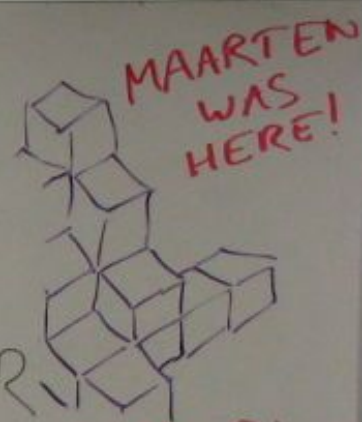
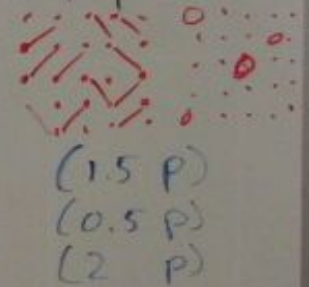


	P1	
	Yes	No
$\forall oc$	$\leftarrow O(n^2)$	is legal



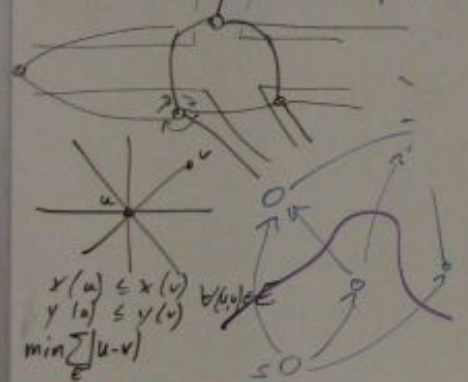
# OPTIMAL 3D ANGULAR -RESOLUTION FOR LOW-DEGREE GRAPHS



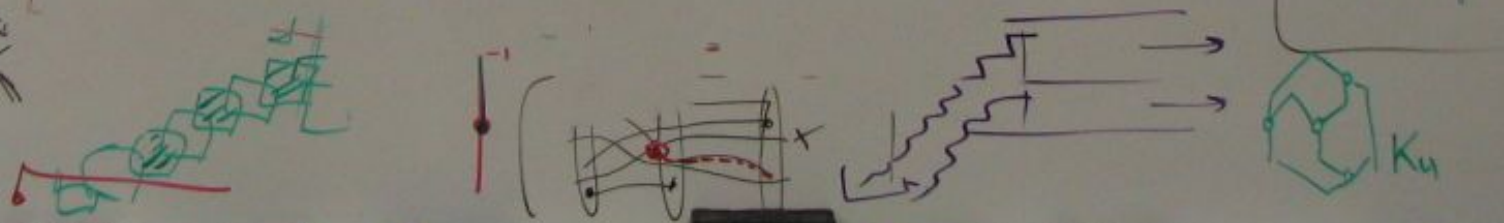
DAVID EPPSTEIN  
MAARTEN LÖFFLER  
ELENA MUMFORD  
MARTIN NÖLLENBURG

1 ALG.  
OUT  
DIM. (3 P)  
LS 5-LEMMA  
IRRUCTION  
LUGIONS (1 P)  
 $\mathcal{P}$  P

DO NOT  
ERASE  
!!!



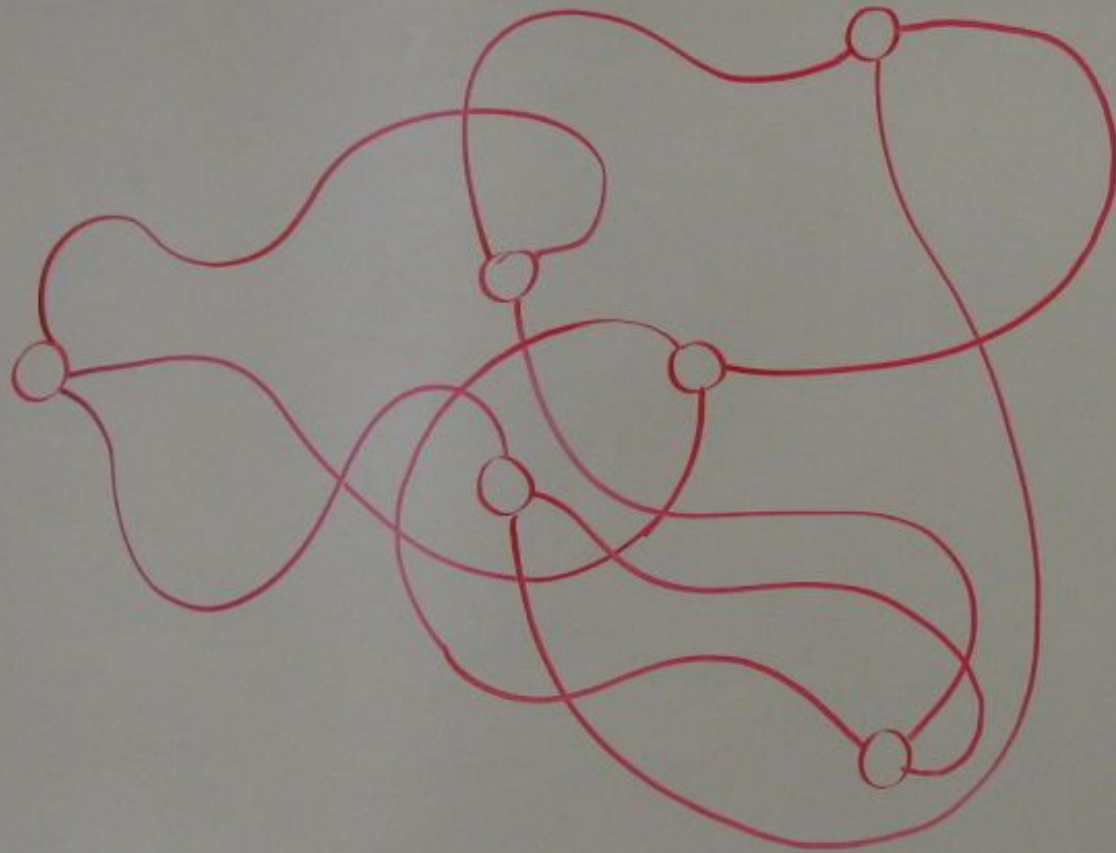
- To Do:
- GD registration
  - book flights
  - slides!!



# DRAWING A GRAPH

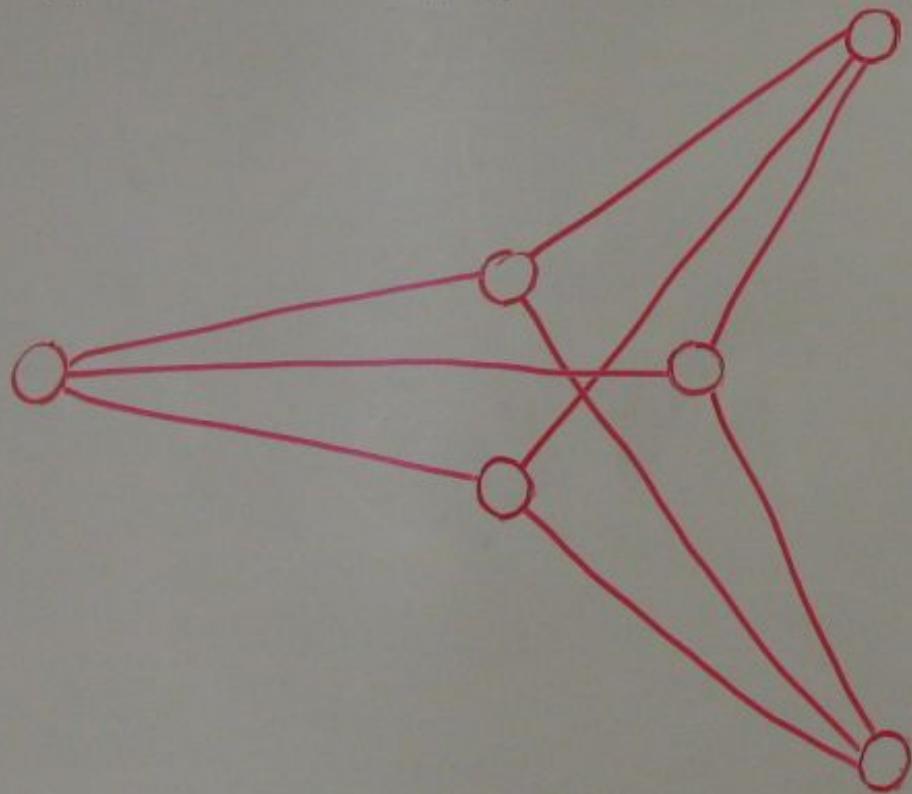
EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG    OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# A GRAPH



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

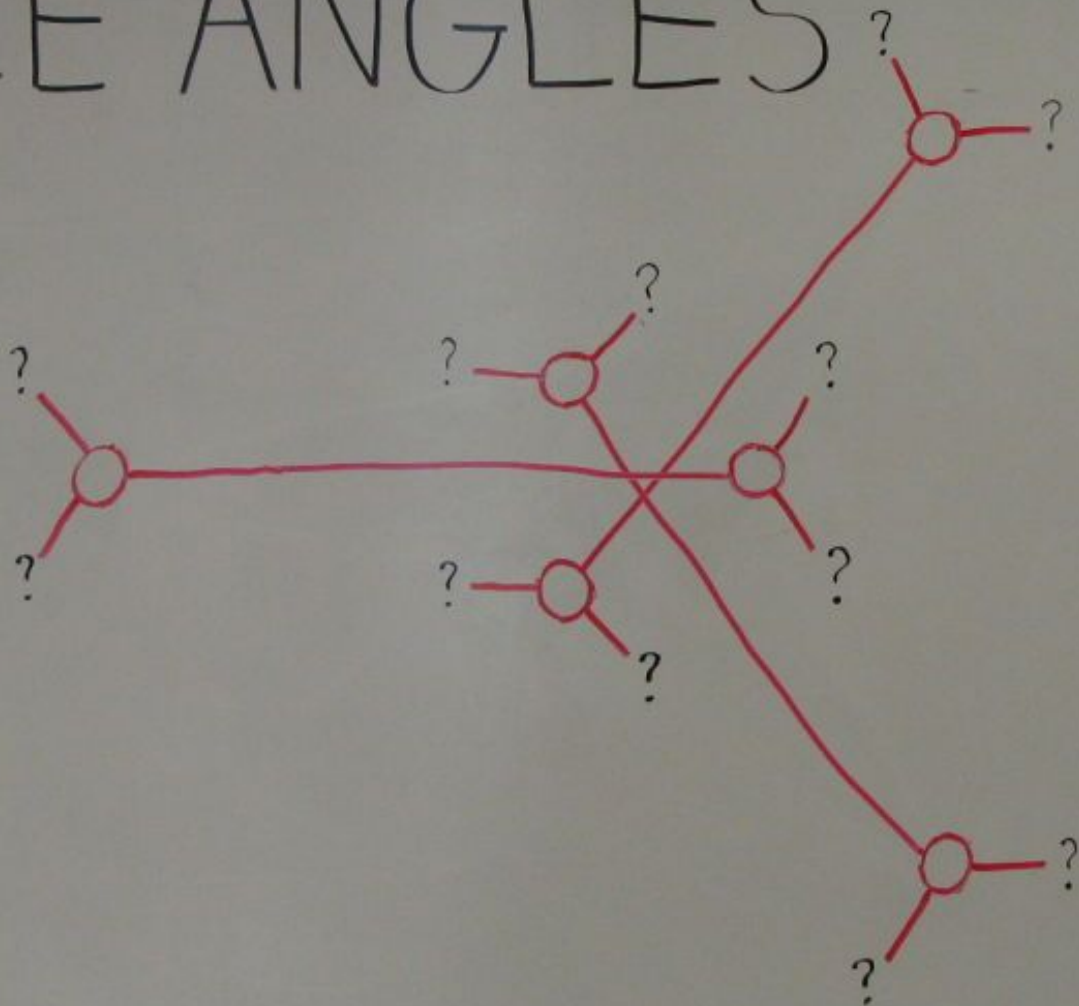
# STRAIGHT EDGES



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

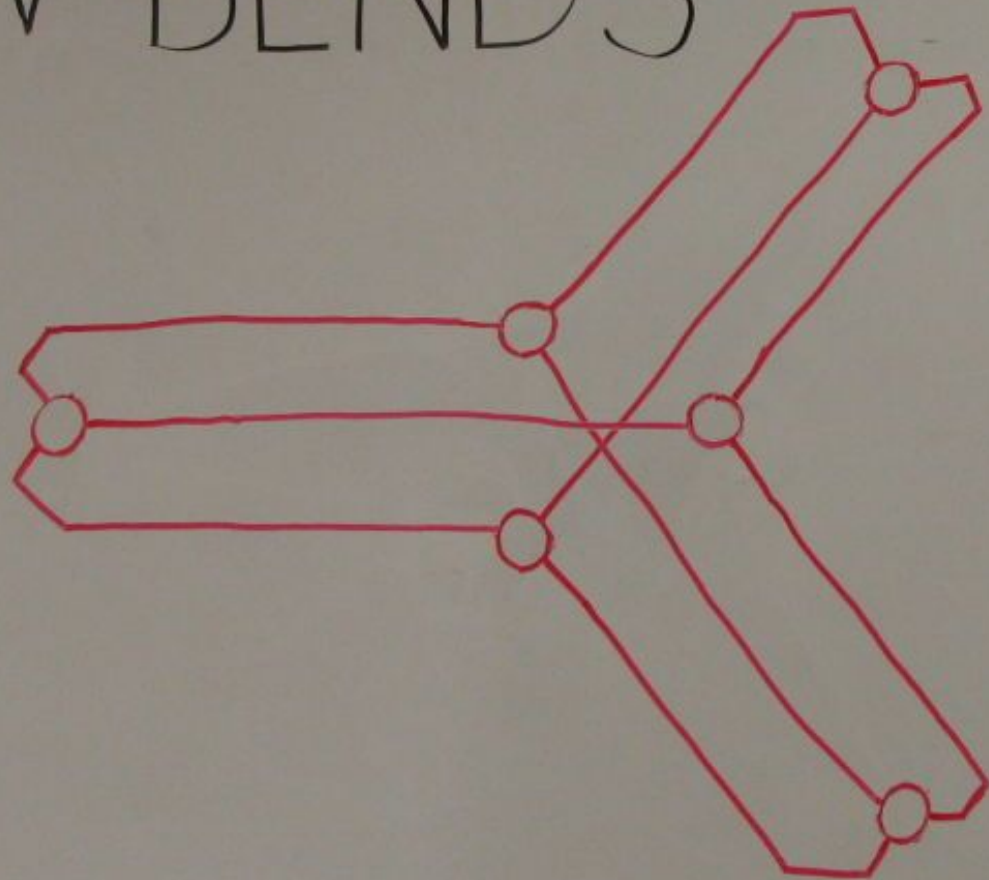
# NICE ANGLES



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

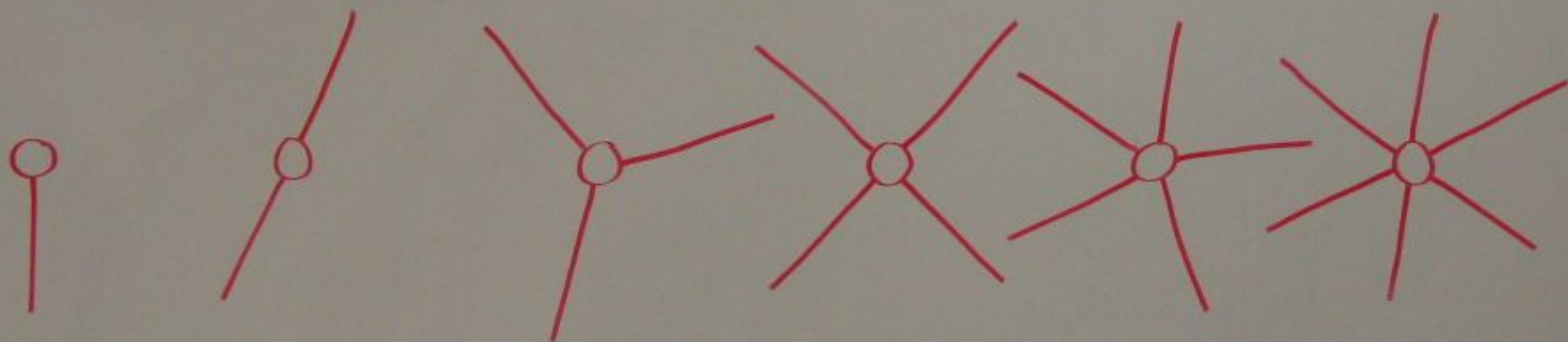
OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# FEW BENDS



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# ANGULAR RESOLUTION



$d=1$

$d=2$

$d=3$

$d=4$

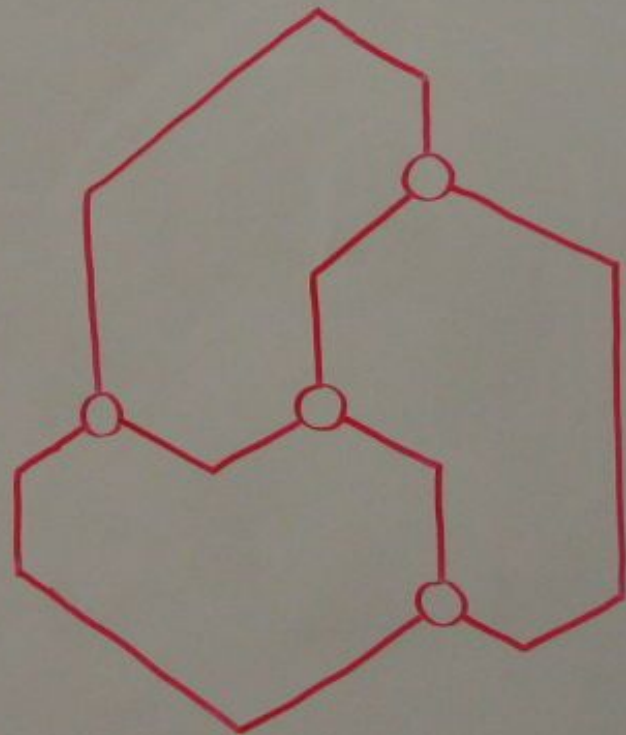
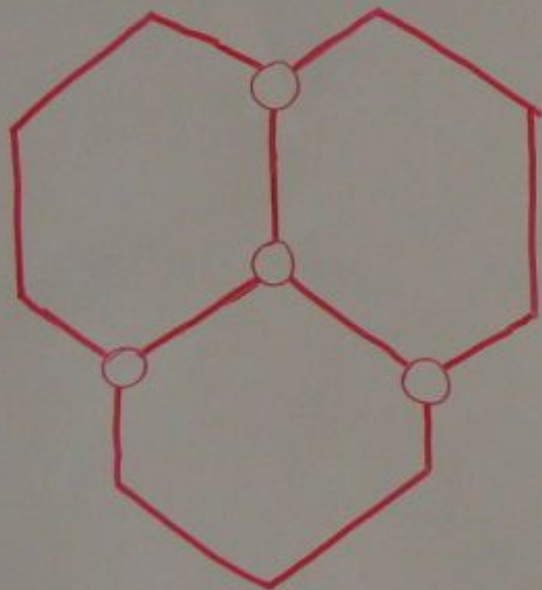
$d=5$

$d=6$

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OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

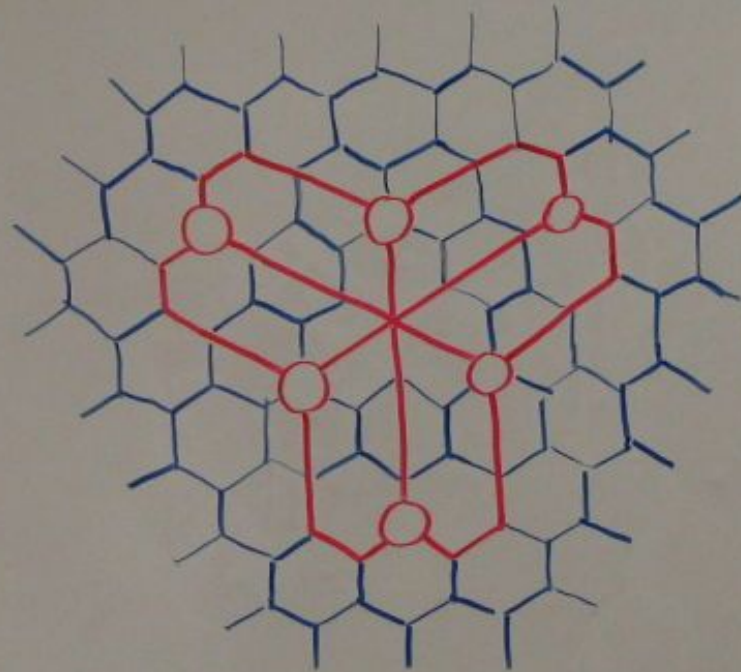
# ALIGNED VERTICES



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# ON A GRID



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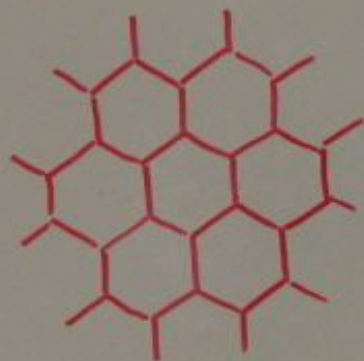
# ON A GRID



$d=1$



$d=2$



$d=3$



$d=4$



$d=5$

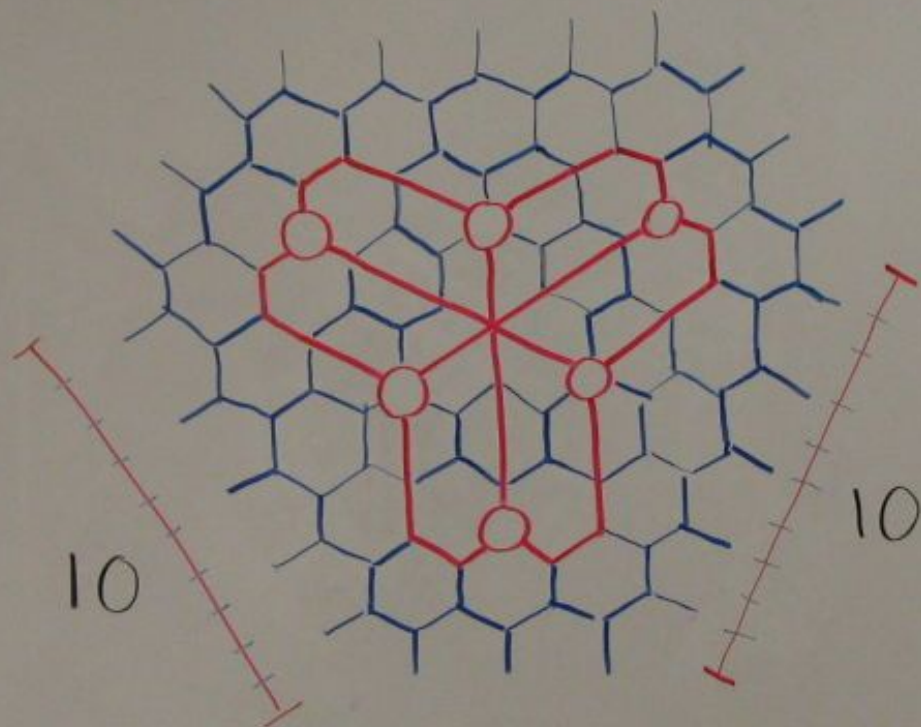


$d=6$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

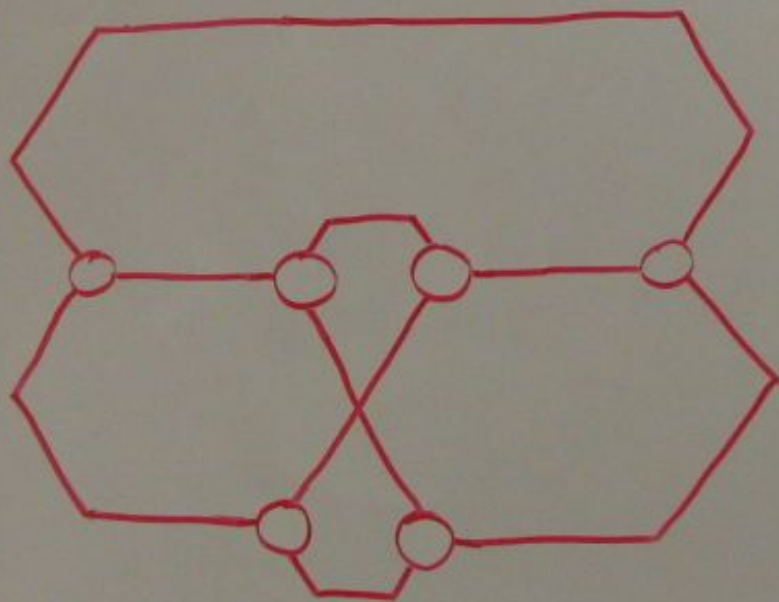
OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# SMALL AREA



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# NO CROSSINGS?



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2D

$d=1$

$d=2$

$d=3$

$d=4$

$d=5$

$d=6$

0 bends

1 bend

2 bends

3 bends

4 bends

5 bends

other

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

2D	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
0 bends	✓ (TRIVIAL)					
1 bend	✓					
2 bends	✓					
3 bends	✓				✓ (TRIVIAL)	✓ (TRIVIAL)
4 bends	✓			✓ (TRIVIAL)	✓	✓
5 bends	✓		✓ (TRIVIAL)	✓	✓	✓
other						

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

2D	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	✗ (CYCLES)	✗	✗	✗ (TRIVIAL)	✗ (TRIVIAL)
1 bend	✓	✗	✗ (U-TURN)	✗ (U-TURN)		
2 bends	✓	✗				
3 bends	✓	✗			✓ (TRIVIAL)	✓ (TRIVIAL)
4 bends	✓	✗		✓ (TRIVIAL)	✓	✓
5 bends	✓	✗	✓ (TRIVIAL)	✓	✓	✓
other						

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OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

2D	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	✗ (CYCLES)	✗	✗	✗ (TRIVIAL)	✗ (TRIVIAL)
1 bend	✓	✗	✗ (U-TURN)	✗ (U-TURN)		
2 bends	✓	✗	✗ (U-TURN)			
3 bends	✓	✗			✓ (TRIVIAL)	✓ (TRIVIAL)
4 bends	✓	✗		✓ (TRIVIAL)	✓	✓
5 bends	✓	✗	✓ (TRIVIAL)	✓	✓	✓
other						

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS



2D	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	✗ (CYCLES)	✗	✗	✗ (TRIVIAL)	✗ (TRIVIAL)
1 bend	✓	✗	✗ (U-TURN)	✗ (U-TURN)		
2 bends	✓	✗	✗ (U-TURN)	✓ [PAPAKOSTAS & TOLLIS, 1998]		
3 bends	✓	✗		✓	✓ (TRIVIAL)	✓ (TRIVIAL)
4 bends	✓	✗		✓ (TRIVIAL)	✓	✓
5 bends	✓	✗	✓ (TRIVIAL)	✓	✓	✓
other						

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

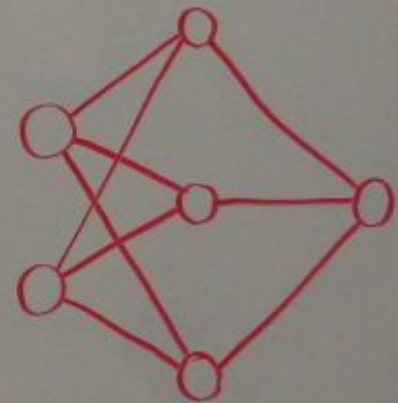
2D	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	✗ (CYCLES)	✗	✗	✗ (TRIVIAL)	✗ (TRIVIAL)
1 bend	✓	✗	✗ (U-TURN)	✗ (U-TURN)		
2 bends	✓	✗	✗ (U-TURN)	✓ [PAPAKOSTAS & TOLLIS, 1998]		
3 bends	✓	✗		✓	✓ (TRIVIAL)	✓ (TRIVIAL)
4 bends	✓	✗		✓ (TRIVIAL)	✓	✓
5 bends	✓	✗	✓ (TRIVIAL)	✓	✓	✓
other		0 BENDS $\alpha=60^\circ$ ✓ (TRIVIAL)	0 BENDS $\alpha=30^\circ$ ✓ [KANT, 1993]			3.5n TOTAL BENDS ✓ [AZIZA & BIEDL, 2004]

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WHY STAY IN THE  
PLANE?

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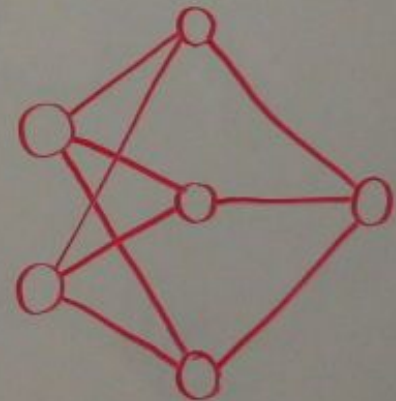
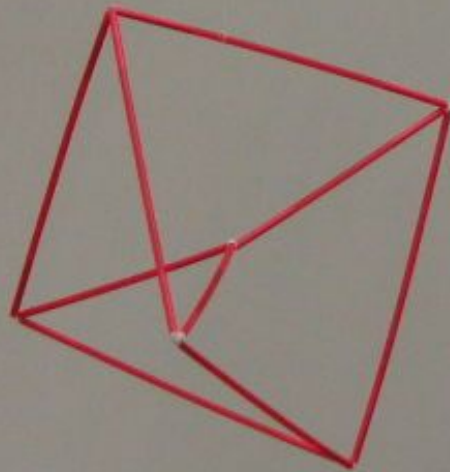
# 3D GRAPH DRAWINGS



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

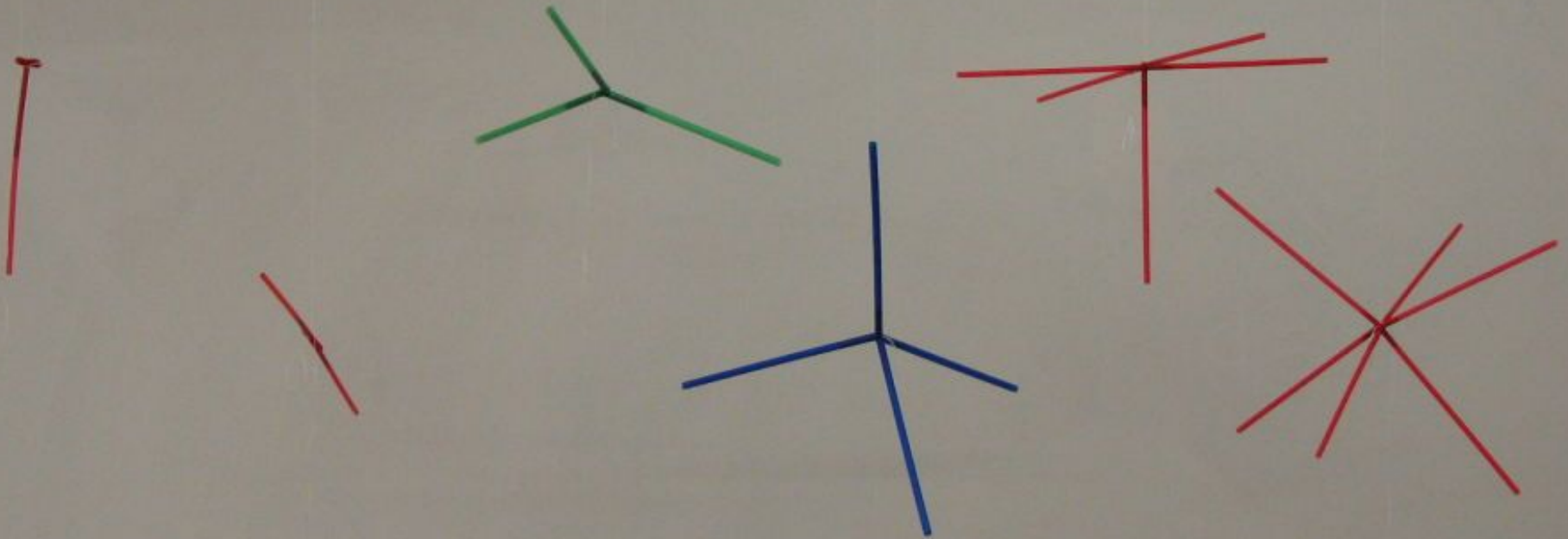
# 3D GRAPH DRAWINGS



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# ANGULAR RESOLUTION



$d=1$

$d=2$

$d=3$

$d=4$

$d=5$

$d=6$

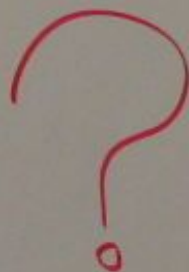
EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

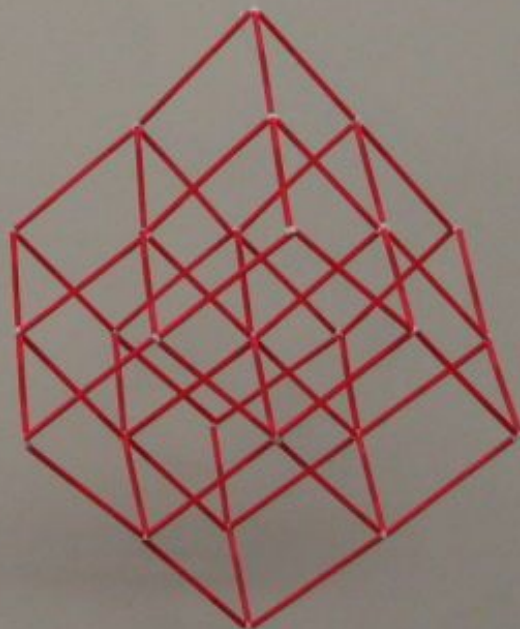
# ON A GRID



$d=1,2$



$d=3,4$

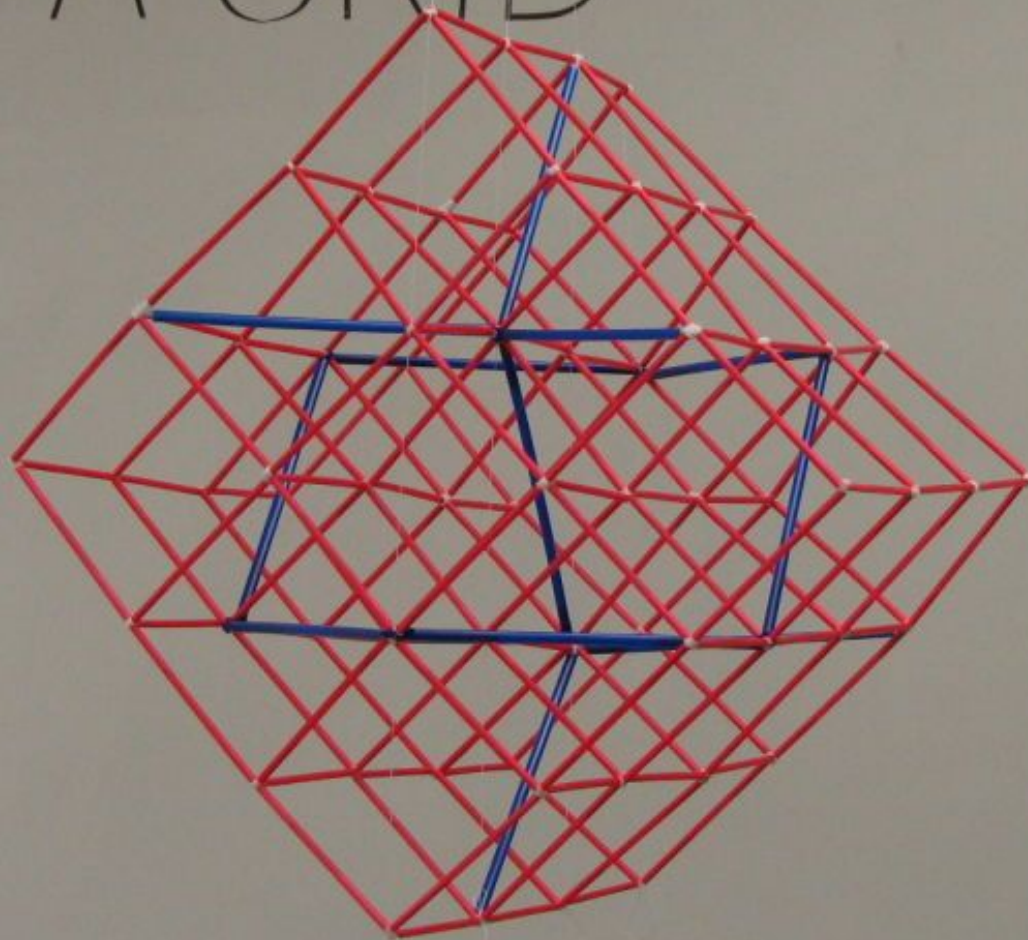


$d=5,6$

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OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# ON A GRID



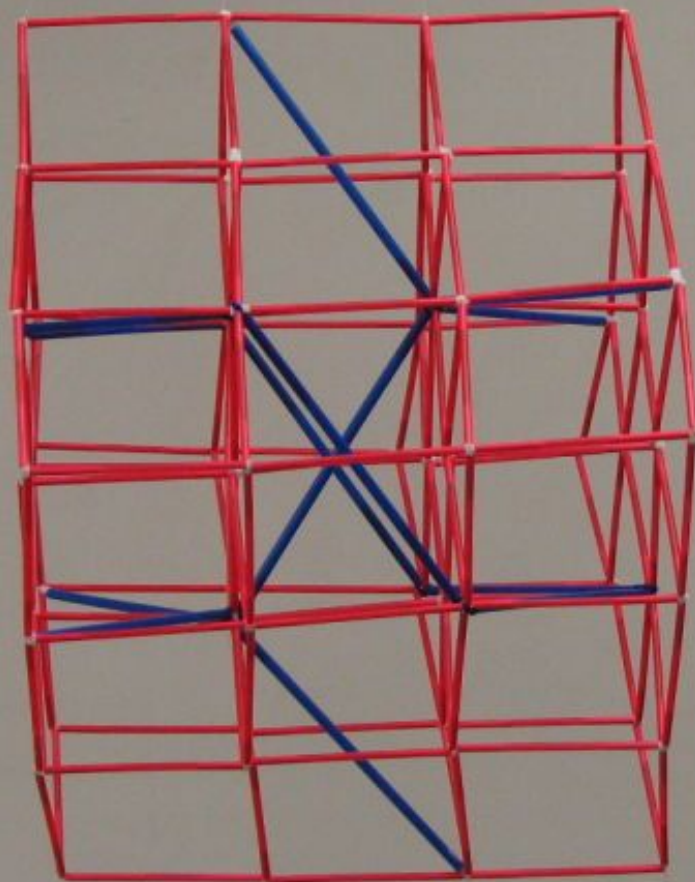
$$d=4$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS



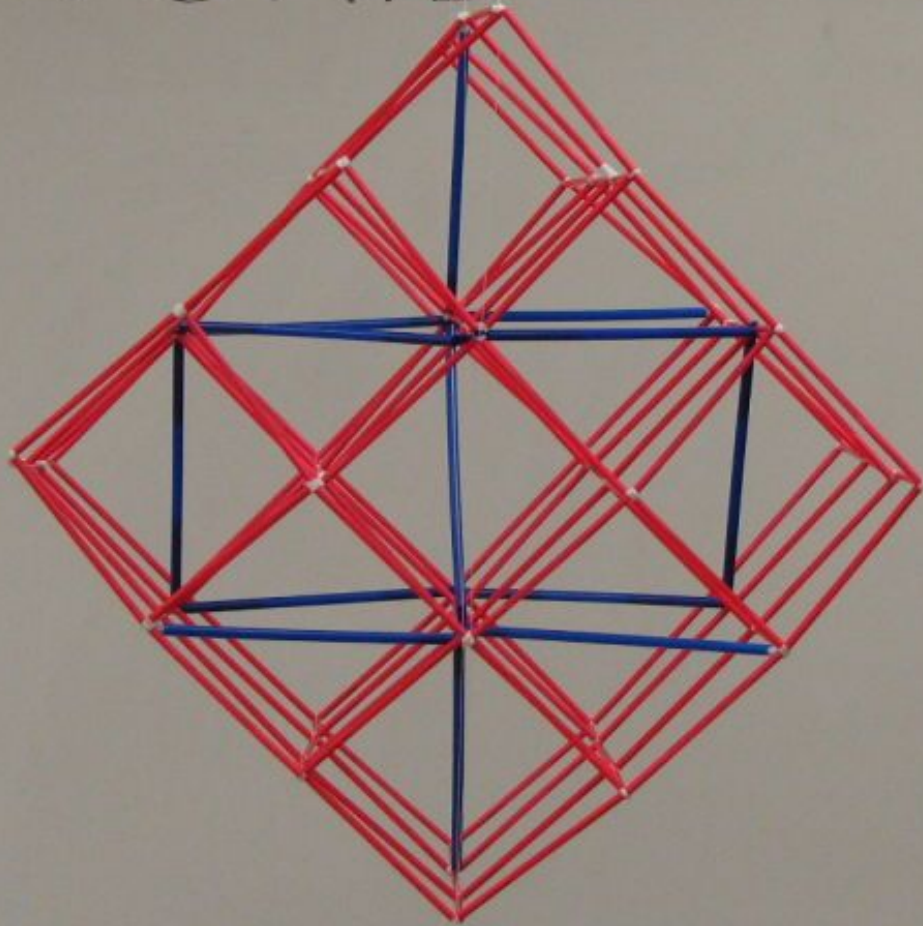
# ON A GRID



$$d=4$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

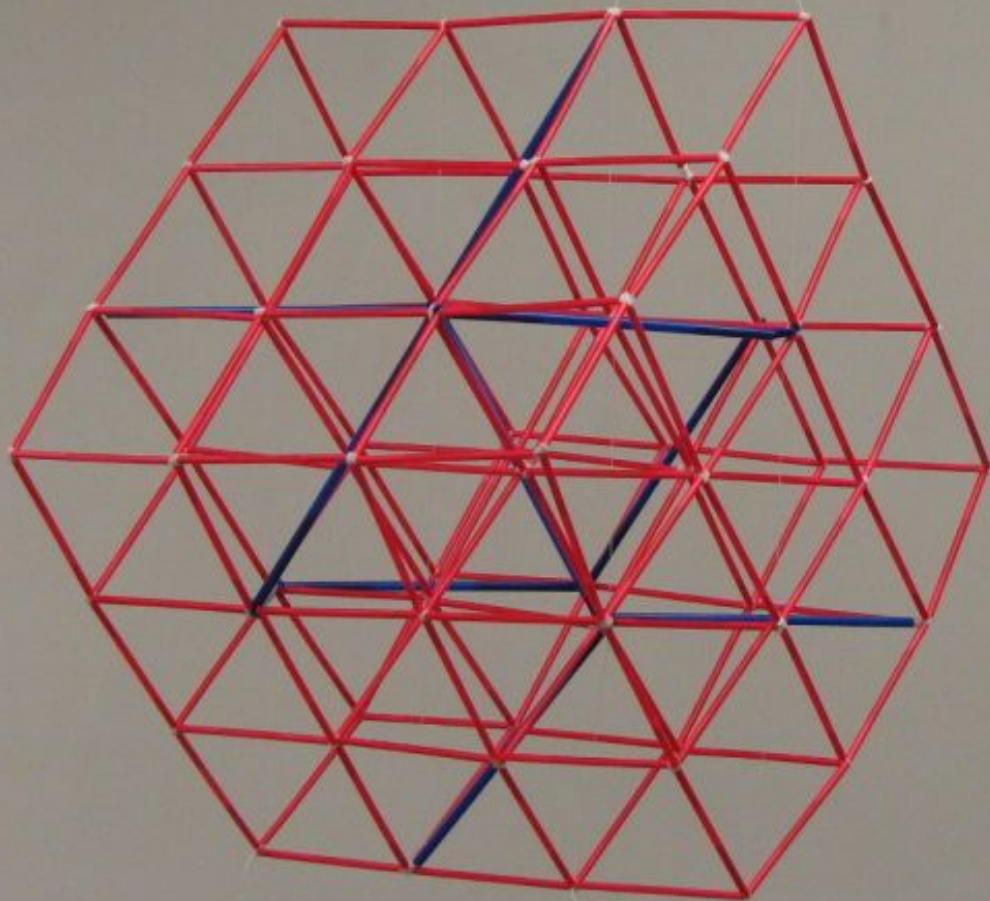
# ON A GRID



$$d=4$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

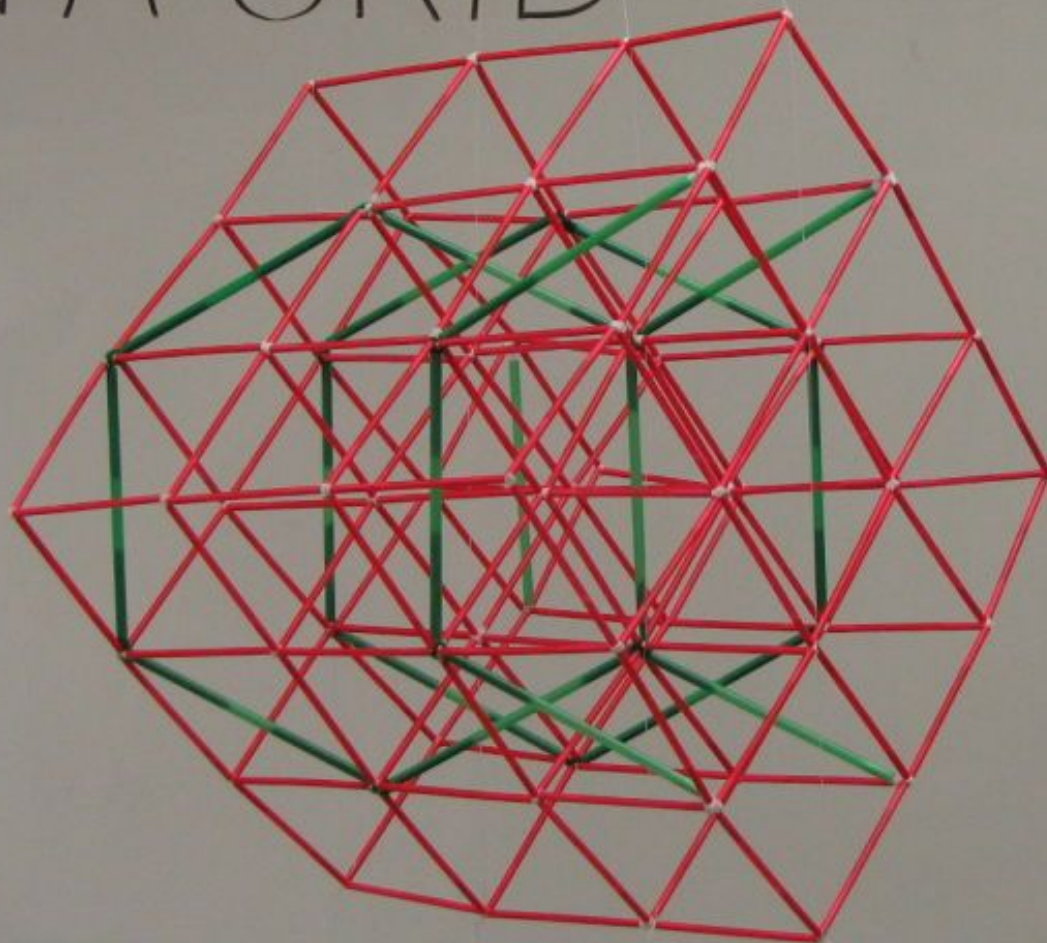
# ON A GRID



$$d=4$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

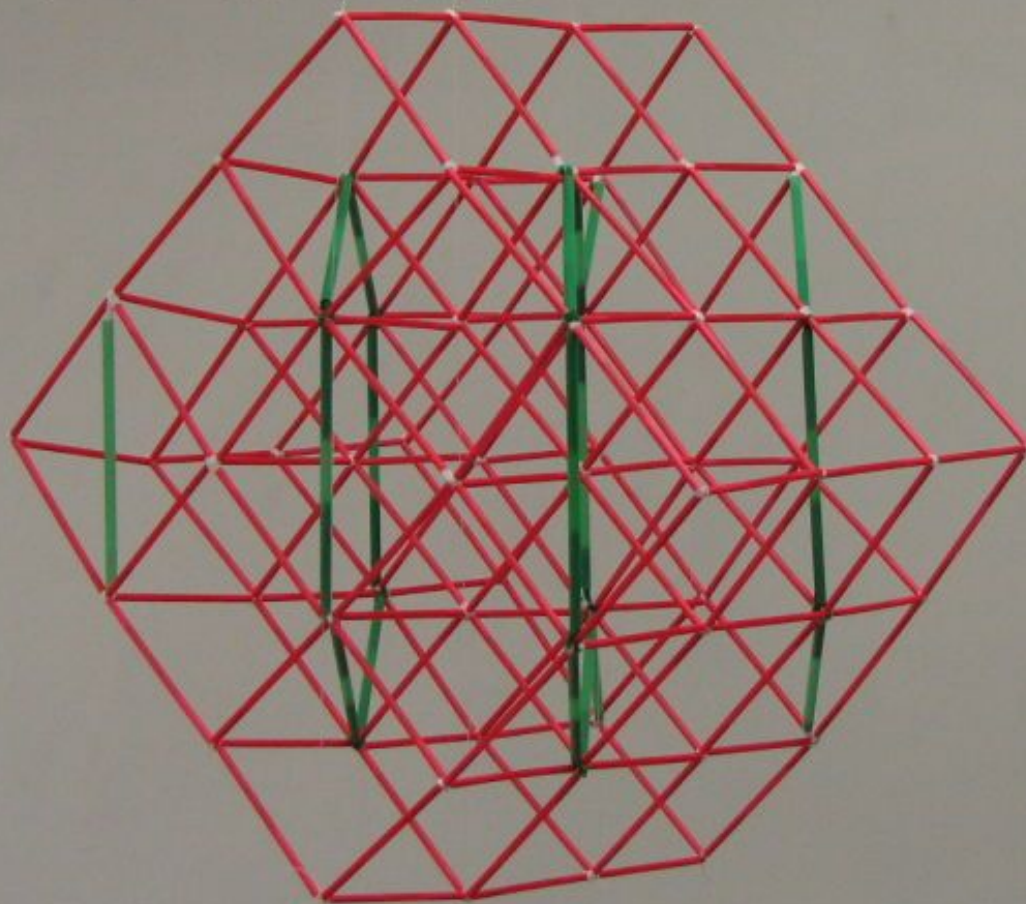
# ON A GRID



$$d=3$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# ON A GRID

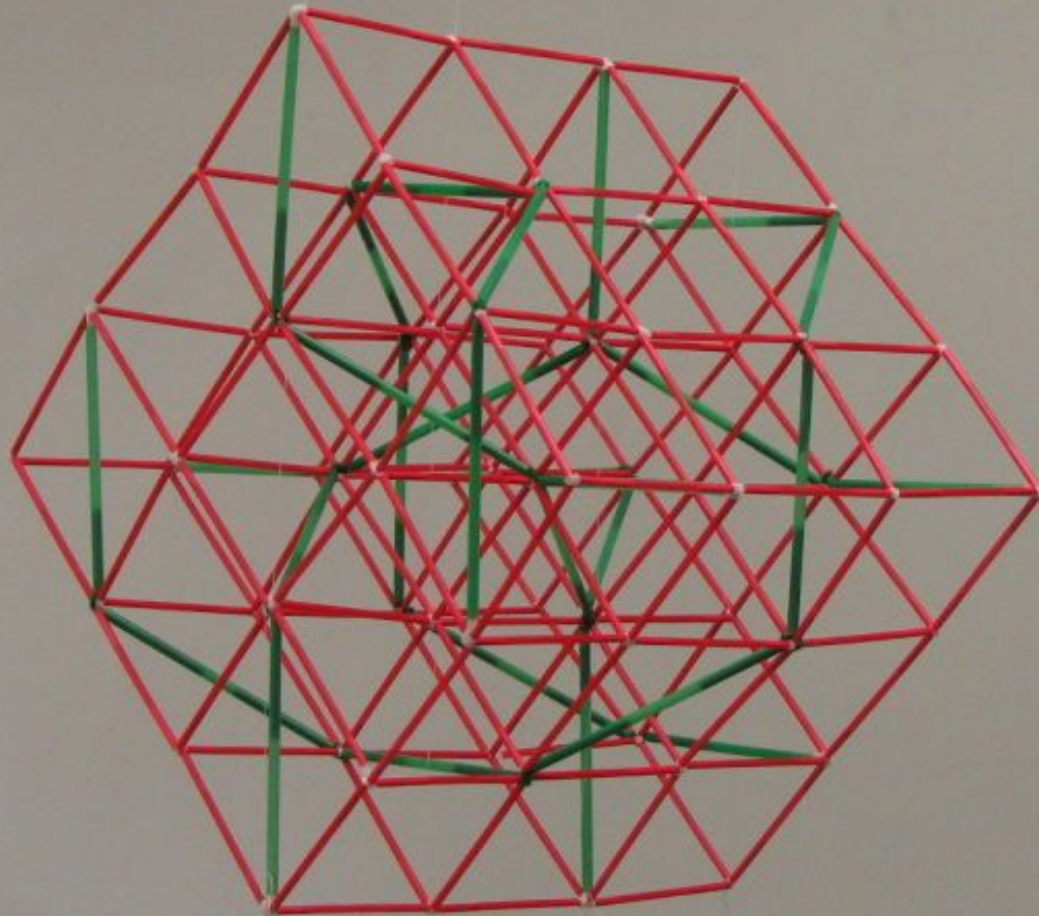


$$d=3$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

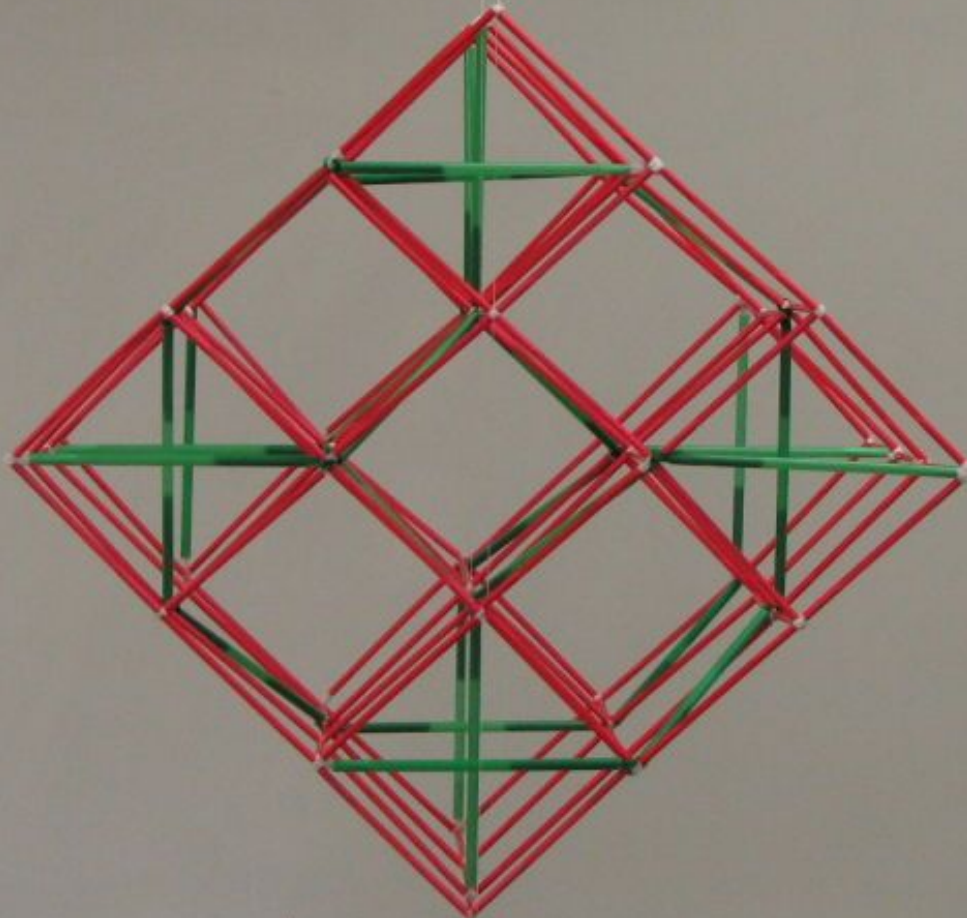
# ON A GRID



$$d=3$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

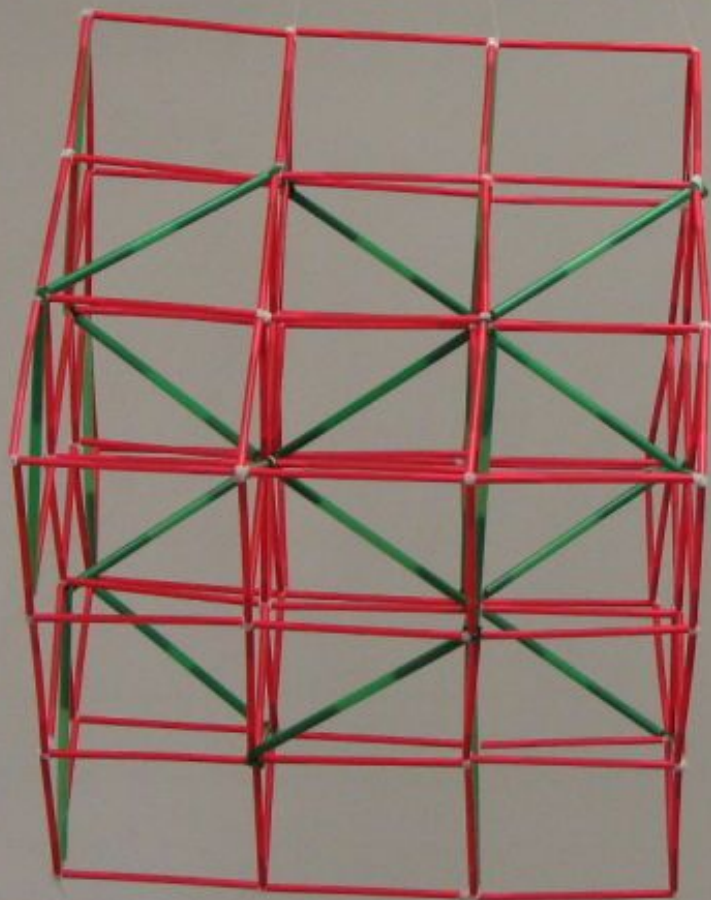
# ON A GRID



$$d=3$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# ON A GRID



$$d=3$$

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS



3D

$d=1$

$d=2$

$d=3$

$d=4$

$d=5$

$d=6$

0 bends

1 bend

2 bends

3 bends

4 bends

5 bends

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

<b>3D</b>	$d=1$	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
0 bends	✓ (TRIVIAL)					
1 bend	✓					
2 bends	✓					
3 bends	✓					
4 bends	✓				✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓		✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

3D	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	X (CYCLES)	X	X	X (TRIVIAL)	X
1 bend	✓	X	X (U-TURN)	X (U-TURN)		X (U-TURN)
2 bends	✓	X				
3 bends	✓	X				
4 bends	✓	X			✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓	X	✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# 3D

	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	✗ (CYCLES)	✗	✗	✗ (TRIVIAL)	✗
1 bend	✓	✗	✗ (U-TURN)	✗ (U-TURN)	✗ (U-TURN)	✗ (U-TURN)
2 bends	✓	✗	✗ (U-TURN)	✗ (U-TURN)		
3 bends	✓	✗				
4 bends	✓	✗			✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓	✗	✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

3D	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	X (CYCLES)	X	X	X (TRIVIAL)	X
1 bend	✓	X	X (U-TURN)	X (U-TURN)	X (U-TURN)	X (U-TURN)
2 bends	✓	X	X (U-TURN)	X (U-TURN)	✓ [WOOD, 2003]	
3 bends	✓	X			✓	✓ [EADES ET AL, 2000]
4 bends	✓	X			✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓	X	✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

<b>3D</b>	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	X (CYCLES)	X	X	X (TRIVIAL)	X
1 bend	✓	X	X (U-TURN)	X (U-TURN)	X (U-TURN)	X (U-TURN)
2 bends	✓	X	✓ X (U-TURN)	X (U-TURN)	✓ [WOOD, 2003]	
3 bends	✓	X	✓	✓	✓	✓ [EADES ET AL 2000]
4 bends	✓	X	✓	✓	✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓	X	✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

<b>3D</b>	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	X (CYCLES)	X	X	X (TRIVIAL)	X
1 bend	✓	X	X (U-TURN)	X (U-TURN)	X (U-TURN)	X (U-TURN)
2 bends	✓	X	✓ X (U-TURN)	X (U-TURN)	✓ [WOOD, 2003]	
3 bends	✓	X	✓	⊙ ✓	✓	✓ [EADES ET AL, 2000]
4 bends	✓	X	✓	✓	✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓	X	✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

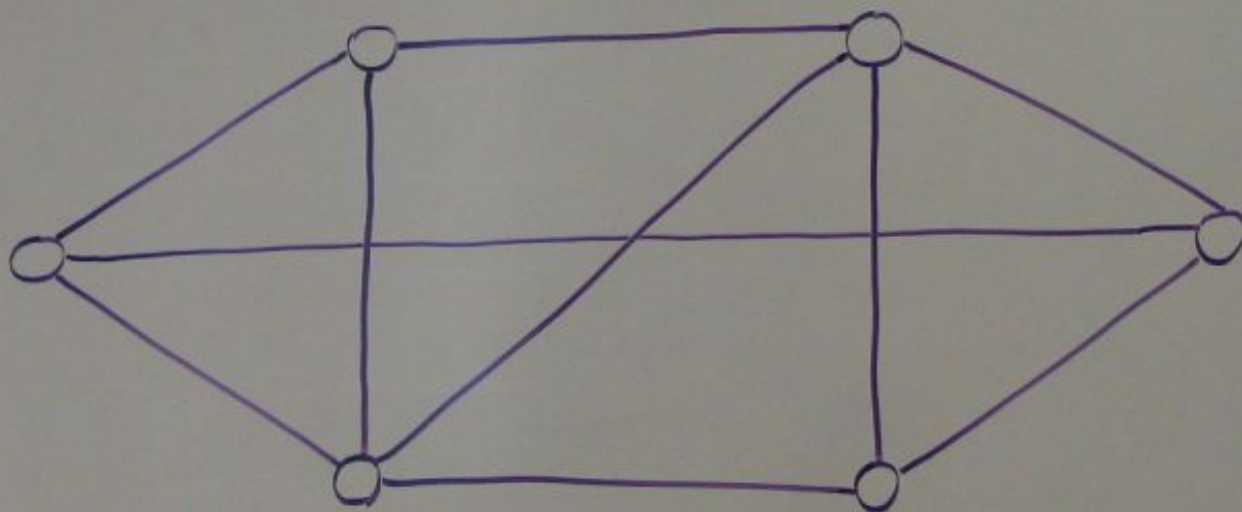
EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# 3-BEND DRAWINGS OF DEGREE-4 GRAPHS

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG    OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS



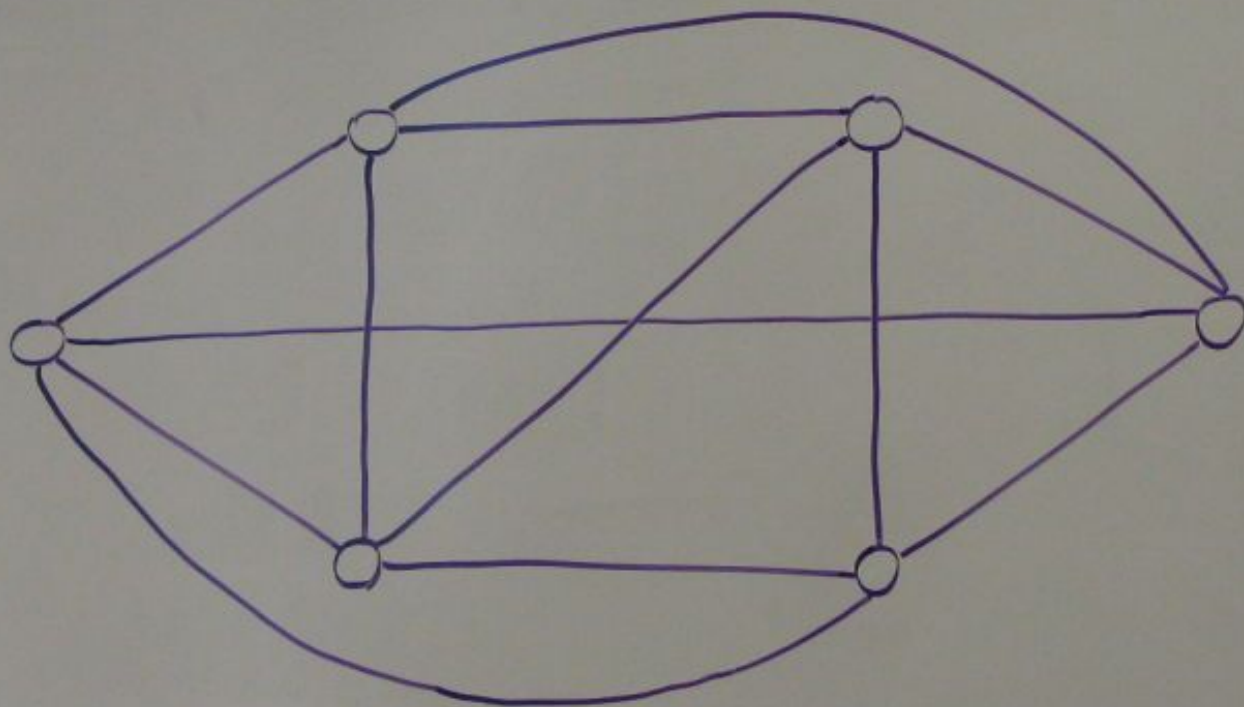
# TAKE A GRAPH



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

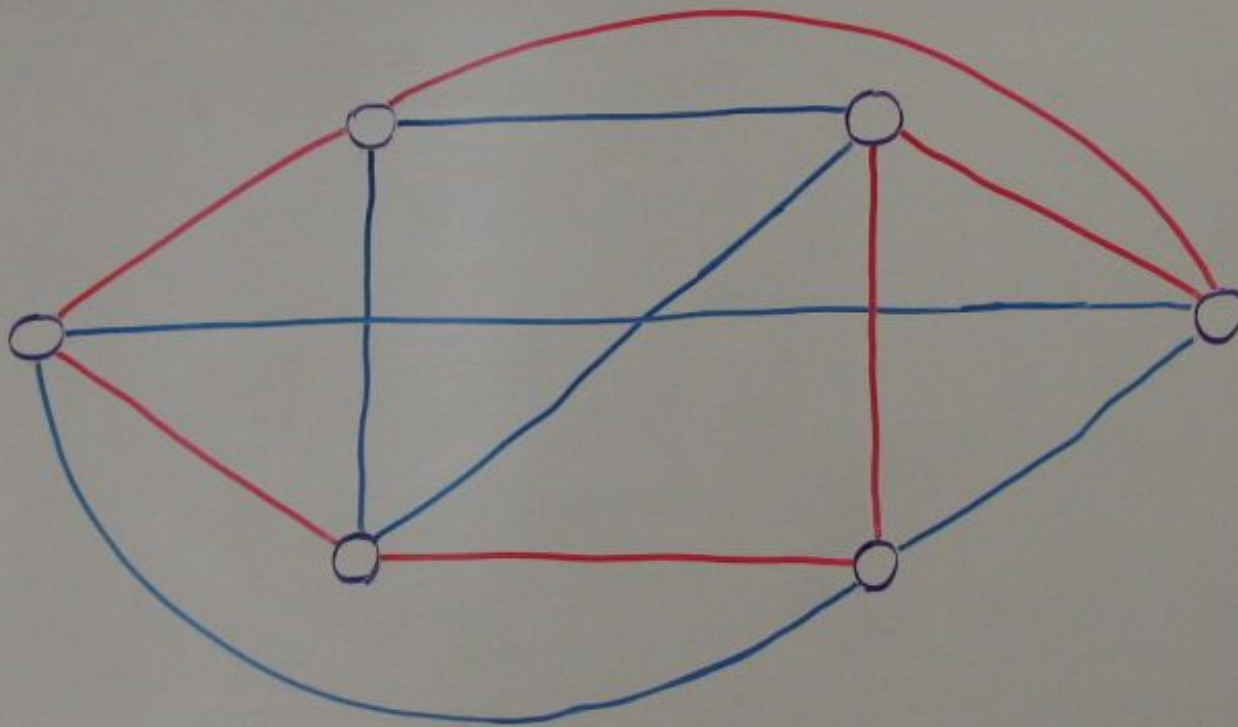
# MAKE 4-REGULAR



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

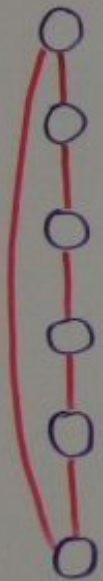
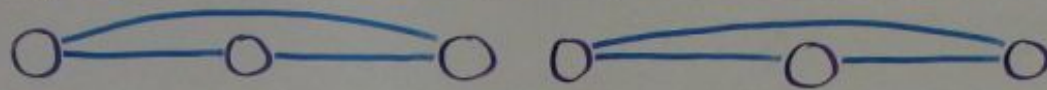
# COLOUR THE EDGES



EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG

OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

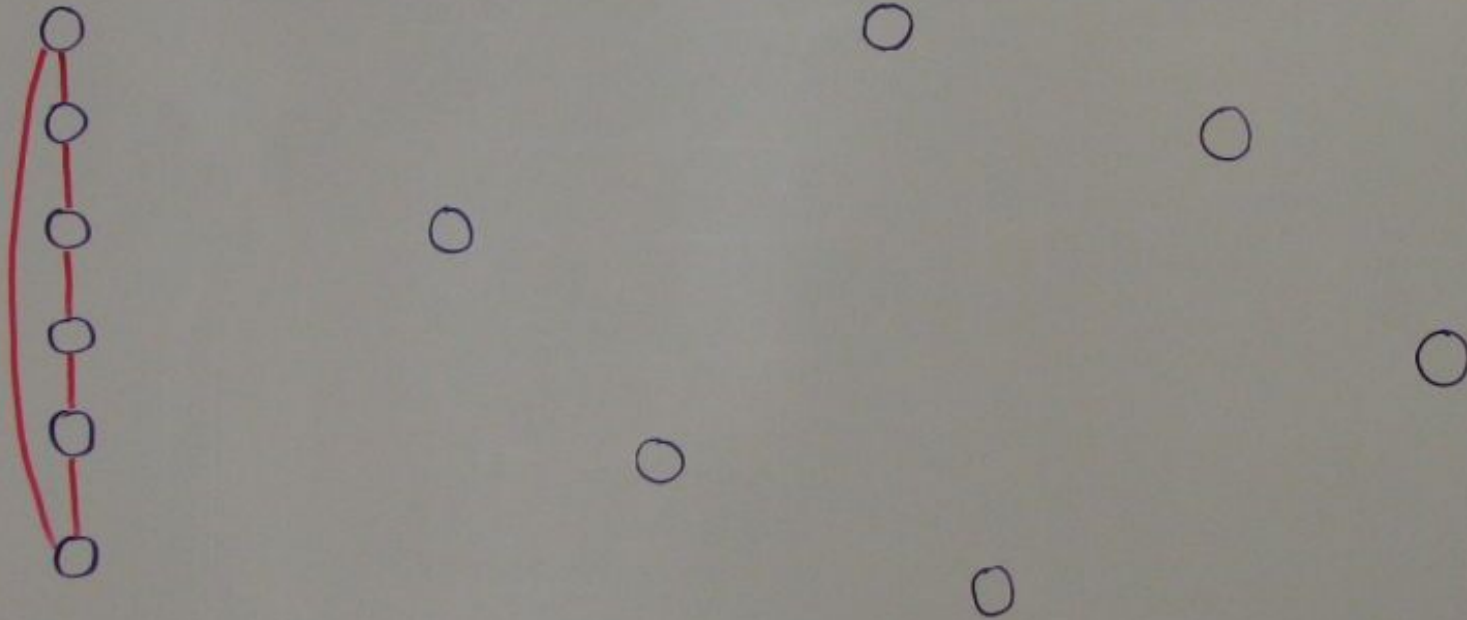
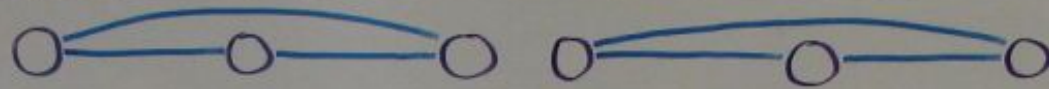
# PLACE VERTICES



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OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

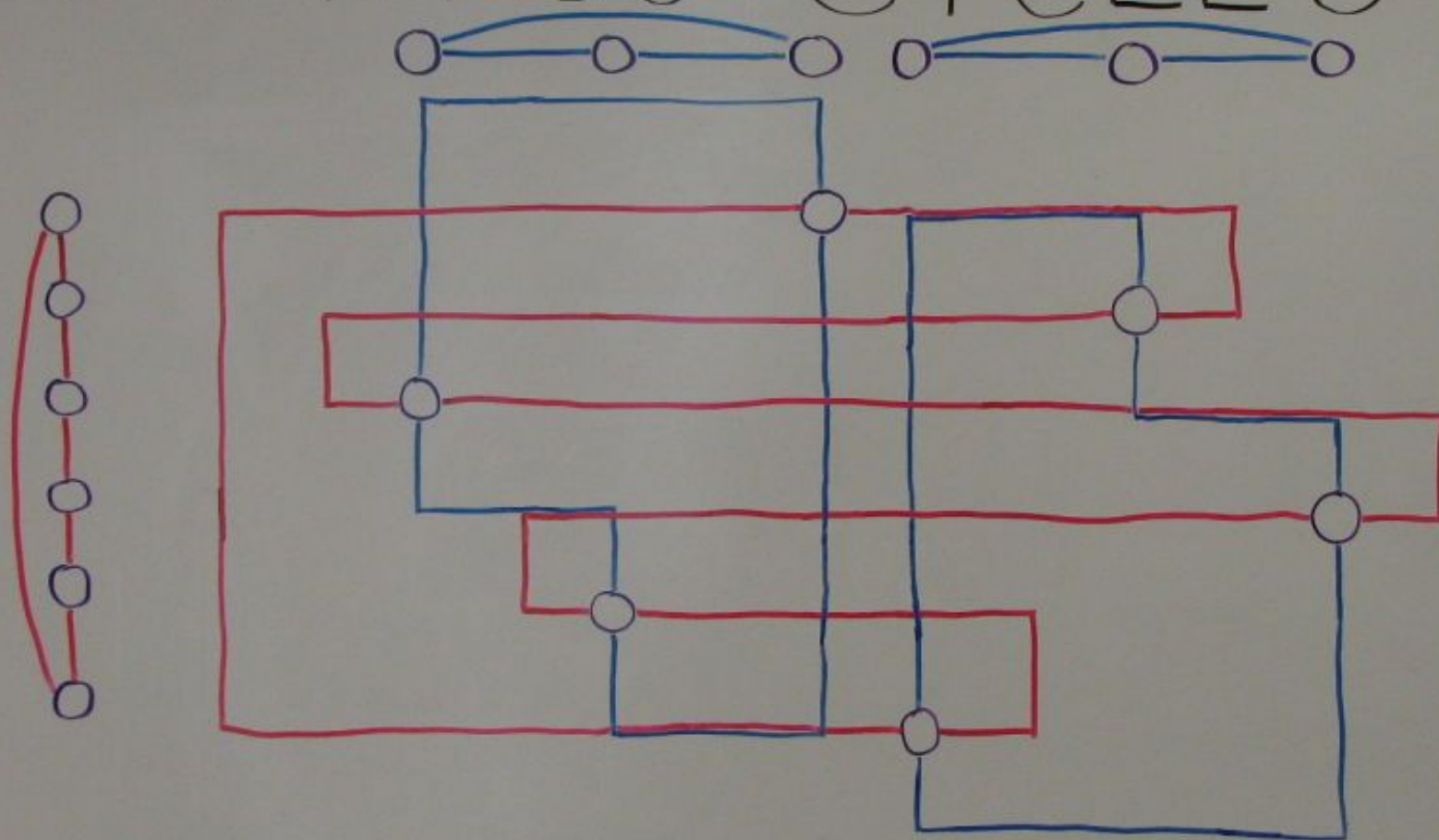
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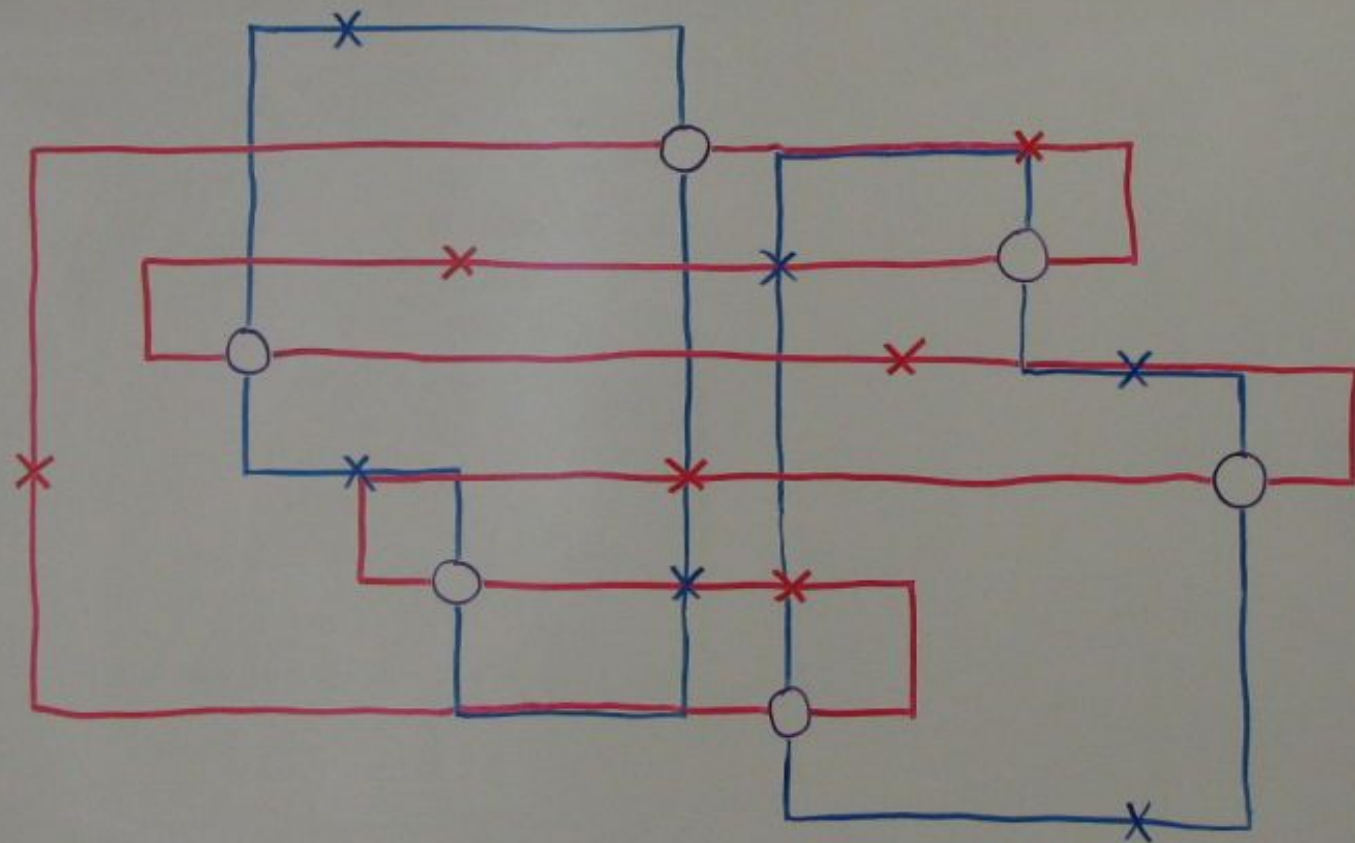
# DRAW $90^\circ$ CYCLES



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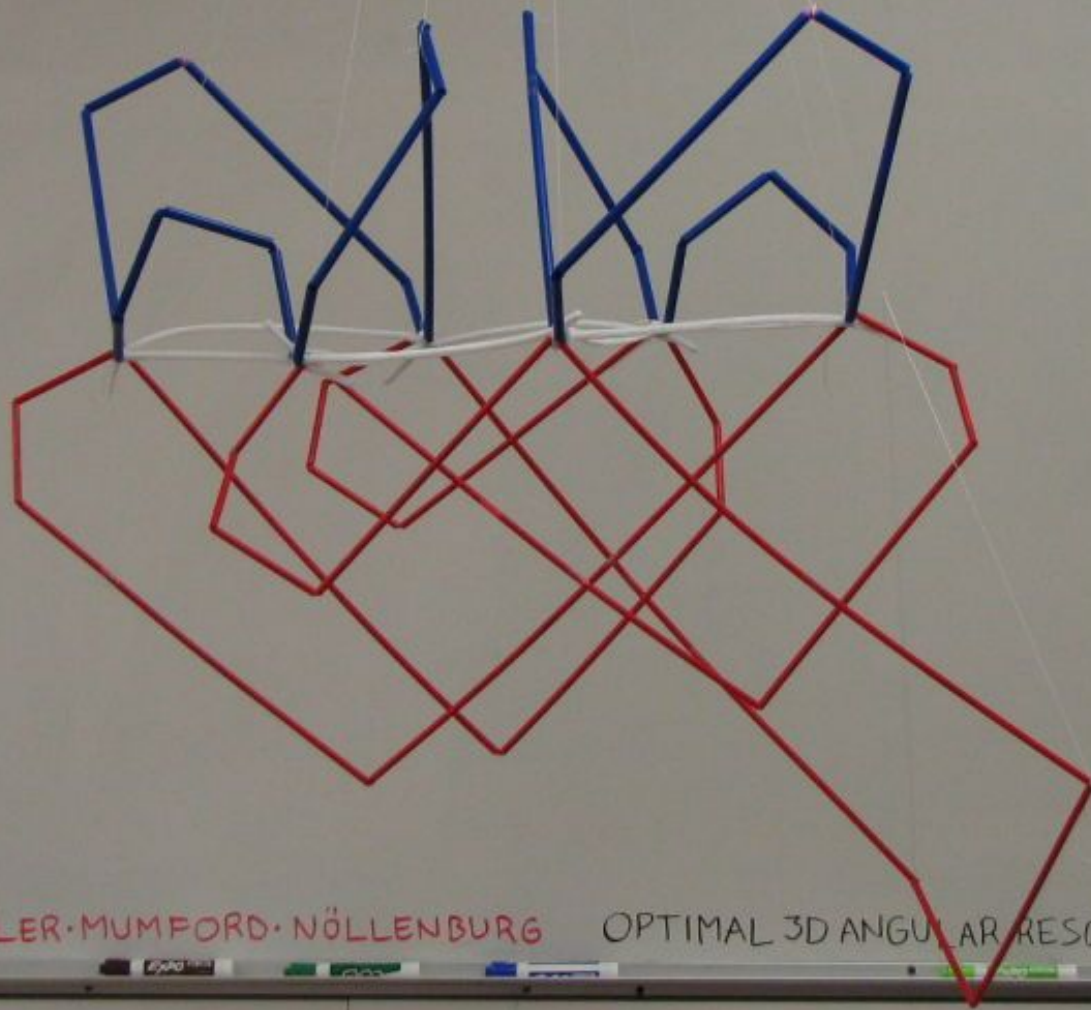
# DRAW $90^\circ$ CYCLES



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# PULL TO 3D

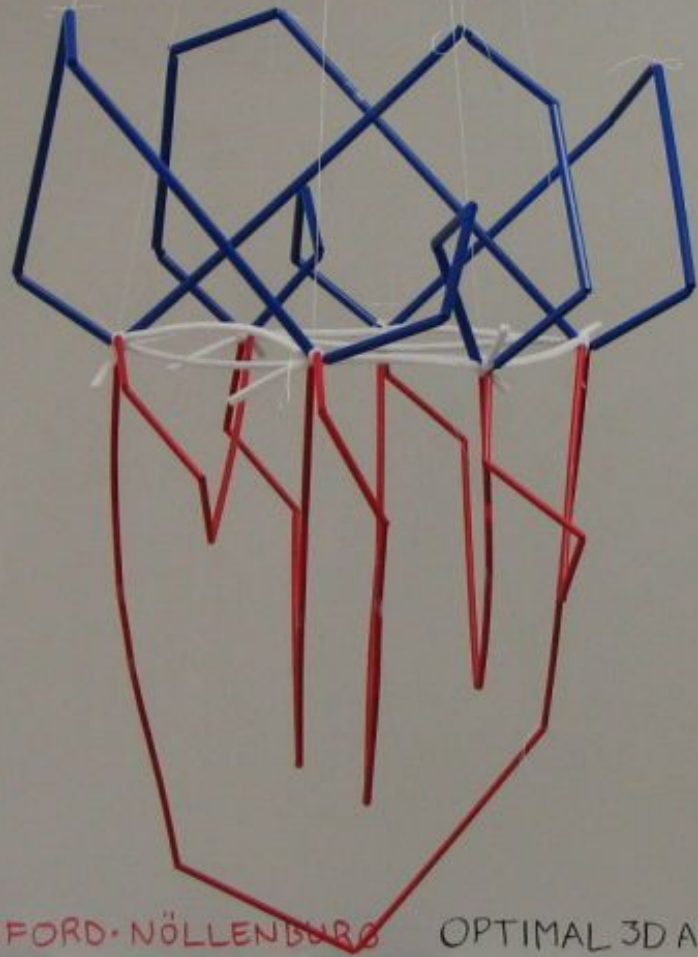


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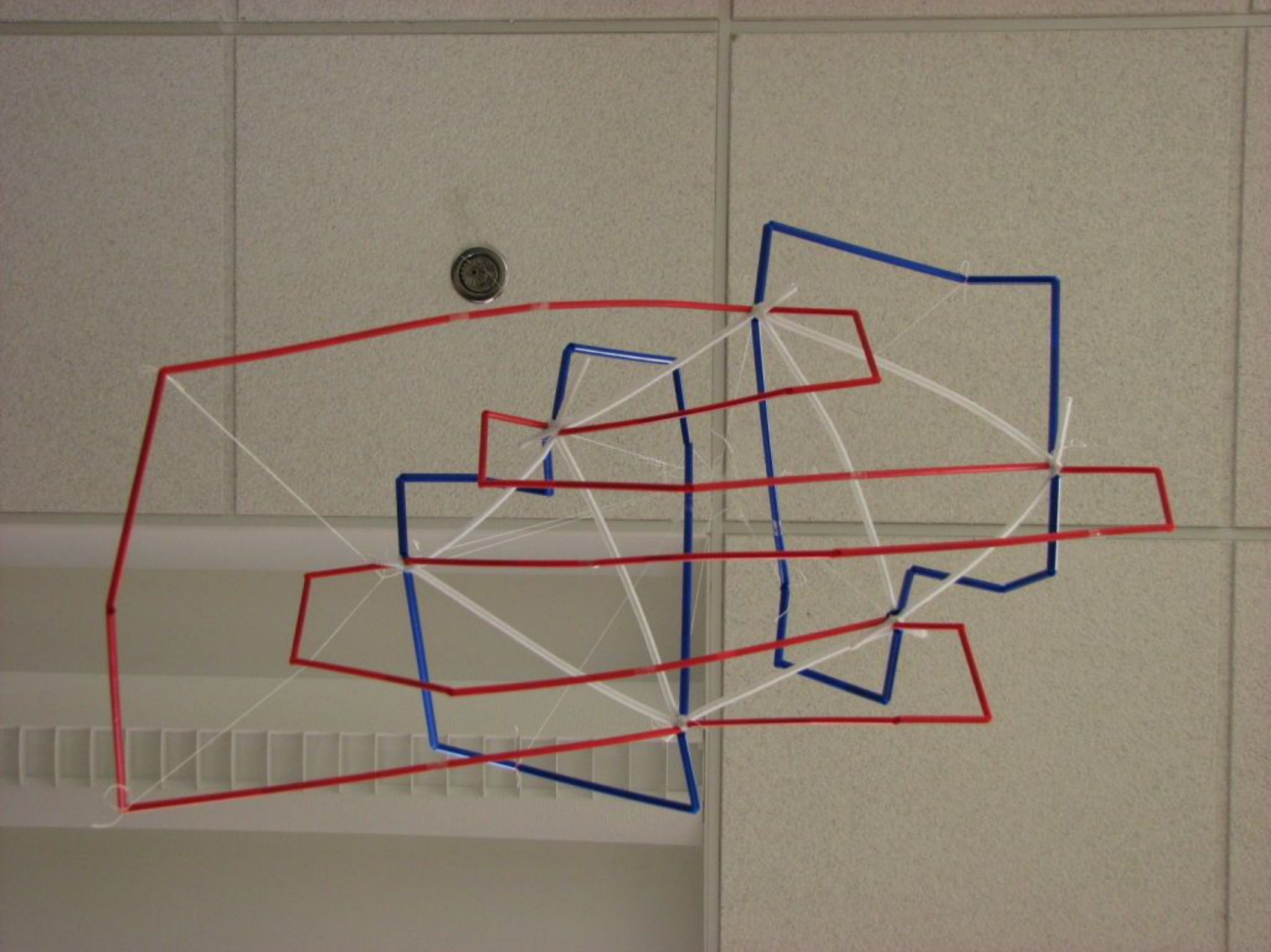


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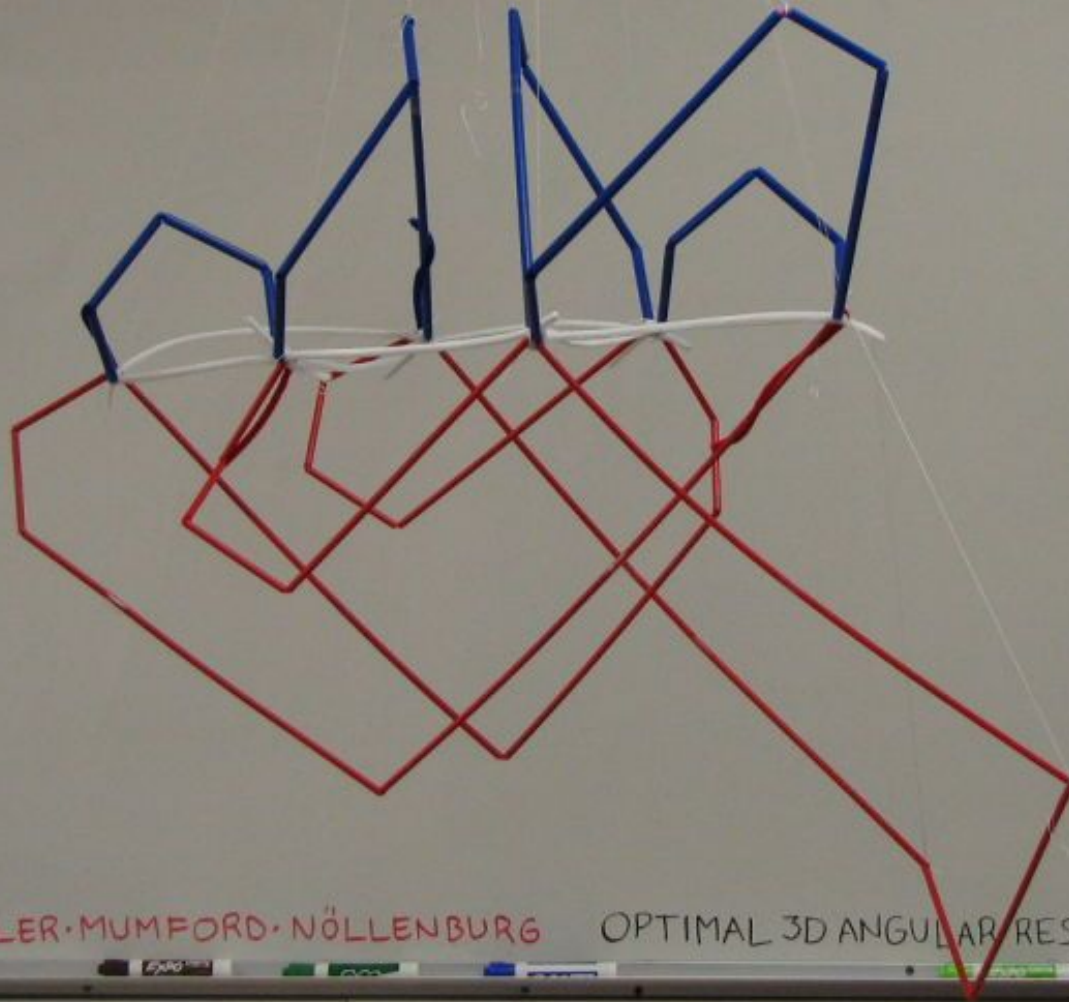


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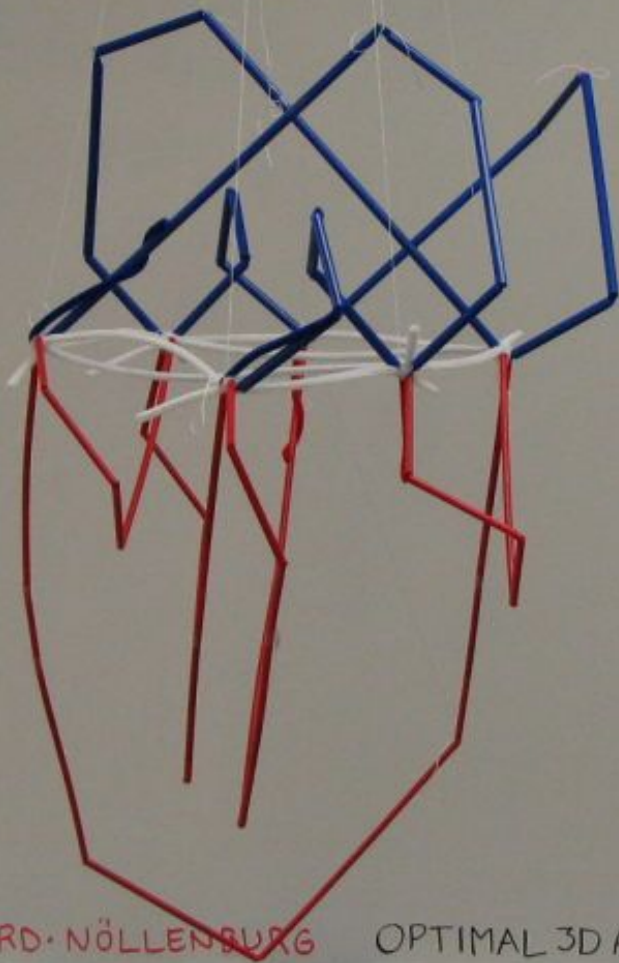
# REMOVE EXTRA EDGES



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OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

# SUMMARY

EPPSTEIN · LÖFFLER · MUMFORD · NÖLLENBURG OPTIMAL 3D ANGULAR RESOLUTION FOR LOW DEGREE GRAPHS

<b>3D</b>	d=1	d=2	d=3	d=4	d=5	d=6
0 bends	✓ (TRIVIAL)	✗ (CYCLES)	✗	✗	✗ (TRIVIAL)	✗
1 bend	✓	✗	✗ (U-TURN)	✗ (U-TURN)	⊙ (U-TURN)	✗ (U-TURN)
2 bends	✓	✗	✓ ⊙ (U-TURN)	⊙ (U-TURN)	✓ [WOOD, 2003]	⊙
3 bends	✓	✗	✓	✓	✓	✓ [EADES ET AL, 2000]
4 bends	✓	✗	✓	✓	✓ (TRIVIAL)	✓ (TRIVIAL)
5 bends	✓	✗	✓ (TRIVIAL)	✓ (TRIVIAL)	✓	✓

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# THANK YOU



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