Towards the Minimization of Global Measures of Congestion Potential for Moving Points

Will Evans
Ivor van der Hoog
David Kirkpatrick
Maarten Löffler

MOVING POINTS

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Location-aware mobile devices

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- Location-aware mobile devices
  - GPS systems

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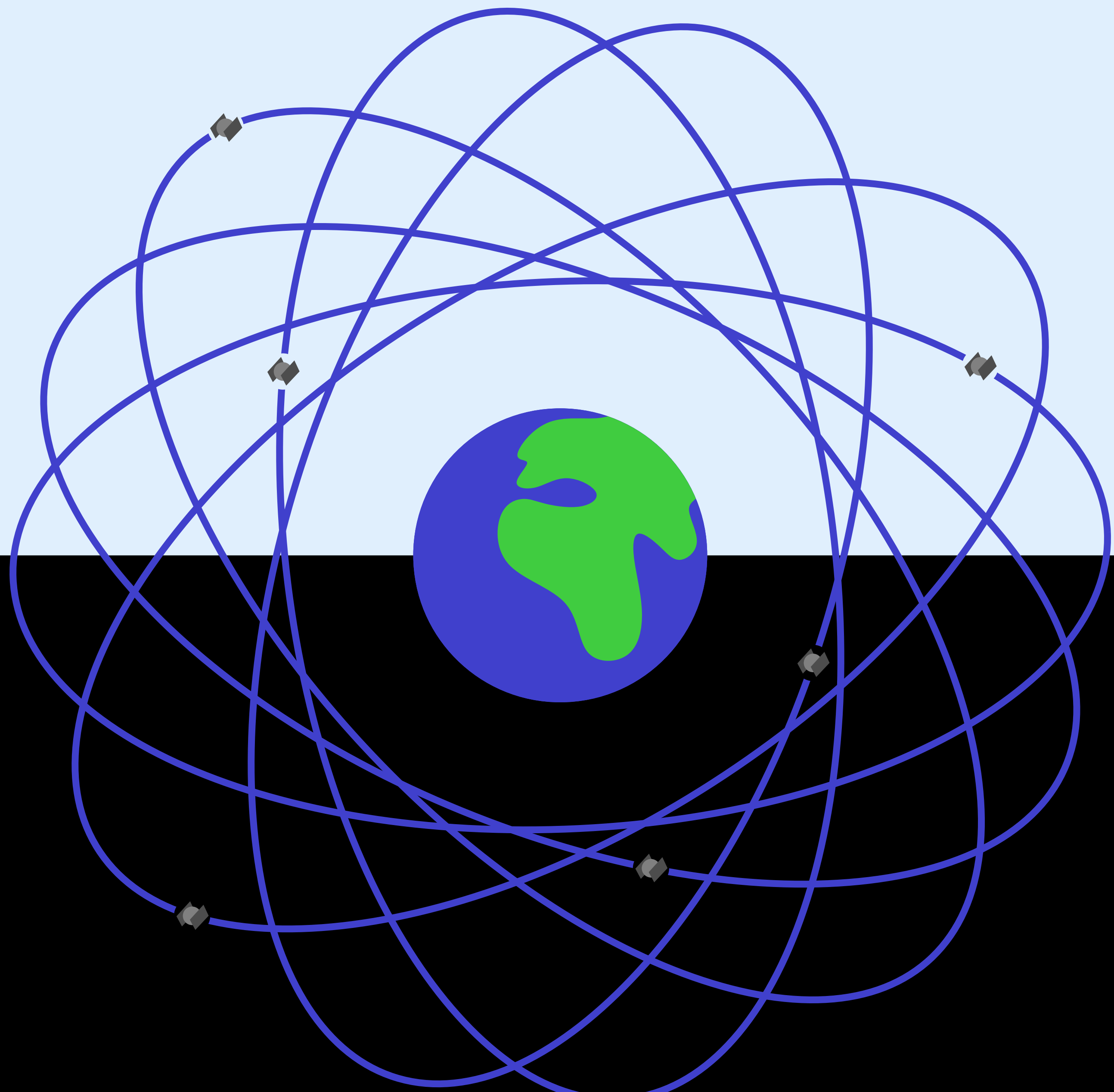
Location-aware mobile devices

GPS systems



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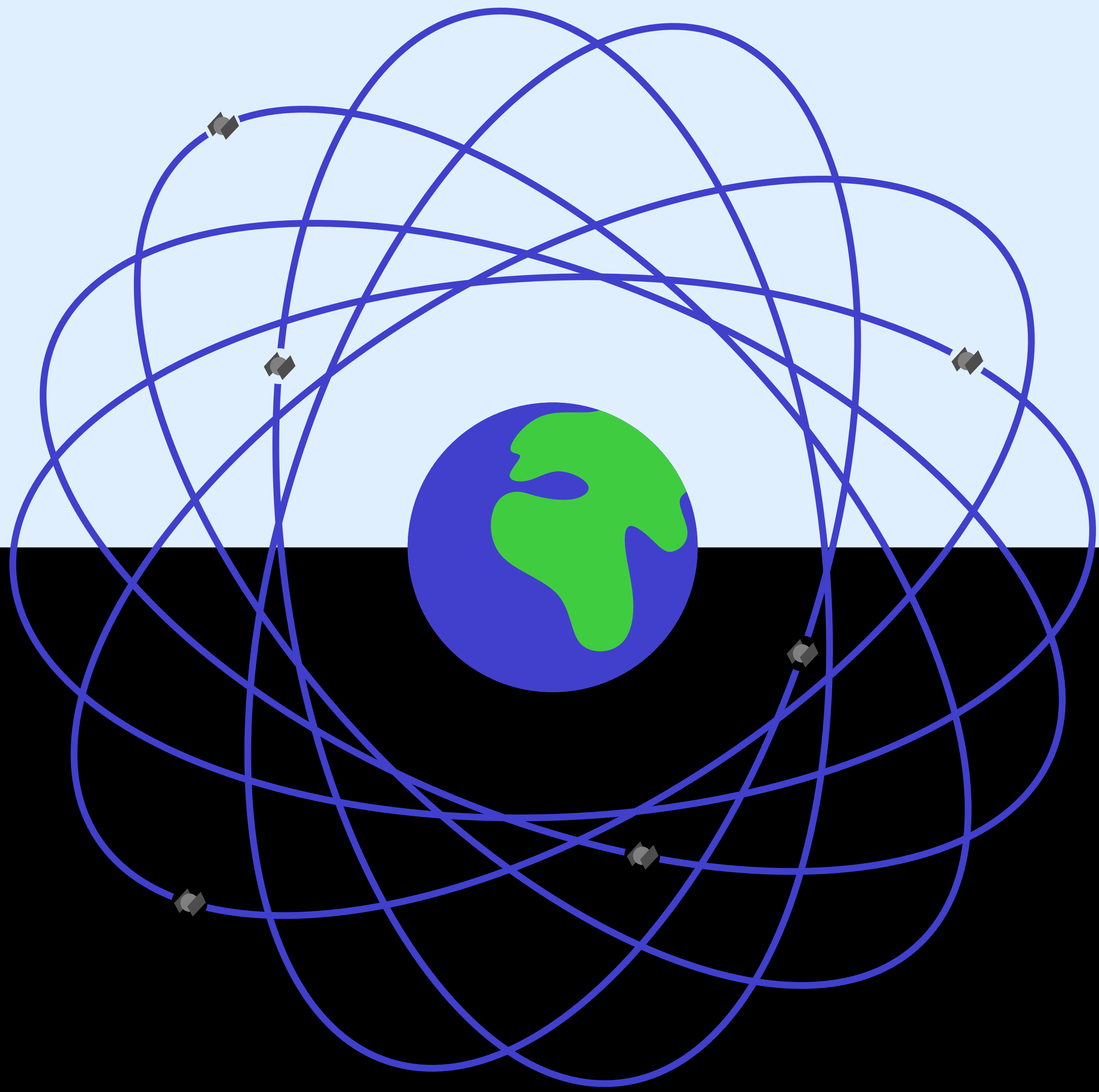

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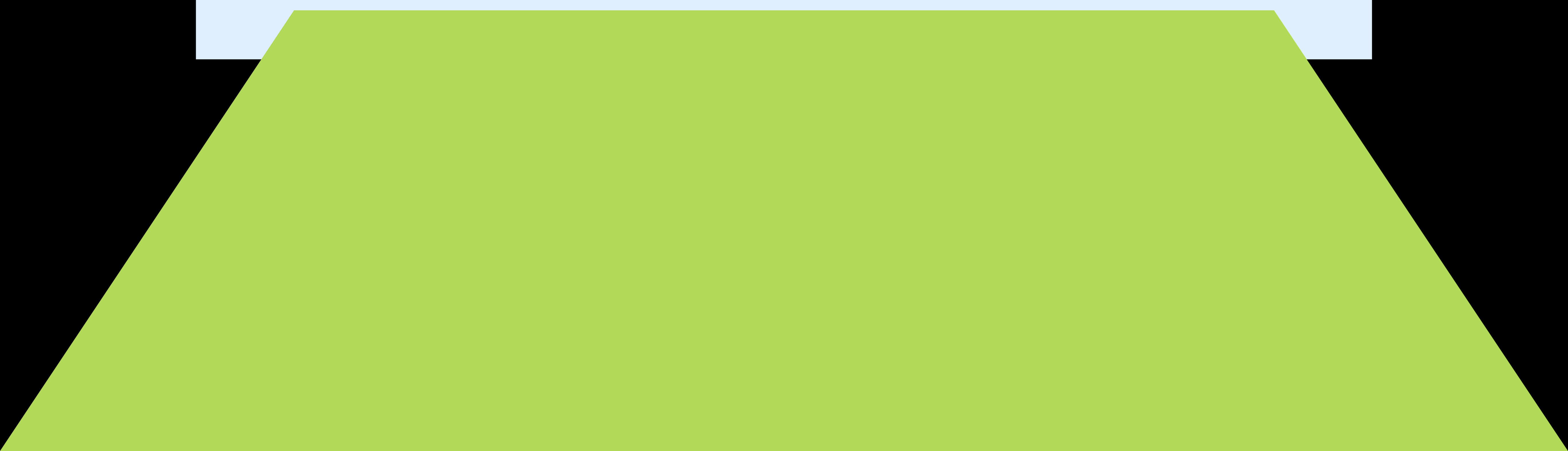

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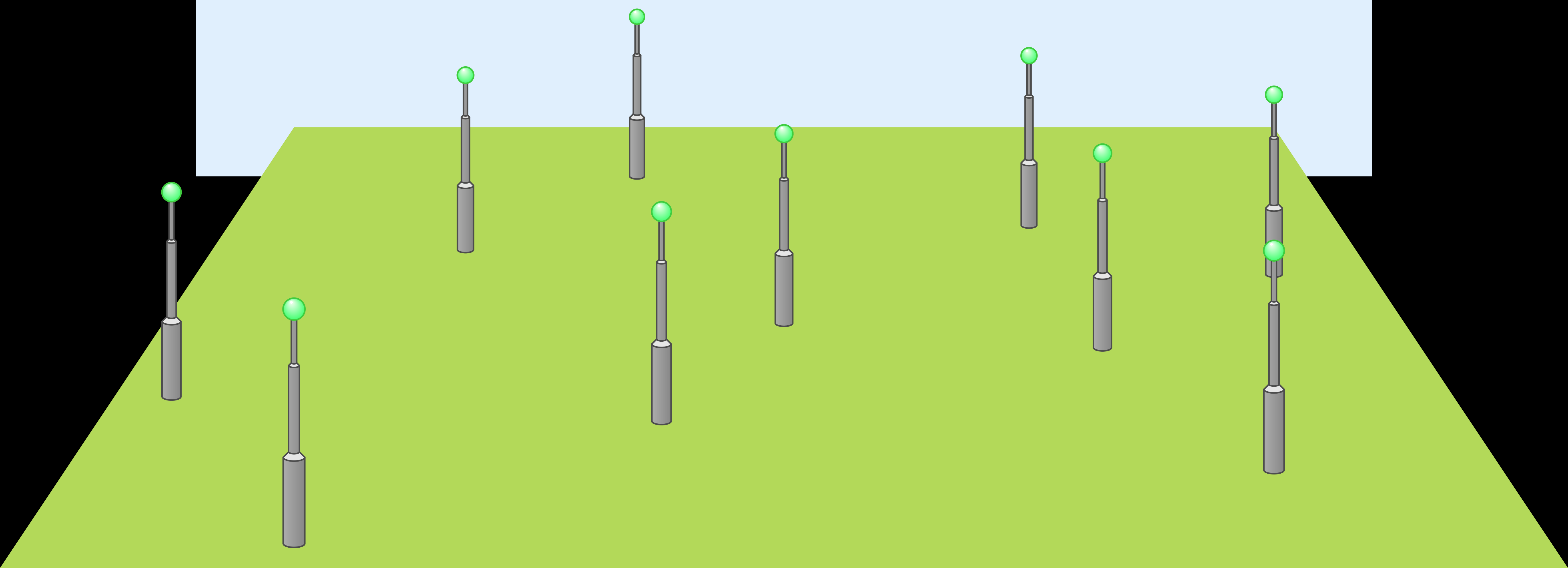

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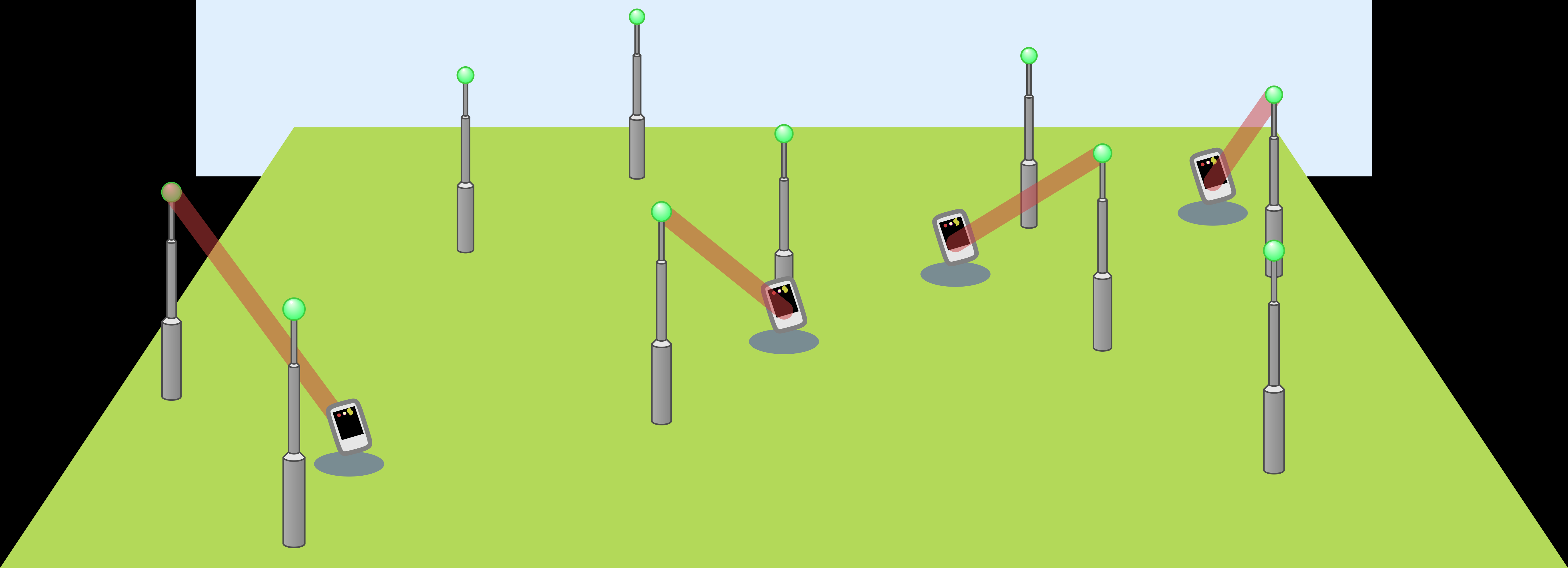

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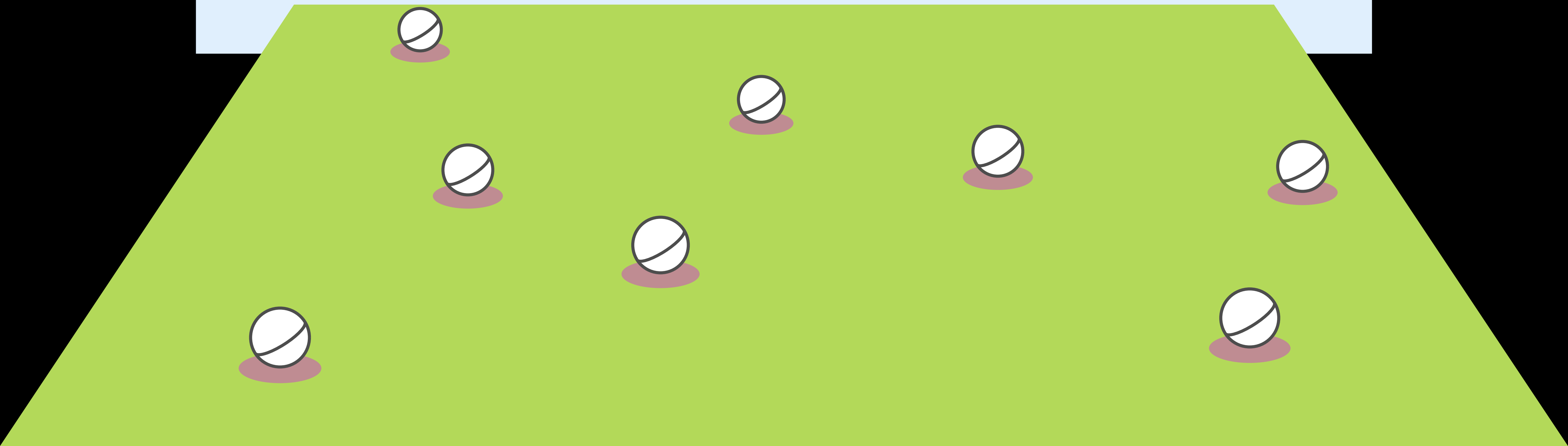

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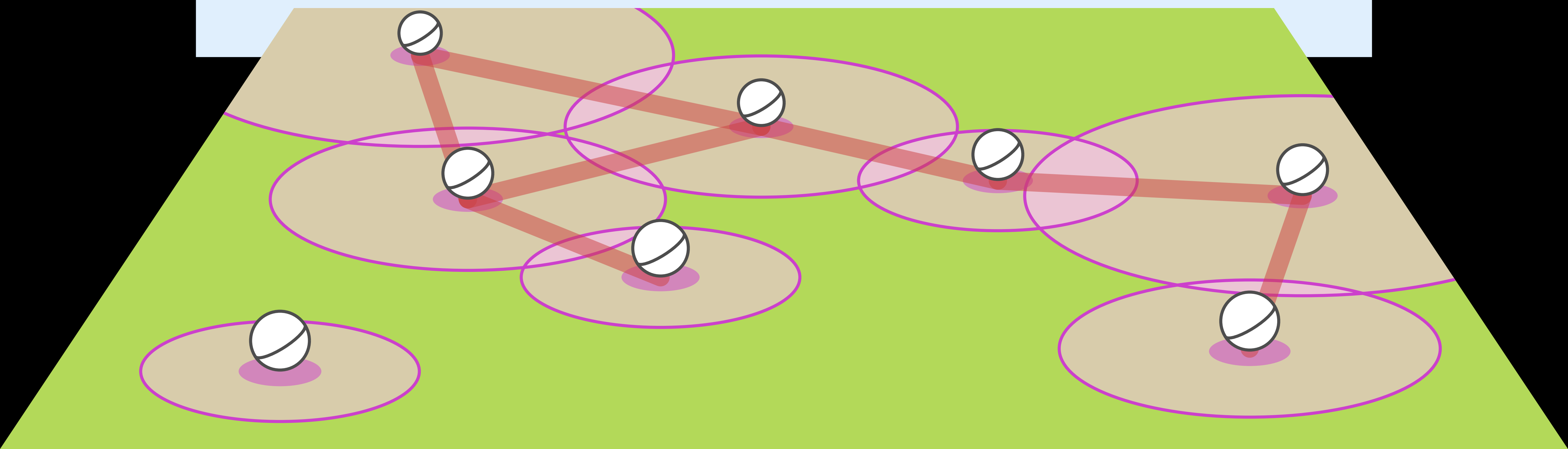

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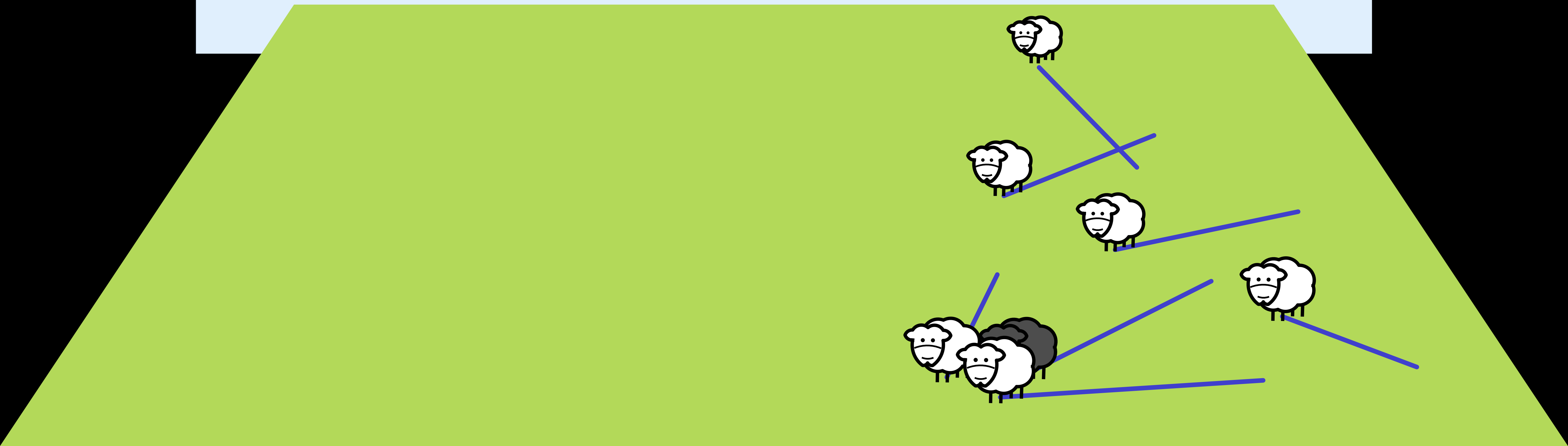

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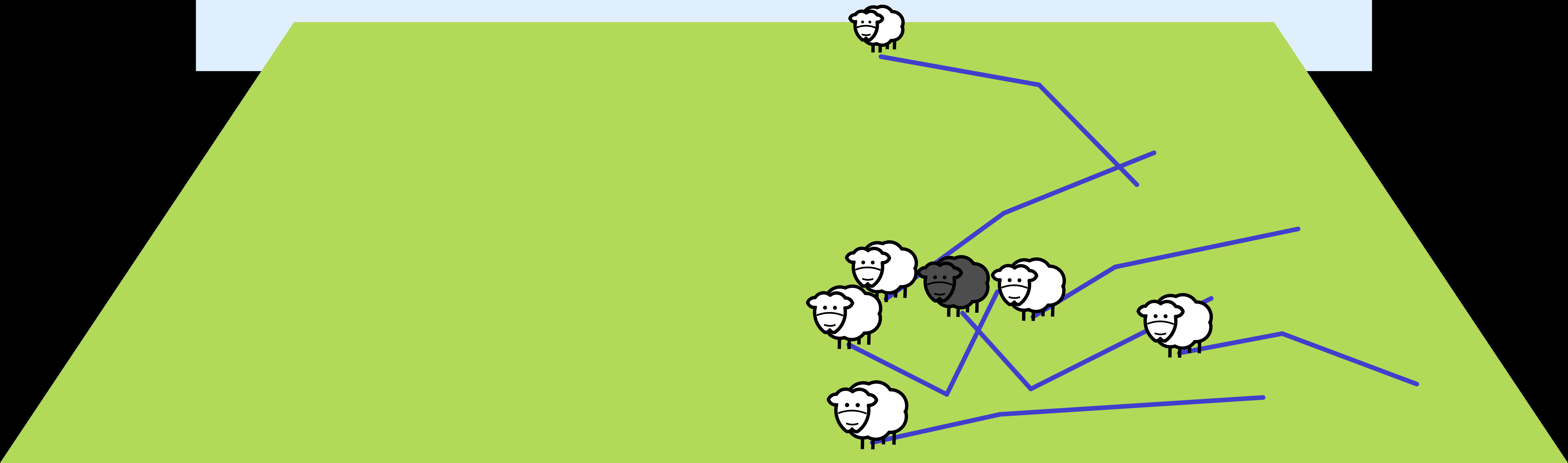

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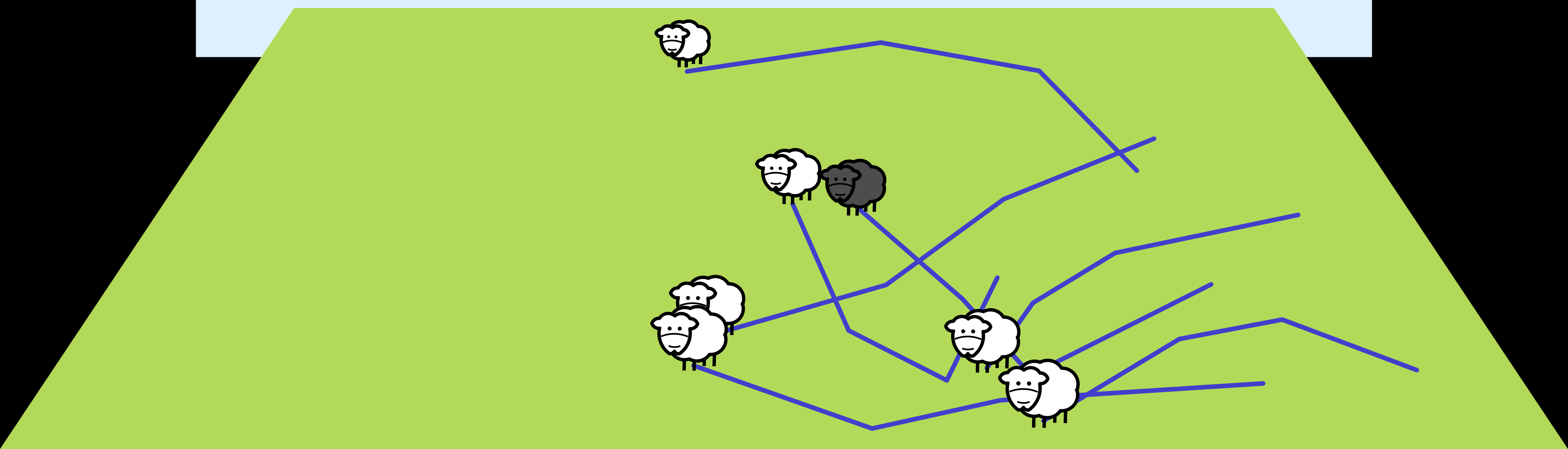

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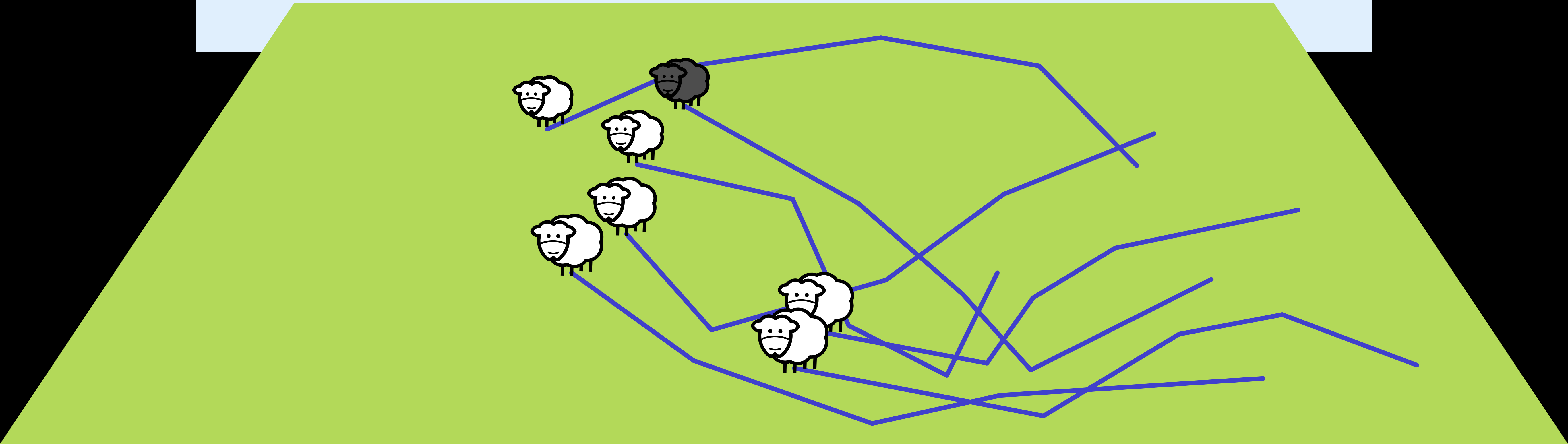

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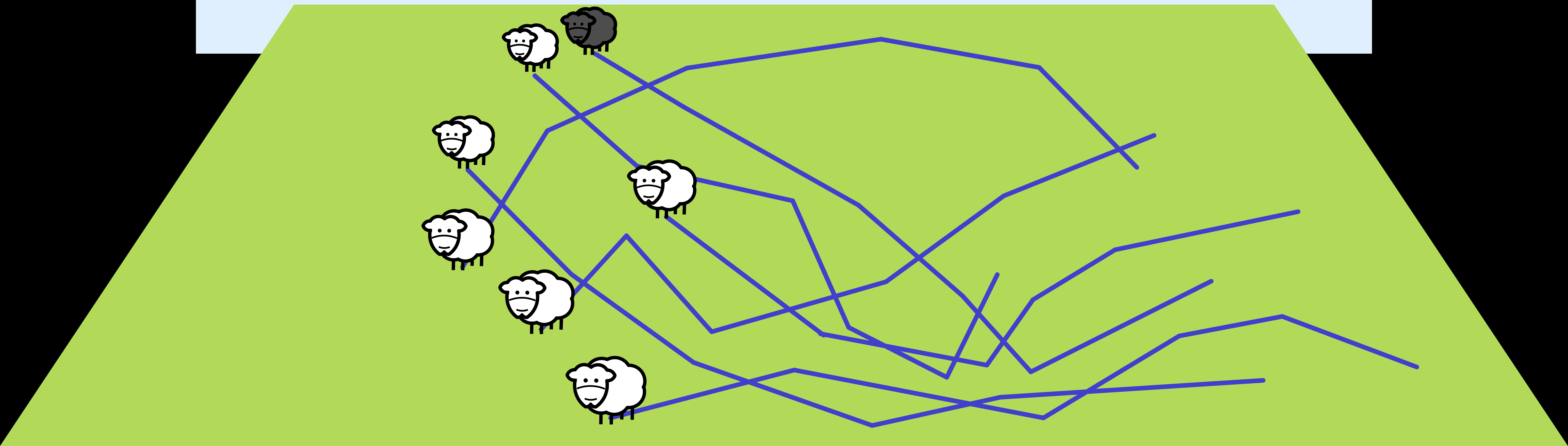

MOVING POINTS

- Location-aware mobile devices
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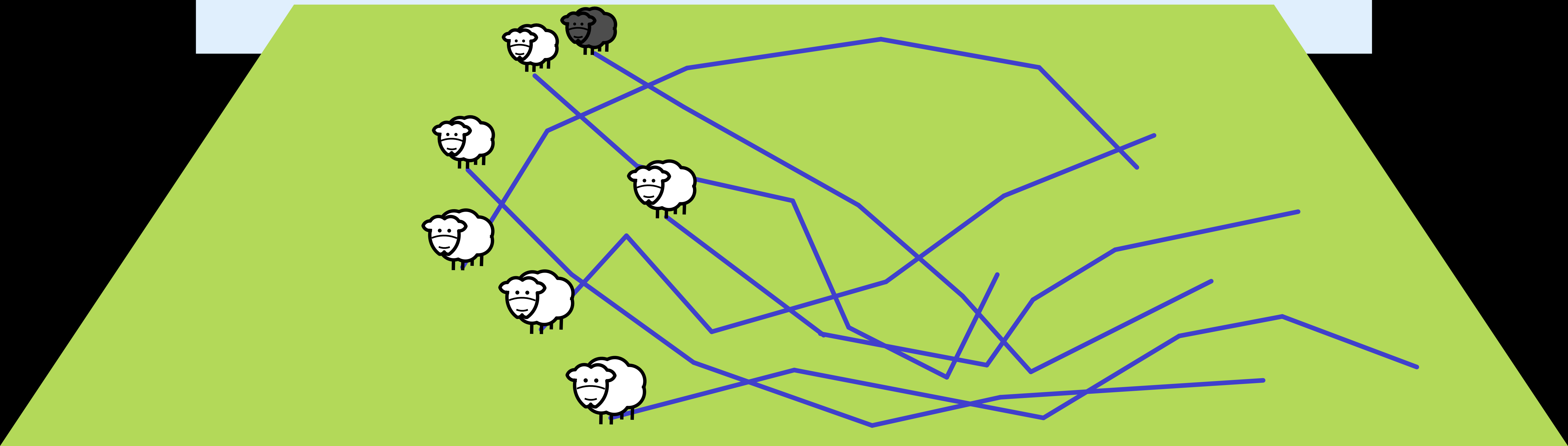
- New geometric data sets



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- Location-aware mobile devices
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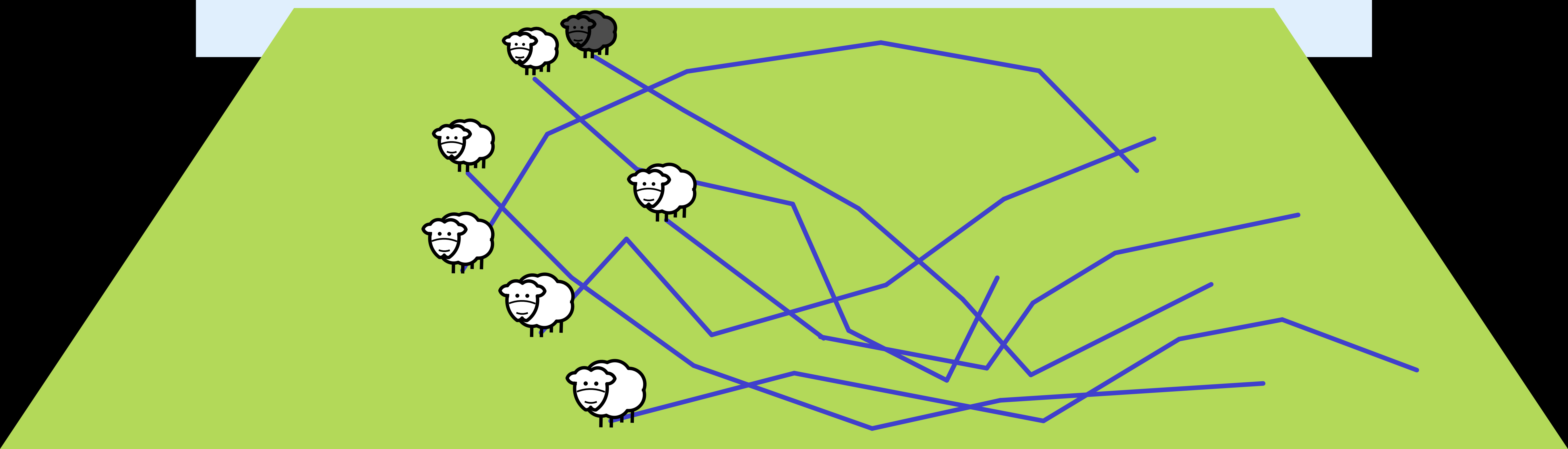
- New geometric data sets
  - Often very large



MOVING POINTS

- Location-aware mobile devices
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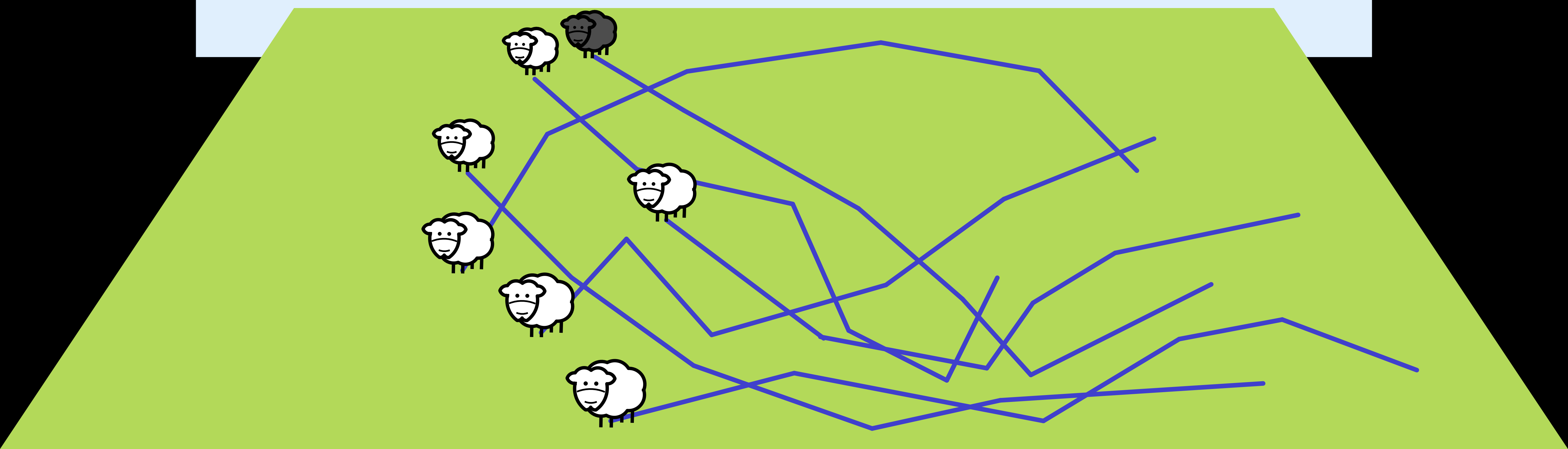
- New geometric data sets
  - Often very large
  - Imprecise in nature



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- Location-aware mobile devices
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  - Wireless sensor networks
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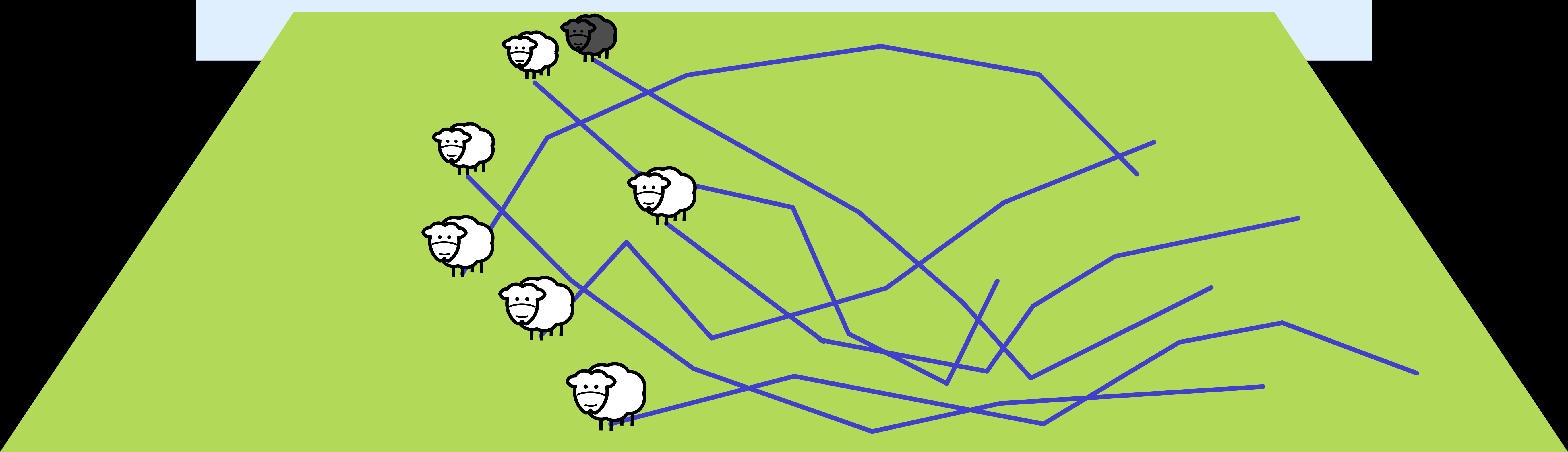
- New geometric data sets
  - Often very large
  - Imprecise in nature
  - Dynamic / unpredictable



UNCERTAIN POINTS

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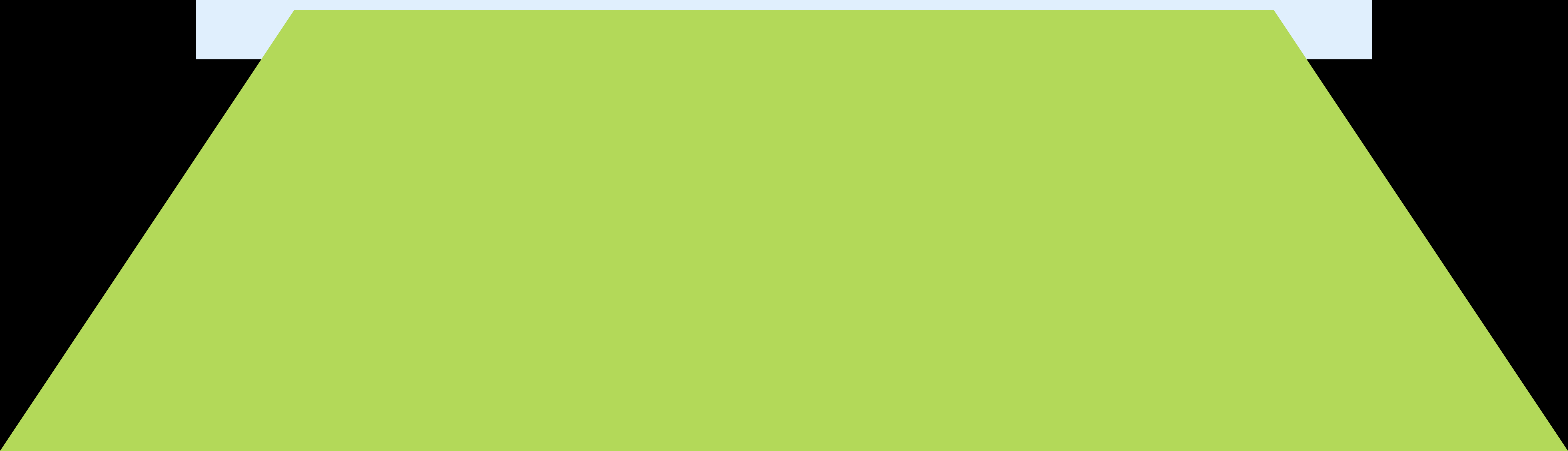
Motion causes imprecision



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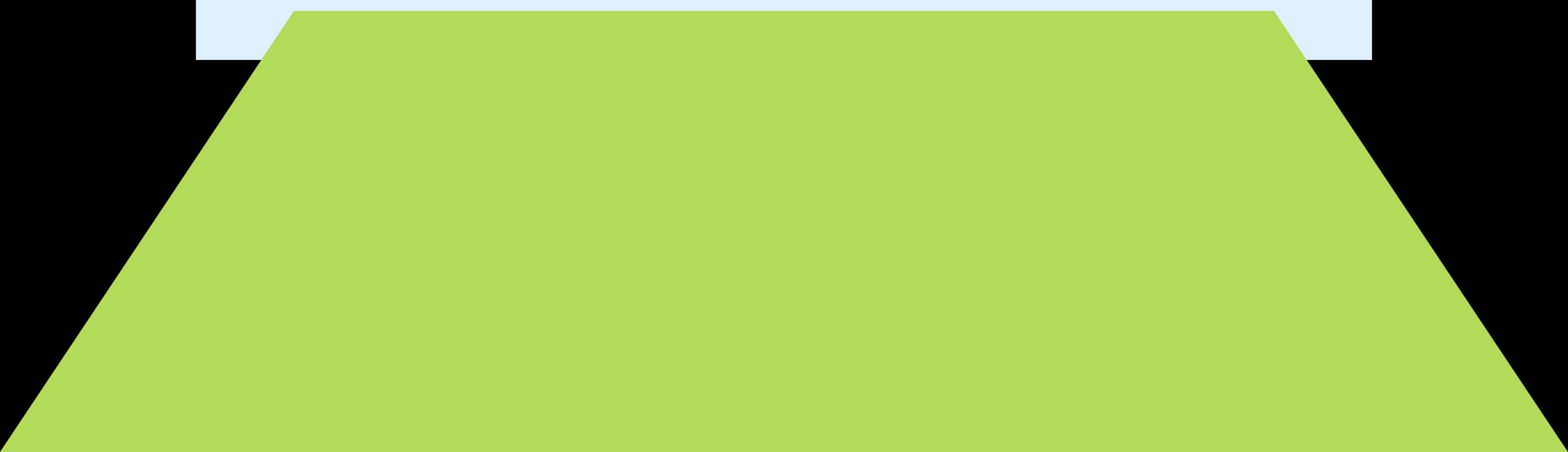

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- Suppose we have a moving point p
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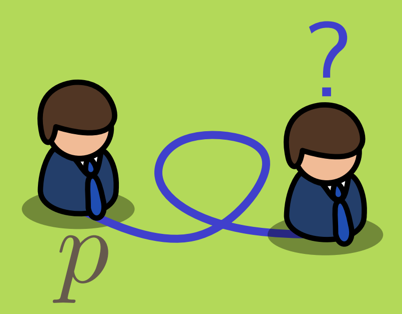

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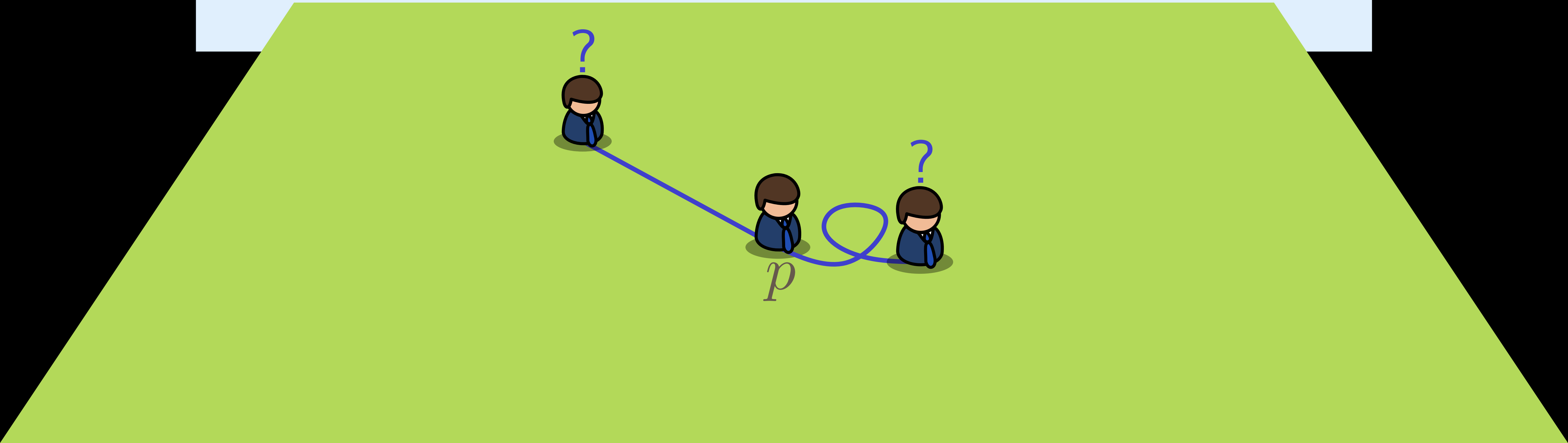

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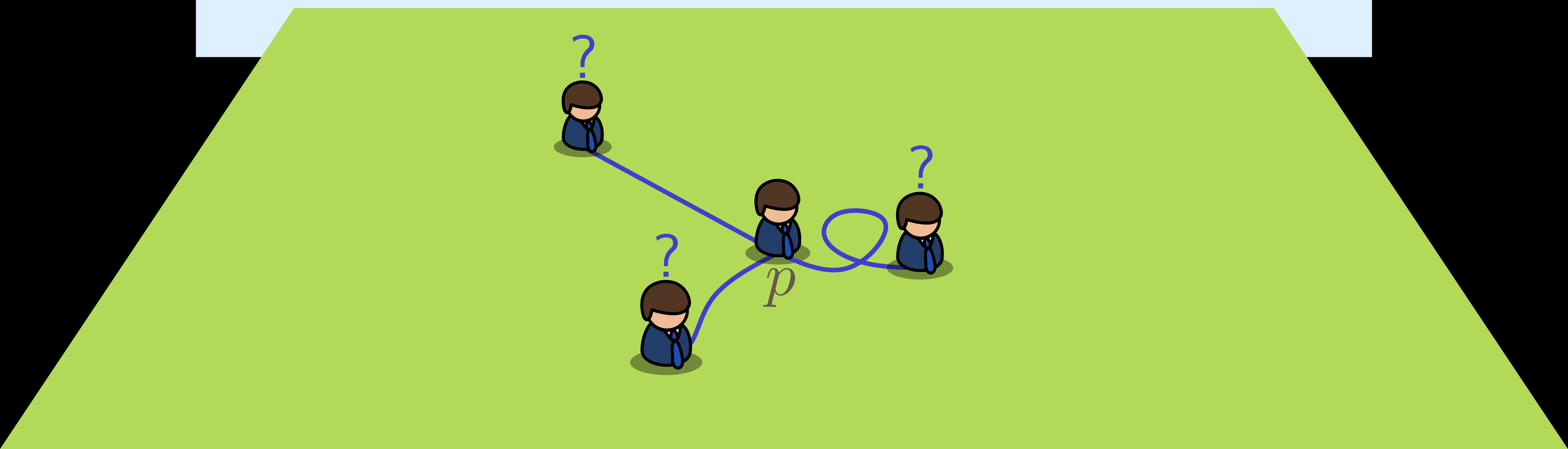

UNCERTAIN POINTS

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- Suppose we have a moving point p
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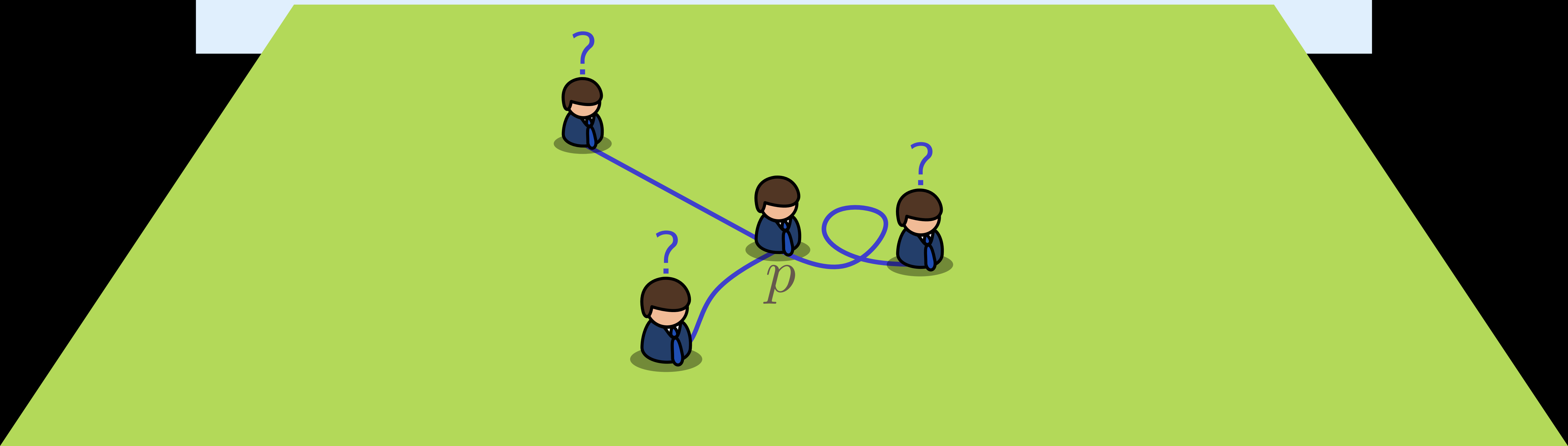
- We model the *uncertainty region* of p in the next time step by a disk of radius d



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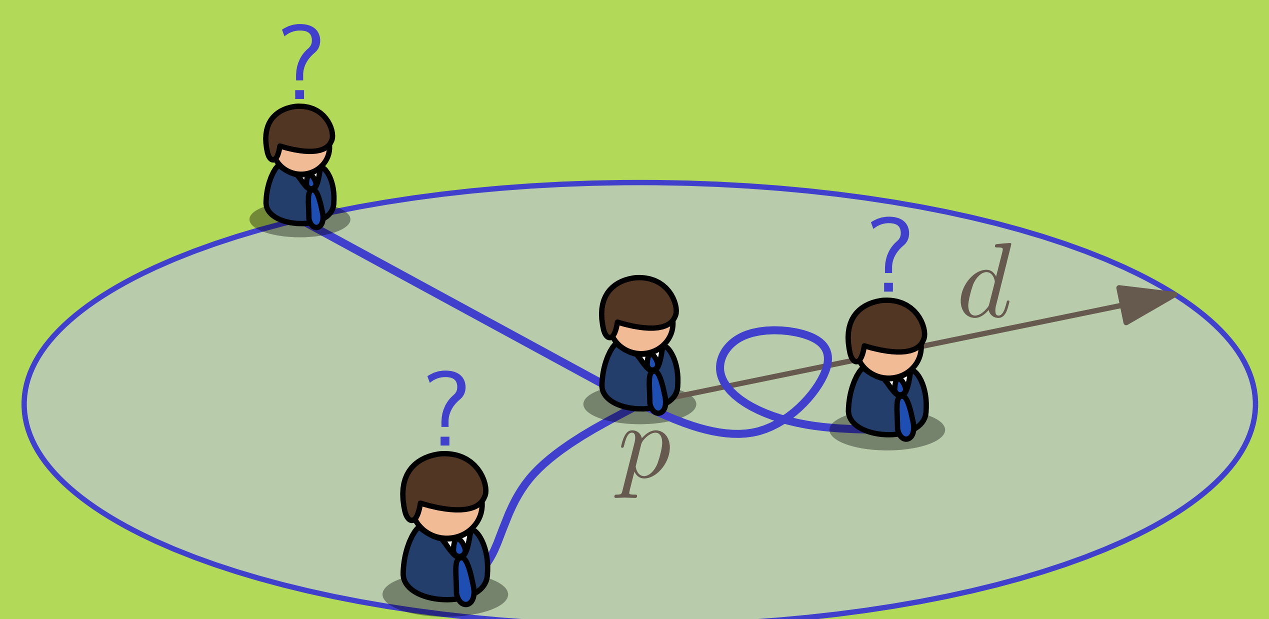

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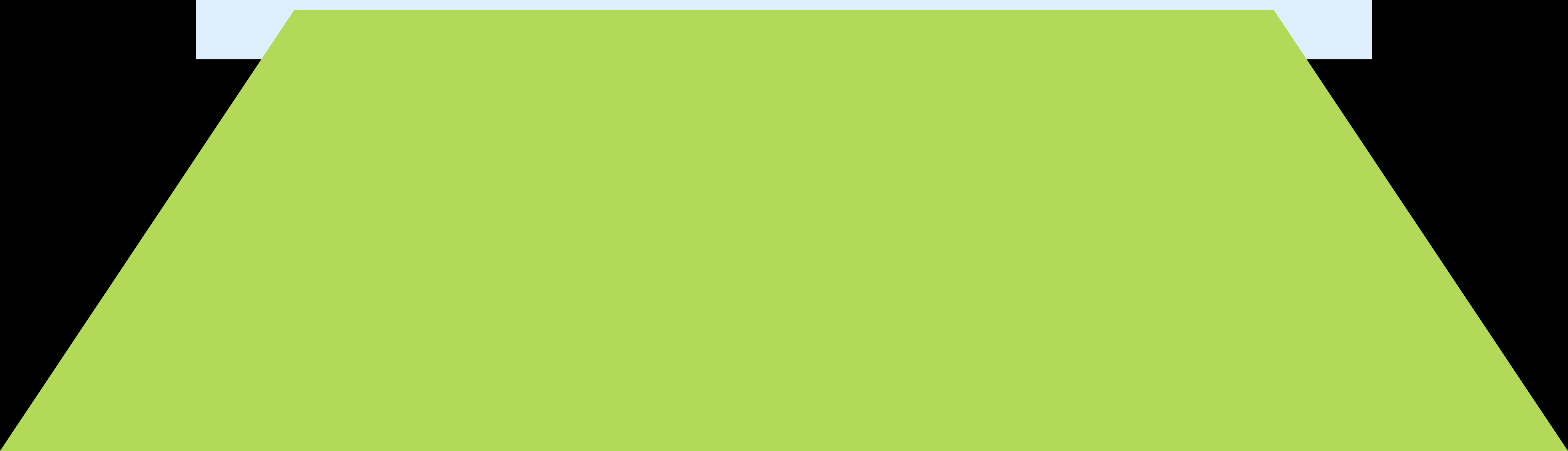

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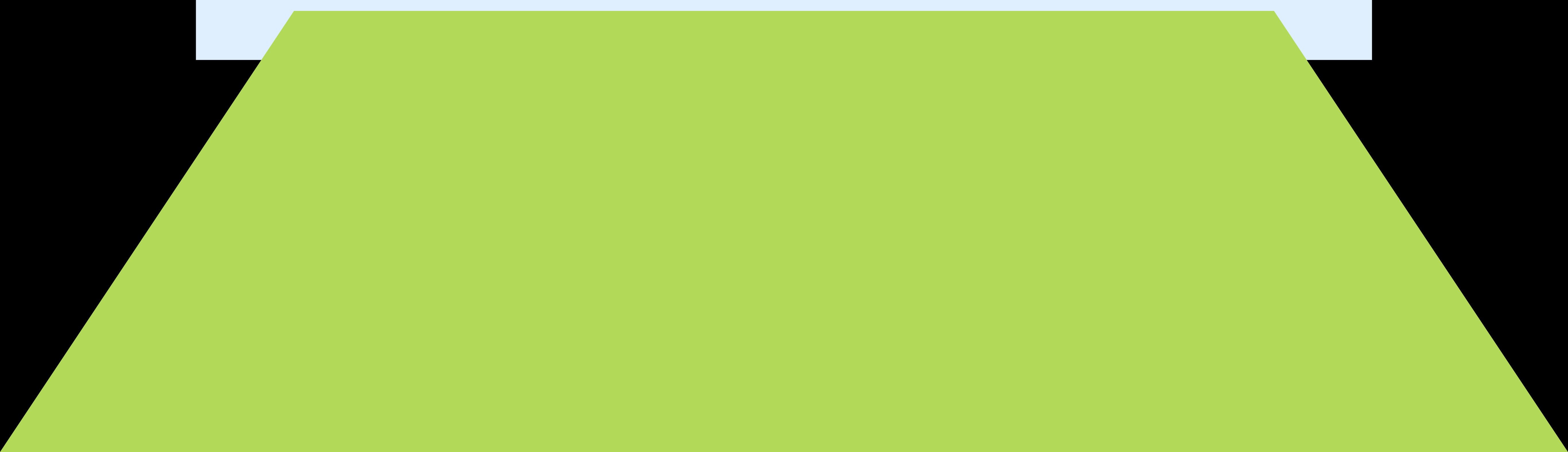

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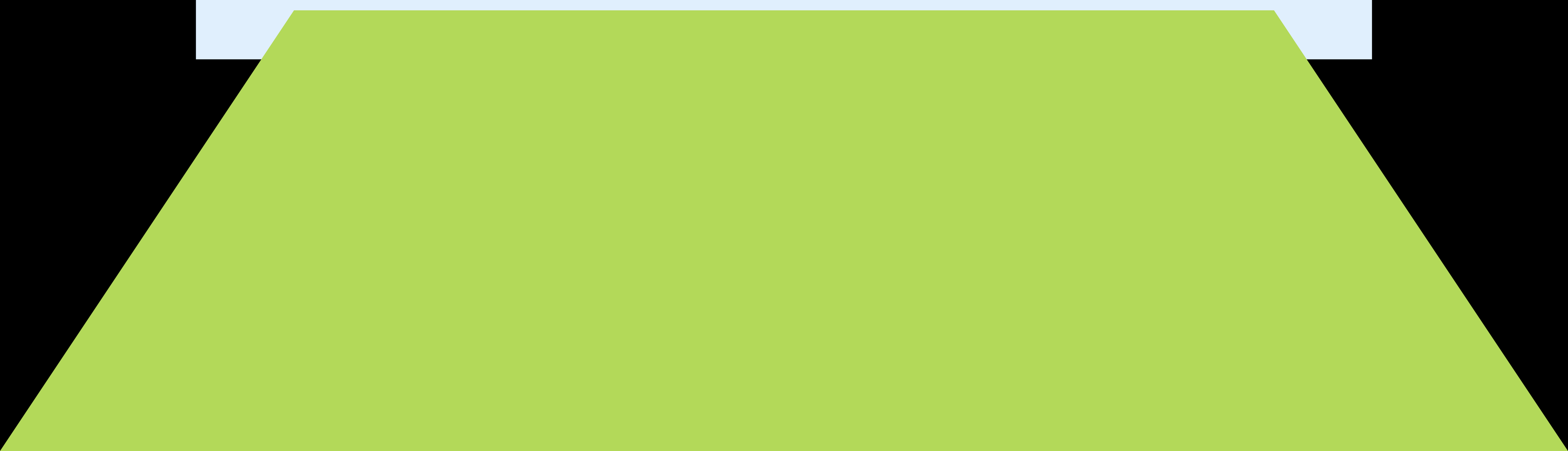
Suppose that for each point we know a *location* p_i and an associated *time stamp* t_i

We can view $(p_i, t - t_i)$ as points in $\mathbb{R}^3$



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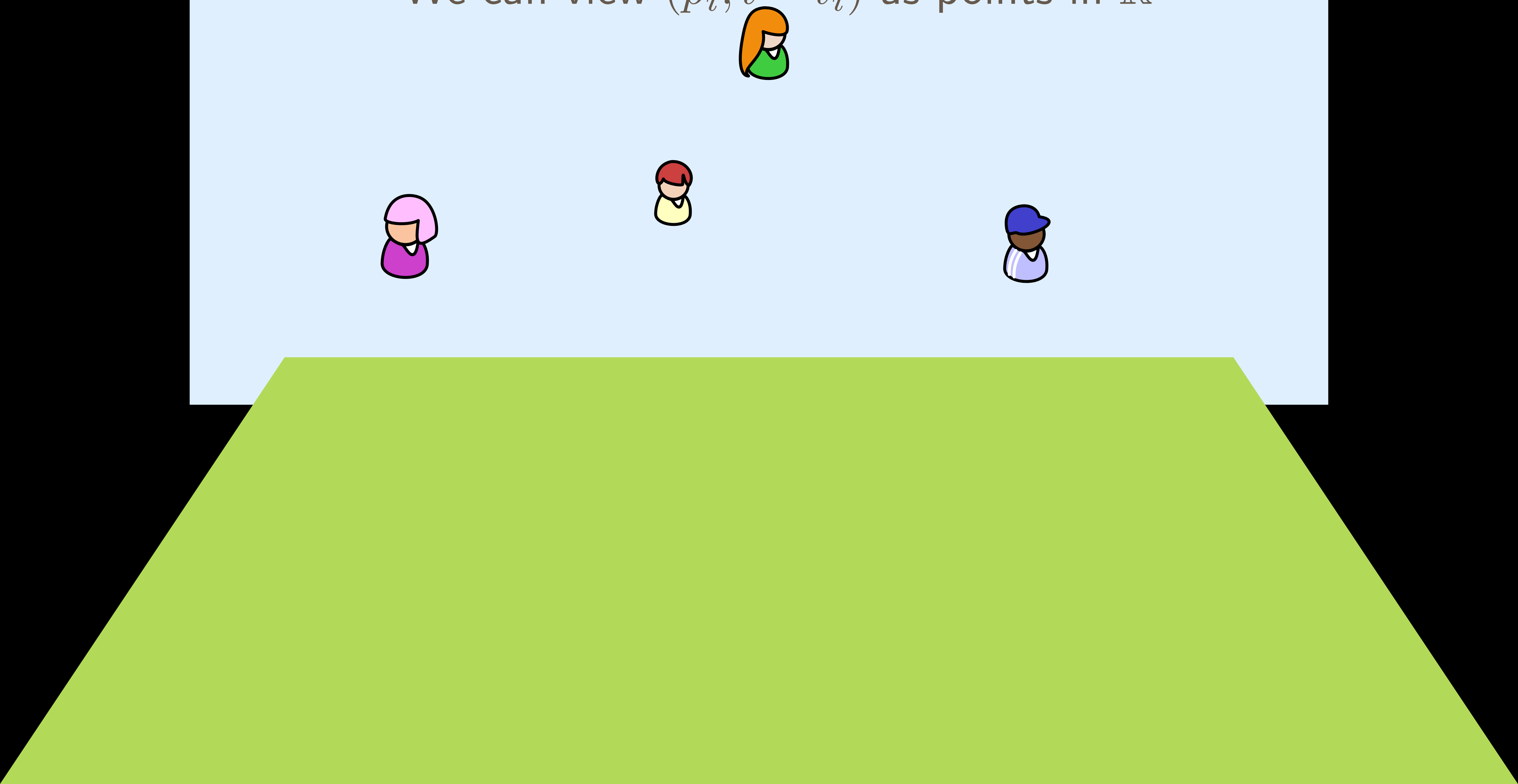

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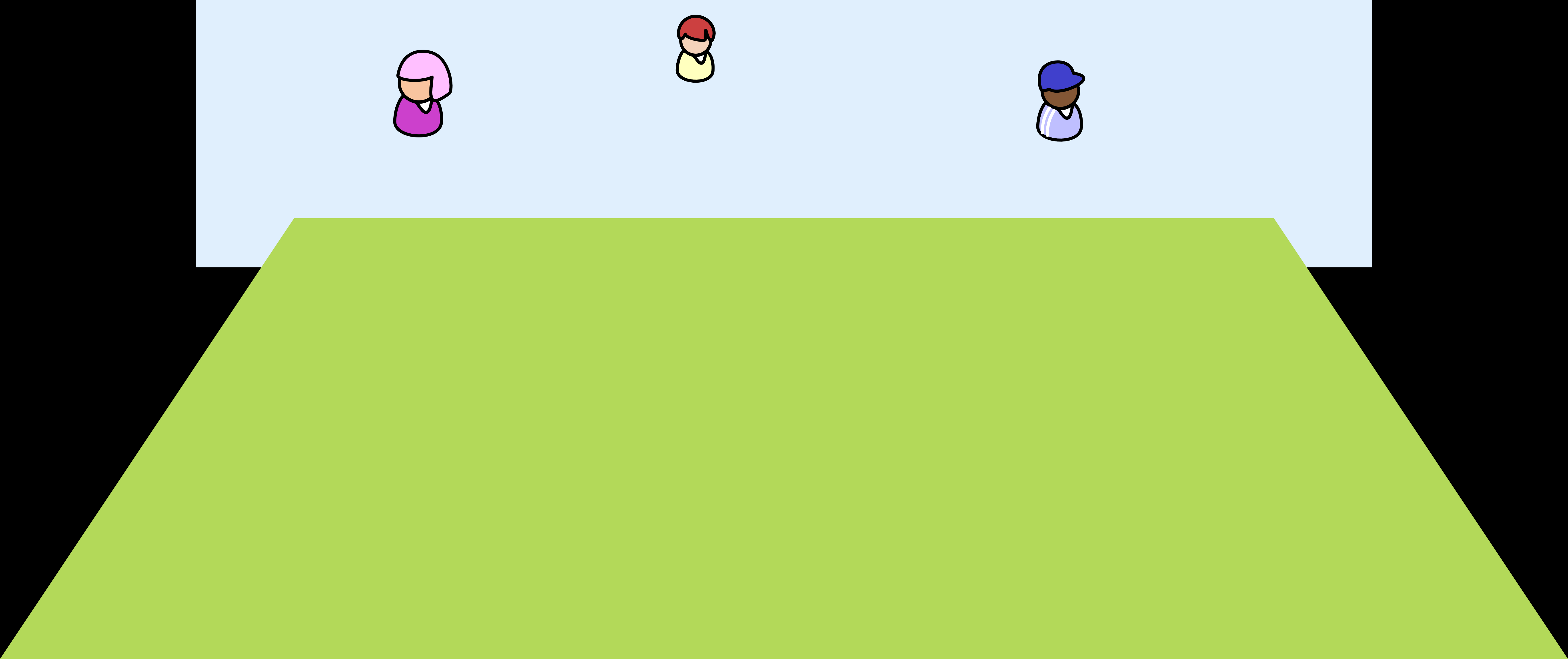
- Suppose that for each point we know a *location* p_i and an associated *time stamp* t_i
- We can view $(p_i, t - t_i)$ as points in $\mathbb{R}^3$
- Points follow some unknown z-monotone trajectory



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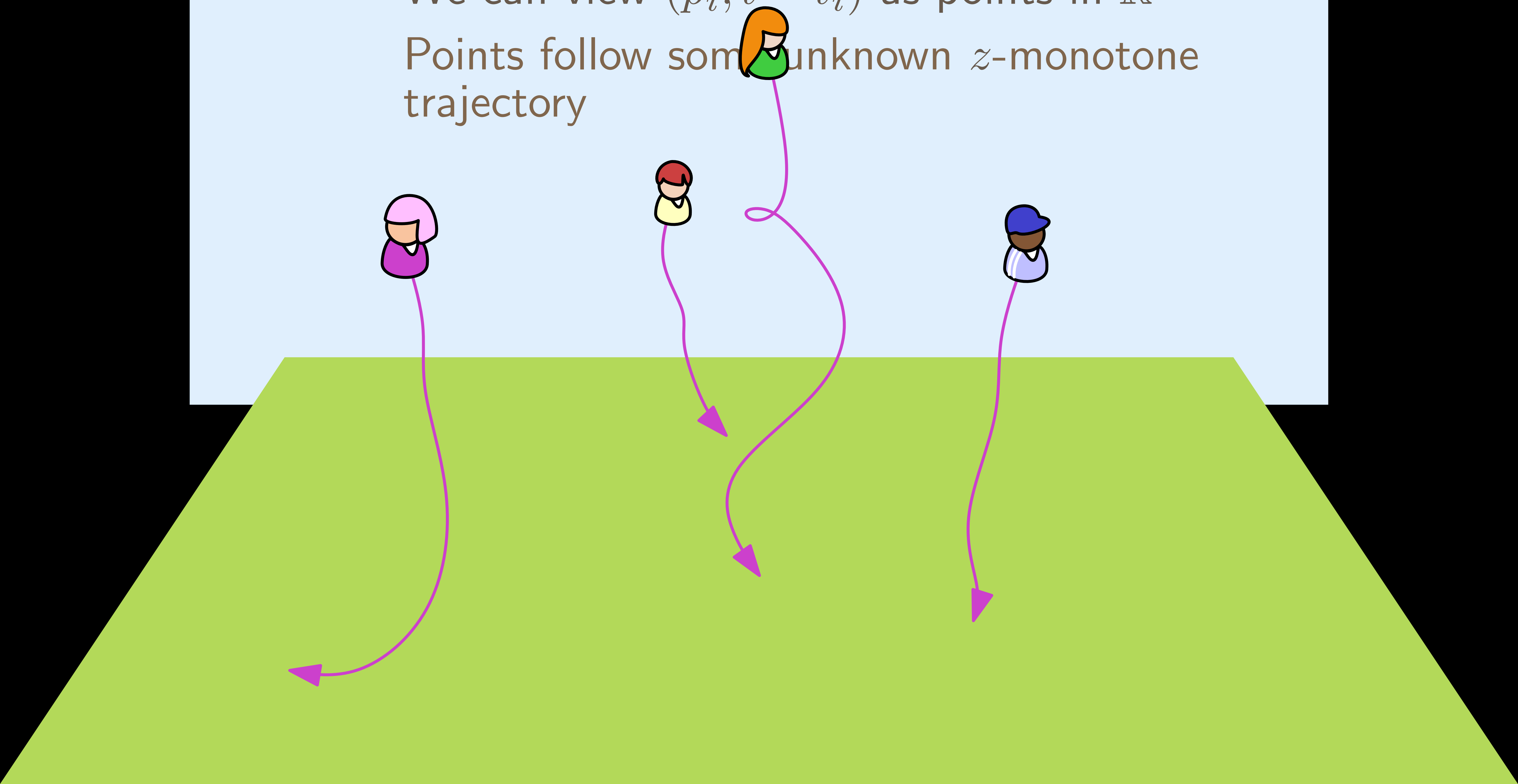

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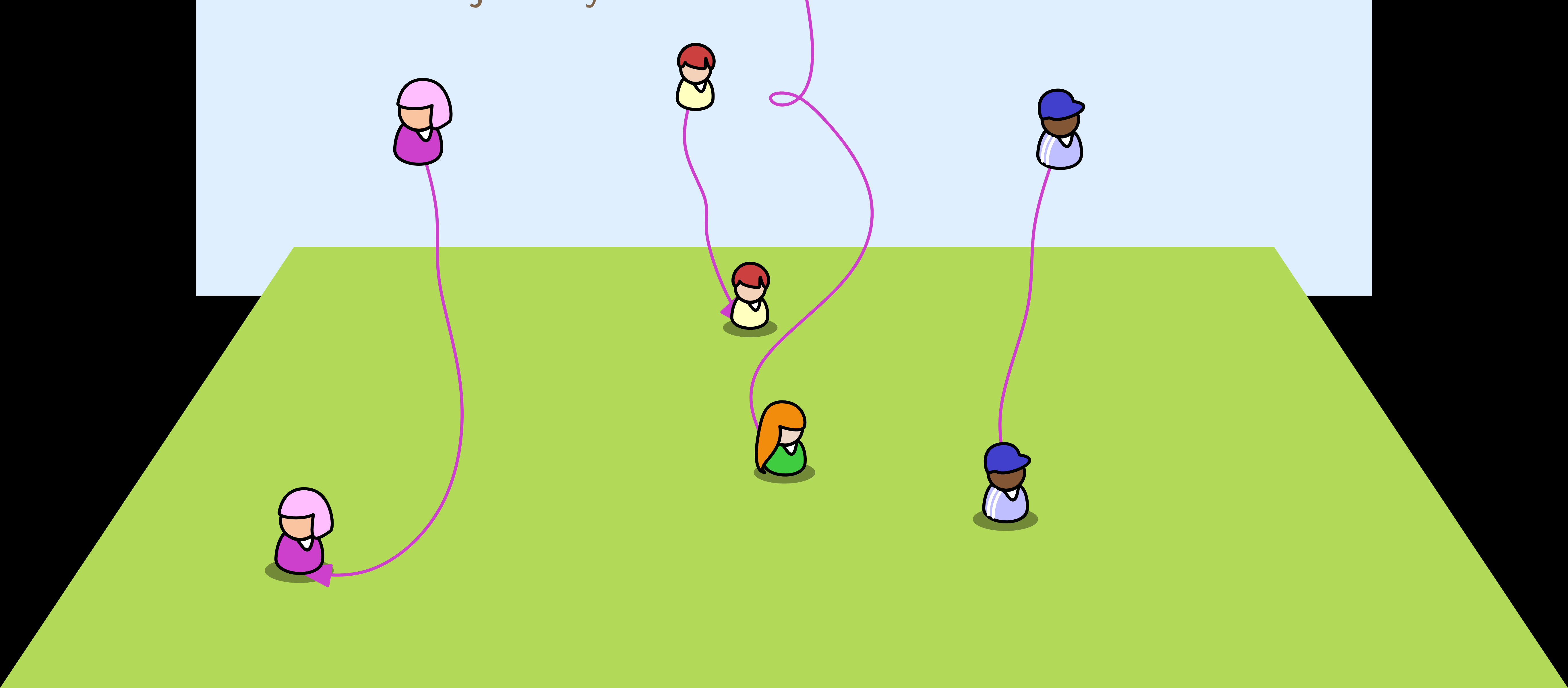

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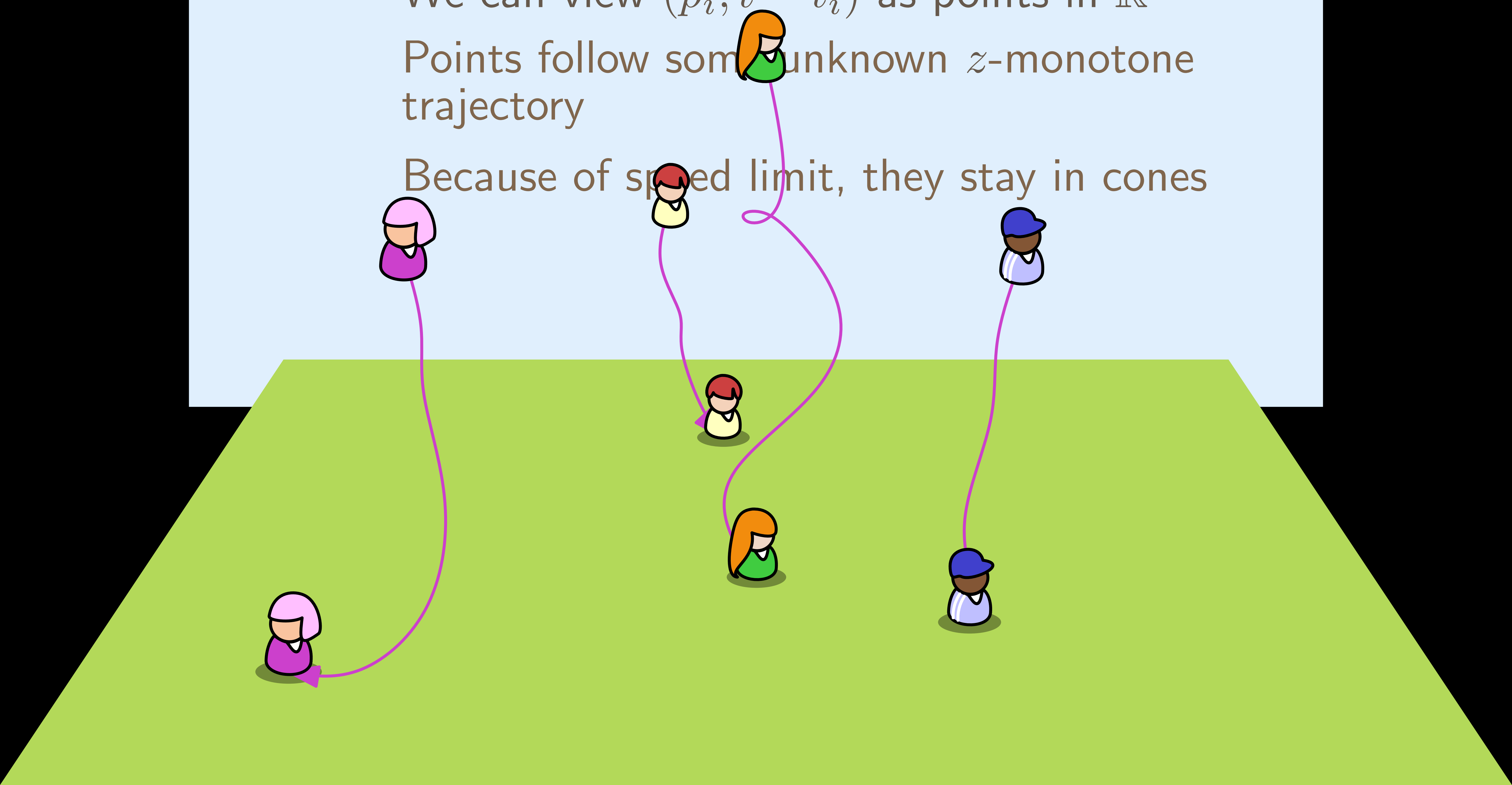
Because of speed limit, they stay in cones



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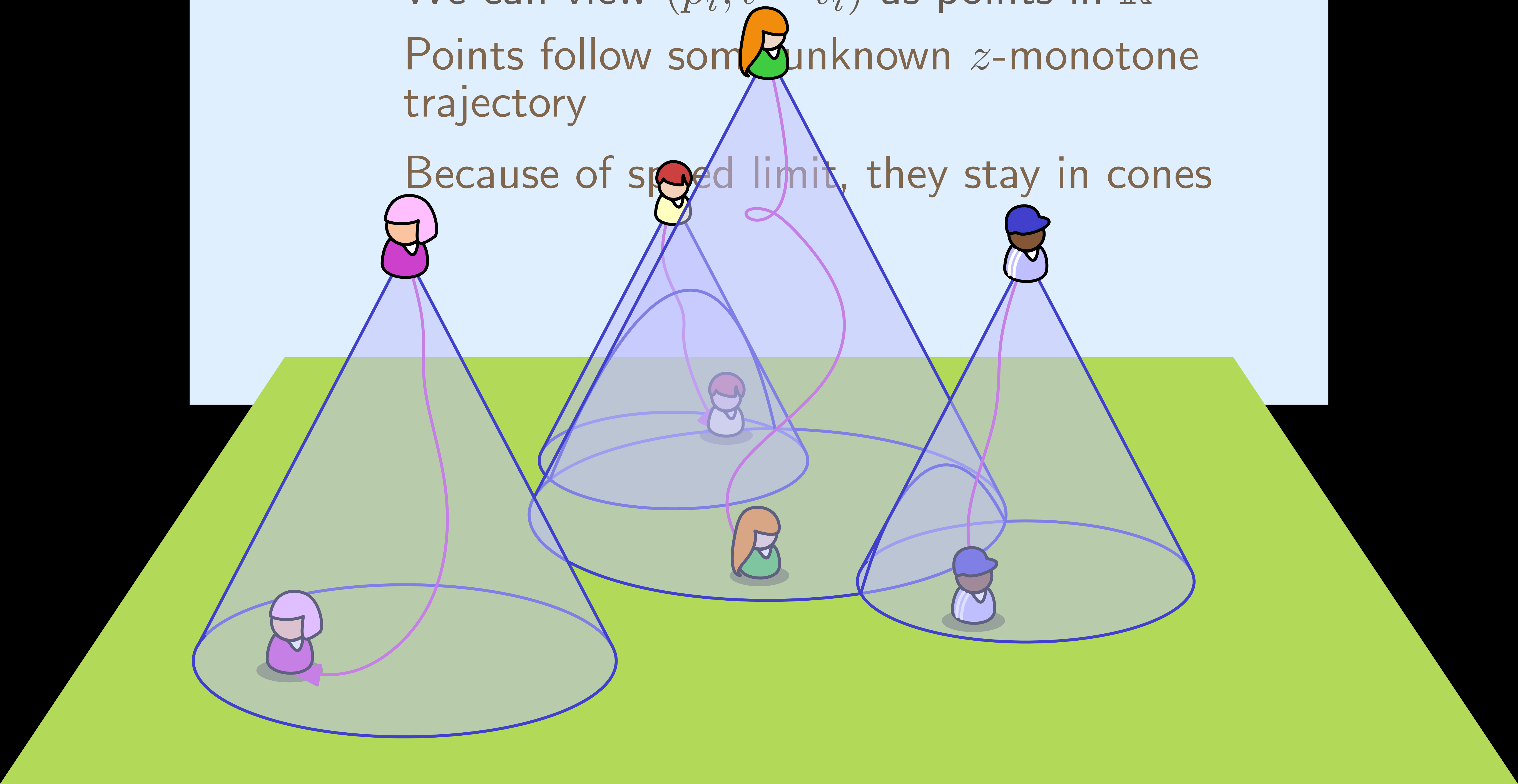

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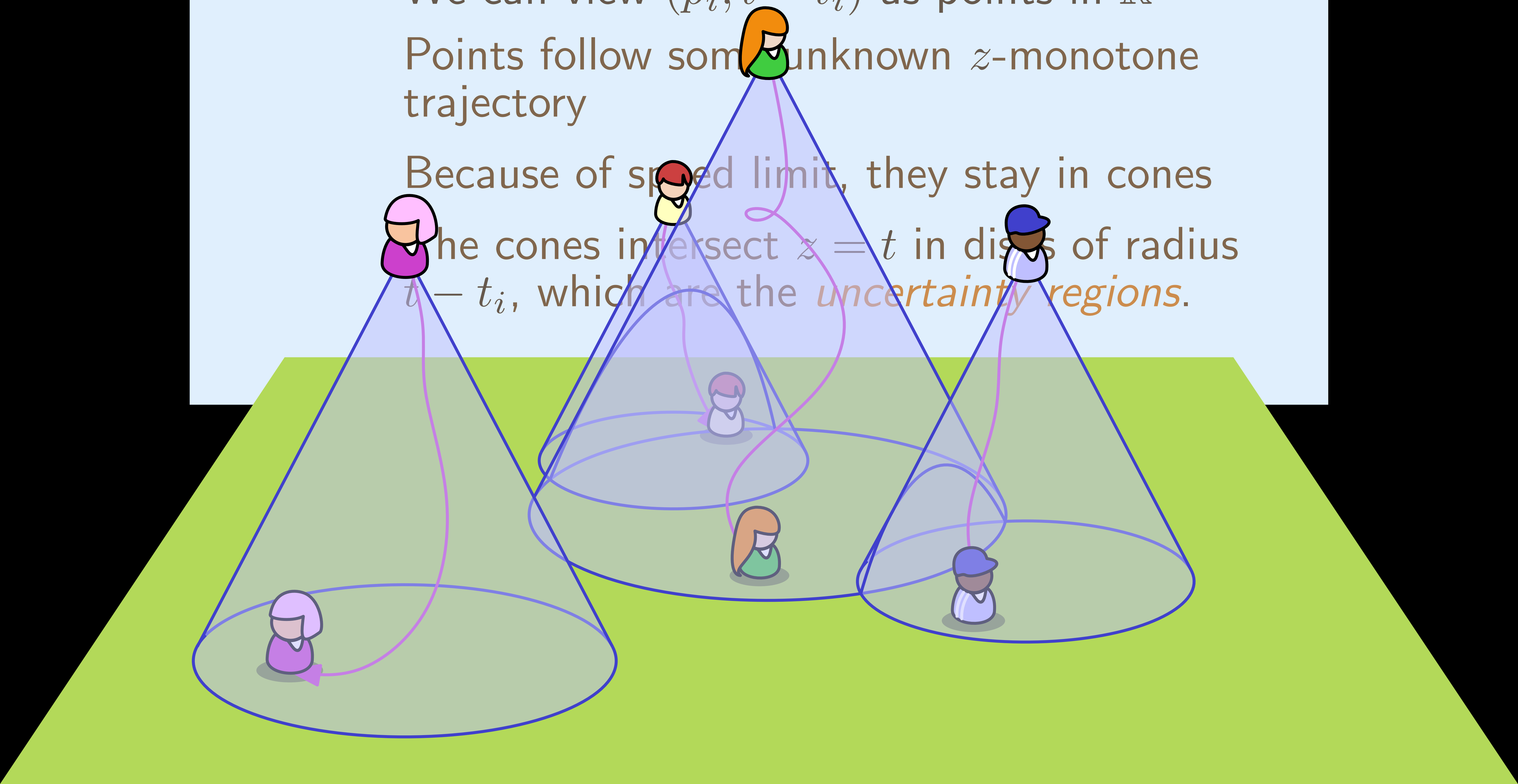
- The cones intersect $z = t$ in discs of radius $t - t_i$, which are the *uncertainty regions*.



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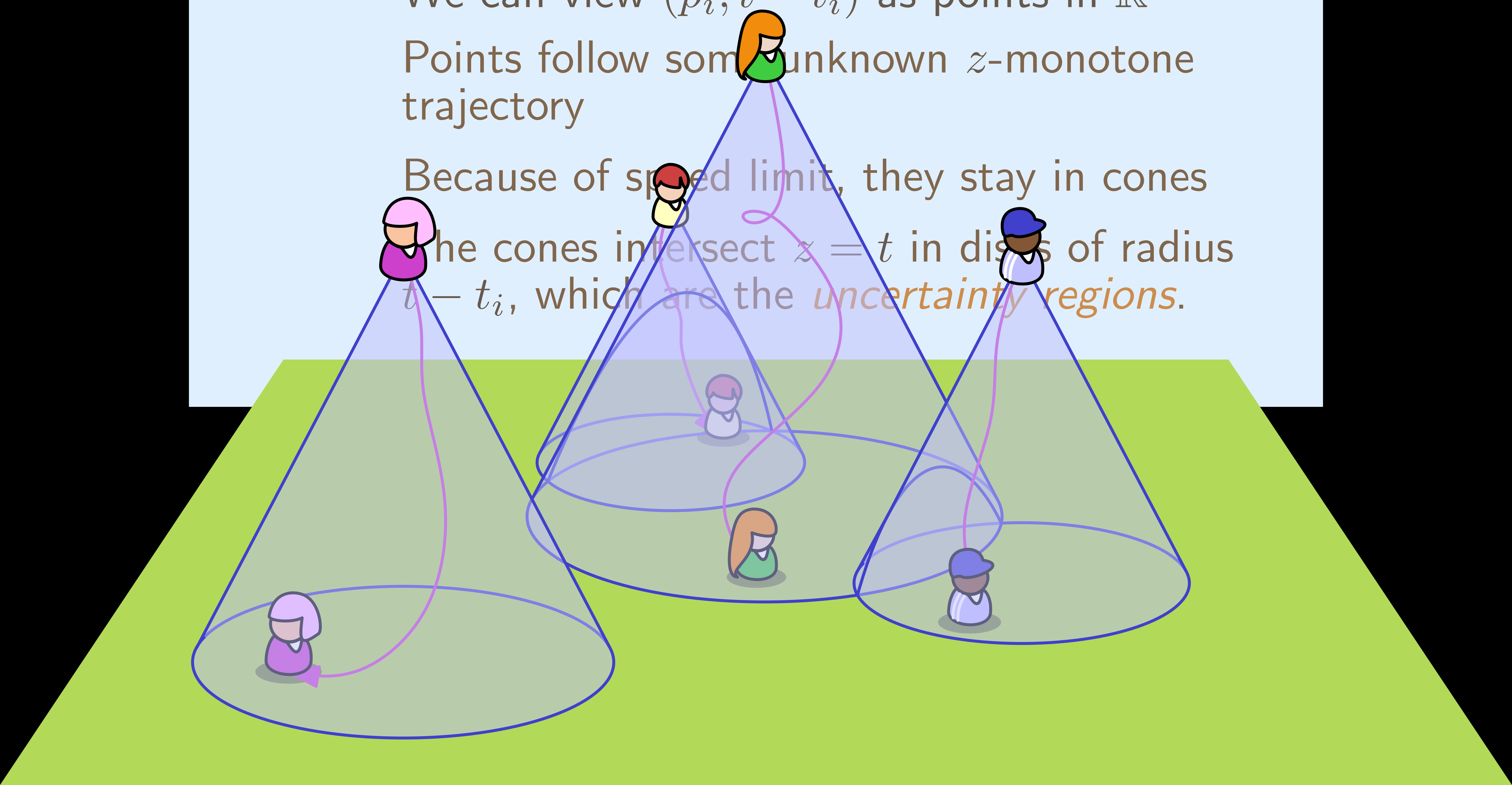

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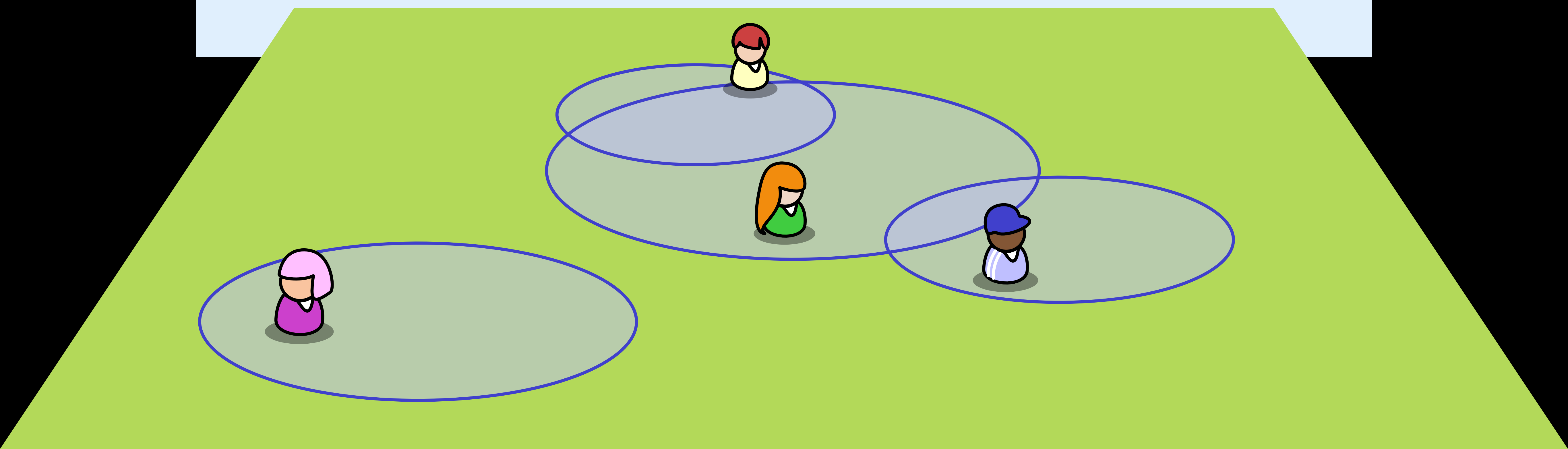

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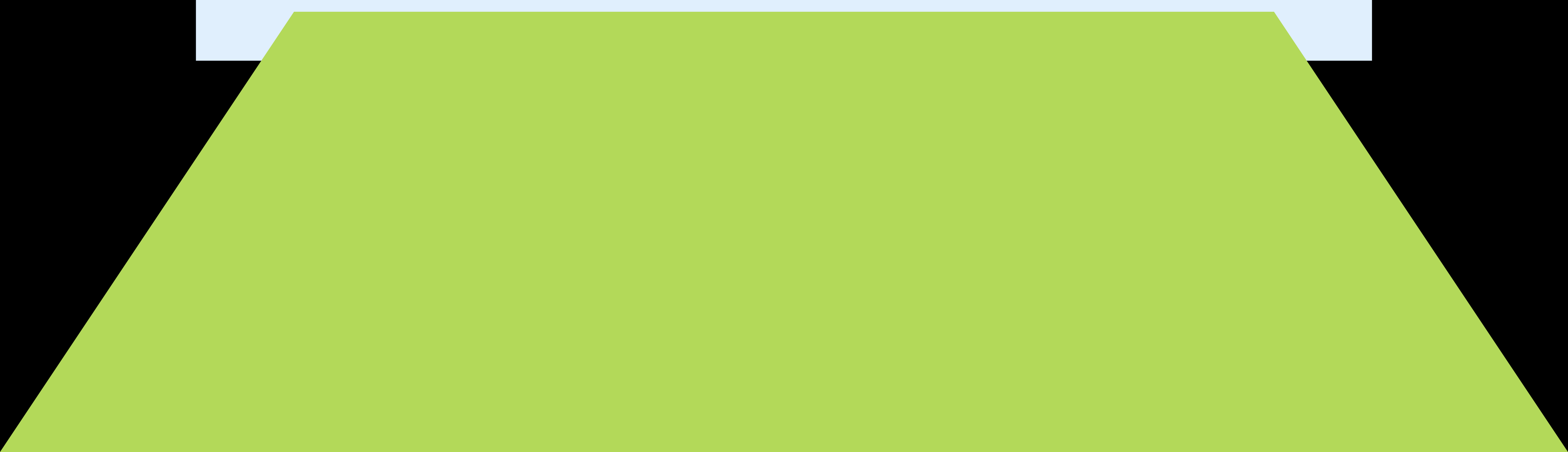

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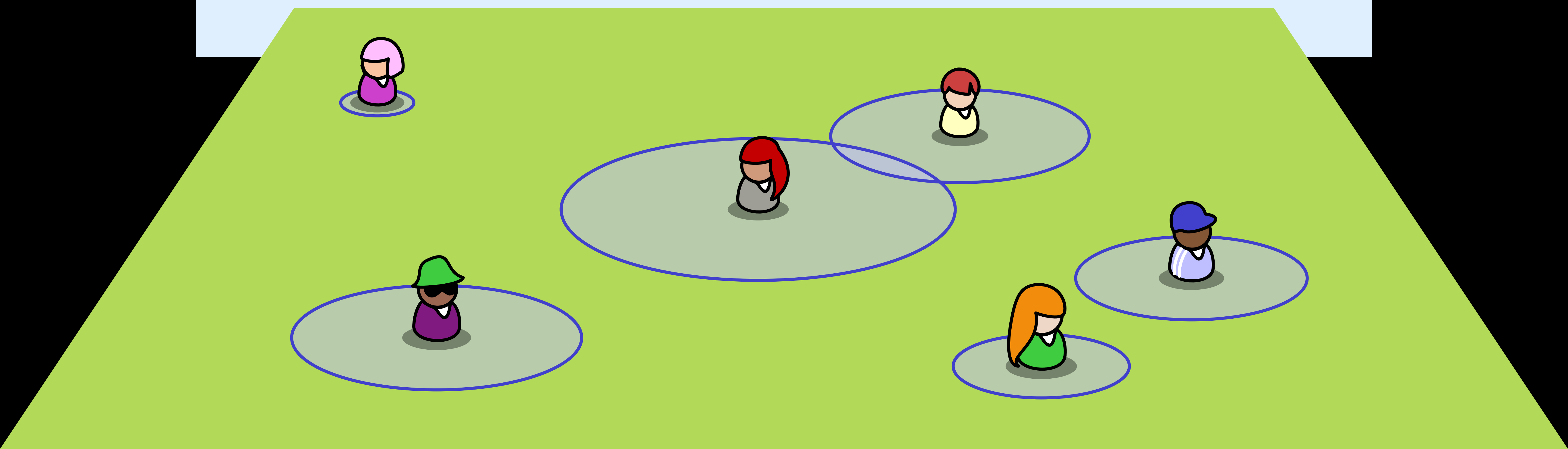

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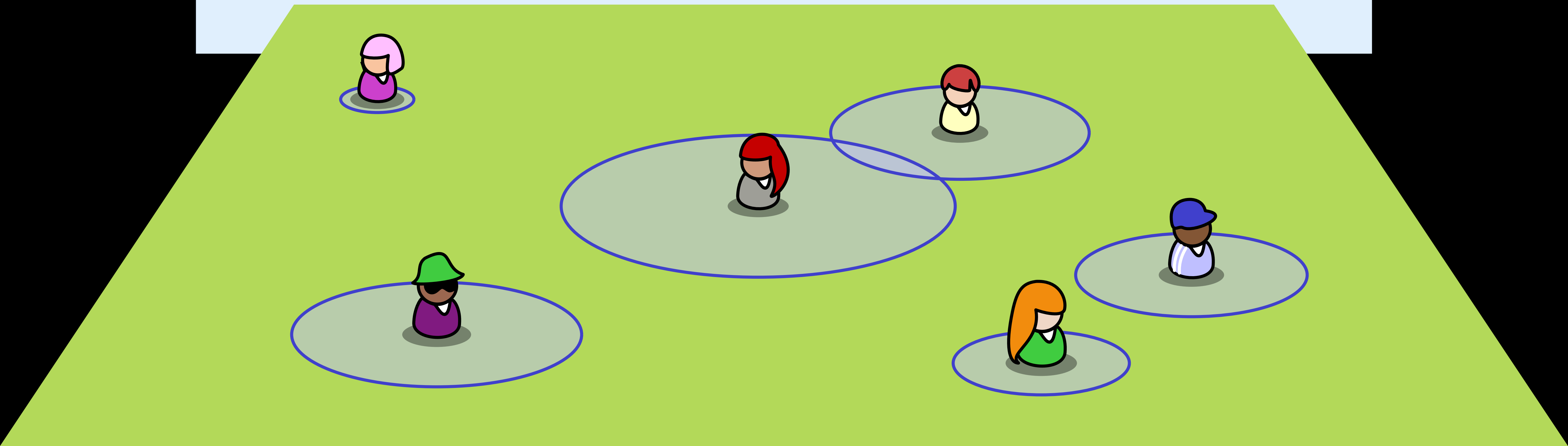
Suppose that for each point we know a *location* p_i and an associated *time stamp* t_i

Then the current uncertainty region of each point is a disk centered at p_i

Each unit of time, all regions grow by d



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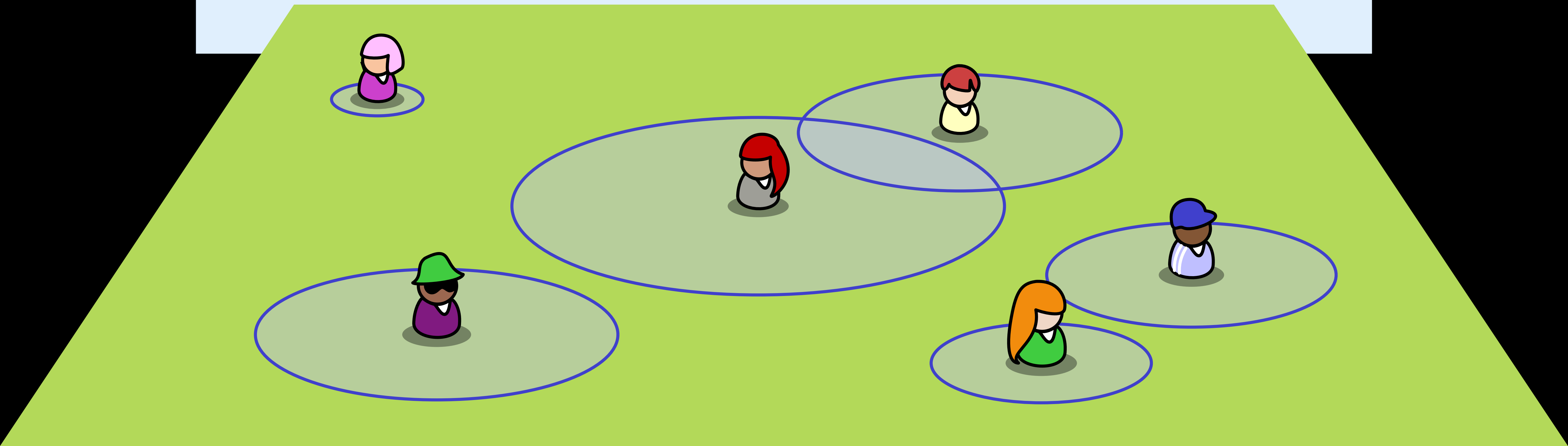

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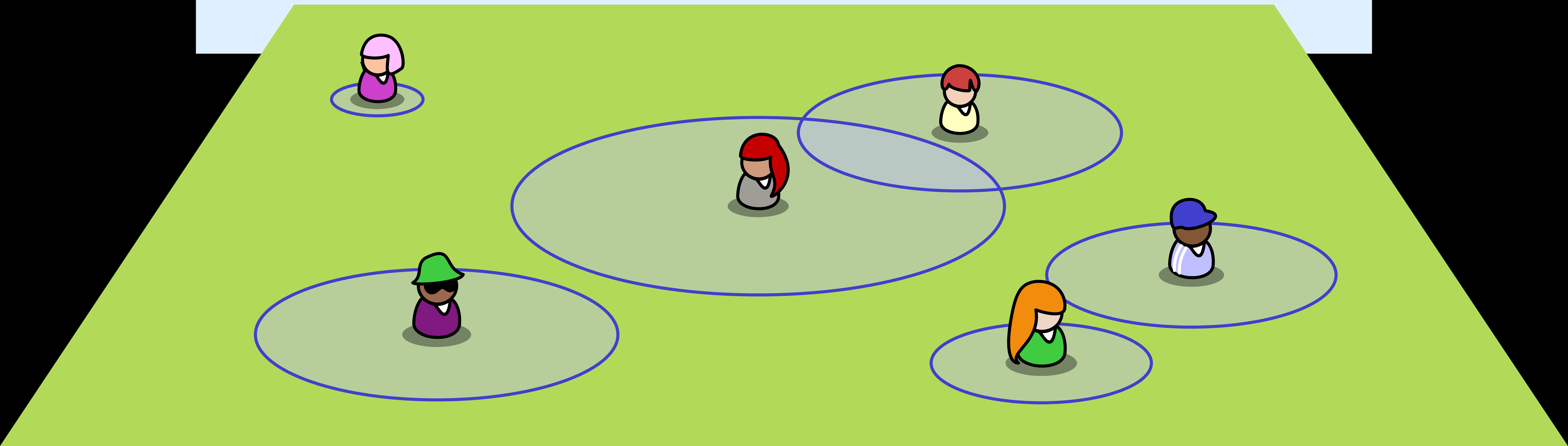
We can *query* a point to update it



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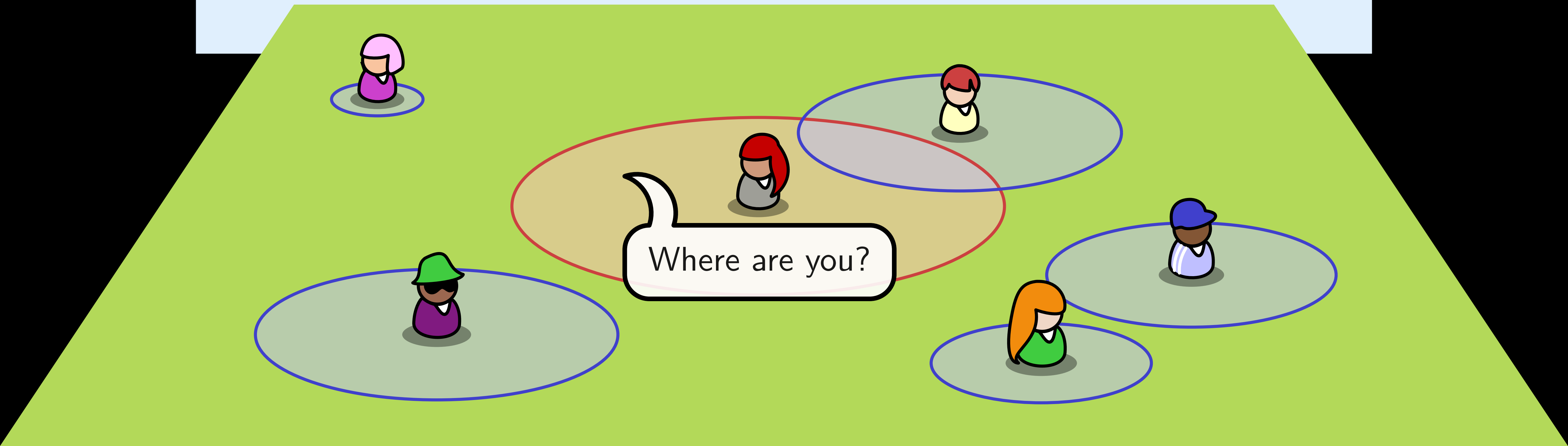

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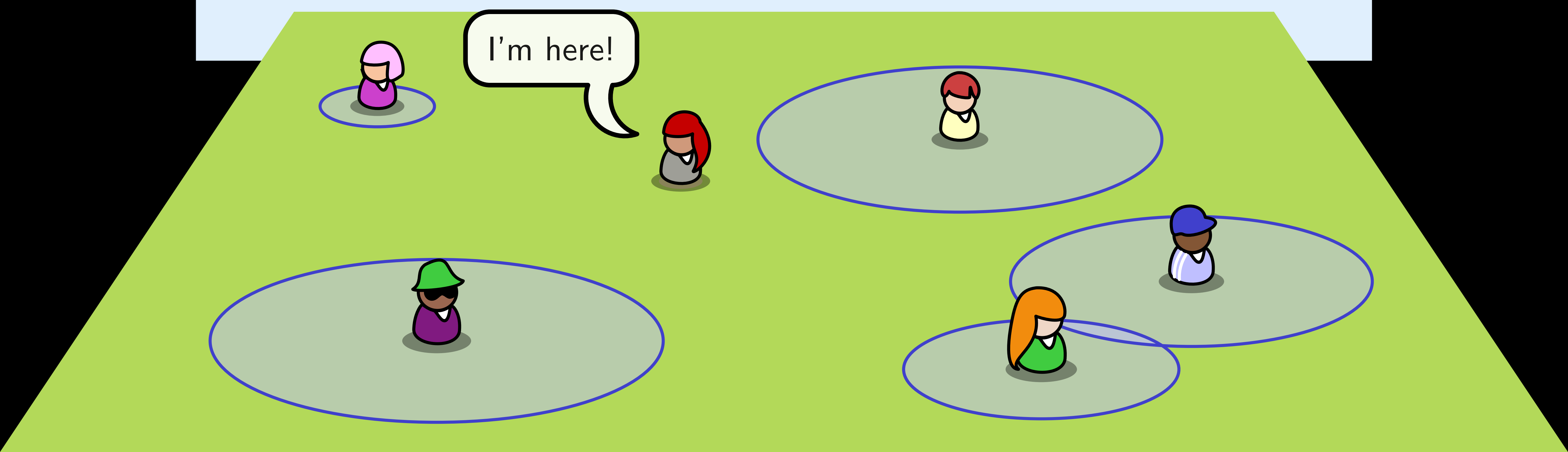

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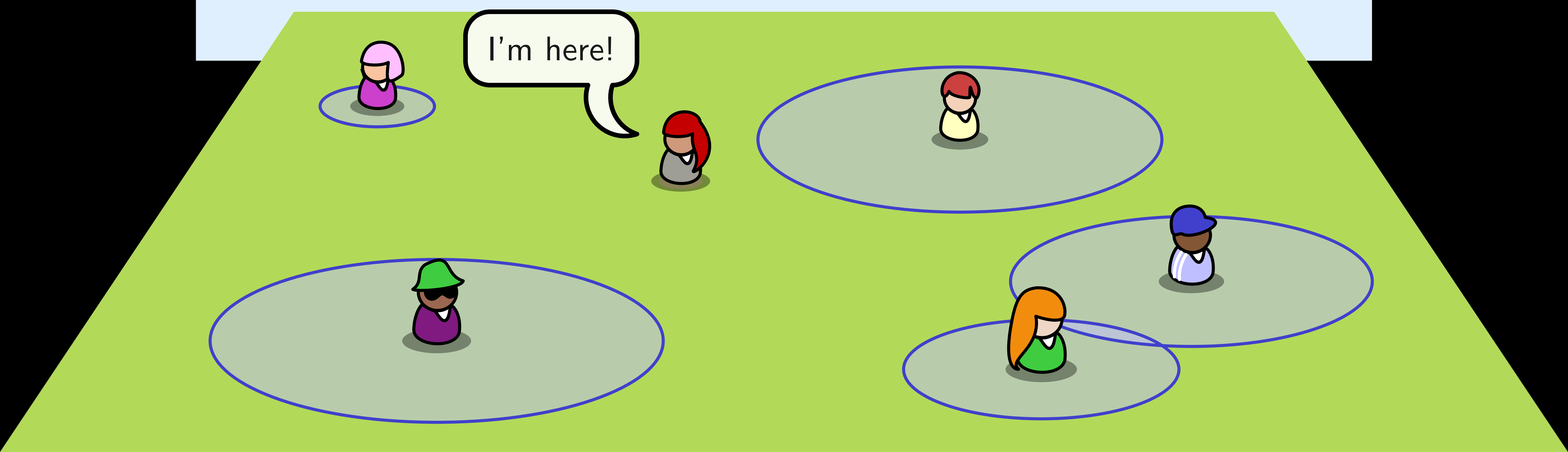
We can *query* a point to update it

- Takes 1 unit of time to execute
- After a query, one region collapses to a point, but all other regions grow



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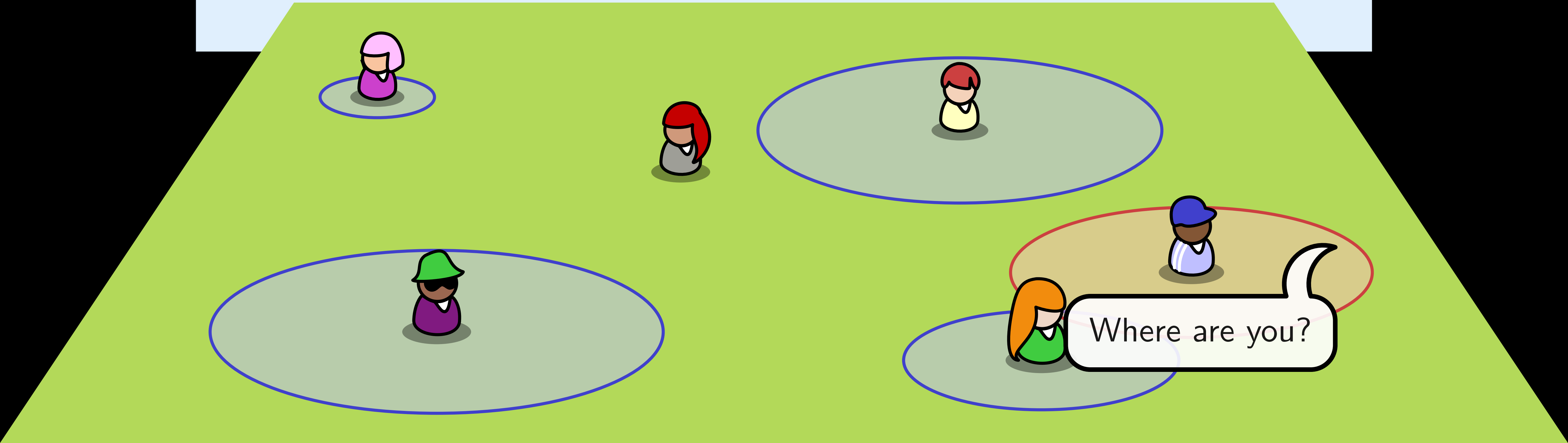

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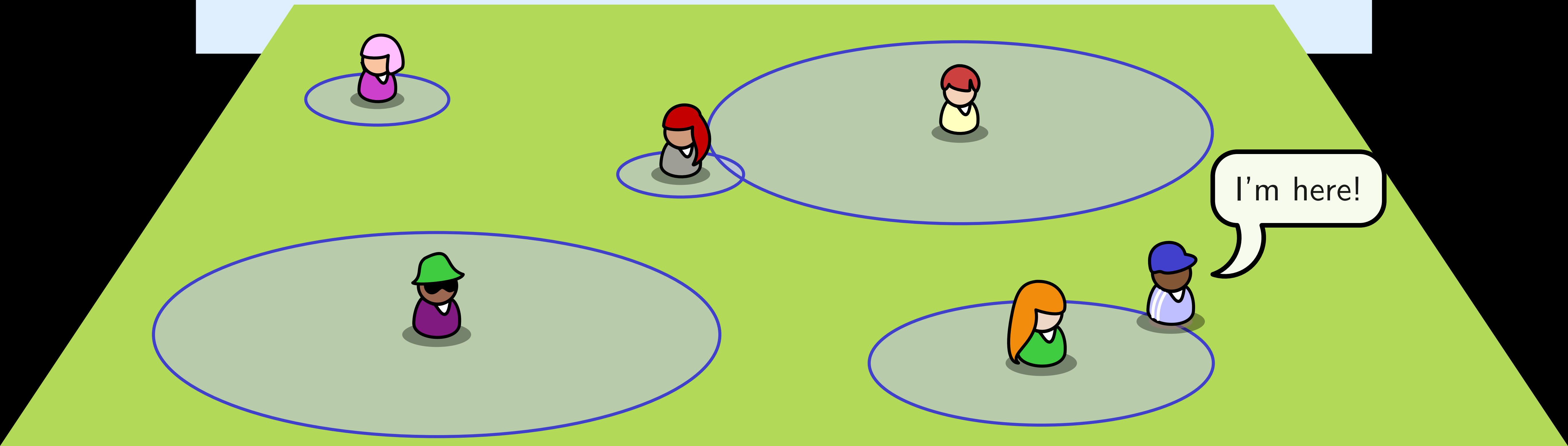

CONGESTION MEASURES

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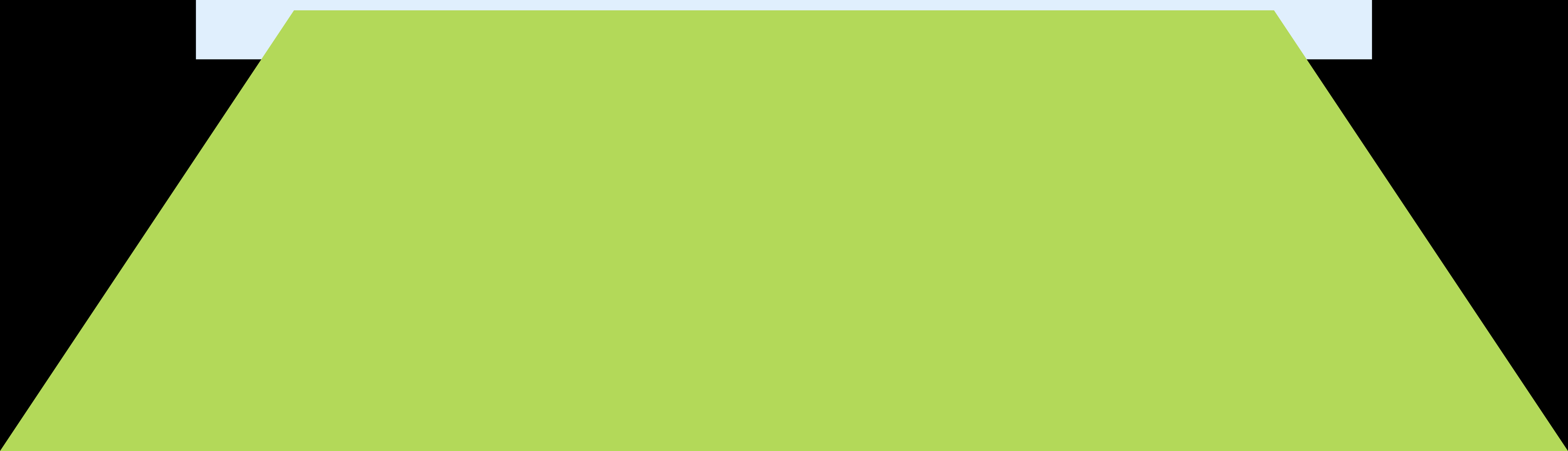
Consider a set of regions



CONGESTION MEASURES

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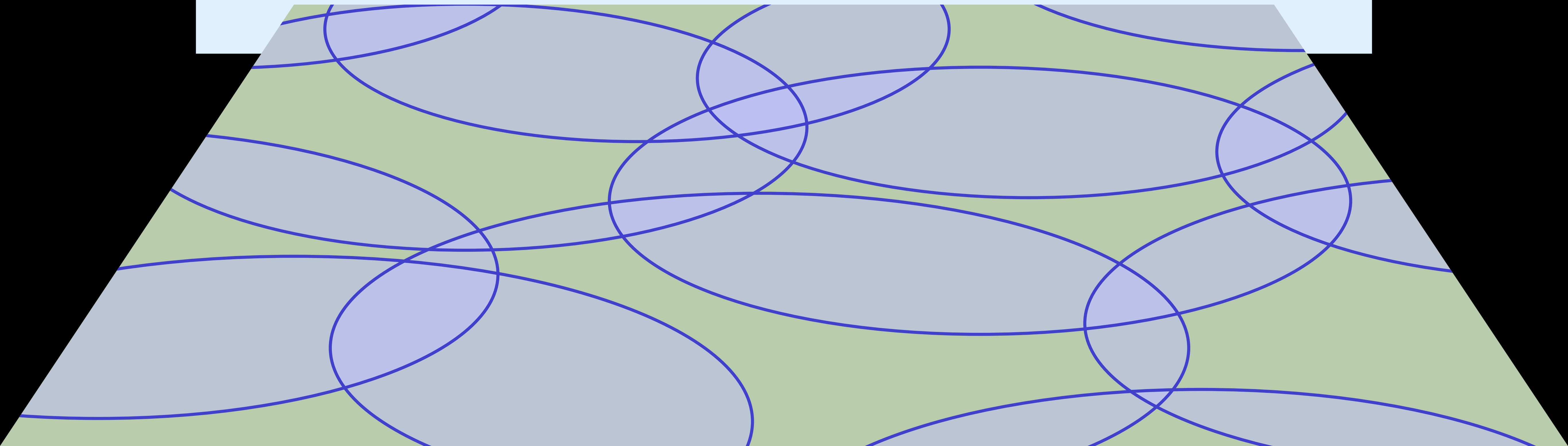
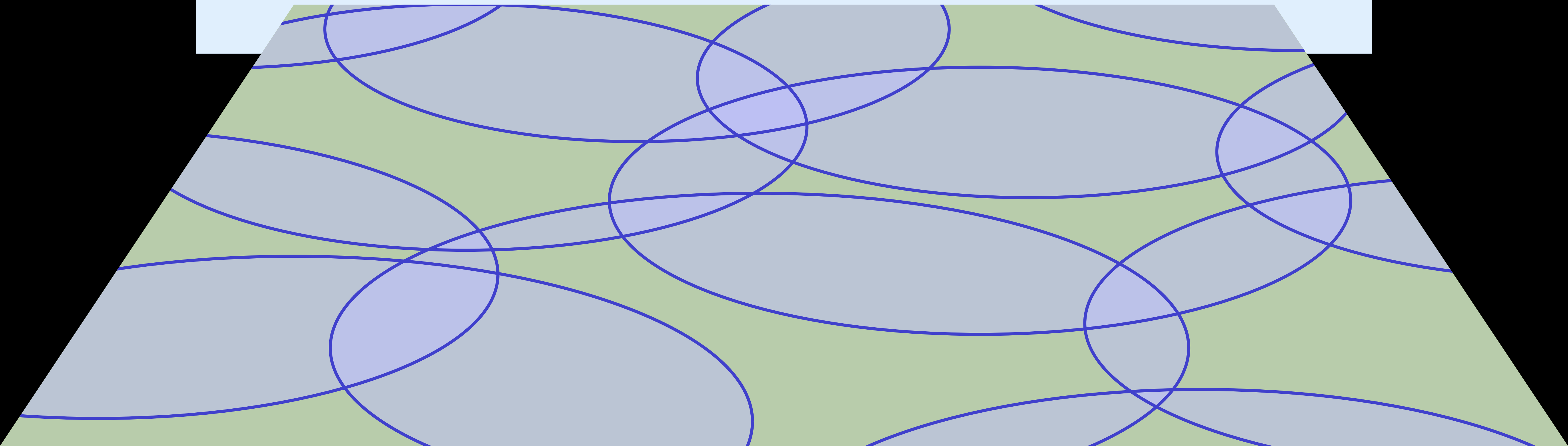


CONGESTION MEASURES

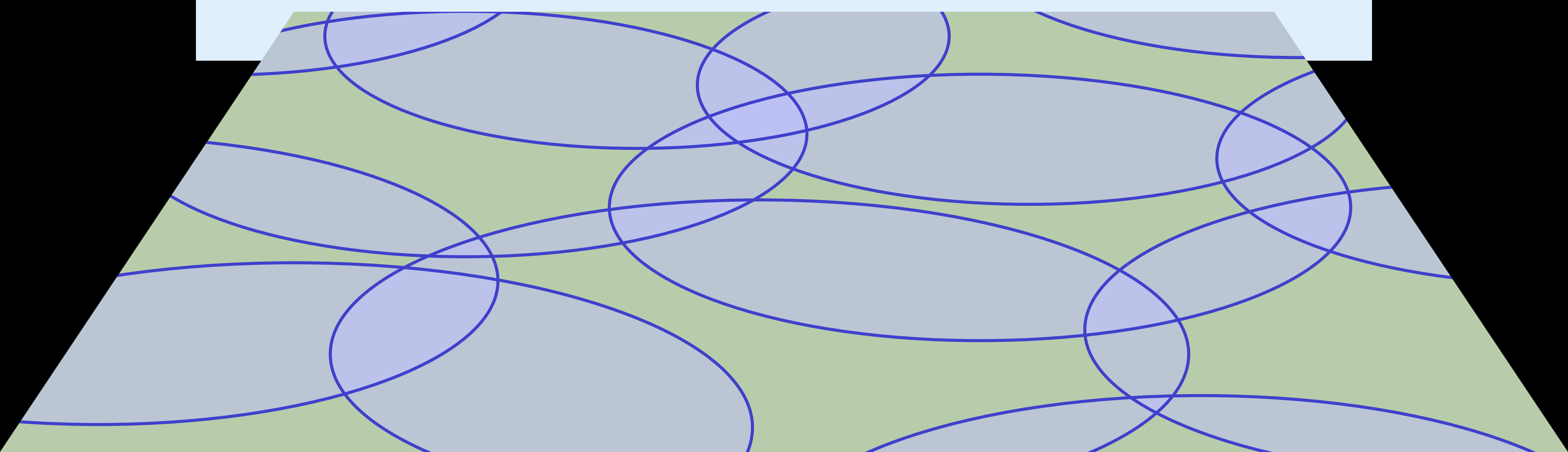
Consider a set of regions

What do they tell us about the point set?



CONGESTION MEASURES

- Consider a set of regions
  - What do they tell us about the point set?
  - The more they overlap, the less information we have.



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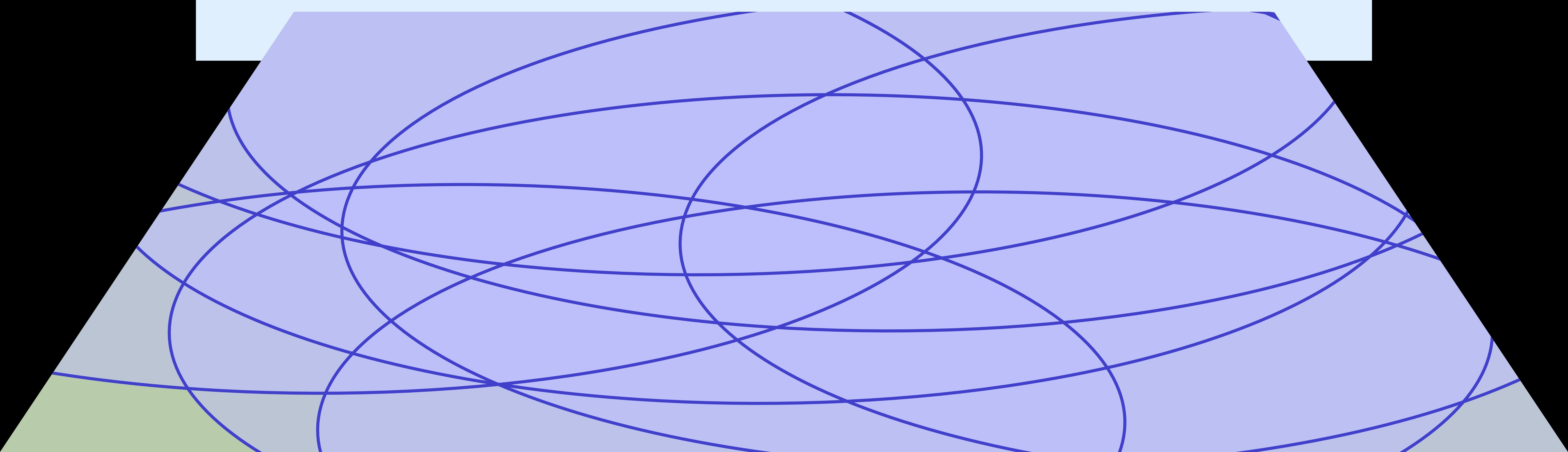

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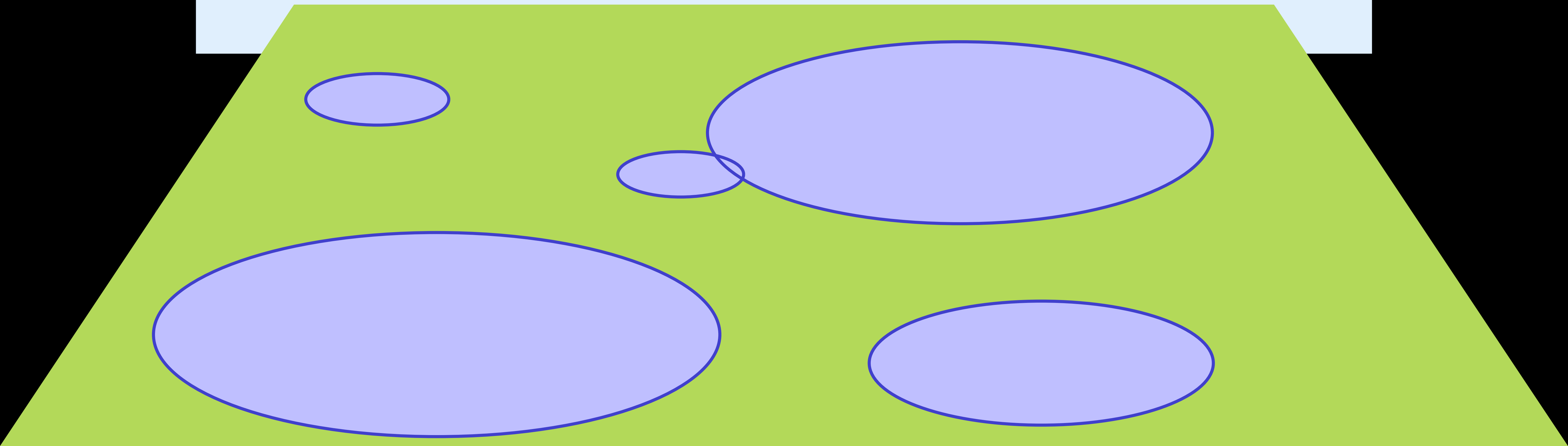

CONGESTION MEASURES

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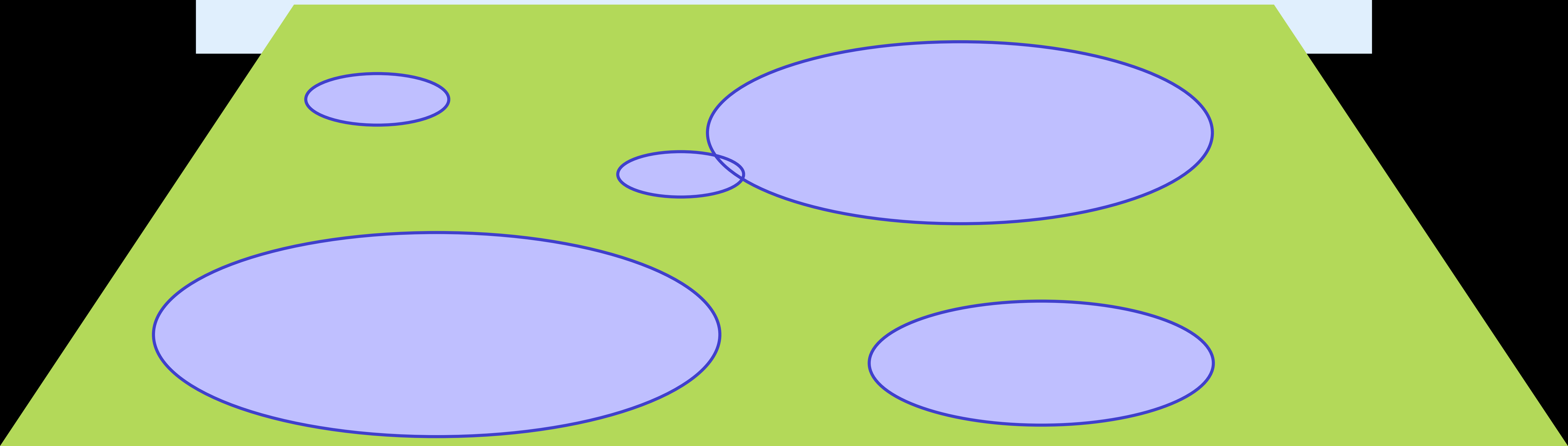
- How do we measure how much a set of regions is "overlapping"?



PLY

PLY

Ply of a point

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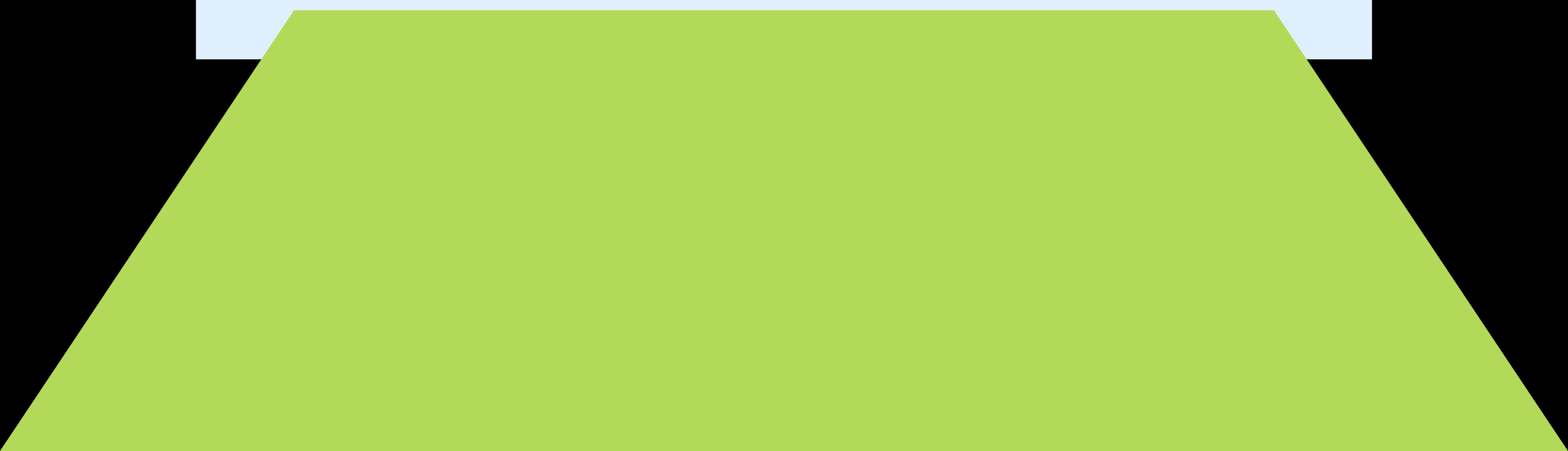

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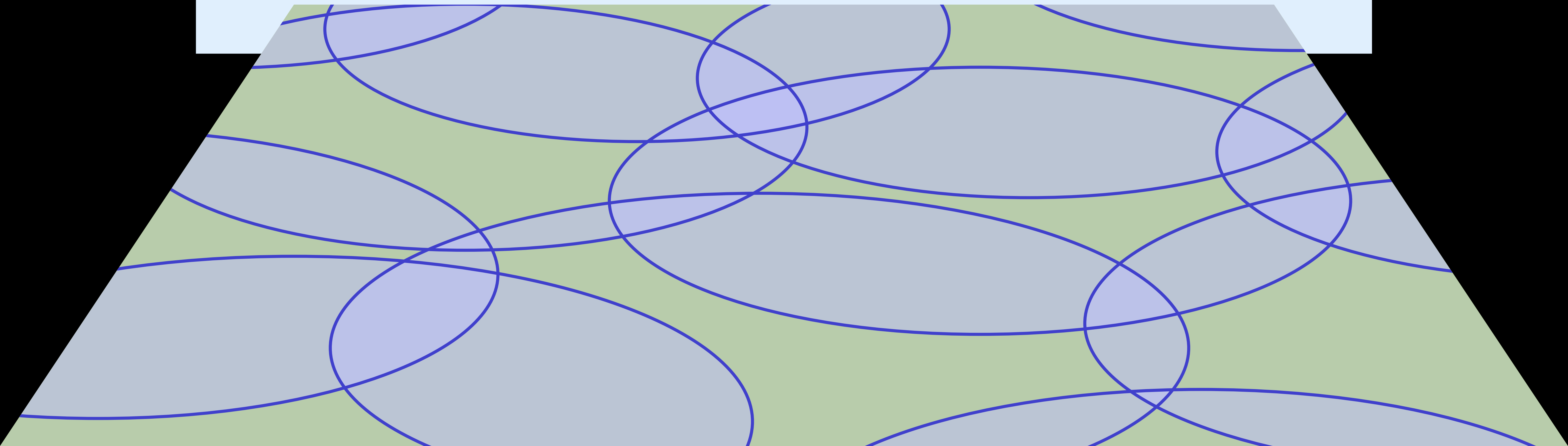

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Given a set of regions

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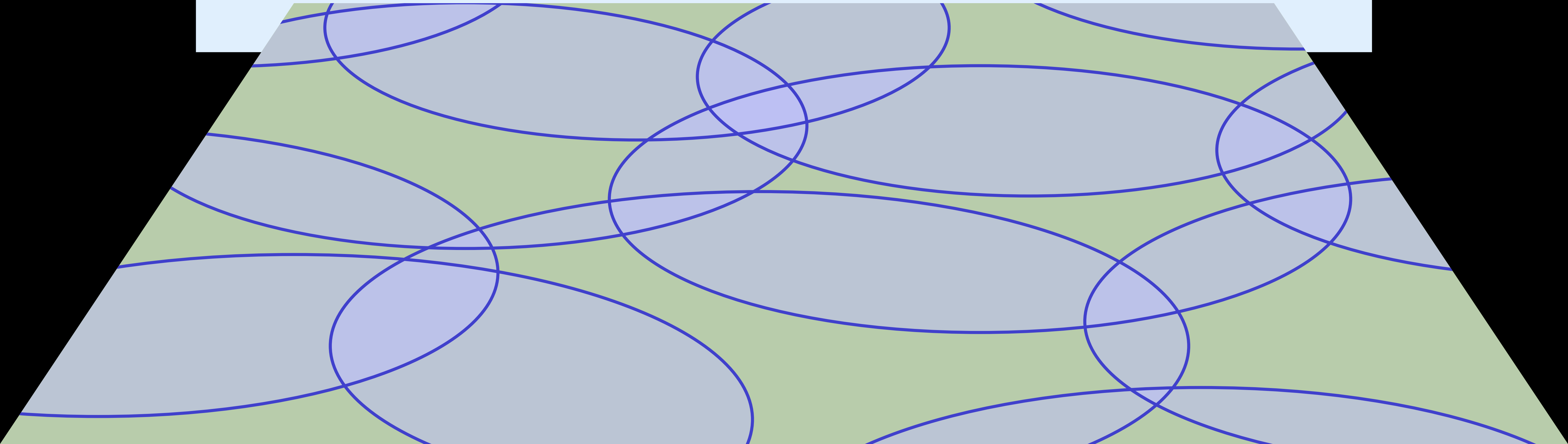

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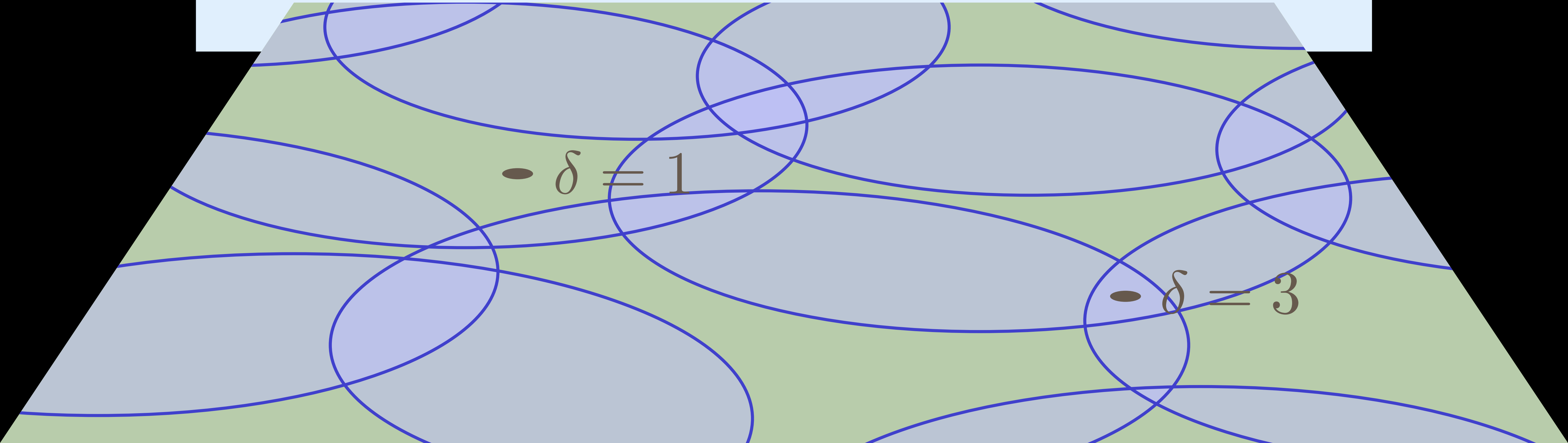

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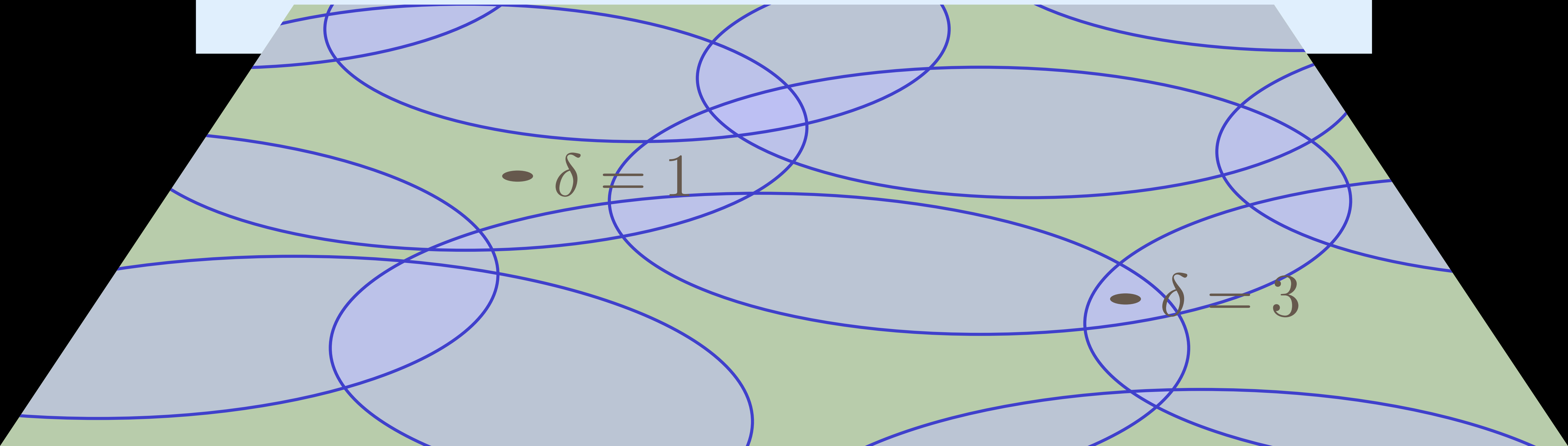

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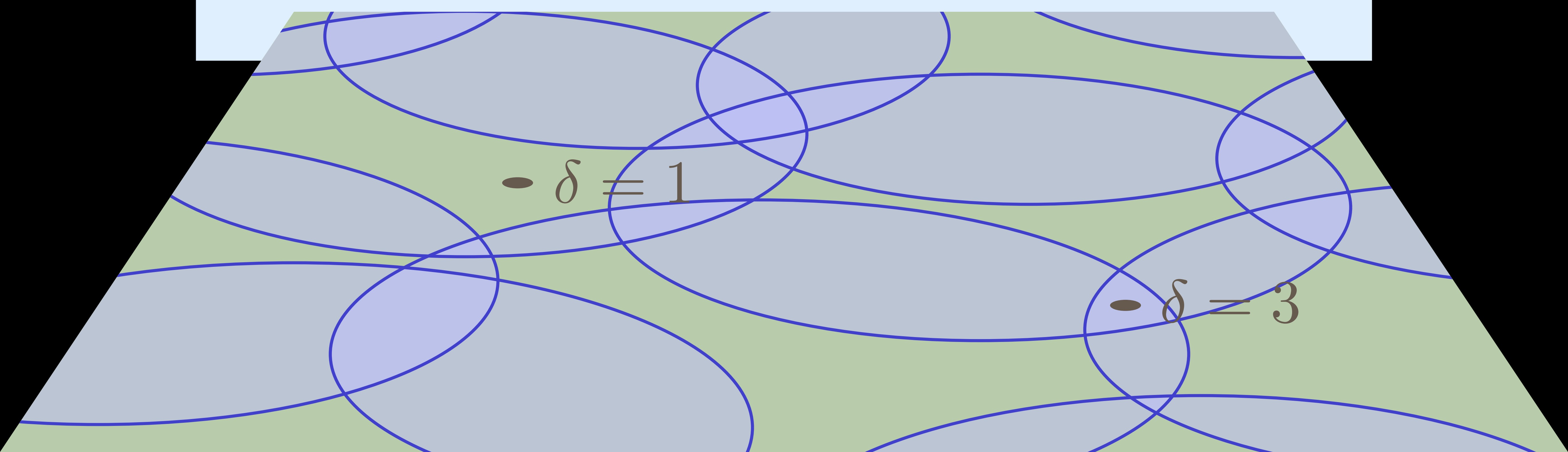

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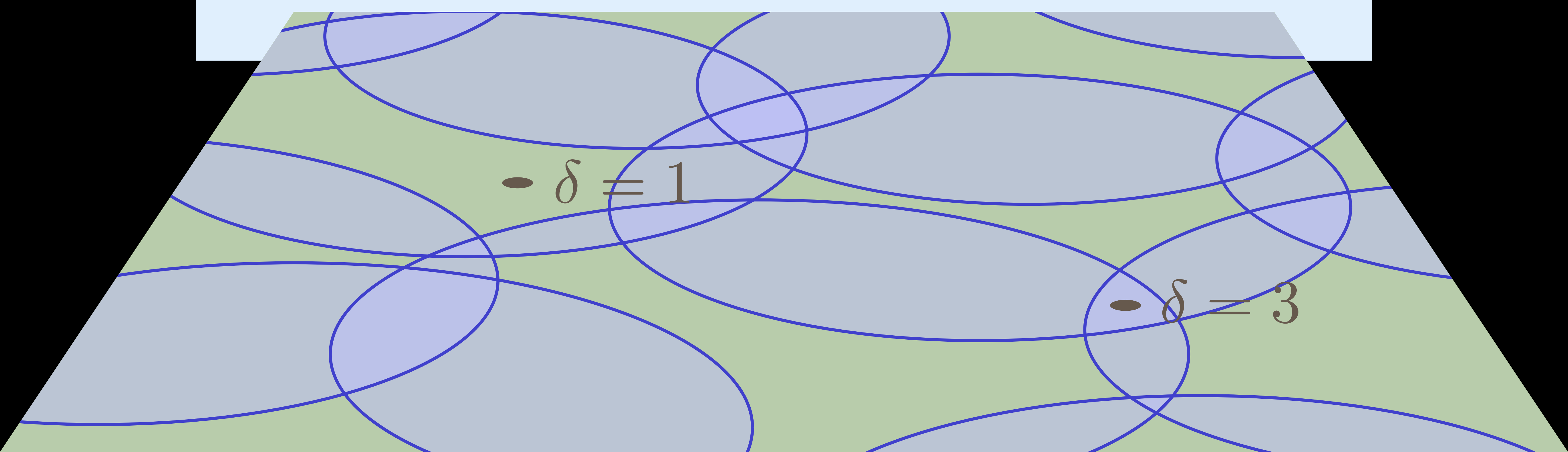

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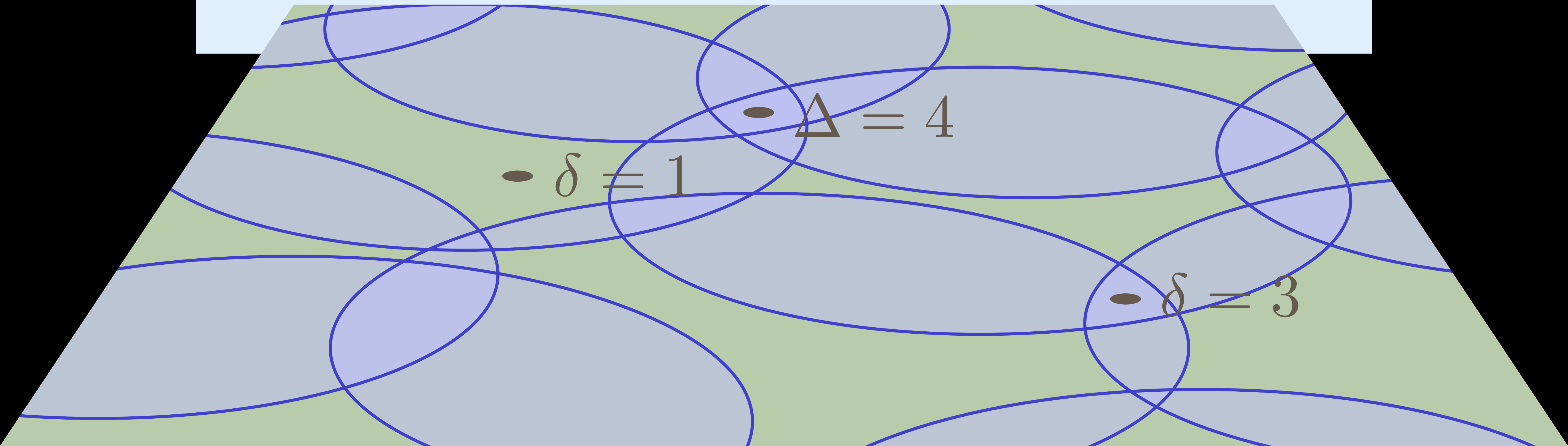

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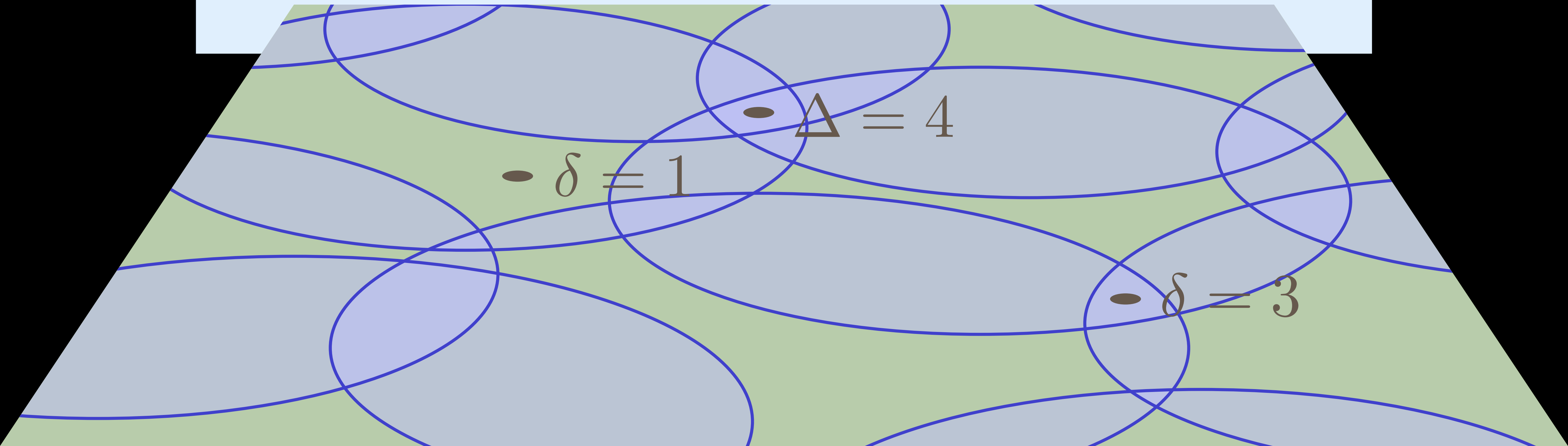

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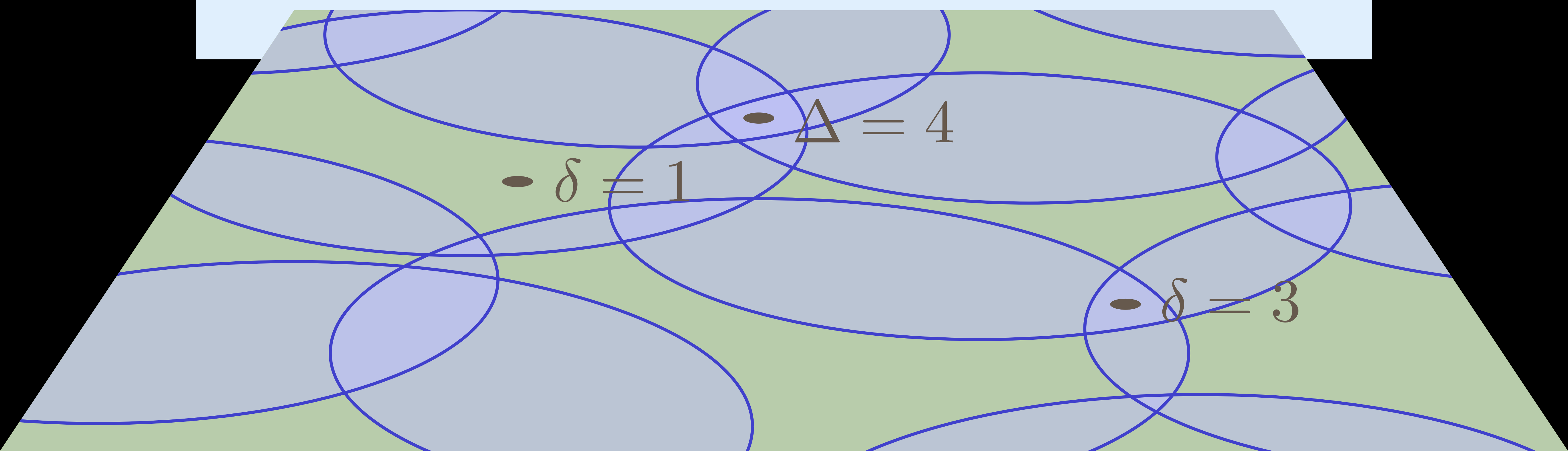

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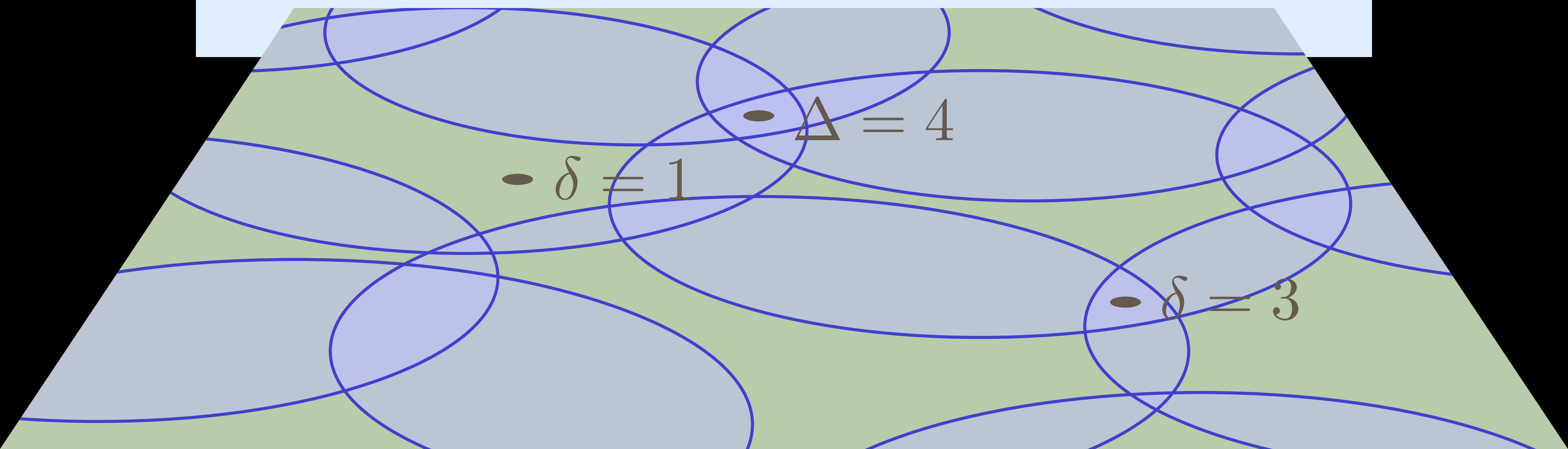
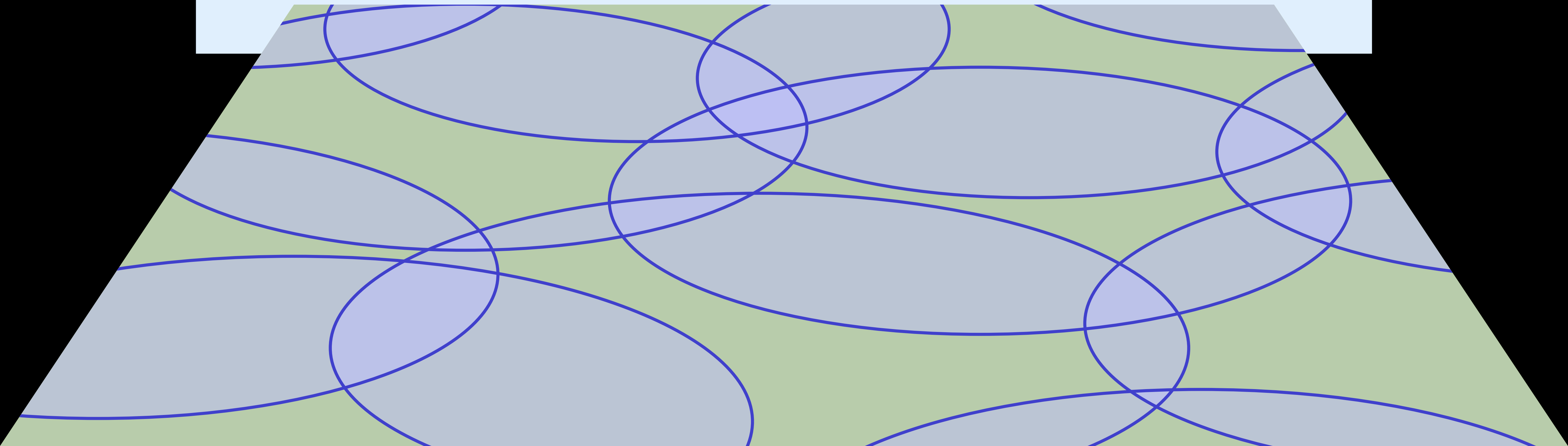
- High ply is not *necessarily* bad.
- Ply is a *local* congestion measure.



TOTAL DEGREE



TOTAL DEGREE

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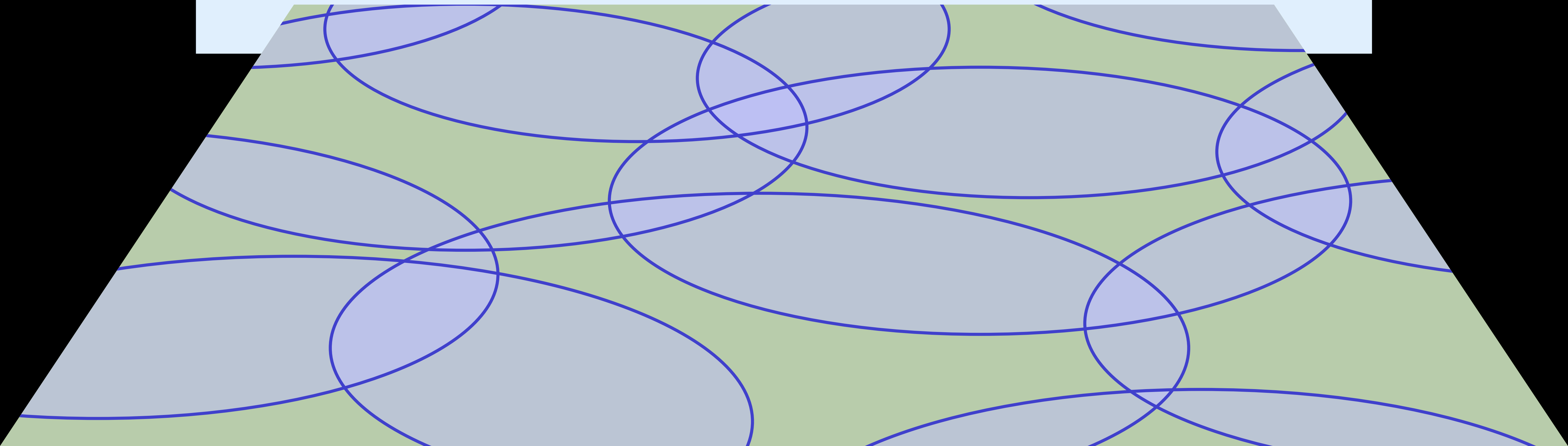

TOTAL DEGREE

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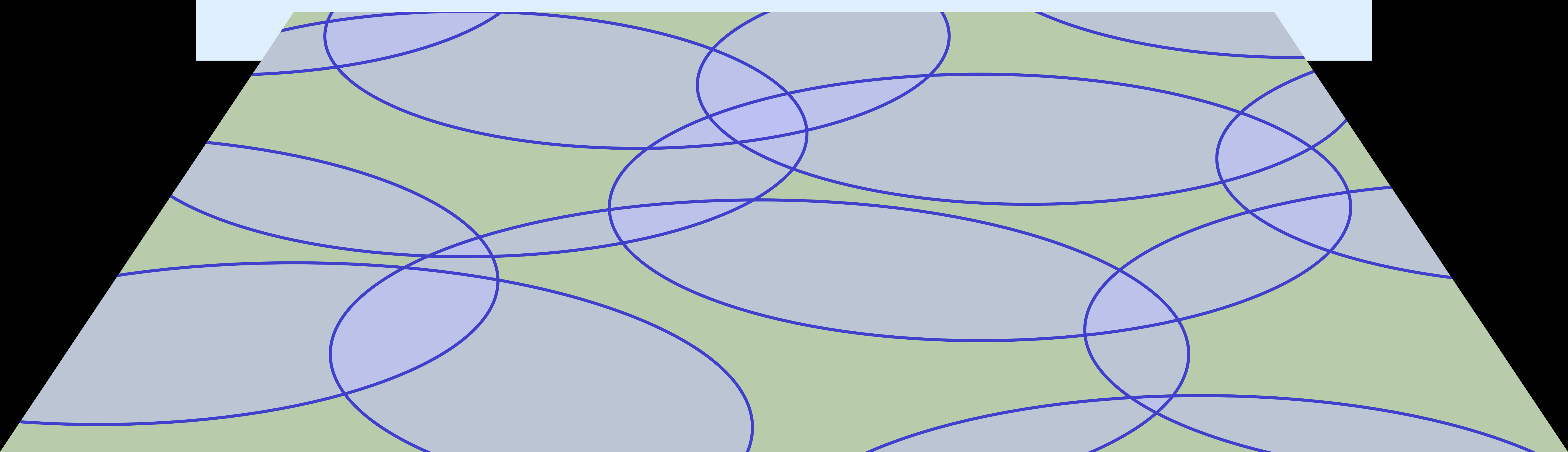

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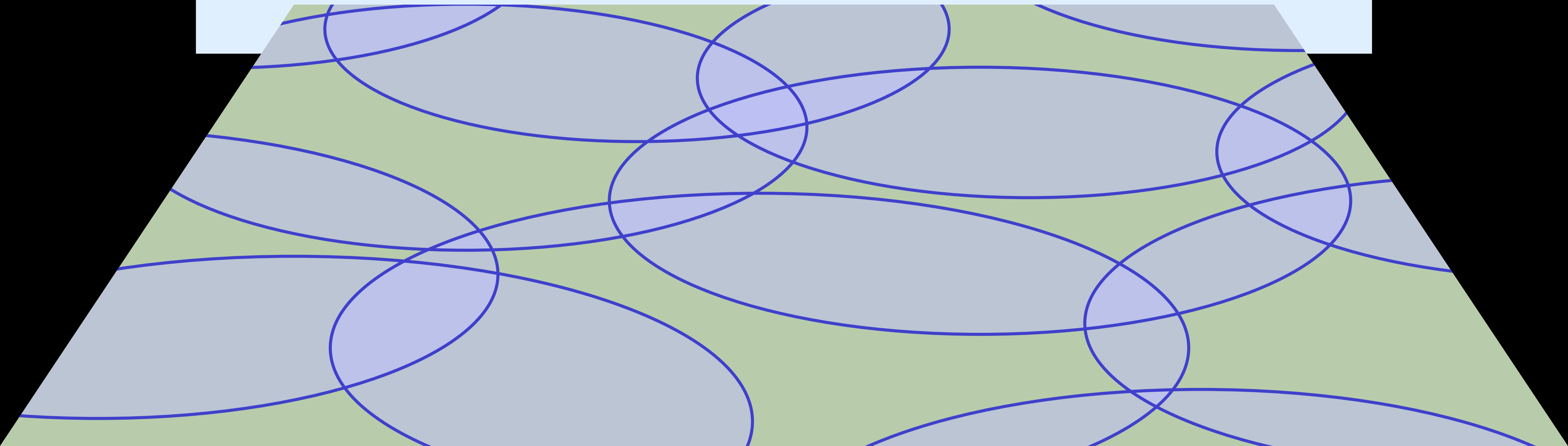

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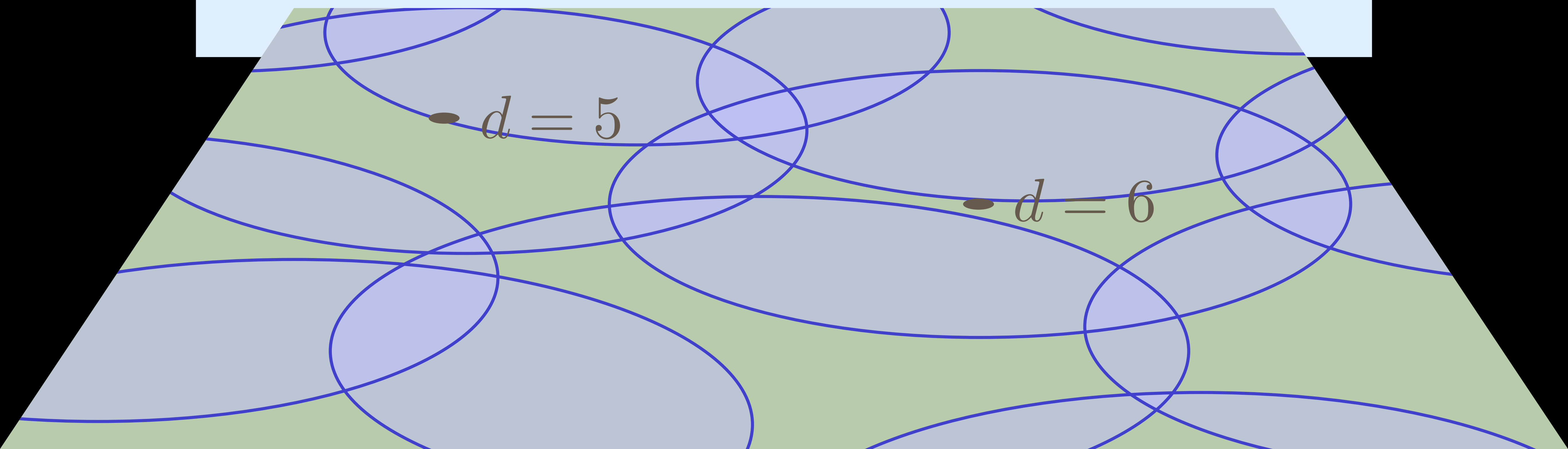

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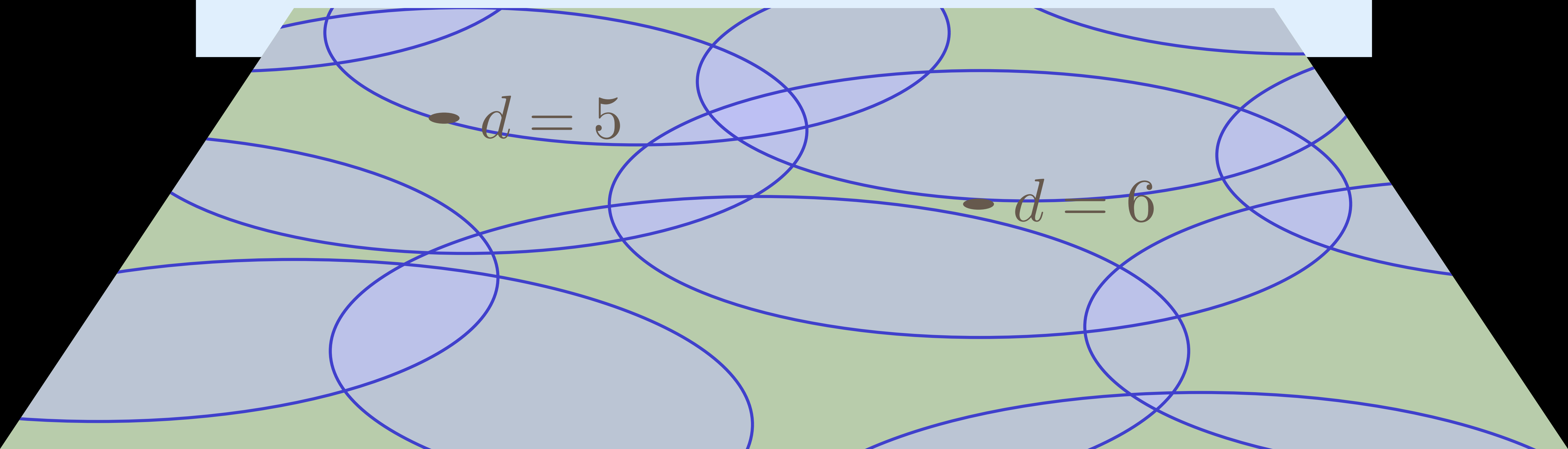

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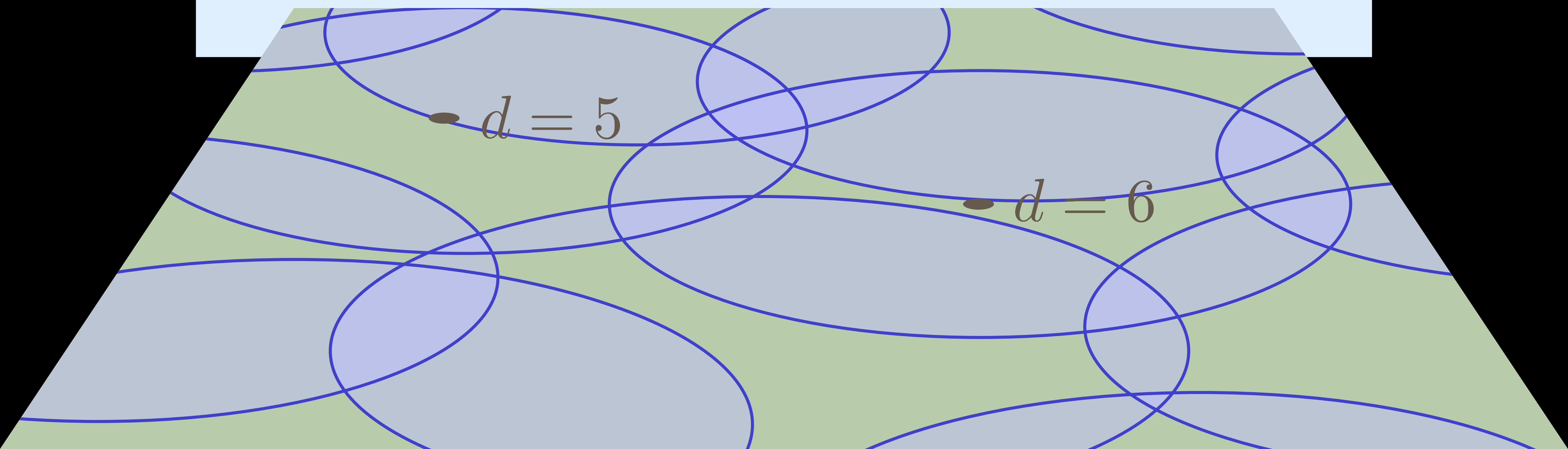

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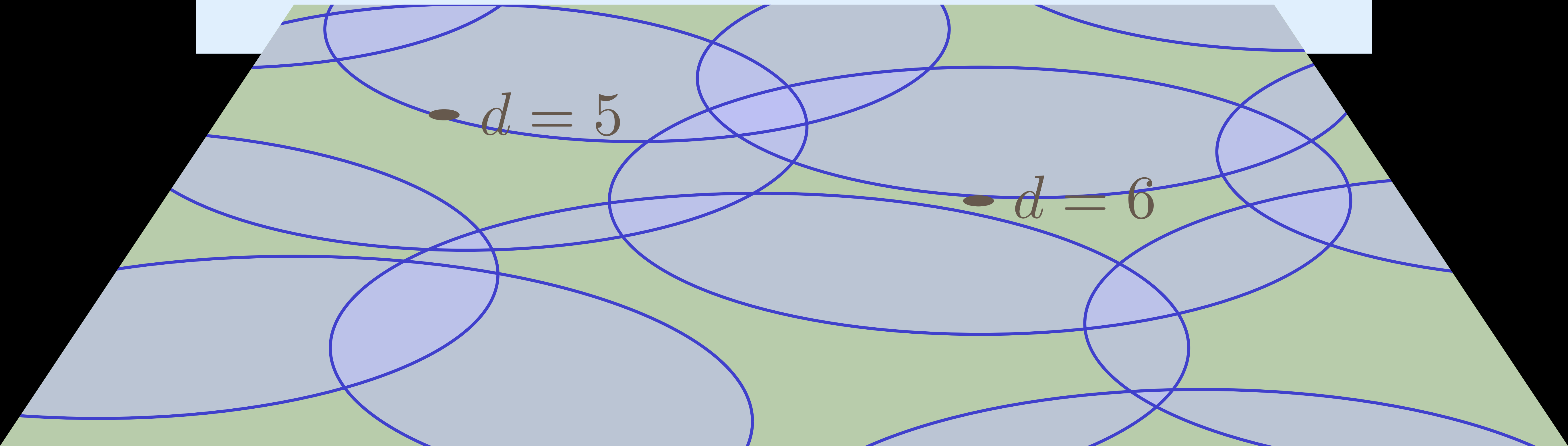

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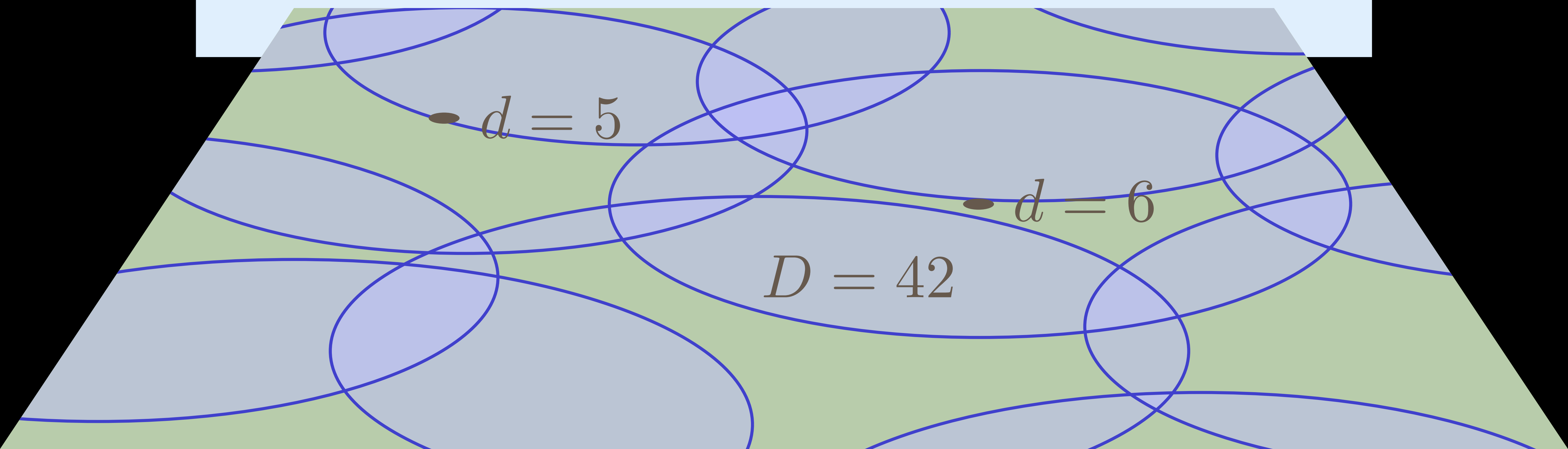

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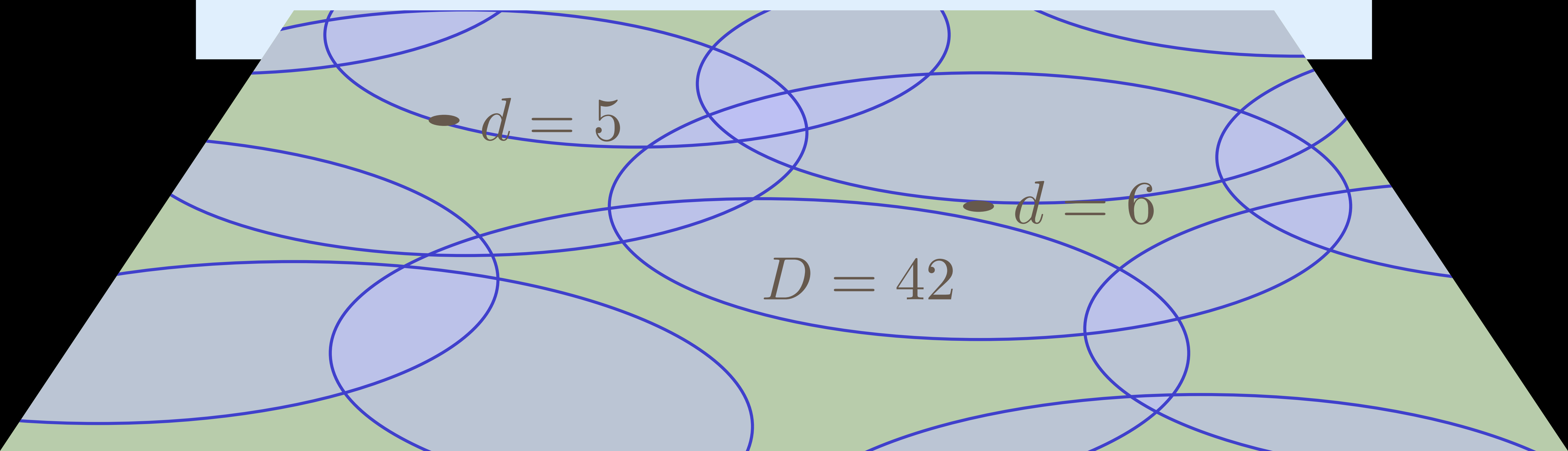
Total degree of a set of regions

Sum of degrees of all regions

$D = \sum_R d(R)$

twice the number of edges in the disk intersection graph



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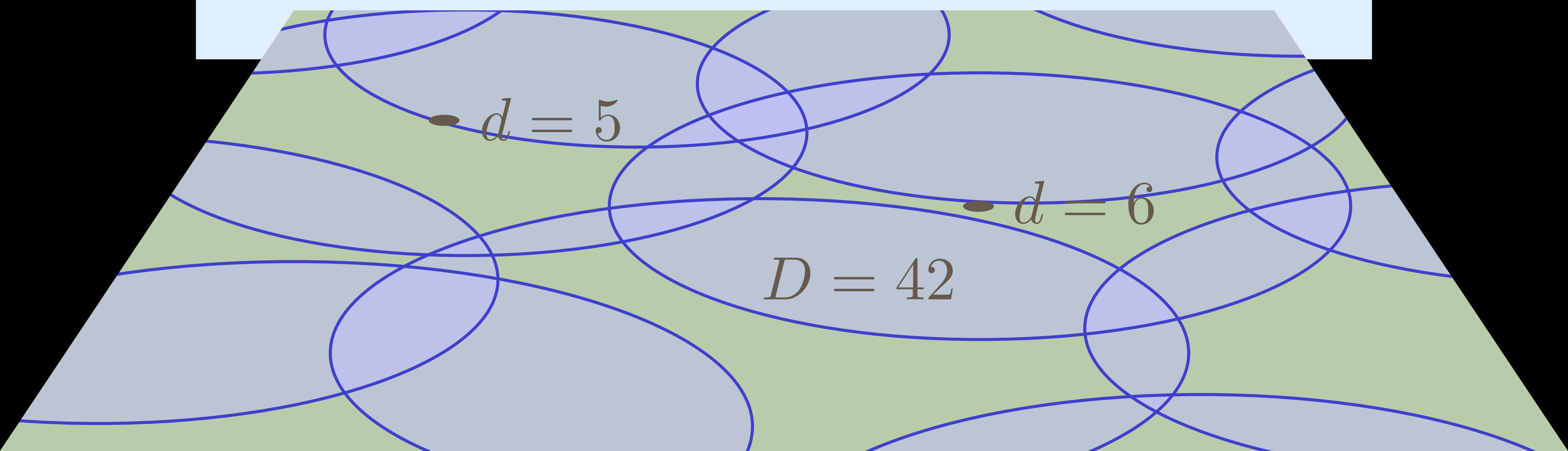
  - $D = \sum_R d(R)$
  - twice the number of edges in the disk intersection graph
- Total degree is a *global* congestion measure.



PROBLEM STATEMENT

PROBLEM STATEMENT

Input

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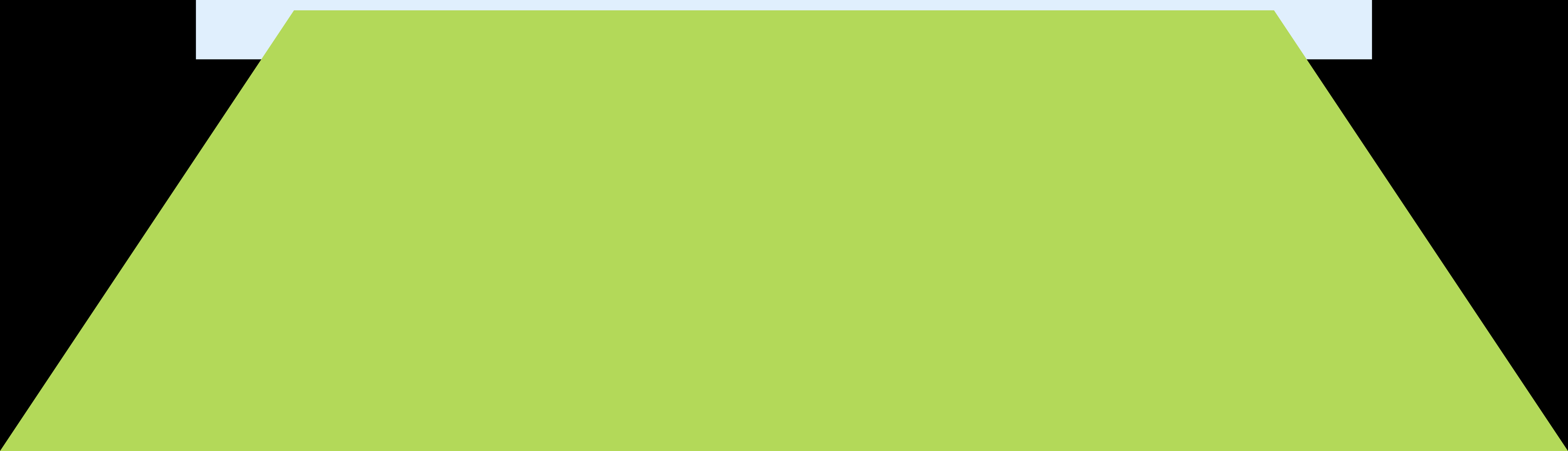
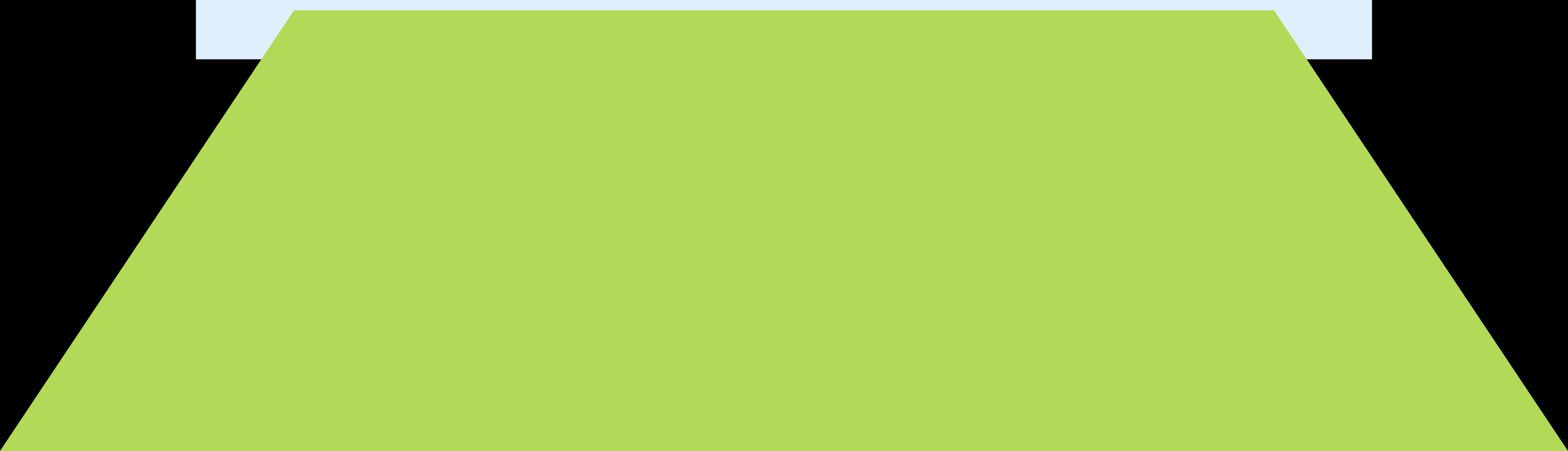
Set of n moving points with initial regions



PROBLEM STATEMENT

Input

- Set of n moving points with initial regions
- Target value τ



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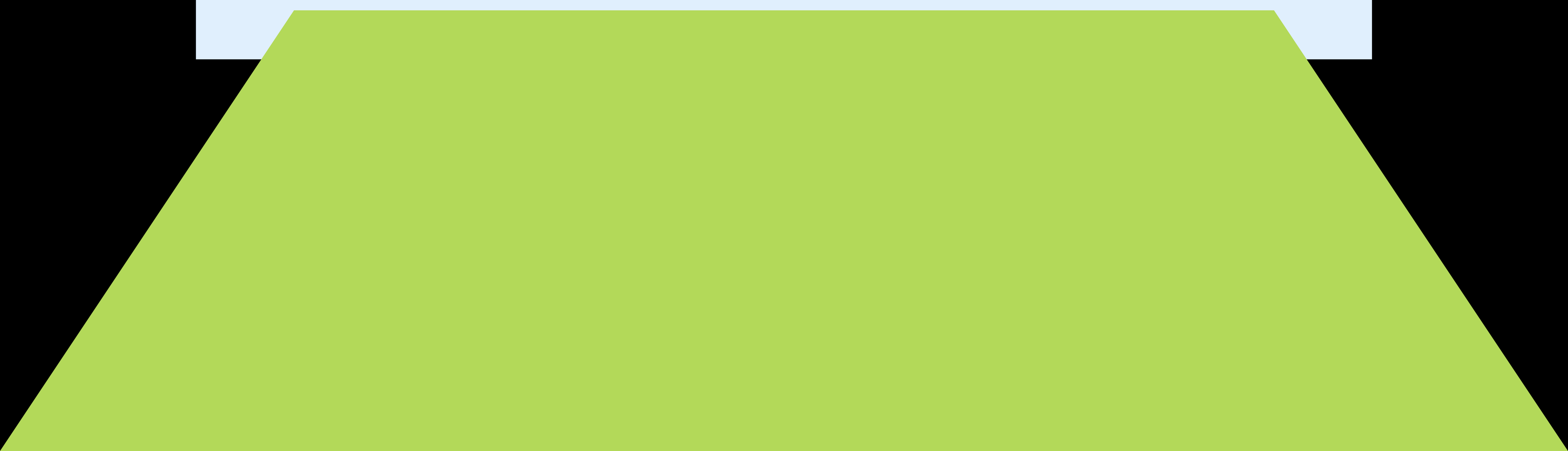
Strategy



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- Set of n moving points with initial regions
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Strategy

- Select one region to query each time step

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- Set of n moving points with initial regions
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- Select one region to query each time step
- Keep the total degree of the intersection graph under τ

PROBLEM STATEMENT

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- Set of n moving points with initial regions
- Target value τ

Strategy

- Select one region to query each time step
- Keep the total degree of the intersection graph under τ

Problem

PROBLEM STATEMENT

Input

Set of n moving points with initial regions
Target value τ

Strategy

Select one region to query each time step
Keep the total degree of the intersection graph under τ

Problem

Decide whether such a strategy exists.

STATIC MOVING POINTS

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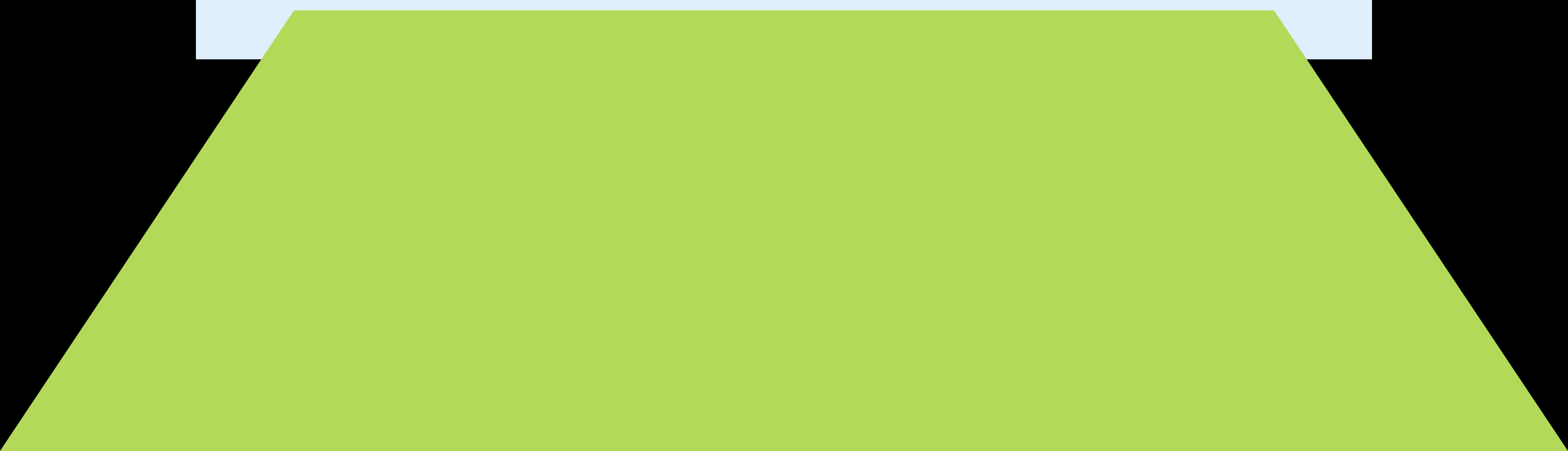
Consider n non-moving points



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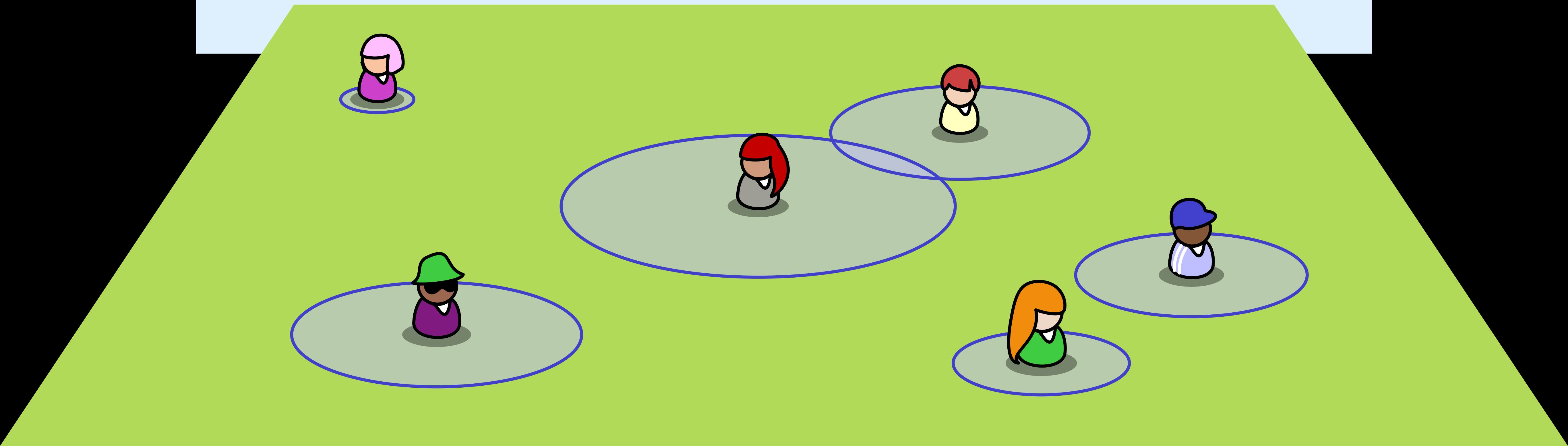

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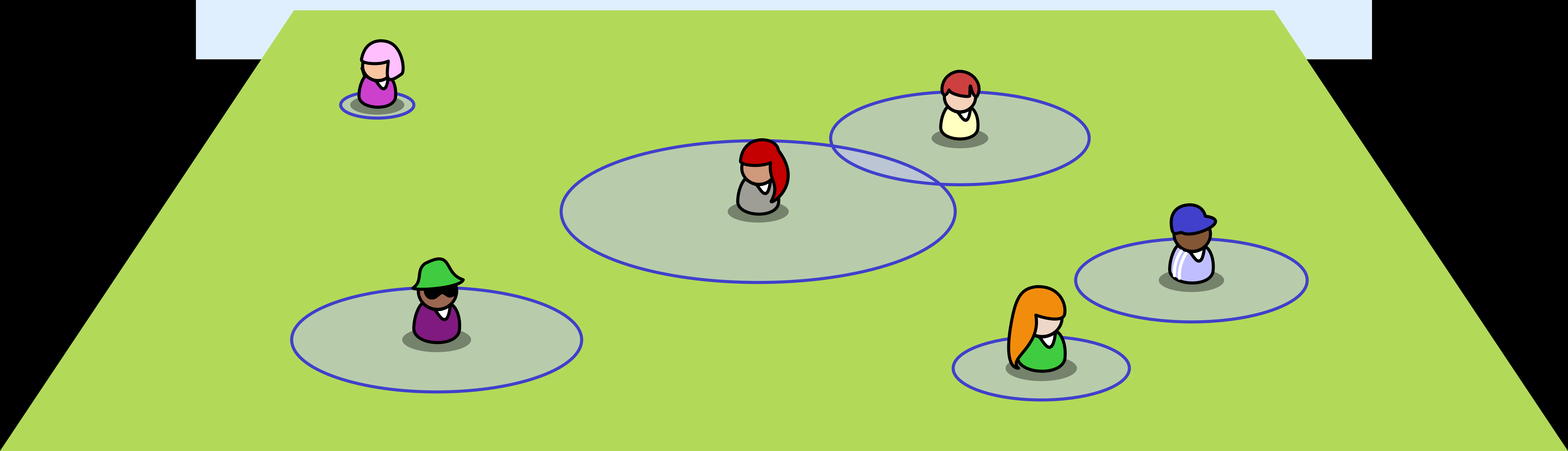
Each unit of time, all regions grow by d



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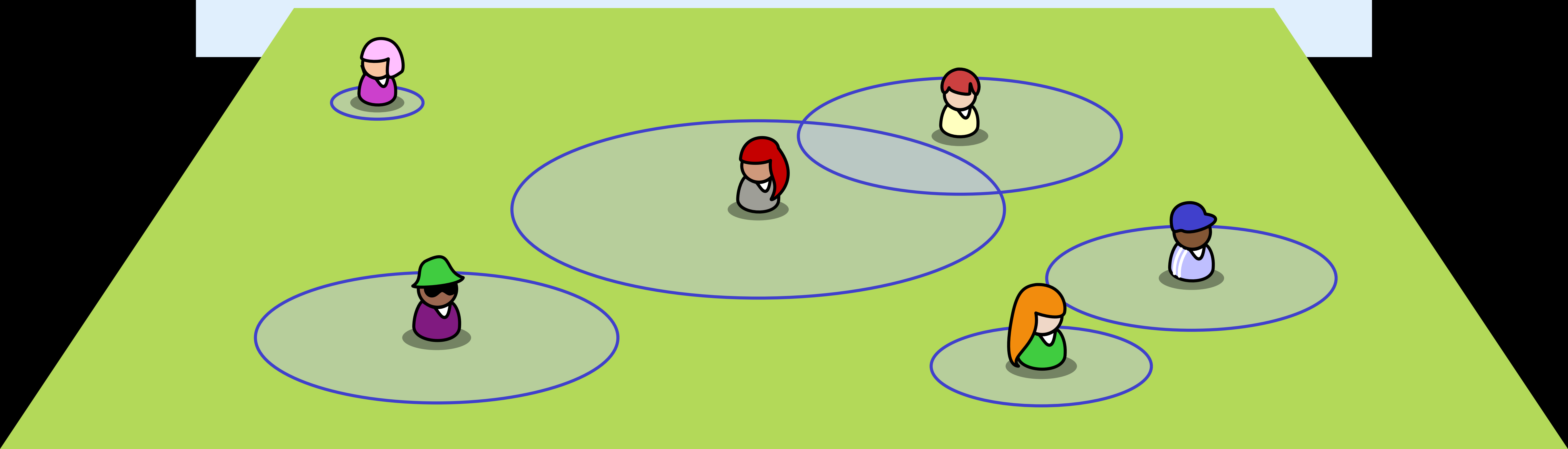

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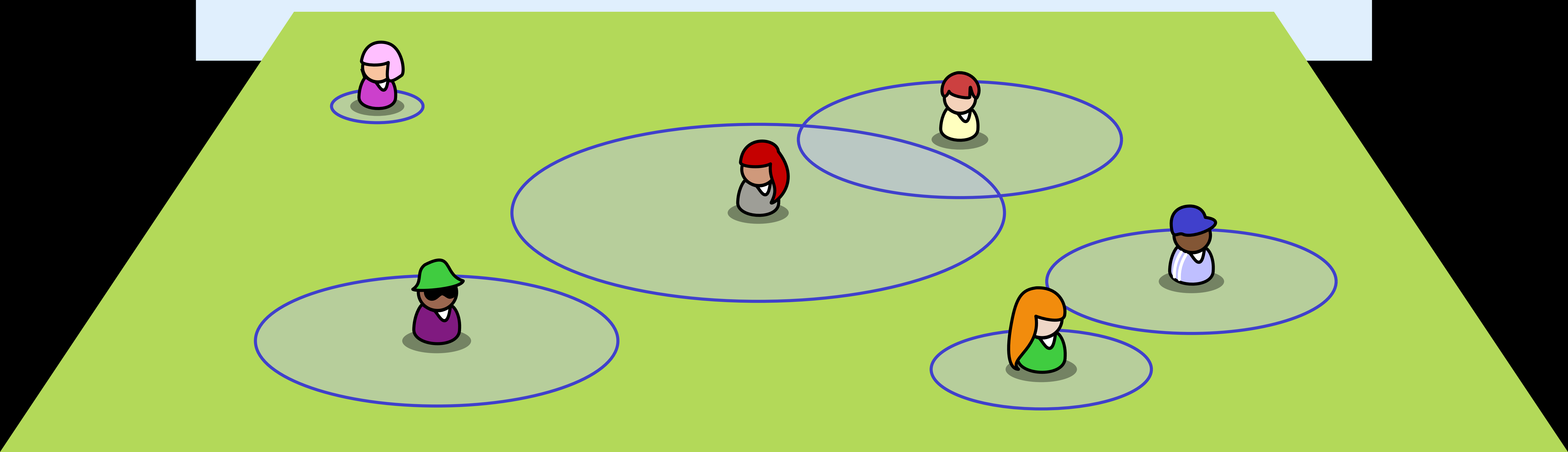
We can *query* a point to update it



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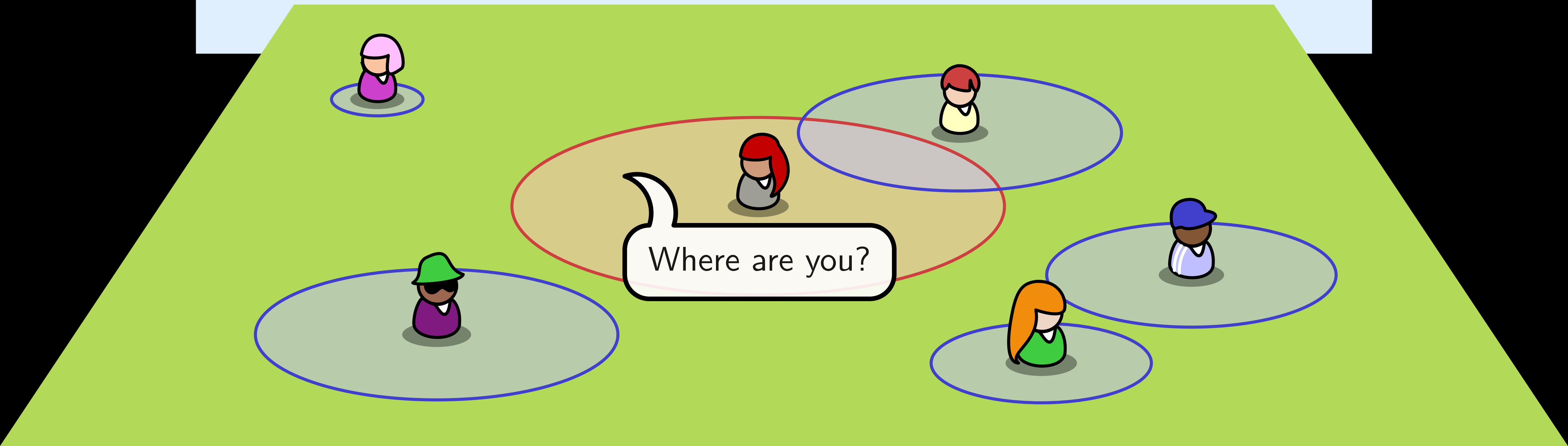

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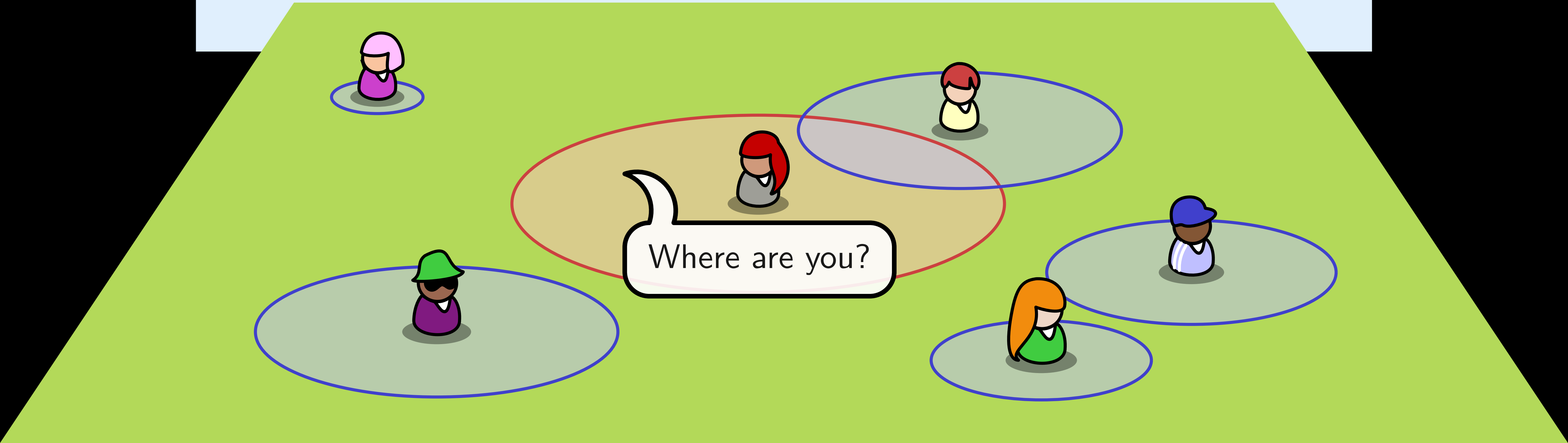

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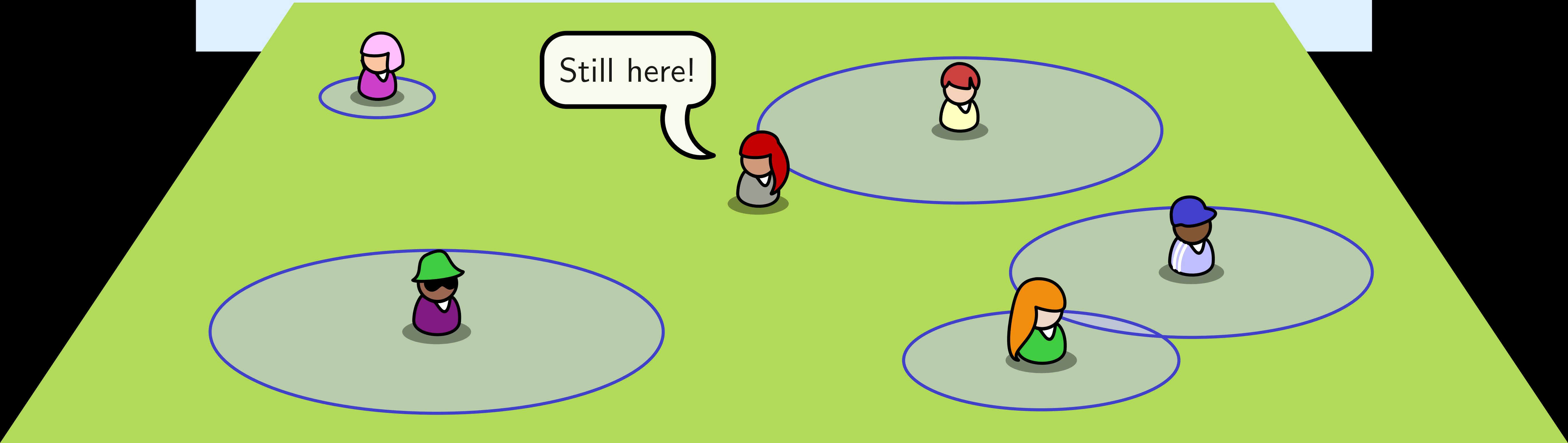

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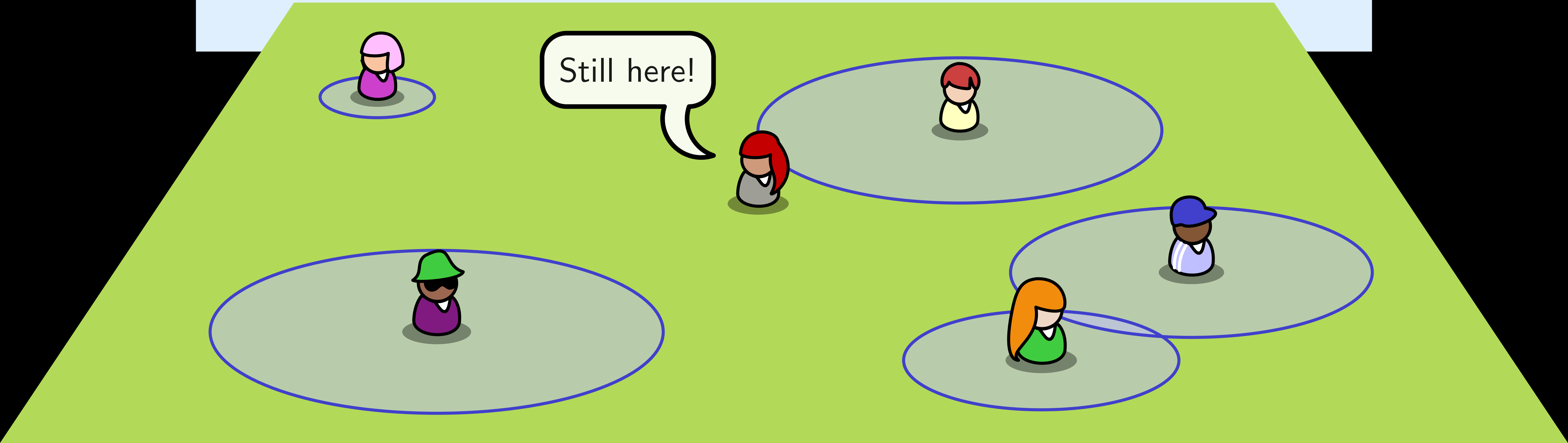

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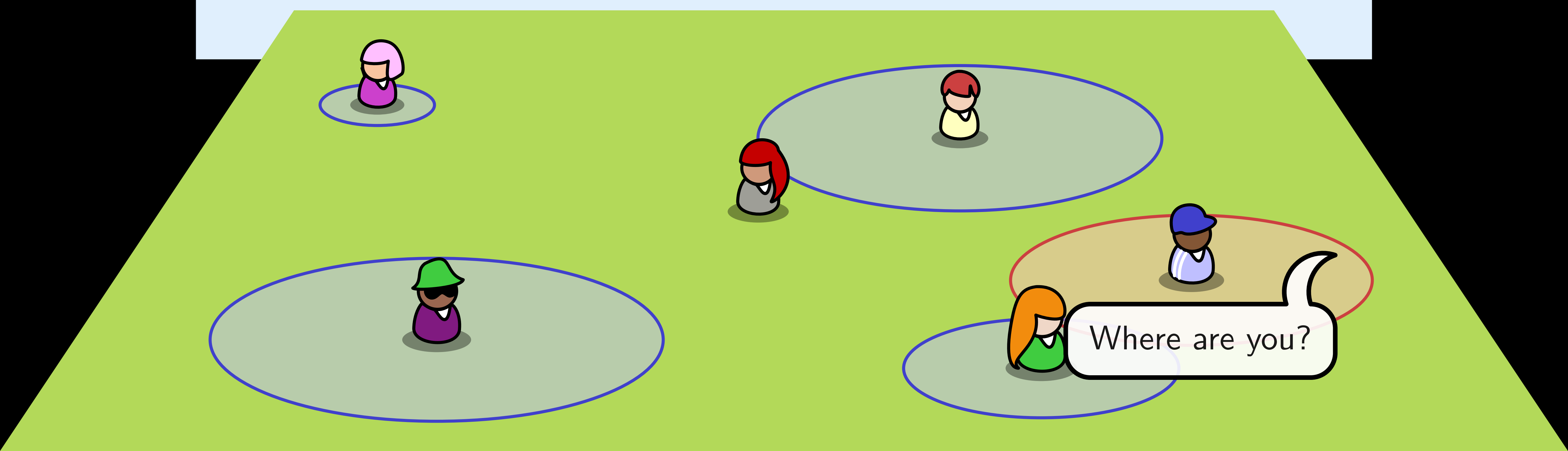

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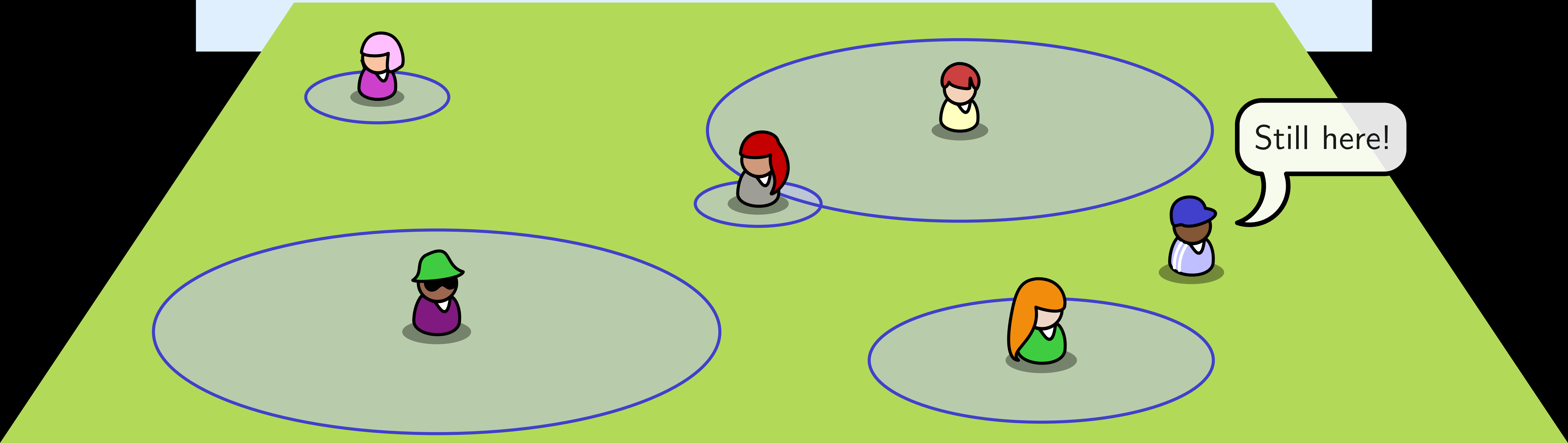

1-DIMENSIONAL POINTS

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Points are on a line

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1-DIMENSIONAL POINTS

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1-DIMENSIONAL POINTS

- Points are on a line
- We play the same game

1-DIMENSIONAL POINTS

Points are on a line

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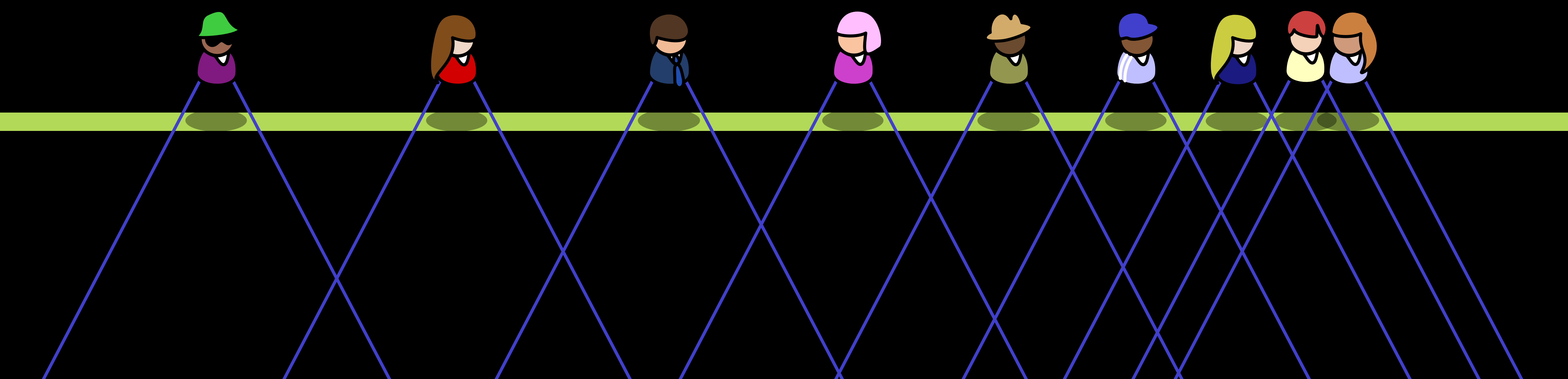

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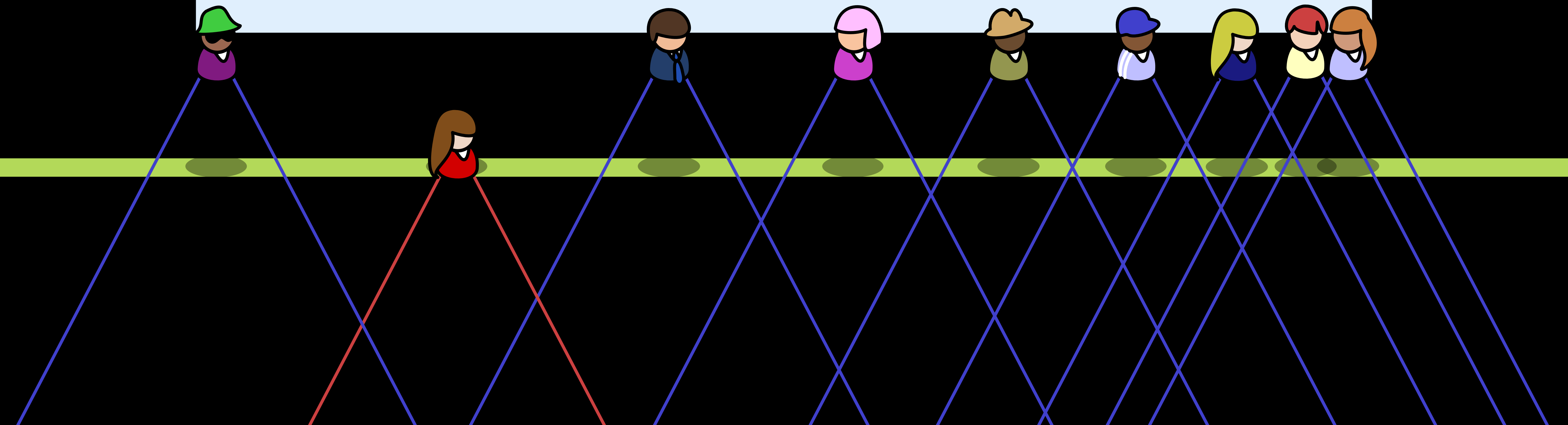

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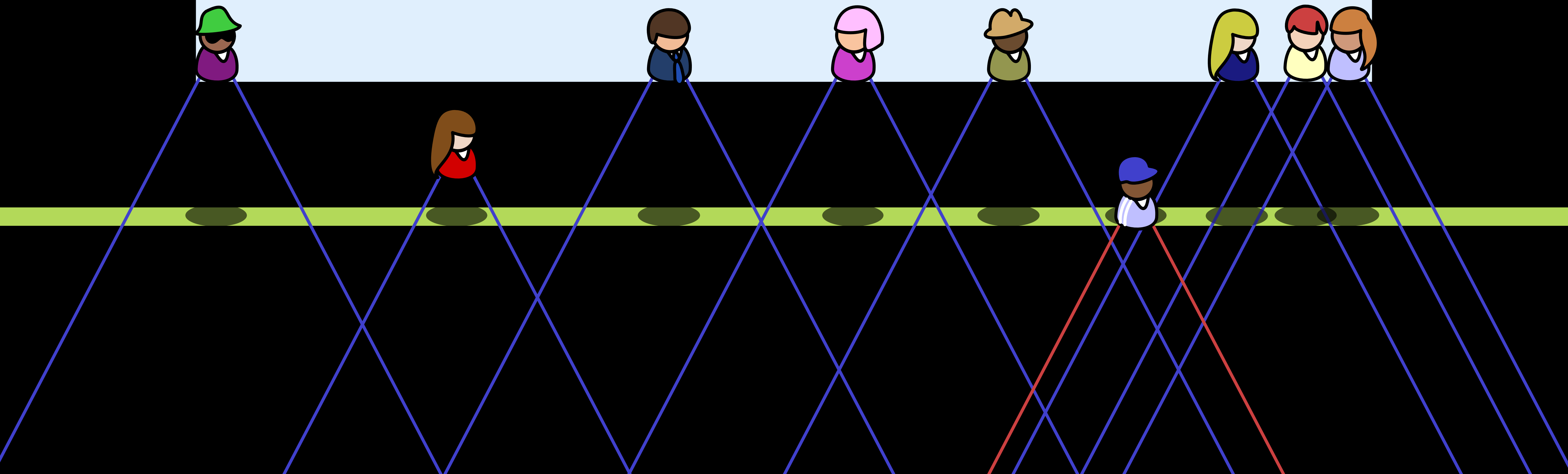

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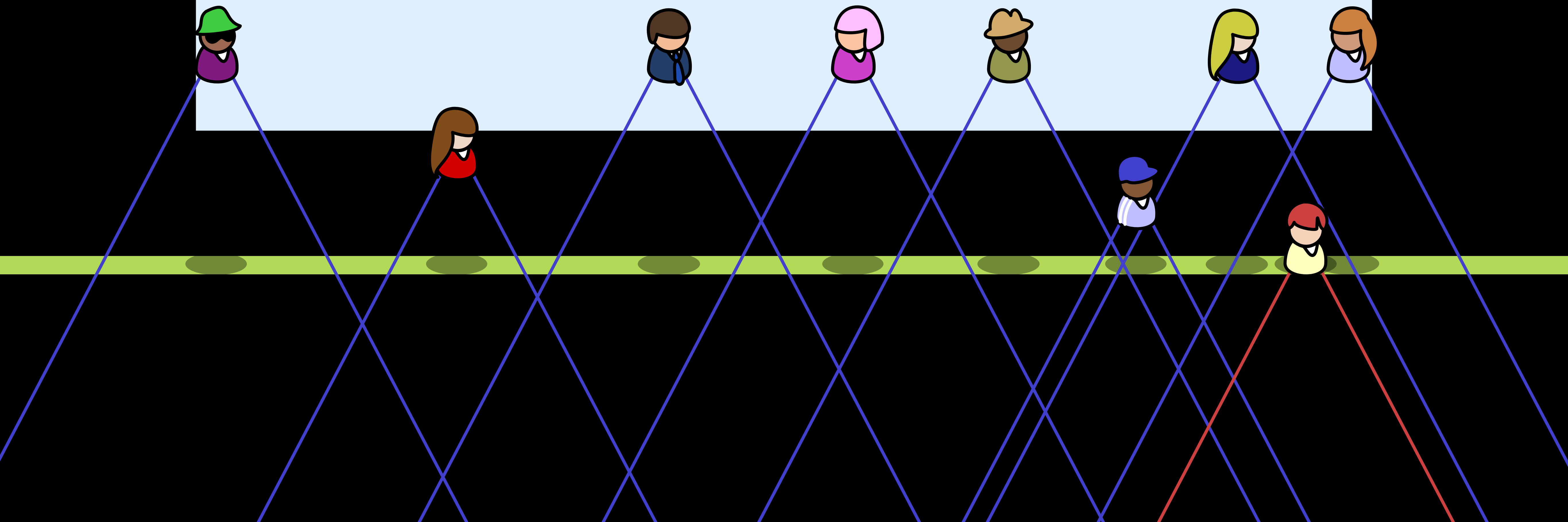

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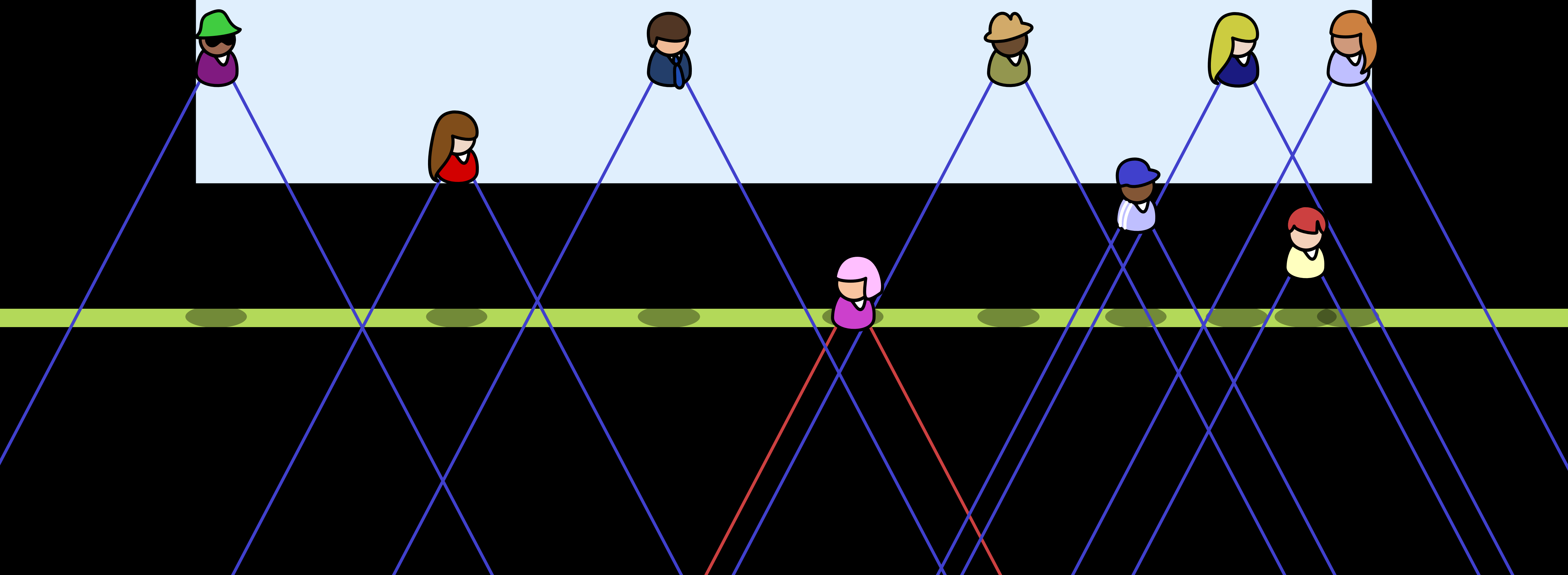

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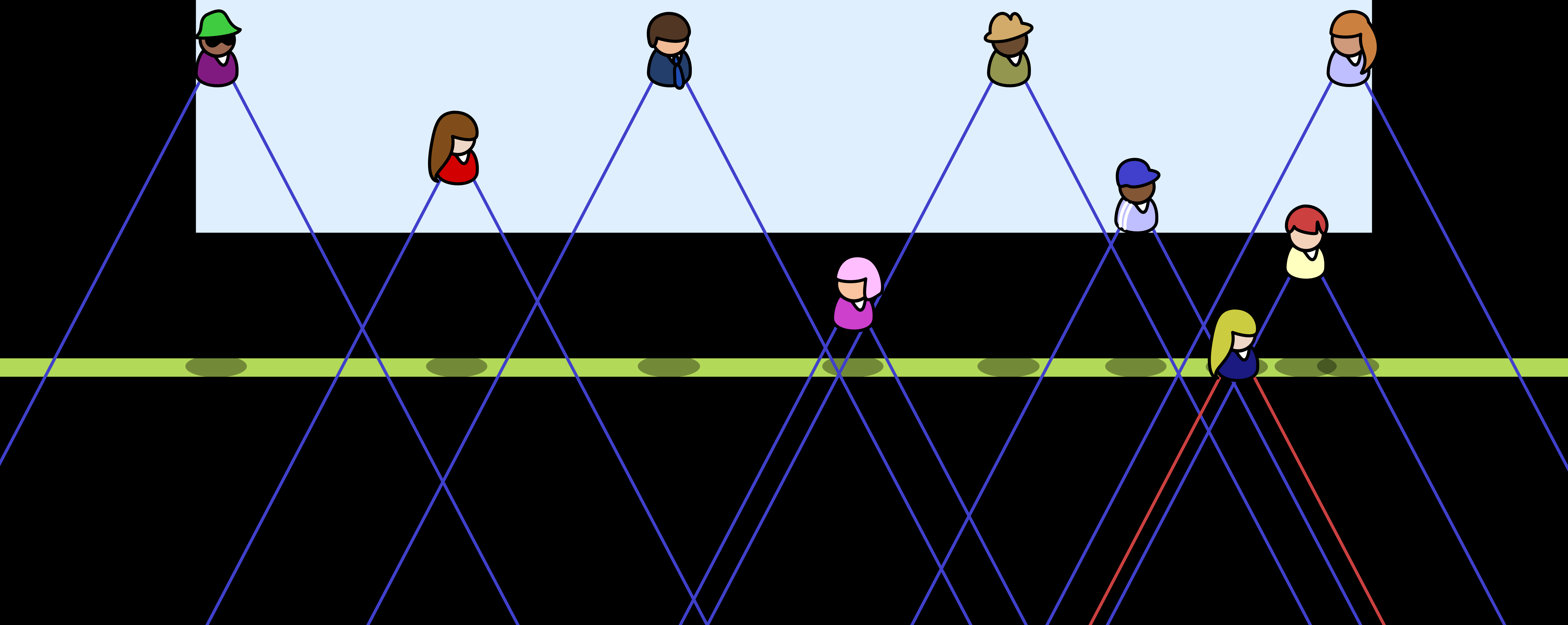

1-DIMENSIONAL POINTS

- Points are on a line
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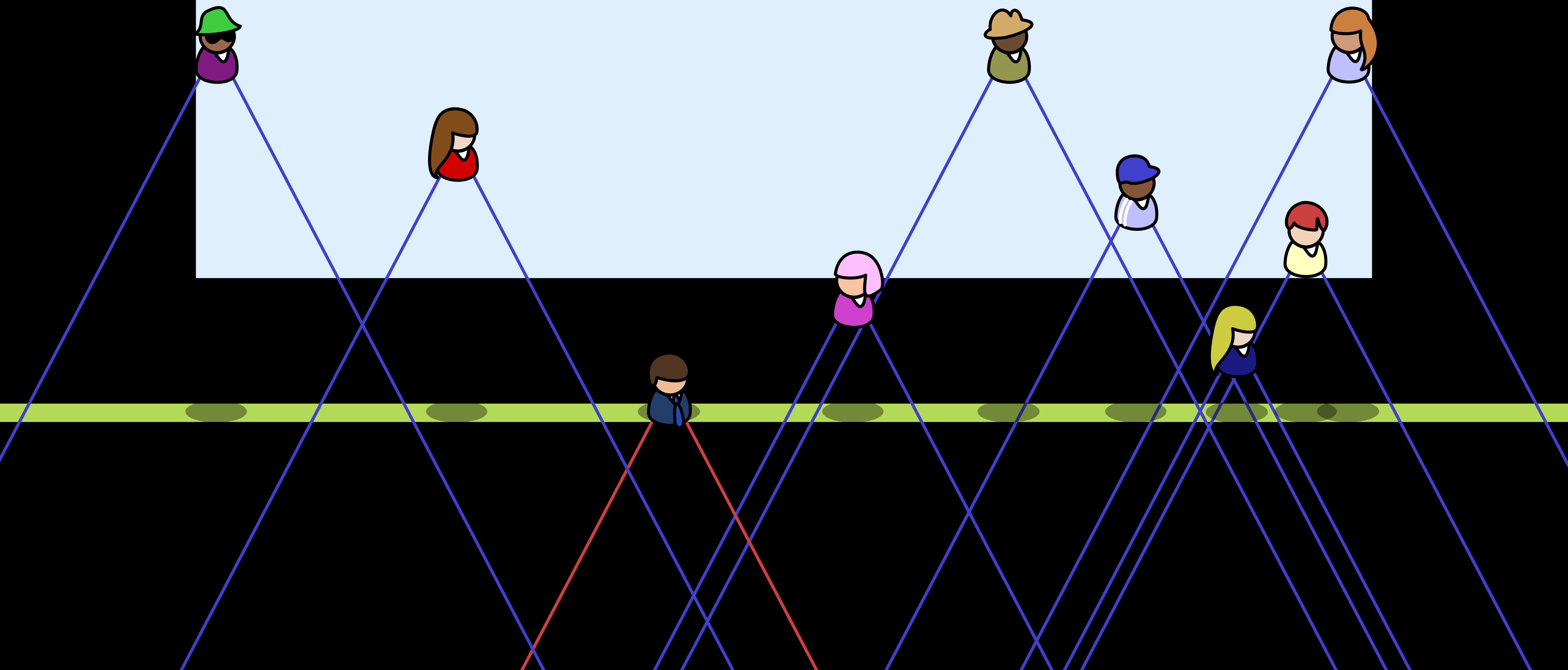

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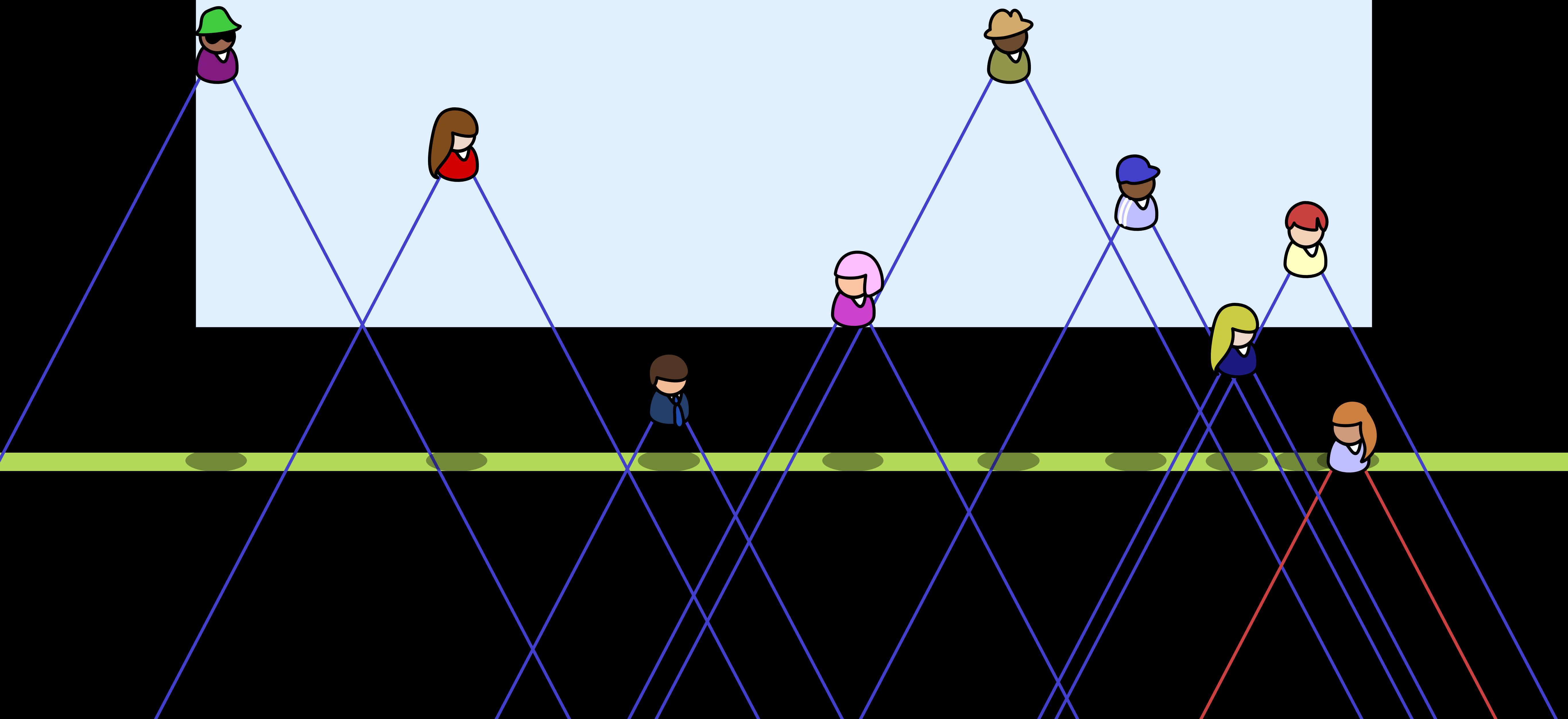

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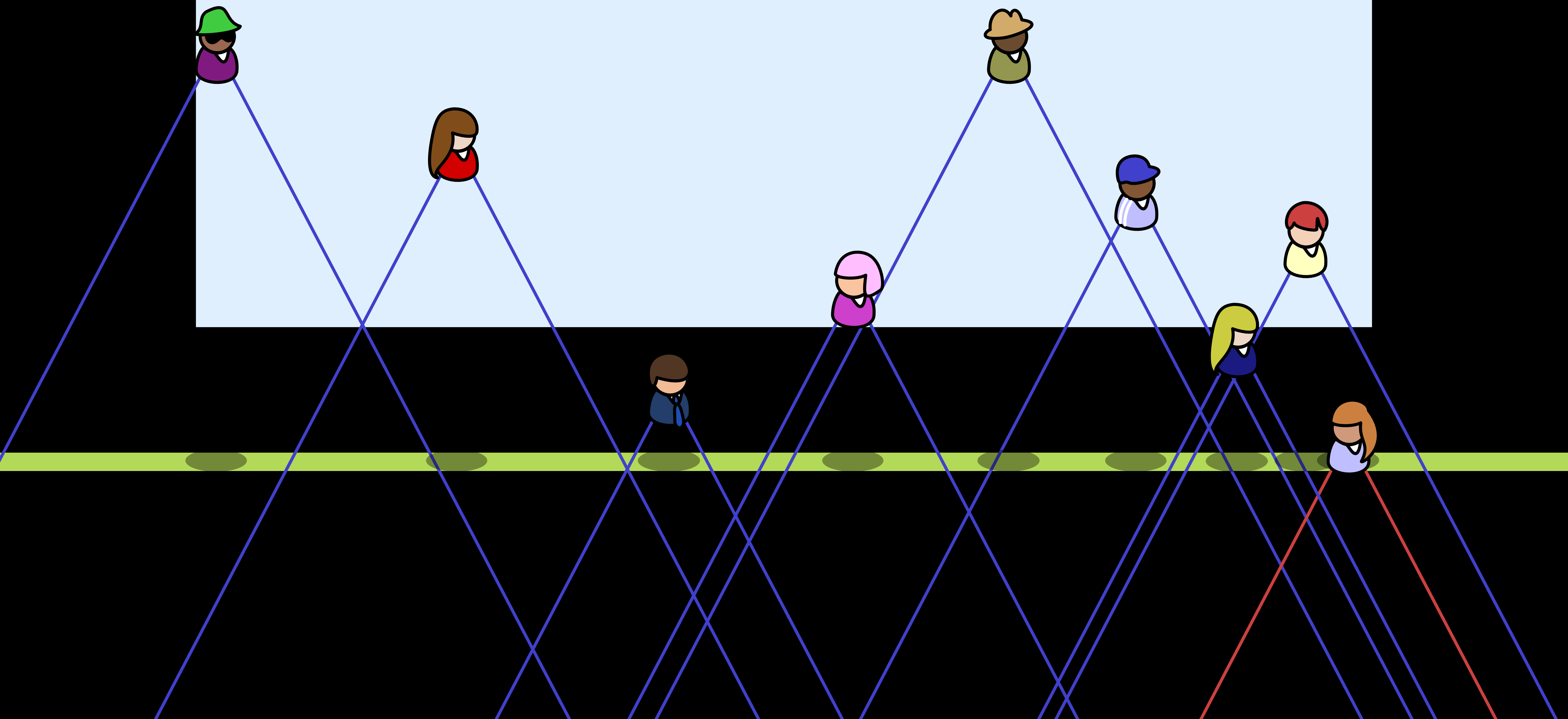
We want a small intersection graph



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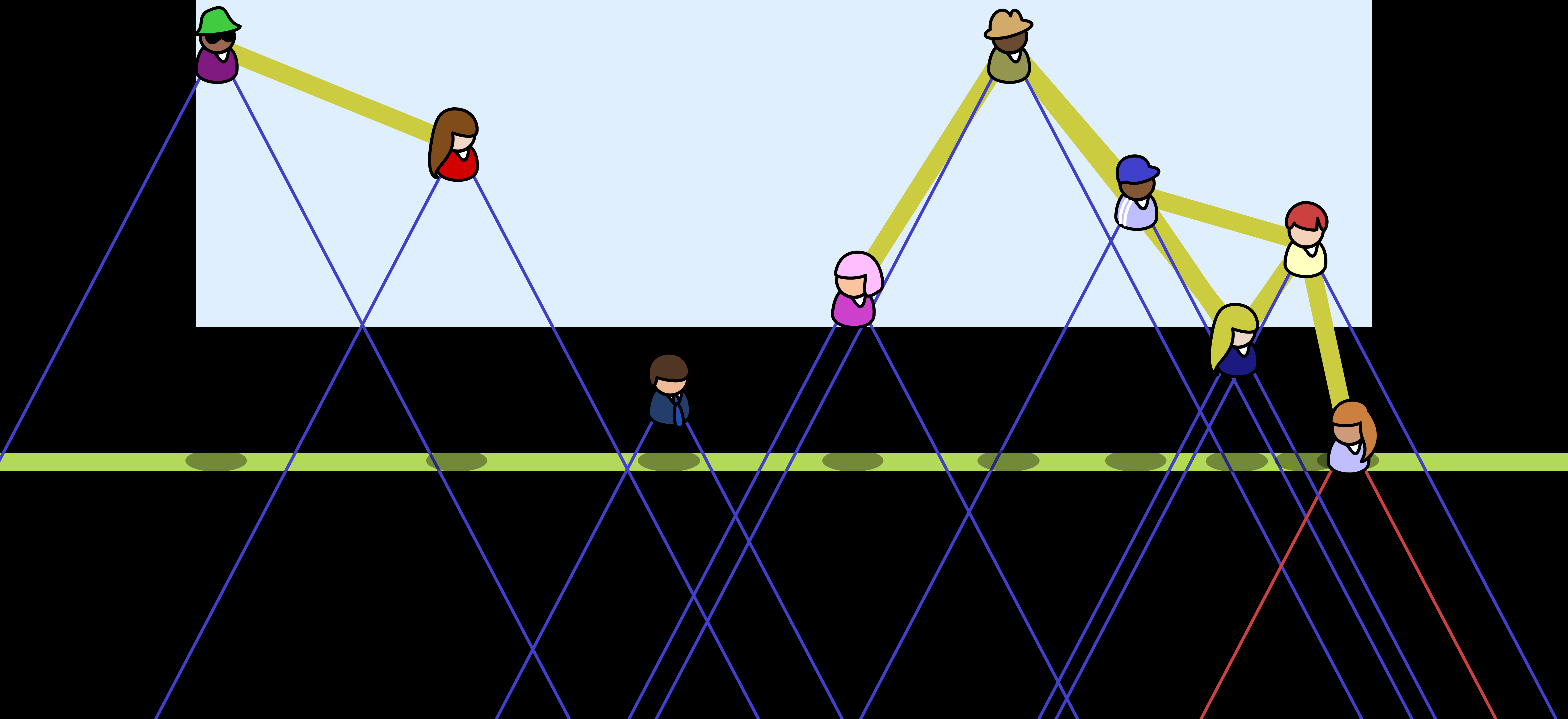

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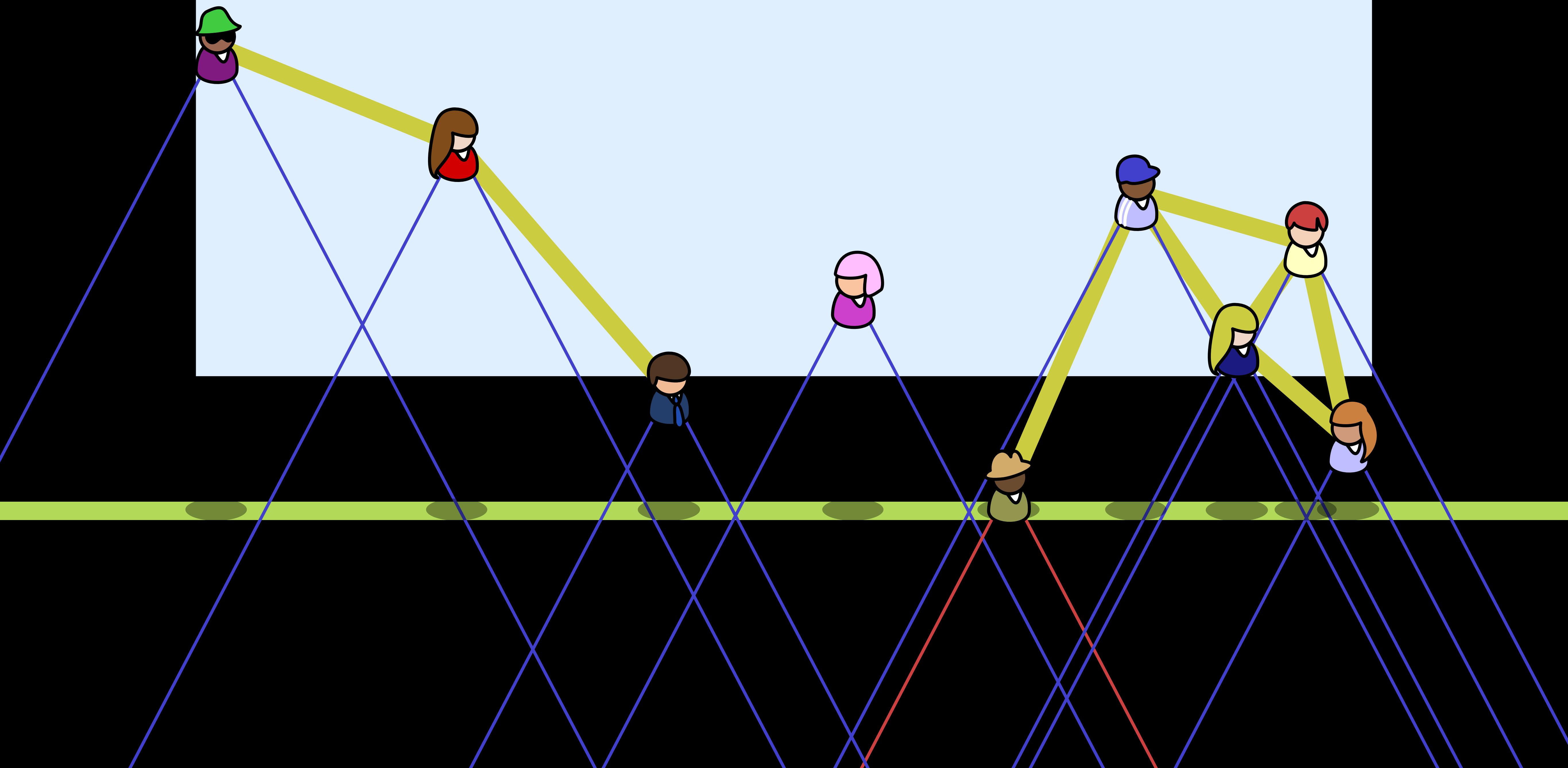

1-DIMENSIONAL POINTS

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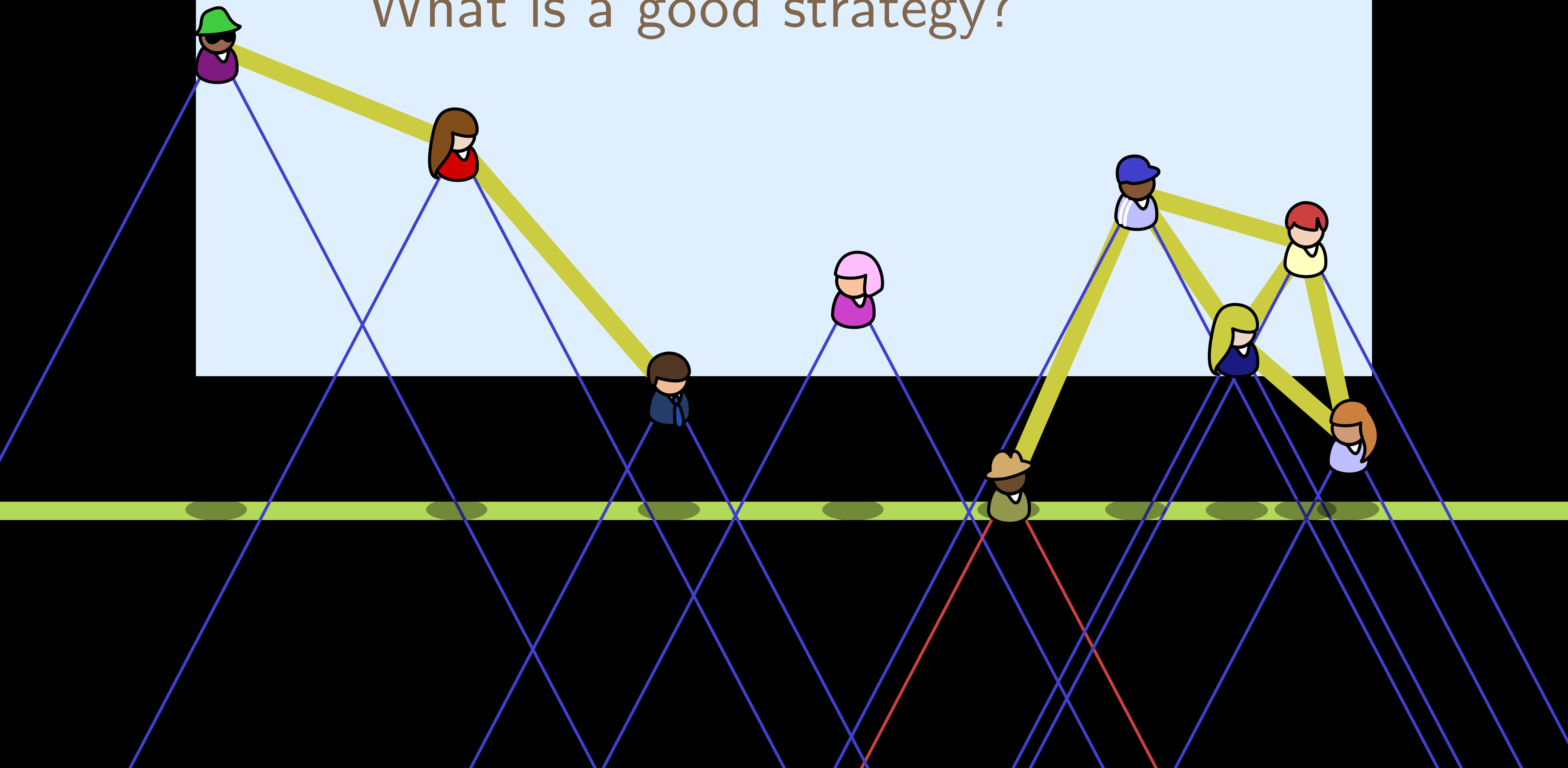
We want a small intersection graph

What is a good strategy?



THEOREM

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For any target value τ, either
(i) any query strategy has uncertainty intervals with intersection graph of degree $\Omega(\tau)$ at some point in any time interval of length τ, or
(ii) a simple query strategy guarantees that the total degree in the intersection graph of the uncertainty intervals is $O(\tau)$ at all times.

THEOREM

For any target value τ, either
(i) ..., or
(ii) a simple query strategy guarantees that the total degree in the intersection graph of the uncertainty intervals is $O(\tau)$ at all times.

> The *smallest* τ for which (ii) applies is the *critical* degree

THE STRATEGY

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Critical radius of point p

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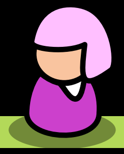

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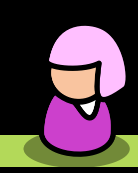
Critical radius of point p

Smallest r such that the ball of radius r around p contains at least $\frac{c\tau}{r}$ other points



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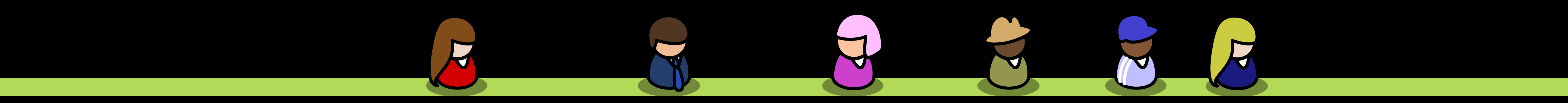

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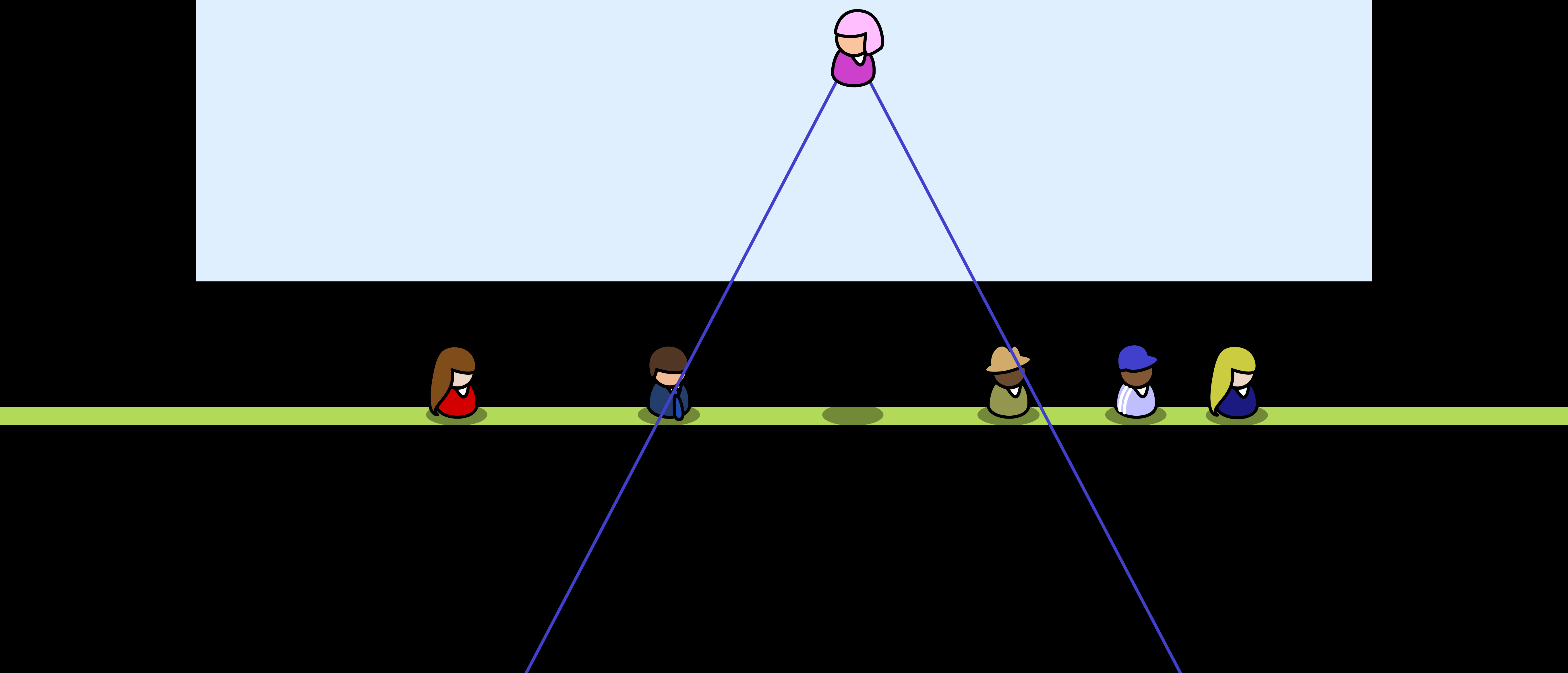

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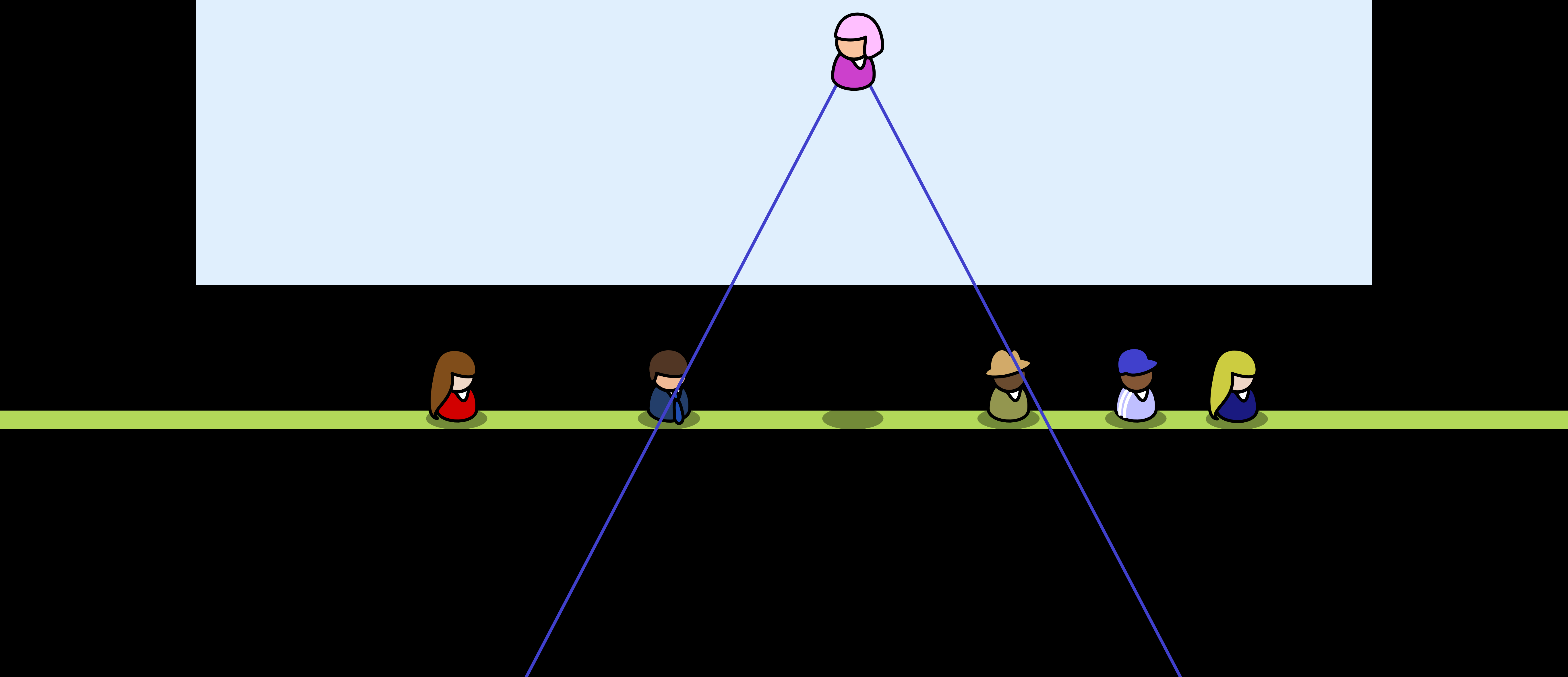
Points in denser areas have smaller r



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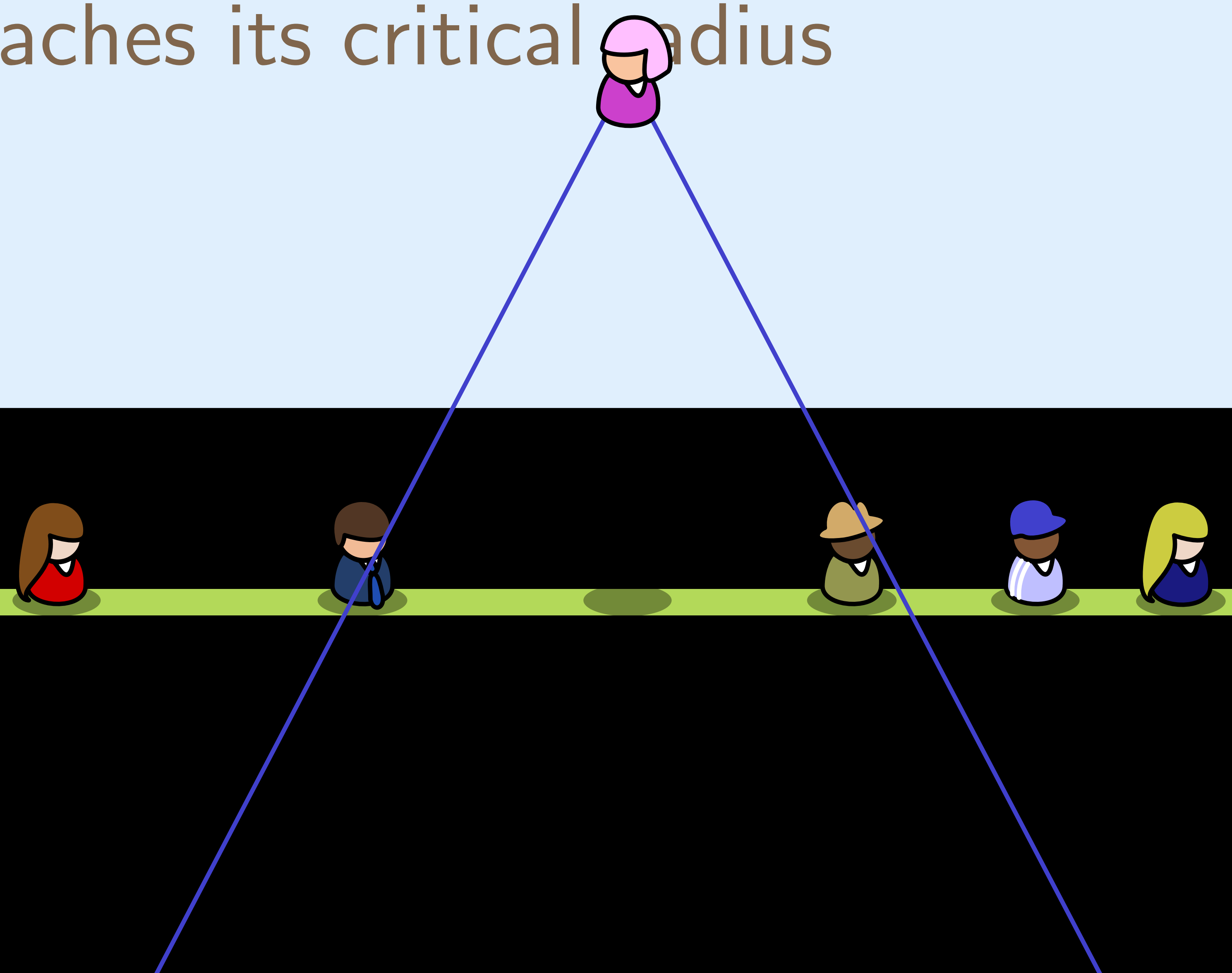
Plan: query each point before it reaches its critical radius



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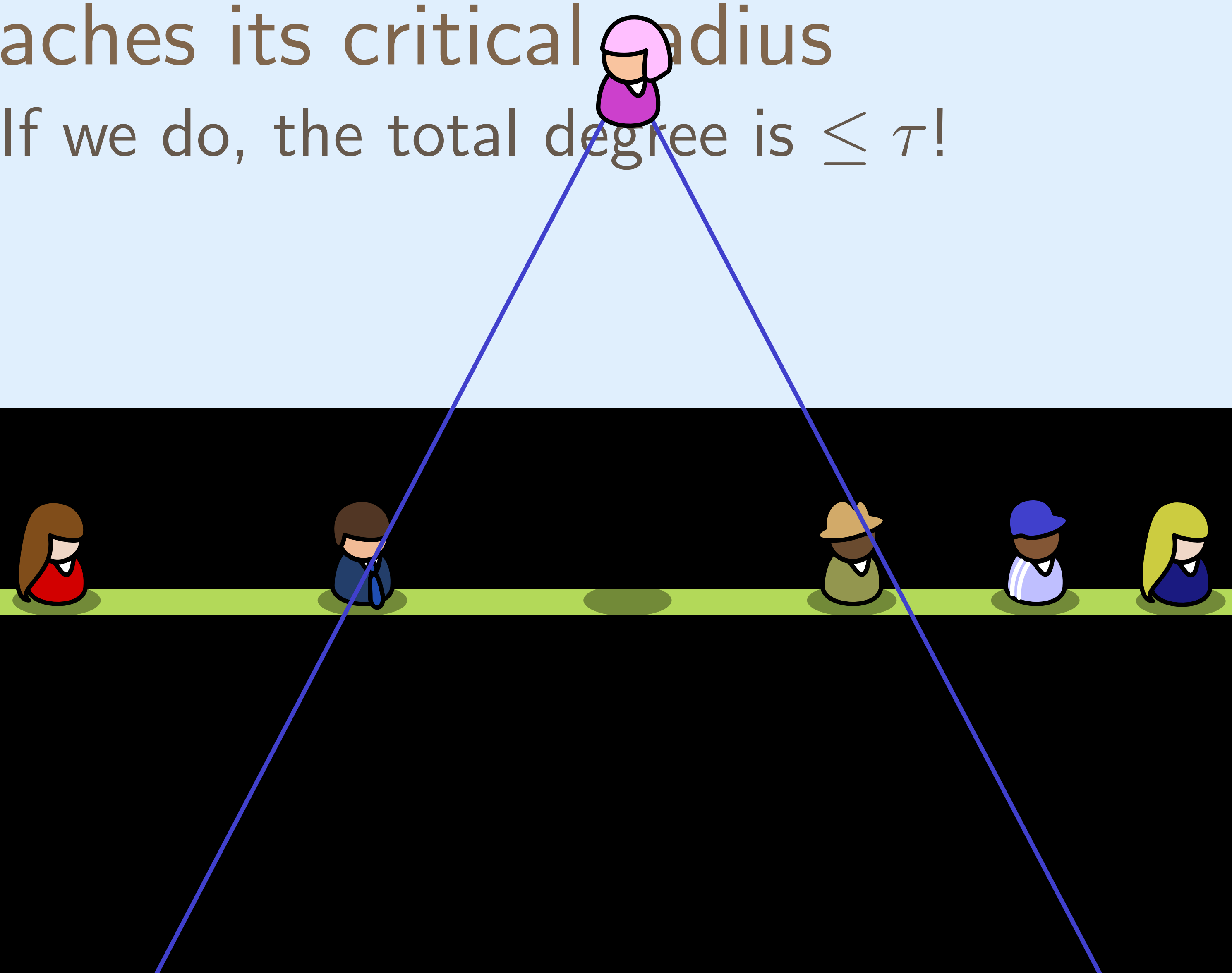
If we do, the total degree is $\leq \tau$!



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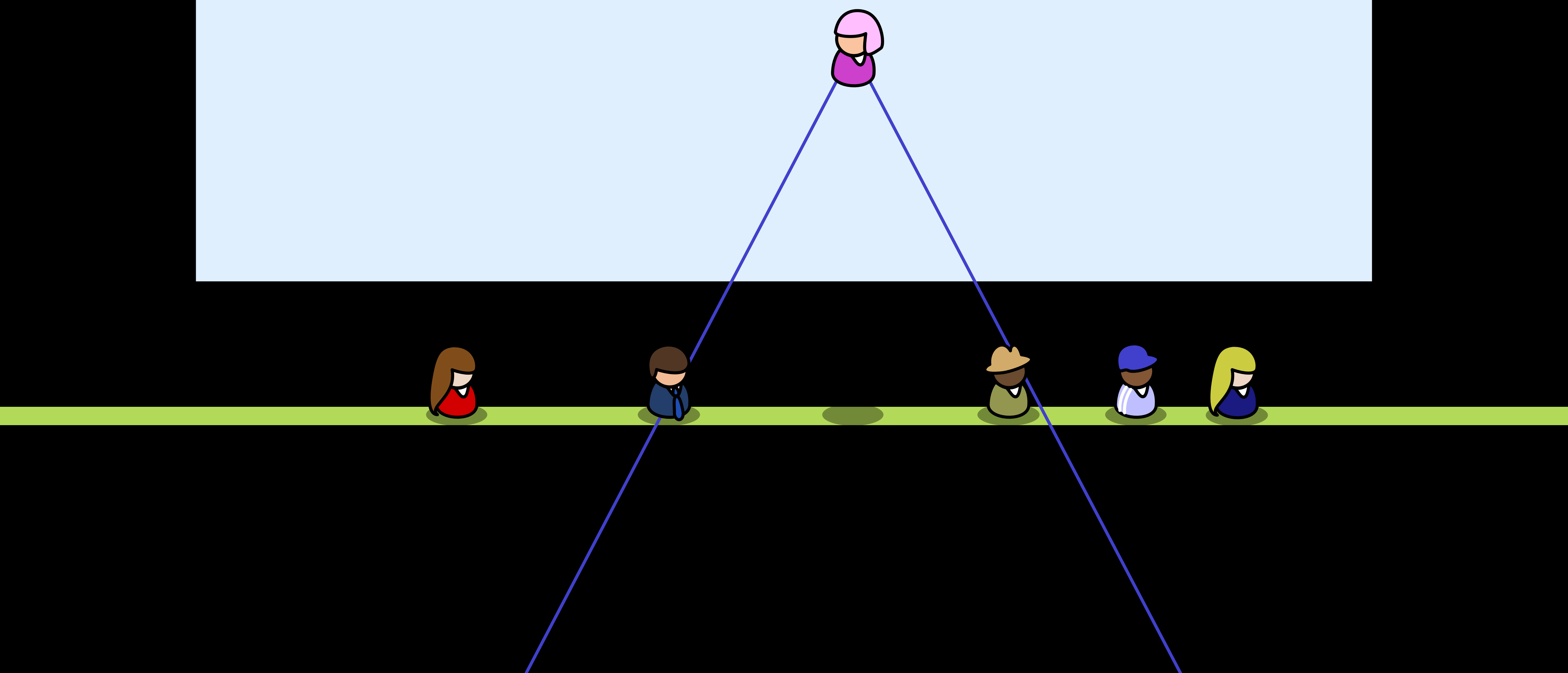

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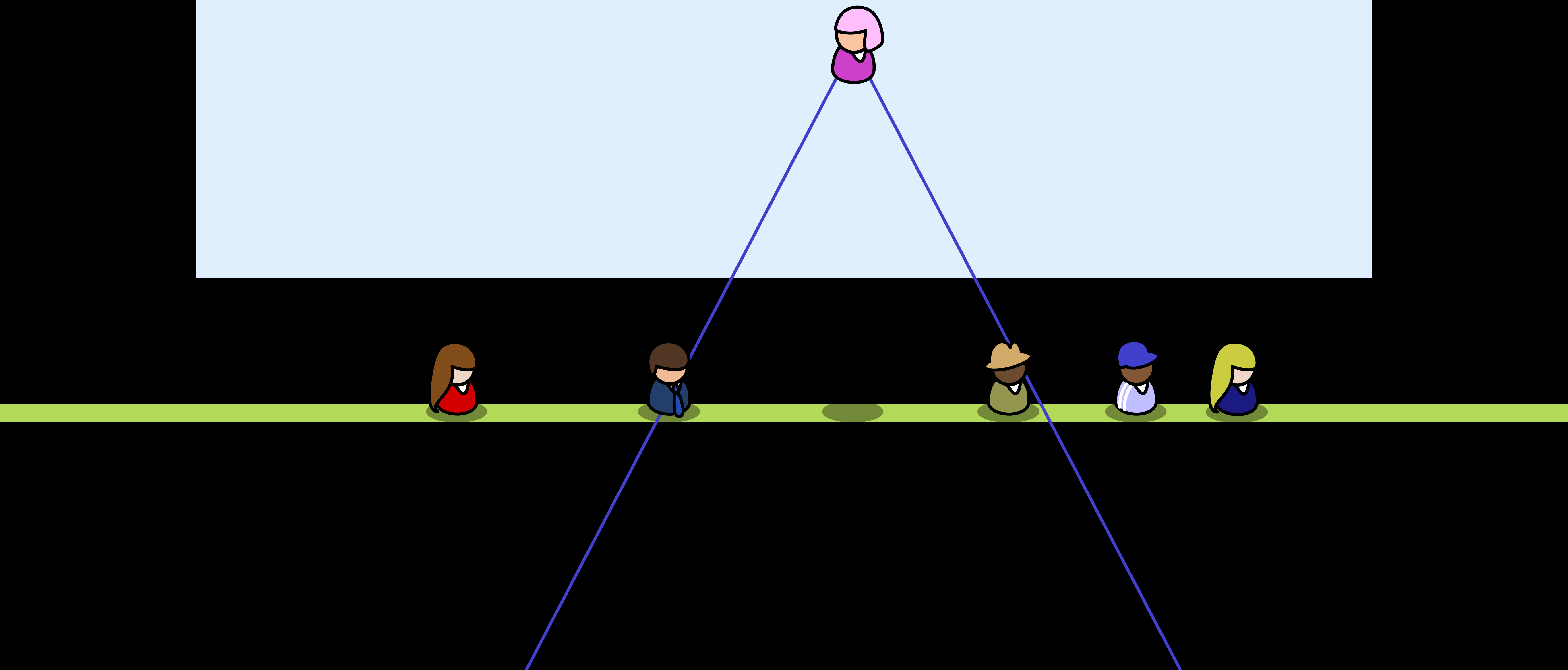
How do we do that?



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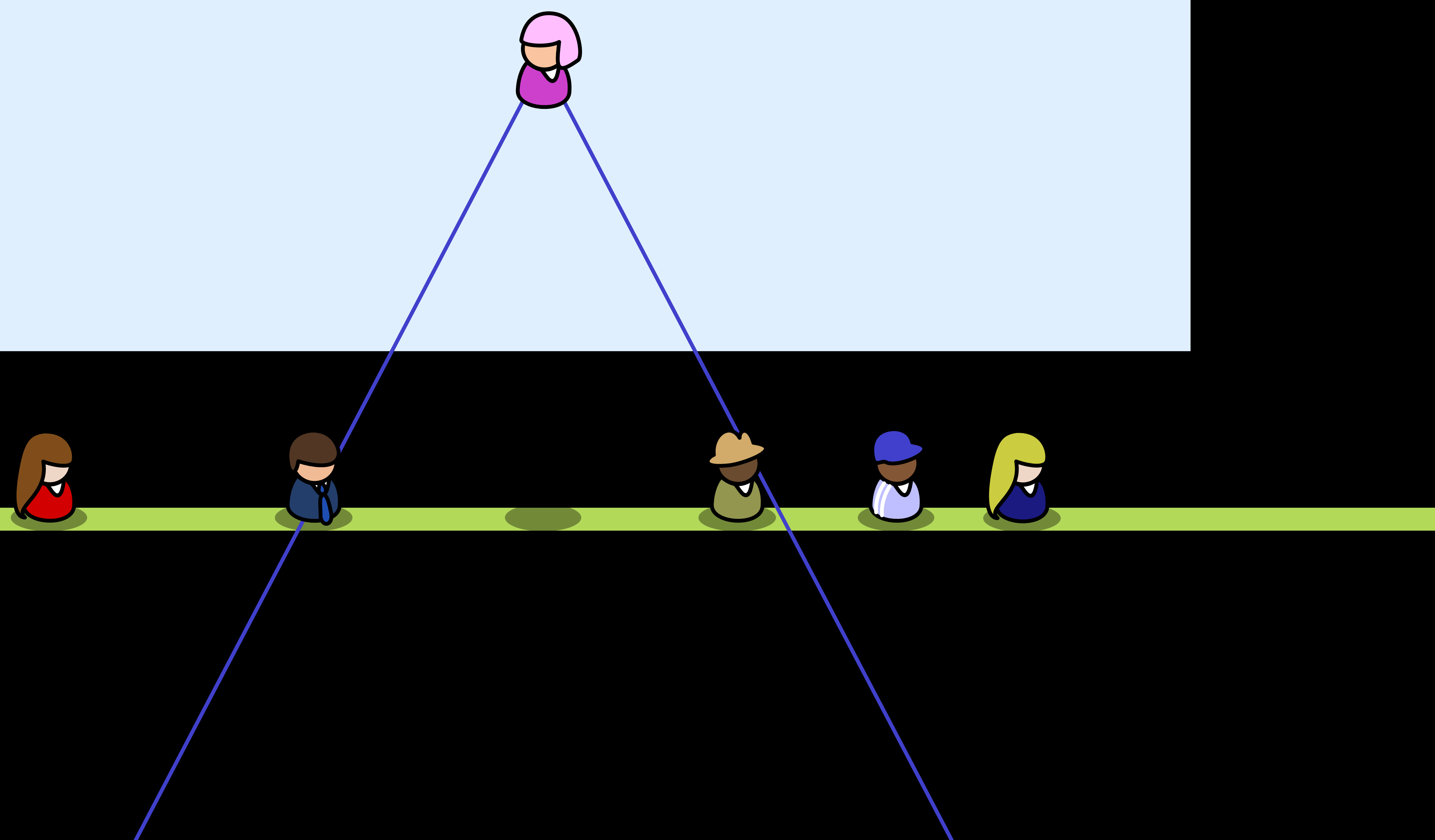
How do we do that?

If $\sum \frac{1}{r_i} > 1$, this is impossible...



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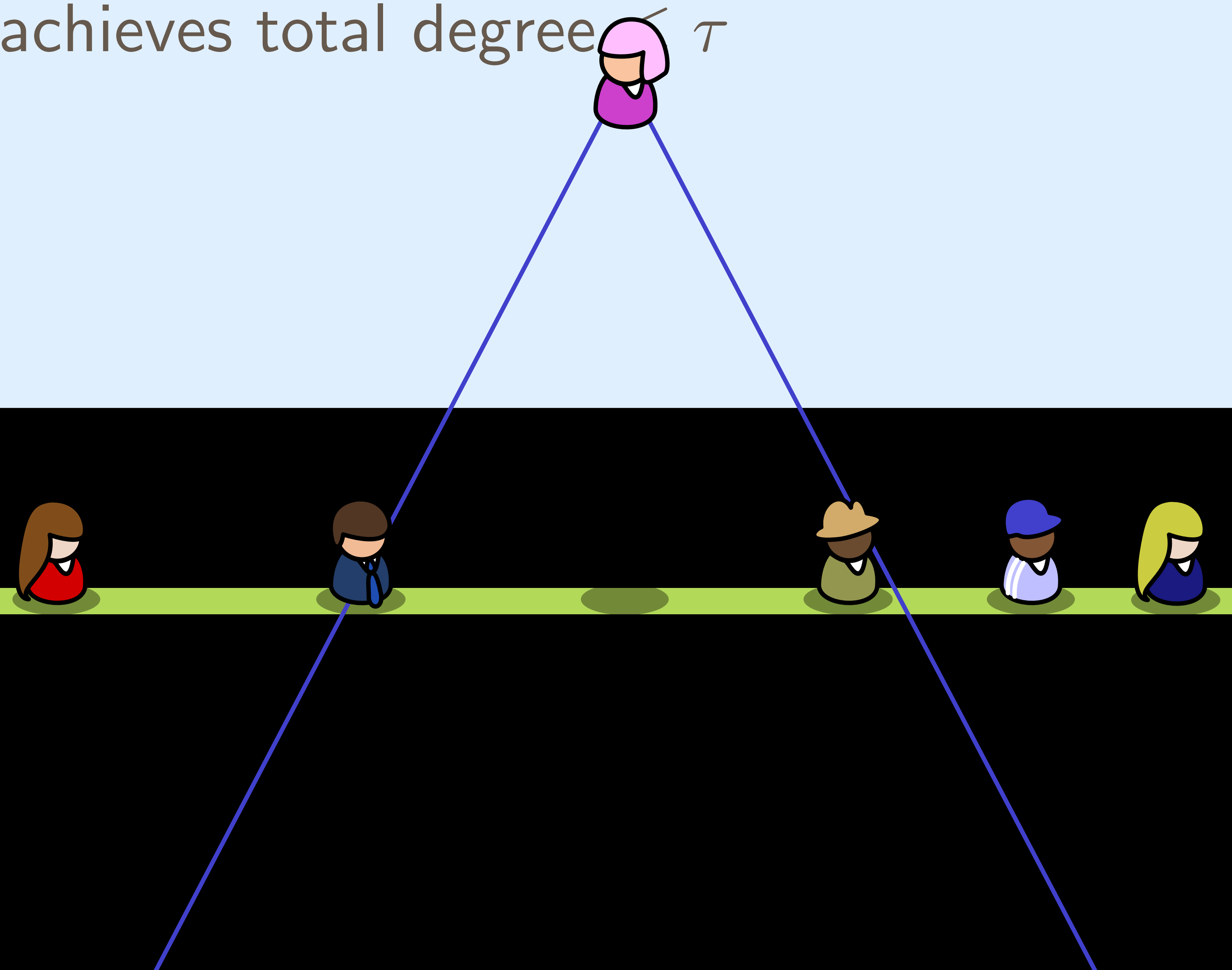

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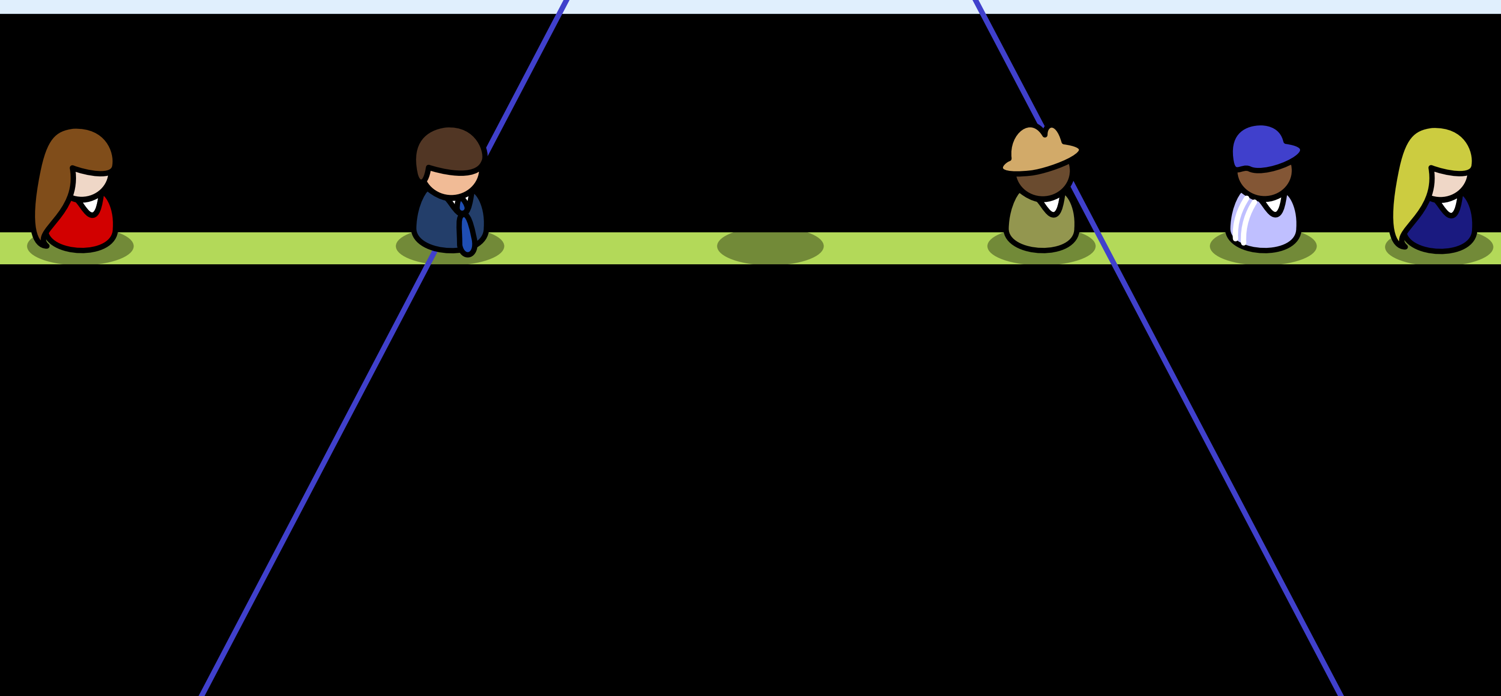

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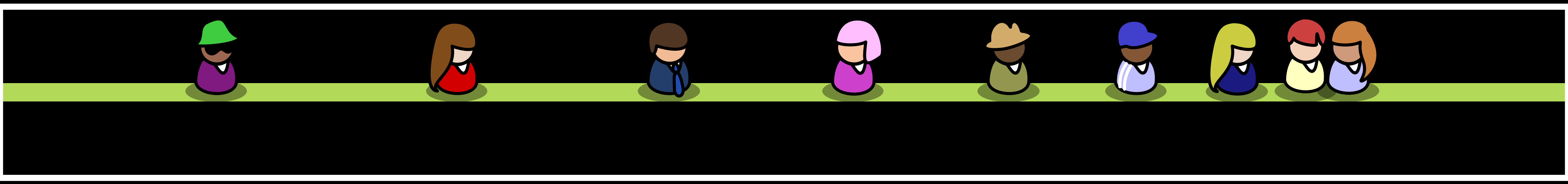

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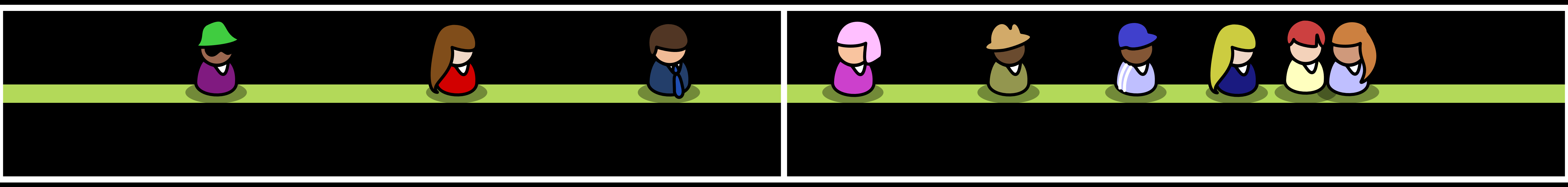

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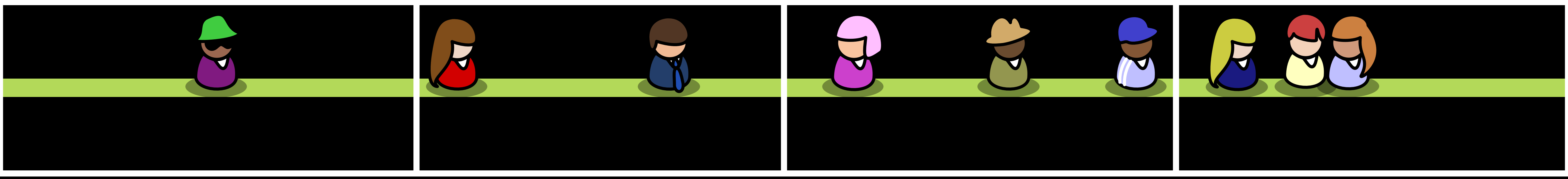

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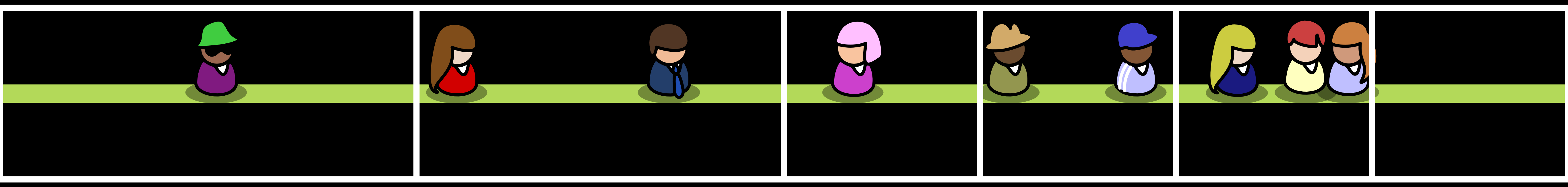

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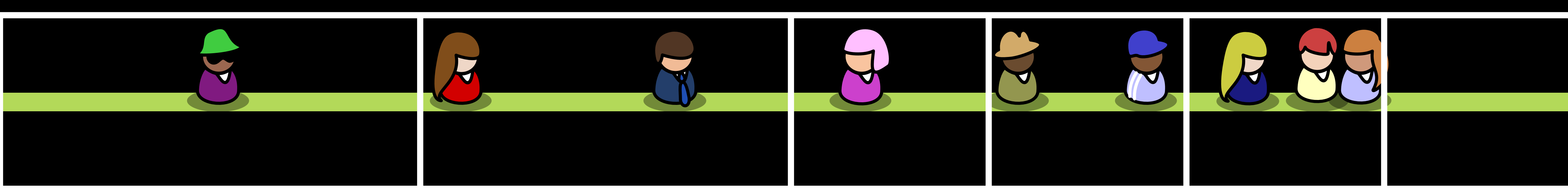
- Points in level ℓ get query frequency $\frac{1}{2^\ell}$



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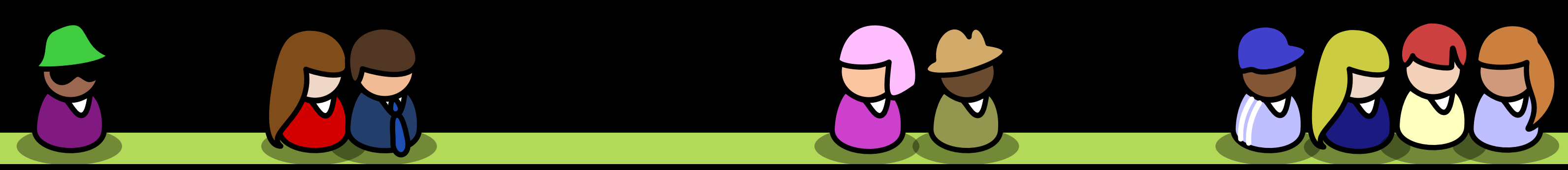

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There is a query strategy such that at *every* time t, the critical degree τ *at time* t is maintained during the time interval $[t, t + c\tau]$.

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THANK YOU!