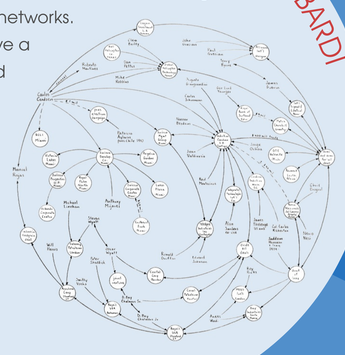
PLANAR LOMBARDI DRAWINGS OF OUTERPATHS

Maarten Löffler Martin Nöllenburg
Utrecht University Karlsruhe Institute of Technology

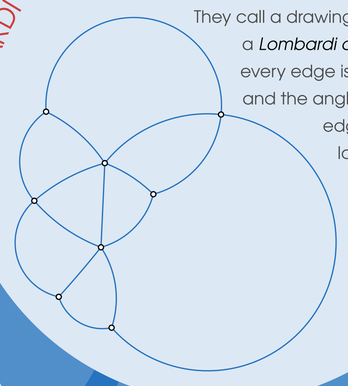
MARK LOMBARDI

Mark Lombardi (1951-2000) was an American artist, known for his drawings of conspiracy networks. His drawings have a distinctive, stylized look.



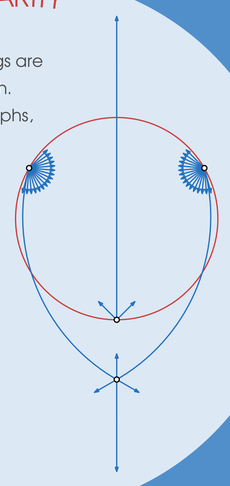
LOMBARDI DRAWINGS

In 2010, Duncan *et al.* suggested a formalization of Lombardi's drawing style. They call a drawing of a graph a ***Lombardi drawing*** when every edge is a circular arc, and the angles at which the edges meet are as large as possible.



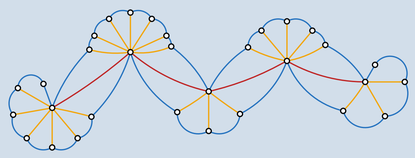
PLANARITY

Lombardi drawings are defined for any graph. However, for ***planar*** graphs, we are particularly interested in ***planar*** Lombardi drawings. In 2011, Duncan *et al.* showed that some planar graphs have no planar Lombardi drawings.



ALGORITHM MAIN IDEA

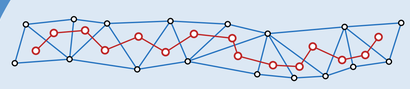
It can be shown that we may assume the outerpath is triangulated.
We subdivide the edges into three groups:
– the ***hull*** edges are indicent to the outer face;
– the ***spine*** edges connect all vertices of degree at least 4;
– the remaining edges form ***flowers*** around the spine vertices.



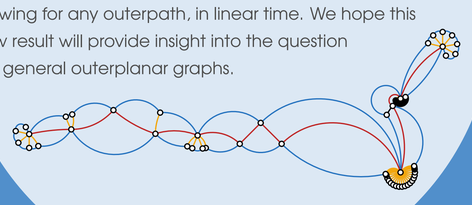
We would like to place the spine vertices on parallel vertical lines, and draw the flowers symmetrically. Usually, this works.

OUTERPATHS

An ***outerpath*** is a graph whose weak dual is a path.

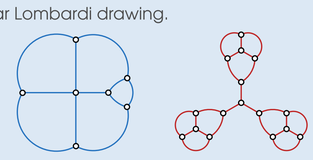


We report an algorithm that produces a planar Lombardi drawing for any outerpath, in linear time. We hope this new result will provide insight into the question for general outerplanar graphs.



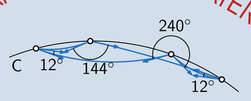
KNOWN RESULTS

Since not every planar graph has a planar Lombardi drawing, we wish to classify the class of graphs that do.
In 2010, Duncan *et al.* showed that every tree has a planar Lombardi drawing.

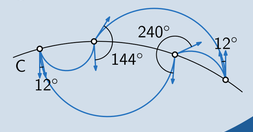


The same is true for Halin graphs.
Eppstein shows in 2012 that subcubic graphs also have planar Lombardi drawings. One class for which existence remains still unknown are the *outerplanar* graphs.

PLANAR ARC QUADRILATERALS

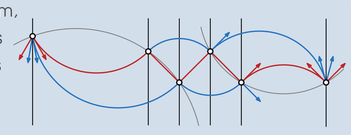


Consider quadrilaterals that consist of four circular arcs. Let $\alpha, \beta, \gamma, \delta$ denote the angles between the arcs. If $\alpha + \gamma - \beta - \delta = 360°$, the vertices lie on a common circle C. To draw it planar two arcs need to be inside, and two outside C. Quadrilaterals with this property may occur in our algorithm: A vertex of degree 5 has a 144° angle between two spine edges. With a 240° flower angle this means the neighbouring vertices have angles that sum to 24°.



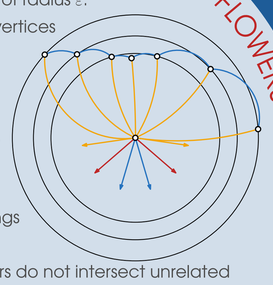
DEGREE 4 CASE

When there are vertices of degree 4 in the spine, the algorithm fails. The reason is that such vertices have no flowers, and there are direct hull edges connecting their neighbours. To solve this problem, we treat sequences of degree-4 vertices as single entities, which are drawn separate from the rest of the graph. If there is only one degree-4 vertex, it may not be possible to keep it symmetric, but it can always be drawn.

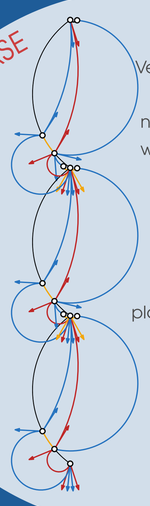


DRAWING THE FLOWERS

Finally, we draw the flowers inside small disks of radius ε. Each pair of petal vertices is drawn on a slightly smaller circle; their locations are uniquely determined by this circle. By keeping the circles sufficiently large, we can make sure that no crossings occur internally.
To make sure the flowers do not intersect unrelated parts of the drawing, we choose ε small enough.



DEGREE 5 CASE



Vertices of degree 5 can usually be drawn normally. The exception is when their neighbours both have very high degree, see "planar arc quadrilaterals". In this case, we switch to a vertical drawing scheme. The high-degree vertices are placed on a single vertical line, and the degree-5 vertices are placed slightly to the left of this line.

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