Delaunay Triangulations of Imprecise Points in Linear Time after Preprocessing

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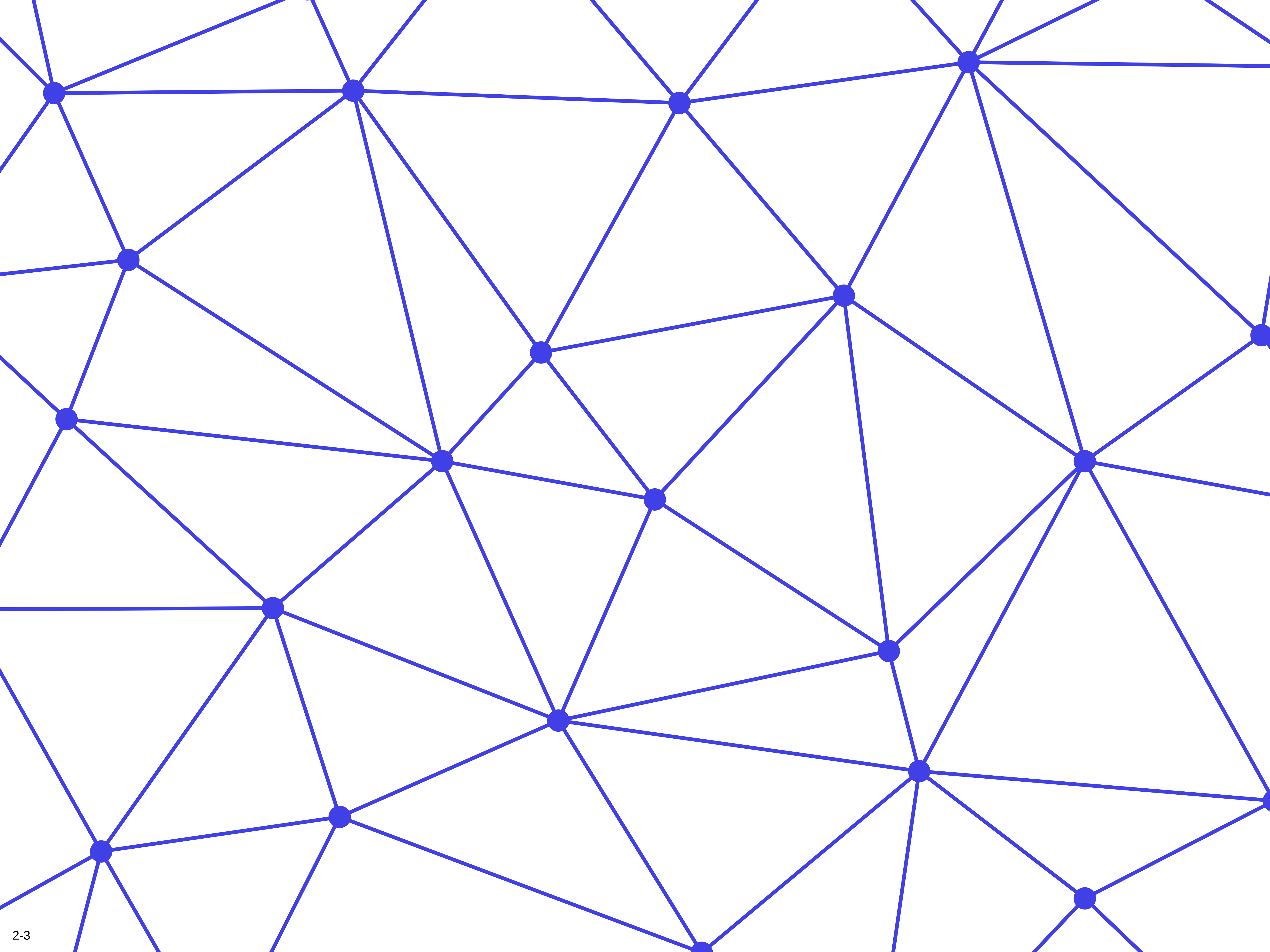
What were *Delaunay triangulations* again?



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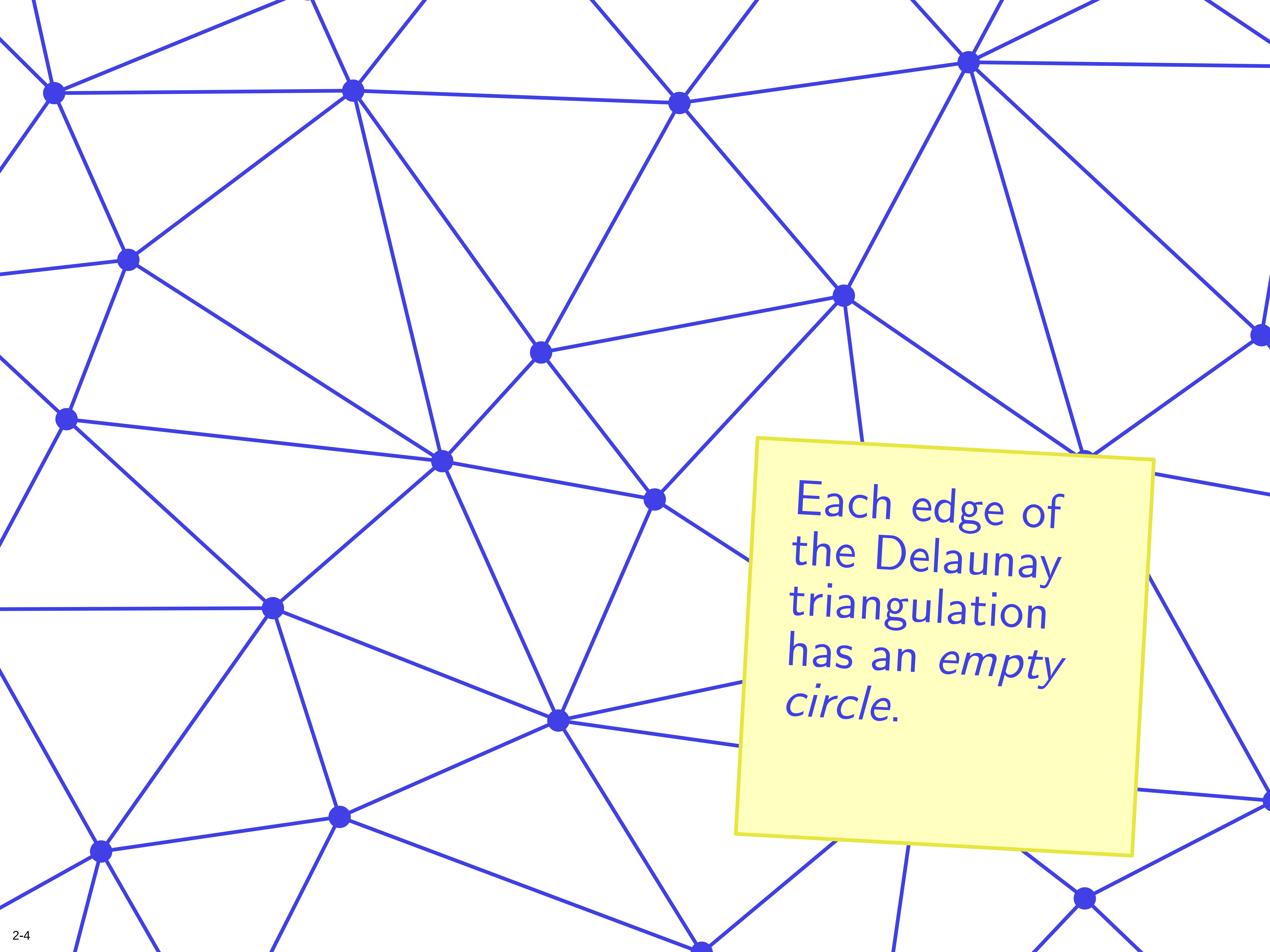
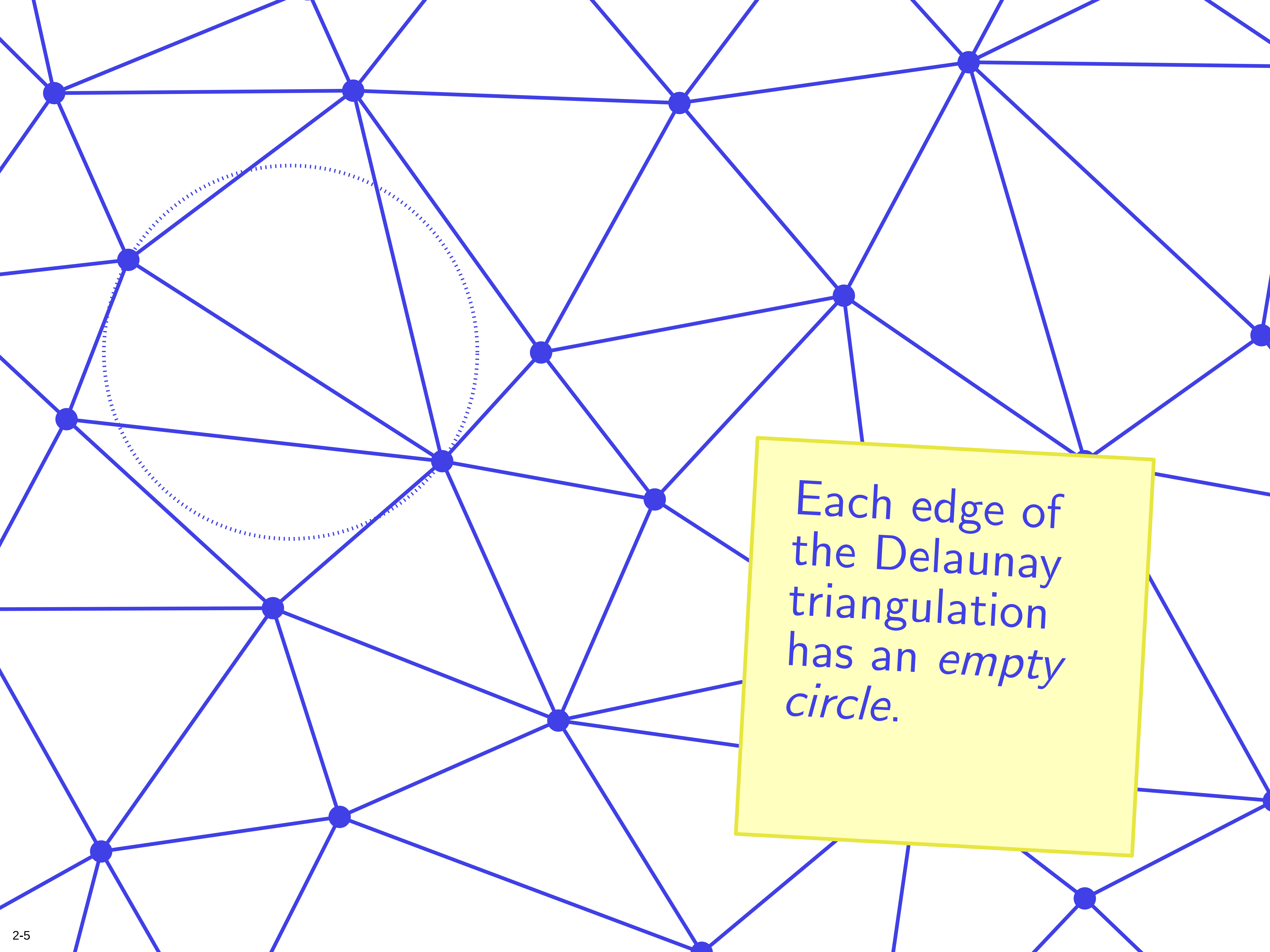


Each edge of the Delaunay triangulation has an *empty circle.*



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So, what exactly are *imprecise points*?

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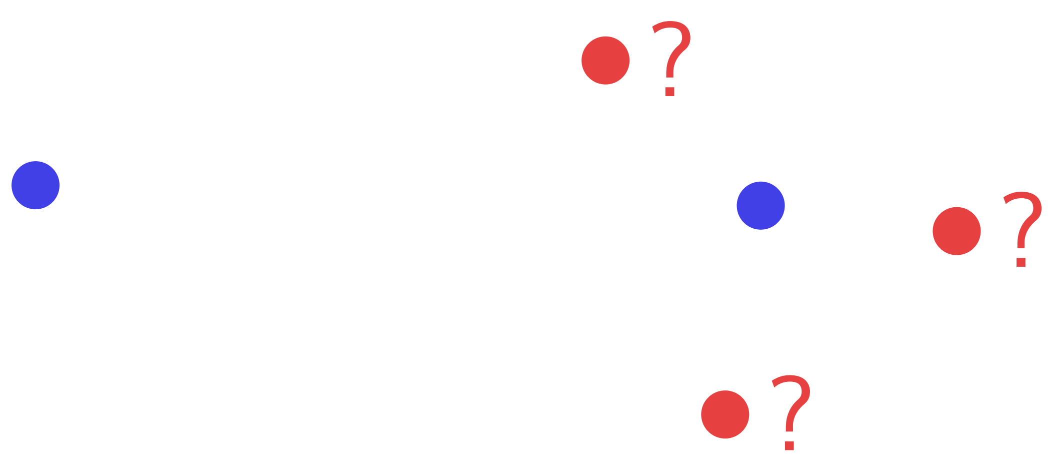

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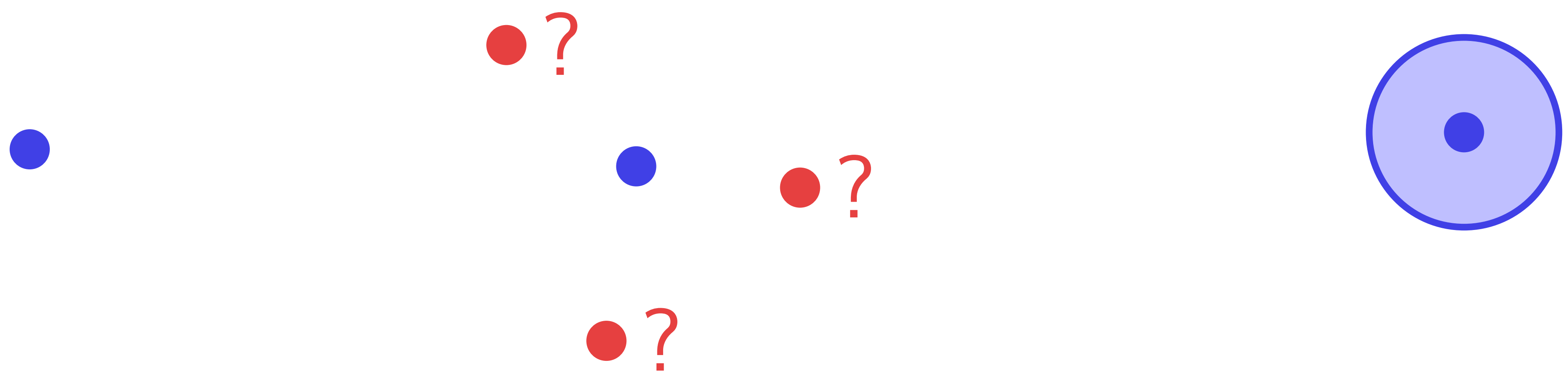

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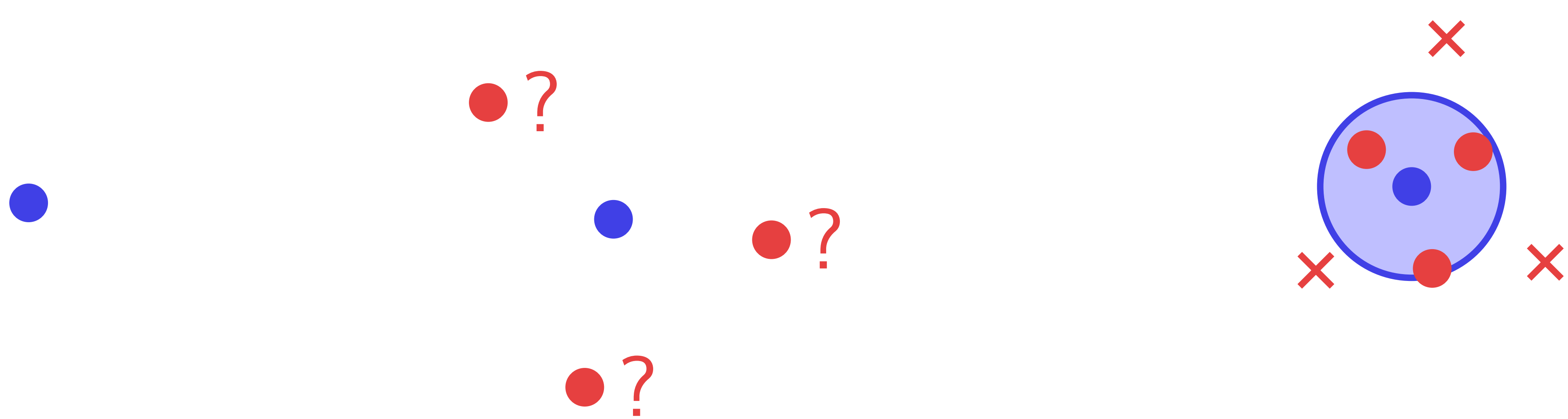
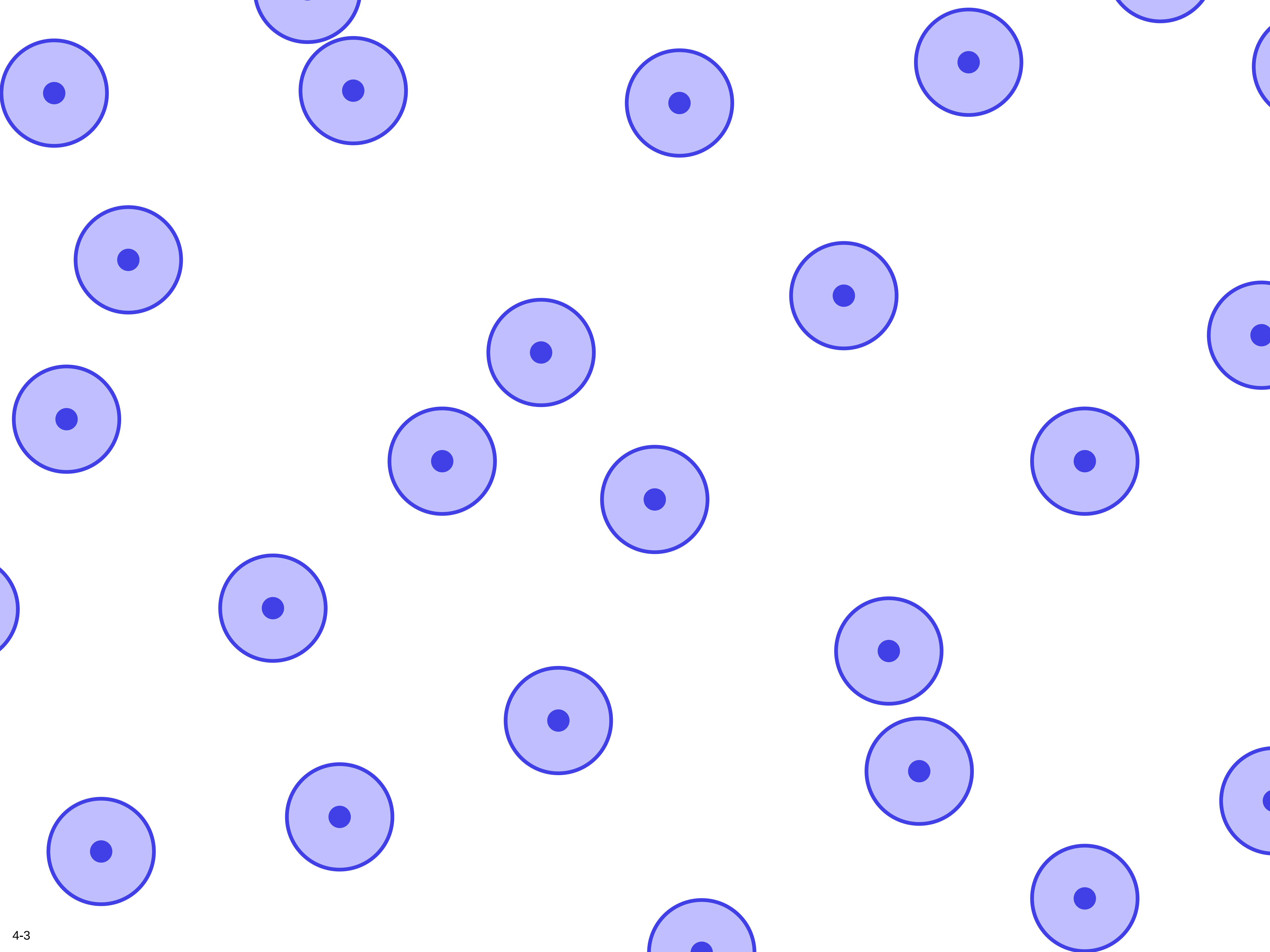
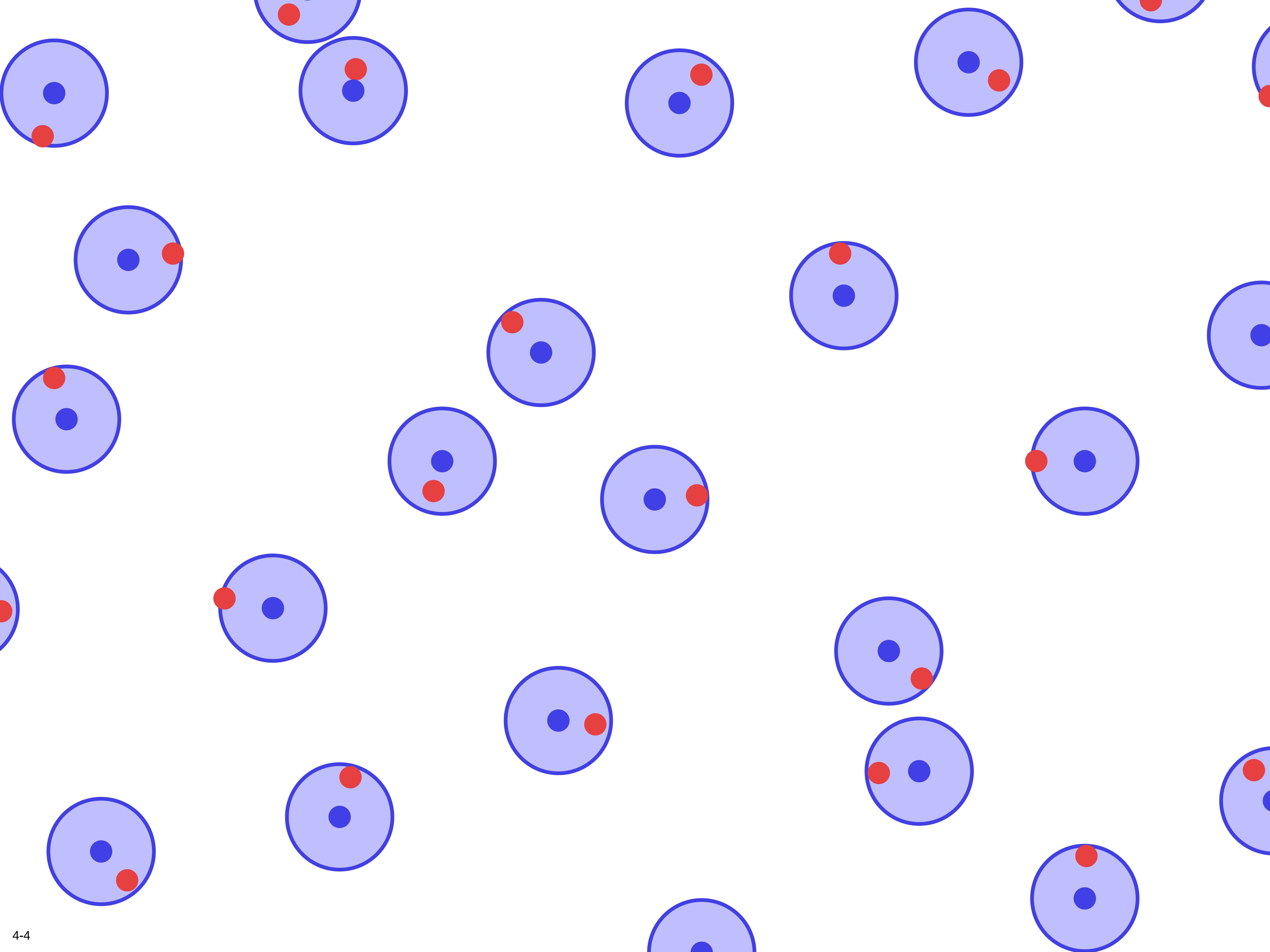
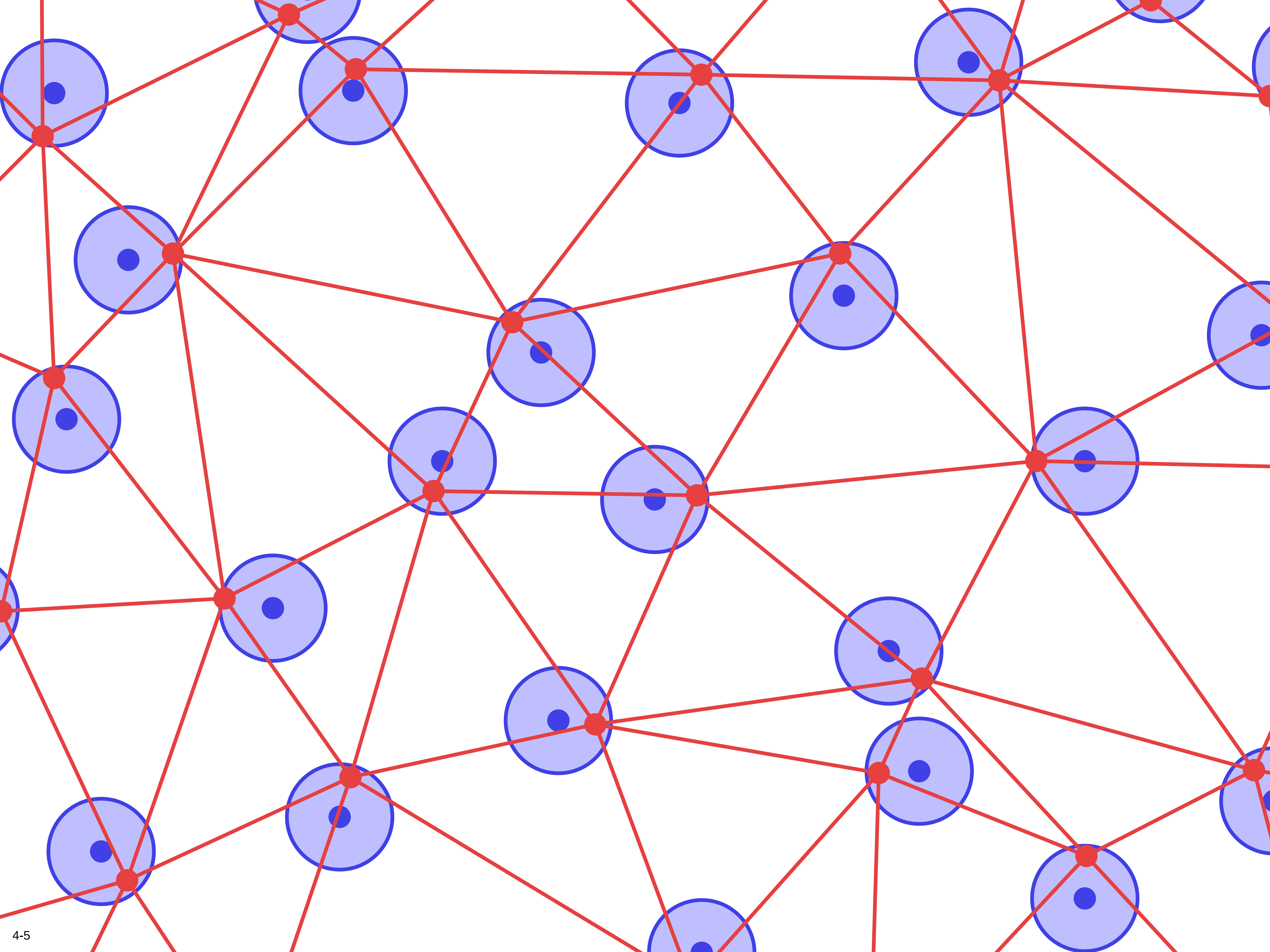
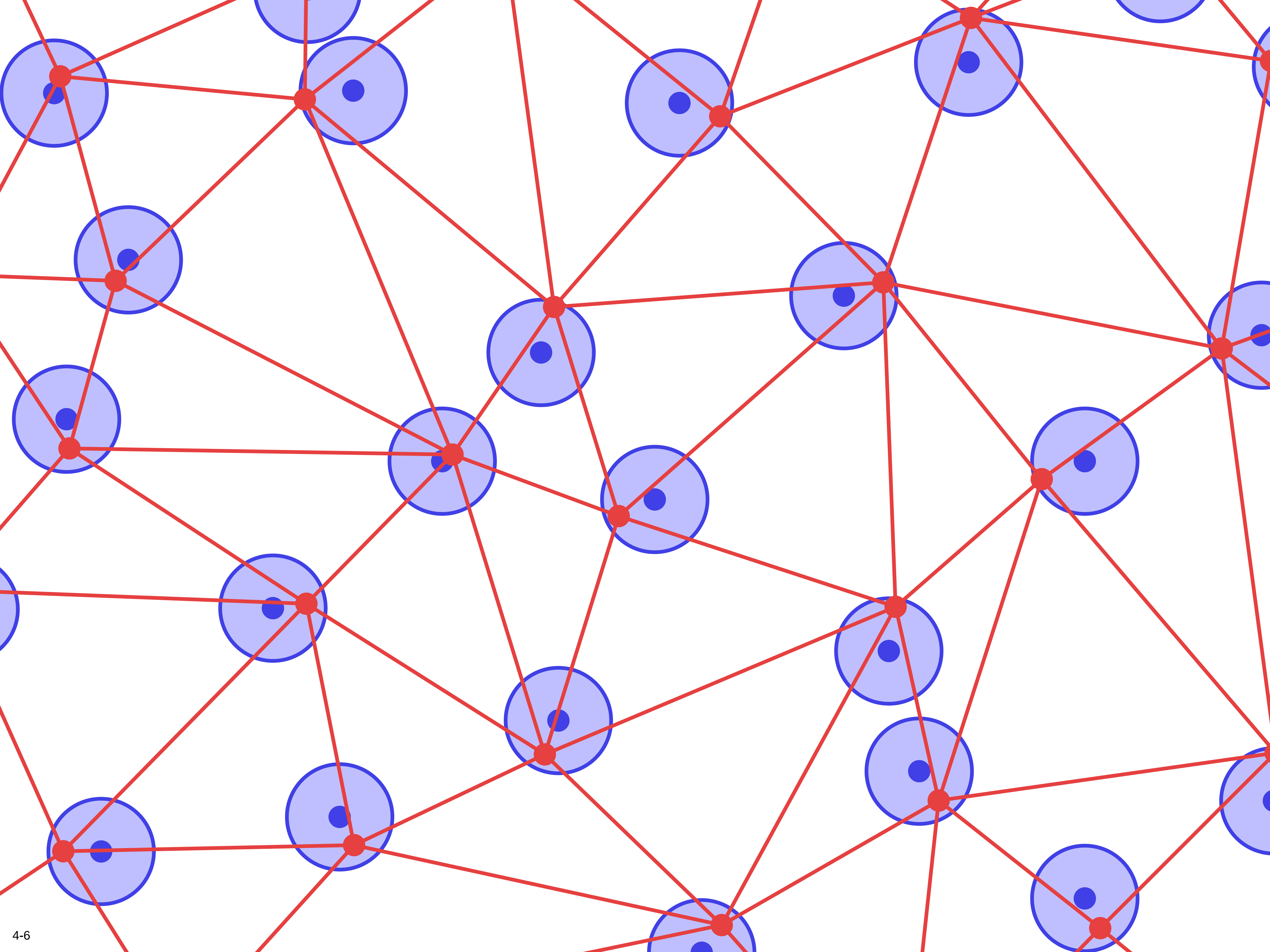


Then, what is the Delaunay triangulation of imprecise points?









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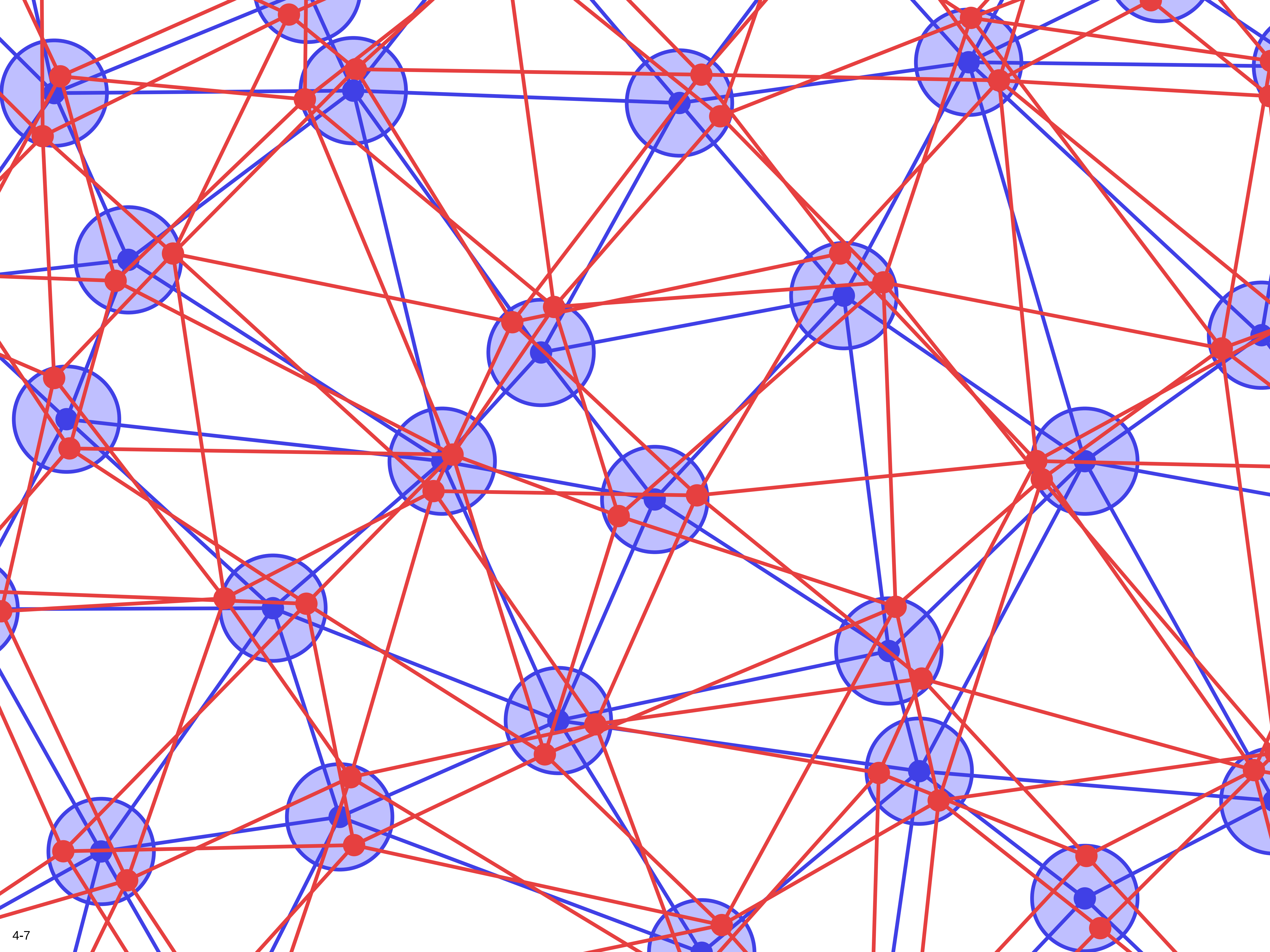
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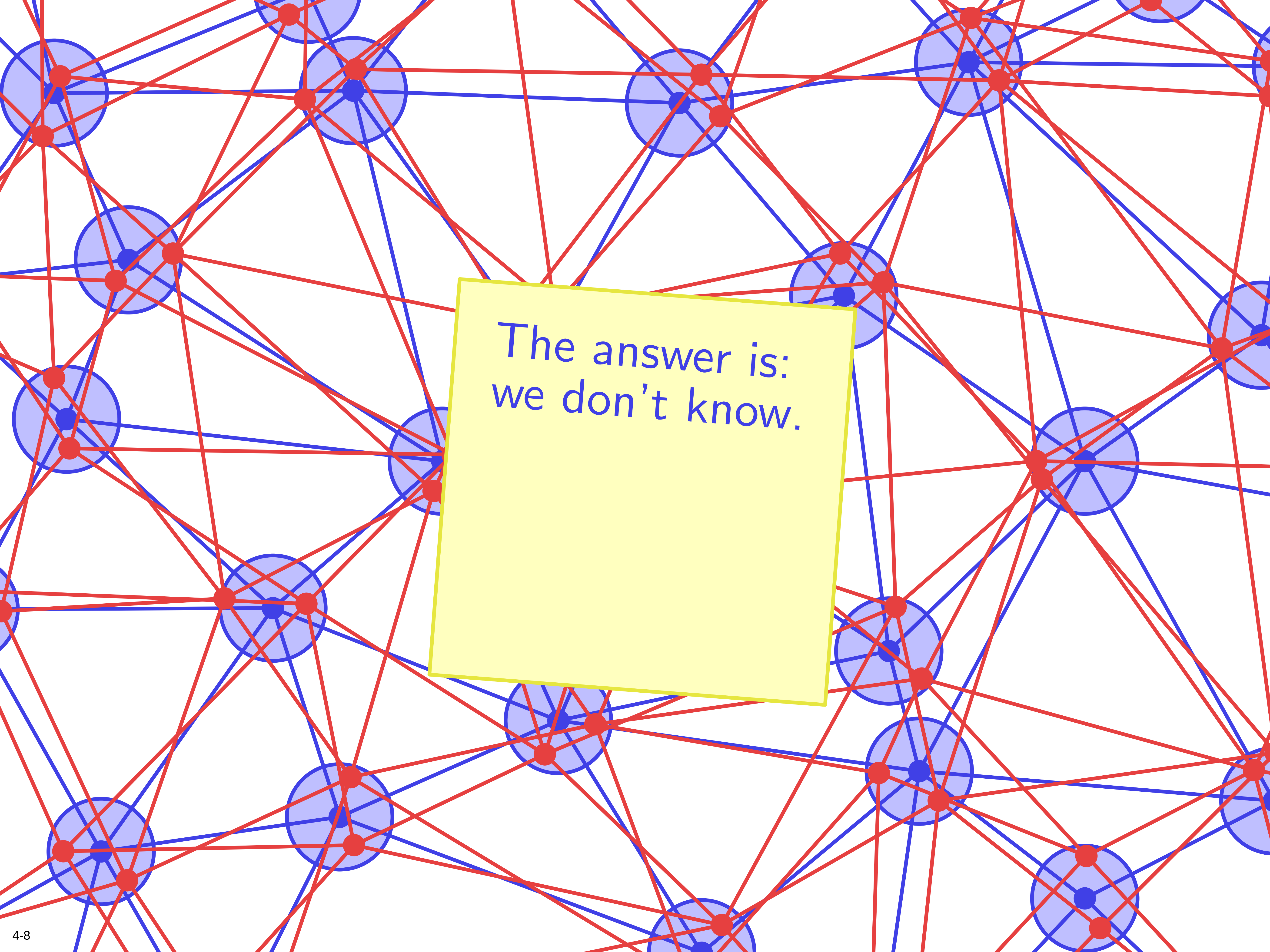

The answer is:
we don't know.

So, we have imprecise input data.

What *can* we do?

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Then, what is the value of the information about the regions that we already have?

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Let's define a *problem statement*.

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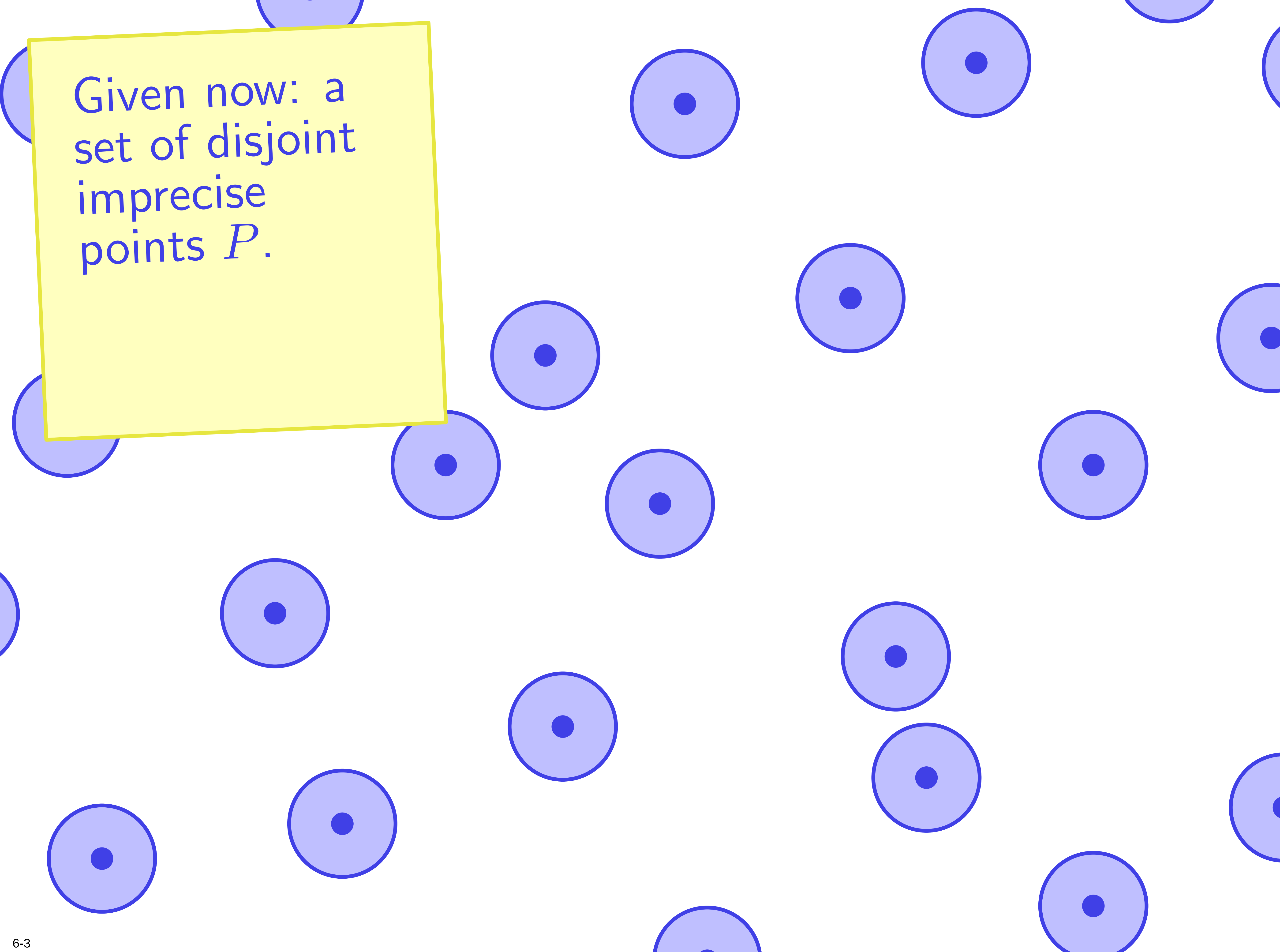

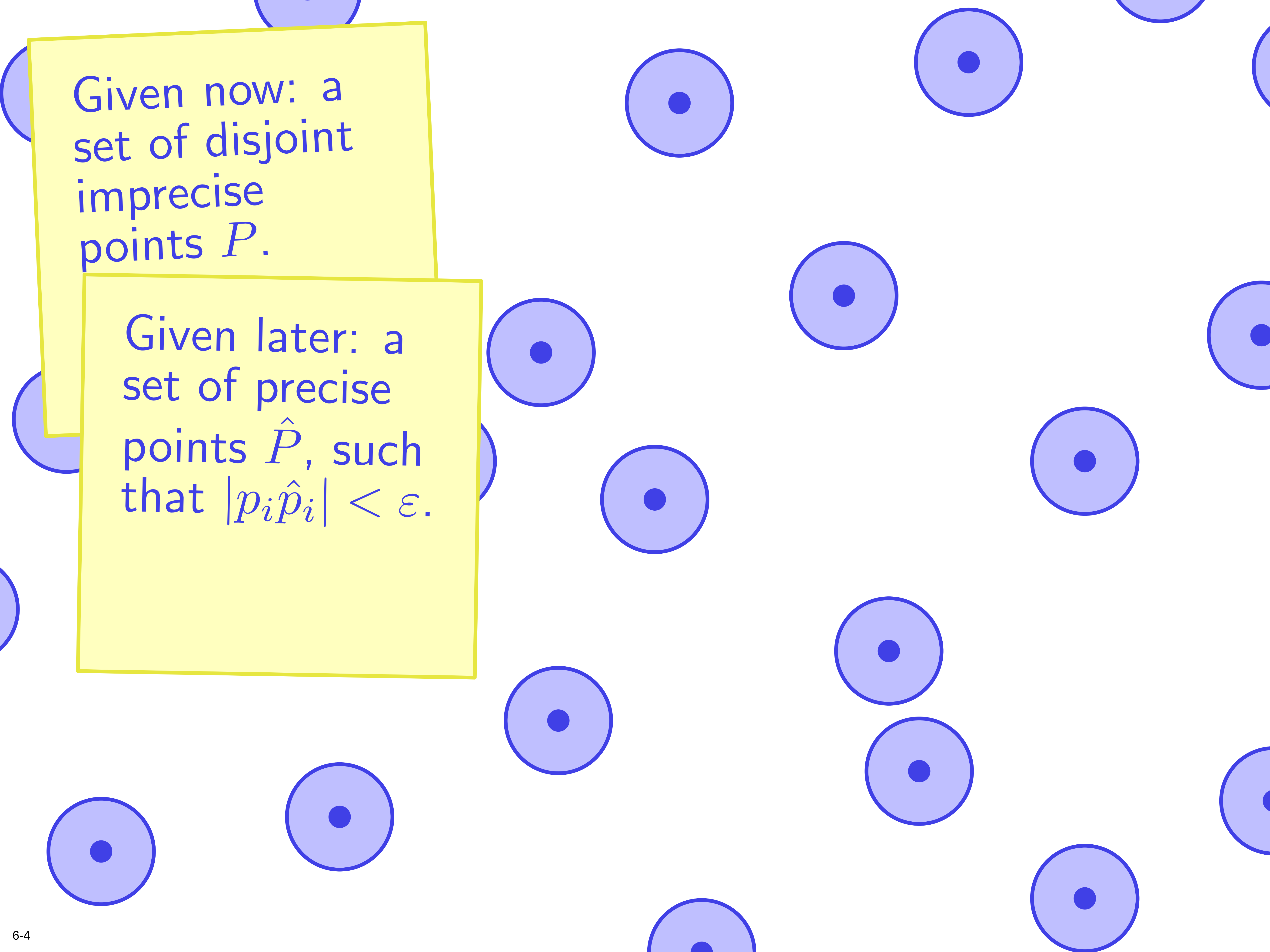

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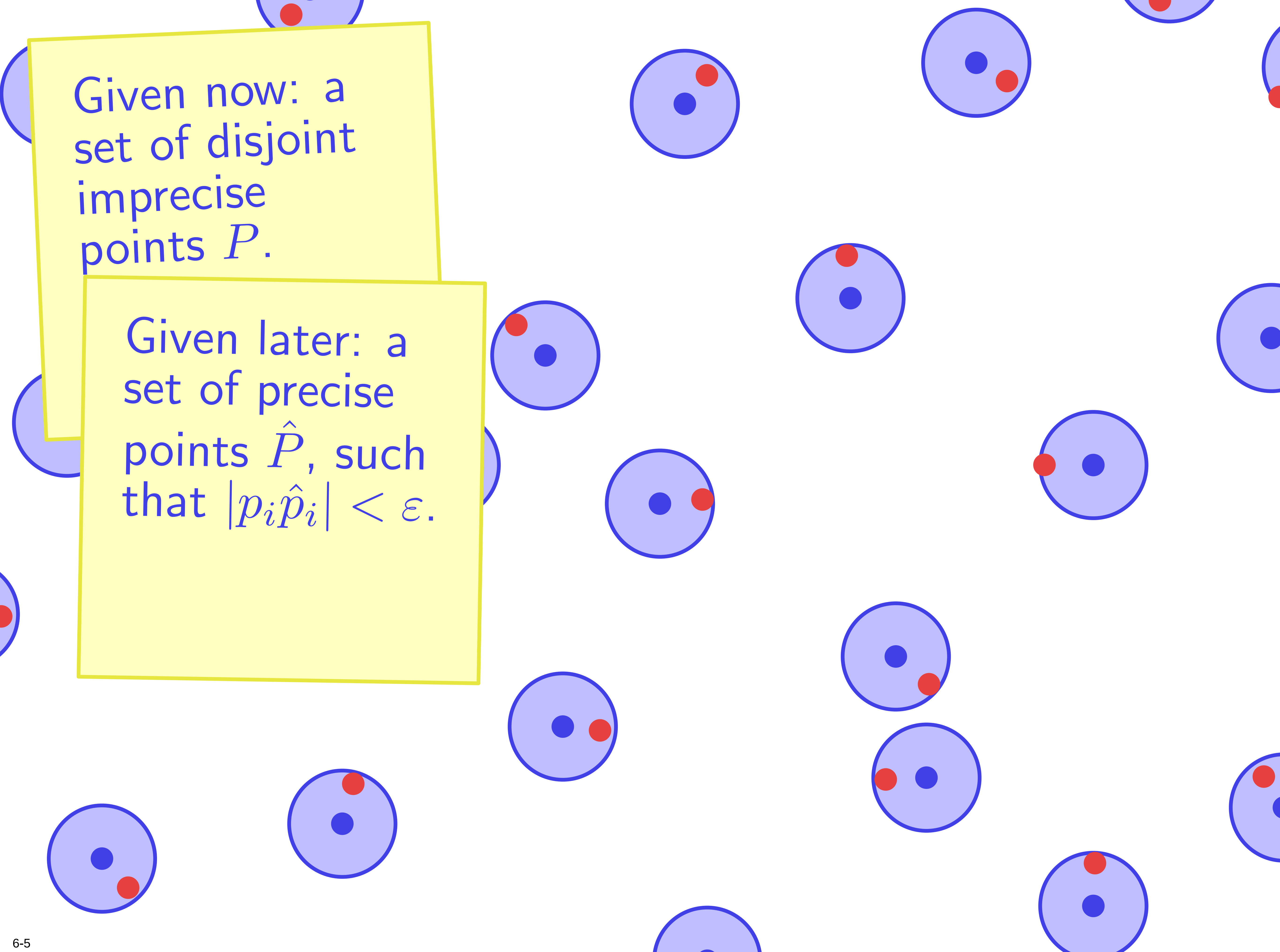
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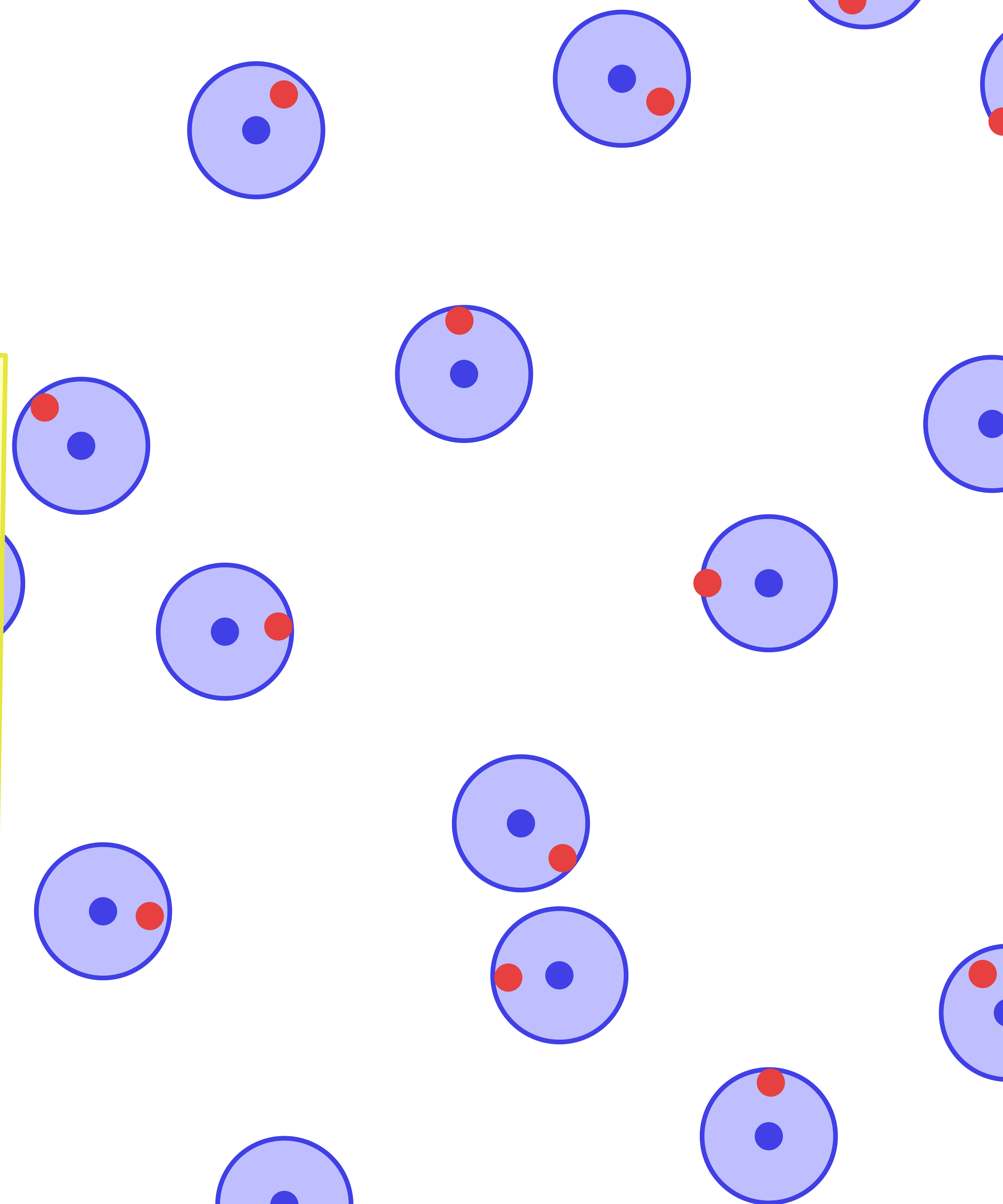

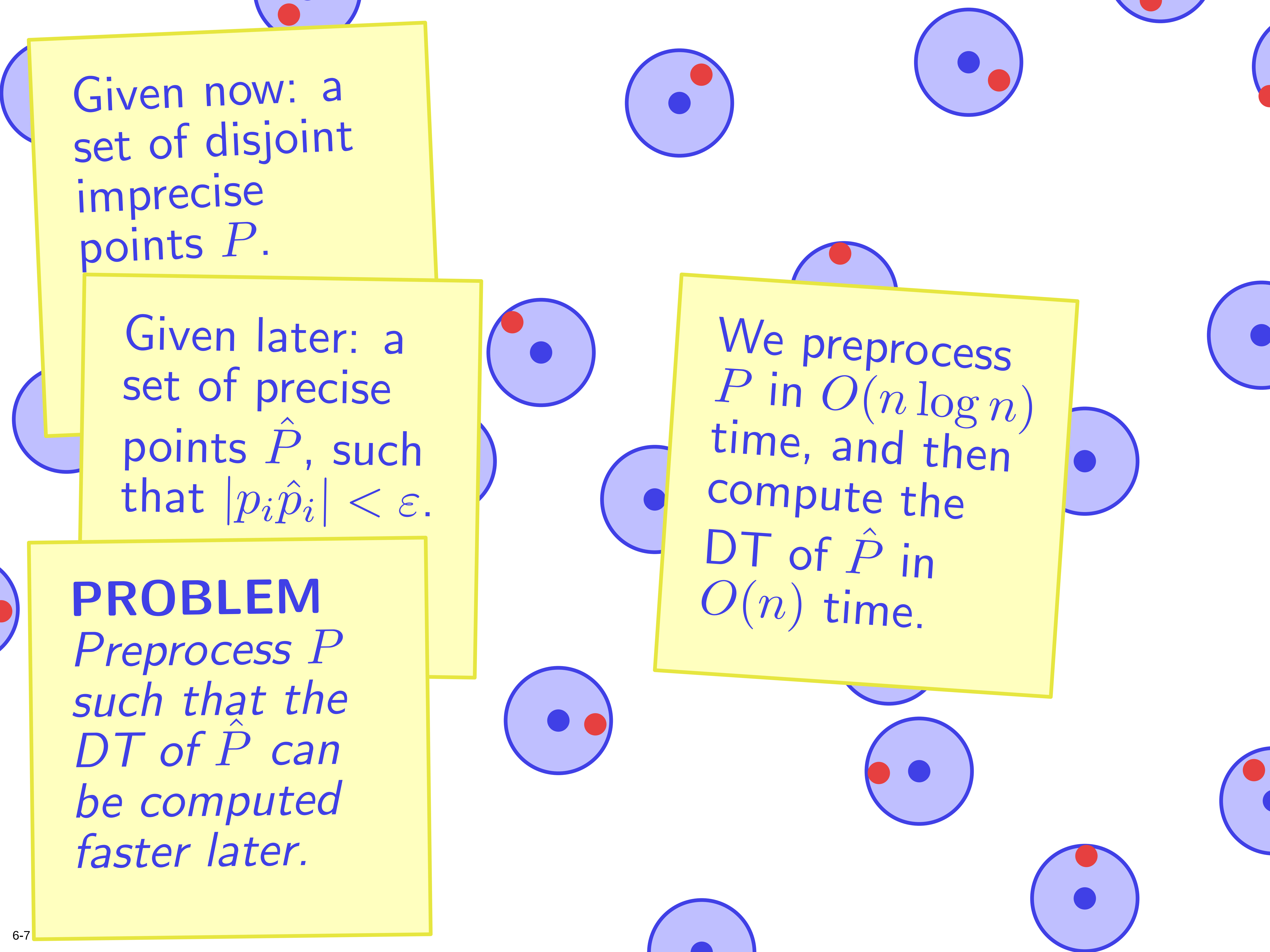

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We preprocess P in $O(n \log n)$ time, and then compute the DT of $\hat{P}$ in $O(n)$ time.

We'll define an important concept: *expanded Gabriel discs*

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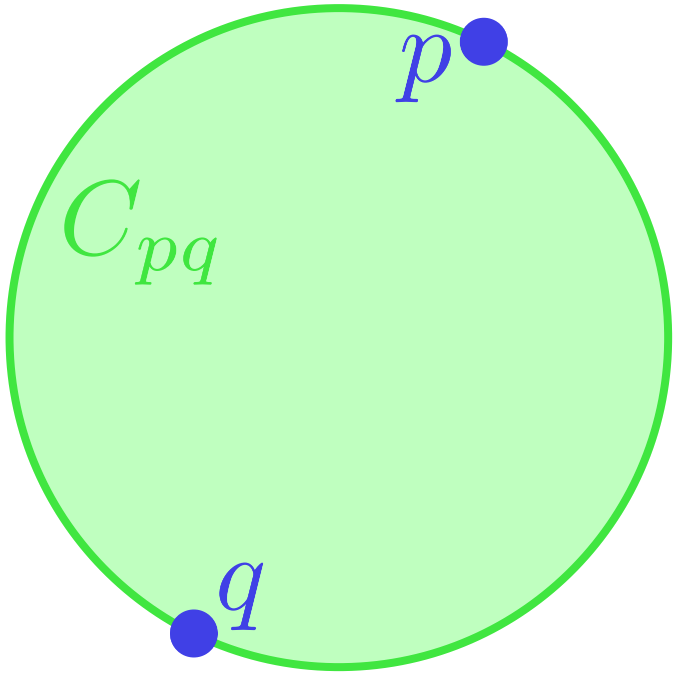

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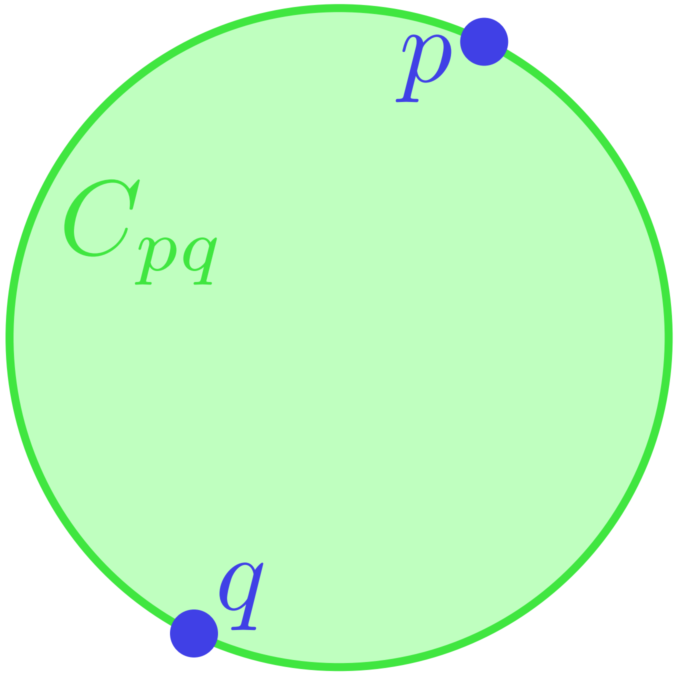

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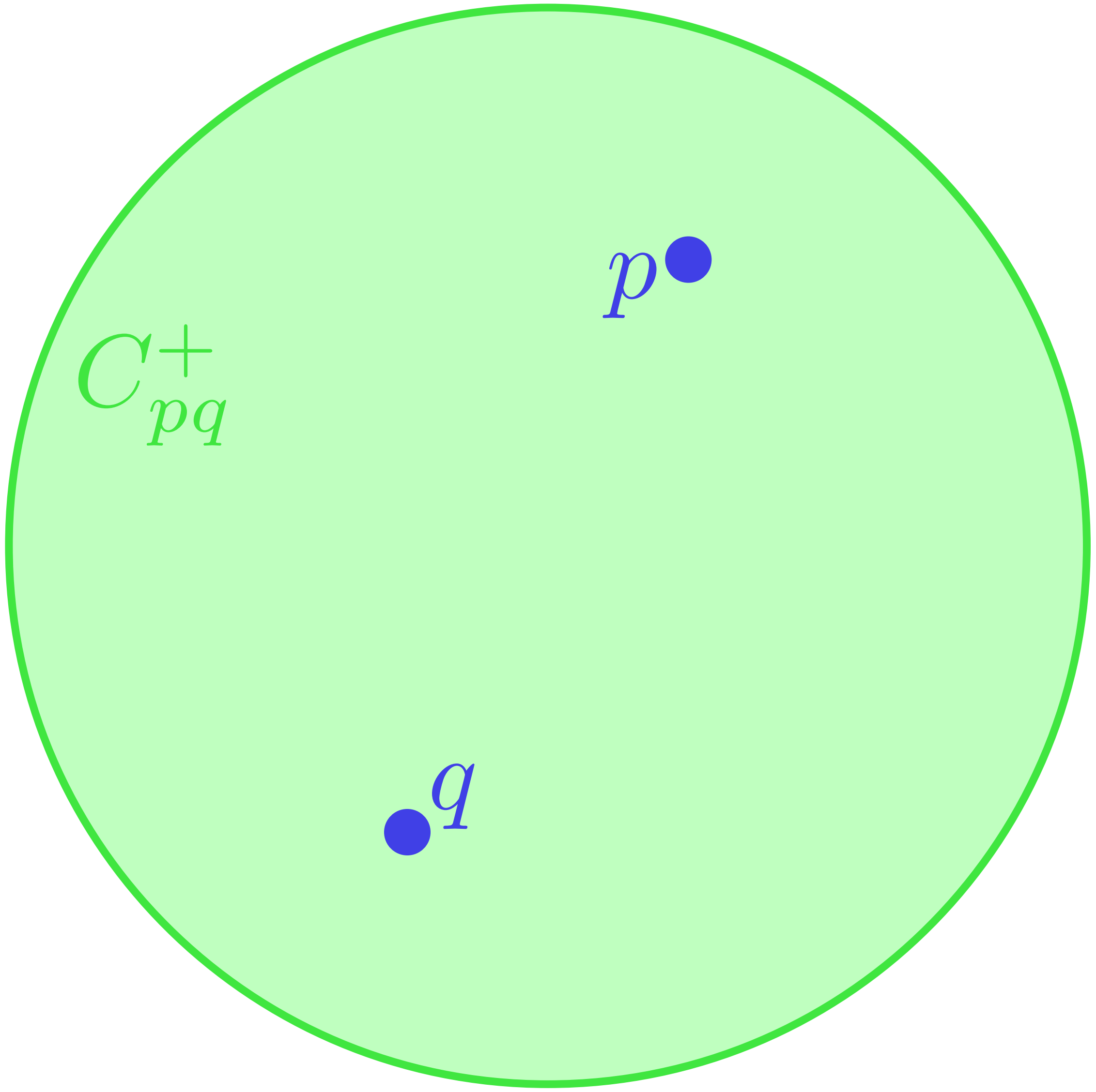

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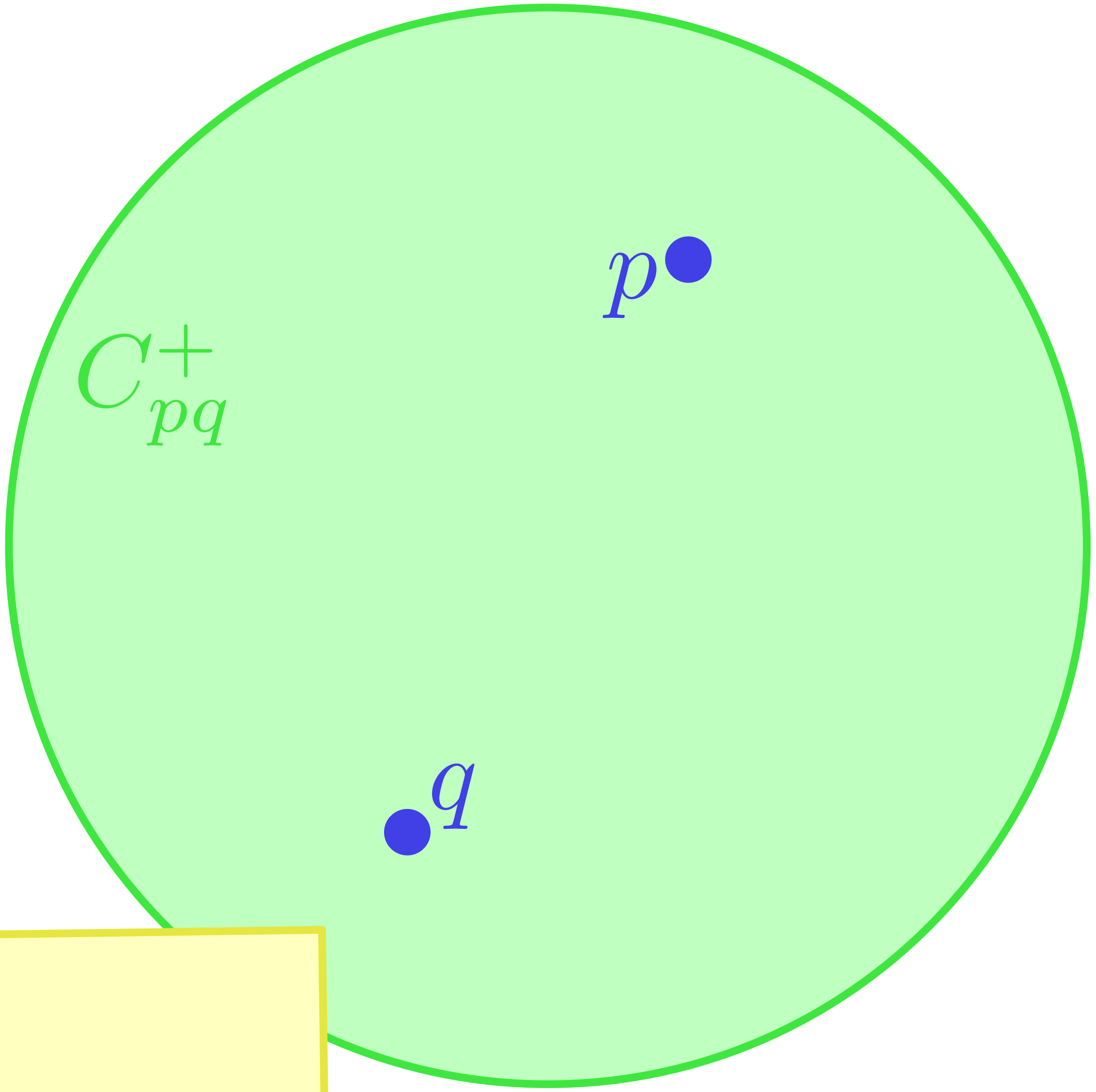

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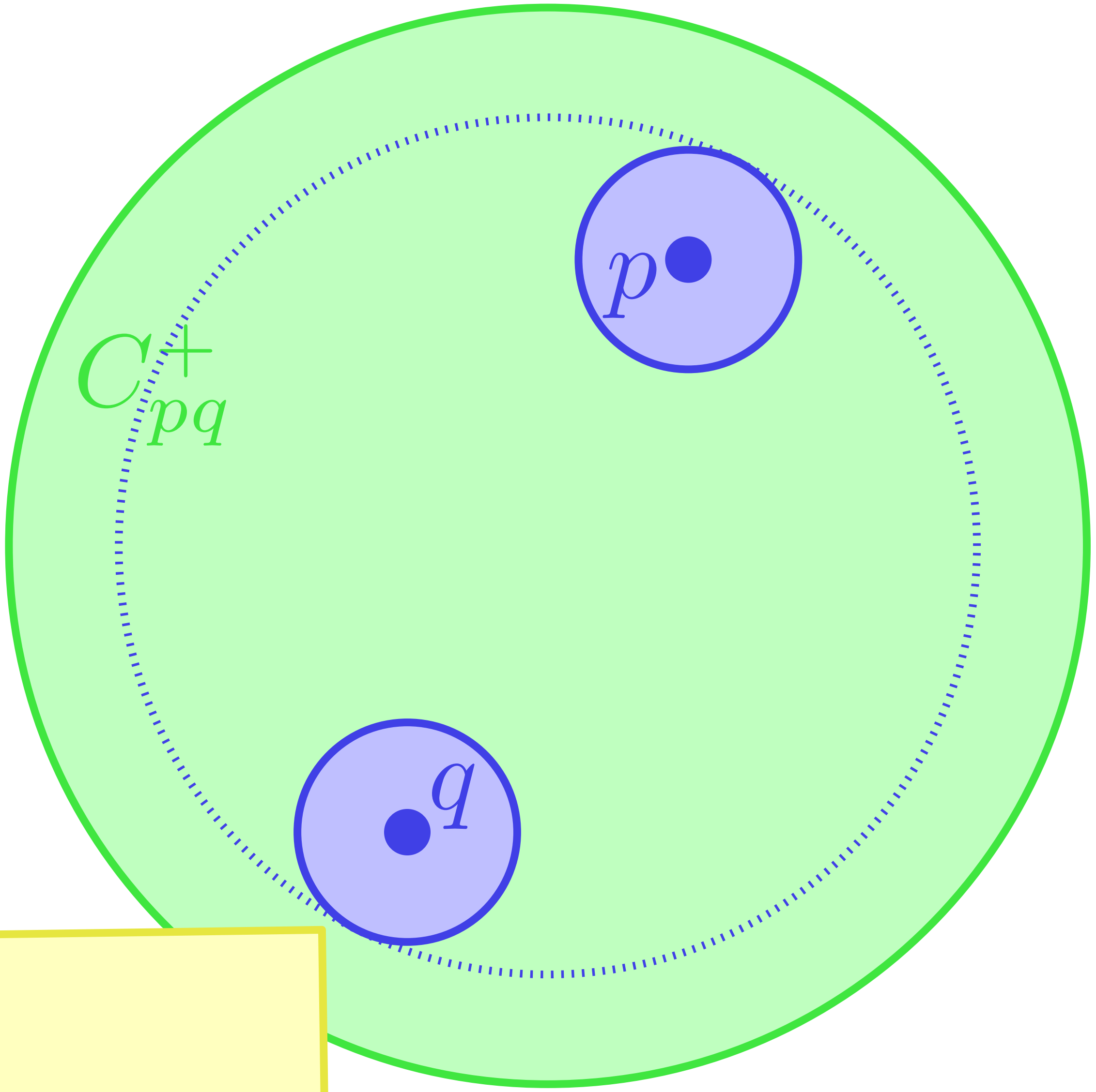

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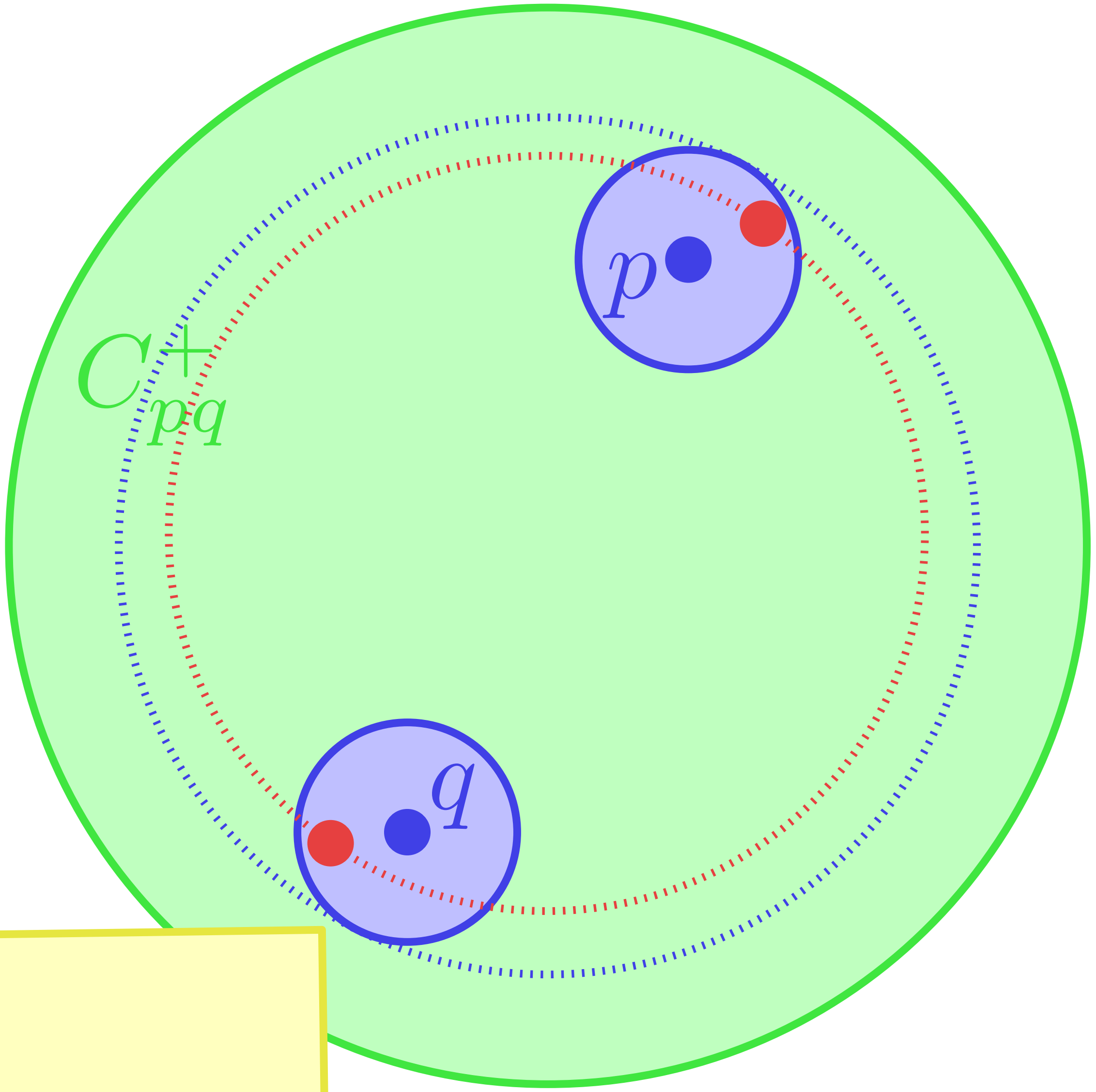

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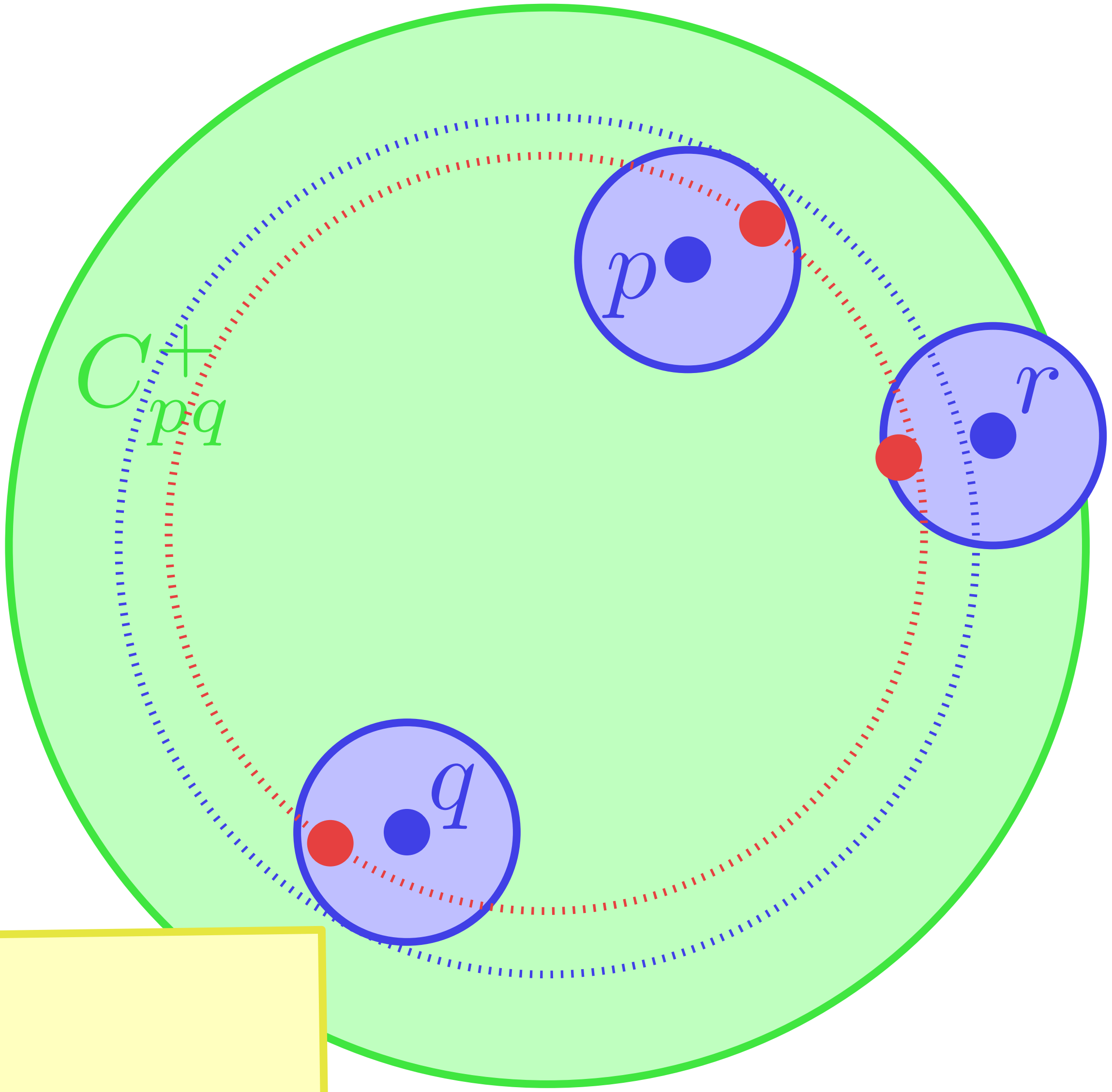

Preprocessing: we're going to compute a *minimum spanning tree*!

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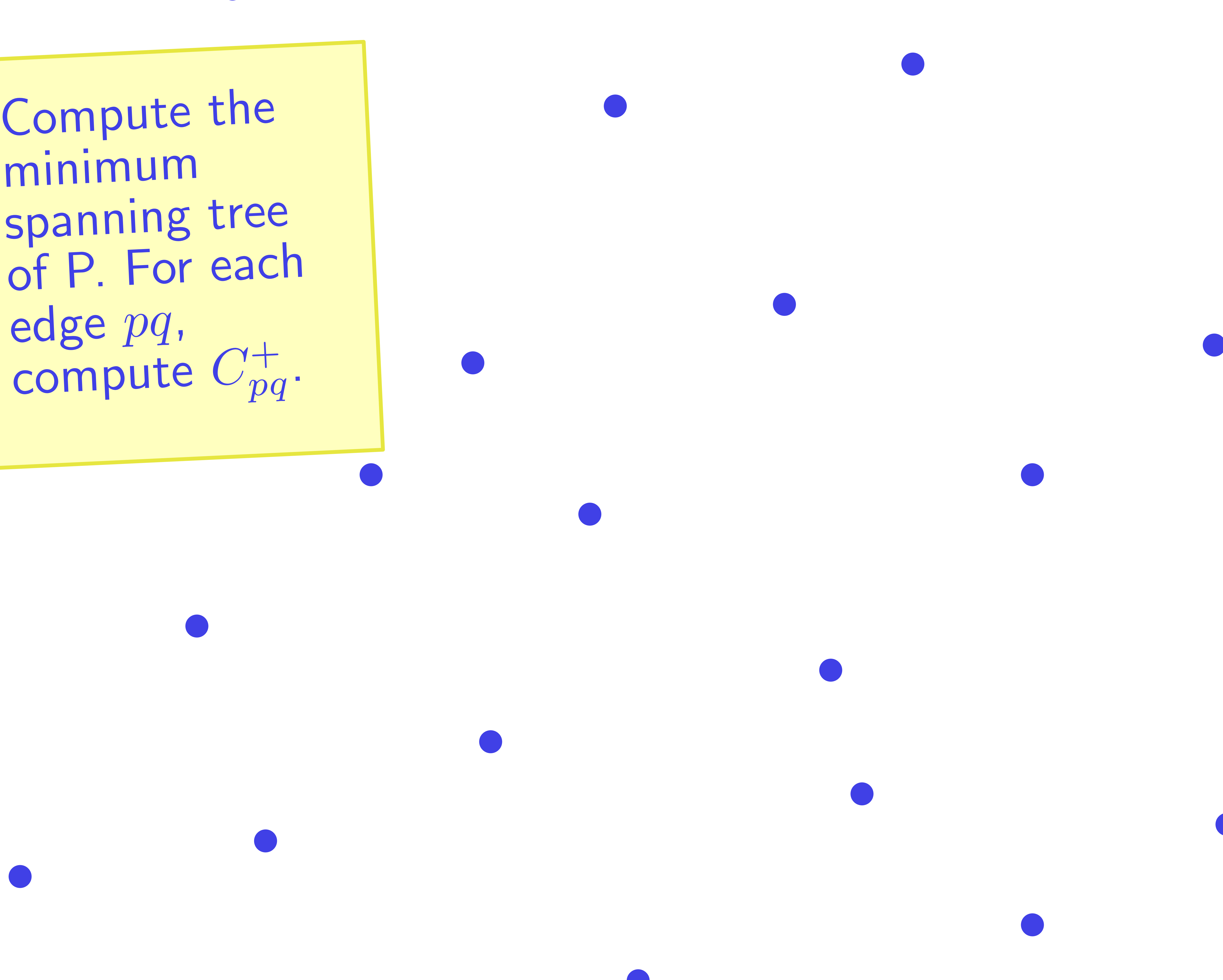

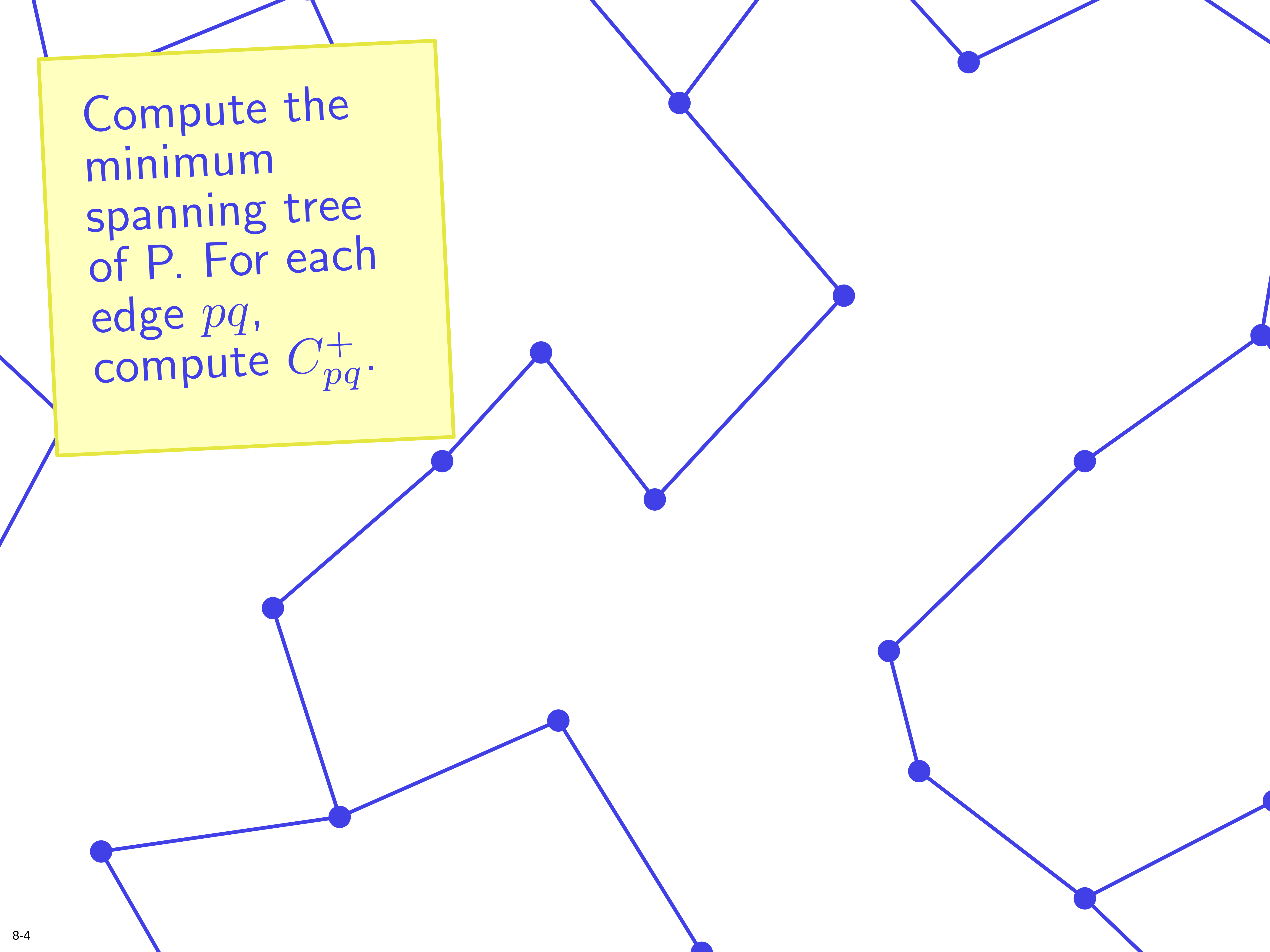

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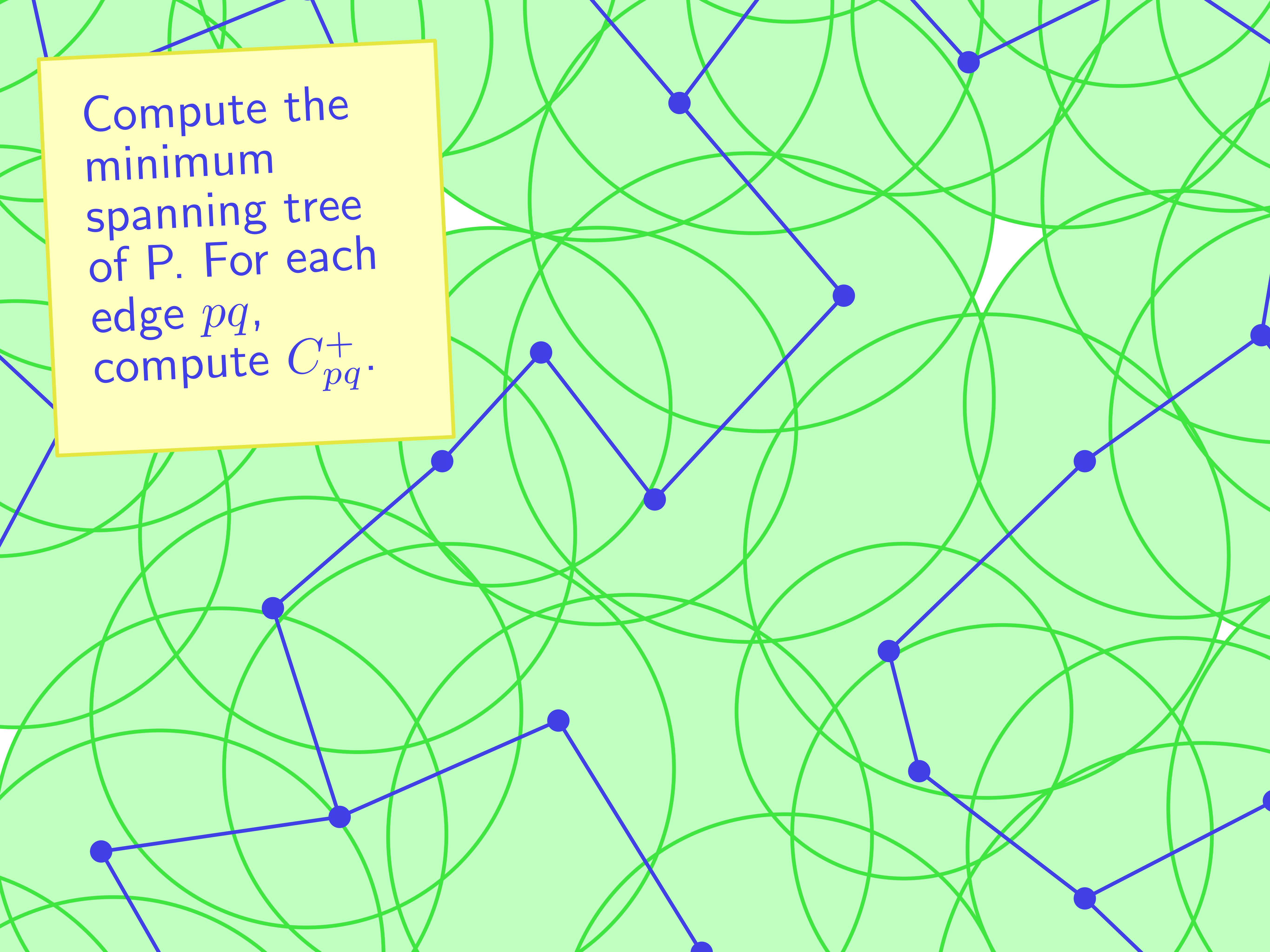

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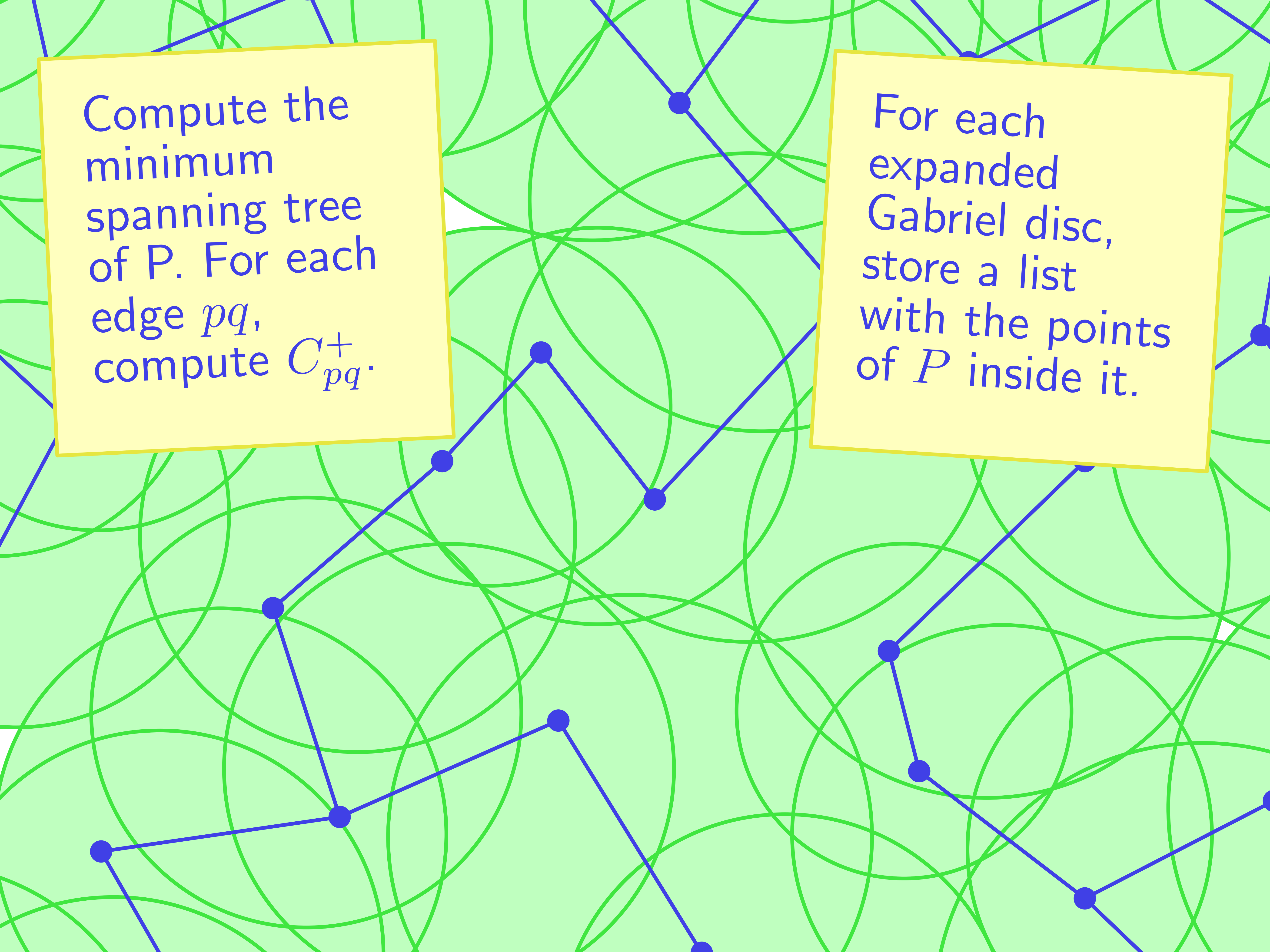
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For each expanded Gabriel disc, store a list with the points of P inside it.

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The total complexity of these lists is $O(n)$.

Let's do a nice small technical lemma.

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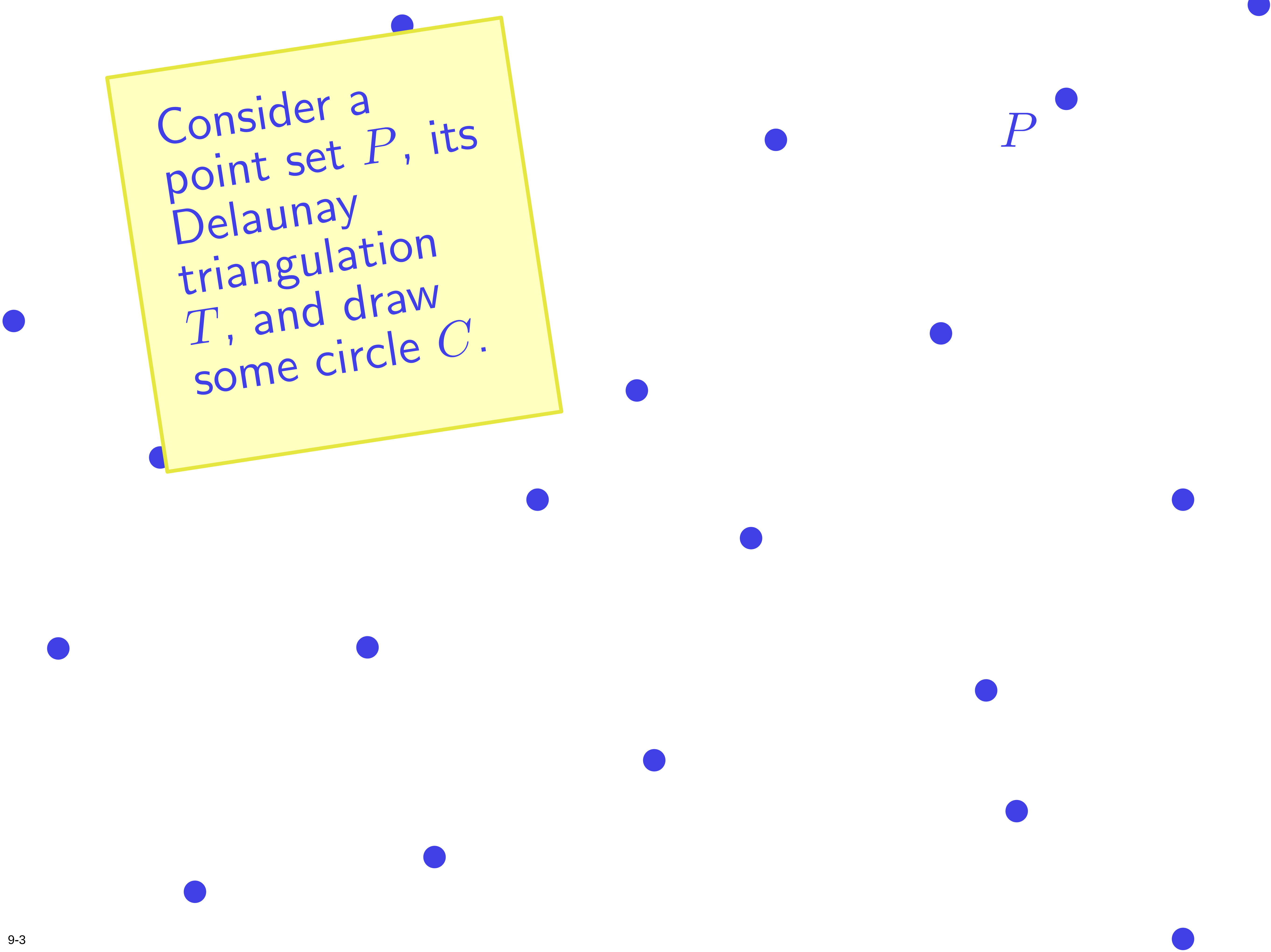

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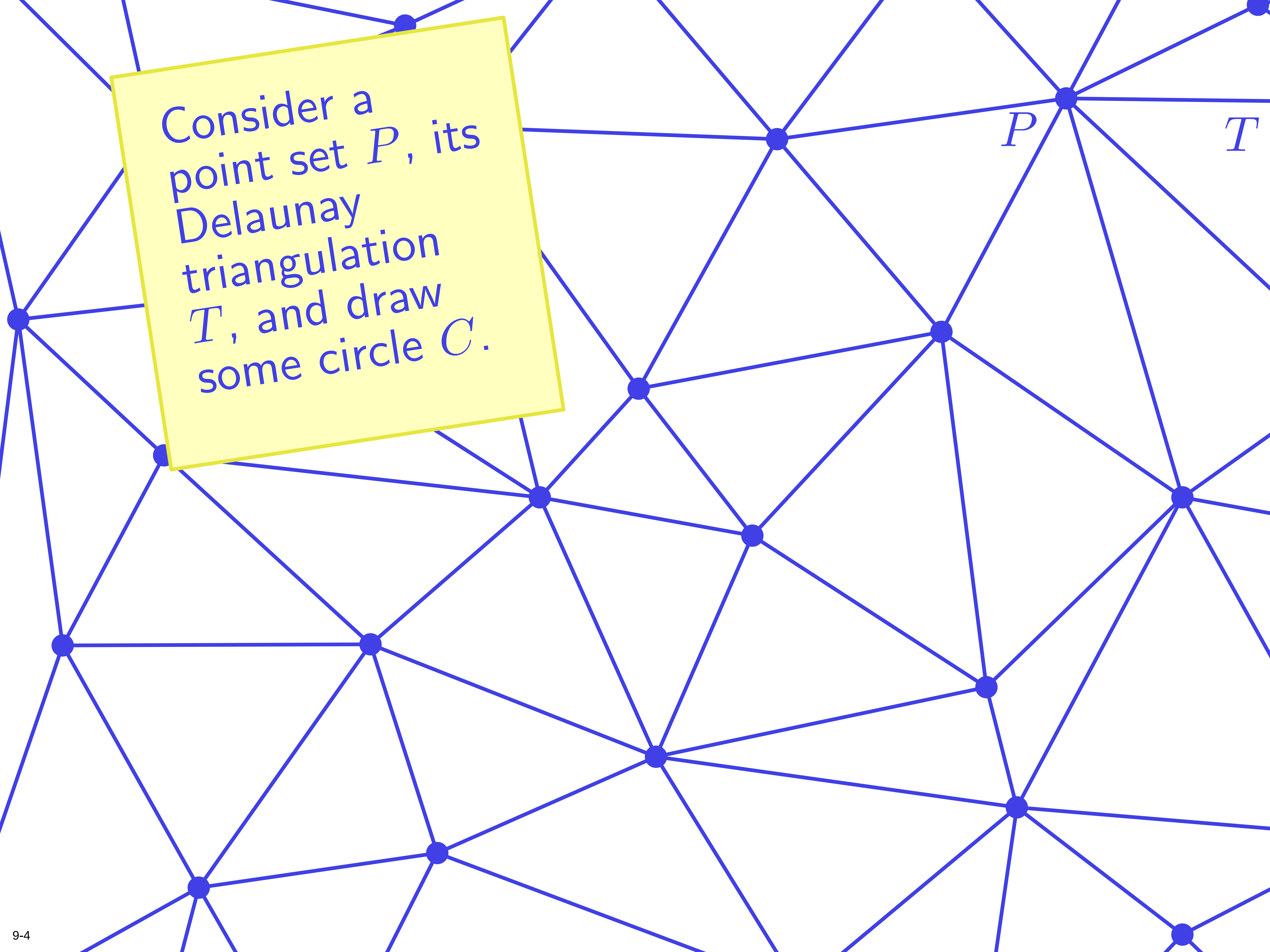

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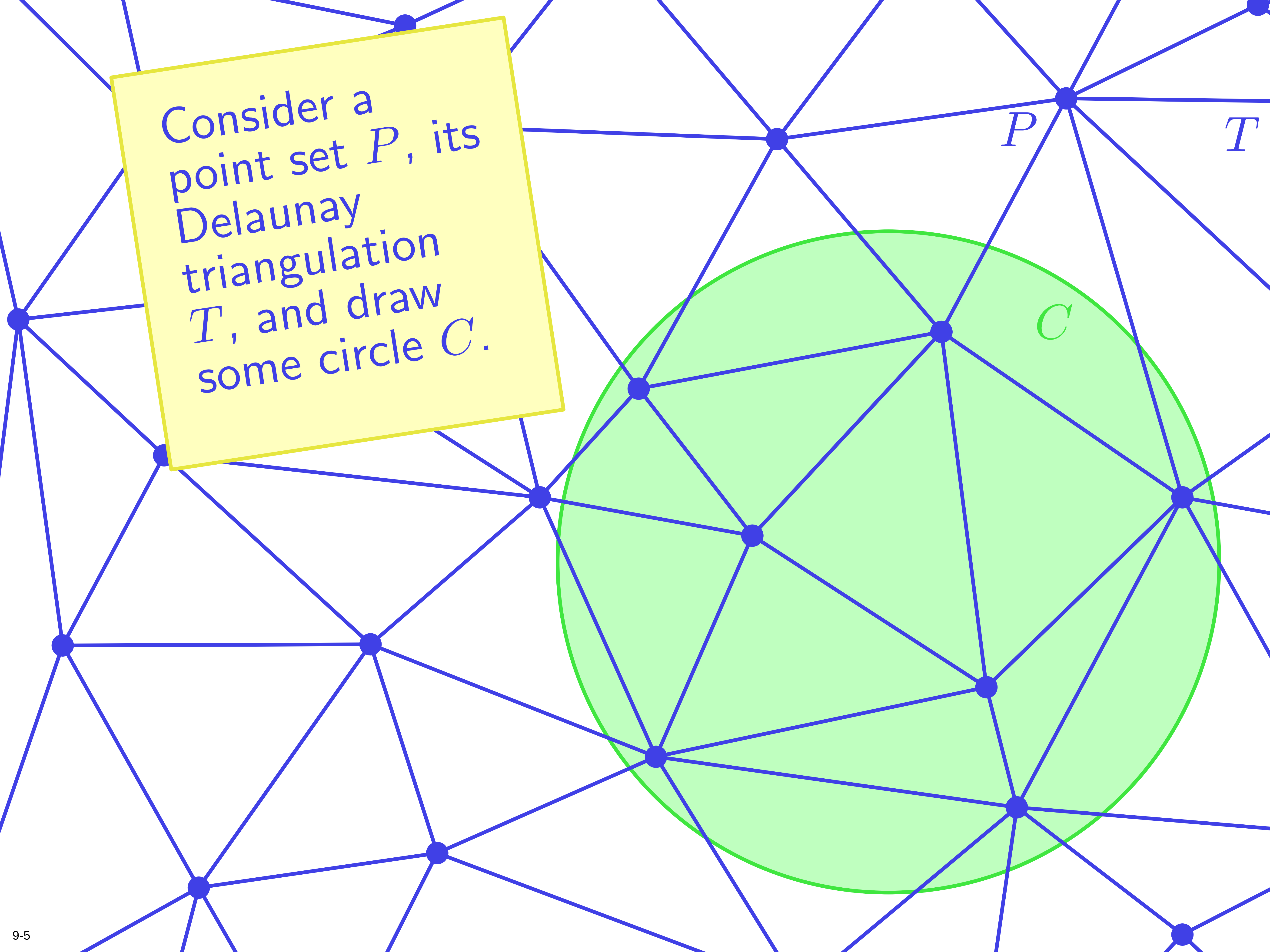

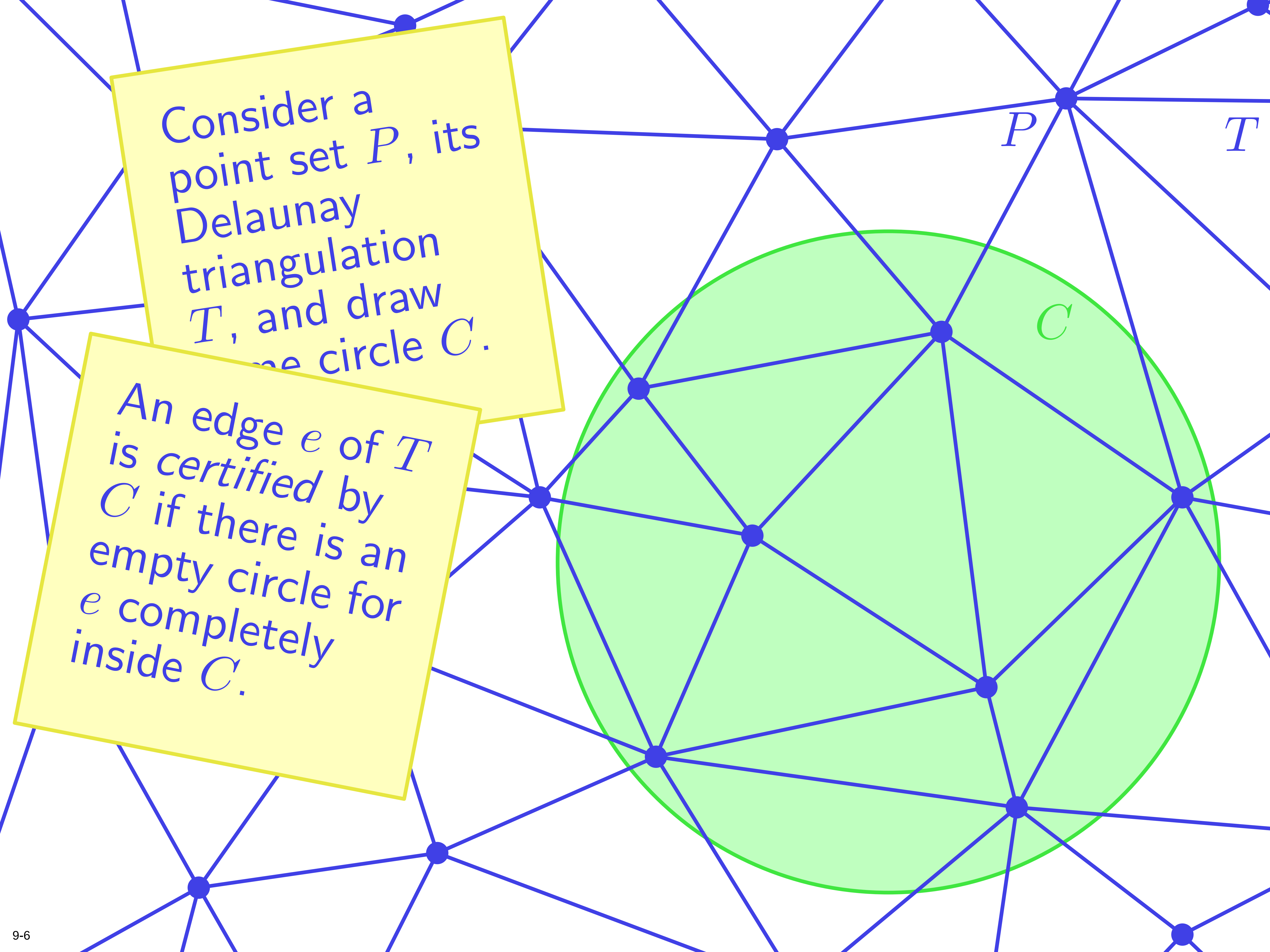

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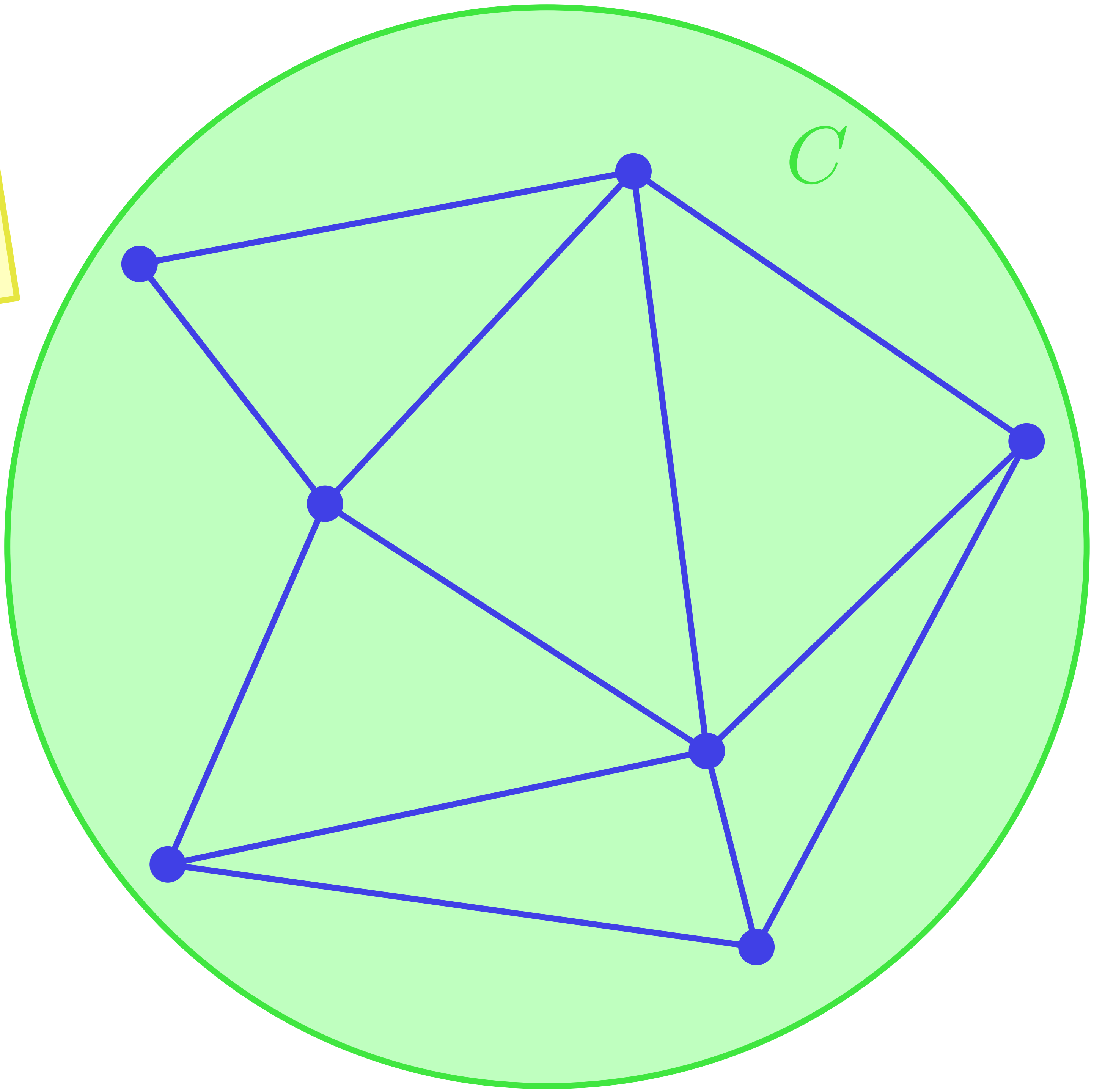

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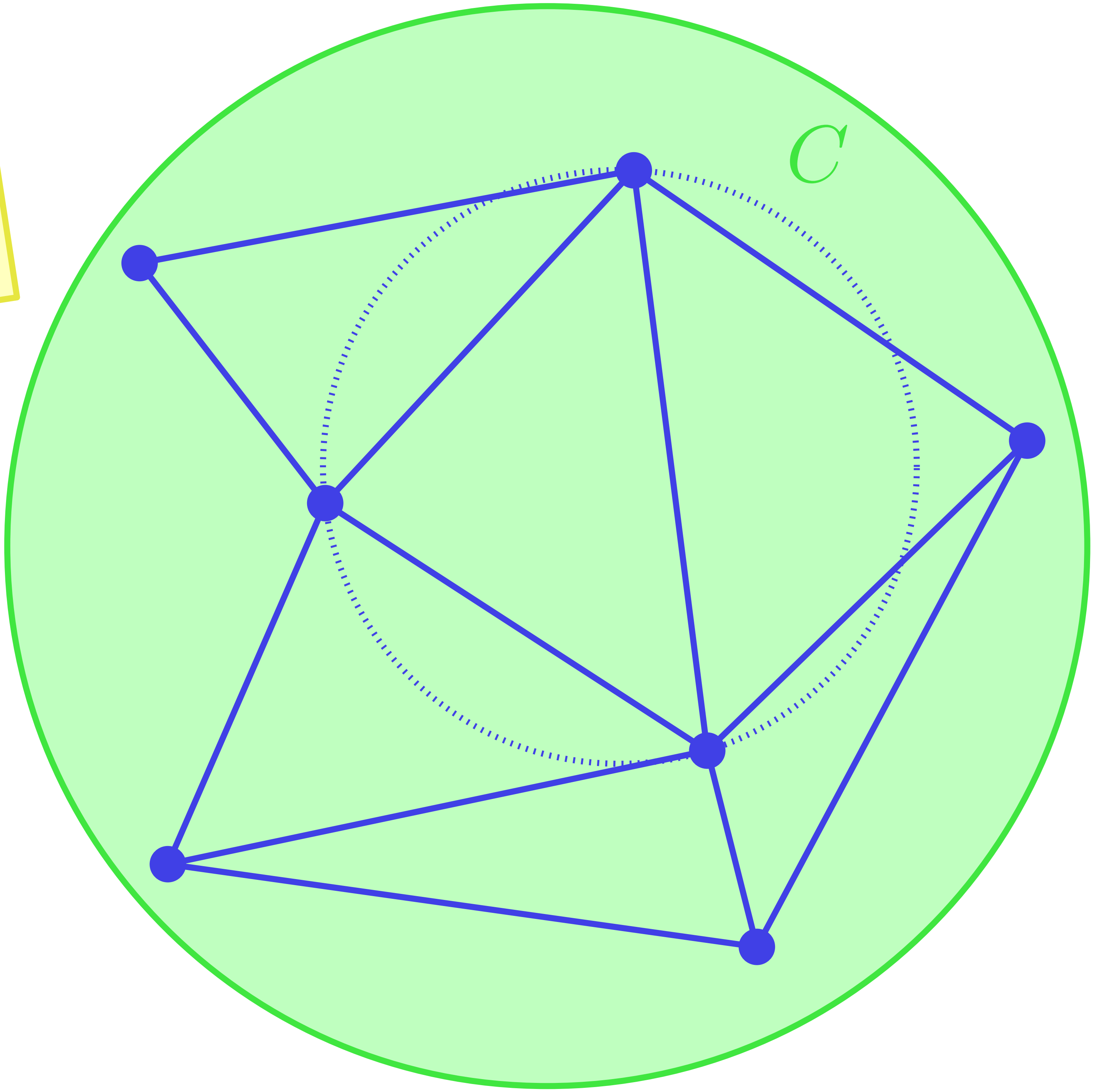

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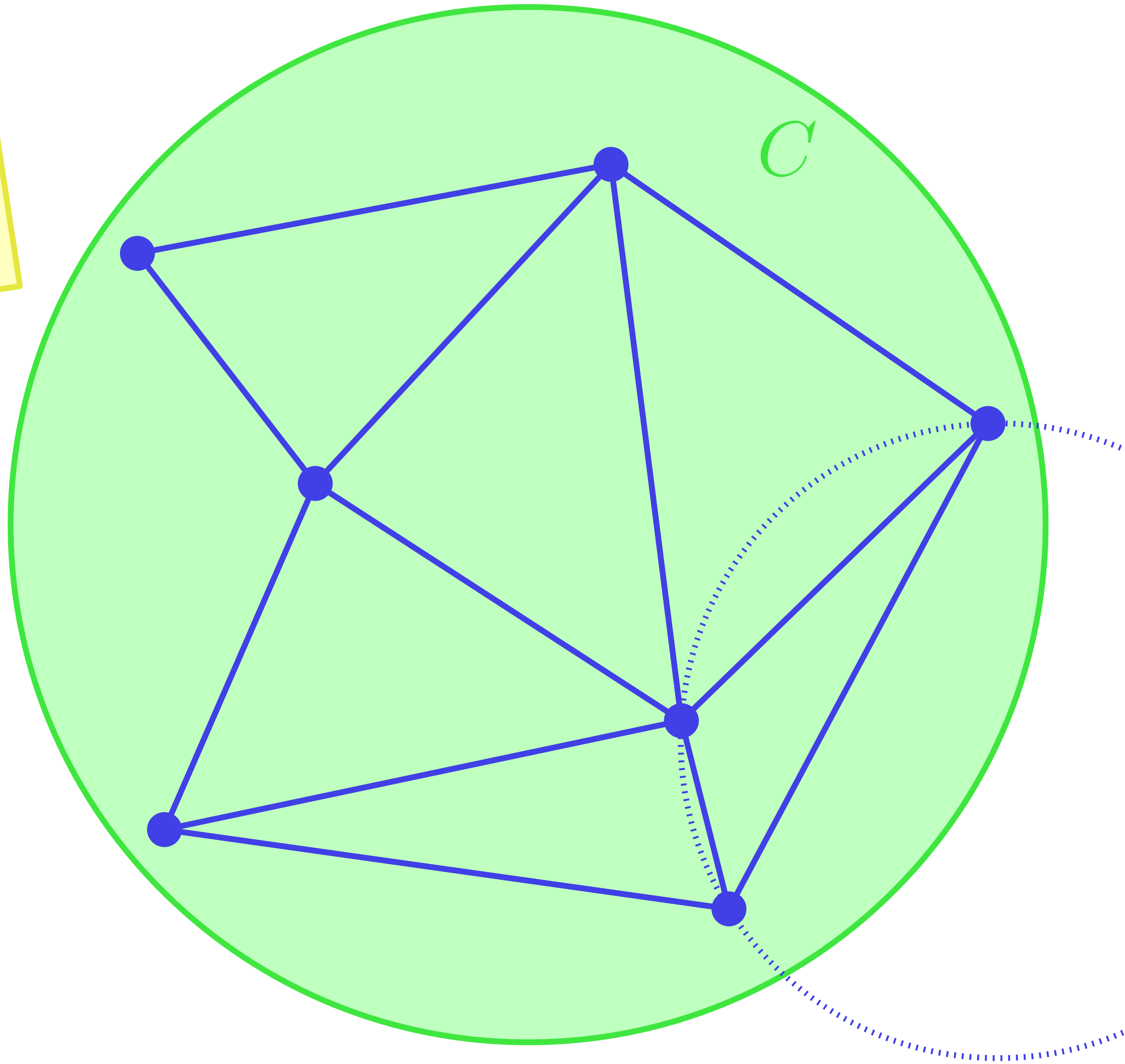

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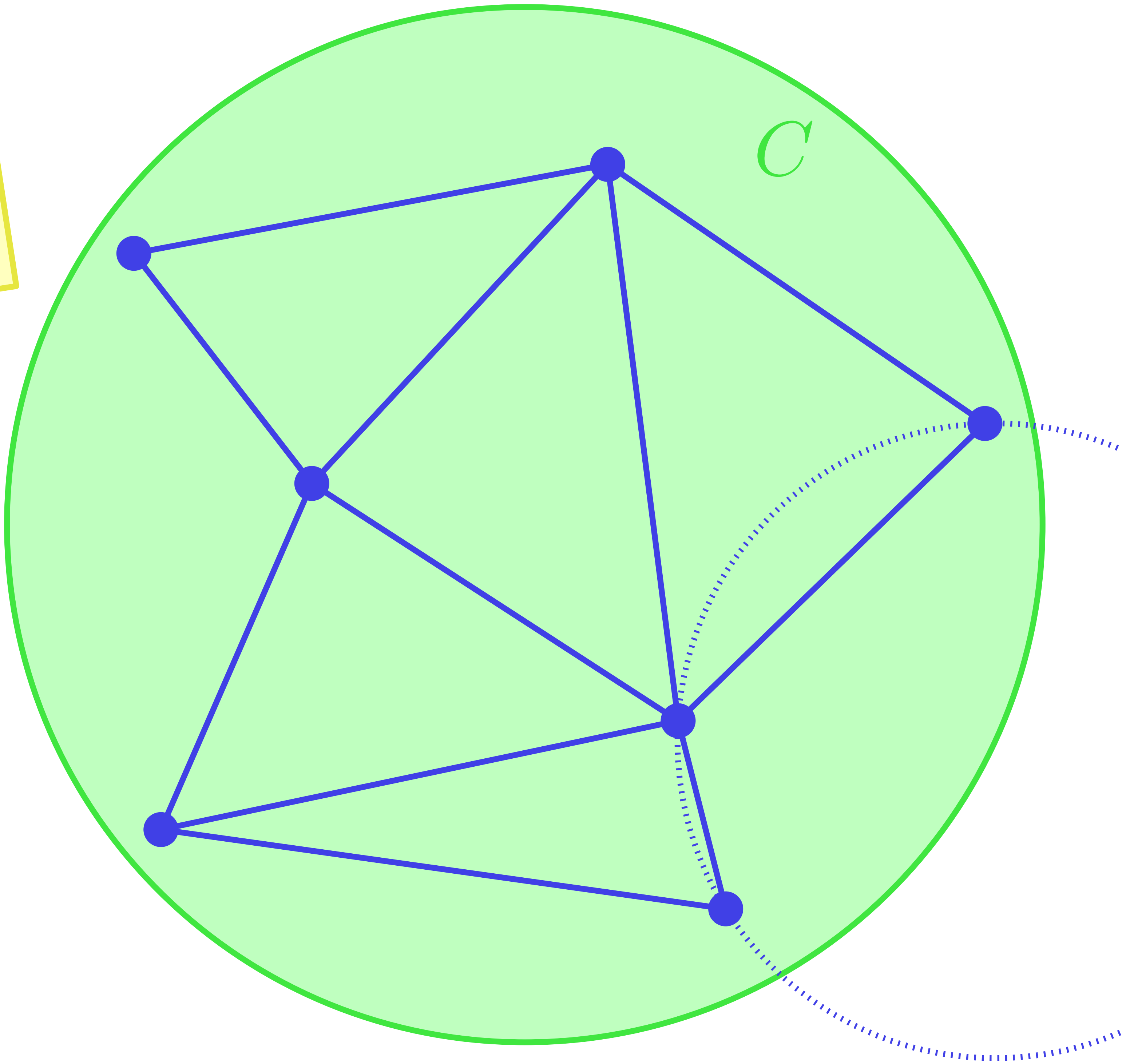

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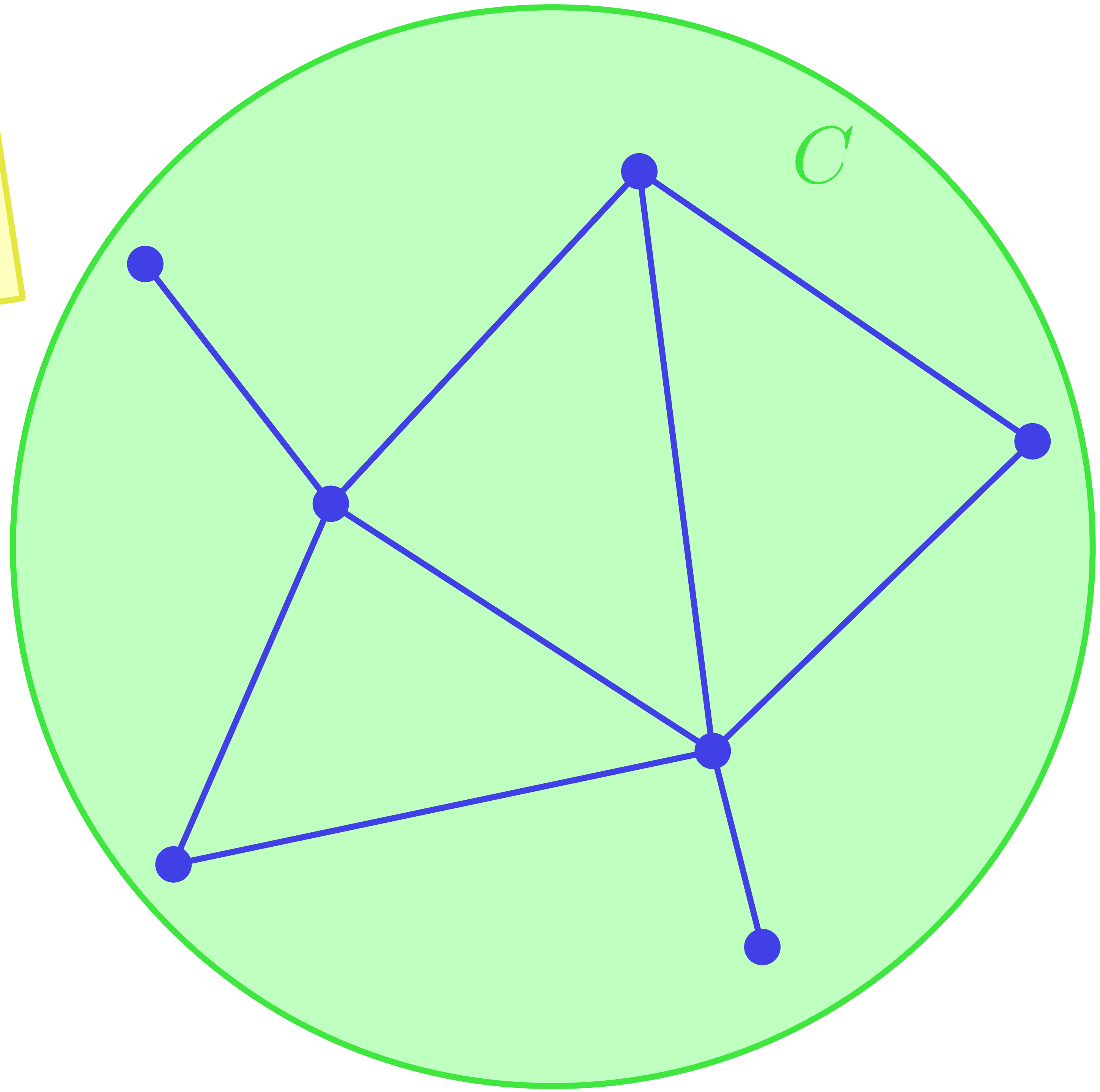

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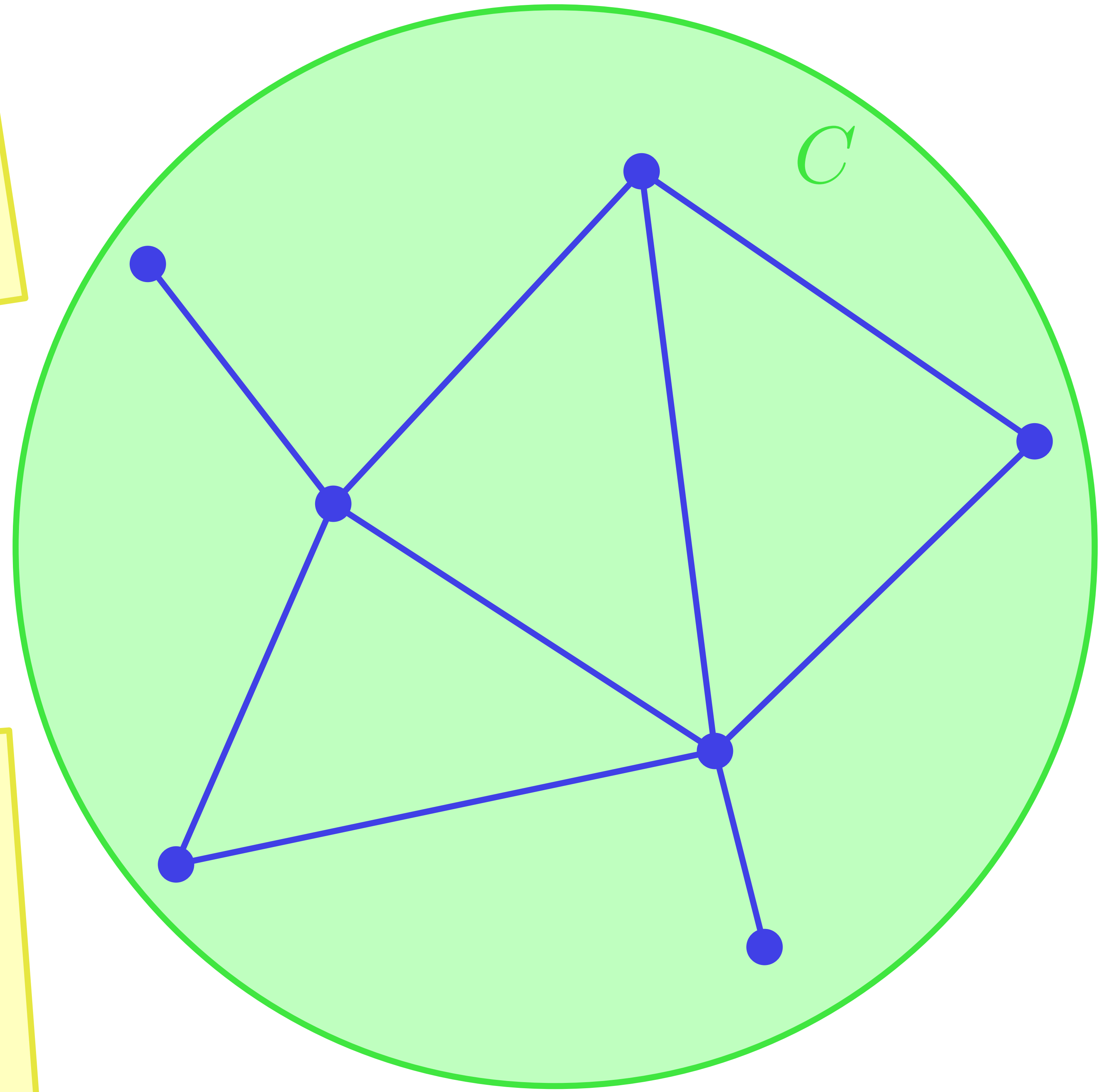

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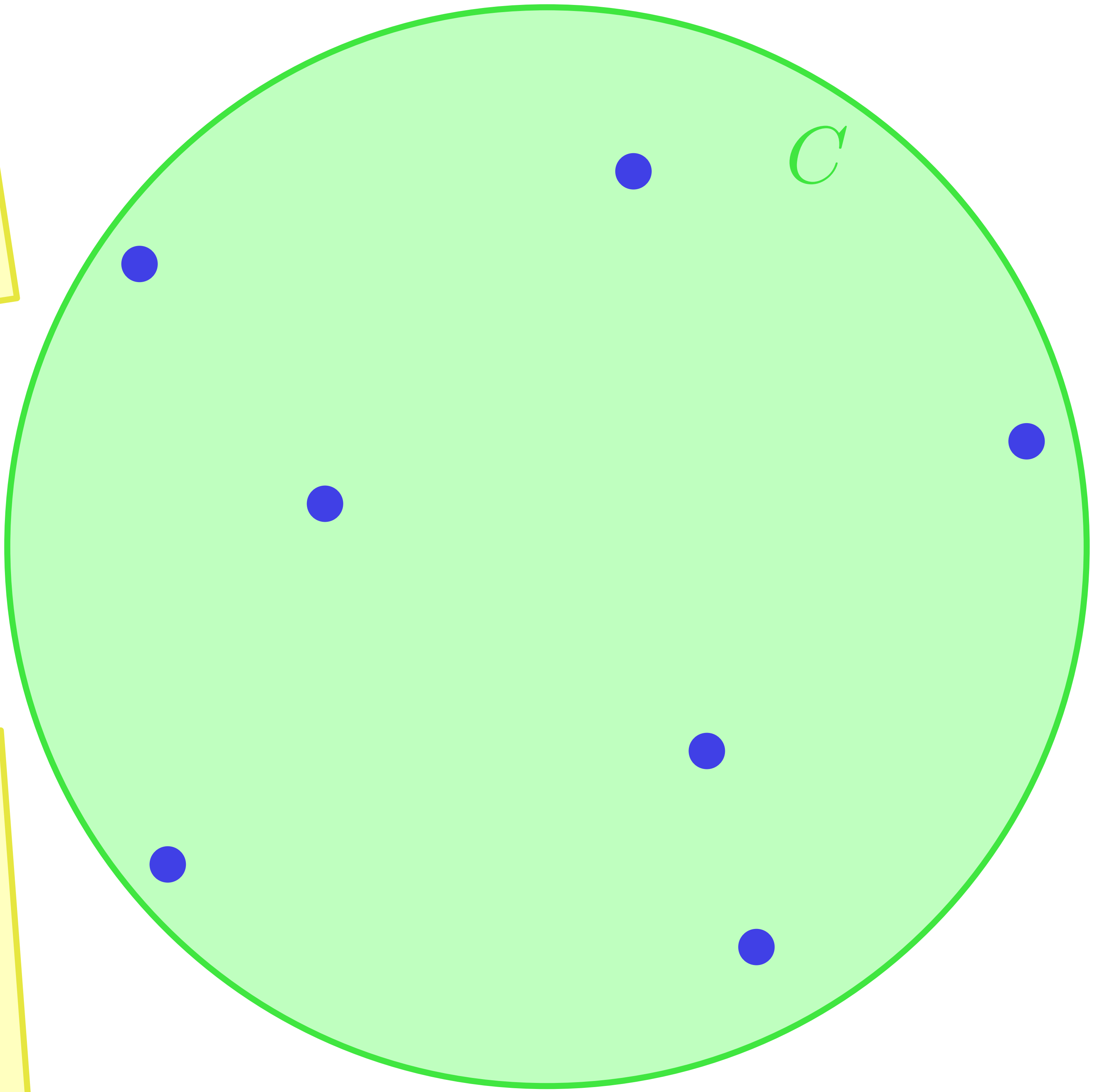

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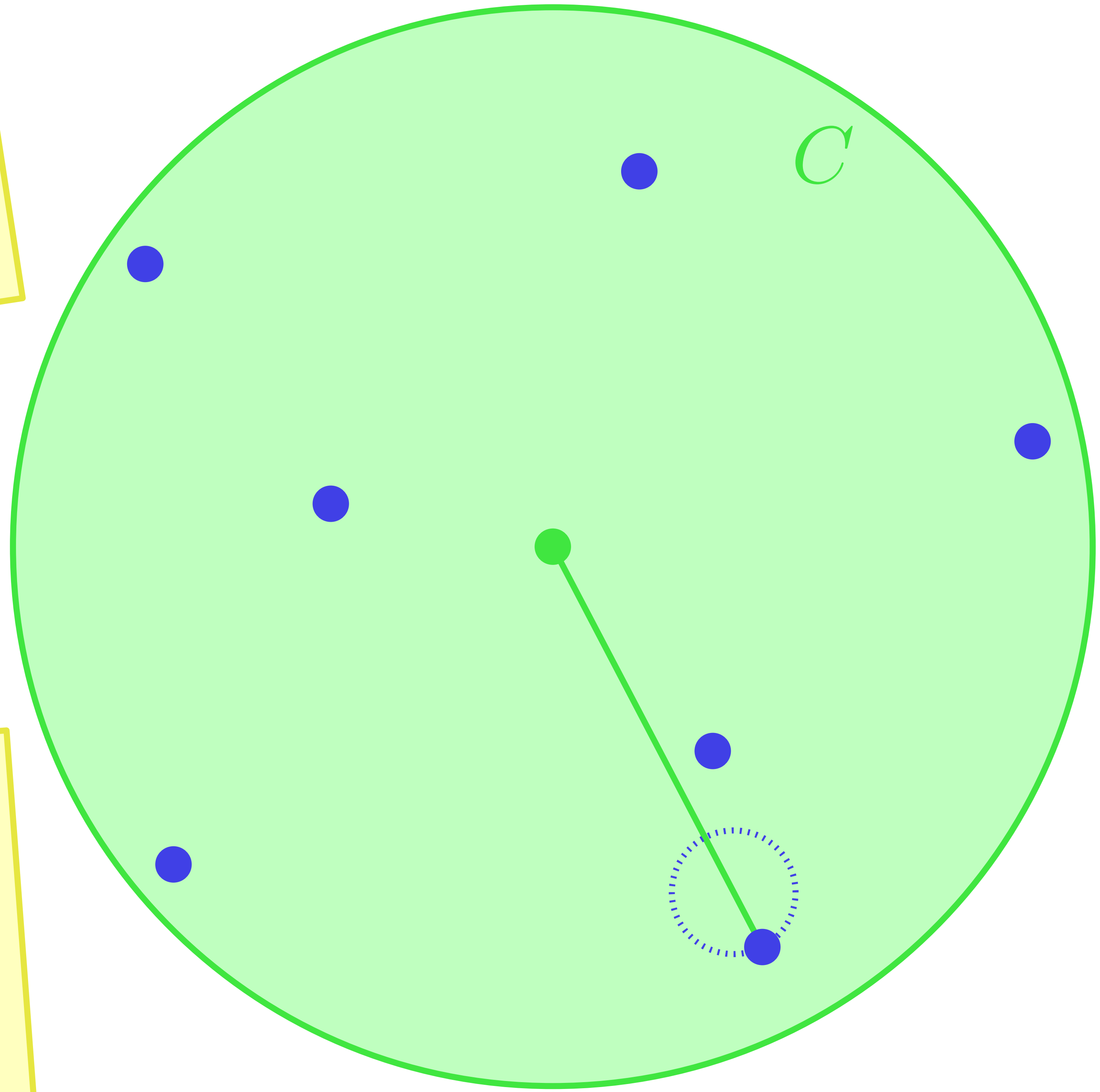

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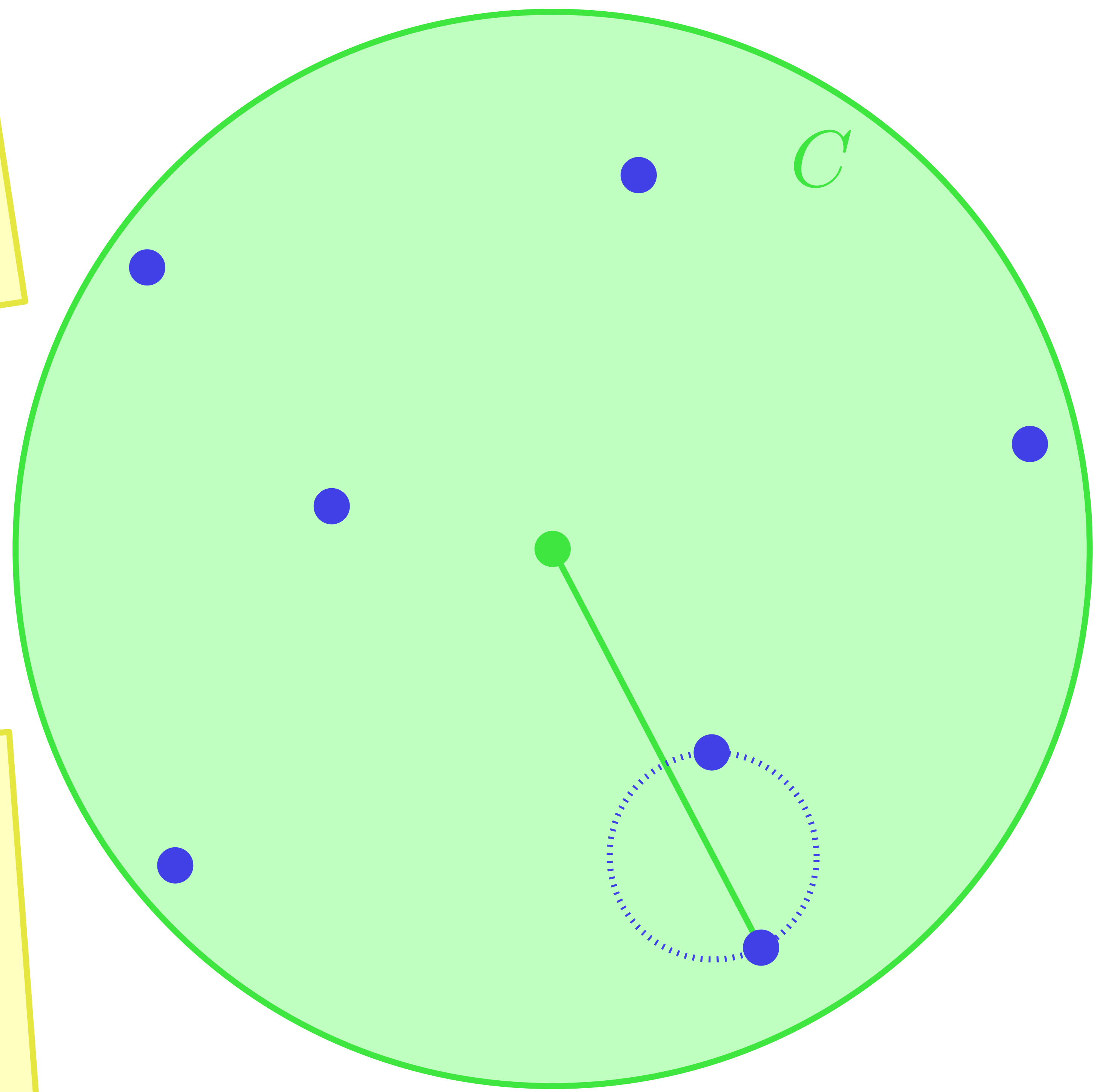

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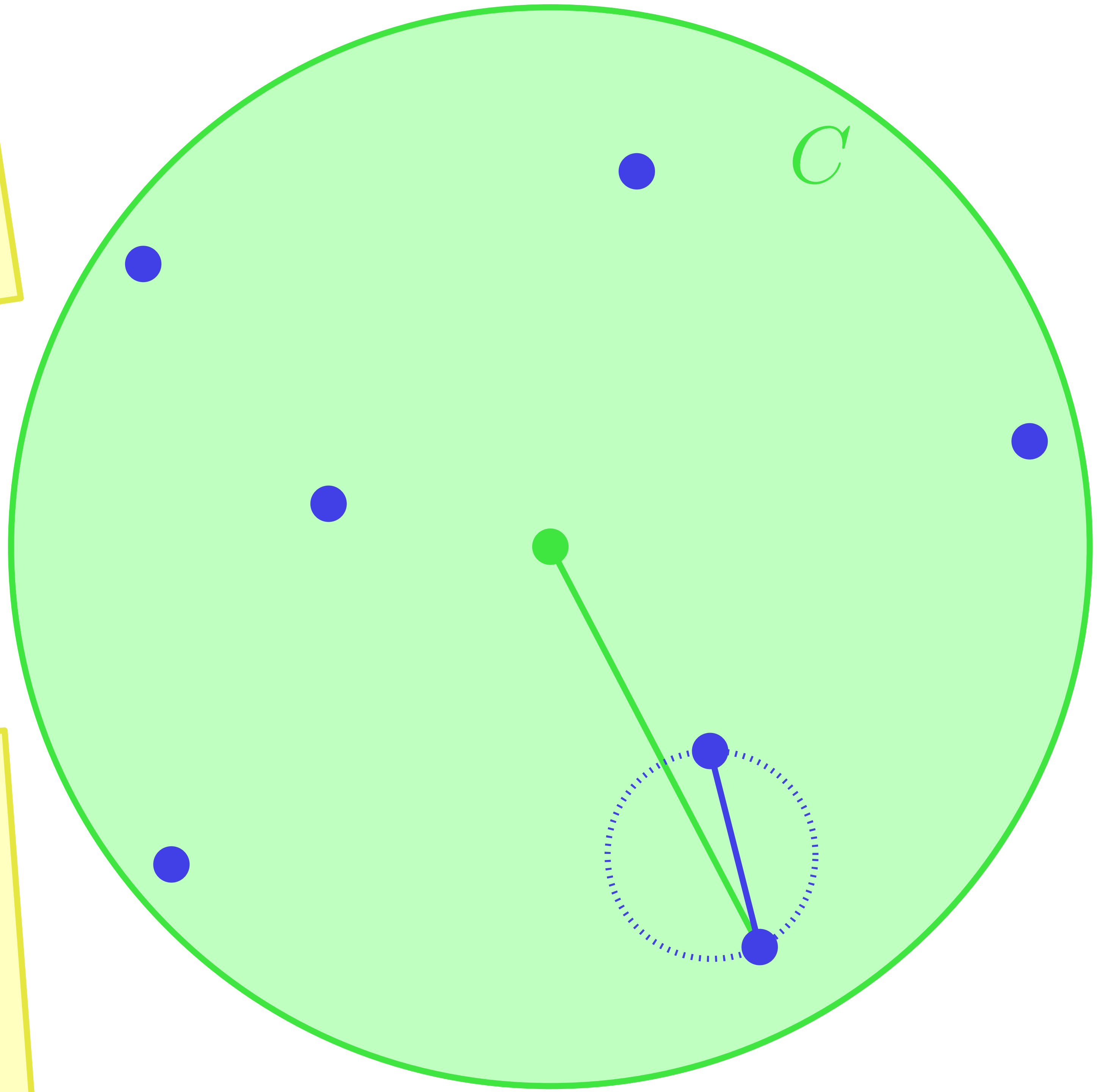

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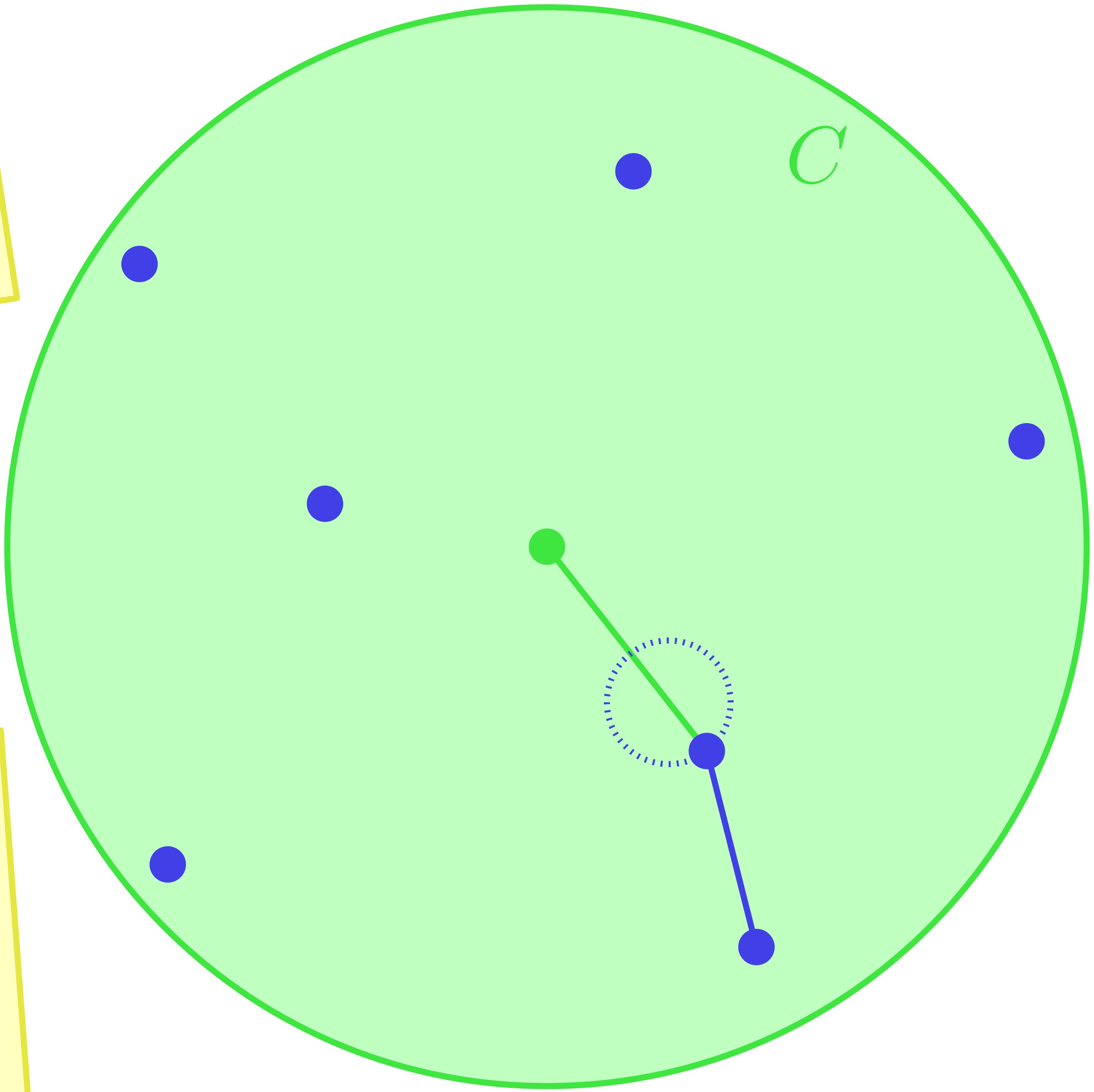

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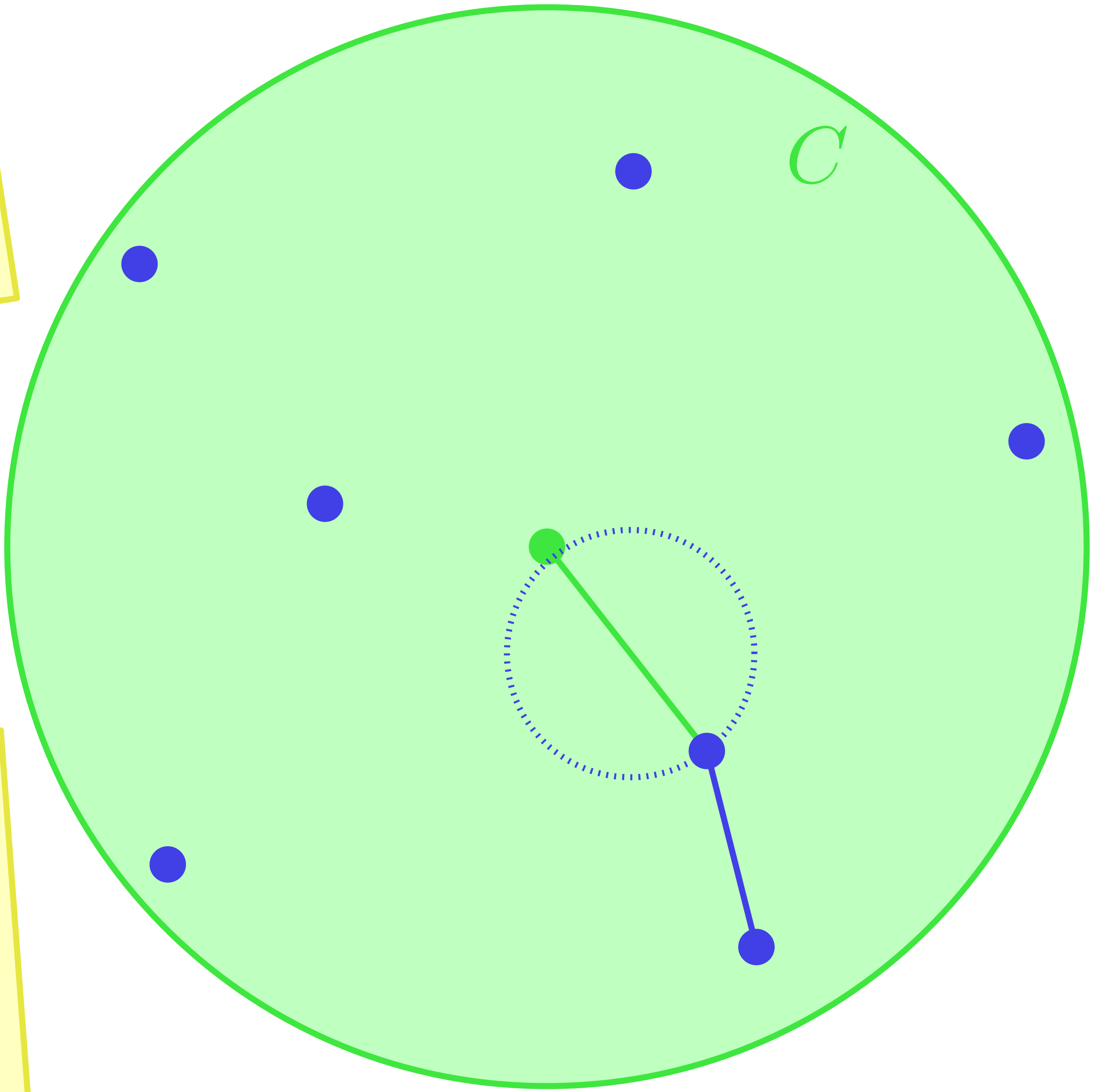

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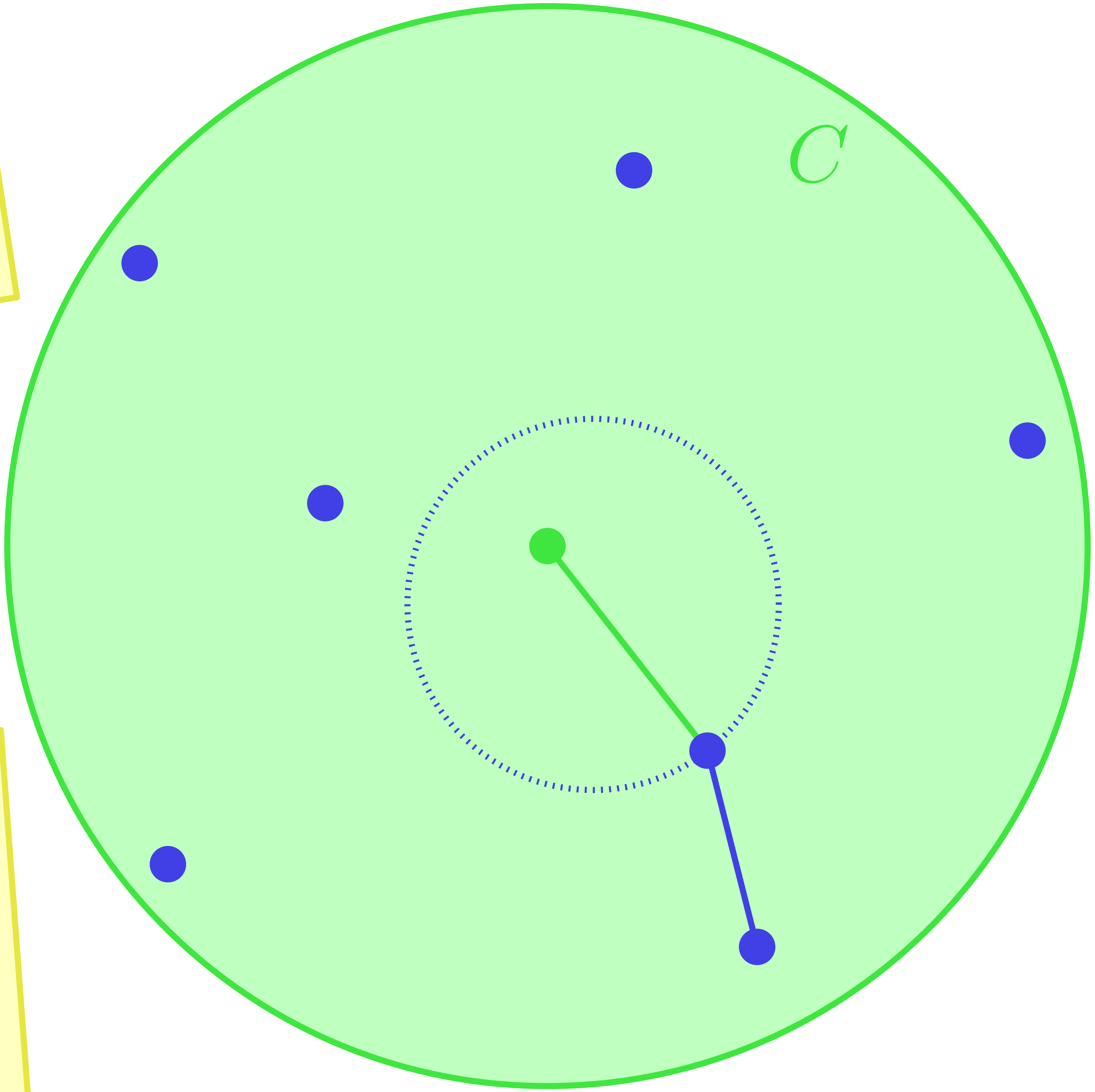

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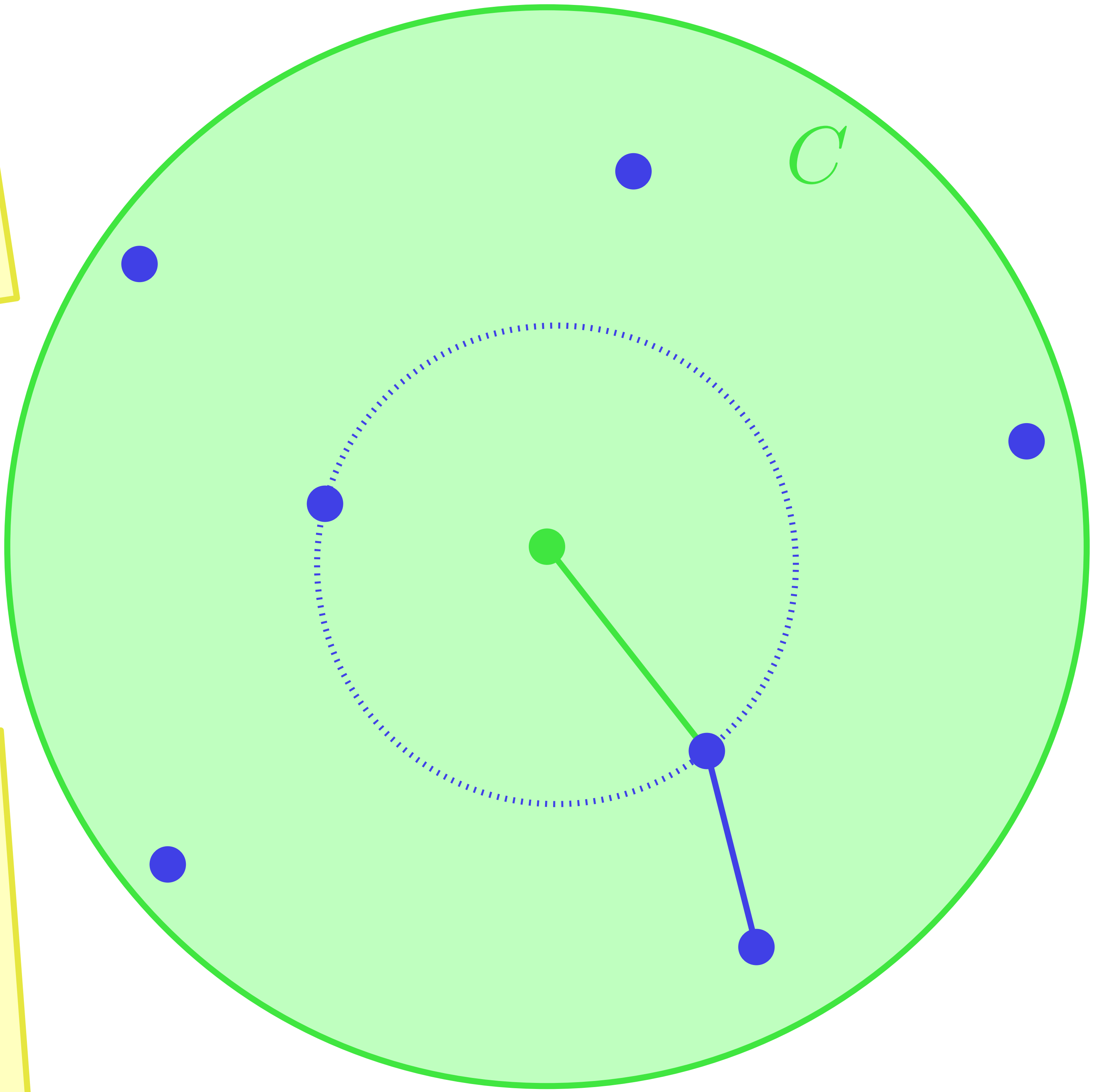

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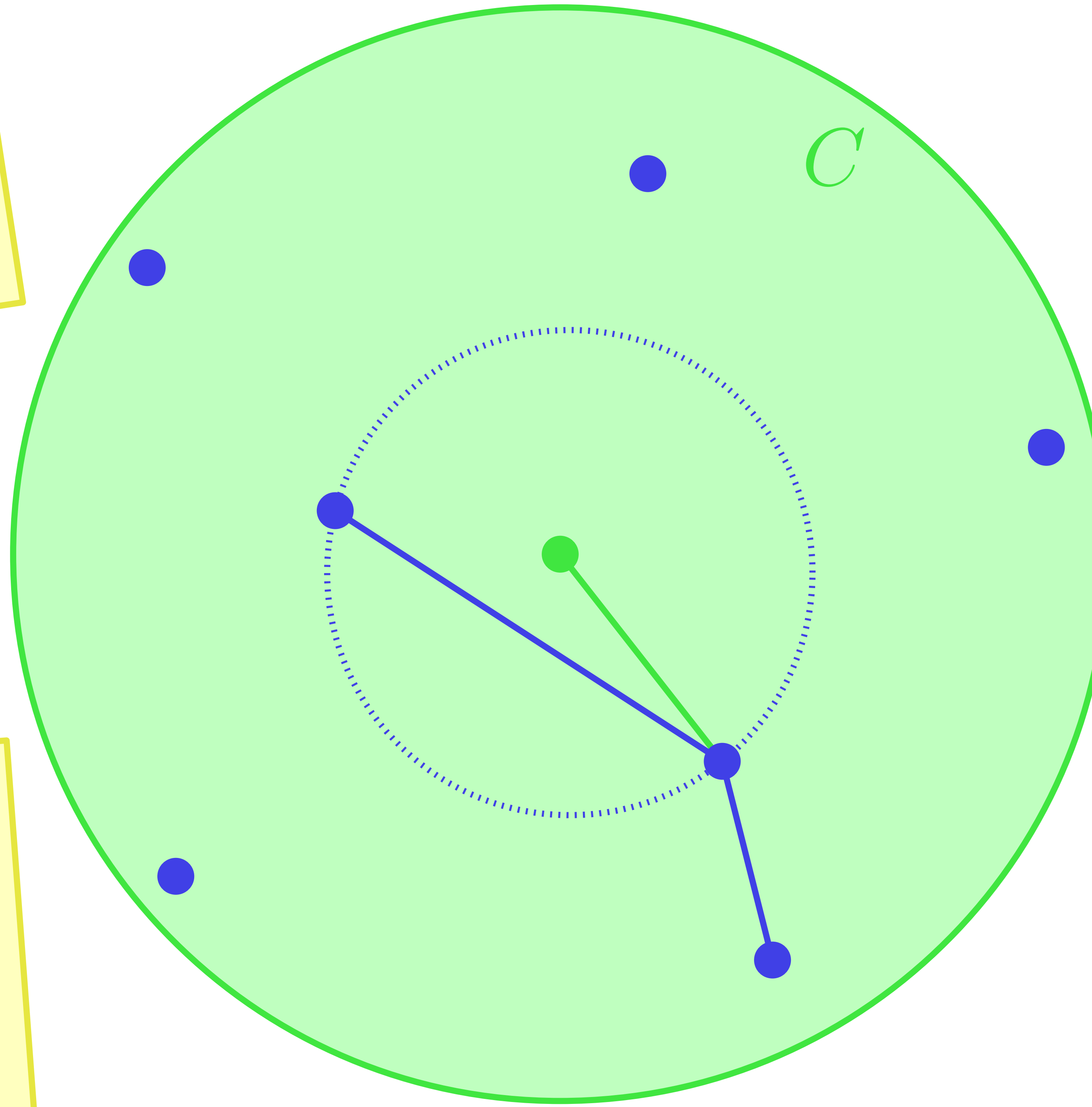

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An edge e of T is certified by C if there is an empty circle for e completely inside C.

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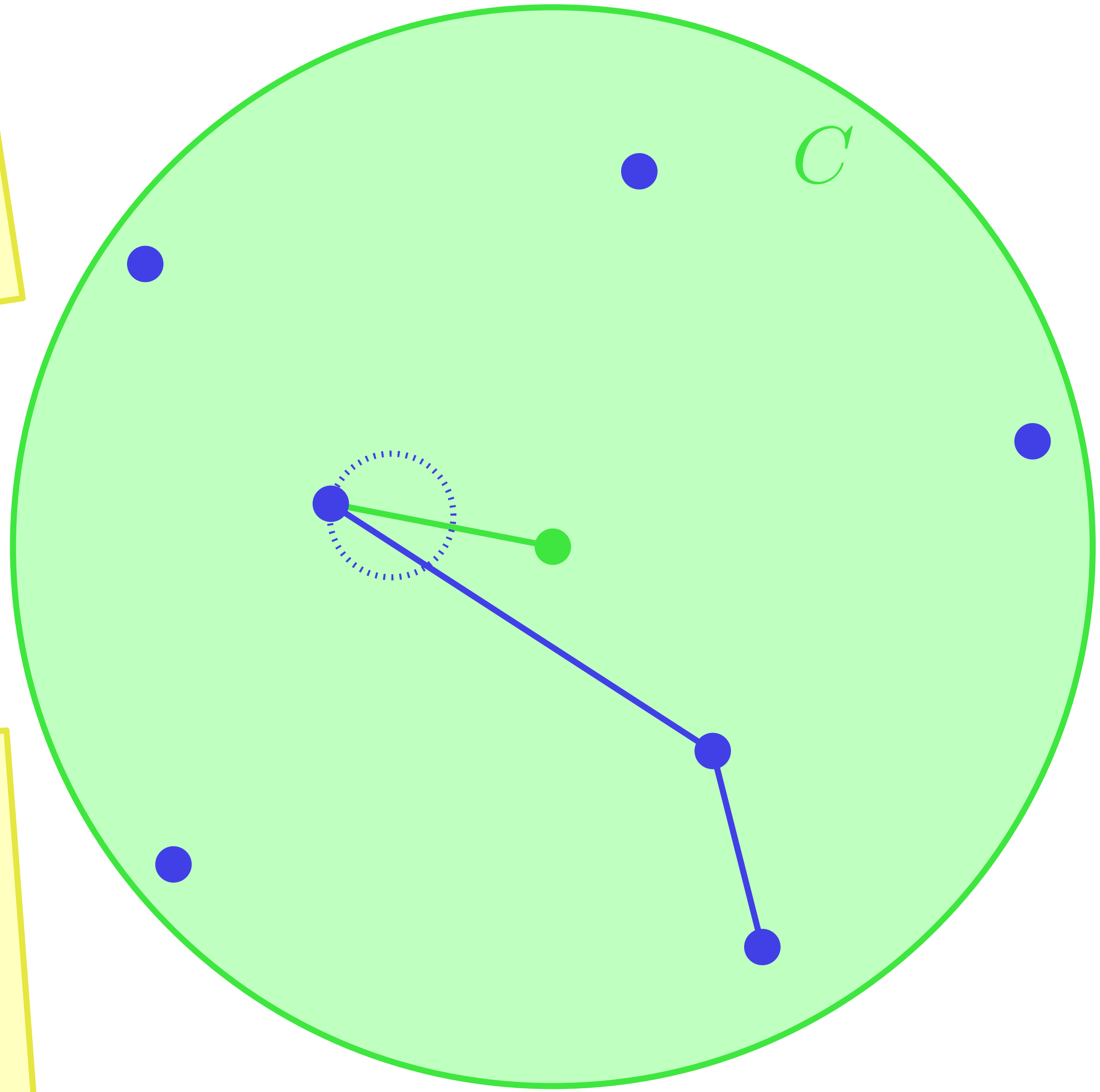

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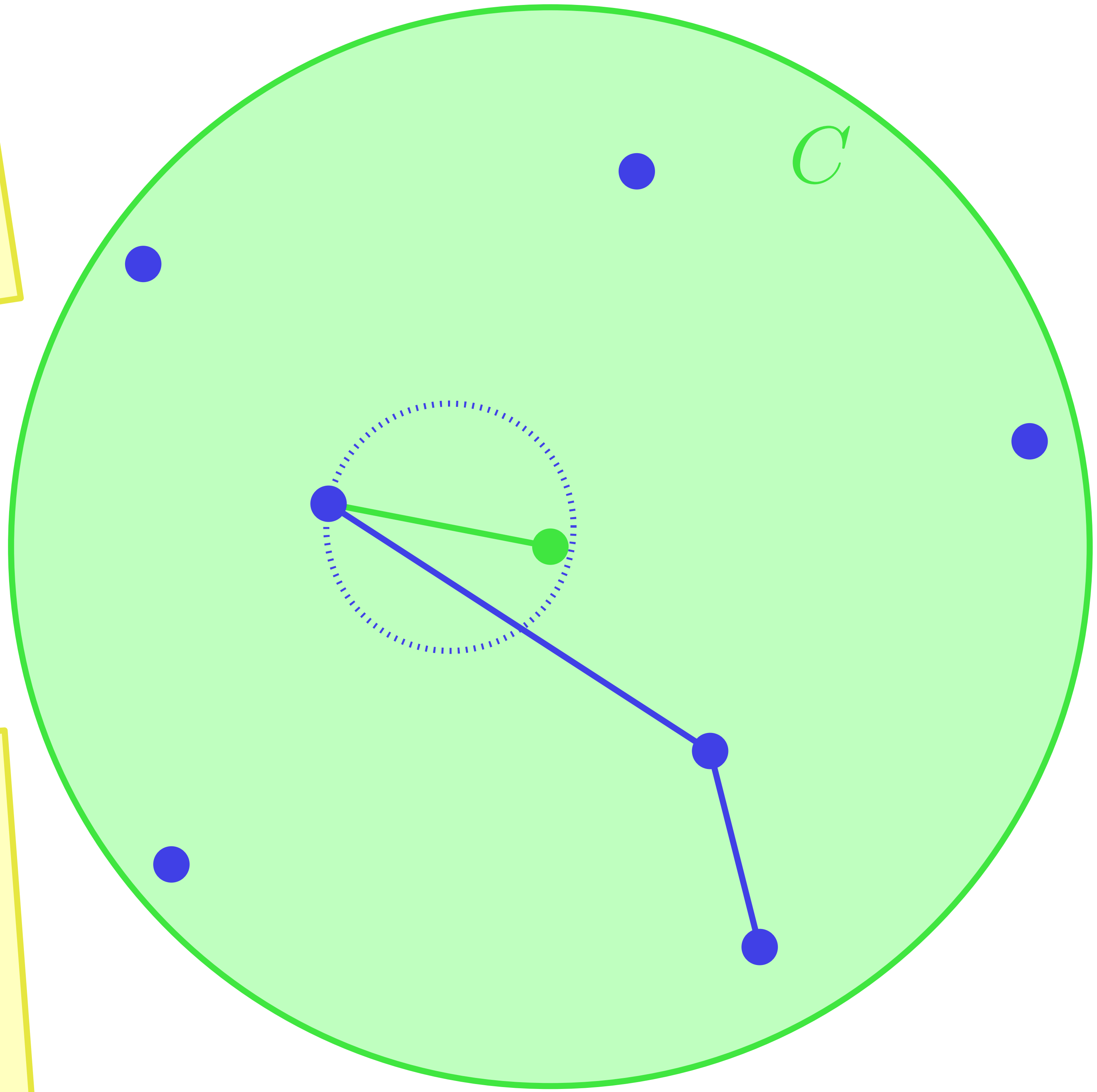

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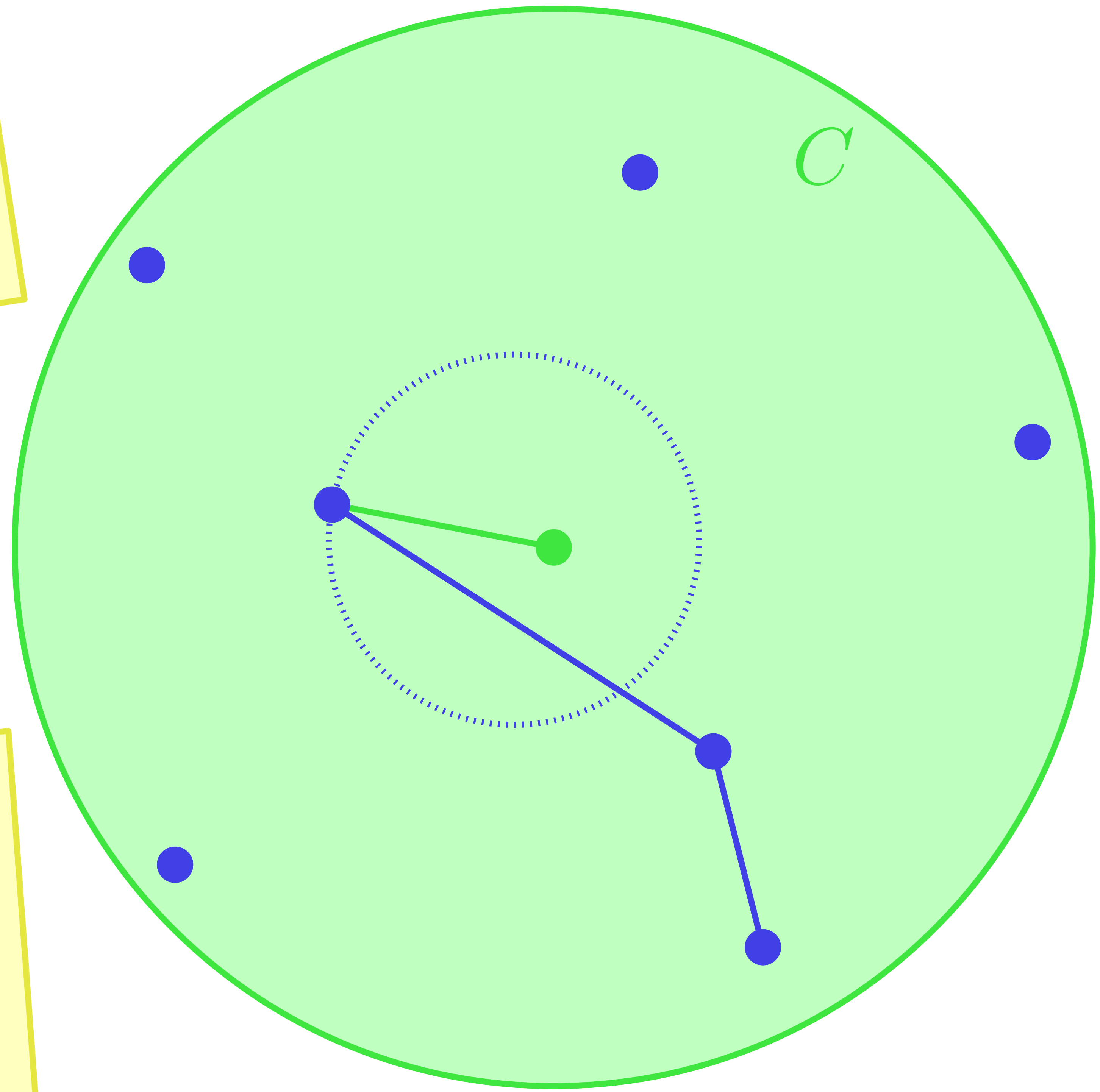

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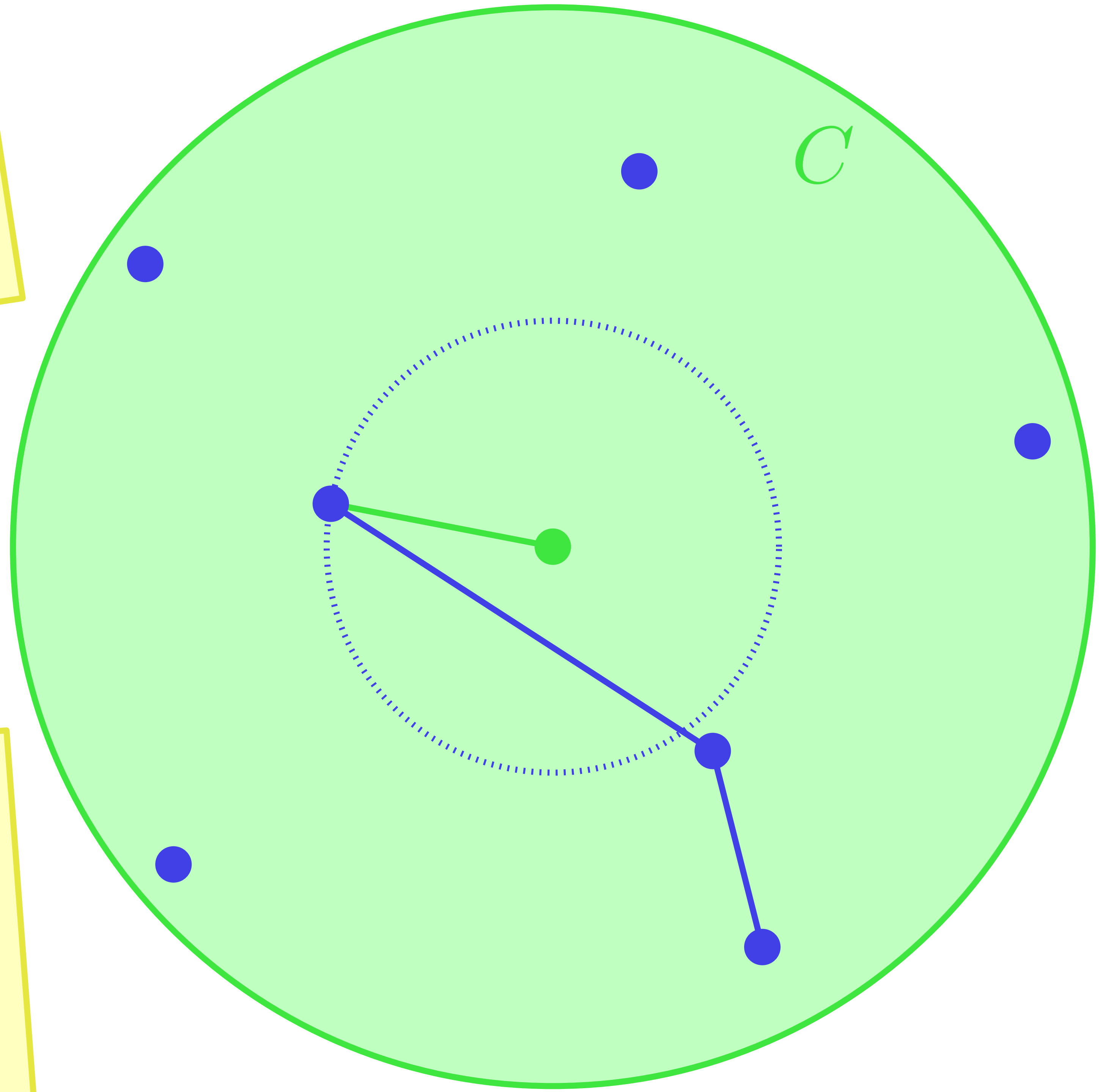

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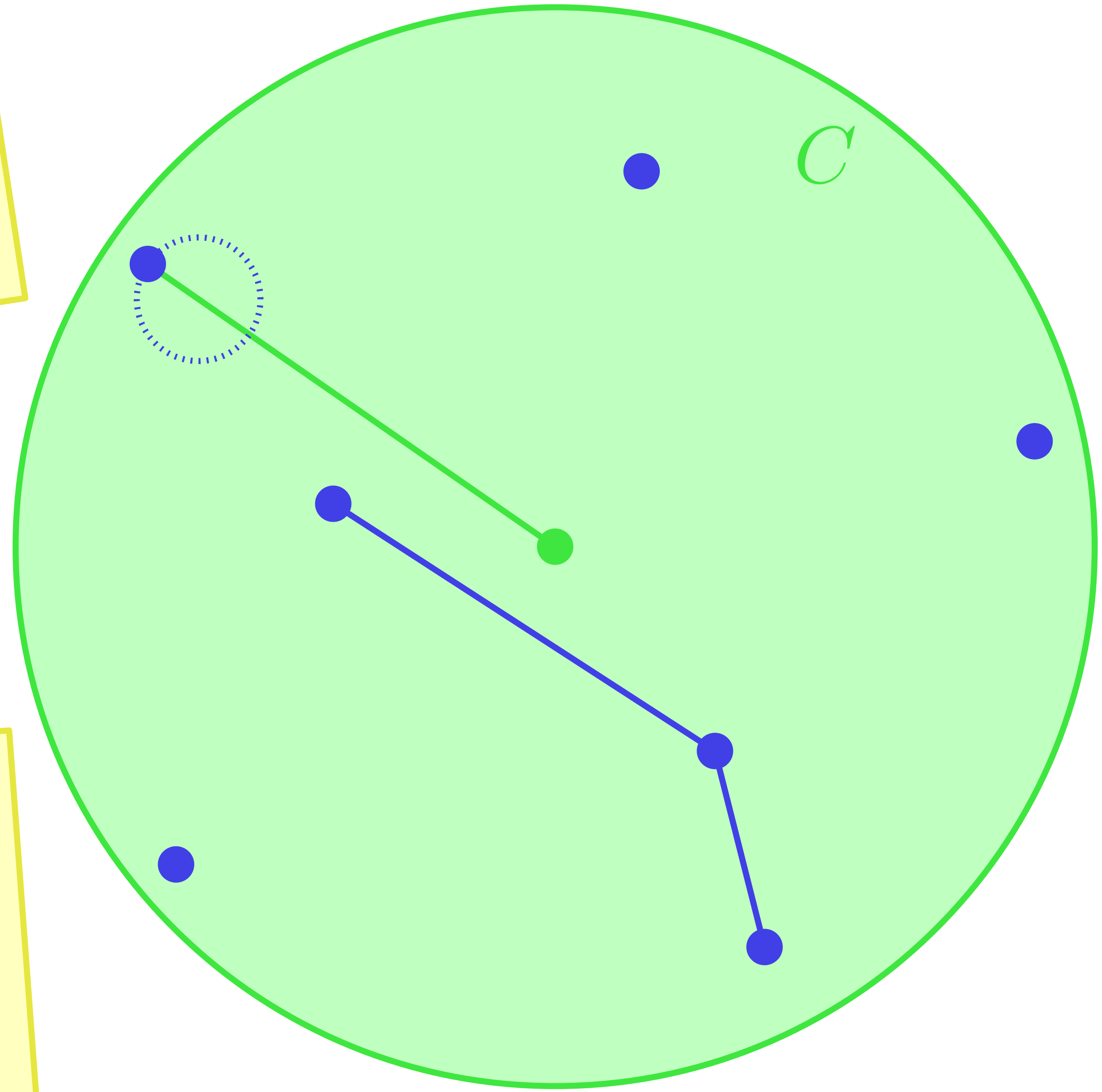

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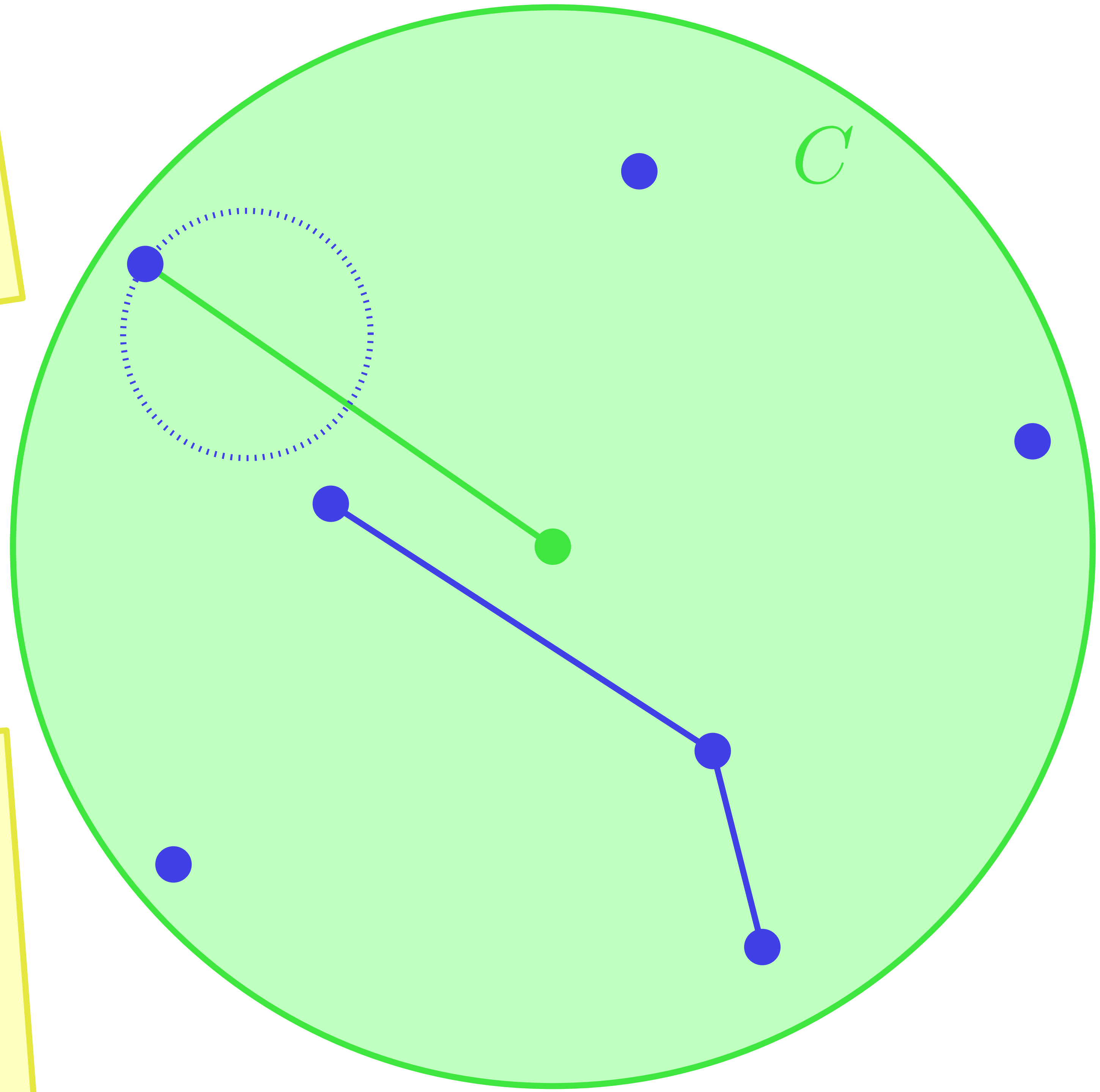

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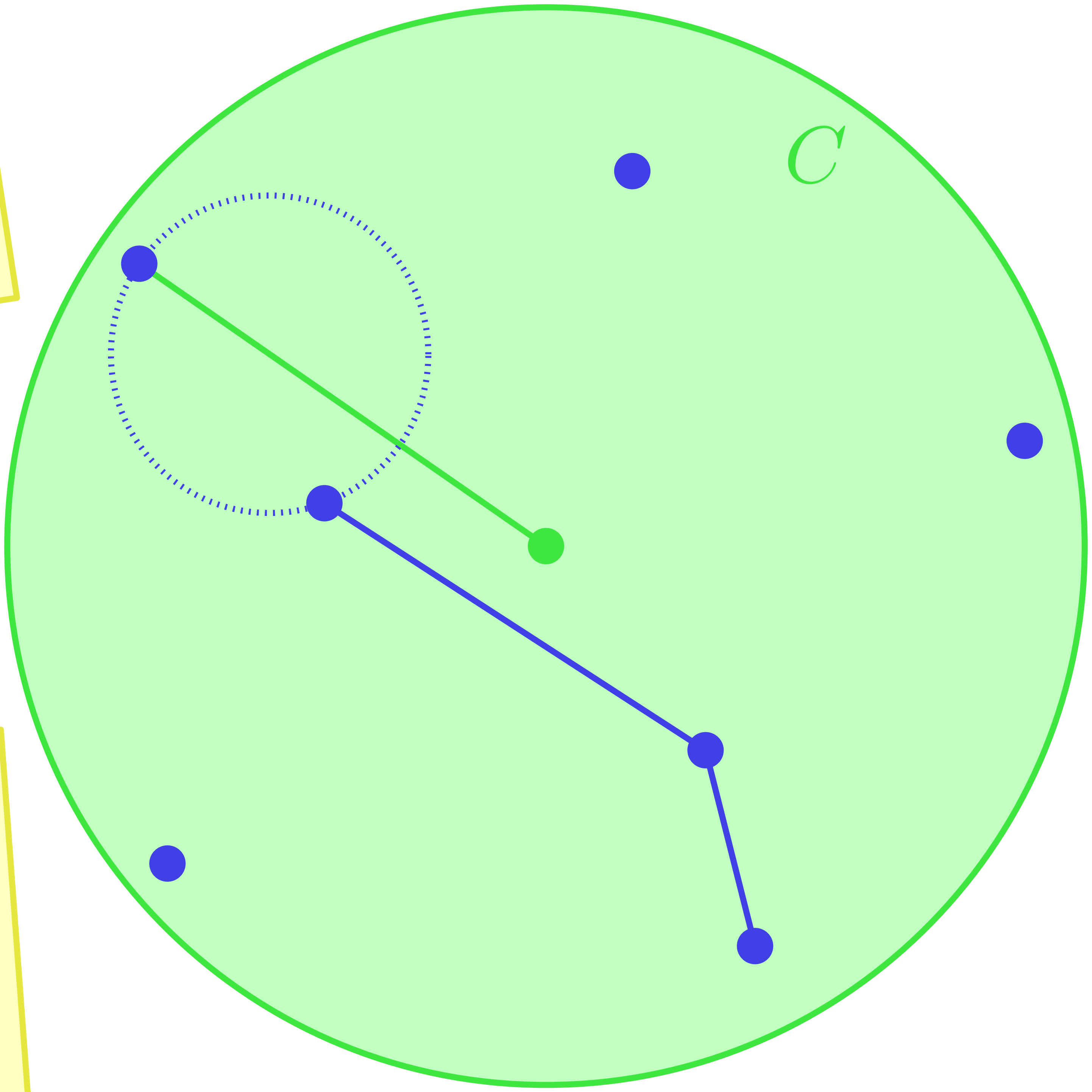

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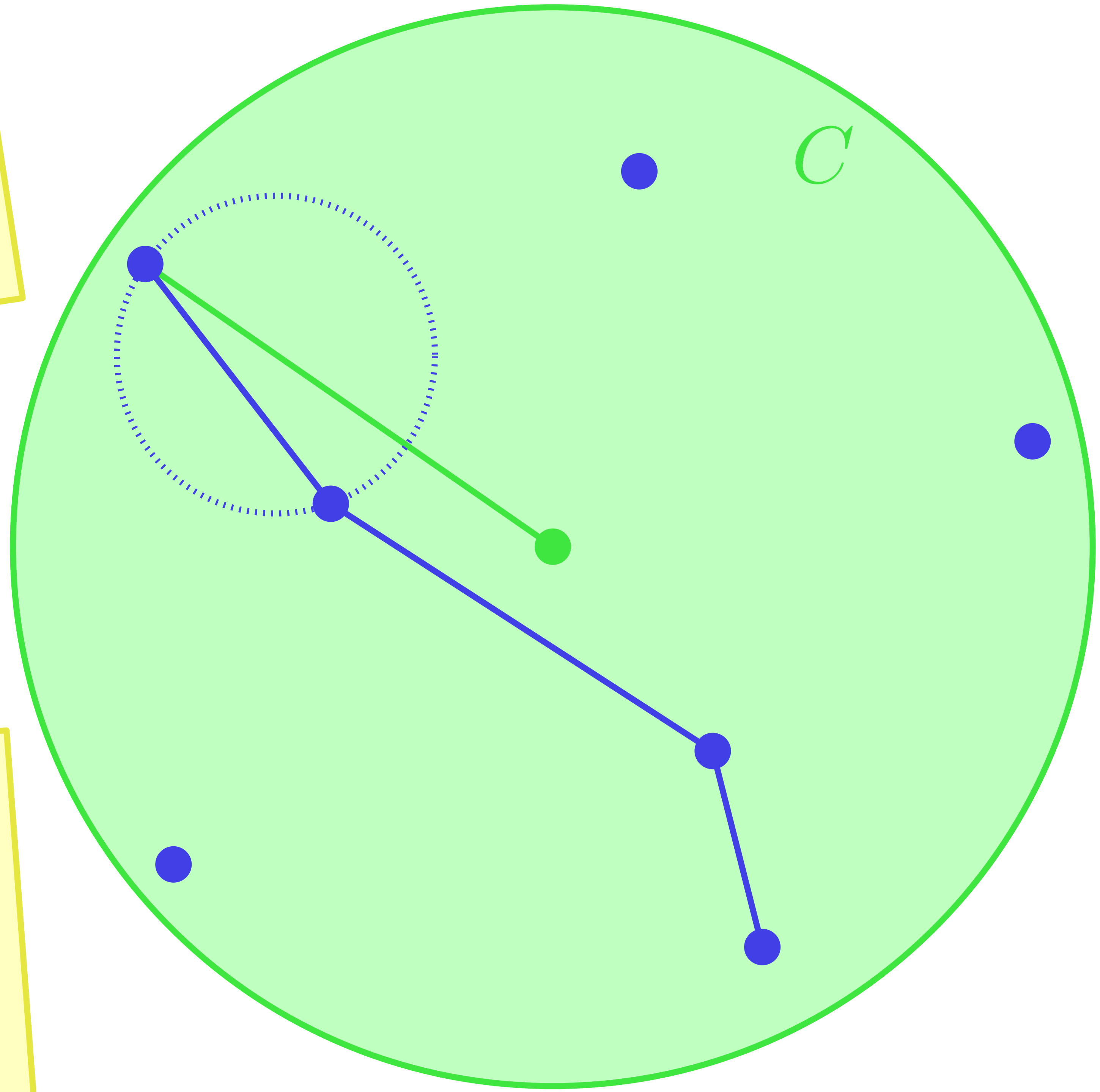

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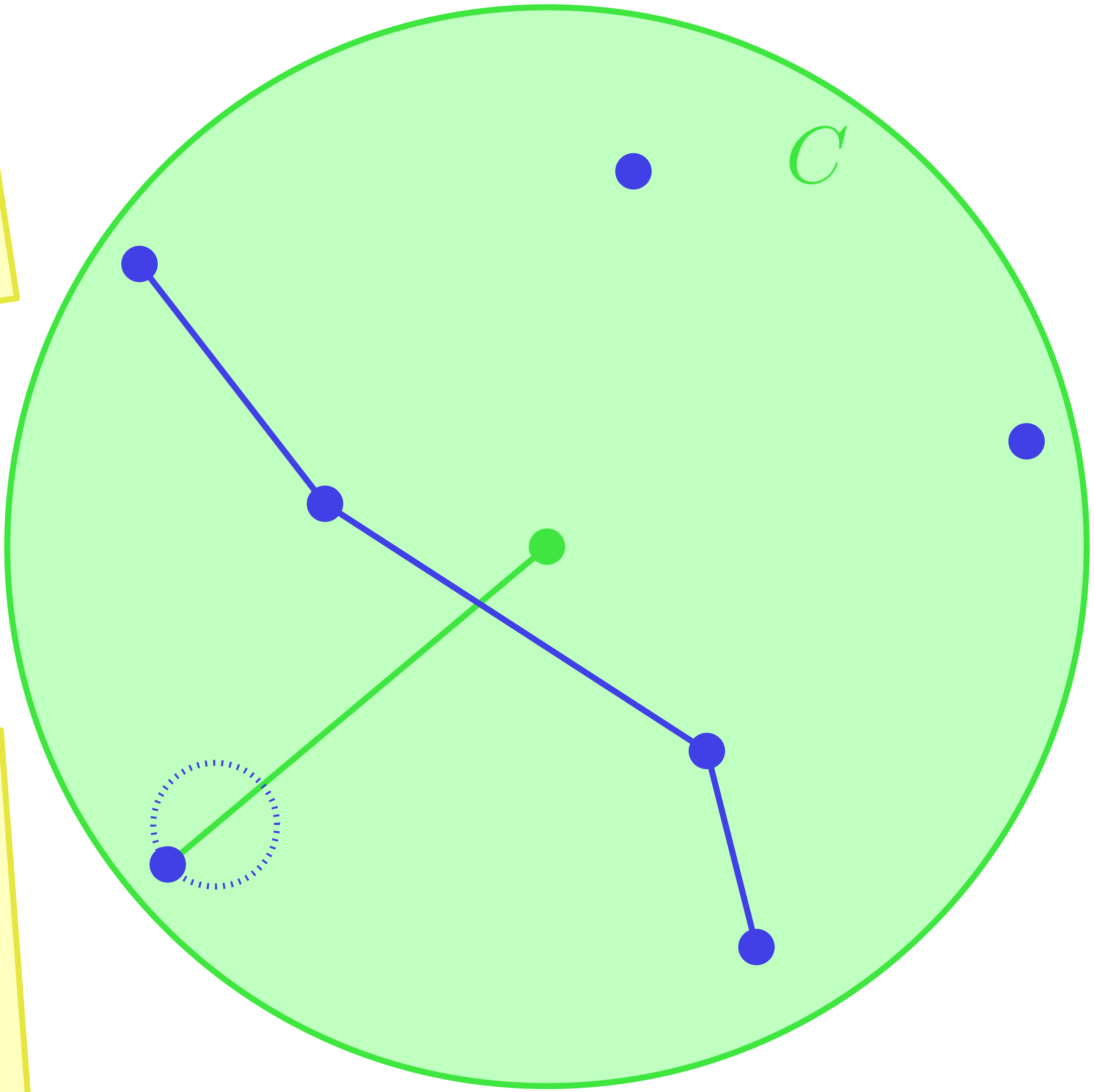

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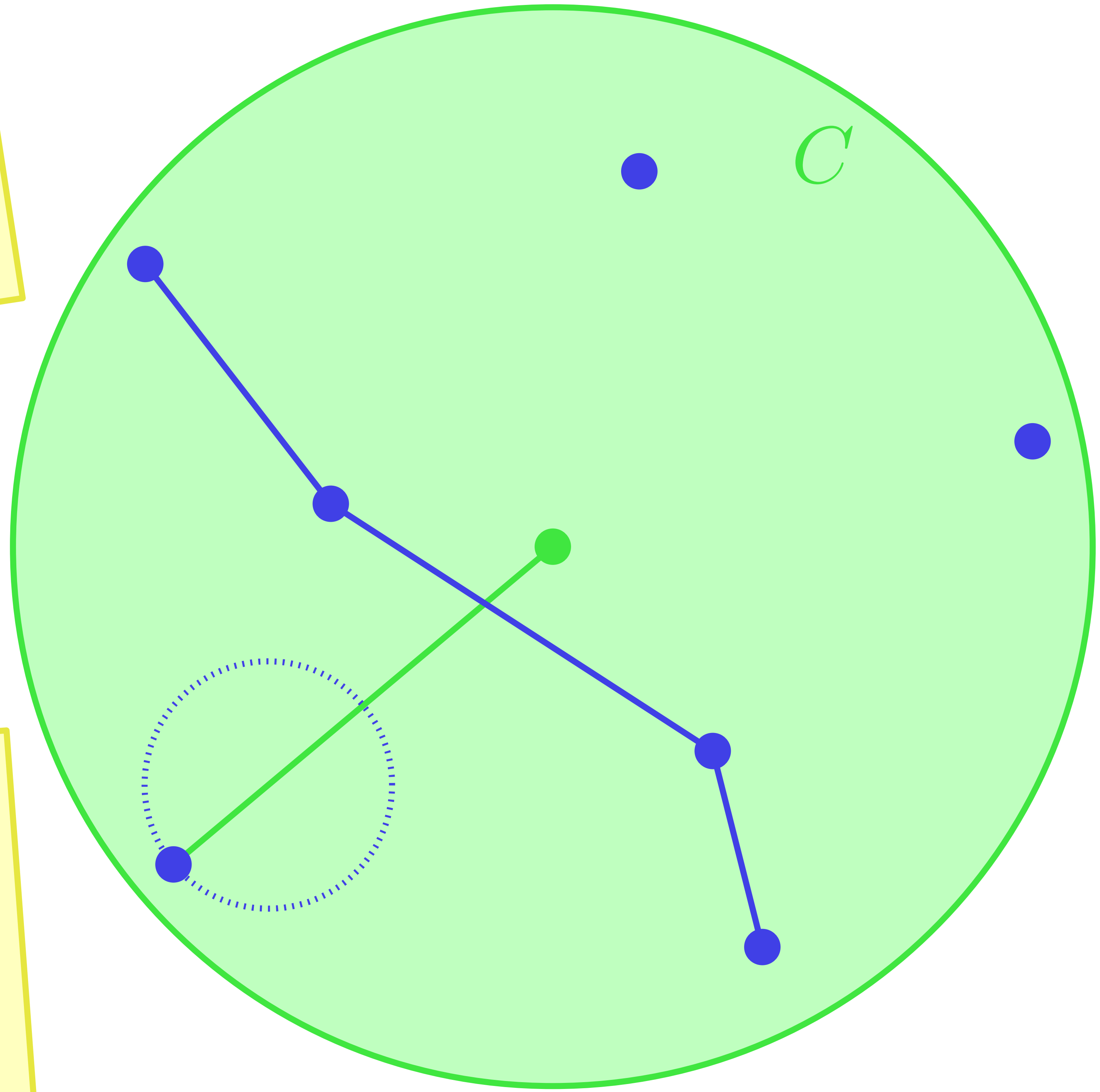

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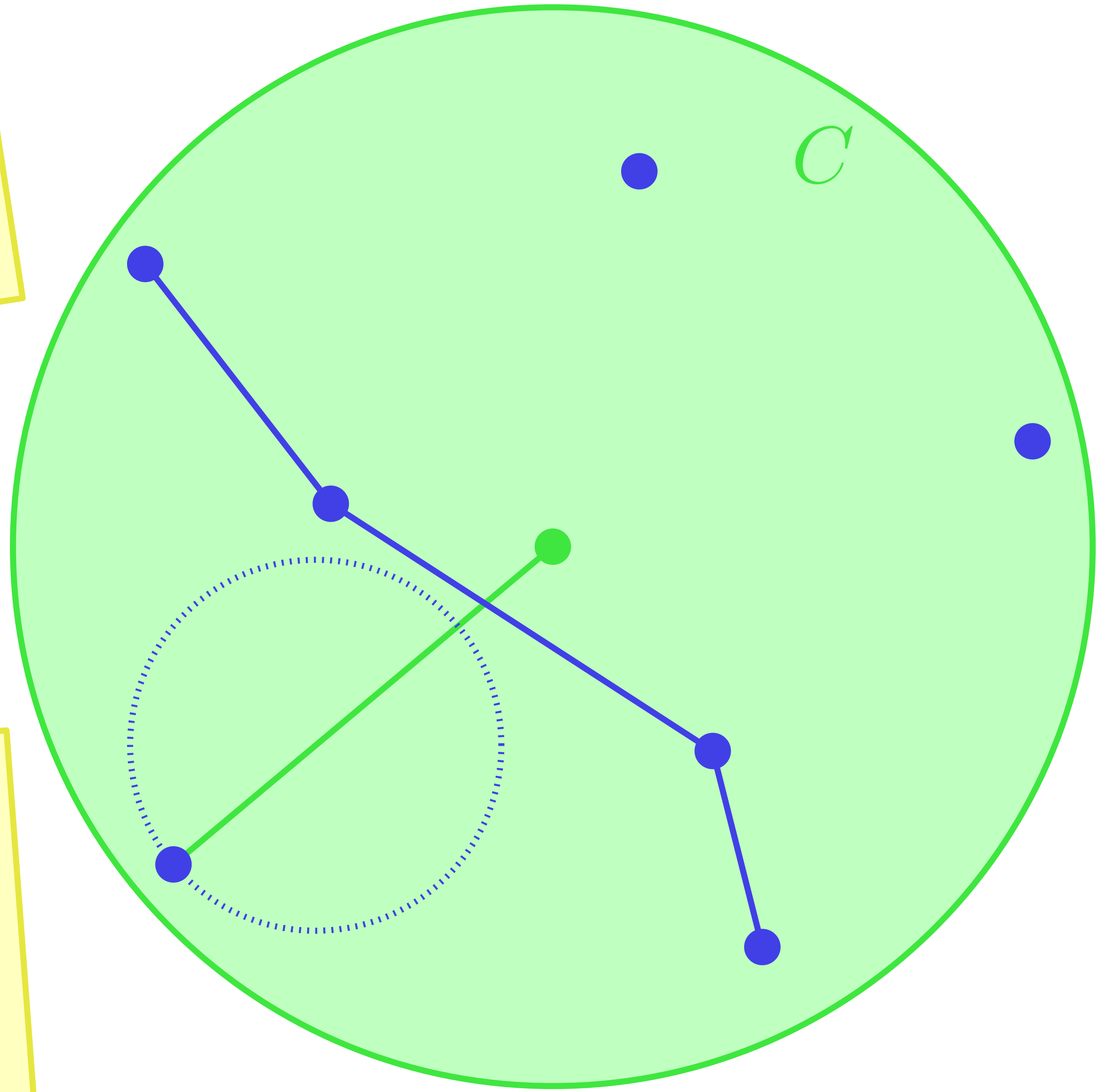

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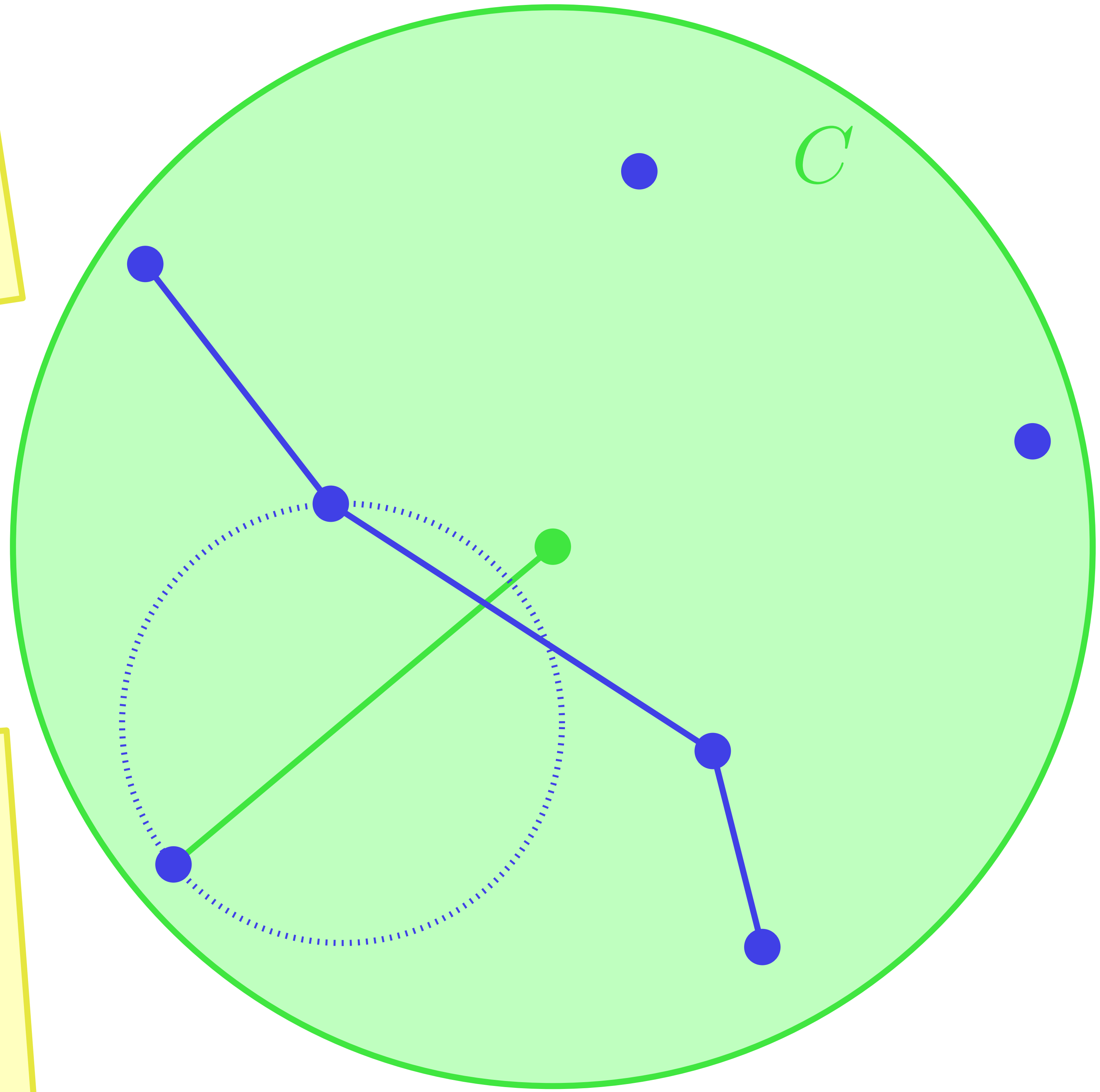

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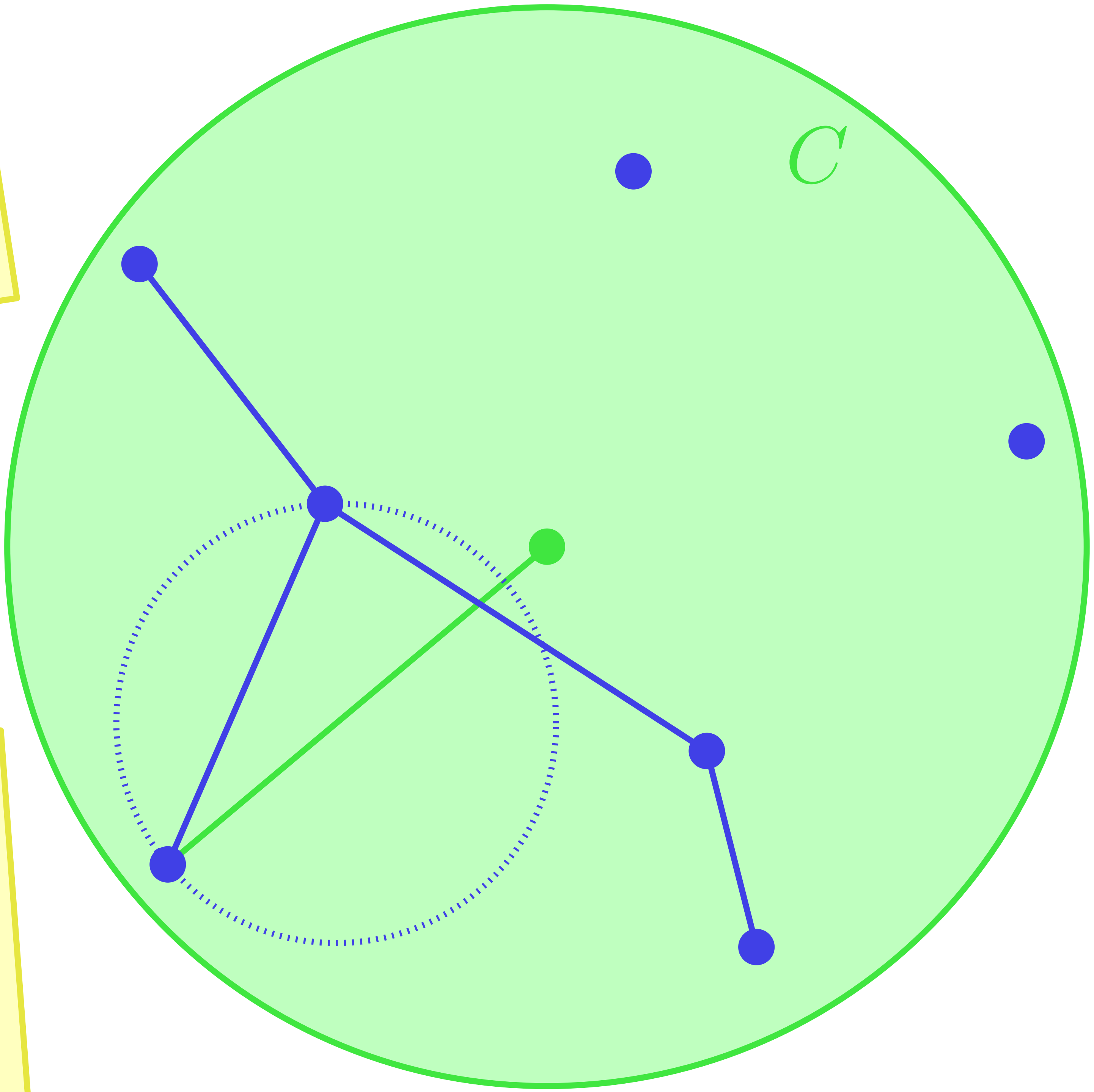

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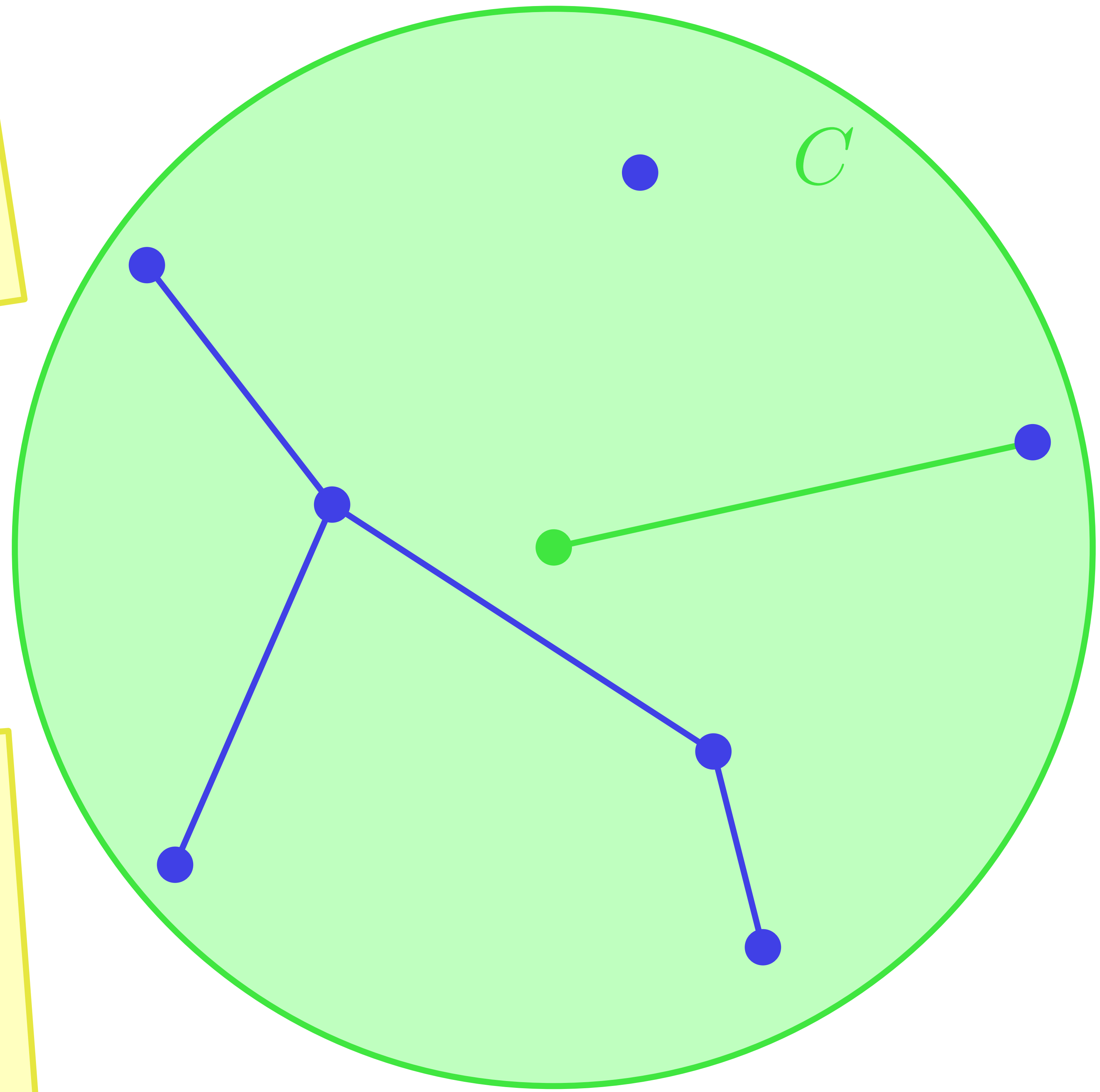

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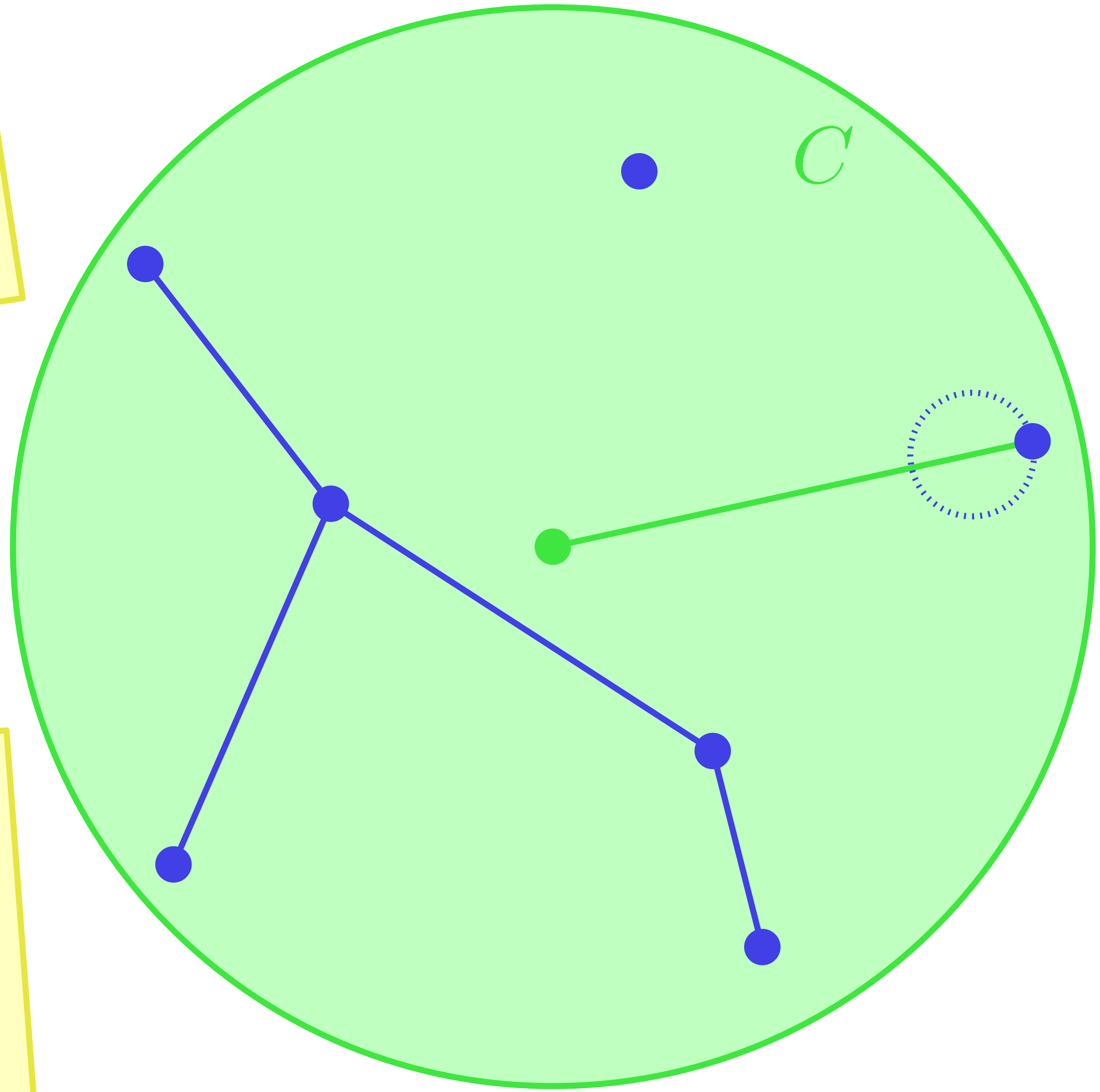

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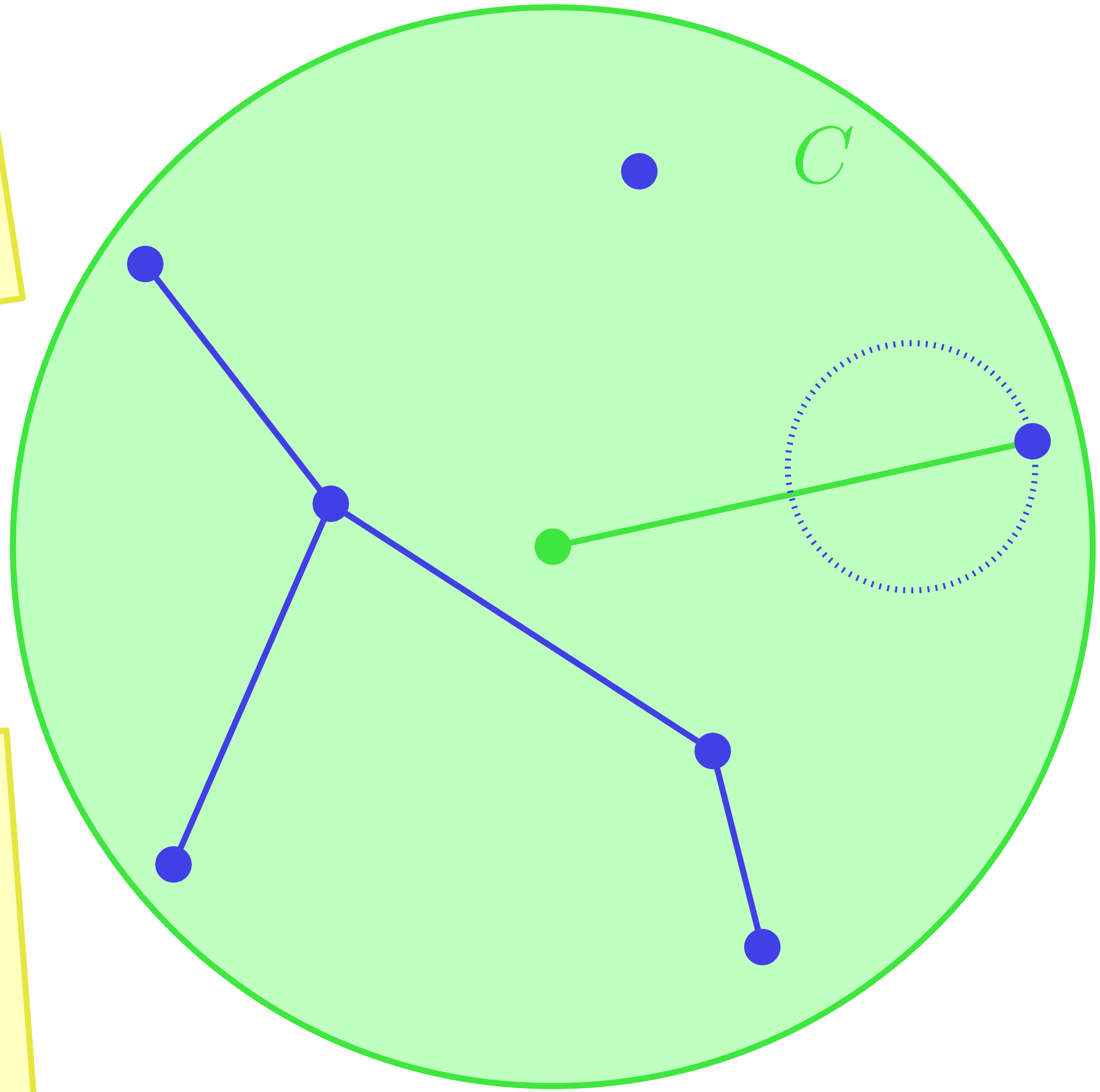

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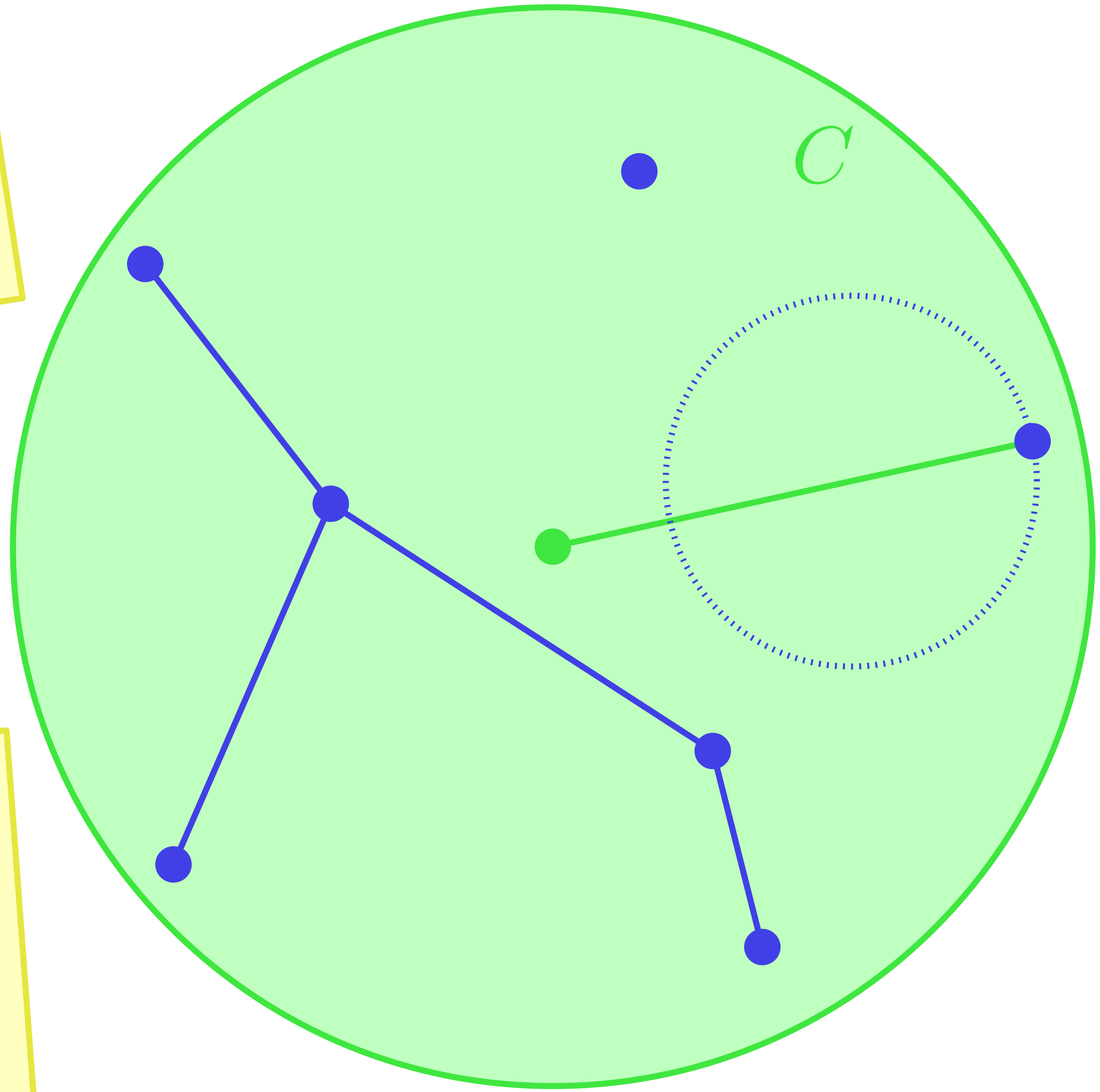

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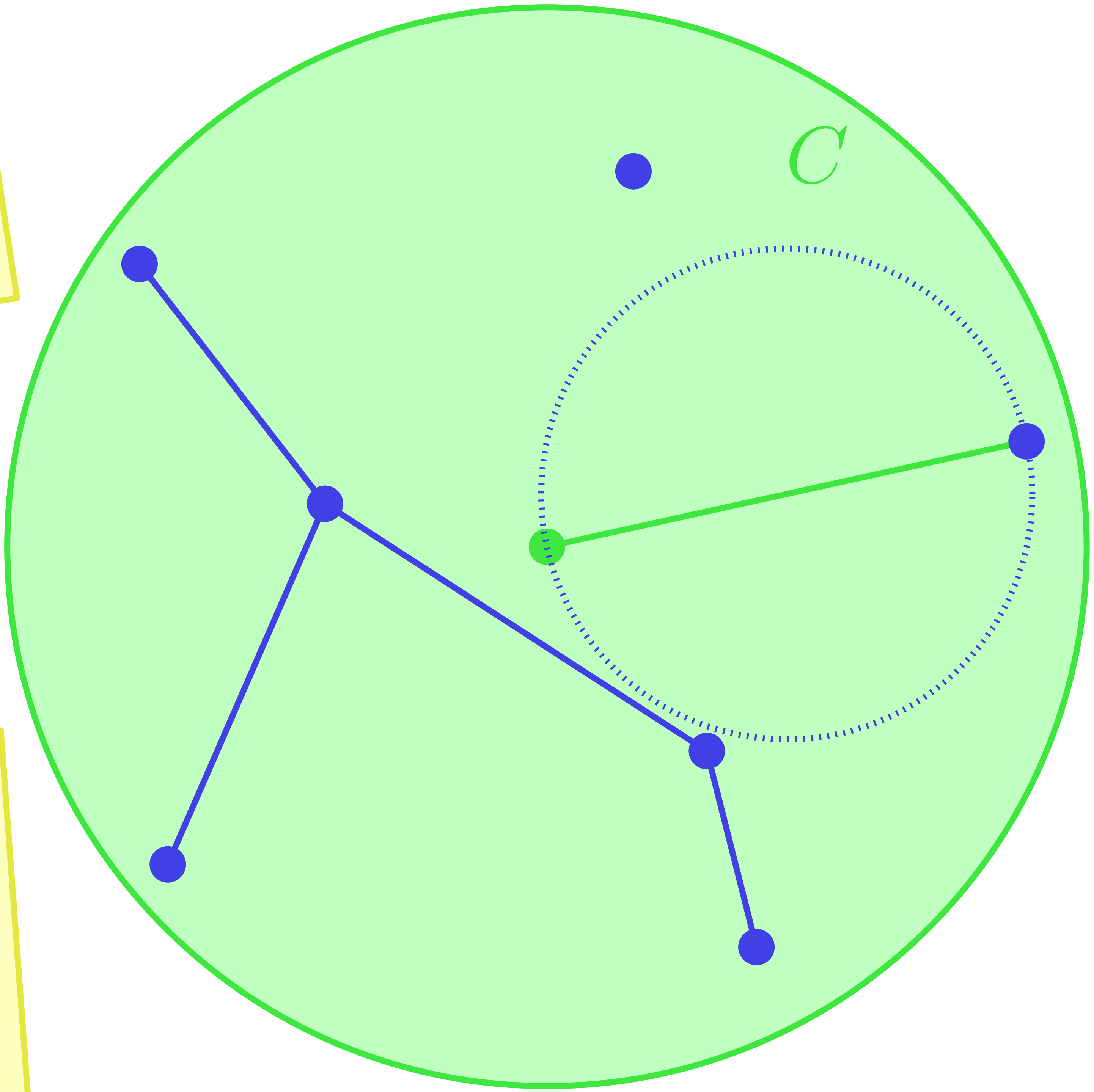

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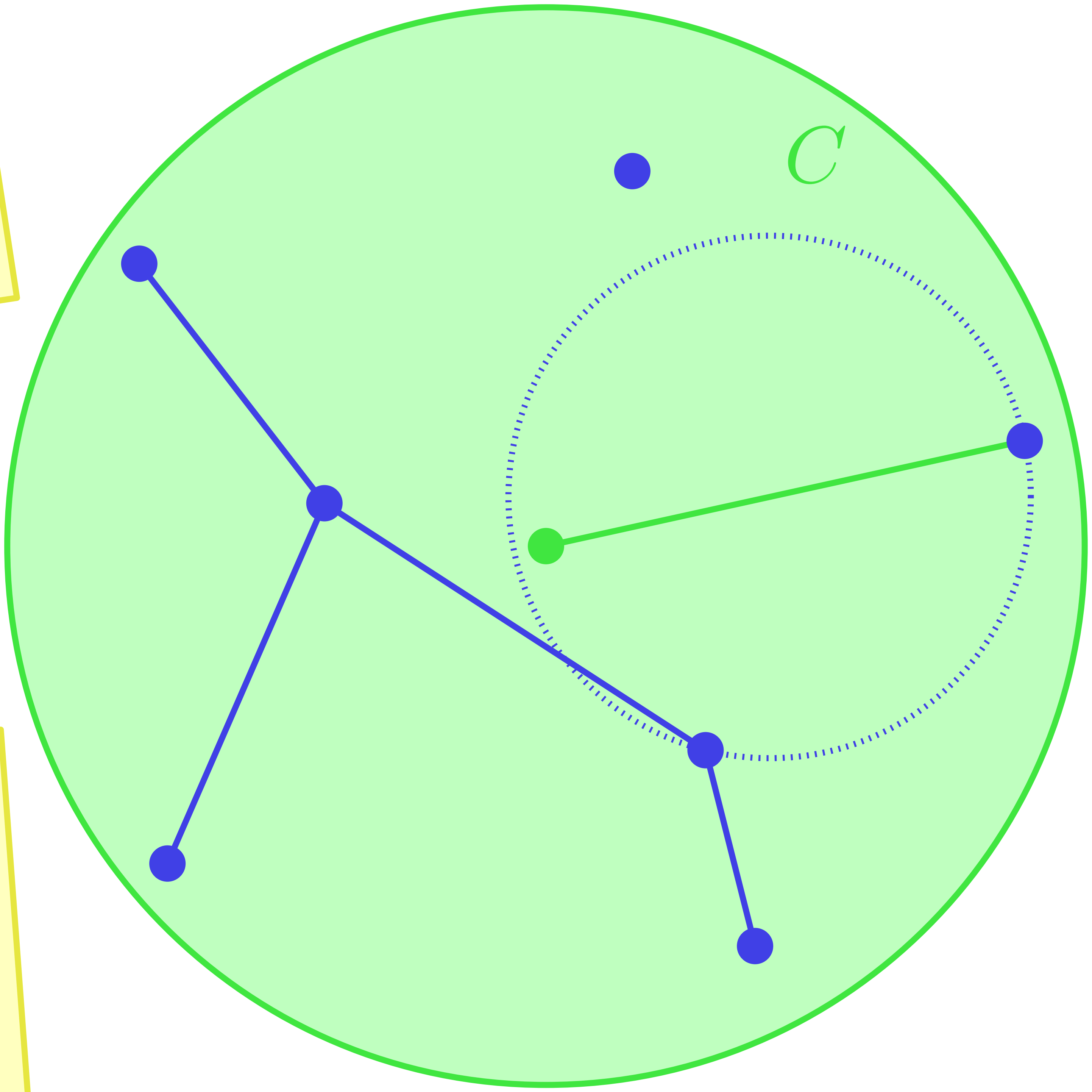

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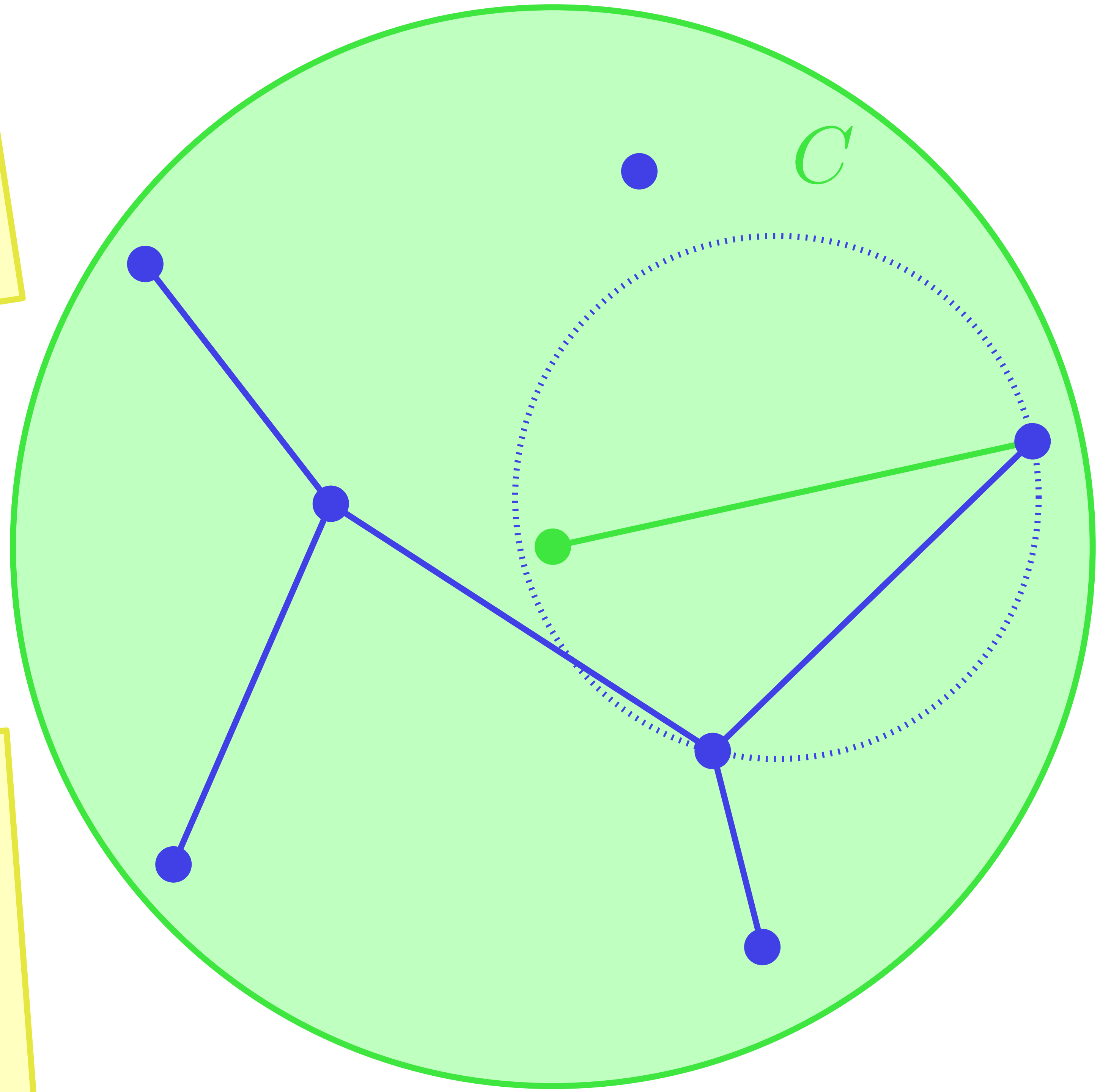

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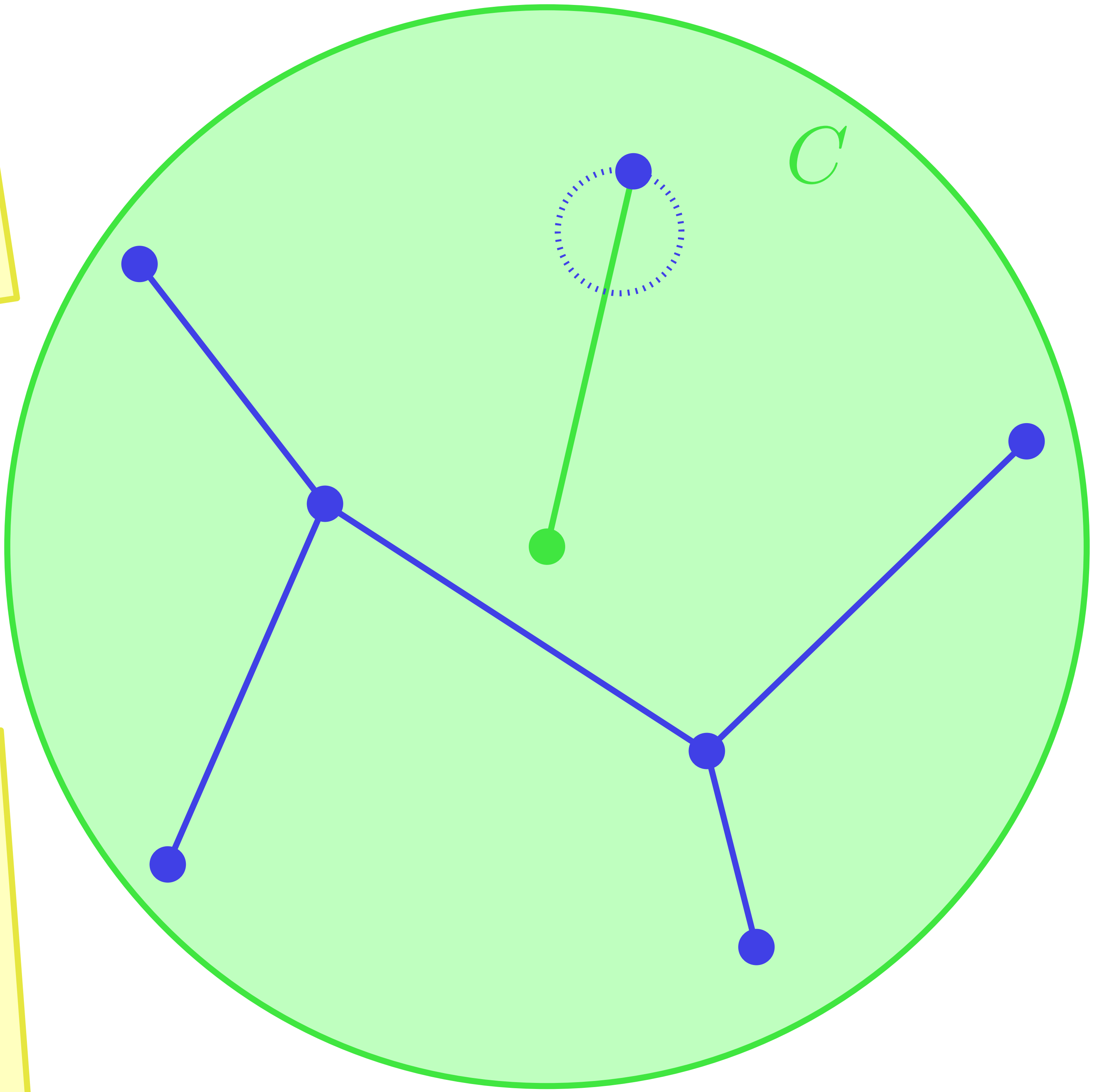

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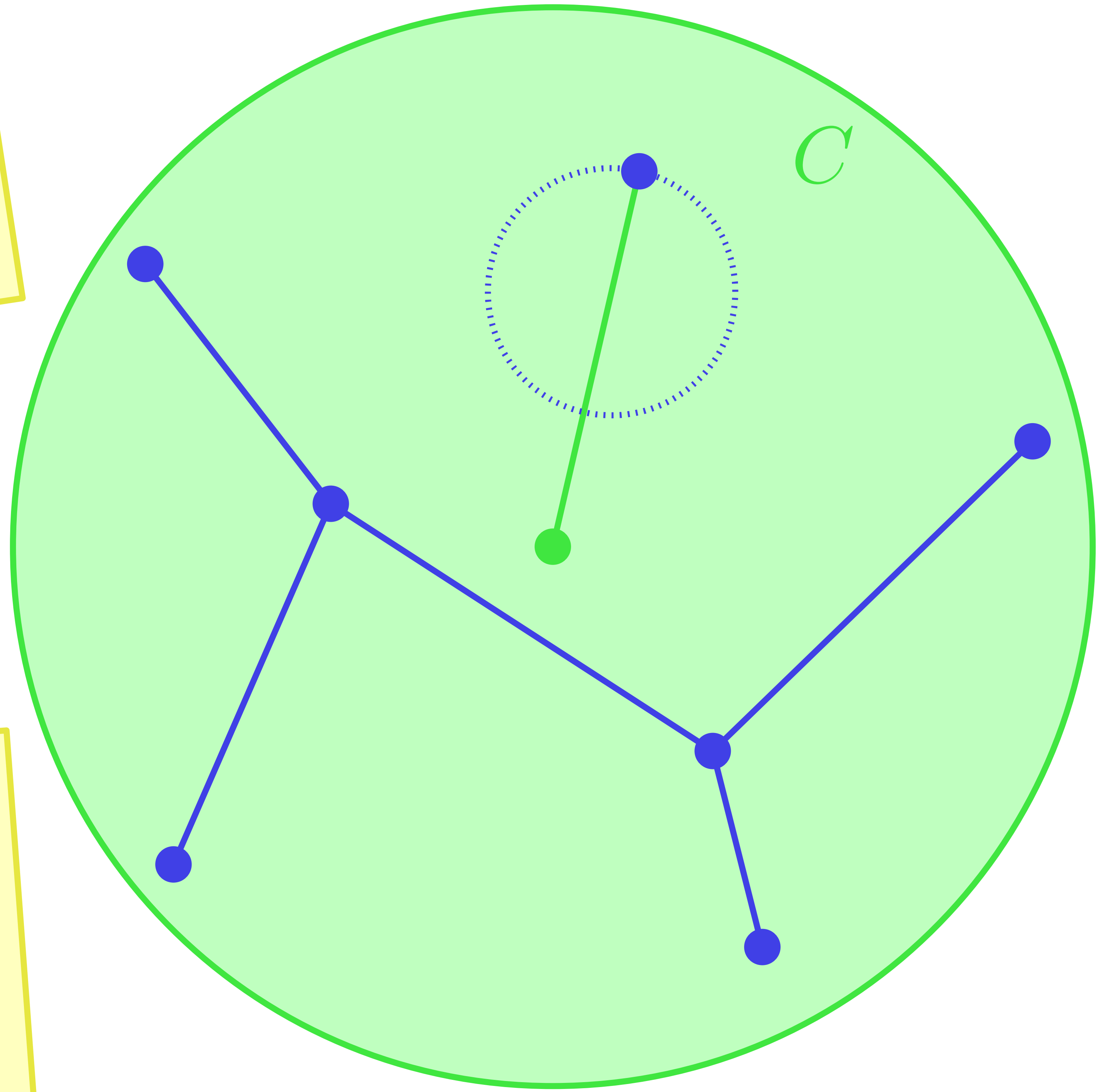

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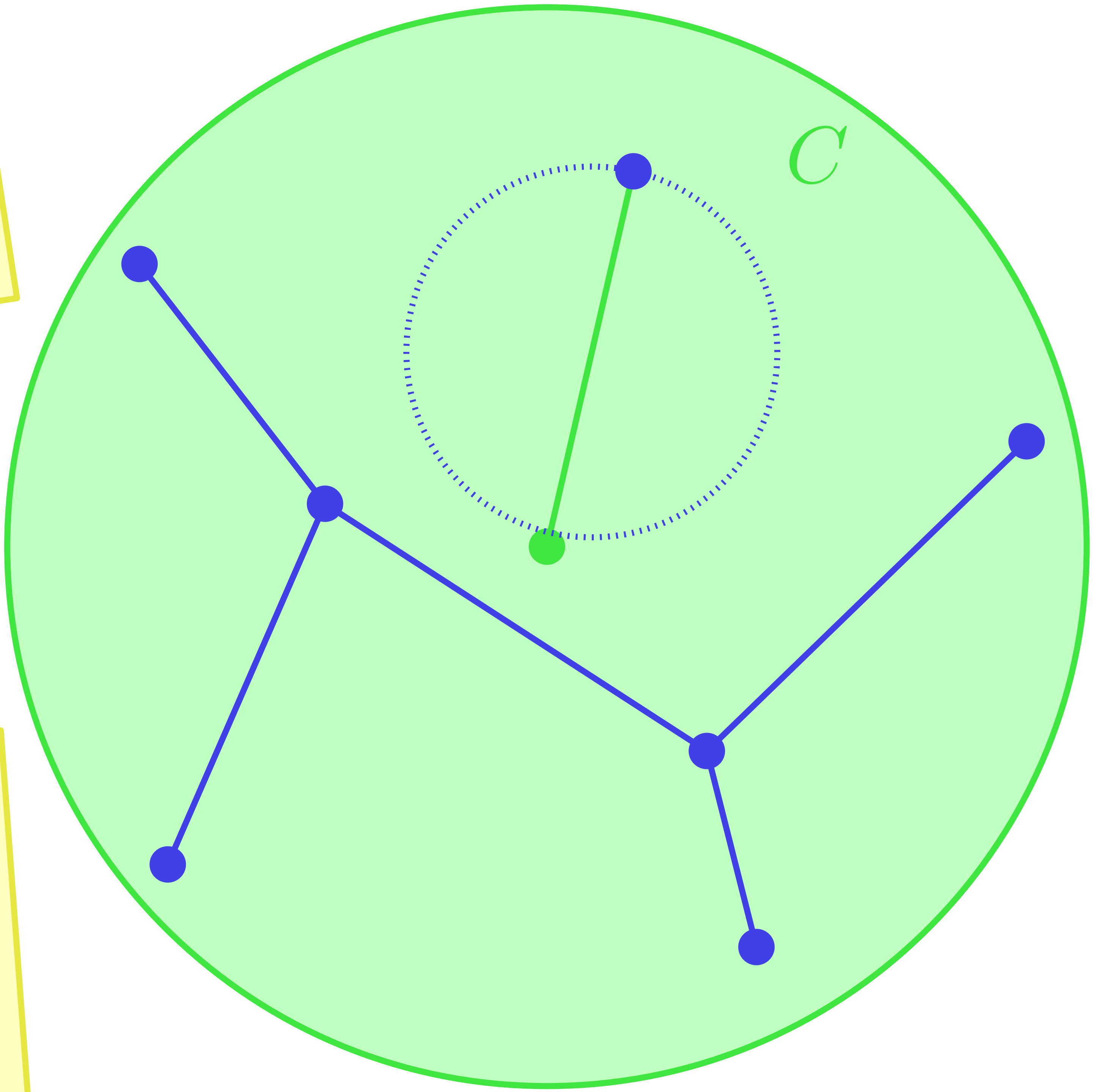

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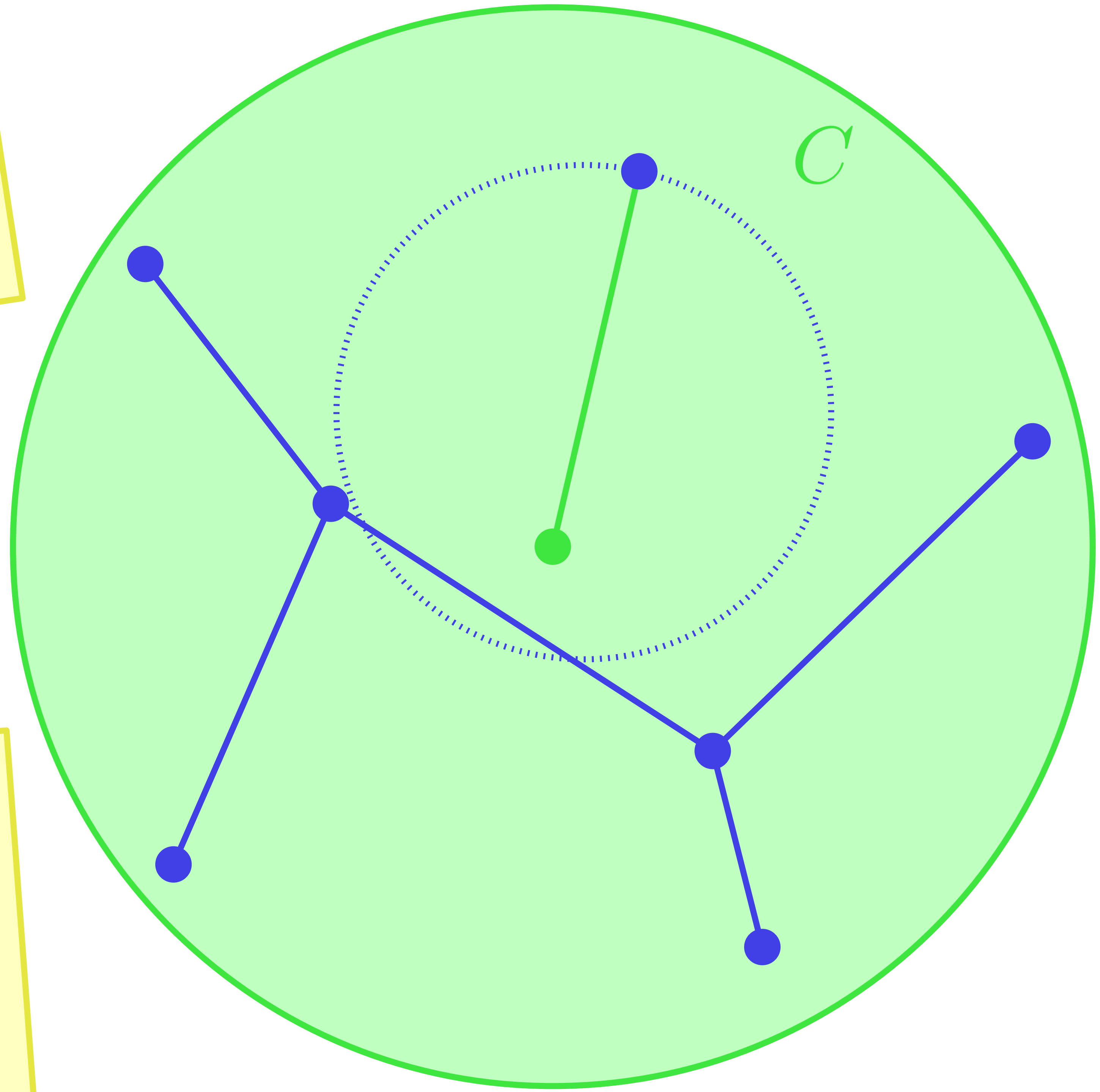

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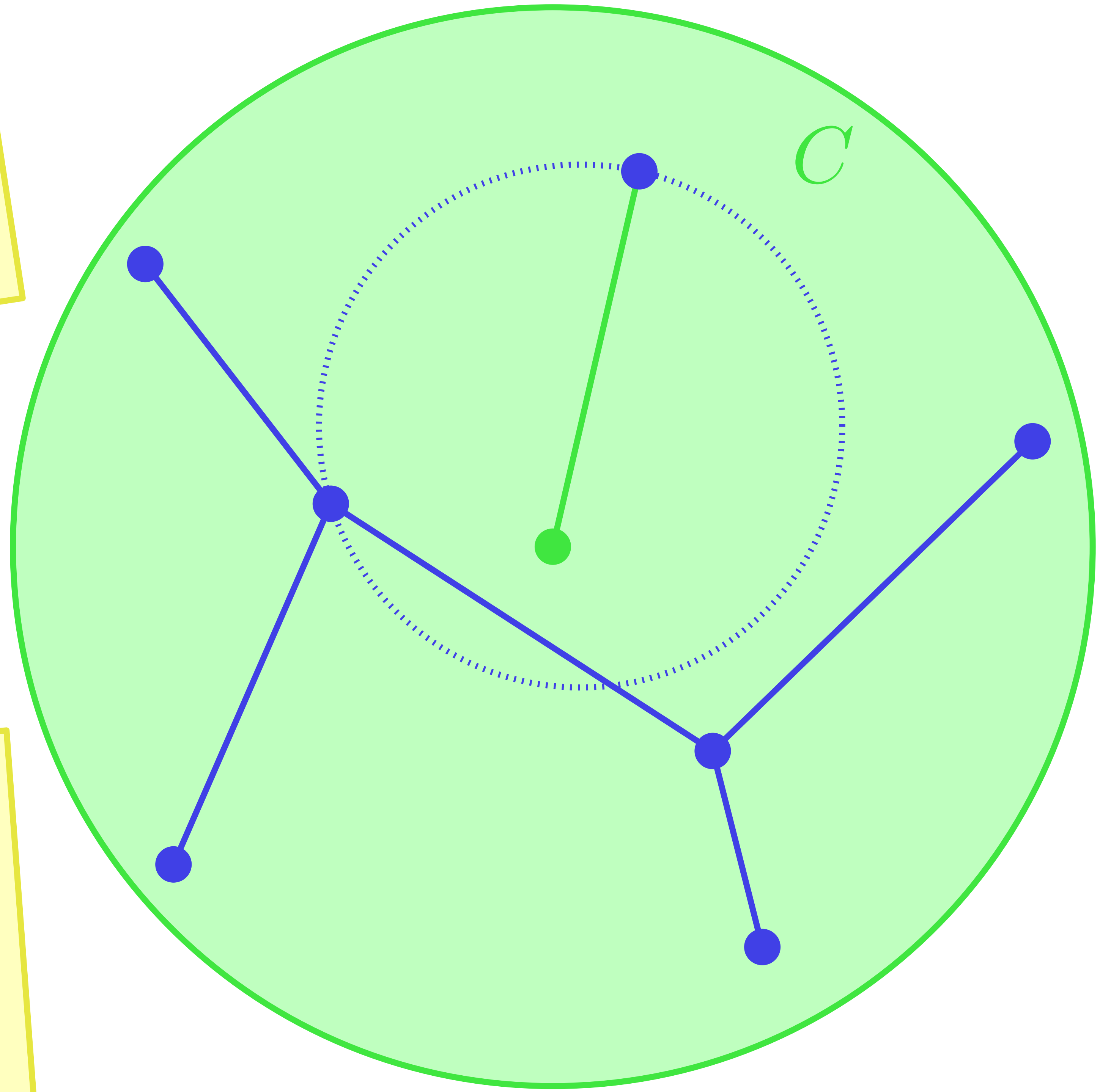

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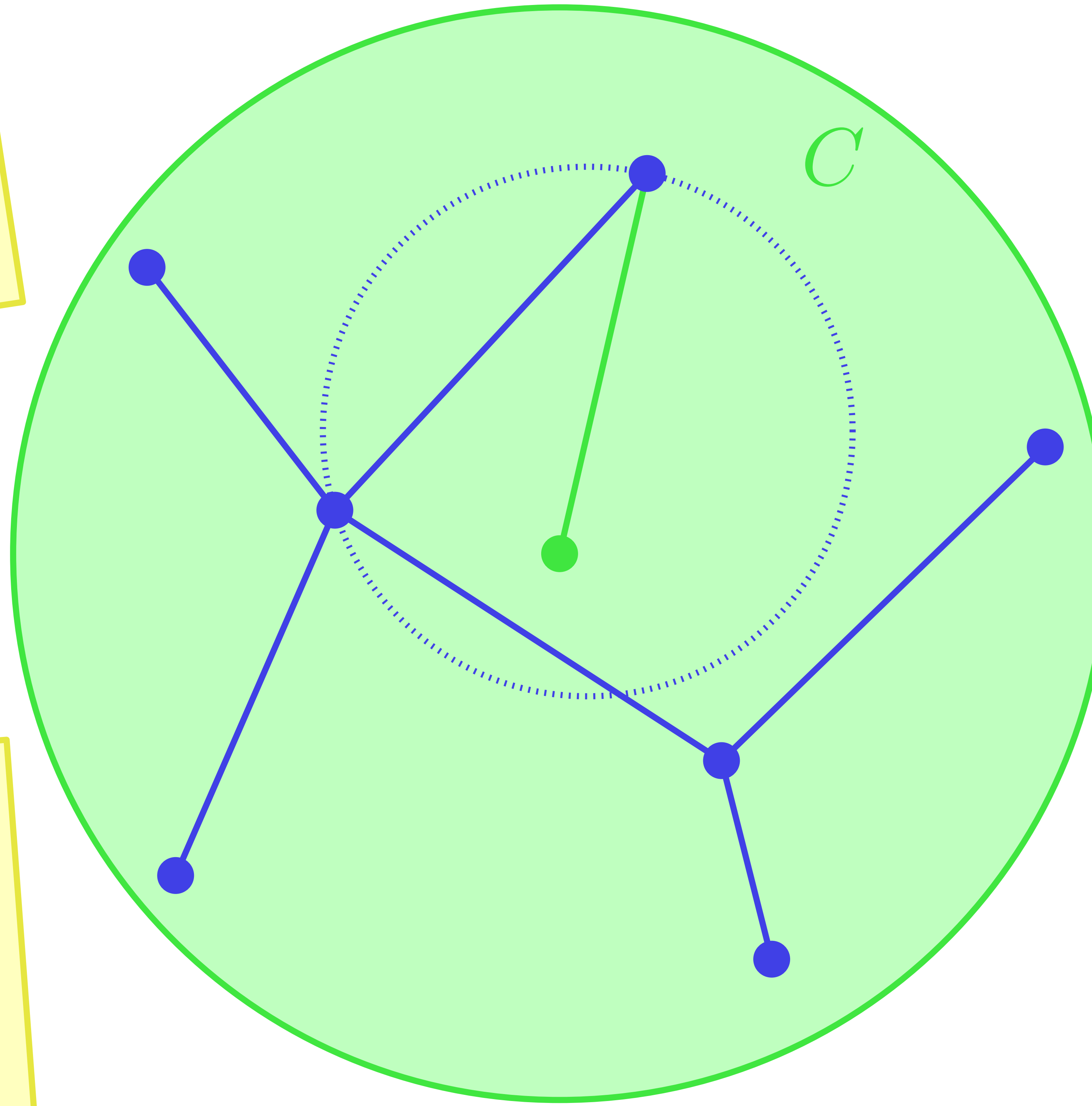

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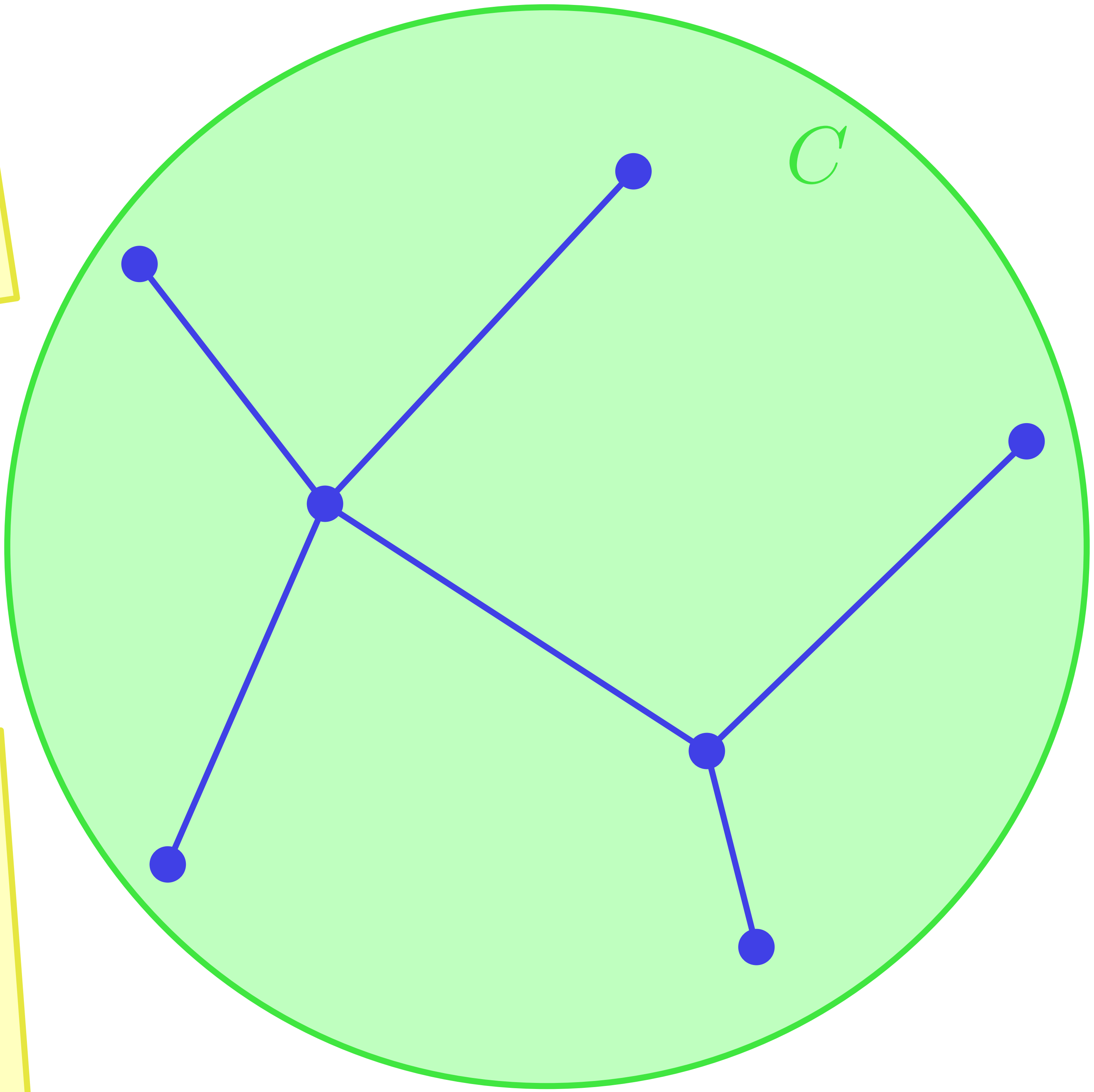

So, how do we reconstruct a spanning tree edge?

We reconstruct connectivity of the MST edges of P by length (increasing).

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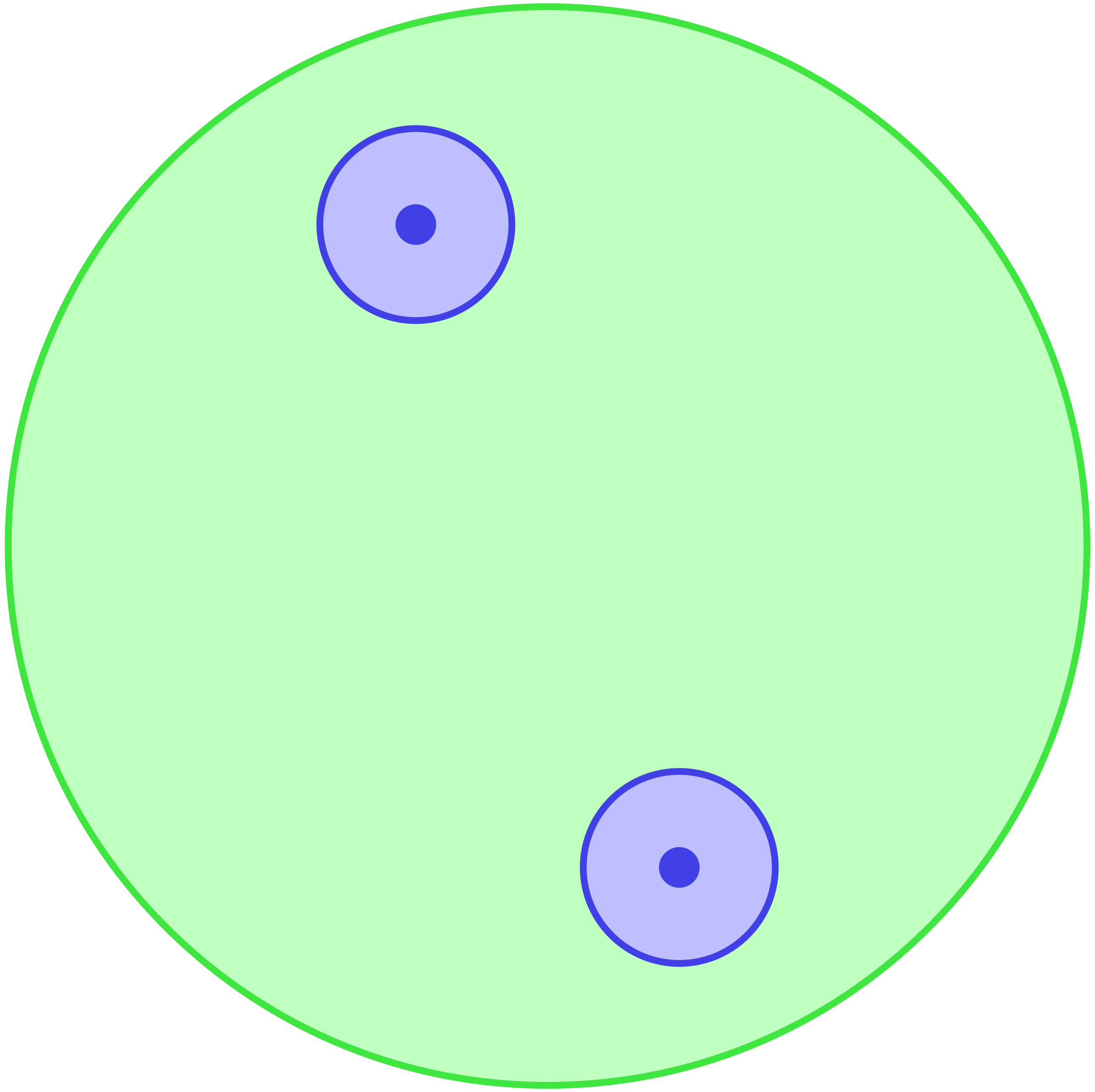

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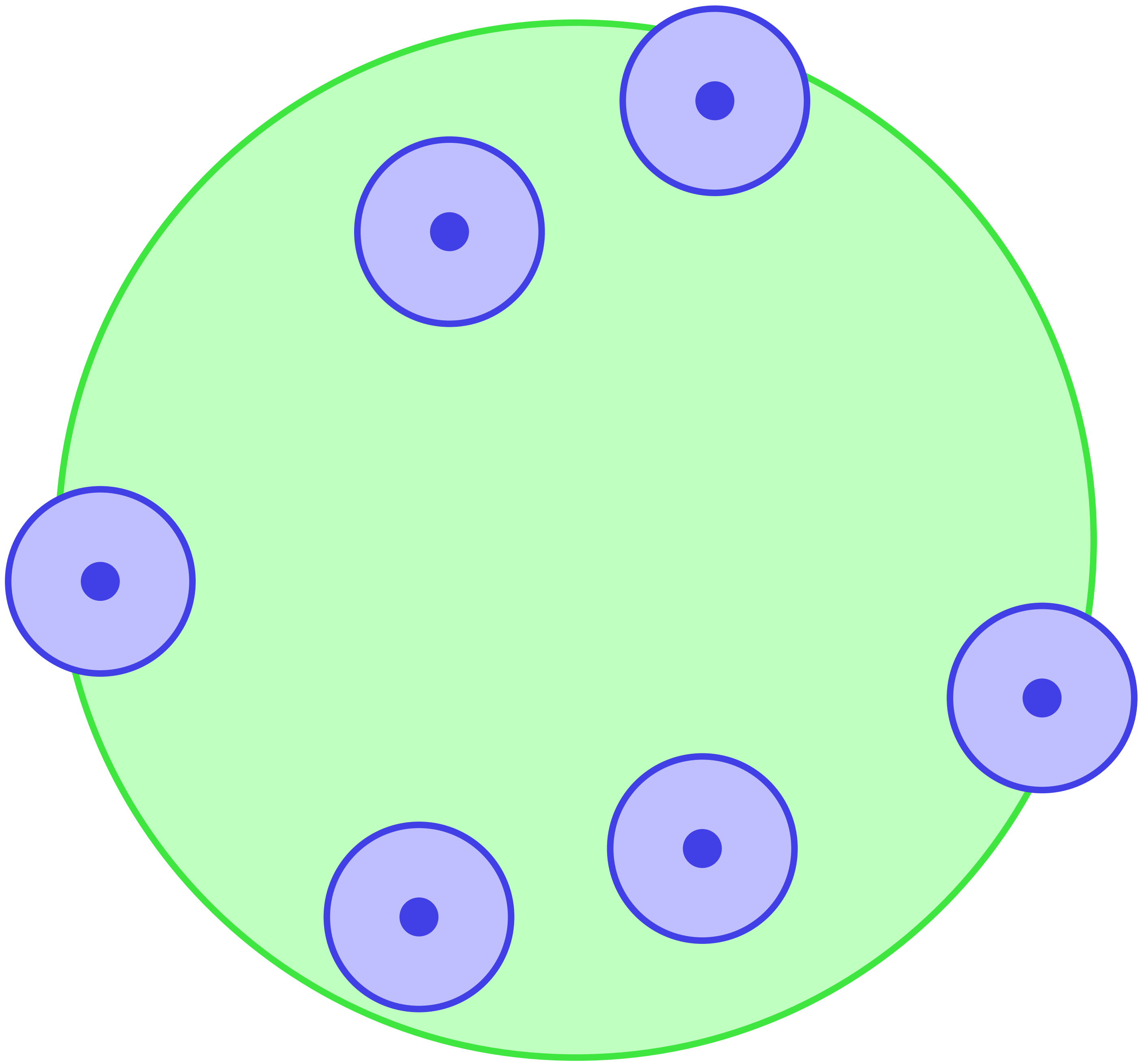

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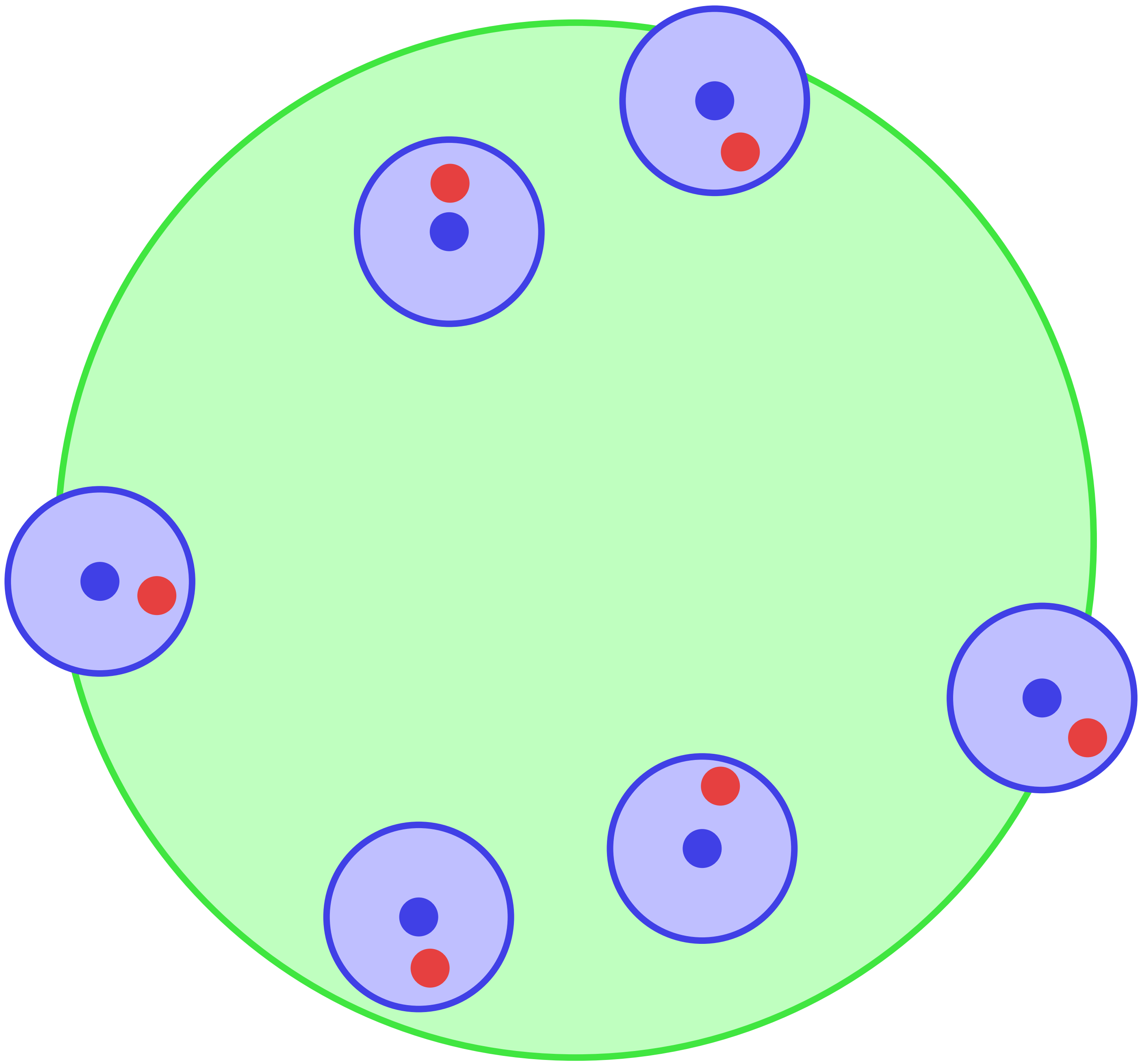

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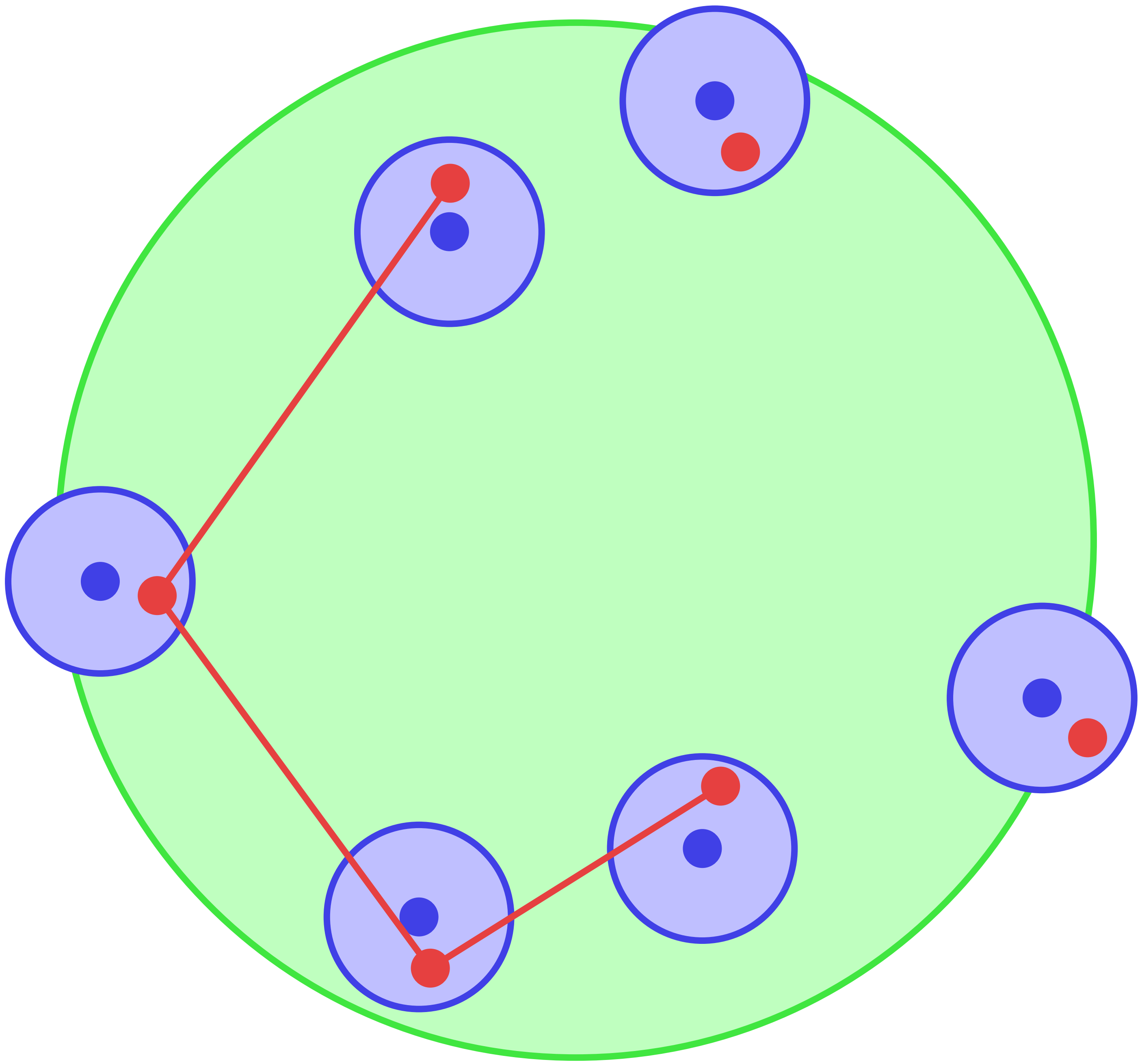
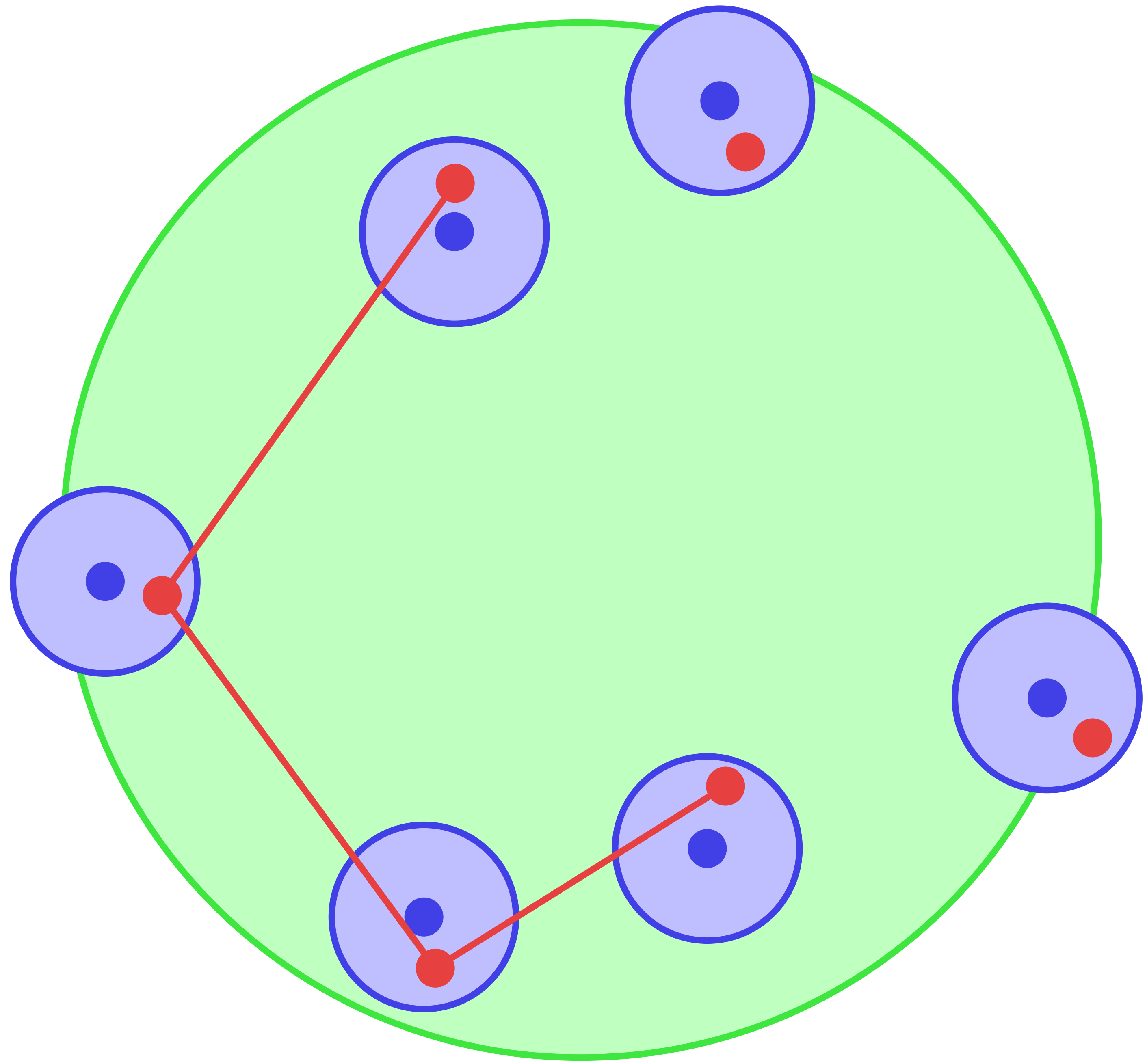


We reconstruct connectivity of the MST edges of P by length (incr

If an edge pq is *short*, the number of other points in C^+_{pq} is constant.



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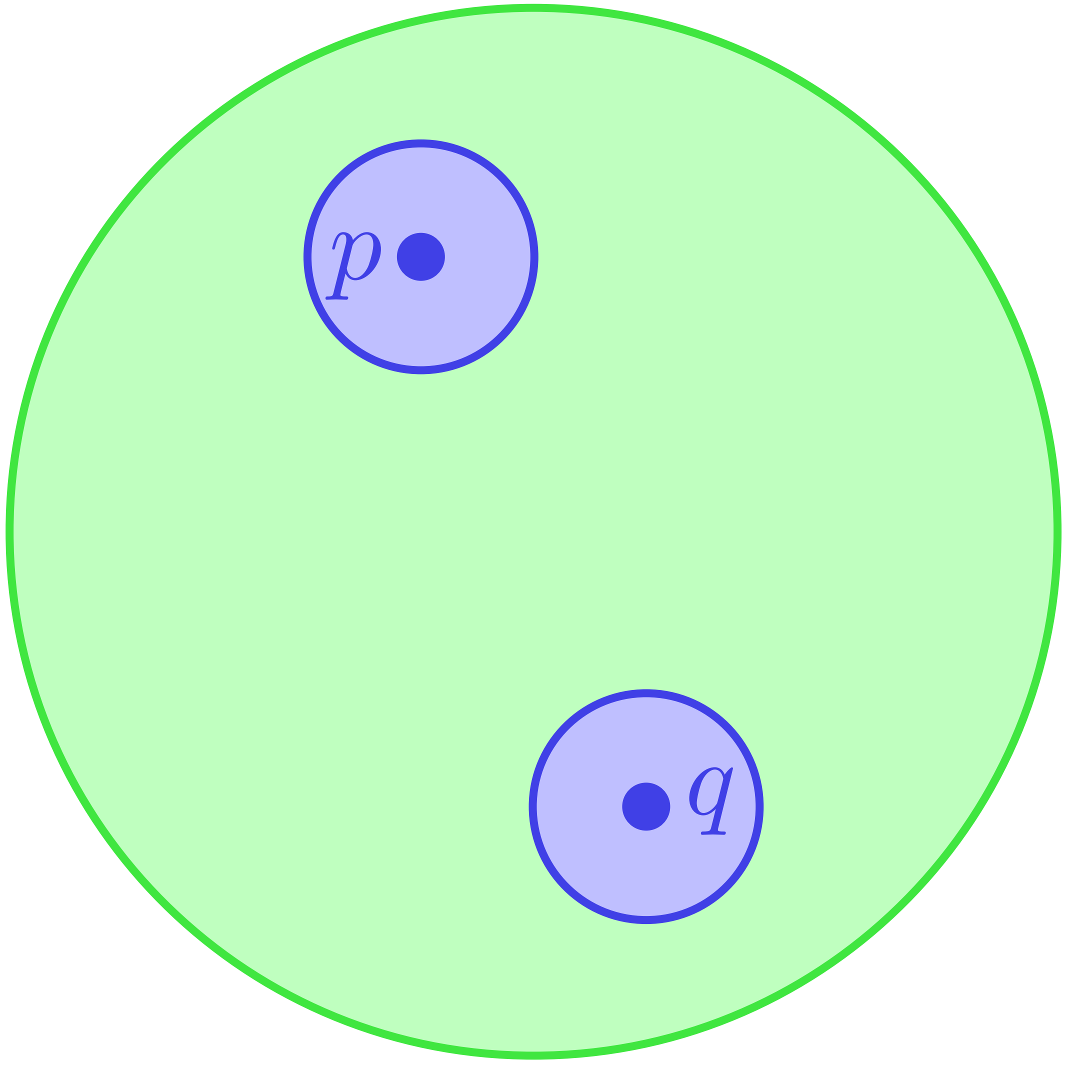

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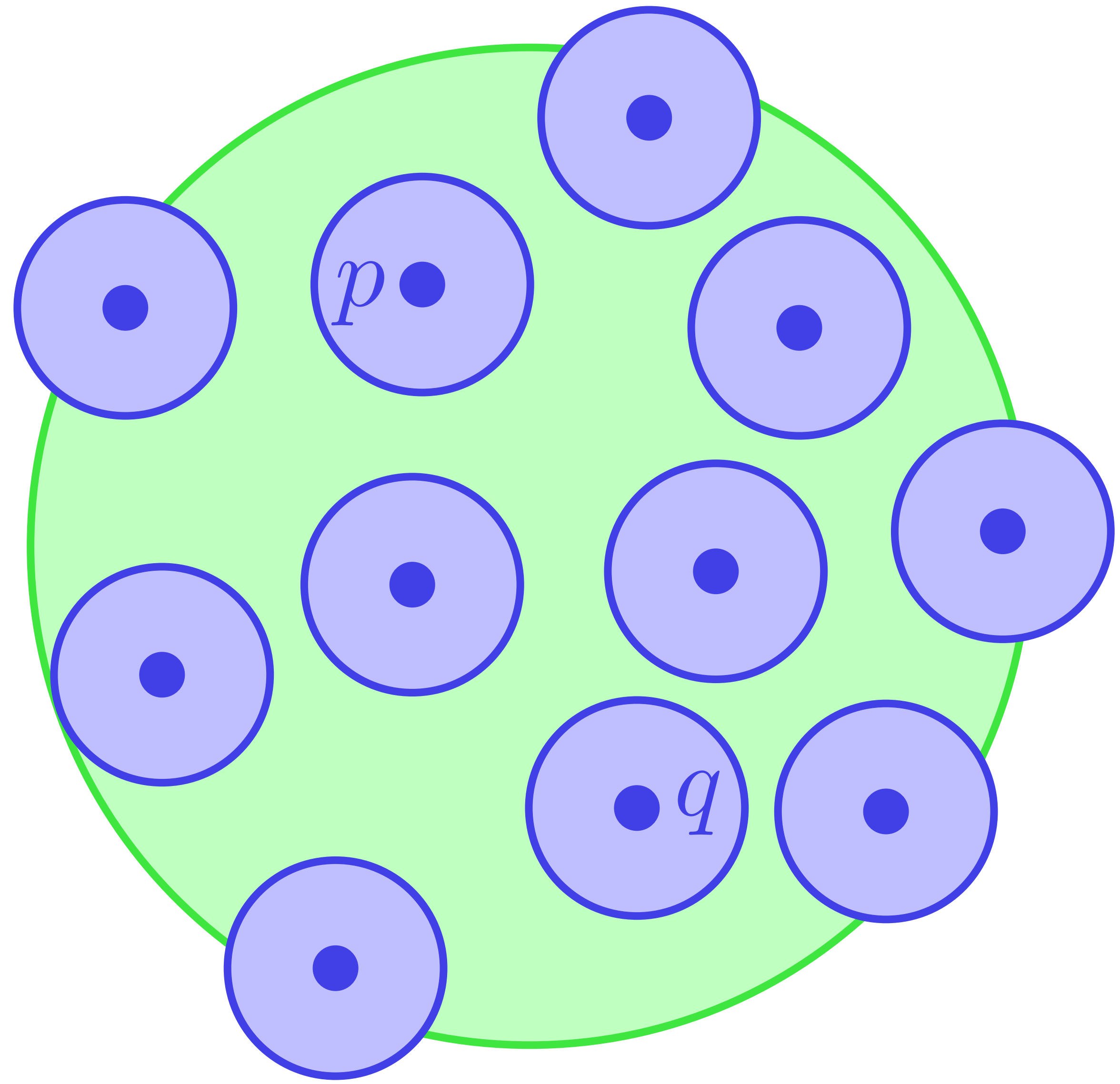

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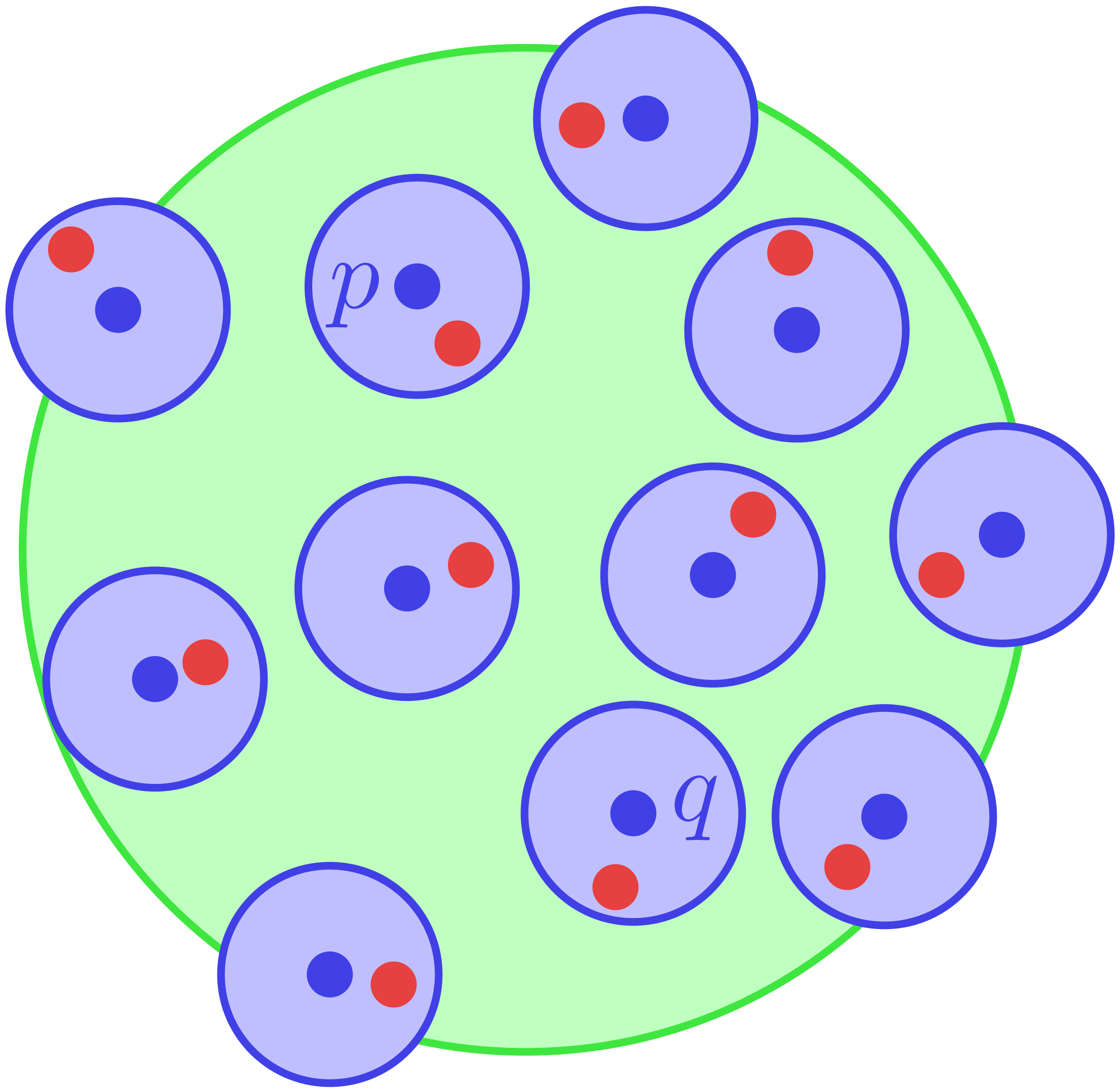

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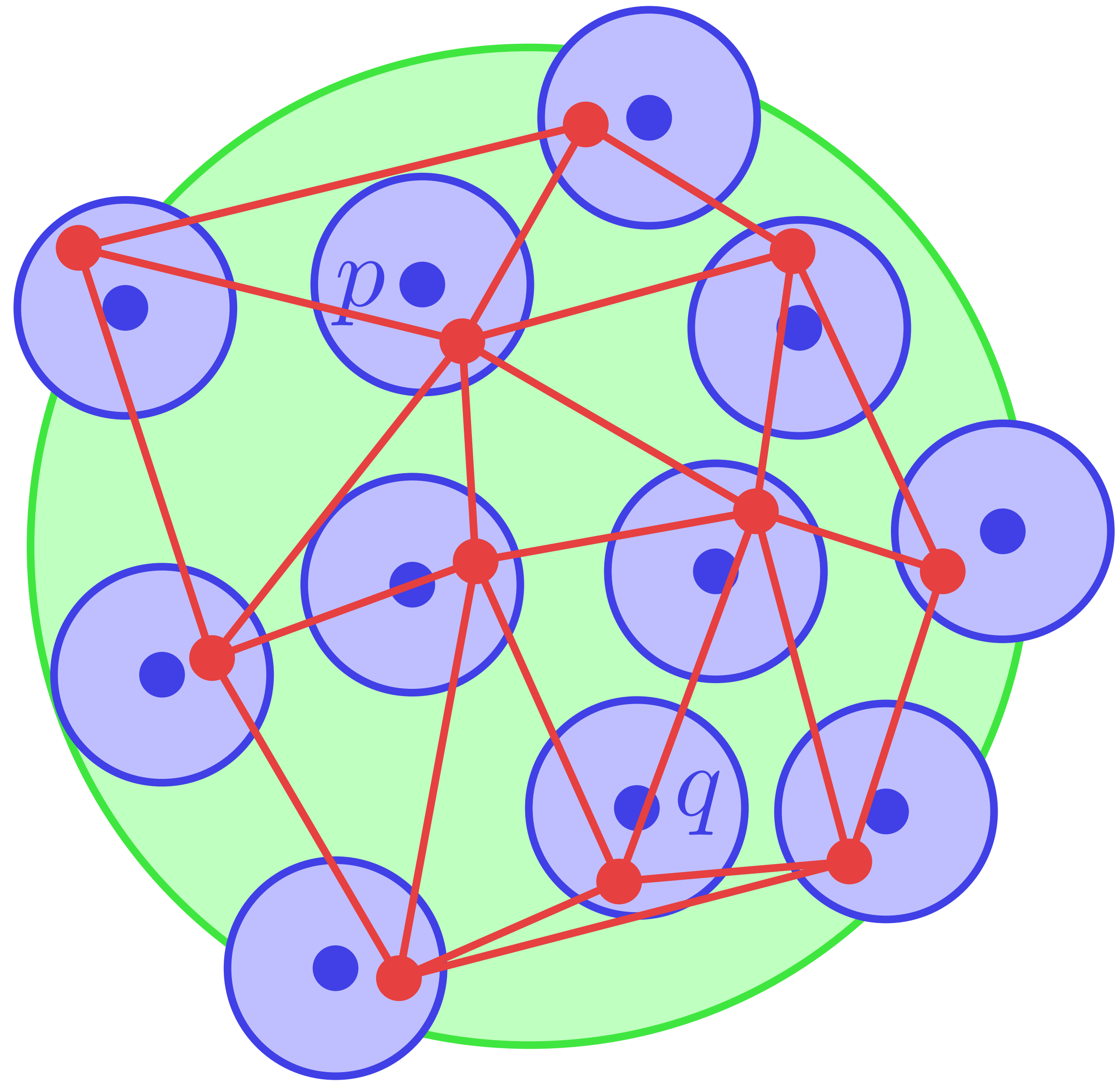

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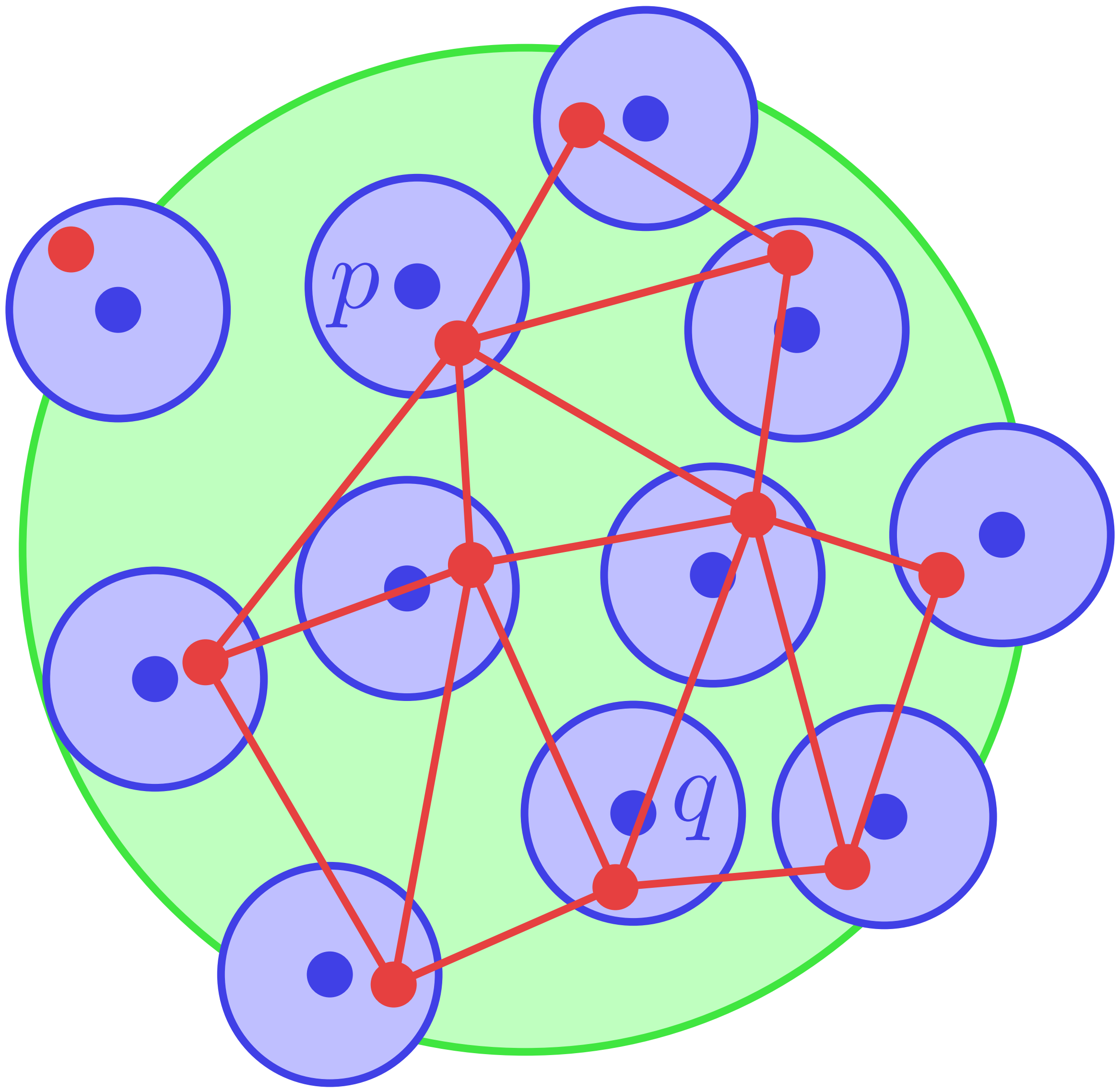

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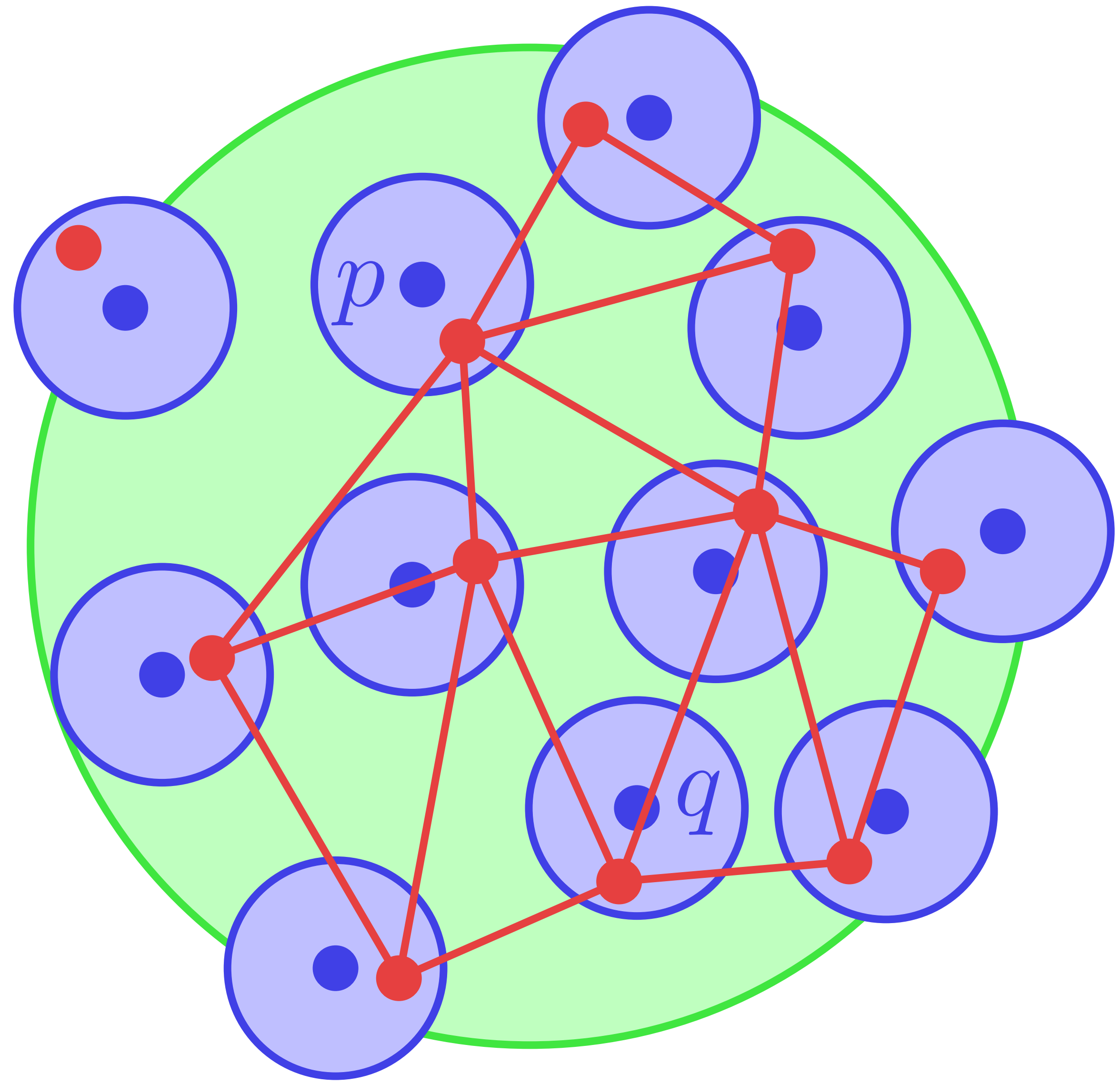

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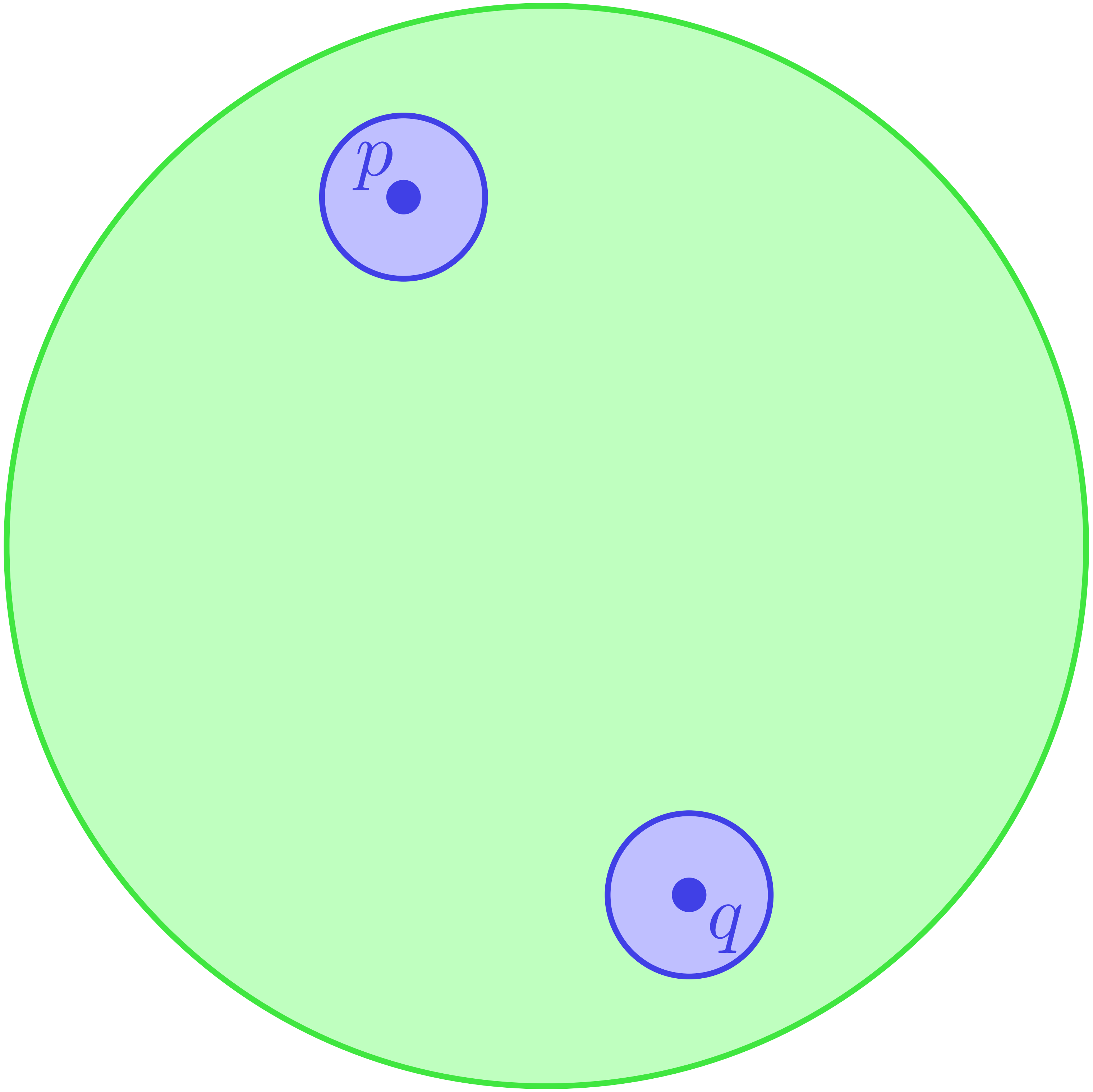

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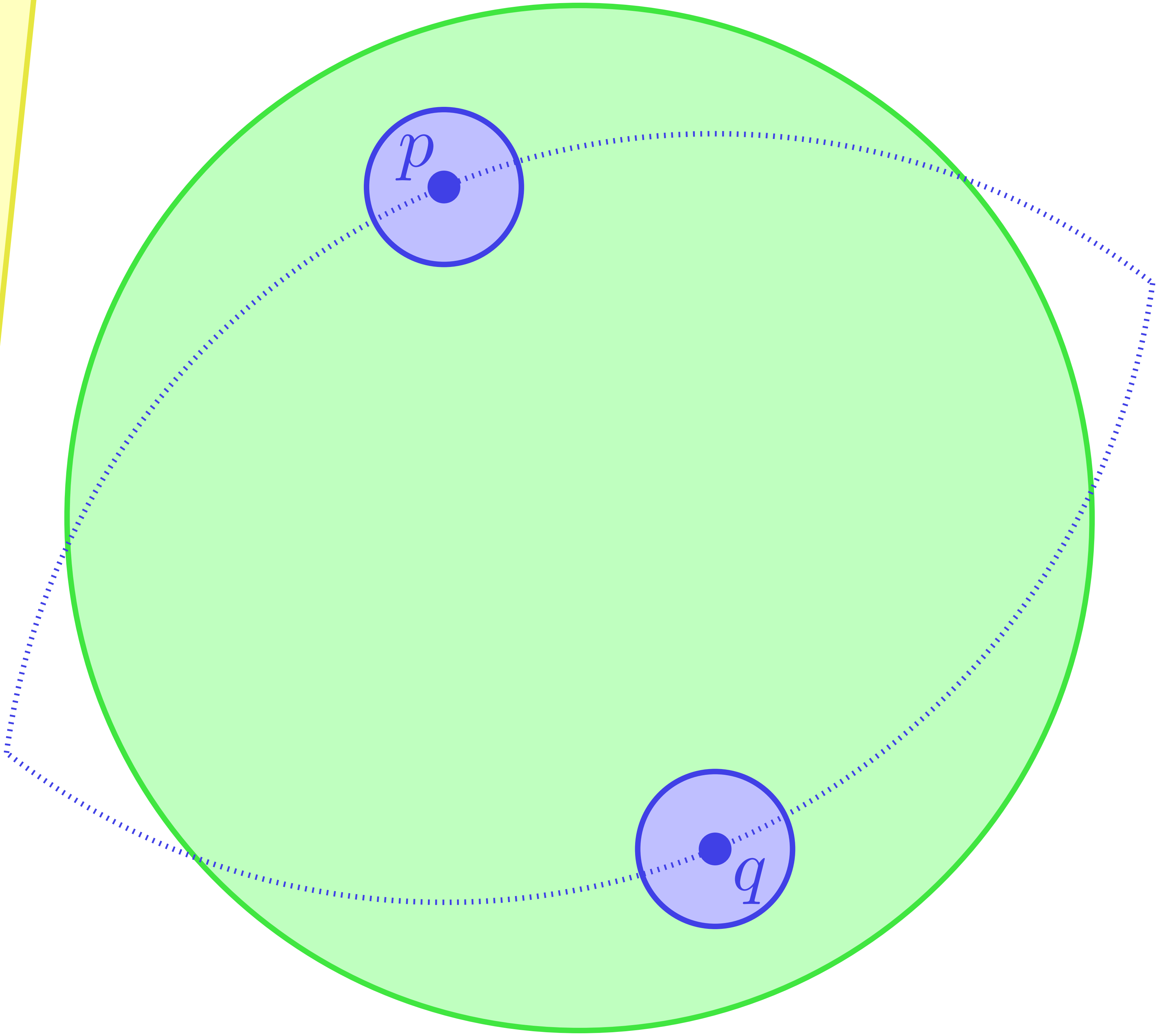

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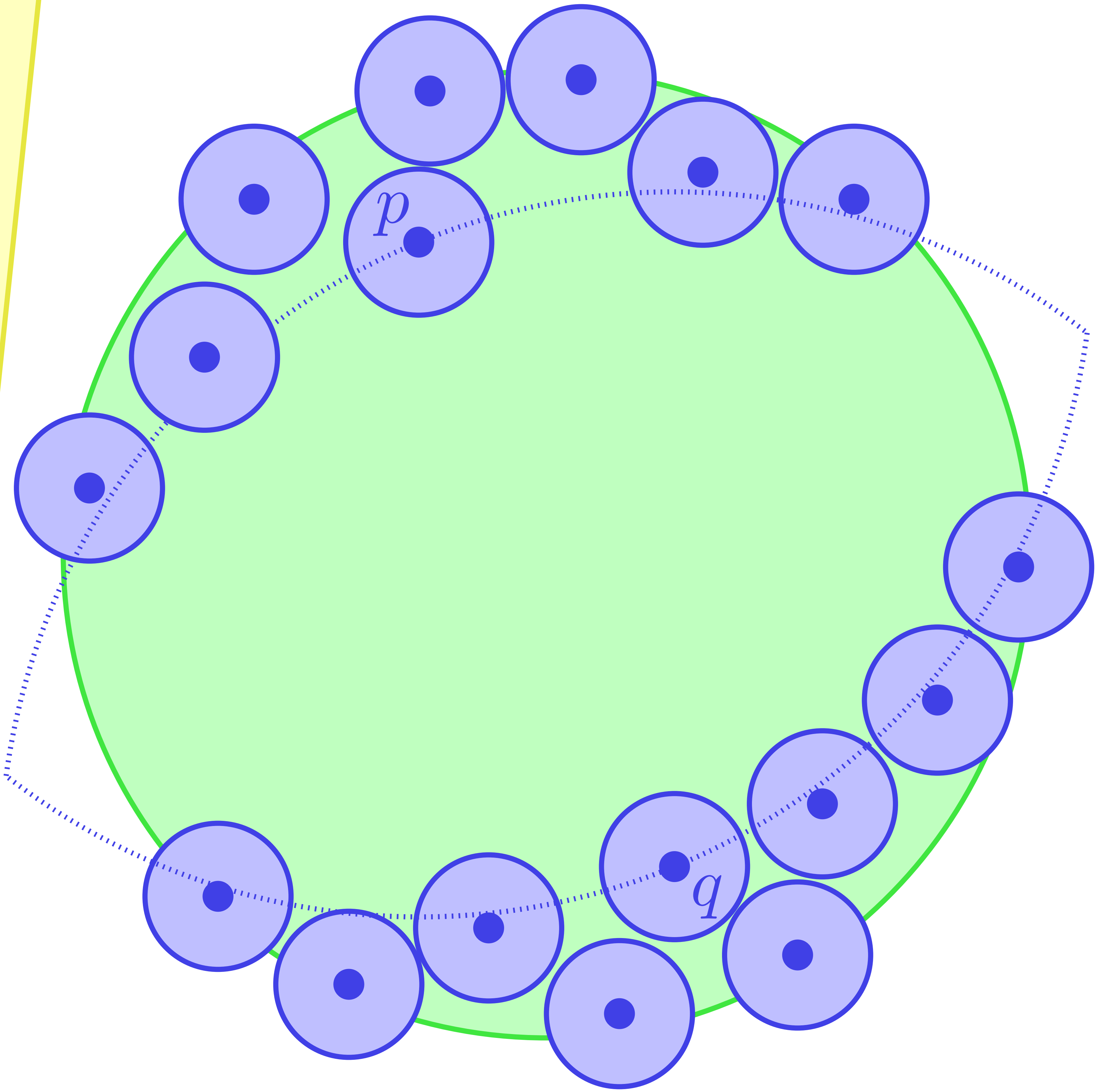

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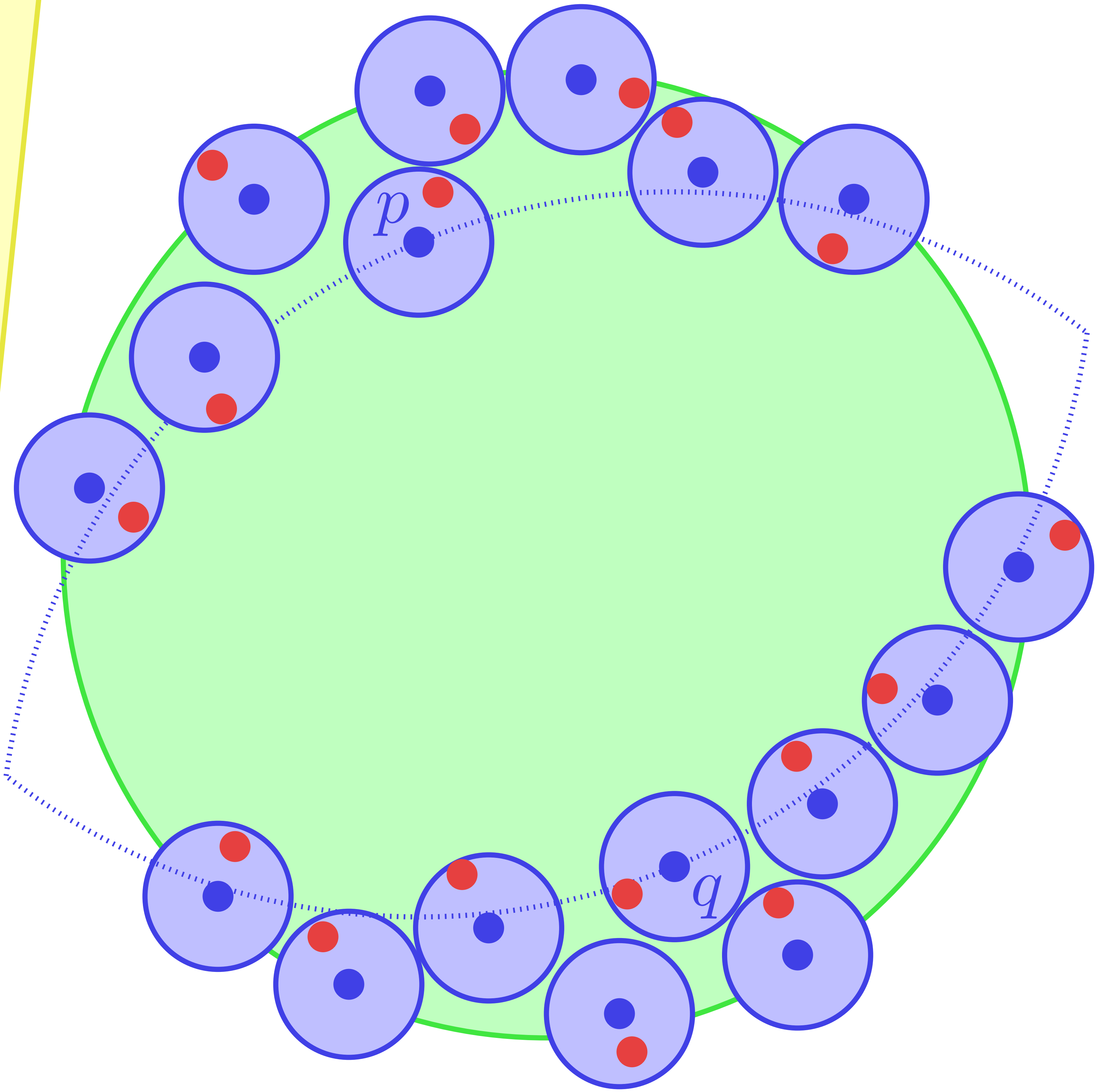

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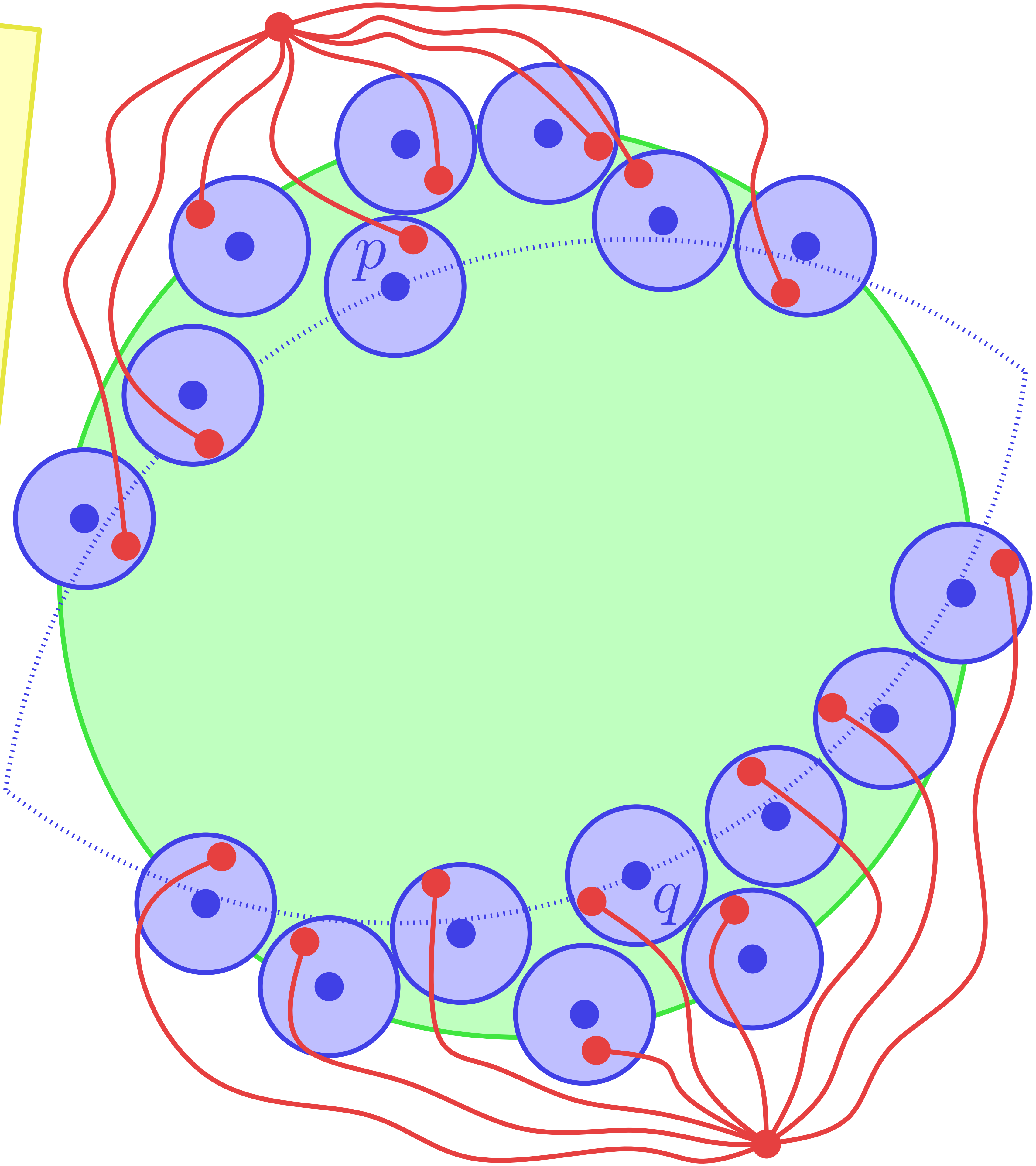

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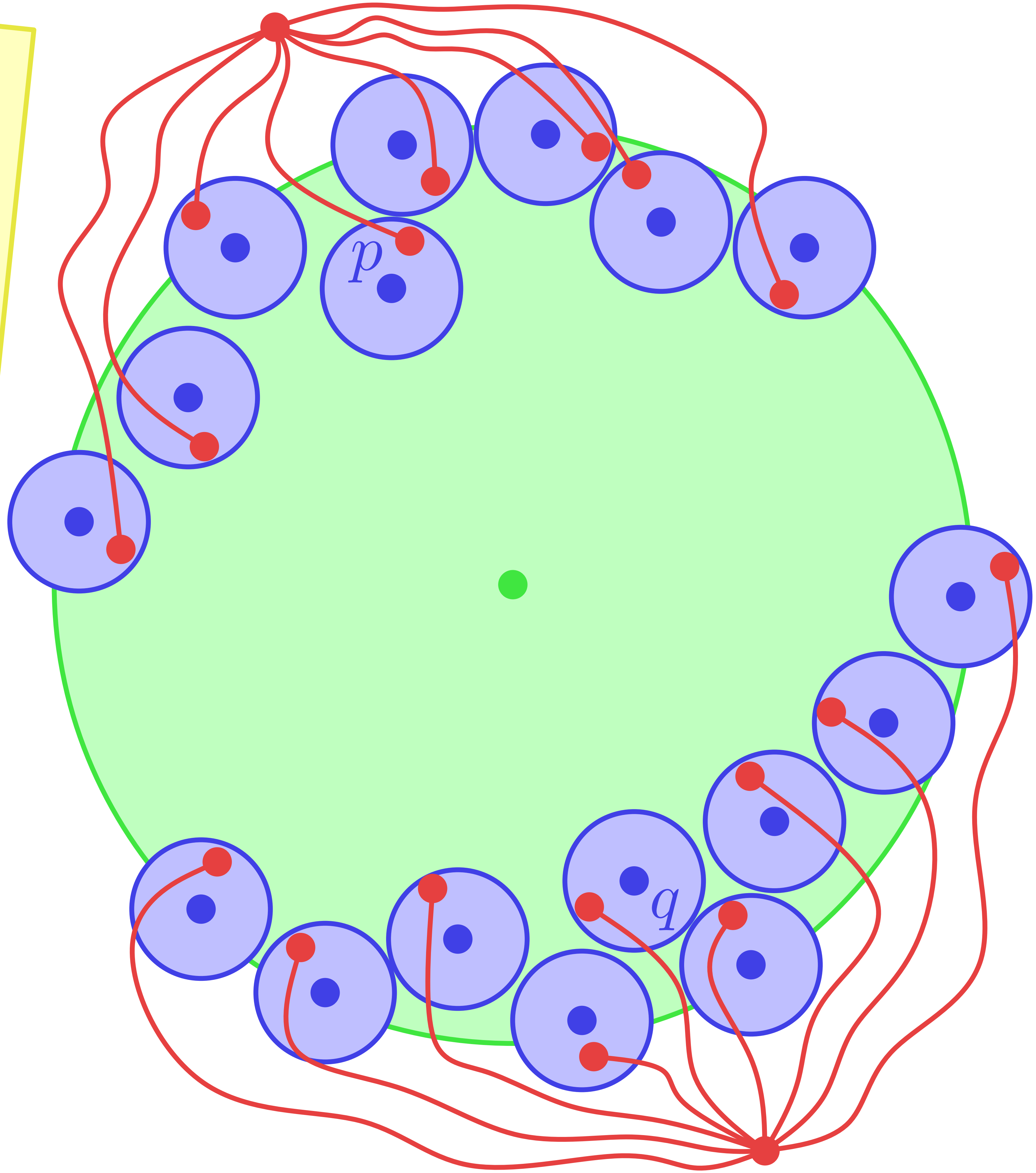

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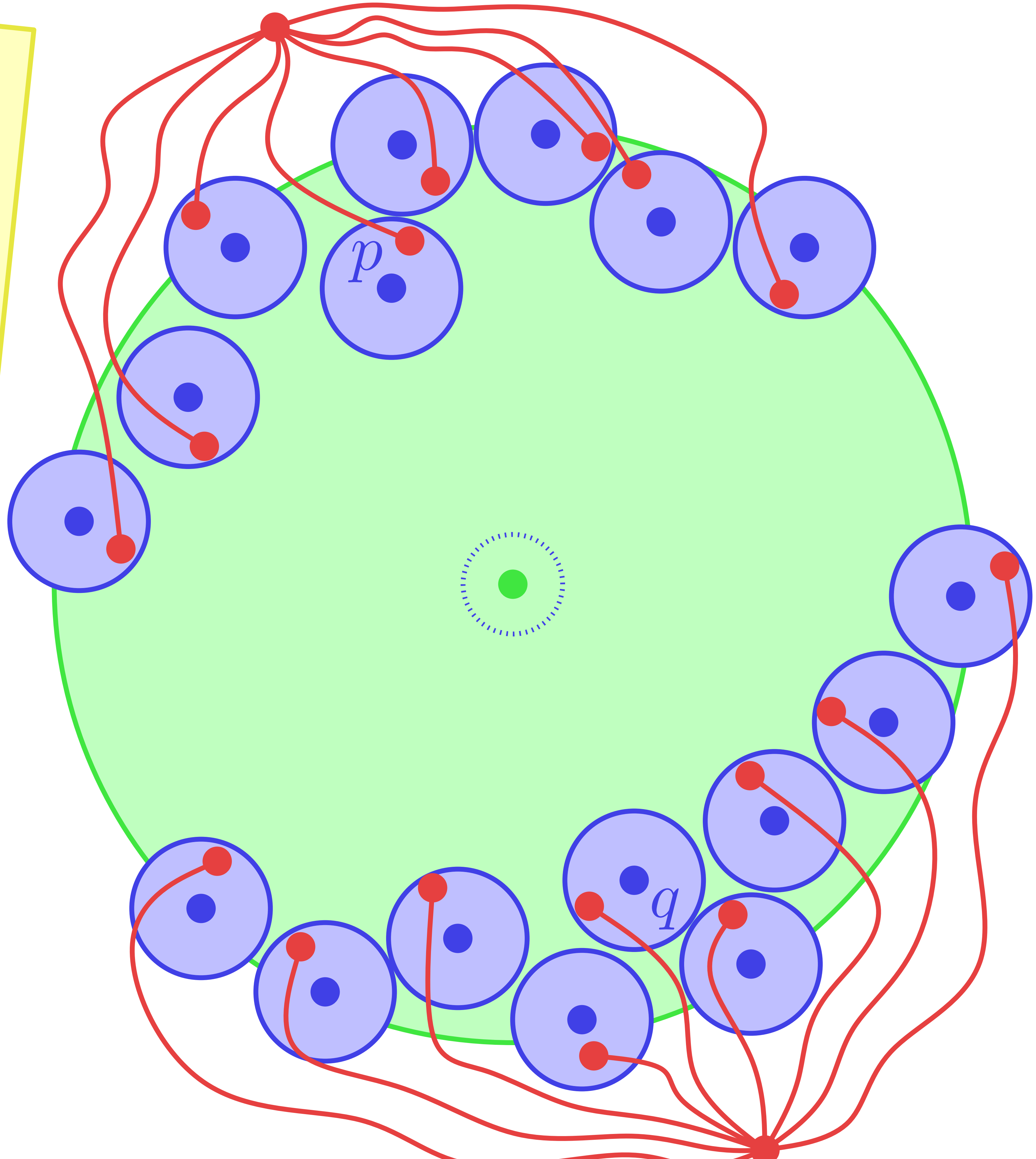

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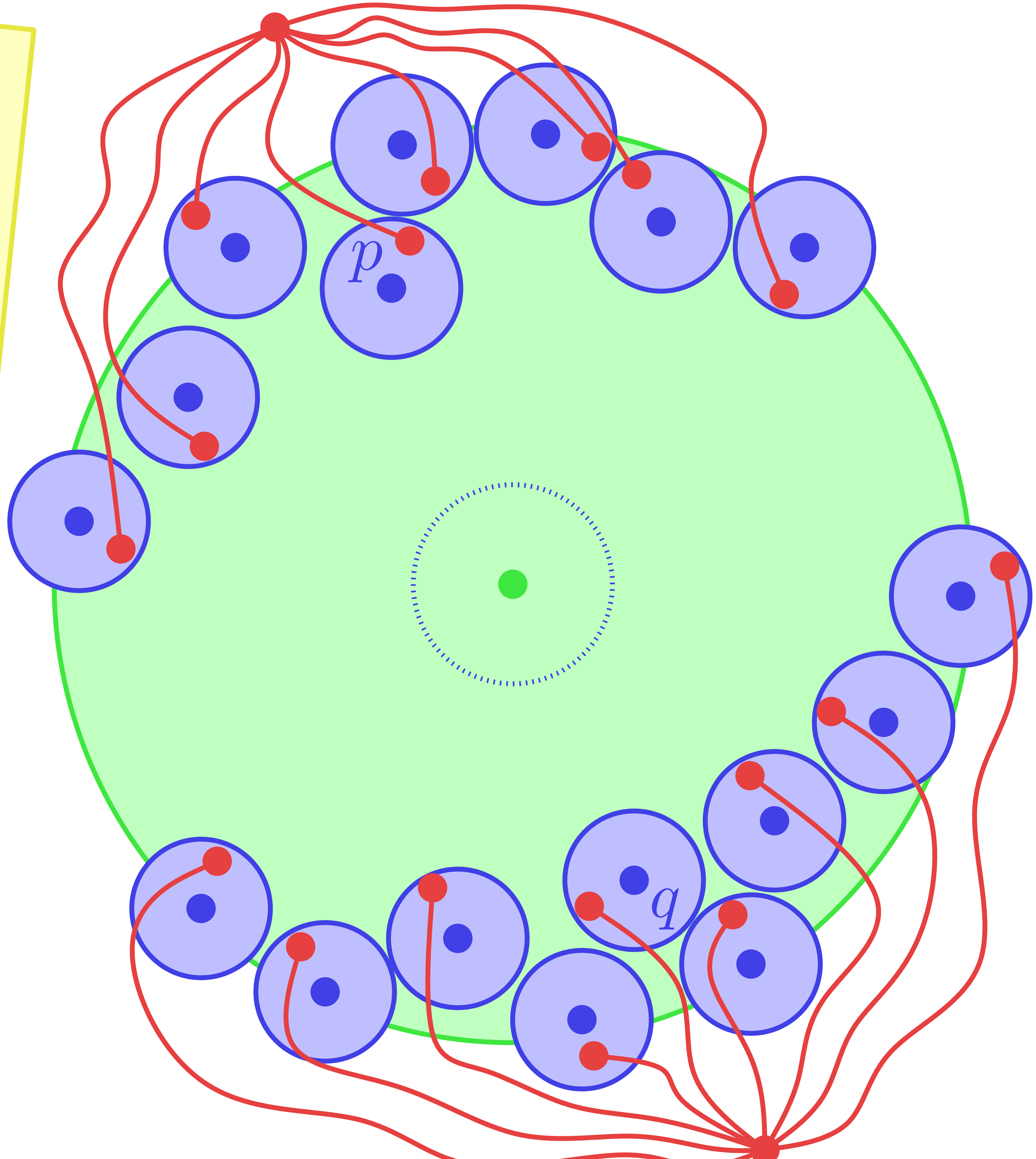

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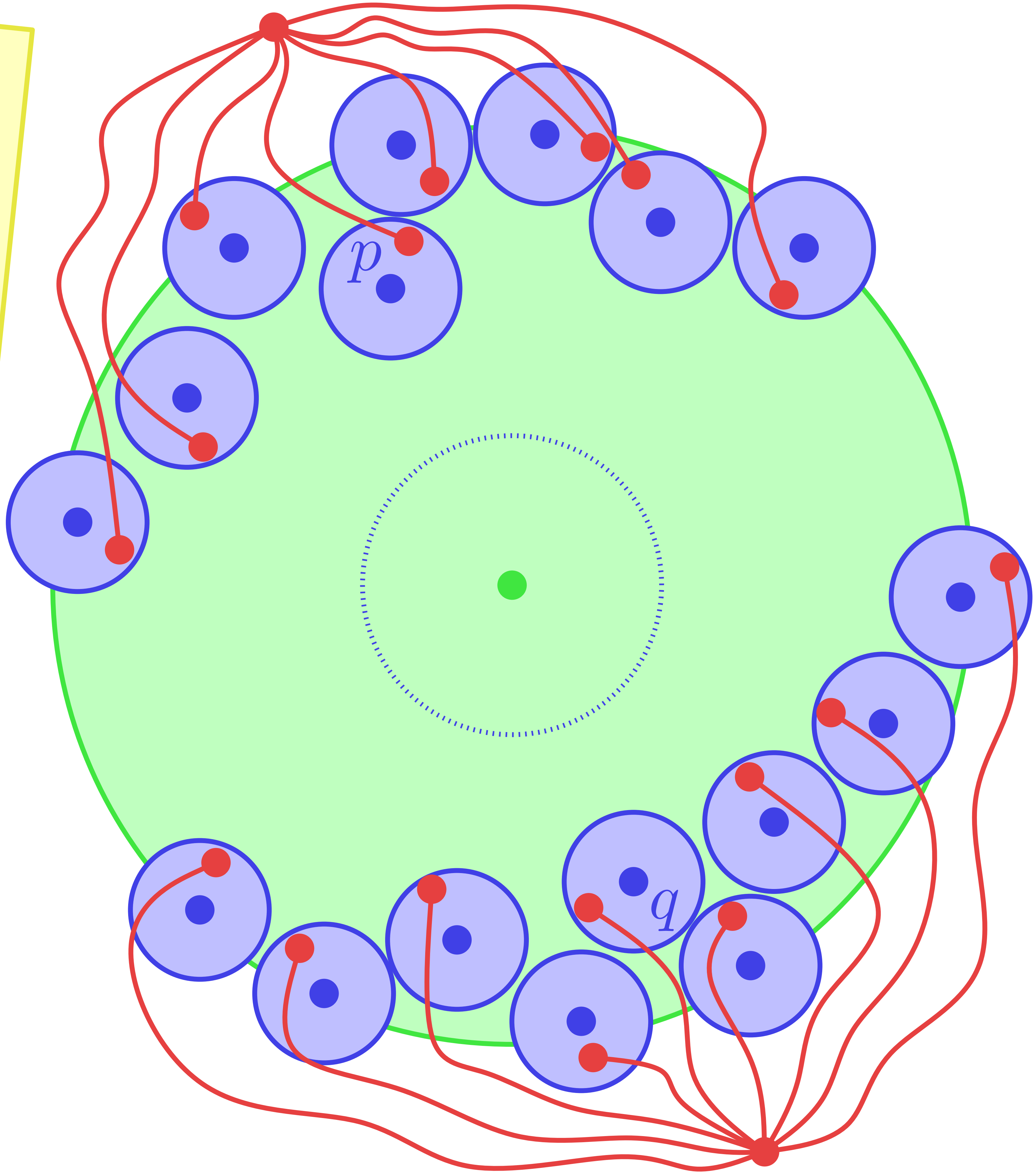

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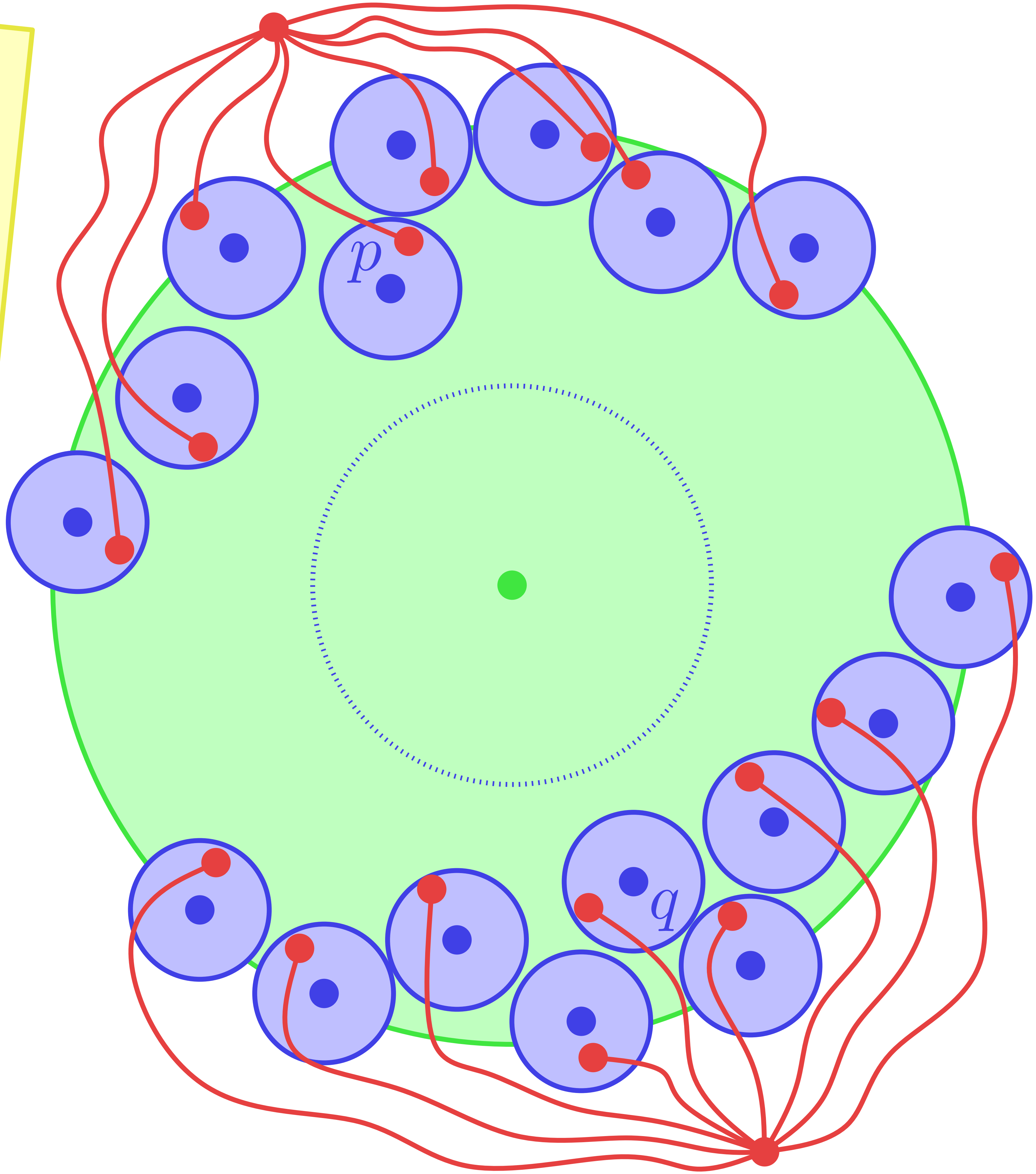

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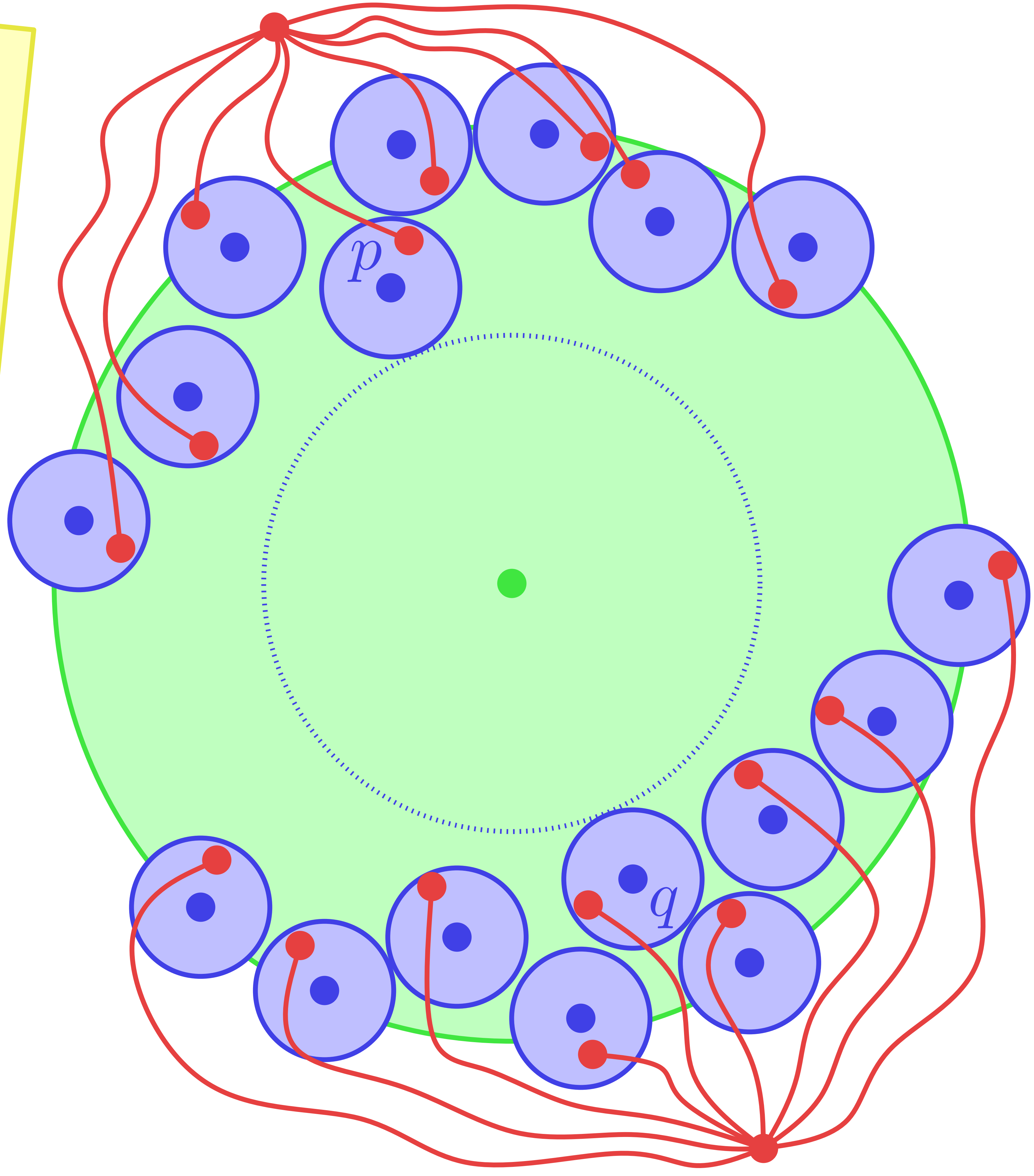

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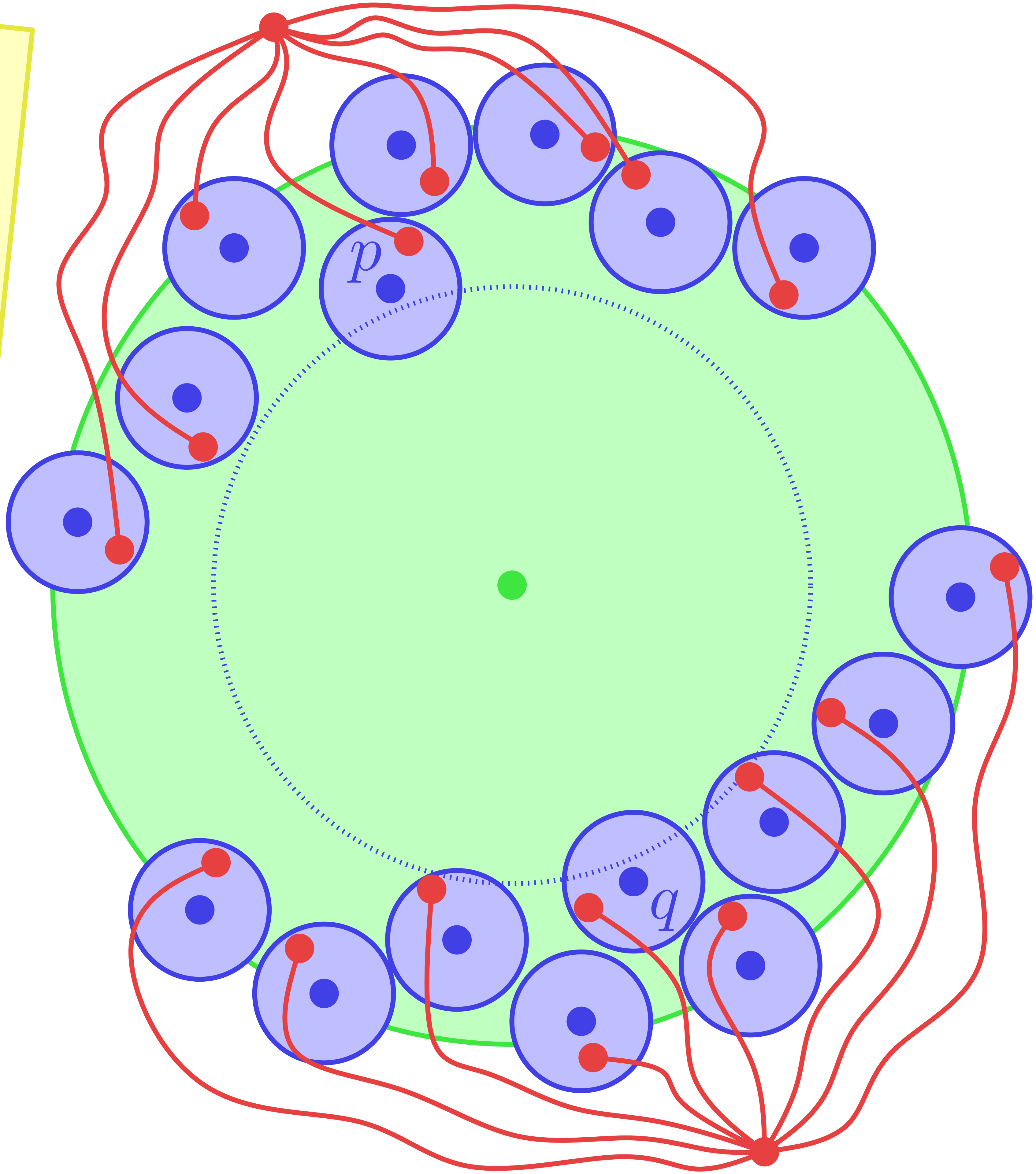

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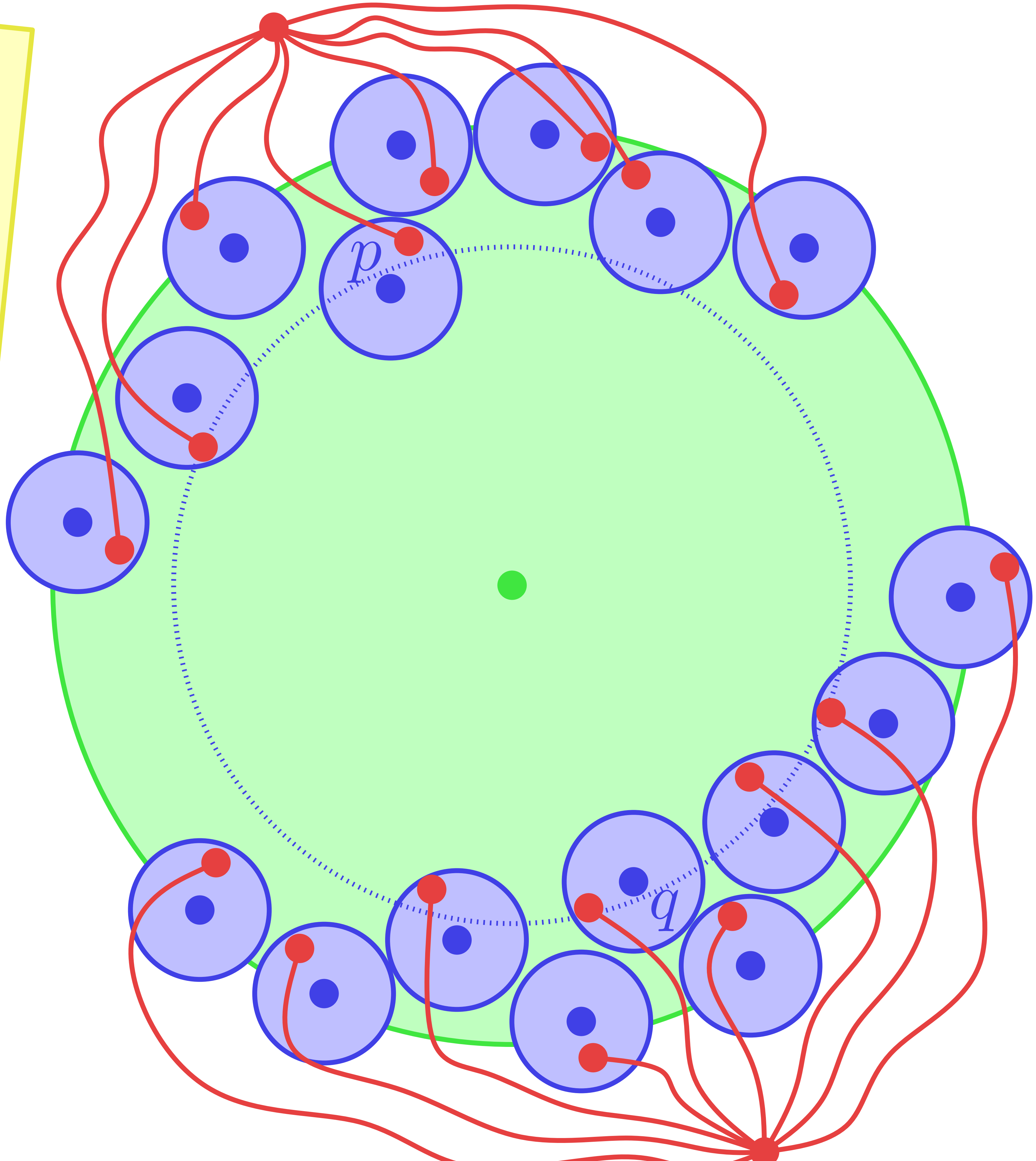
If an edge pq is *short*, the number of other points in

If an edge pq is *long*, all other points in C_{pq}^{+} are already connected to either p or q.



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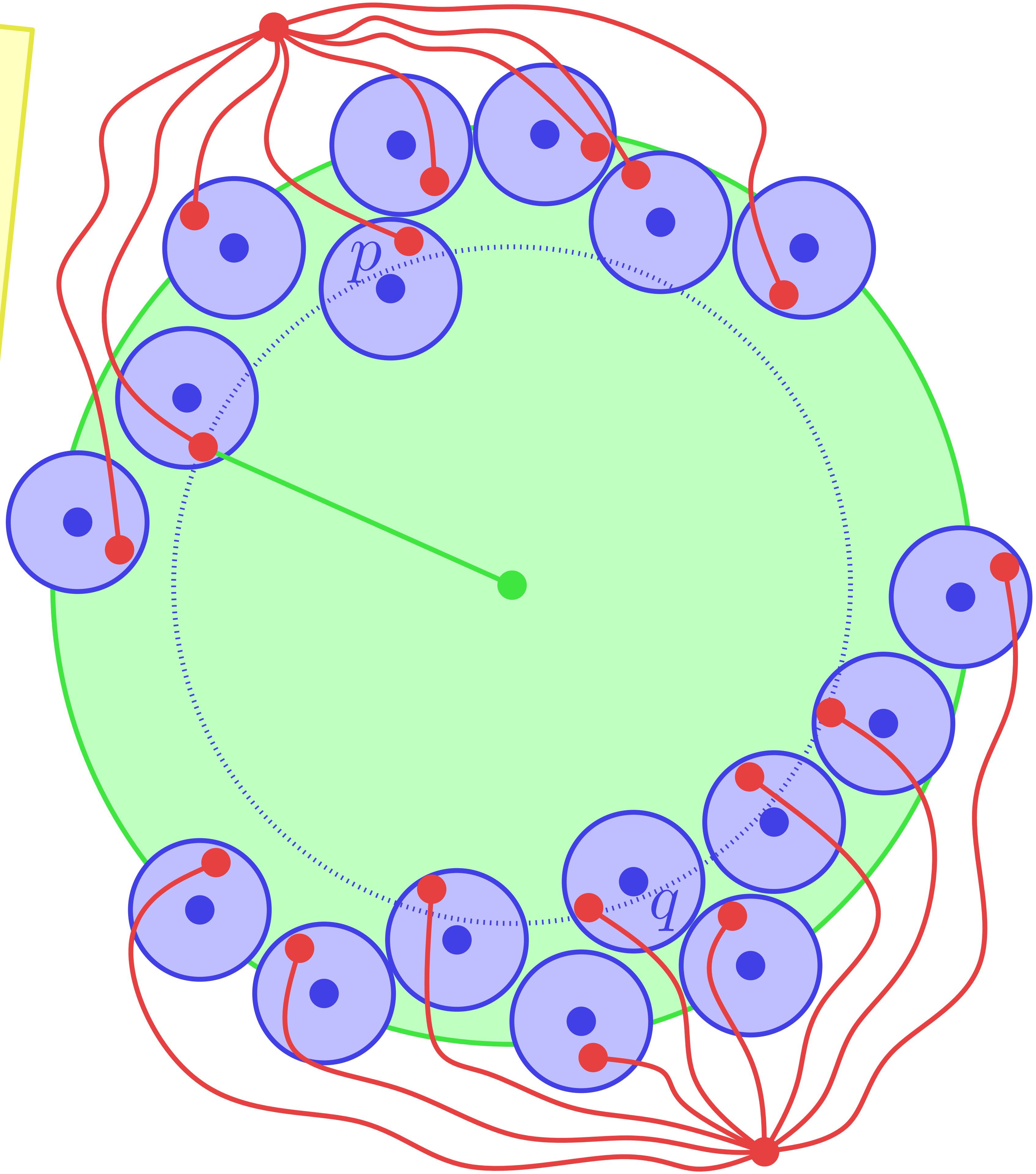

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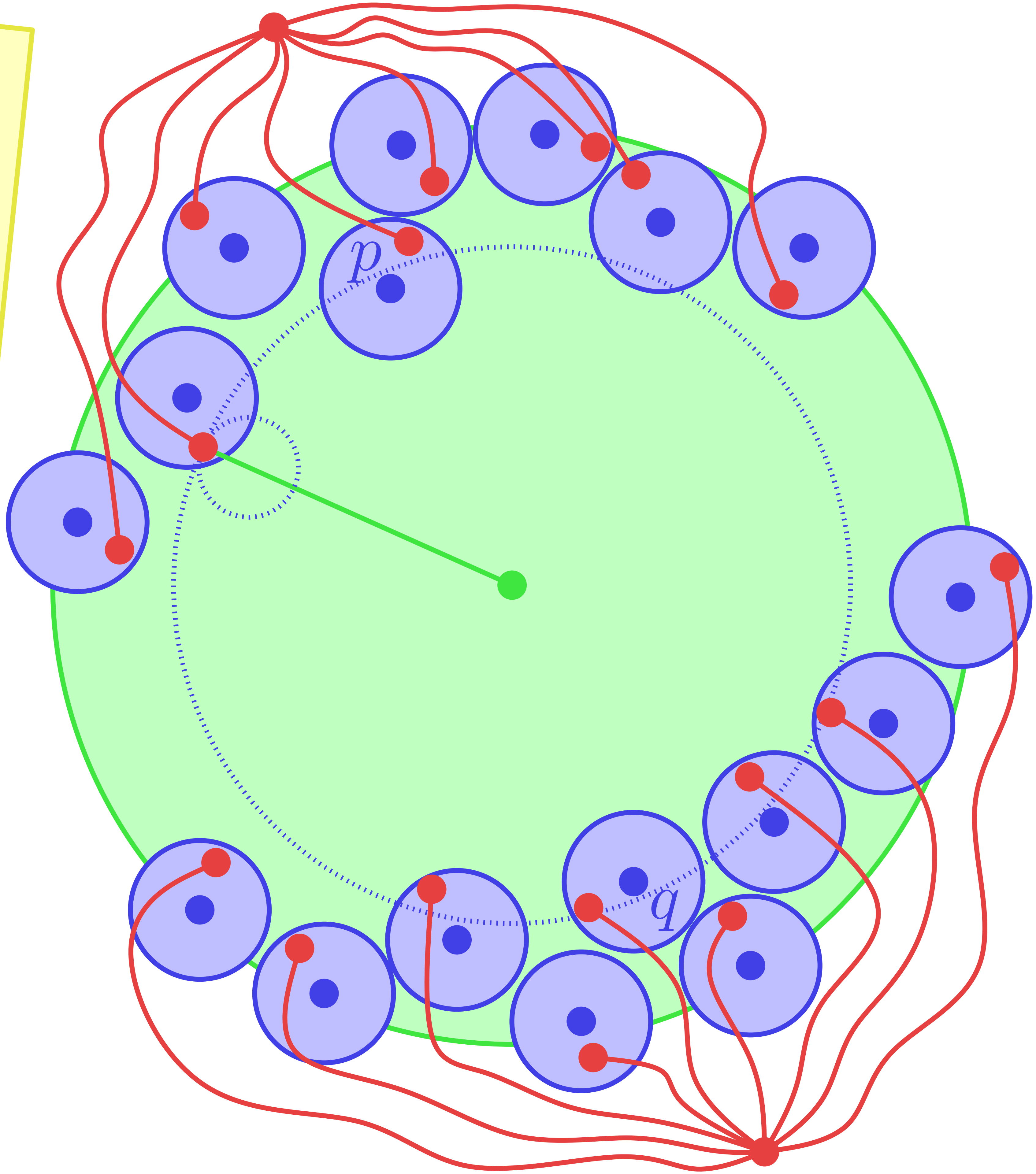

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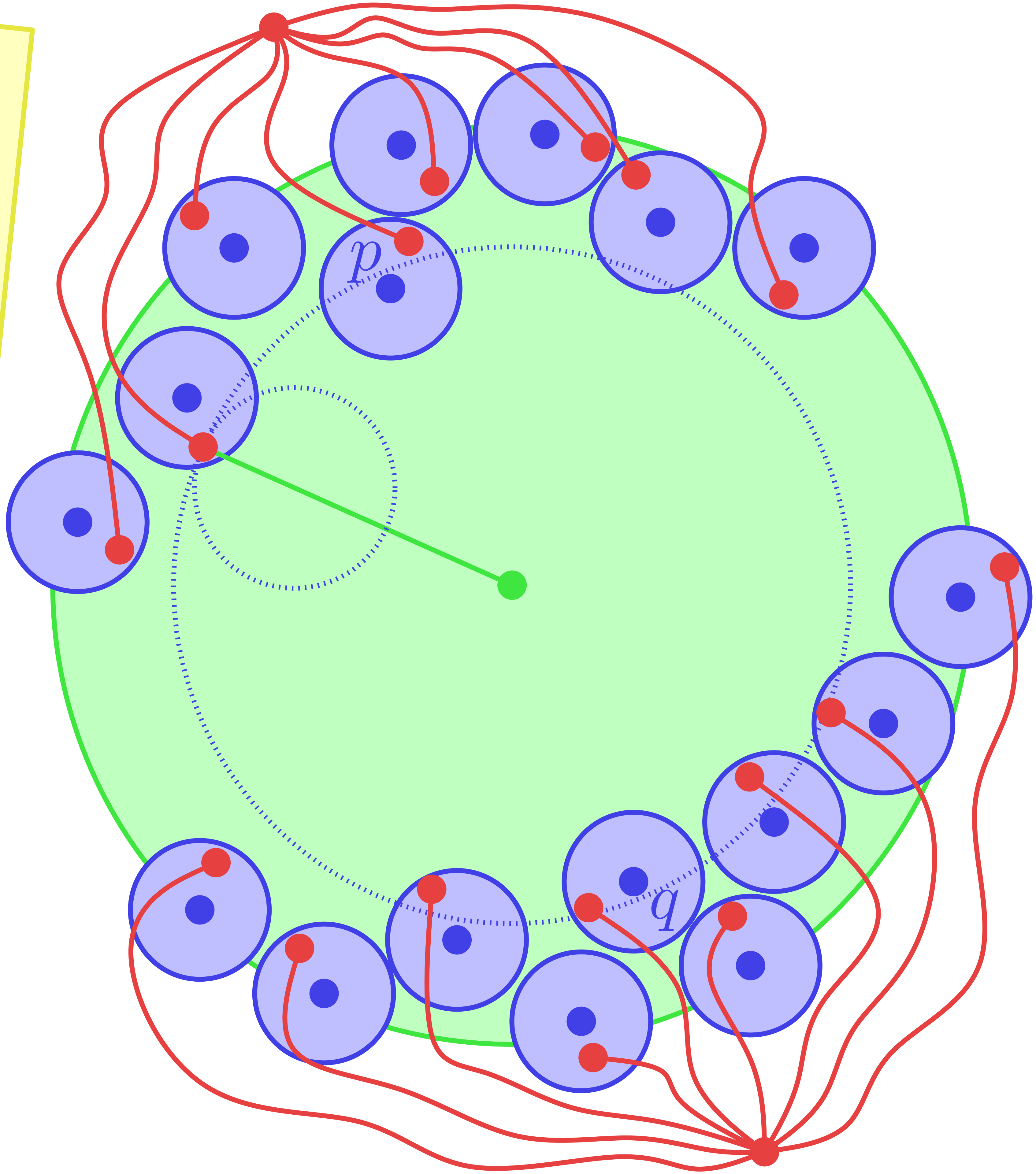

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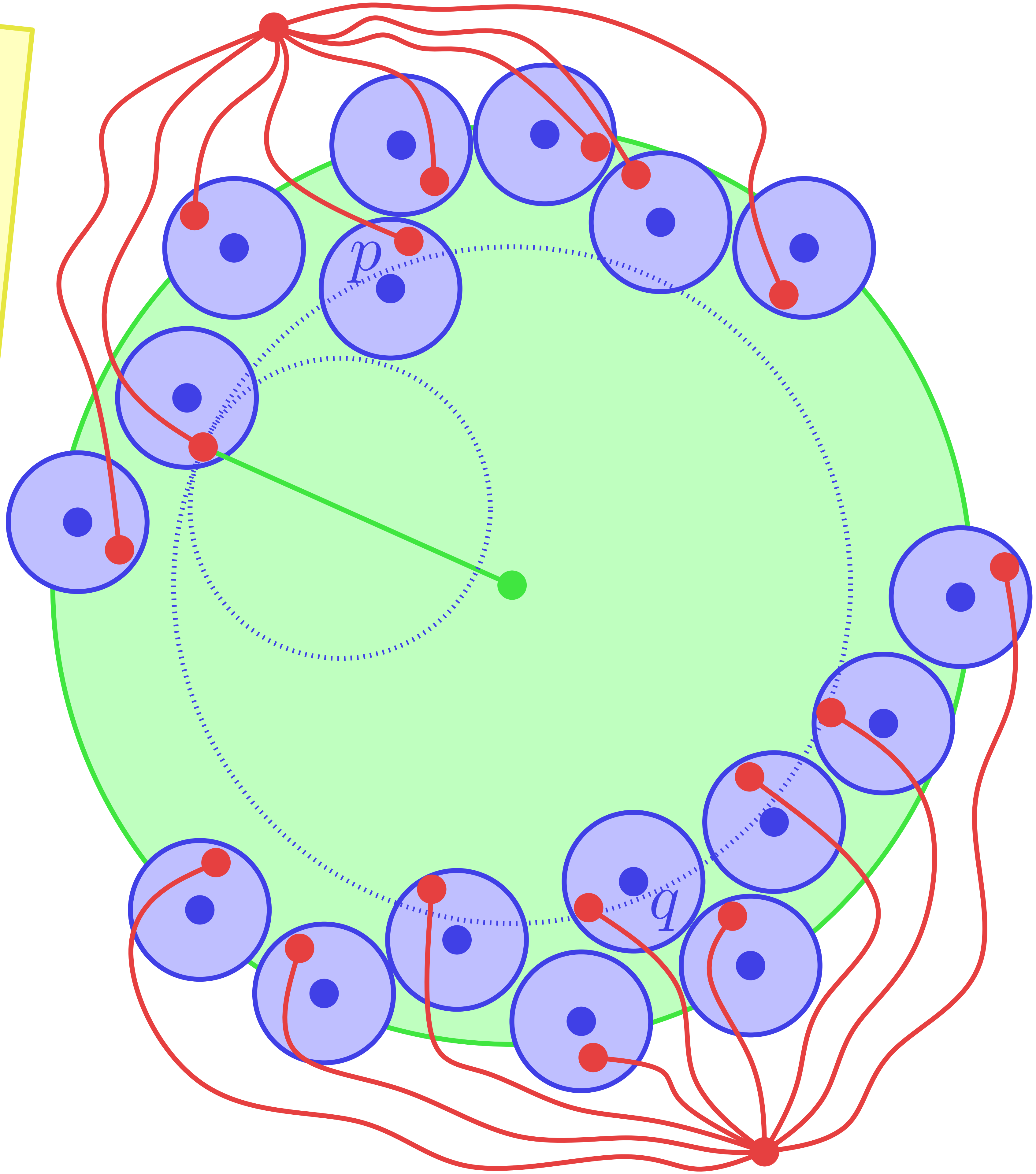

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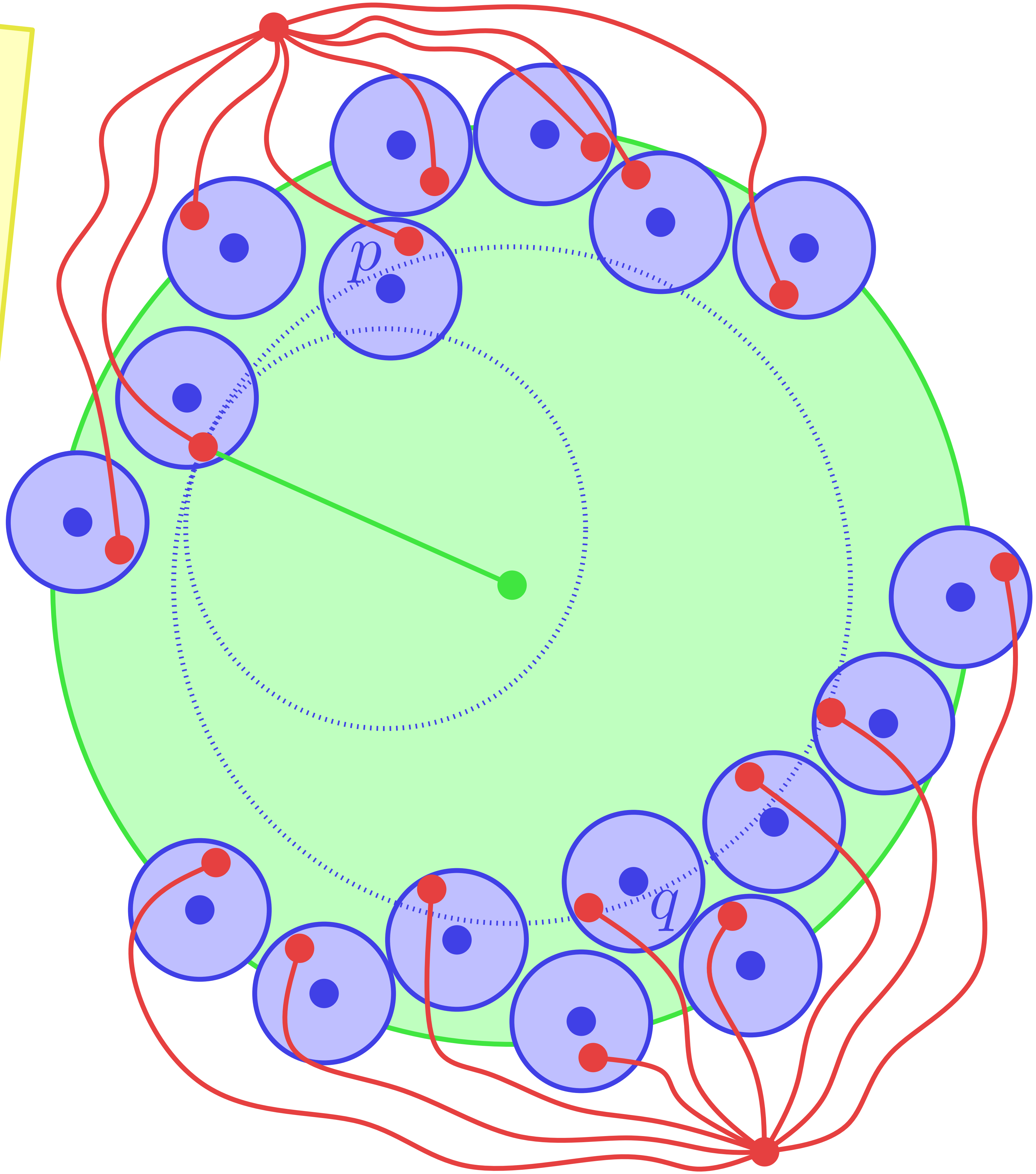

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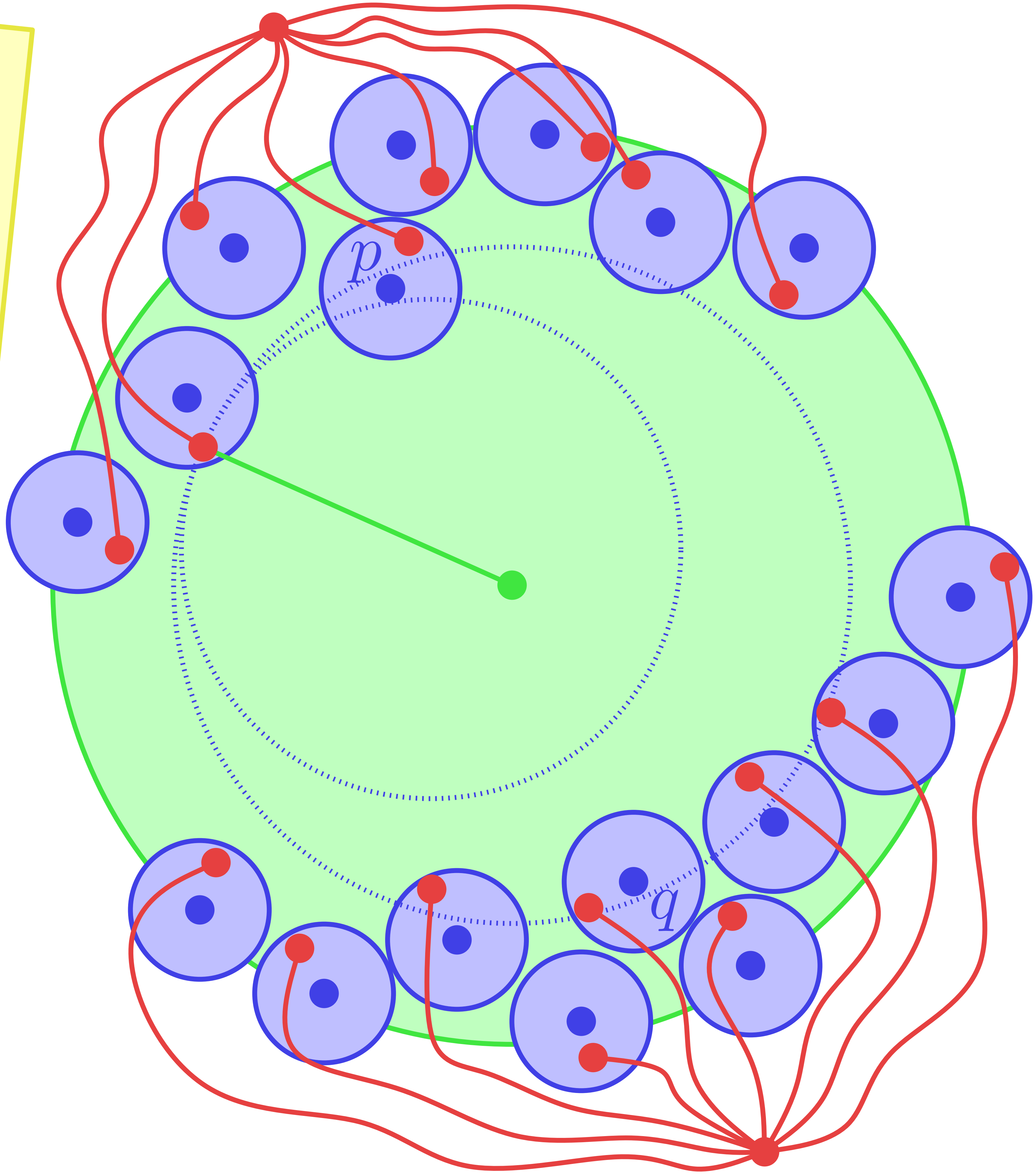

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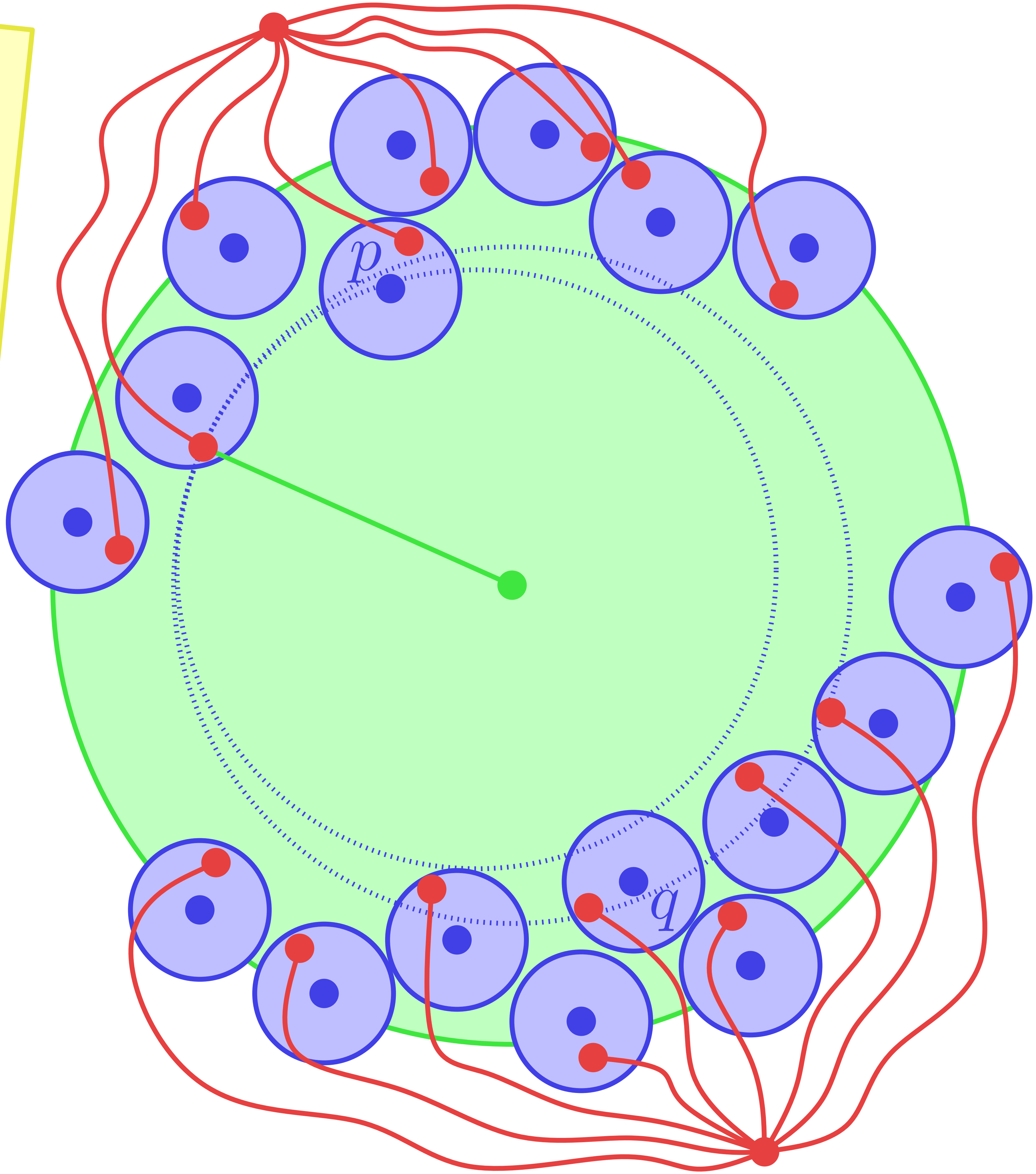

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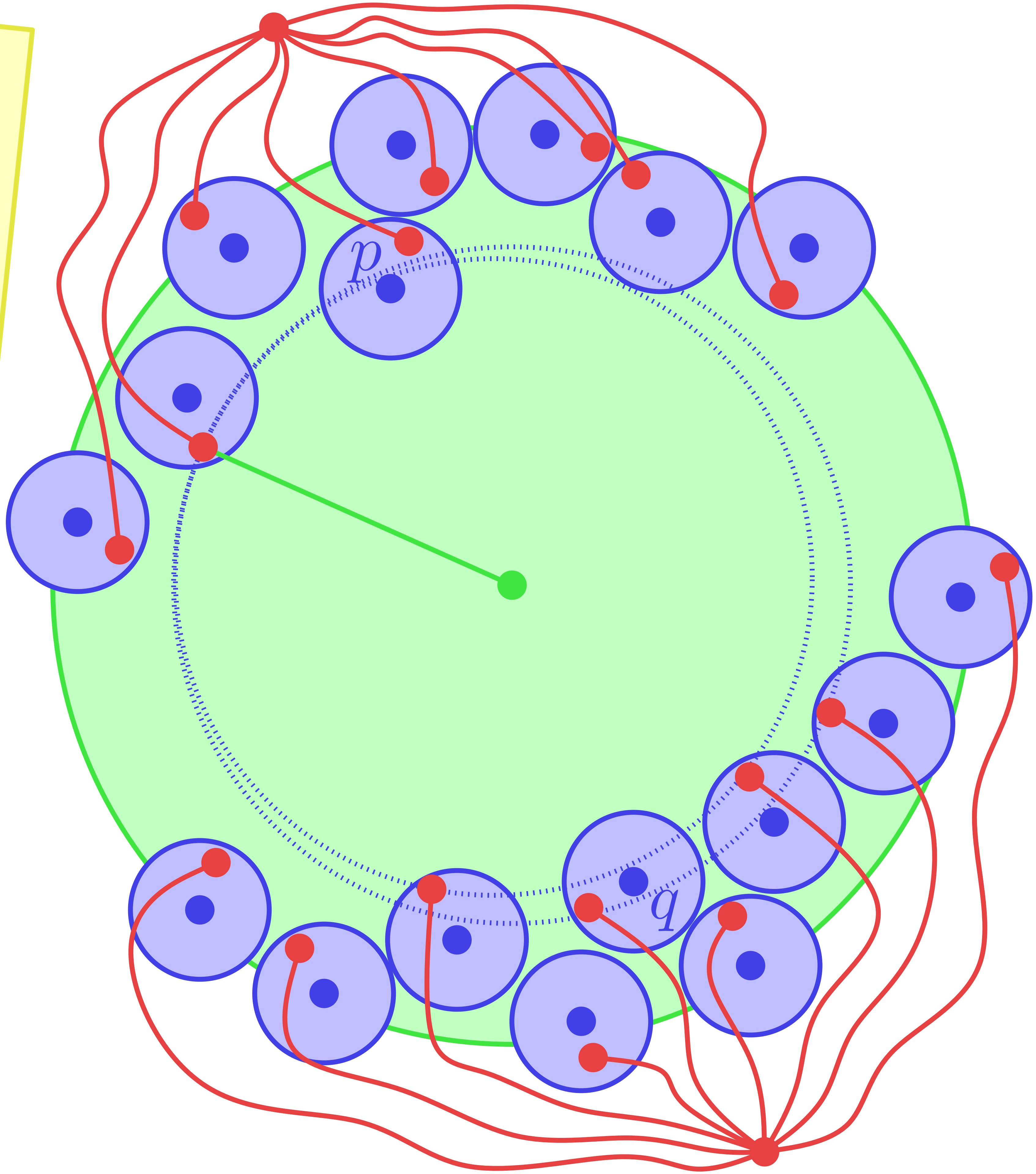

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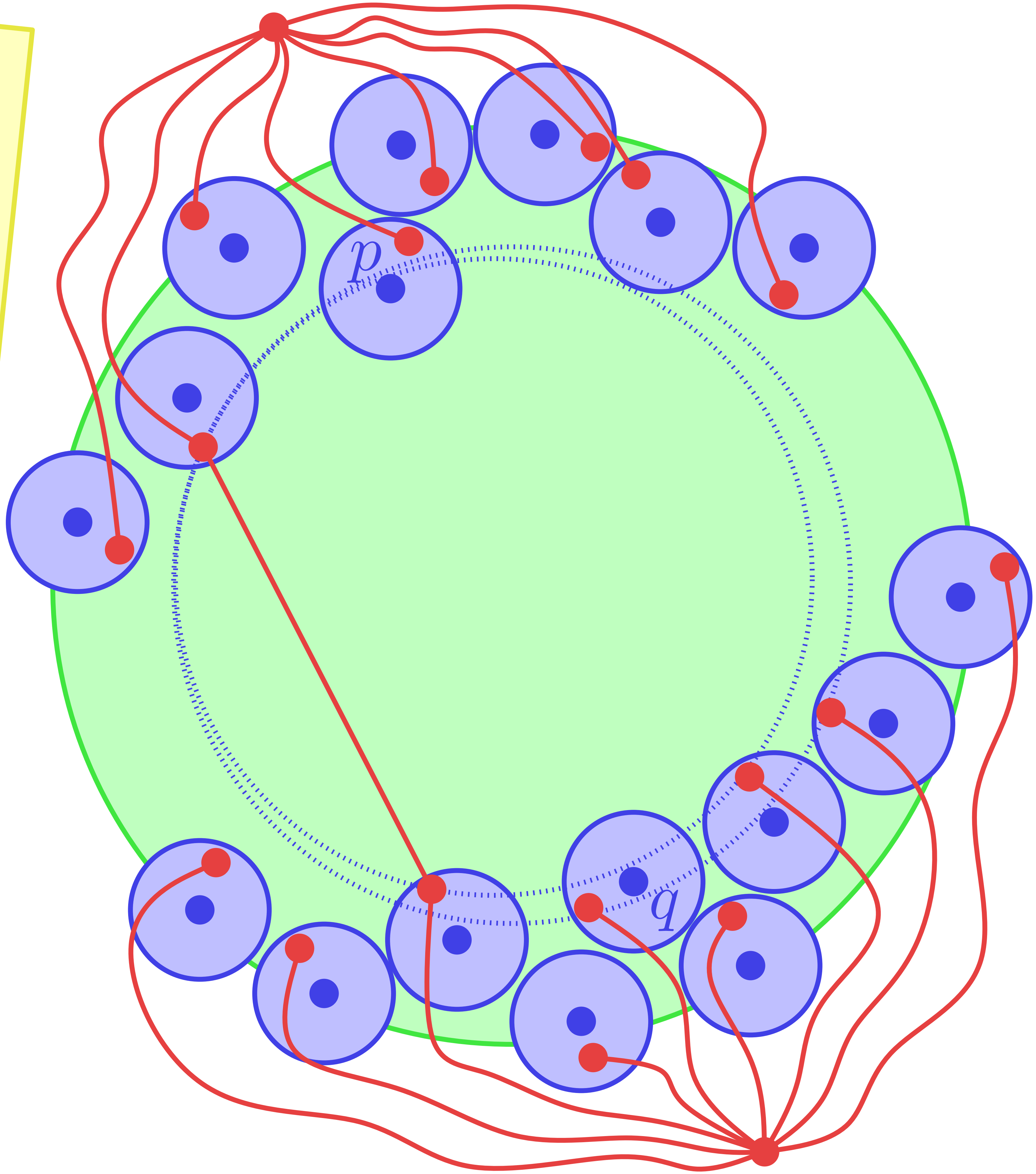
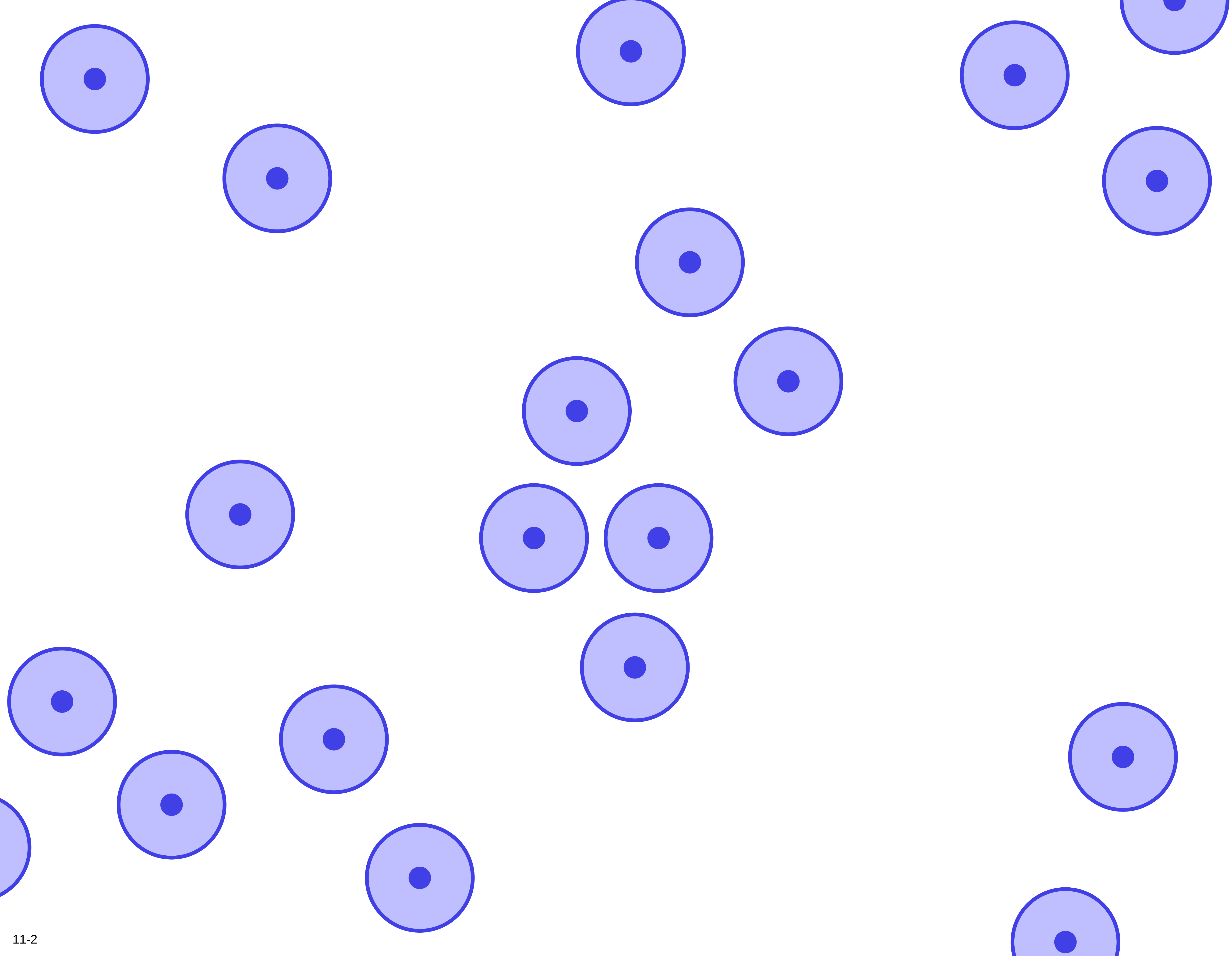


Let's put it together and reconstruct the Delaunay triangulation!



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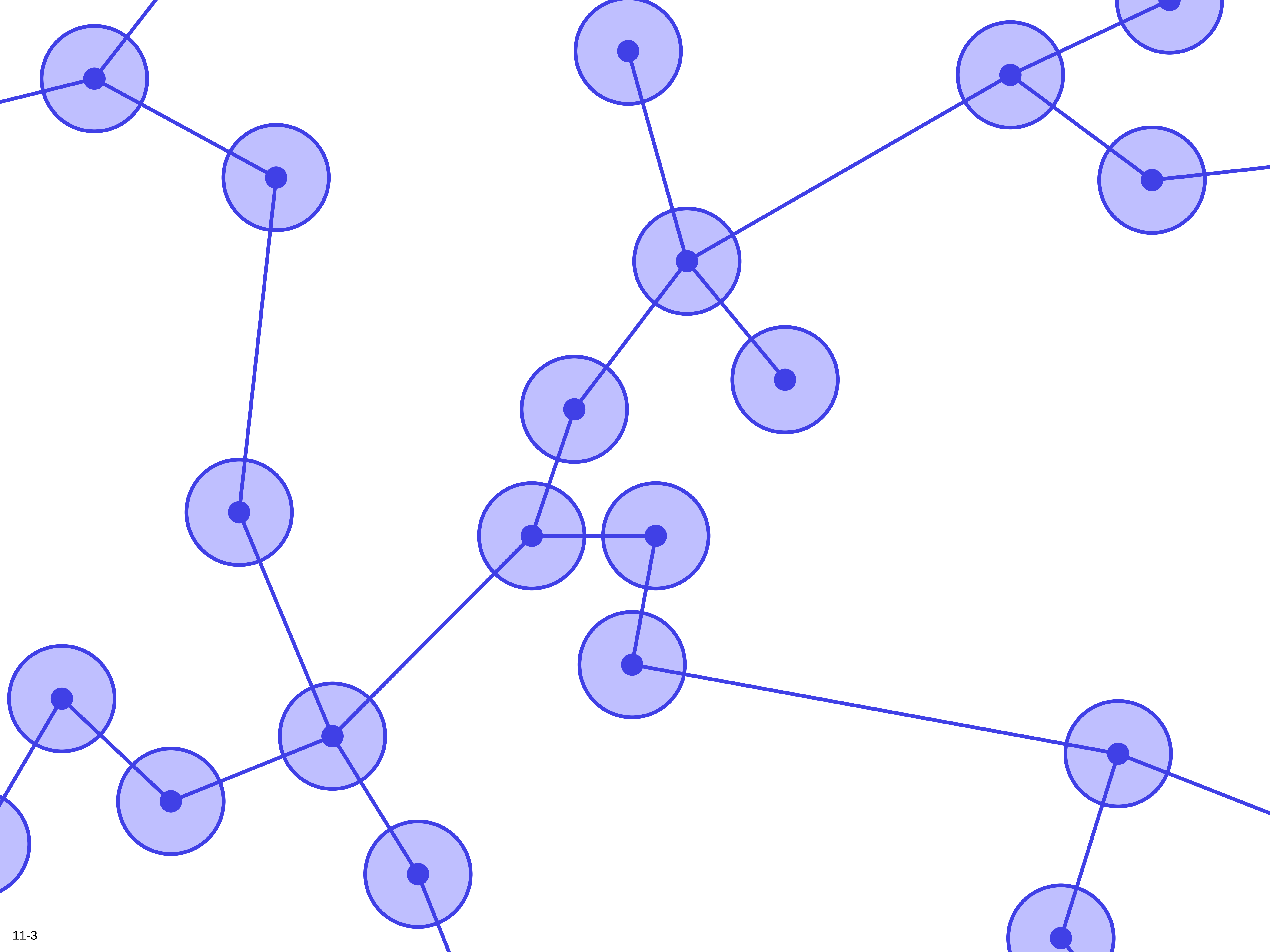

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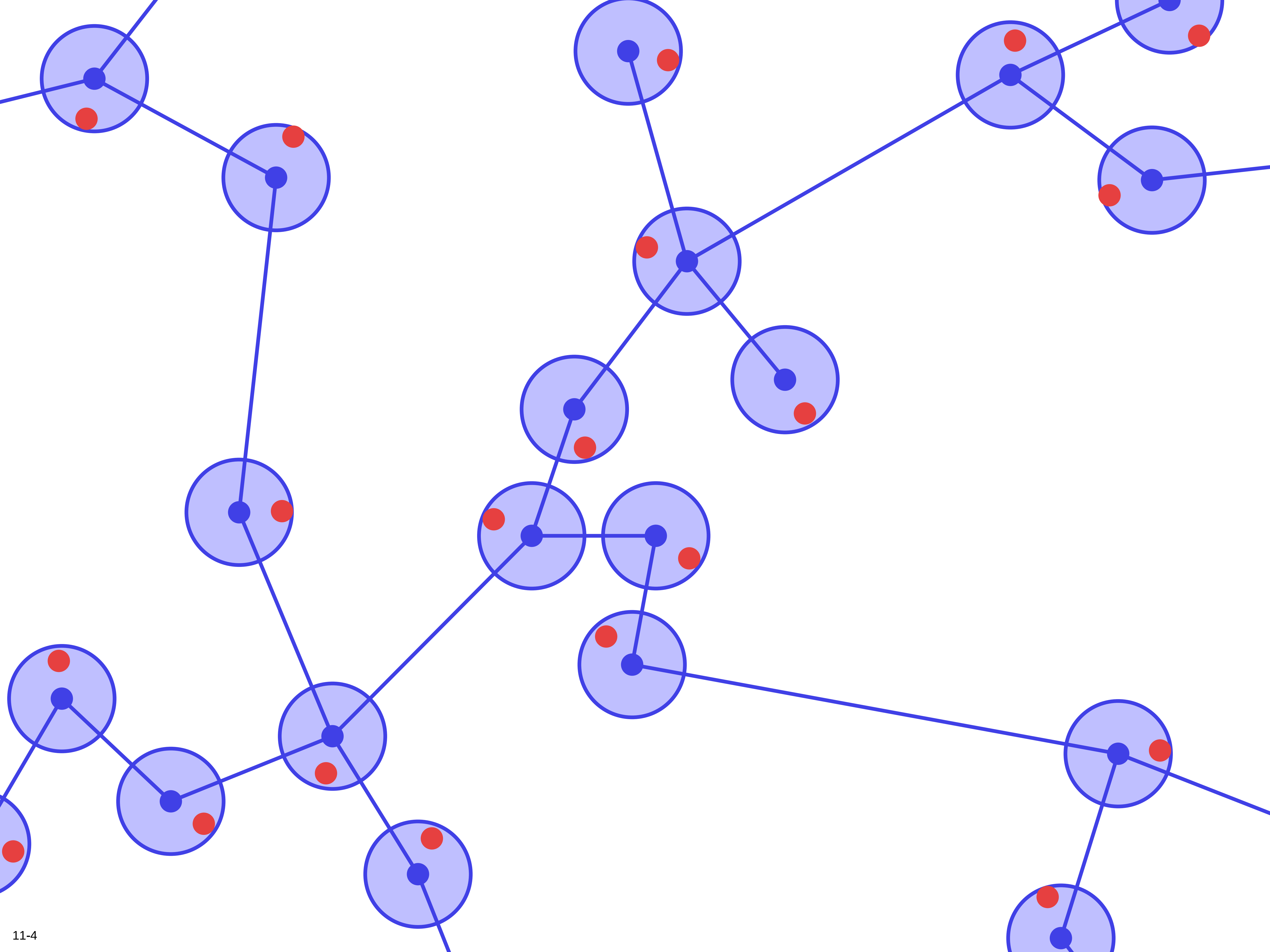

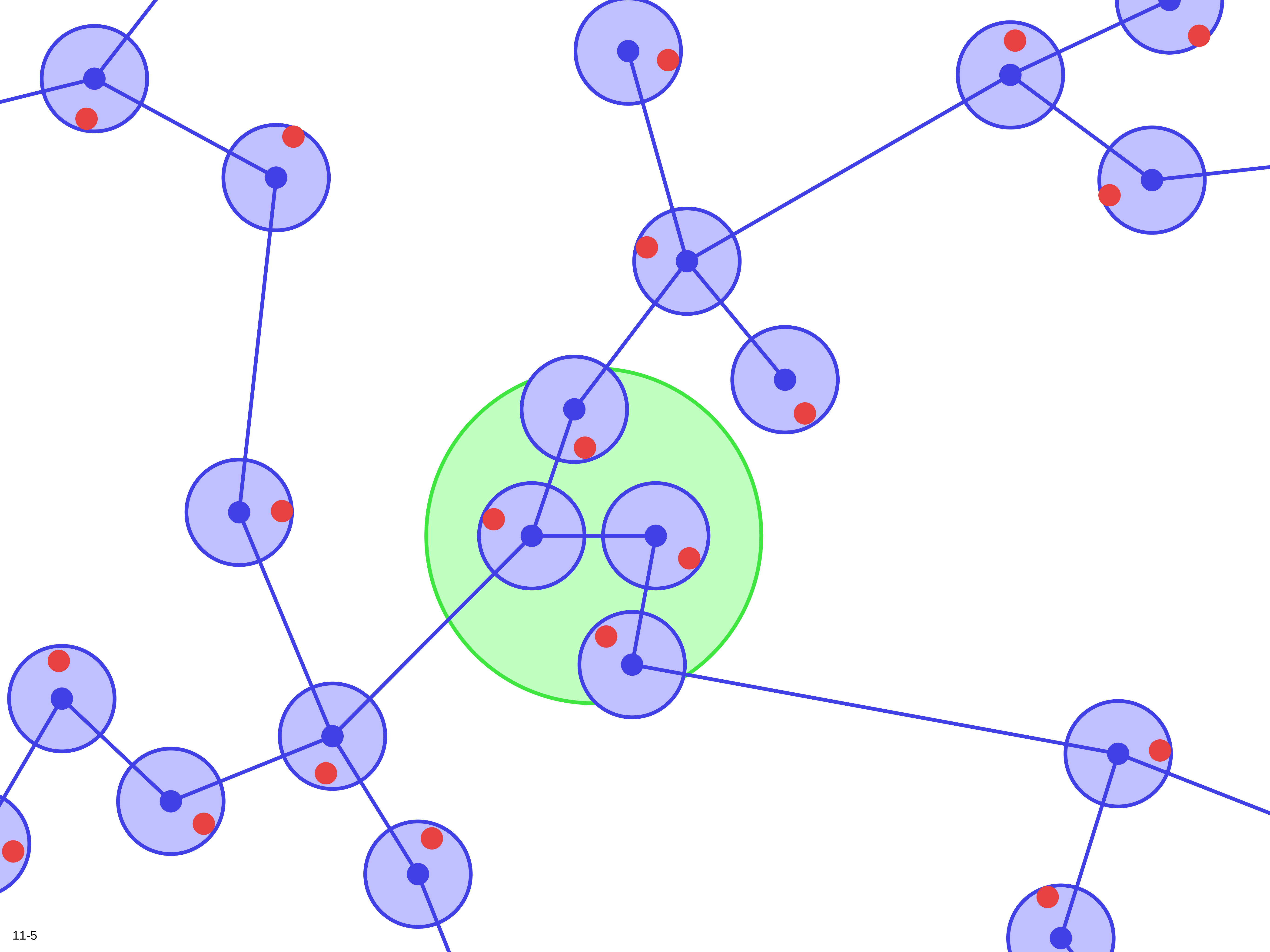

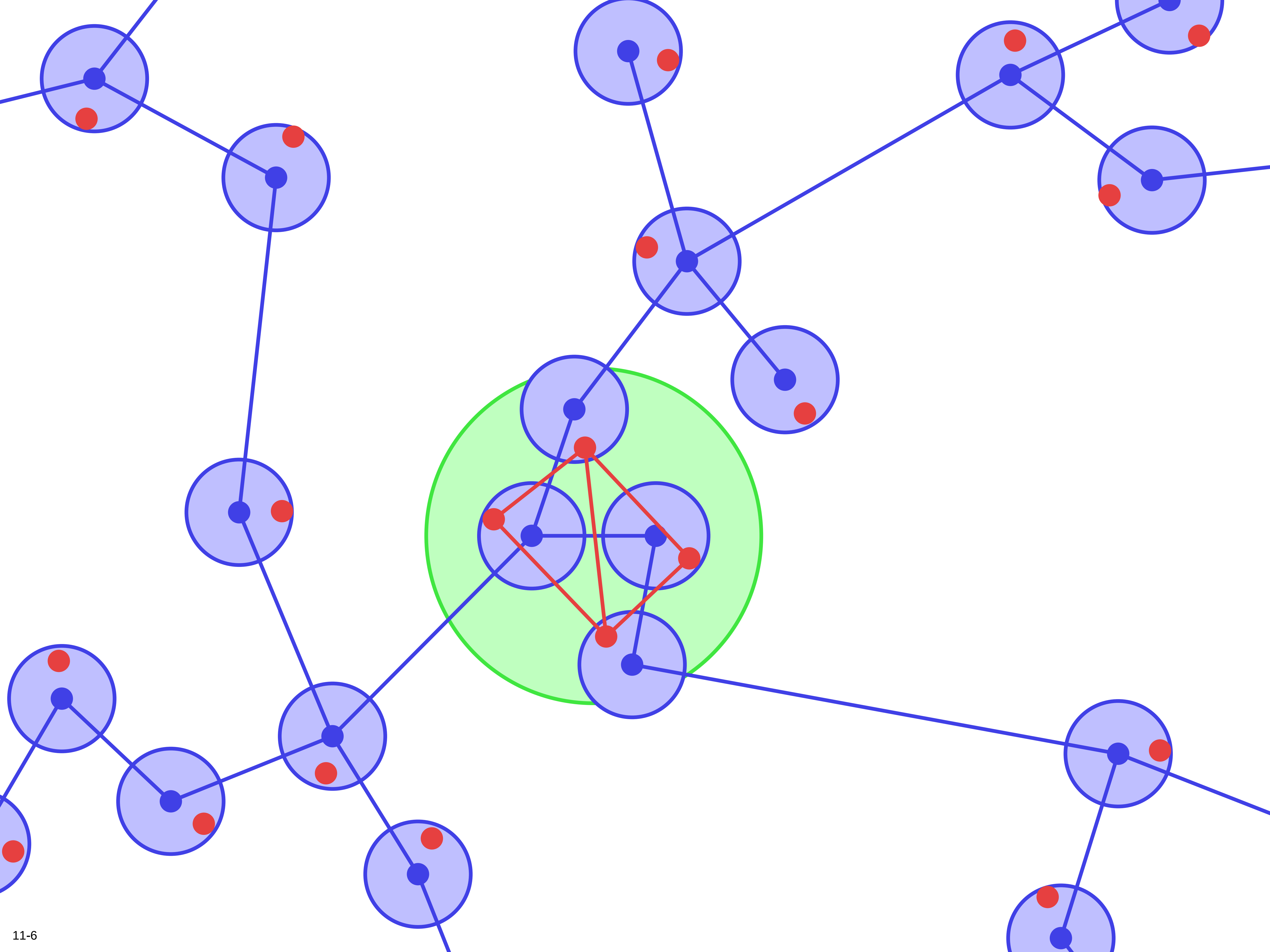

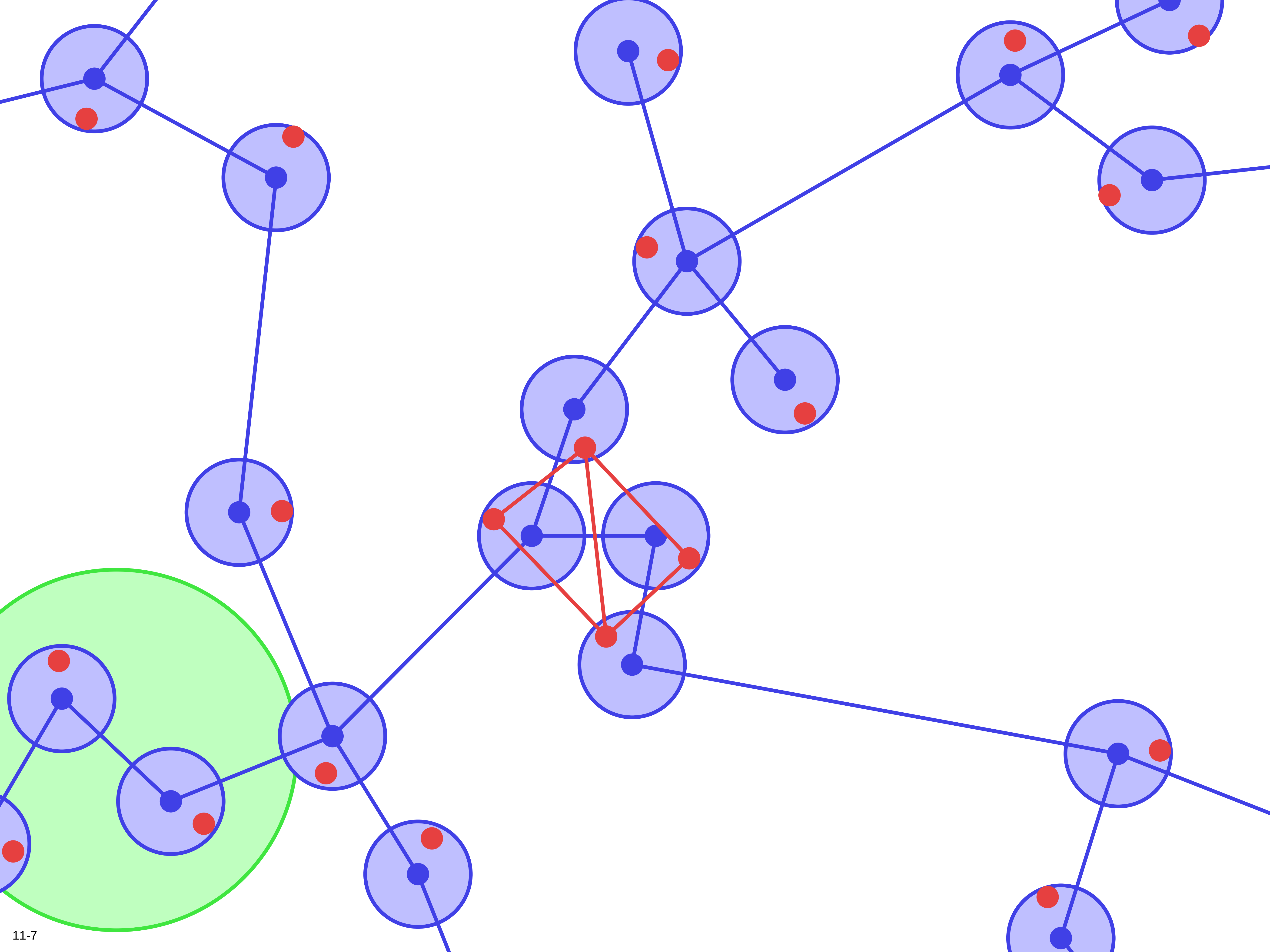
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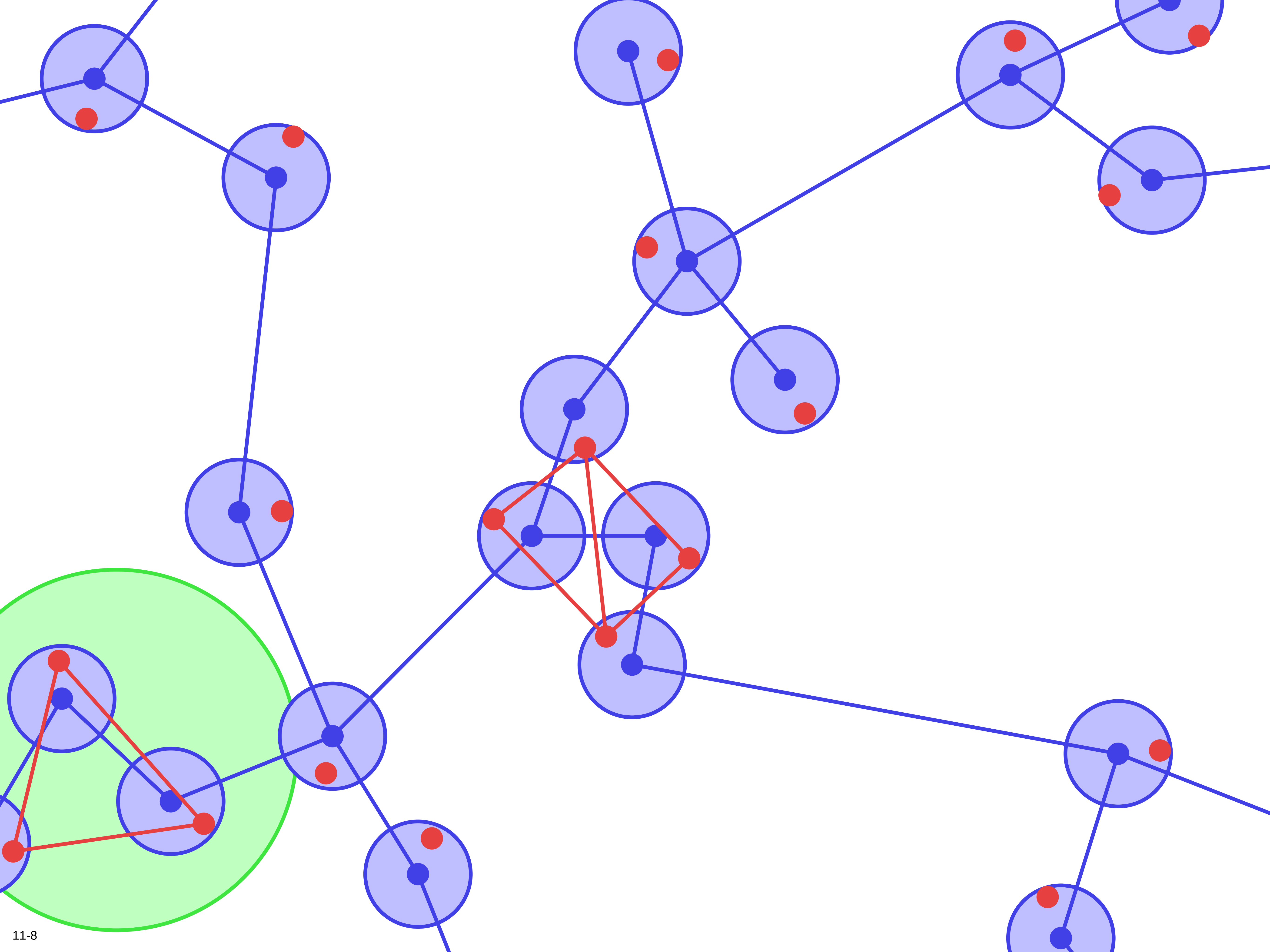

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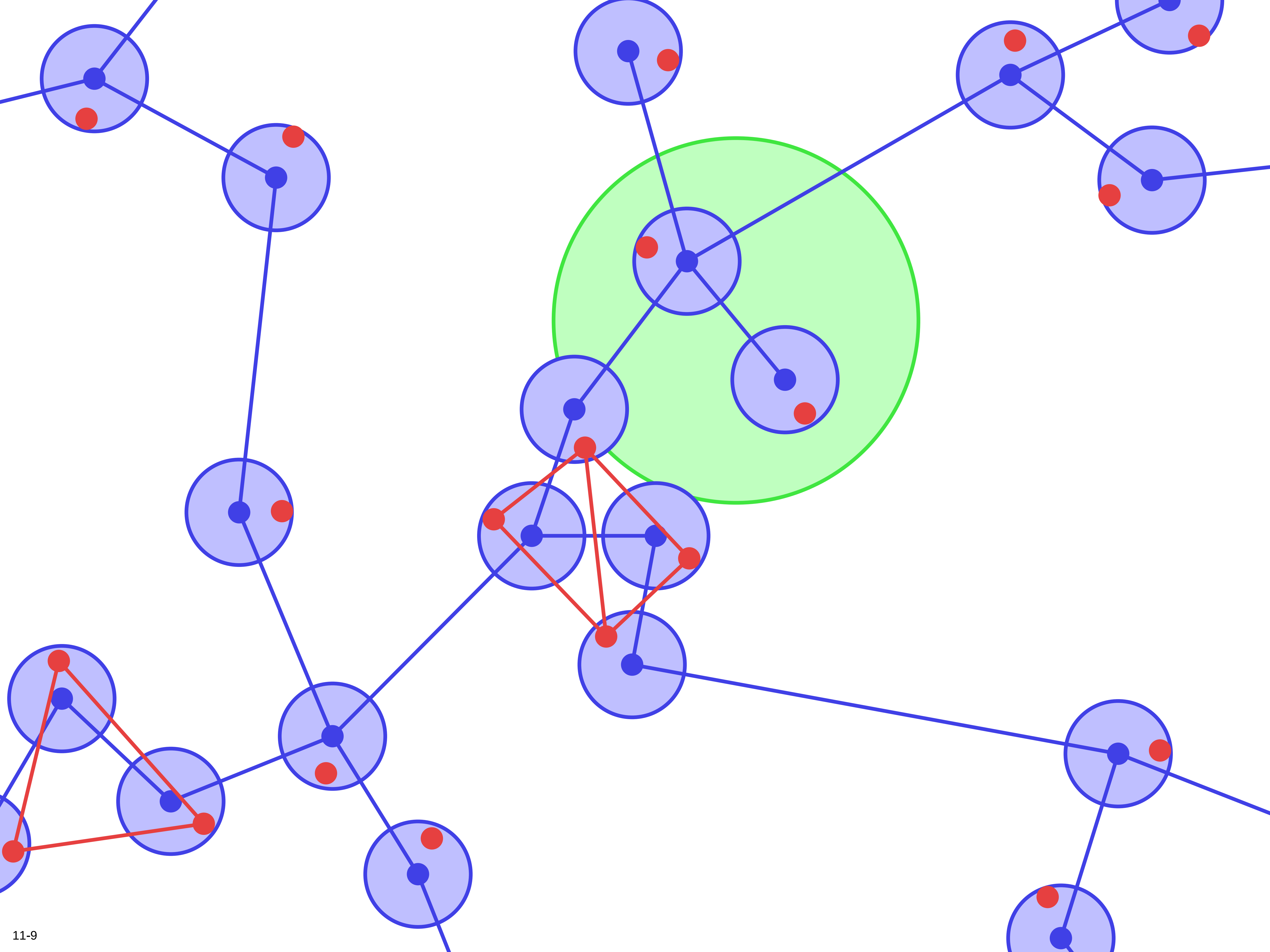

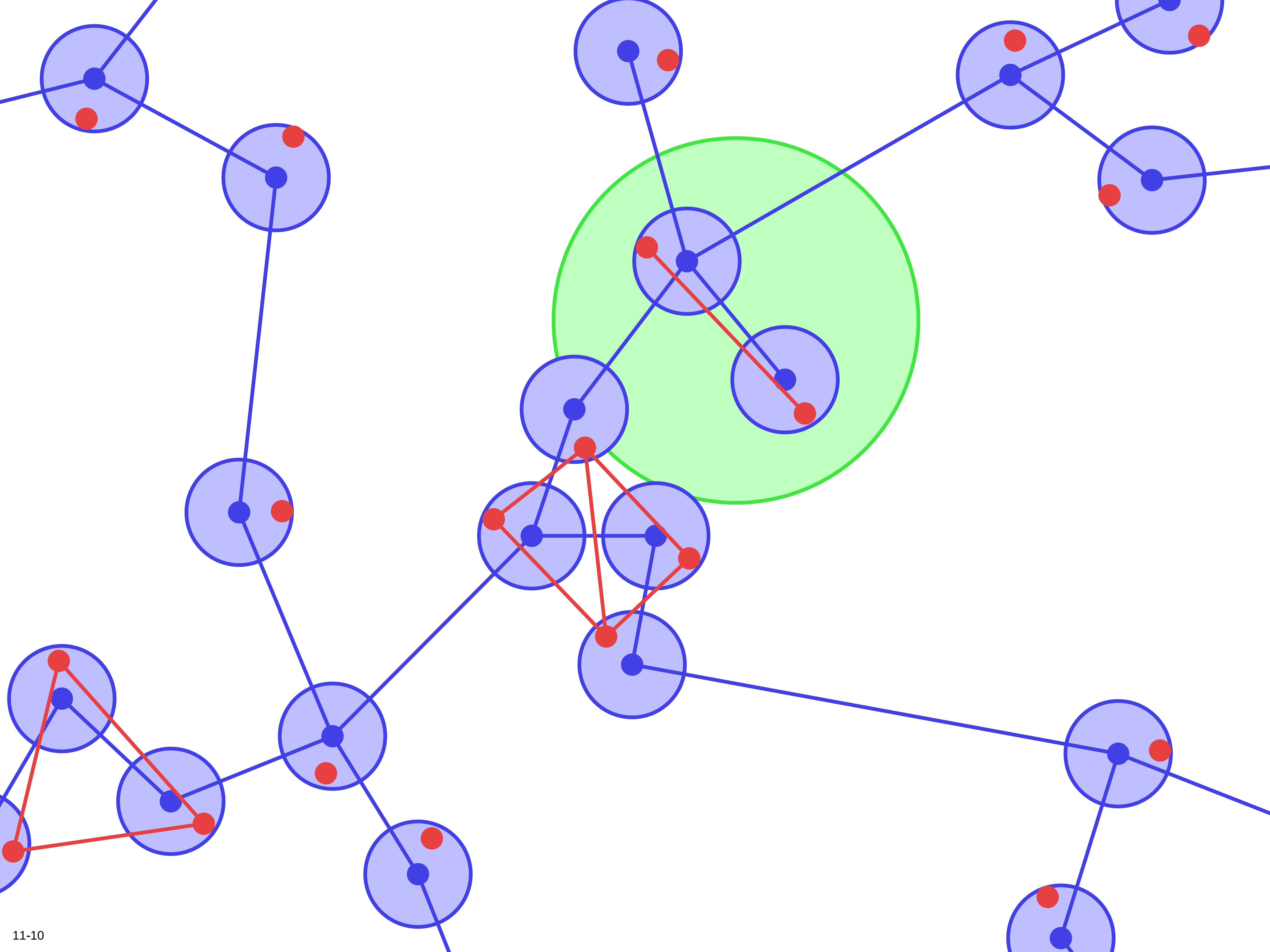

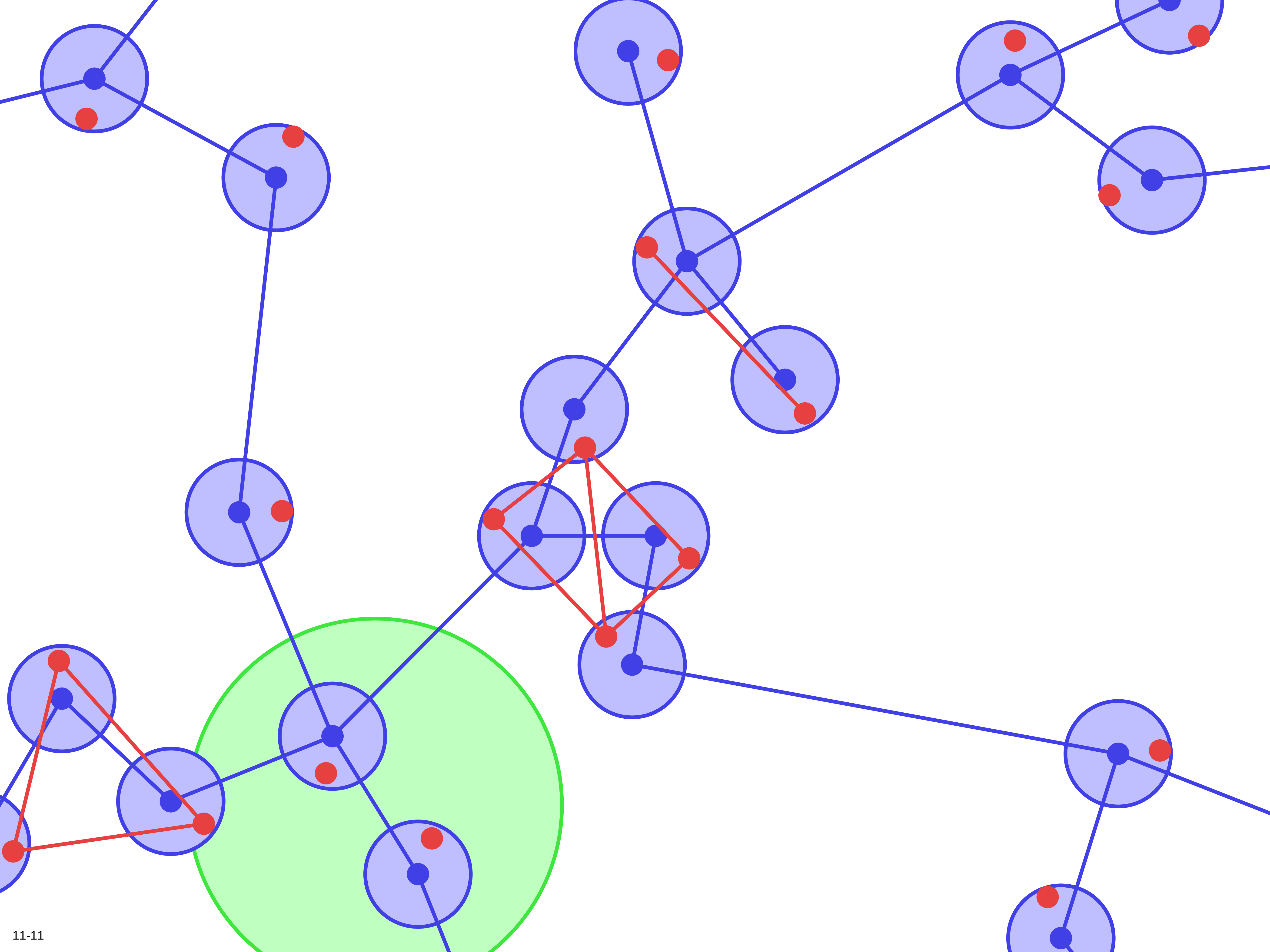

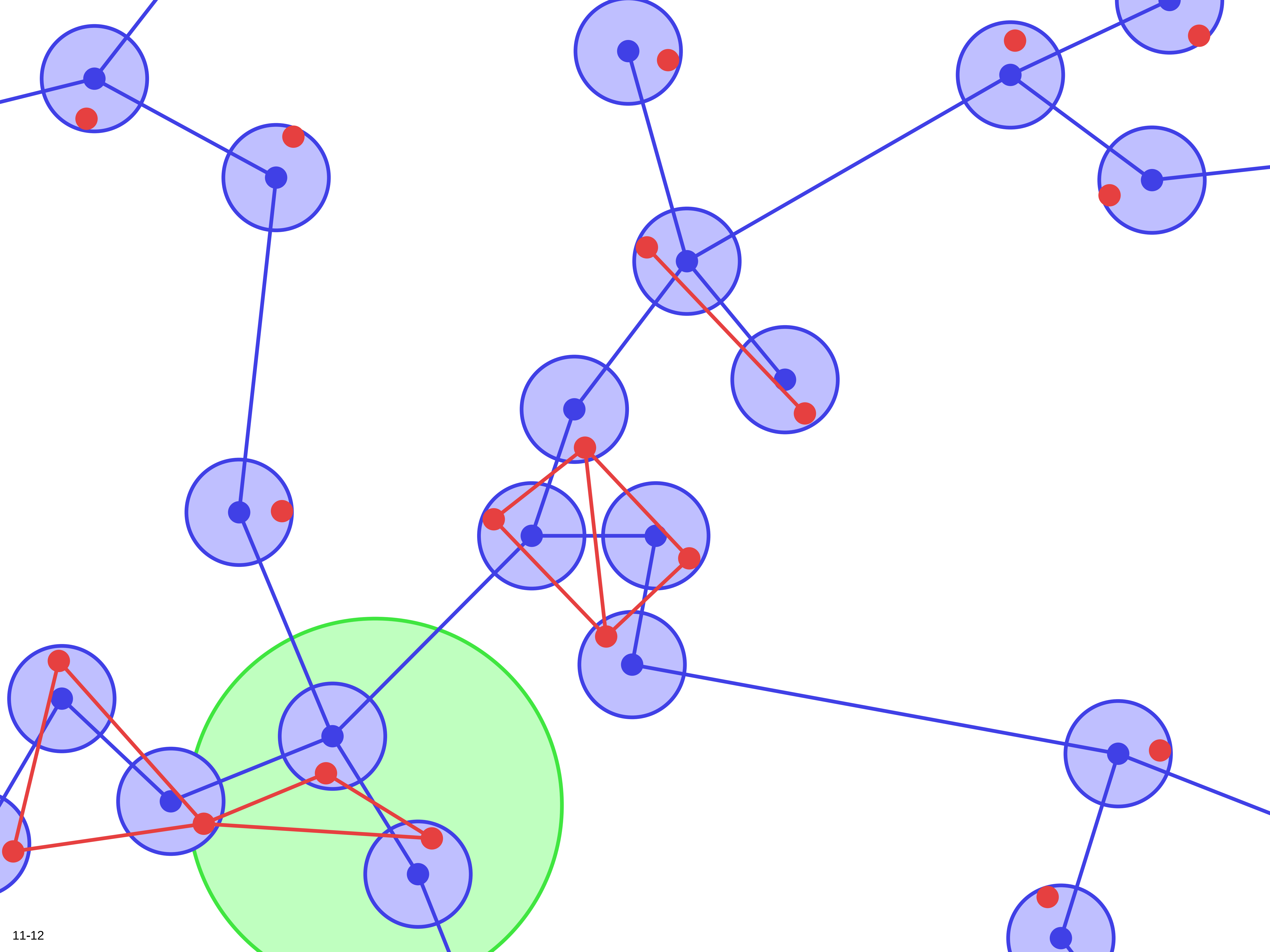

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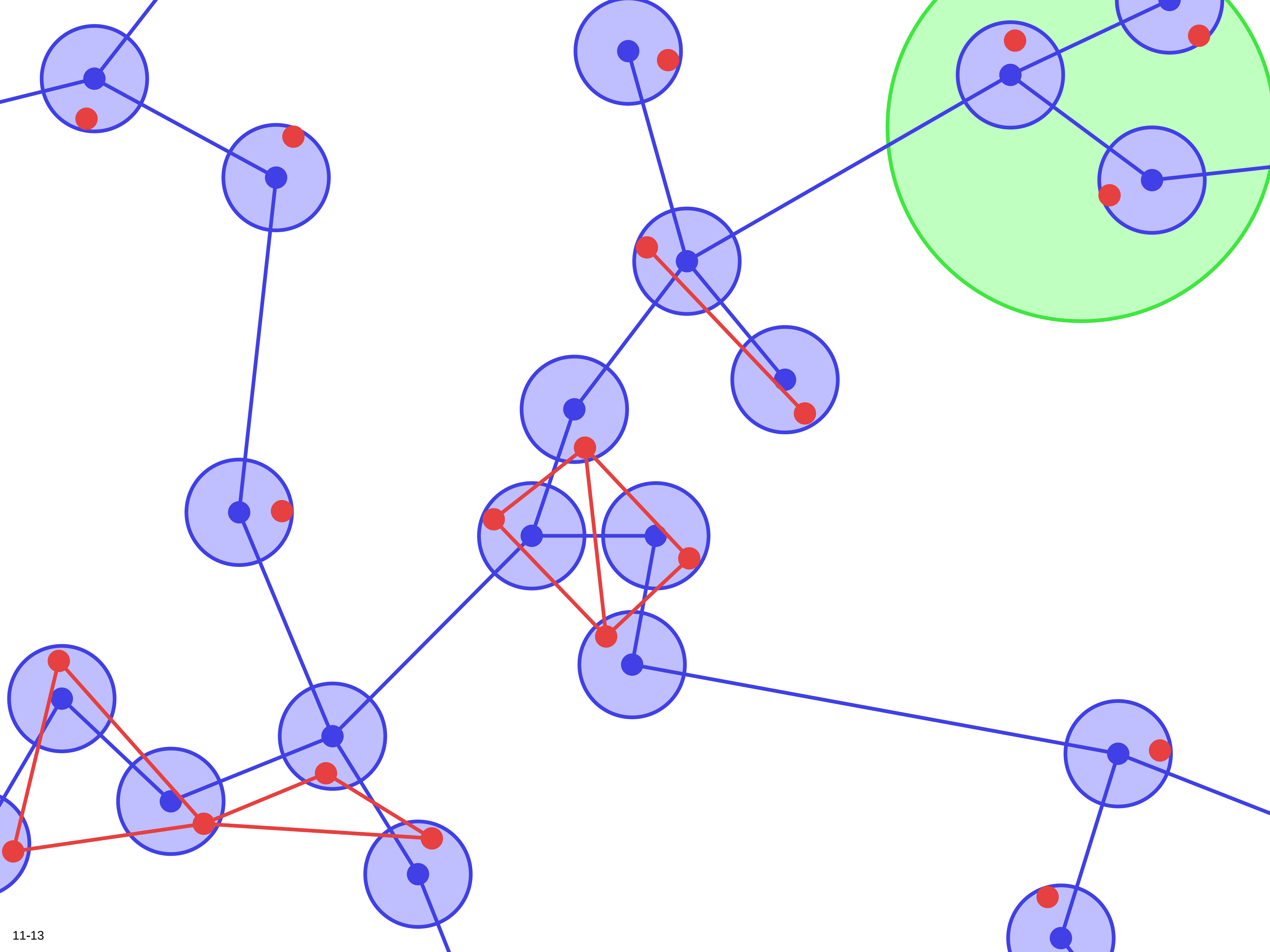

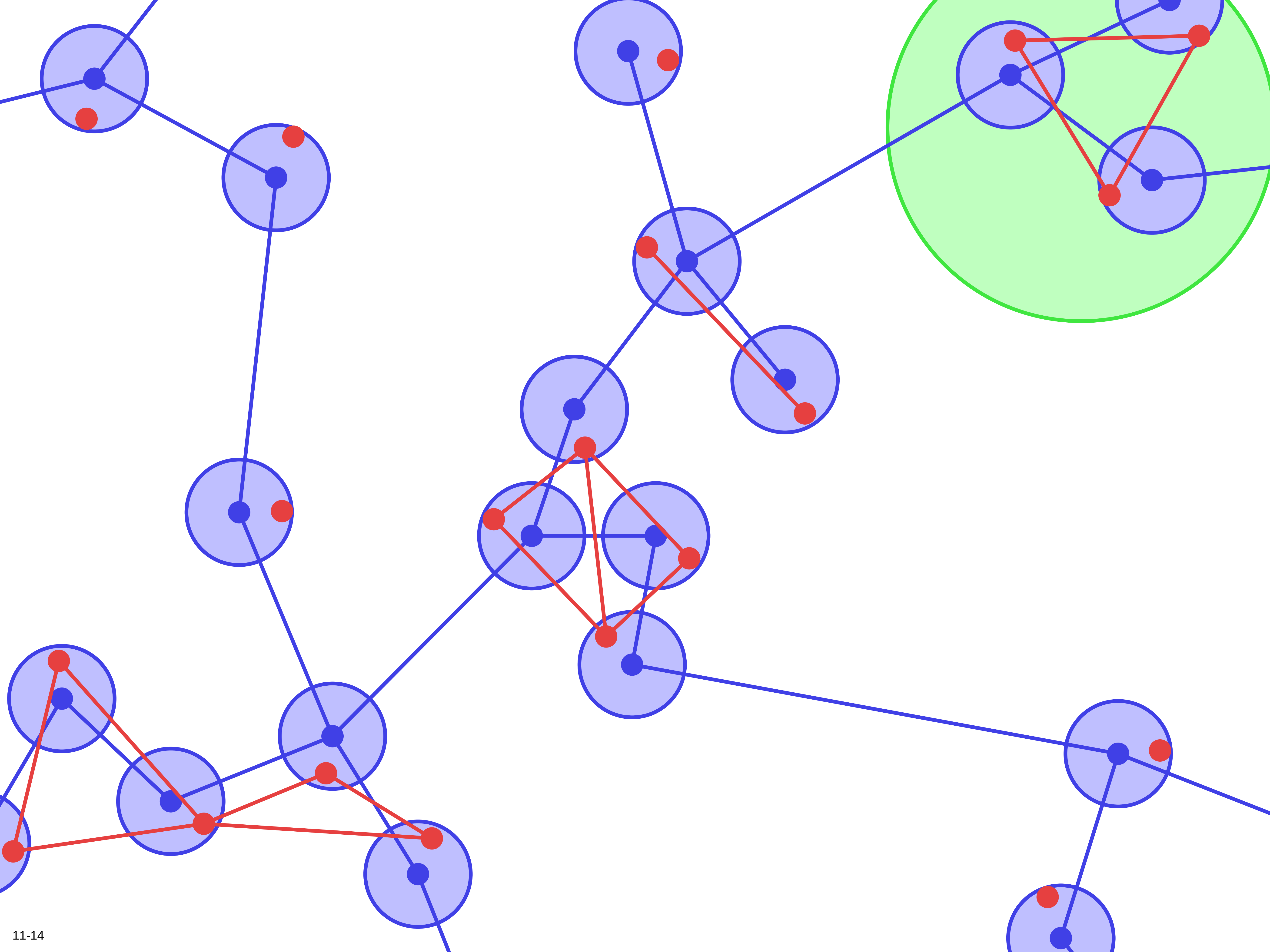

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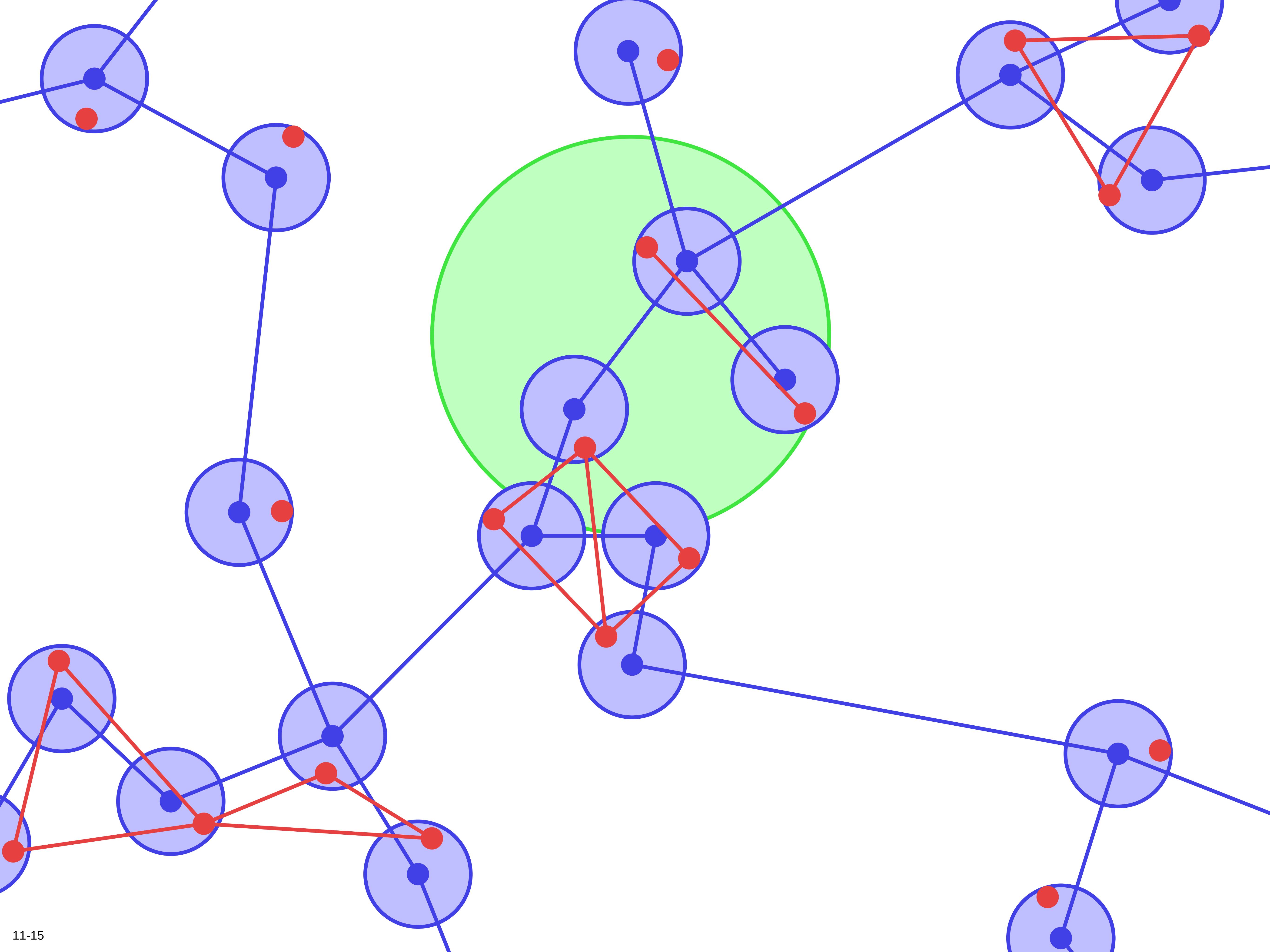

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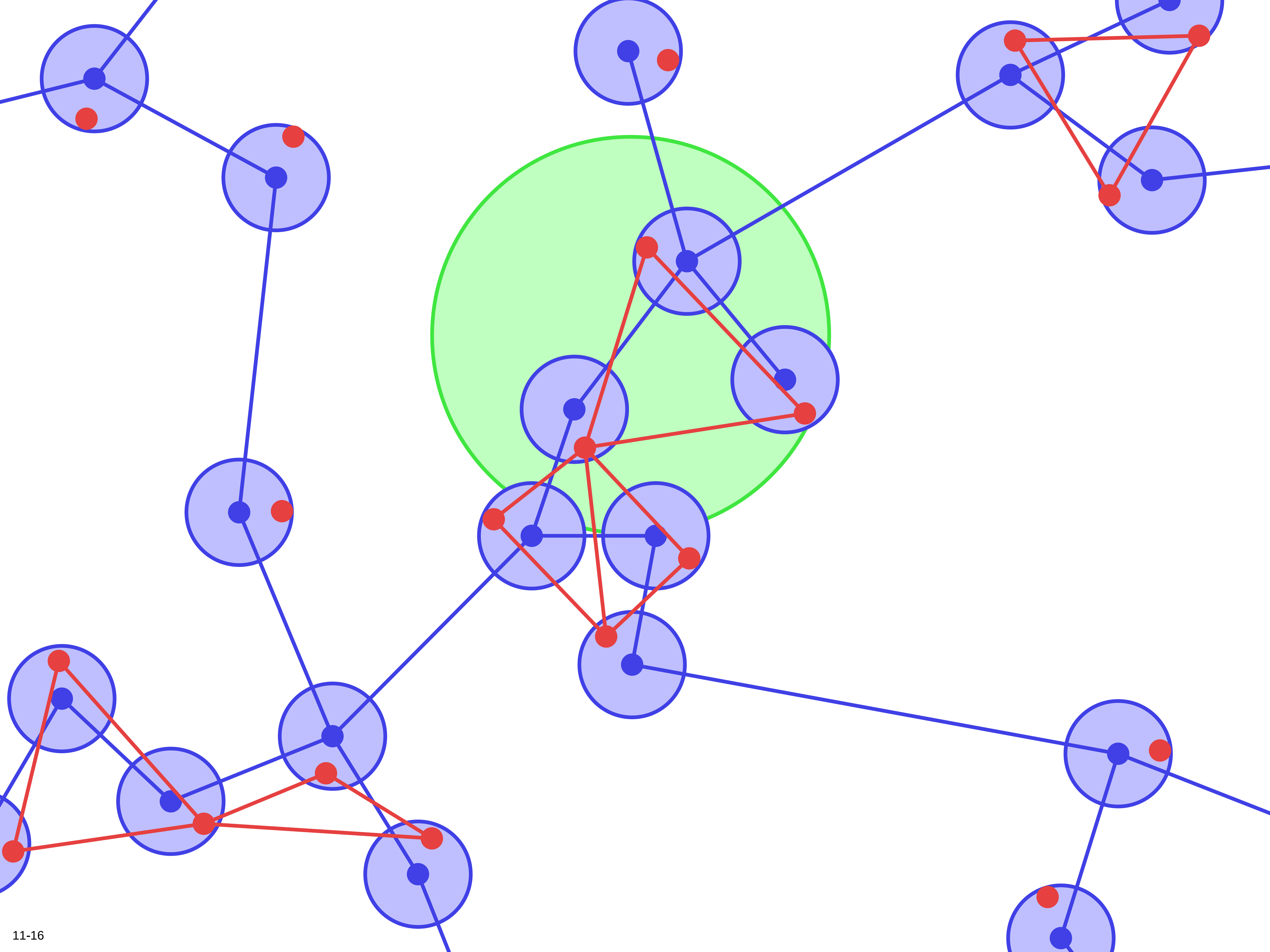

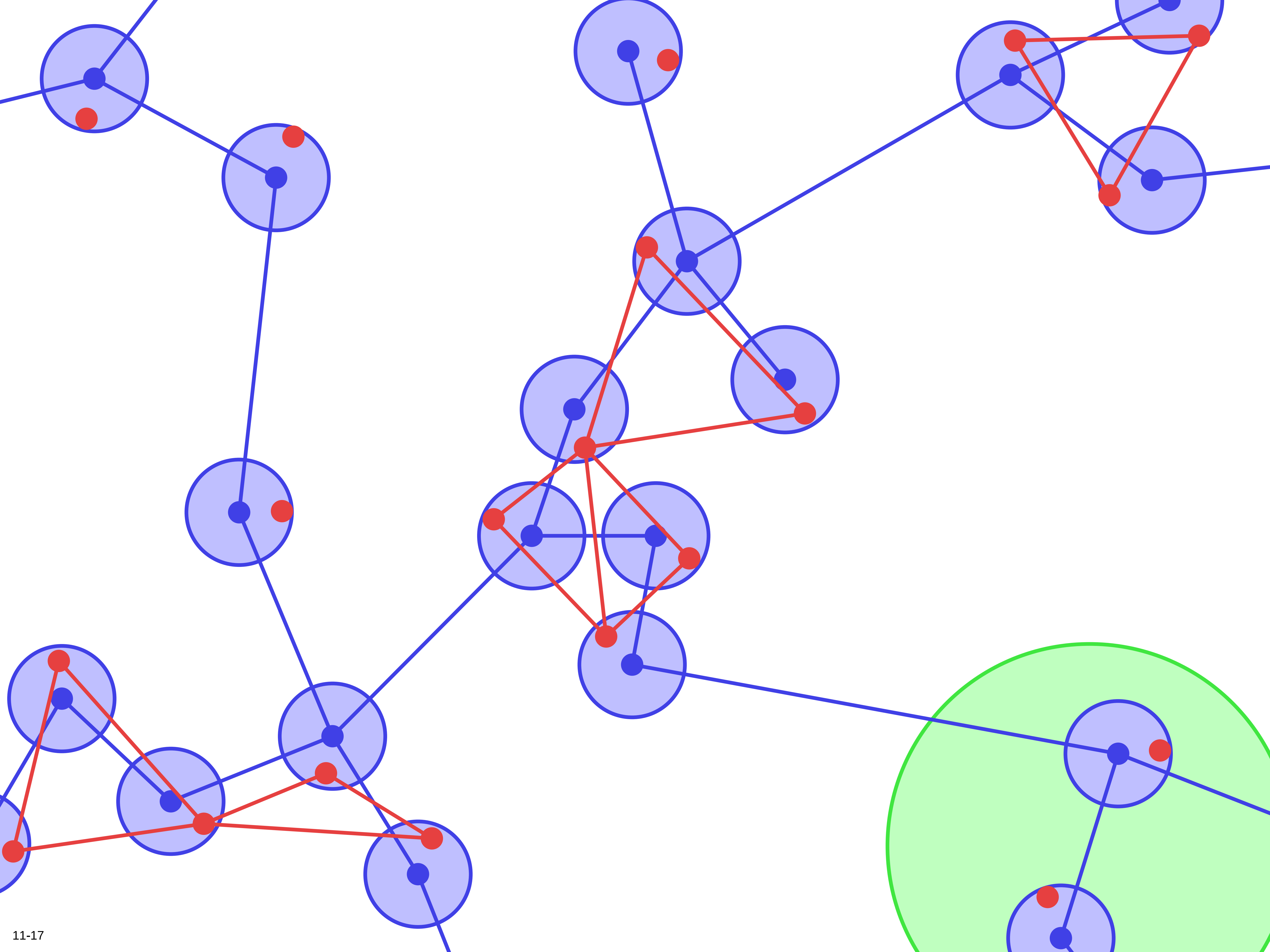

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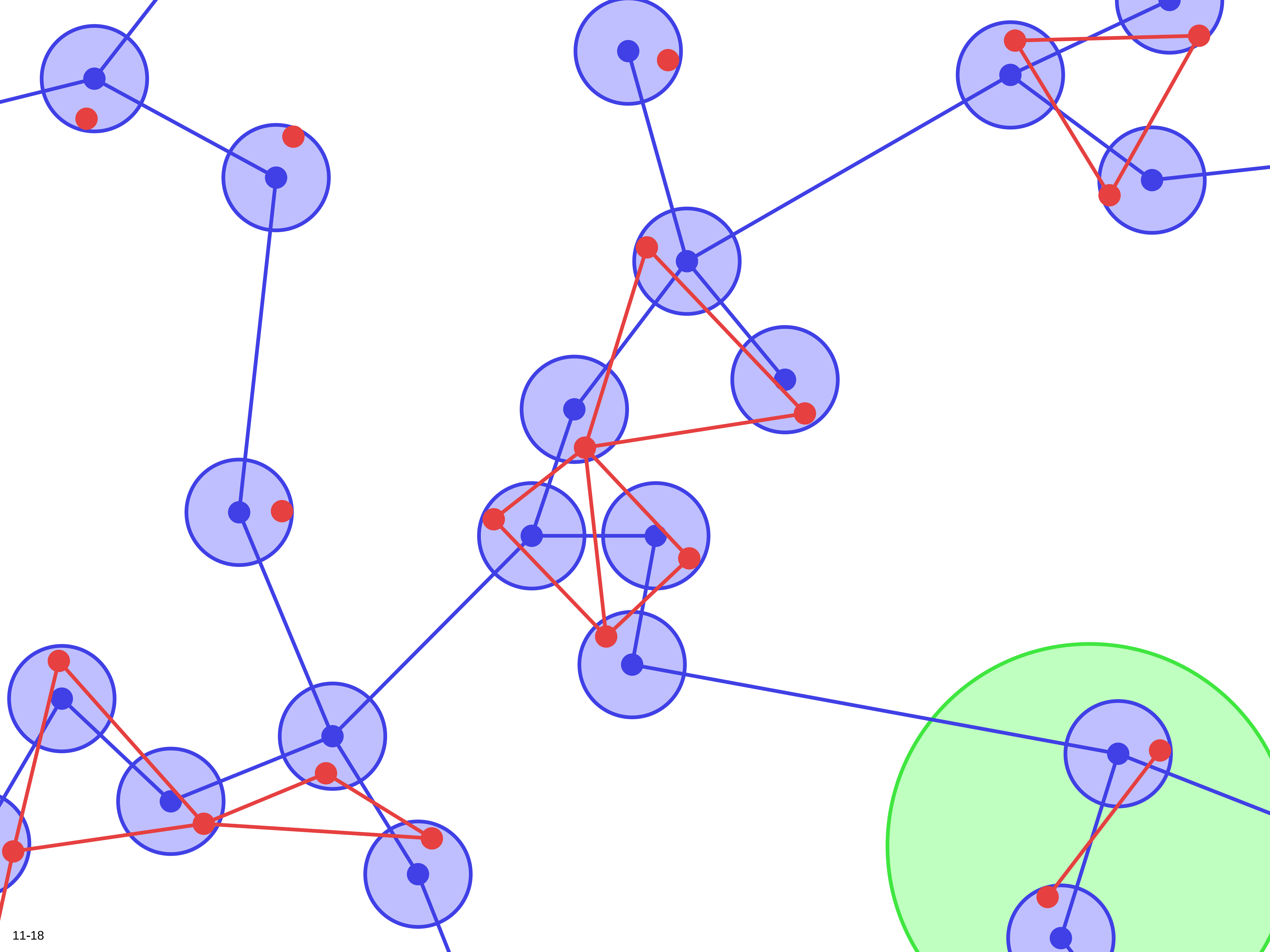

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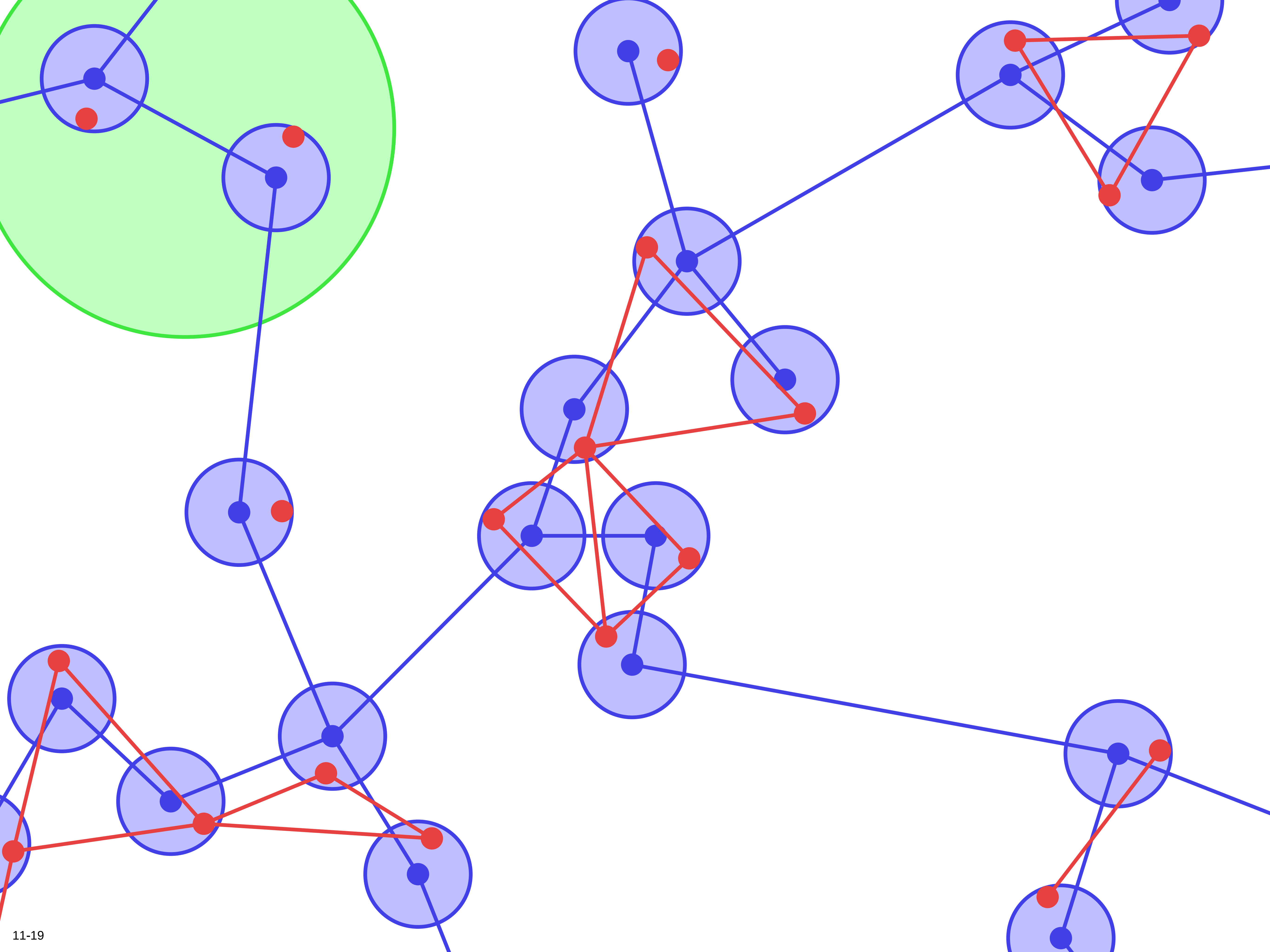

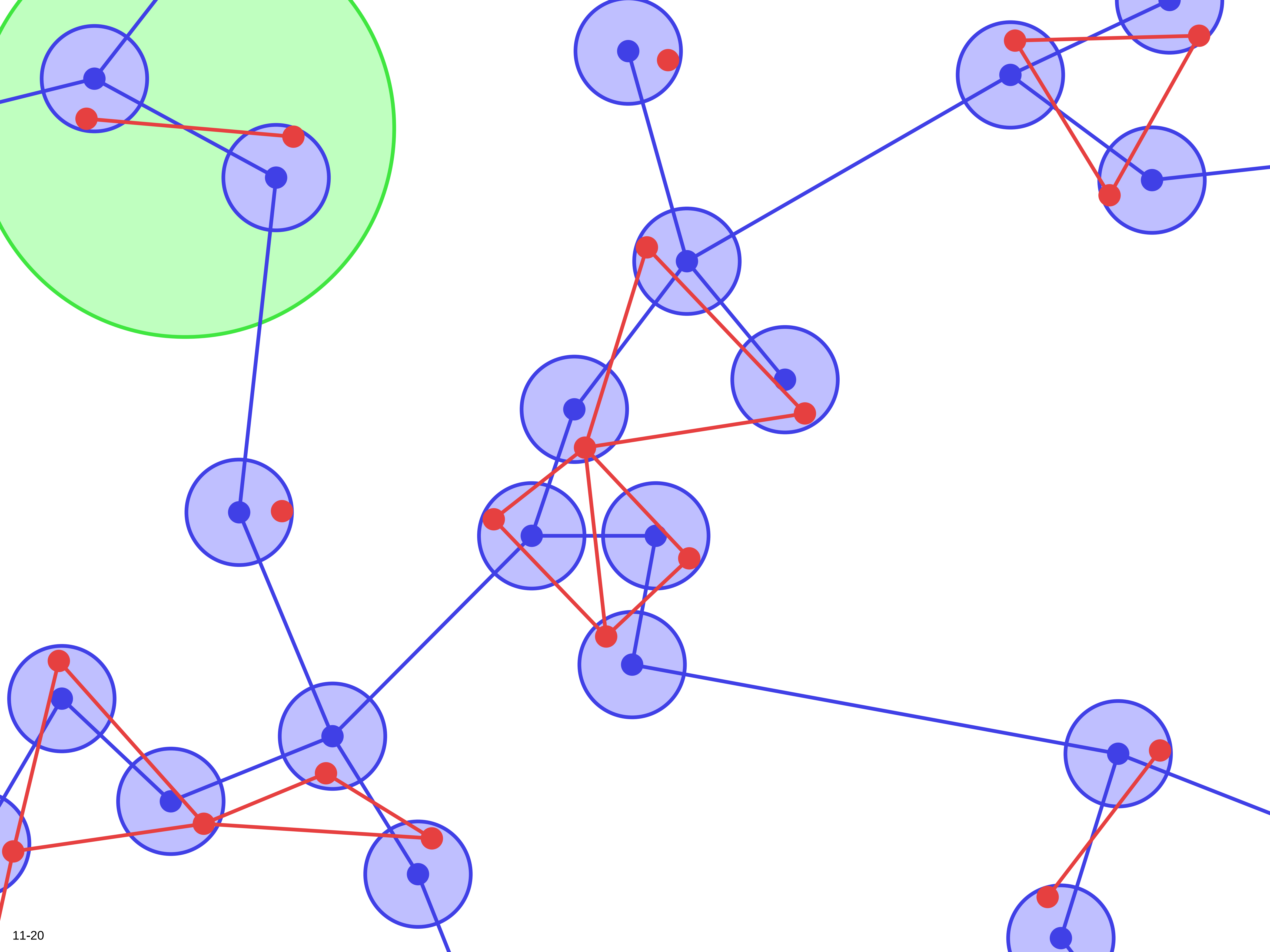

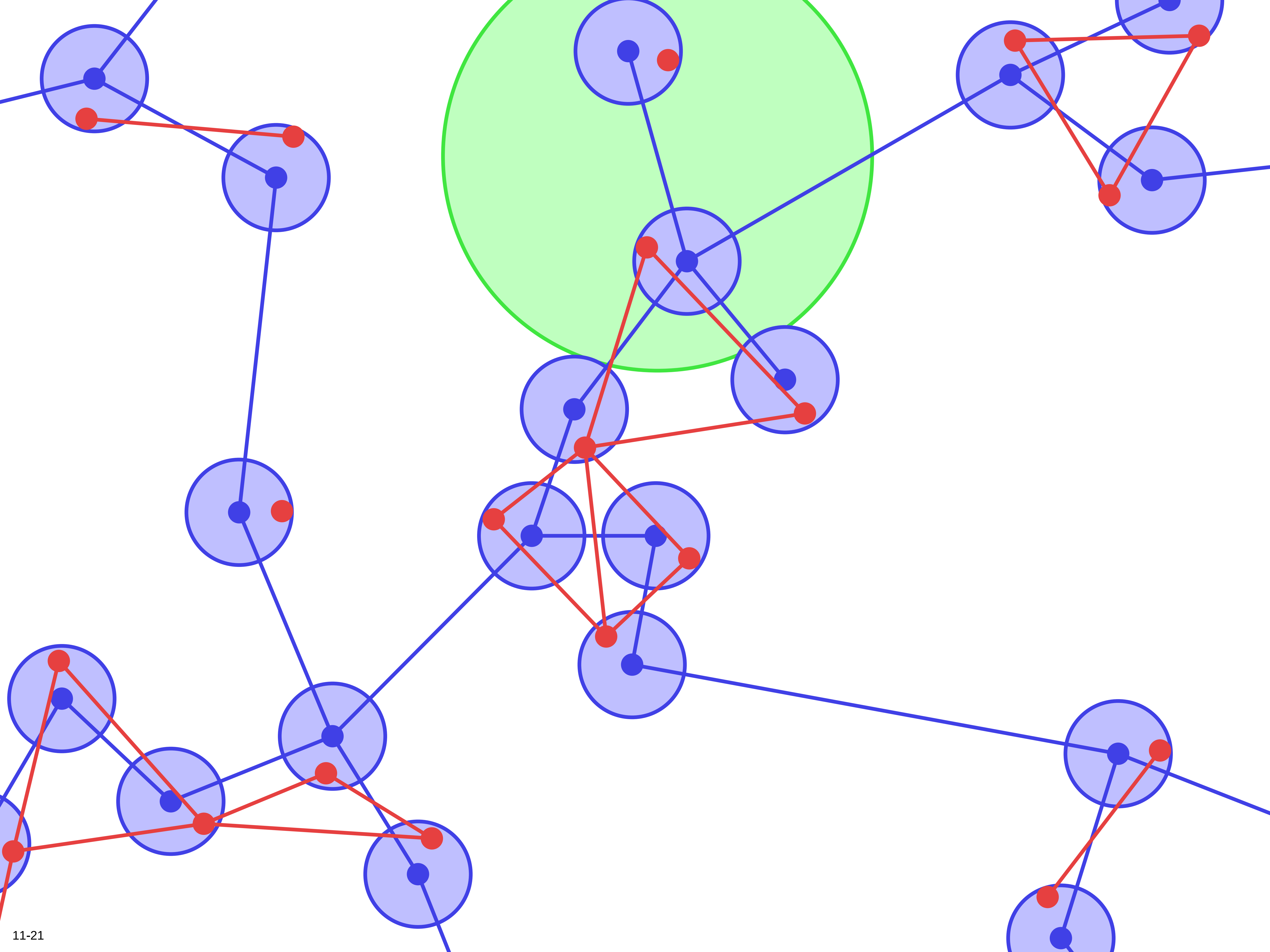

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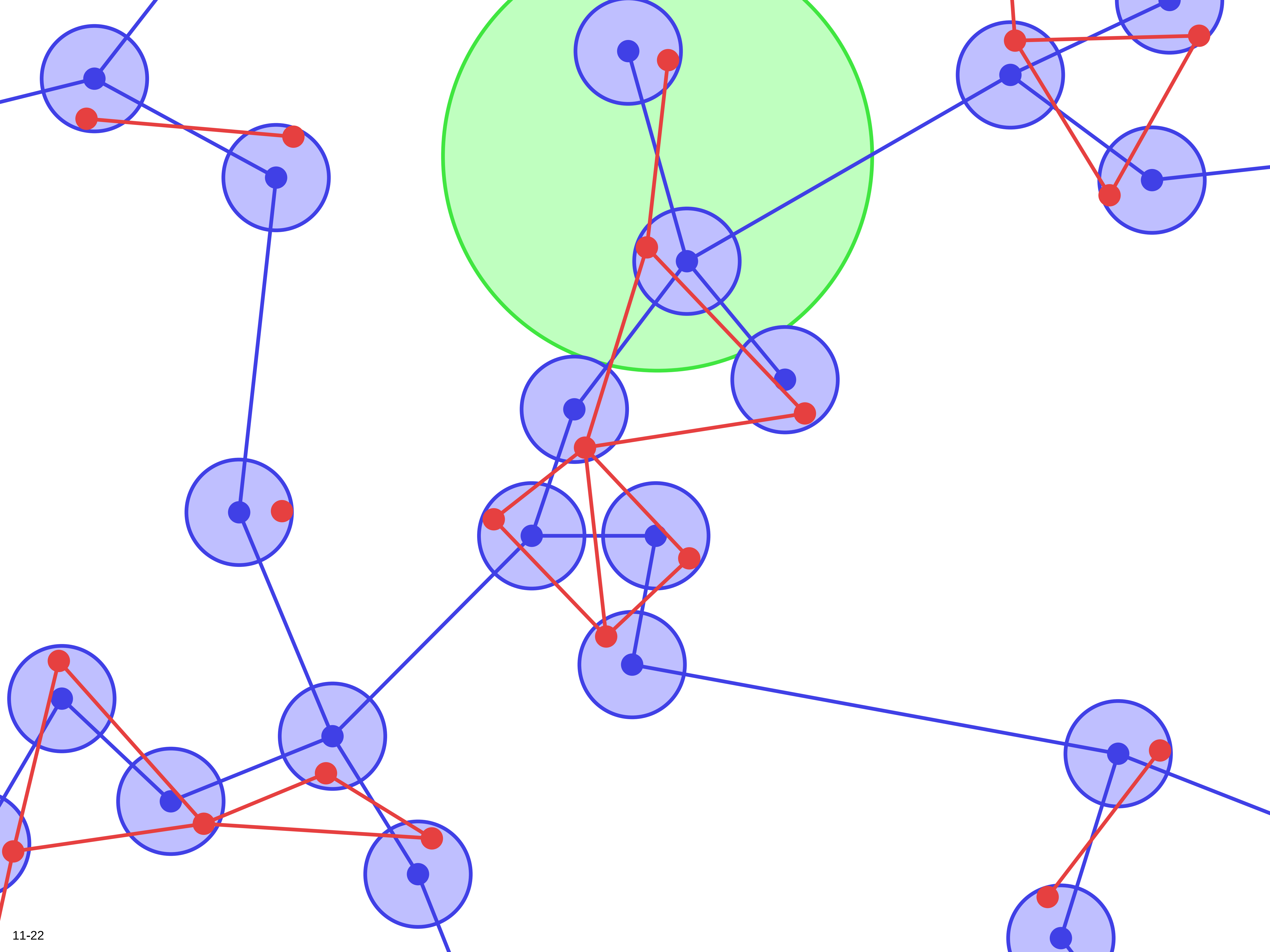

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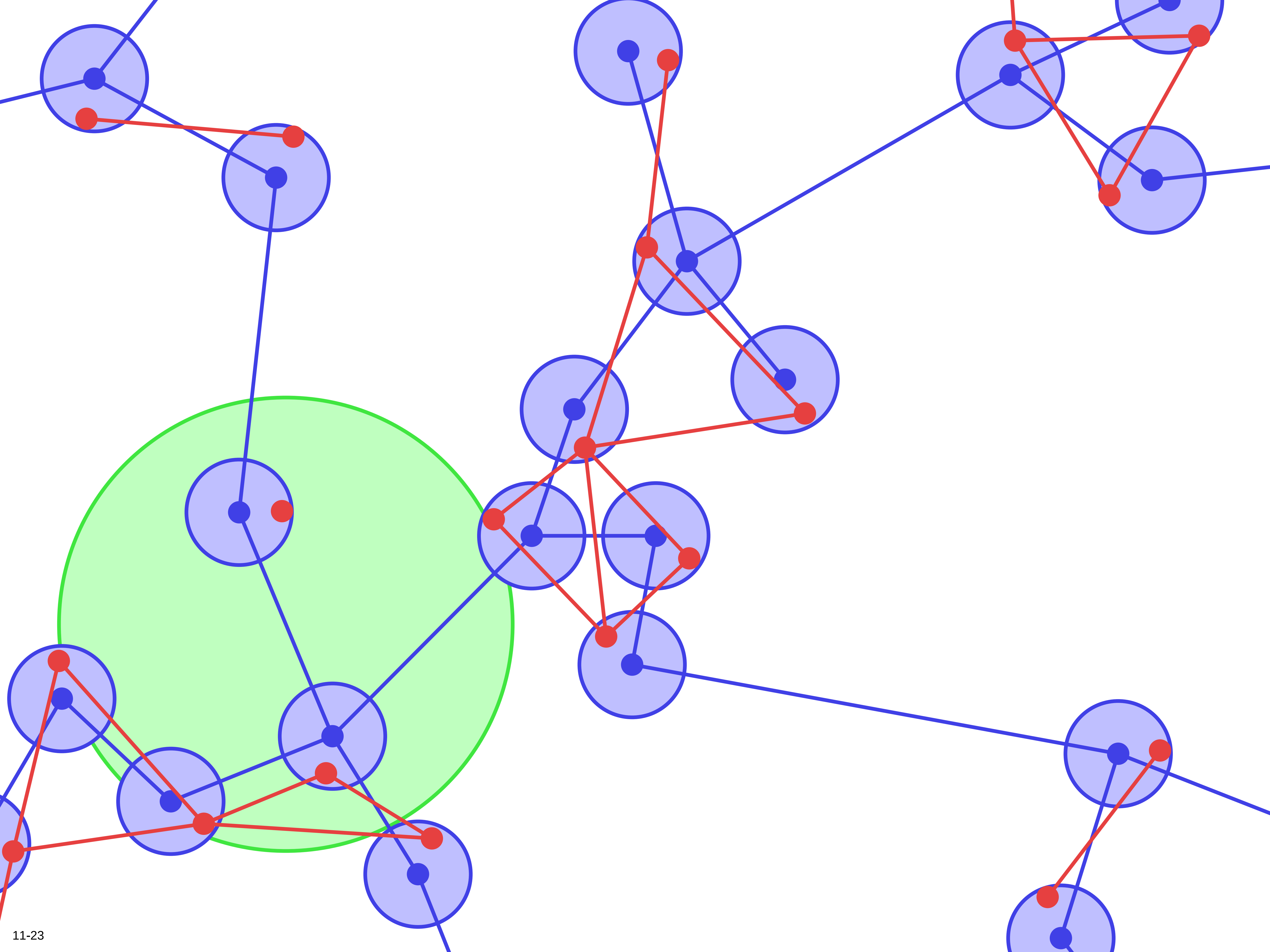

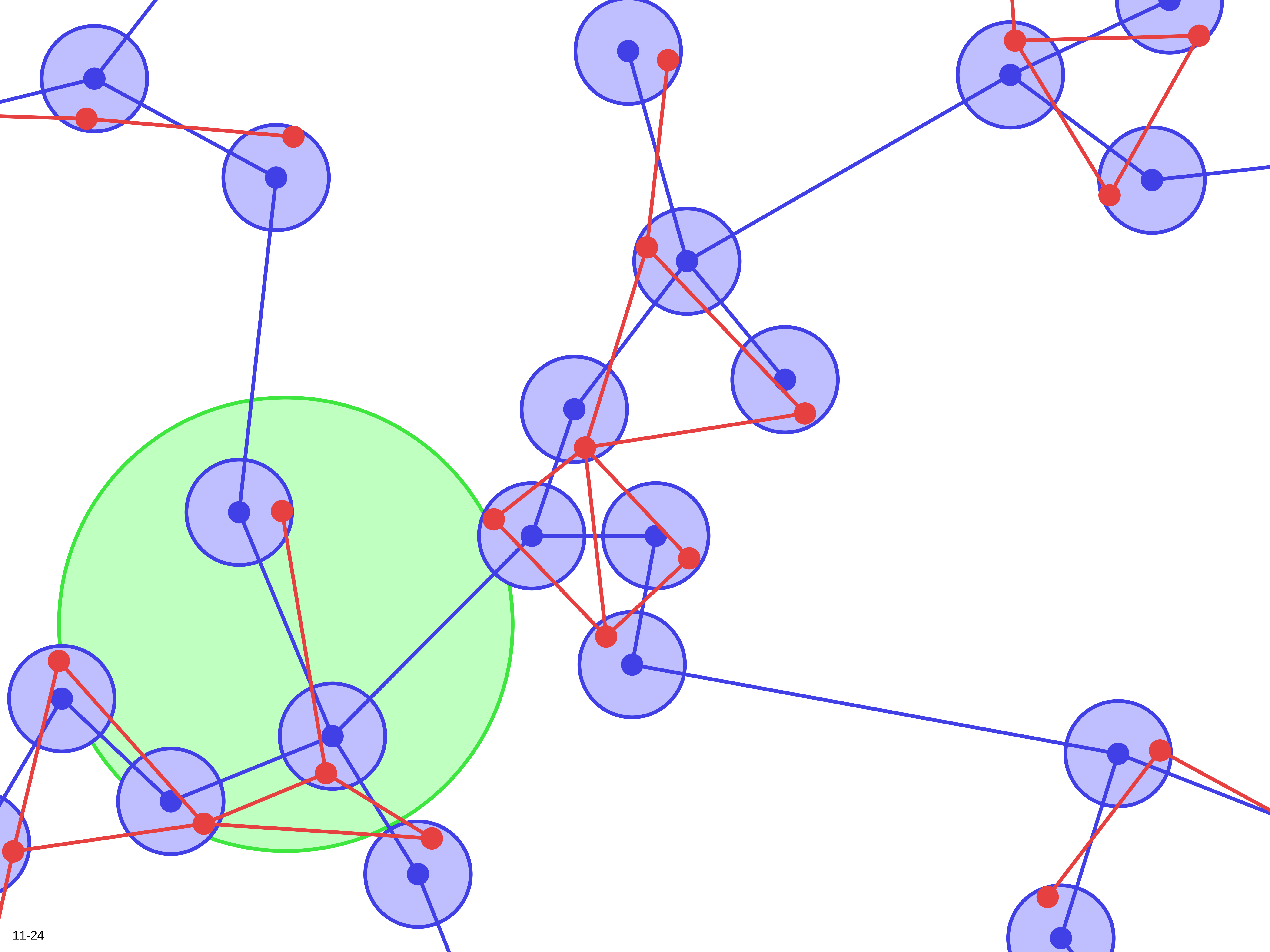

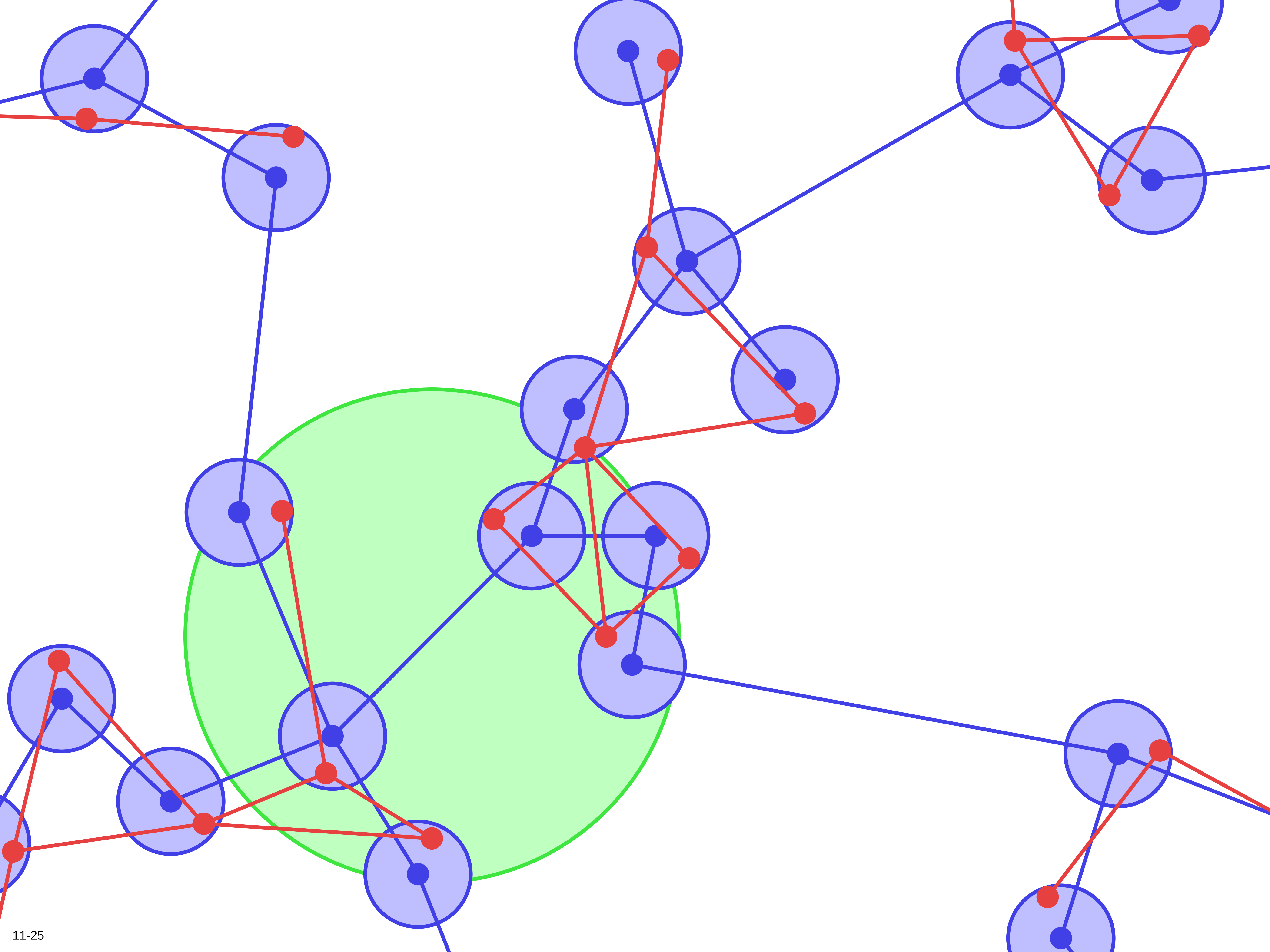

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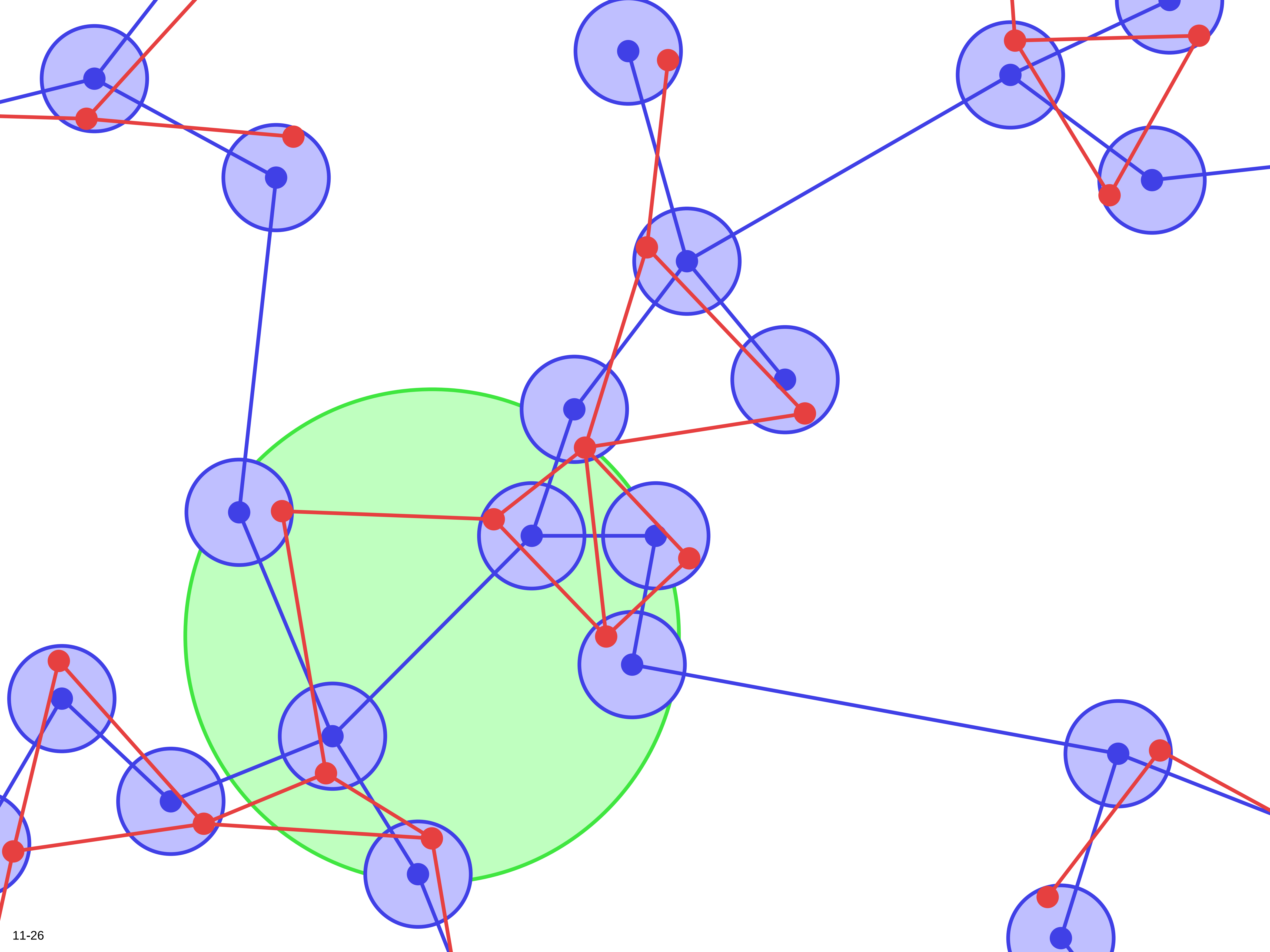

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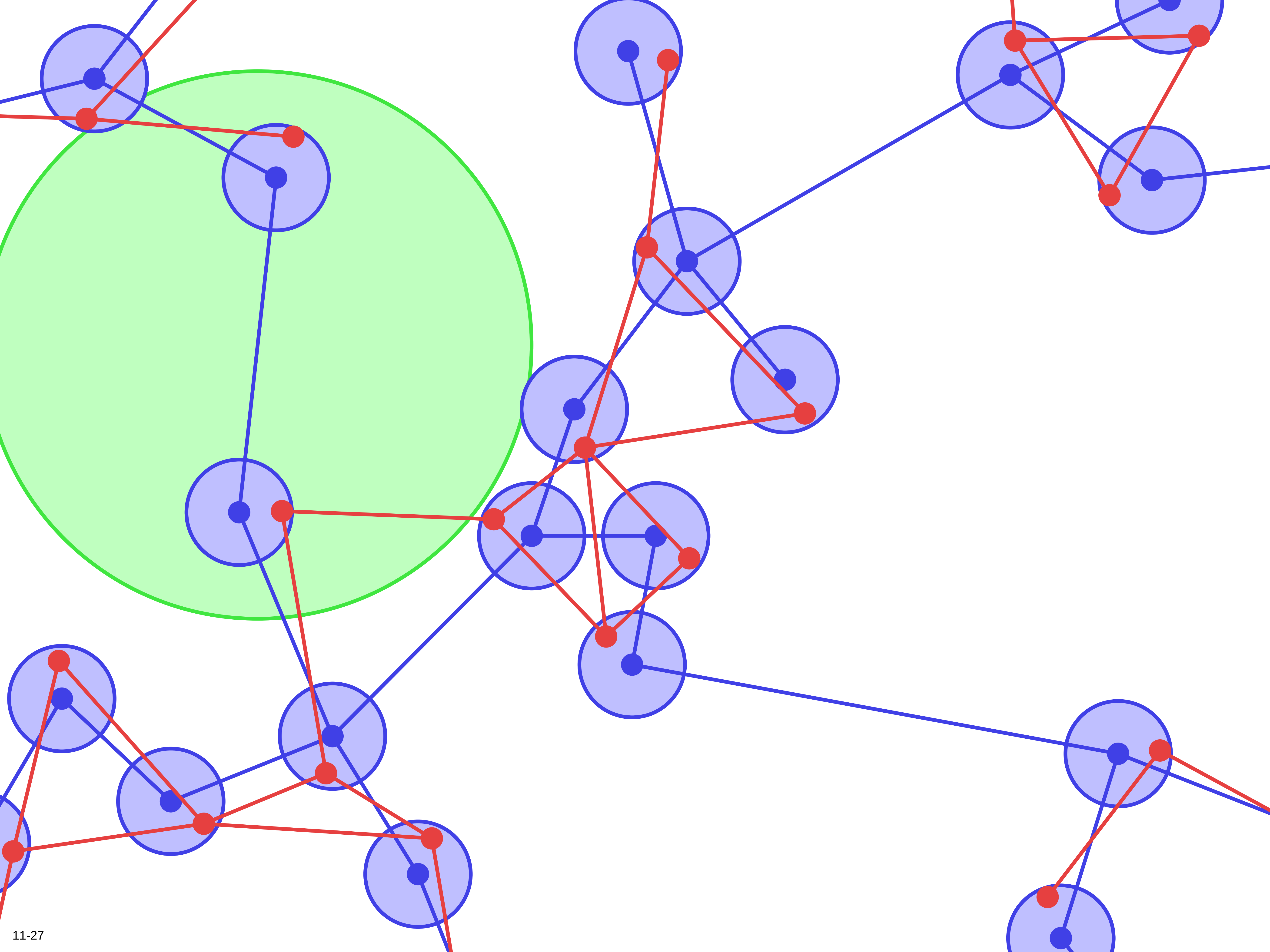

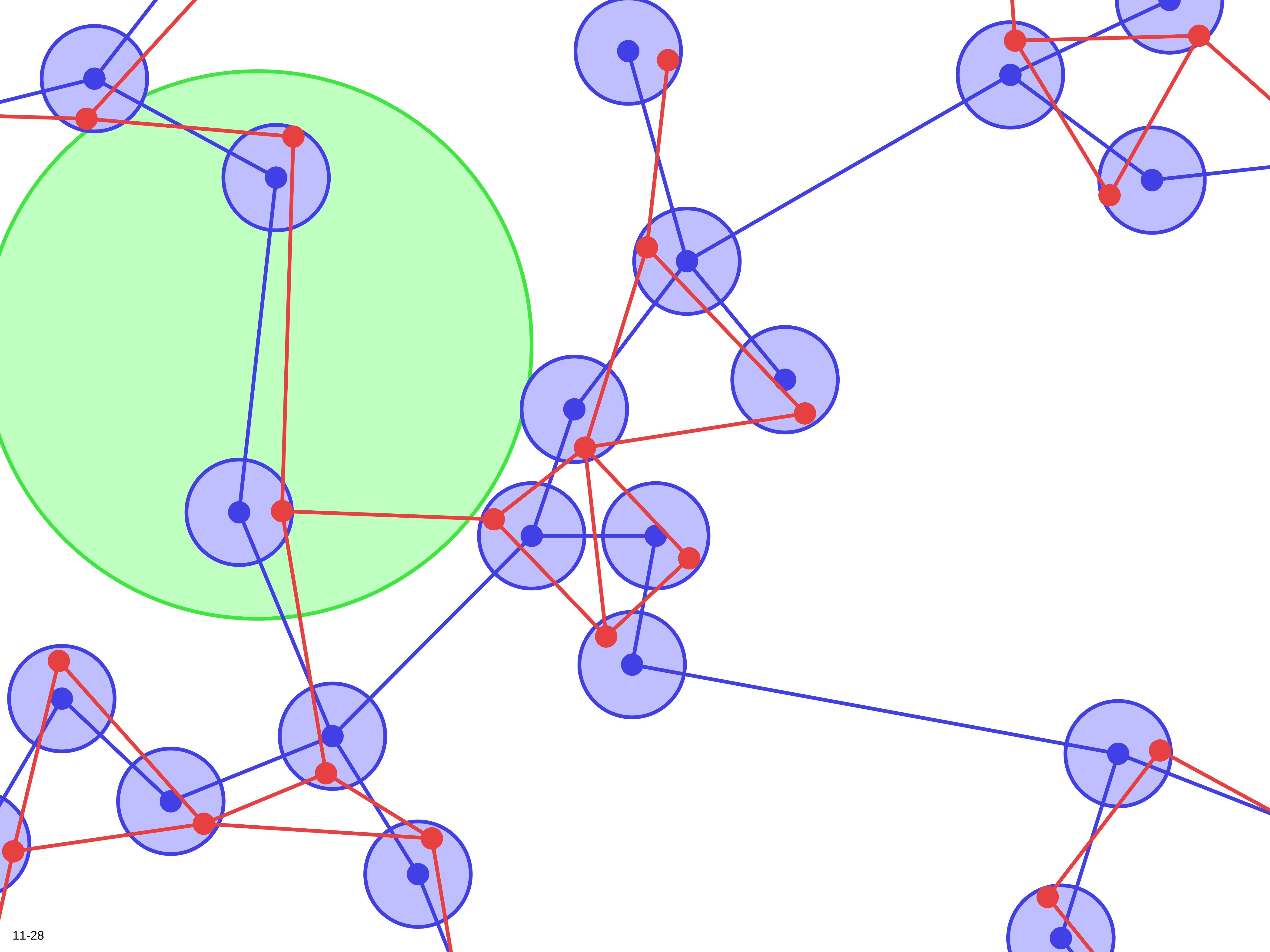

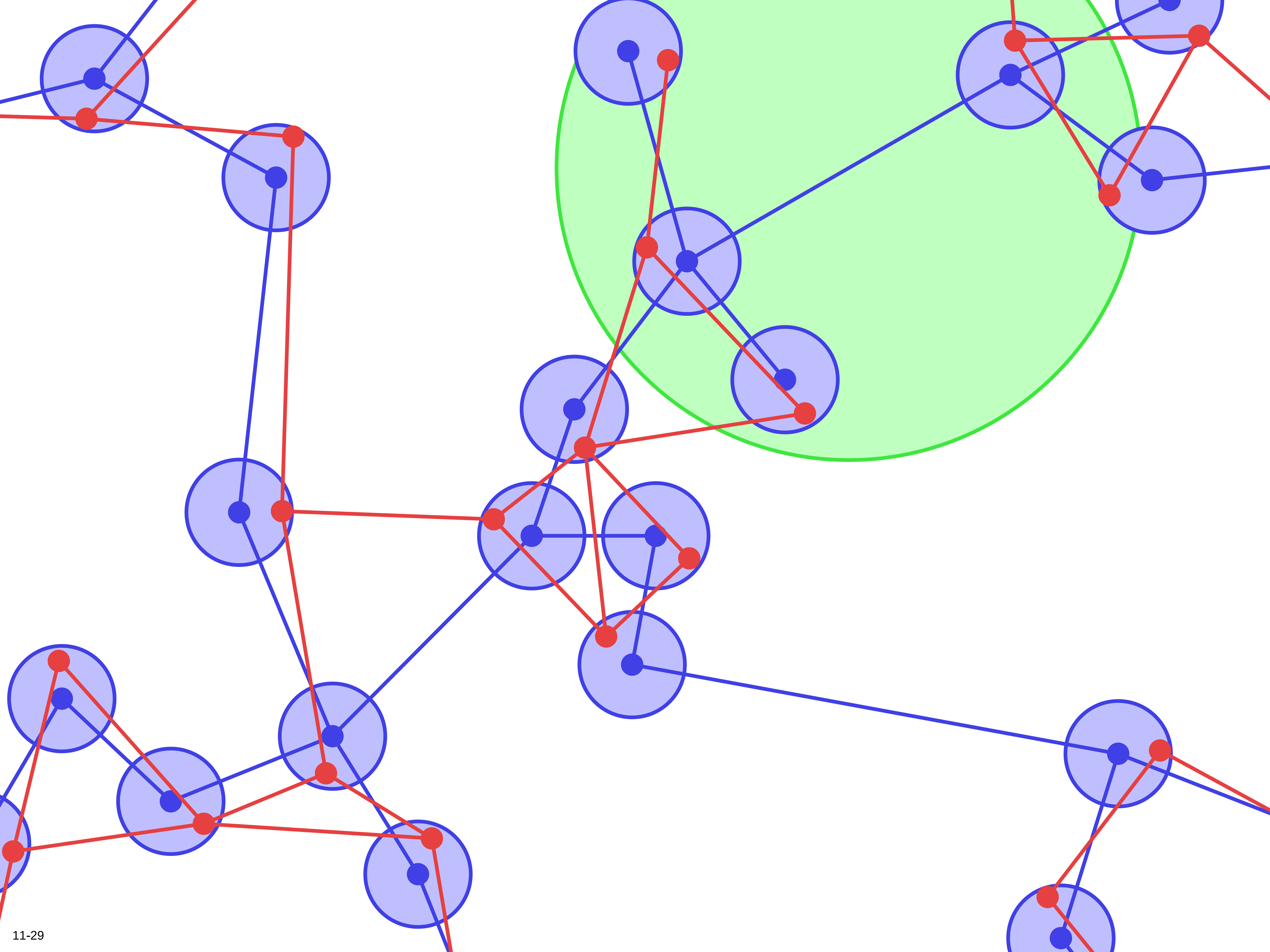

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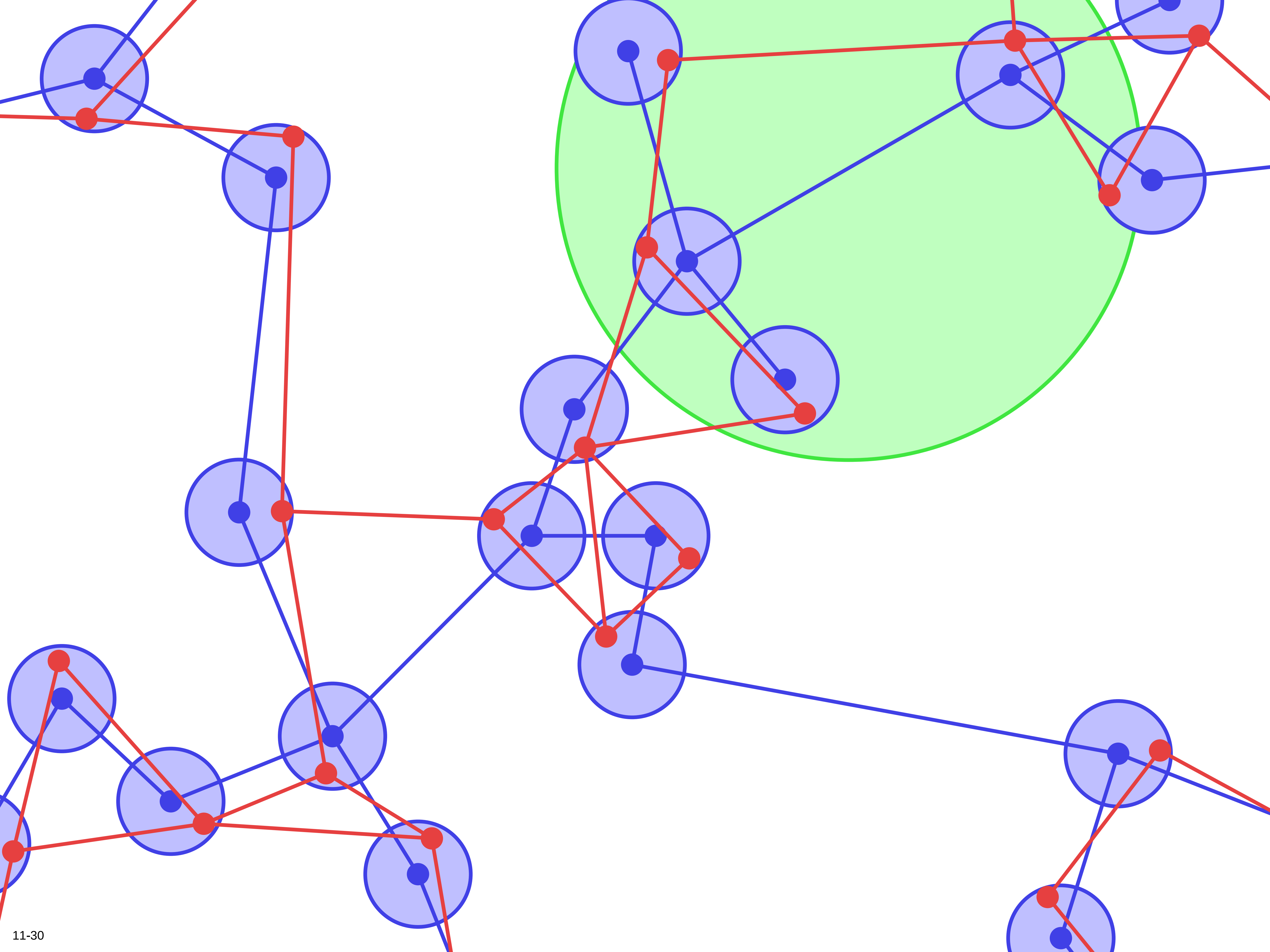

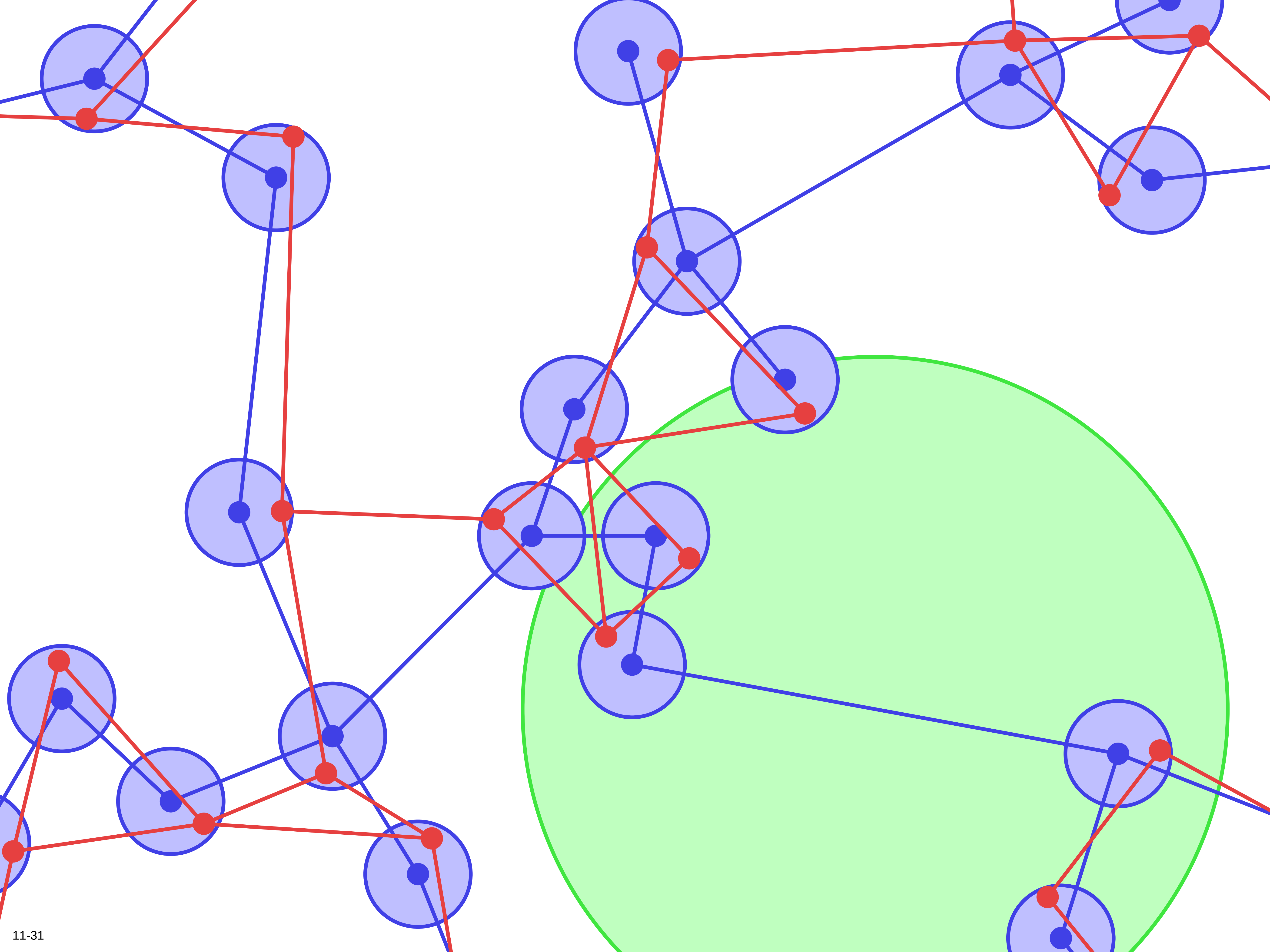

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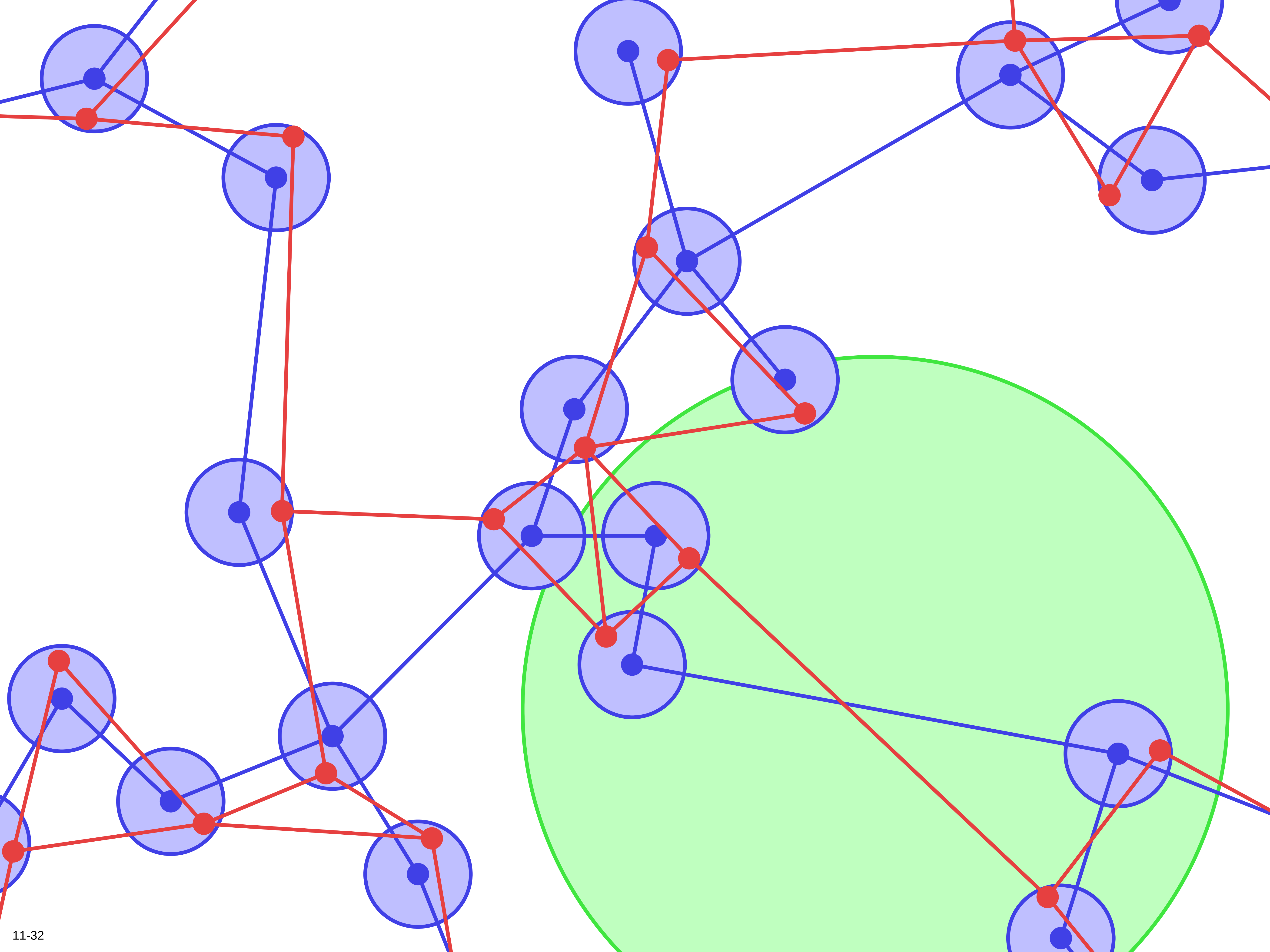

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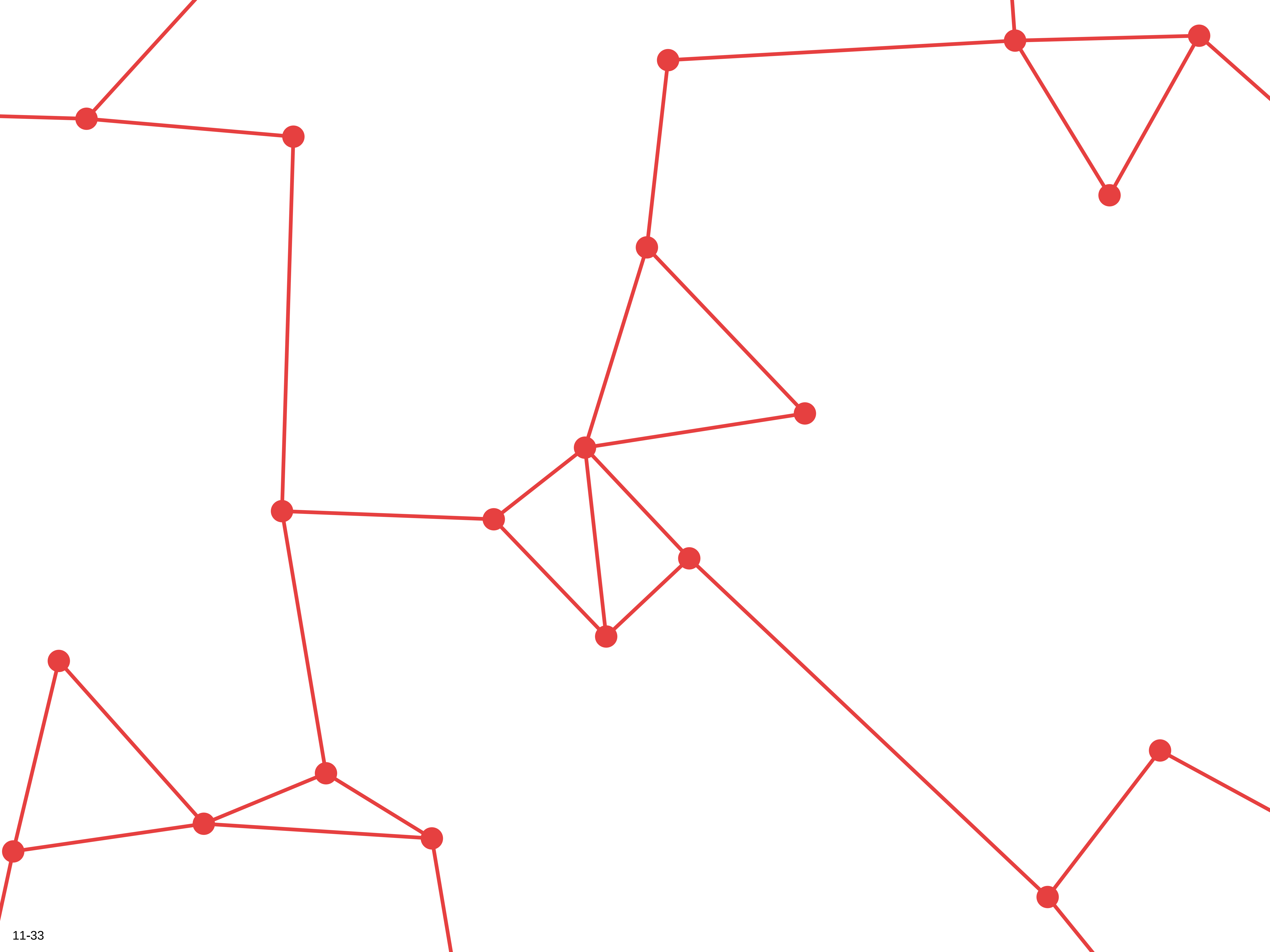
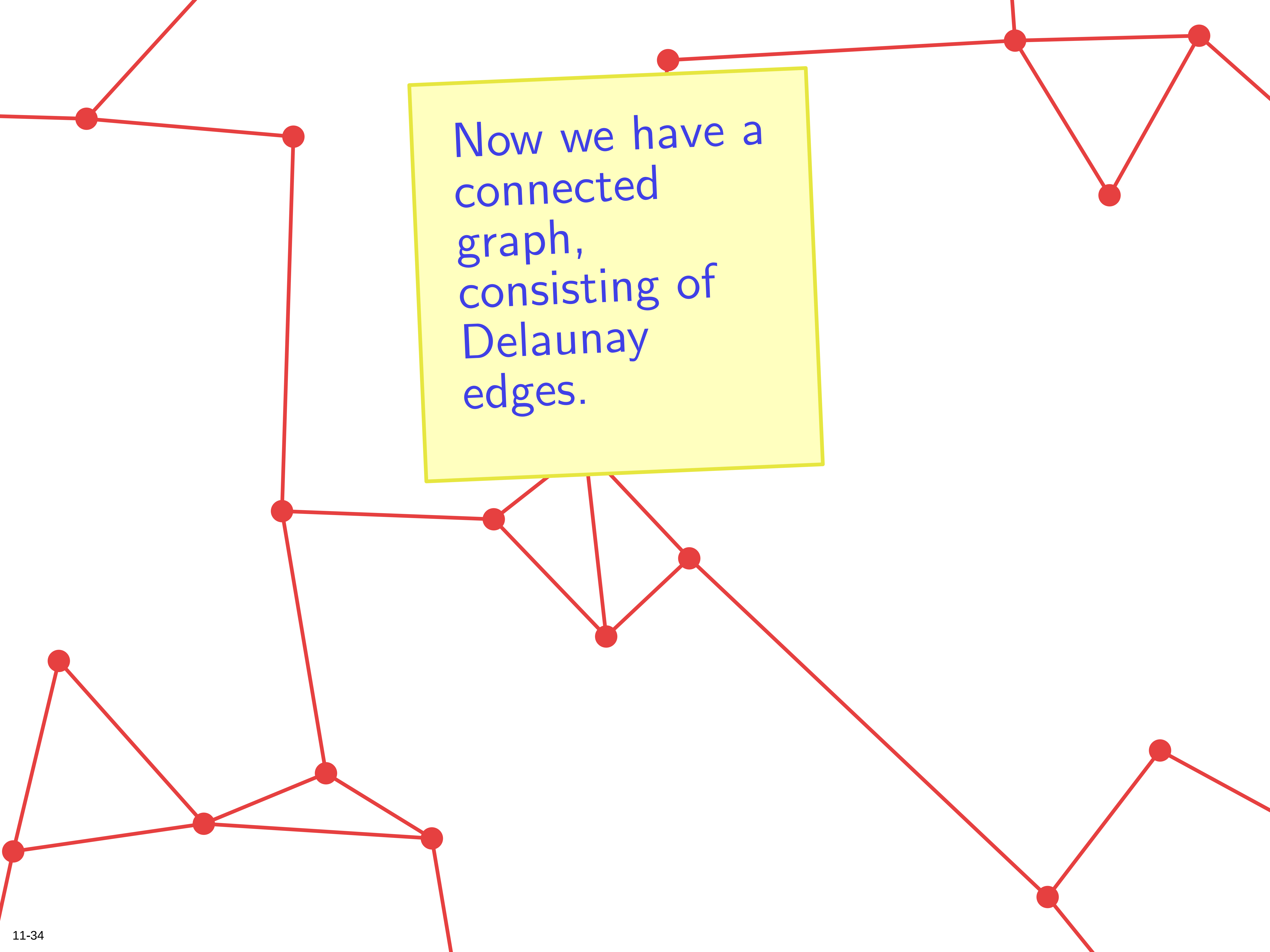
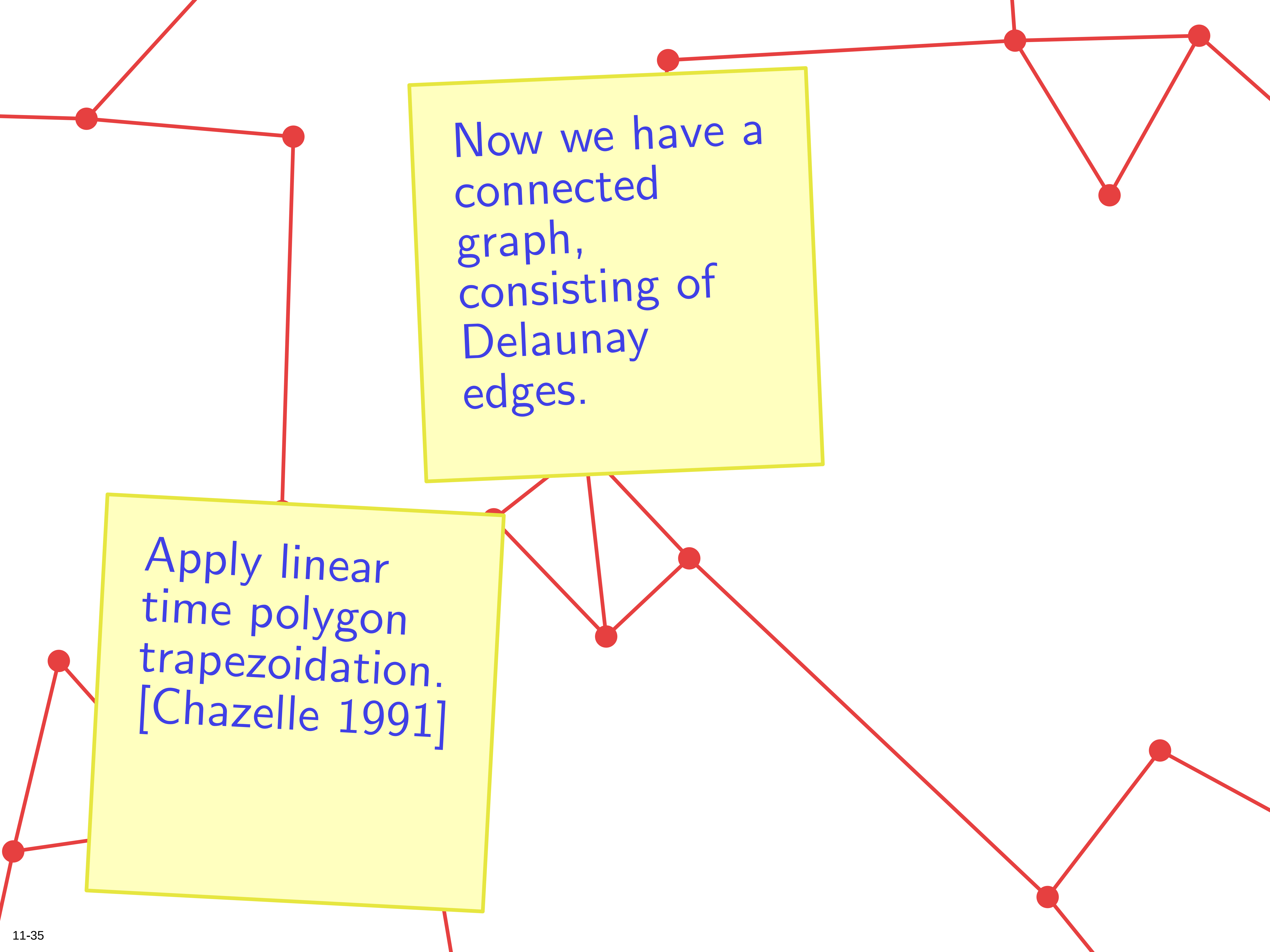




Now we have a connected graph, consisting of Delaunay edges.



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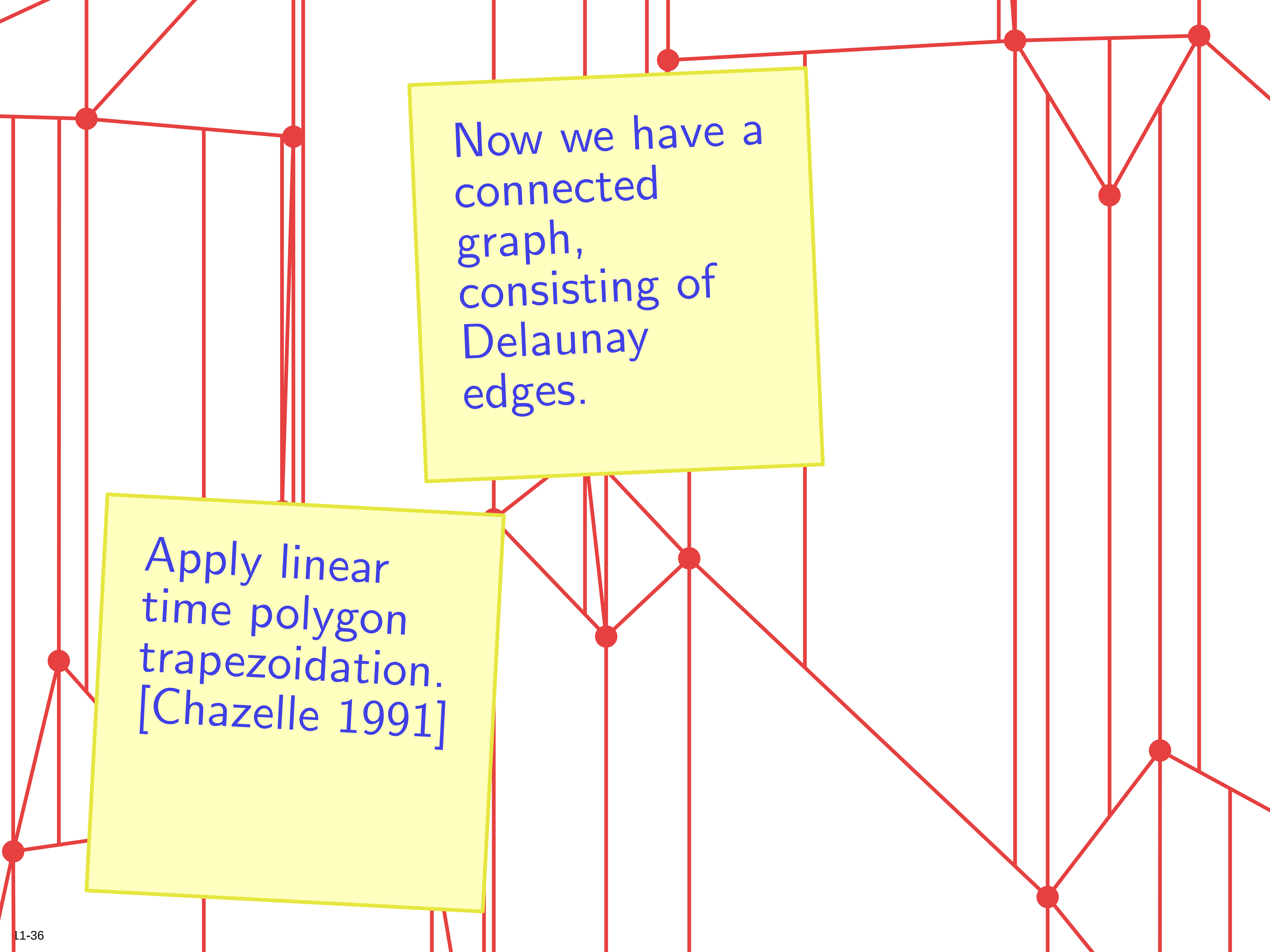
Apply linear time polygon trapezoidation. [Chazelle 1991]



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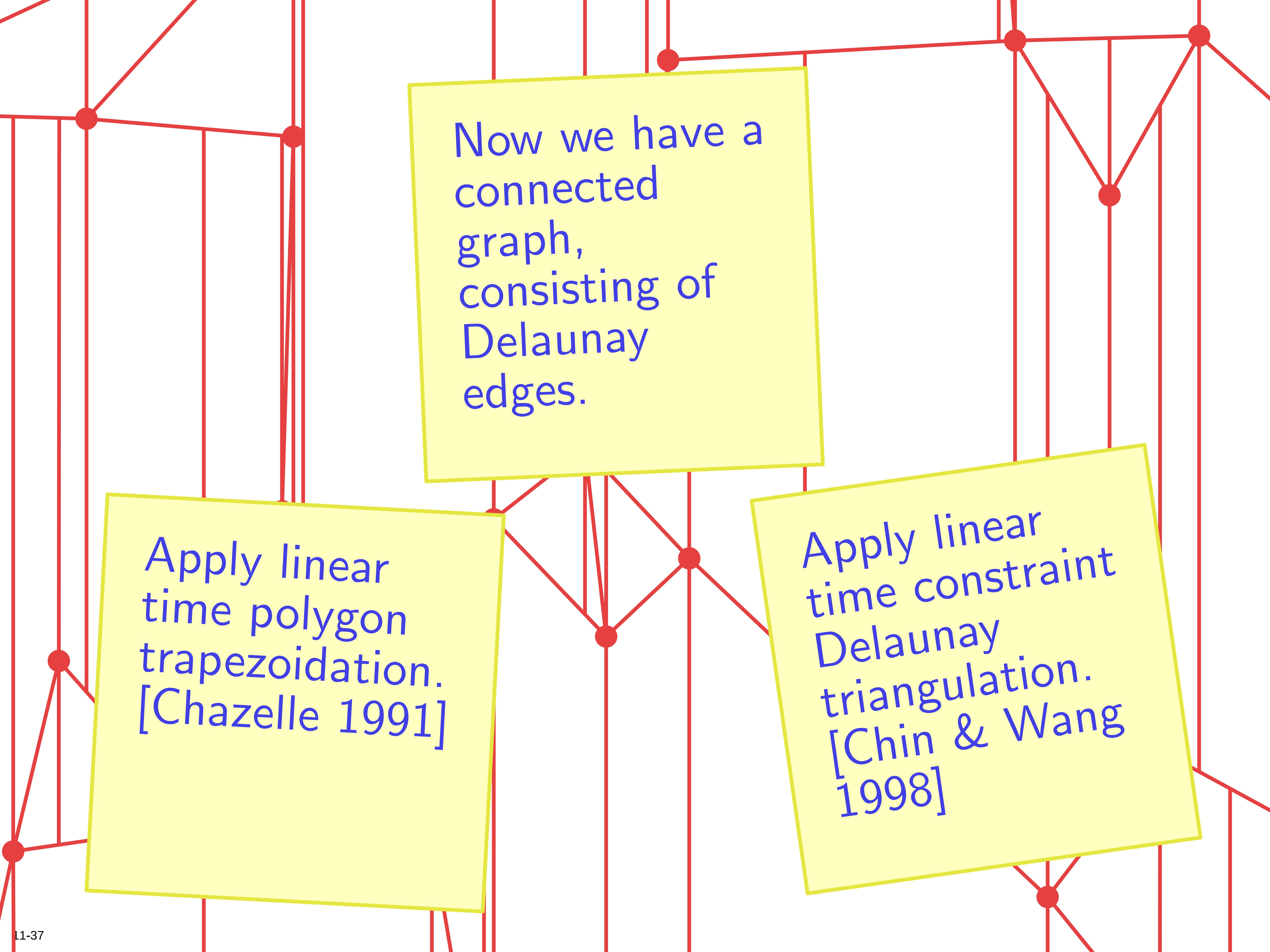

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Done!

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Time to conclude.

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Future work: can we compute more structure, to avoid using Chin & Wang?

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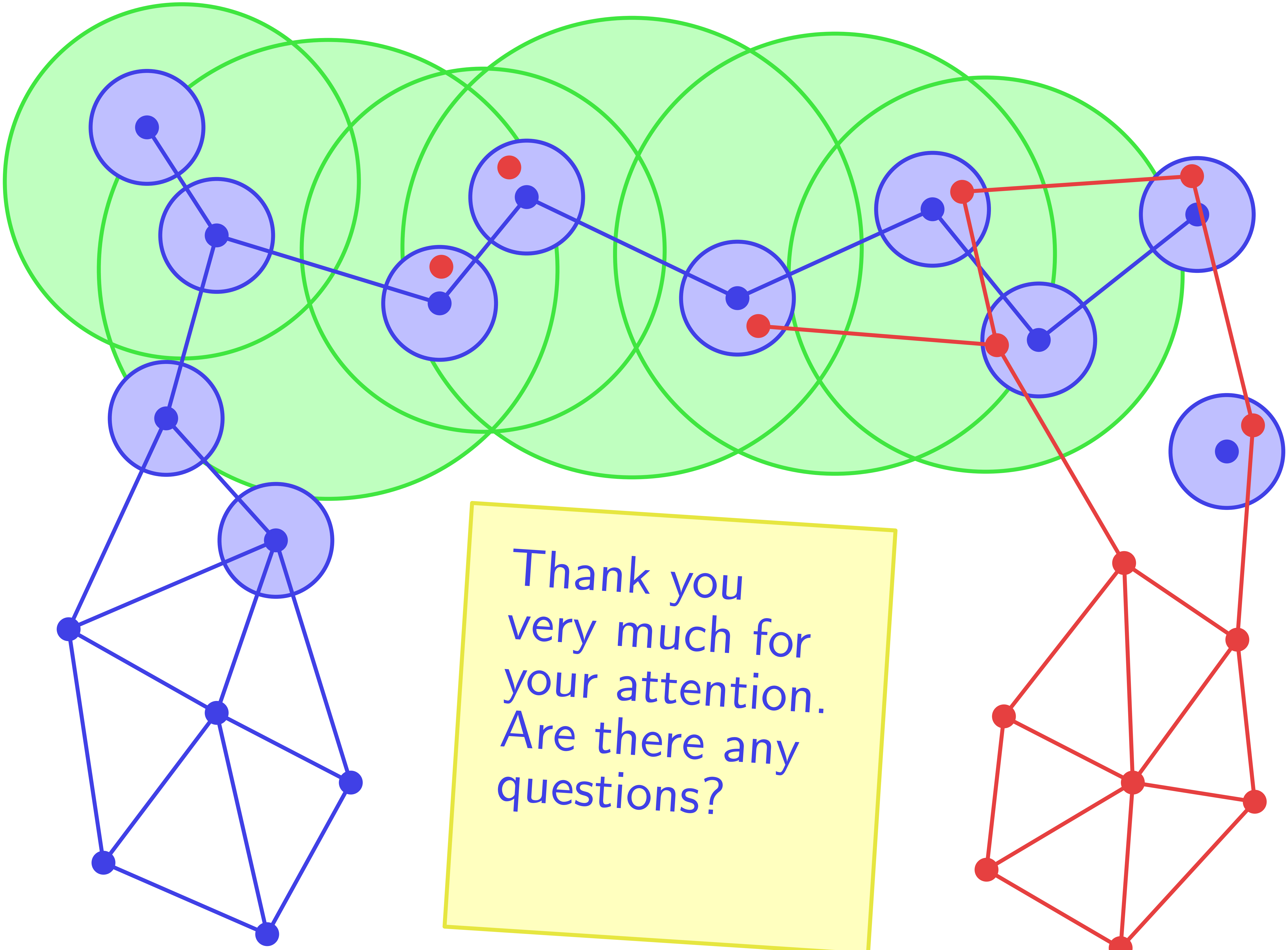
Future work: can we compute more structure, to avoid using Chin & Wang?

Future work: can our techniques be applied in settings with moving points?


Thank you
very much for
your attention.
Are there any
questions?