COMPETITIVE QUERY STRATEGIES FOR MINIMISING THE PLY OF THE POTENTIAL LOCATIONS OF MOVING POINTS

Will Evans
David Kirkpatrick
Frank Staals
Maarten Löffler

A BRIEF ELUCIDATION OF THE TITLE

“MOVING POINTS”

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- Location-aware mobile devices

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  - GPS systems

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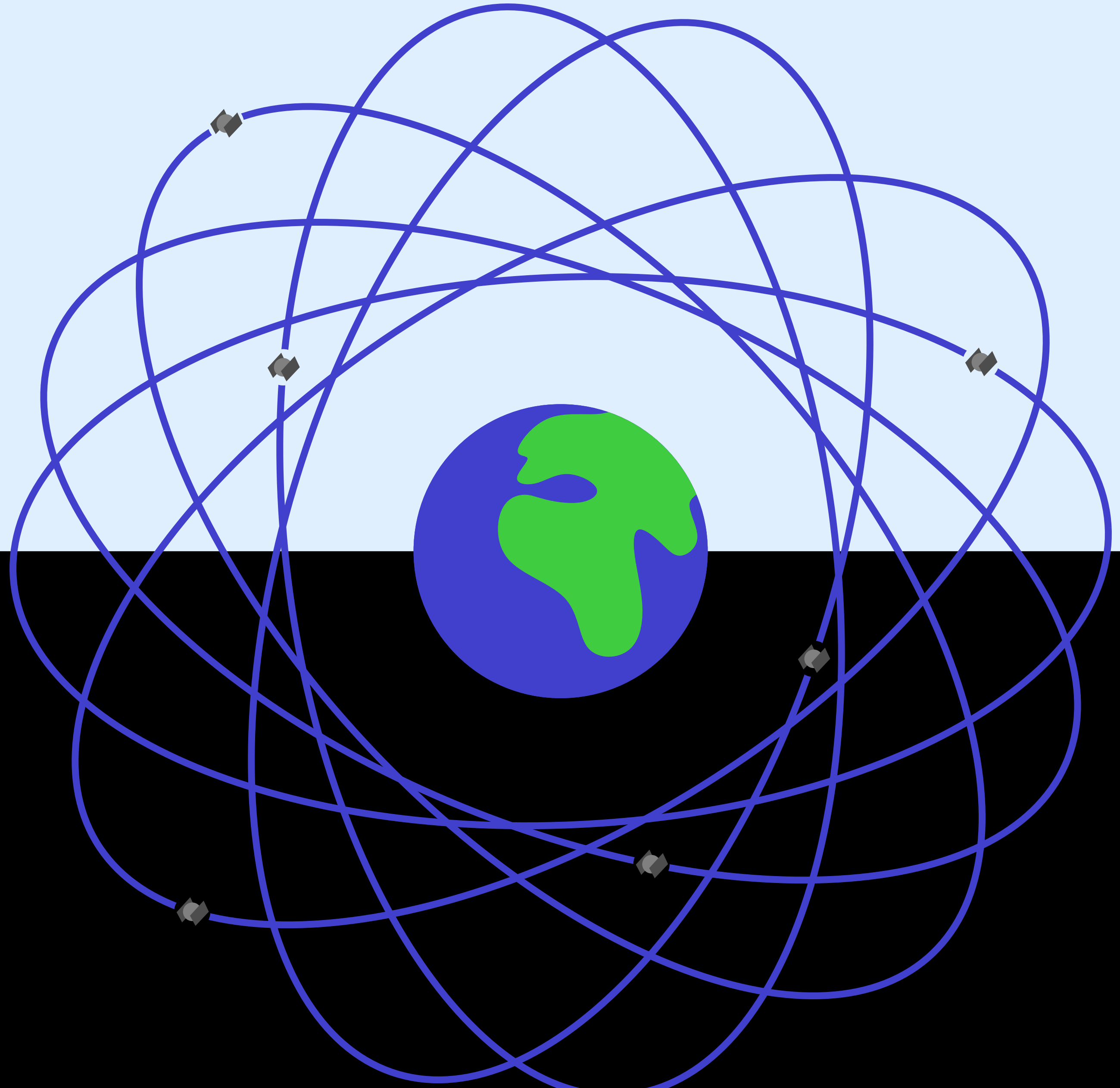

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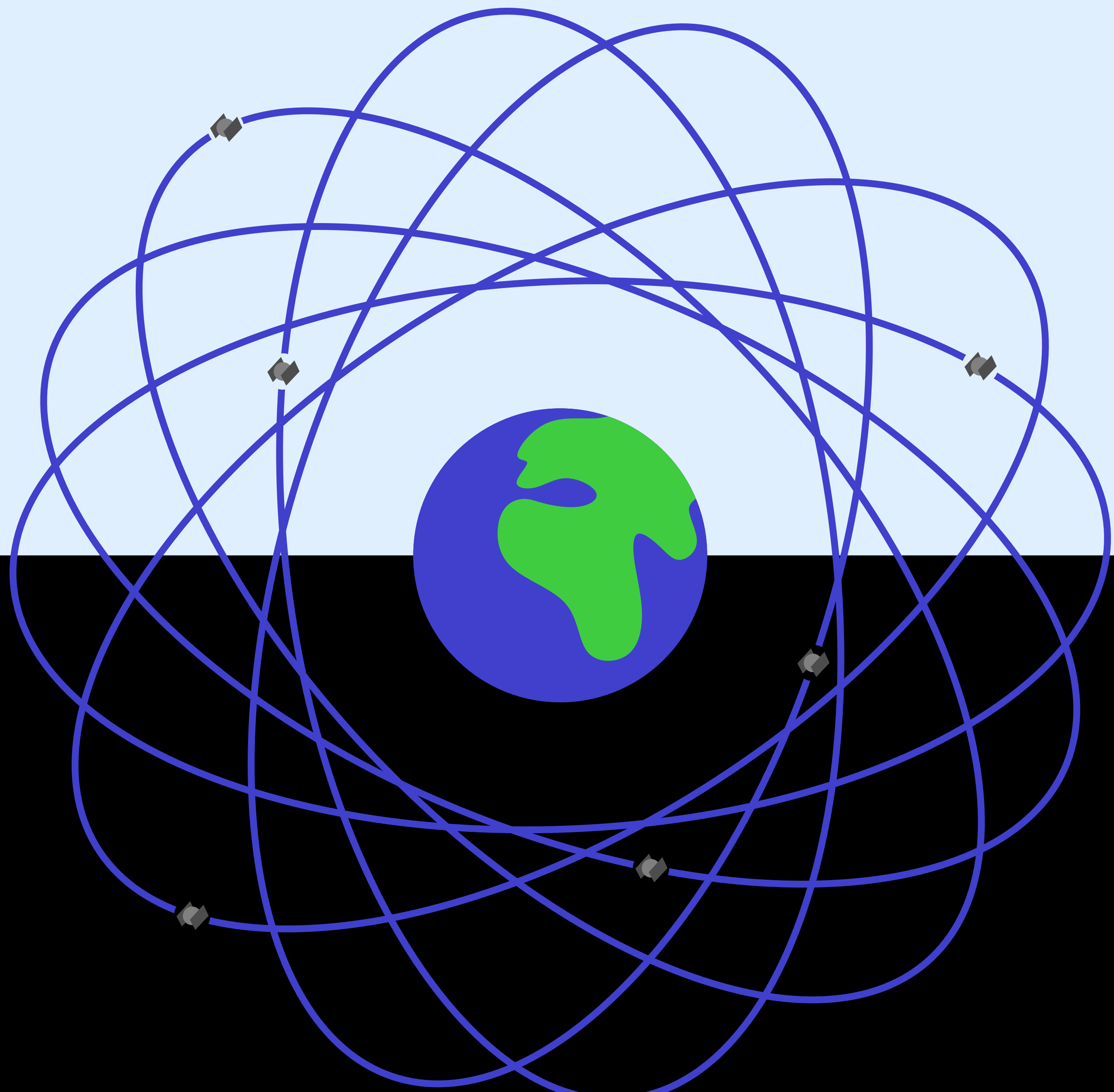

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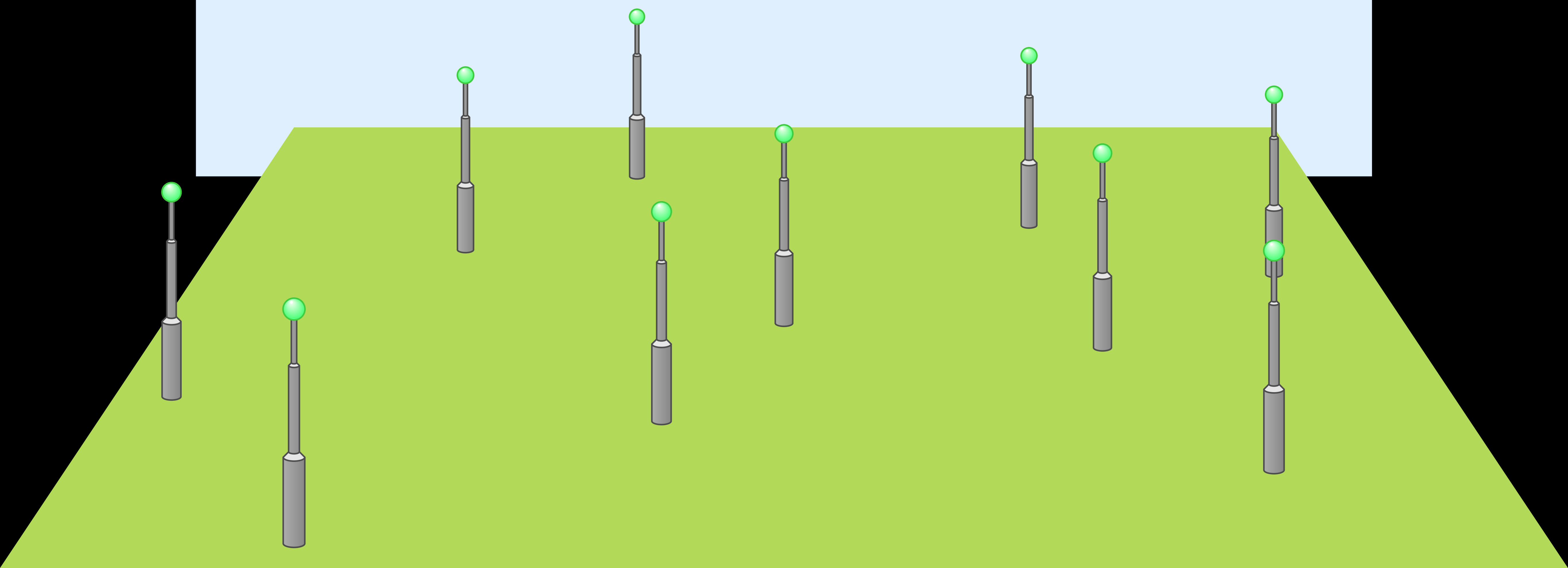

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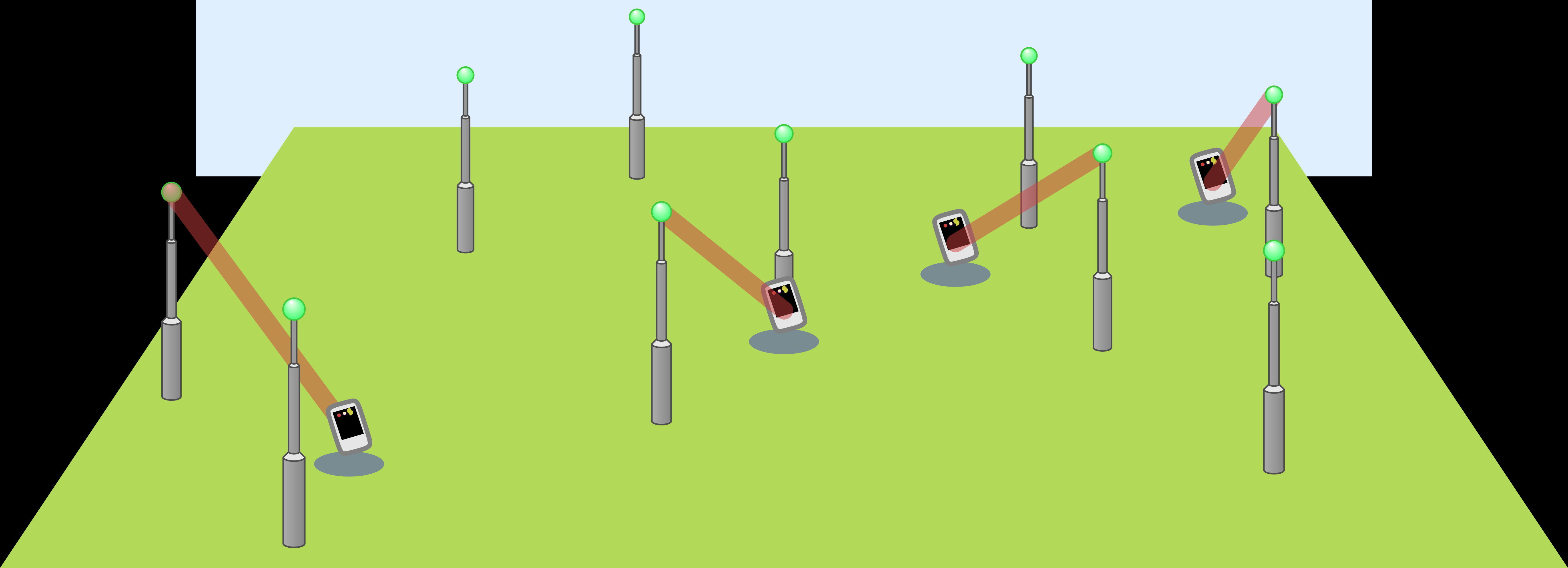

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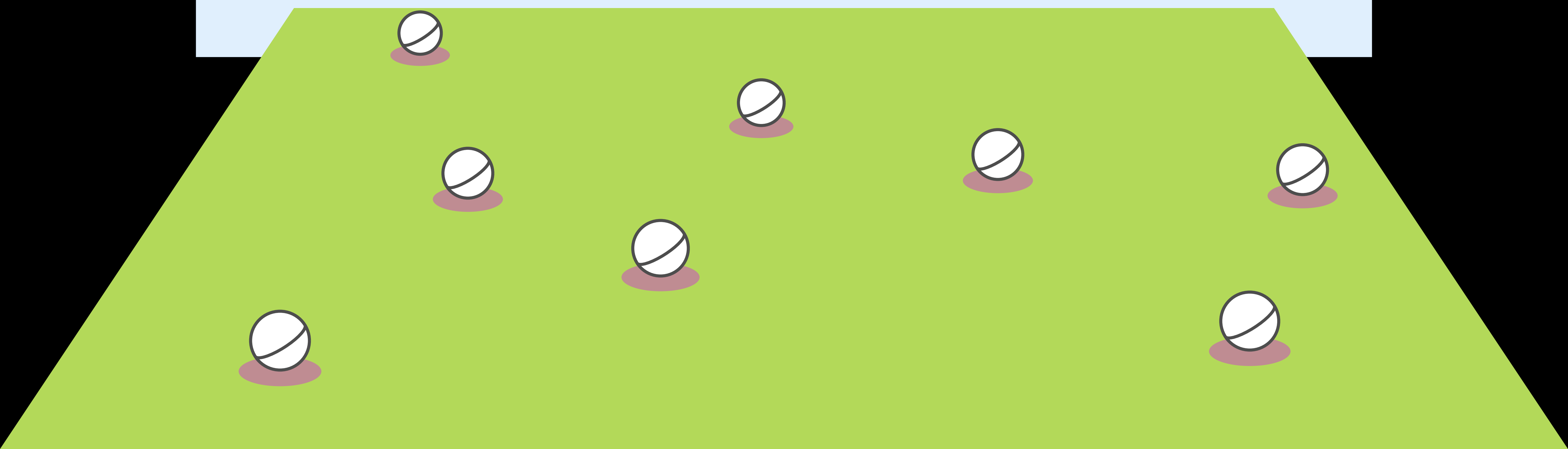

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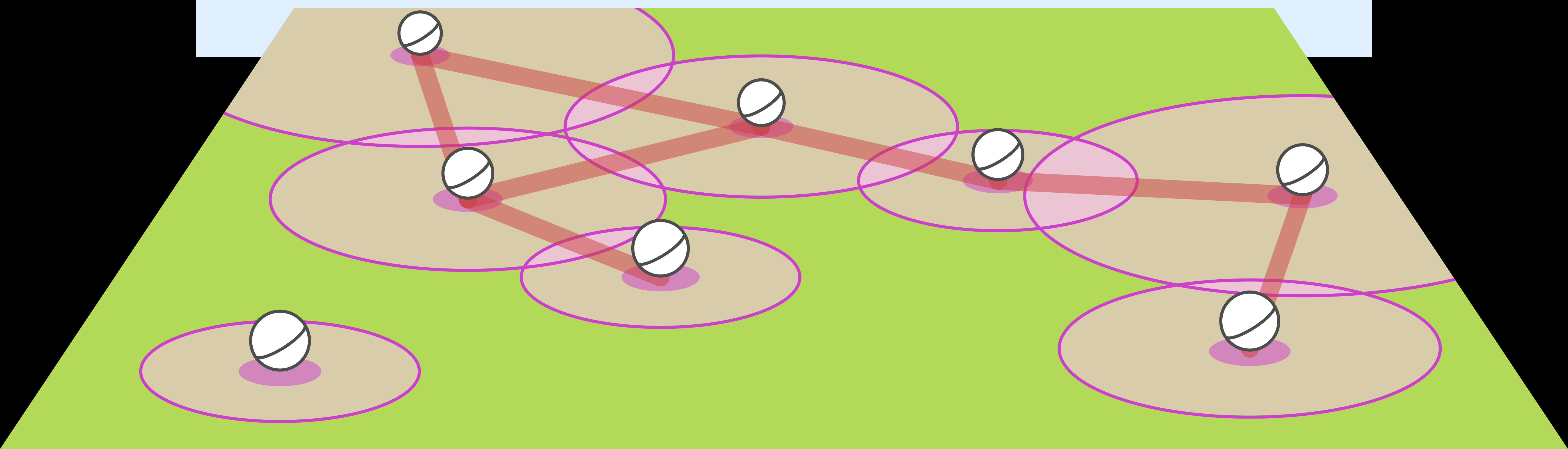

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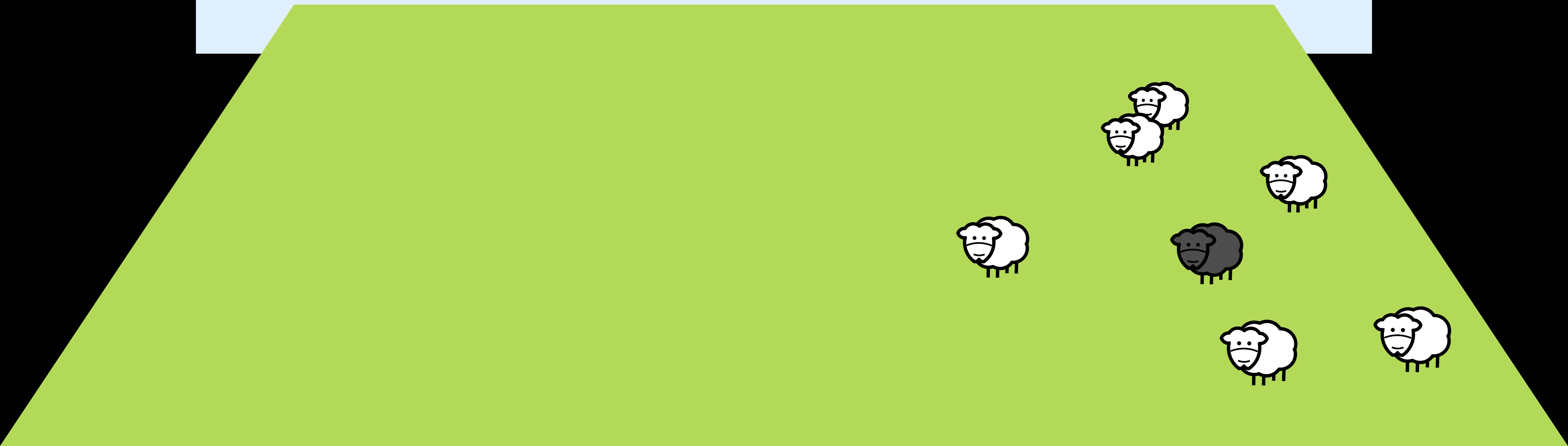

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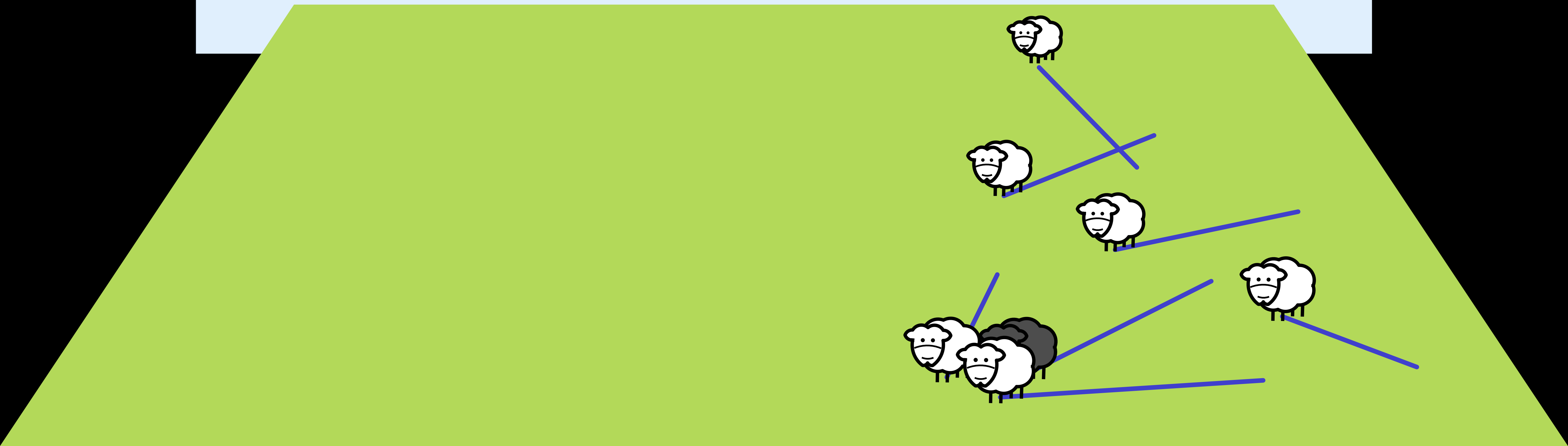

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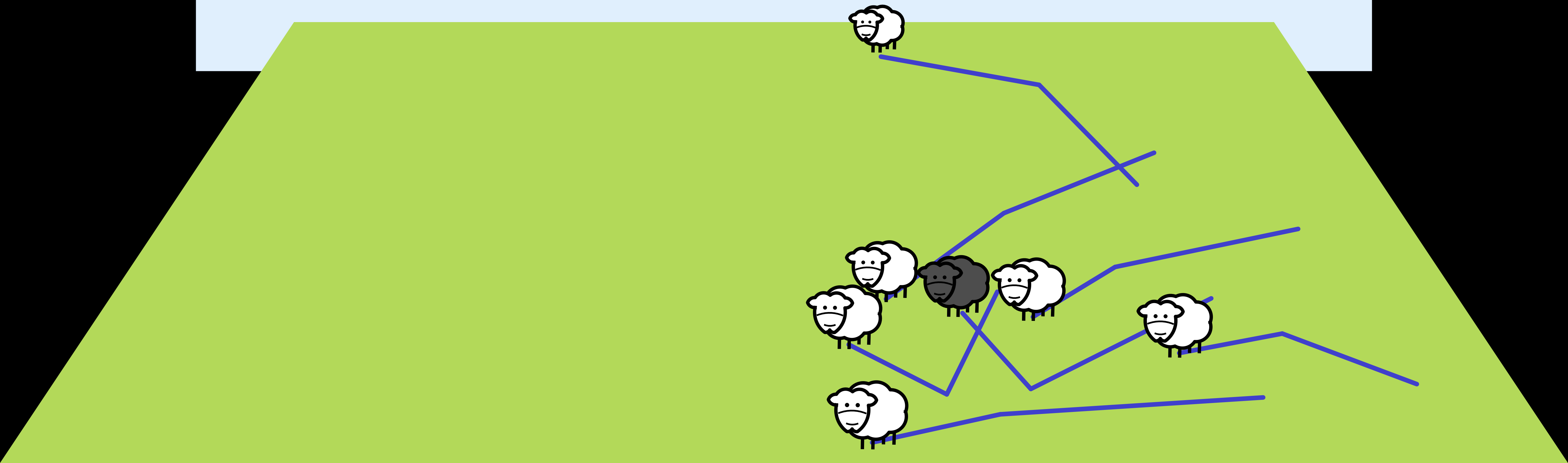

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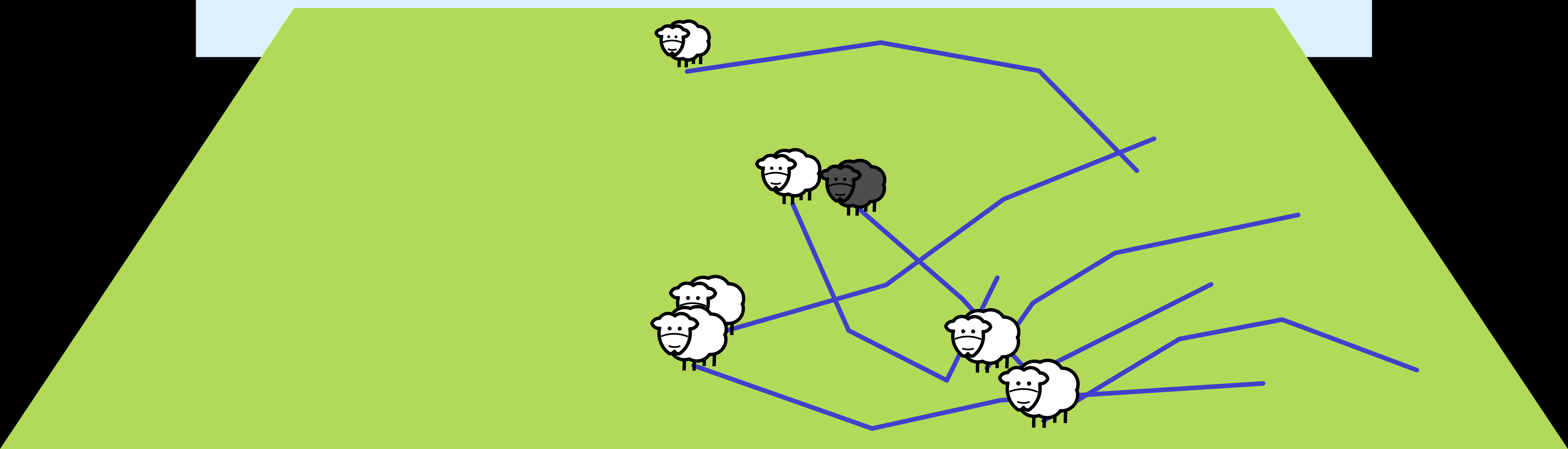

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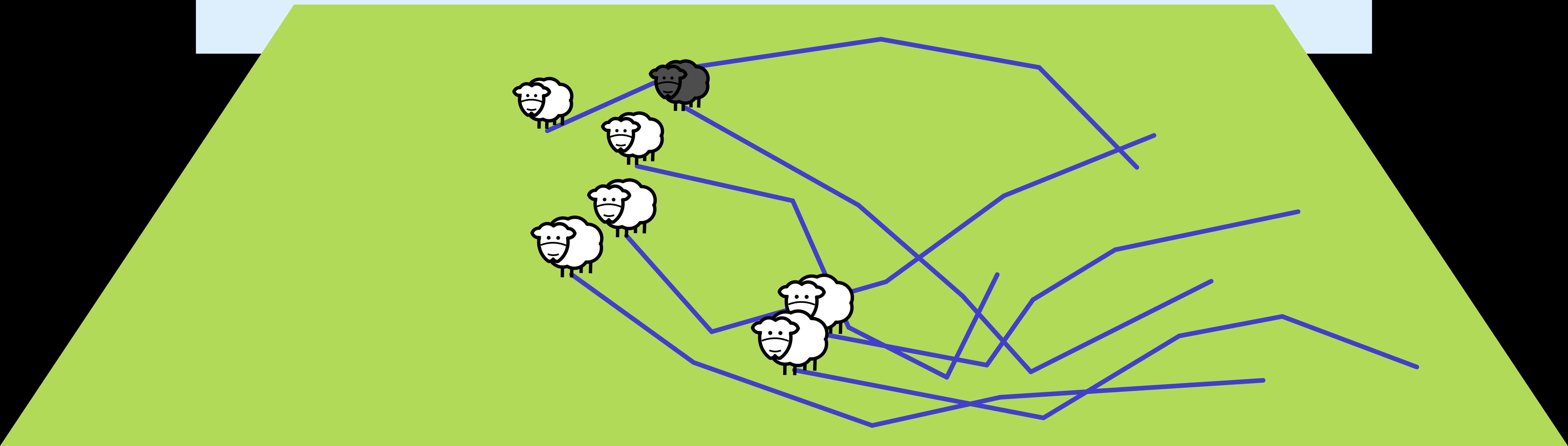

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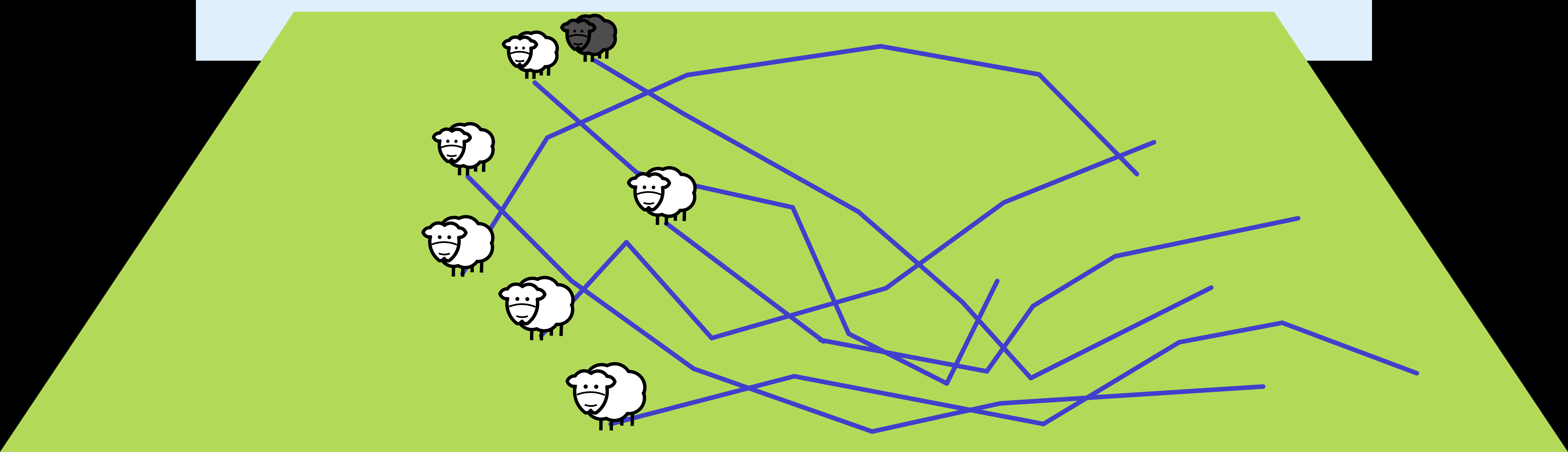

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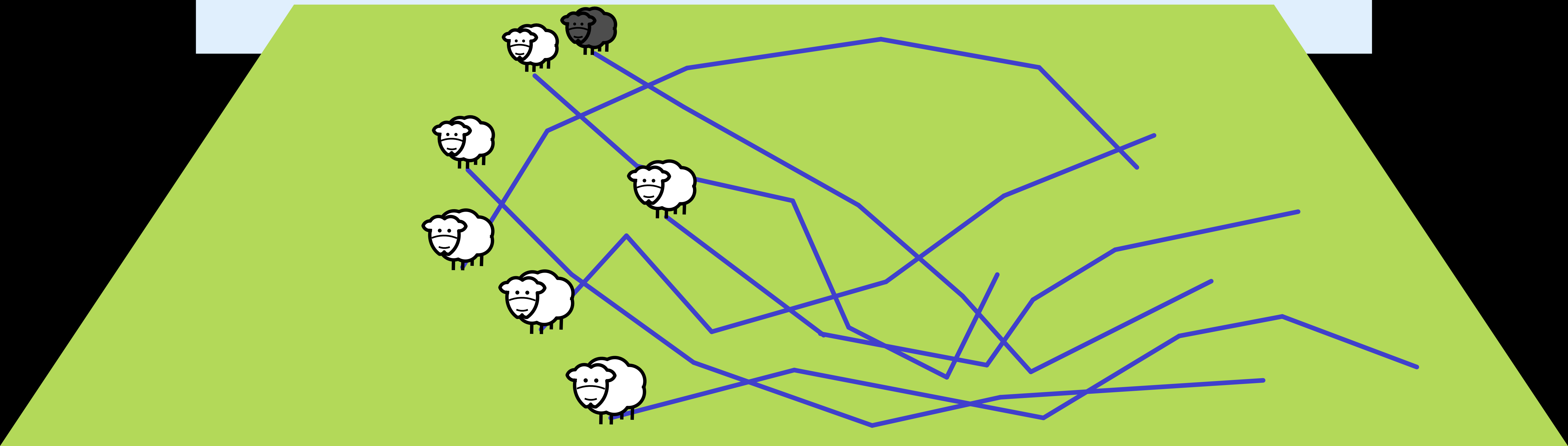
- New geometric data sets



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- New geometric data sets
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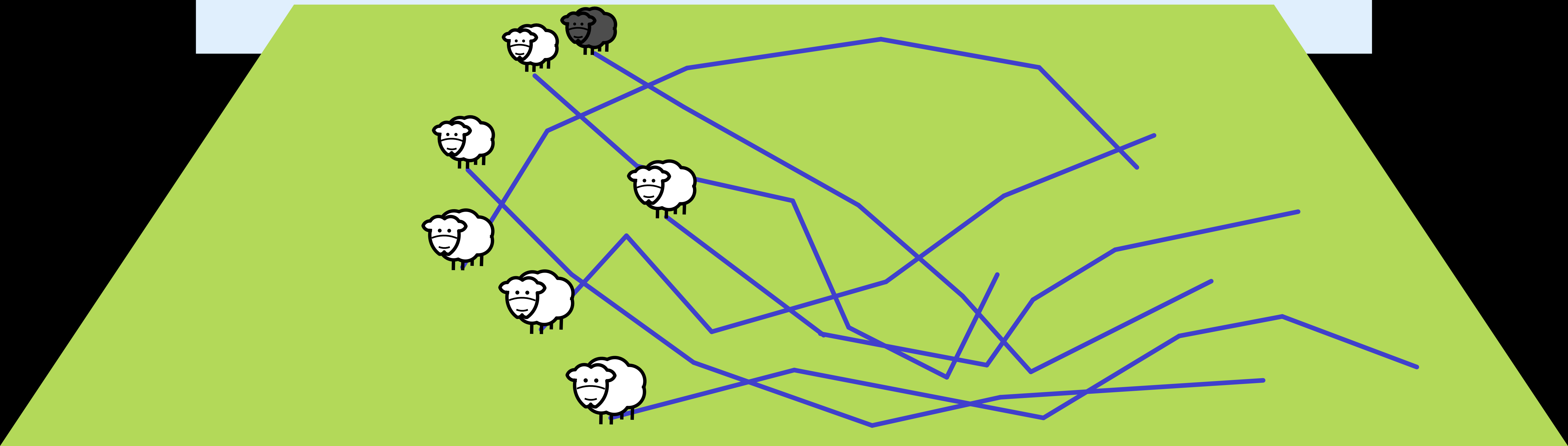

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- New geometric data sets
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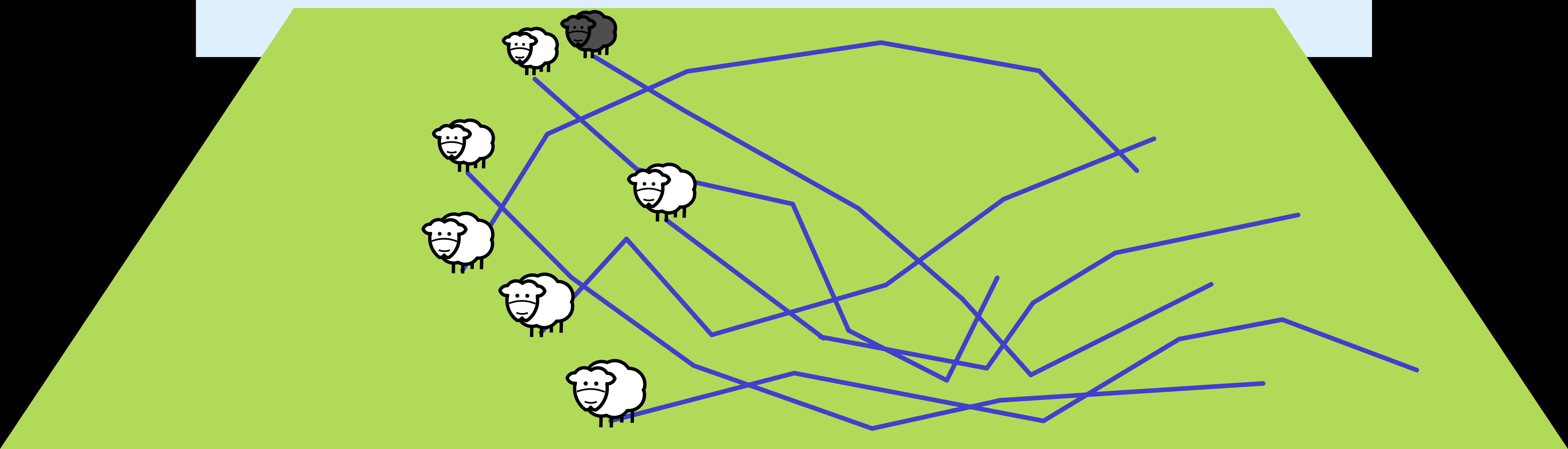
  - Imprecise in nature



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- New geometric data sets
  - Often very large
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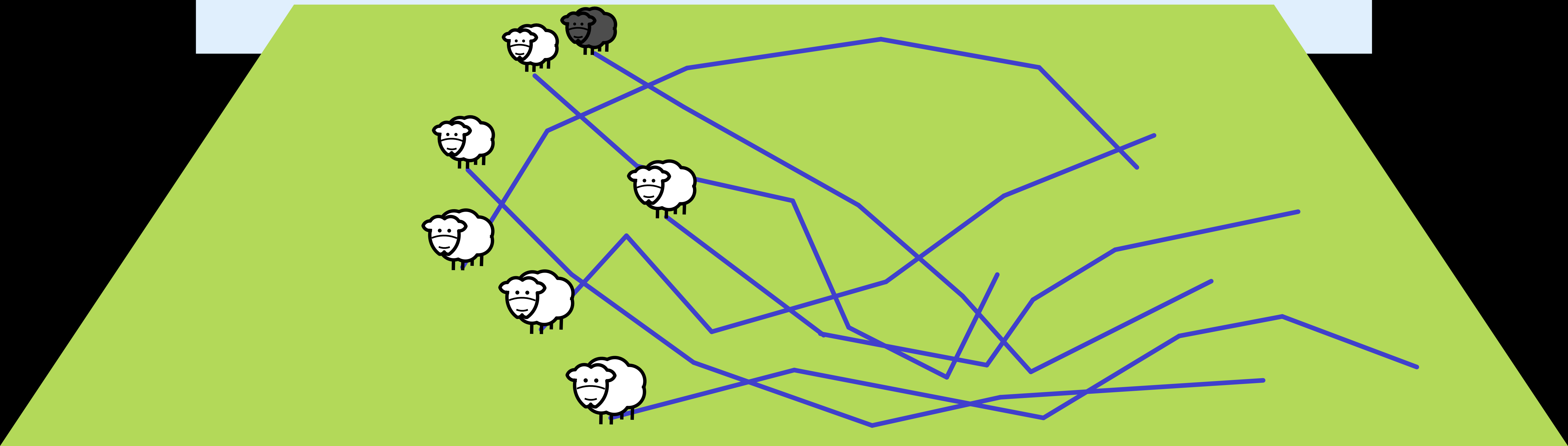
  - Dynamic / unpredictable



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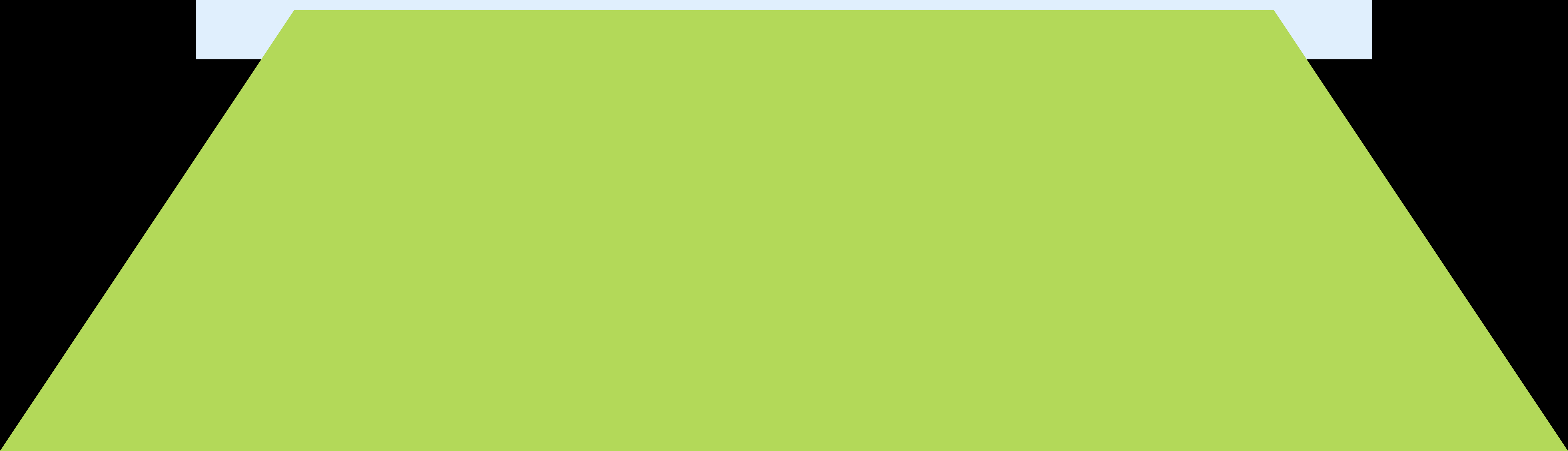

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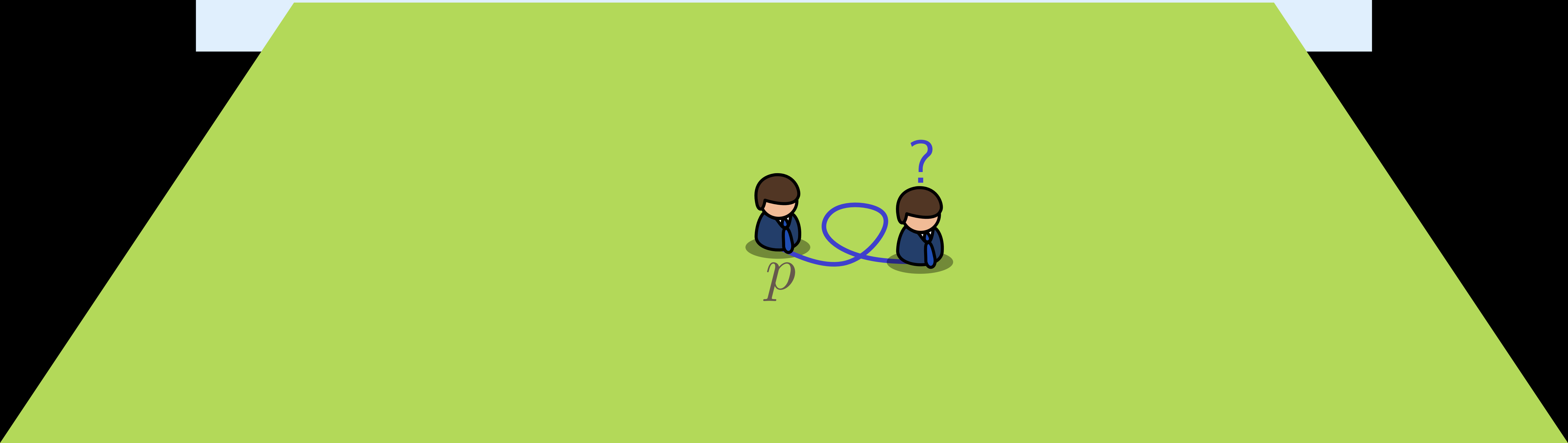

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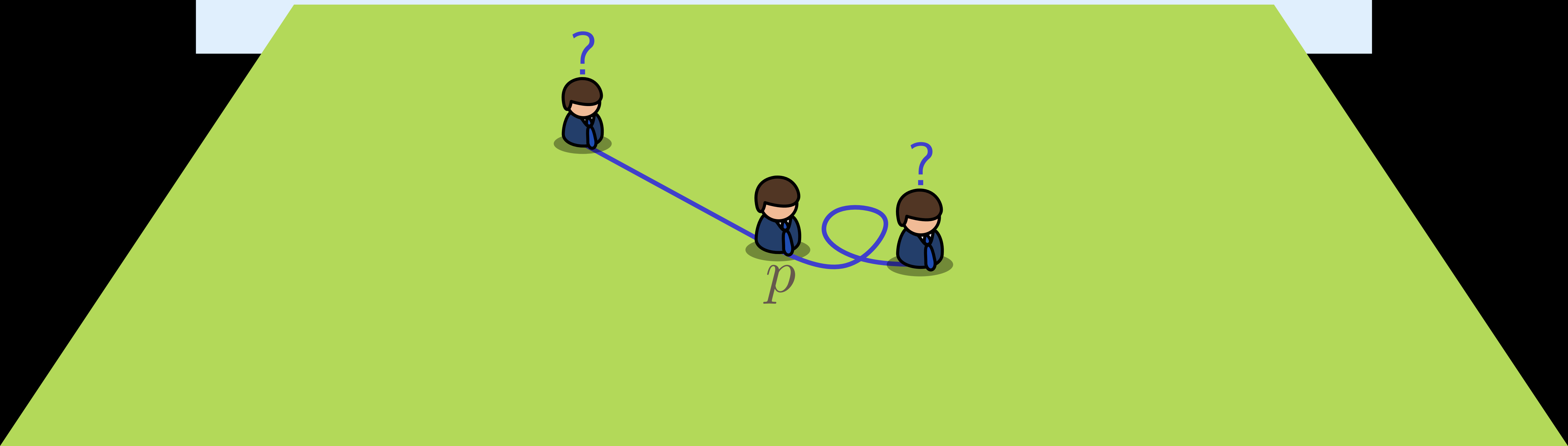

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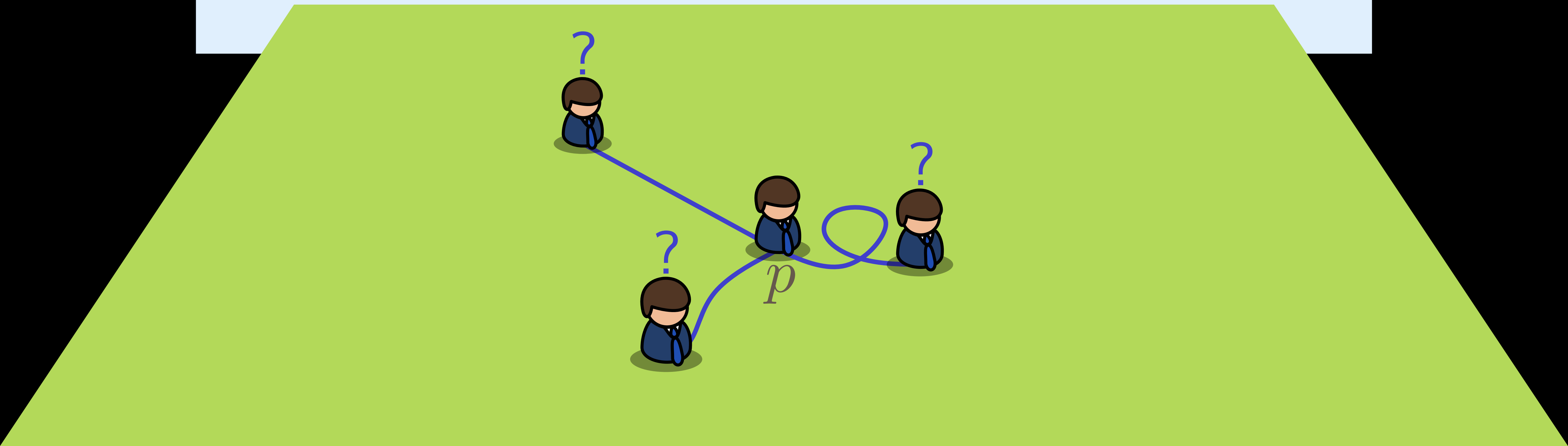

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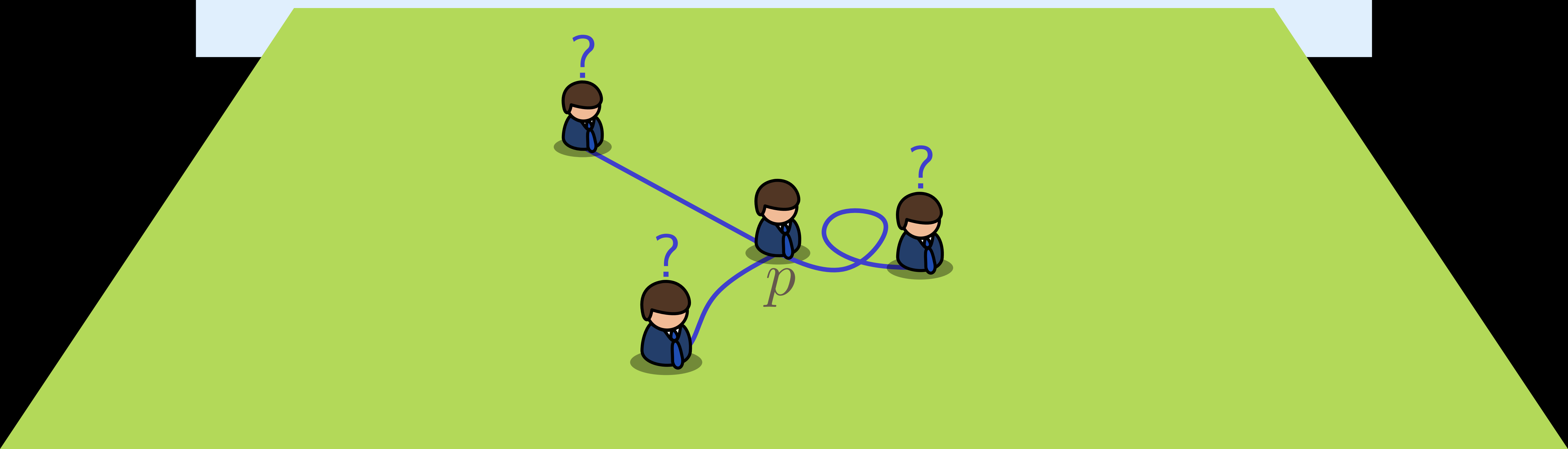

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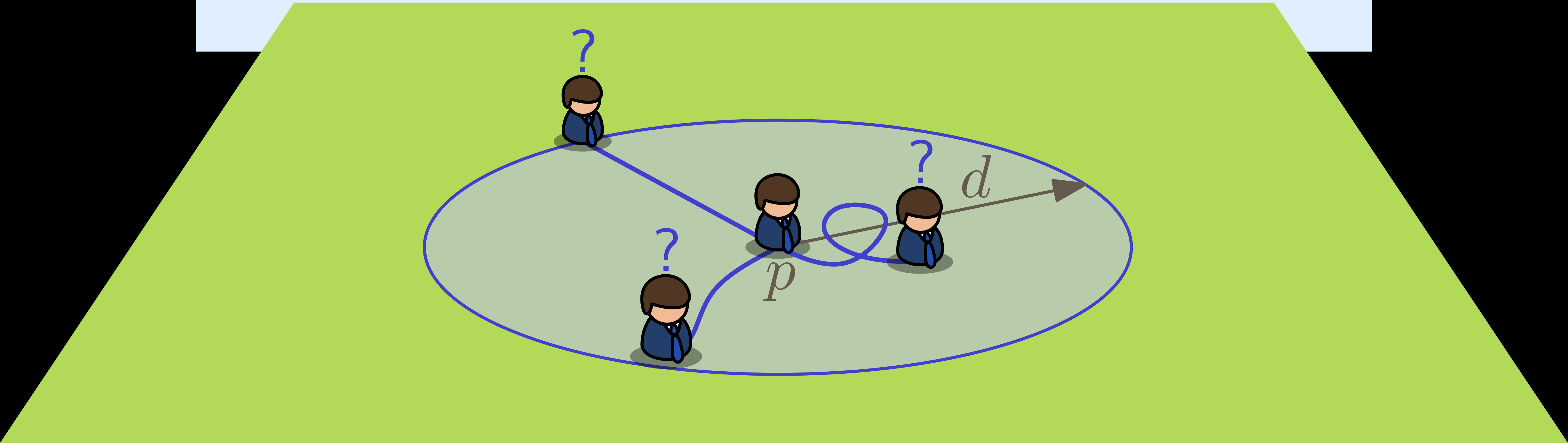

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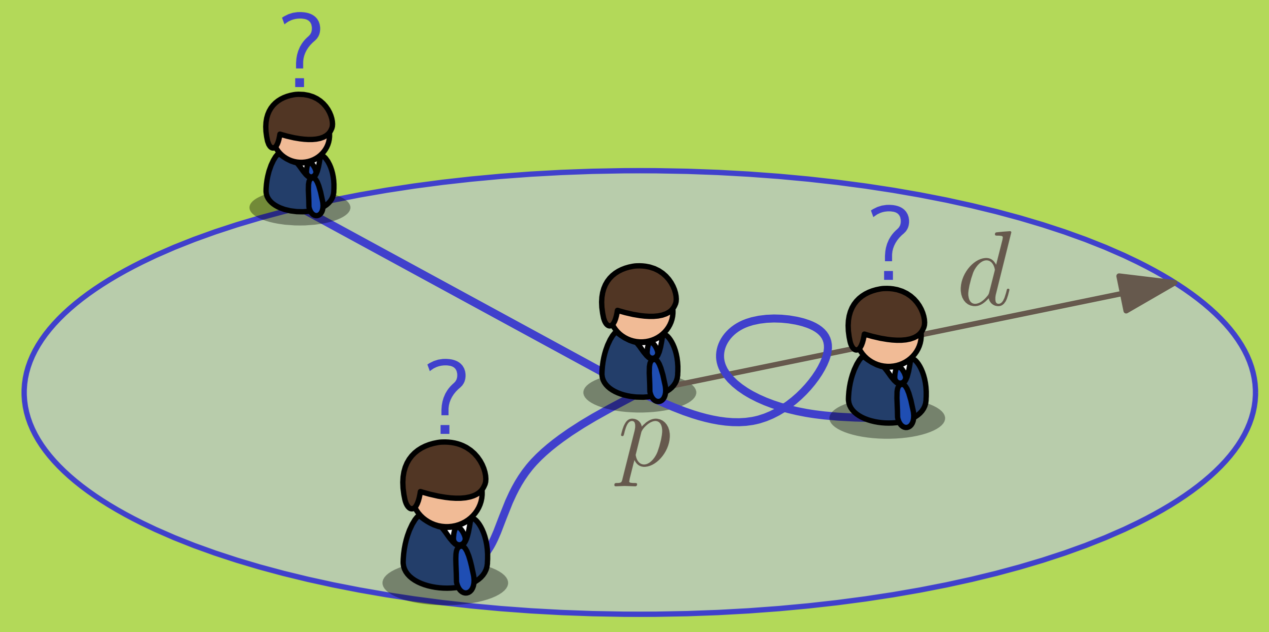
OBSERVATION: *If we knew where p was t time ago, we know it is now somewhere in a disk of radius td*



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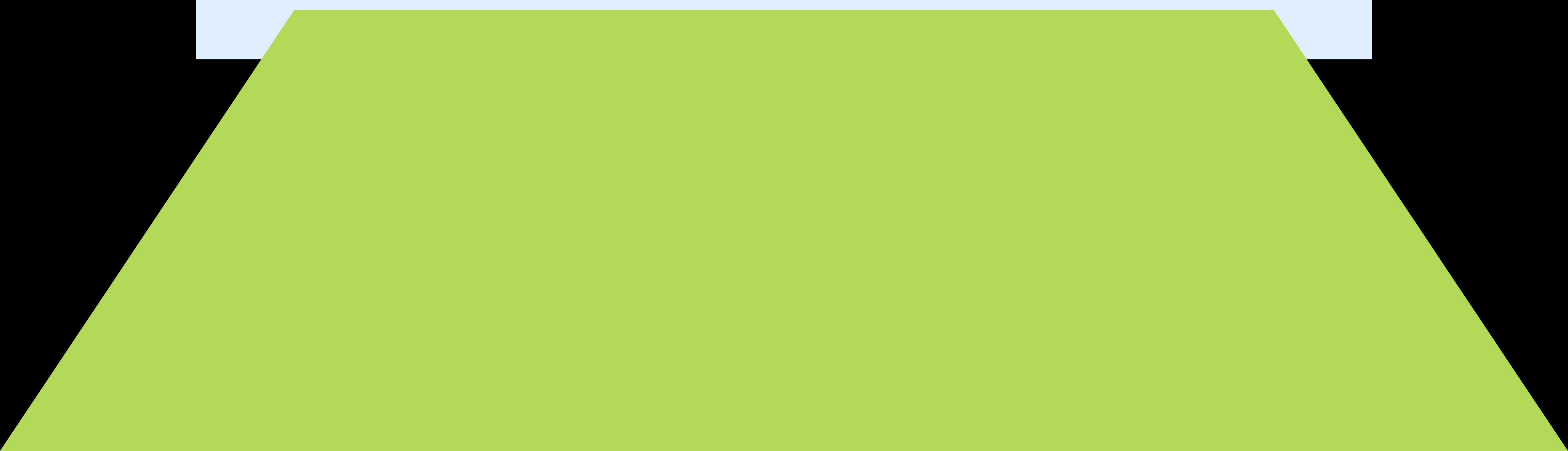
- Consider n moving points



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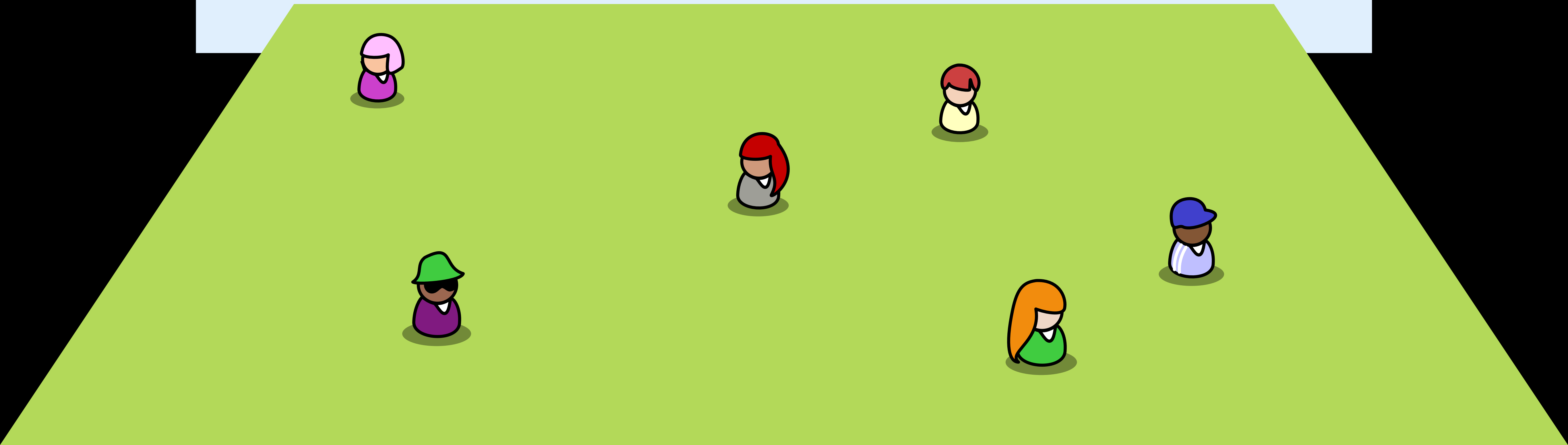

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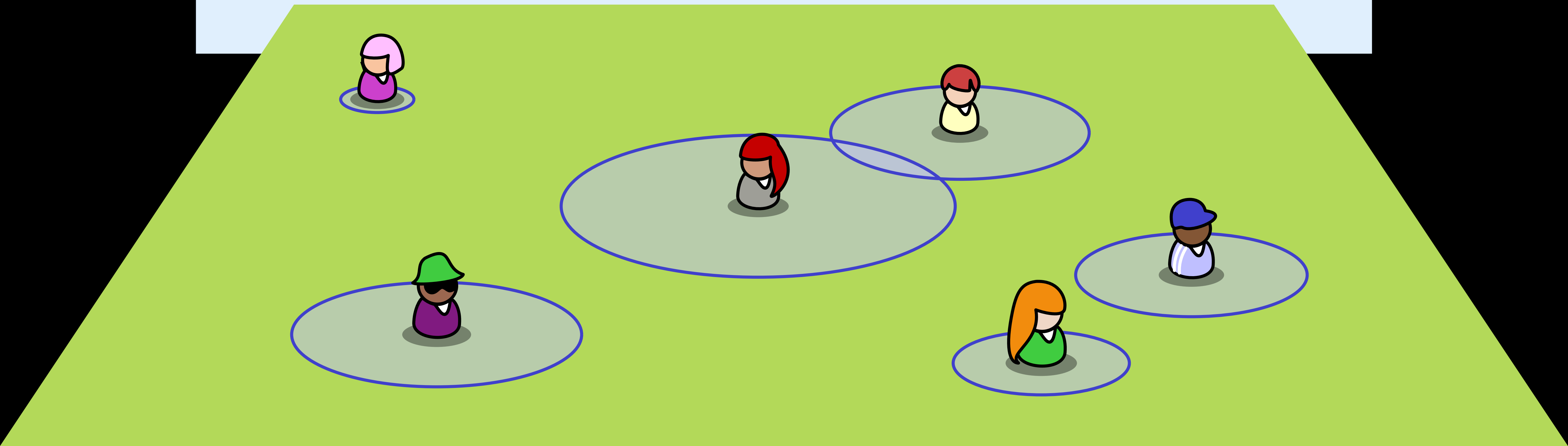

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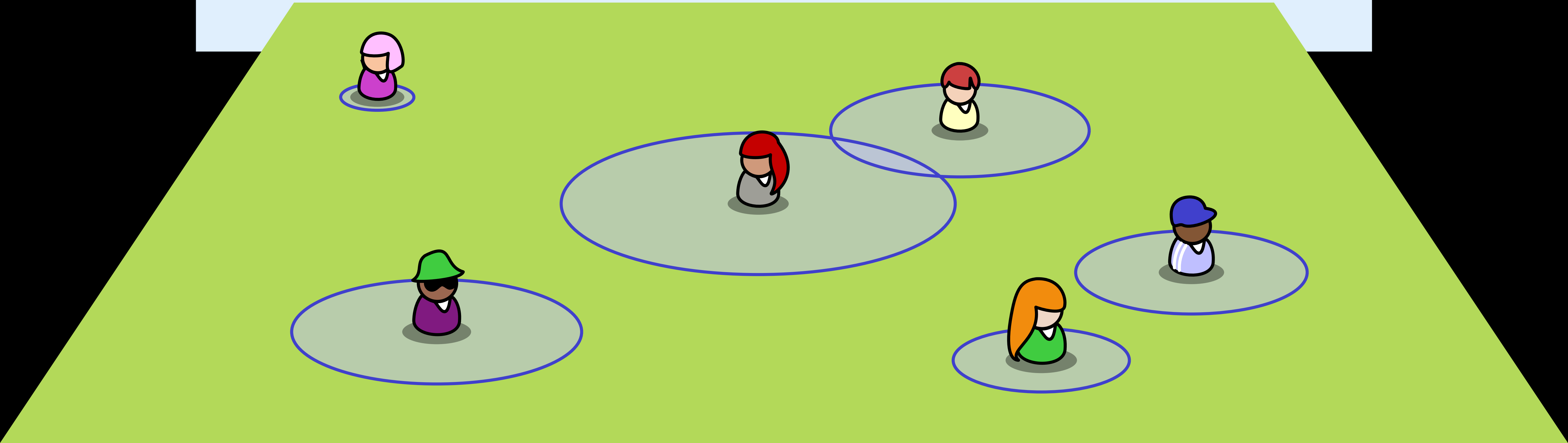

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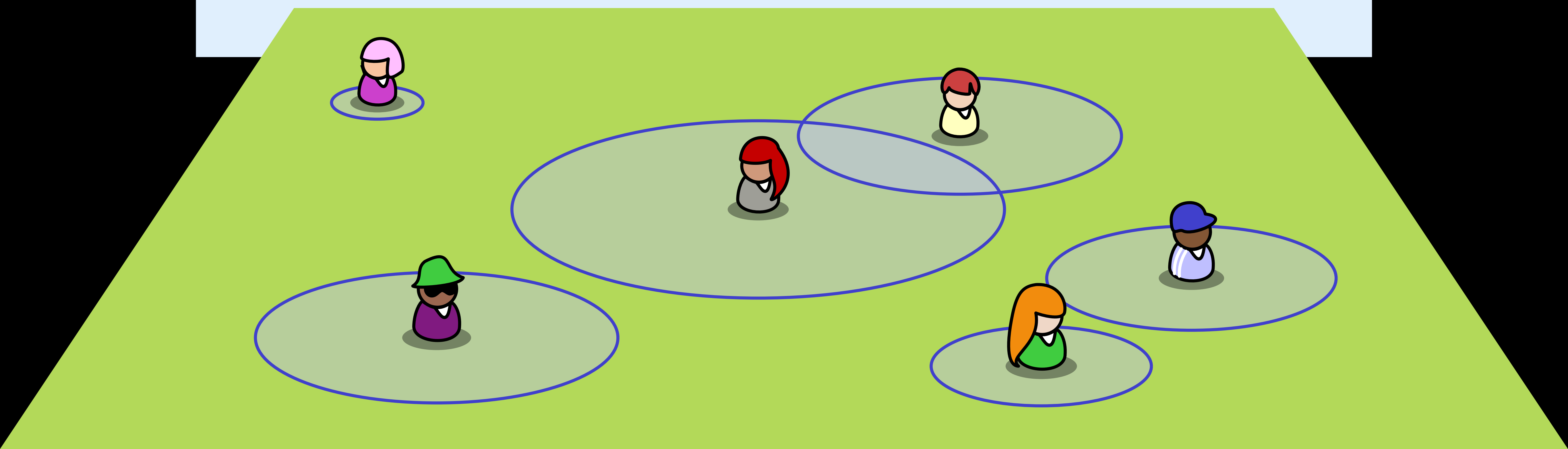

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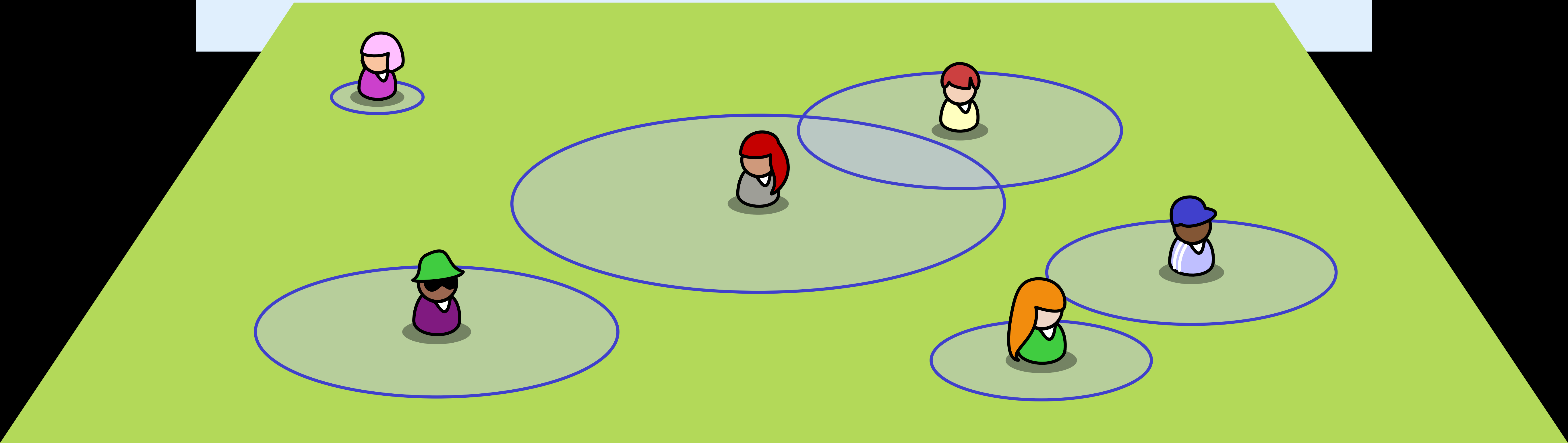

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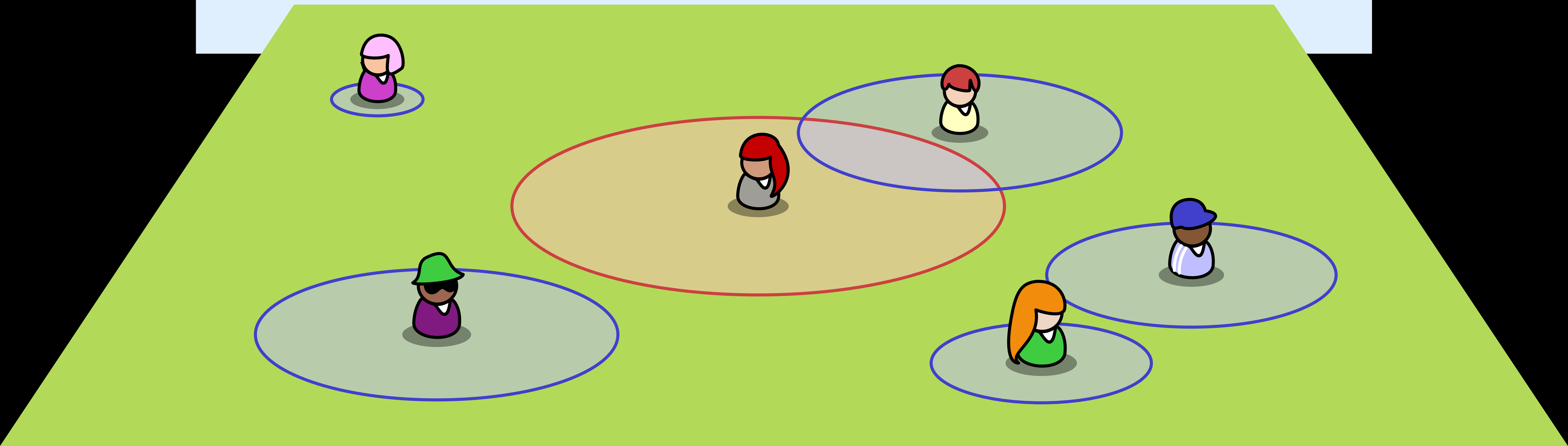

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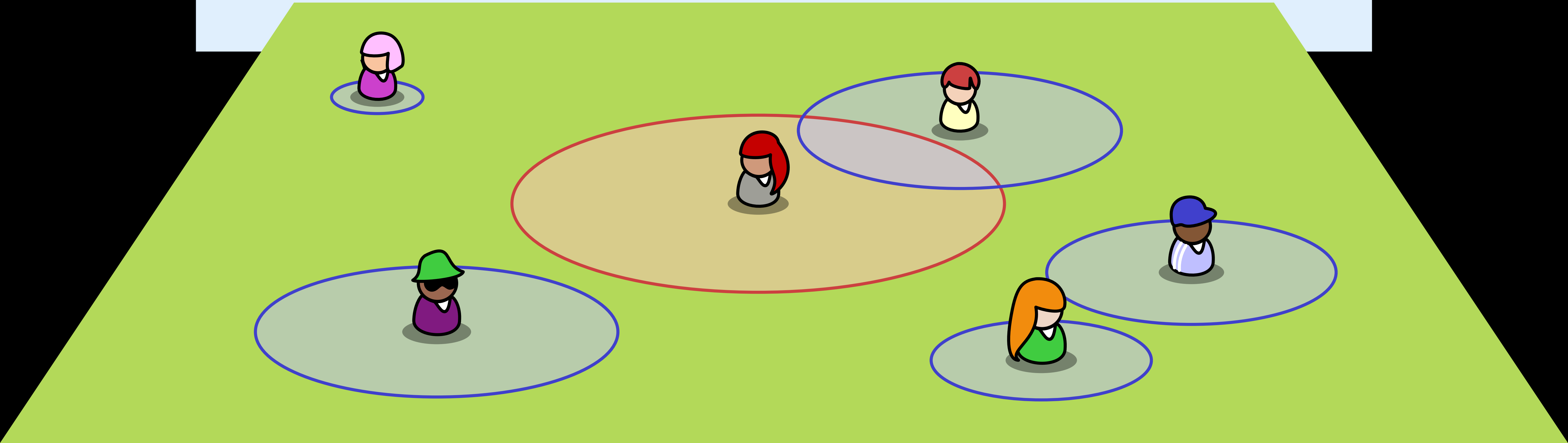

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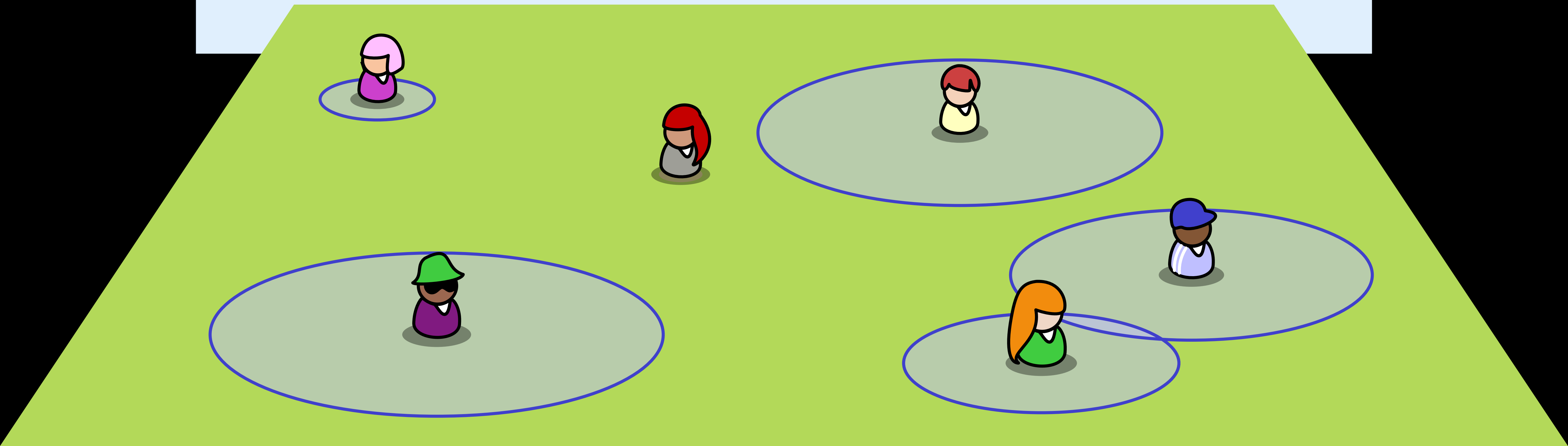

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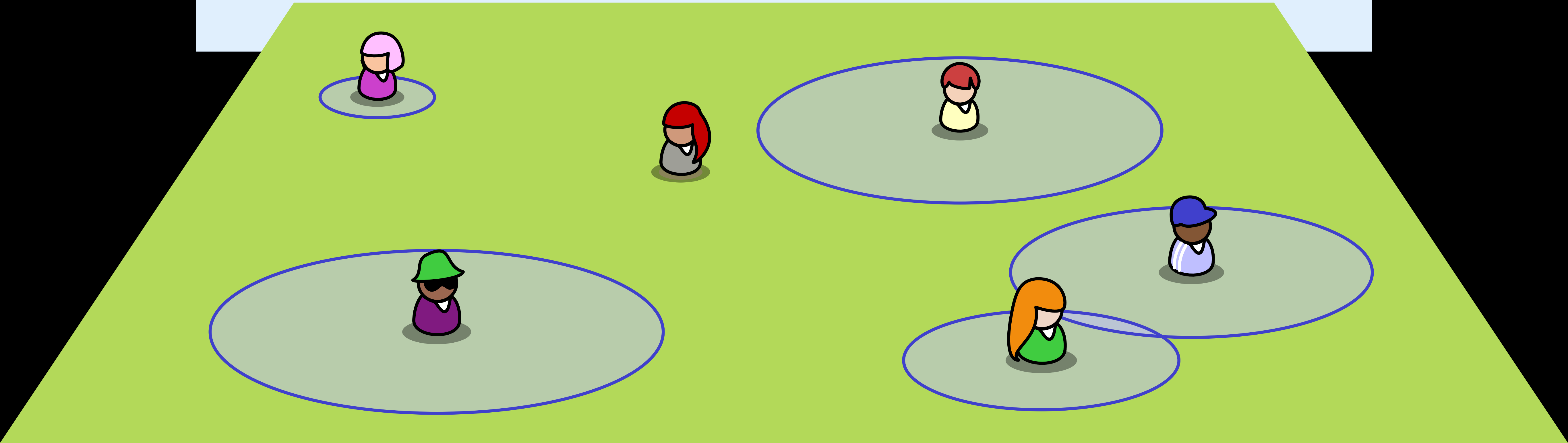
  - After a query, one region collapses to a point, but all other regions grow



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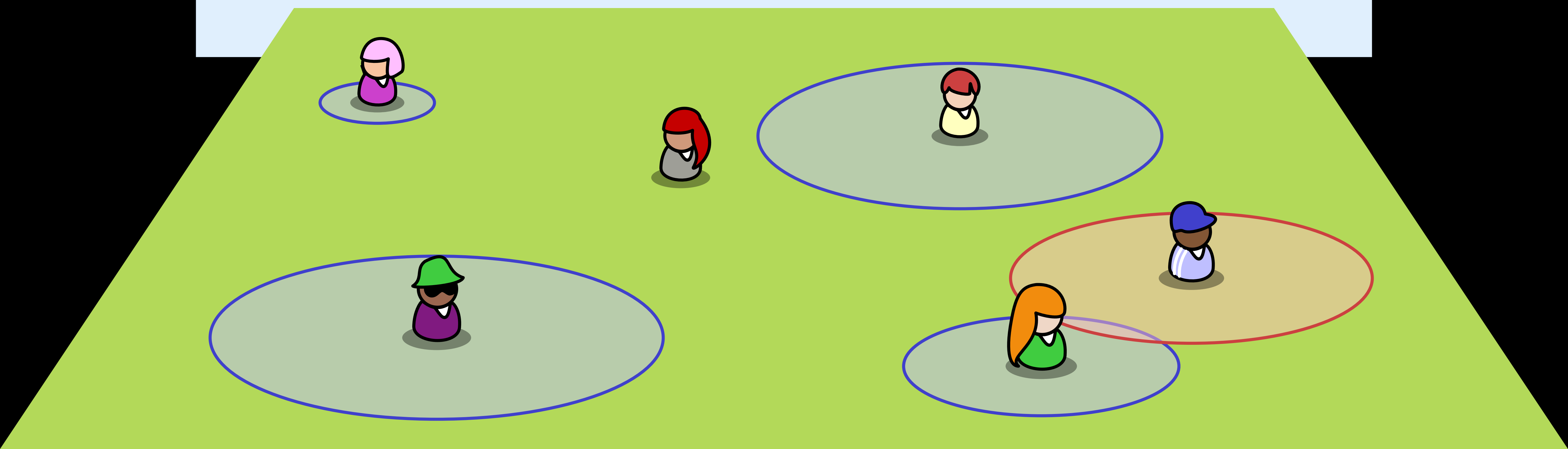

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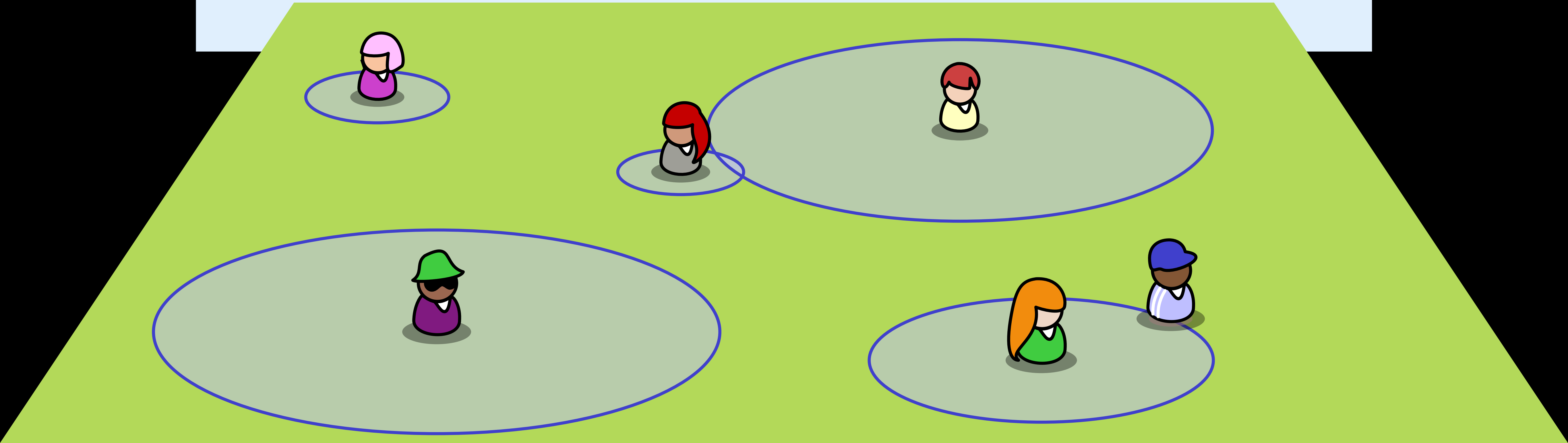

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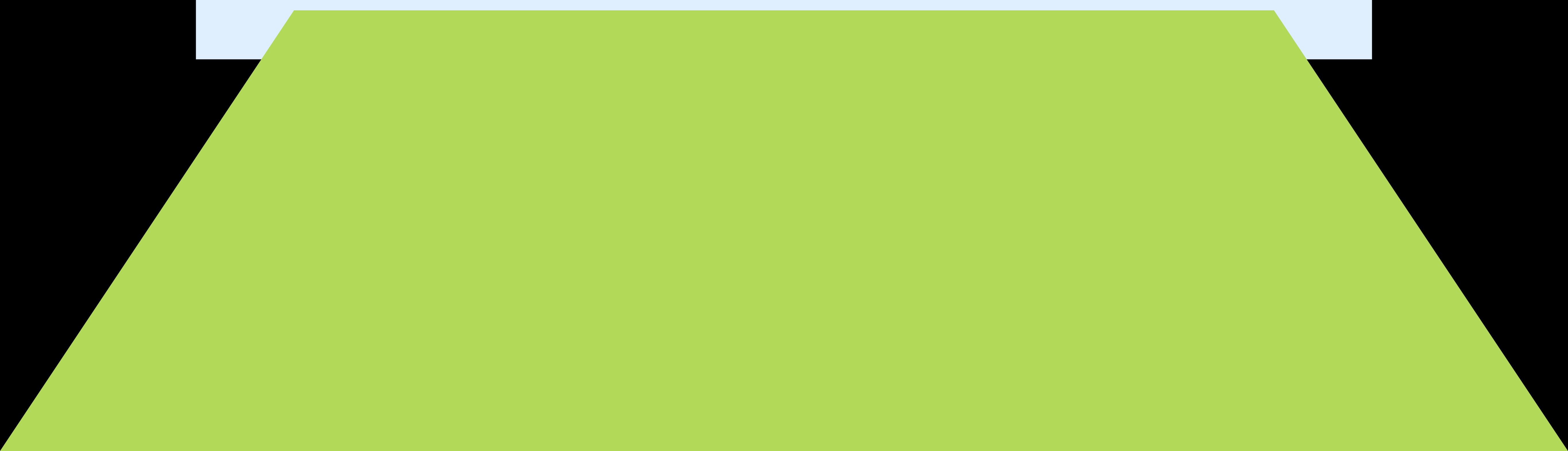

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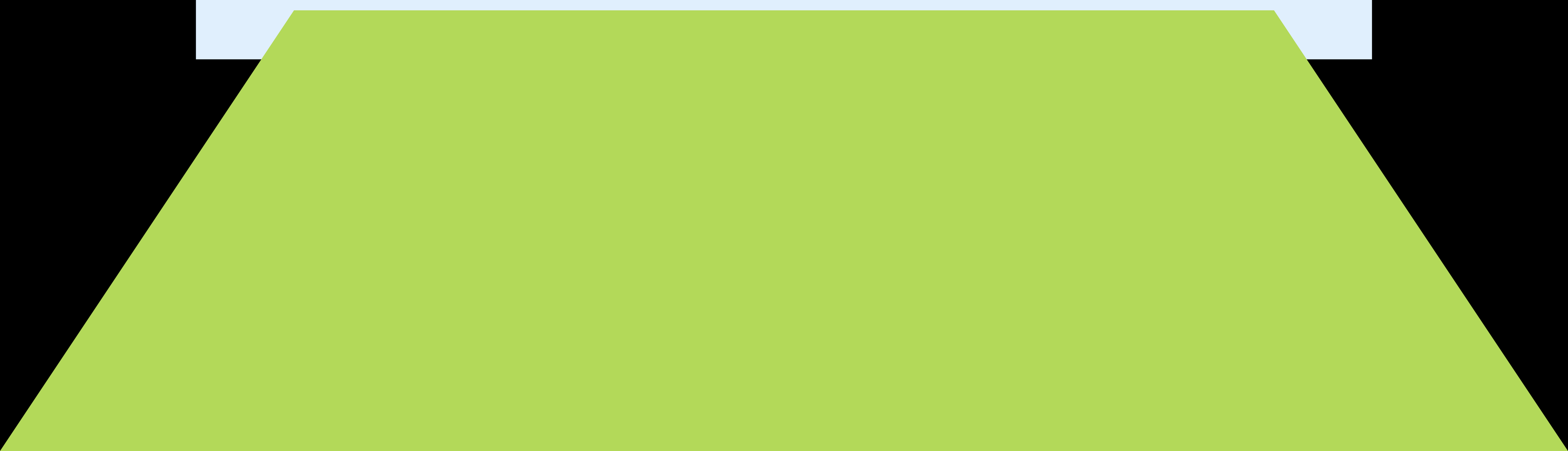

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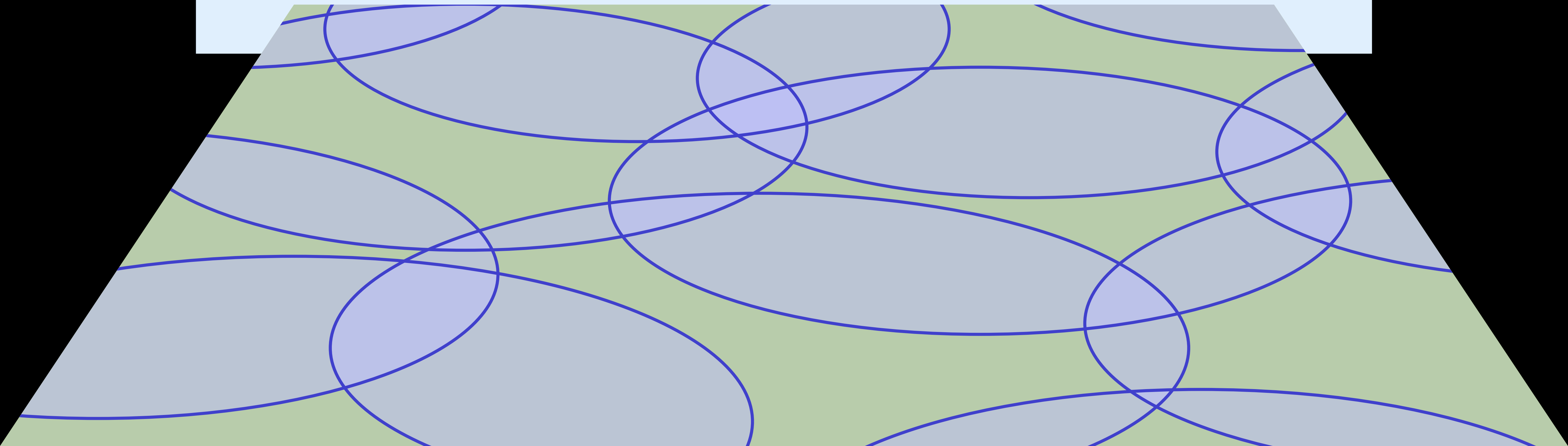

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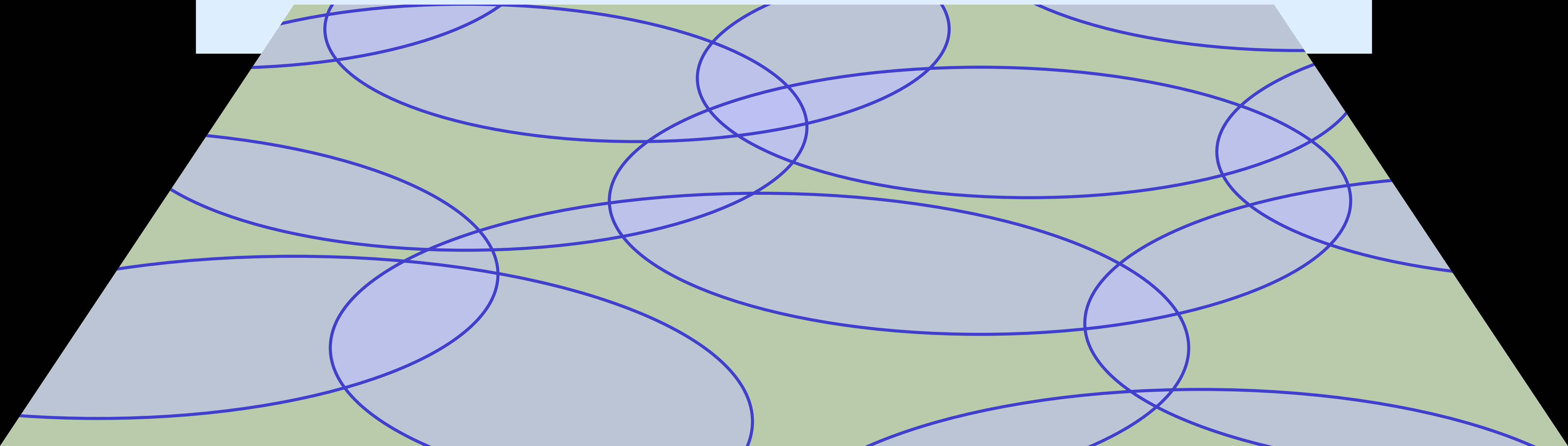
  - $\delta(p) = |\{R : p \in R\}|$



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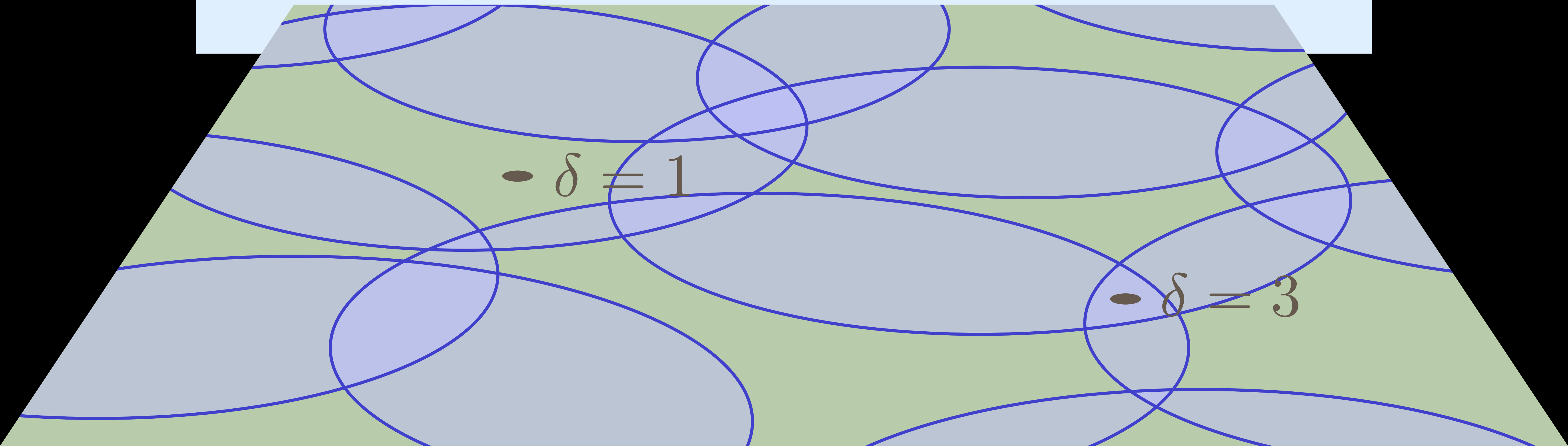

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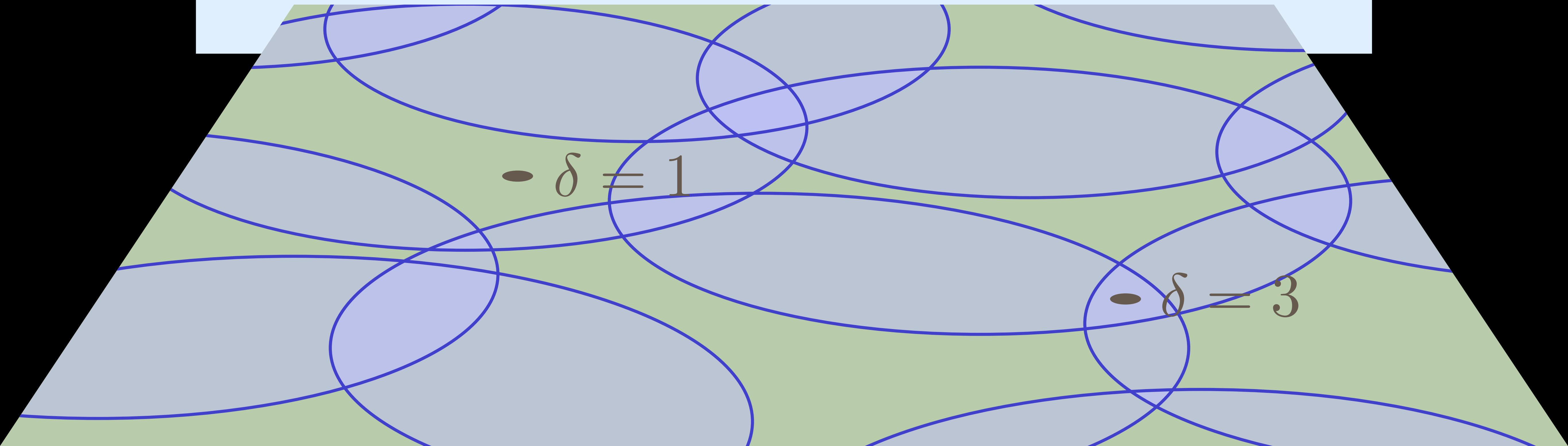

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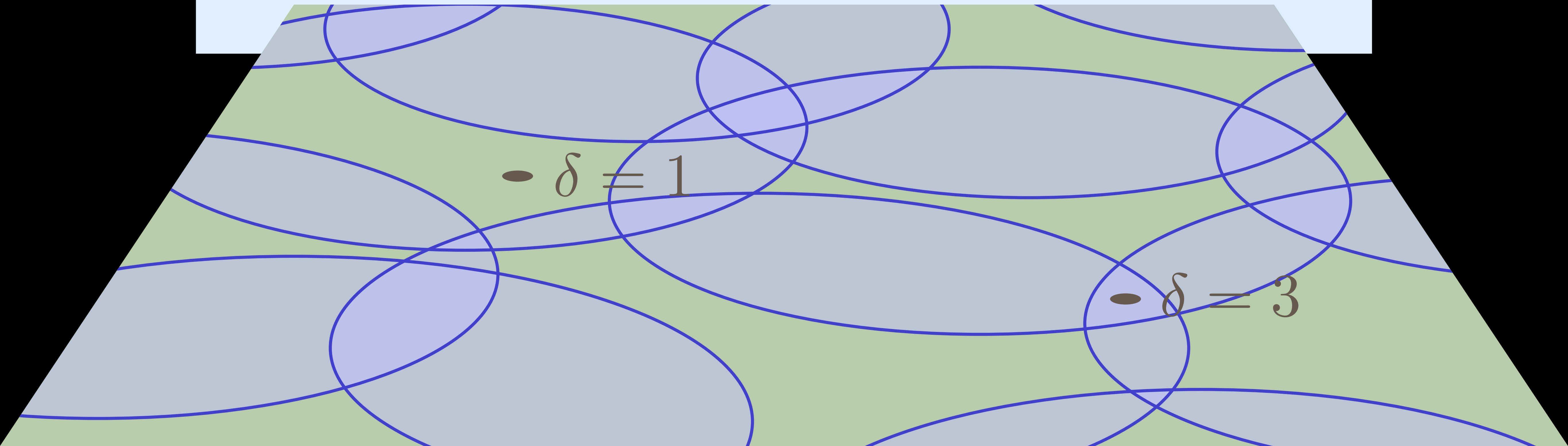
  - $\delta(p) = |\{R : p \in R\}|$
- Ply of the set of regions
  - Maximum ply of all points



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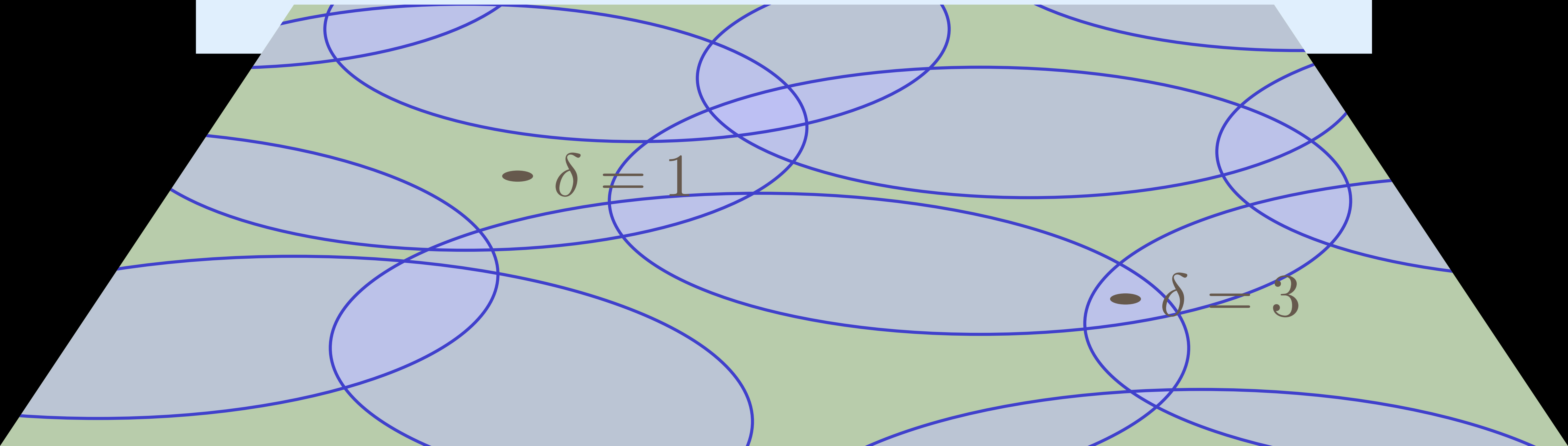

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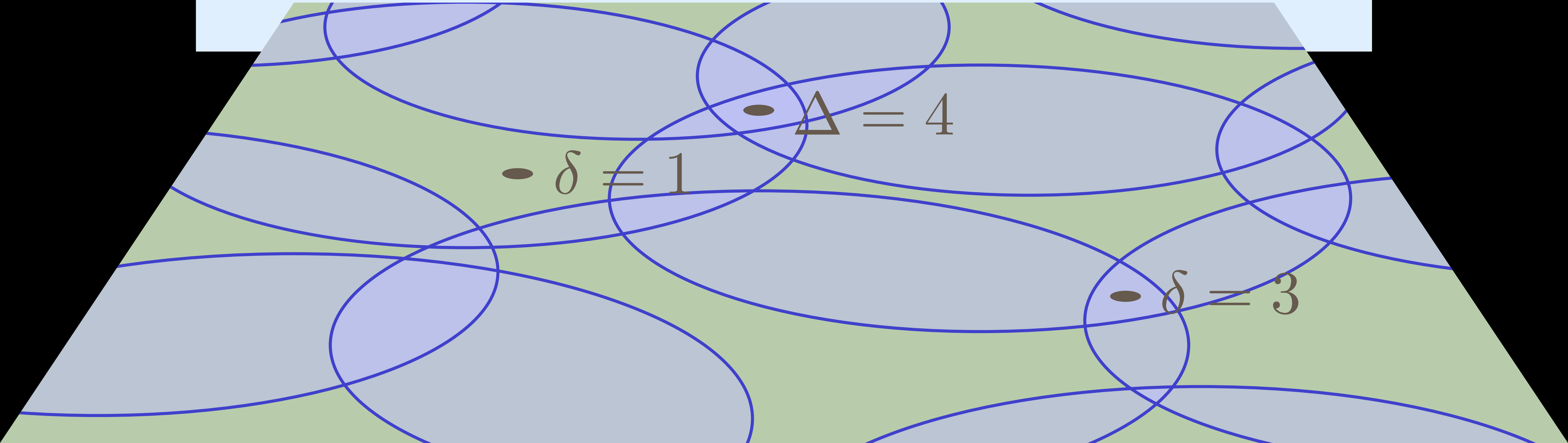

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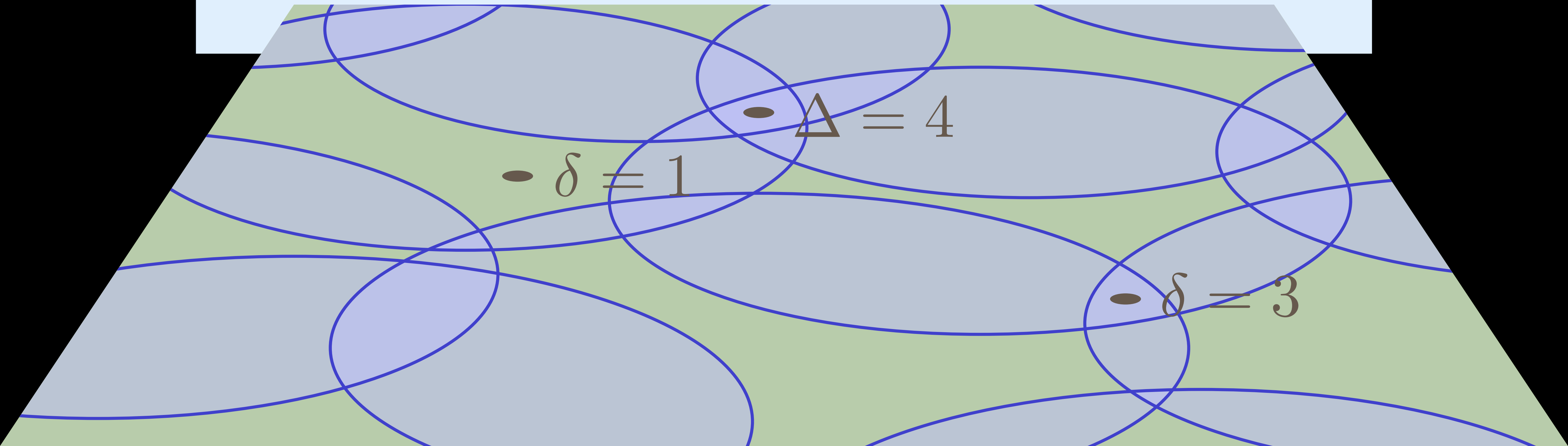
POSTULATION: *Low ply is good!*



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- Competitive ratio
  - Performance measure for online algorithms

“COMPETITIVE”

- Classical, “offline” algorithm
  - Receives the whole input at once
- Online algorithm
  - Receives a sequence of input *events*
  - Needs to react to each event before knowing what the next event will be
- Competitive ratio
  - Performance measure for online algorithms
  - Cost of online algorithm divided by cost of best possible offline algorithm

COMPETITIVE QUERY STRATEGIES FOR MINIMISING THE PLY OF THE POTENTIAL LOCATIONS OF MOVING POINTS

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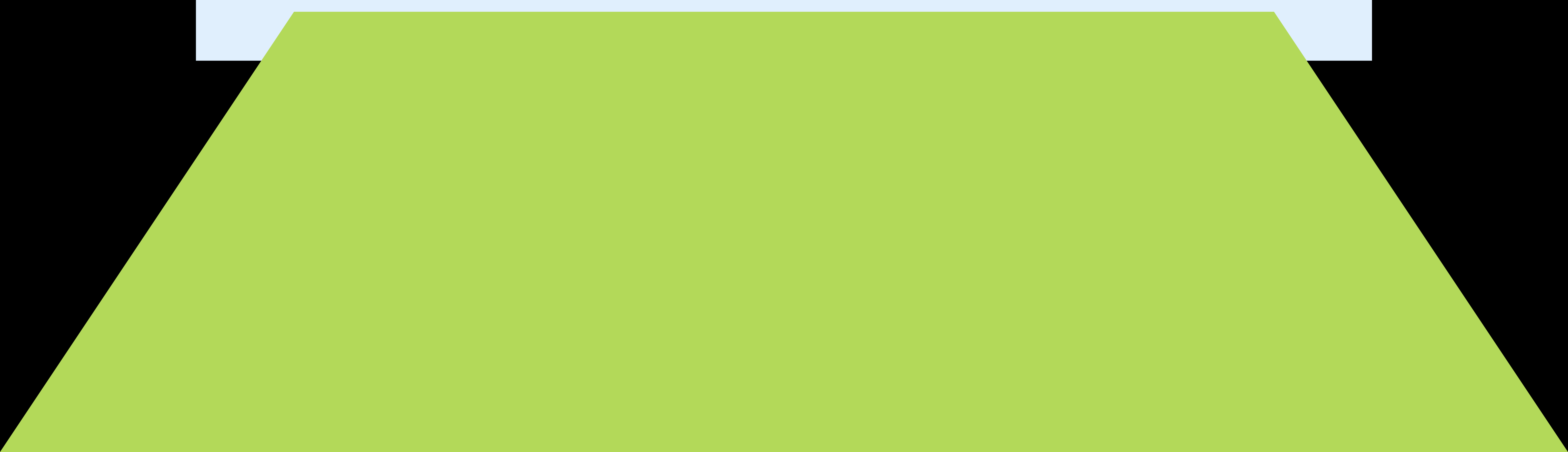
  - Select one region to query each time step
  - Minimize the ply of the regions at time t^*
- Competitive analysis
  - Compare against adversary that knows the full trajectories of the points in advance

TECHNICAL STUFF

PROJECTED REGIONS



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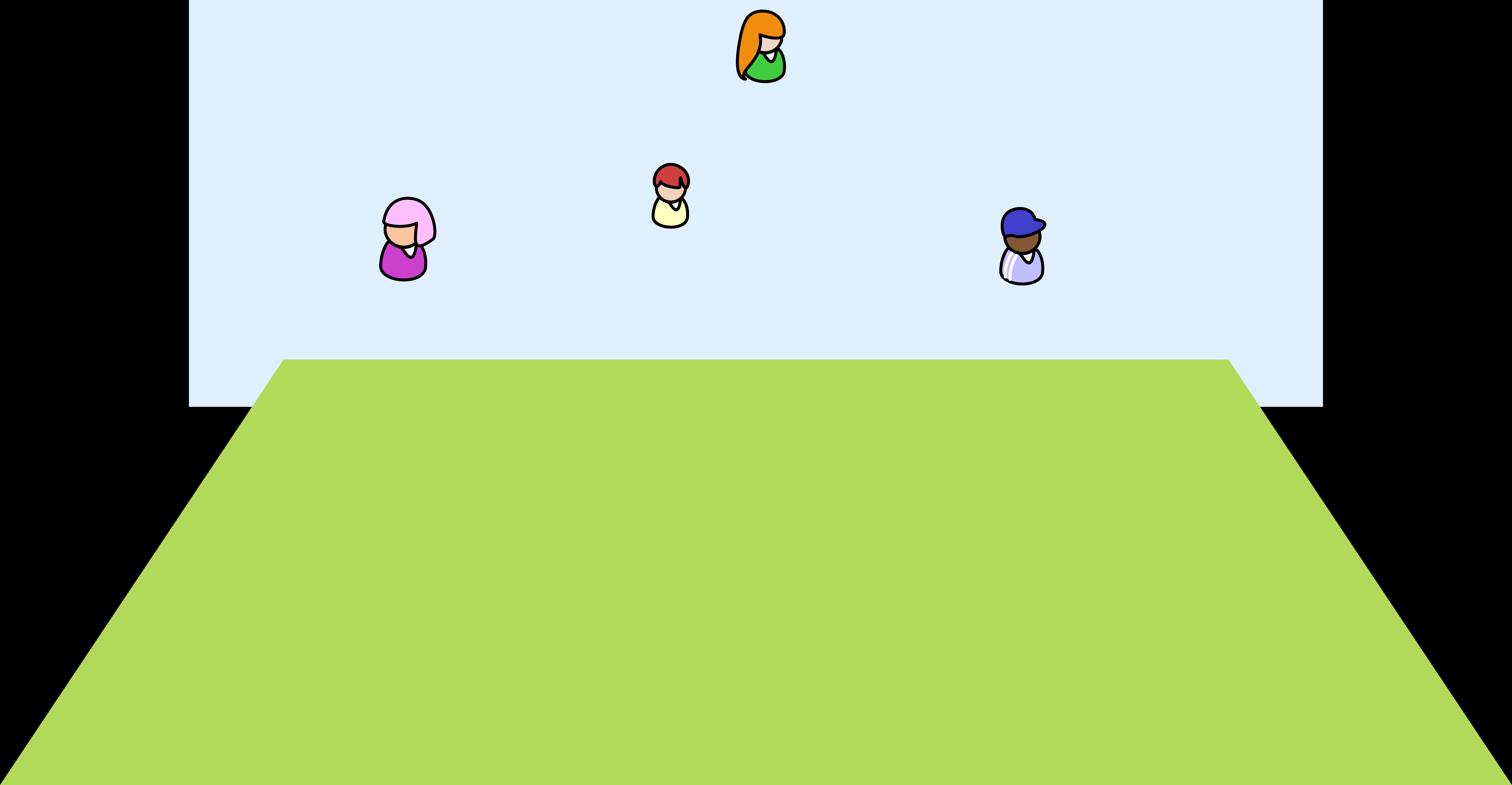

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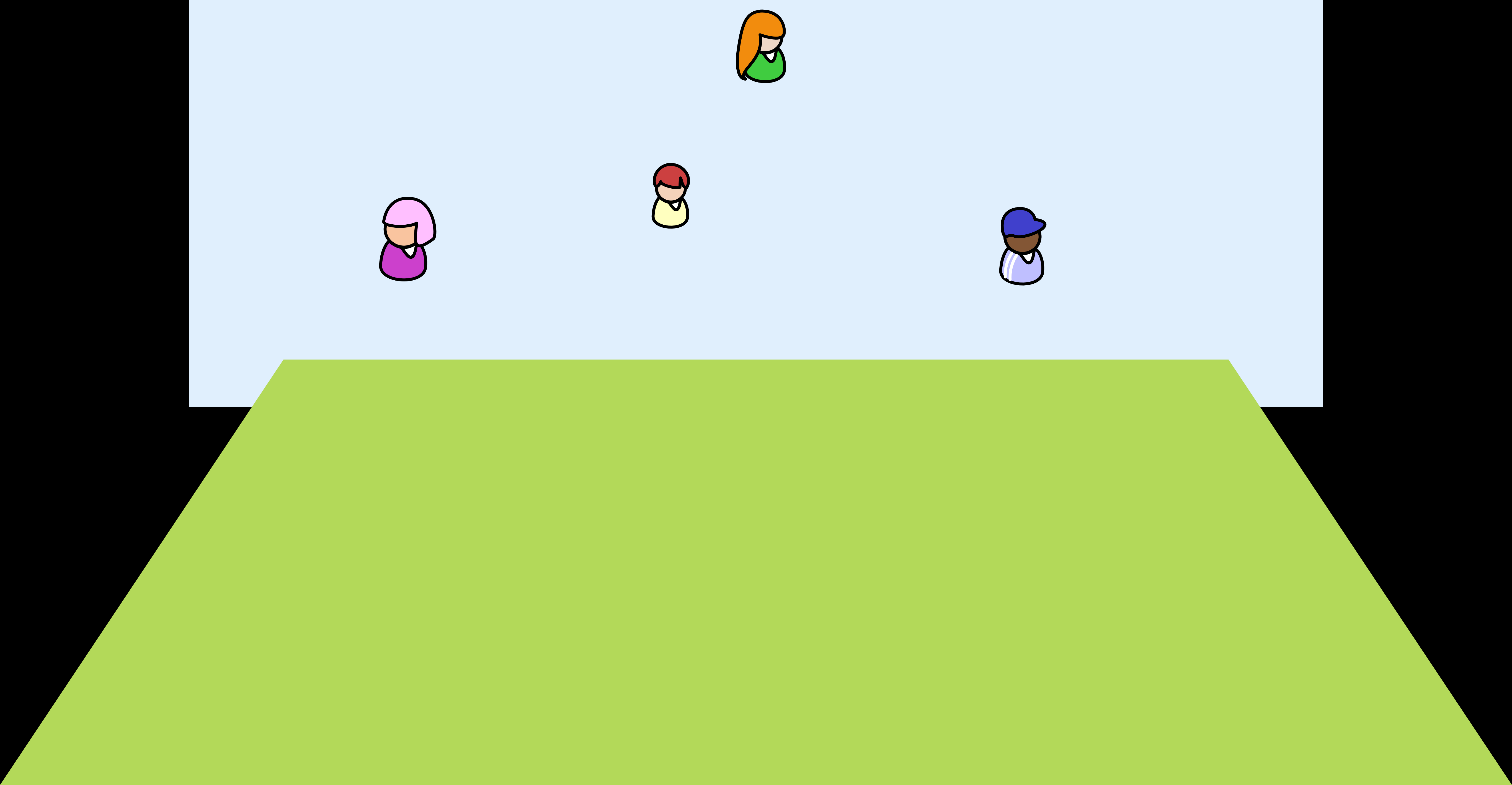

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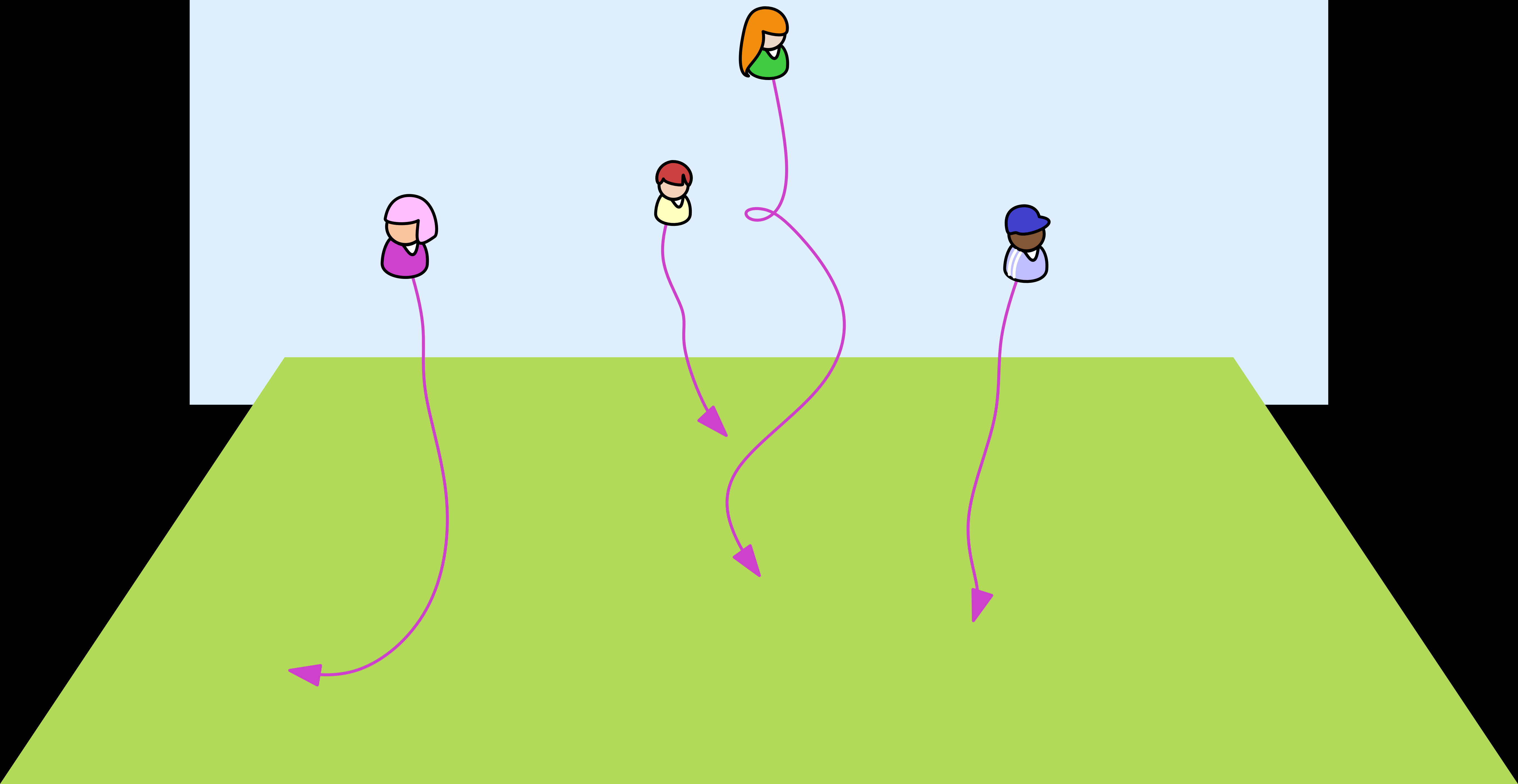

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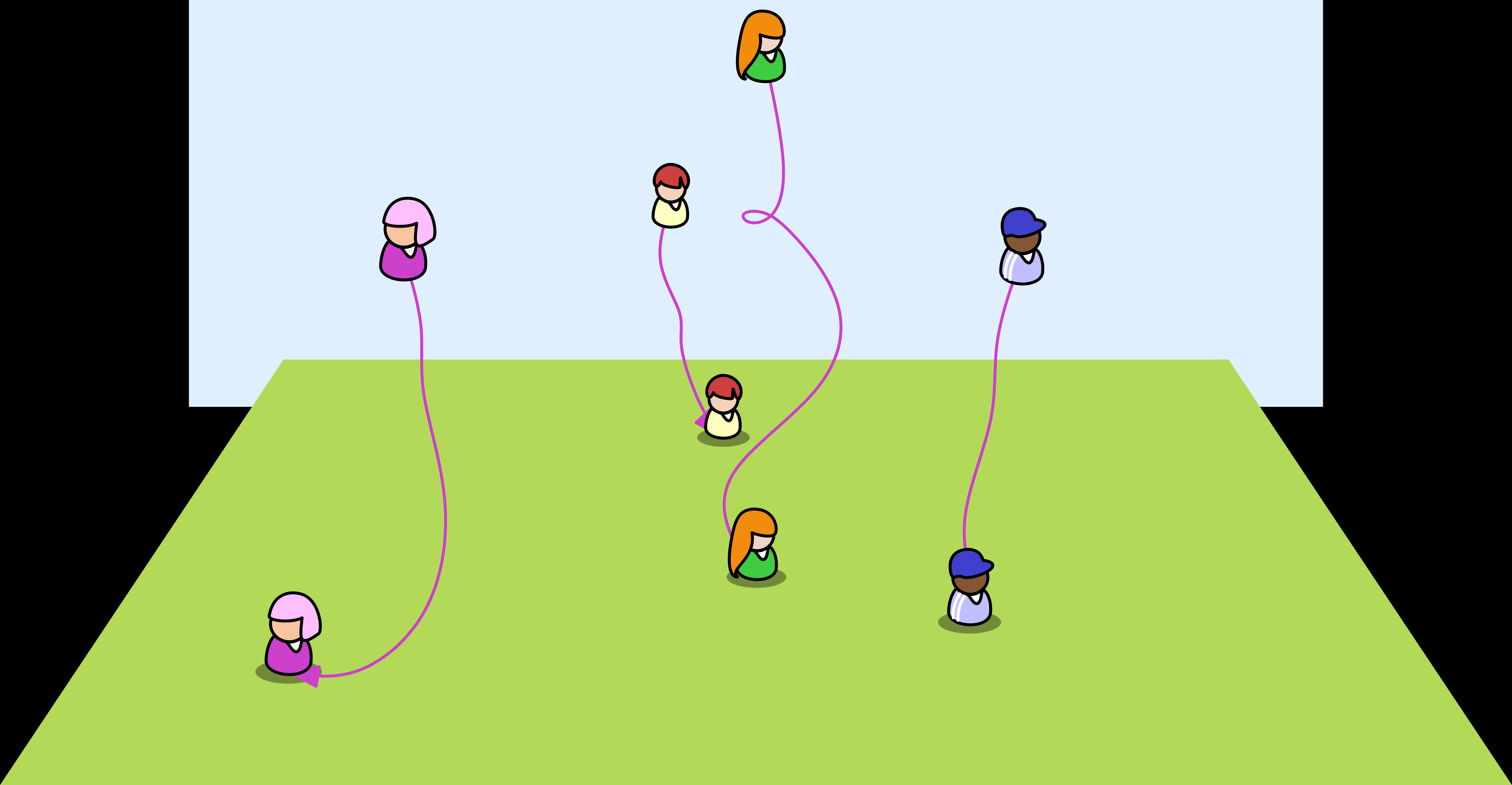

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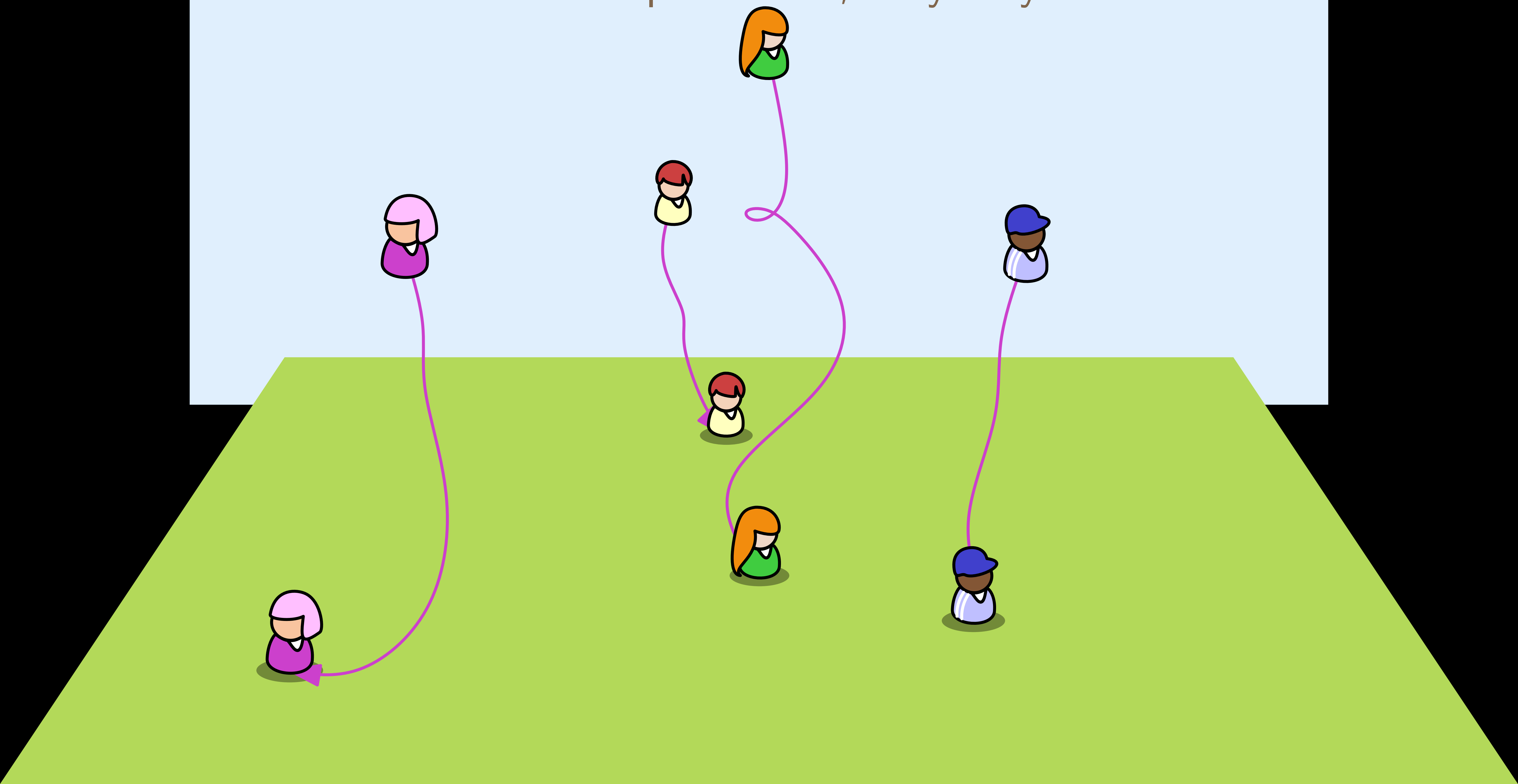

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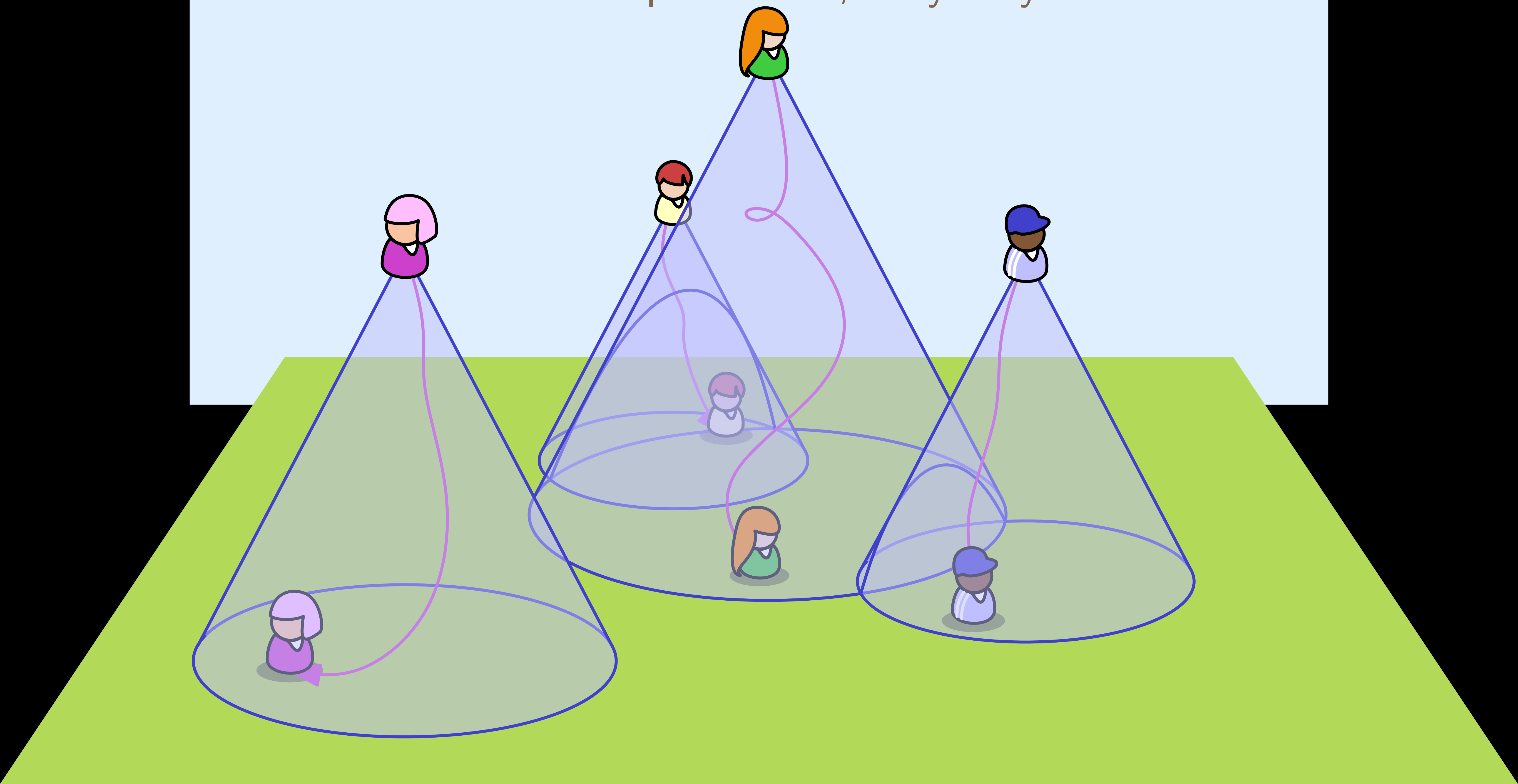

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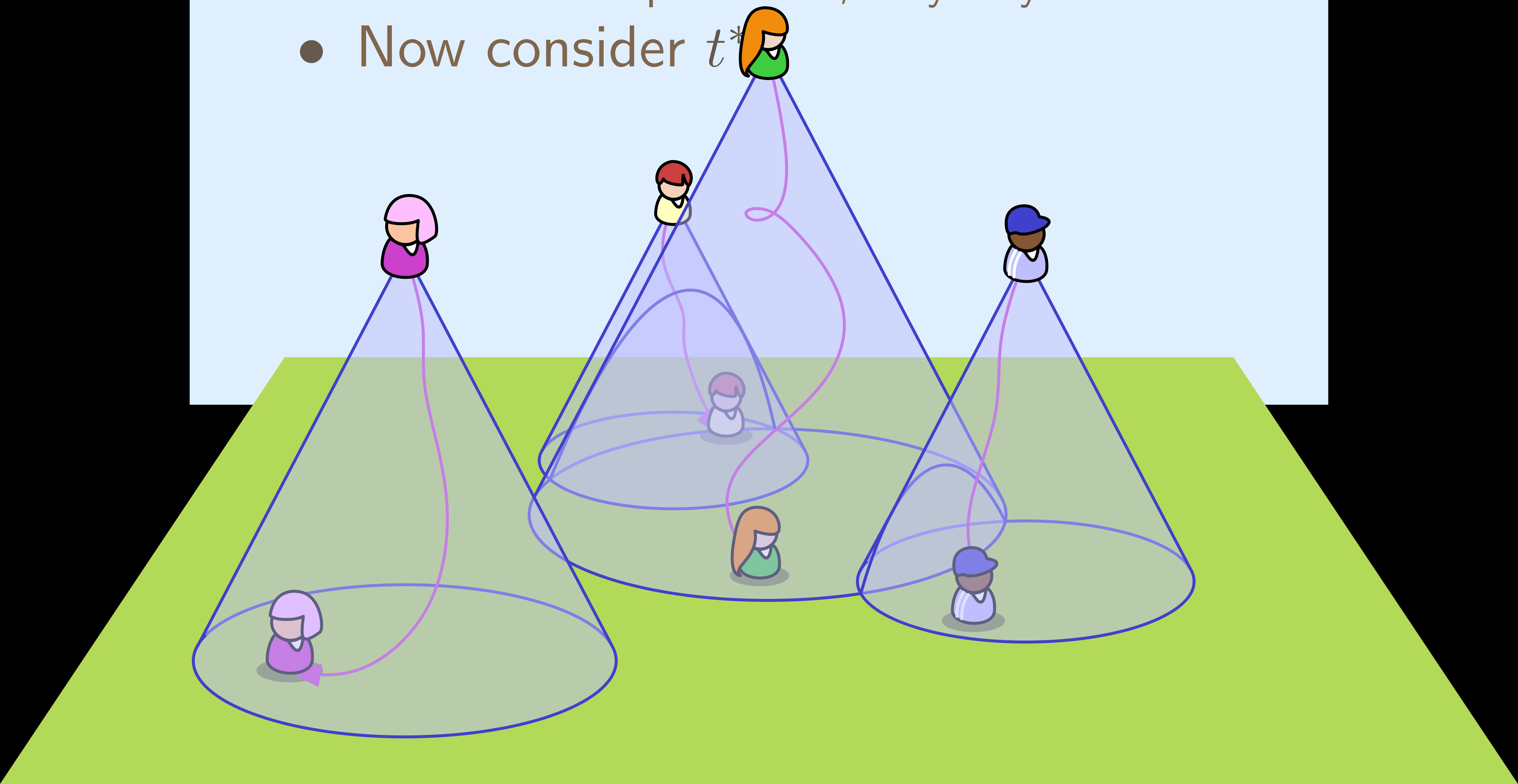
- Now consider t^*



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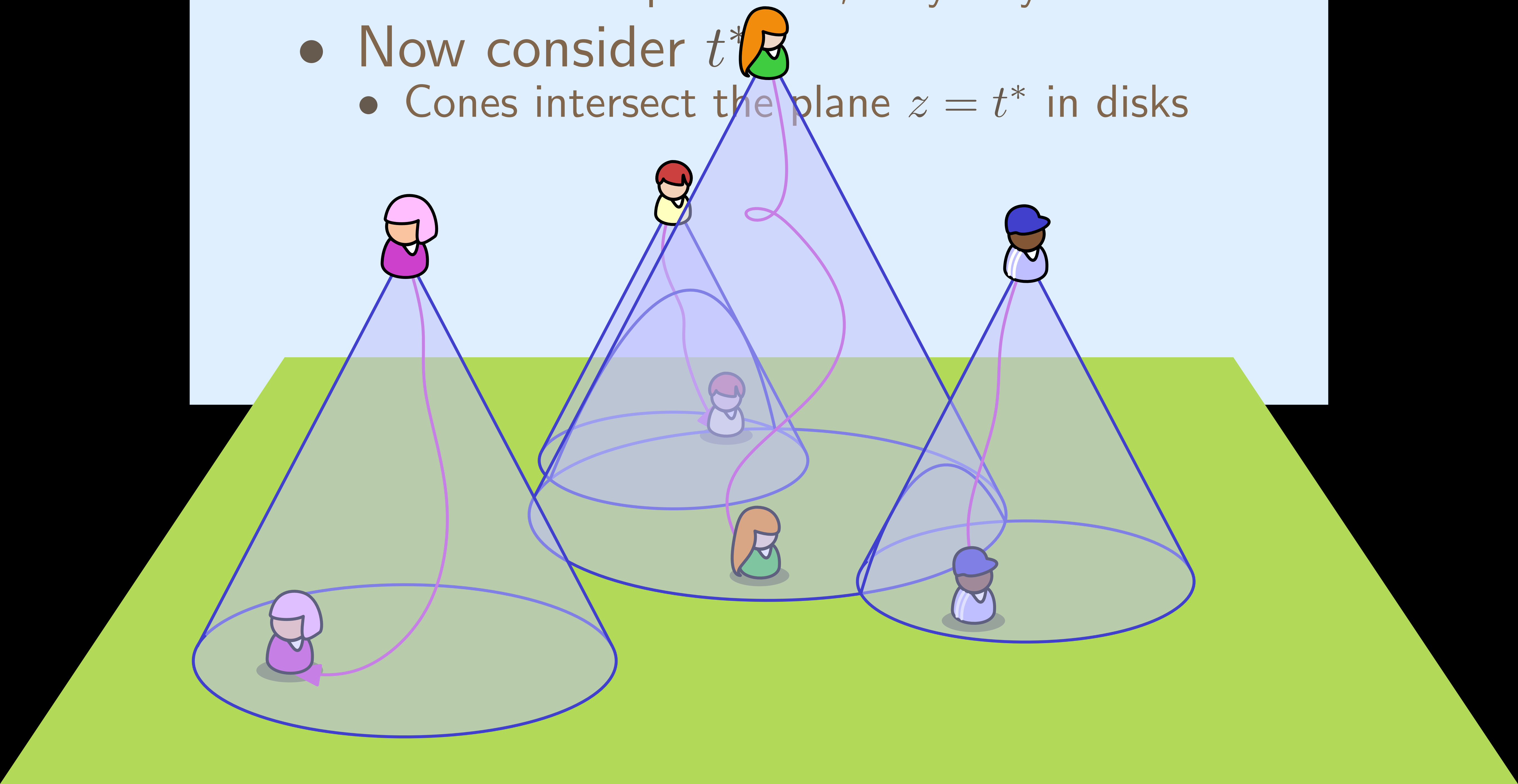
- Now consider t^*
  - Cones intersect the plane $z = t^*$ in disks



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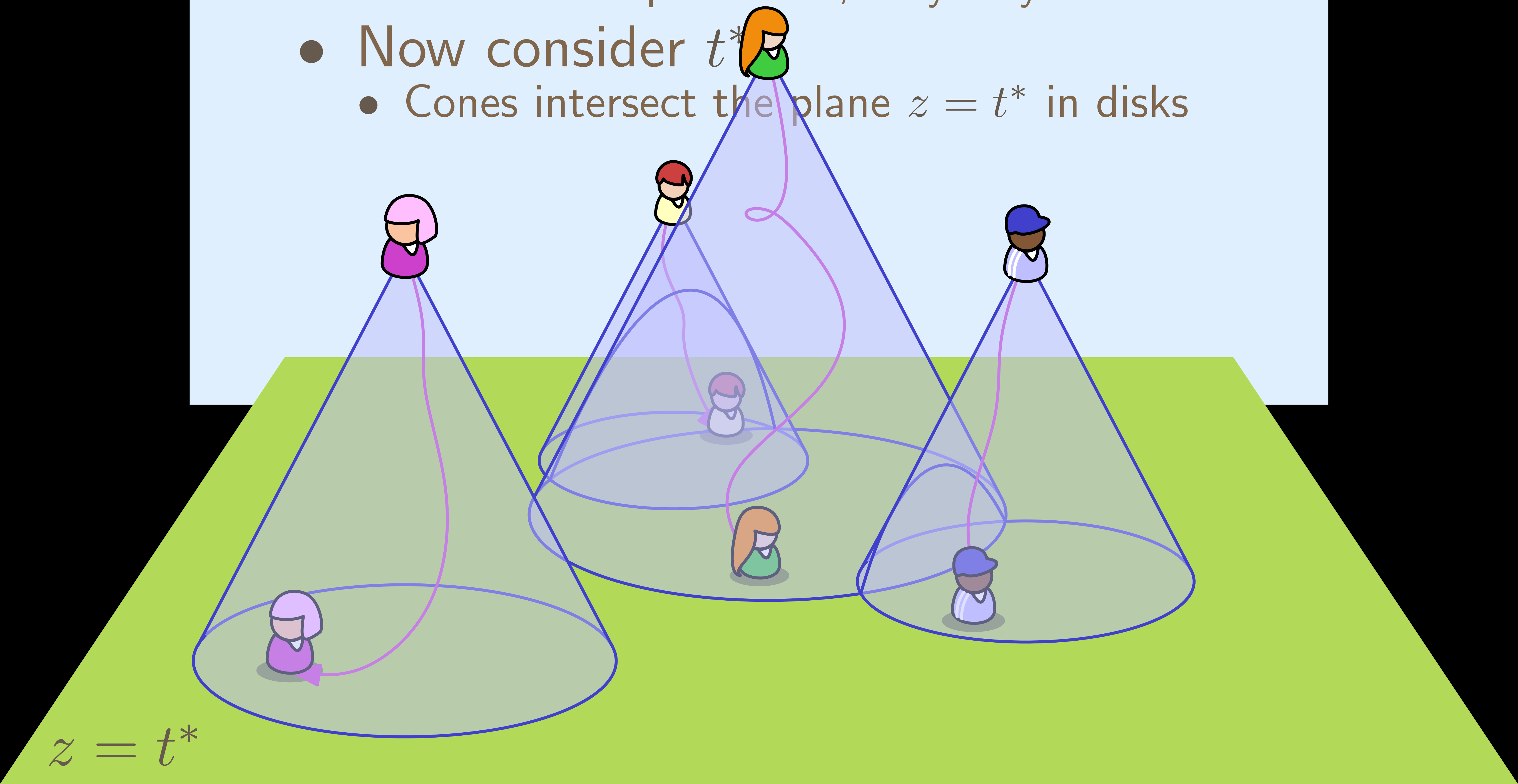

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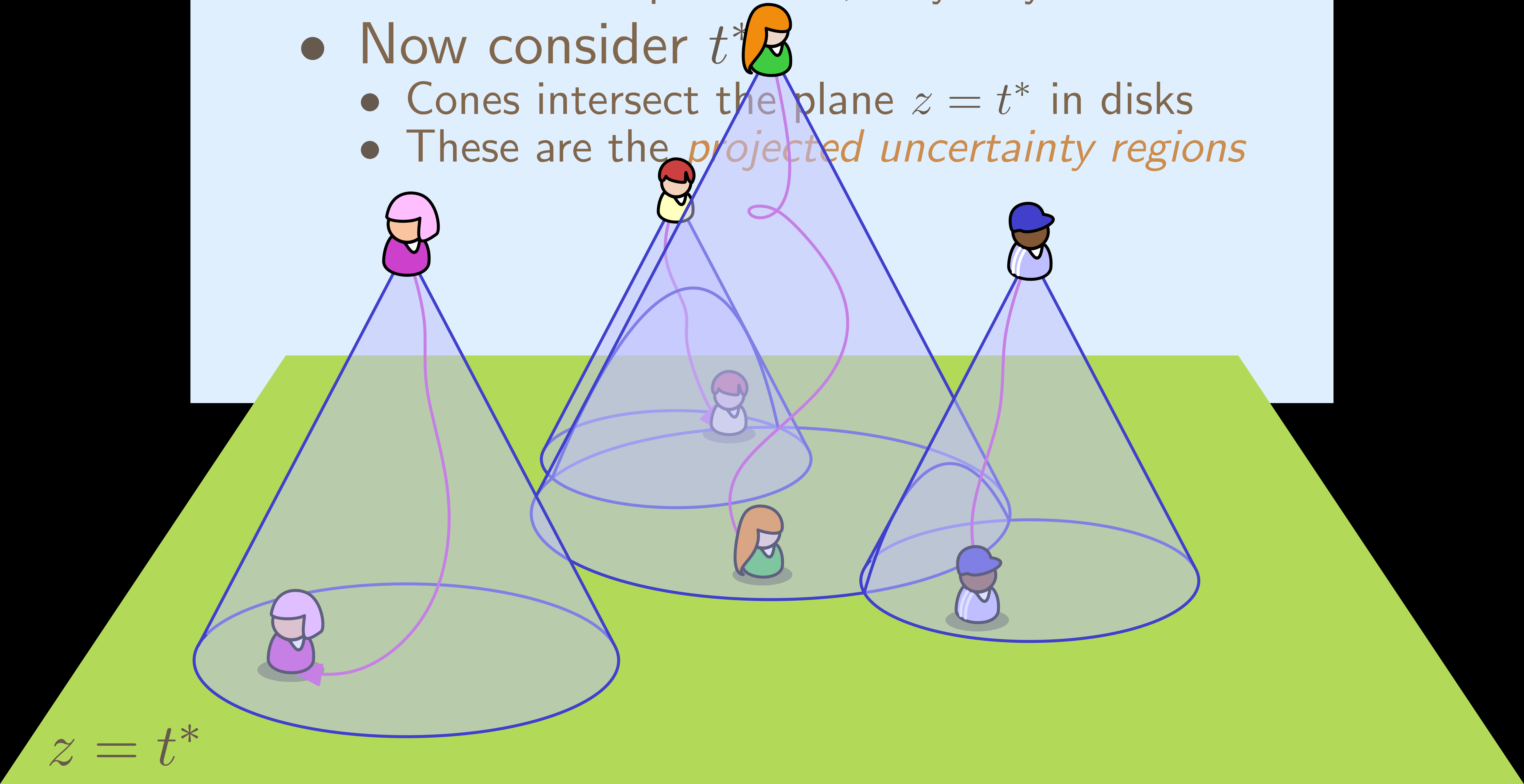

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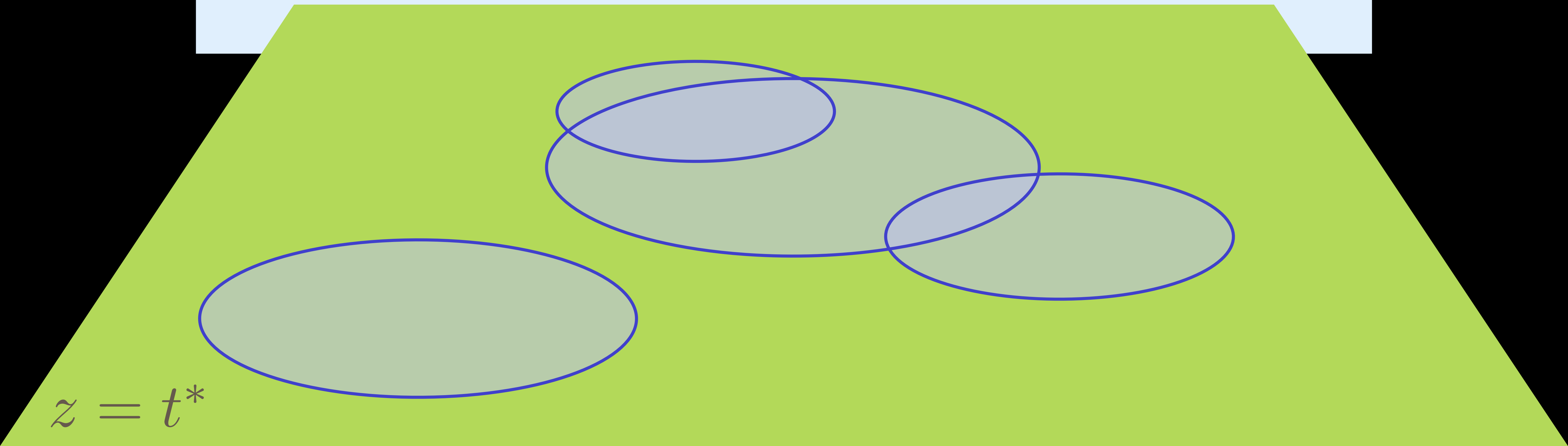

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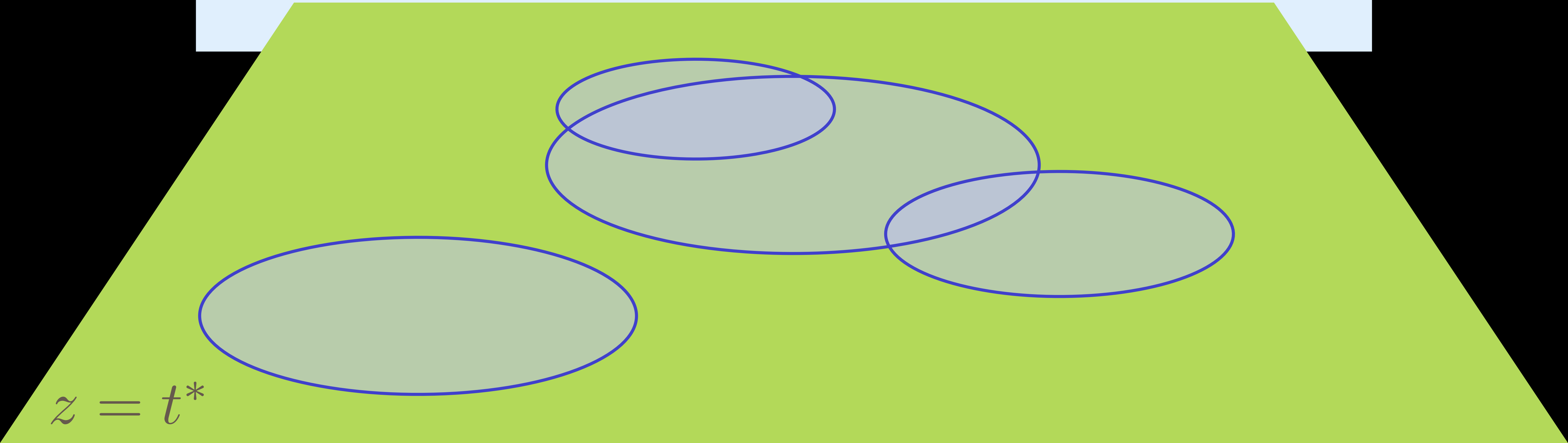
  - Projected regions never grow, and they shrink to radius $t^* - t$ when we query them



STRATEGY ATTEMPT 1

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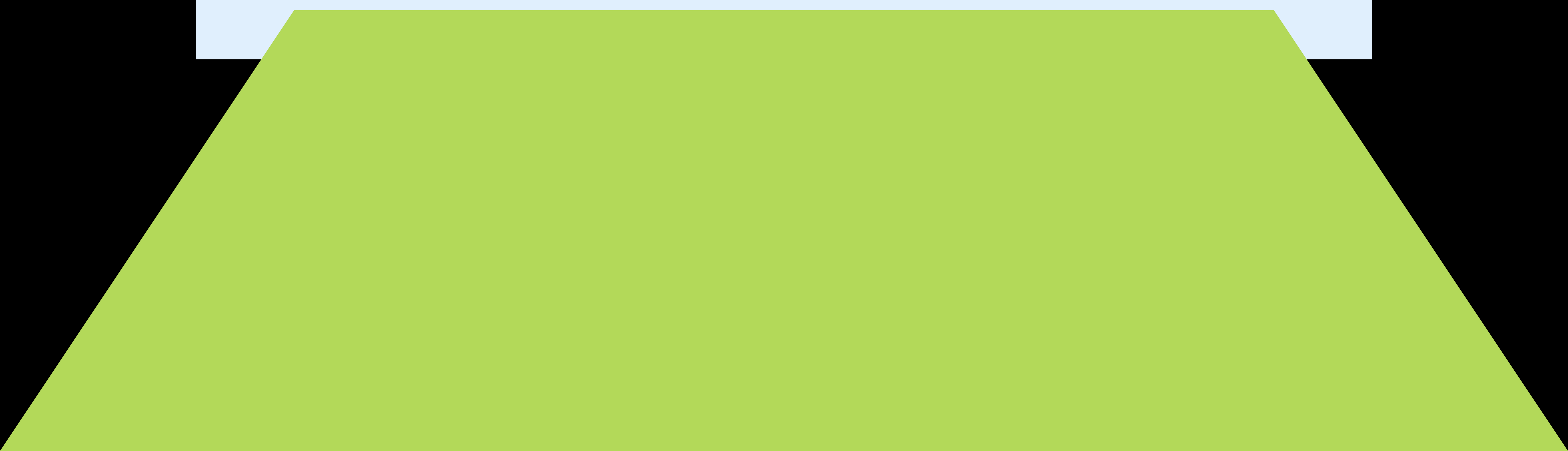

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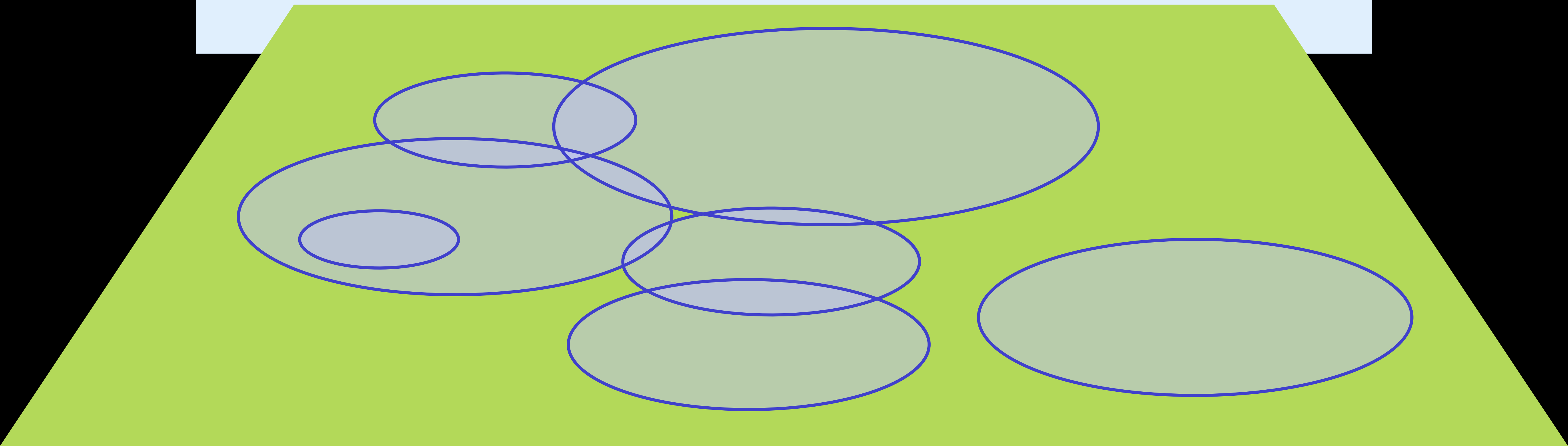

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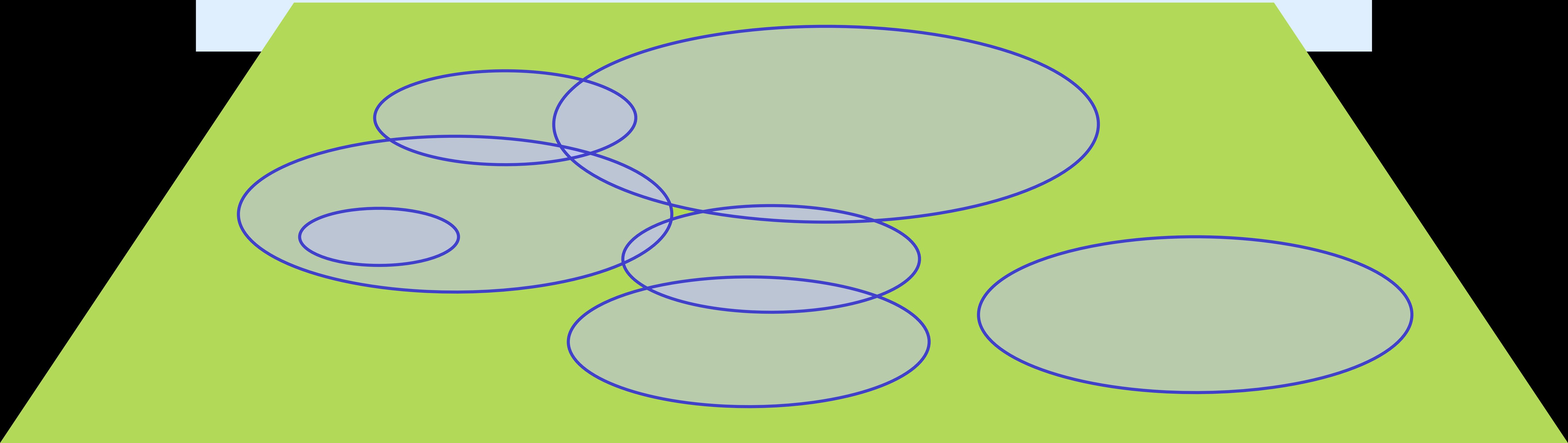

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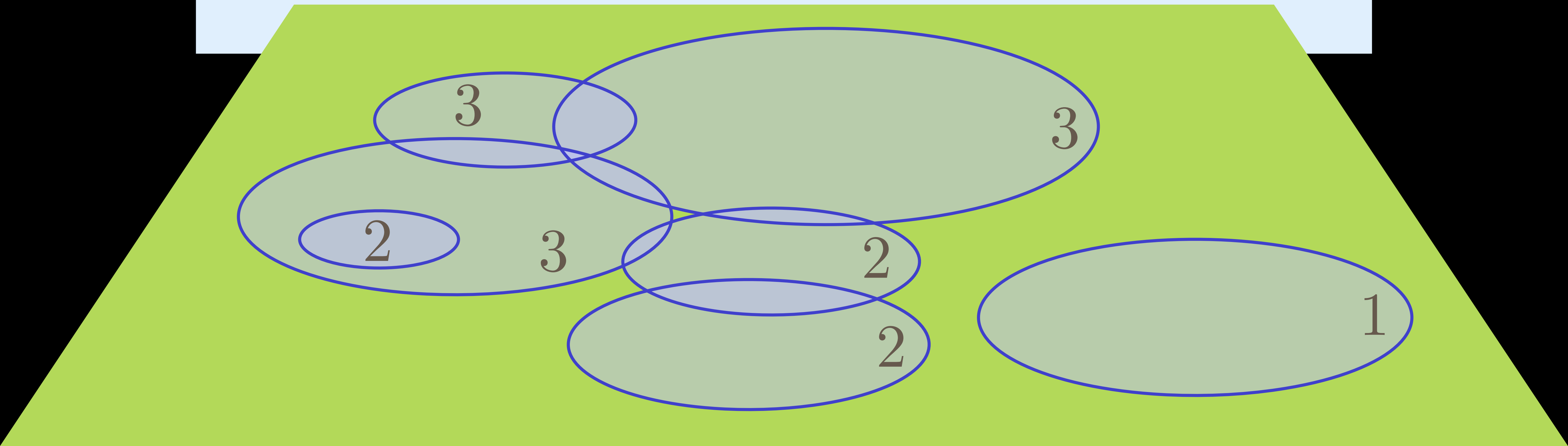

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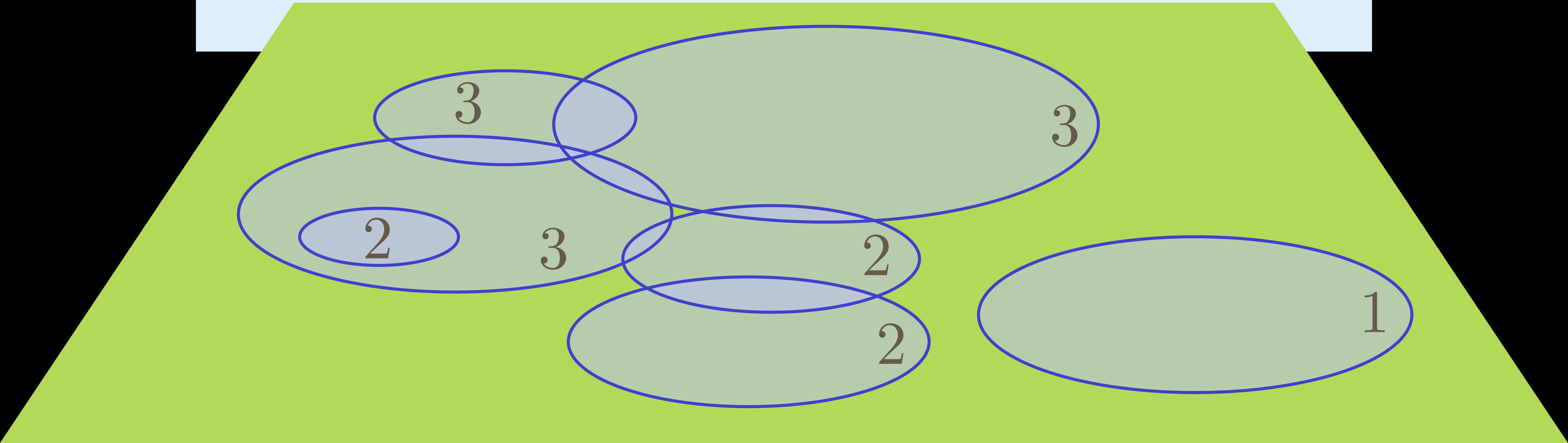

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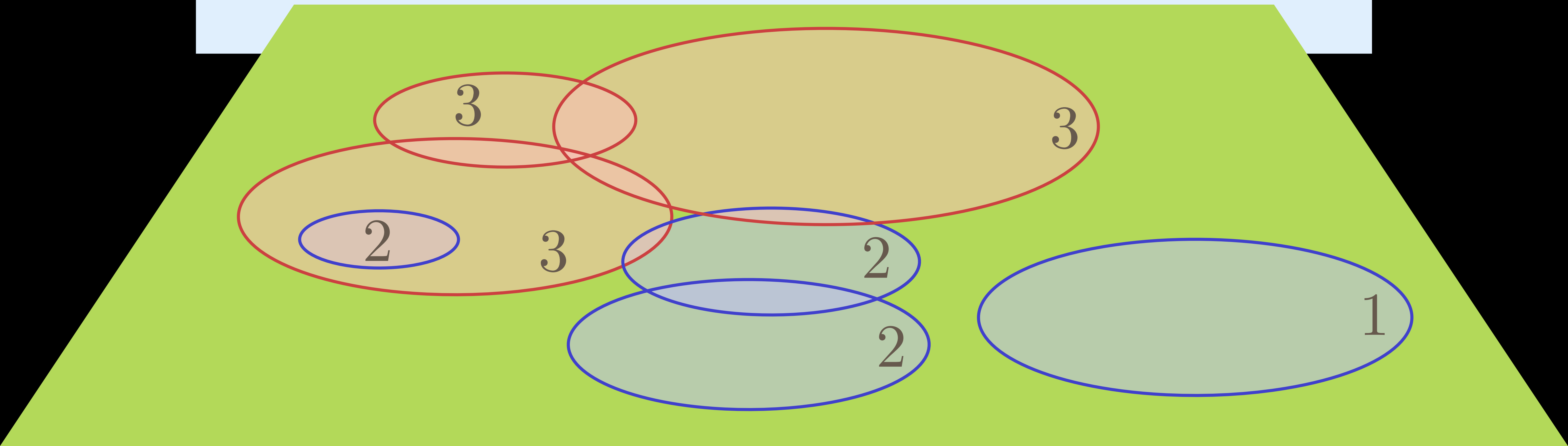

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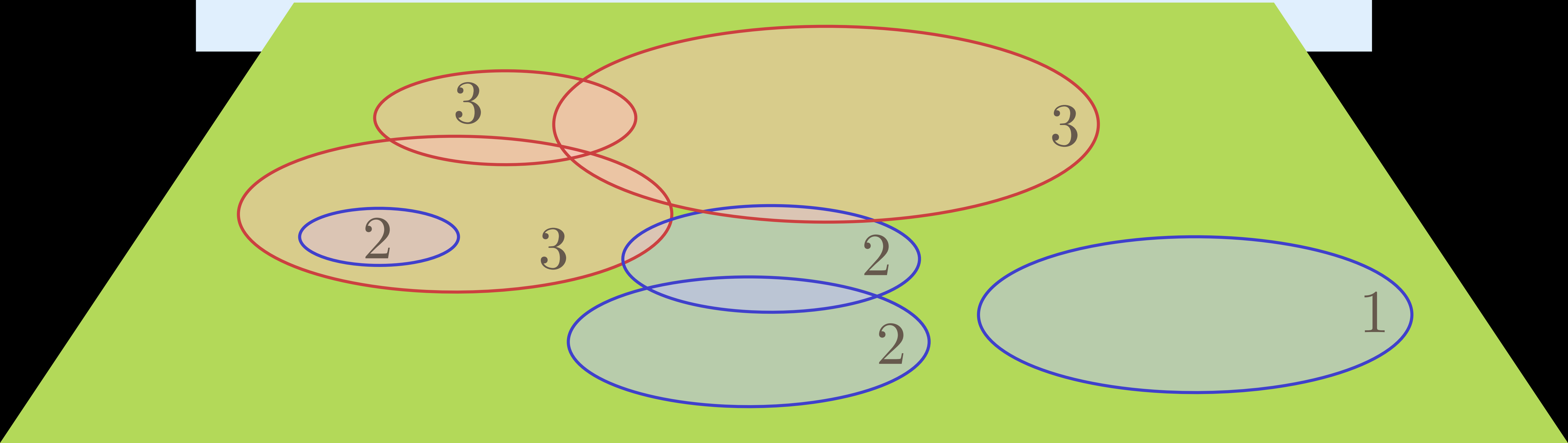

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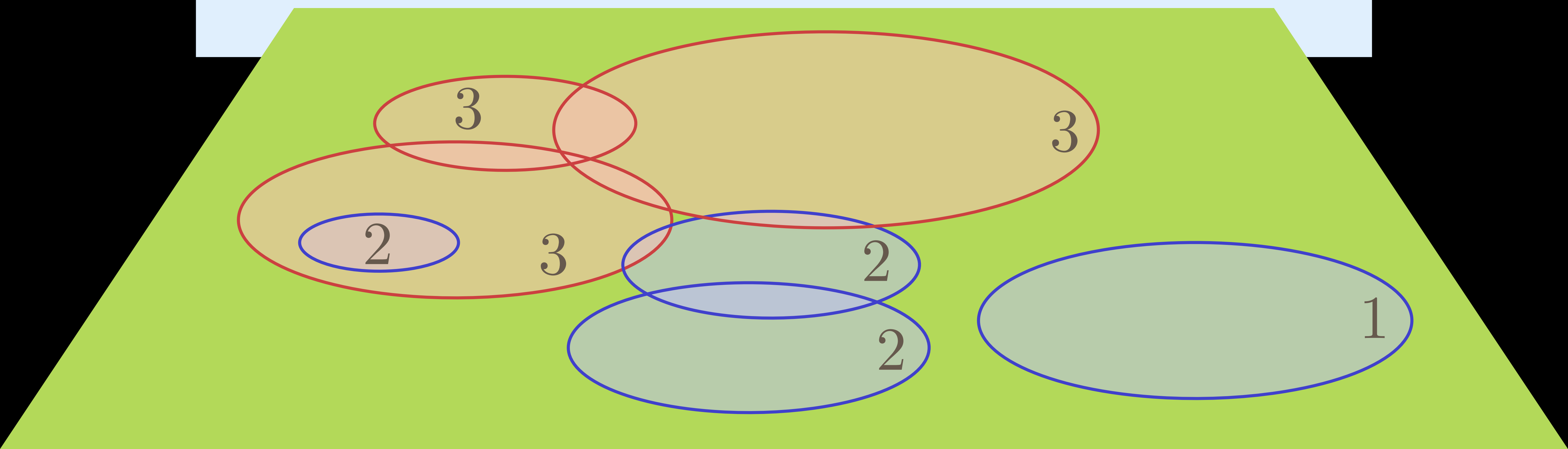

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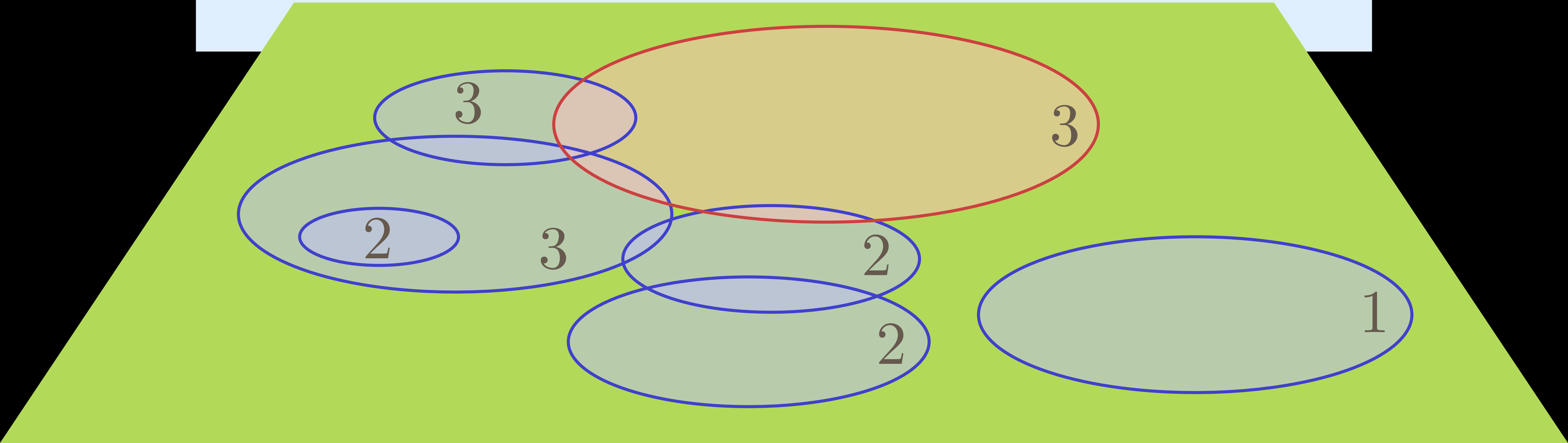

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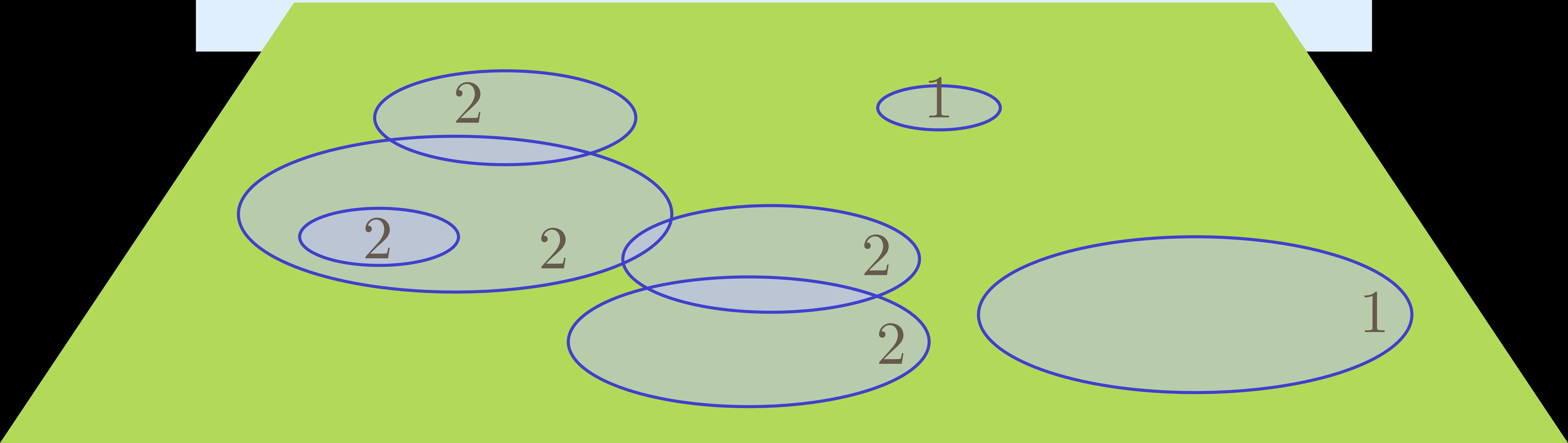

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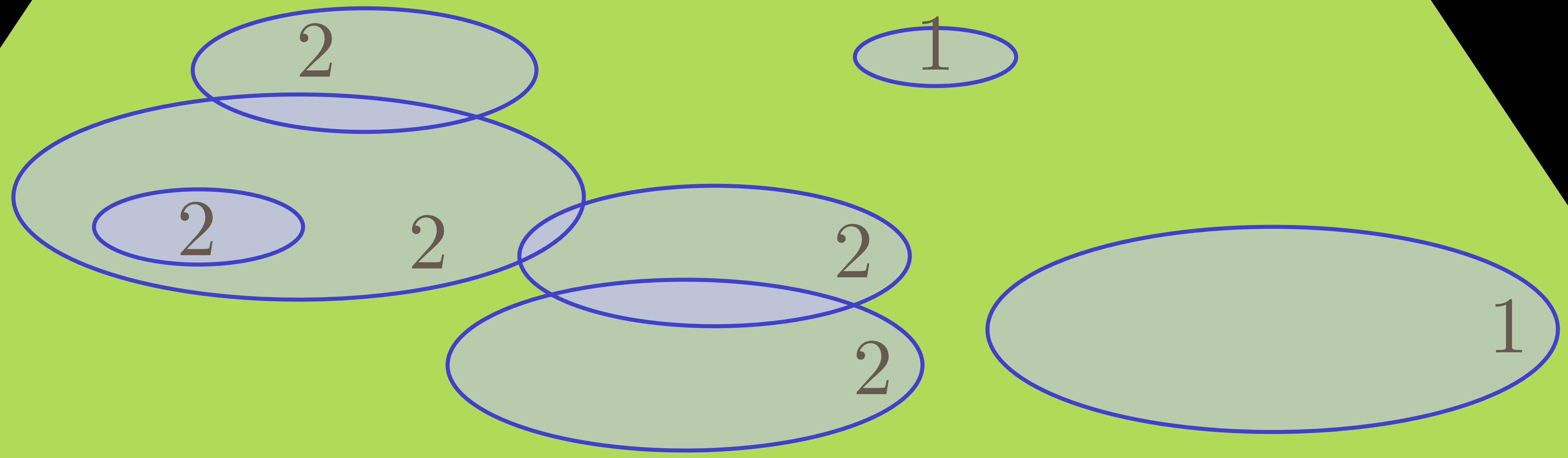

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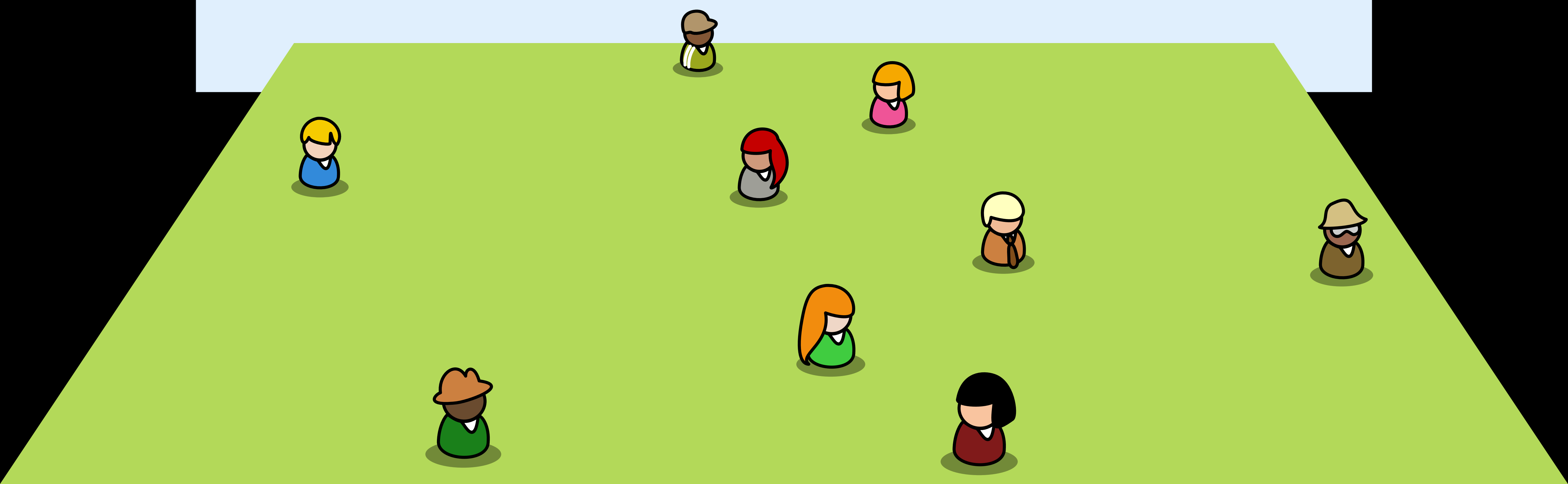

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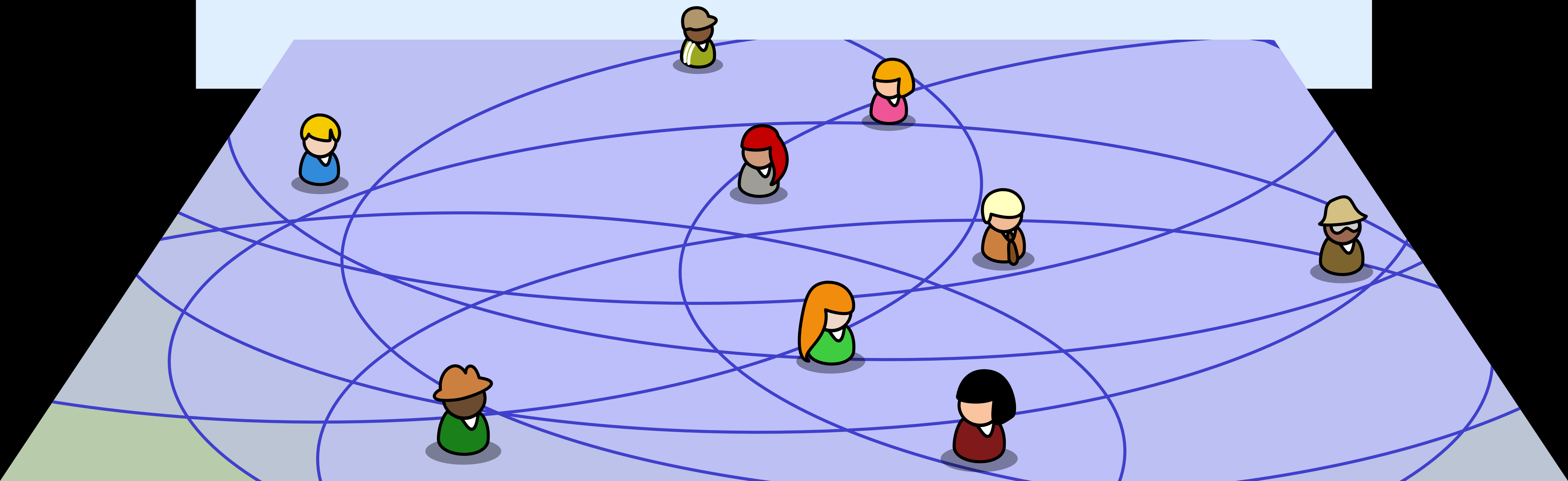

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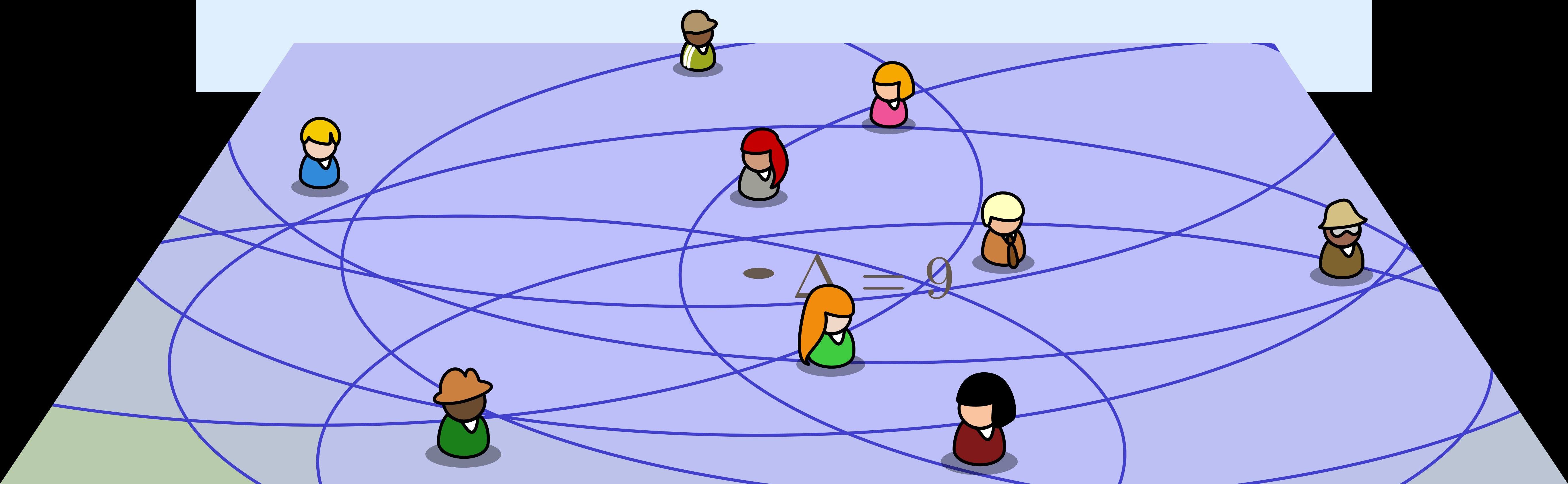

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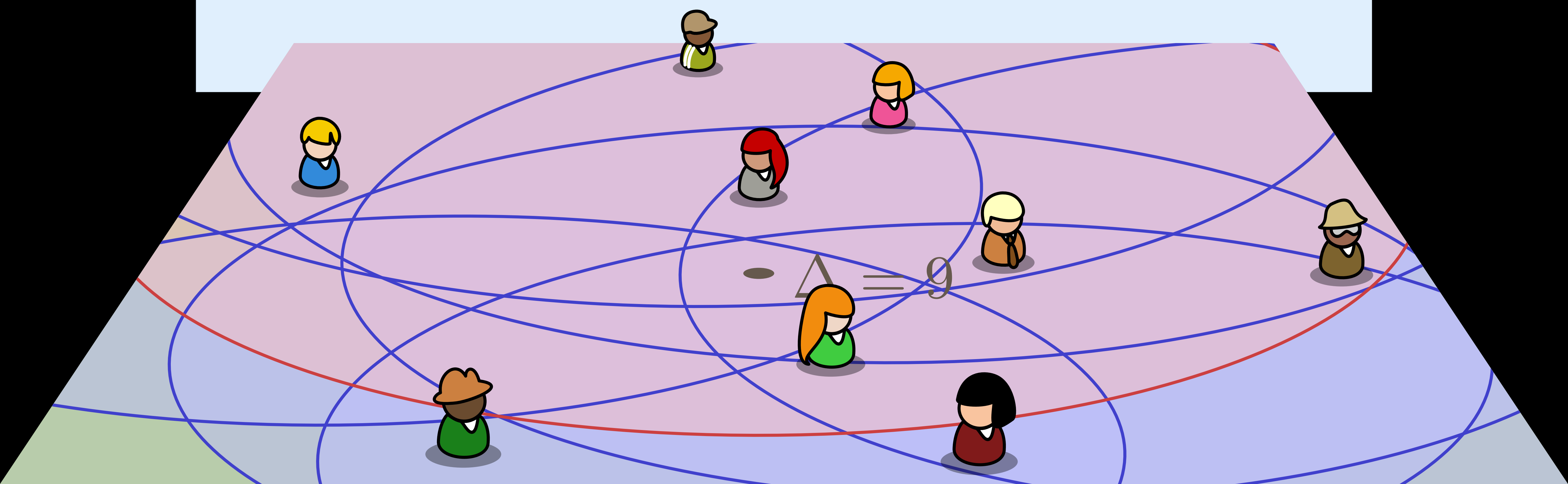

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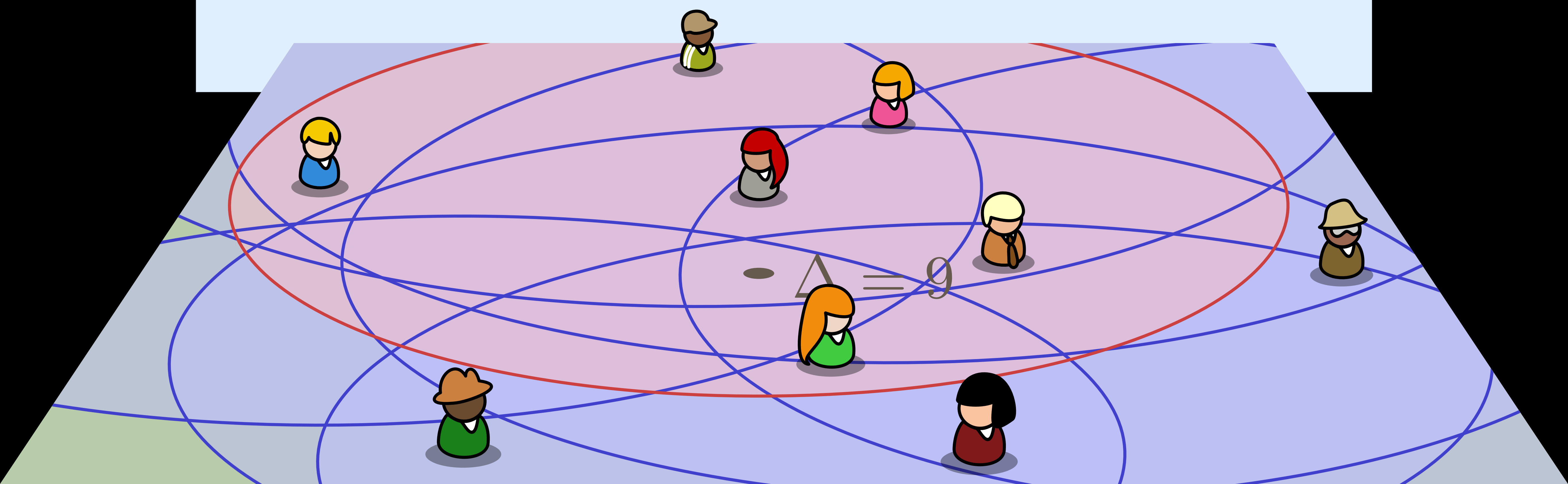

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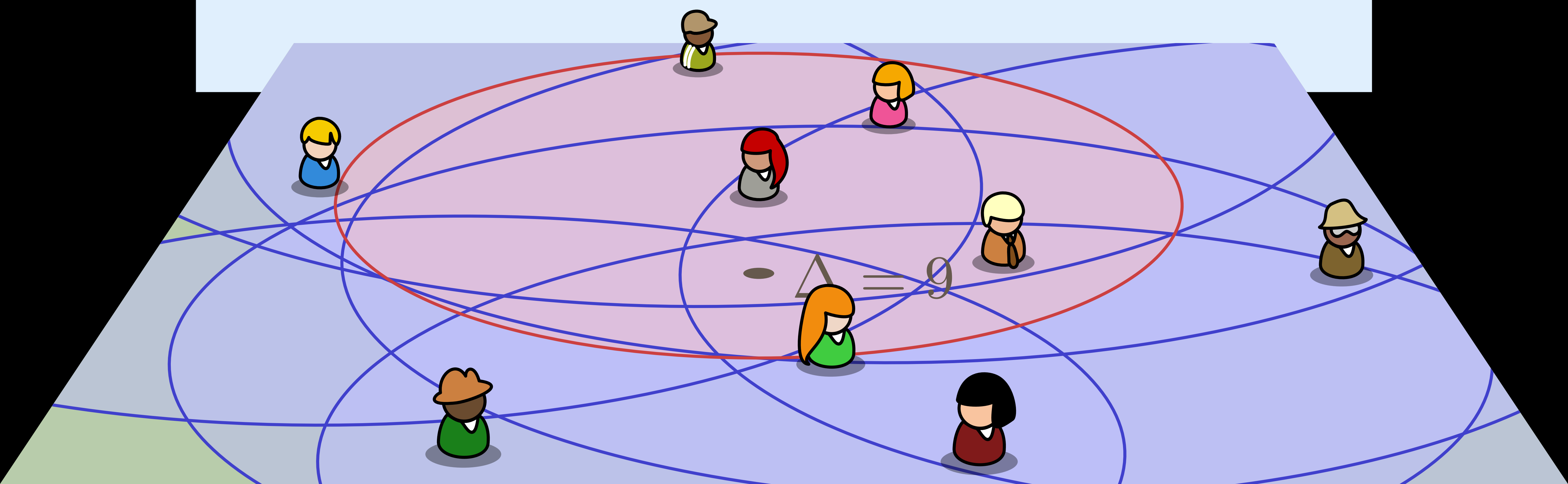

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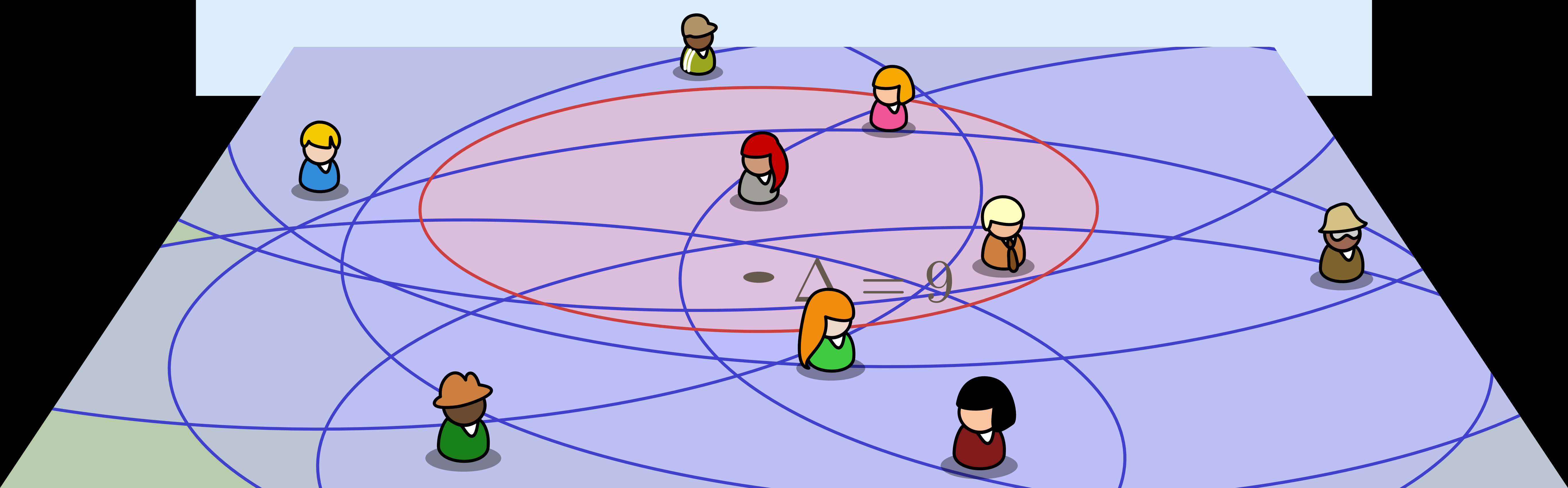

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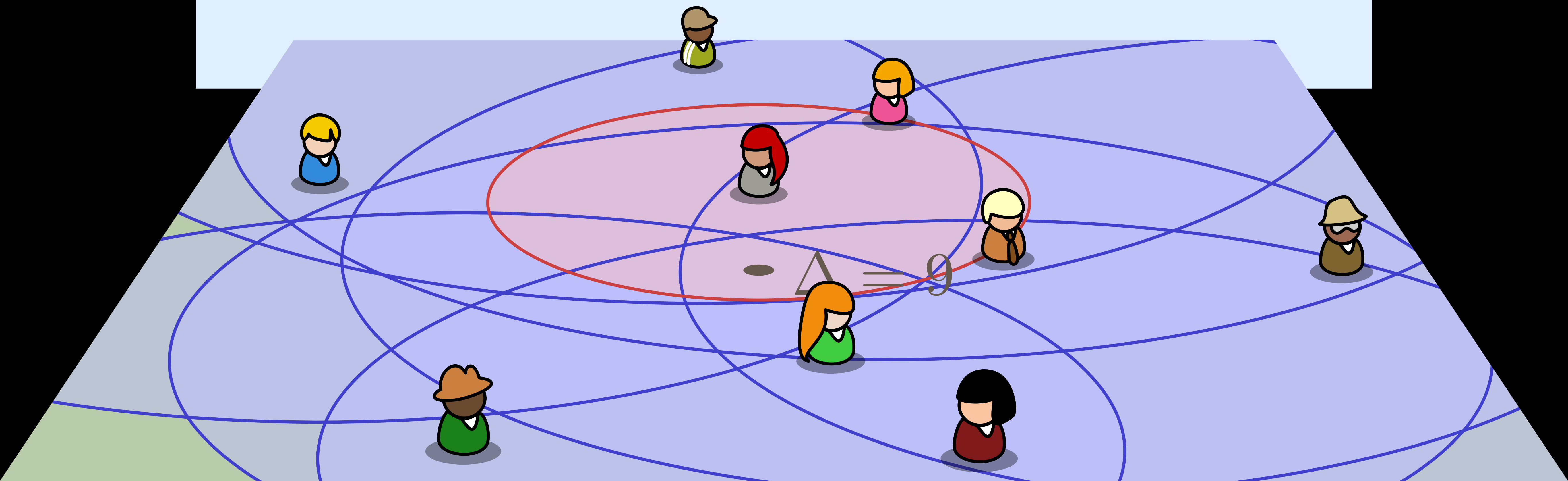

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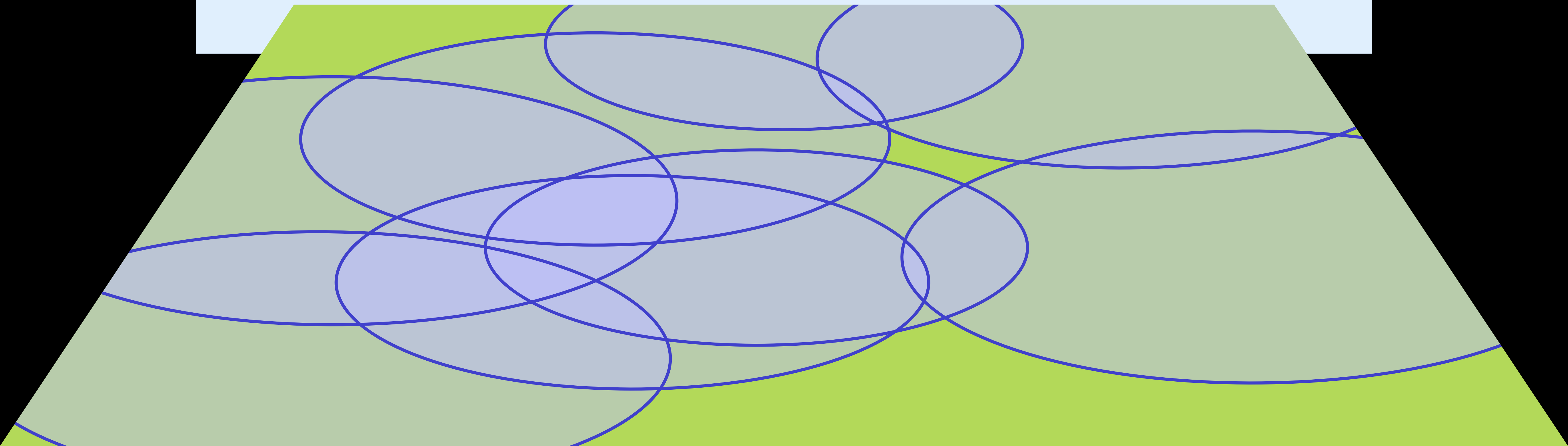

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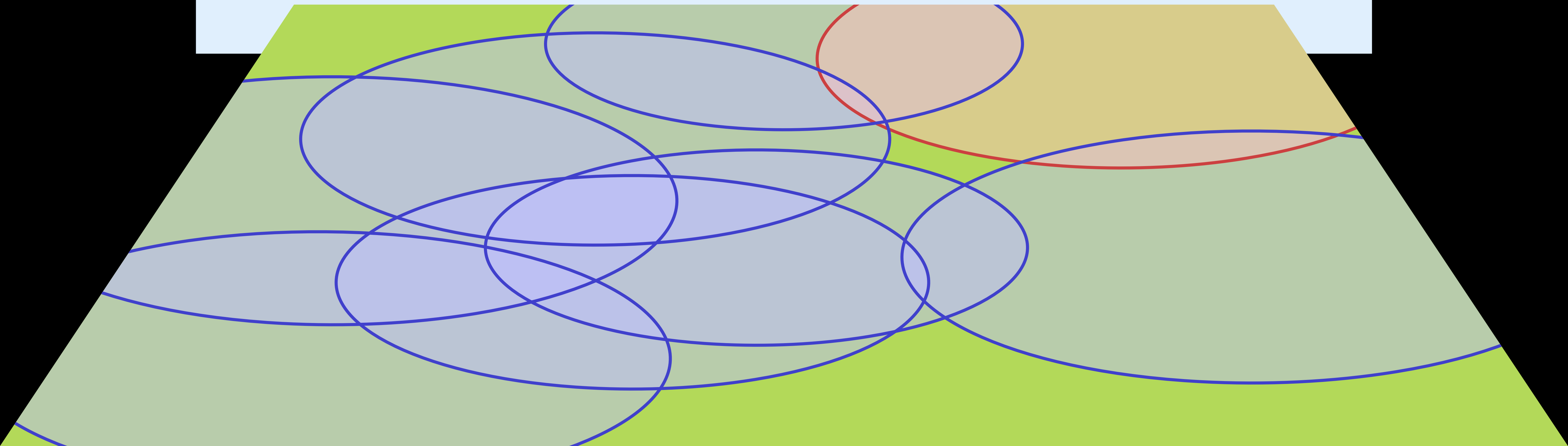

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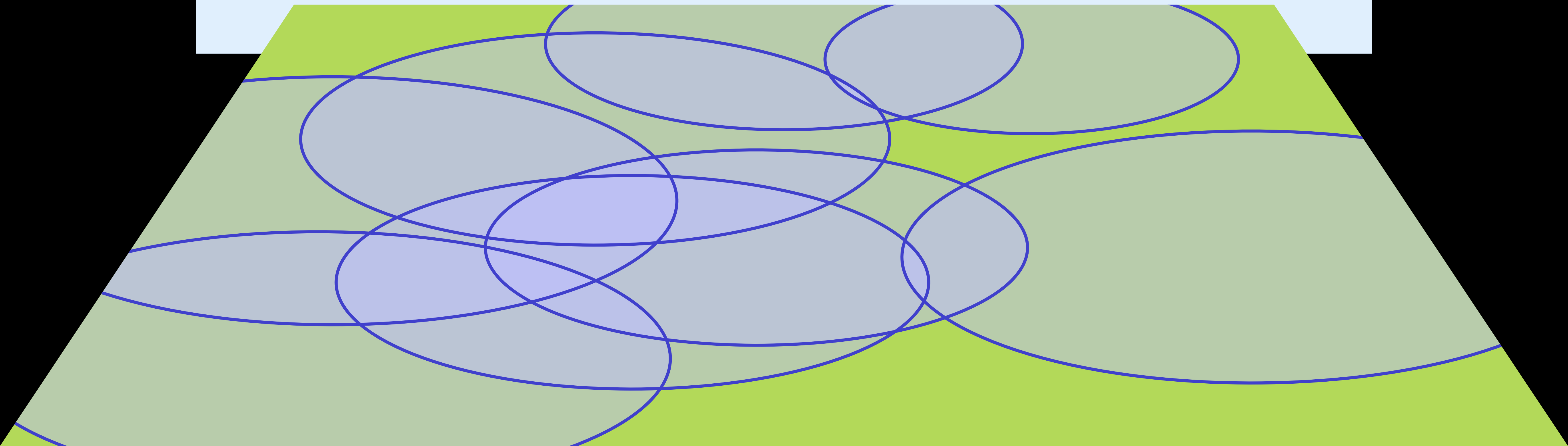

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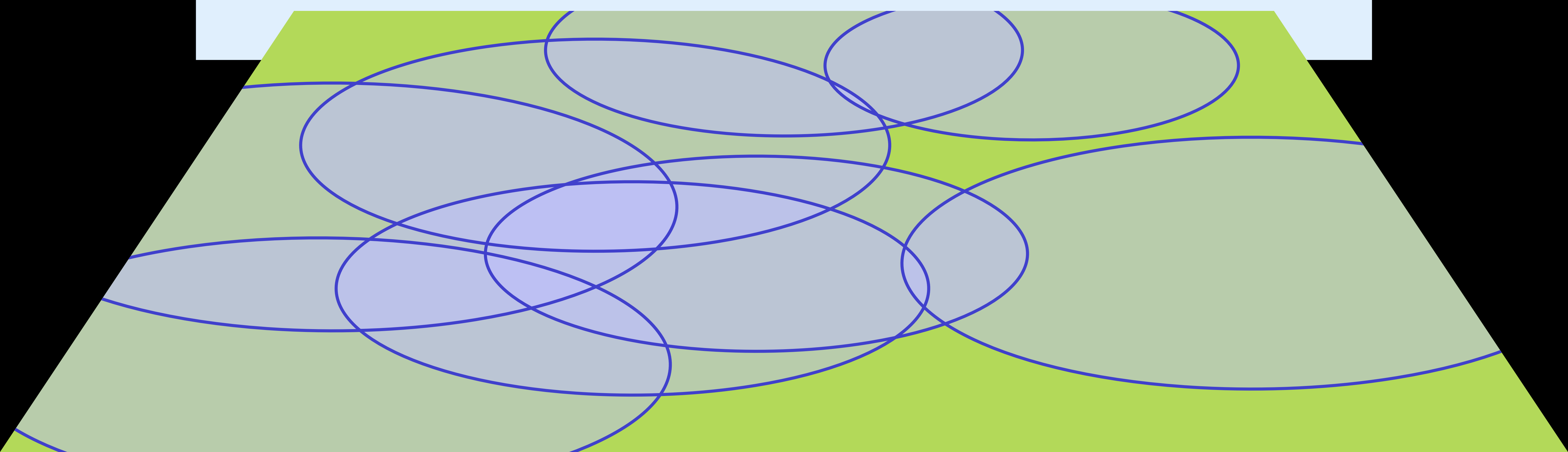

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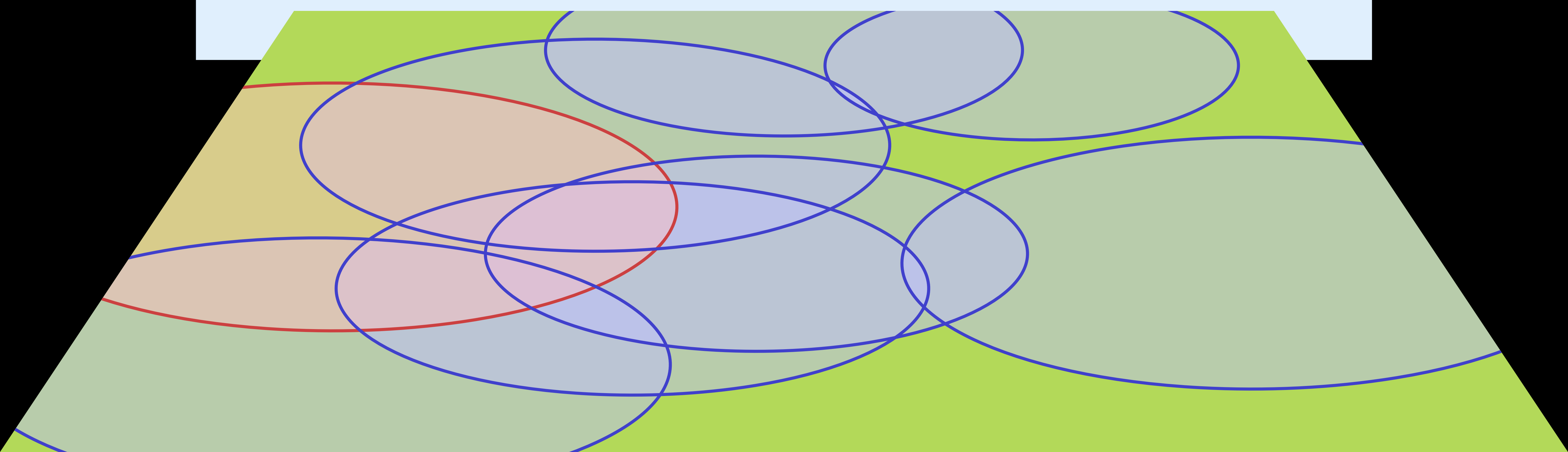

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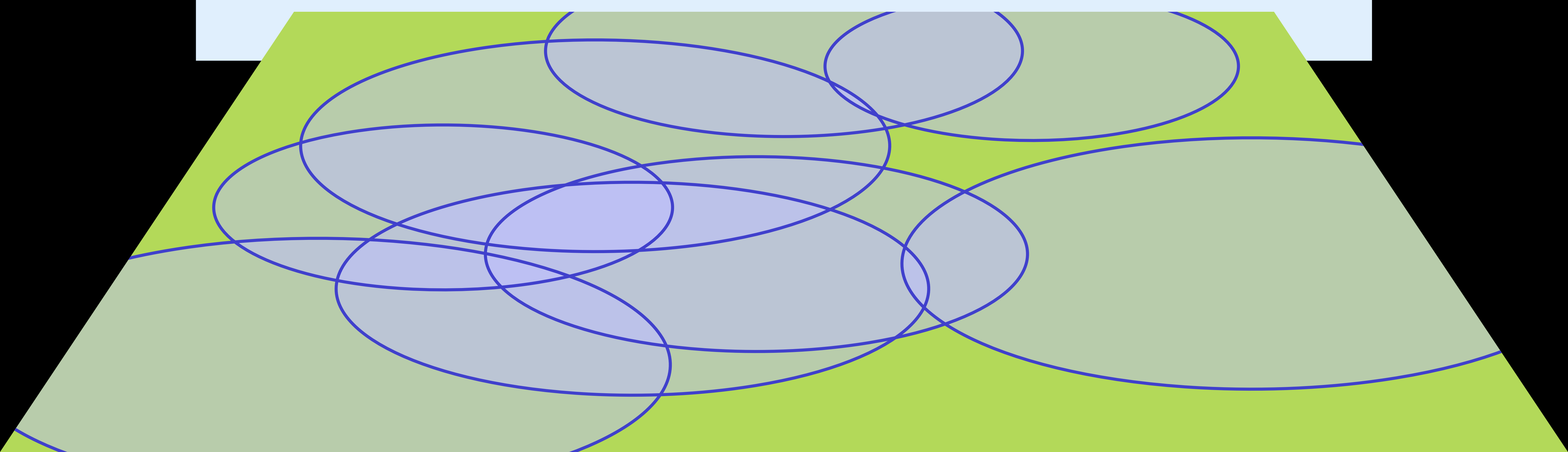

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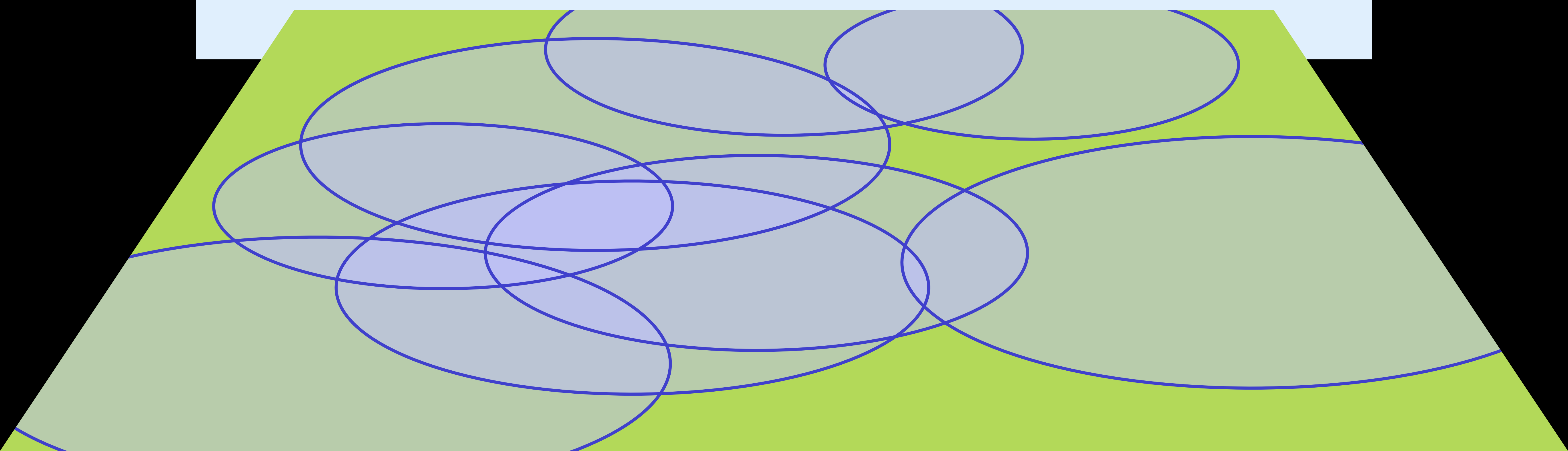

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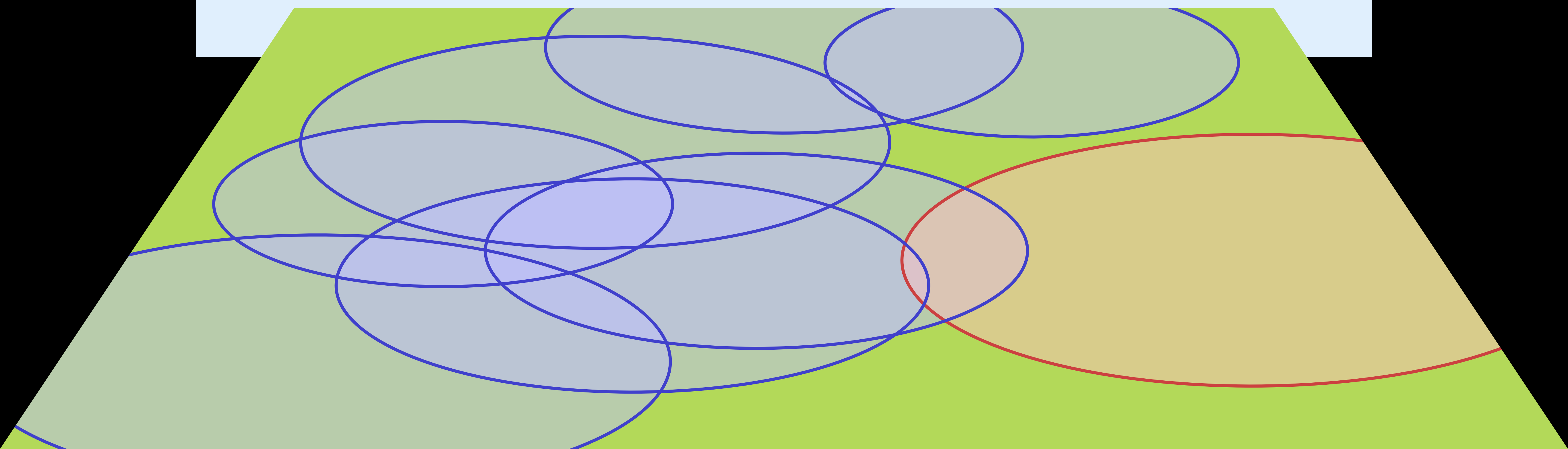

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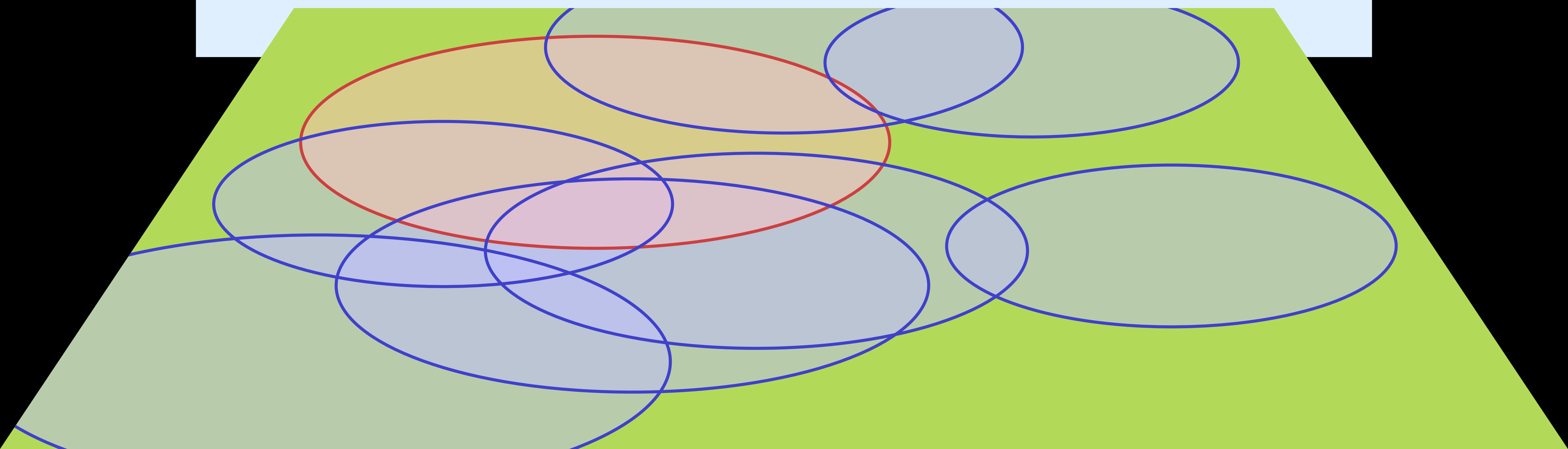

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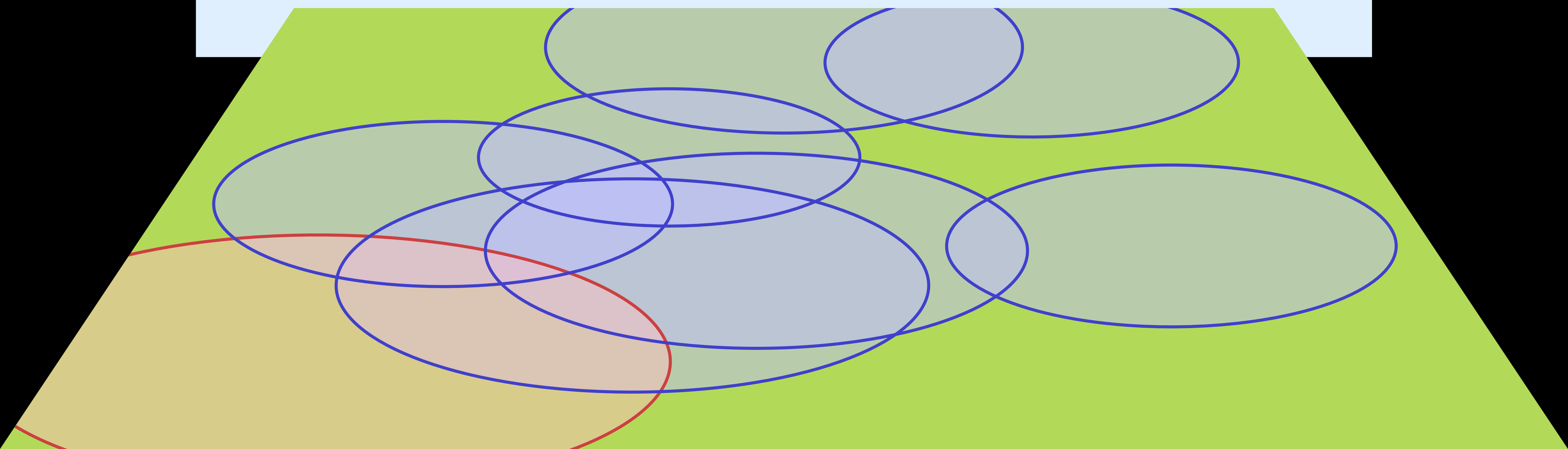

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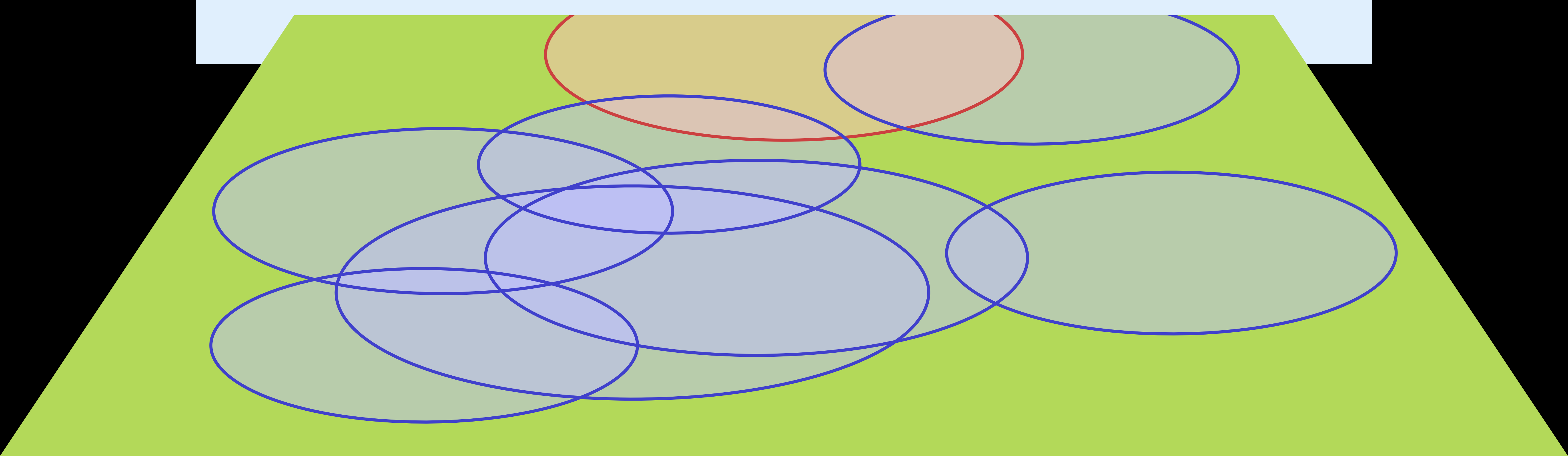

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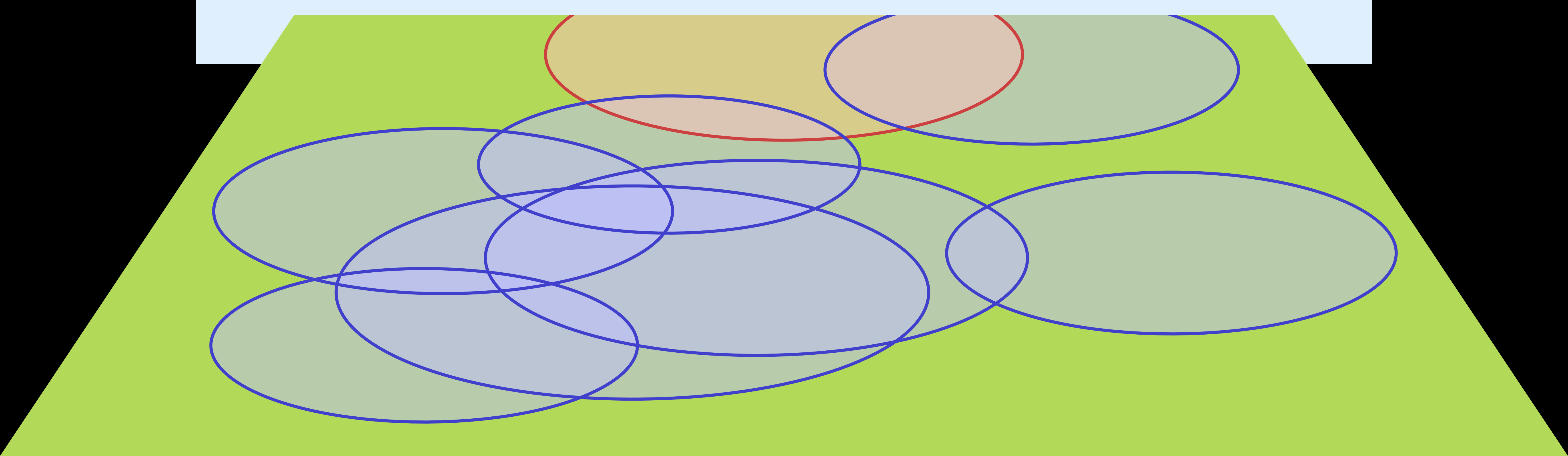

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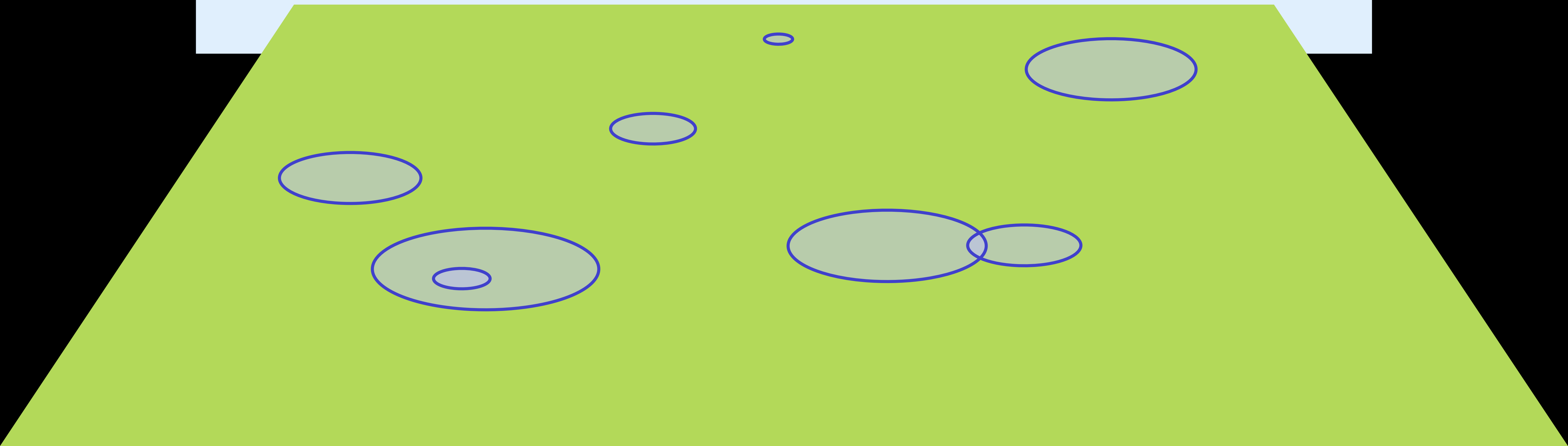

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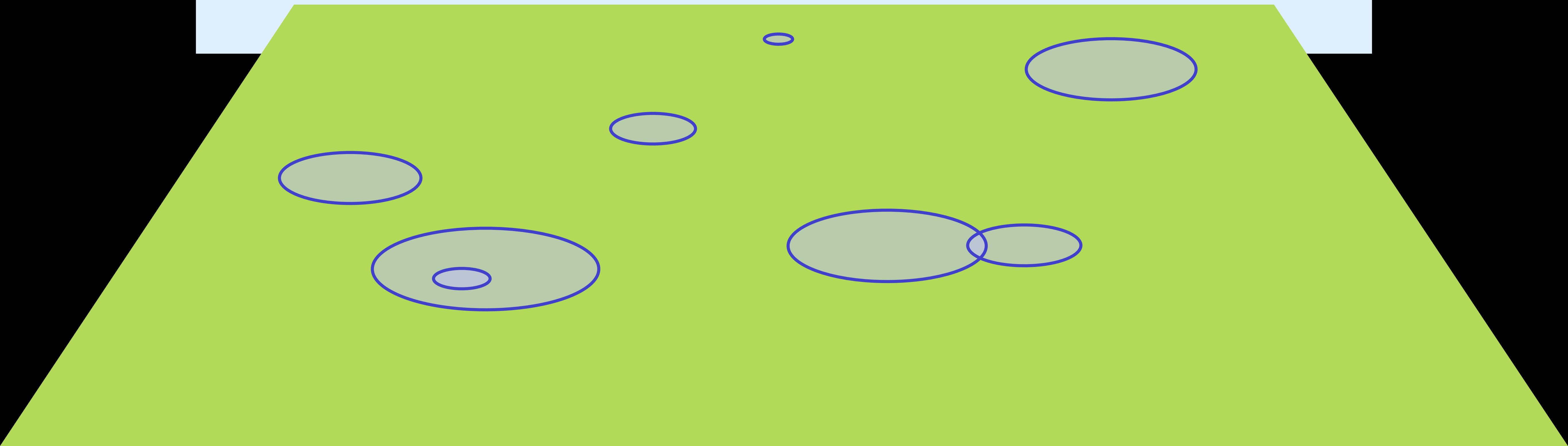

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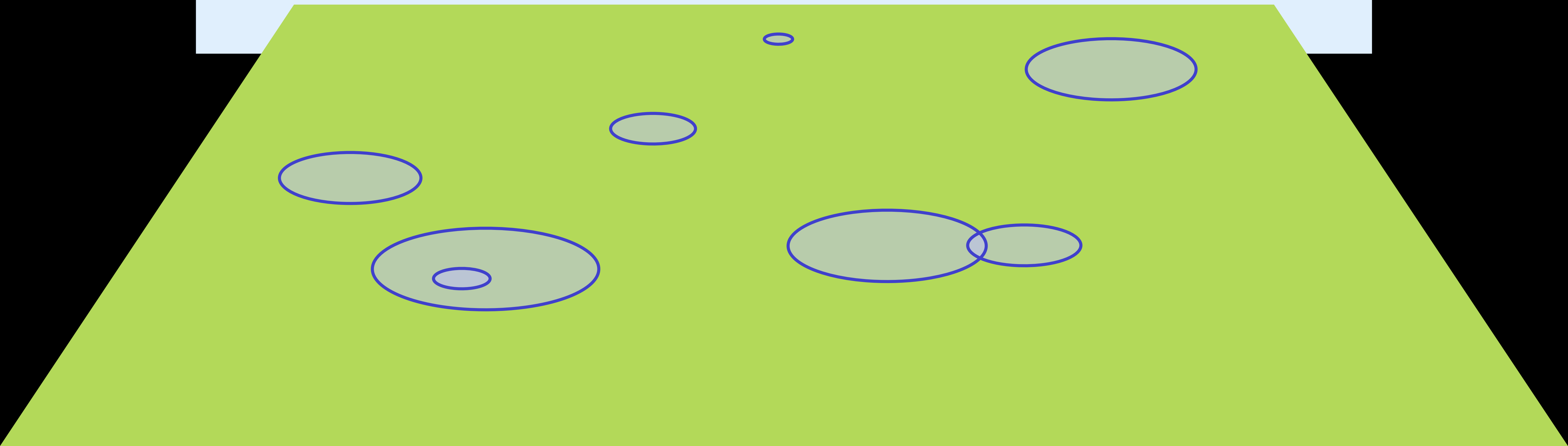

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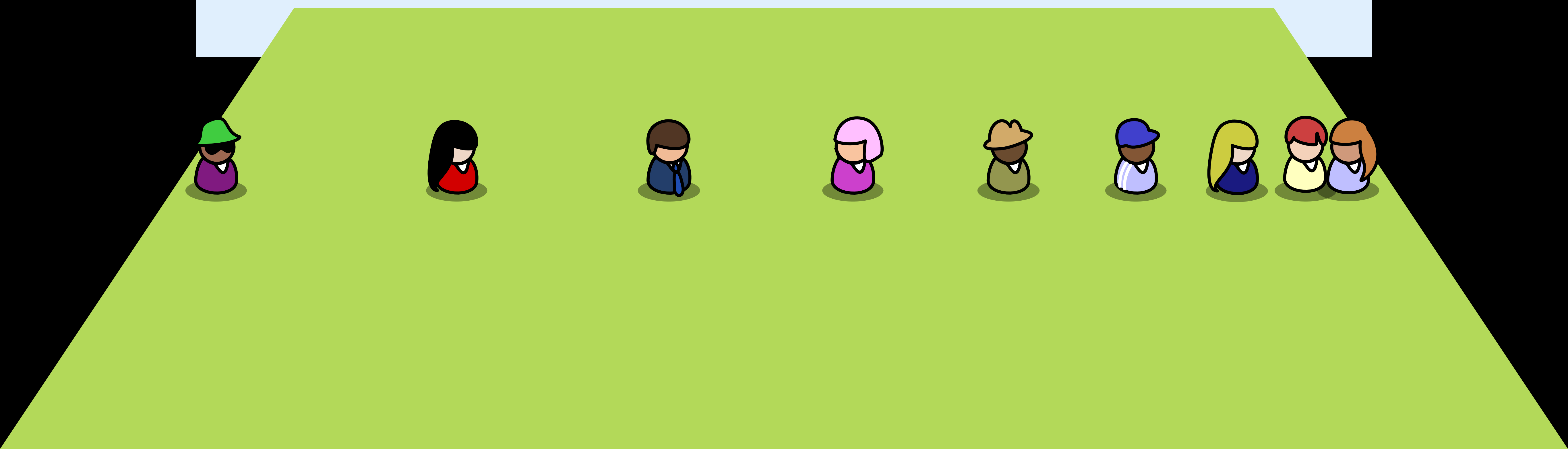

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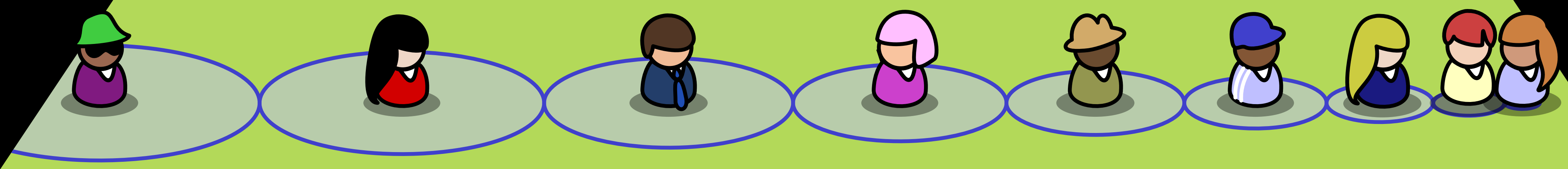

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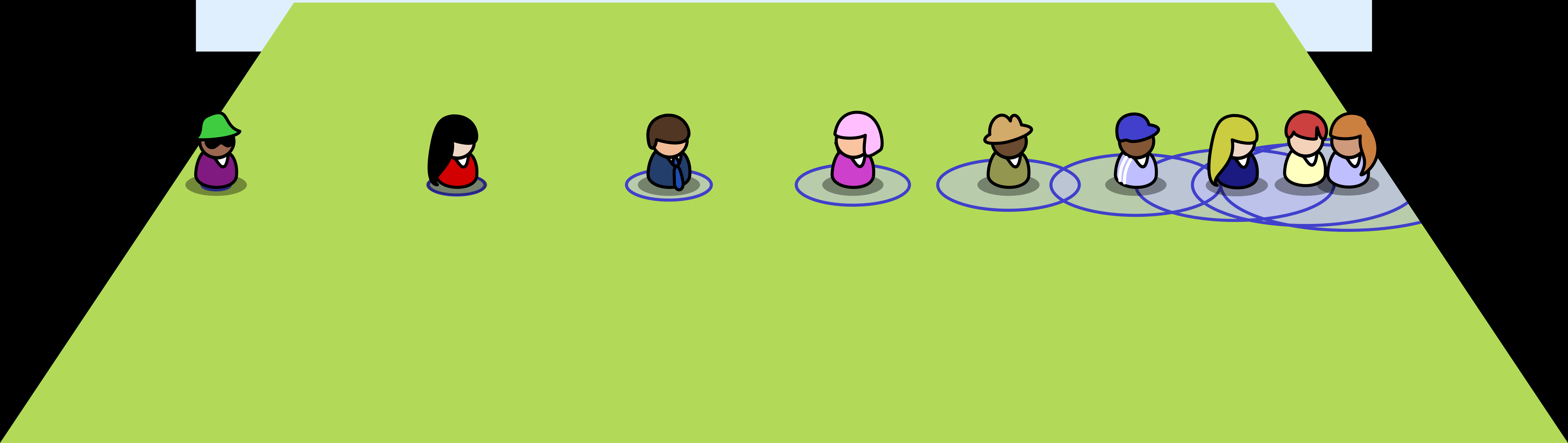

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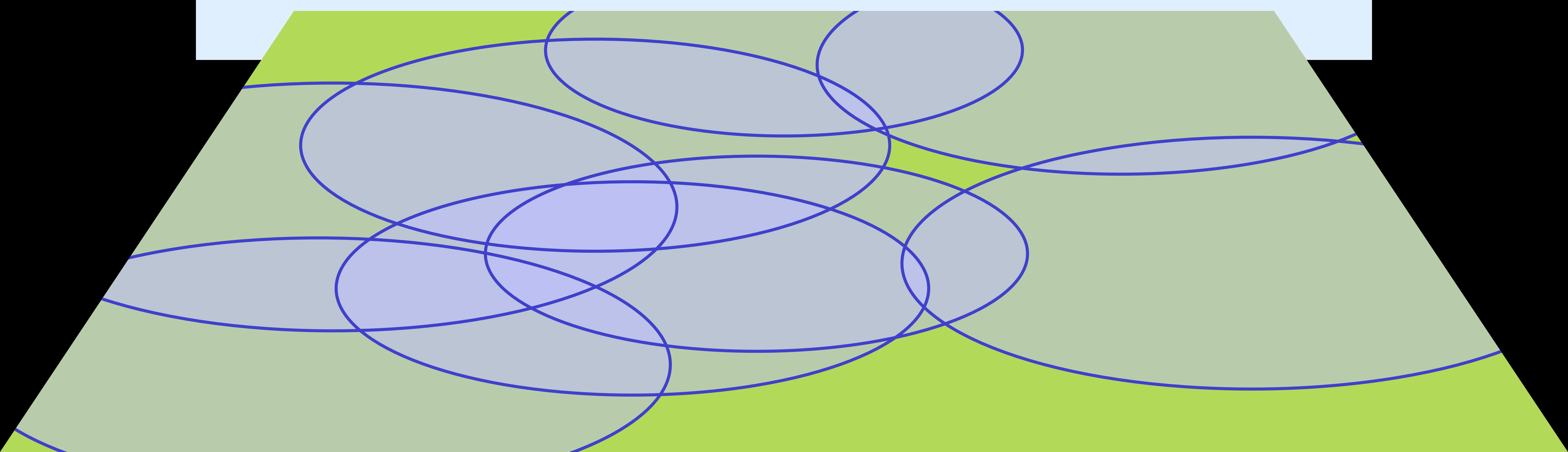

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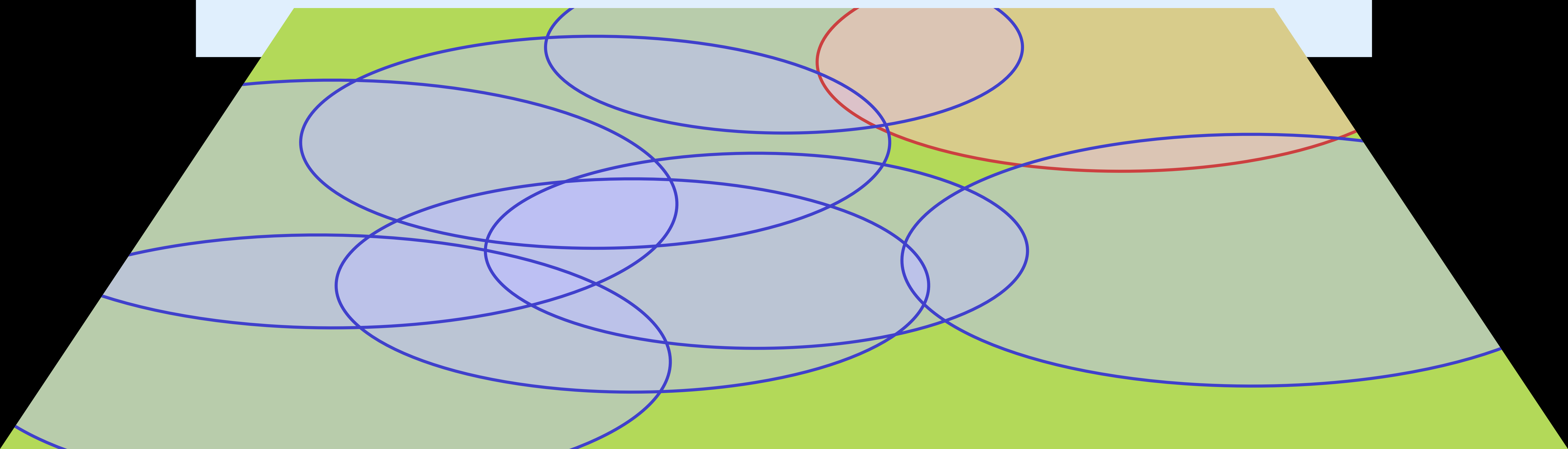

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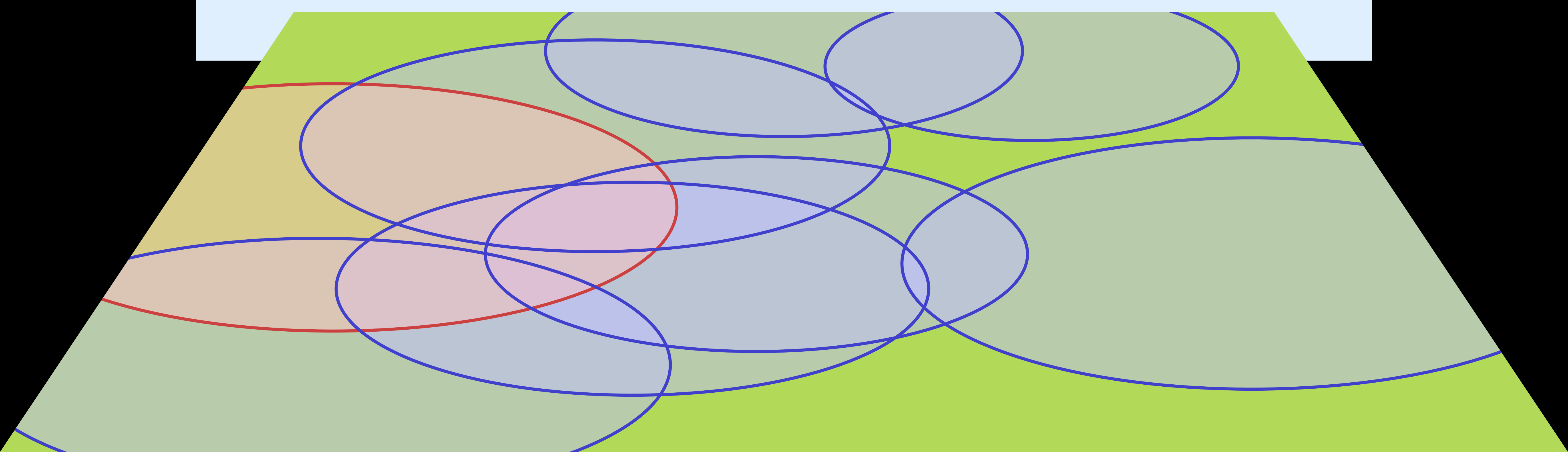

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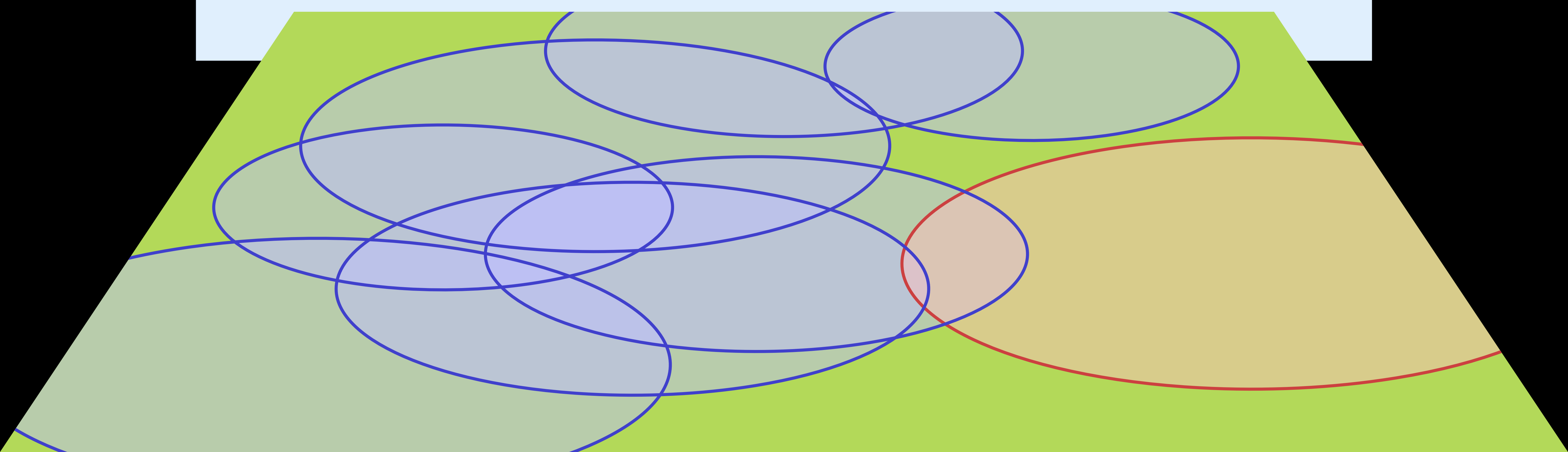

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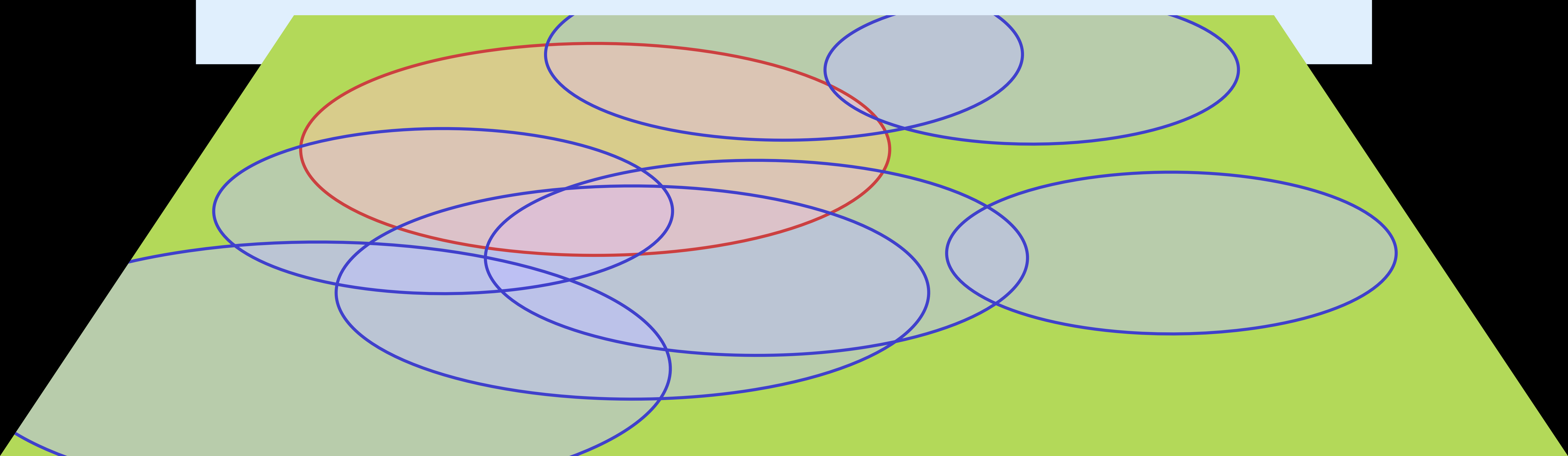

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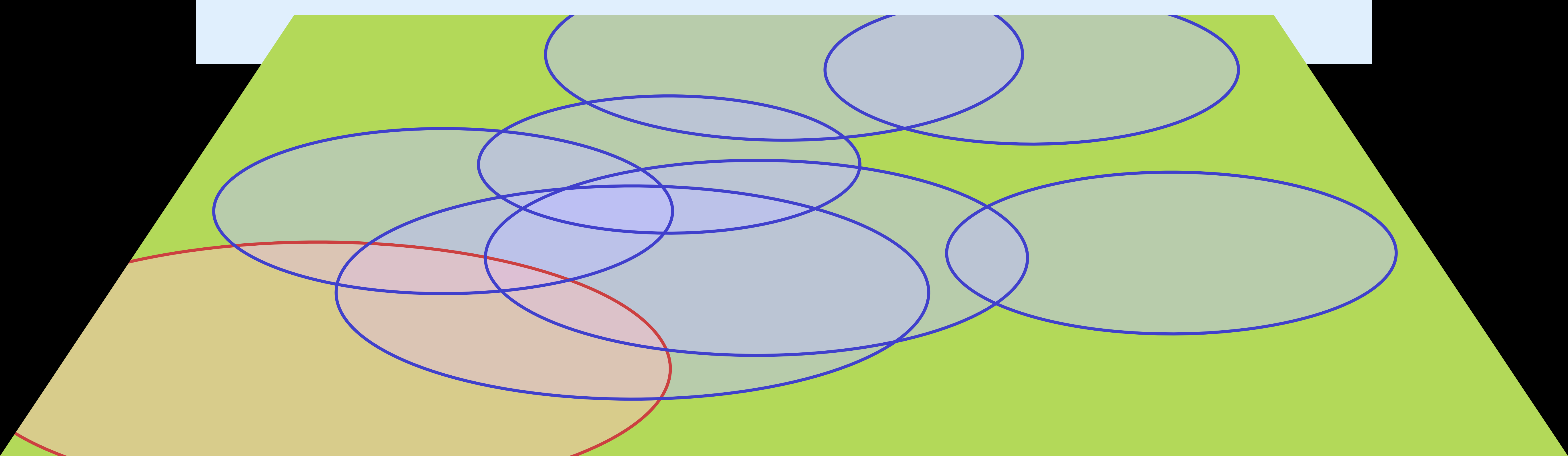

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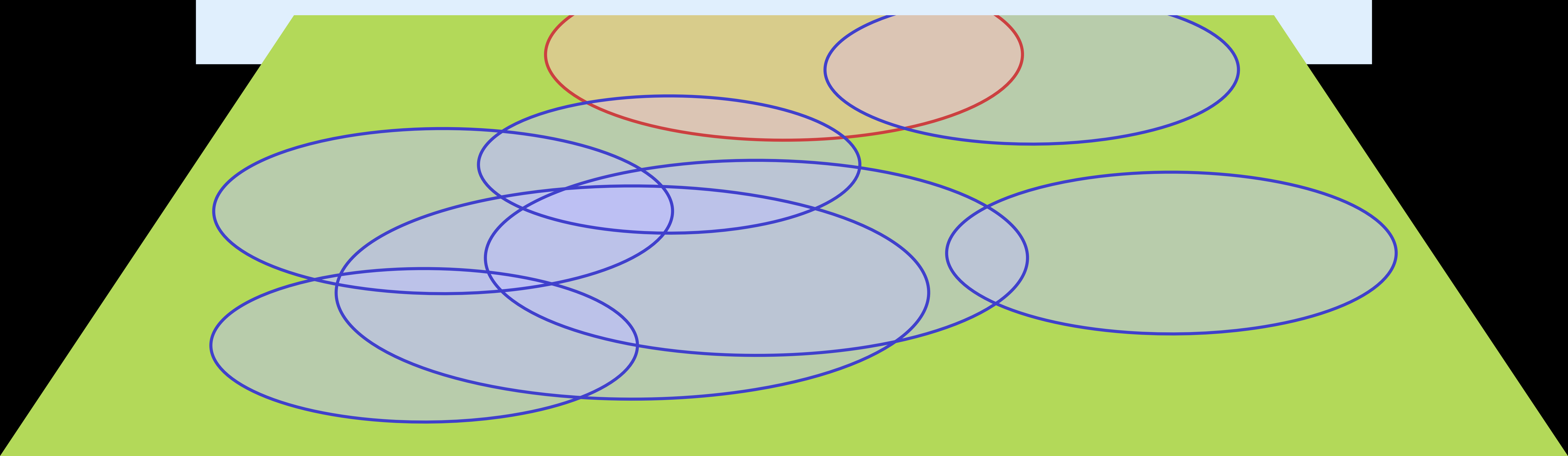

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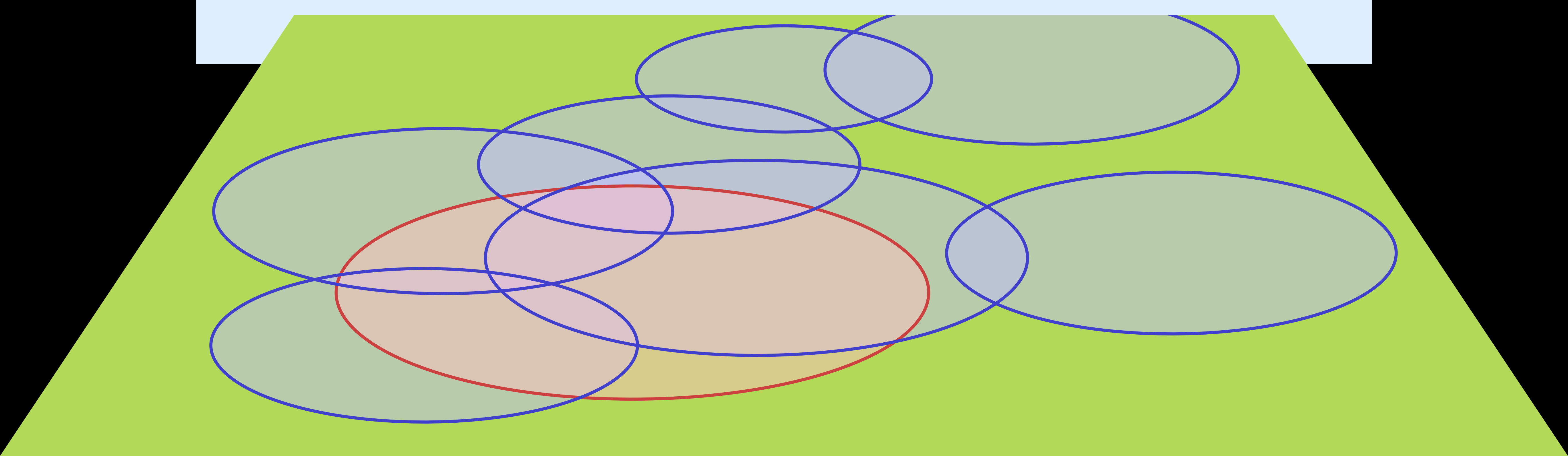

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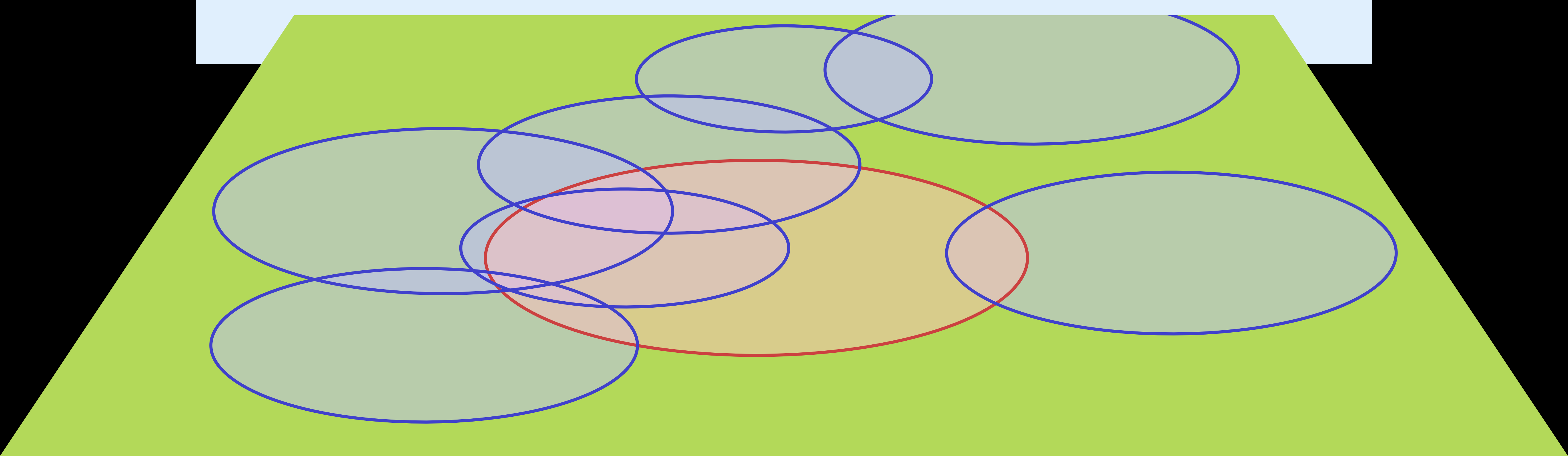

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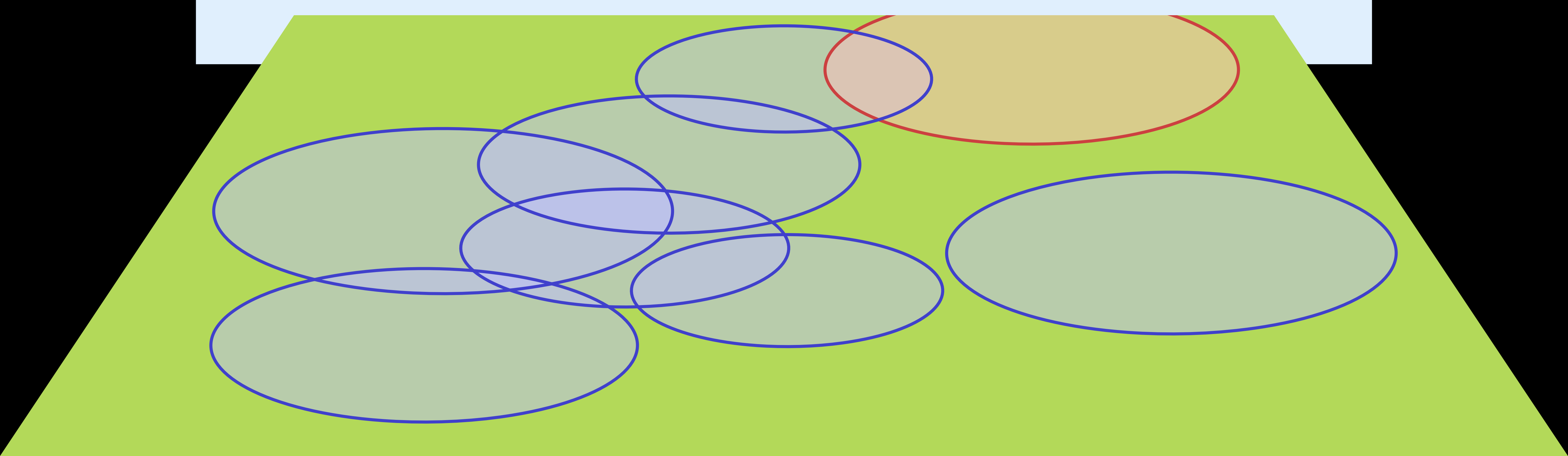

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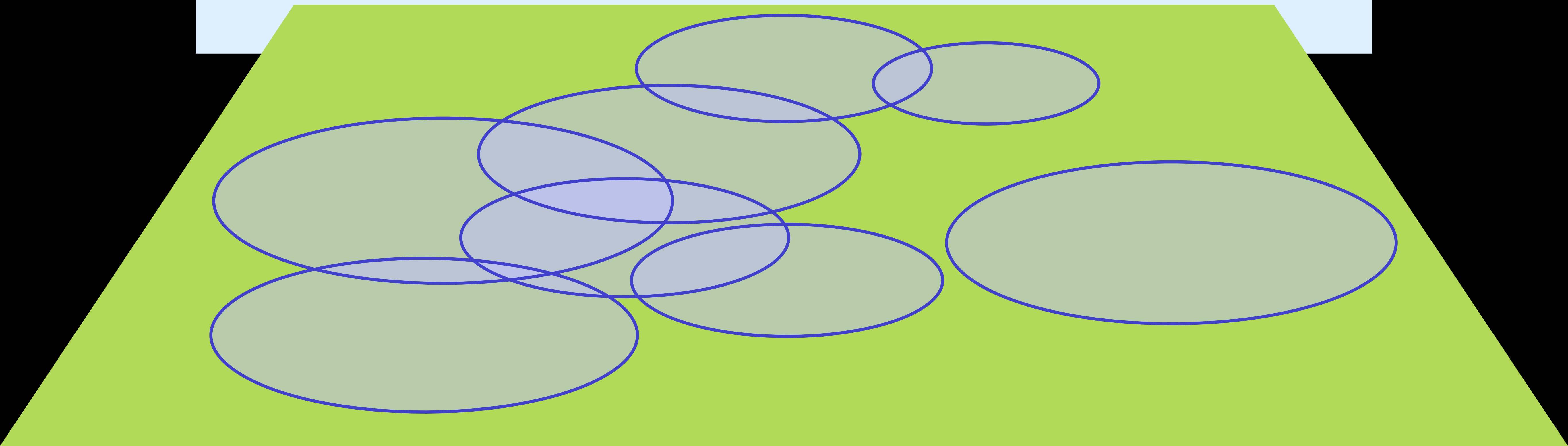

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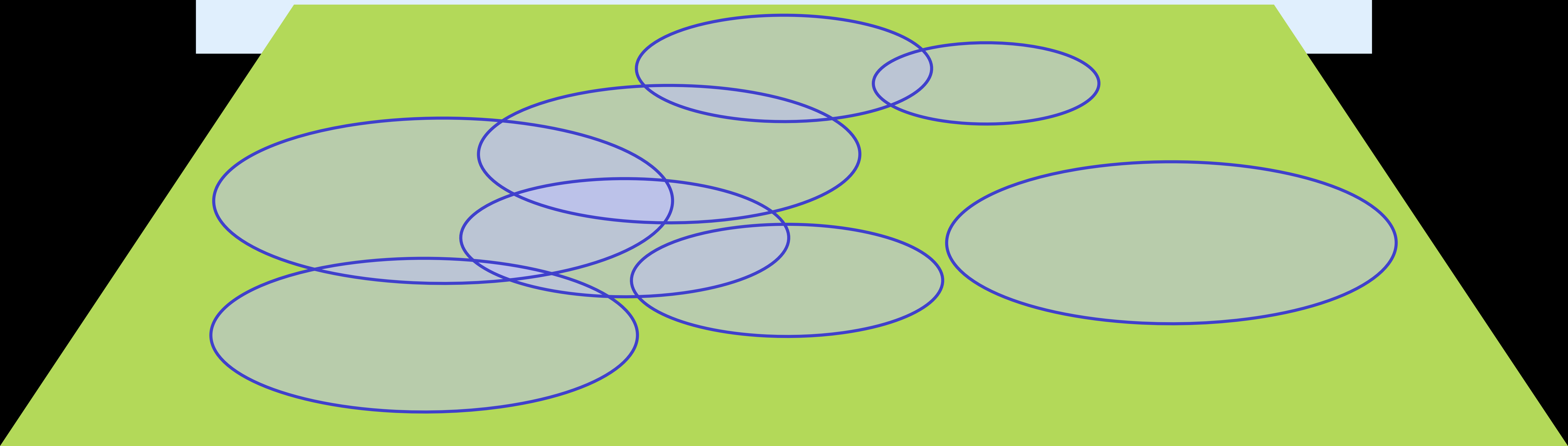

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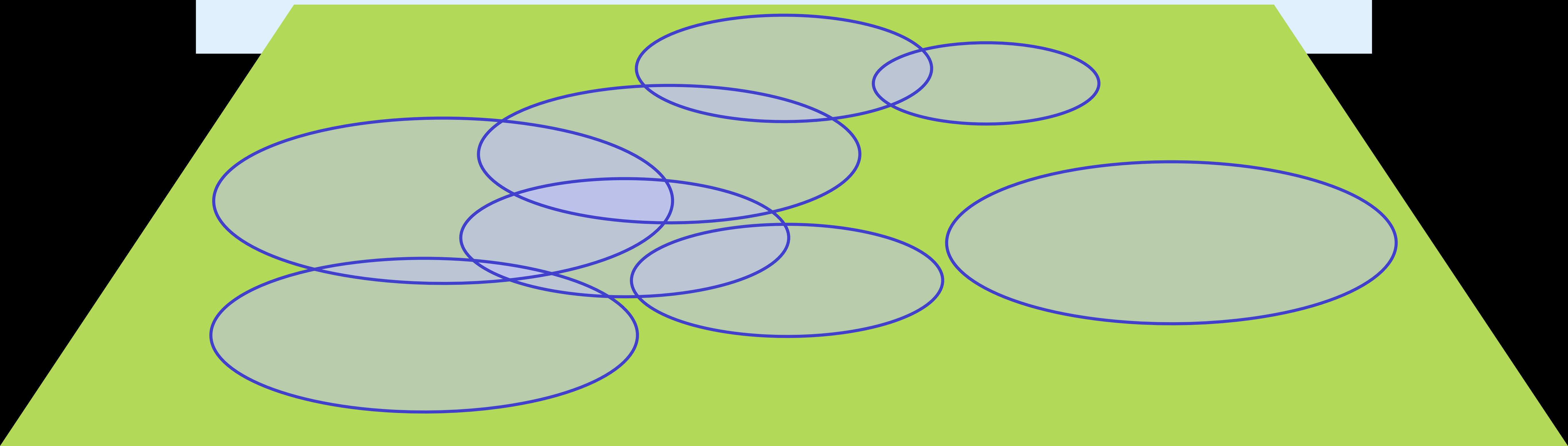

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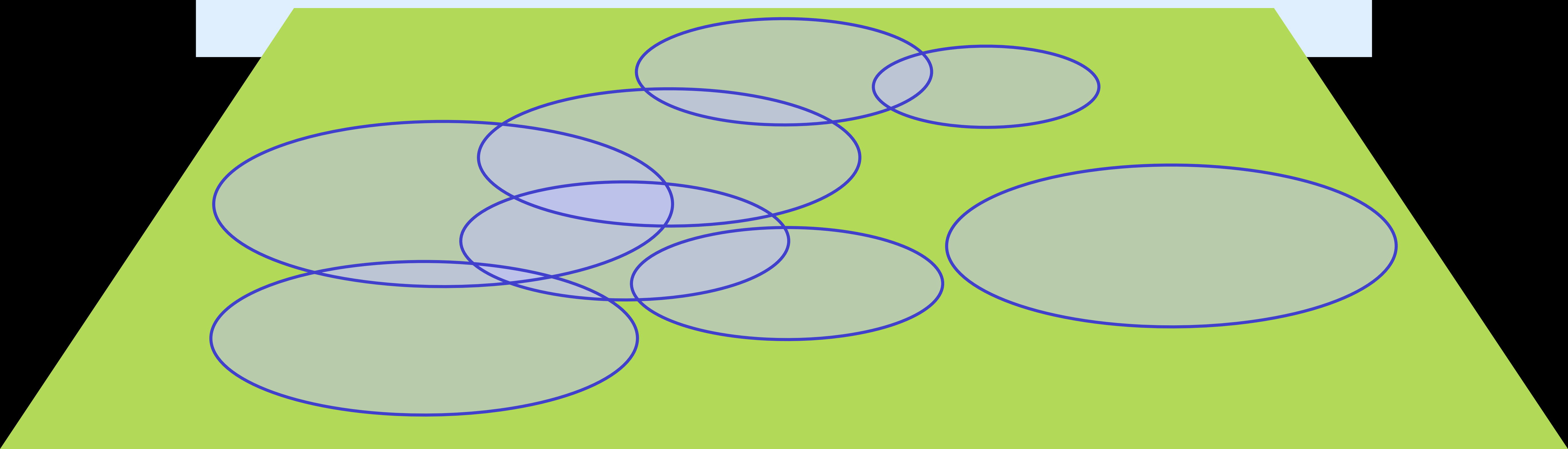

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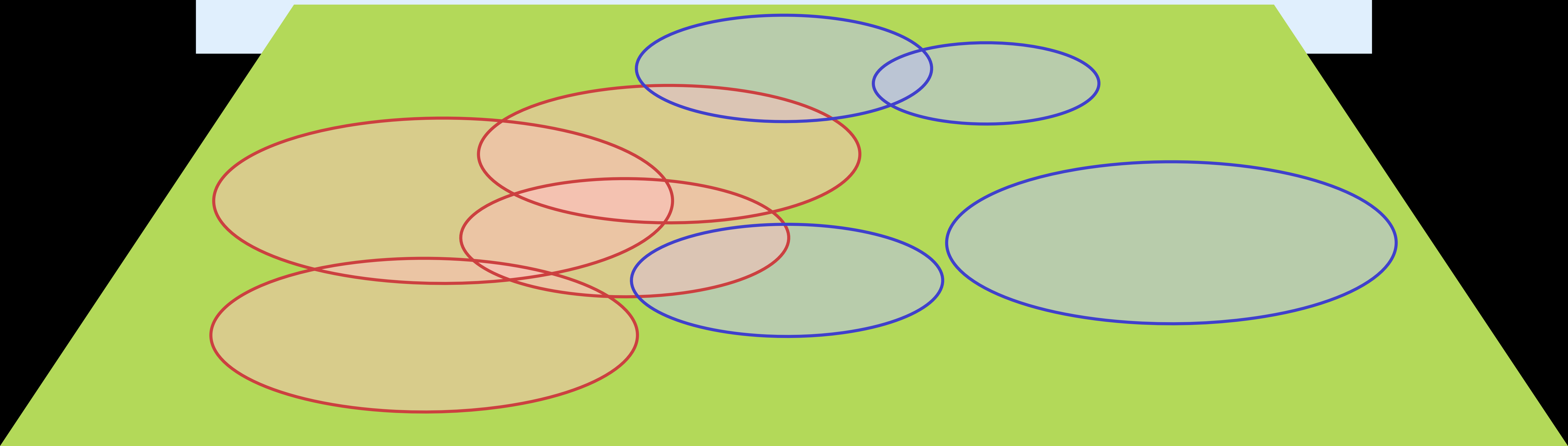

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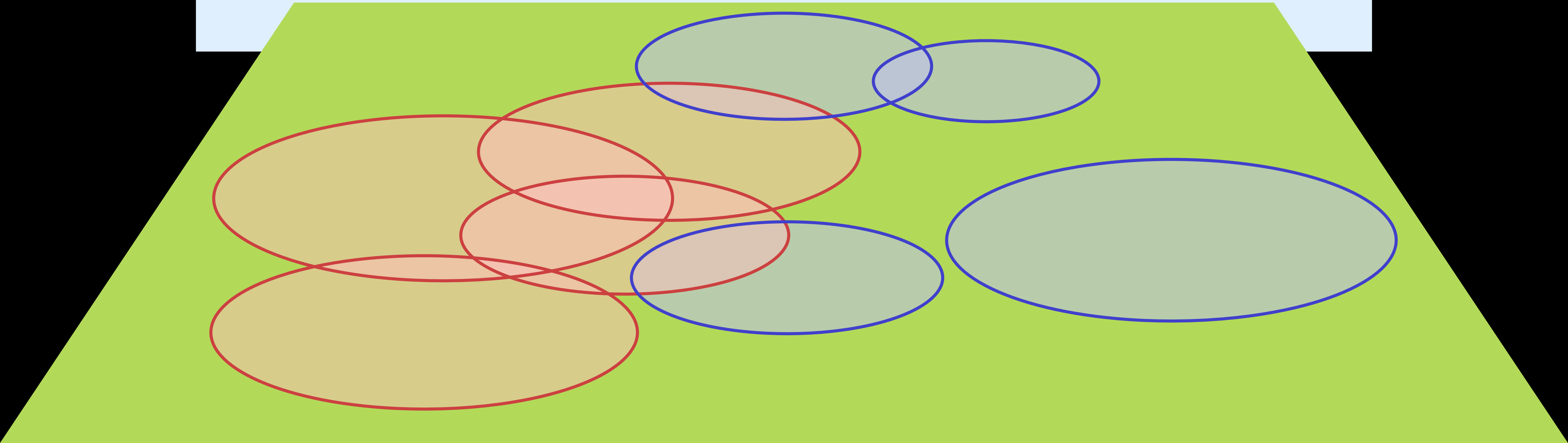

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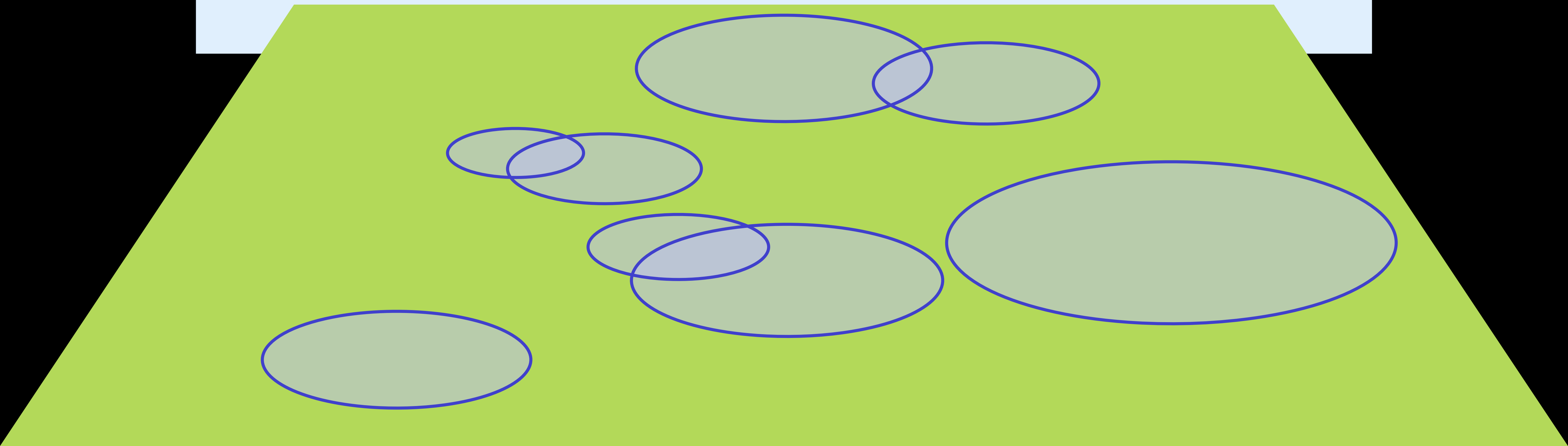

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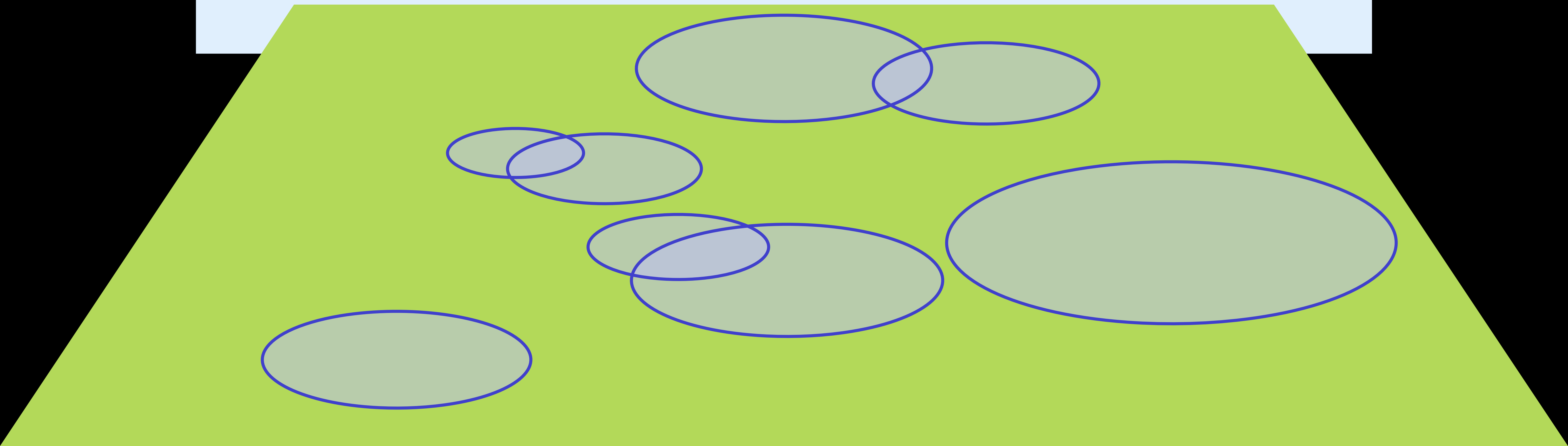

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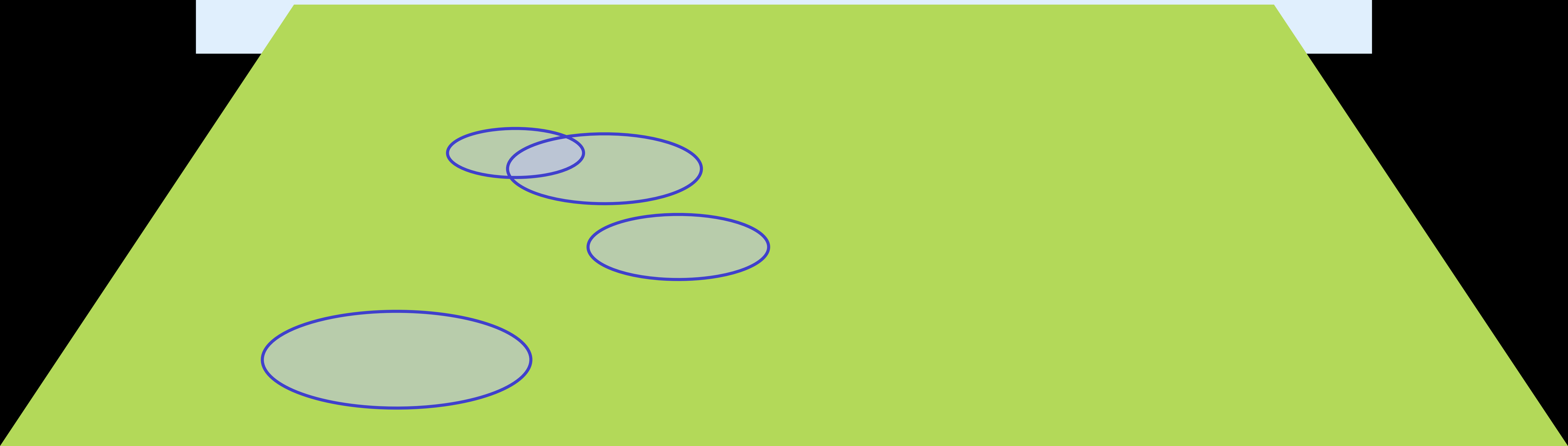

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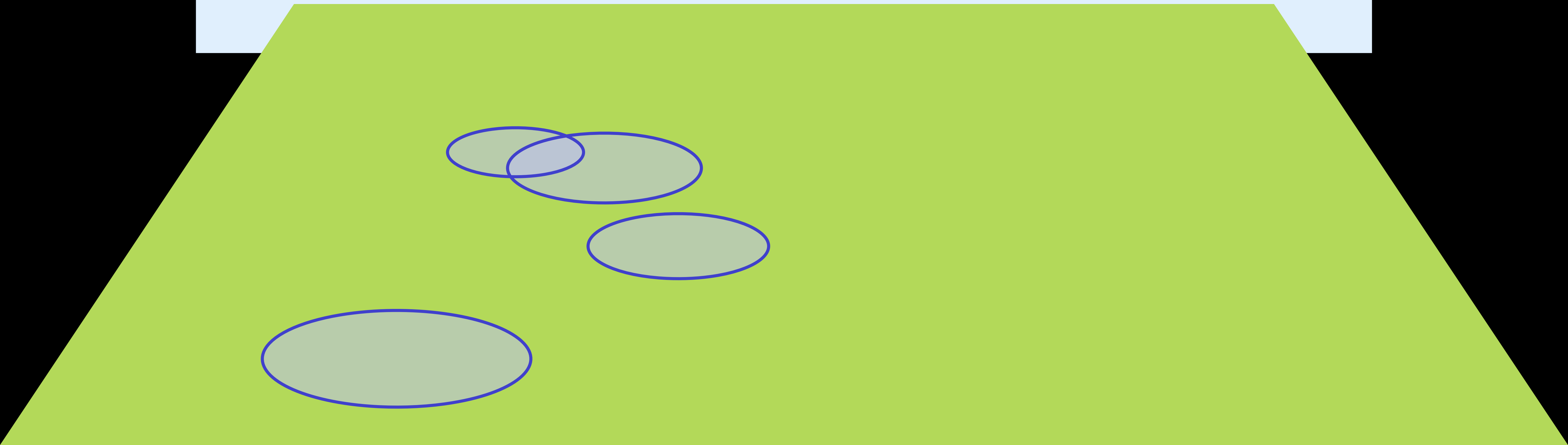

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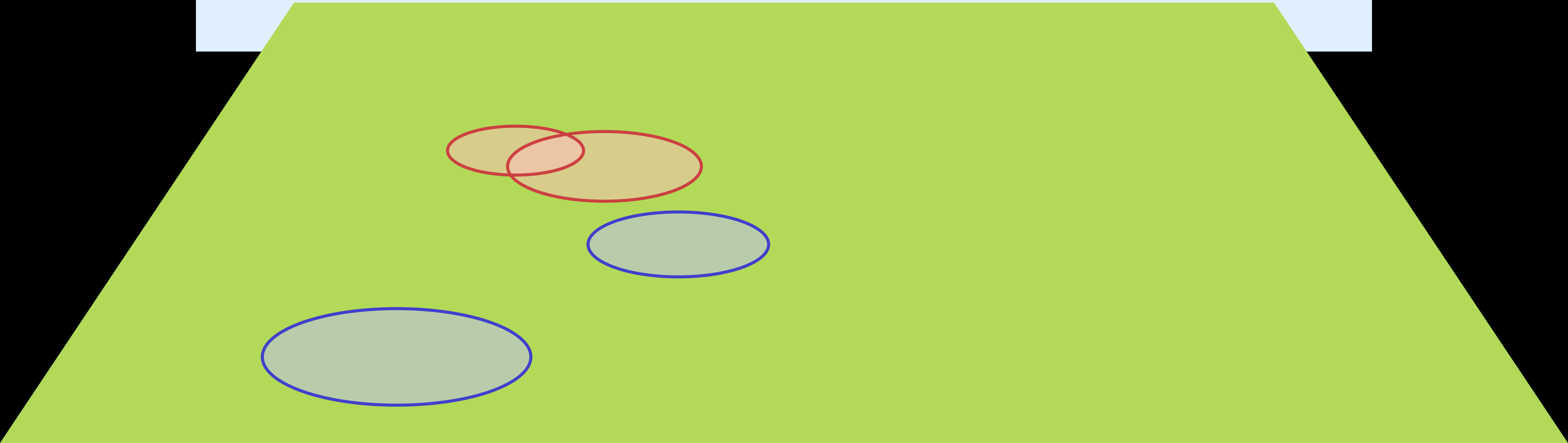

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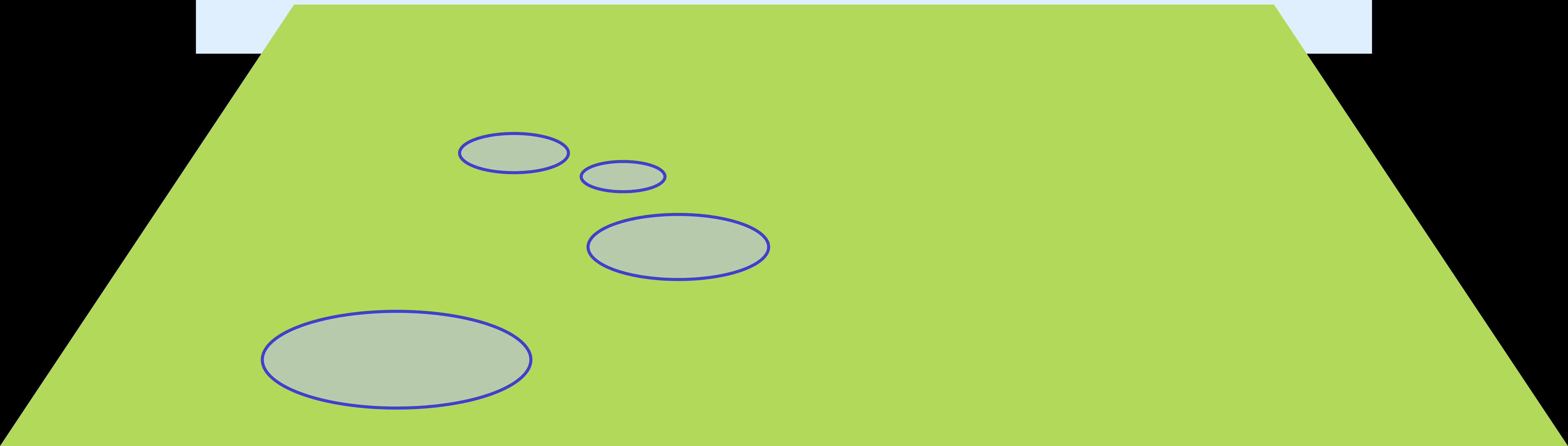

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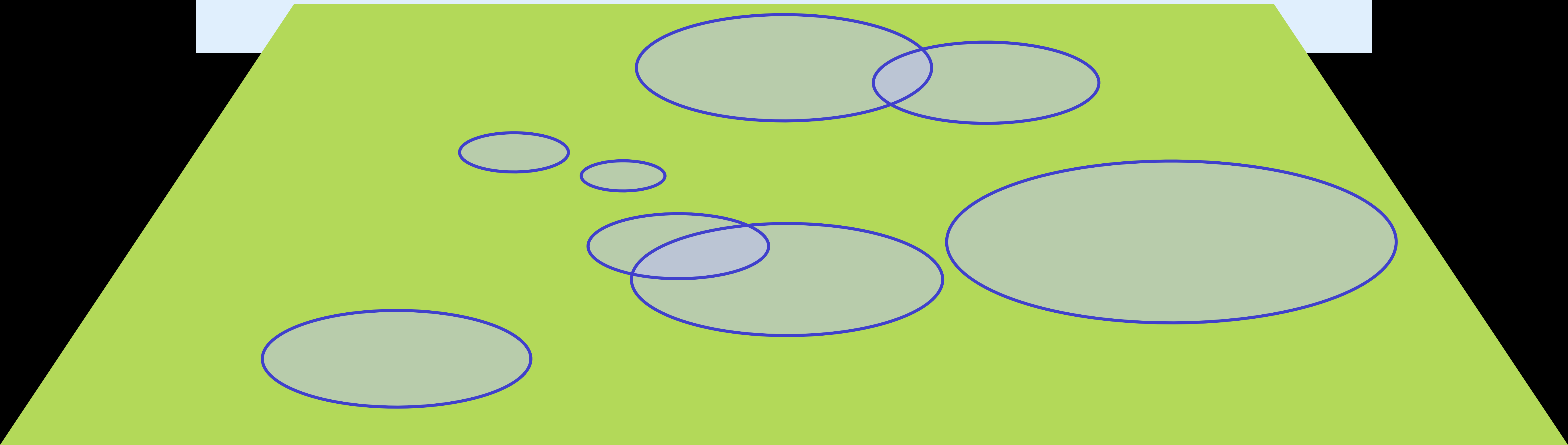

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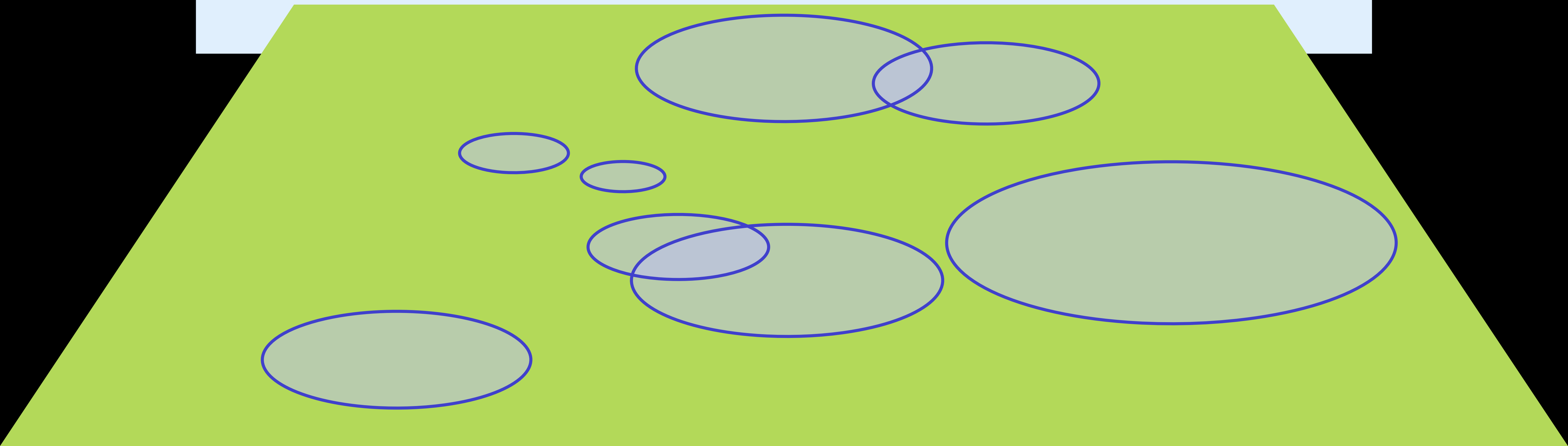

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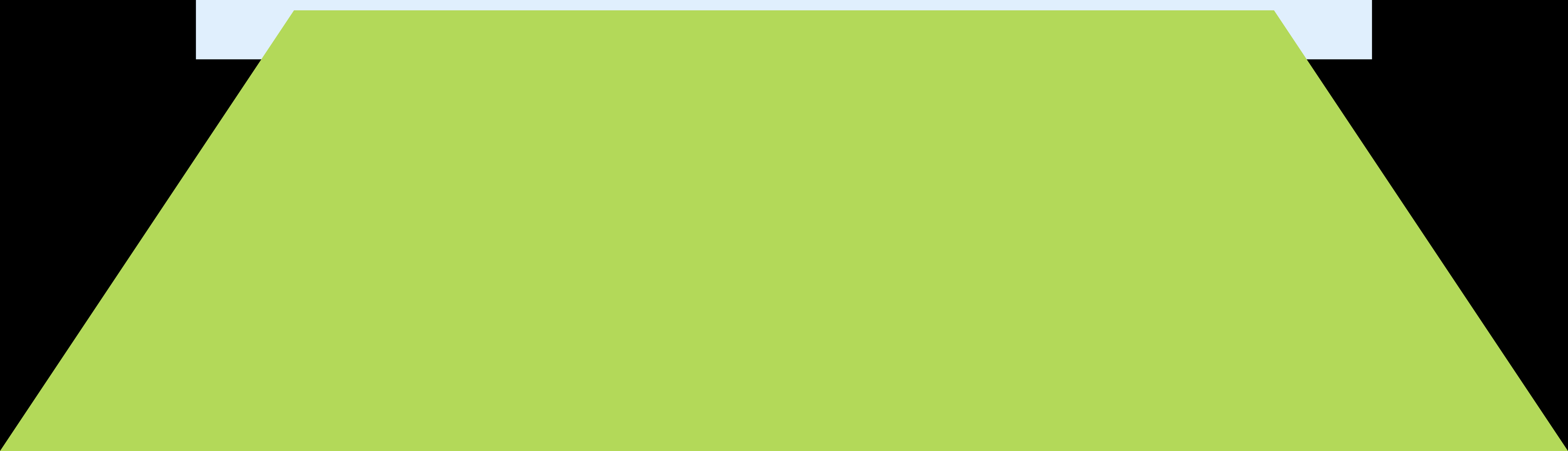

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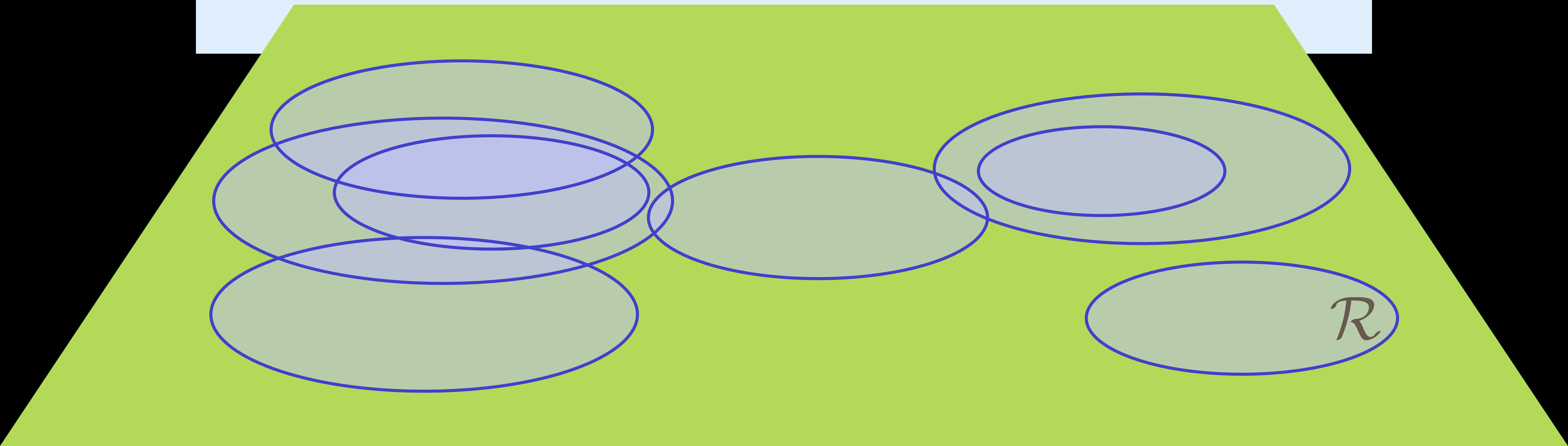

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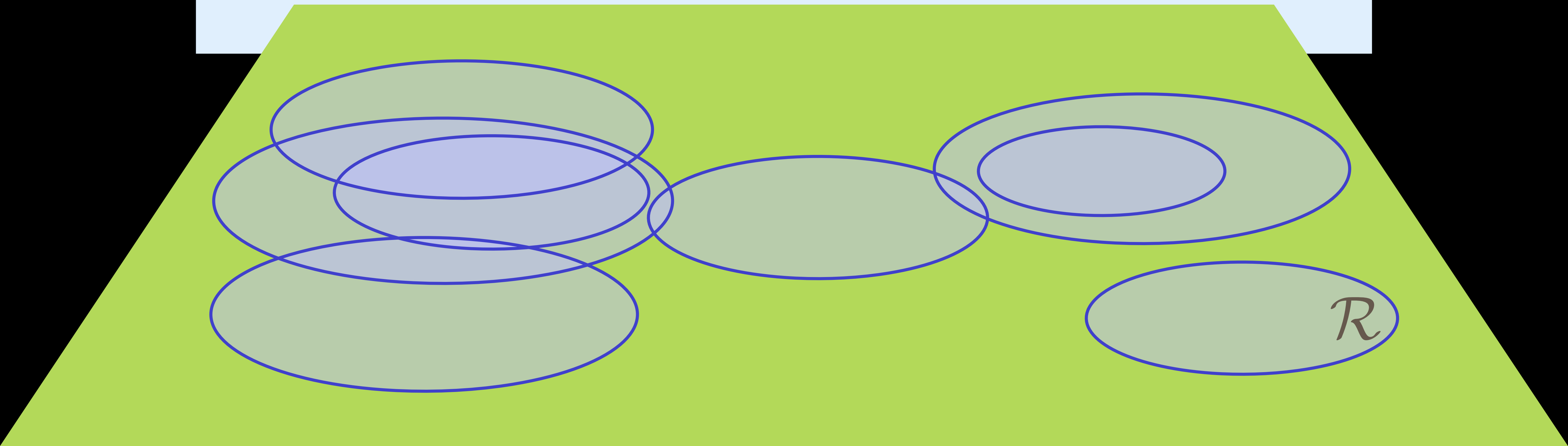

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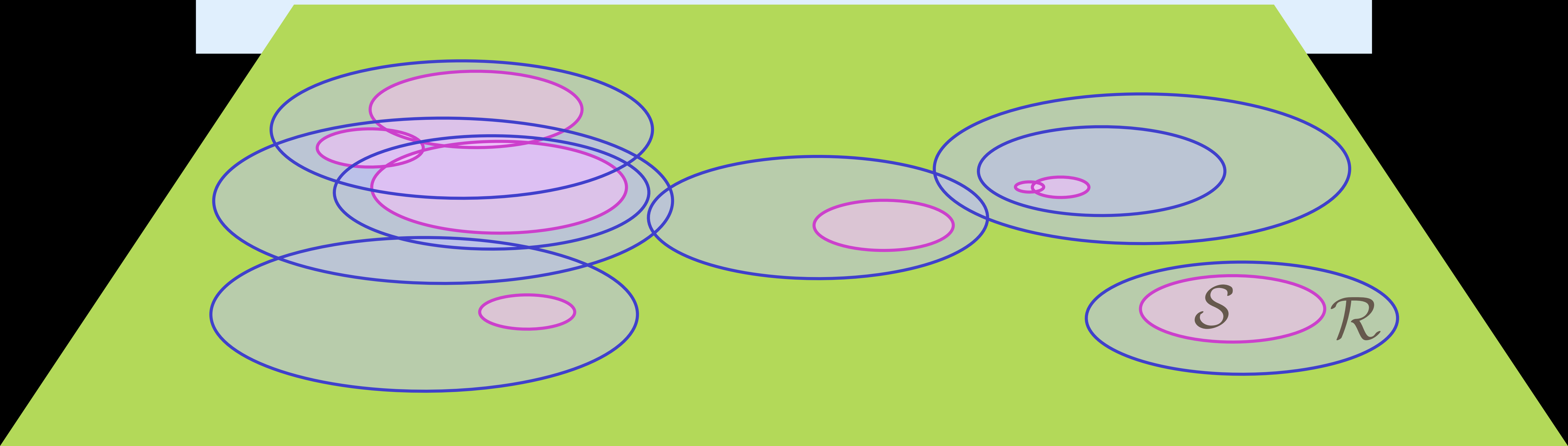

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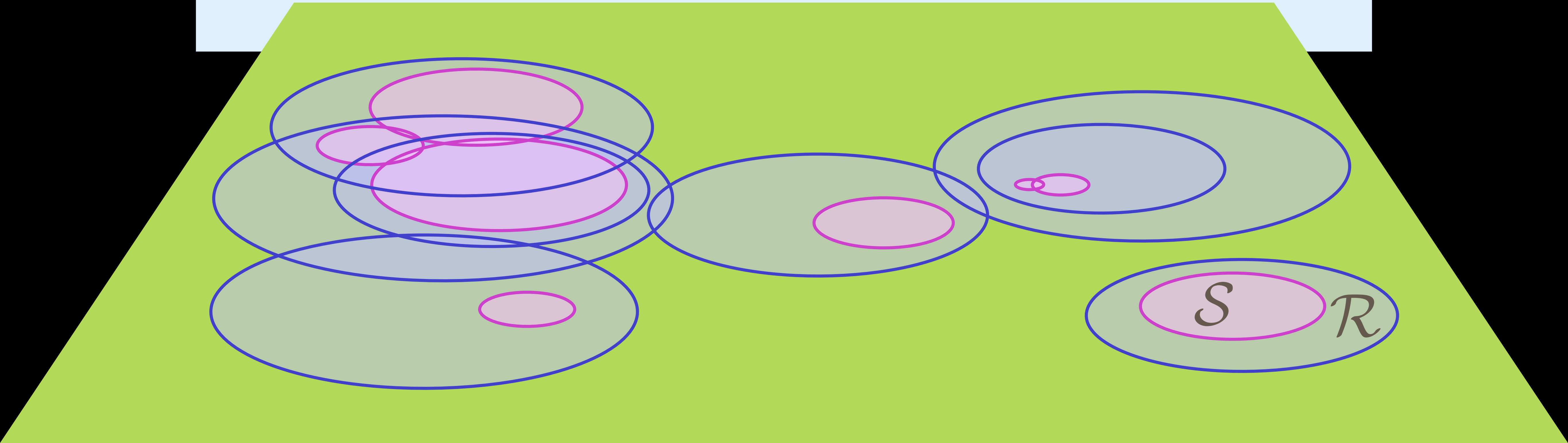

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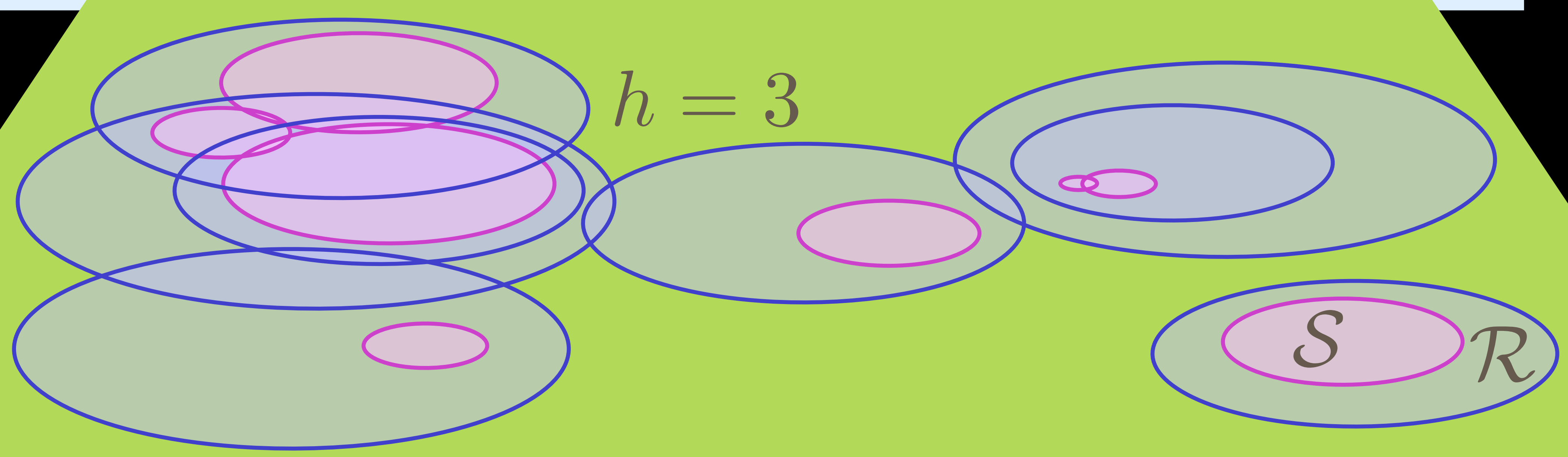

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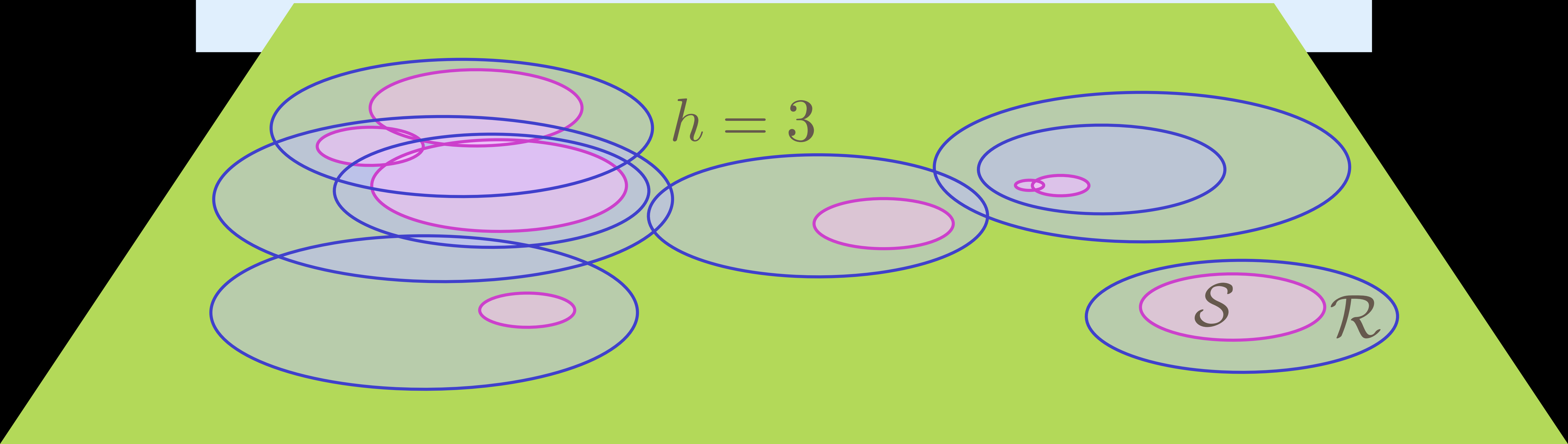

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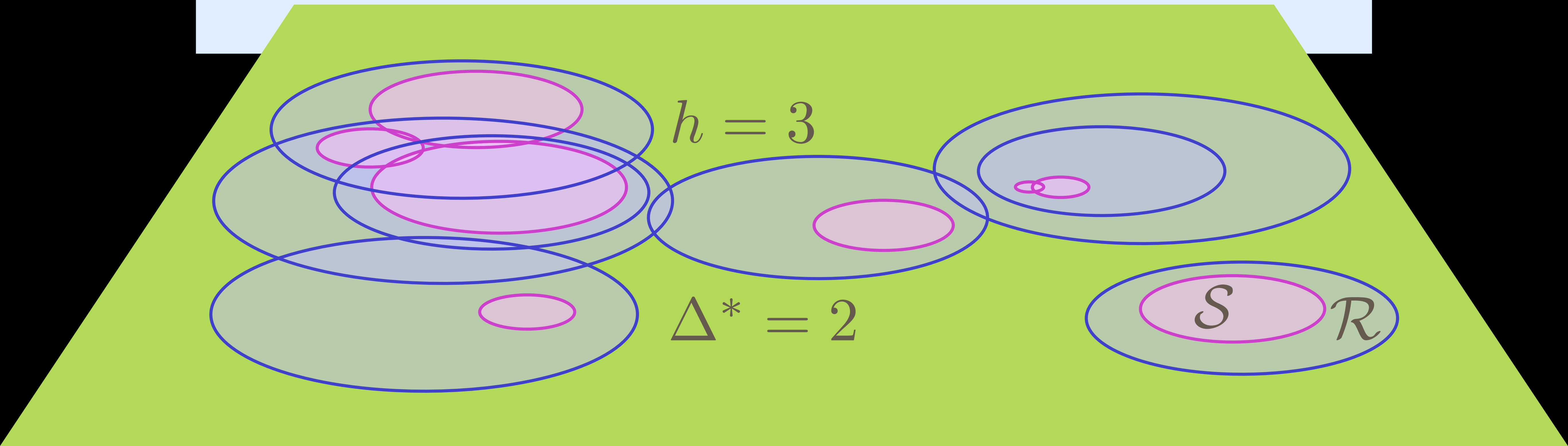

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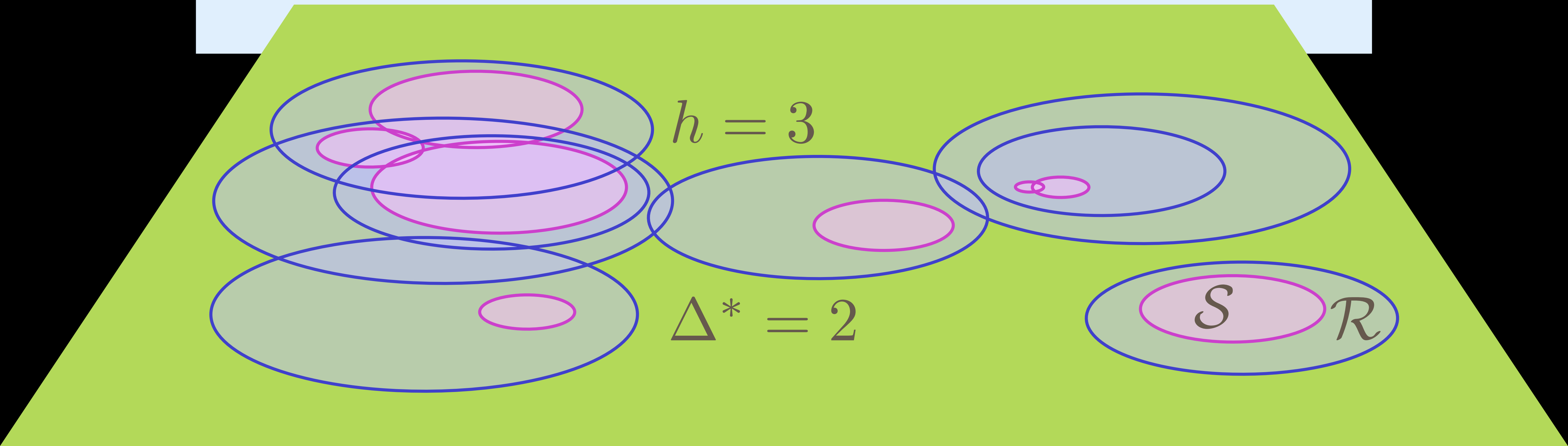

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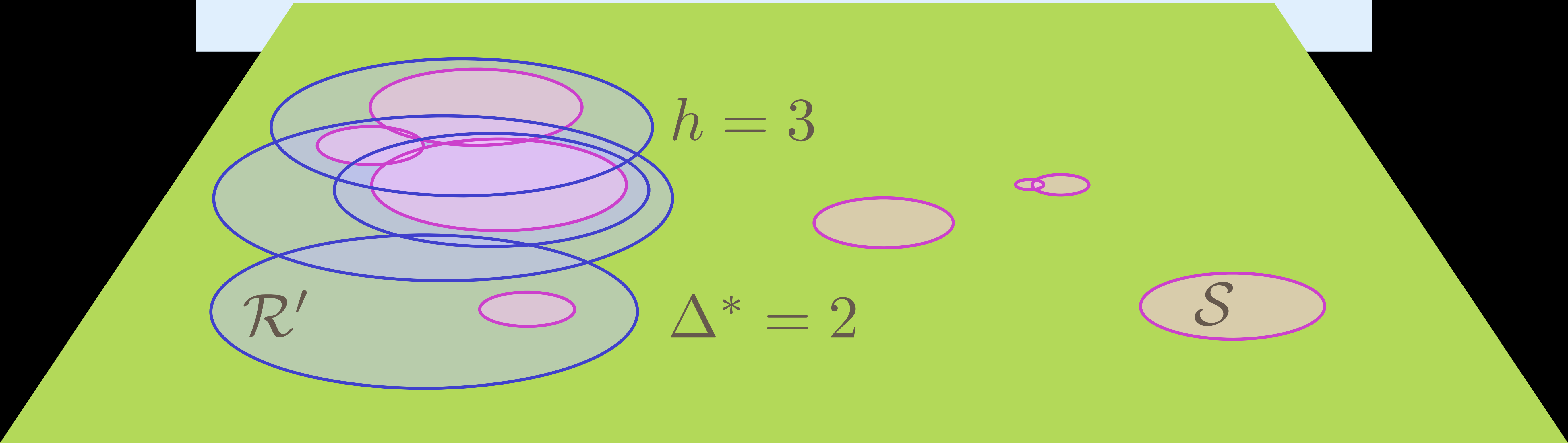

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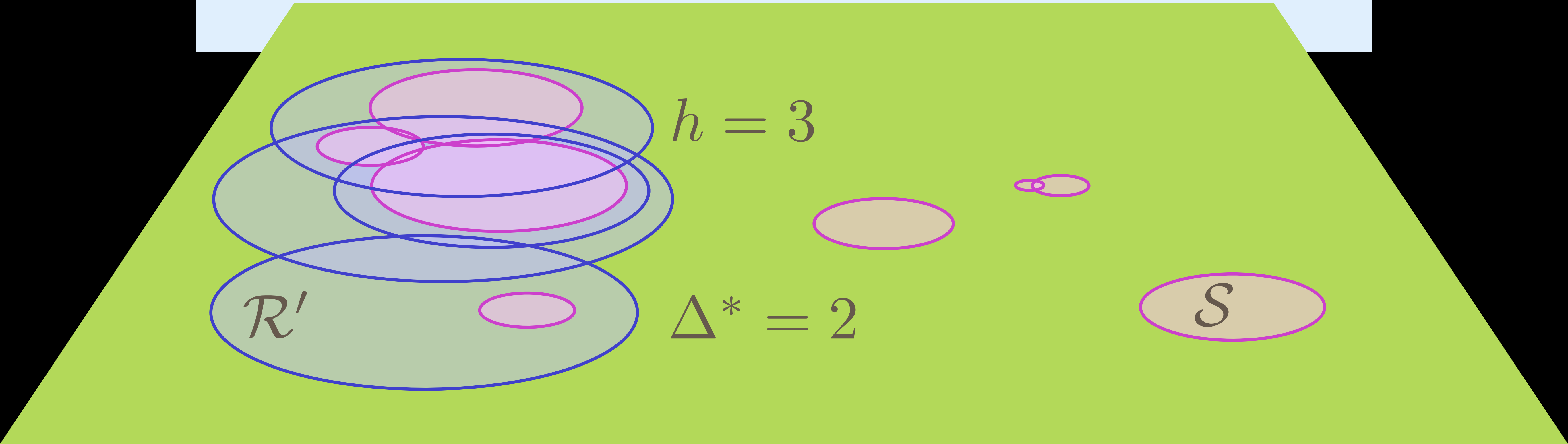

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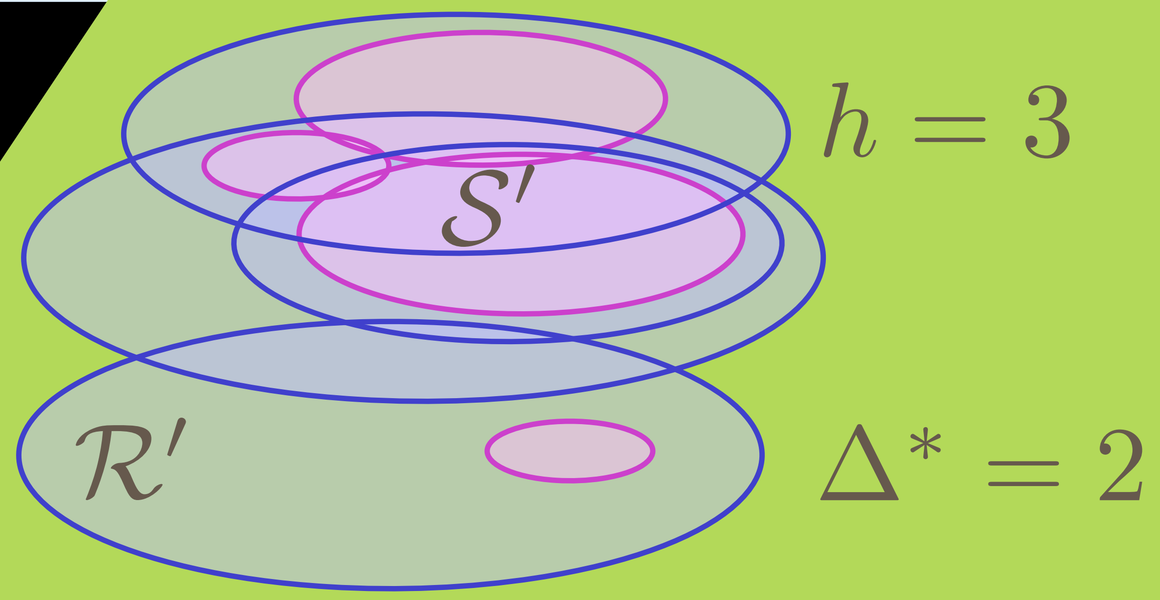

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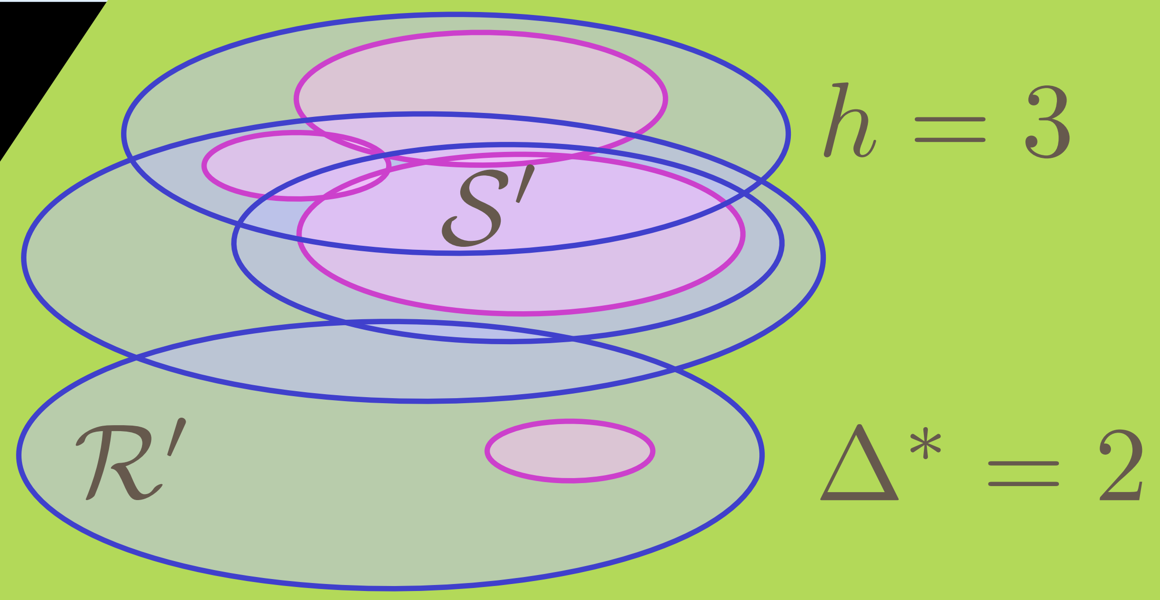

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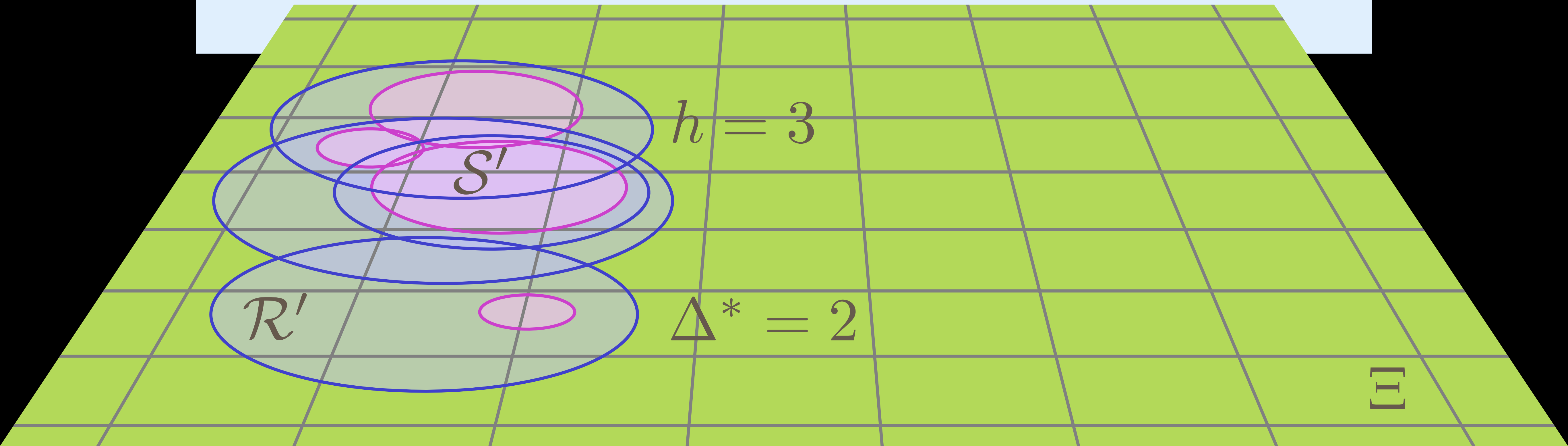

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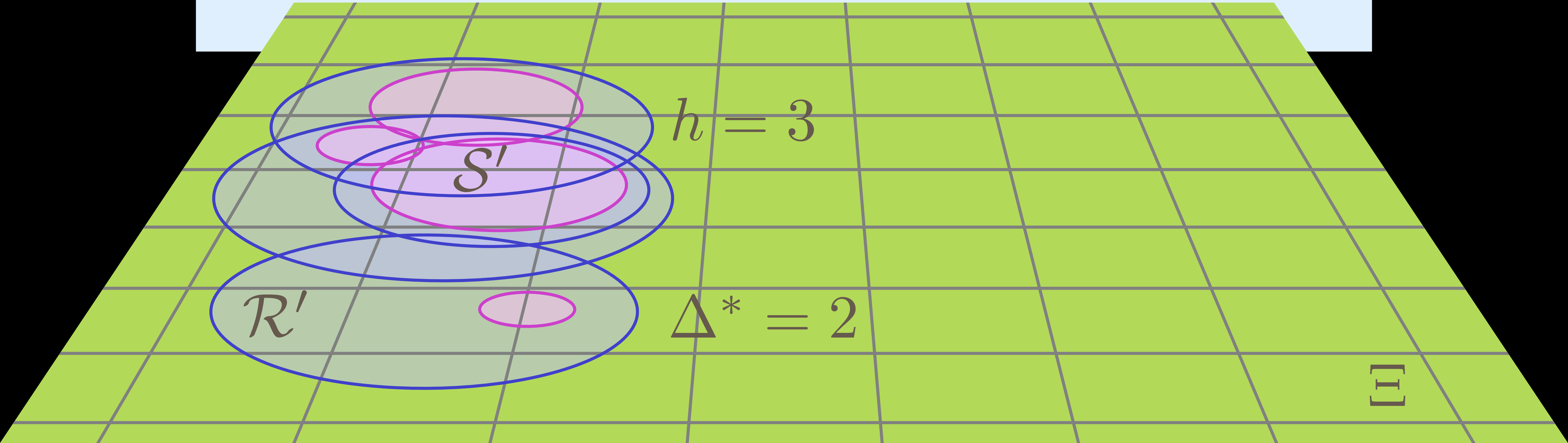

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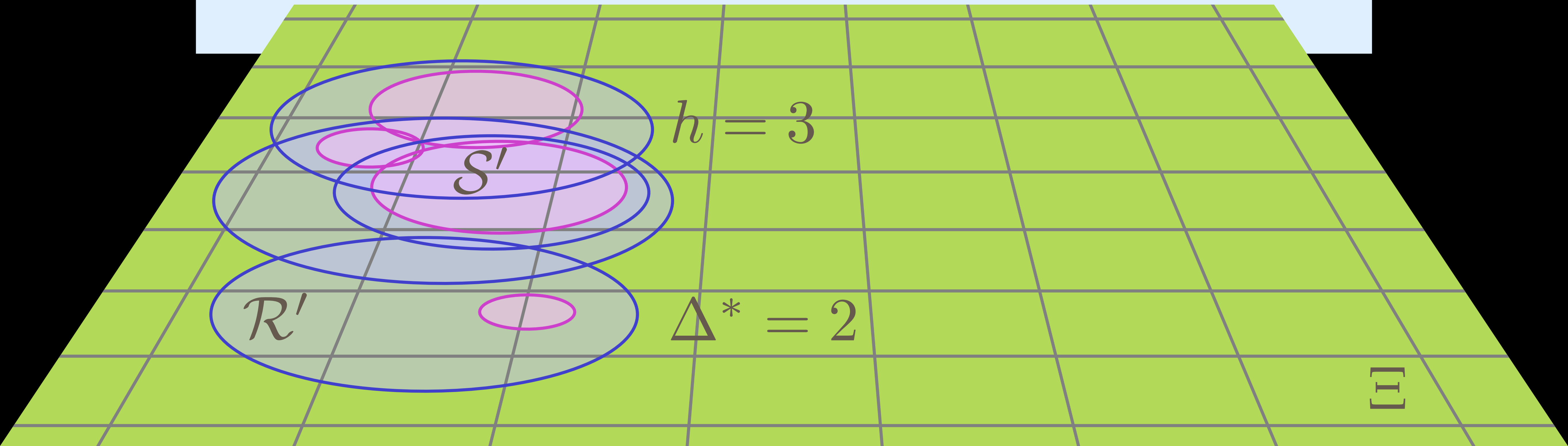

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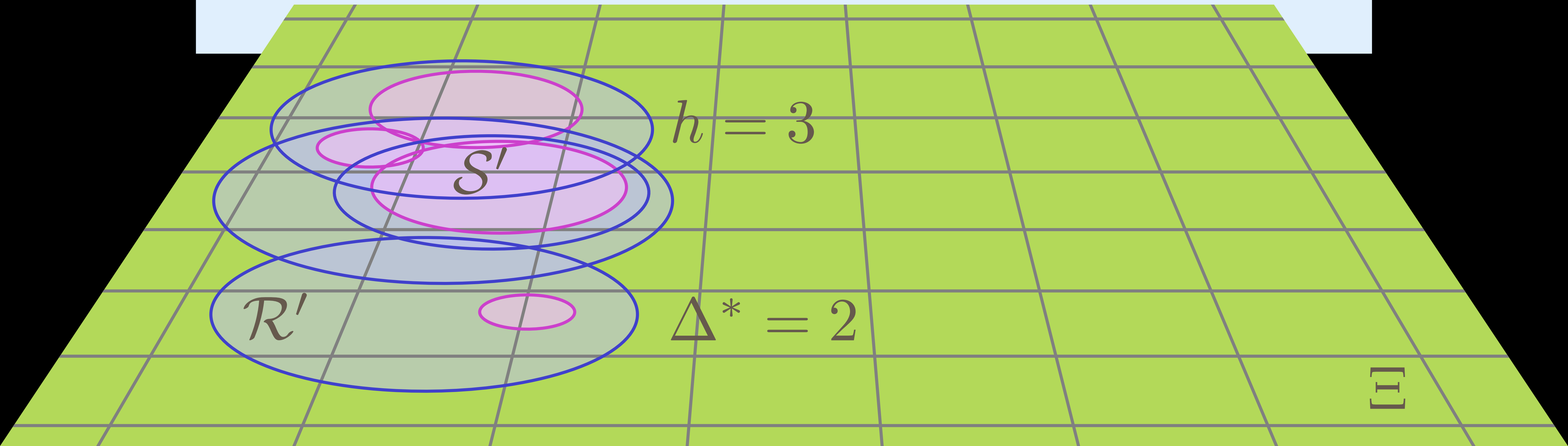

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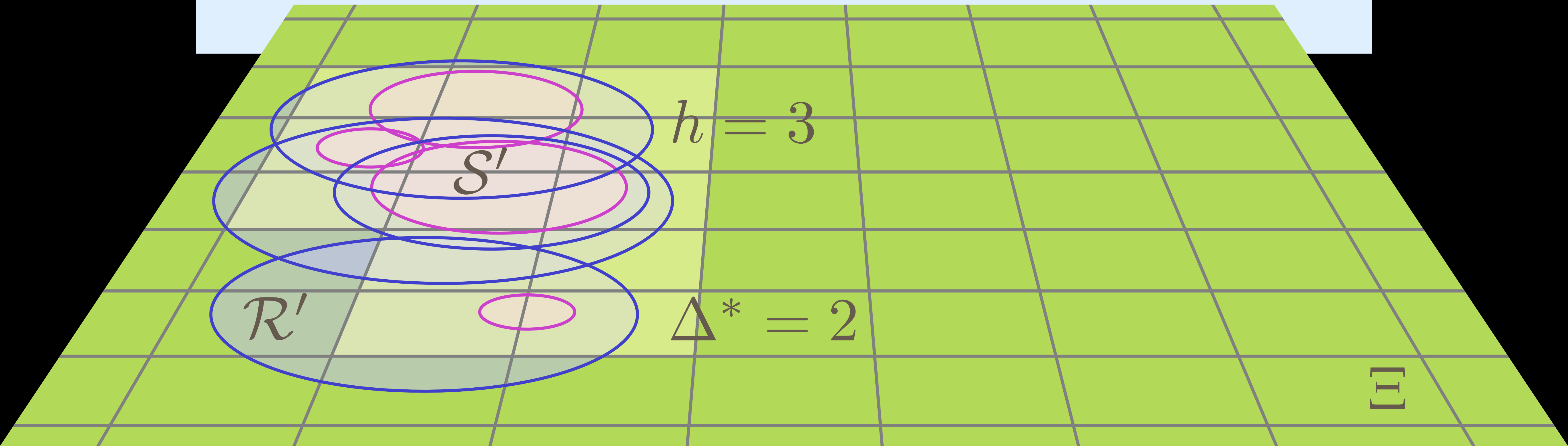

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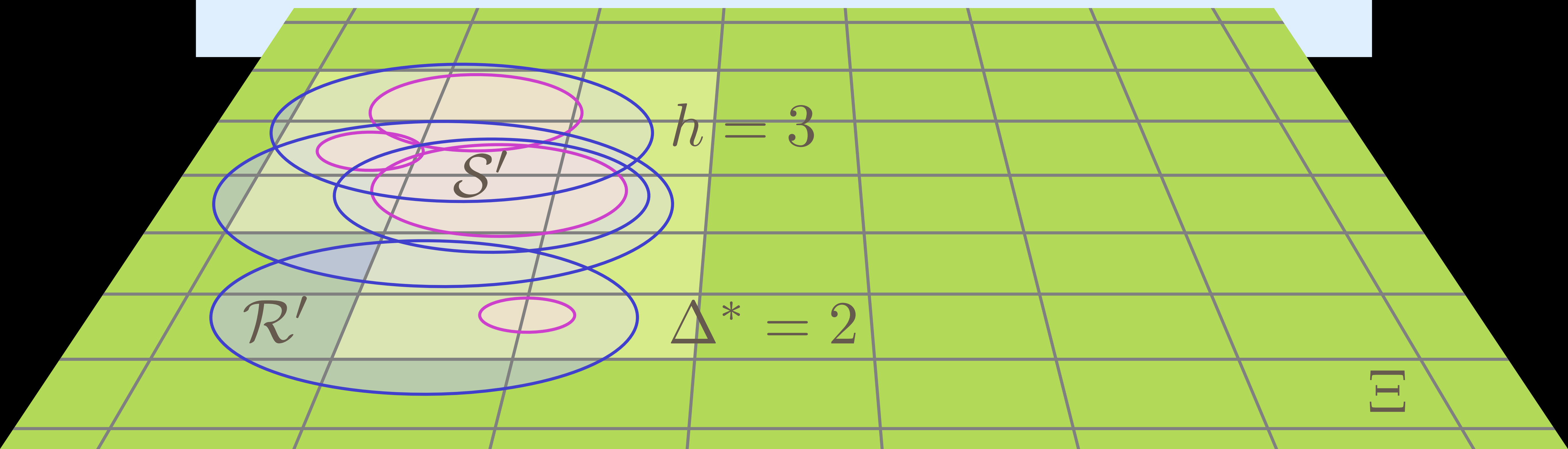

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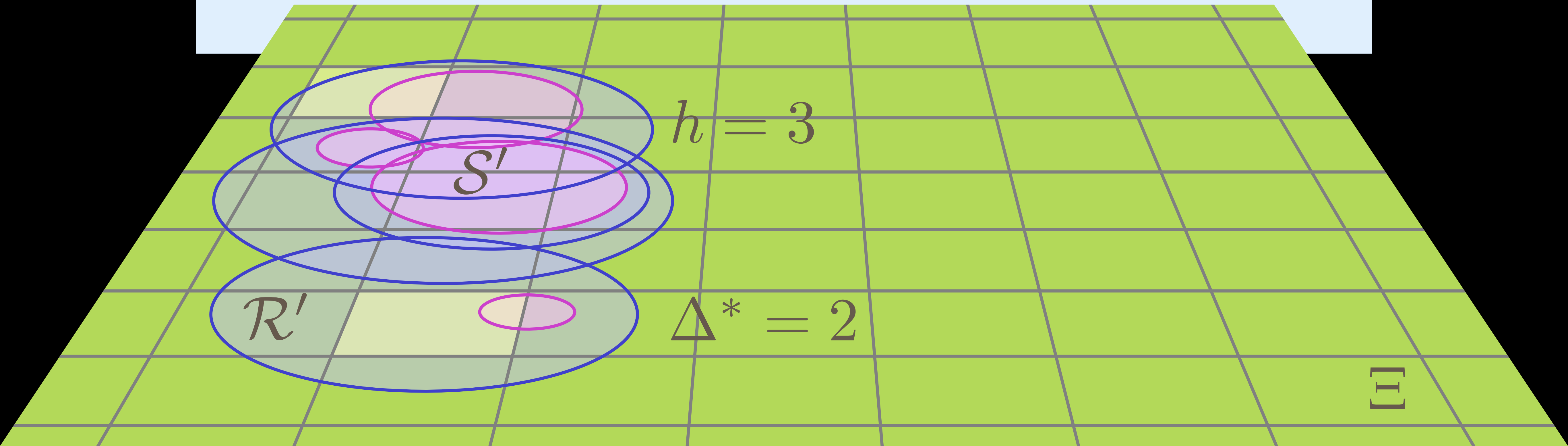

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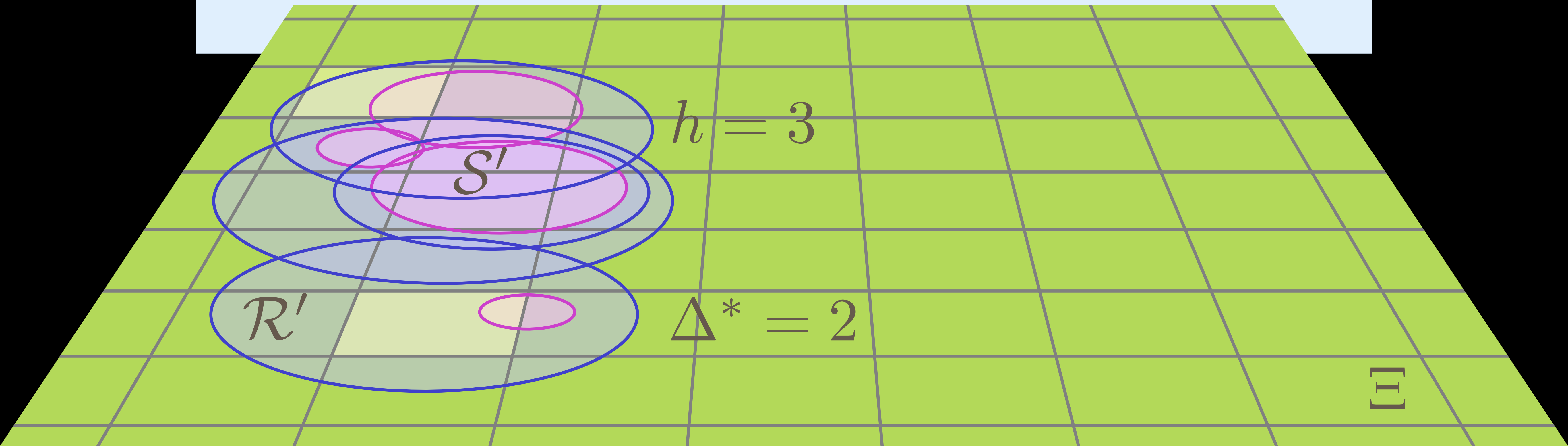

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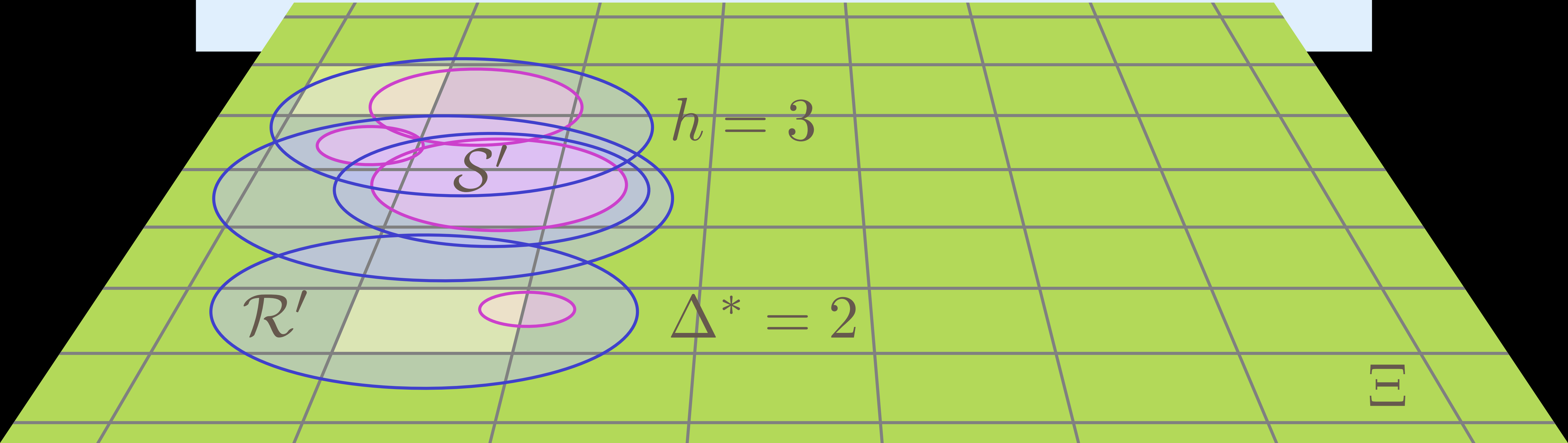

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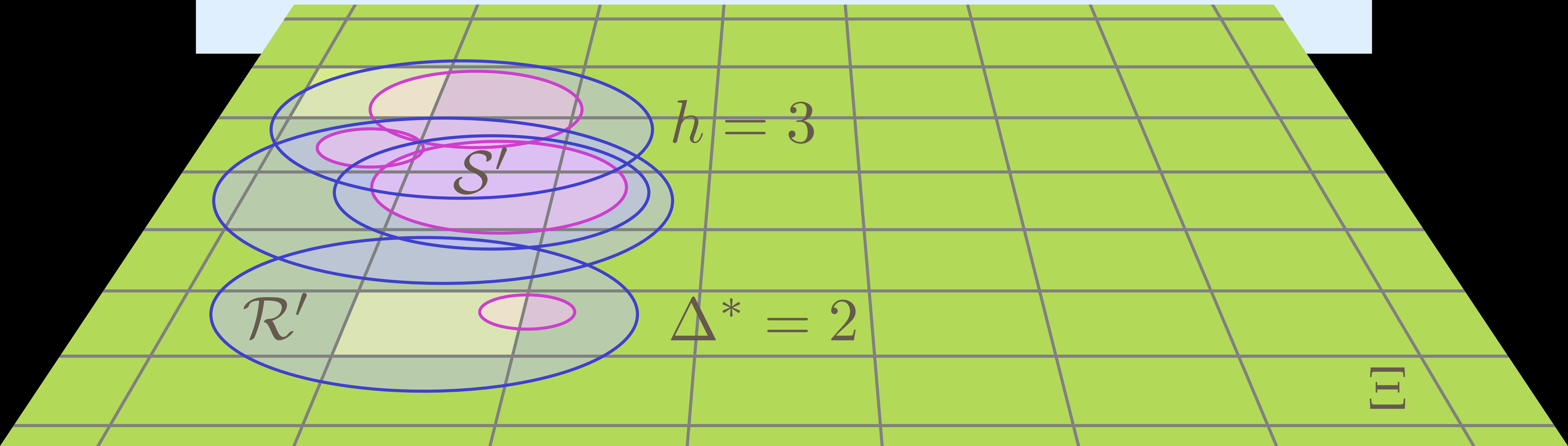

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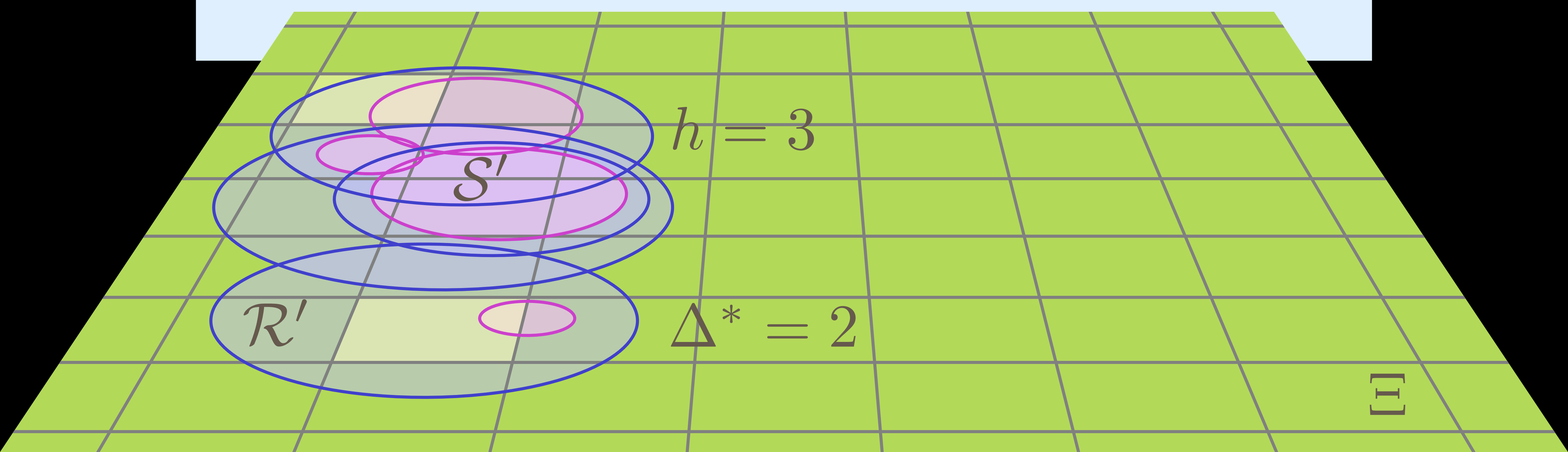

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THANK YOU!