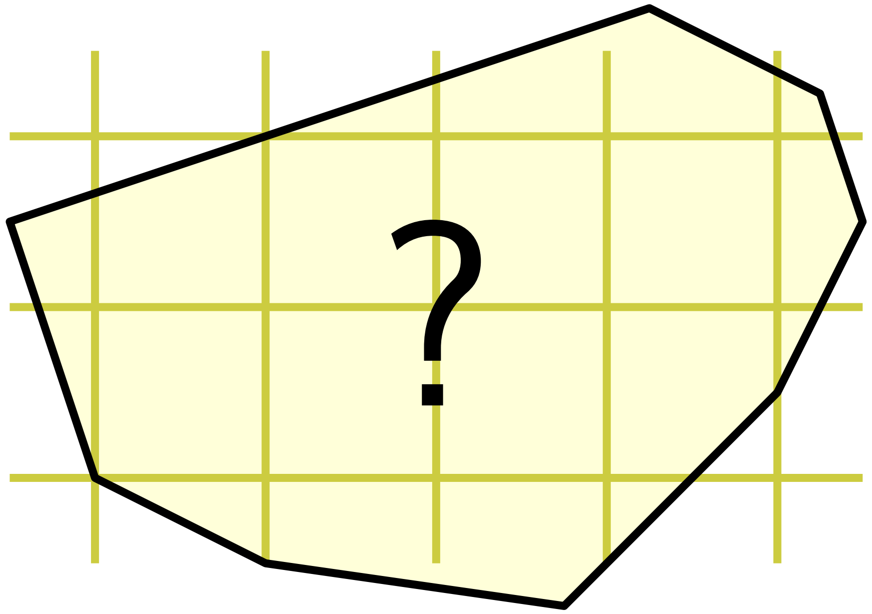
DRAWING GRAPHS IN THE PLANE WITH A PRESCRIBED OUTER FACE AND POLYNOMIAL AREA



Erin Chambers

David Eppstein

Michael Goodrich

Maarten Löffler

FACT

Every planar graph can be drawn with straight edges!

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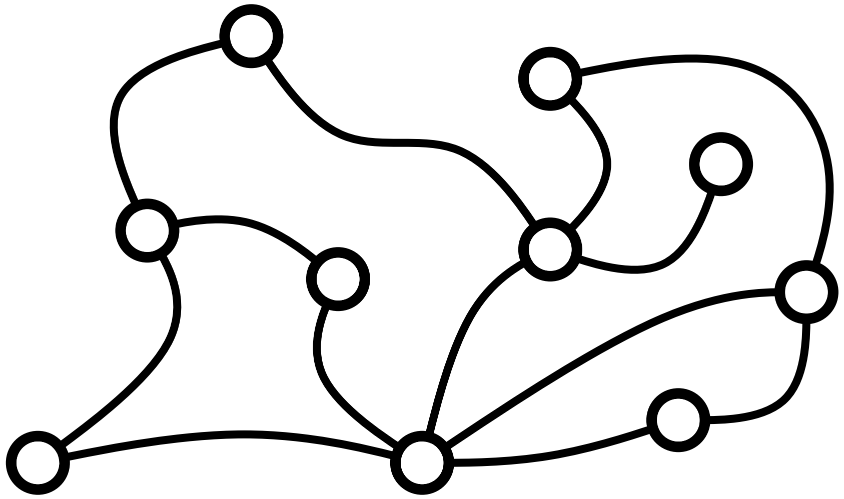

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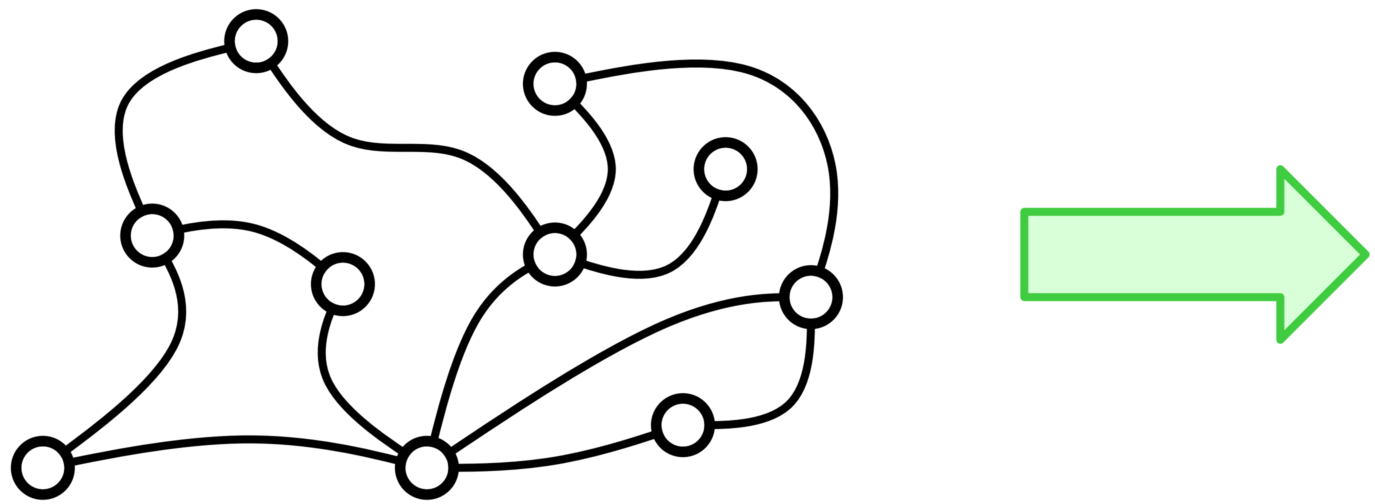

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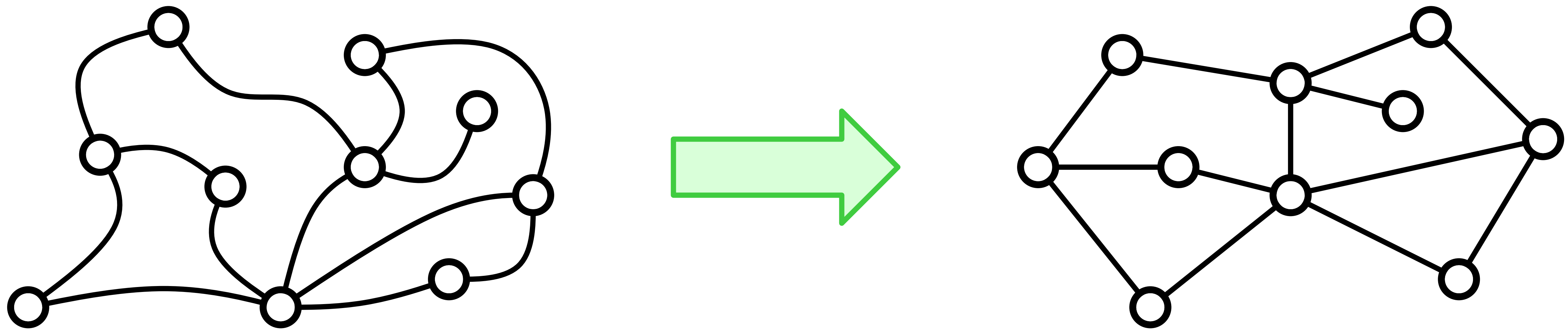

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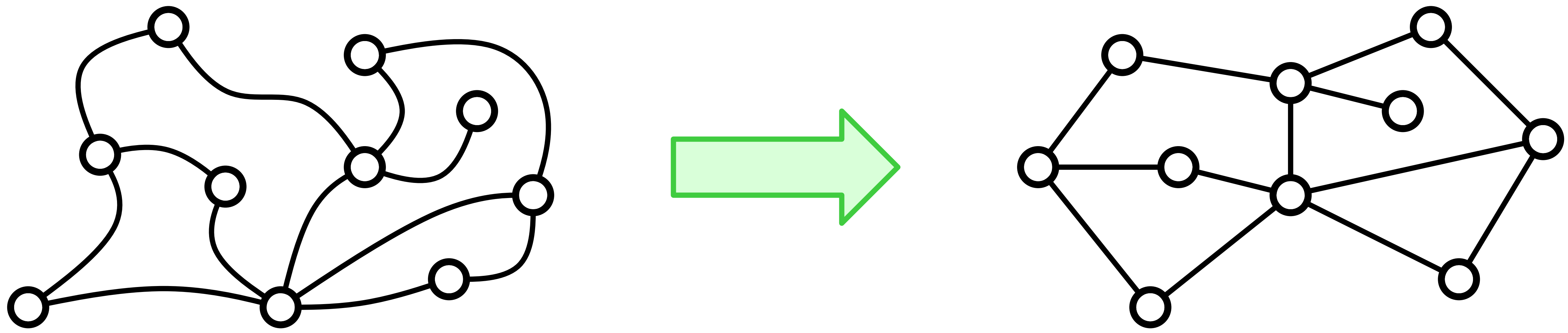
[Wagner, 1936] [Fáry, 1948] [Stein, 1951]



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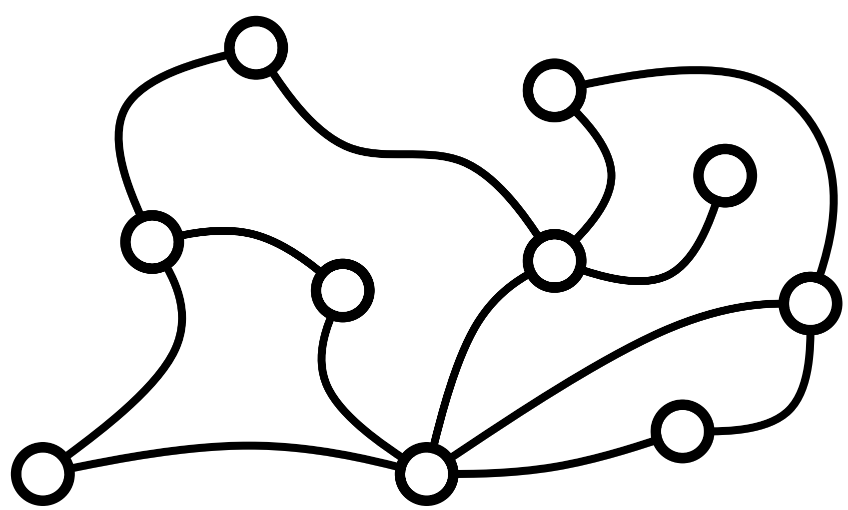

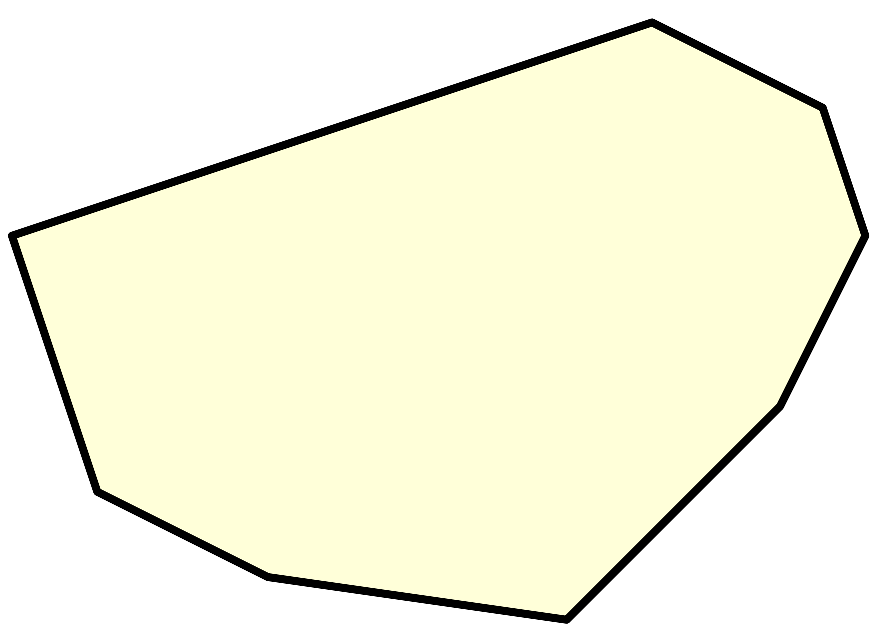

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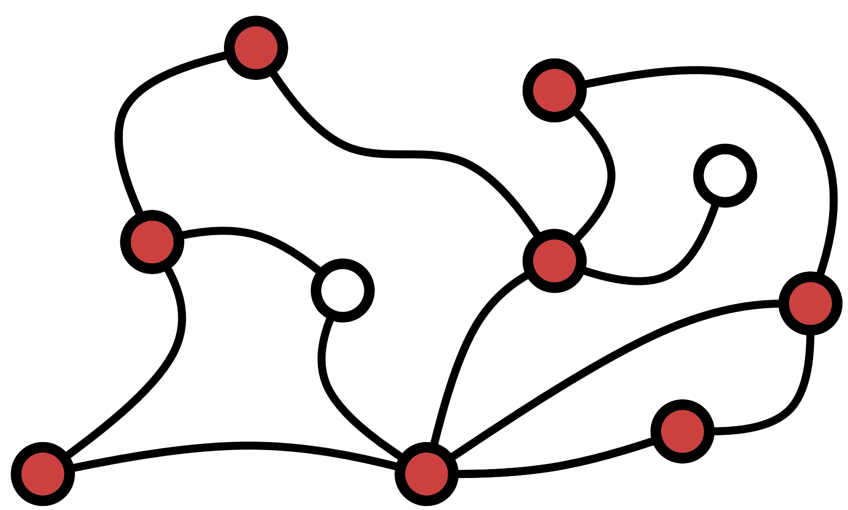

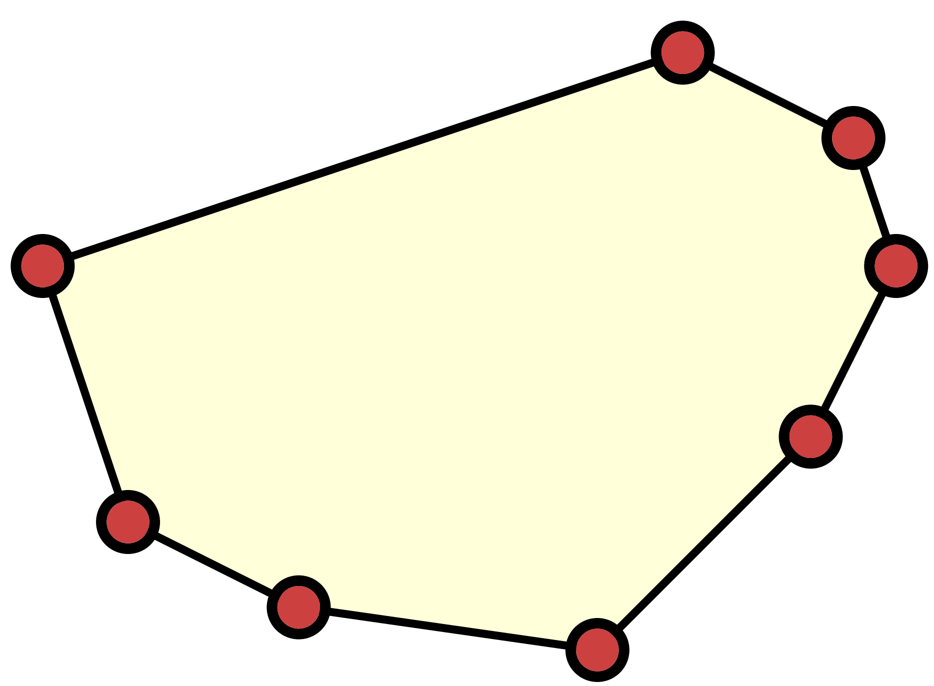

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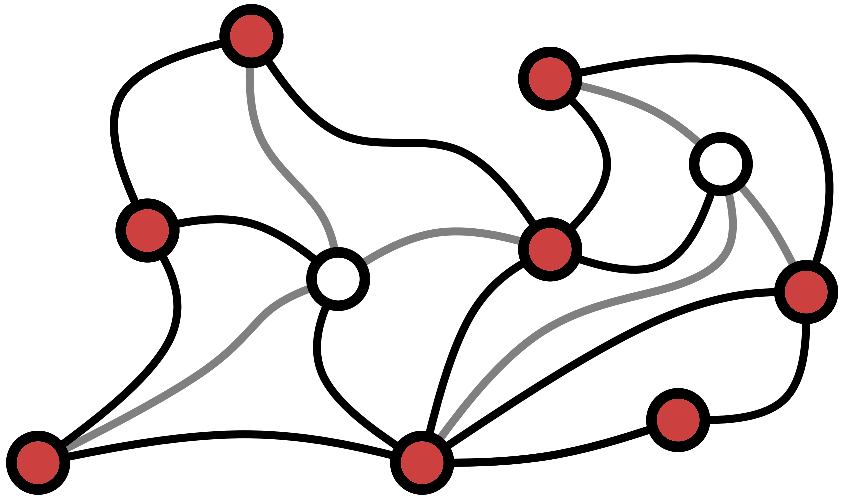

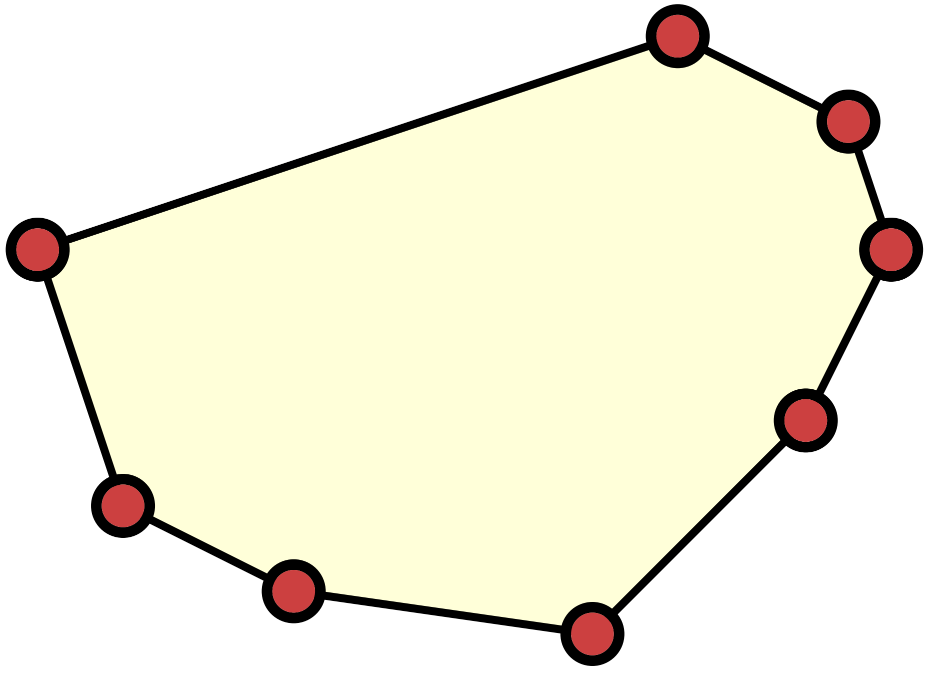

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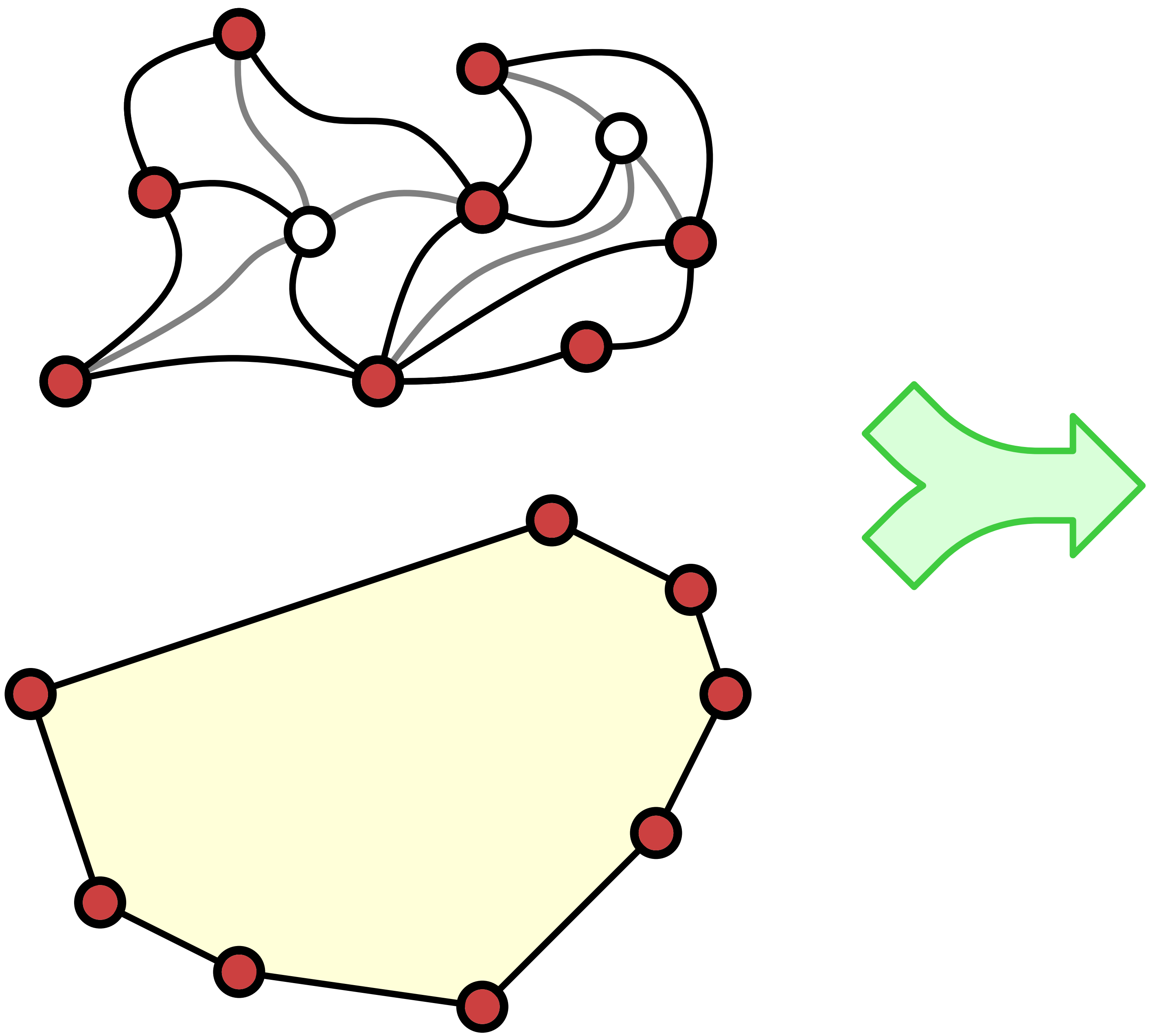

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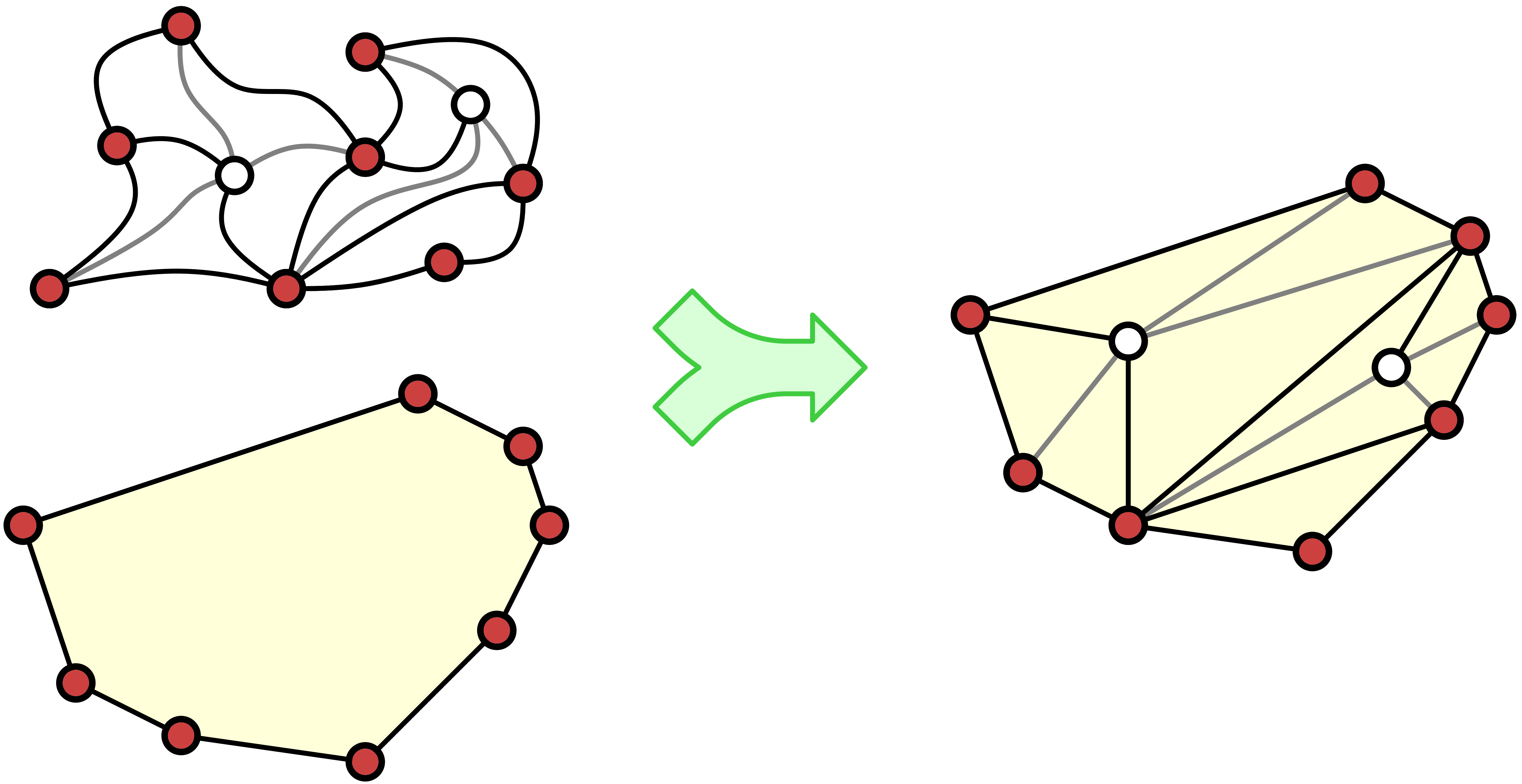

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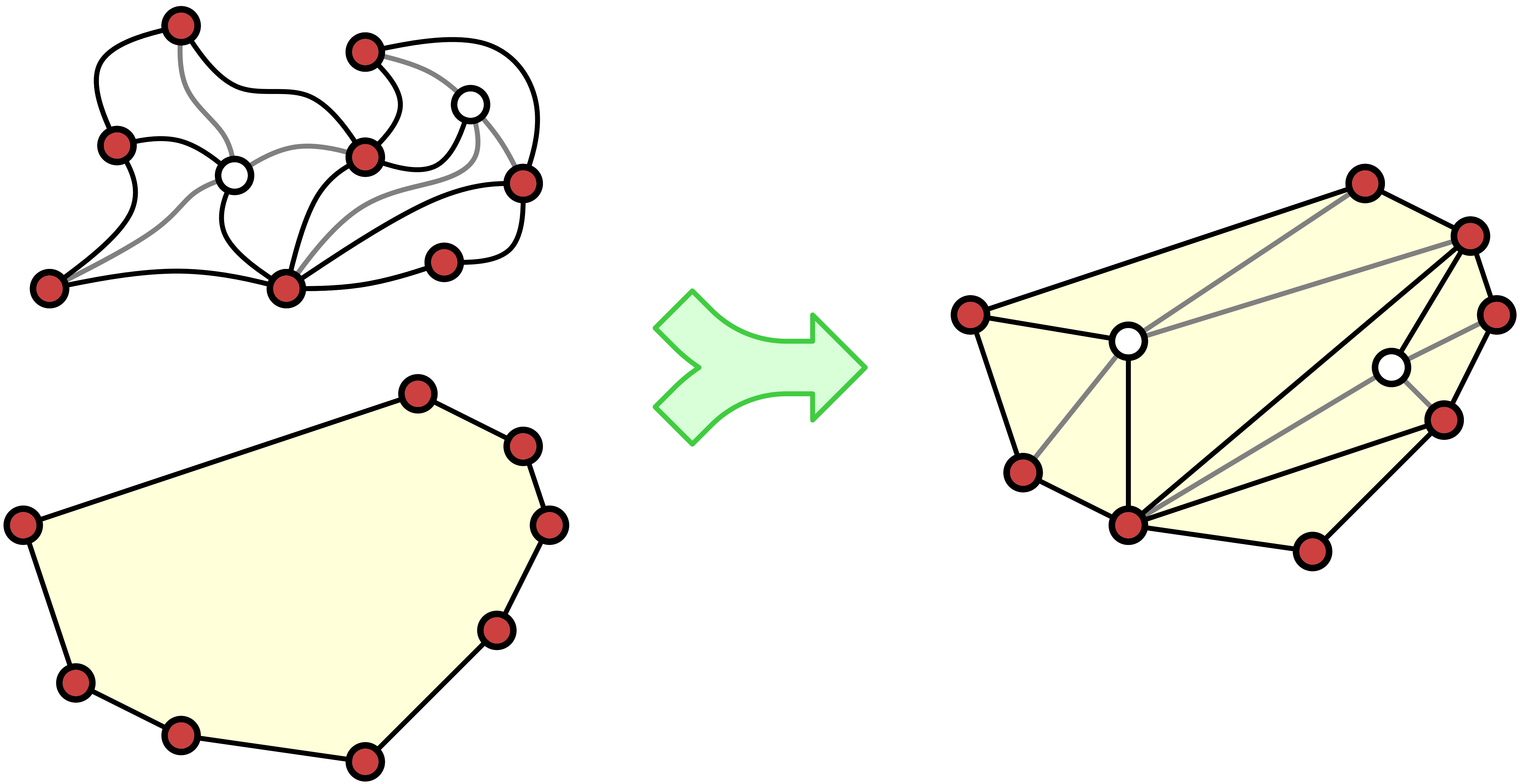

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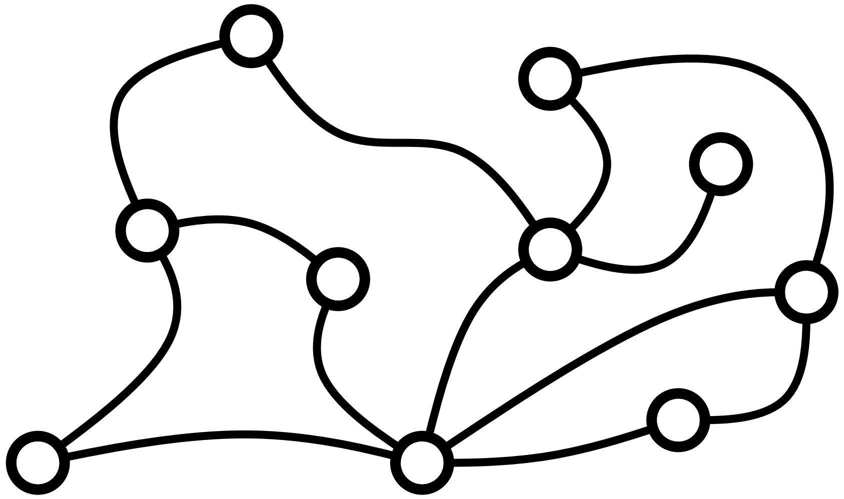

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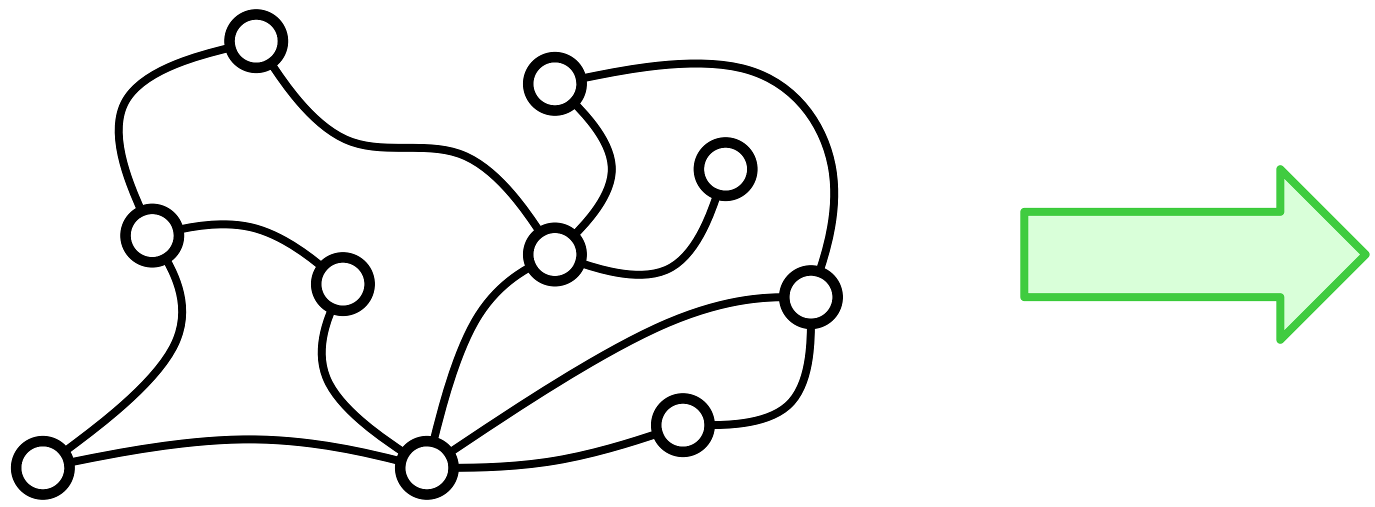

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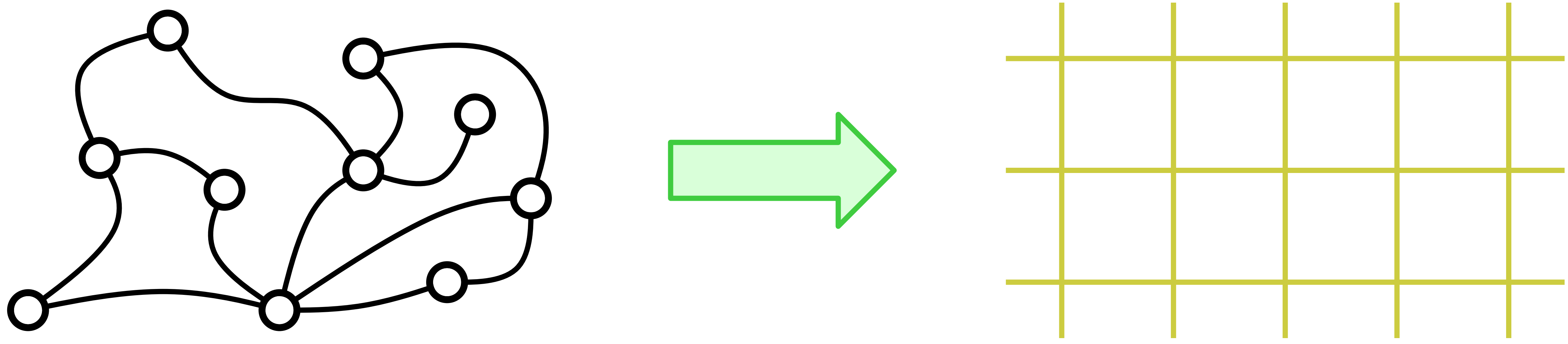

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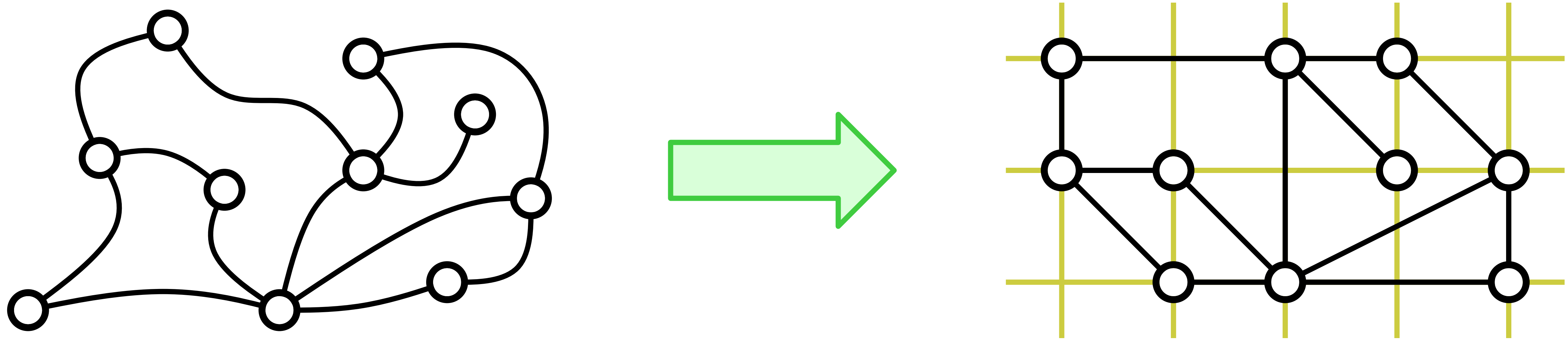

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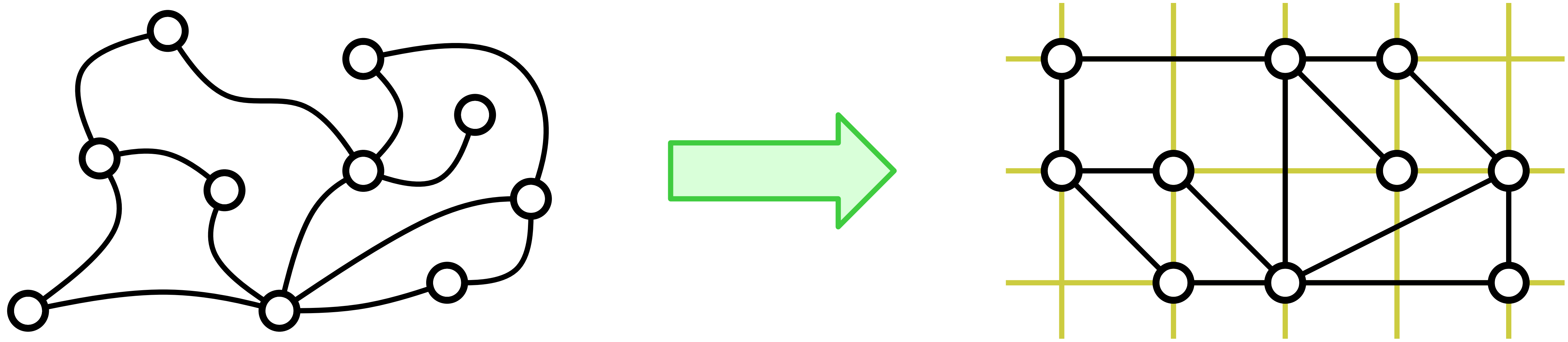
Every planar graph can be drawn with straight edges *on a polynomial grid*. [Schnyder, 1990] [de Fraysseix & Pach & Pollack, 1990]



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Can every planar graph be drawn with straight edges *inside a given convex polygon on a polynomial grid*?

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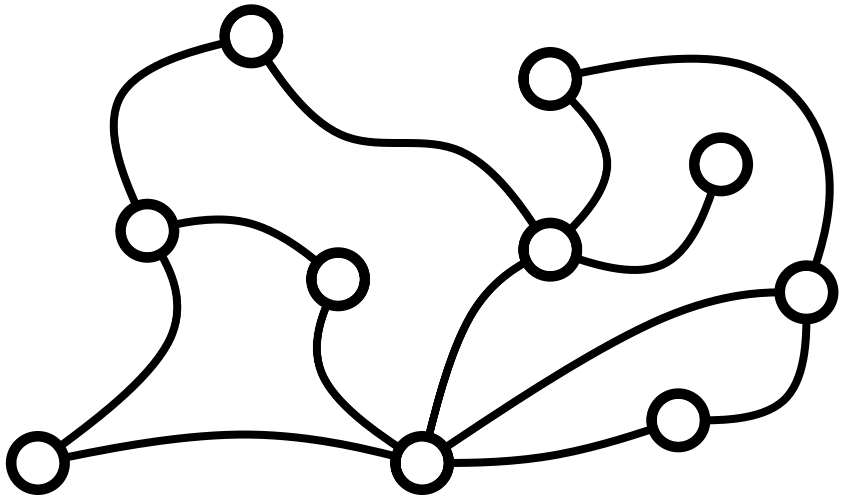

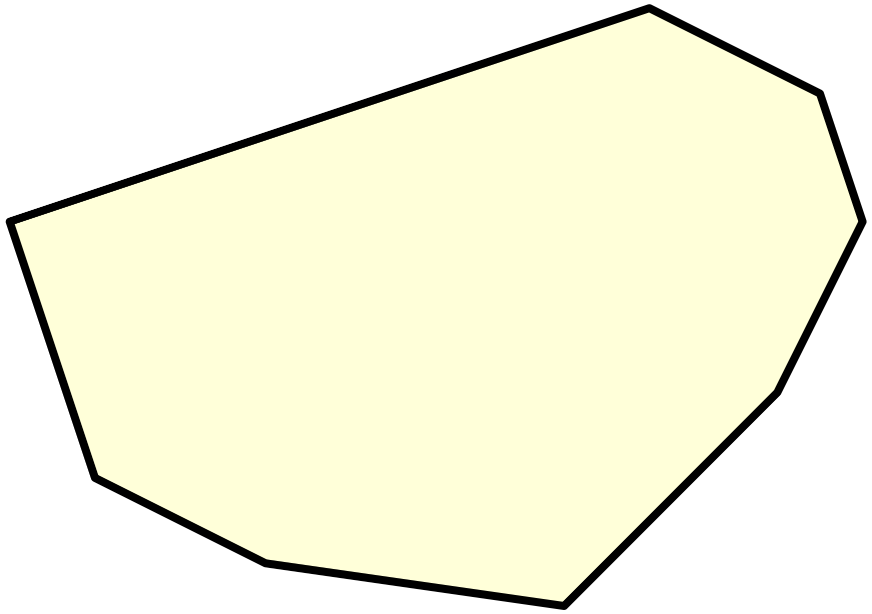

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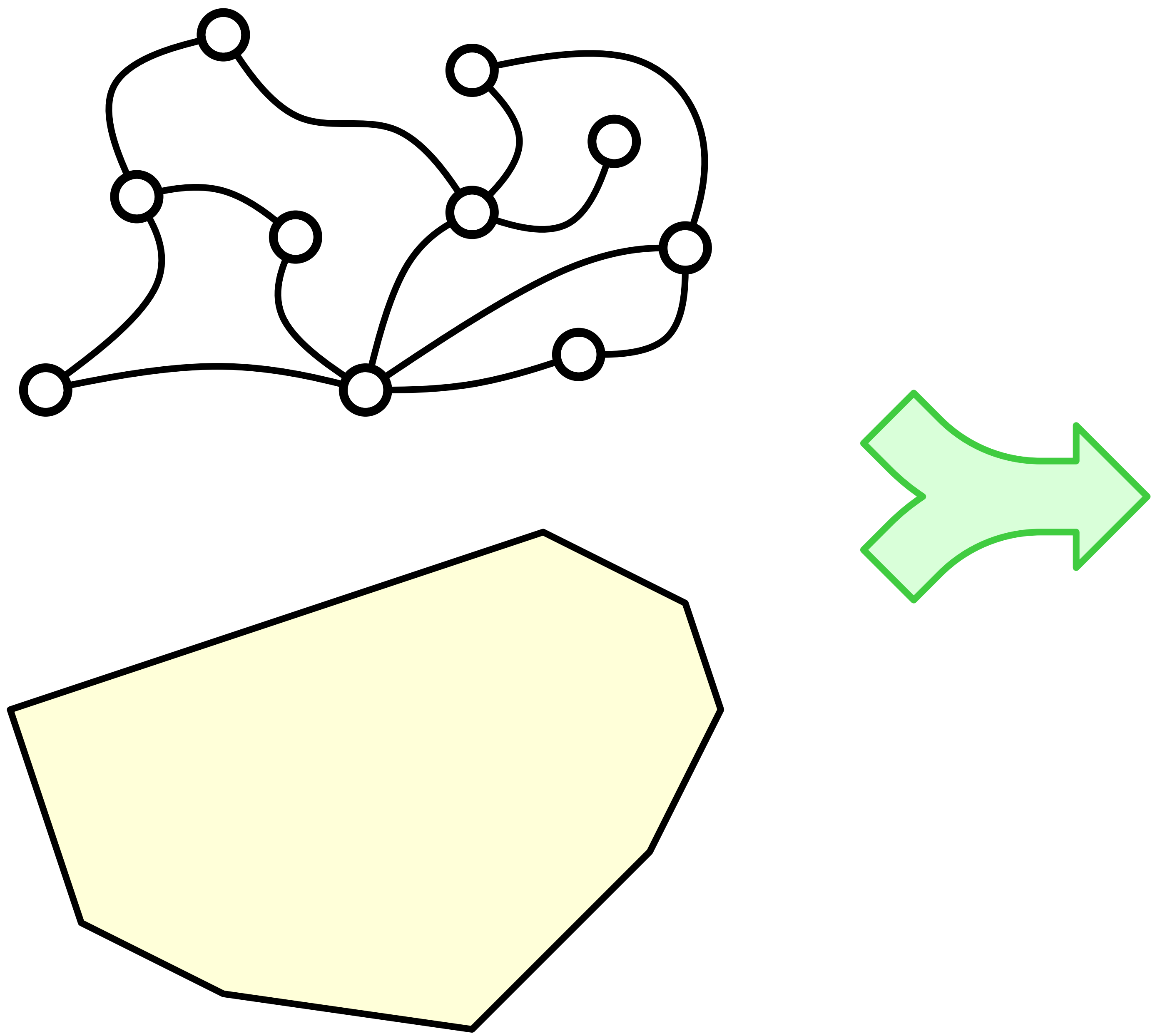

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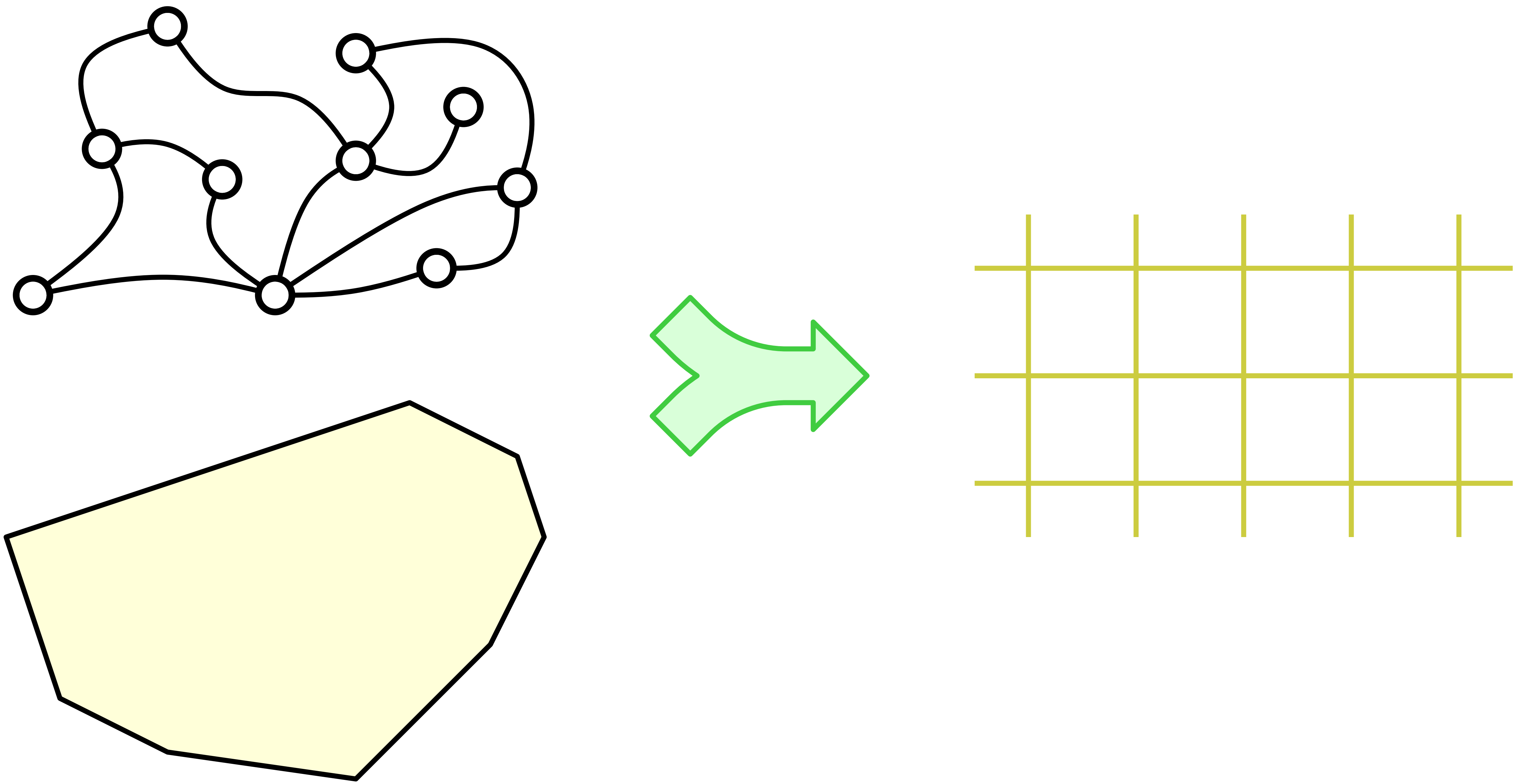

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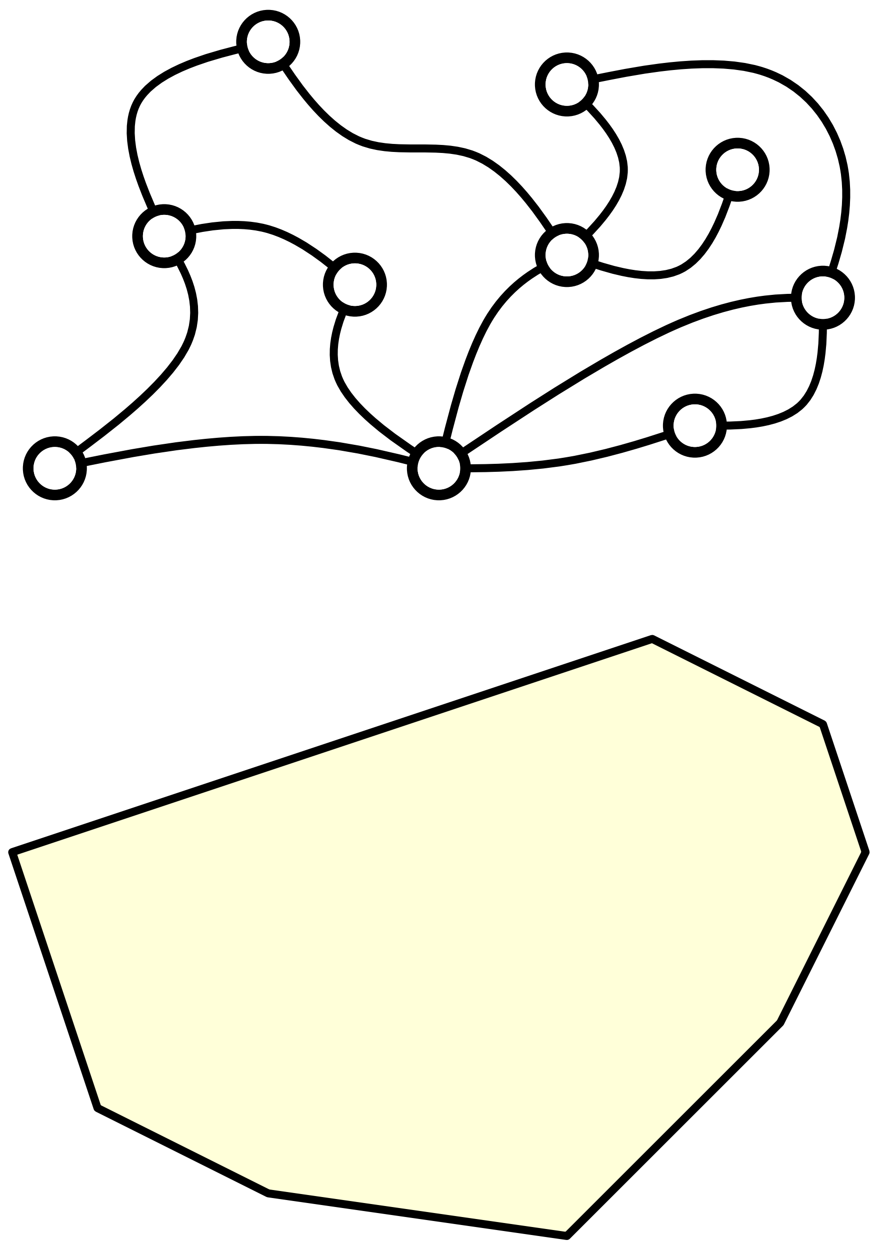

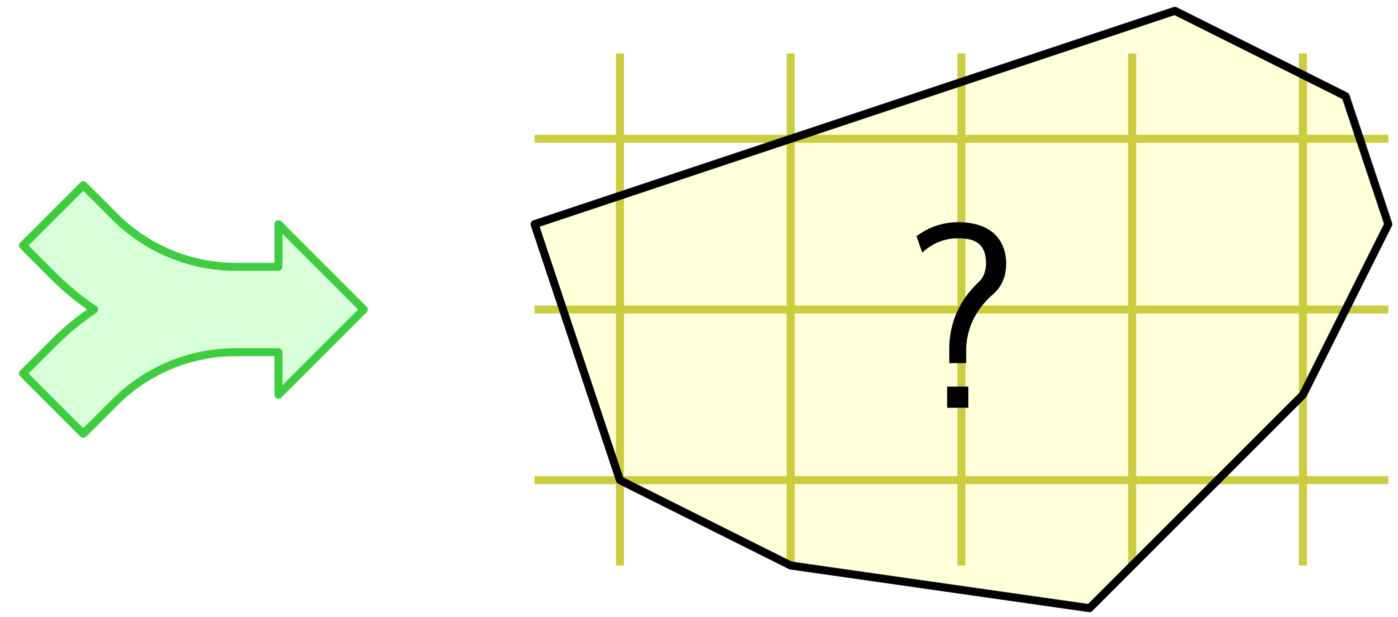

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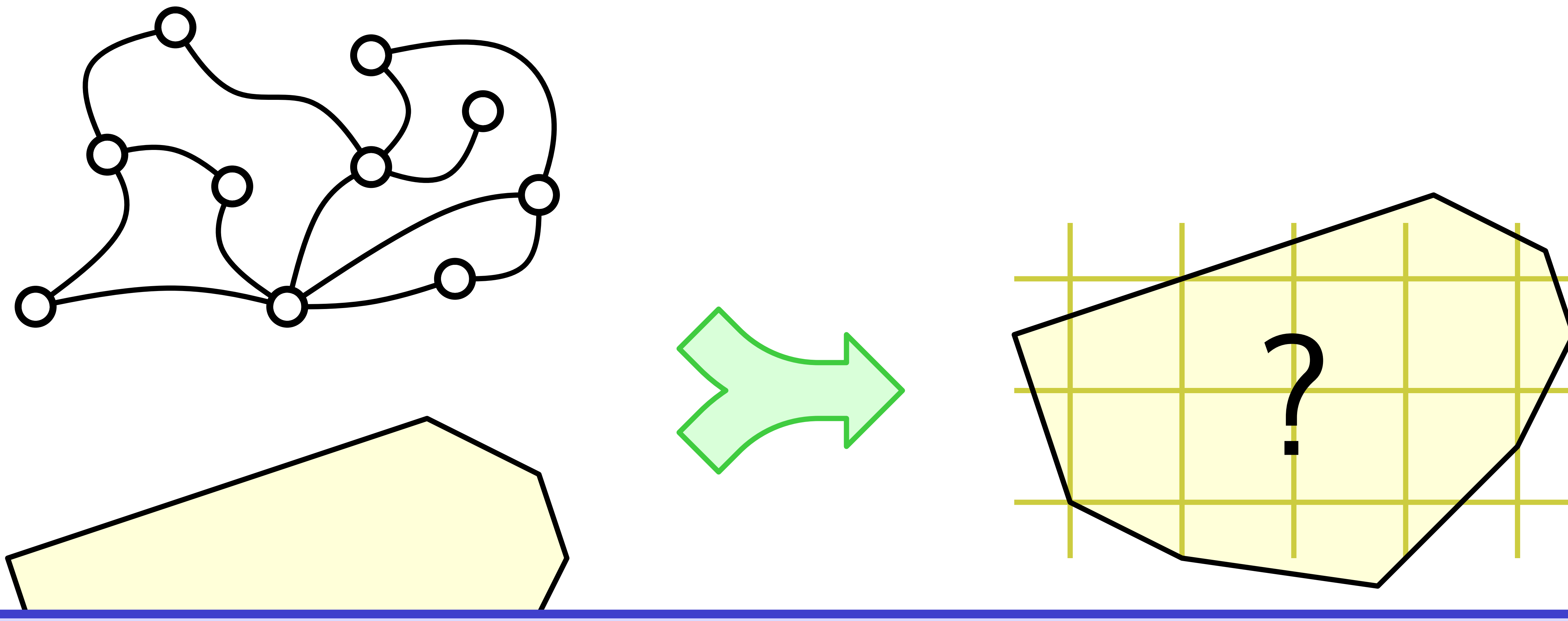

ANSWER

Obviously not...

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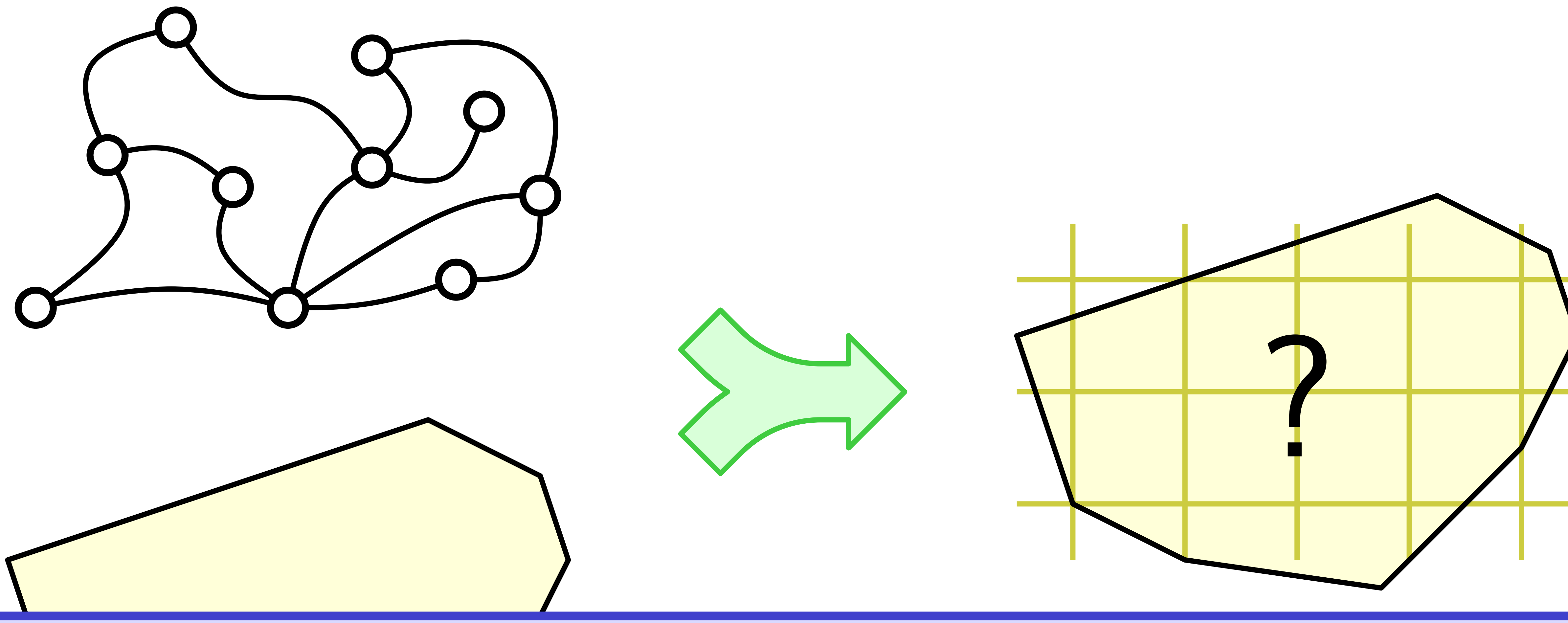
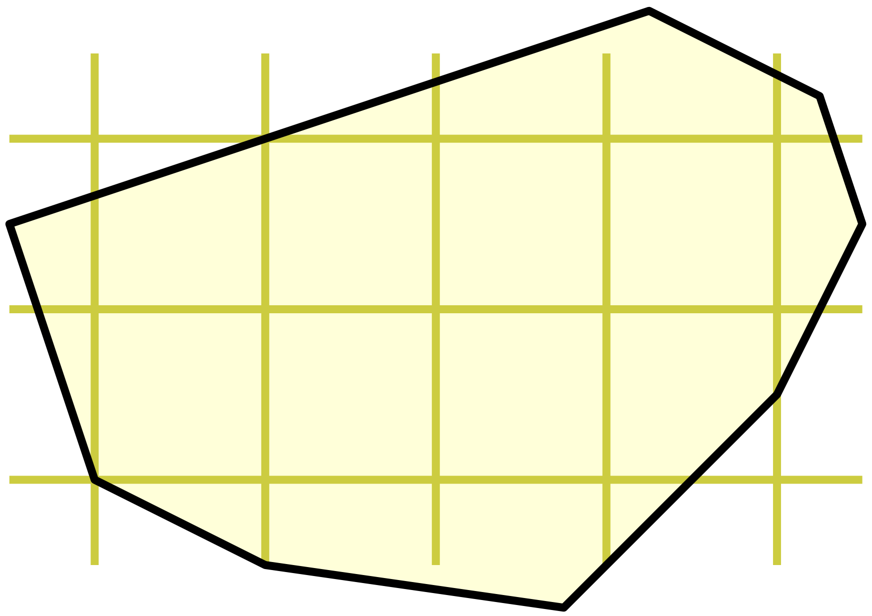
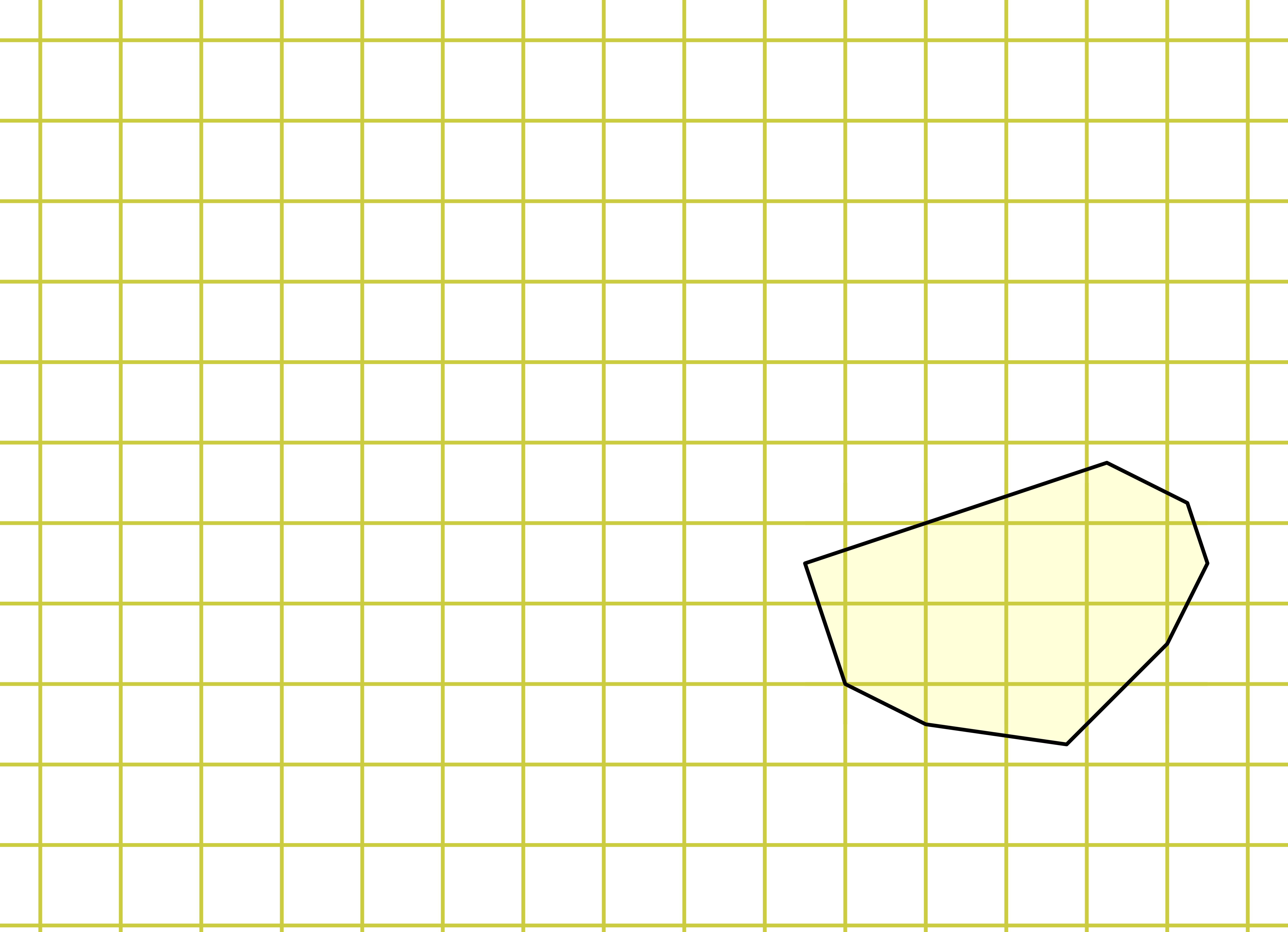
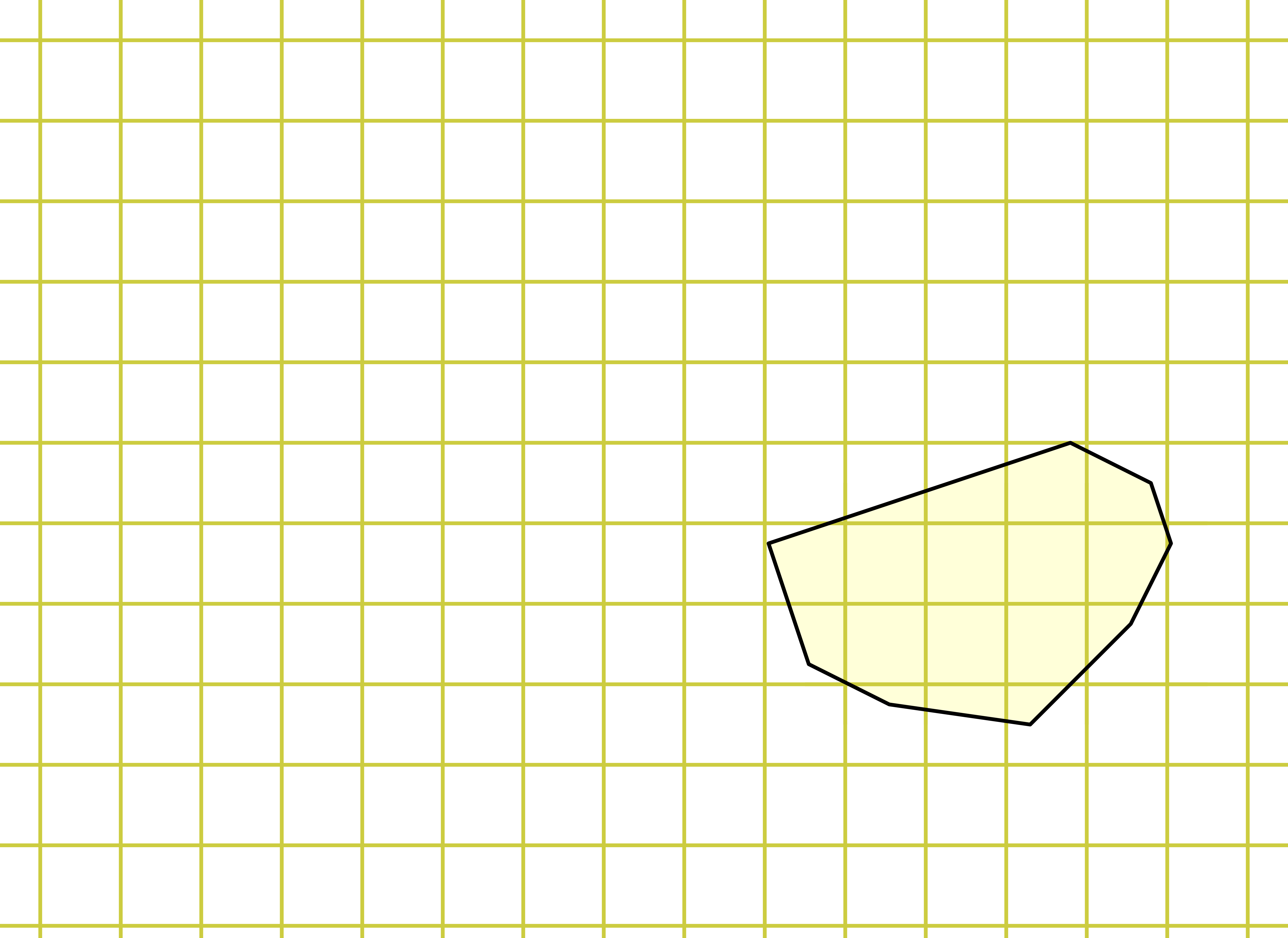


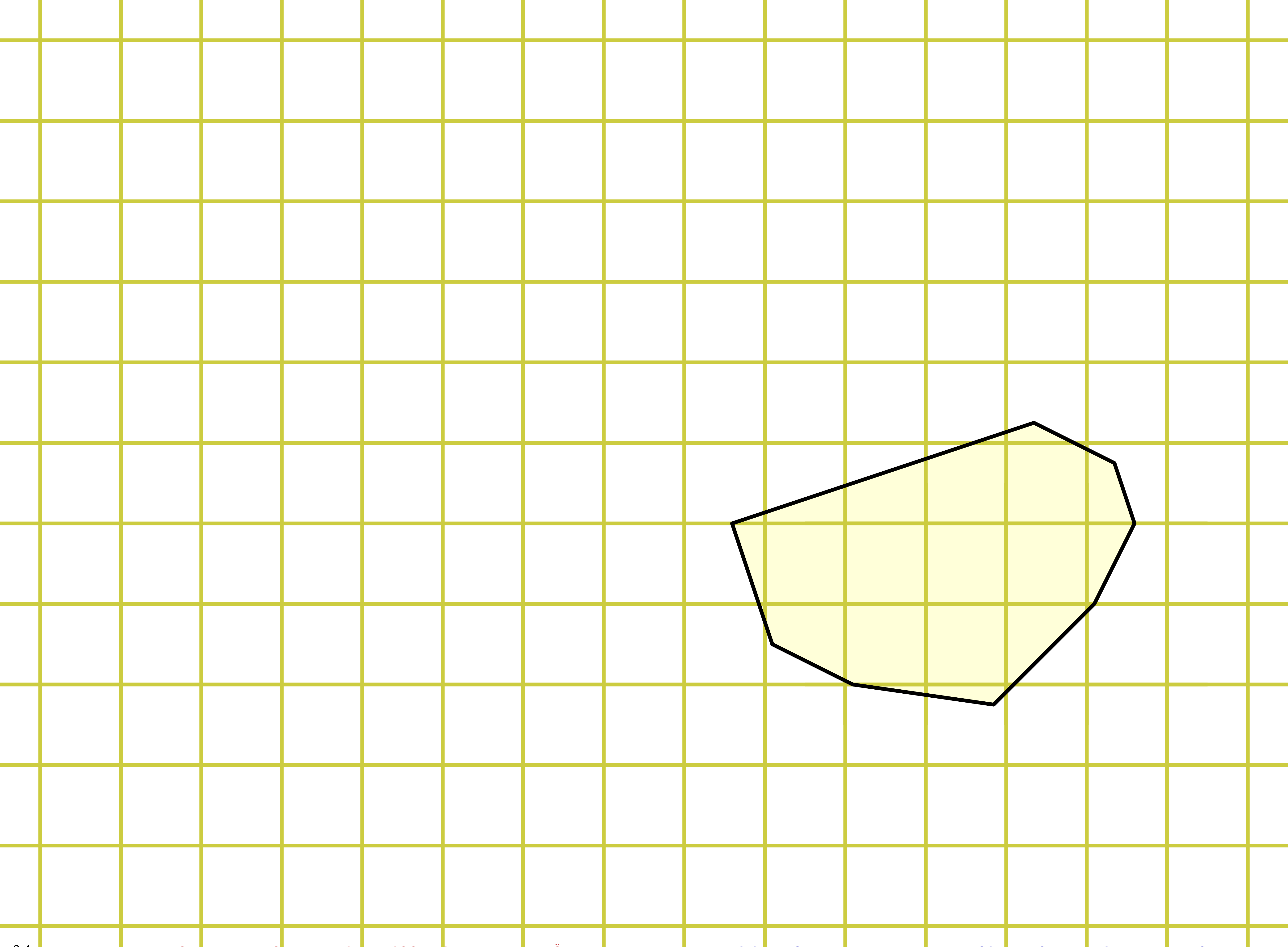
ANSWER

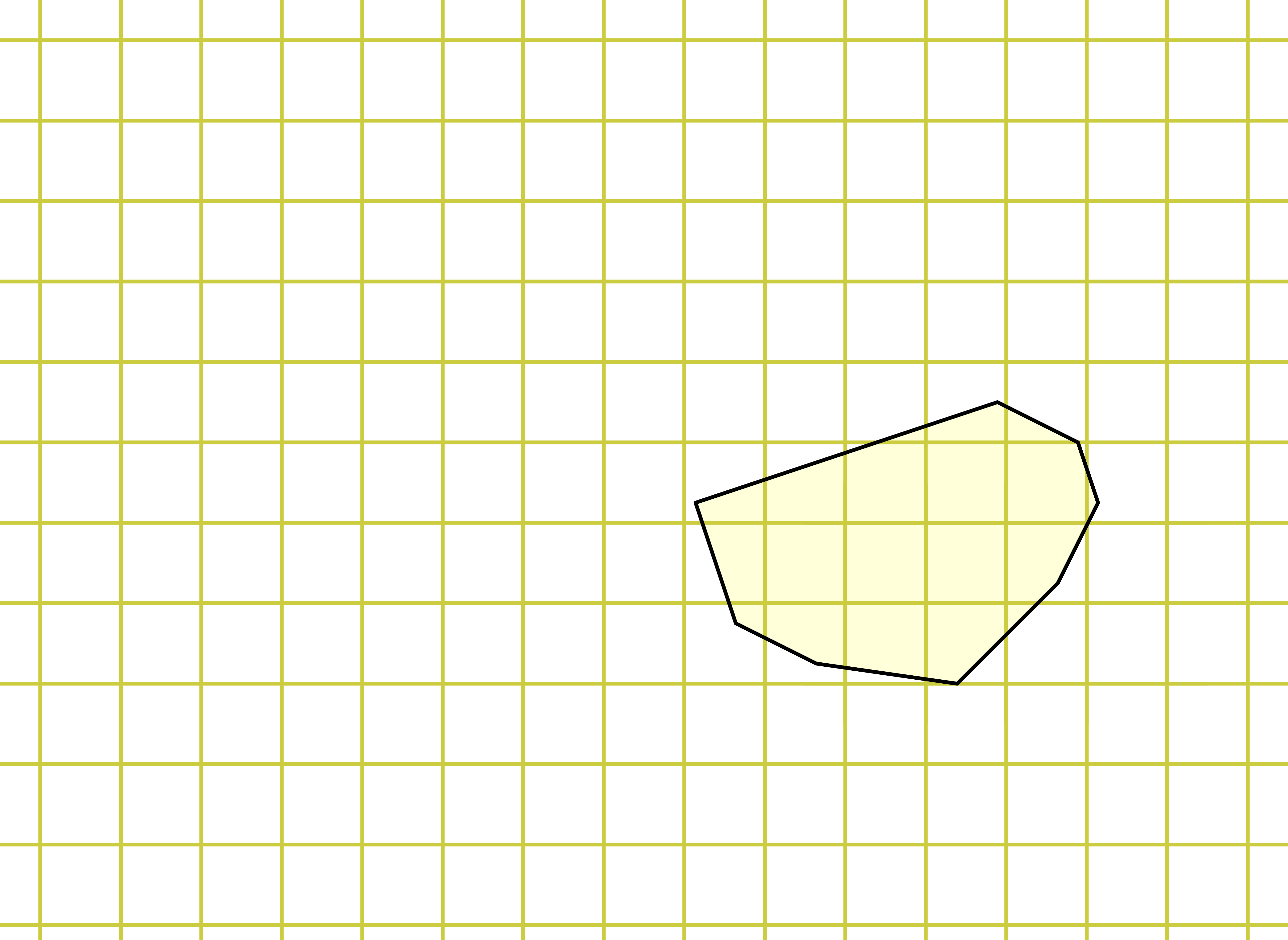
Obviously not... ...butbutbut!

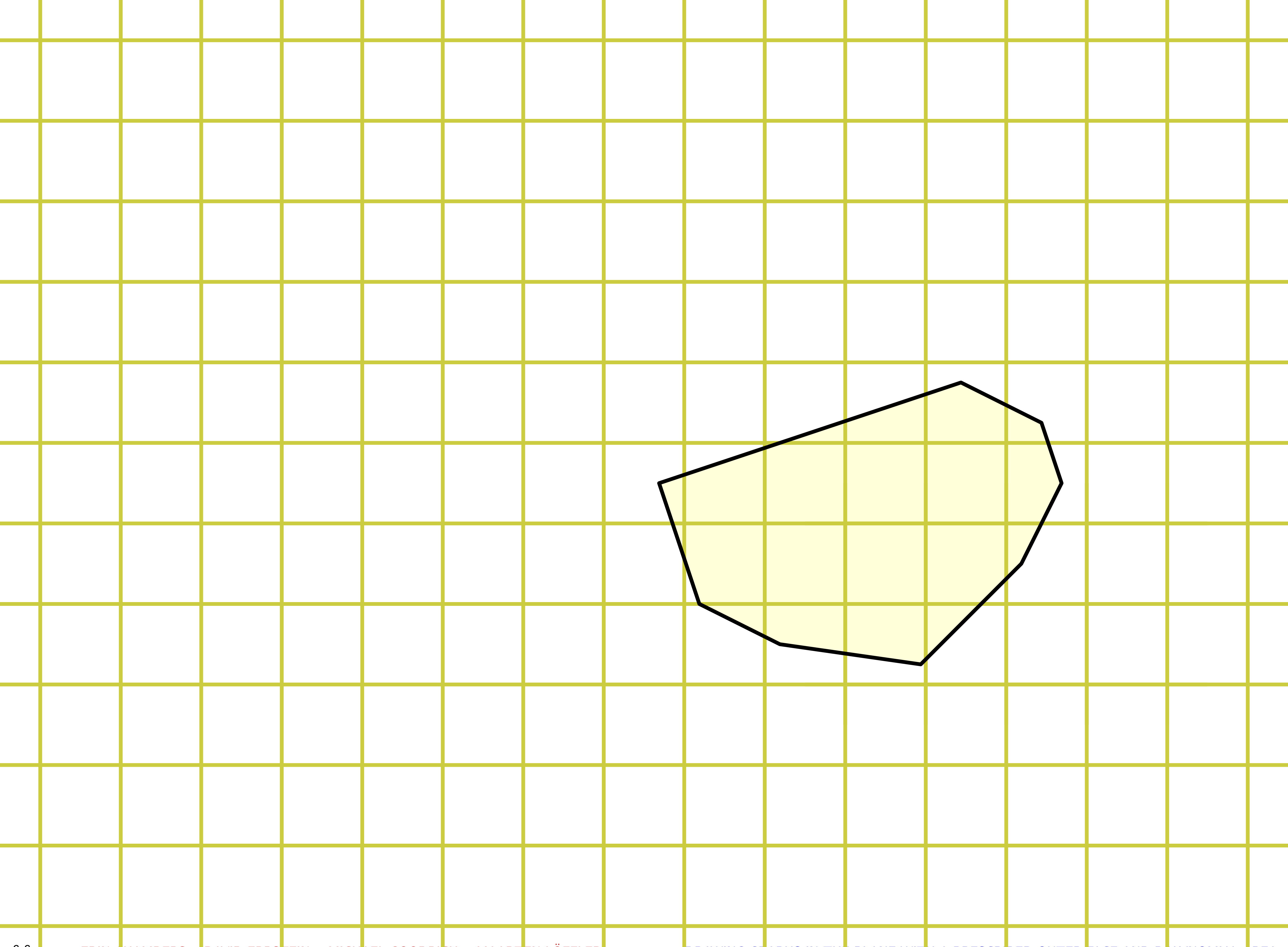


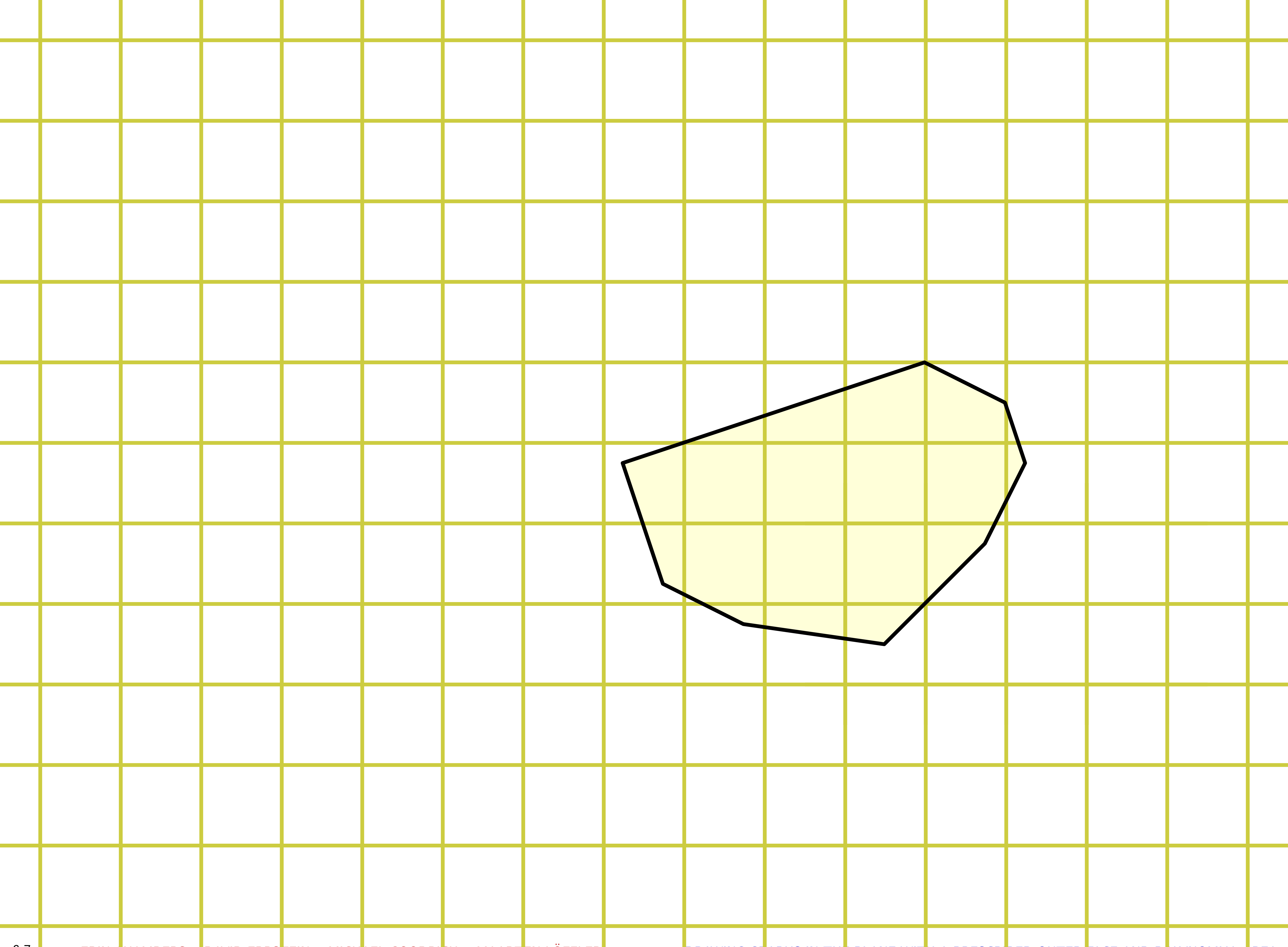


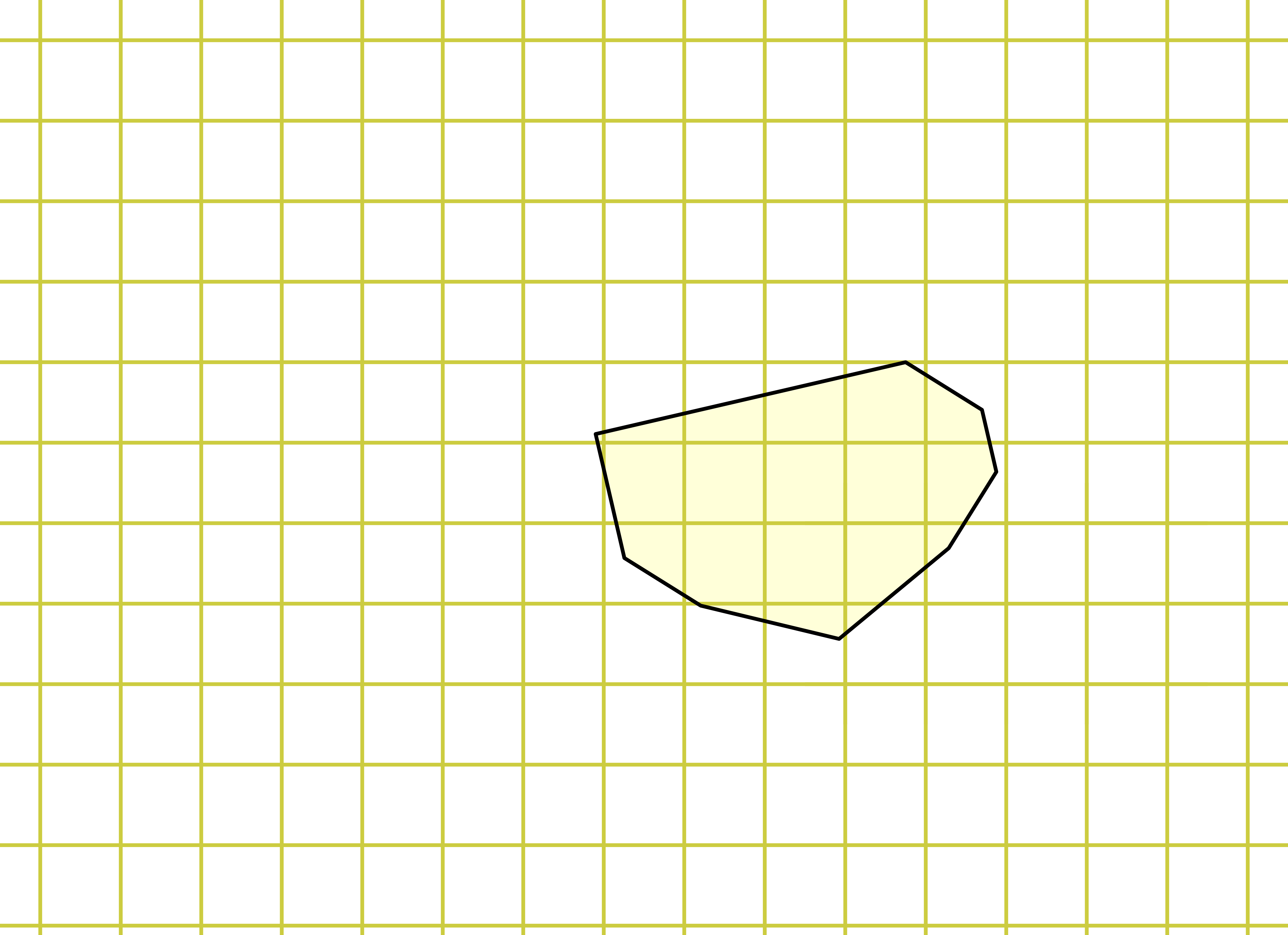


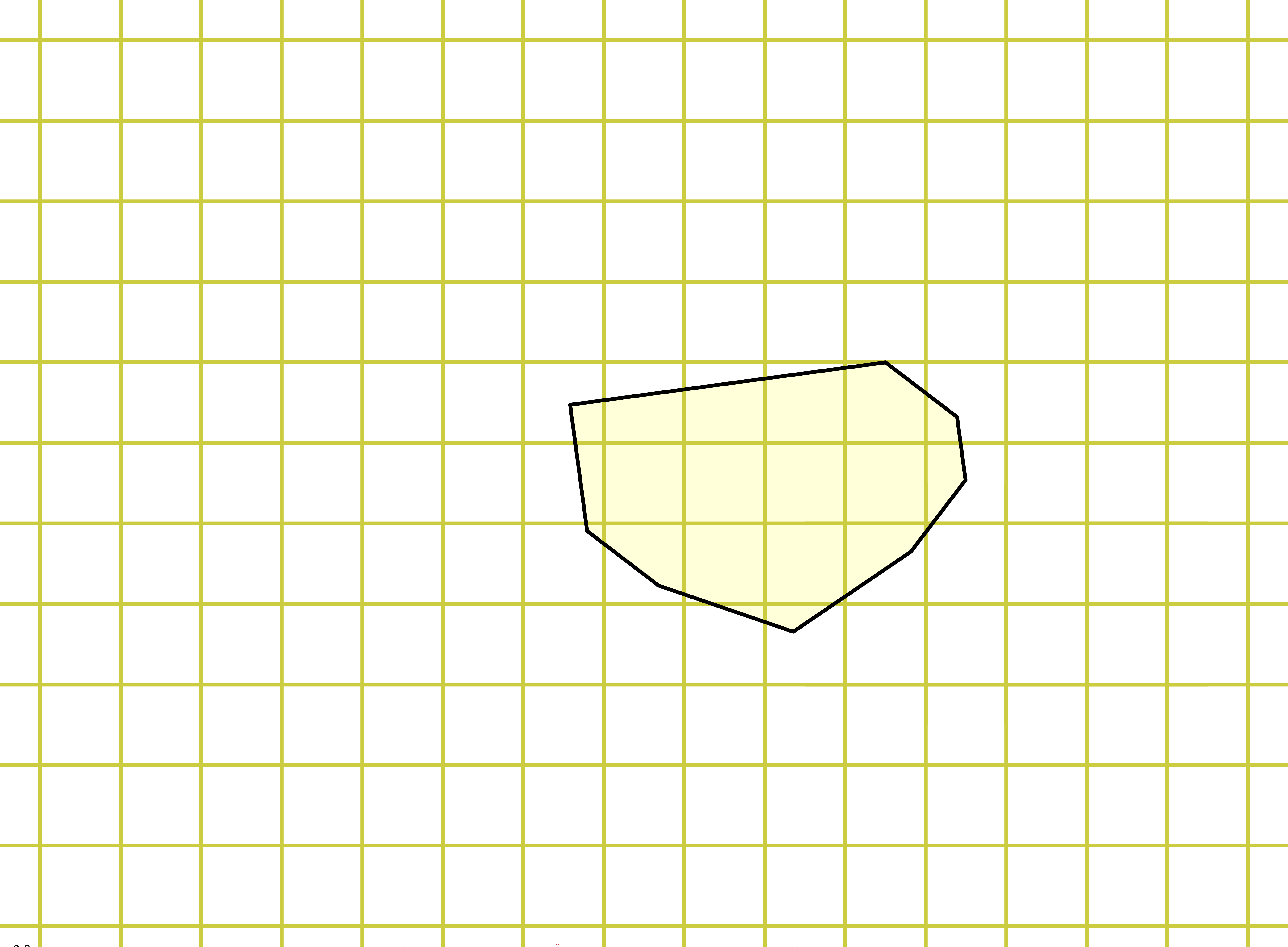


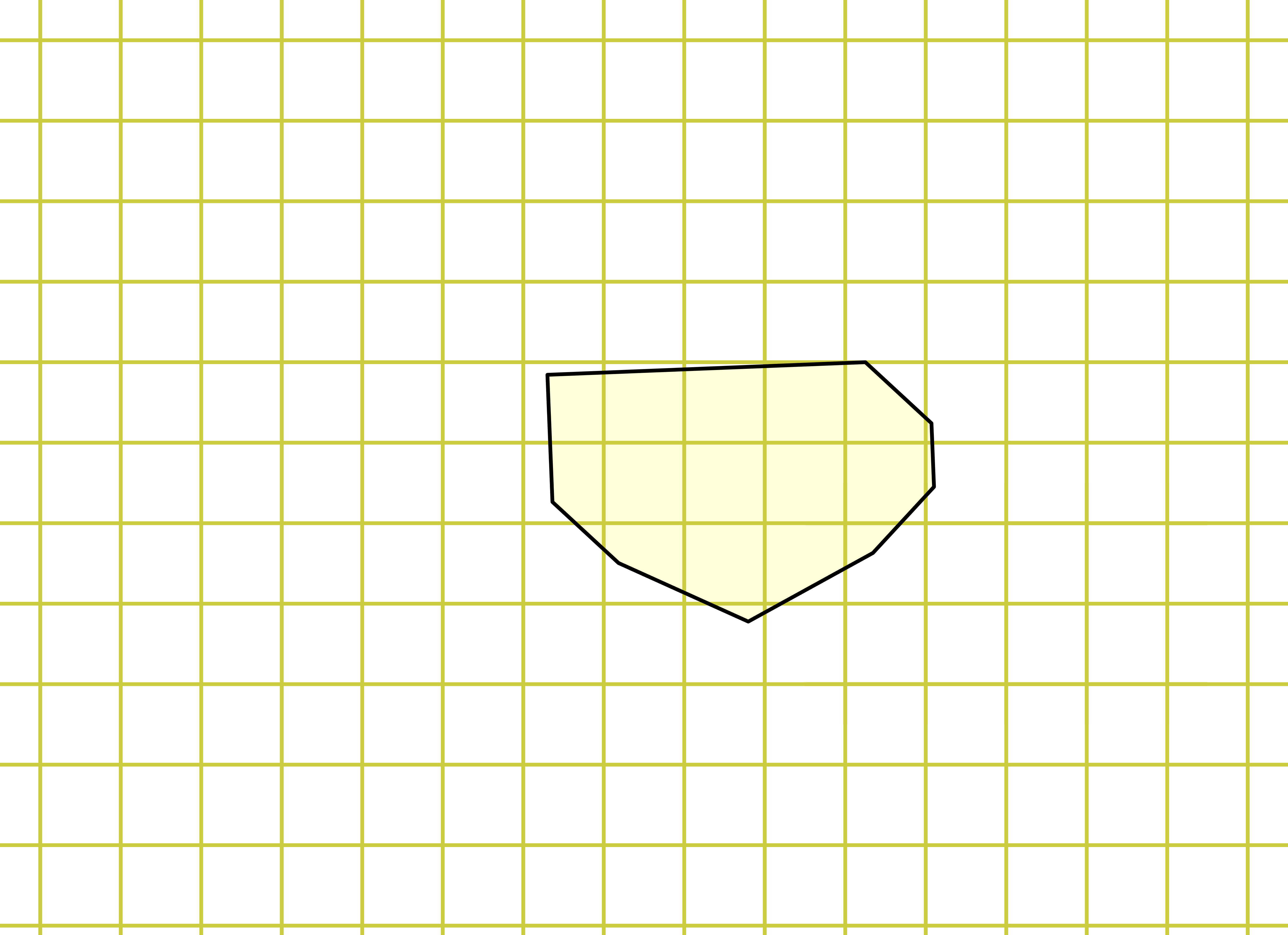


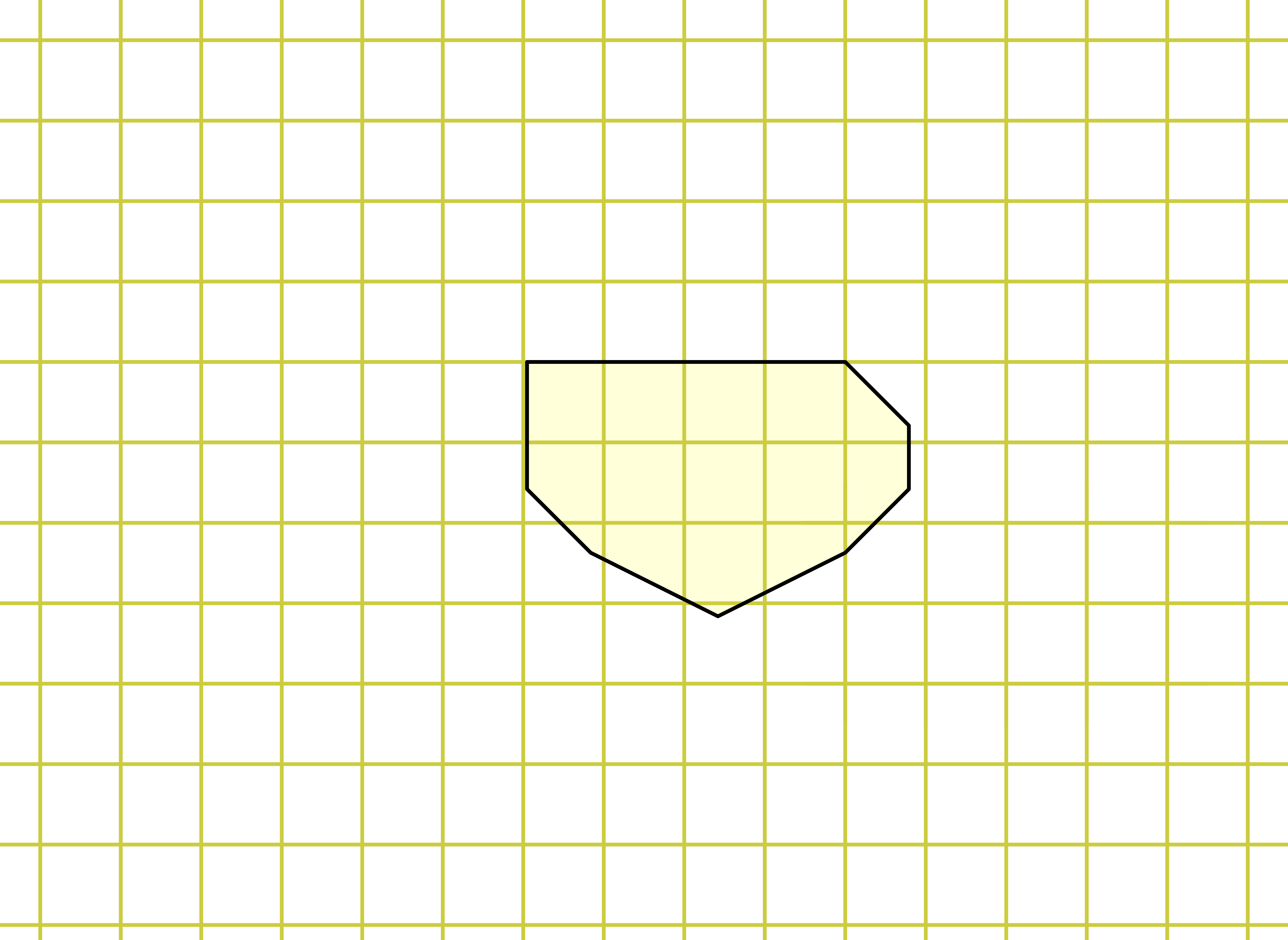


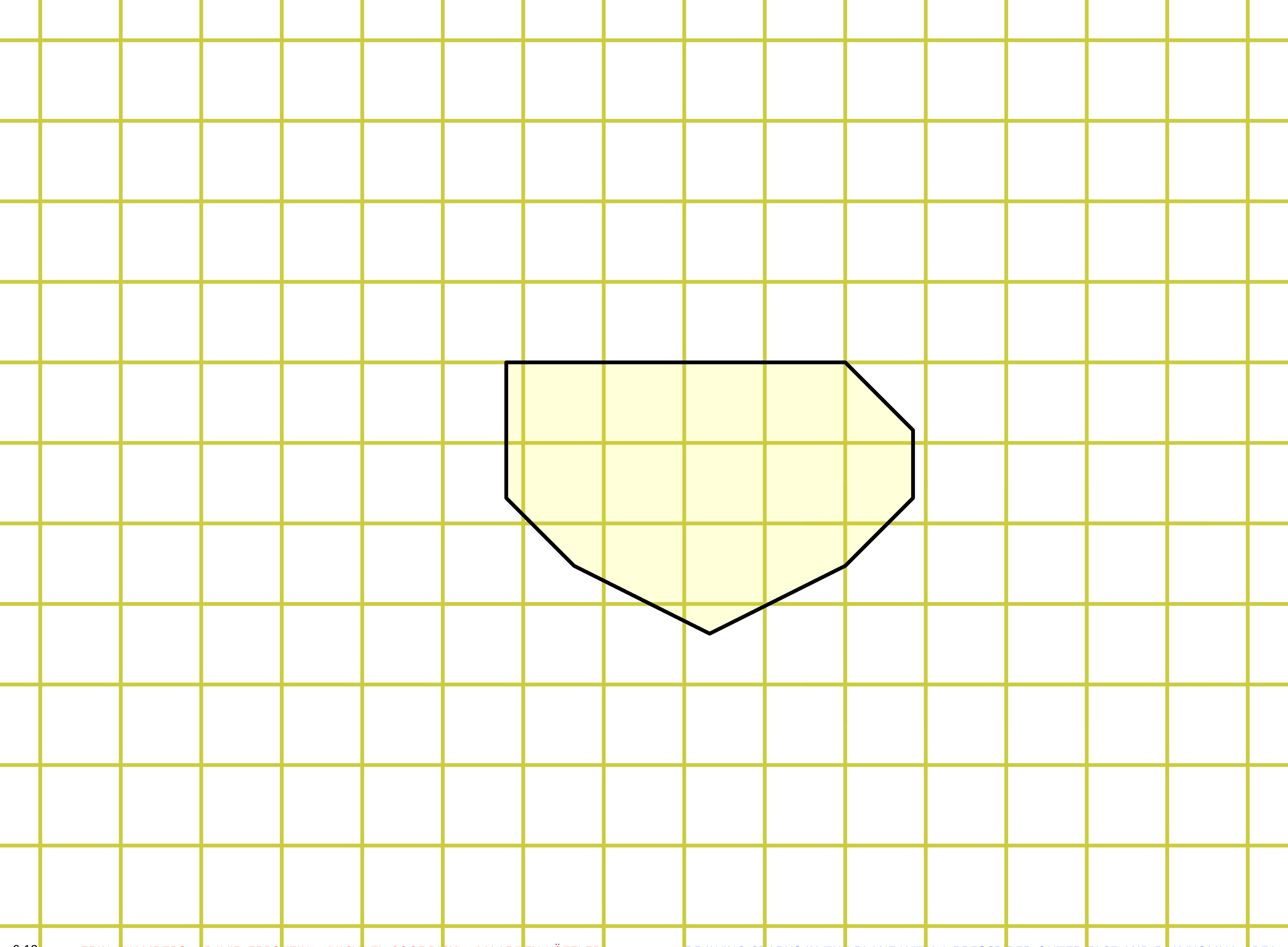


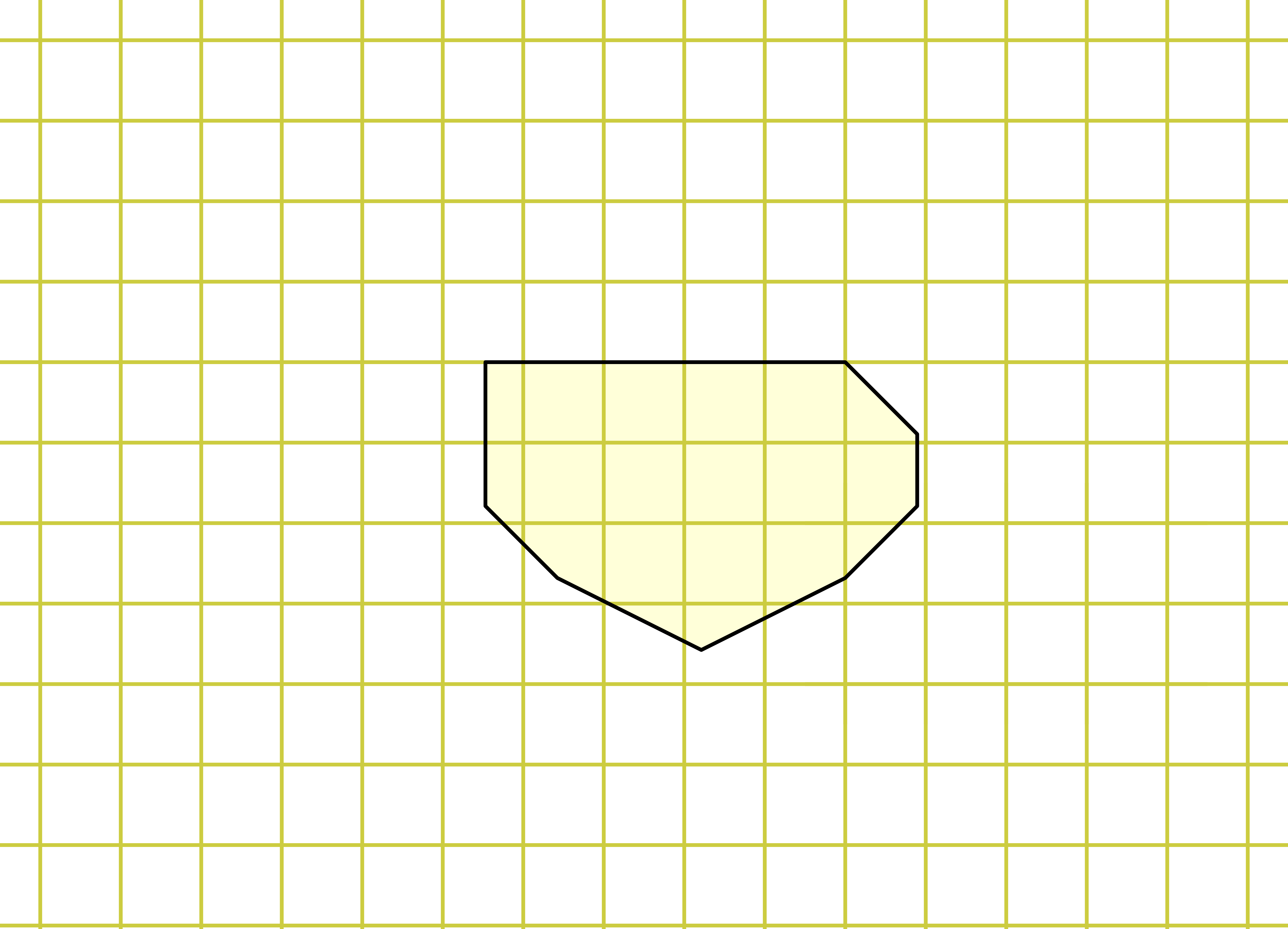


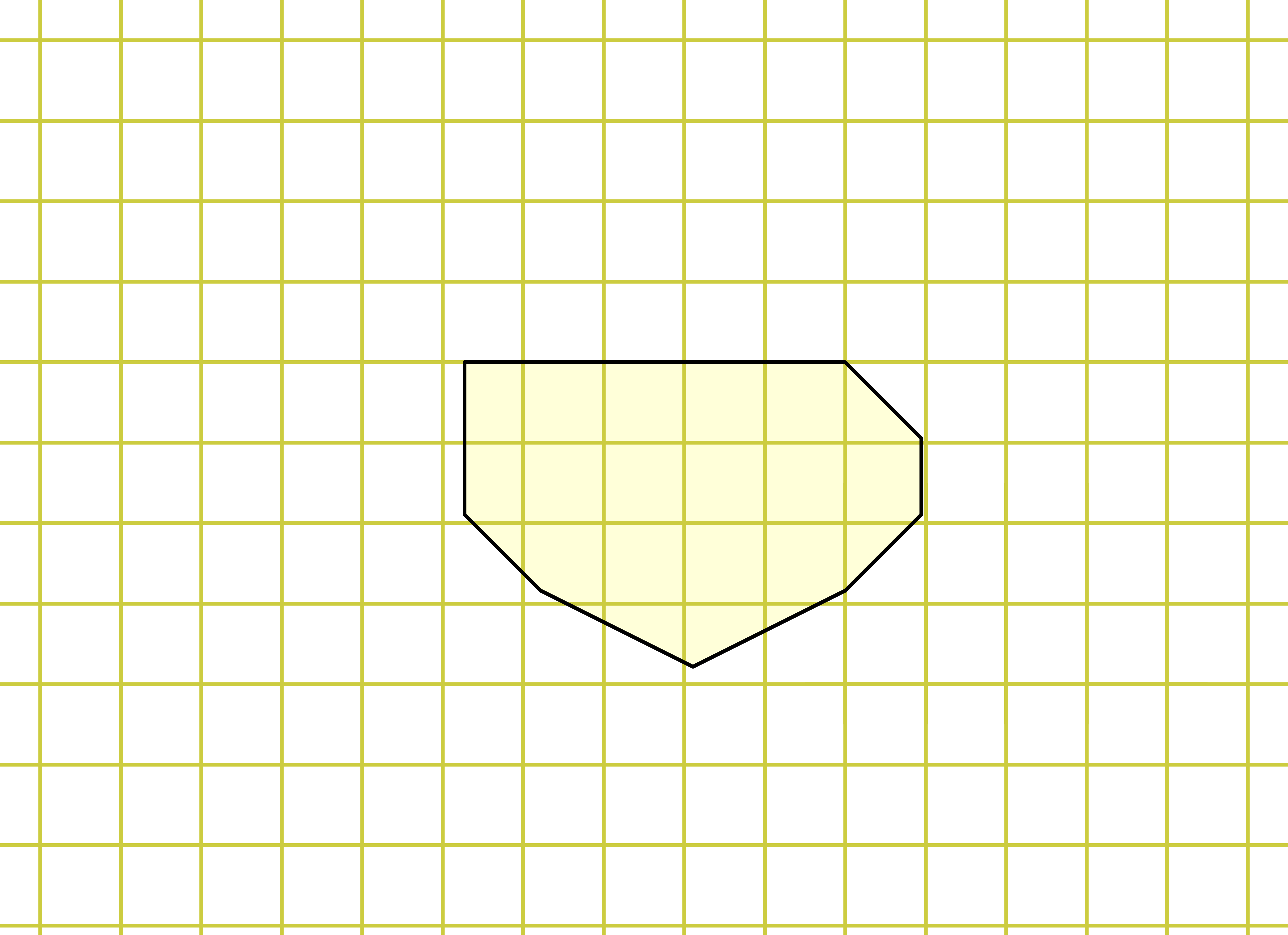


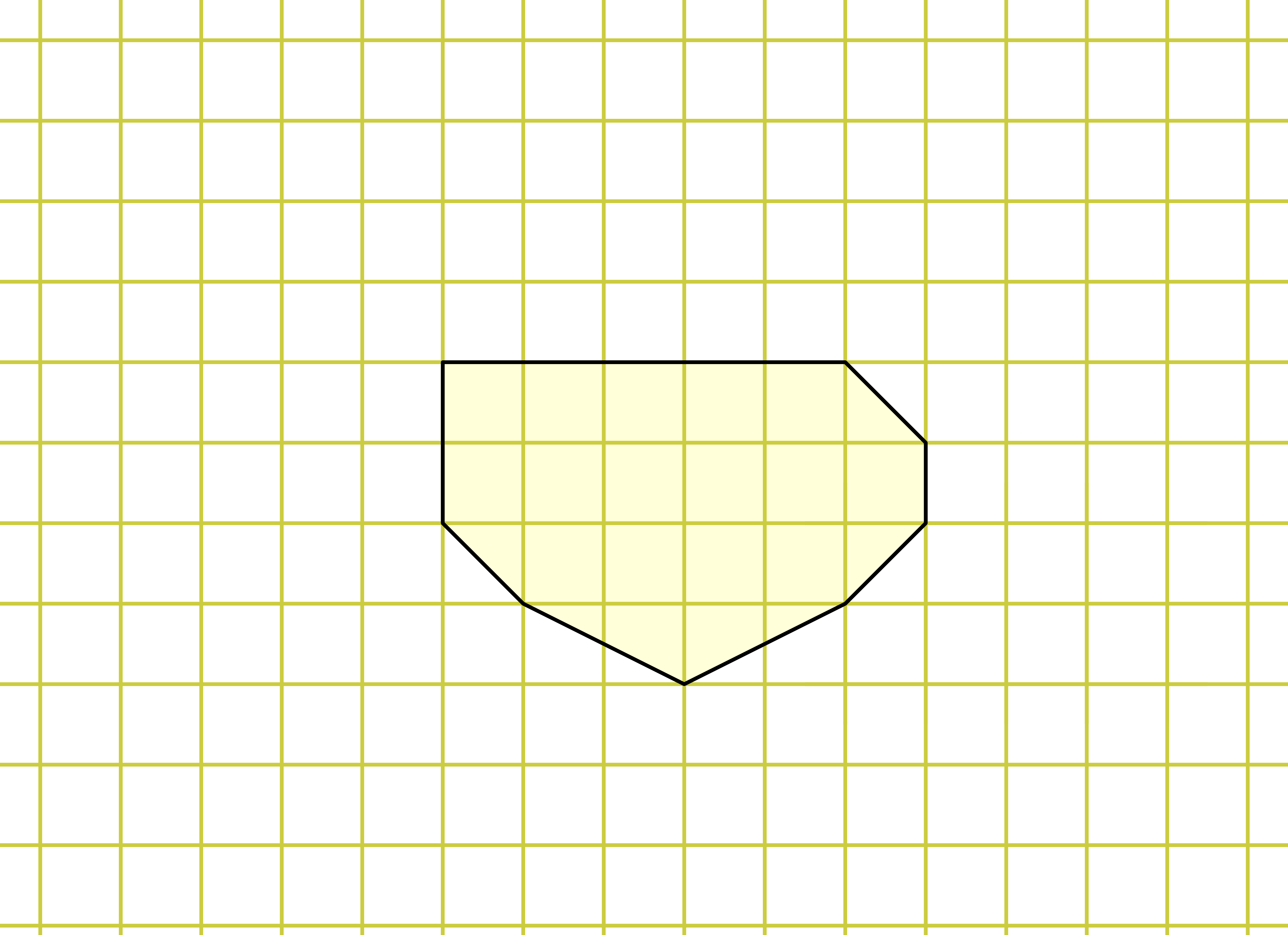


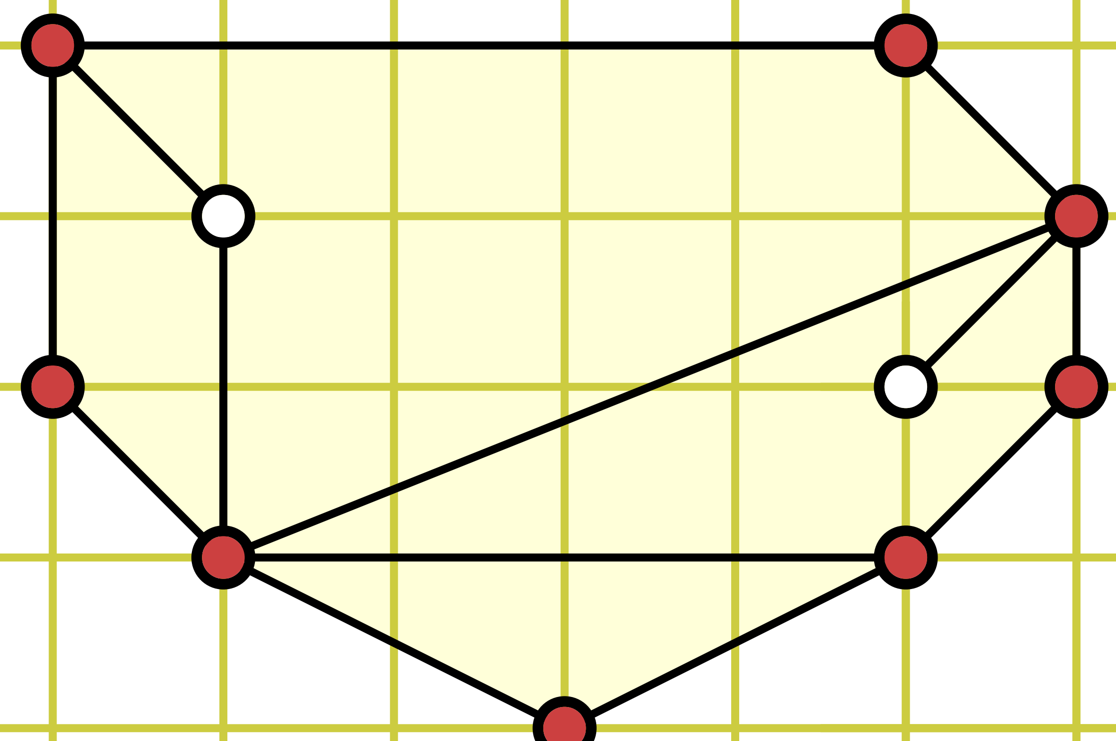












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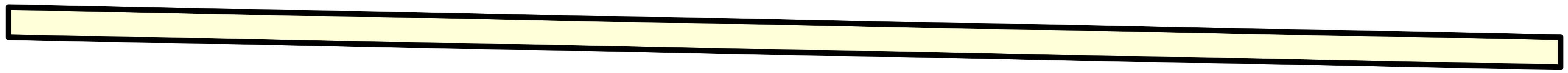

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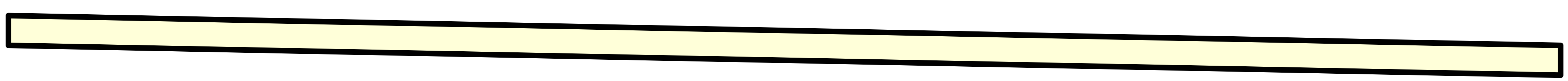

ANSWER

Nope.

TWICE REVISED QUESTION

Can every planar graph be drawn with straight edges inside a polygon that is similar to a given convex polygon *of bounded resolution* on a polynomial grid?

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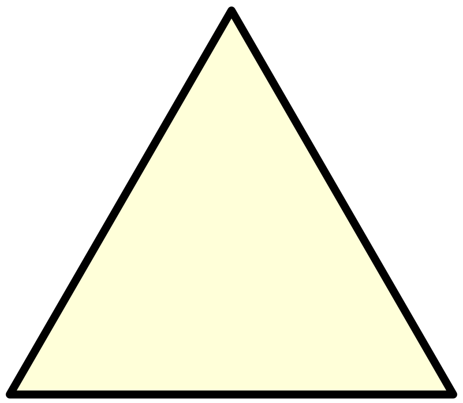

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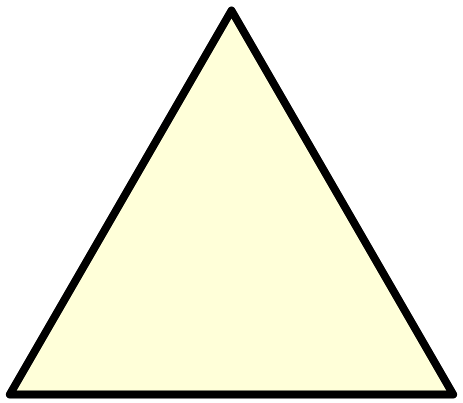

ANSWER

Still no.

THRICE REVISED QUESTION

Can every planar graph be drawn with straight edges inside a polygon that is *almost* similar to a given convex polygon of bounded resolution on a polynomial grid?

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Can every planar graph be drawn with straight edges inside a polygon that is *almost* similar to a given convex polygon of bounded resolution on a polynomial grid?

ANSWER

Yes!

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Can every planar graph be drawn with straight edges inside a polygon that is *almost* similar to a given convex polygon of bounded resolution on a polynomial grid?

ANSWER

Yes! [Us, Here, Now]

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The *resolution* of a drawing is the ratio between the diameter and the shortest distance between two non-incident features of the drawing.

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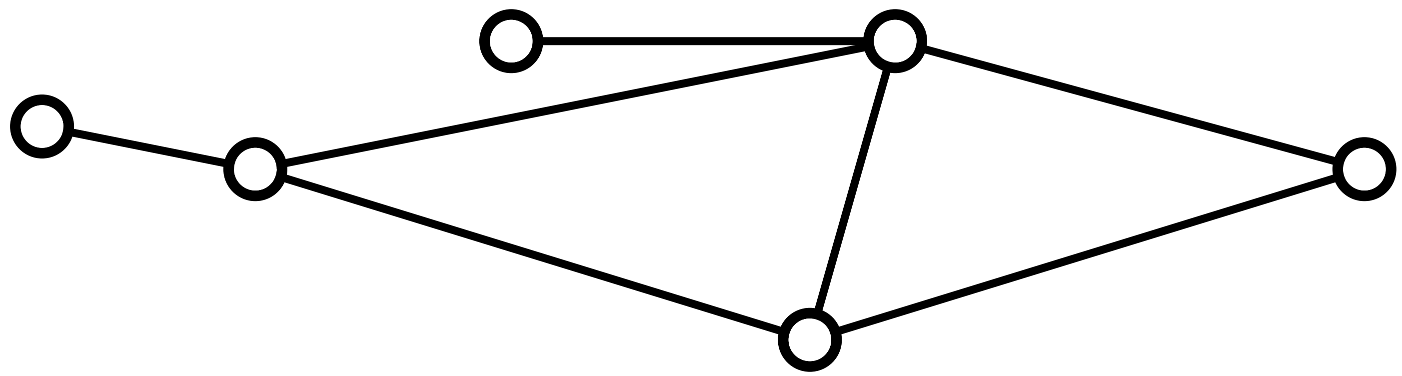

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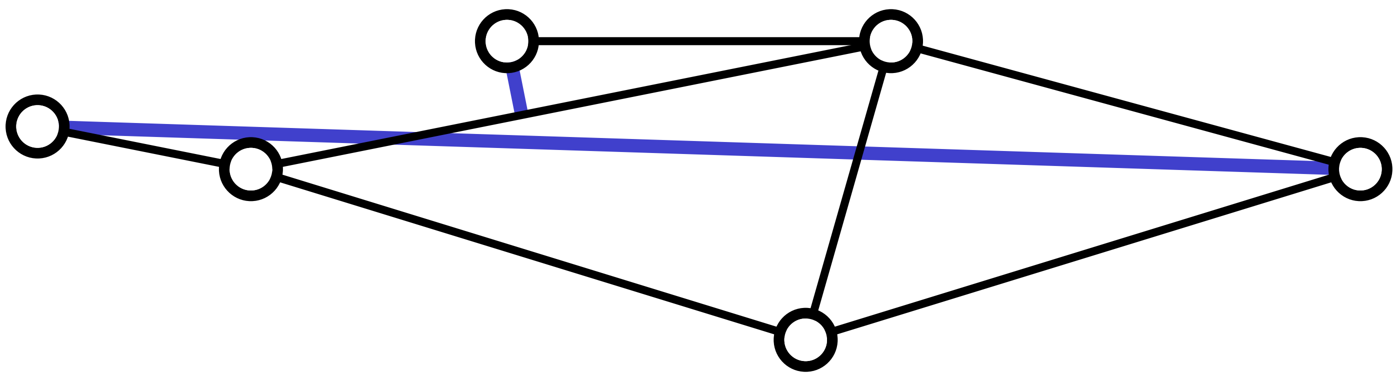

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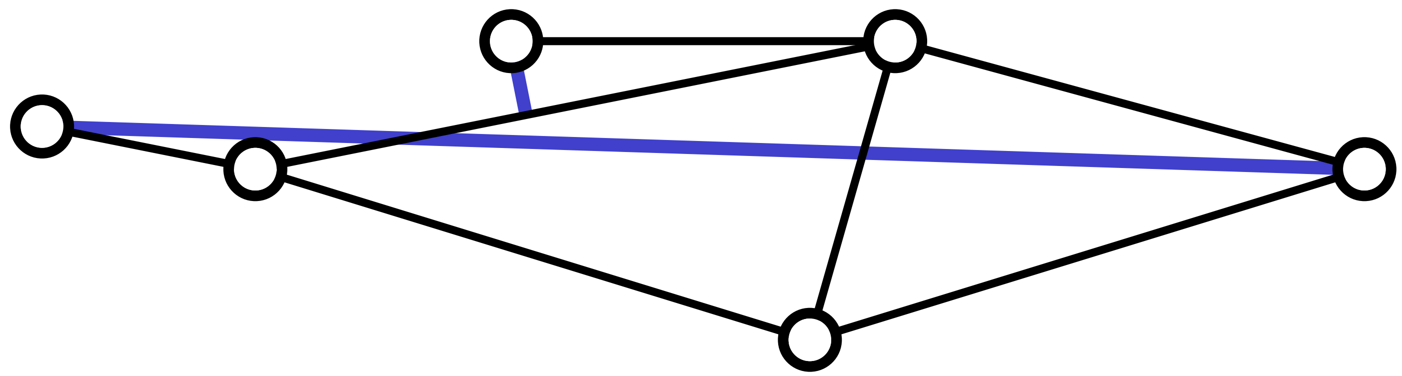

LEMMA

A drawing with polynomial resolution can be transformed into a drawing on a polynomial grid.

THEOREM

Given a planar graph G with n vertices and a convex polygon P with resolution d, there exists a drawing of G inside P with resolution $\Omega(d/n^3)$.

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PROOF

Just use divide & conquer...

LEMMA *(geometric split lemma)*
Every convex polygon with resolution d can be split through any pair of edges into smaller polygons with resolutions d_1 and d_2 such that $d_1 + d_2 \geq d$.

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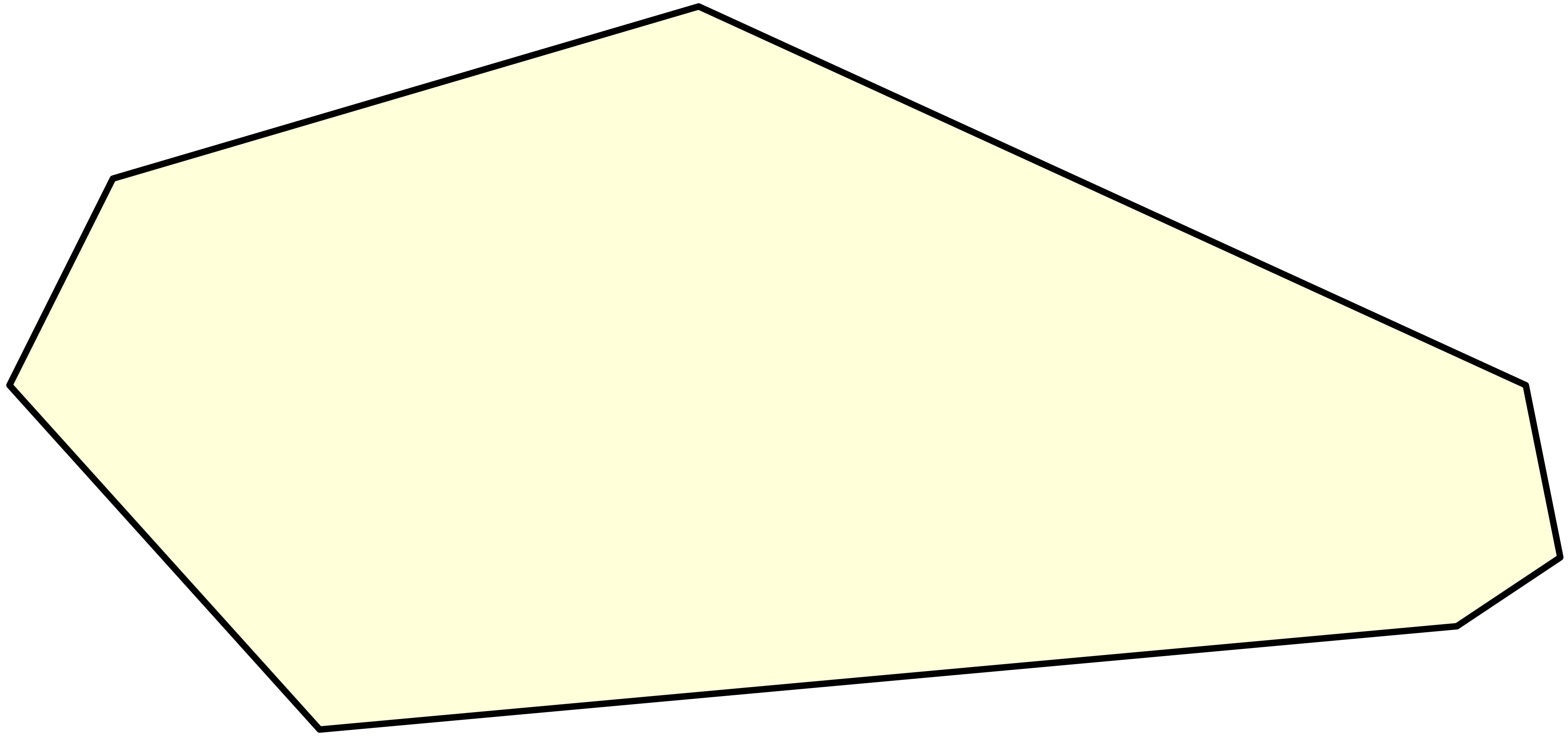

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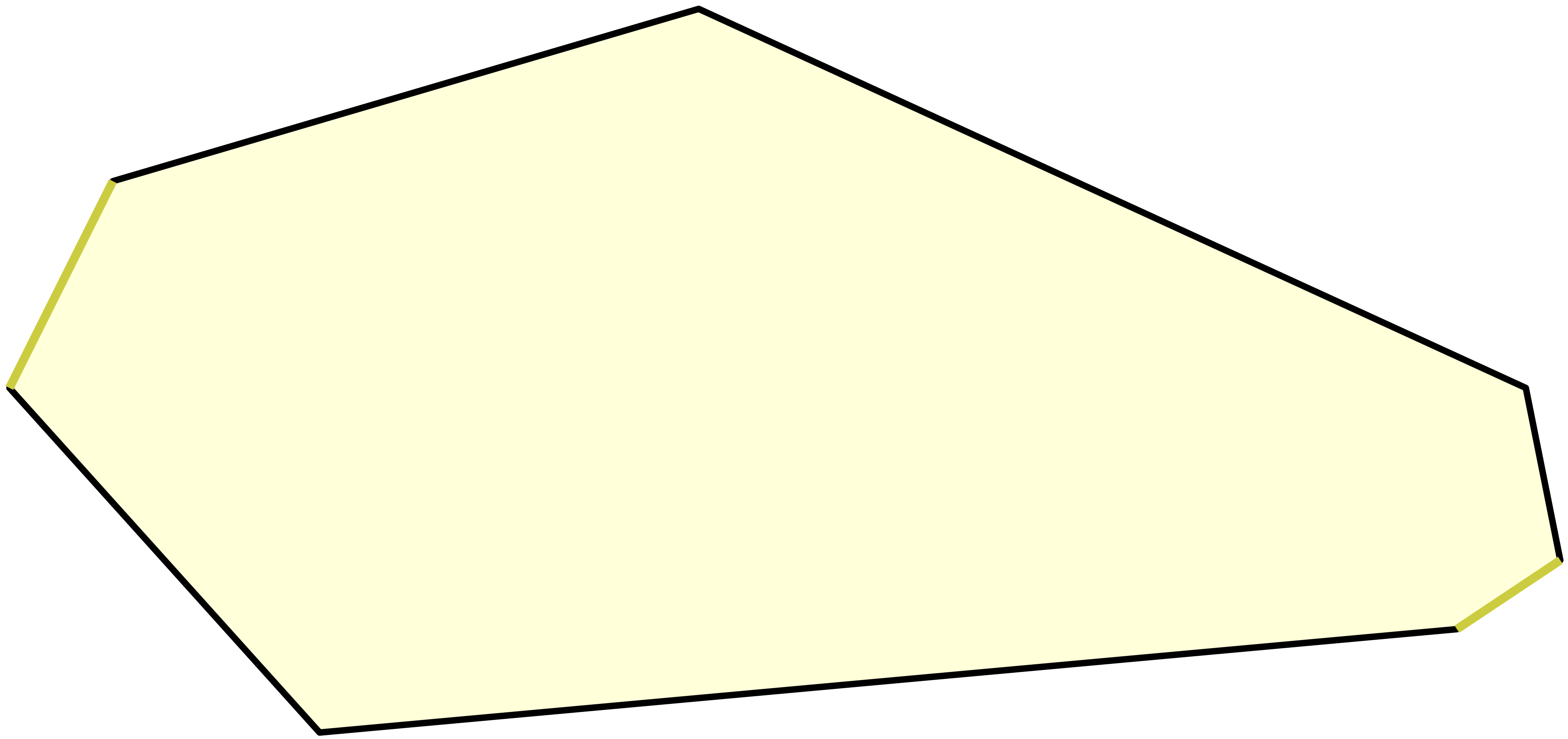

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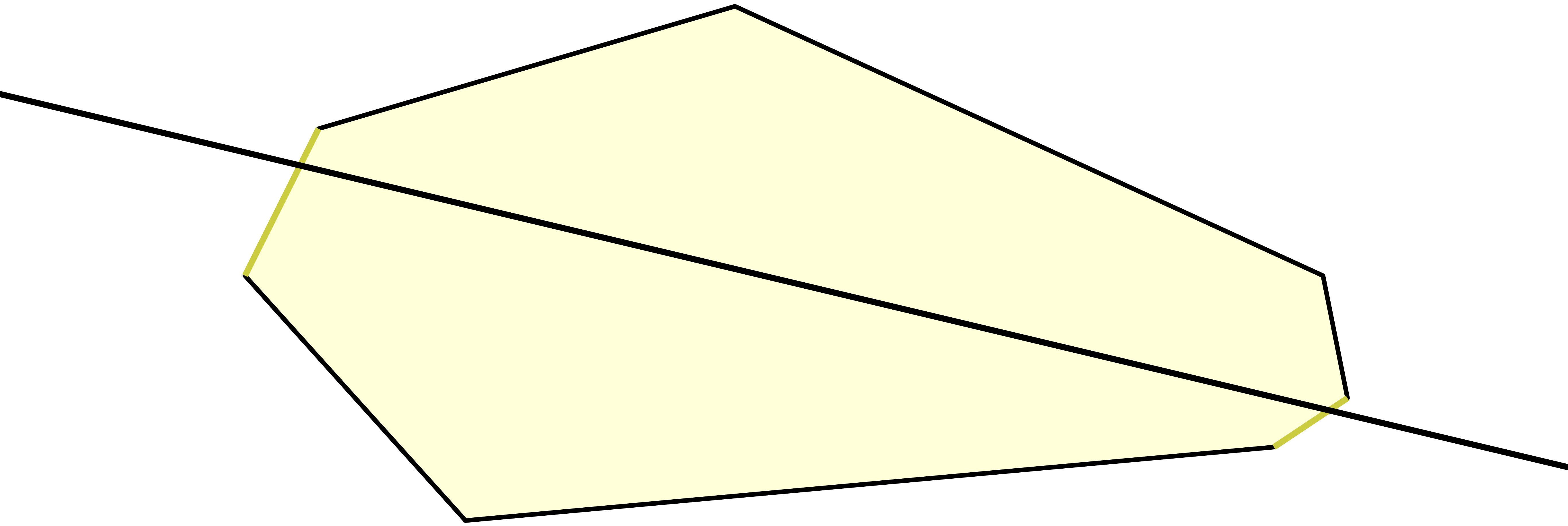

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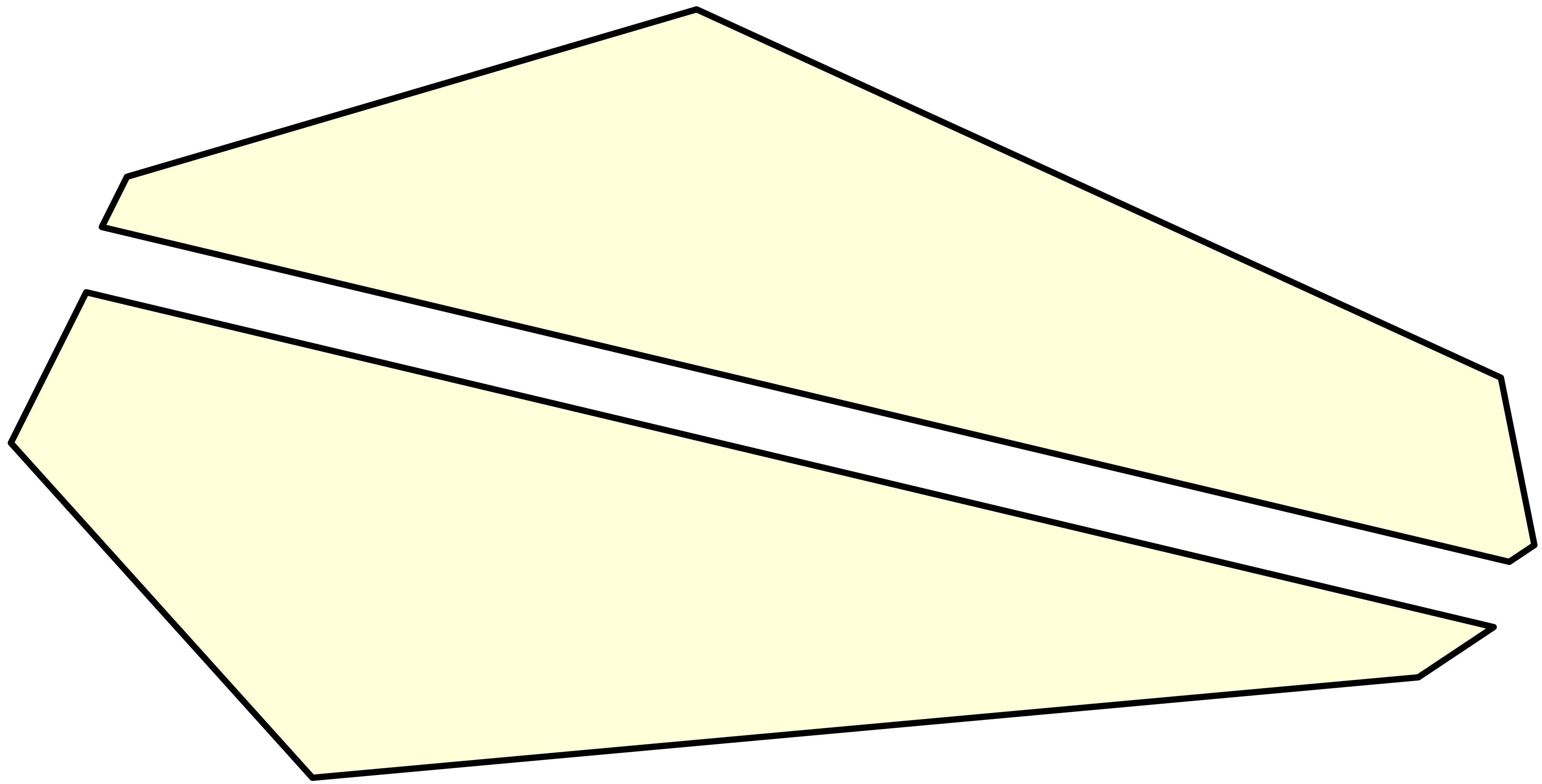

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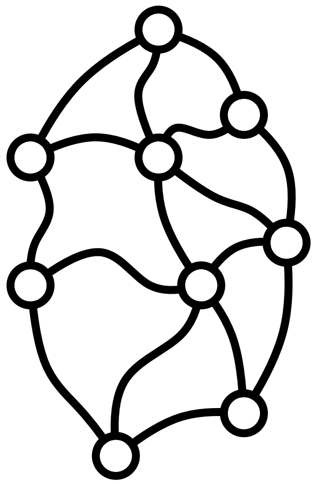

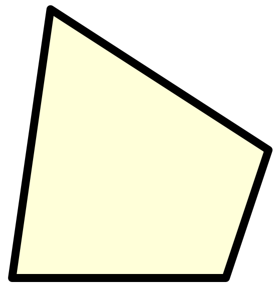

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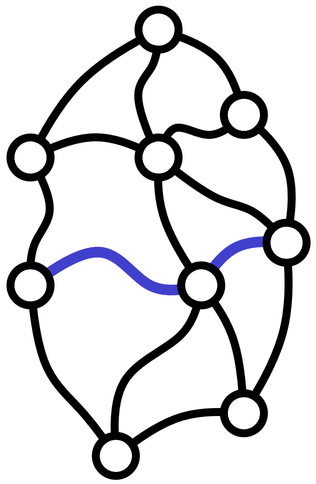

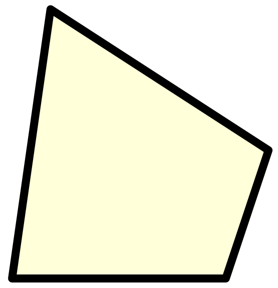

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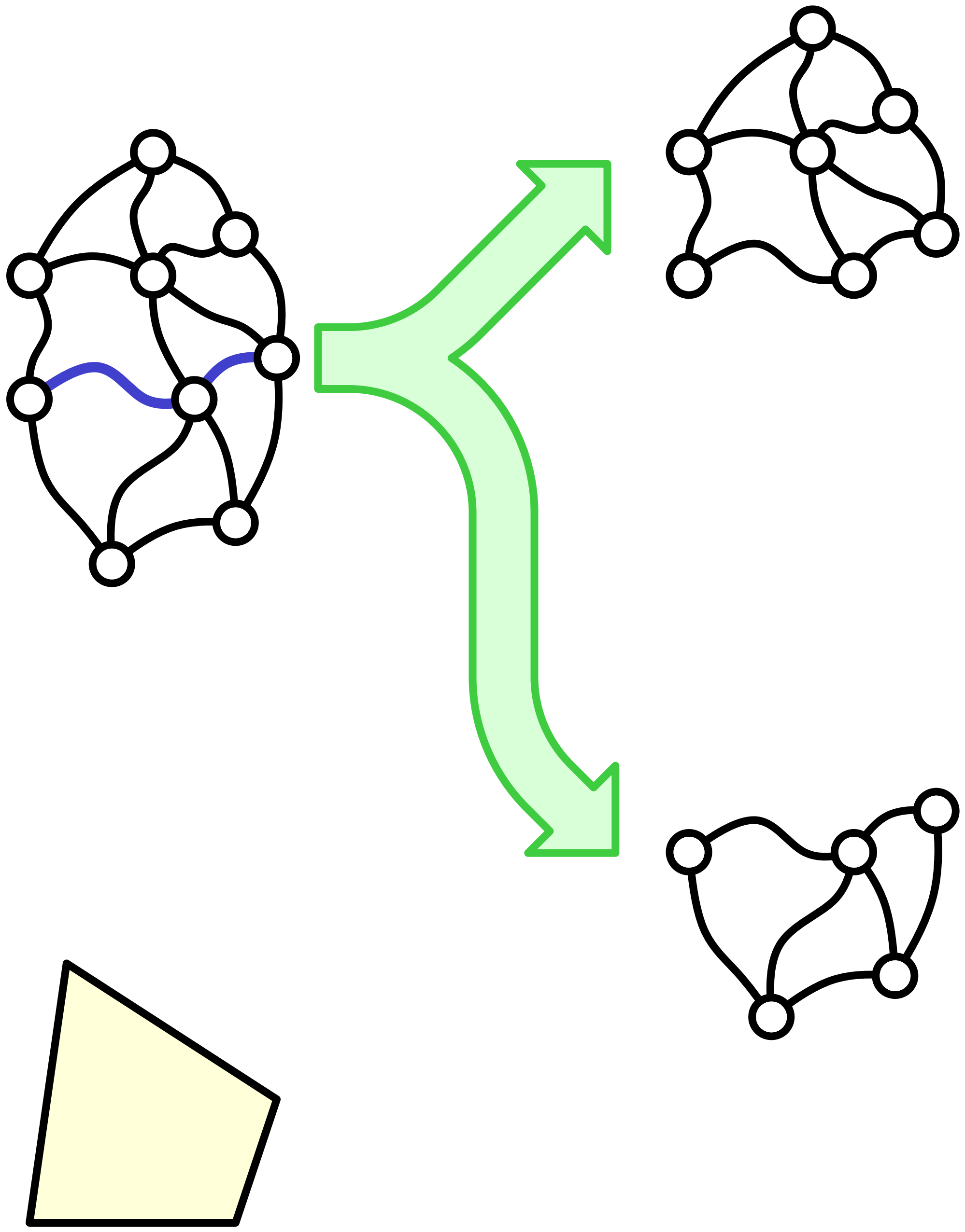

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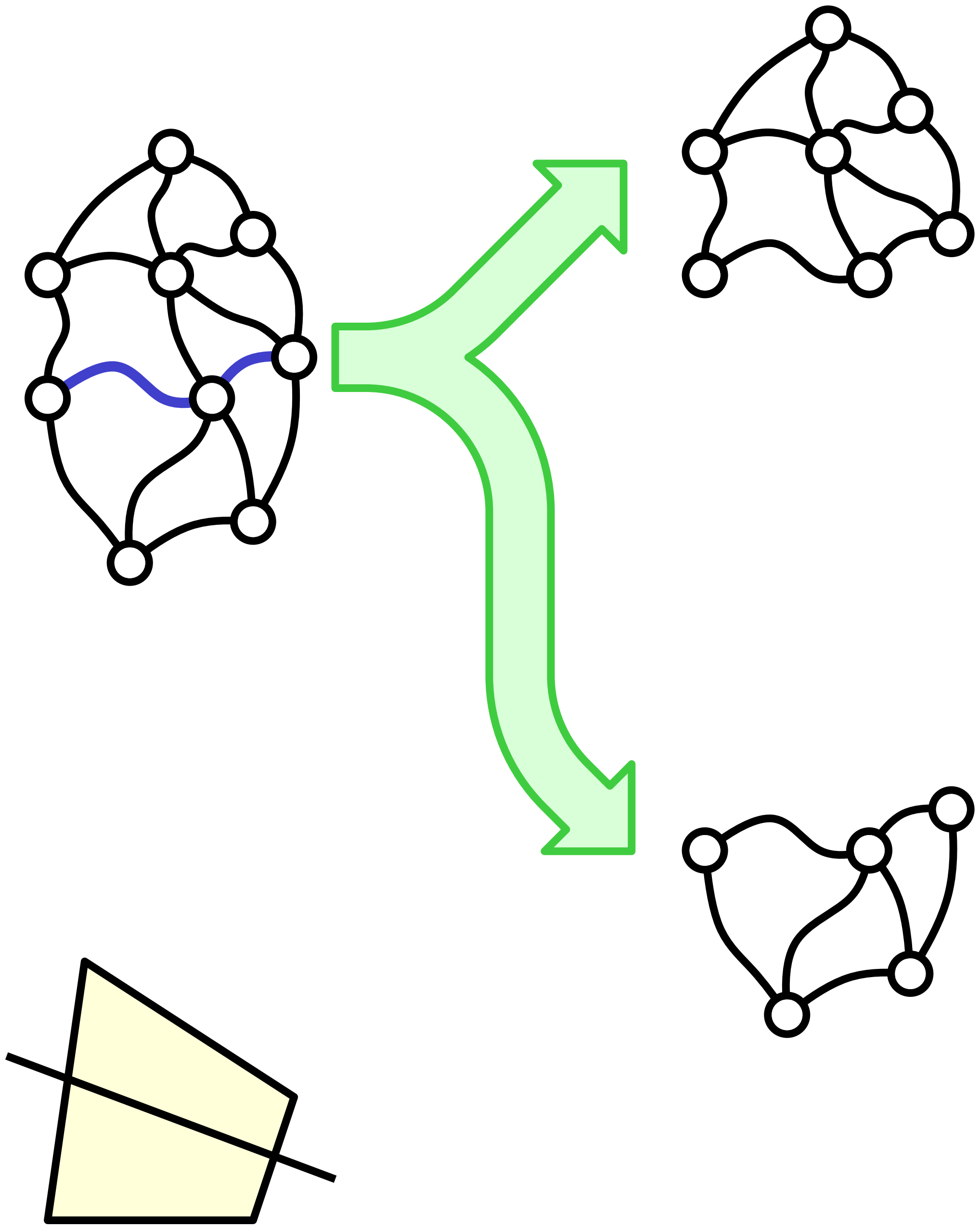

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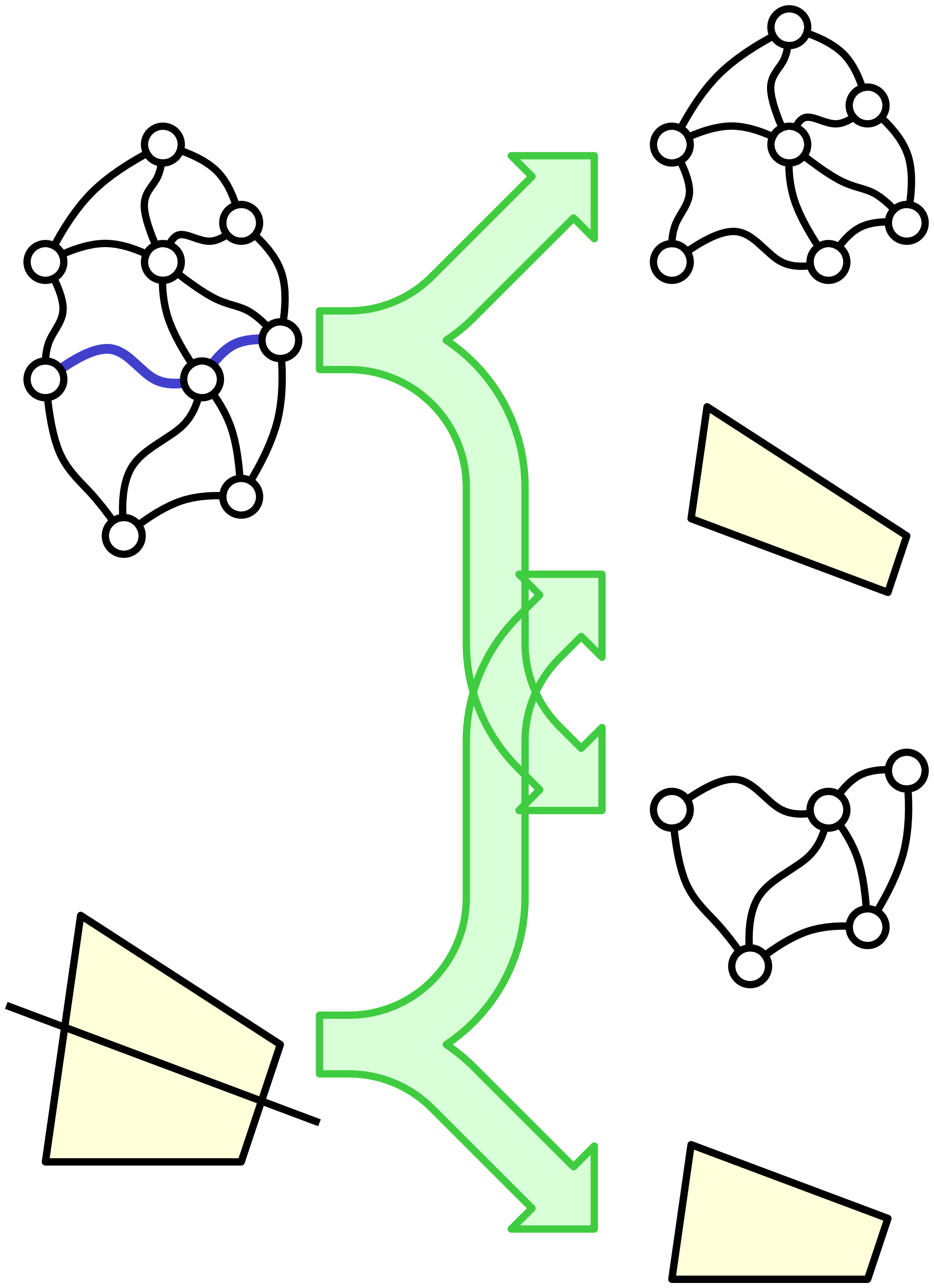

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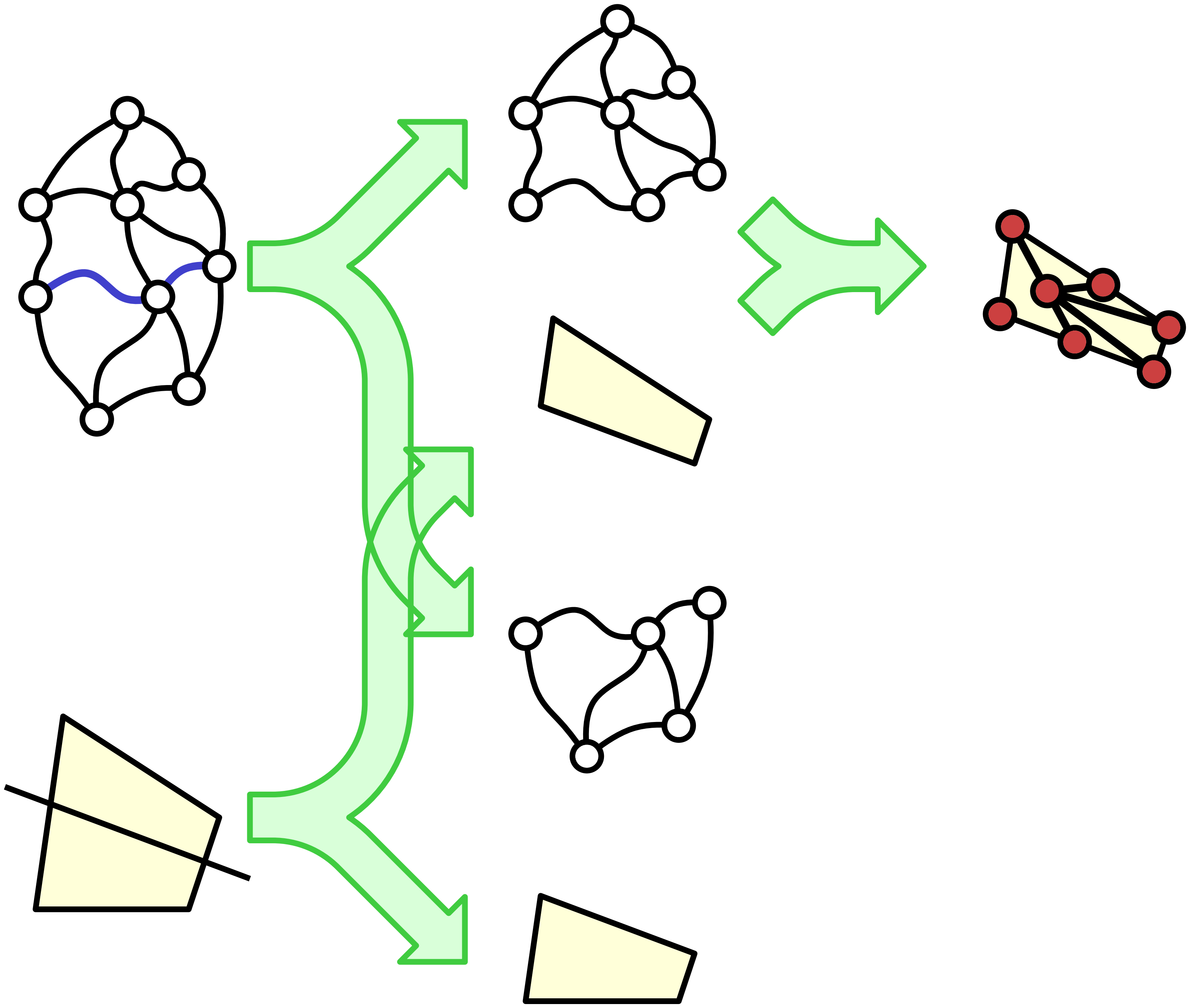

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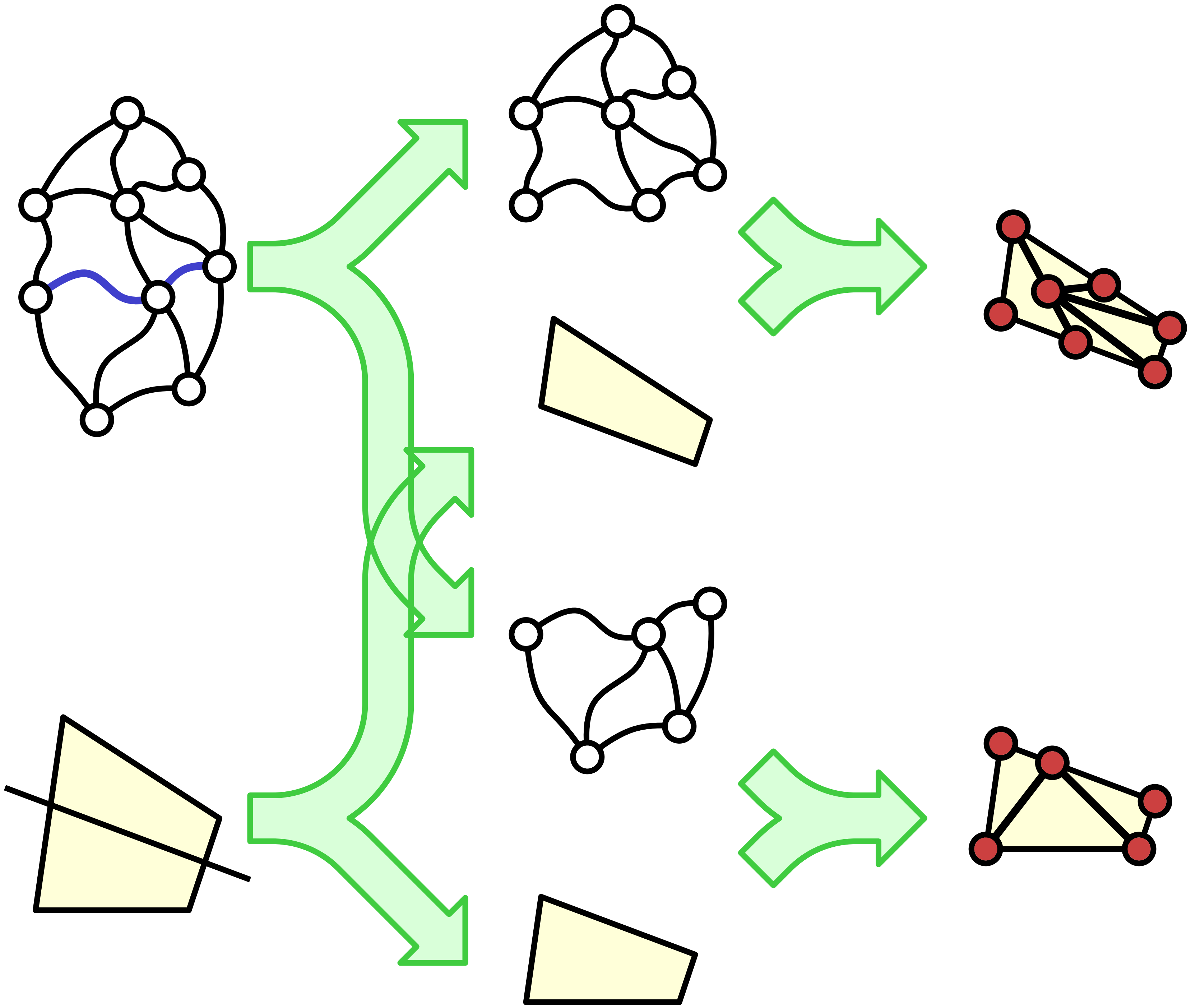

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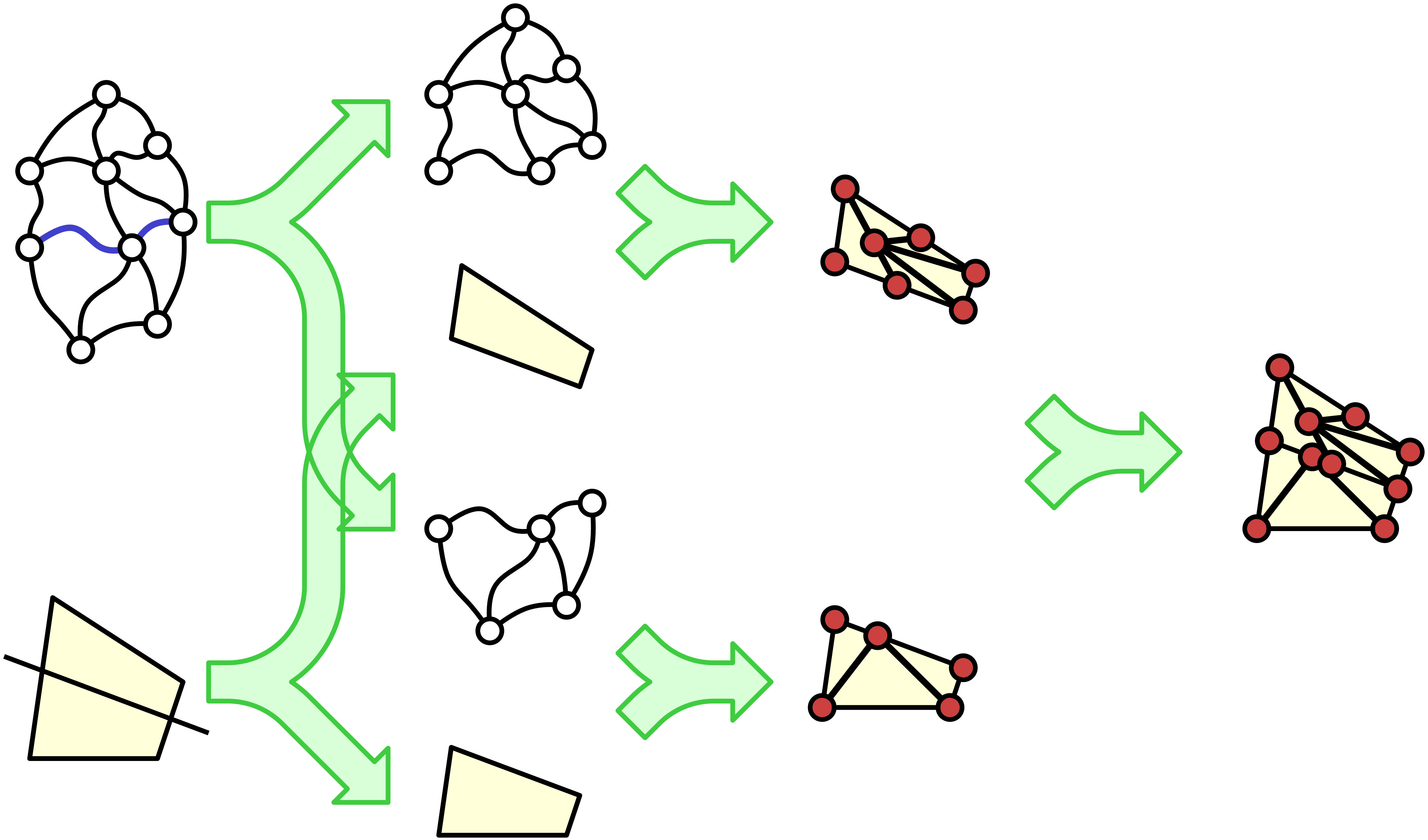

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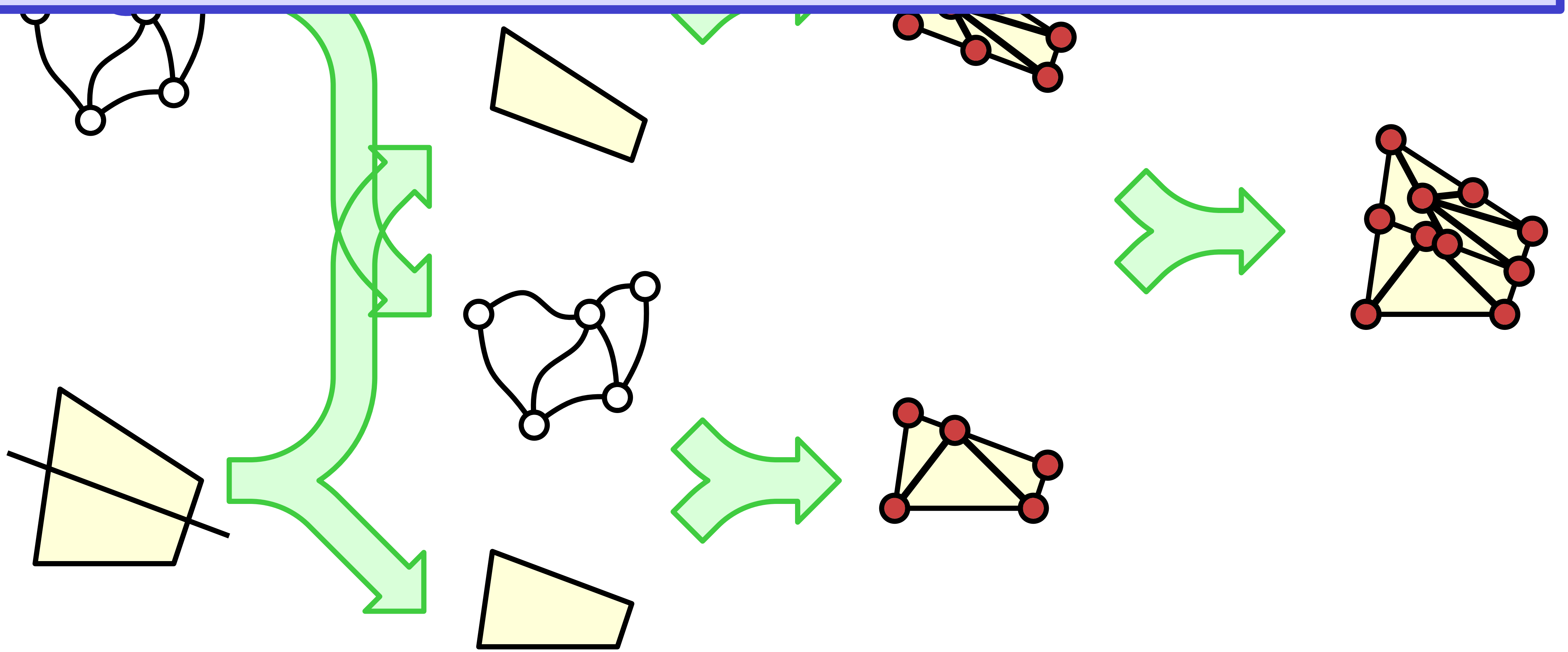
SLIGHT PROBLEM

The vertices along the cut need not be drawn in the same place by the subproblems.



IDEA

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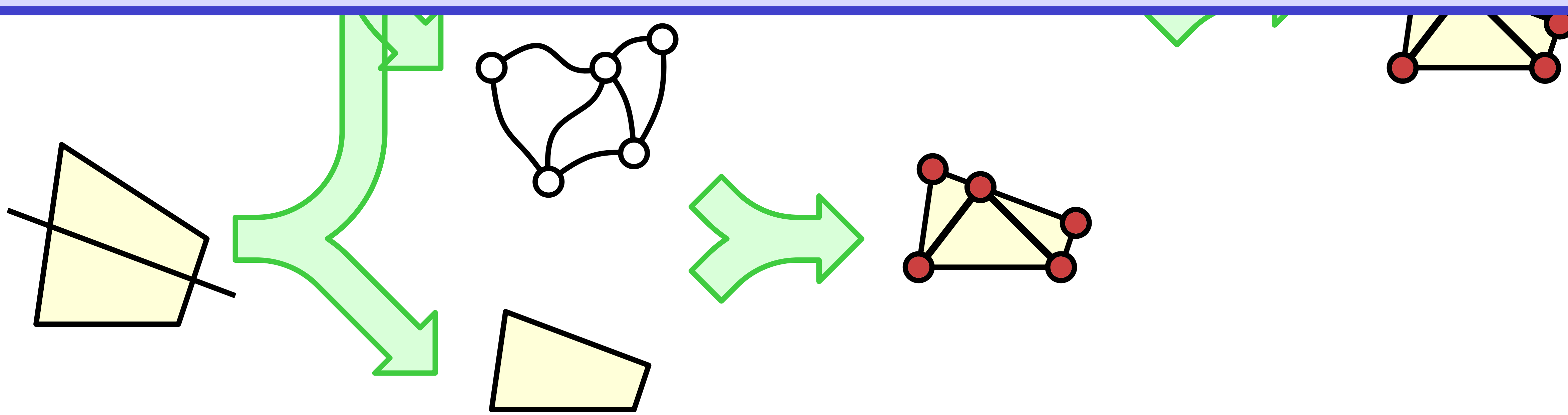
QUESTION

What ever shall we do?



LEMMA *(combinatorial split lemma)*

Every embedded triangulated graph can be split through any pair of edges by a simple connected strip of faces.

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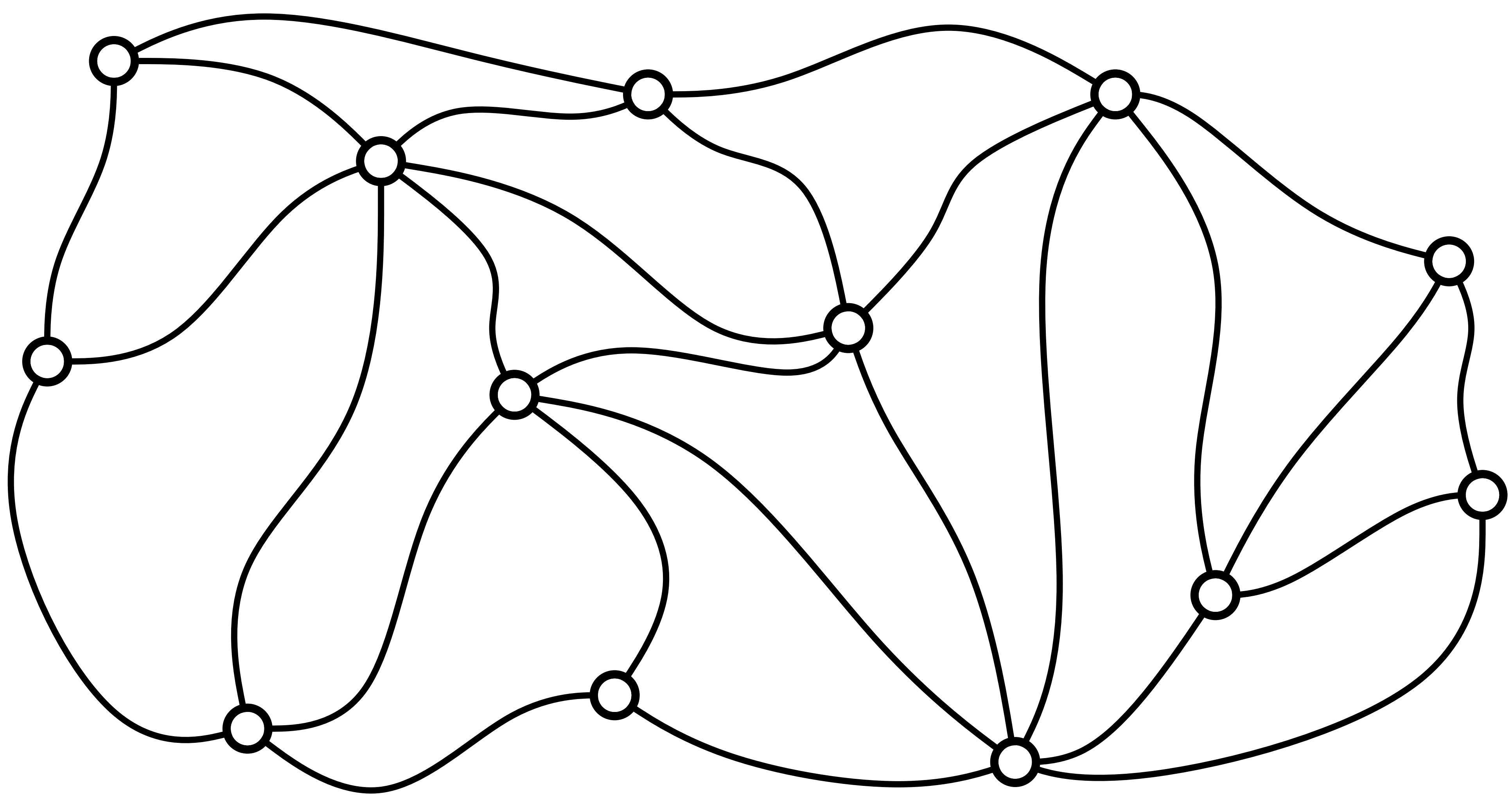

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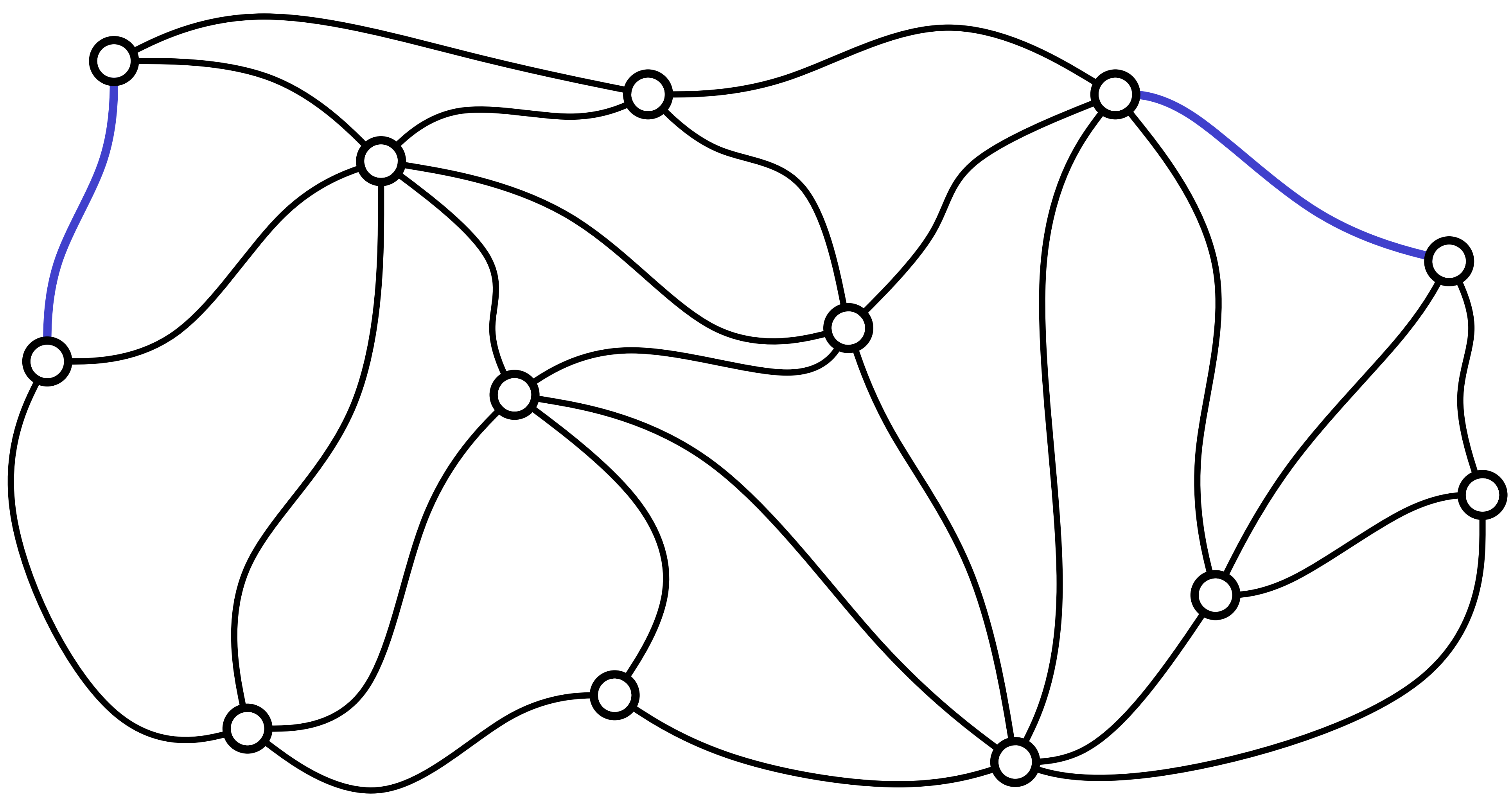

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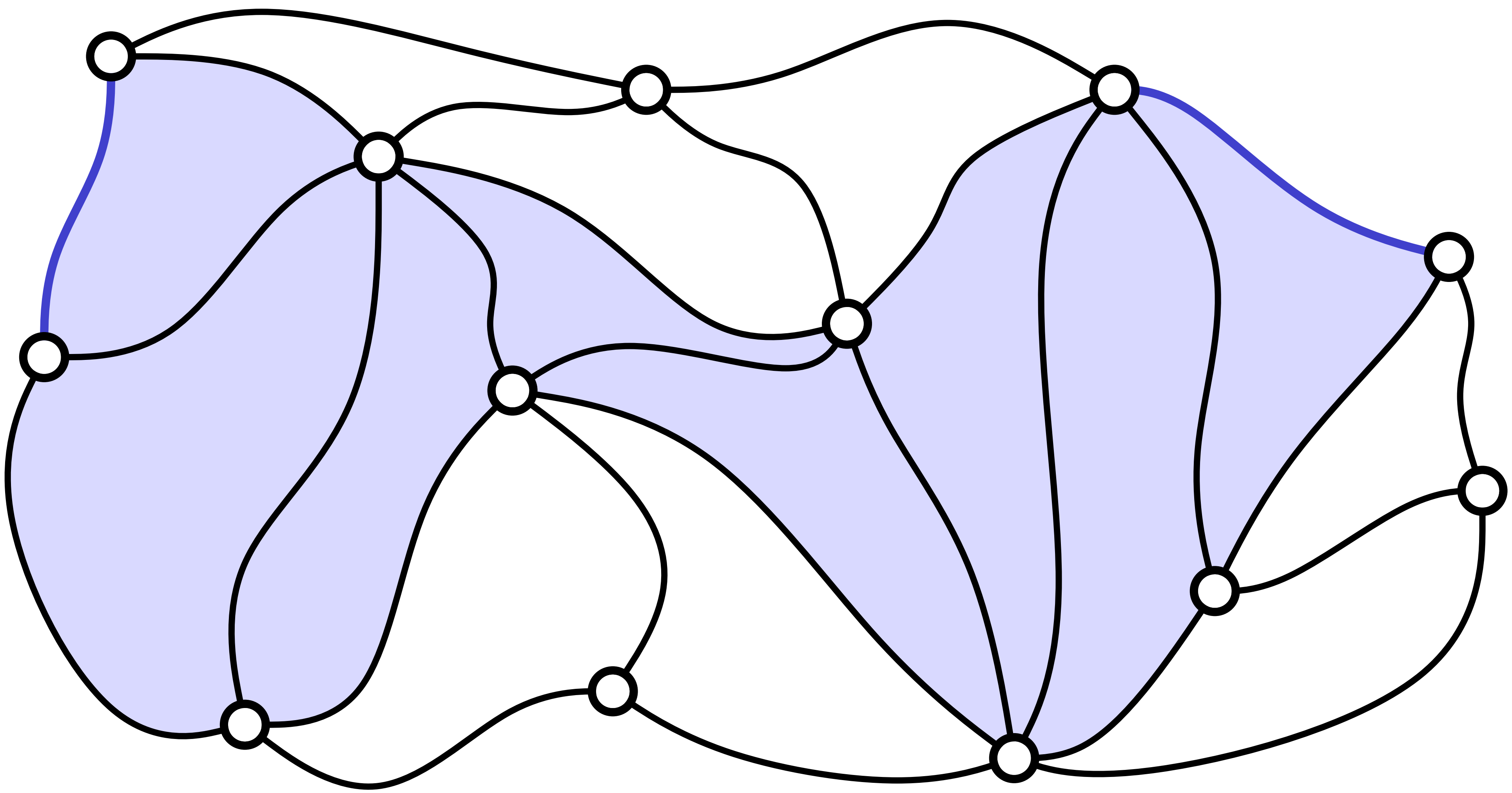

PROPOSITION

Let's try that again.

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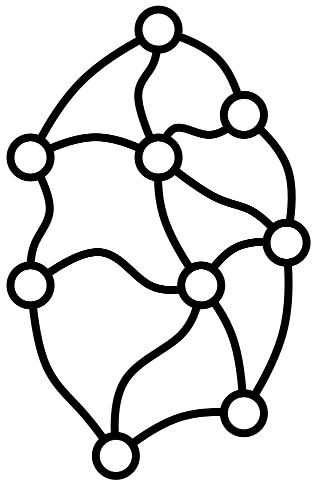

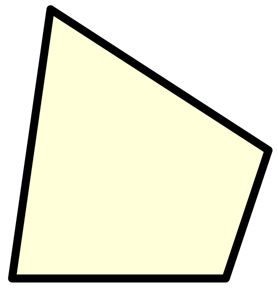

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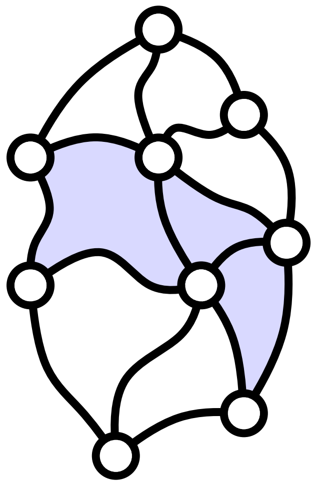

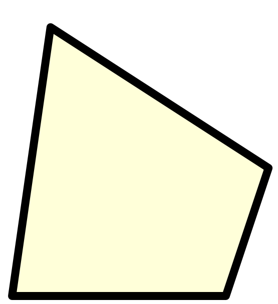

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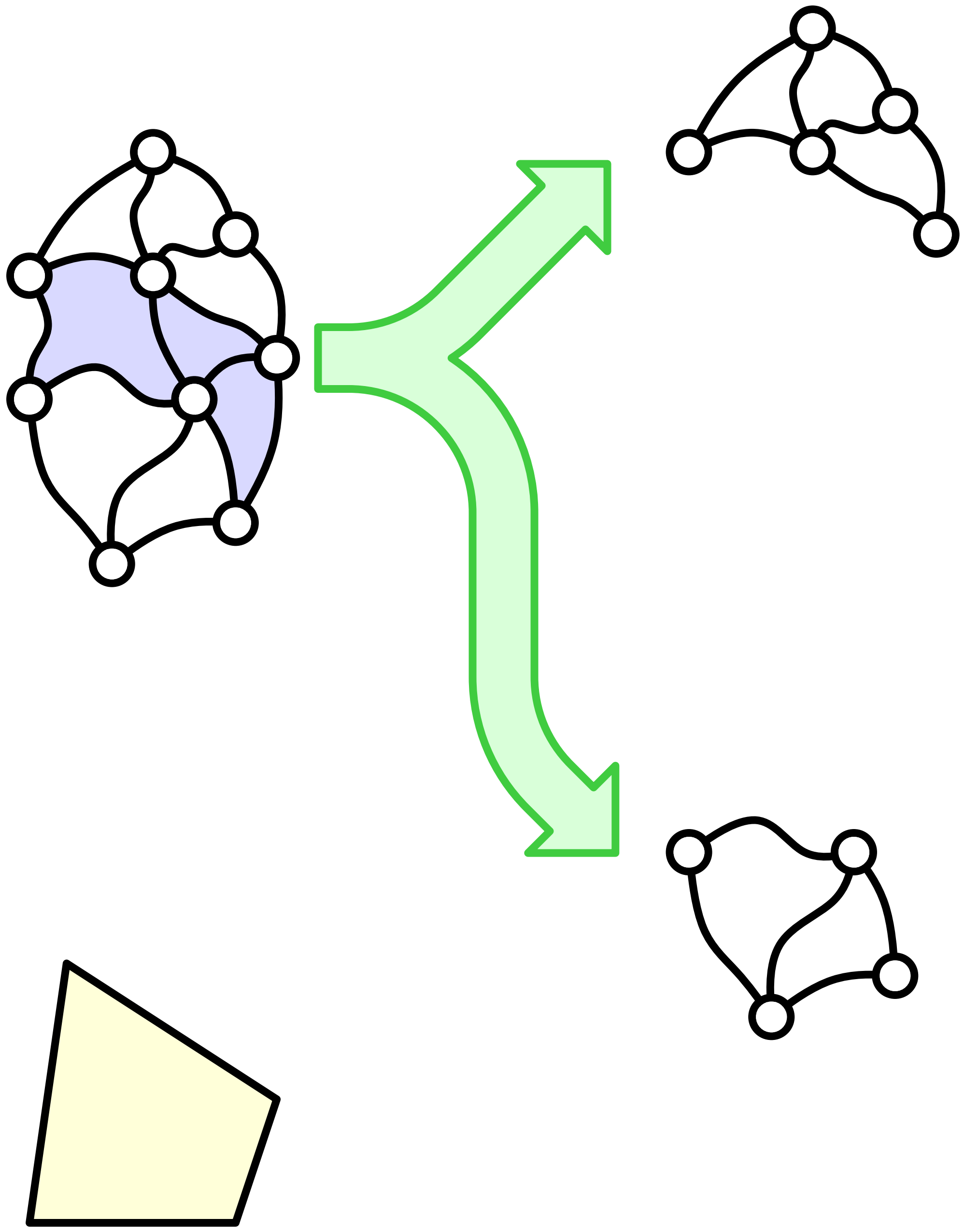

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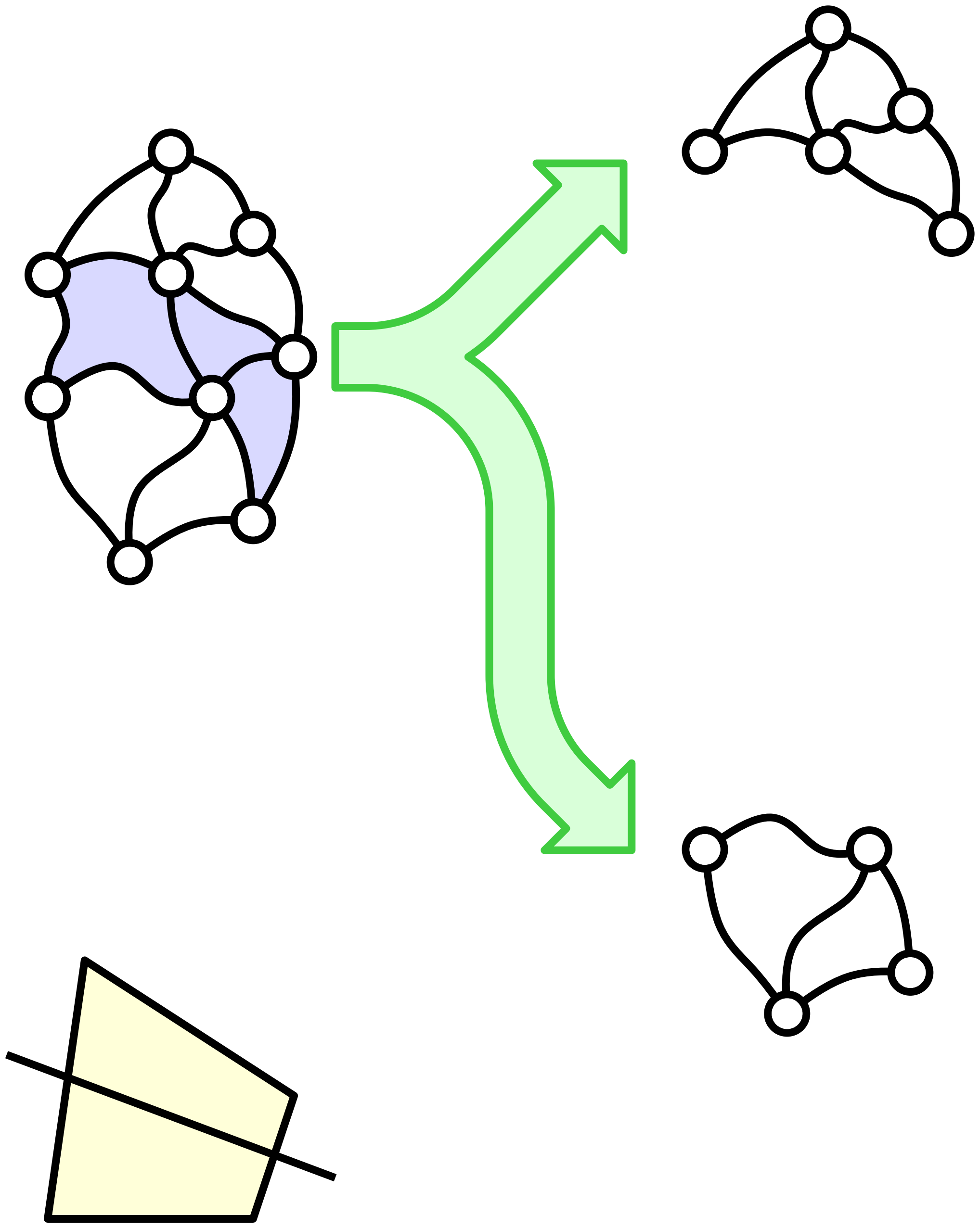

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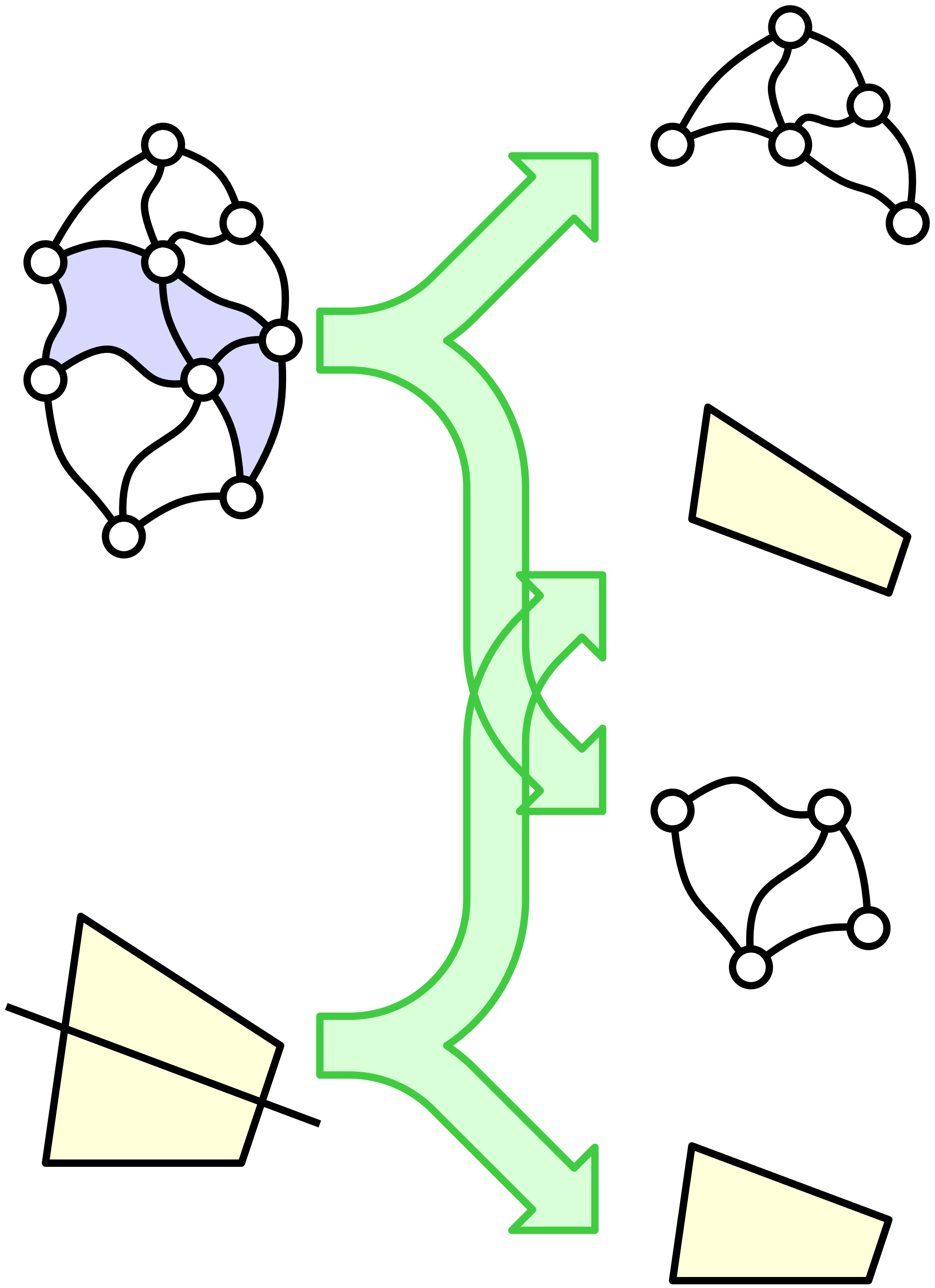

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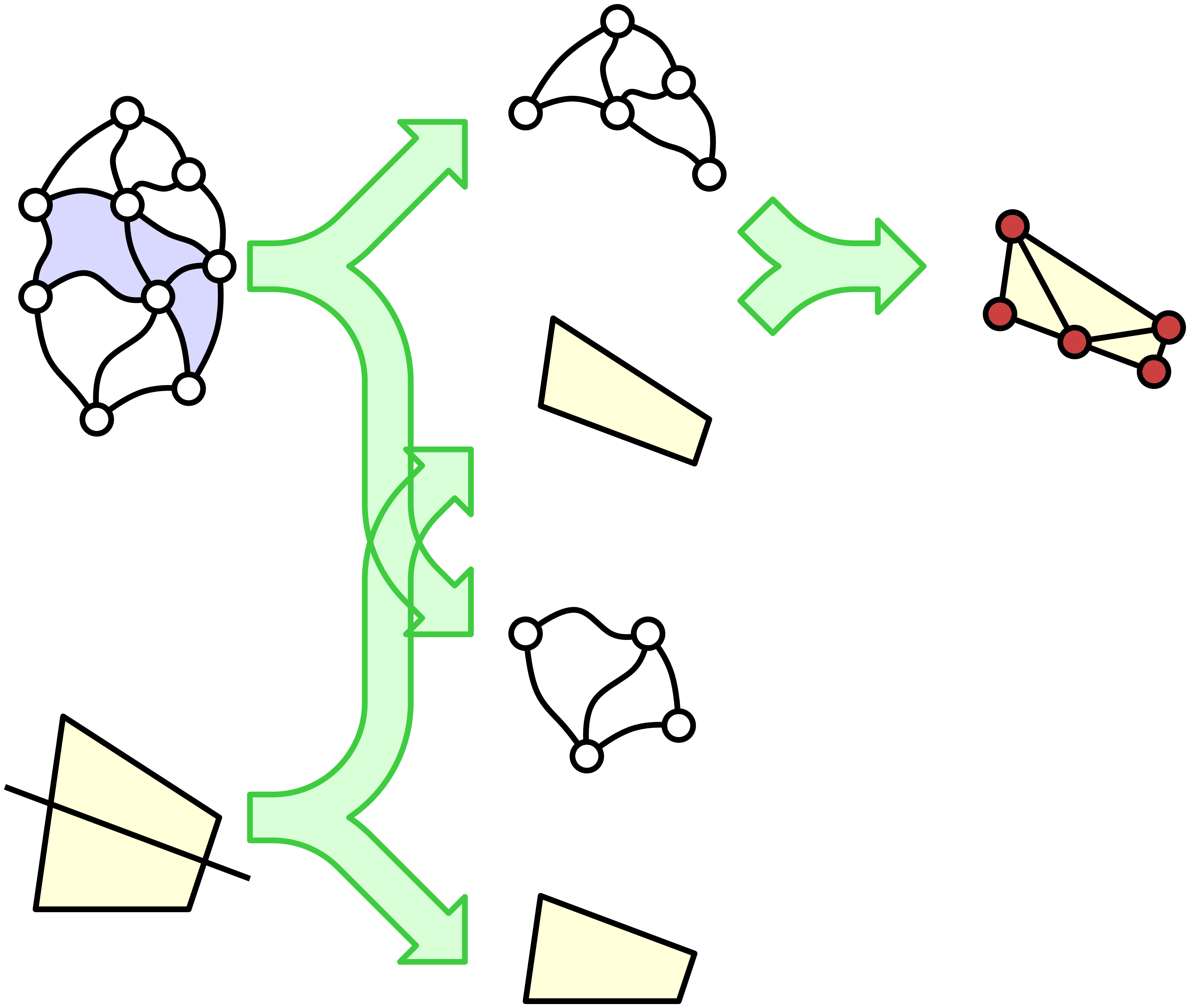

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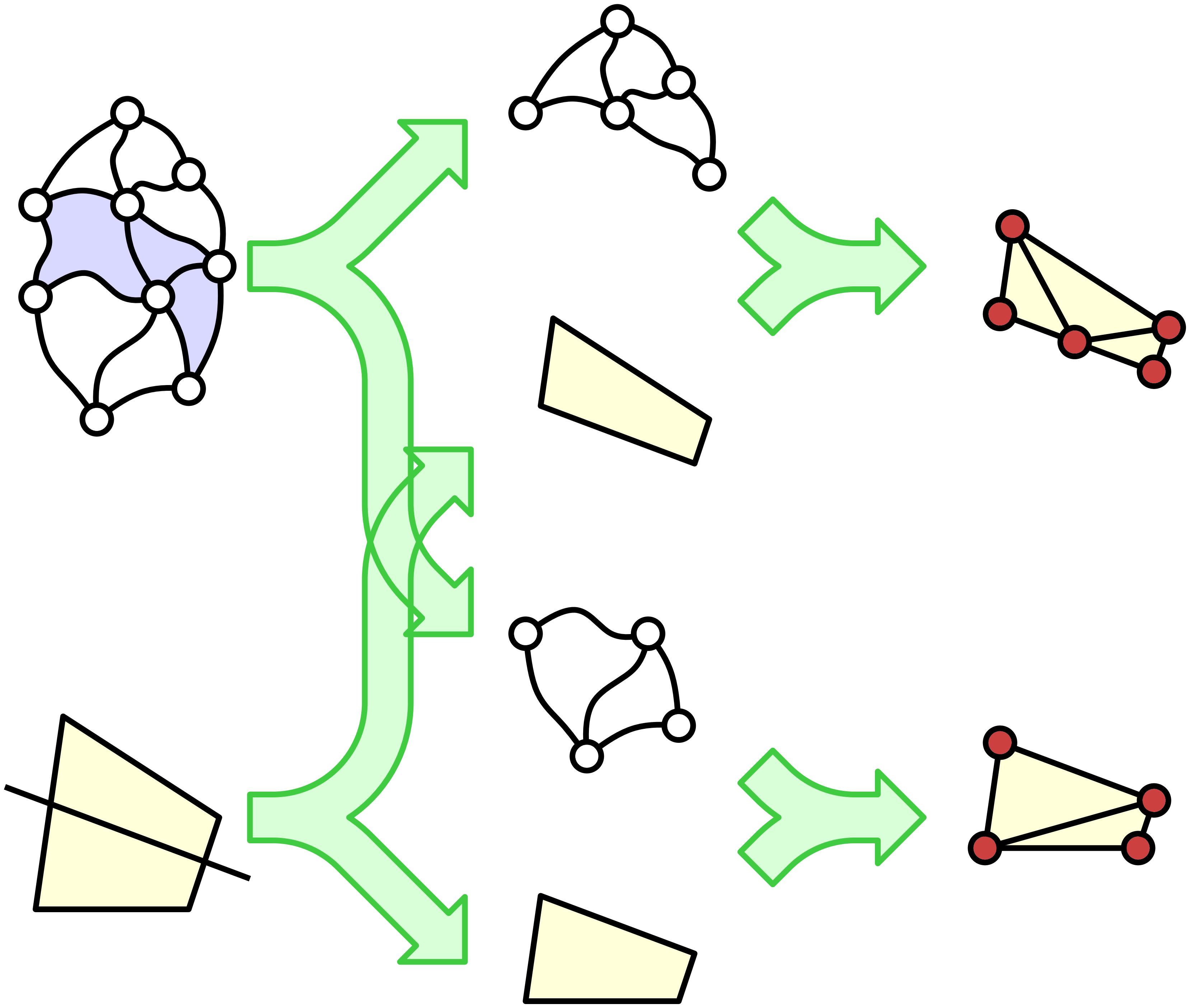

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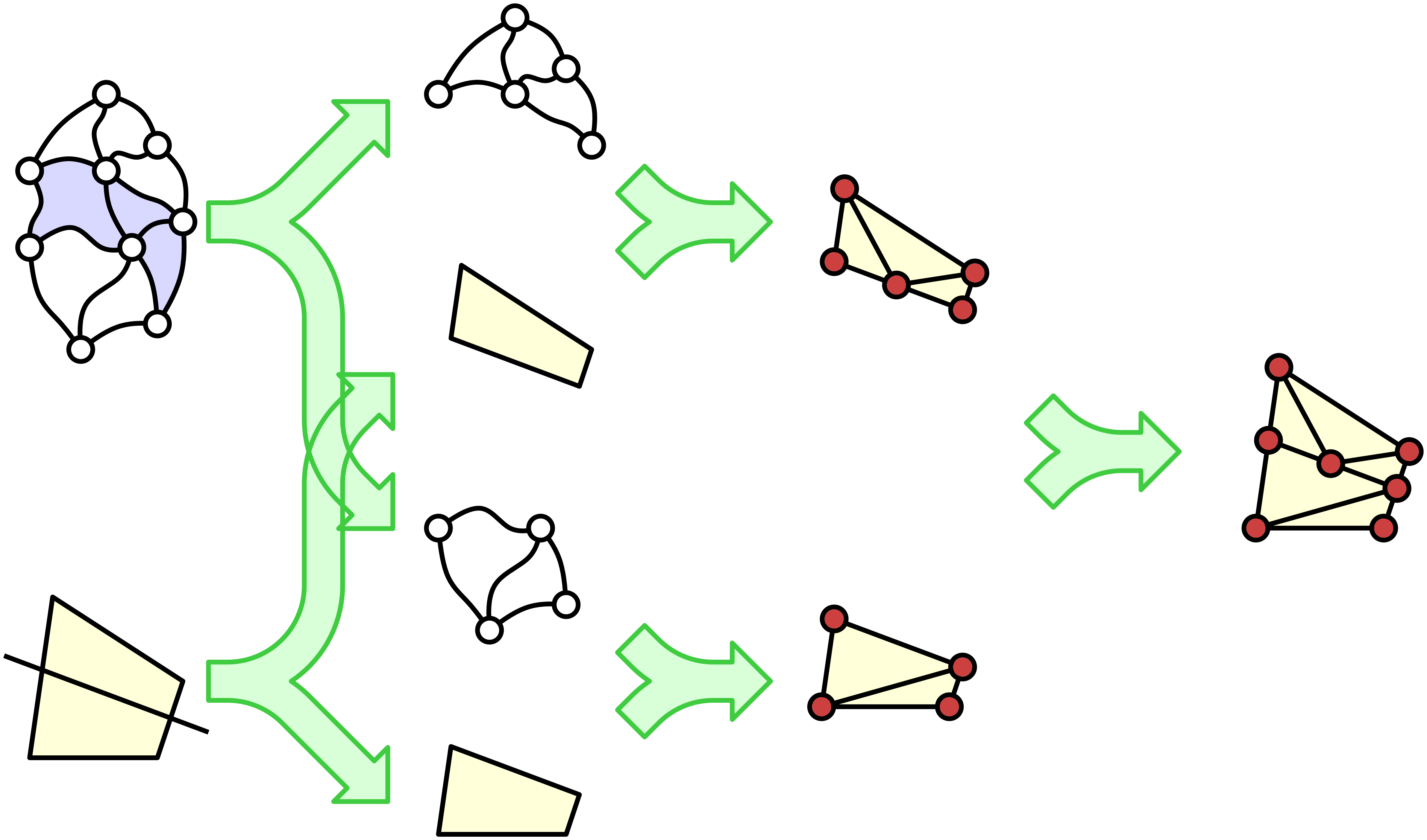
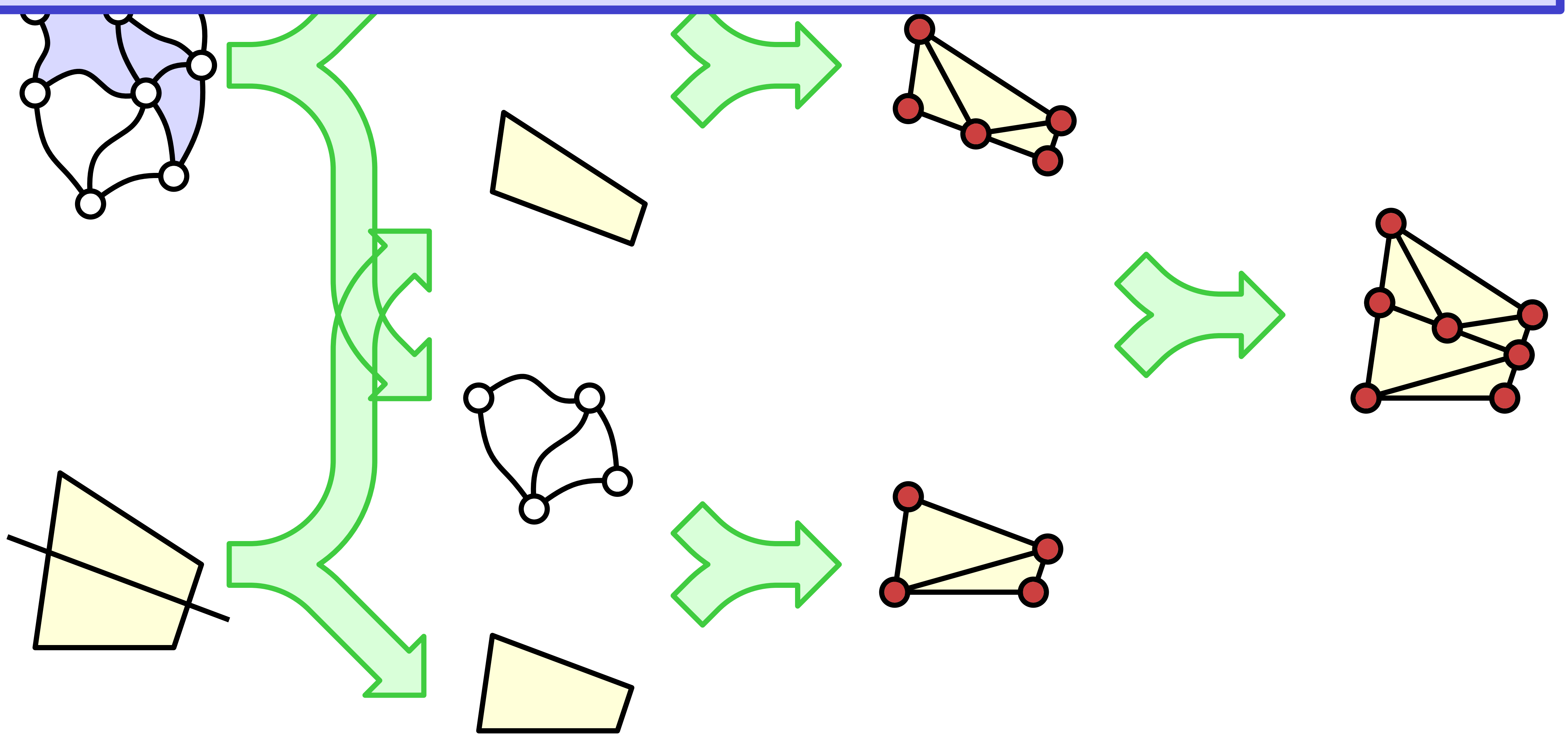


PROPOSITION

Let's try that again.

OBSERVATION

Well, that didn't work.



PROPOSITION

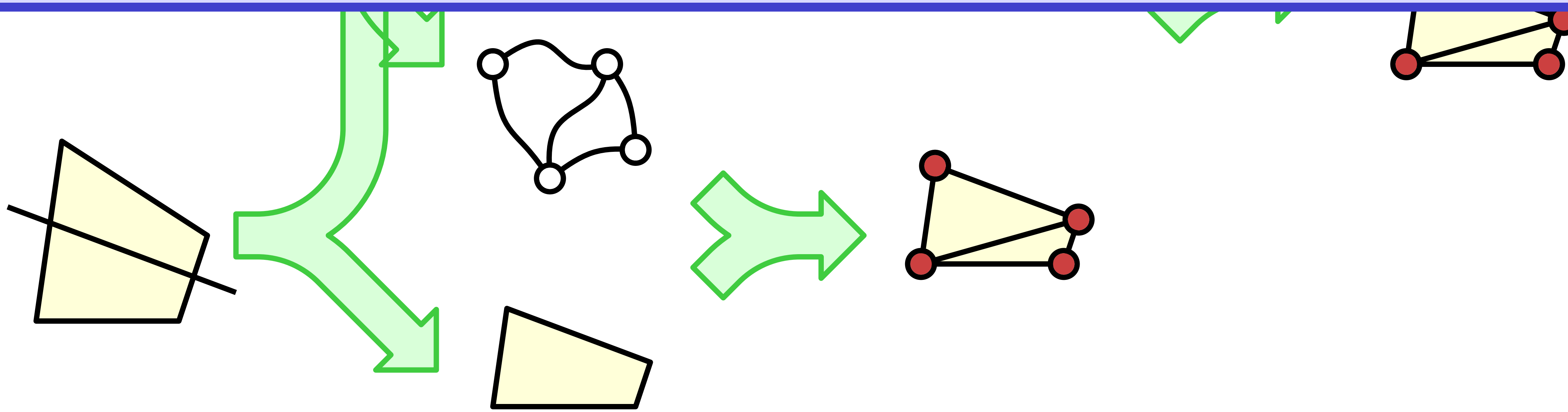
Let's try that again.

OBSERVATION

Well, that didn't work.

JUST A THOUGHT

Maybe it would have worked better if we had left some space for that strip of faces...



PROVERB

Third time lucky.

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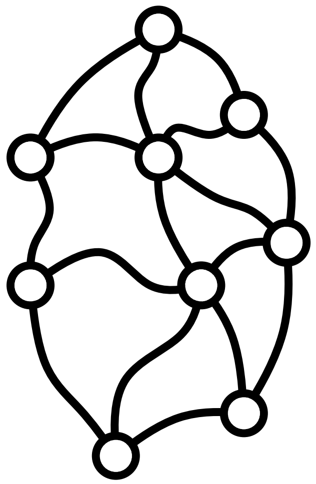

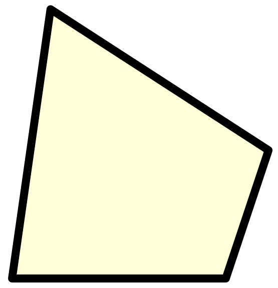

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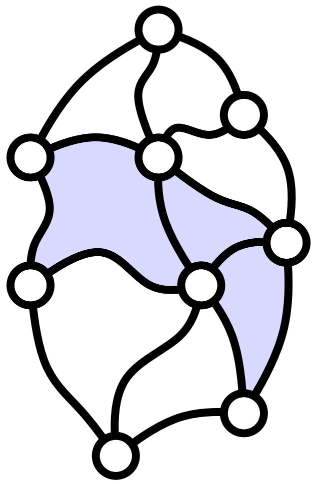

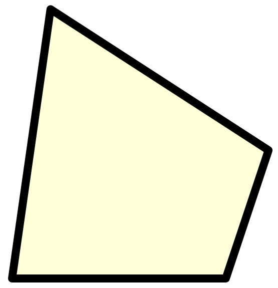

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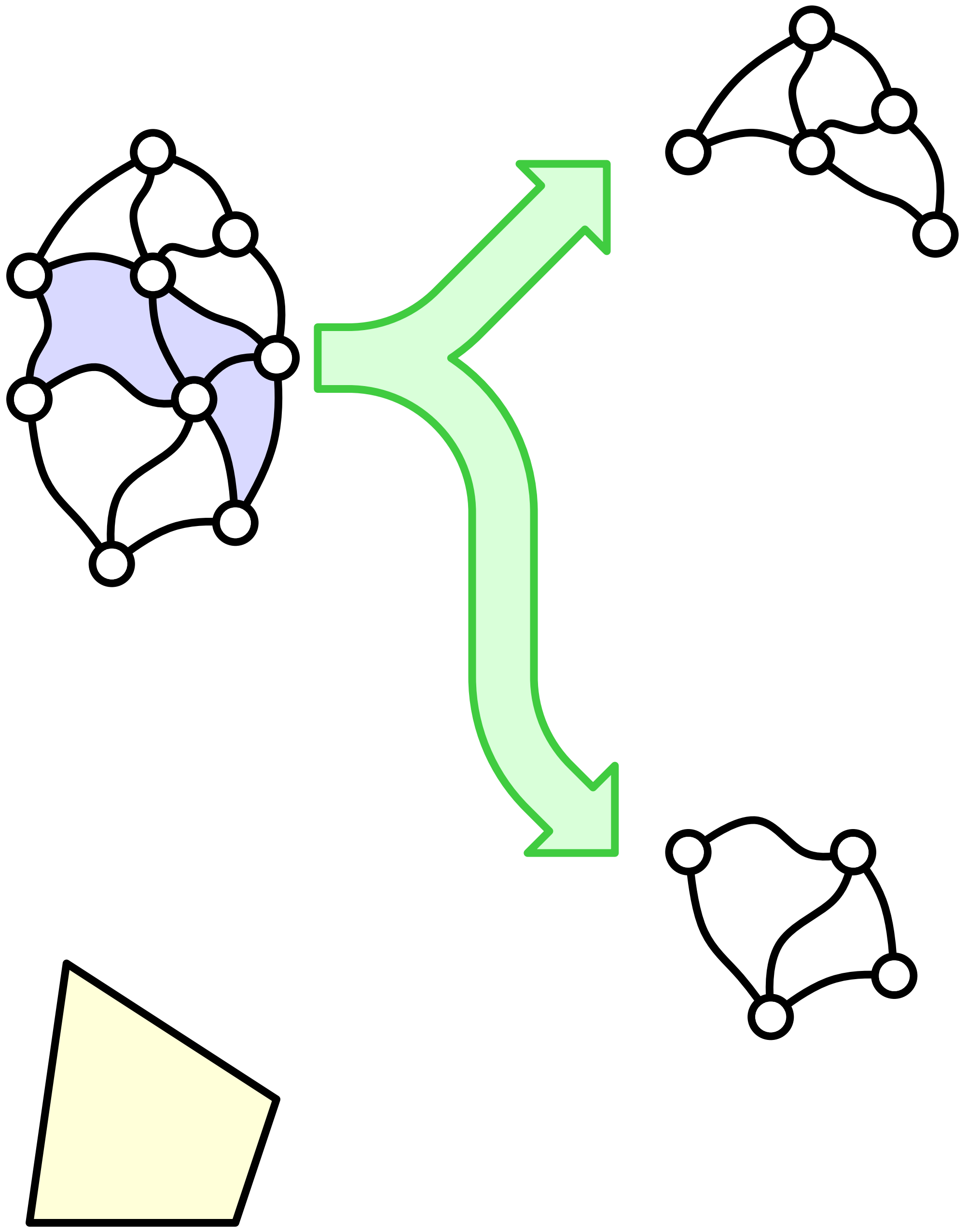

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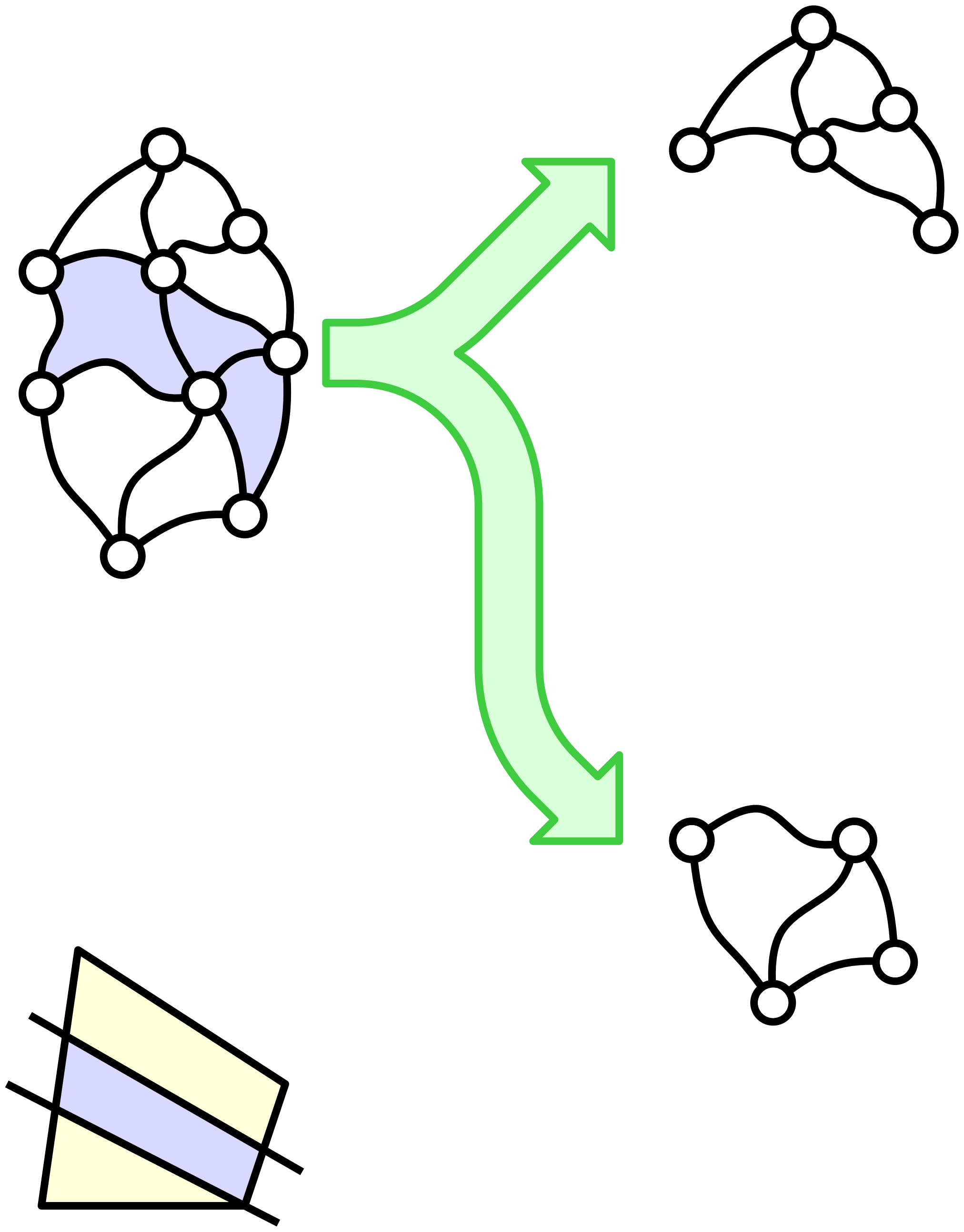

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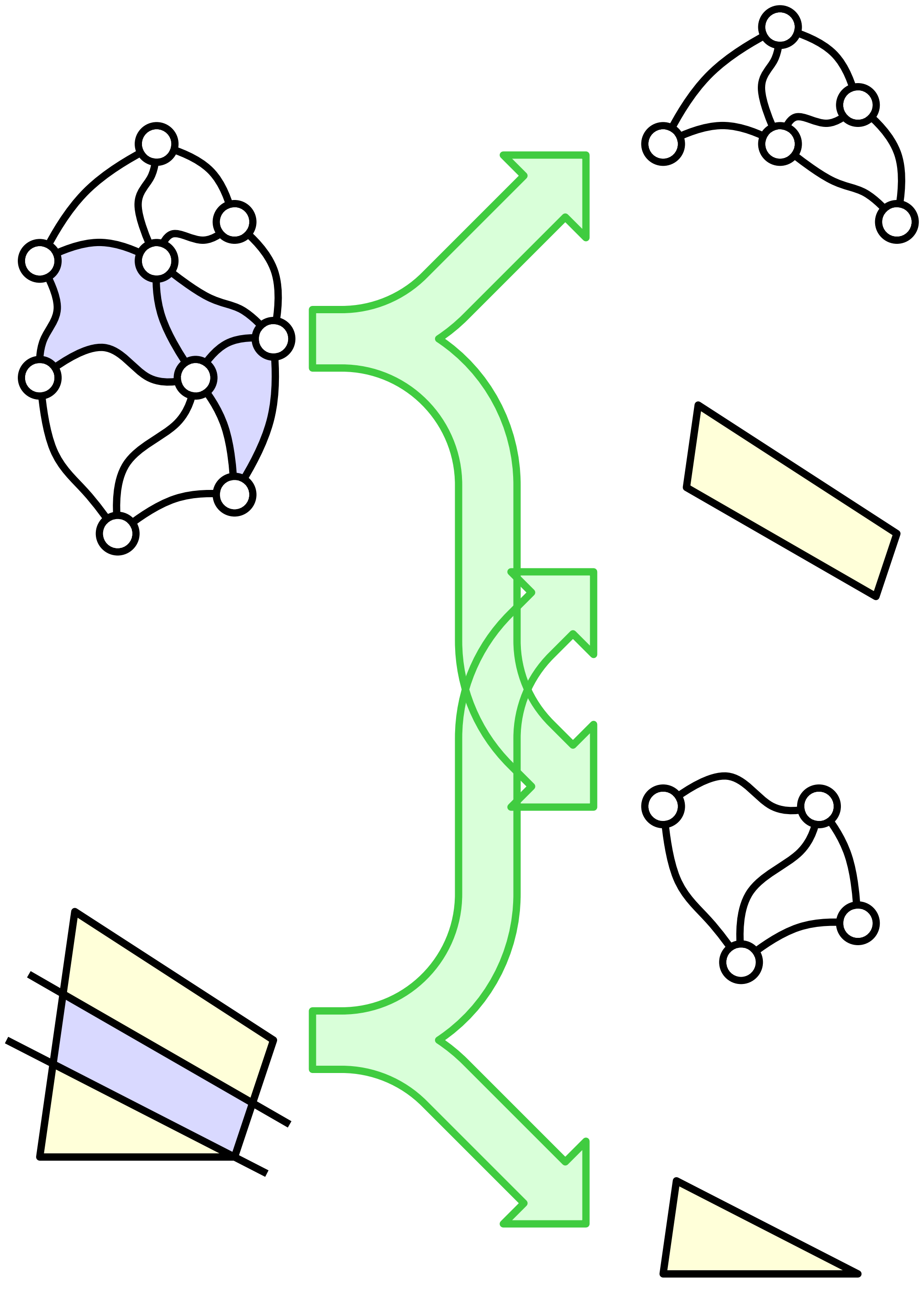

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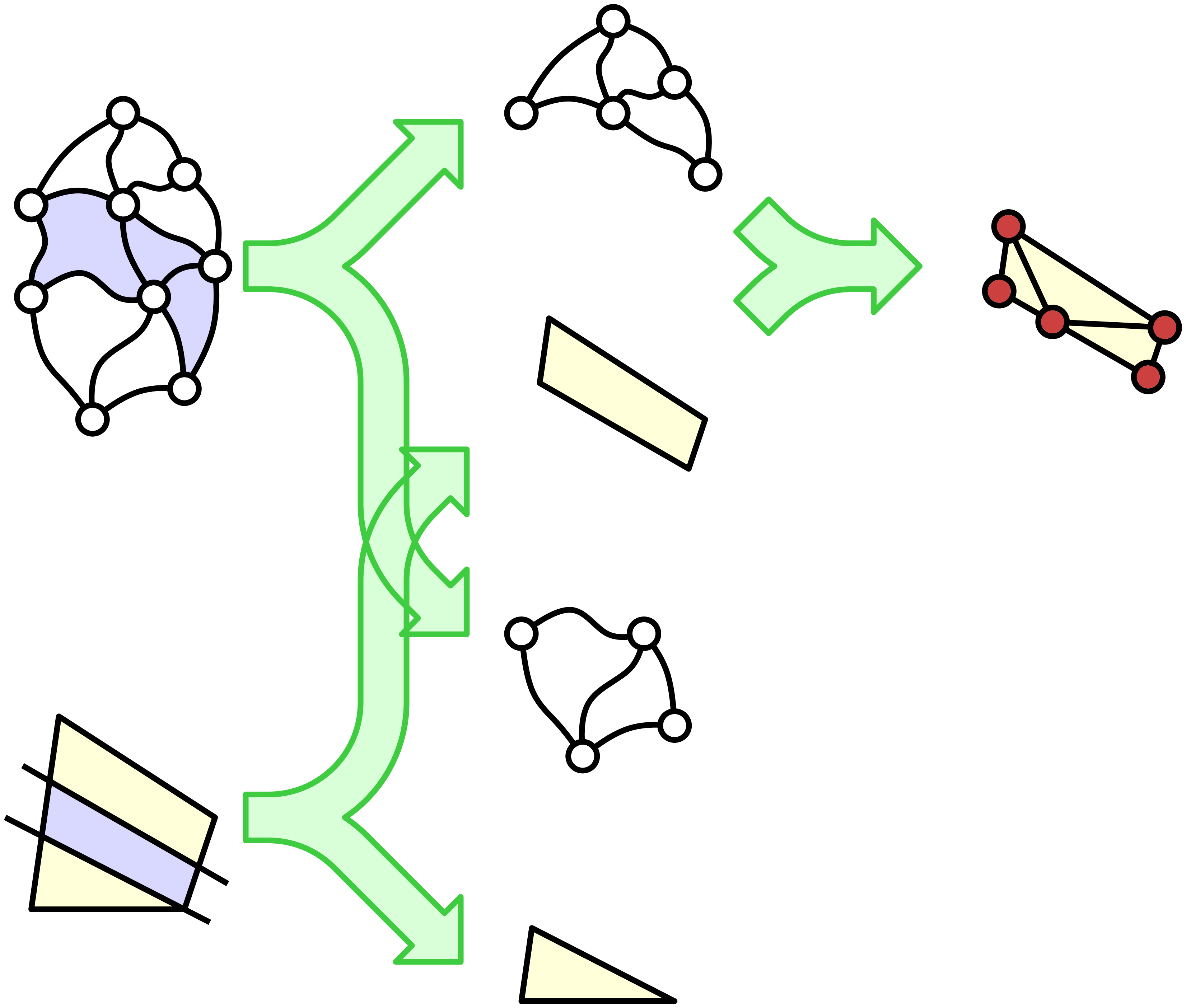

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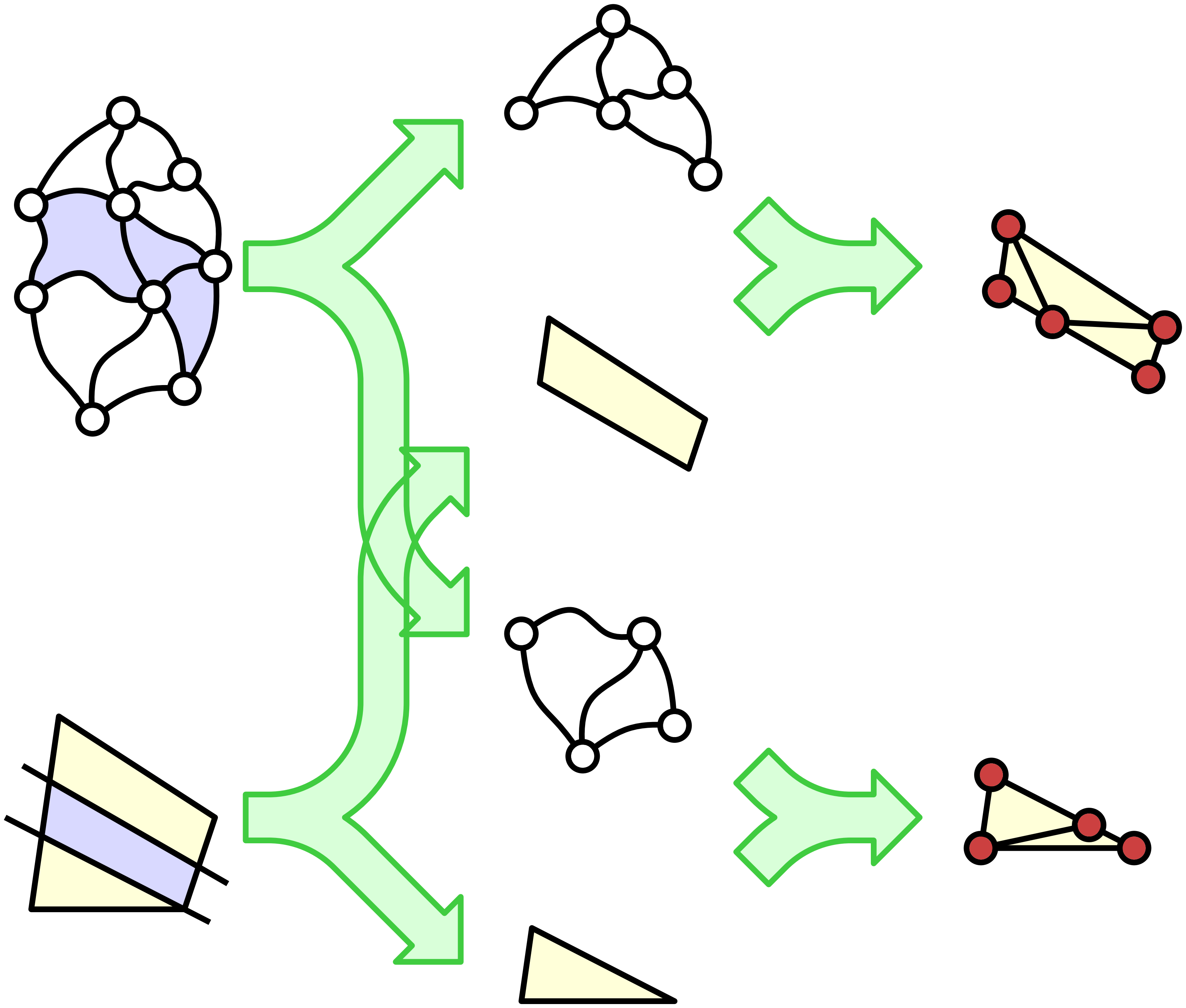

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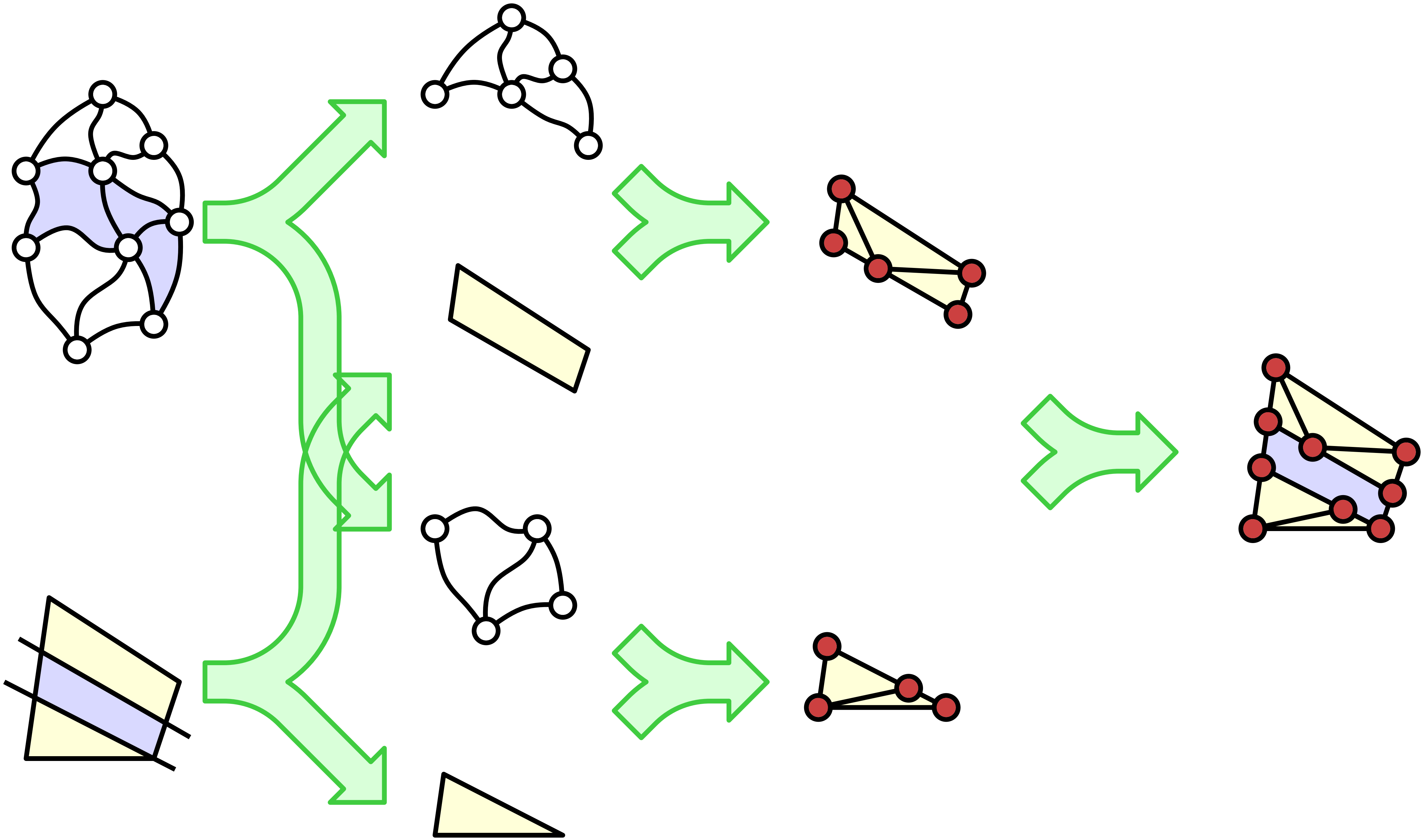

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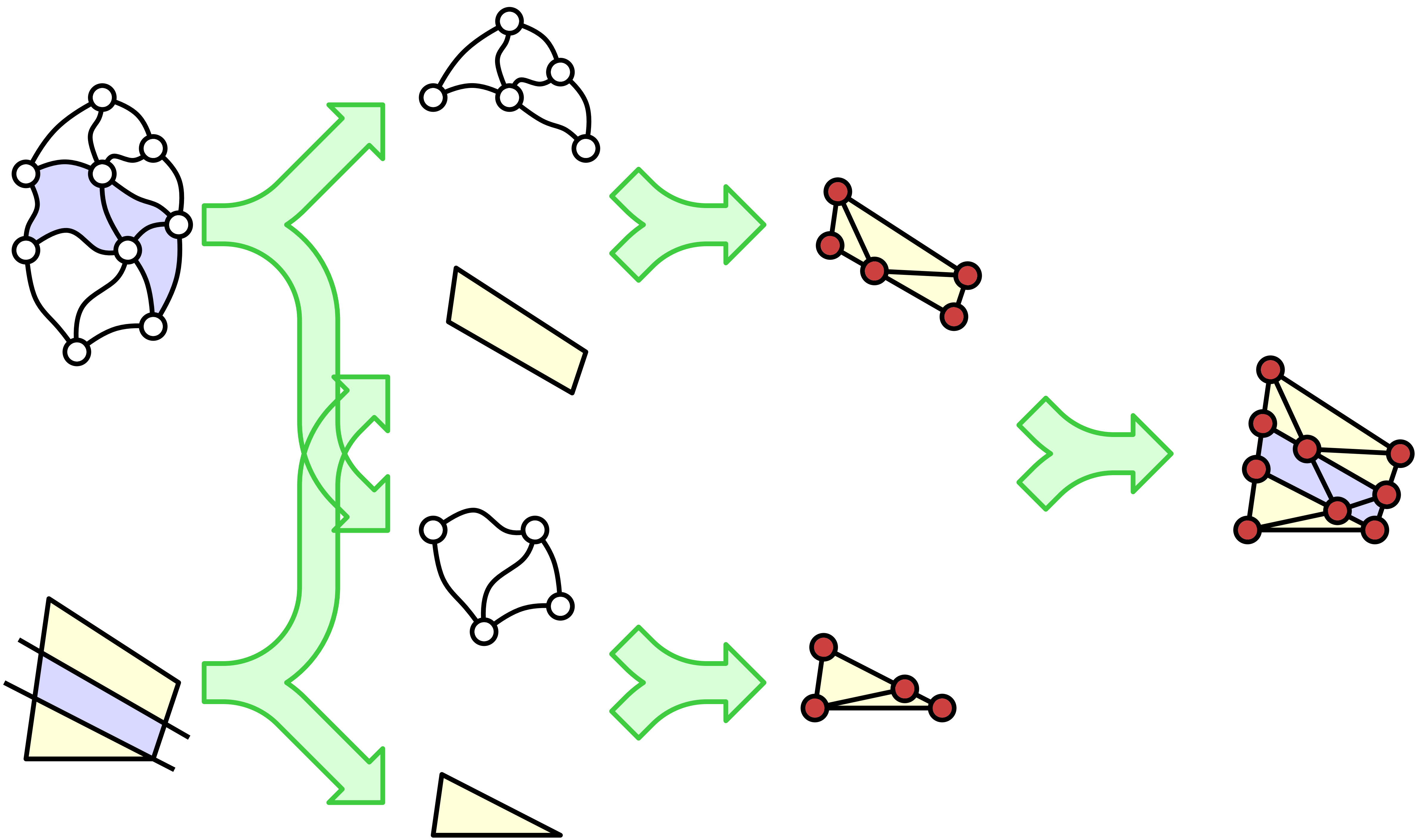

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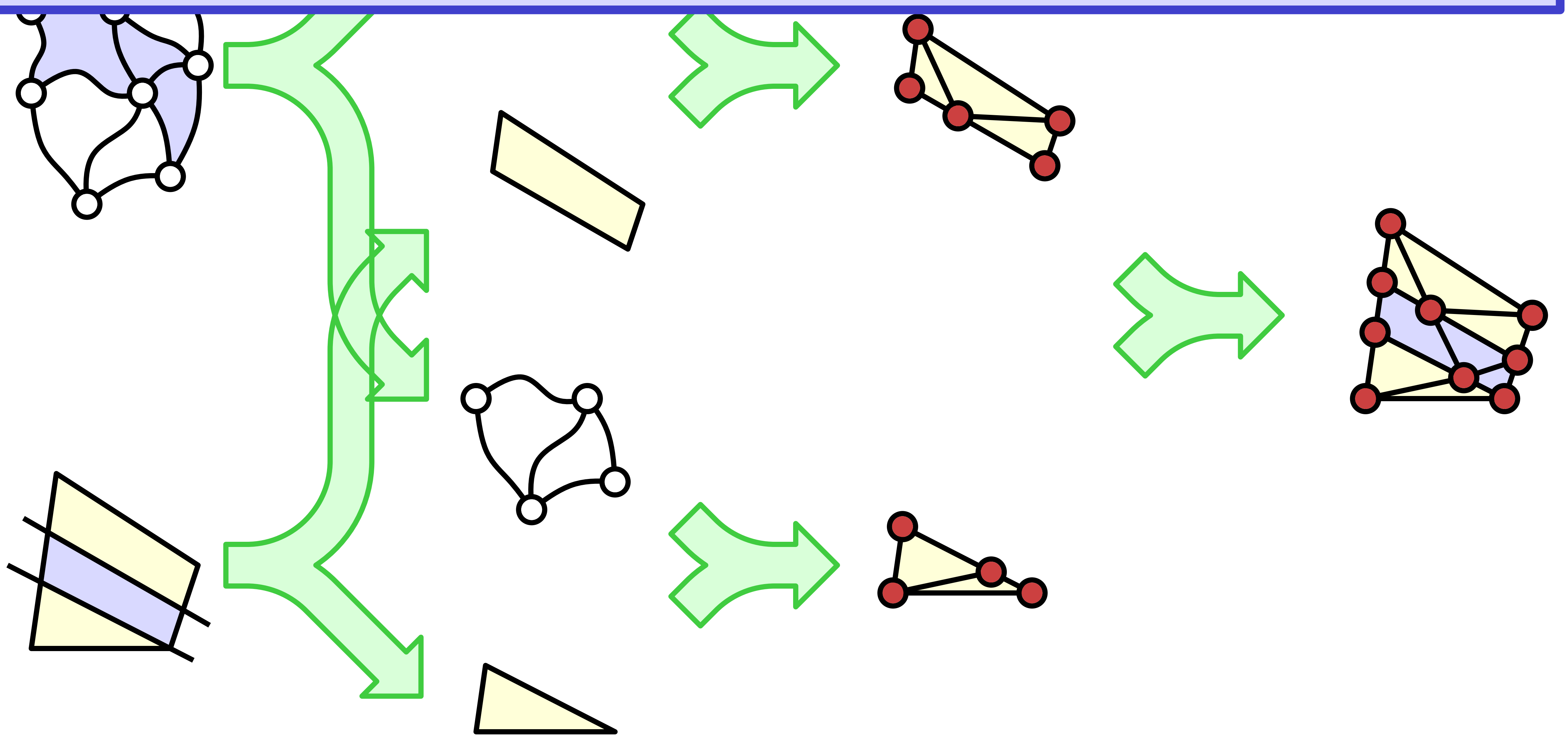
EXCLAMATION

Success!



PROVERB

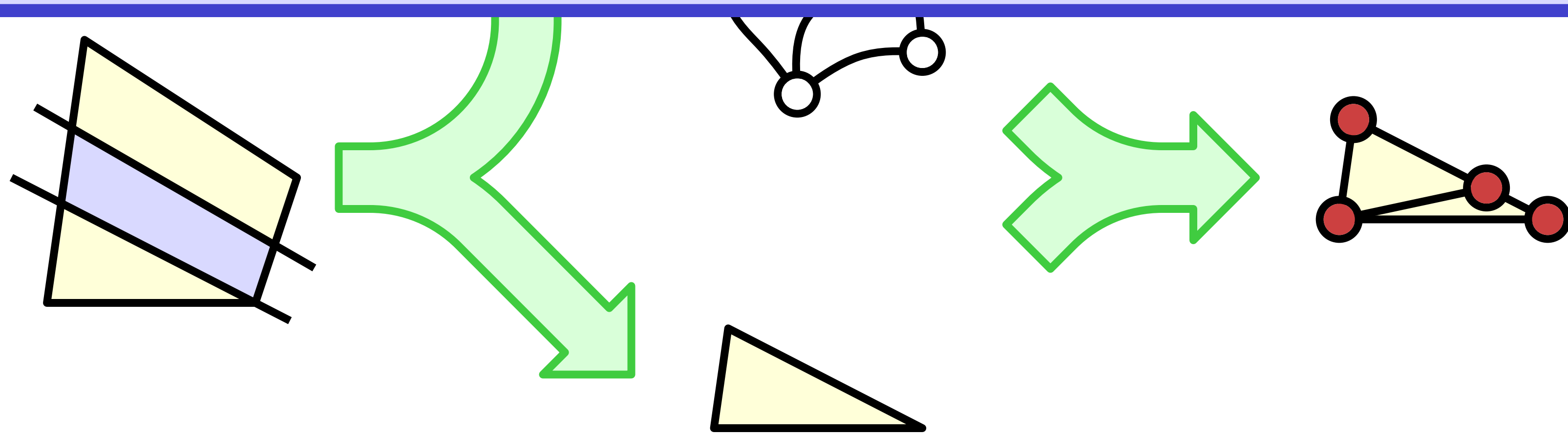
Third time lucky.

EXCLAMATION

Success!

DISCLAIMER

Several details have been swept under the carpet here. The geometric split lemma was only applied partially on one side. It is possible that the combinatorial split lemma generates more than two subproblems. Also, we may need to include extra faces inside the strip to avoid chords on the boundary. Oh, and I failed to mention that the resolution of a polygon is not exactly what we need either – in the paper you will find the definition of the *potential* resolution of a polygon. In short, don't try this at home. At least not until you read the paper. No people or animals have been hurt in the production of this presentation.



TENACIOUS INTERROGATION

Ok, so then how does it *really* work?

ALGORITHM

Just follow these easy steps:

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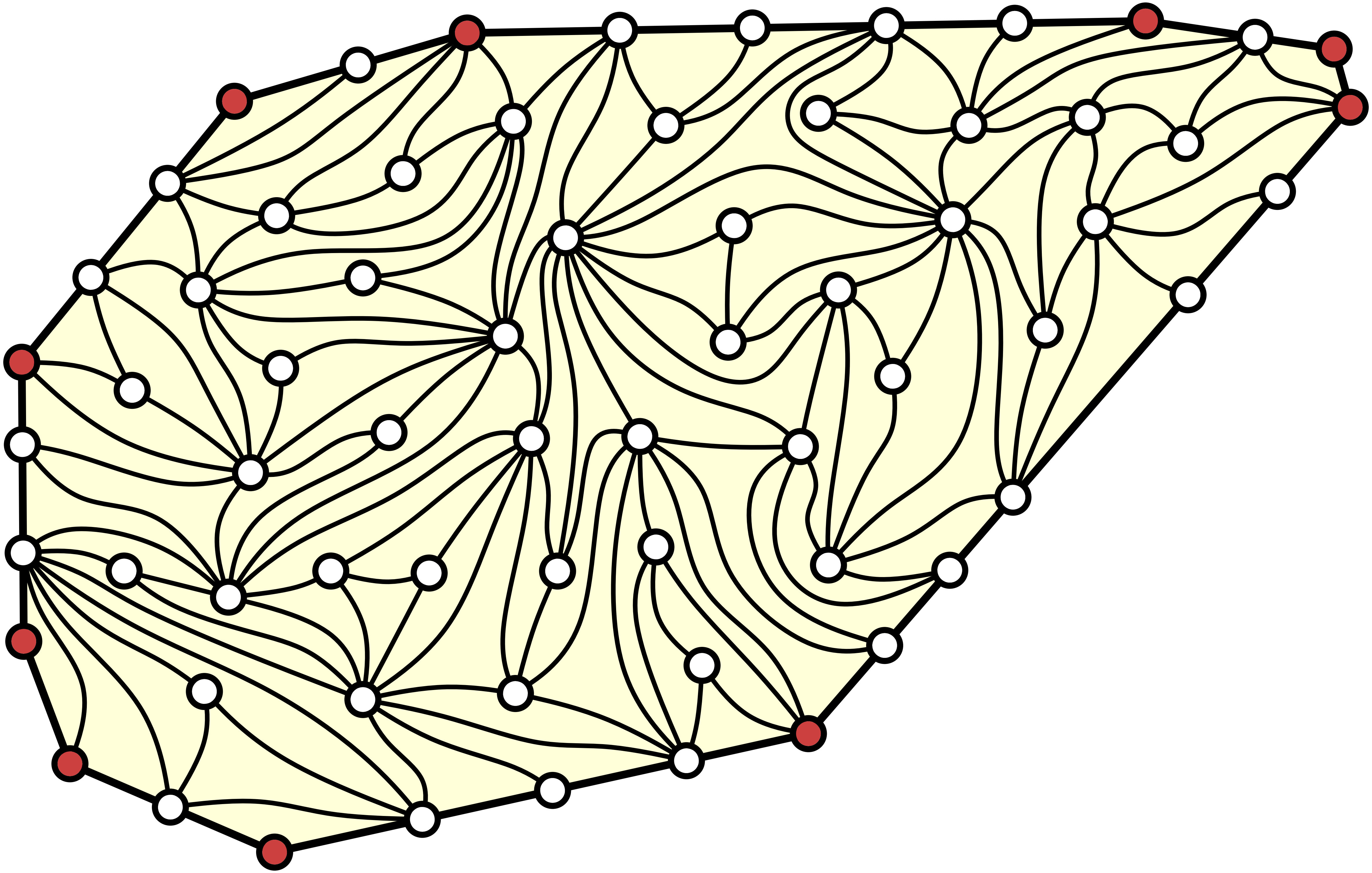

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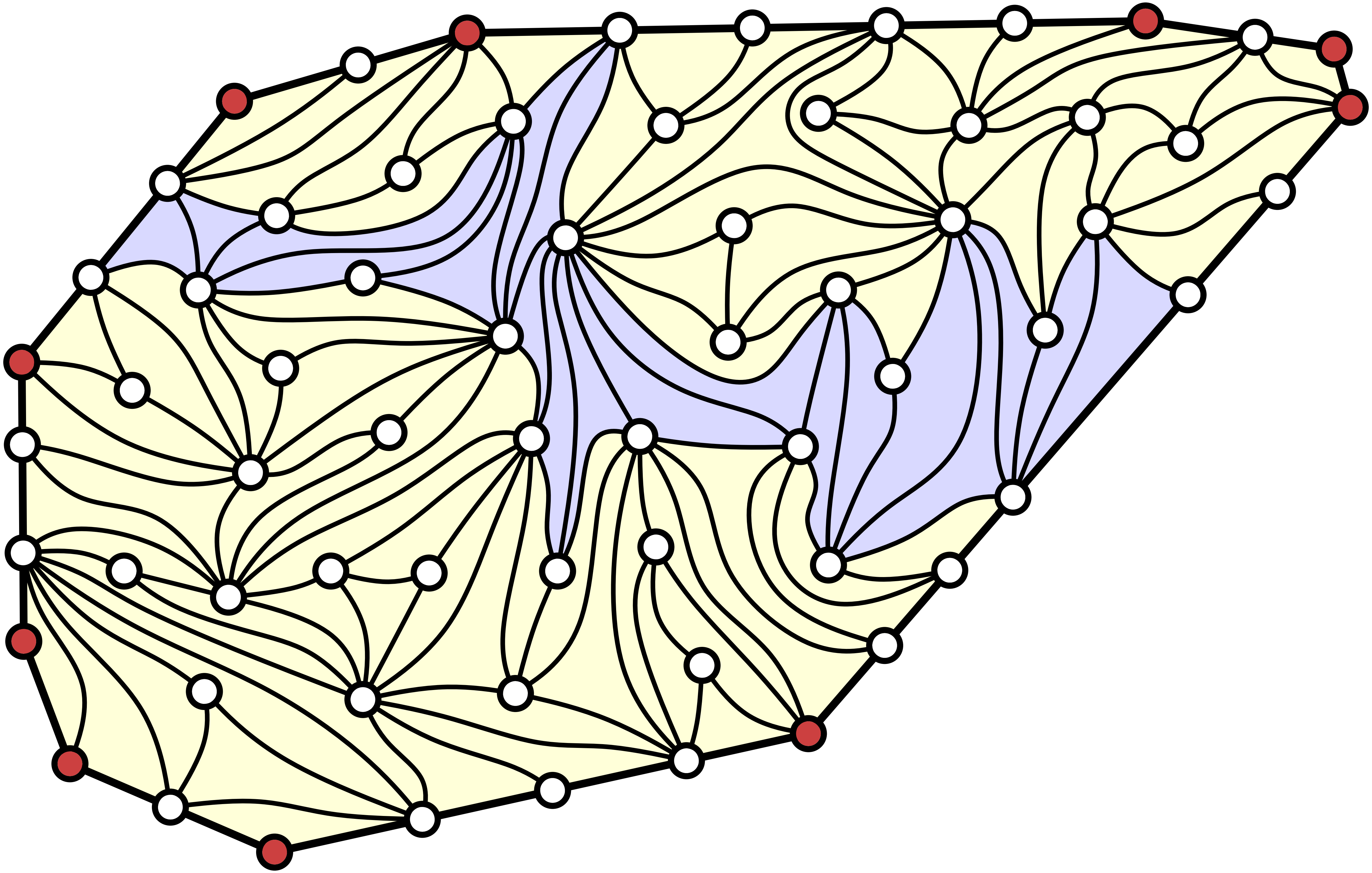

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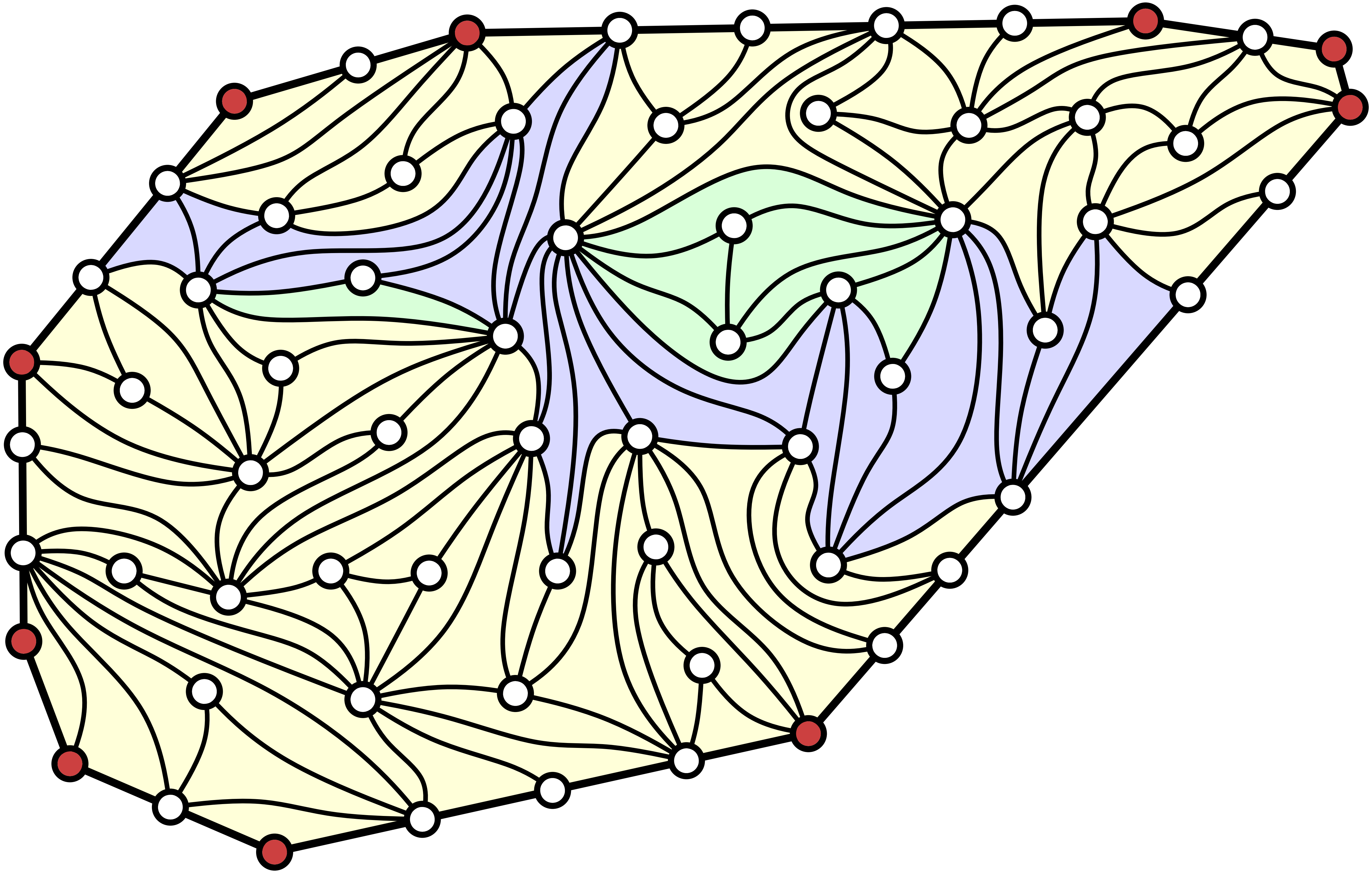

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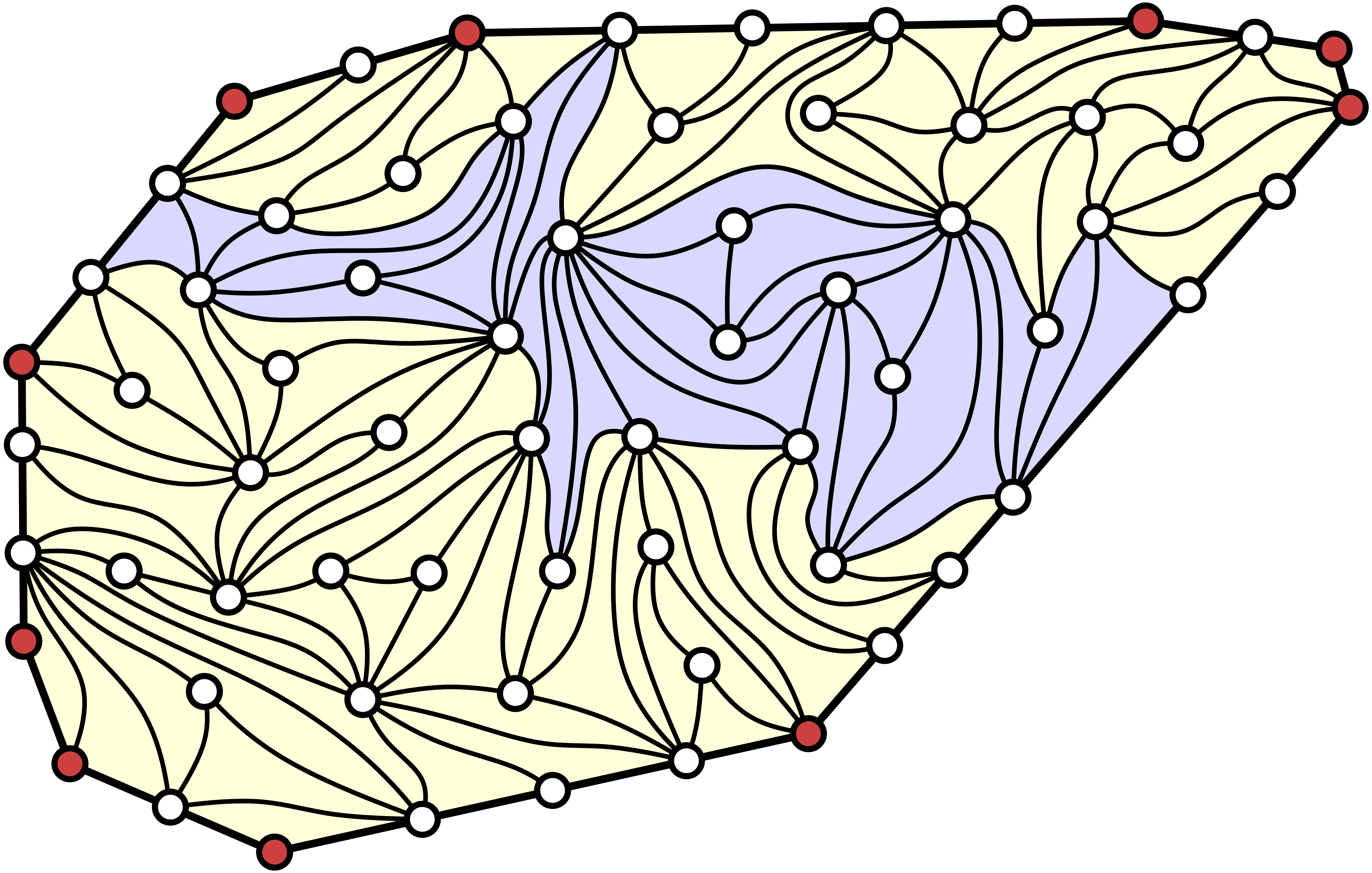

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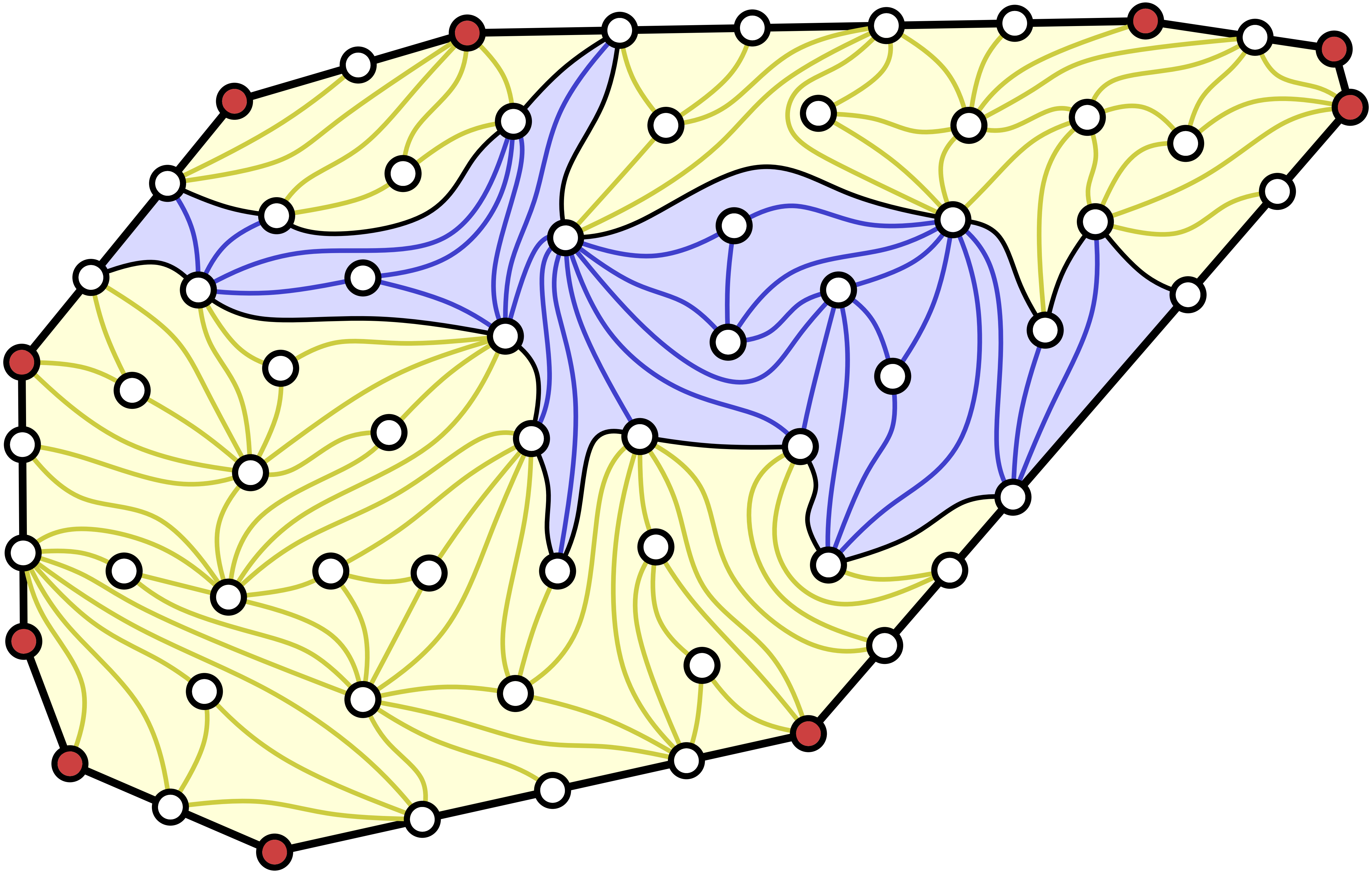

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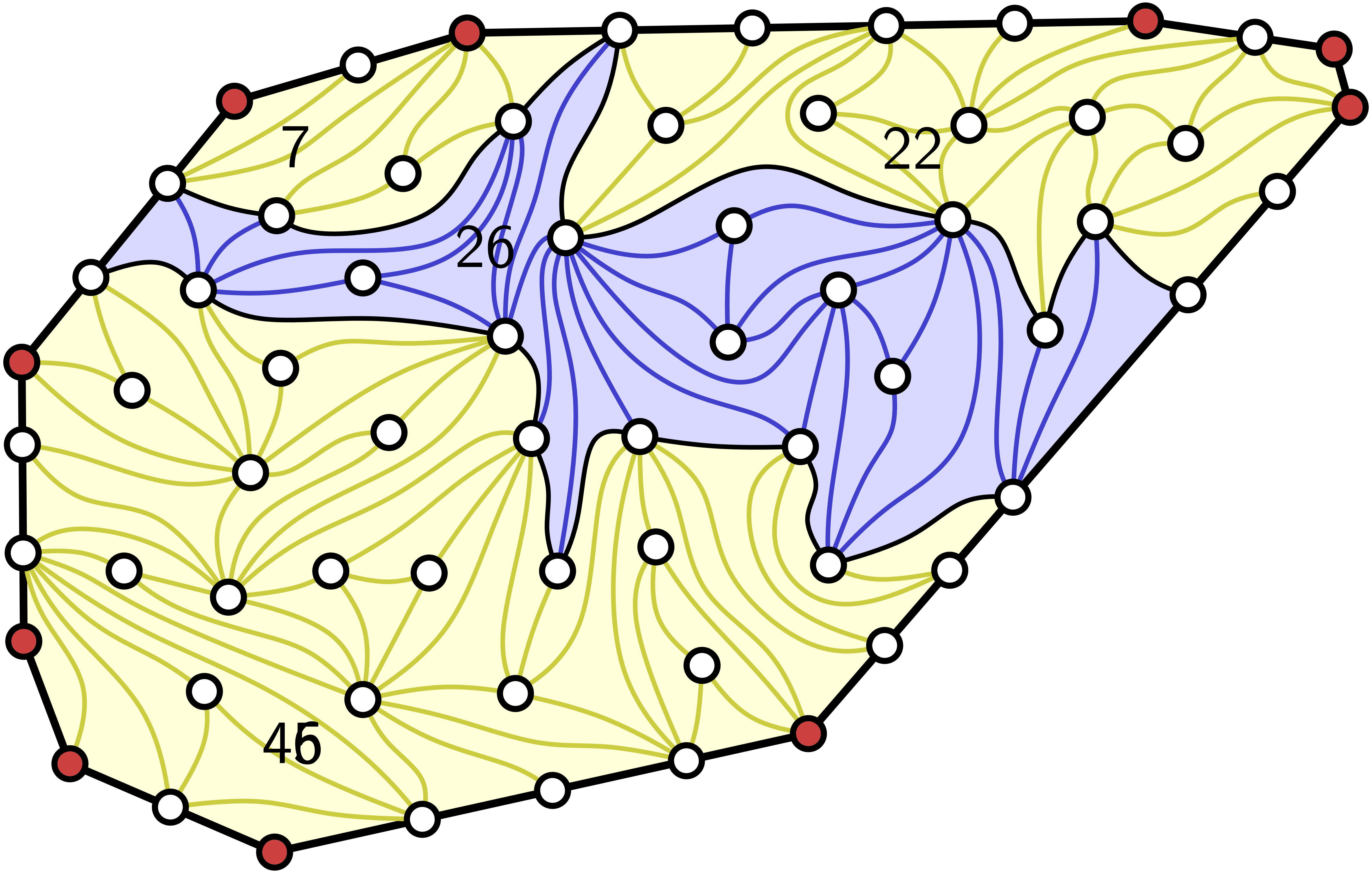

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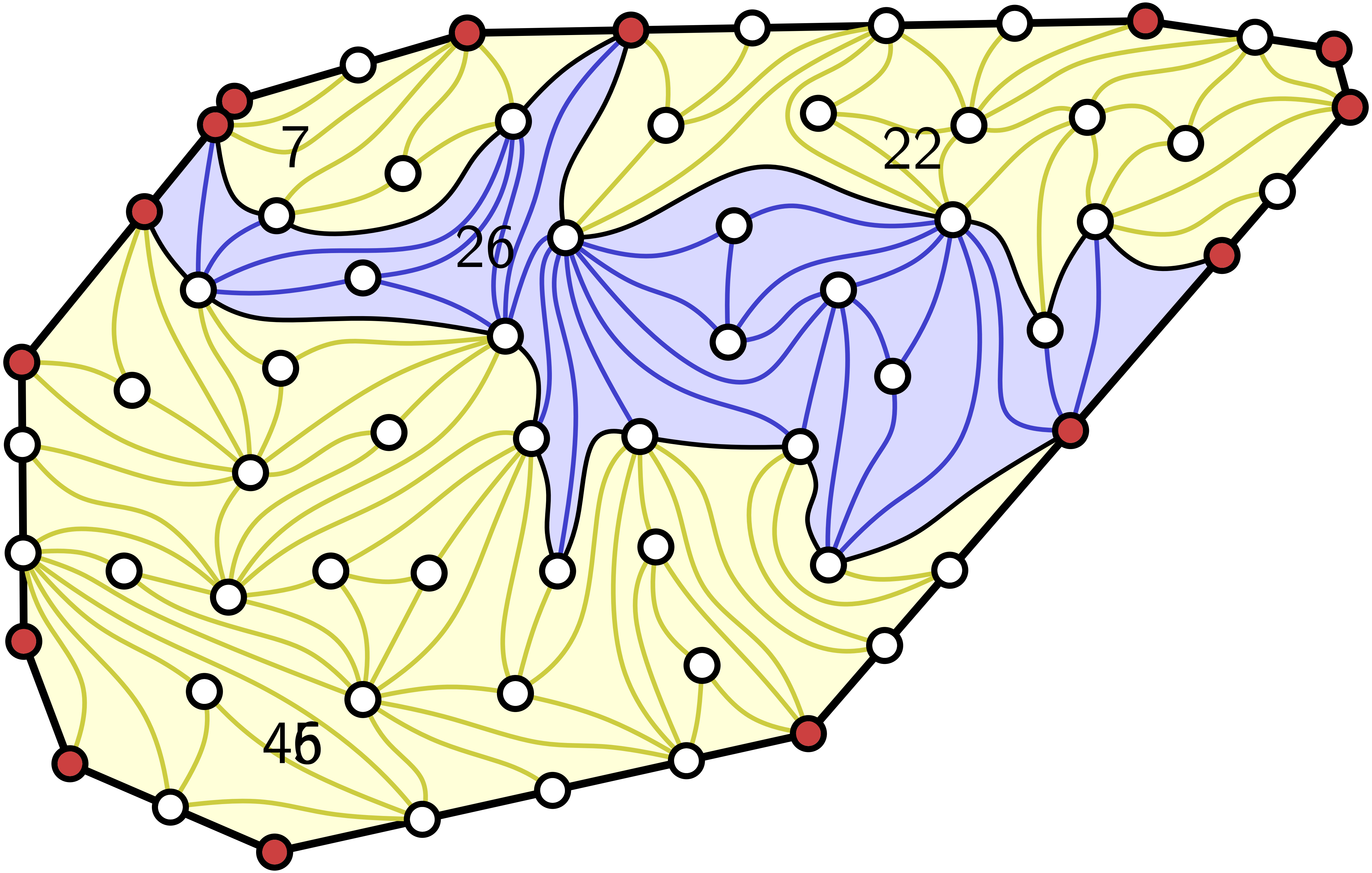

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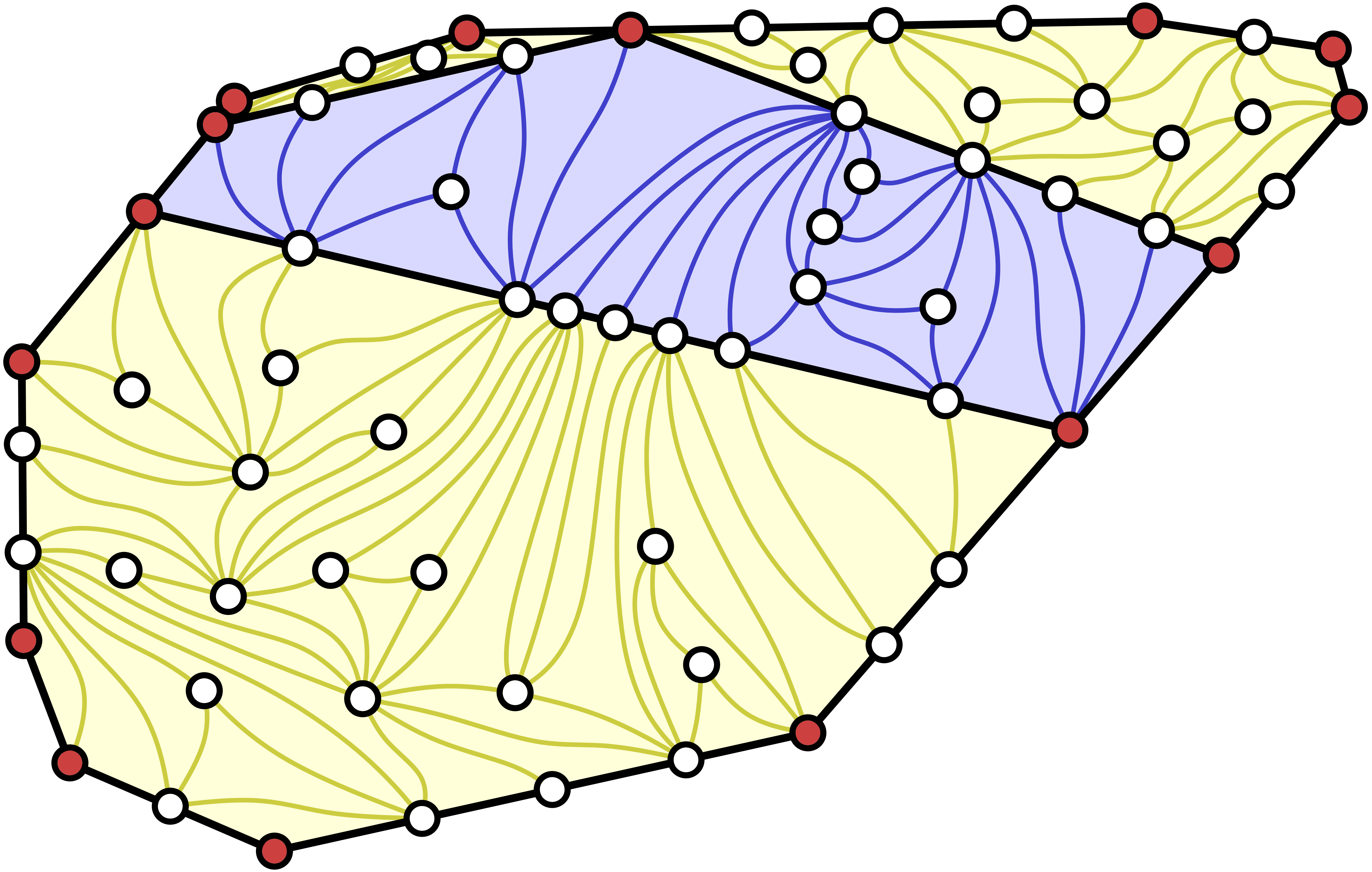

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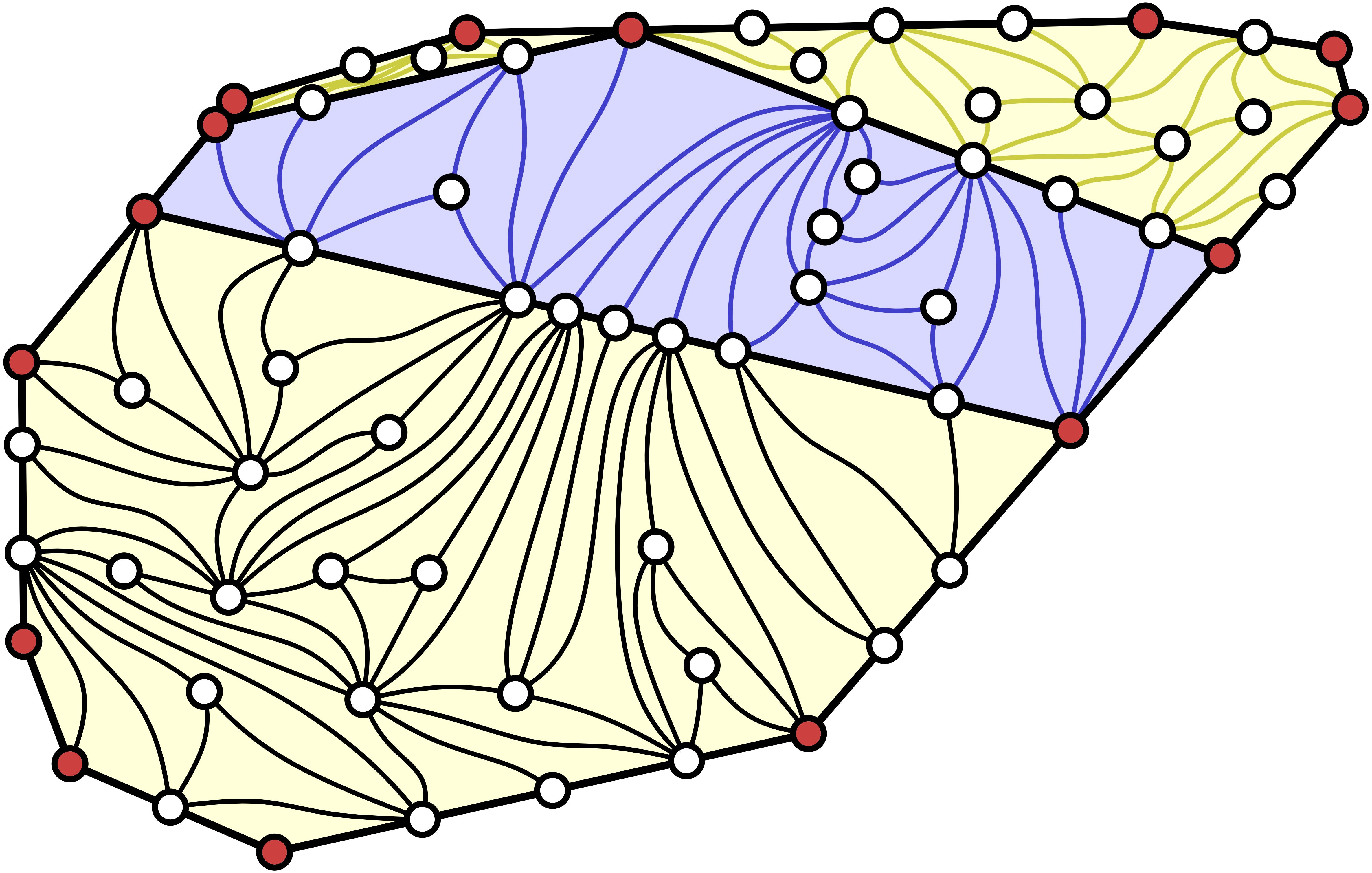

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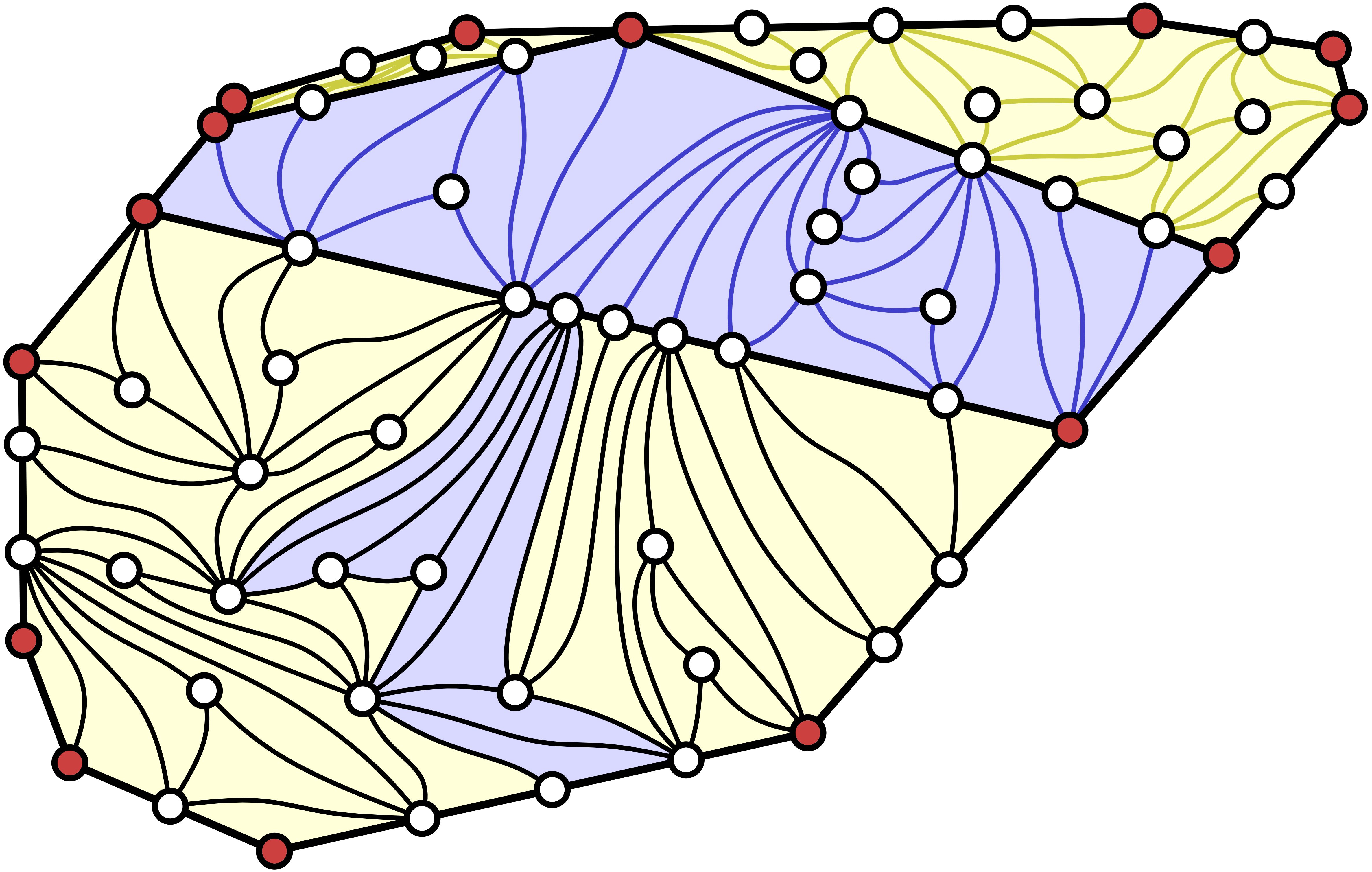

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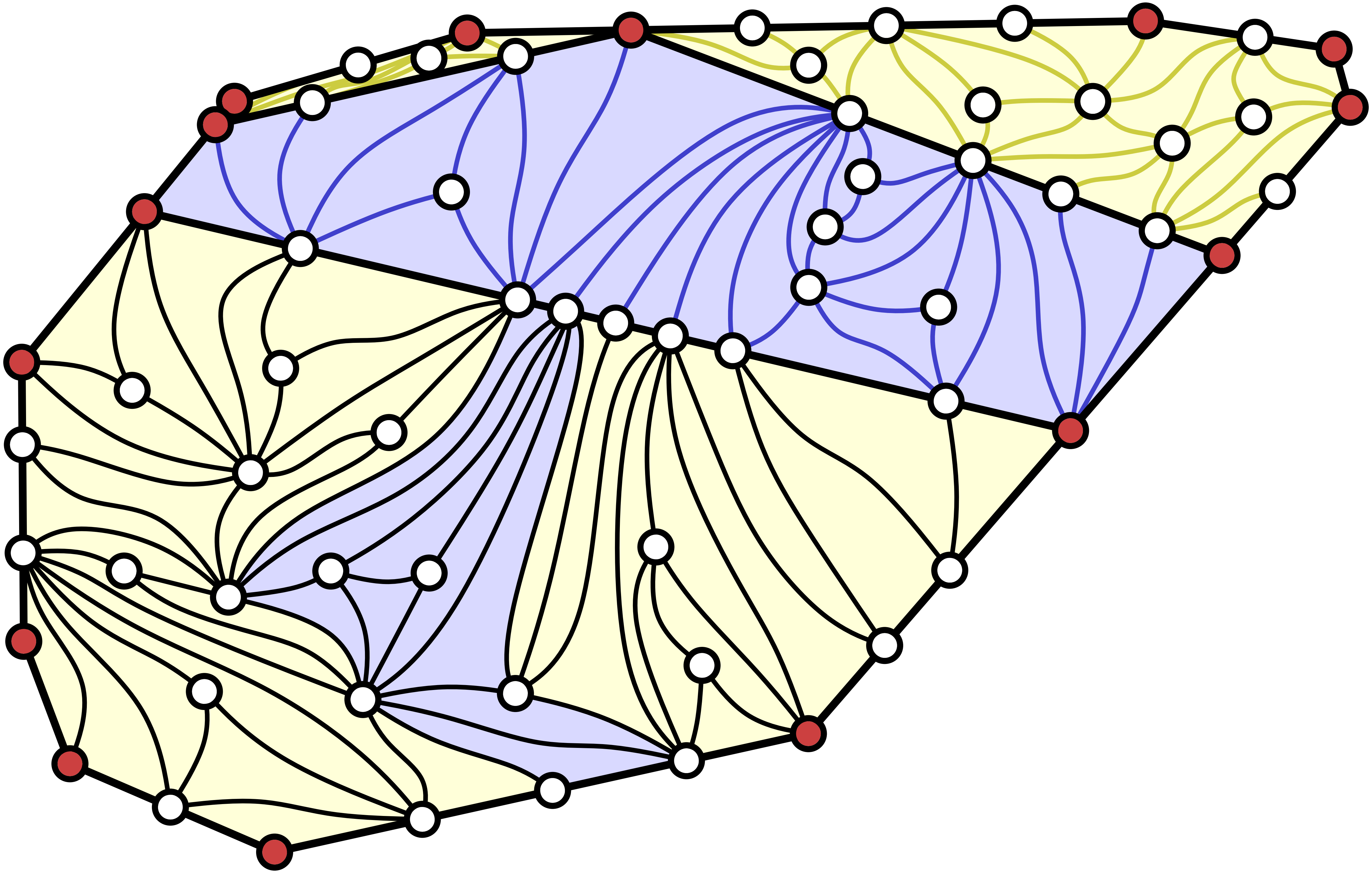

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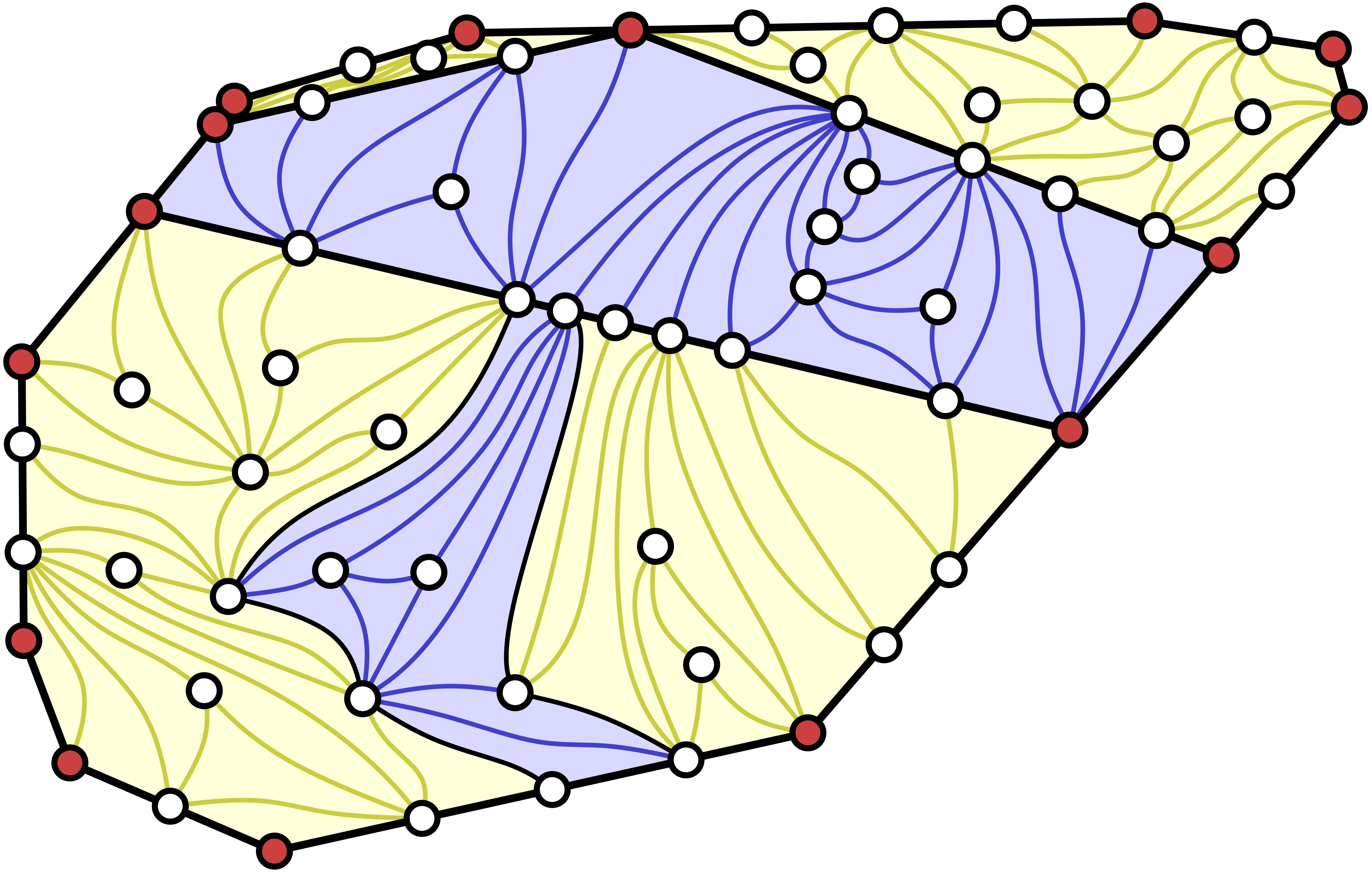

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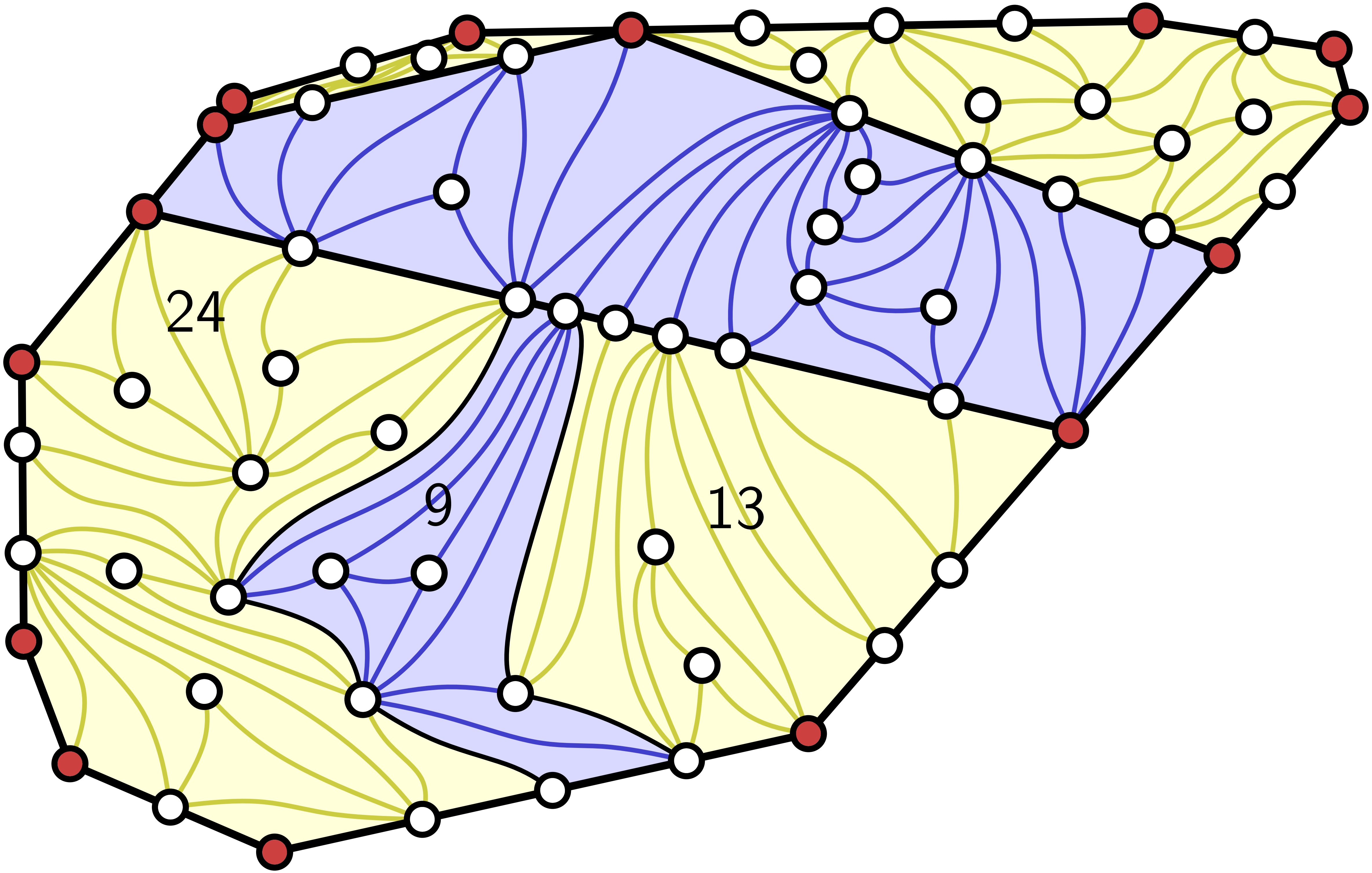

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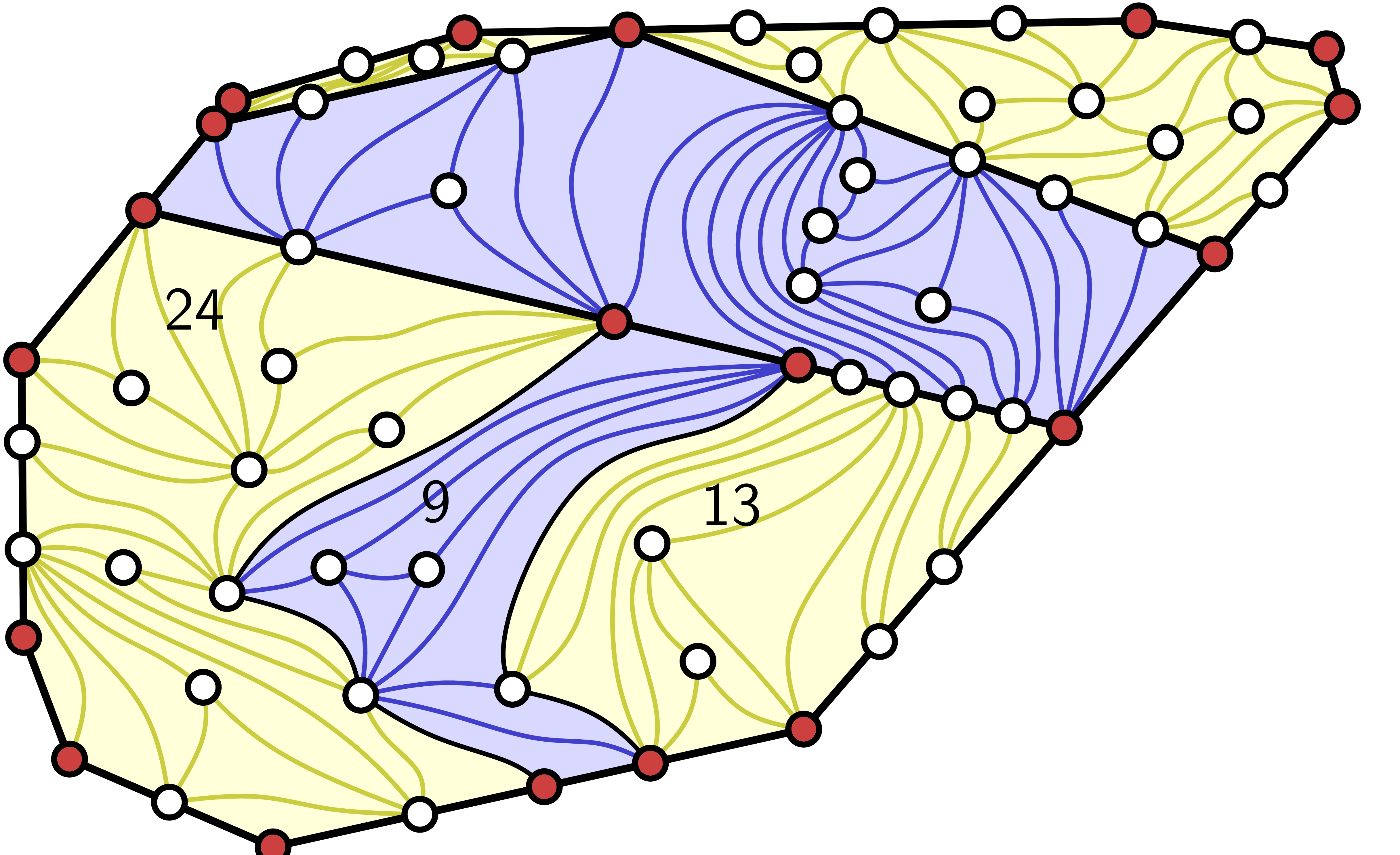

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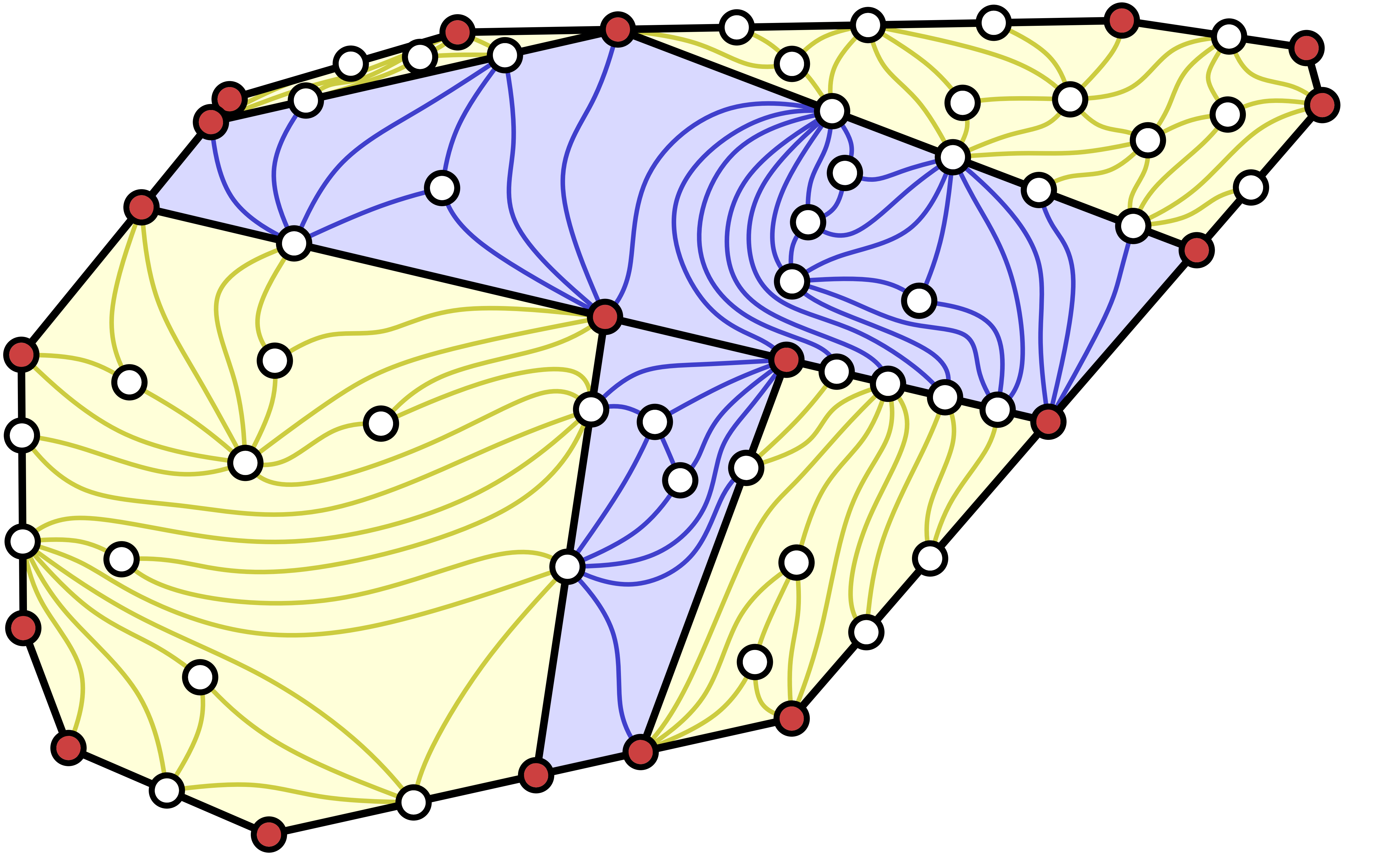

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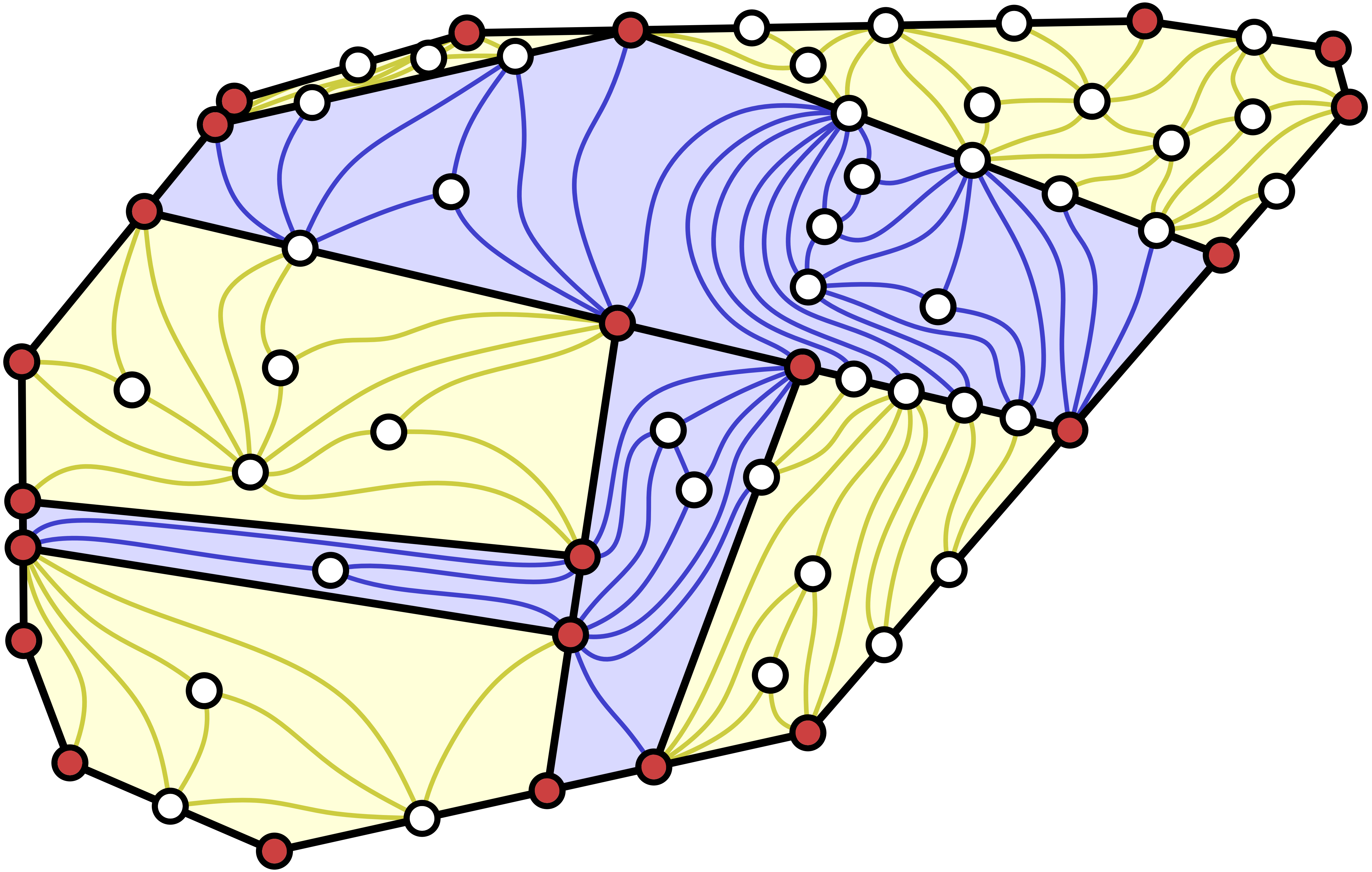

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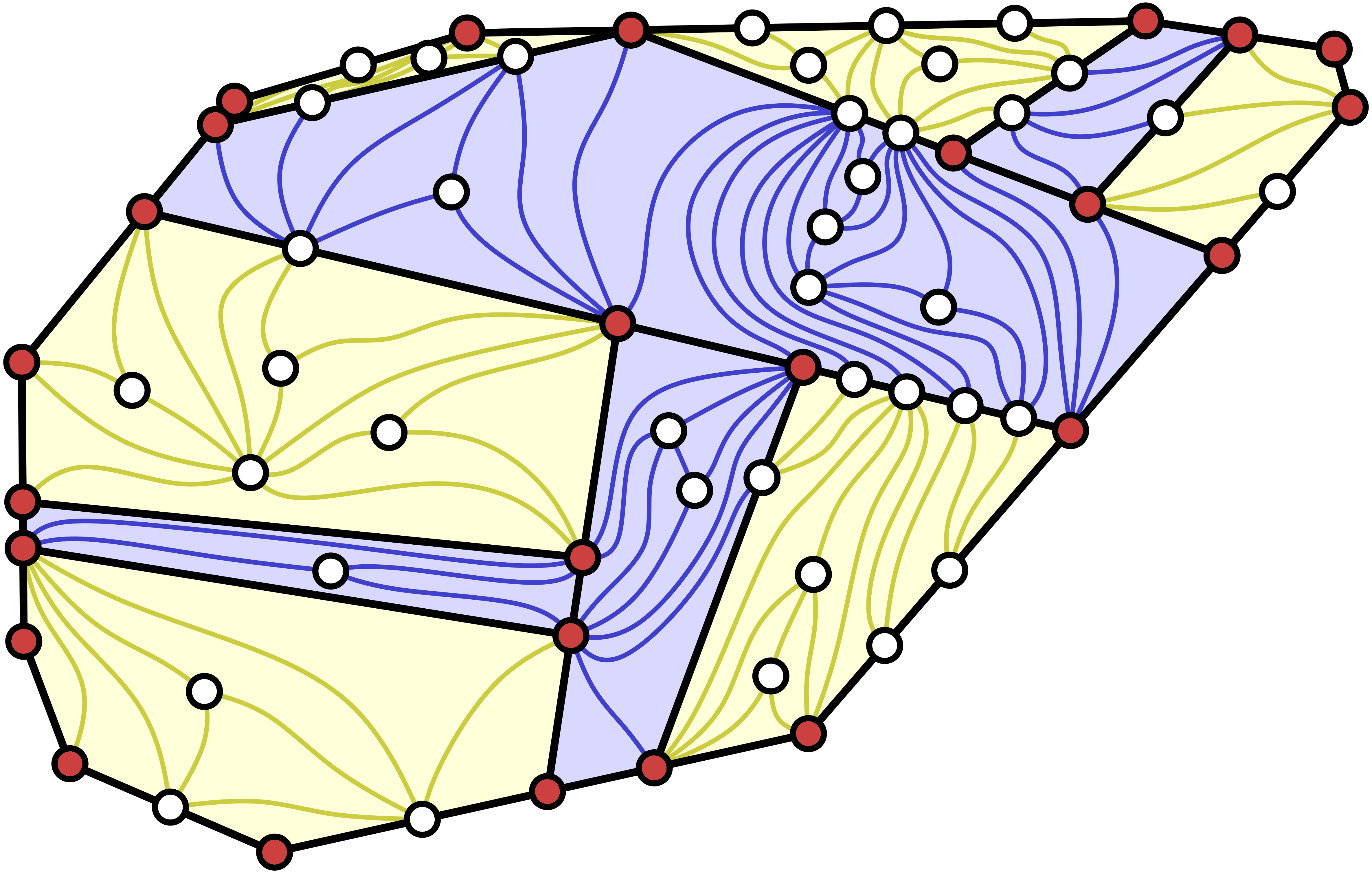

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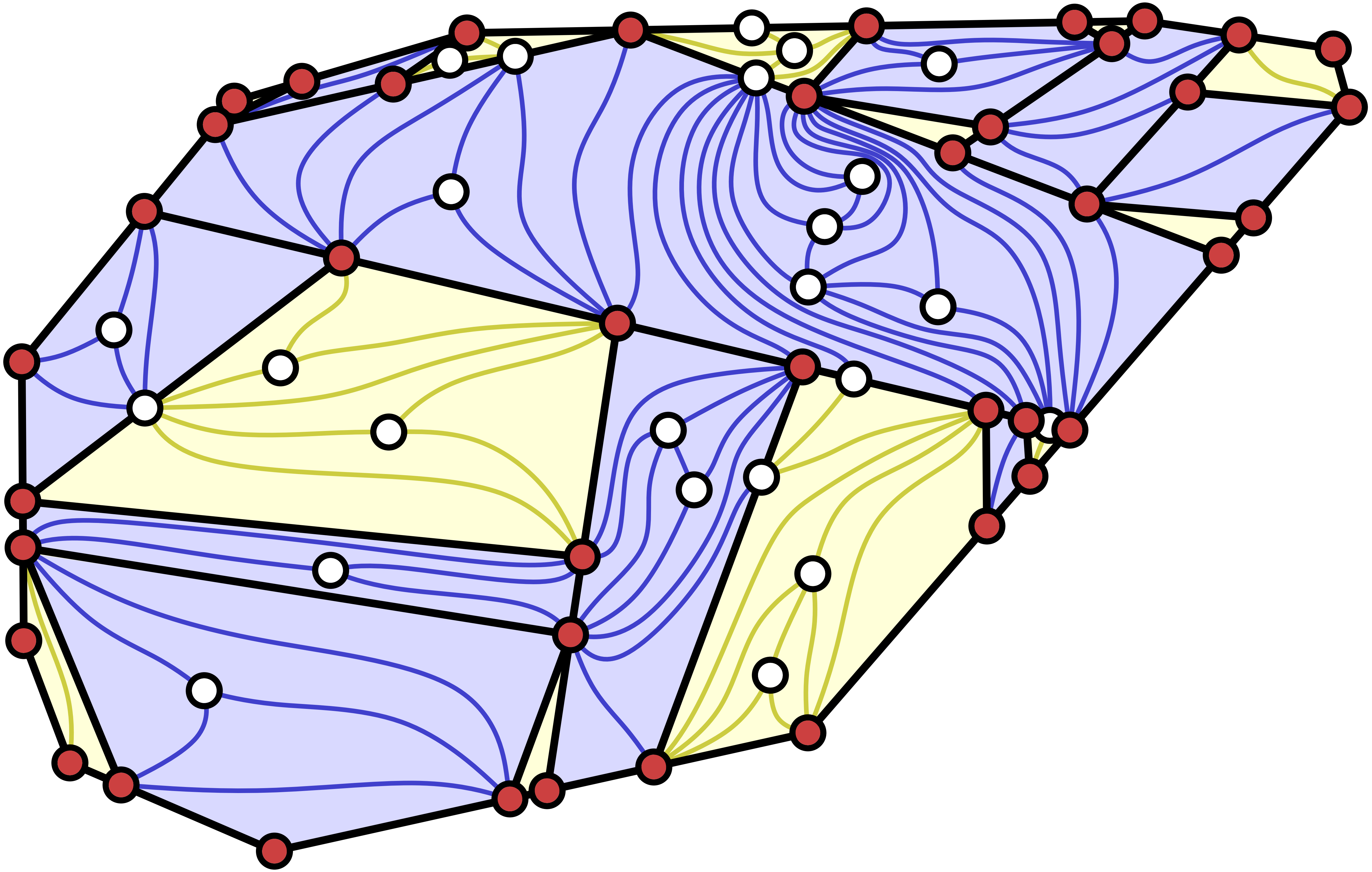

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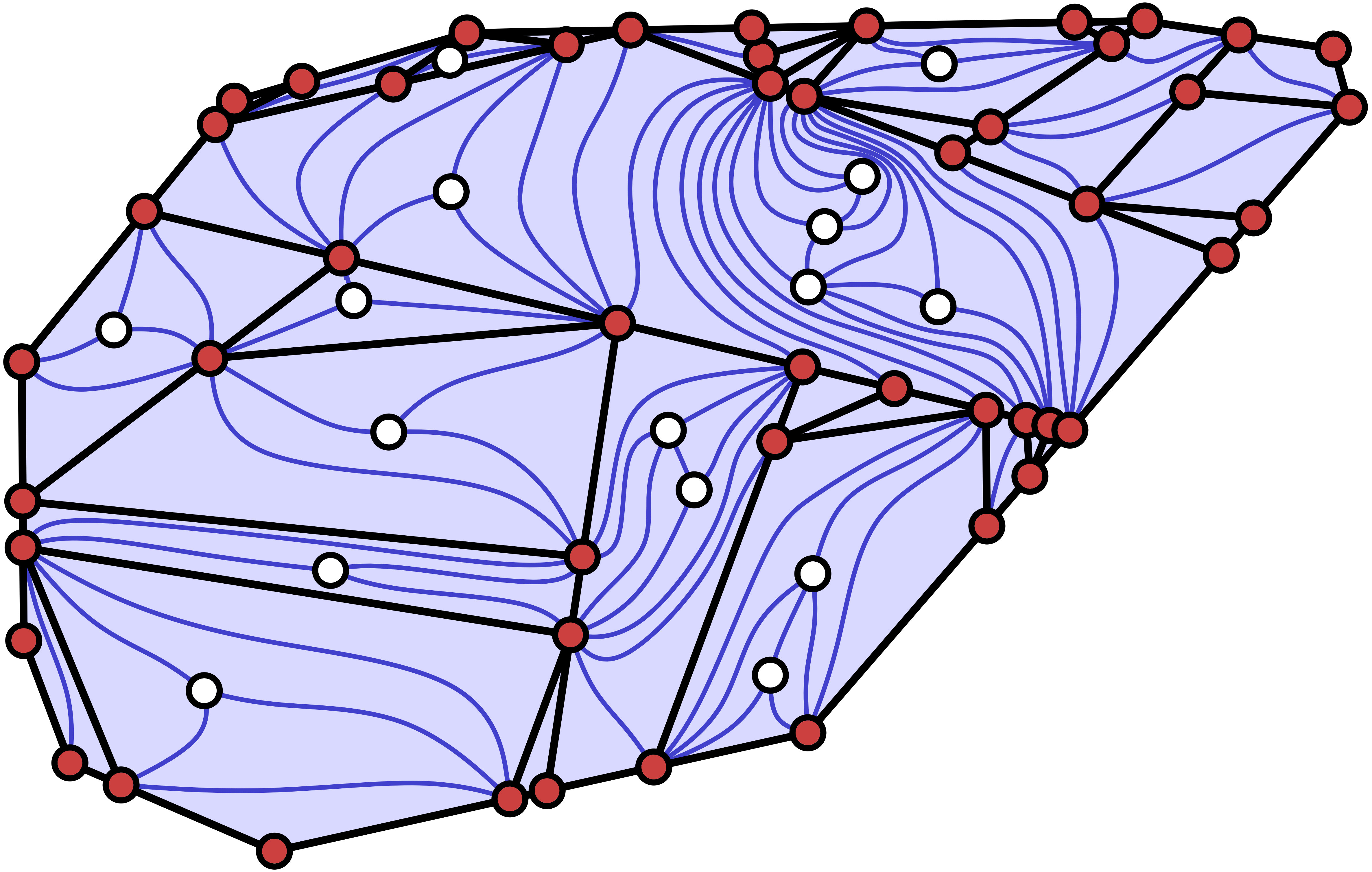

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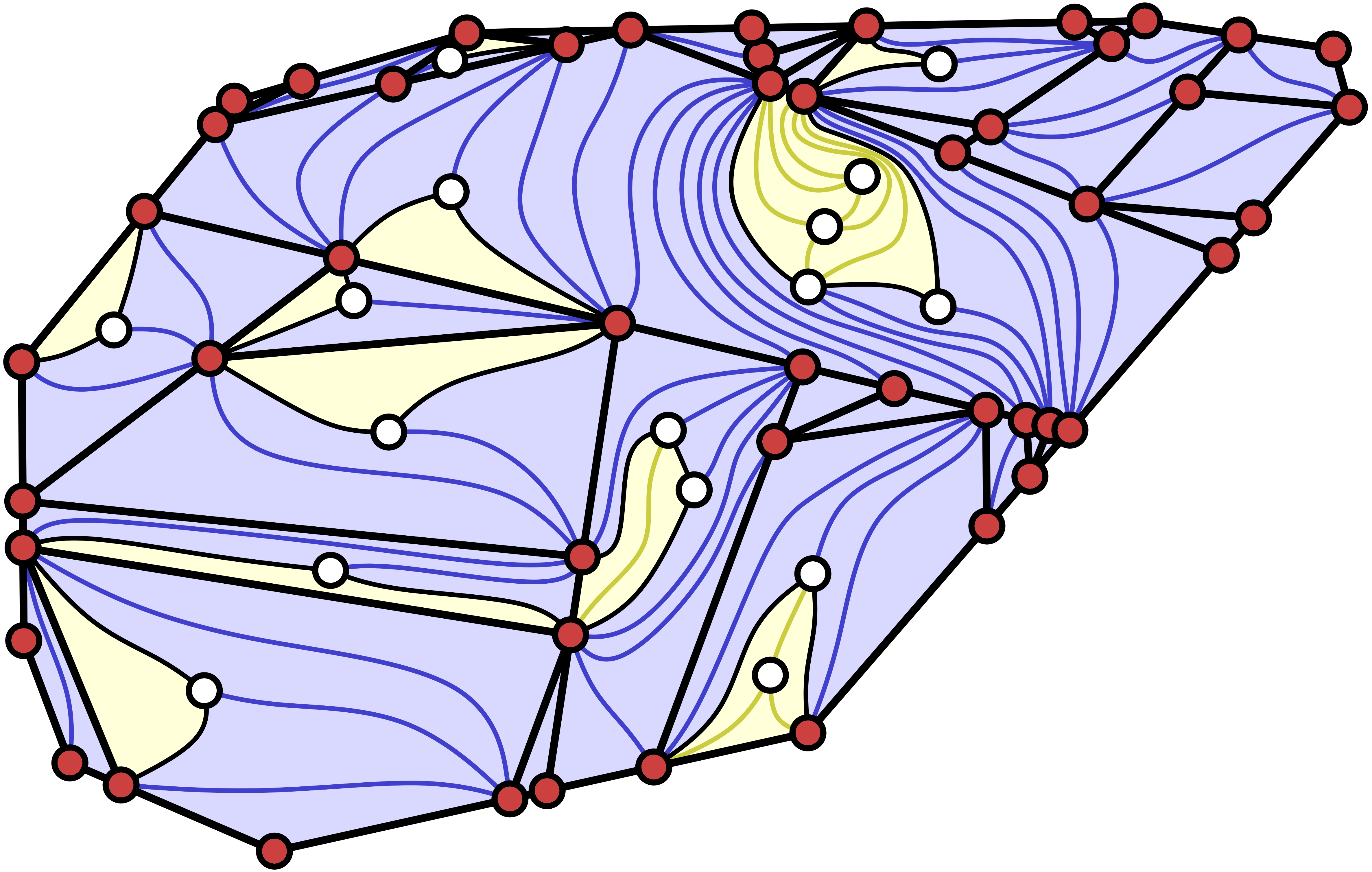

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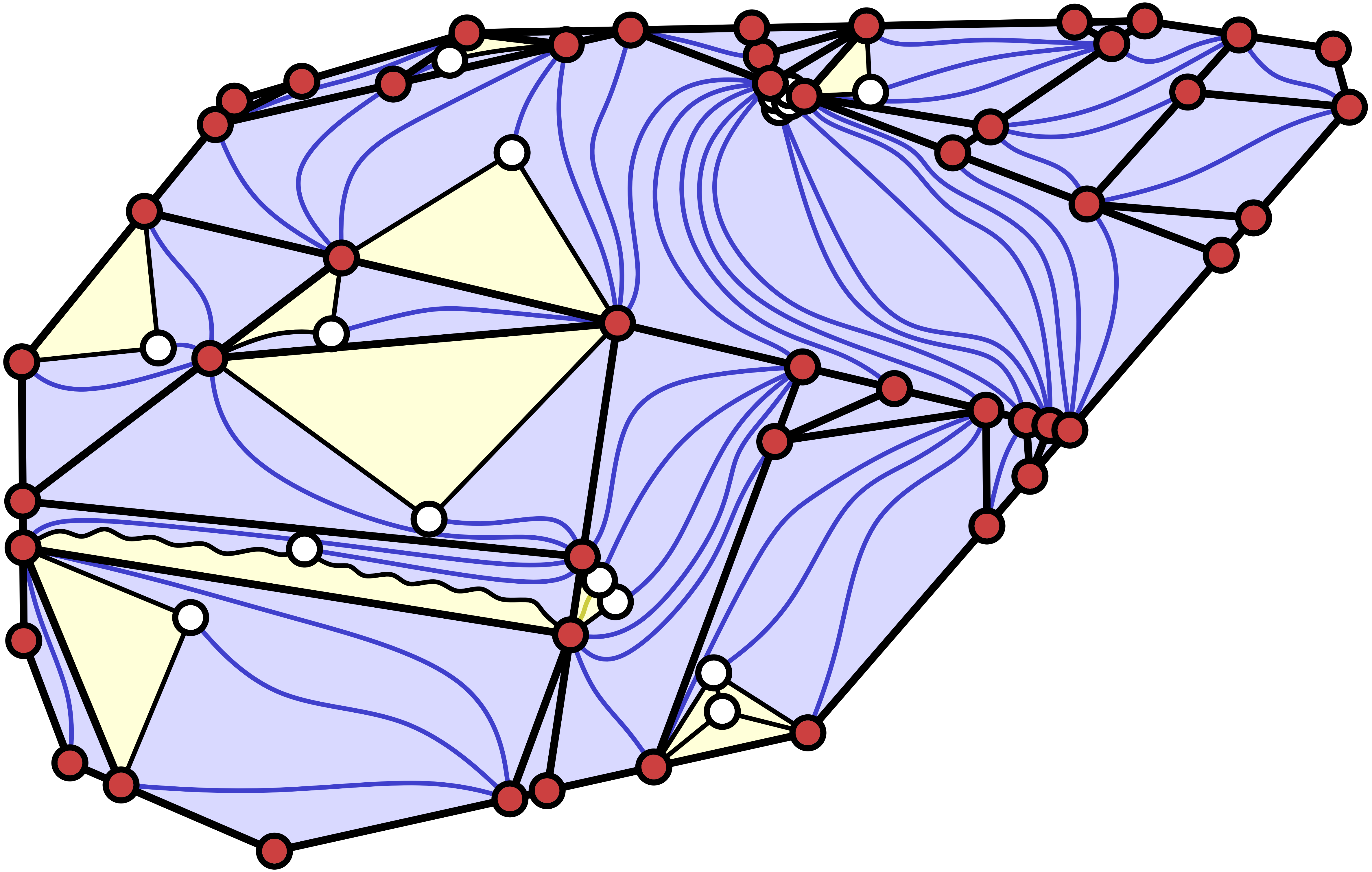

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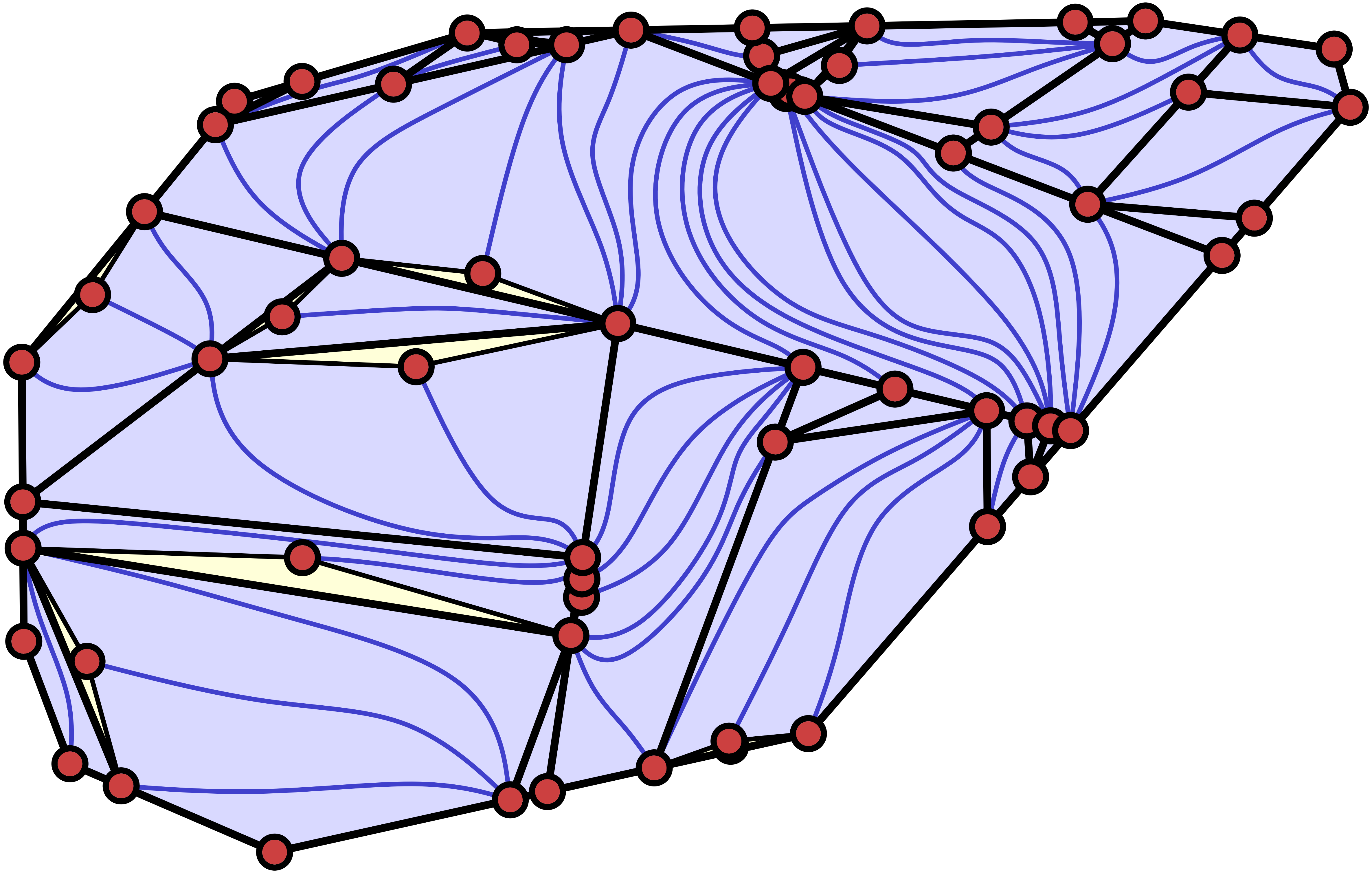

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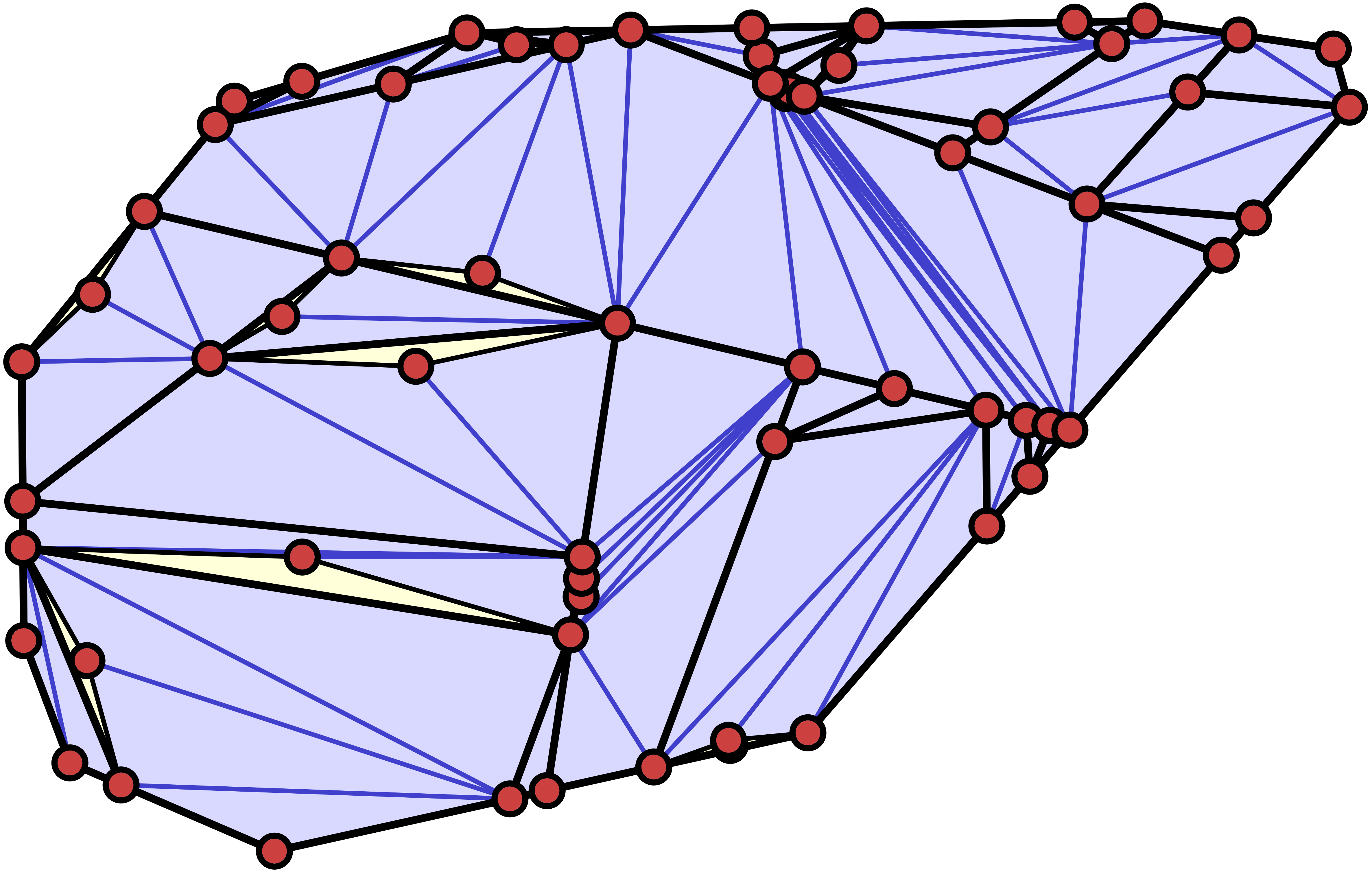

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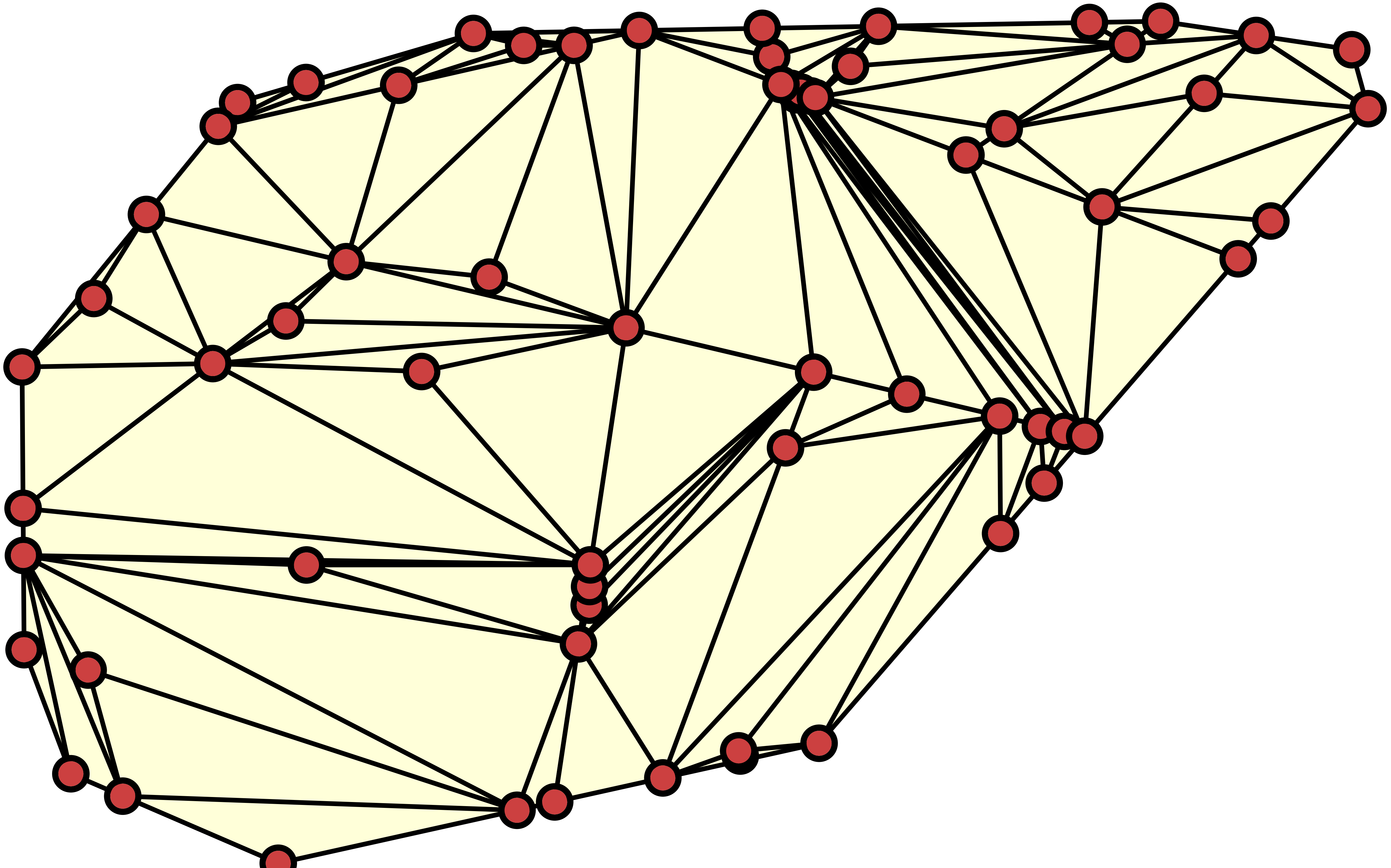

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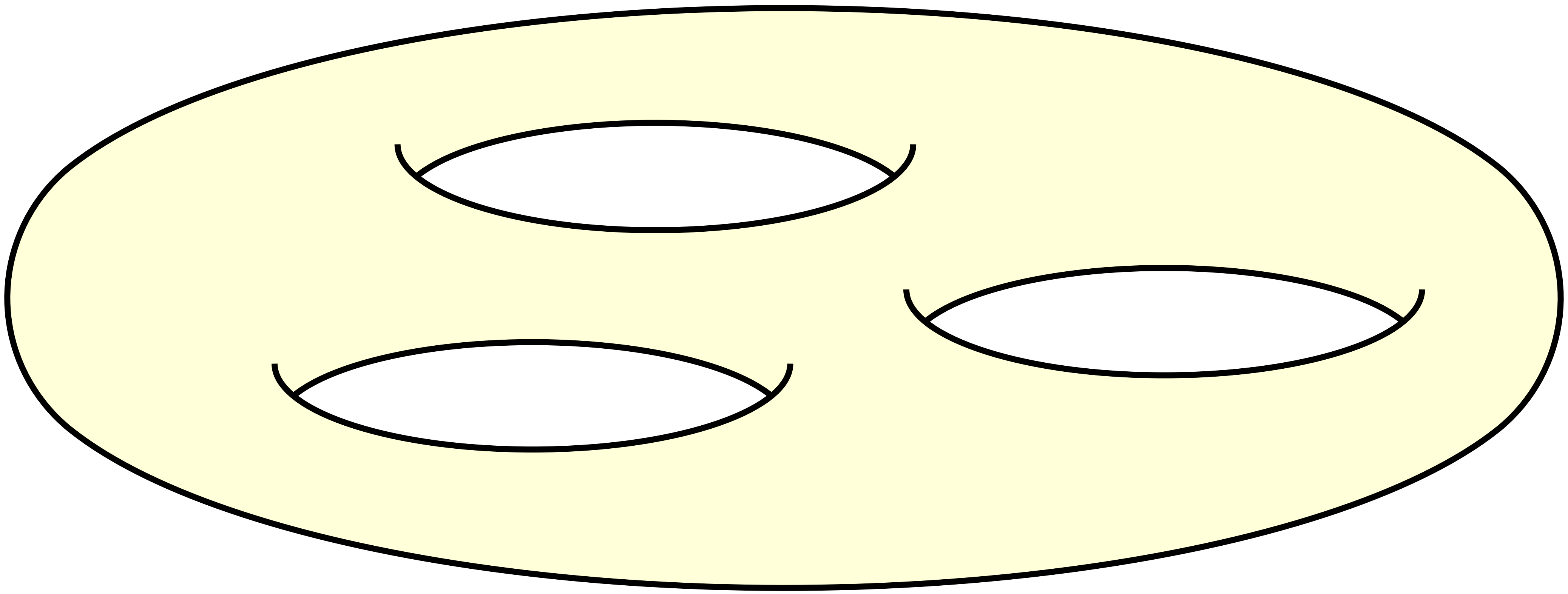

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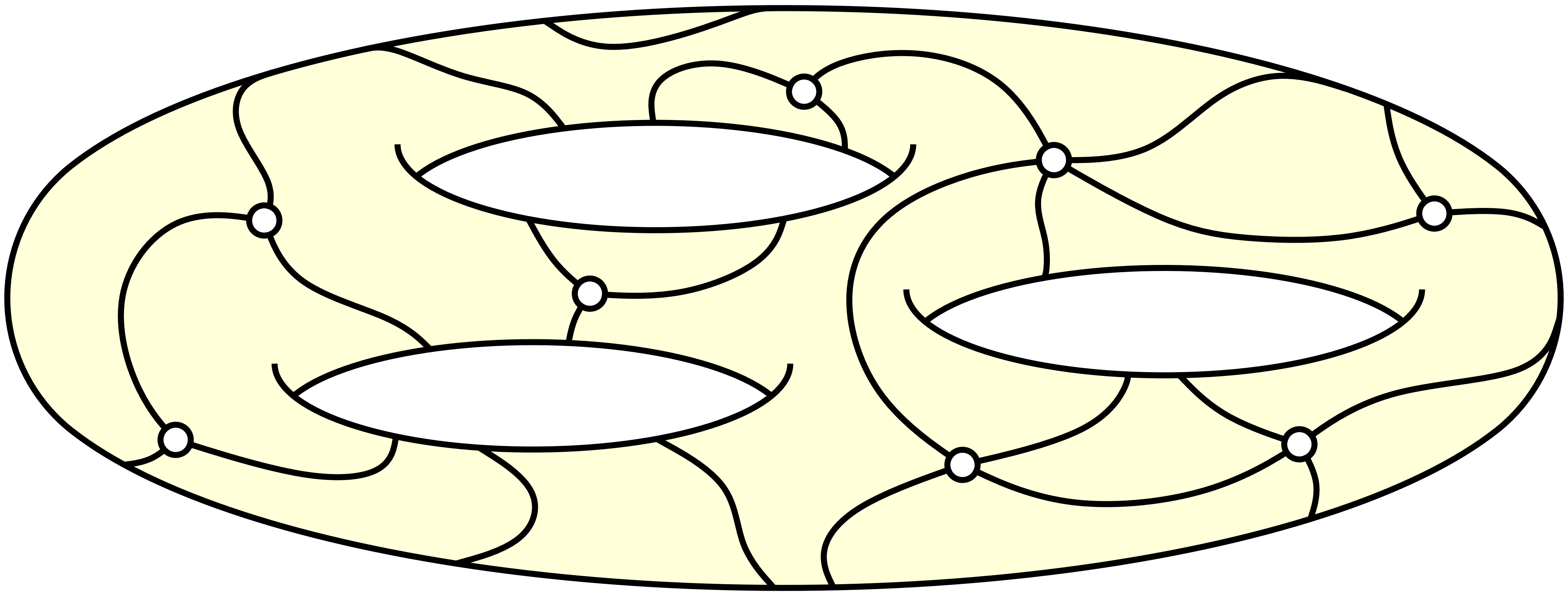

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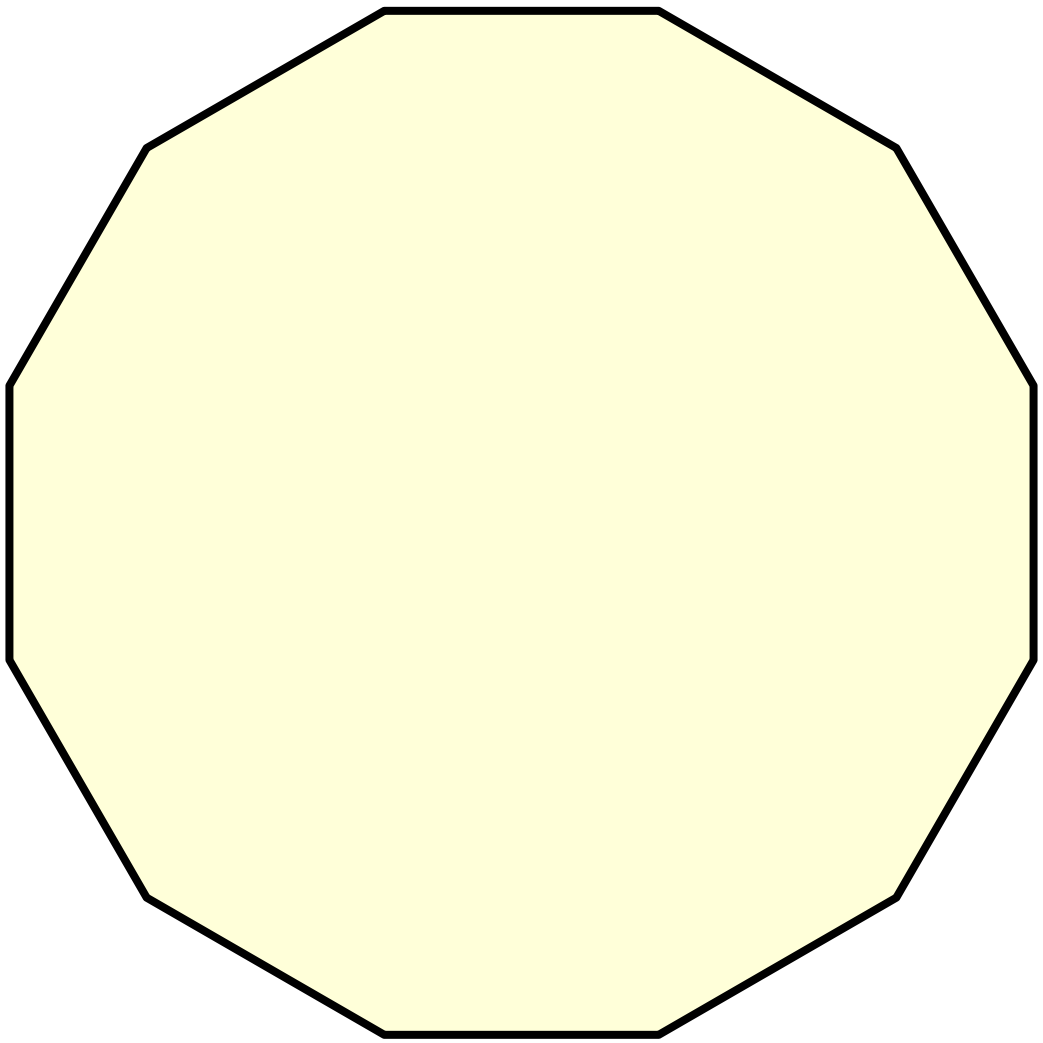

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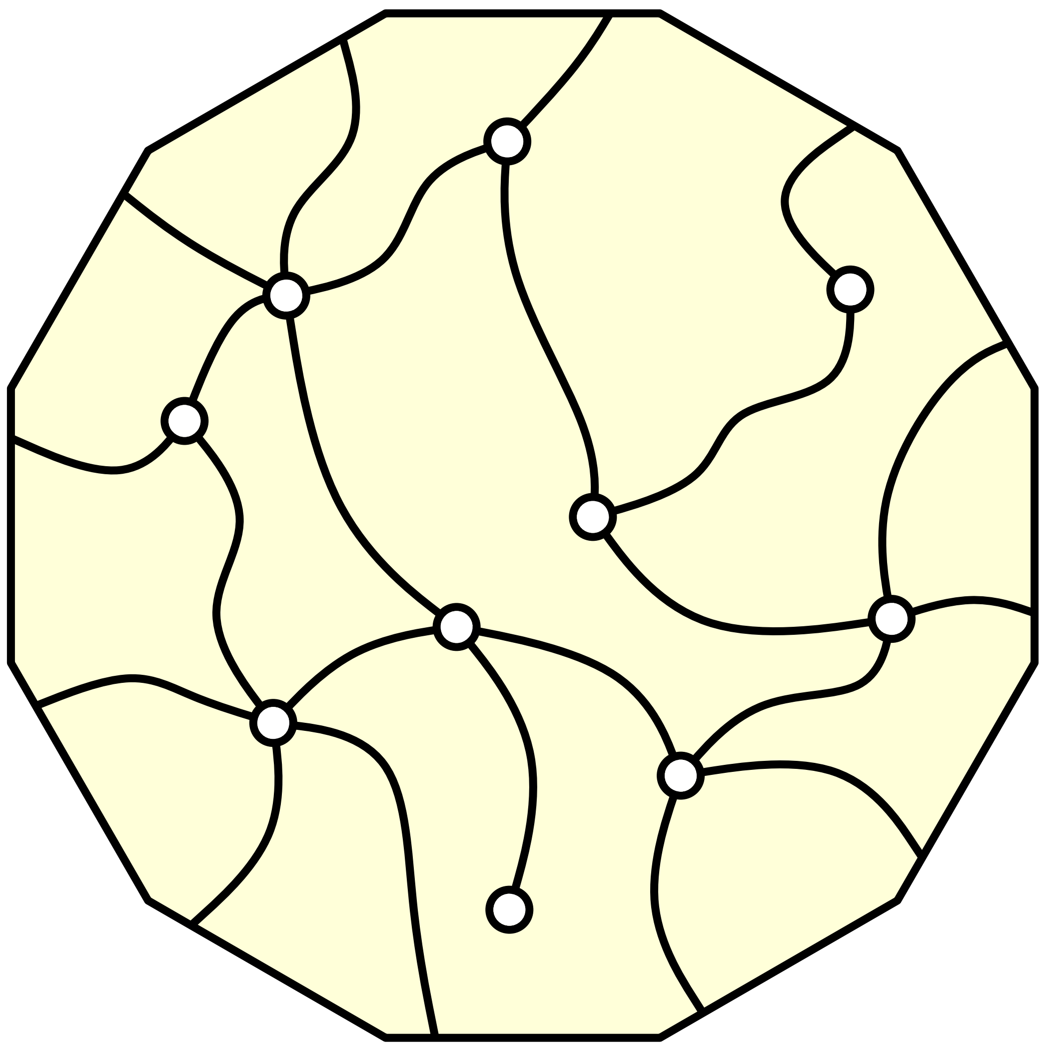

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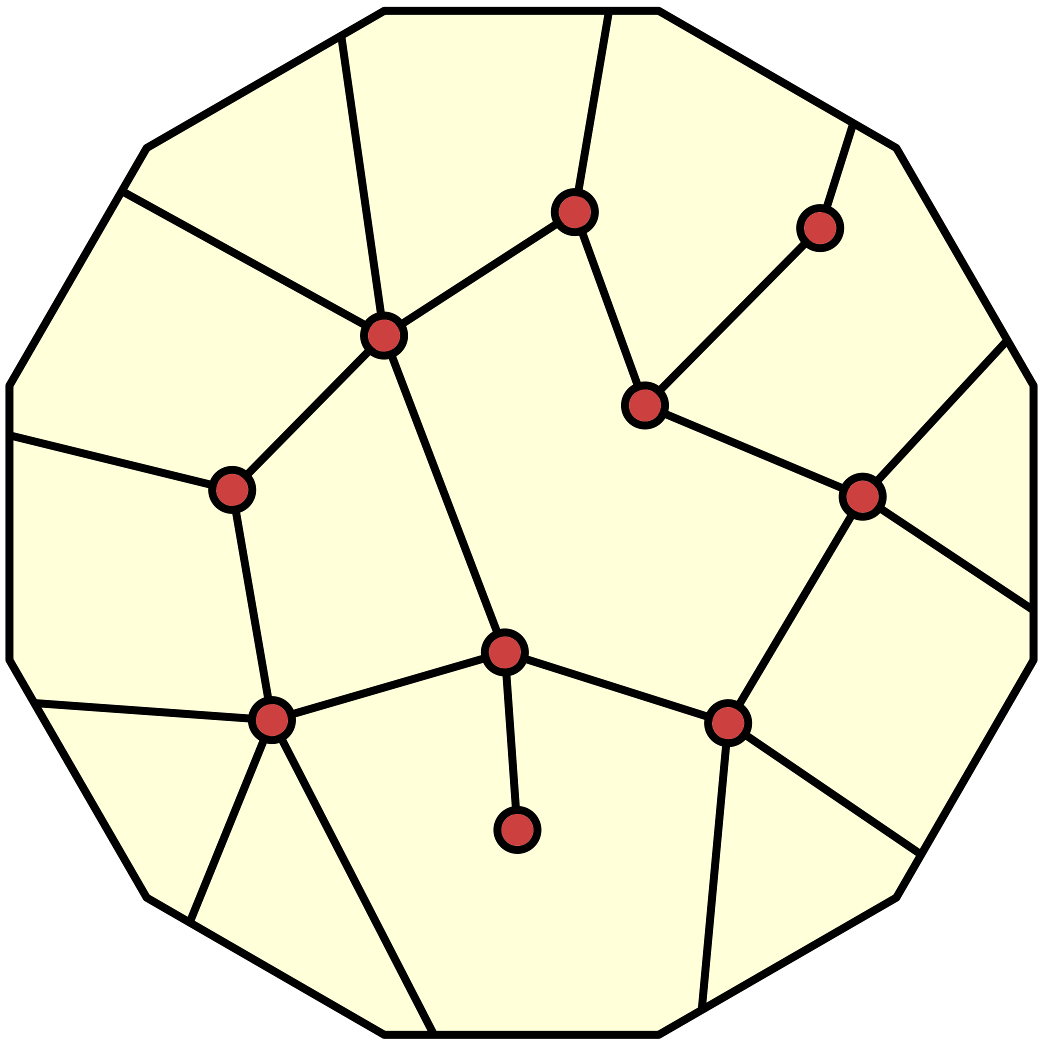

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COROLLARY

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OPEN QUESTION

Can the bound of $\Omega(d/n^3)$ be improved?

ONE MORE QUESTION

Do *you* have any questions?