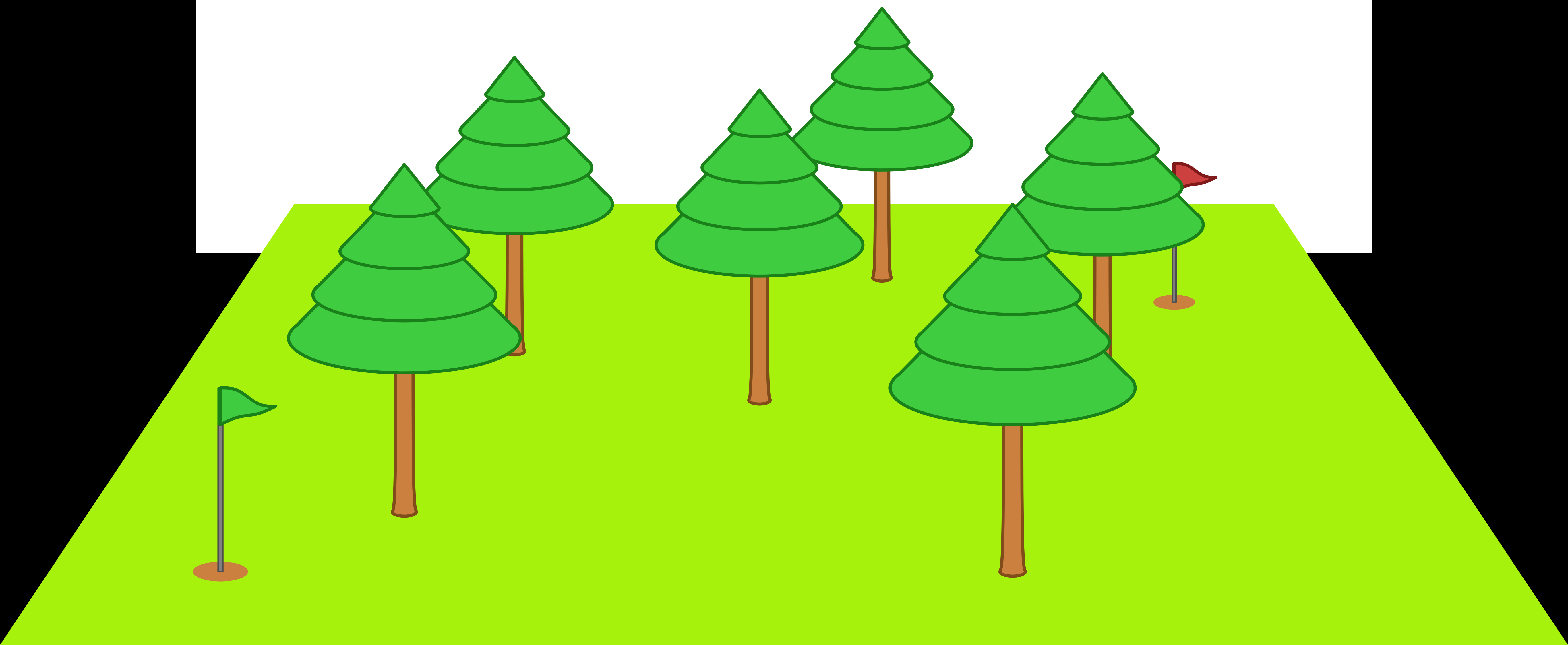

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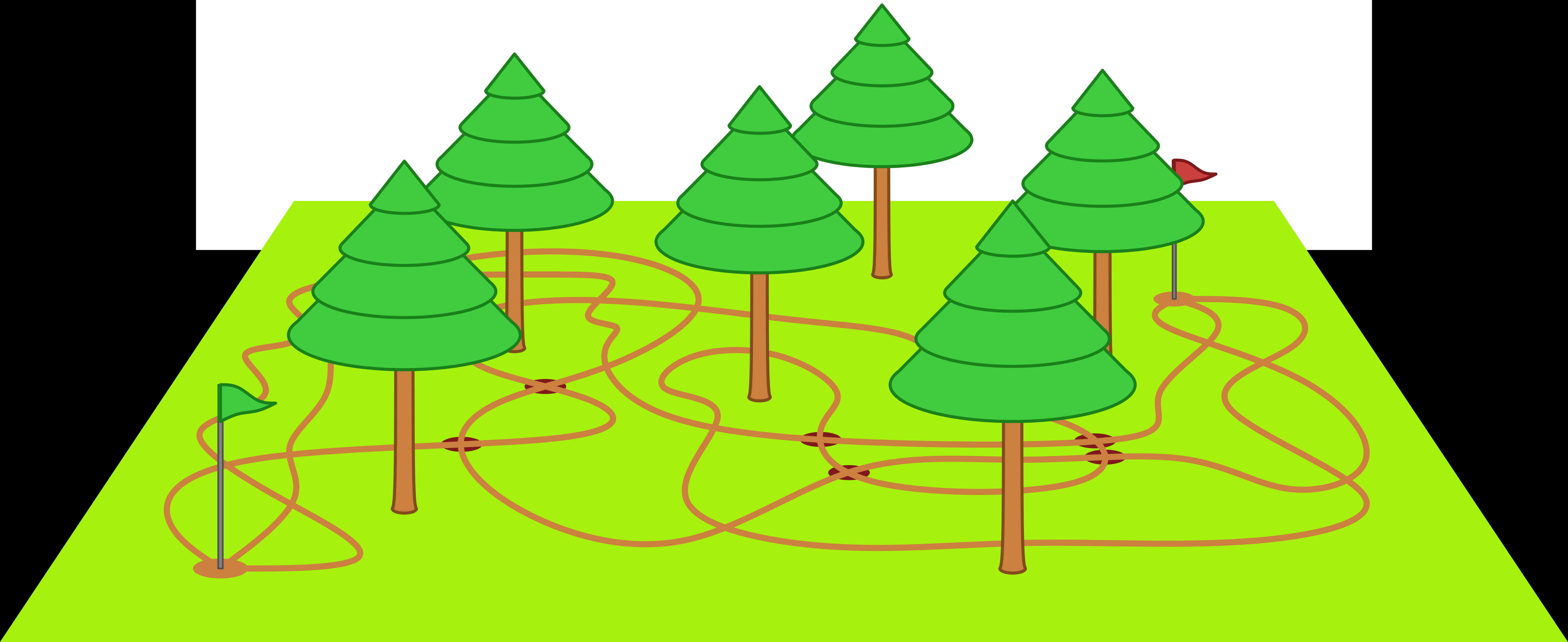

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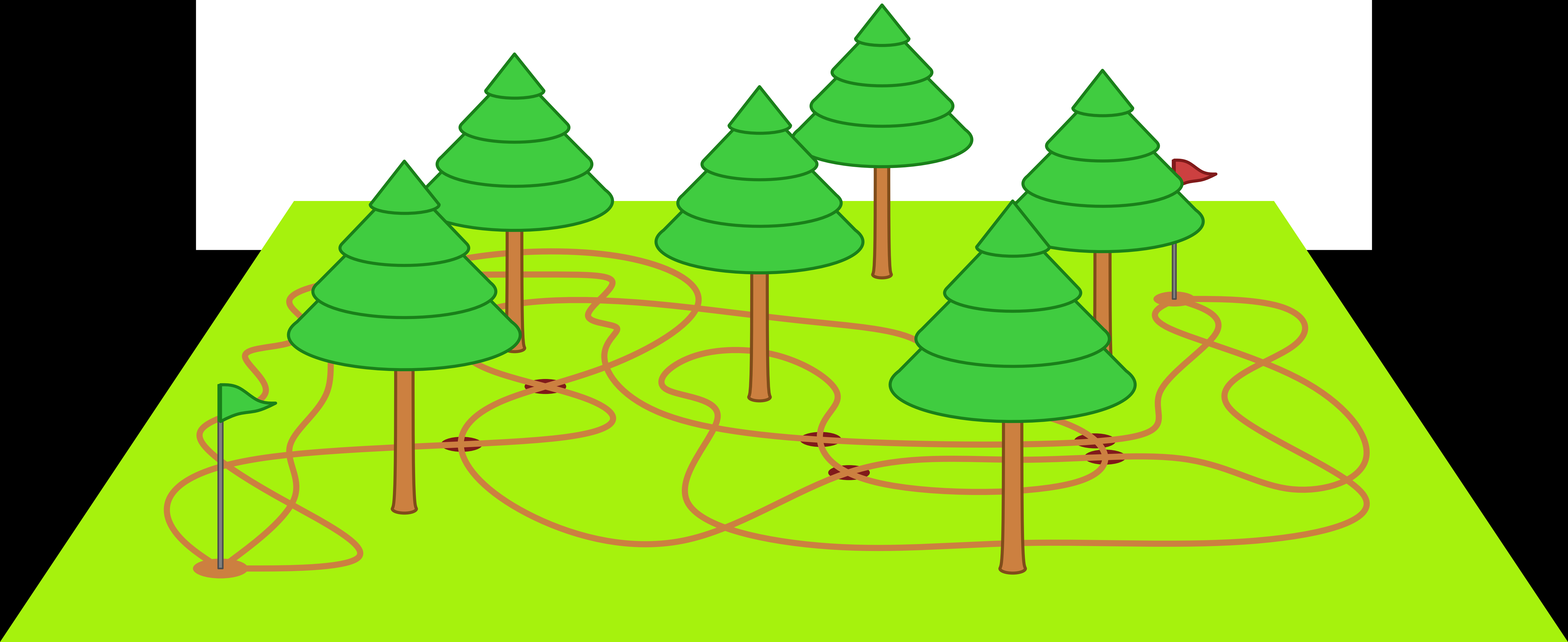
  - The *homotopic median* is just the simple median in the covering space



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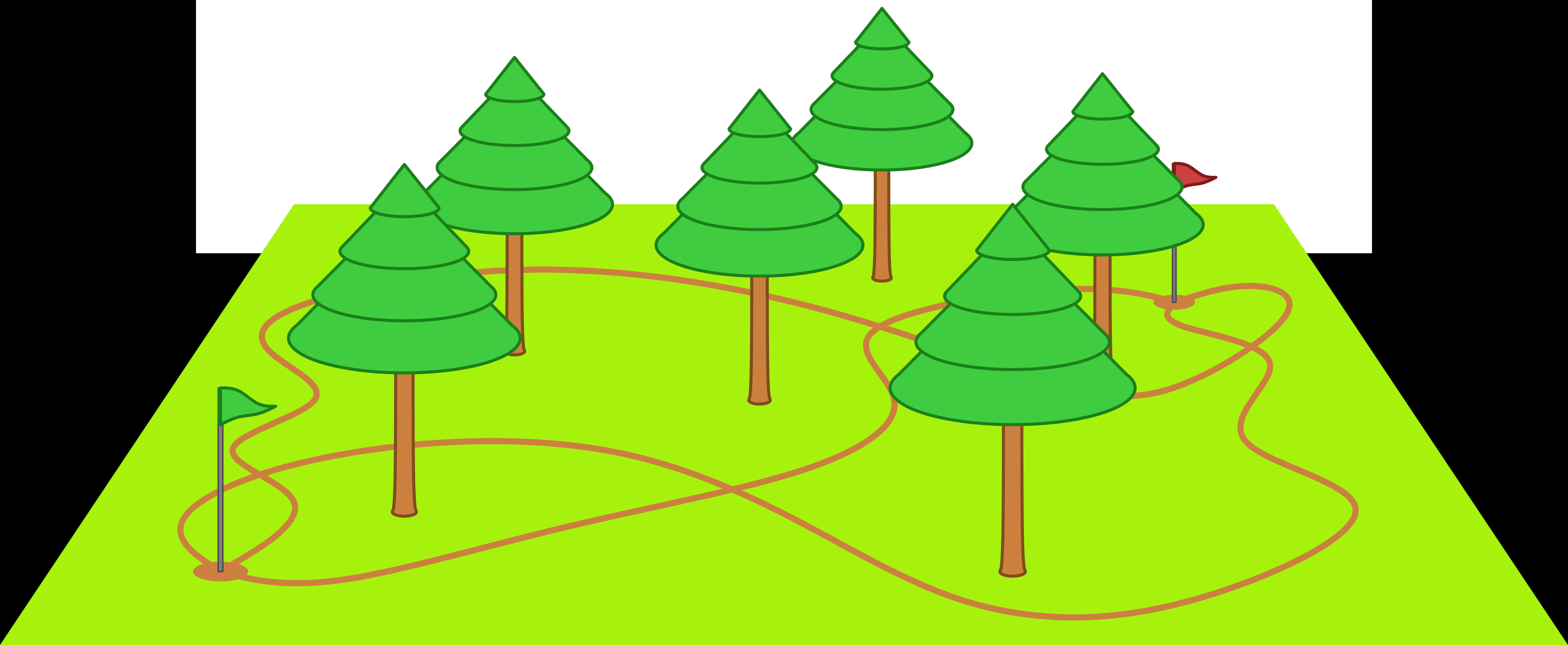

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- For non-homotopic trajectories...
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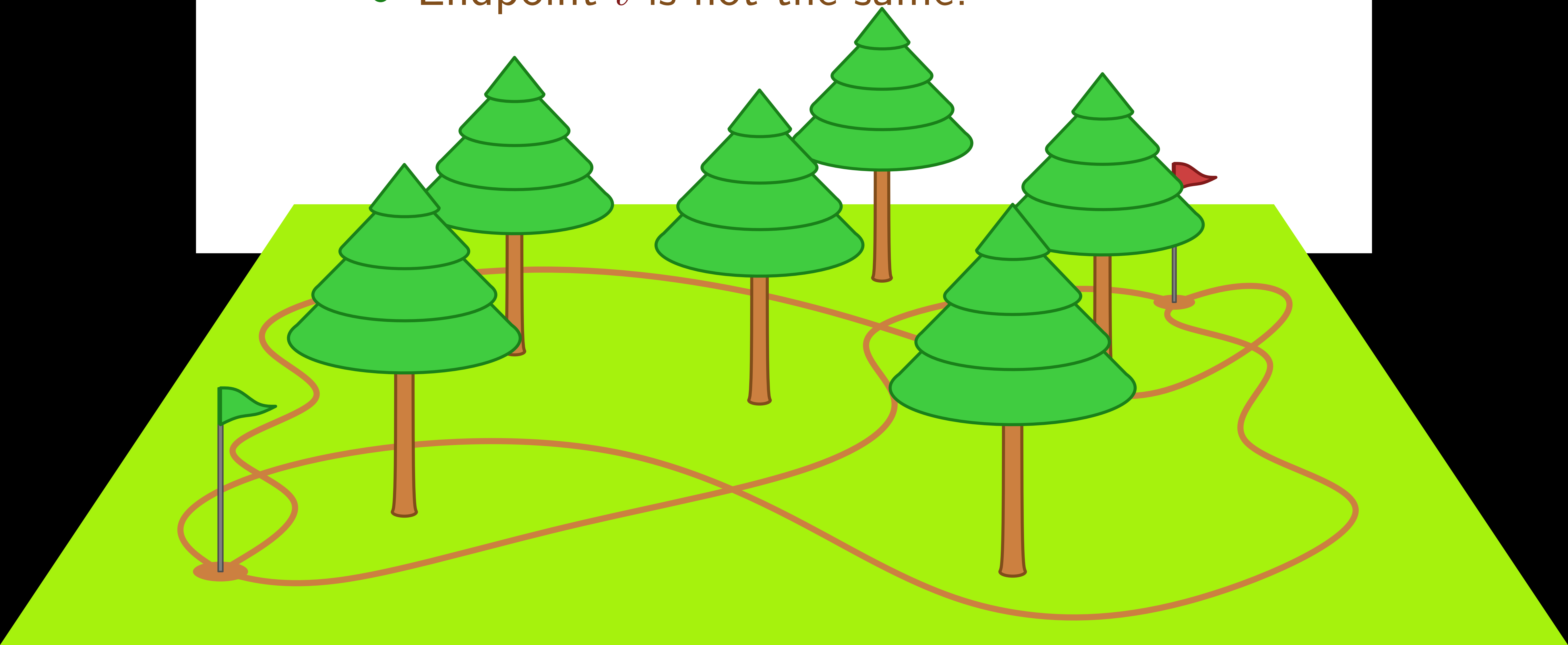

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  - It doesn't work

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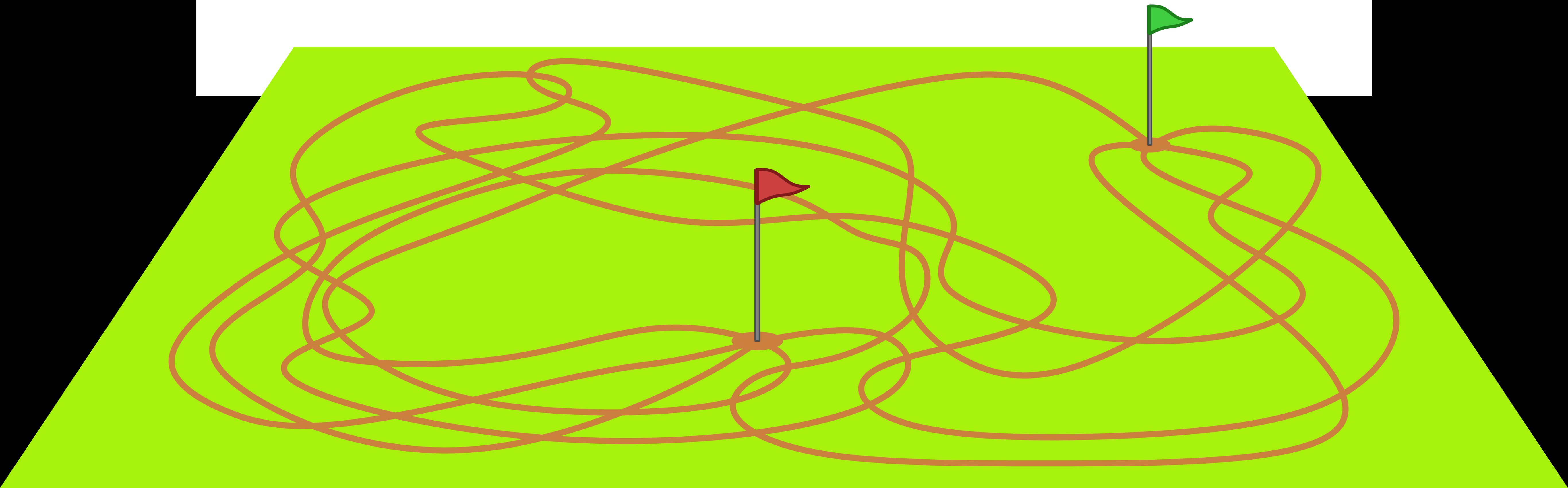

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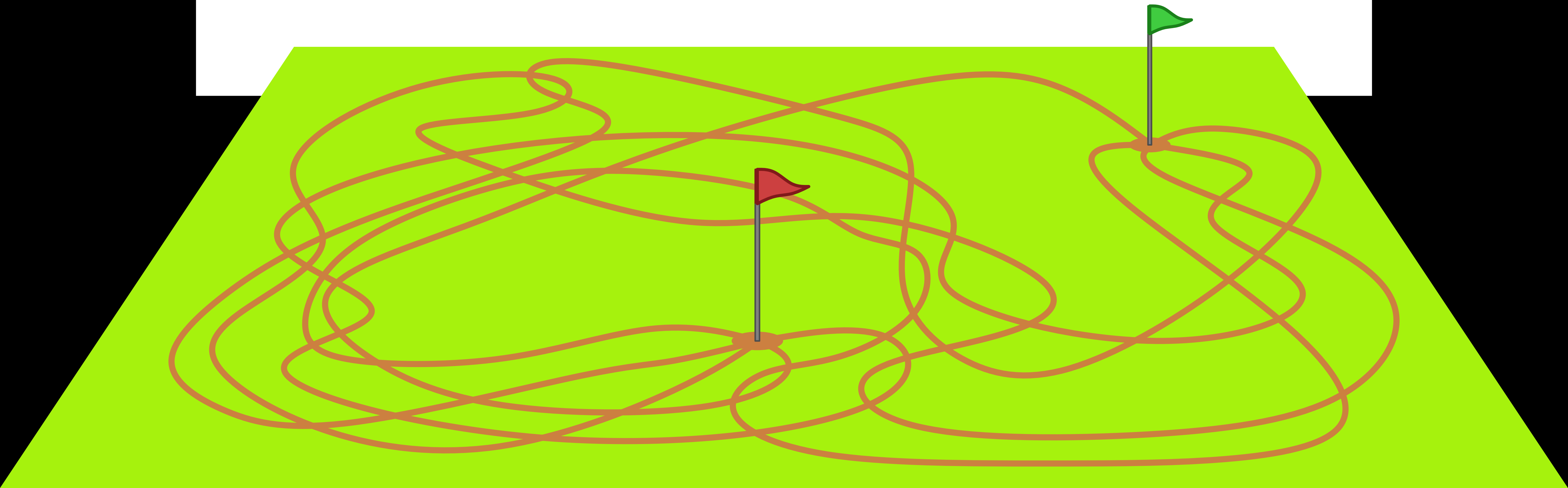
- Given a set of trajectories, can we compute a reasonable set of poles?
- Place poles in...



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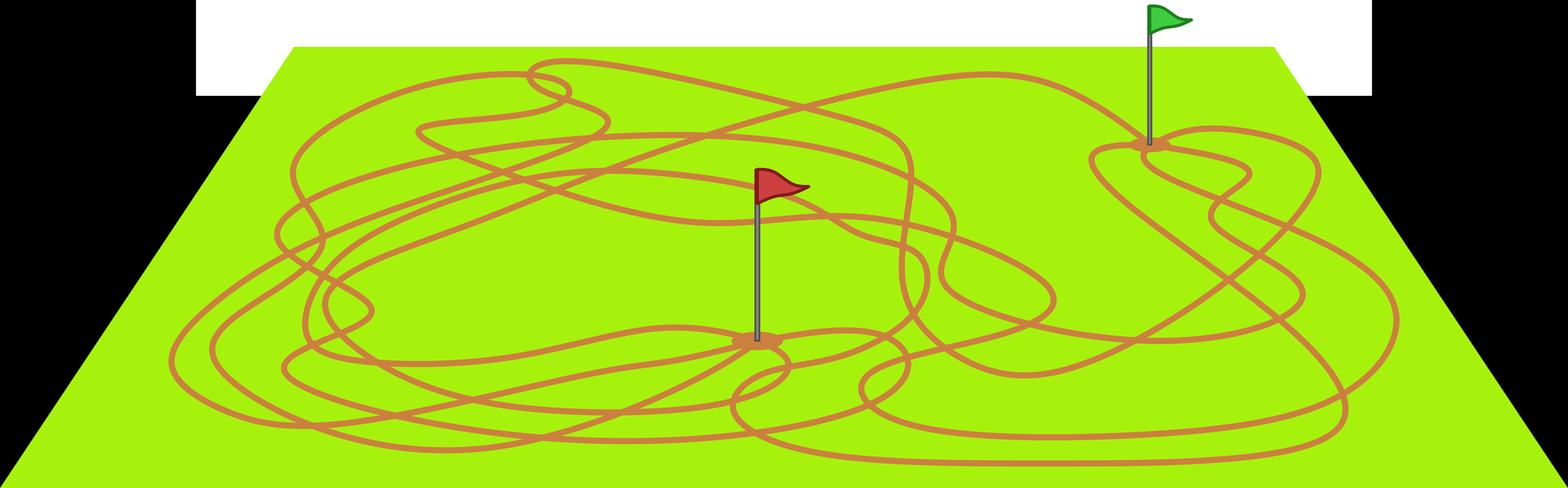

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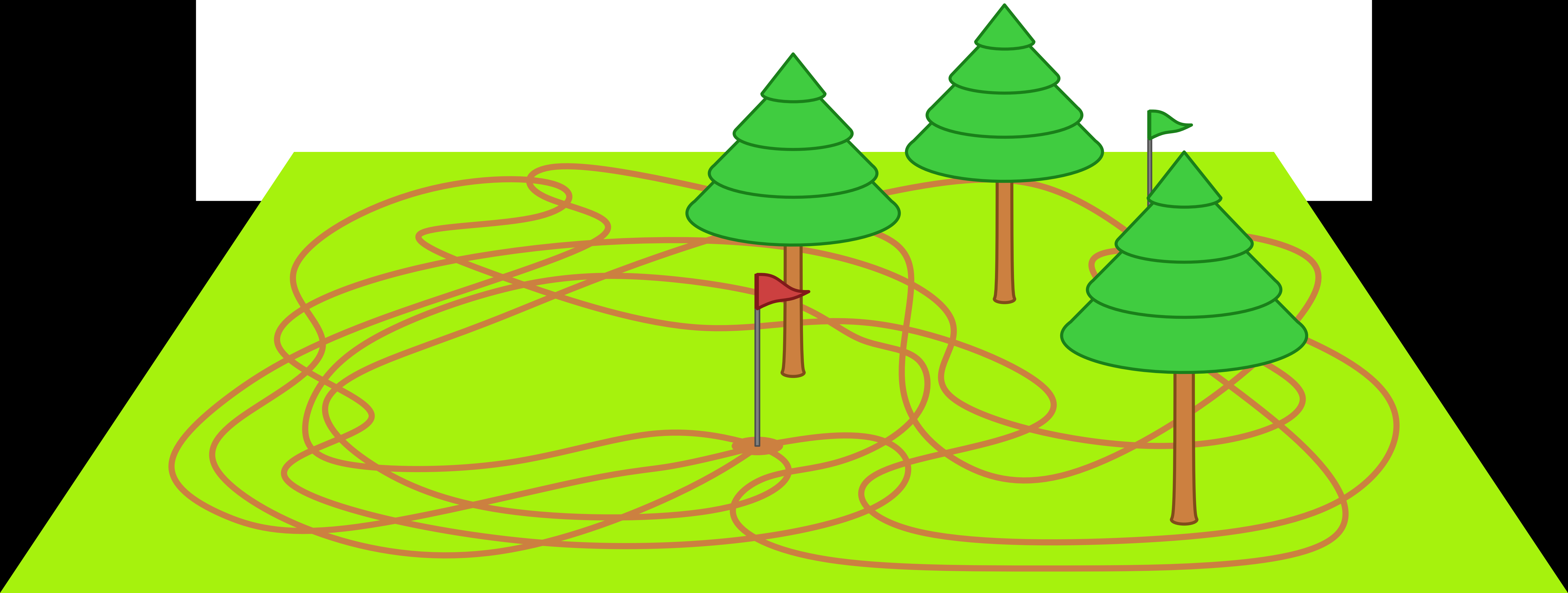

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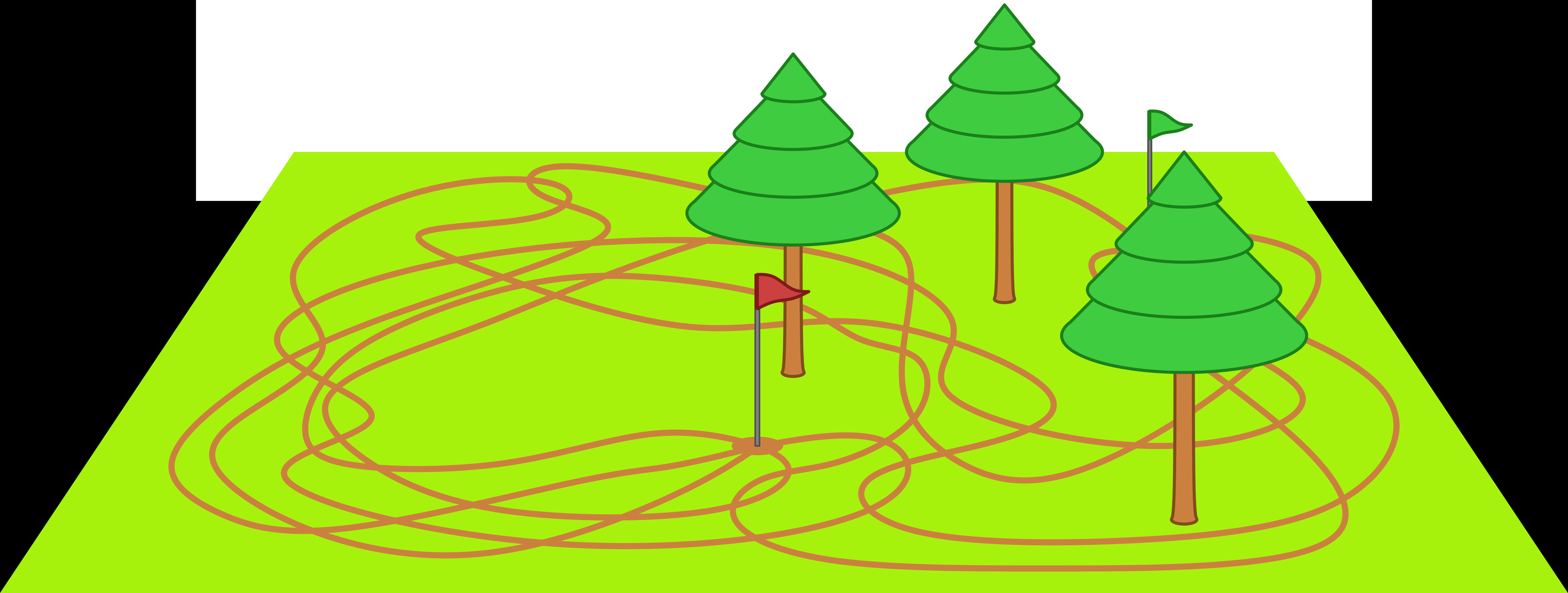

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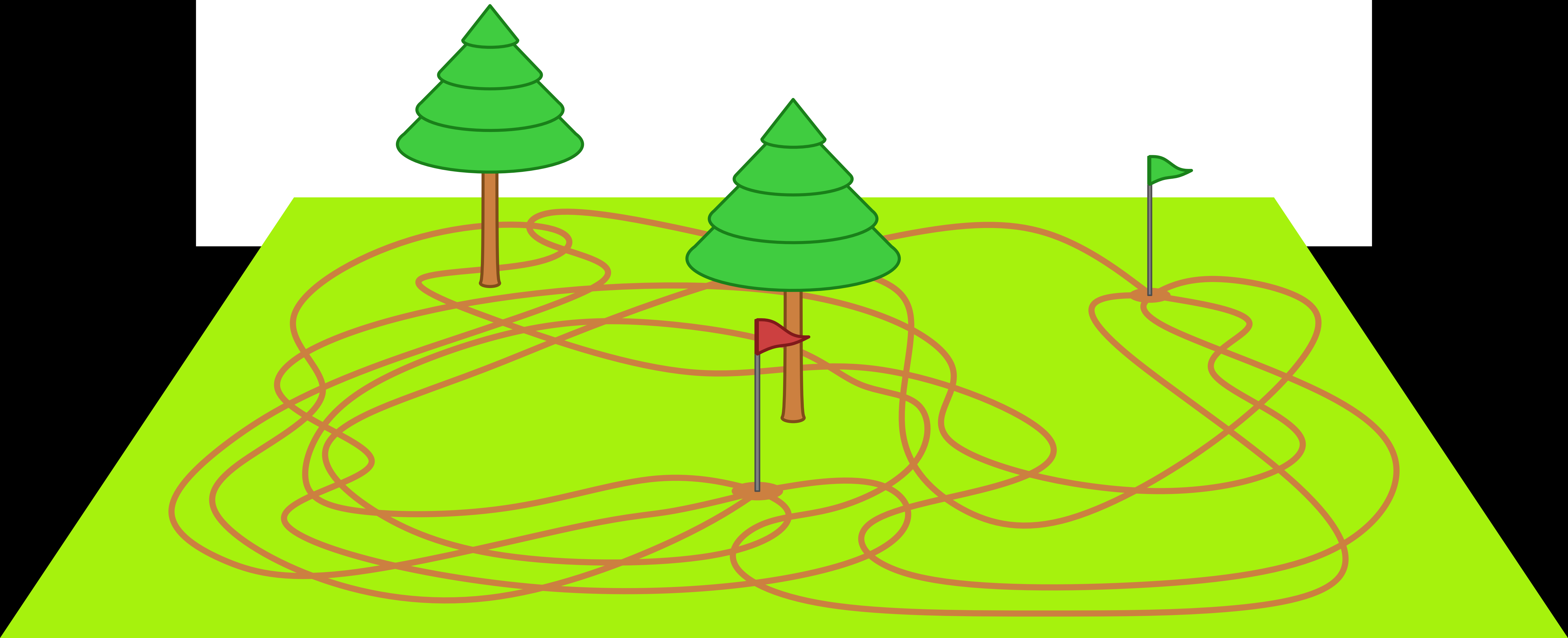

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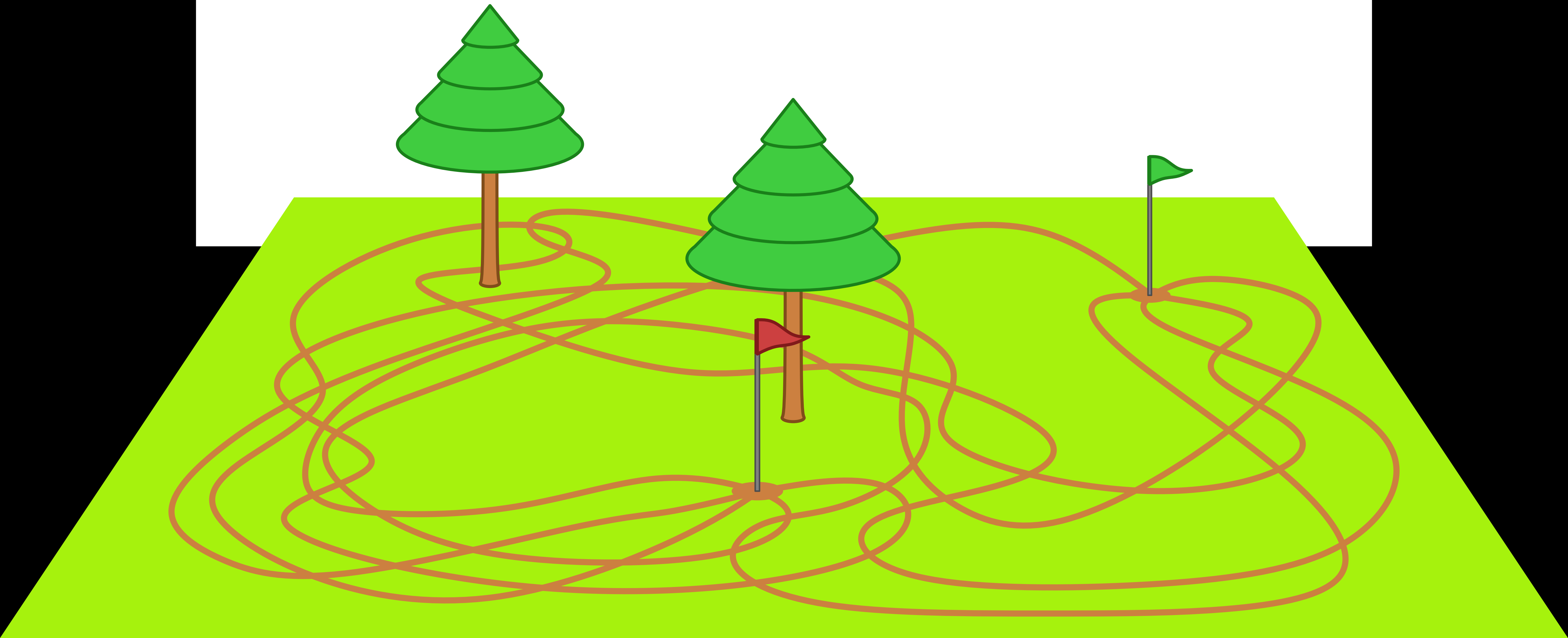

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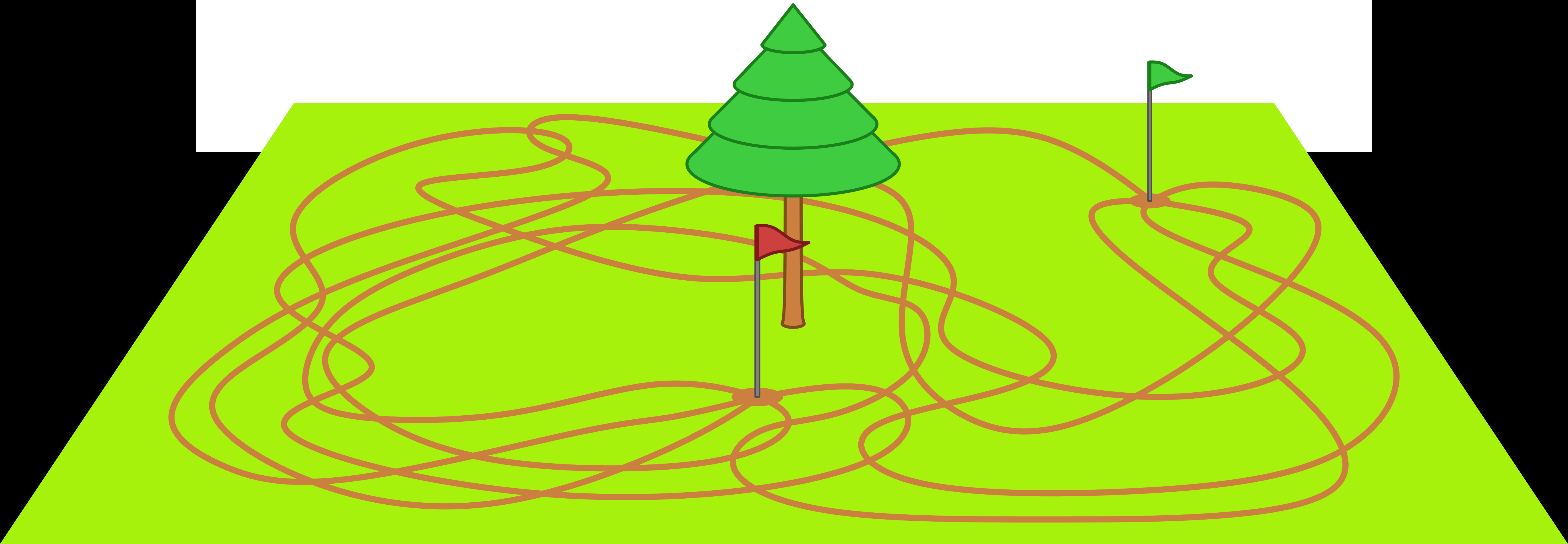

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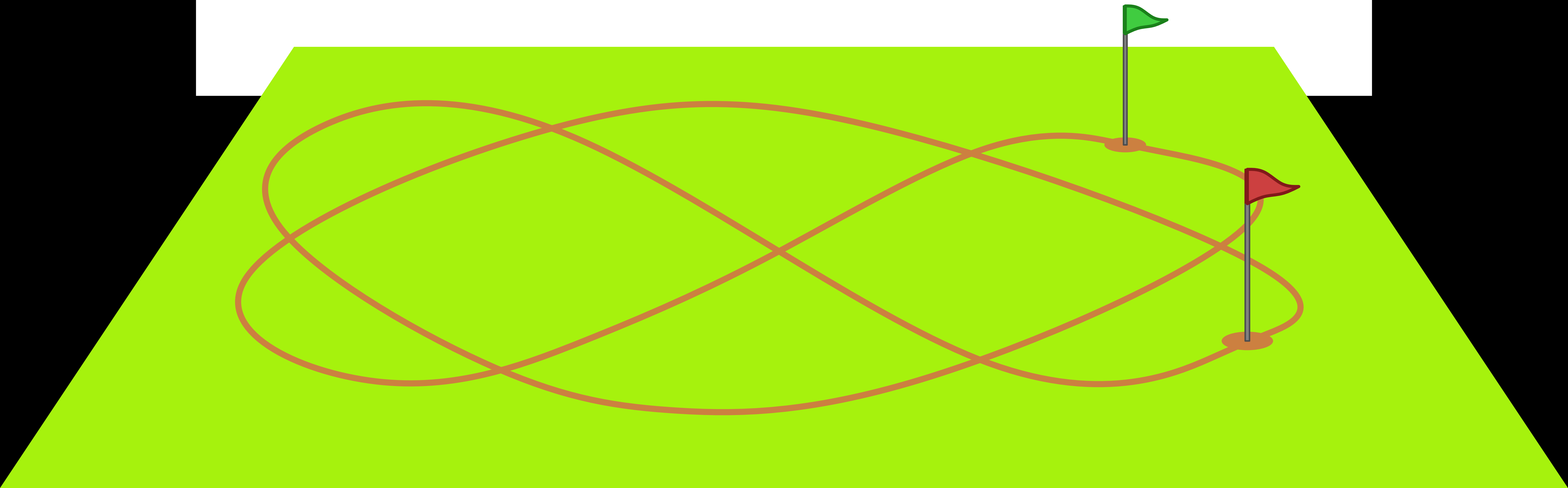

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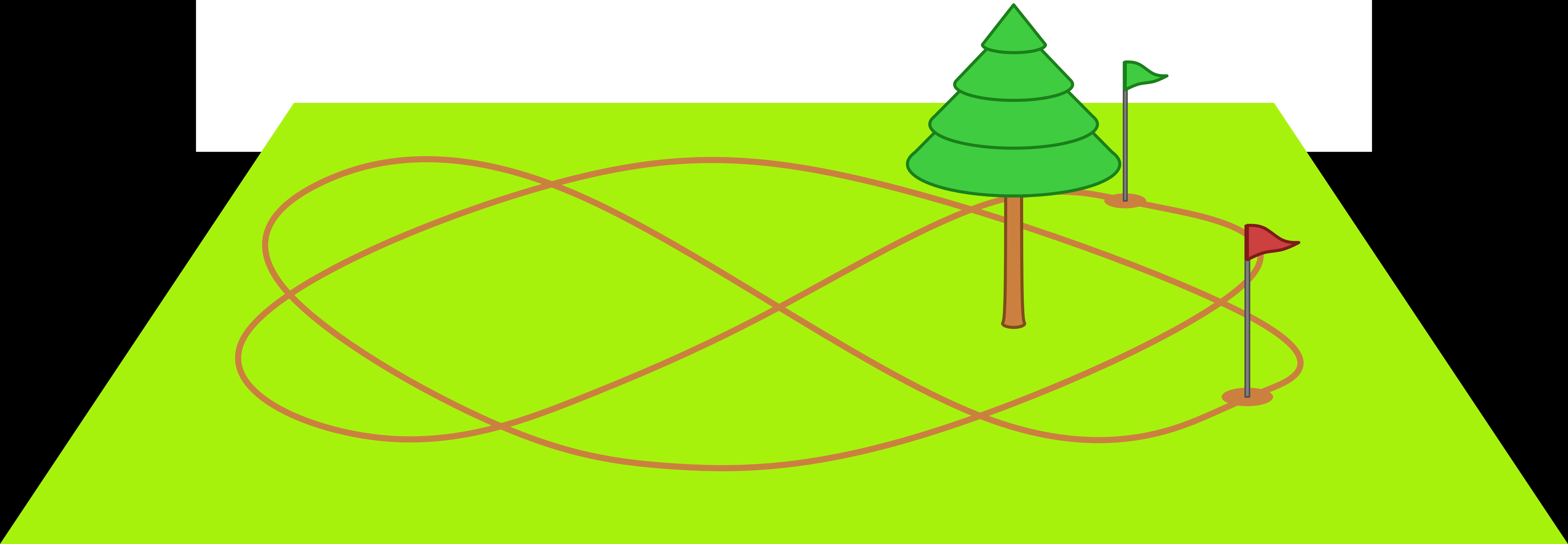

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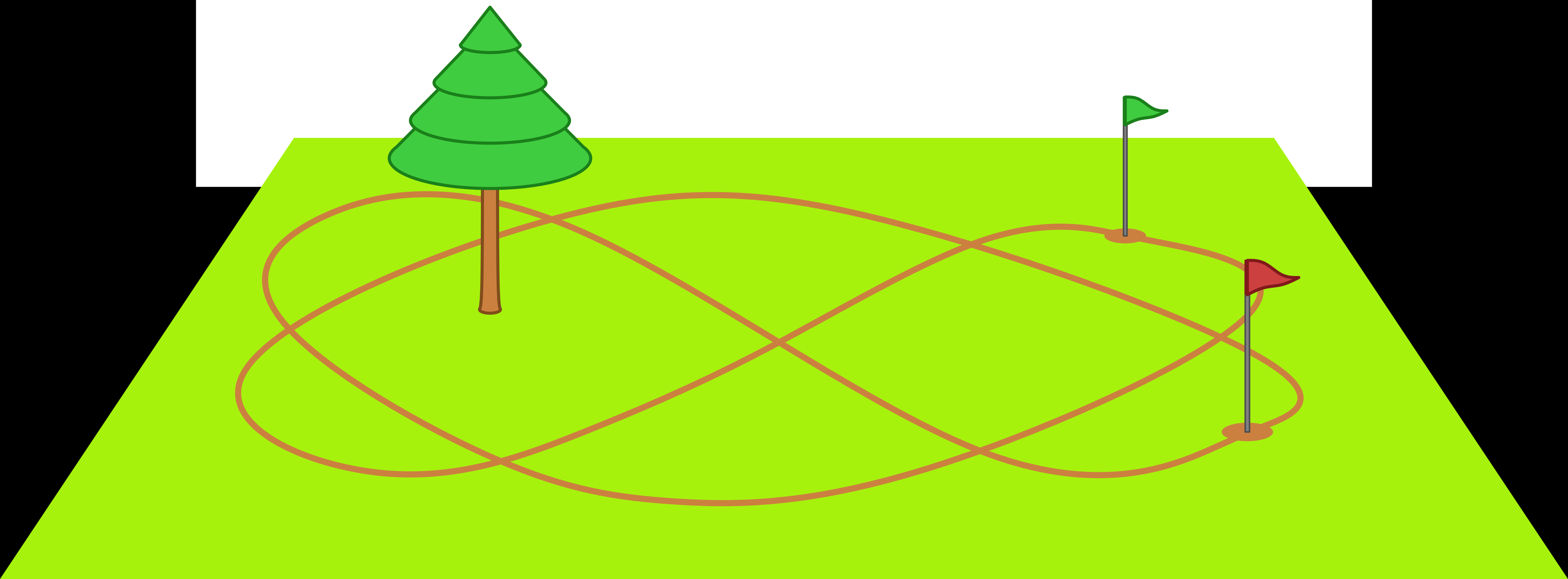

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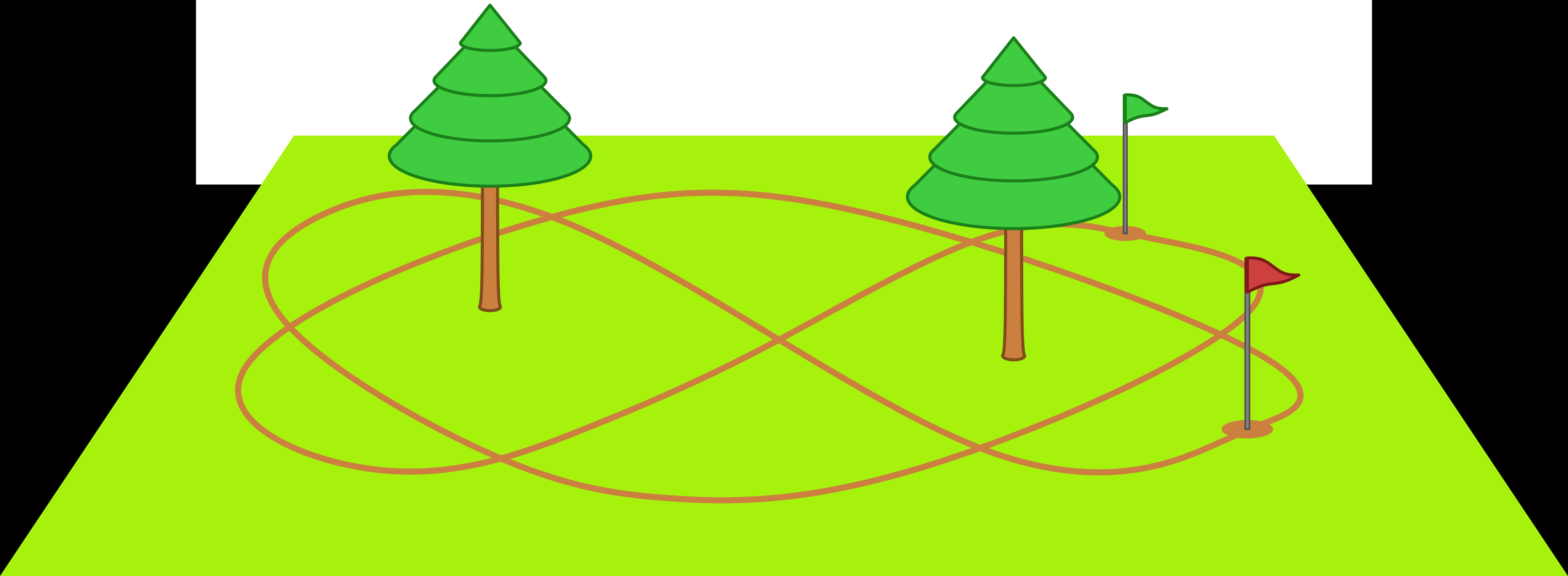

III RESULTS

COMPUTING THE MEDIAN

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- n: Number of input vertices
- h: Number of input poles
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  - $O((n\sqrt{h}+k)\alpha(n) \log n + h^{1+\varepsilon} + A)$ time

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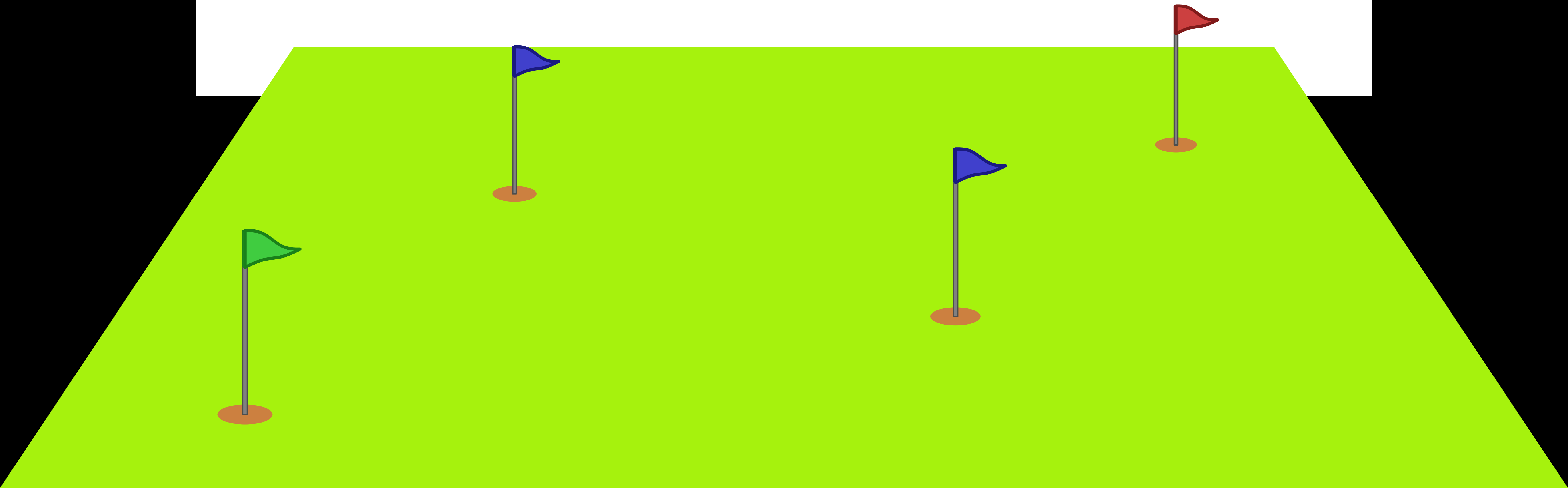

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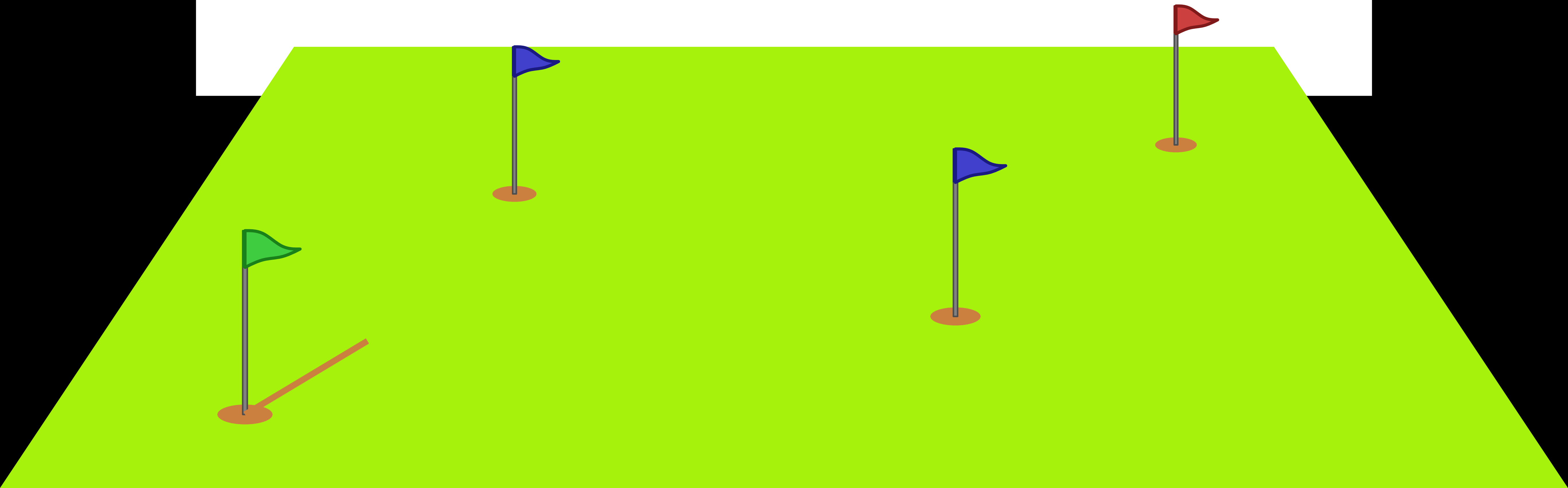

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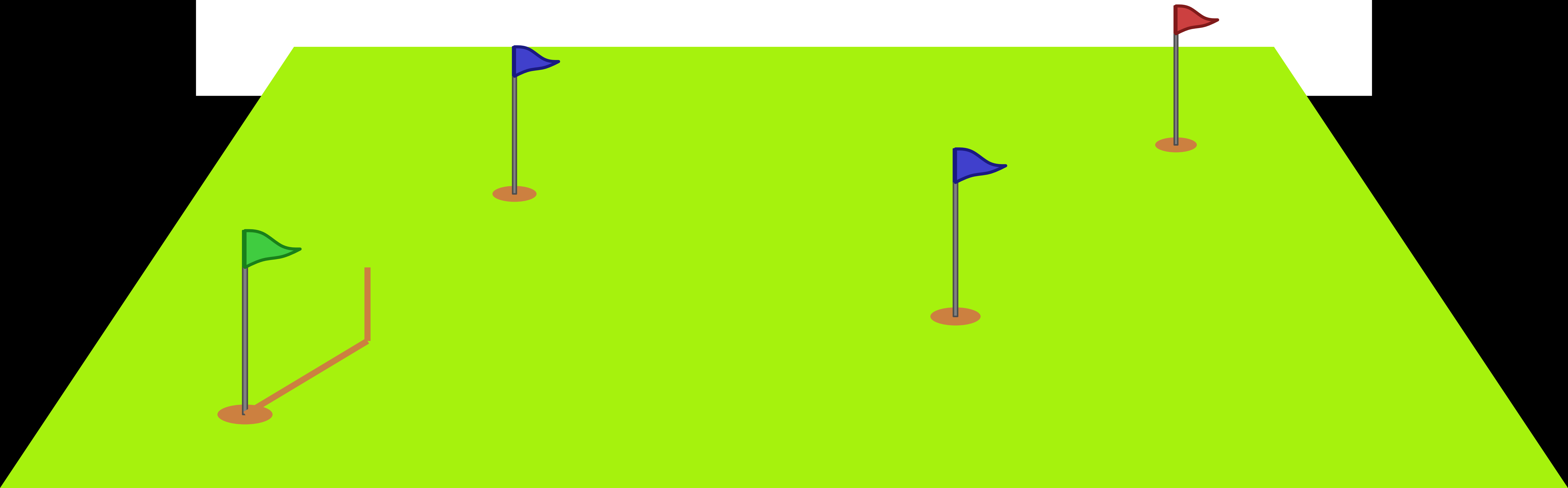

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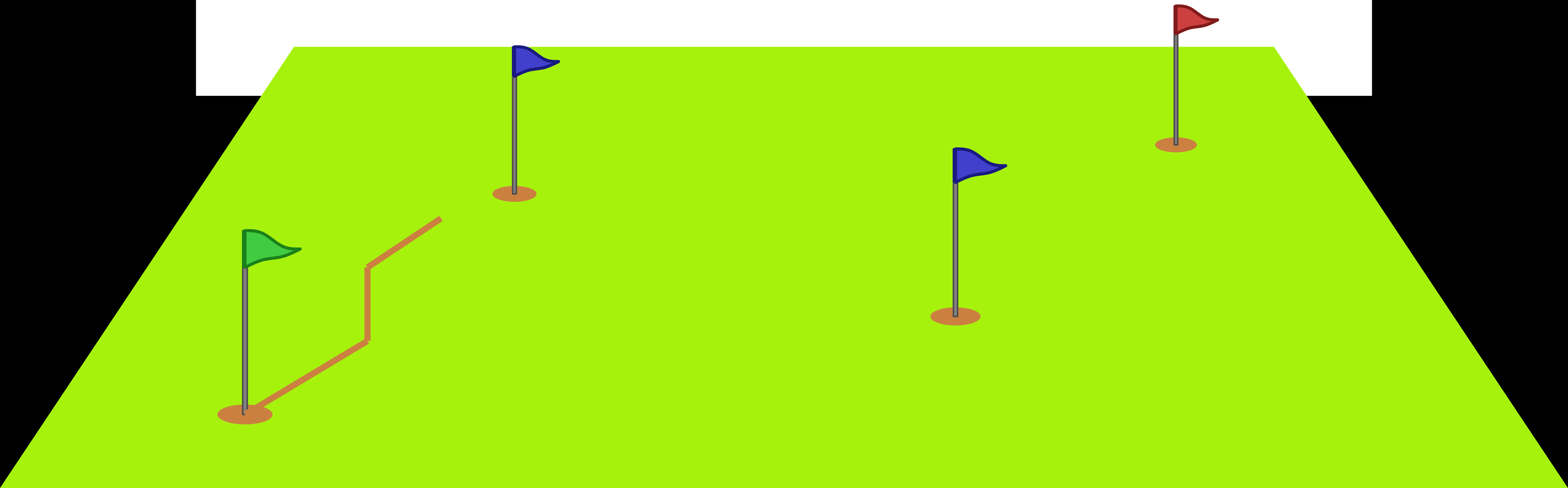

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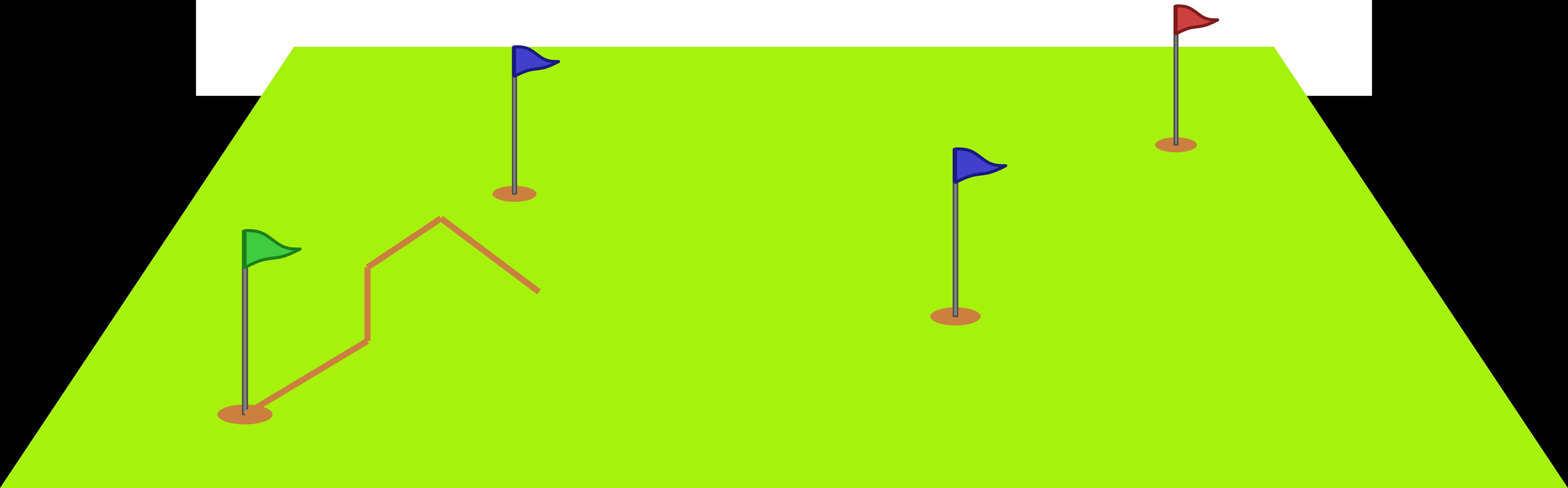

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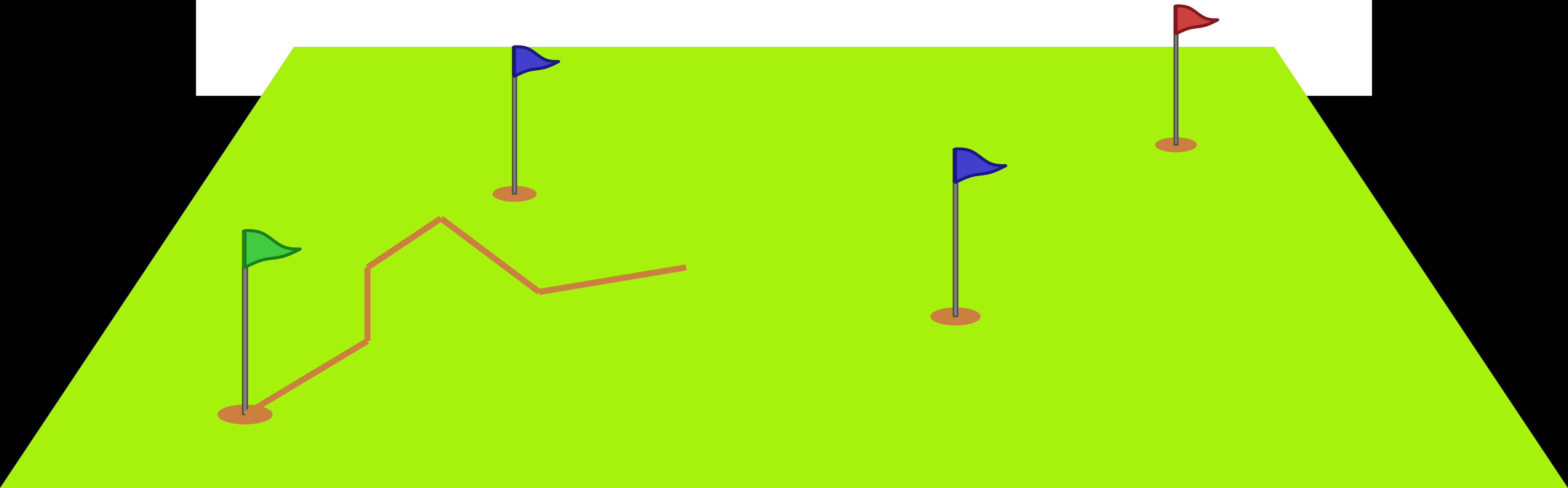

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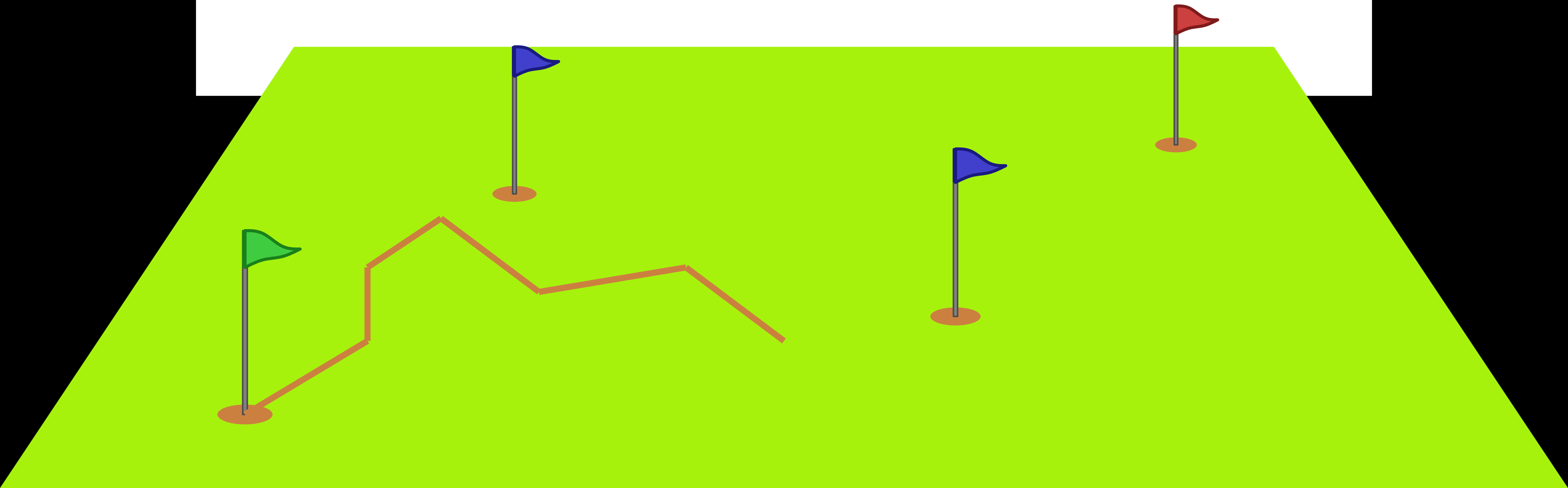

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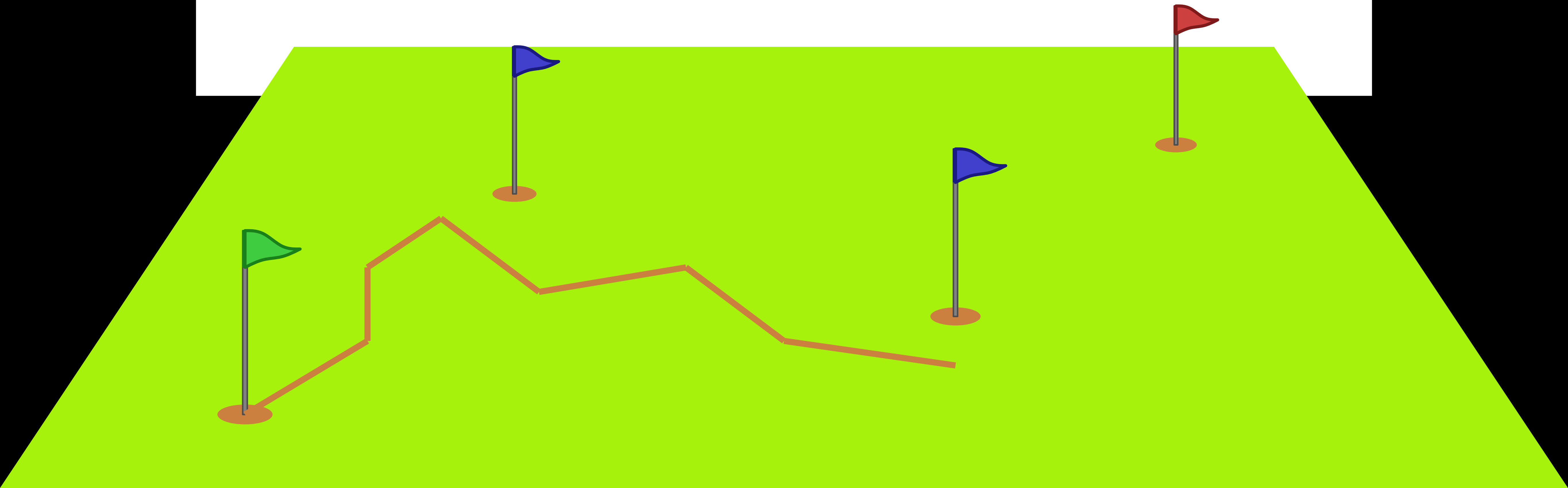

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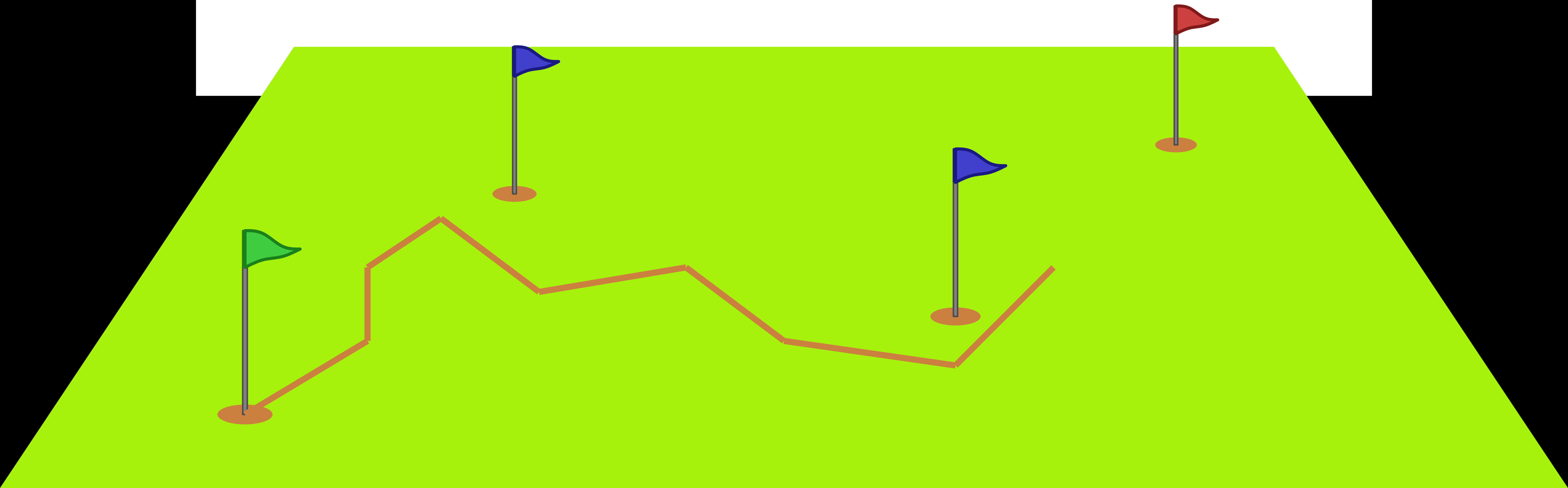

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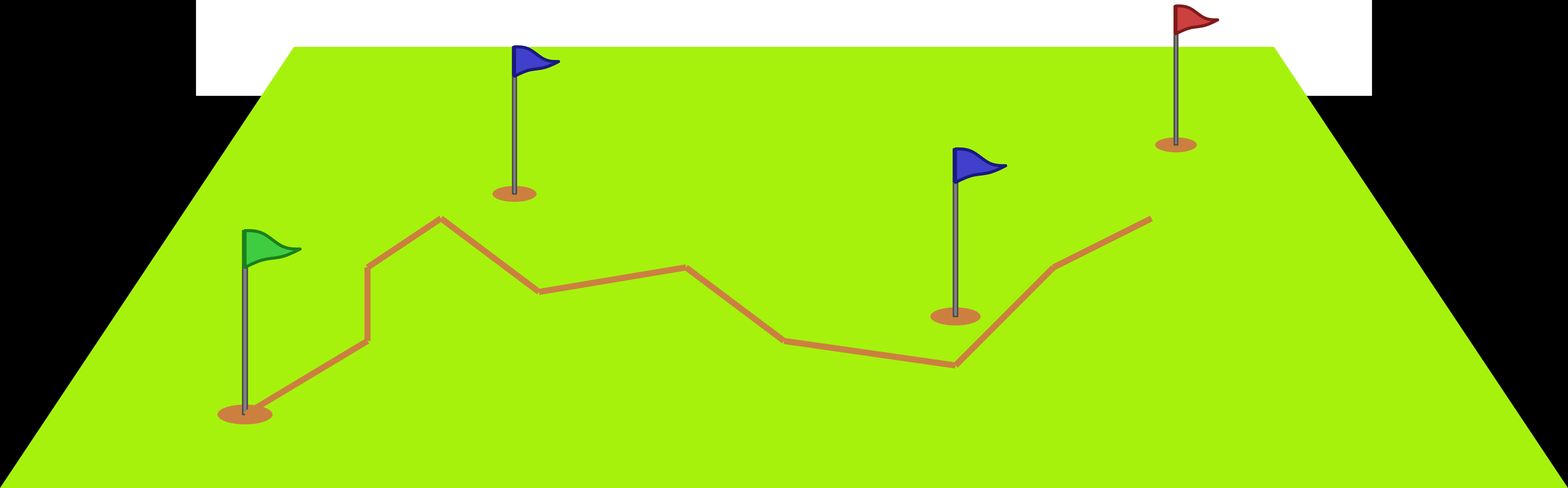

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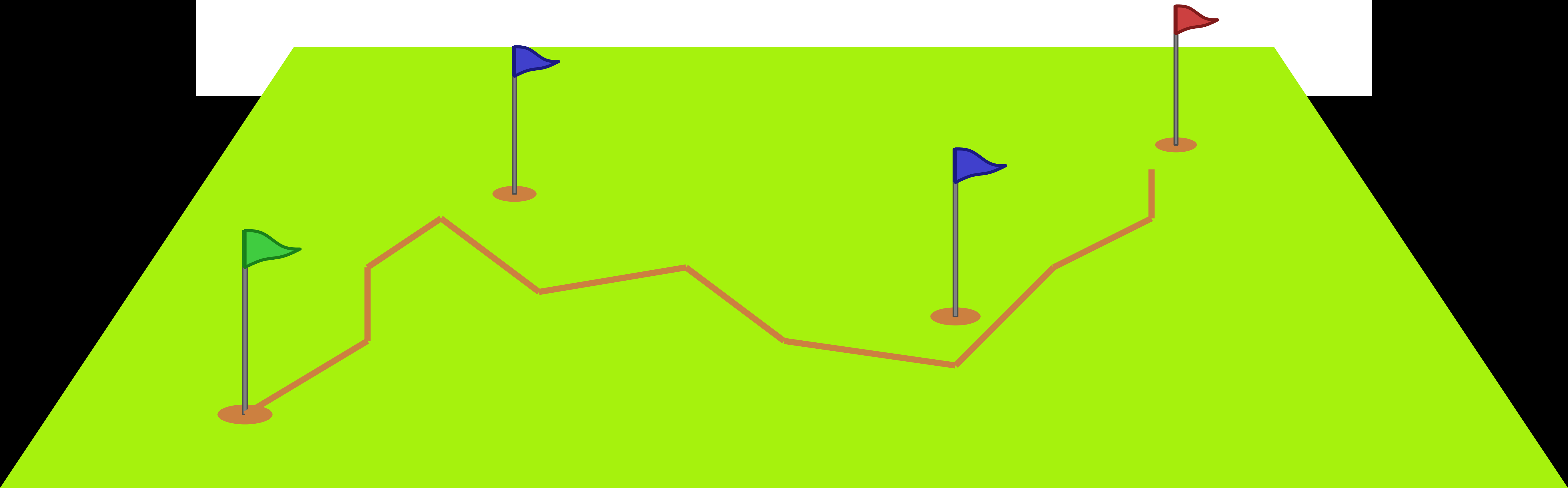

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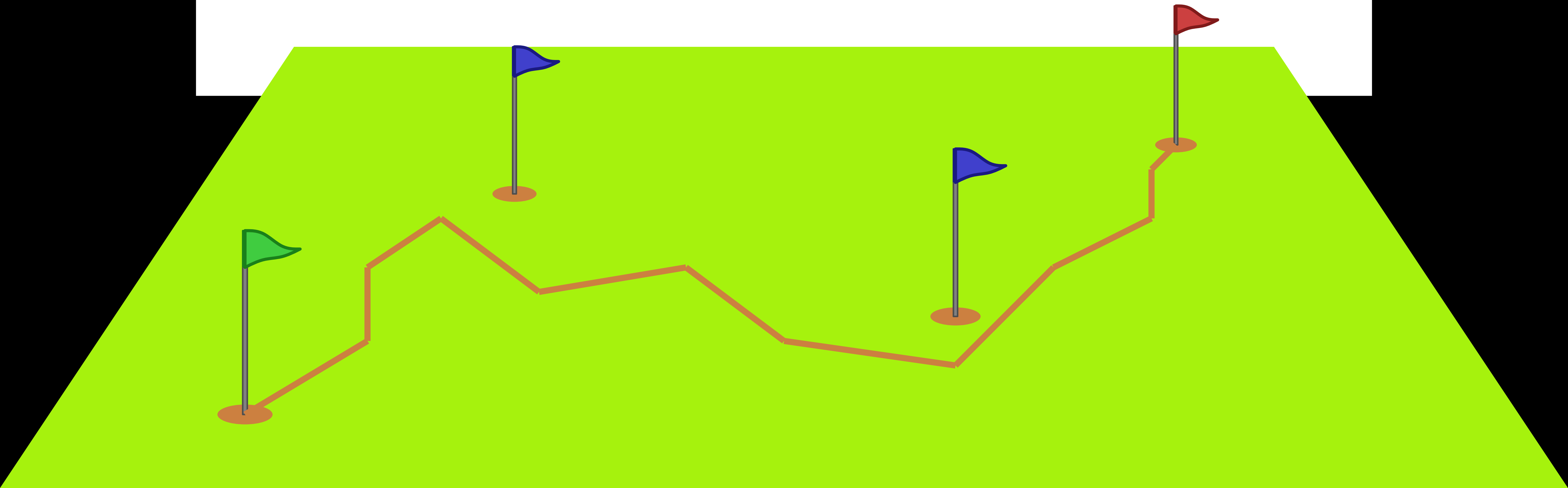

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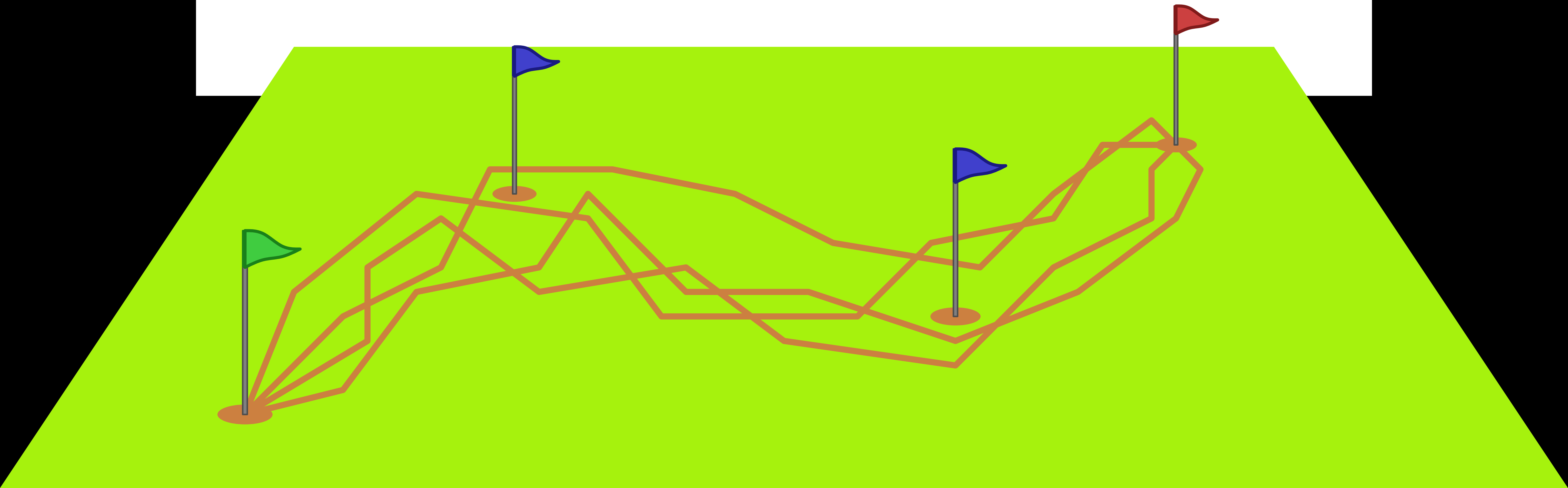
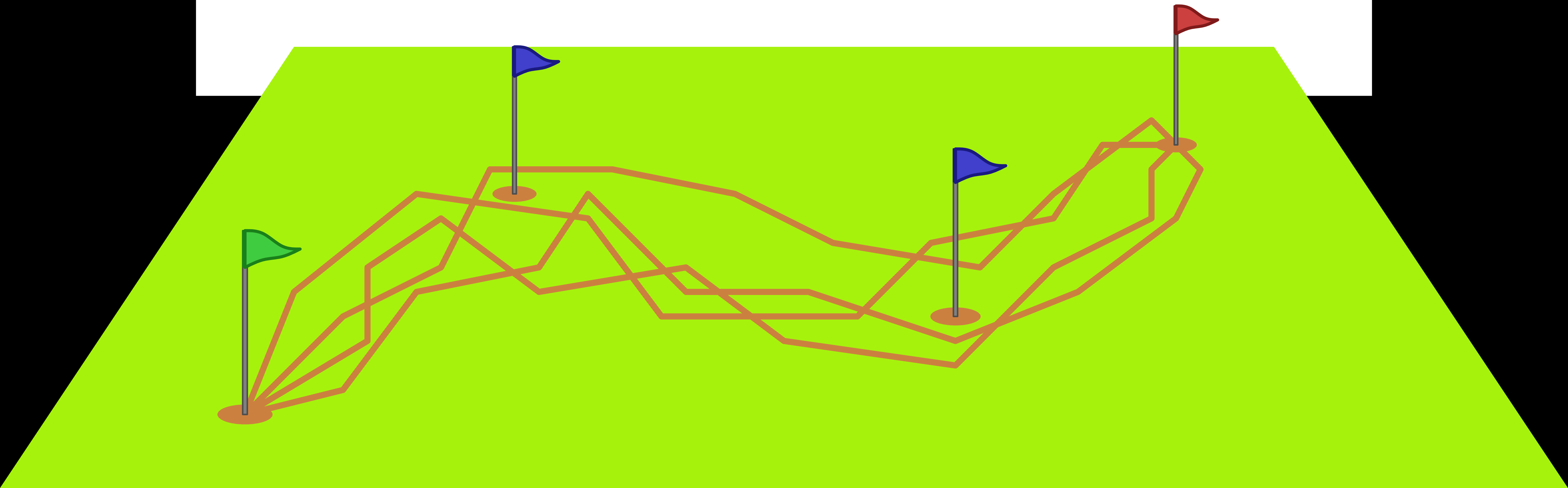


EXPERIMENTAL SETUP

- Trajectory generator
  - Random walk towards a series of waypoints
  - Simple or self-intersecting trajectories



EXPERIMENTAL SETUP

- Trajectory generator
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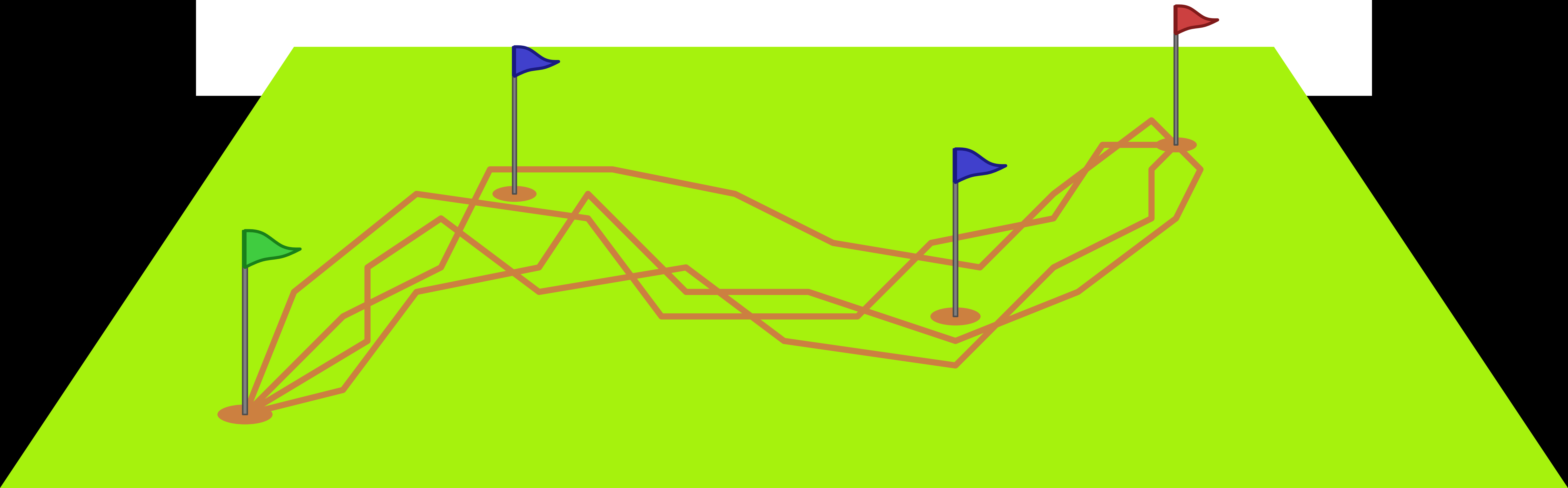
  - Simple or self-intersecting trajectories
- Pole generator



EXPERIMENTAL SETUP

- Trajectory generator
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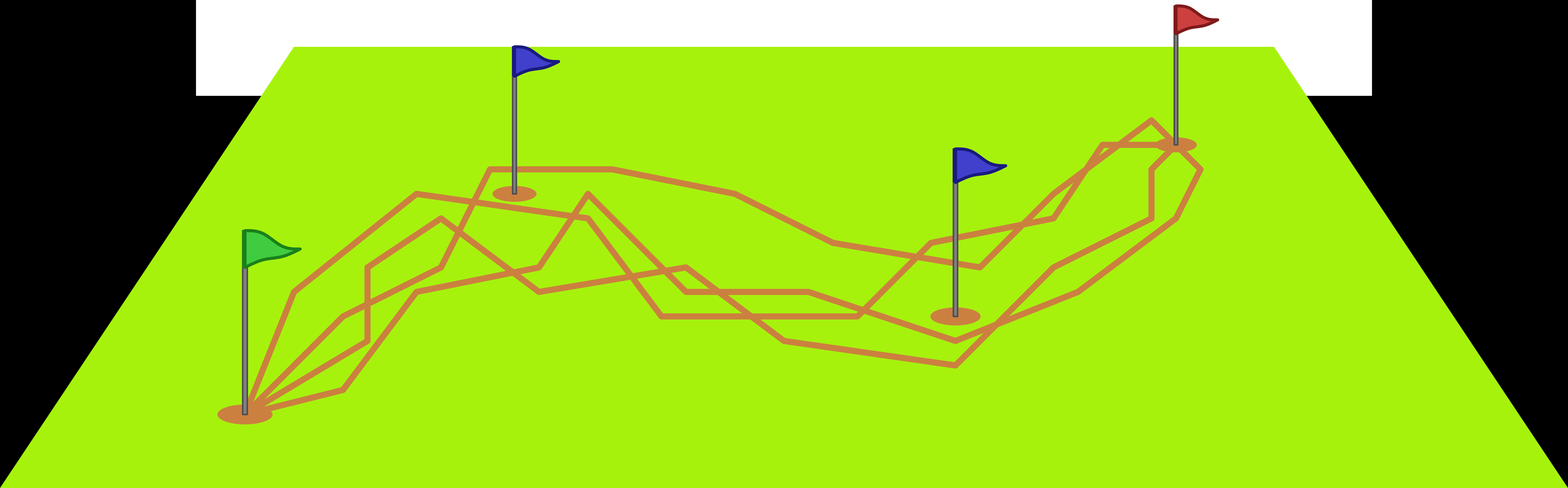
  - Simple or self-intersecting trajectories
- Pole generator
  - Places poles in faces that are large enough



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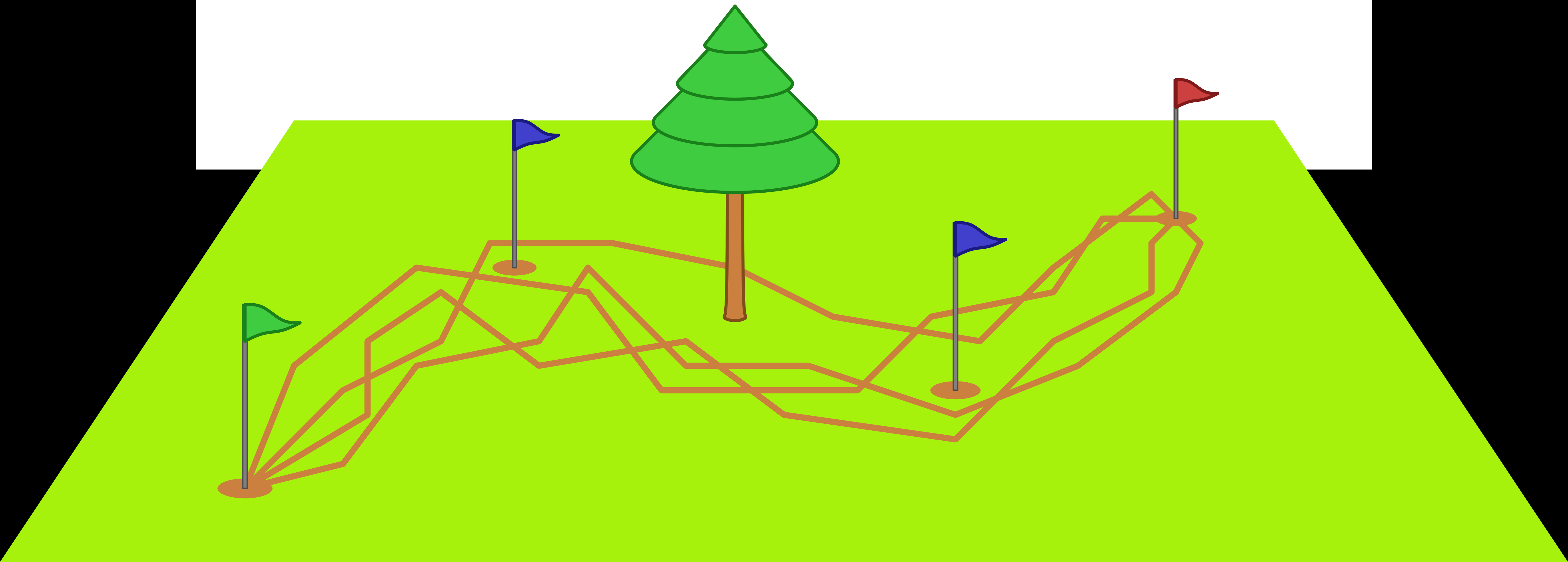
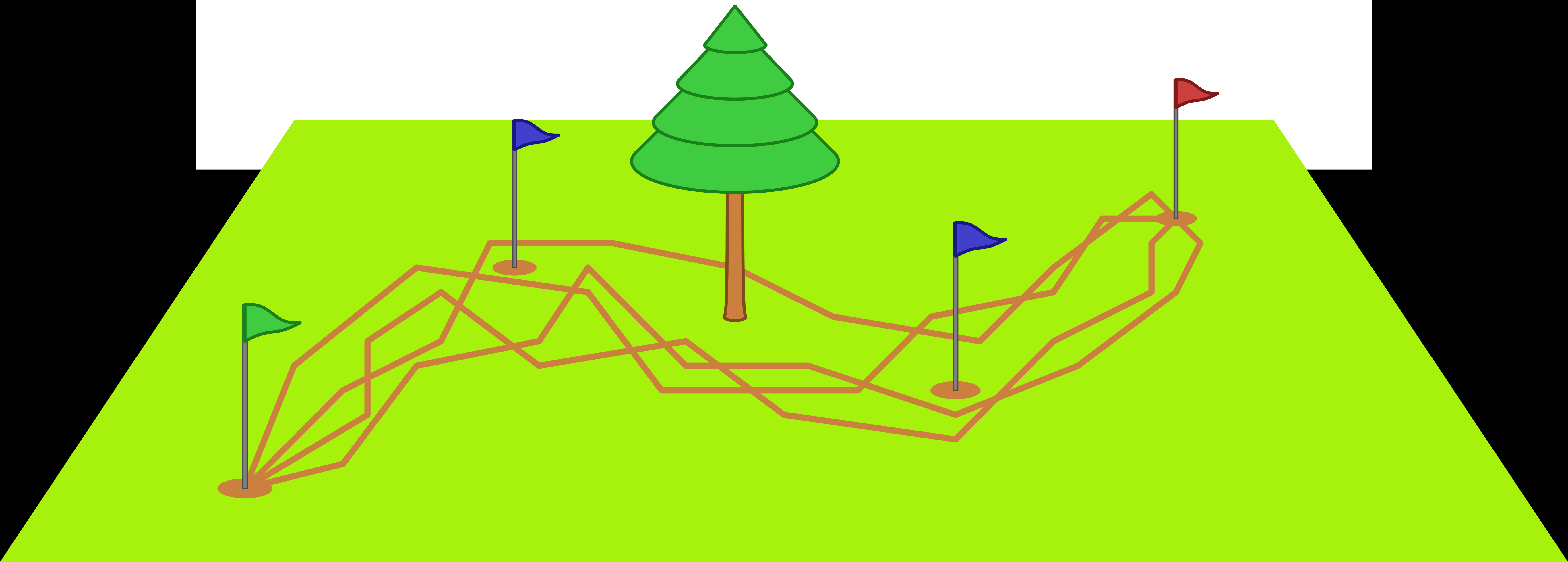
  - Places poles in faces that are large enough



EXPERIMENTAL SETUP

- Trajectory generator
  - Random walk towards a series of waypoints
  - Simple or self-intersecting trajectories
- Pole generator
  - Places poles in faces that are large enough
  - Discards non-homotopic trajectories



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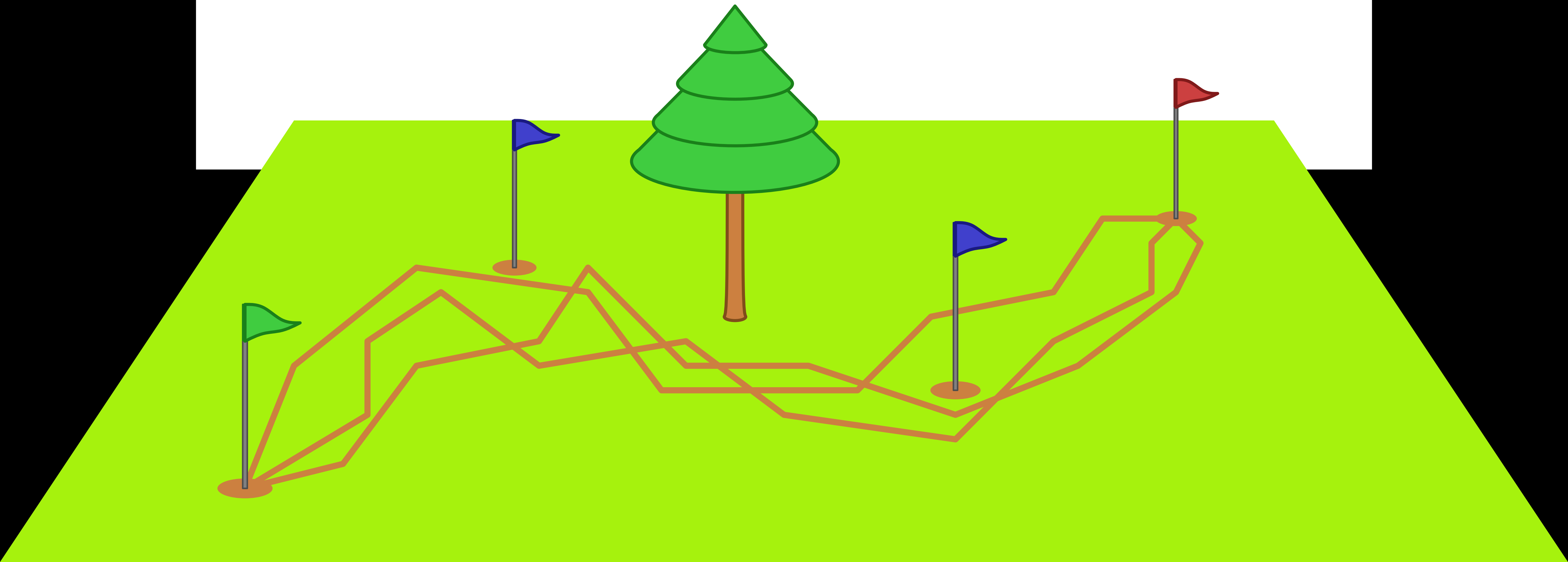
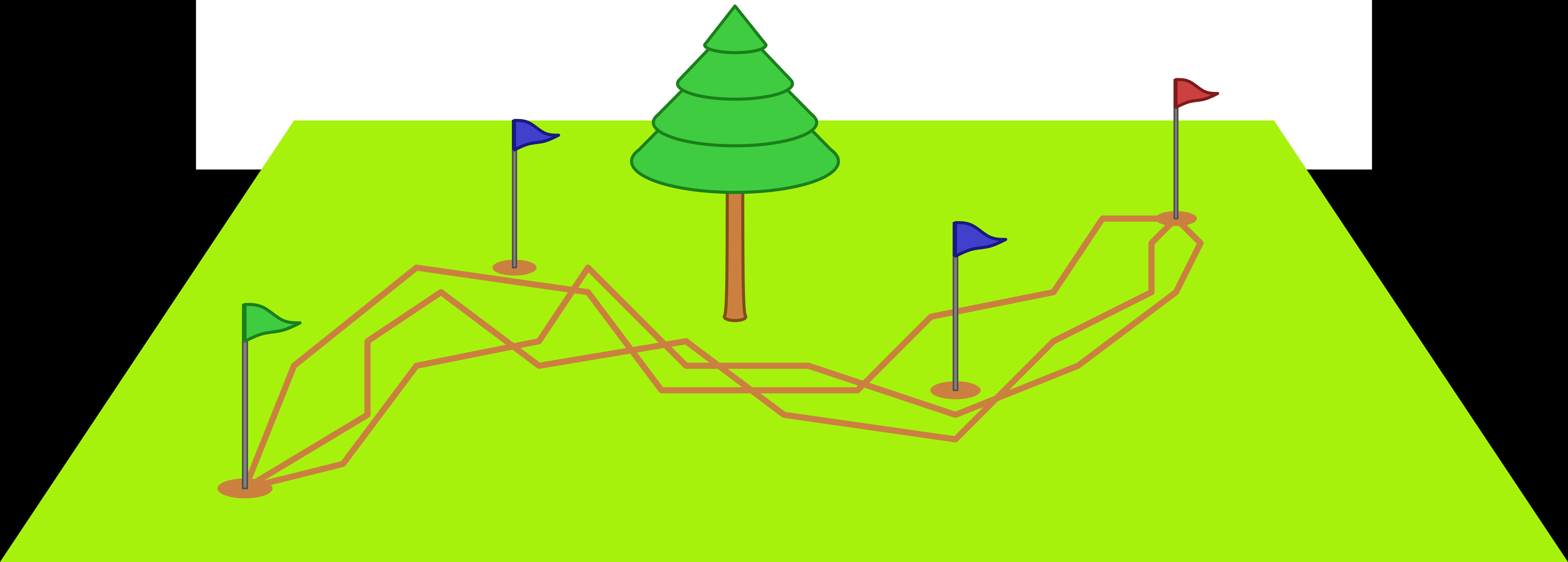


EXPERIMENTAL SETUP

- Trajectory generator
  - Random walk towards a series of waypoints
  - Simple or self-intersecting trajectories
- Pole generator
  - Places poles in faces that are large enough
  - Discards non-homotopic trajectories
- Measures of interest



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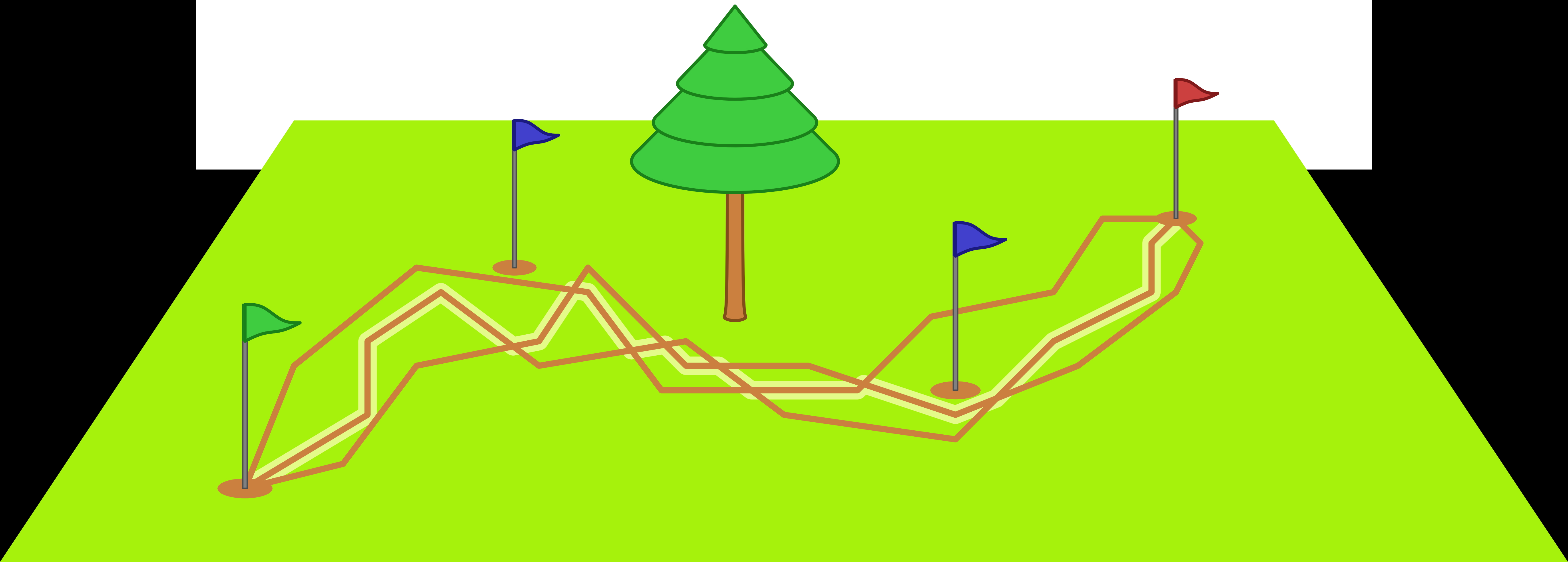
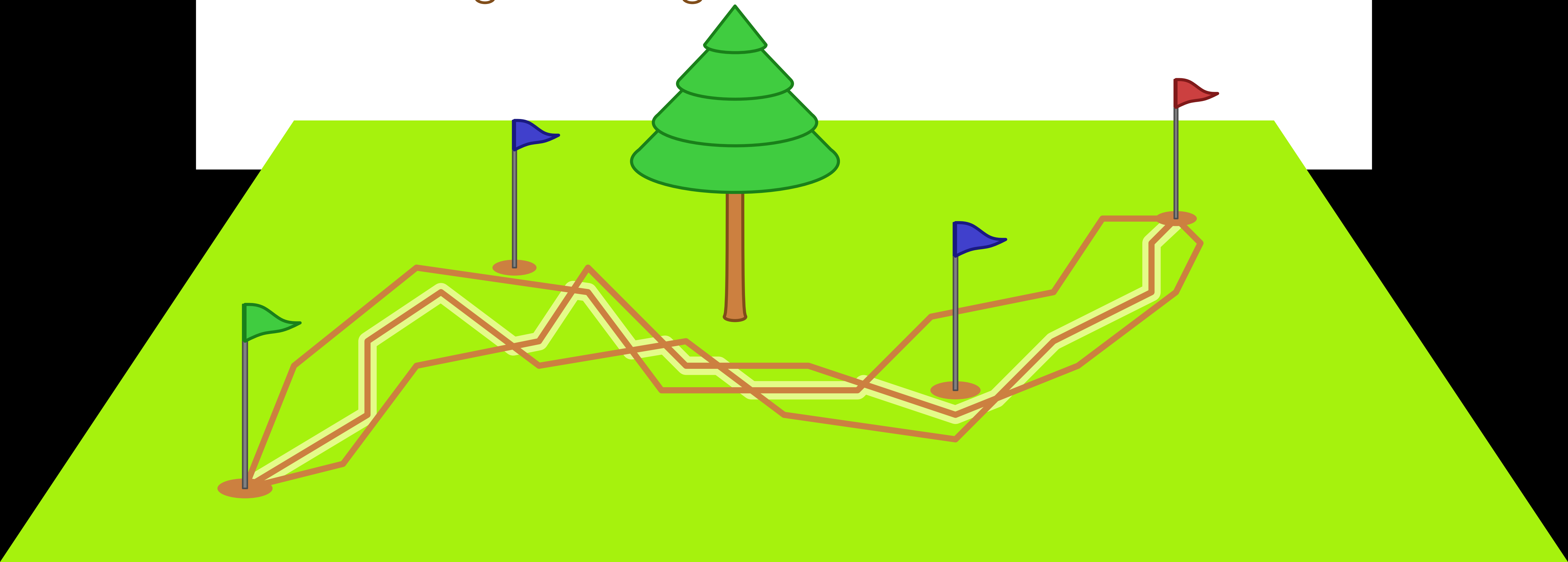


EXPERIMENTAL SETUP

- Trajectory generator
  - Random walk towards a series of waypoints
  - Simple or self-intersecting trajectories
- Pole generator
  - Places poles in faces that are large enough
  - Discards non-homotopic trajectories
- Measures of interest
  - Angular change

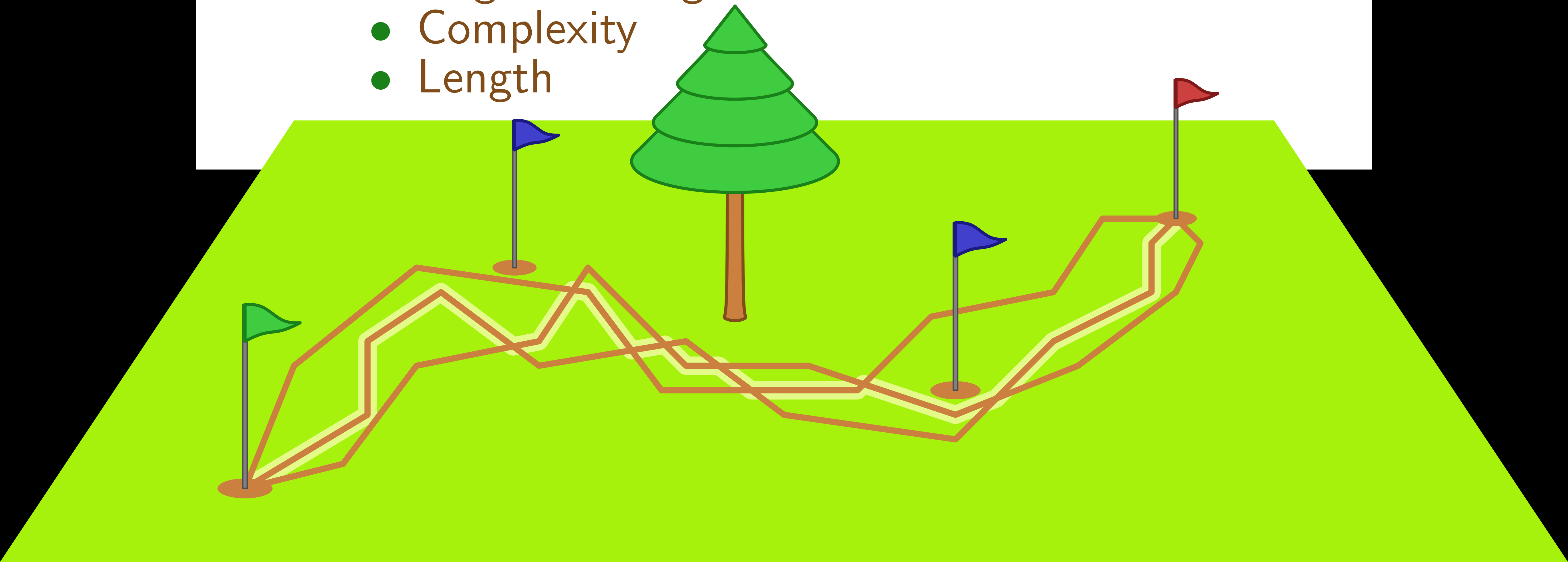


EXPERIMENTAL SETUP

- Trajectory generator
  - Random walk towards a series of waypoints
  - Simple or self-intersecting trajectories
- Pole generator
  - Places poles in faces that are large enough
  - Discards non-homotopic trajectories
- Measures of interest
  - Angular change
  - Complexity

EXPERIMENTAL SETUP

- Trajectory generator
  - Random walk towards a series of waypoints
  - Simple or self-intersecting trajectories
- Pole generator
  - Places poles in faces that are large enough
  - Discards non-homotopic trajectories
- Measures of interest
  - Angular change
  - Complexity
  - Length



EXPERIMENTAL RESULTS

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	S	H
• Complexity		
• No self-intersections	345%	319%
• Self-intersections	207%	343%

EXPERIMENTAL RESULTS

	S	H
Complexity		
• No self-intersections	345%	319%
• Self-intersections	207%	343%
Length		
• No self-intersections	96%	99%
• Self-intersections	51%	96%

EXPERIMENTAL RESULTS

	S	H
• Complexity		
• No self-intersections	345%	319%
• Self-intersections	207%	343%
• Length		
• No self-intersections	96%	99%
• Self-intersections	51%	96%
• Angular change		
• No self-intersections	539%	468%
• Self-intersections	381%	466%

CONCLUSION

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- This work

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  - Two definitions for median trajectory
  - Efficient algorithms for computing them
  - Quantitative evaluation on generated data

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CONCLUSIONS

- This work
  - Two definitions for median trajectory
  - Efficient algorithms for computing them
  - Quantitative evaluation on generated data
- Future work
  - More intelligent automatic pole placement?
  - Evaluation on real-world data?
  - Understand better what makes a us accept a certain curve as a good median

THANK YOU!