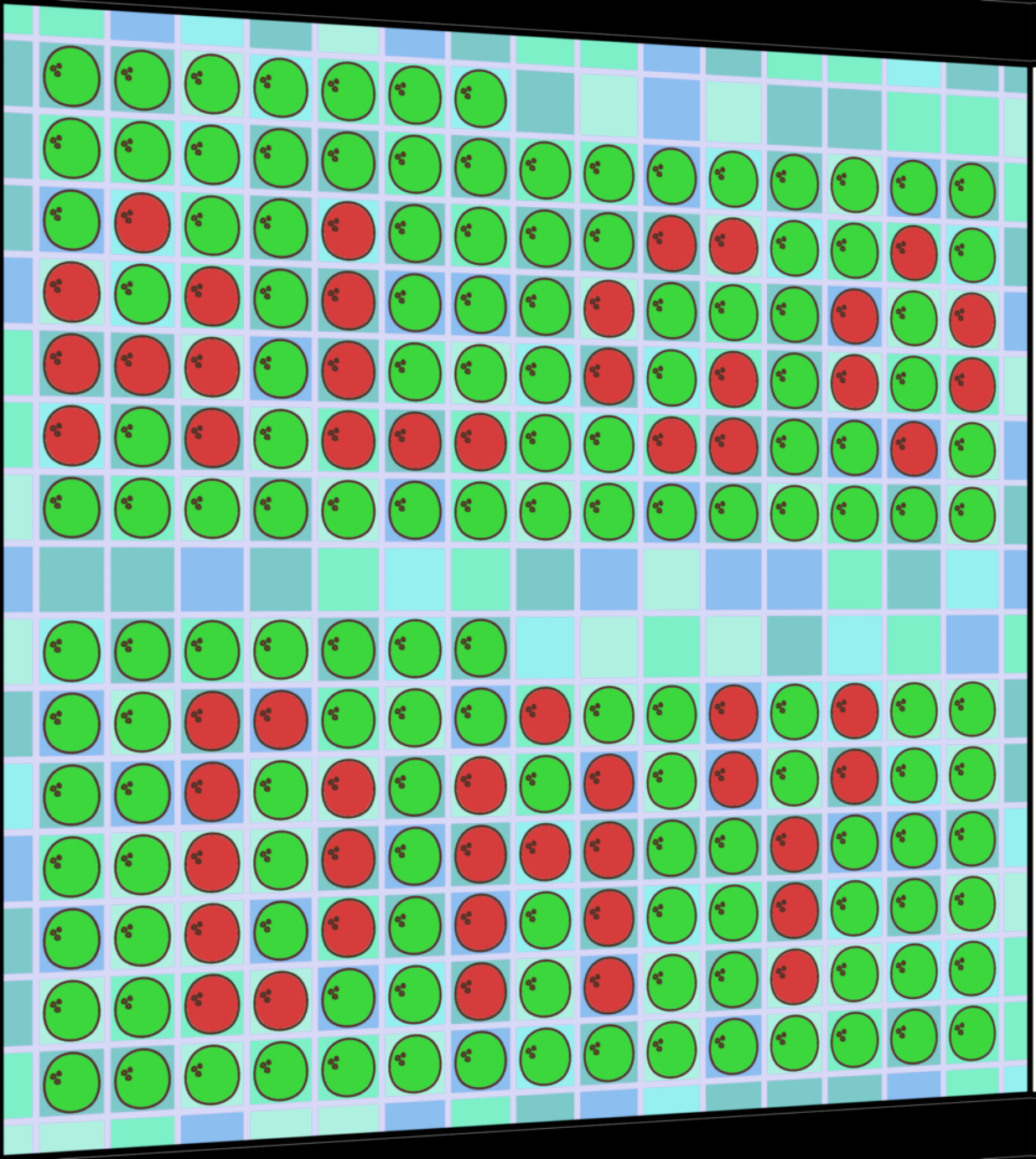
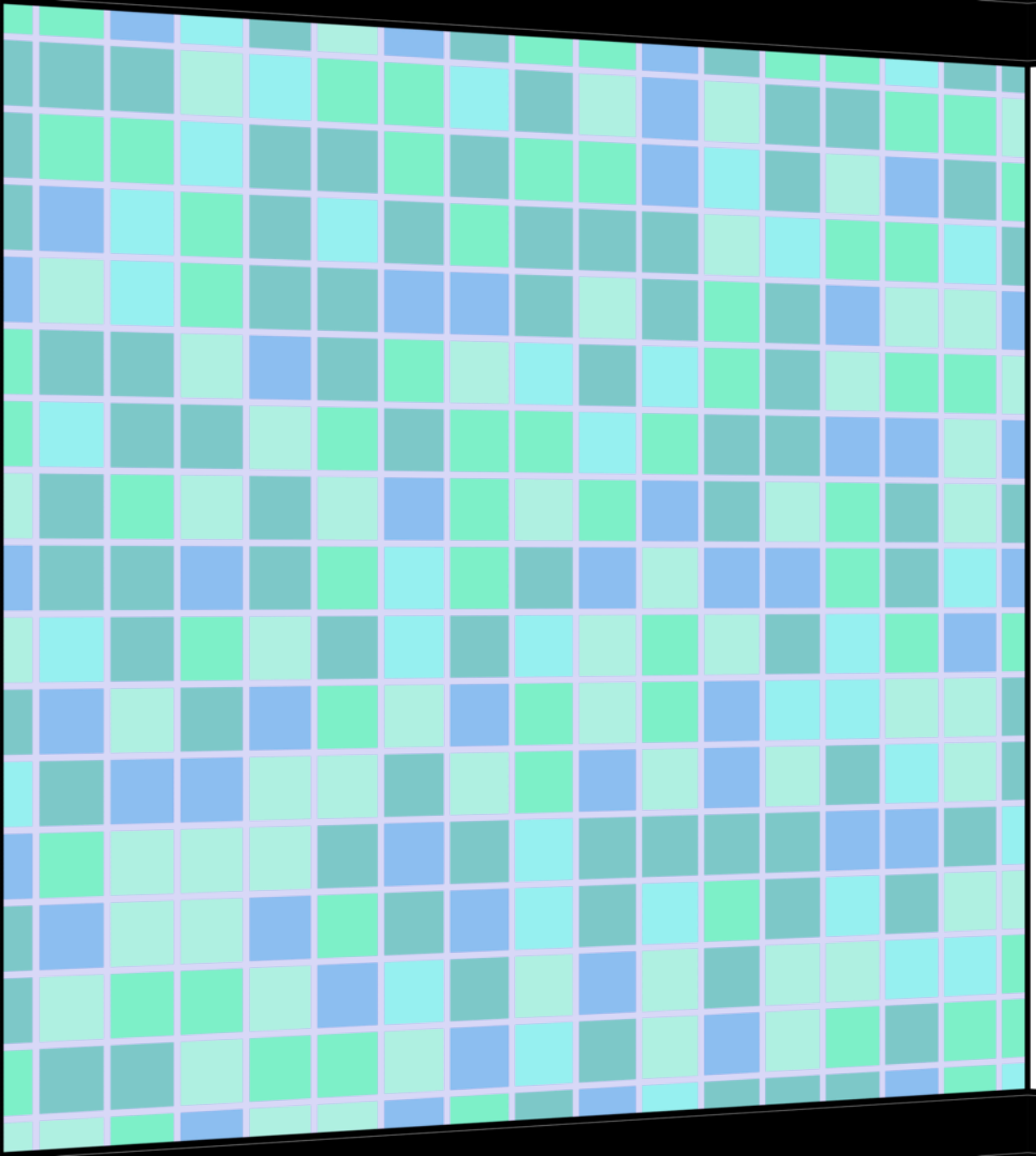


Labeled Reconfiguration by Compaction

Maarten Löffler

Based on joint work with

Hugo Akitaya
Greg Aloupis
Anika Rounds
Giovanni Viglietta



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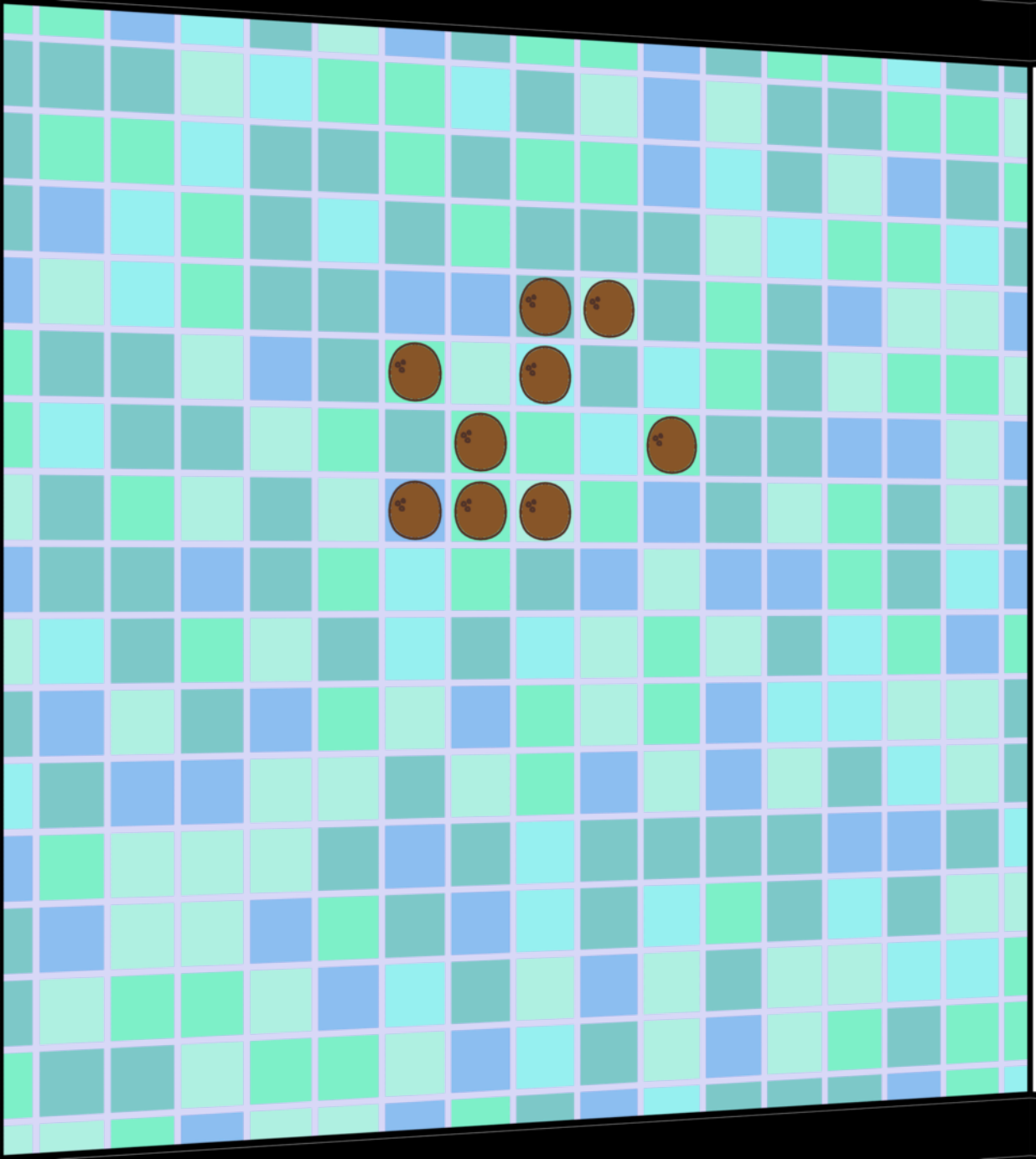

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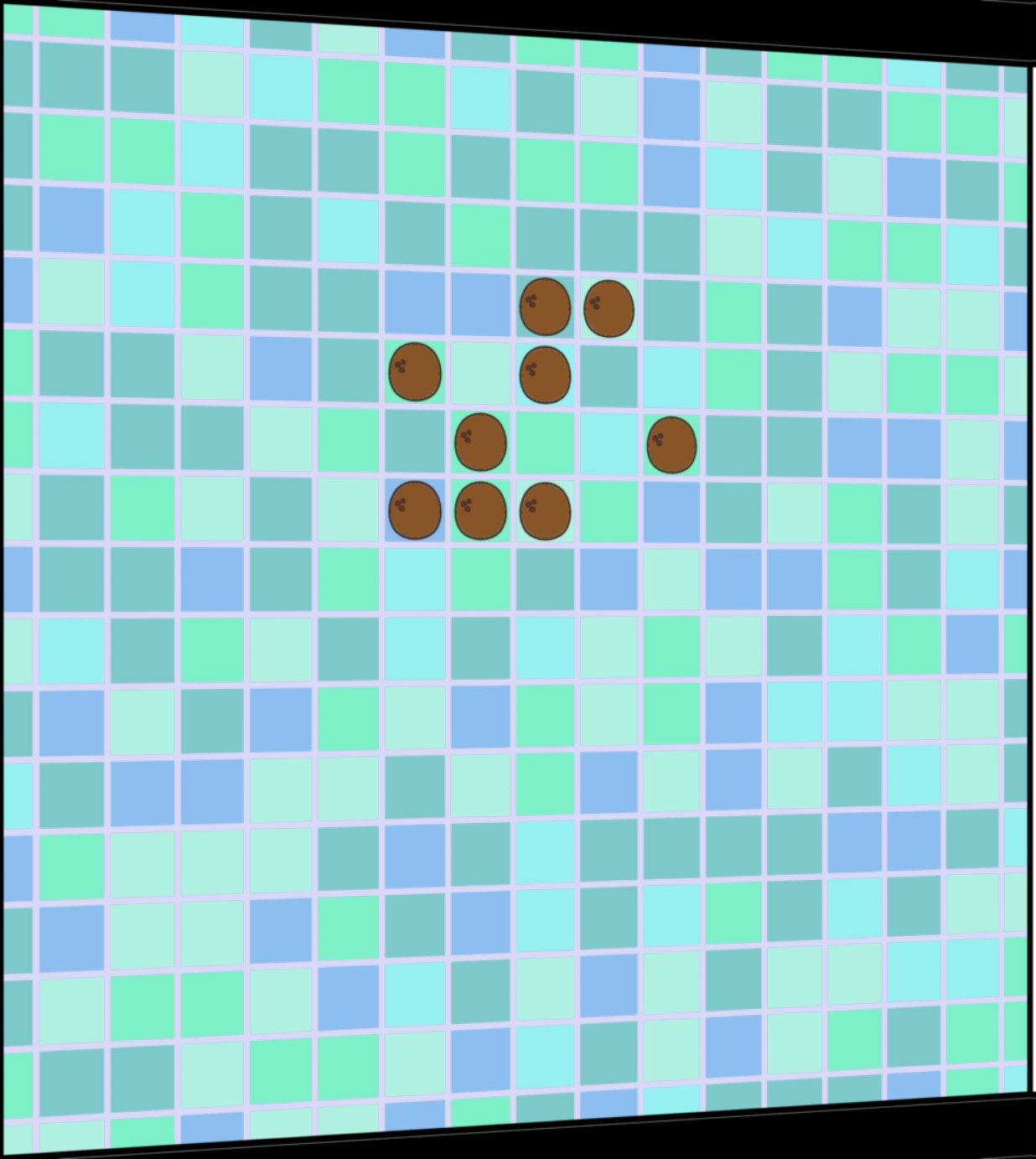

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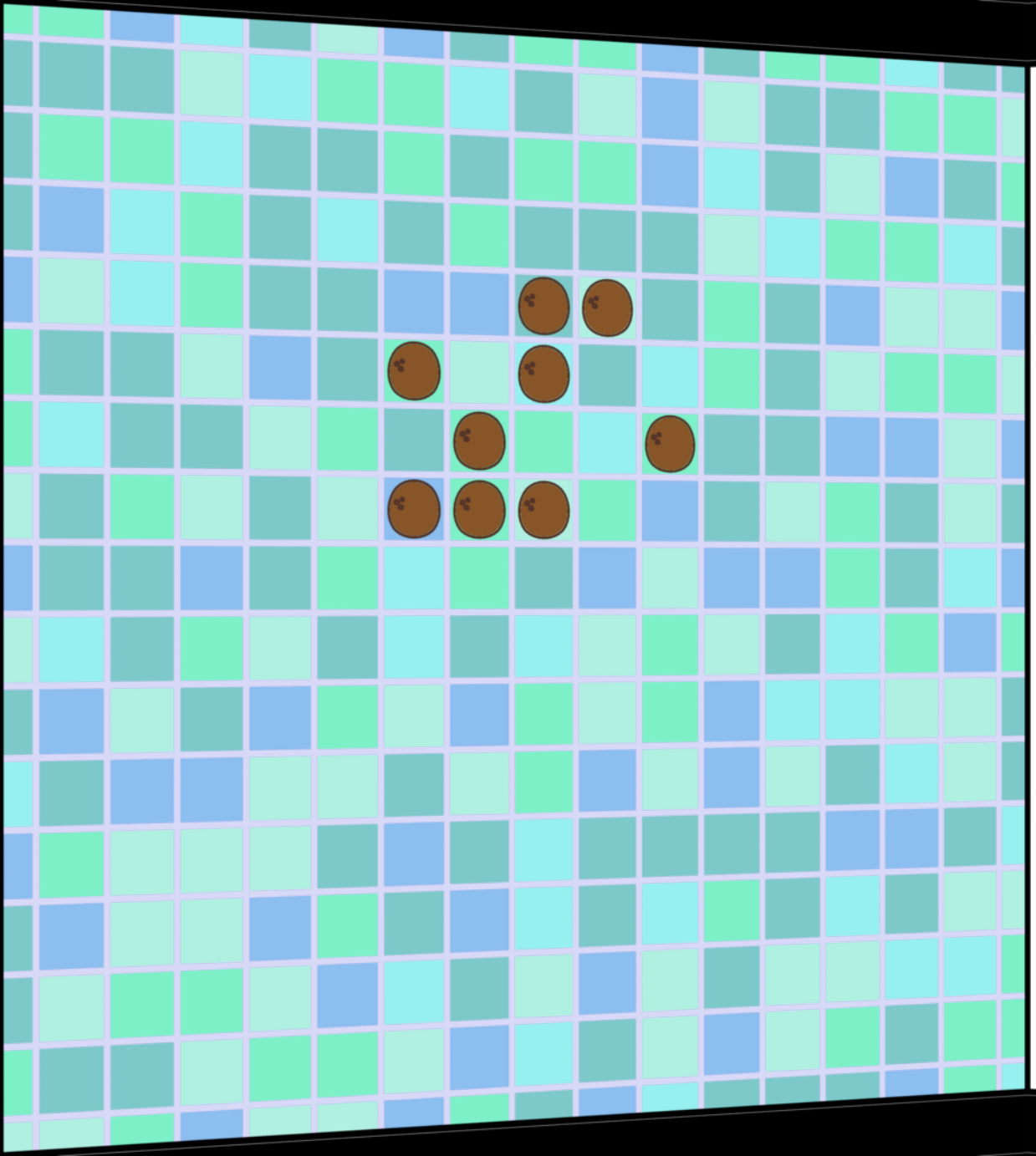

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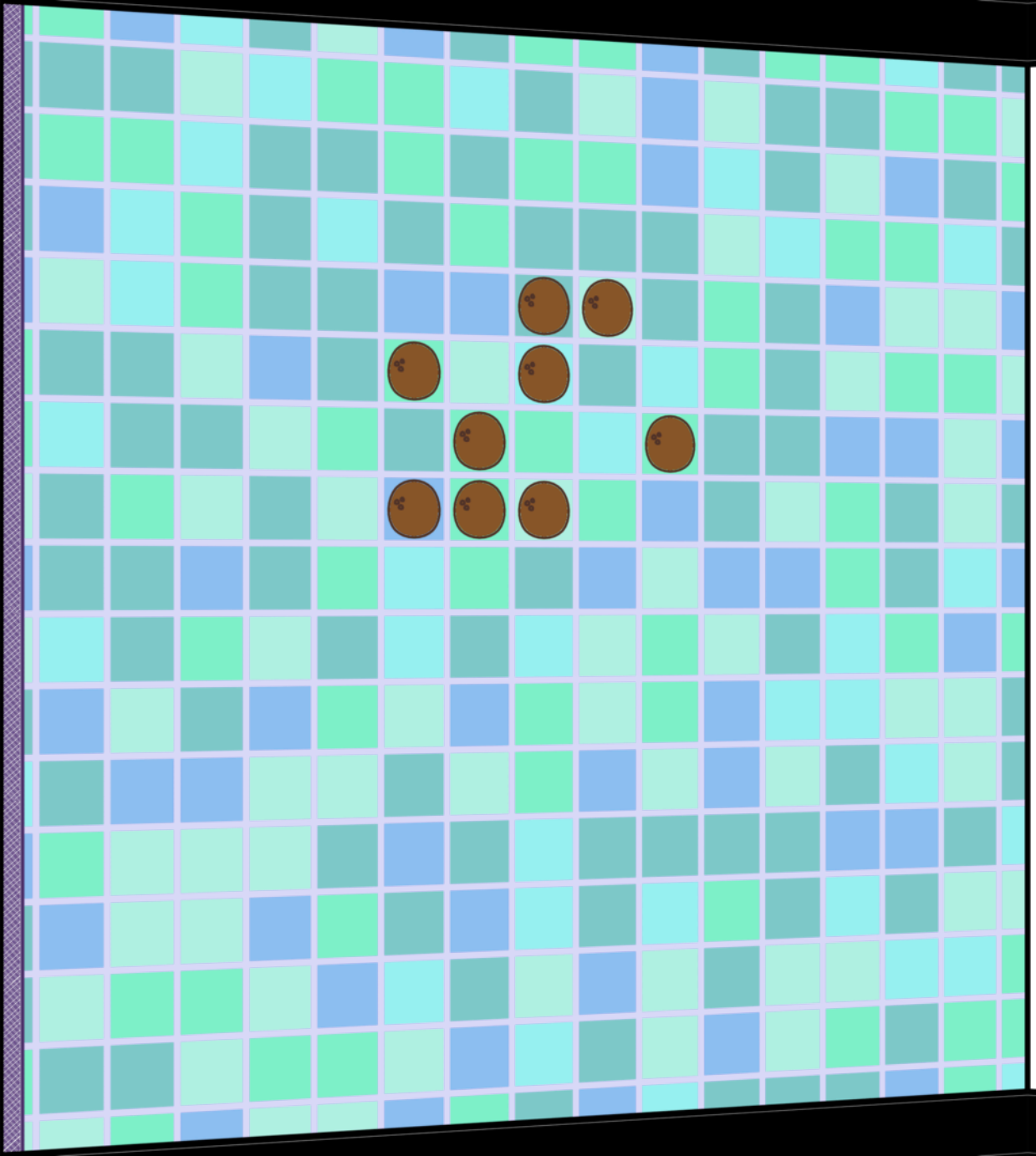

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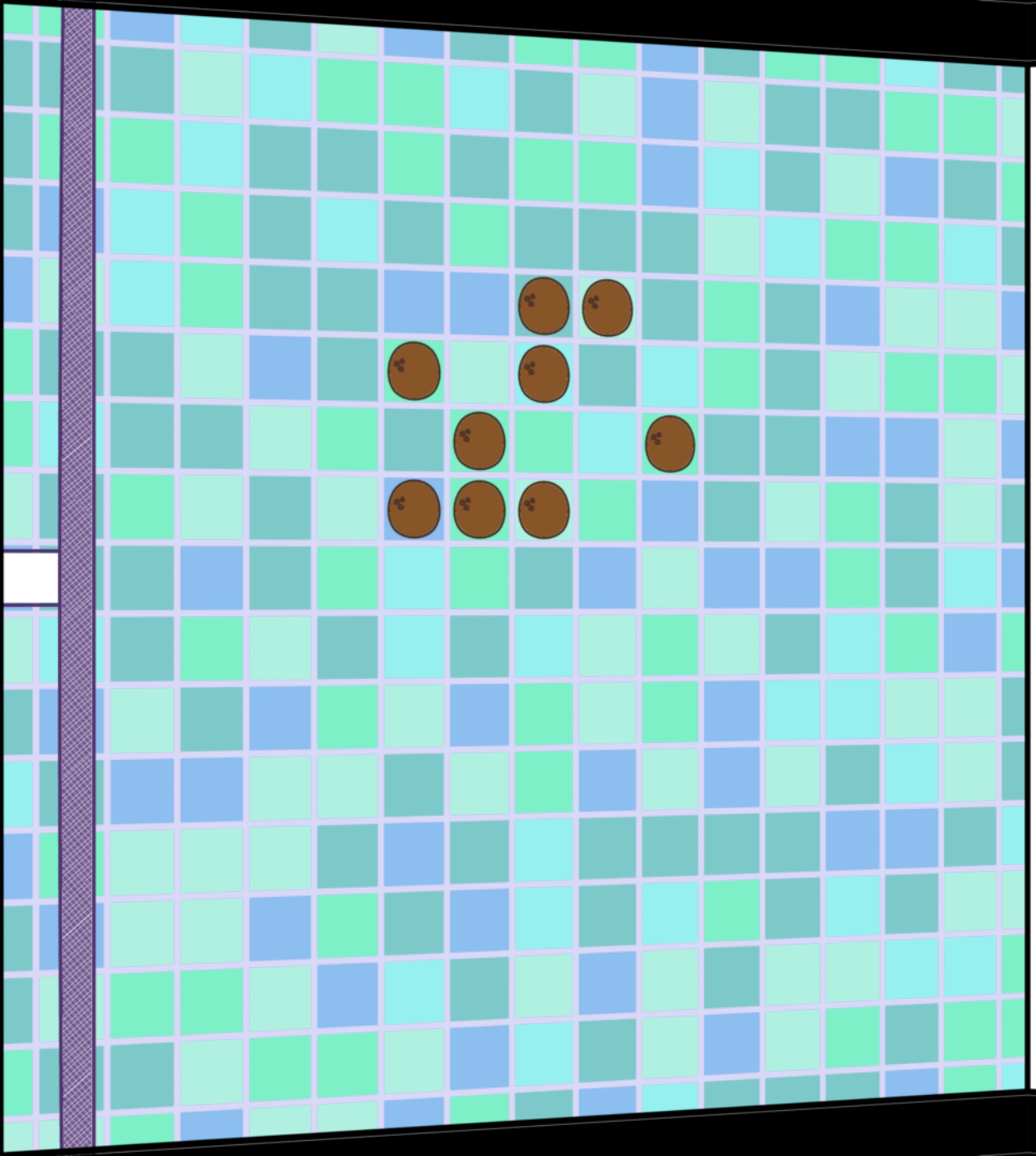

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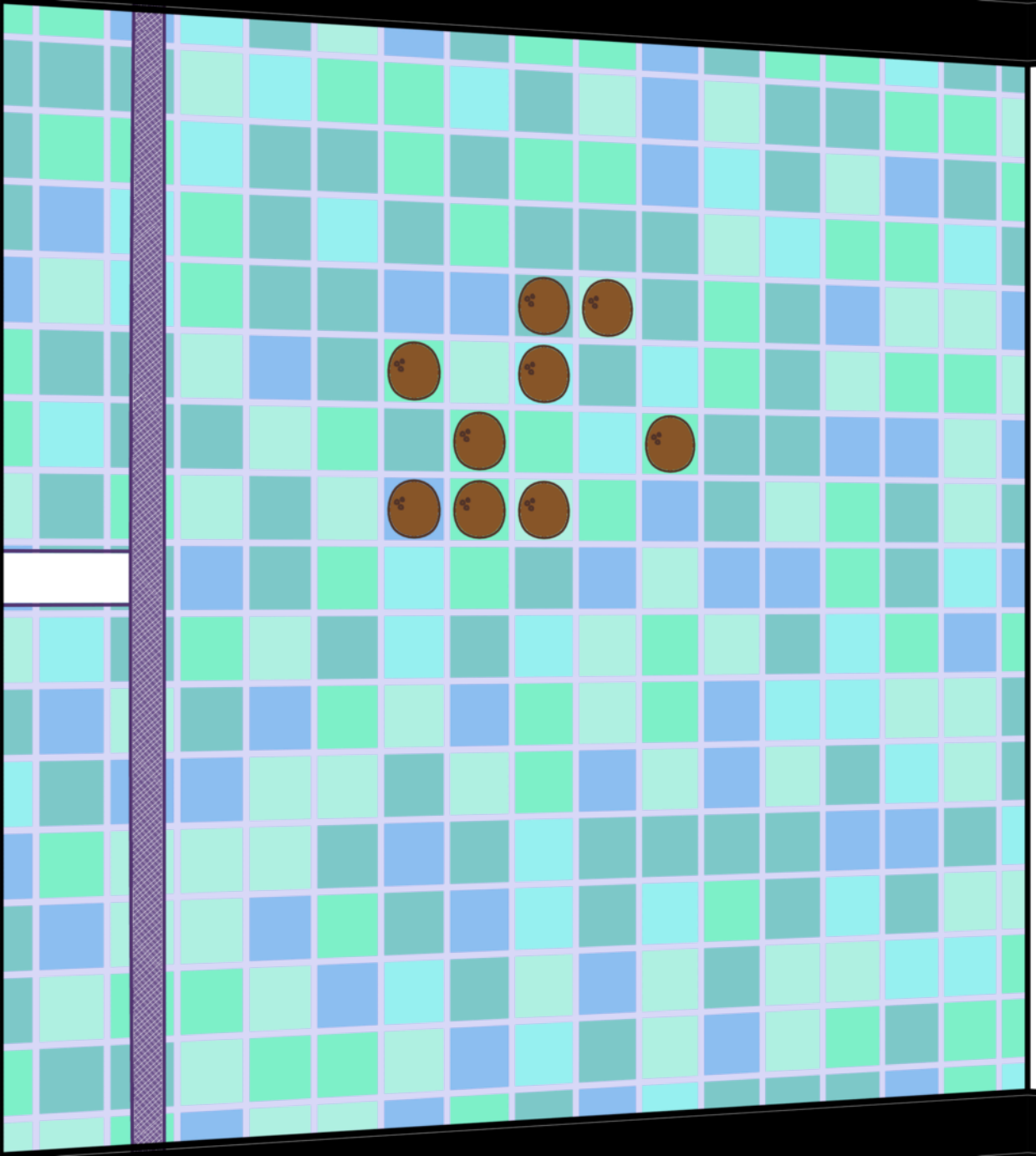

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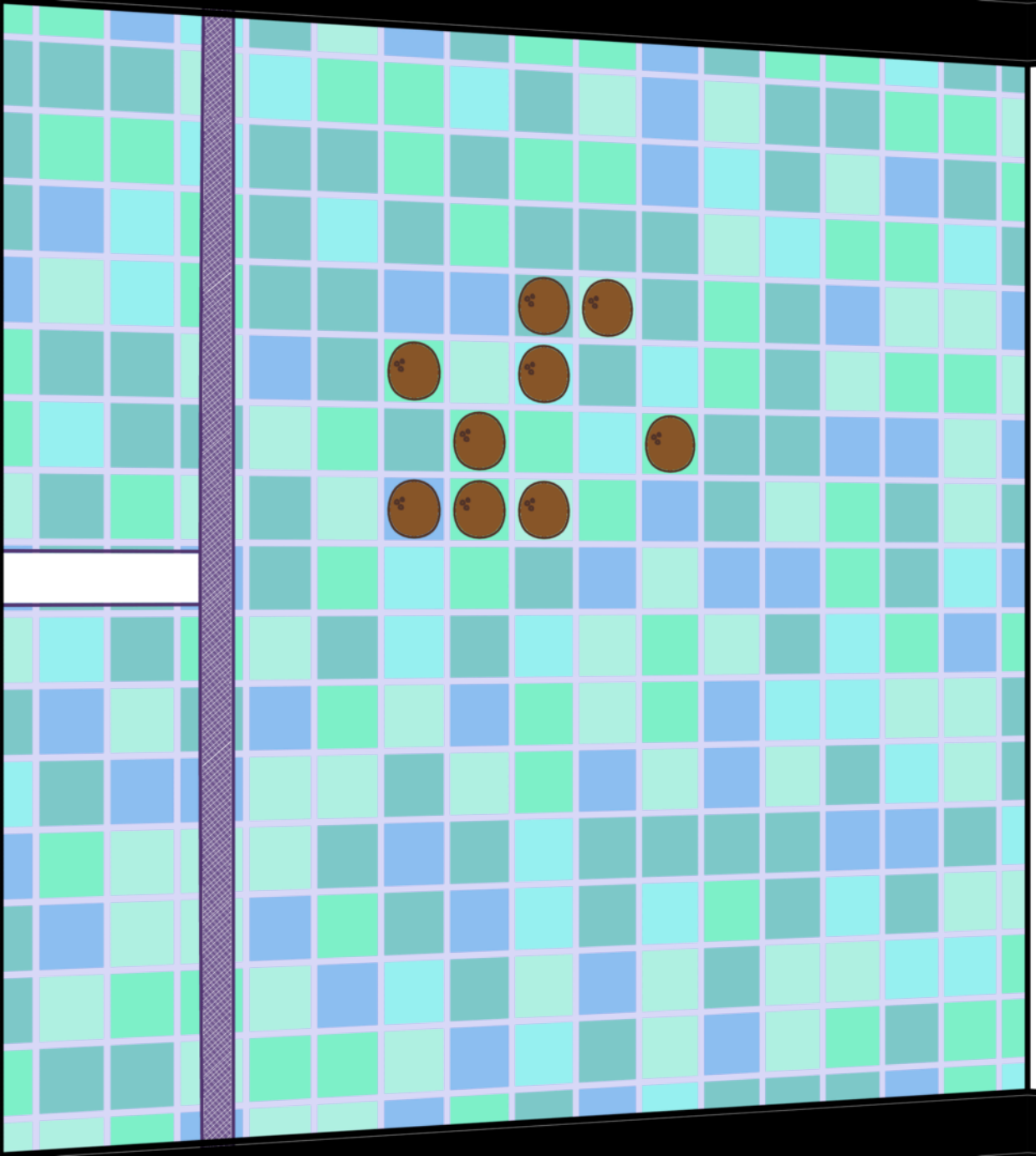

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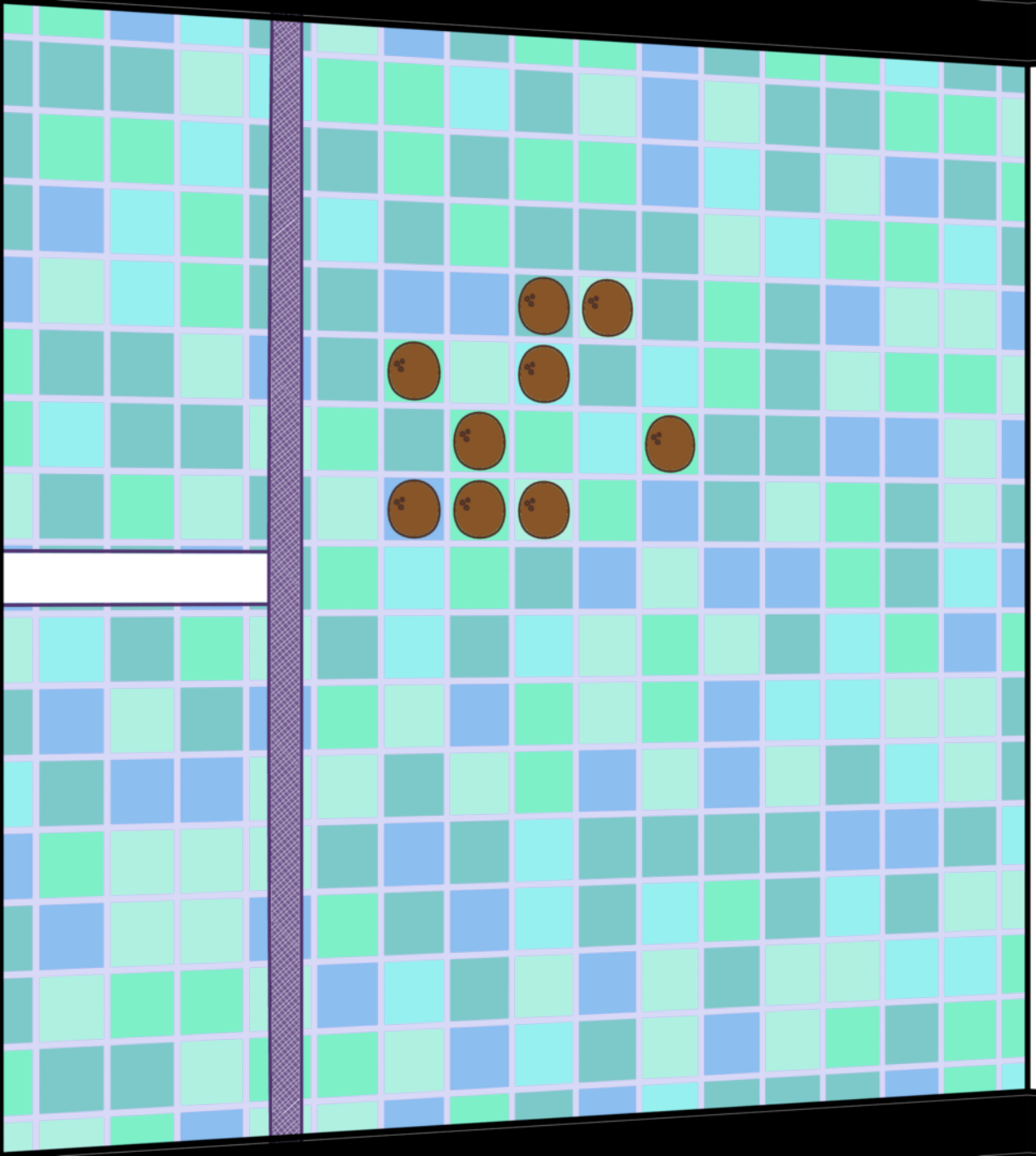

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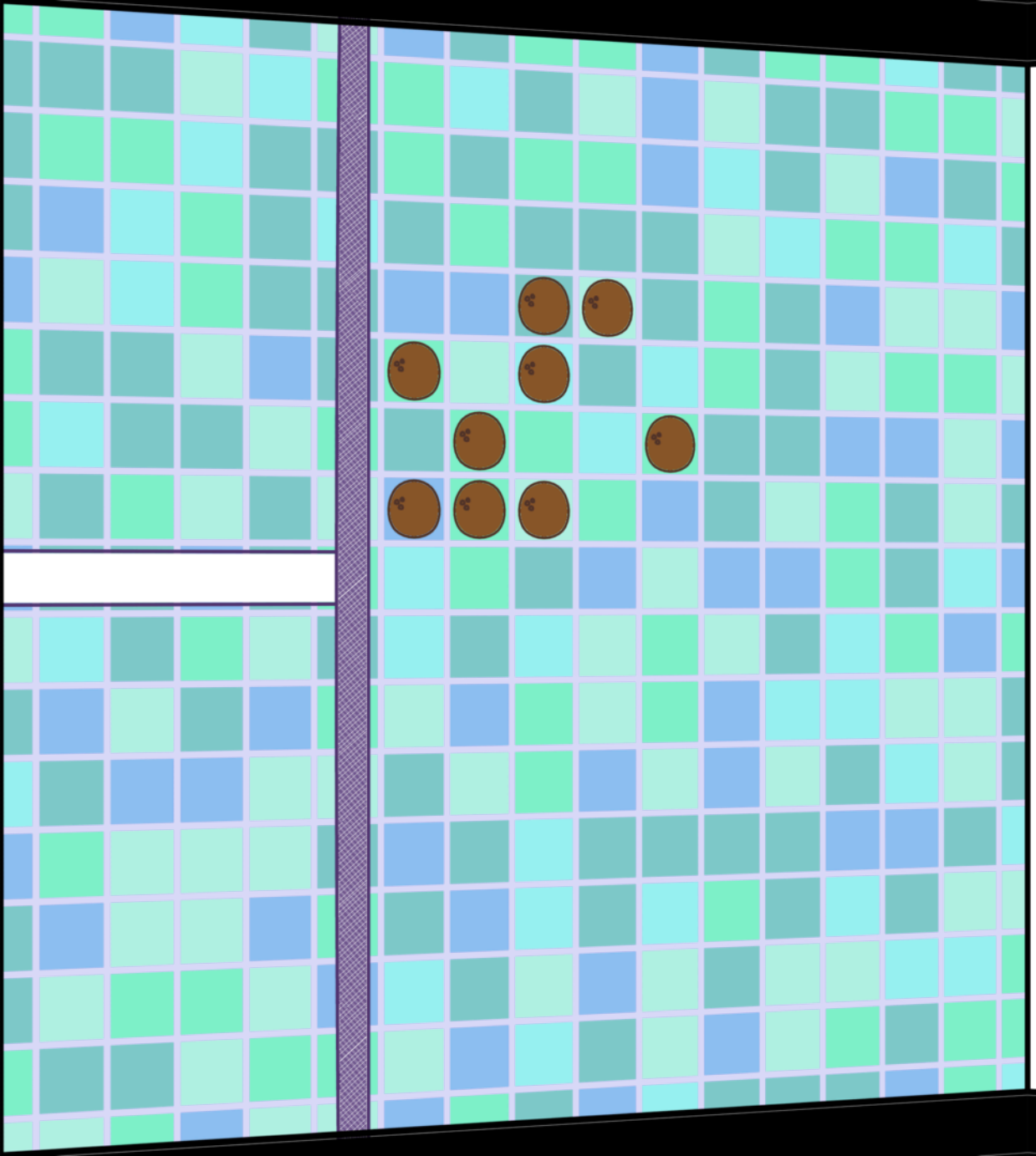

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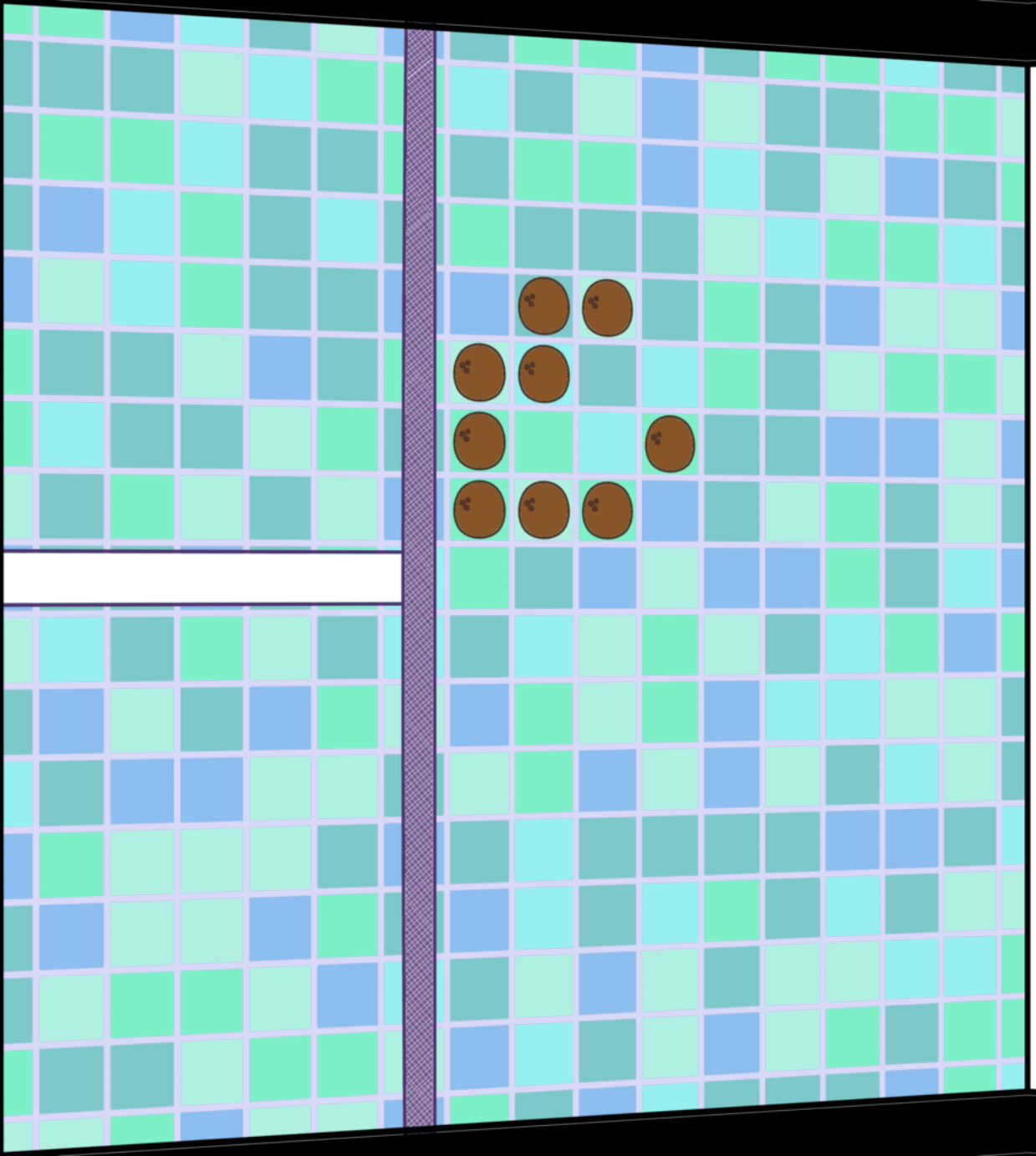

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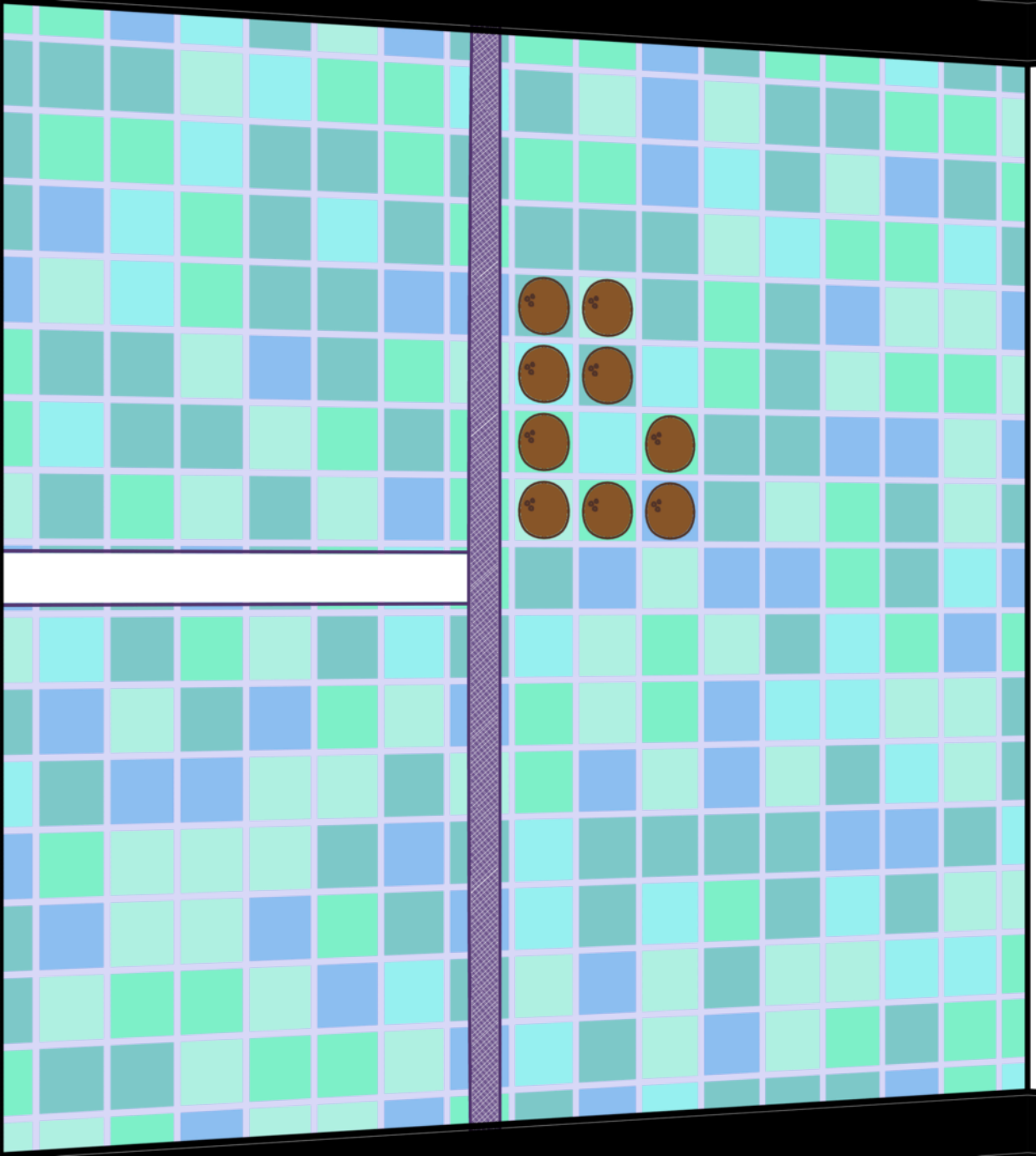

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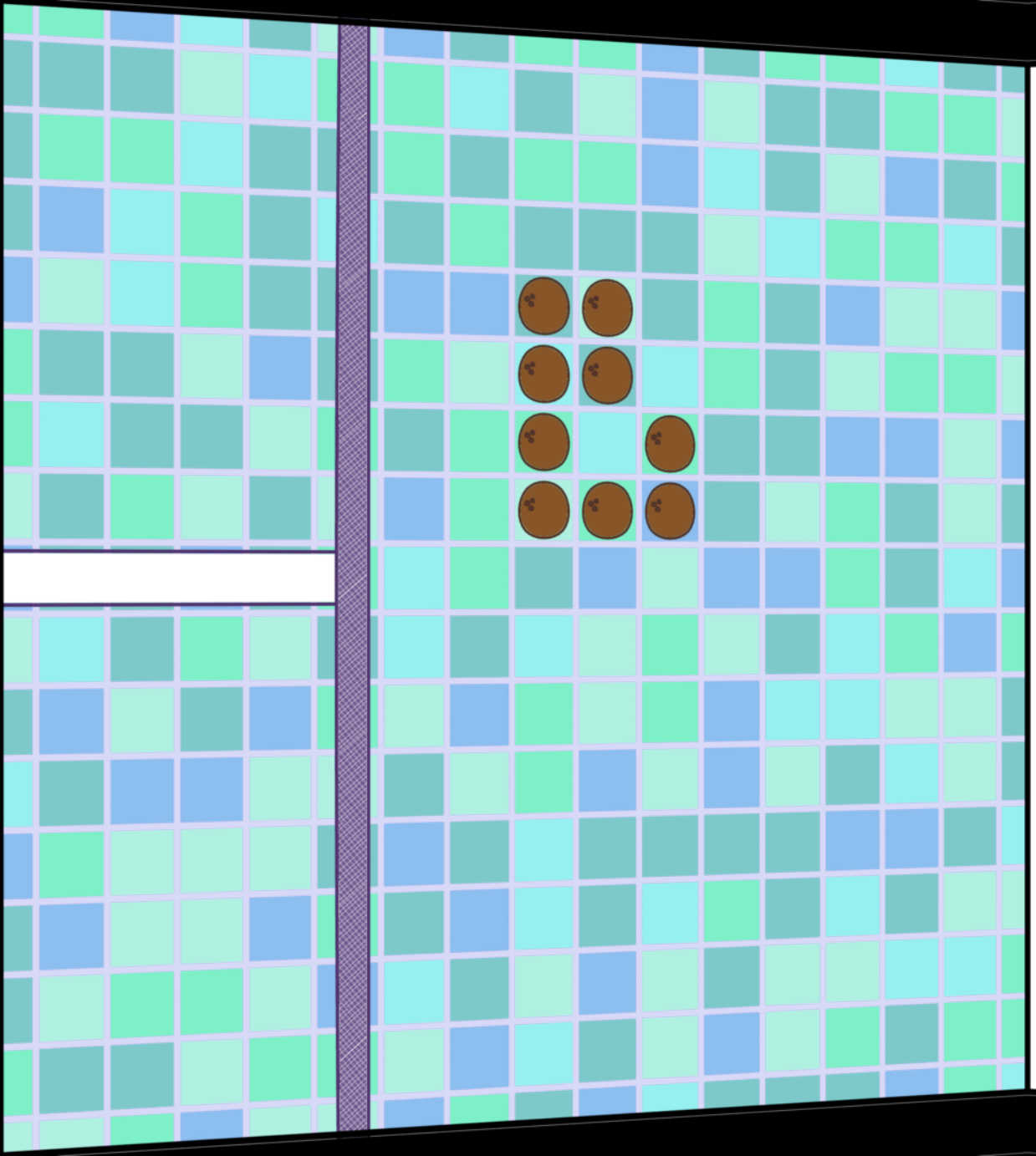

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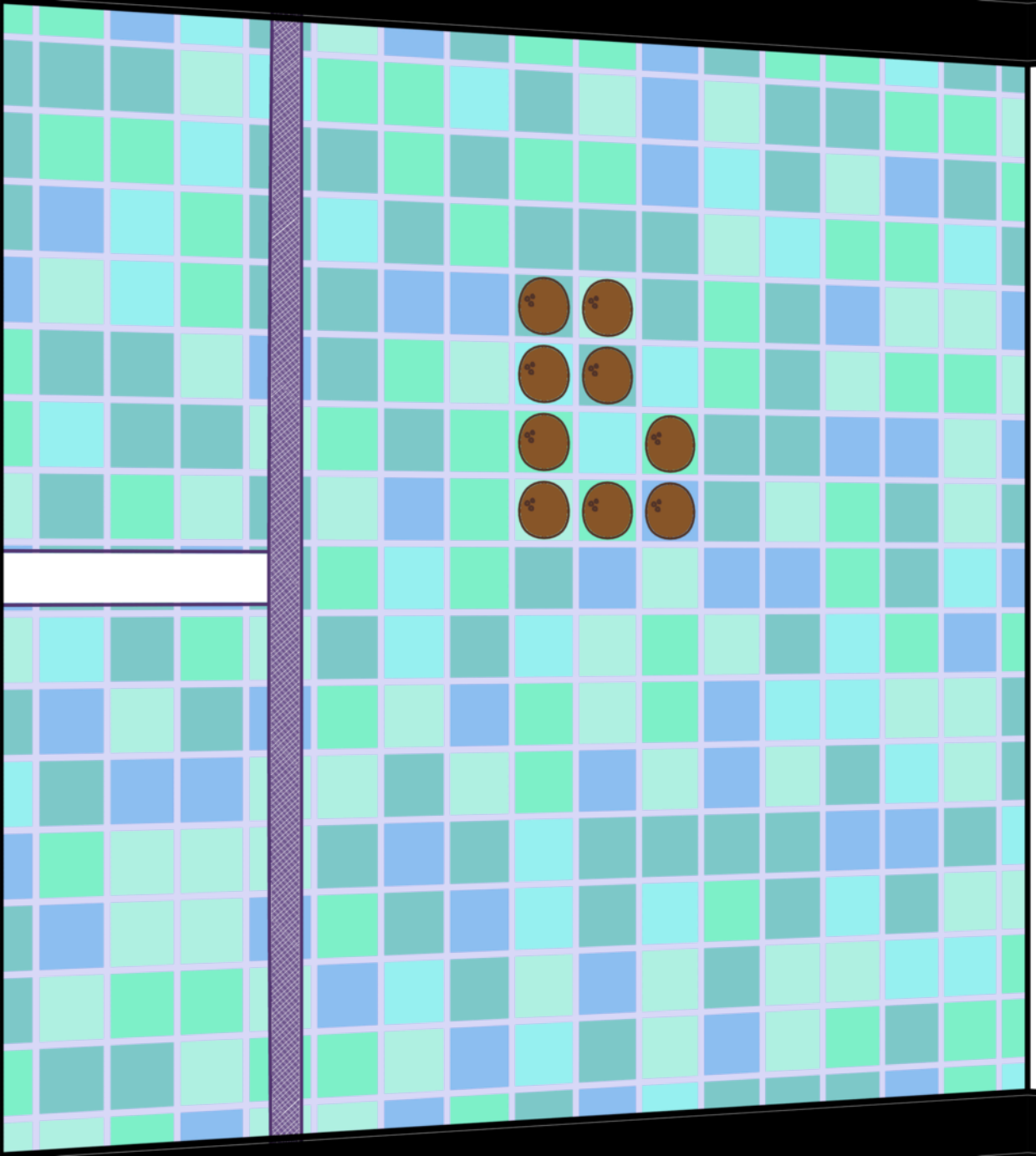

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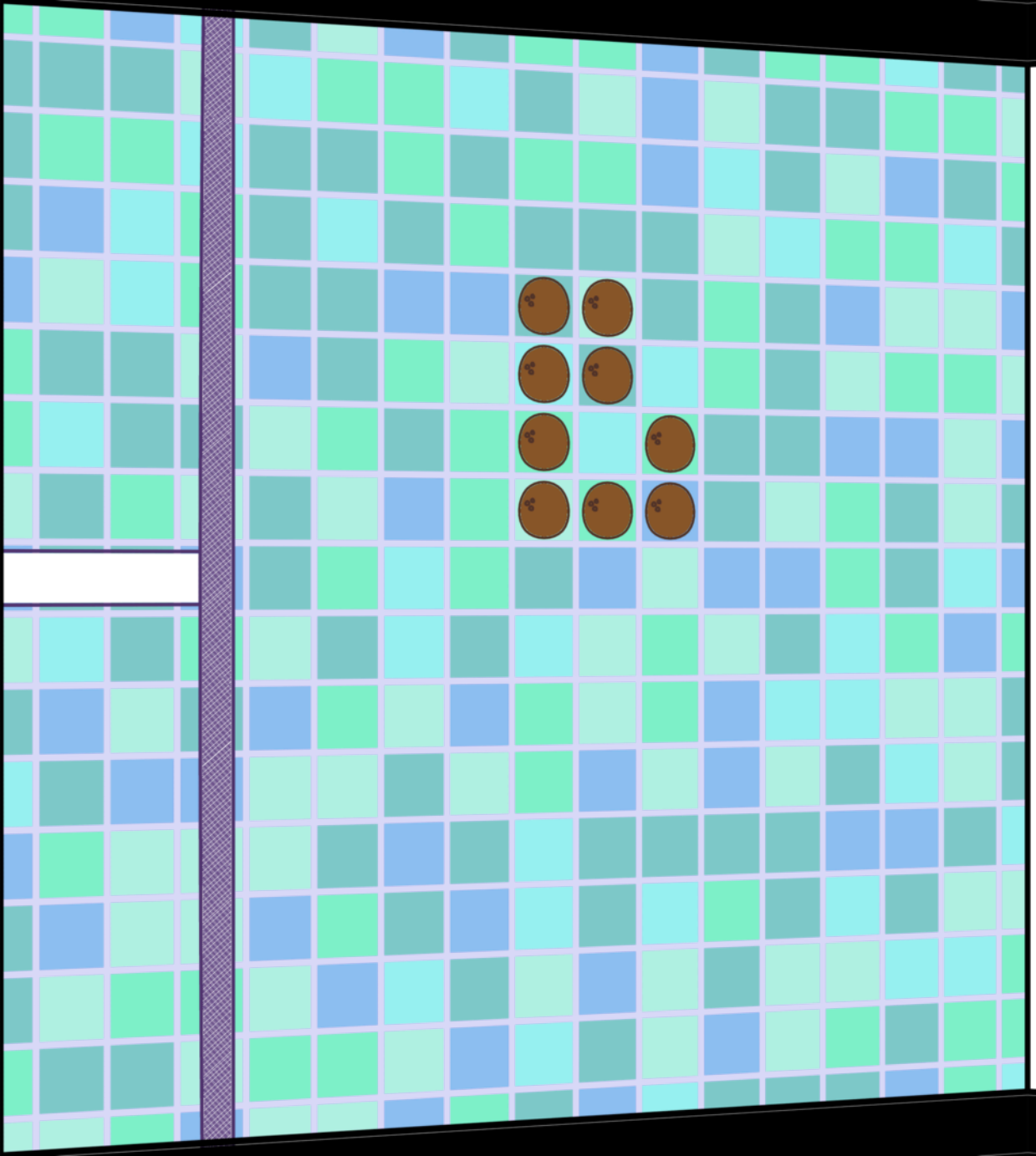

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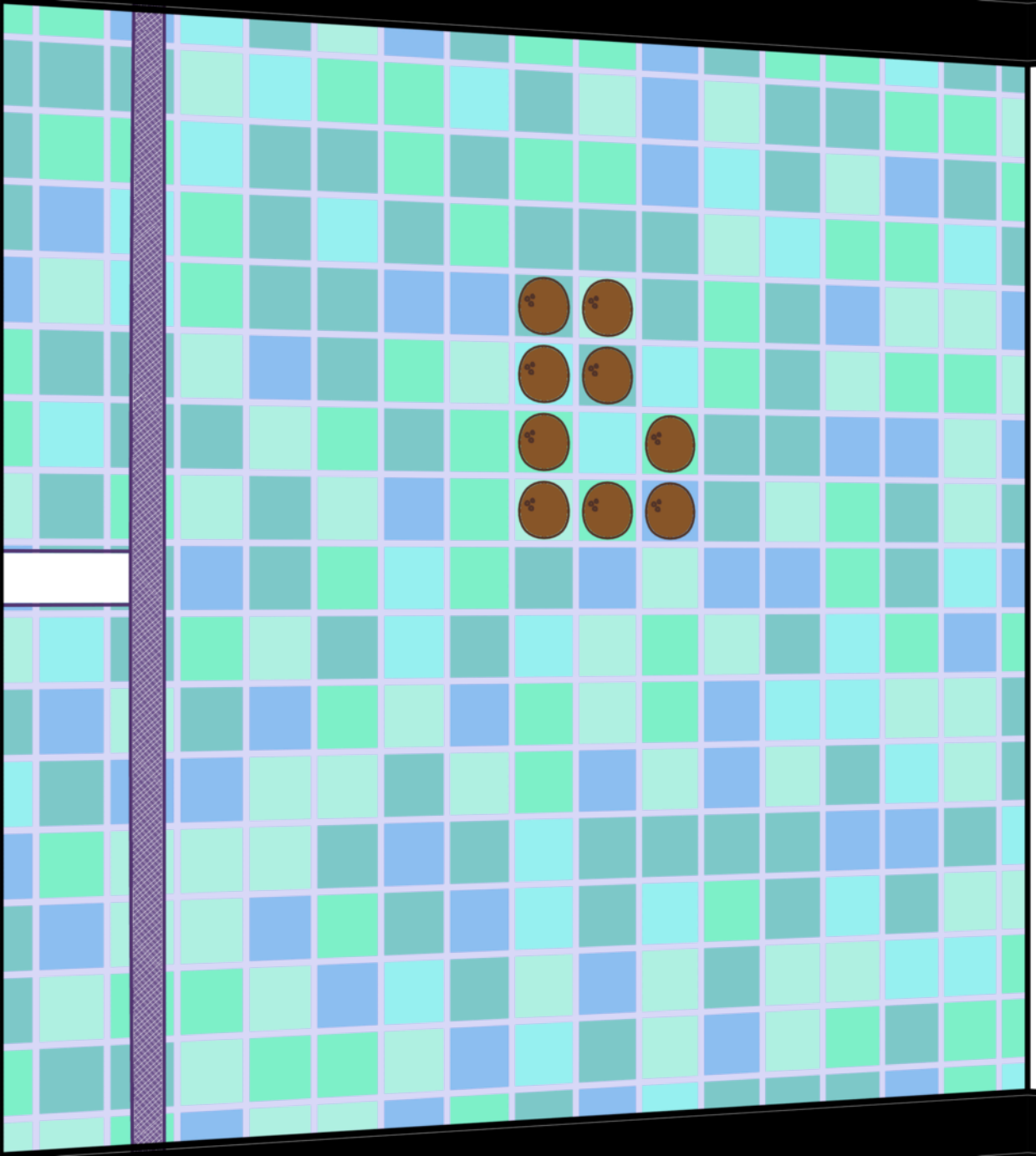

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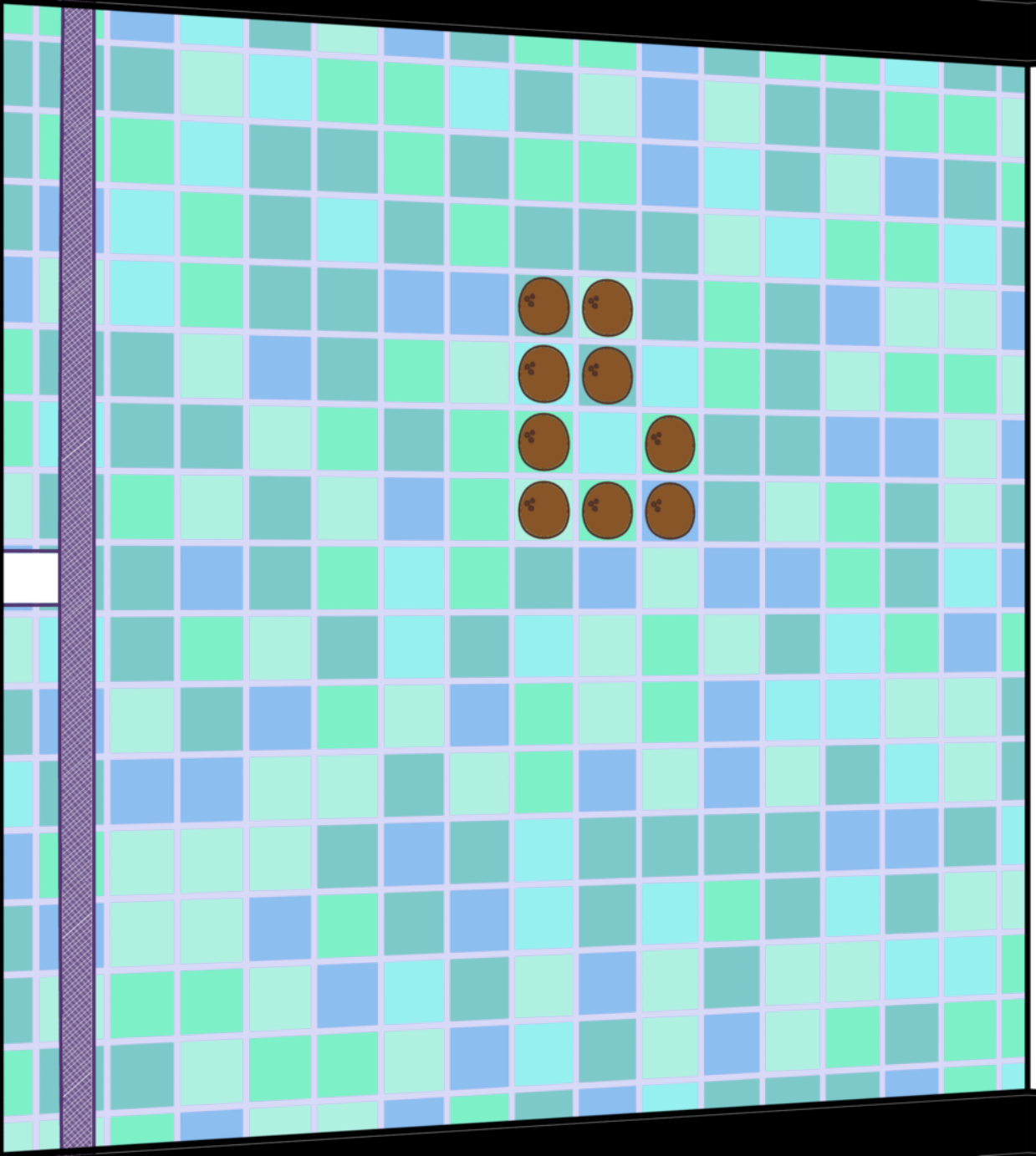

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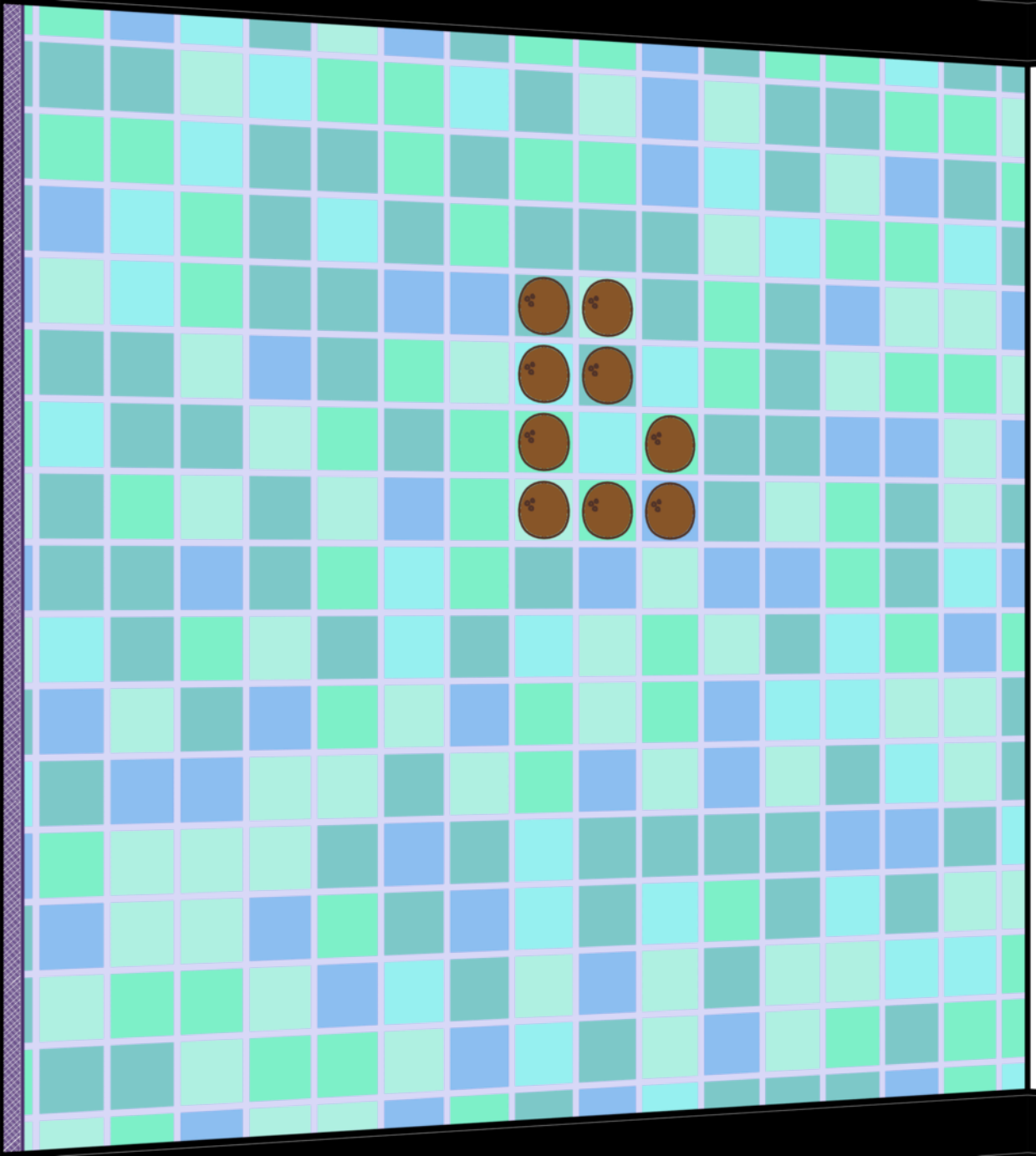

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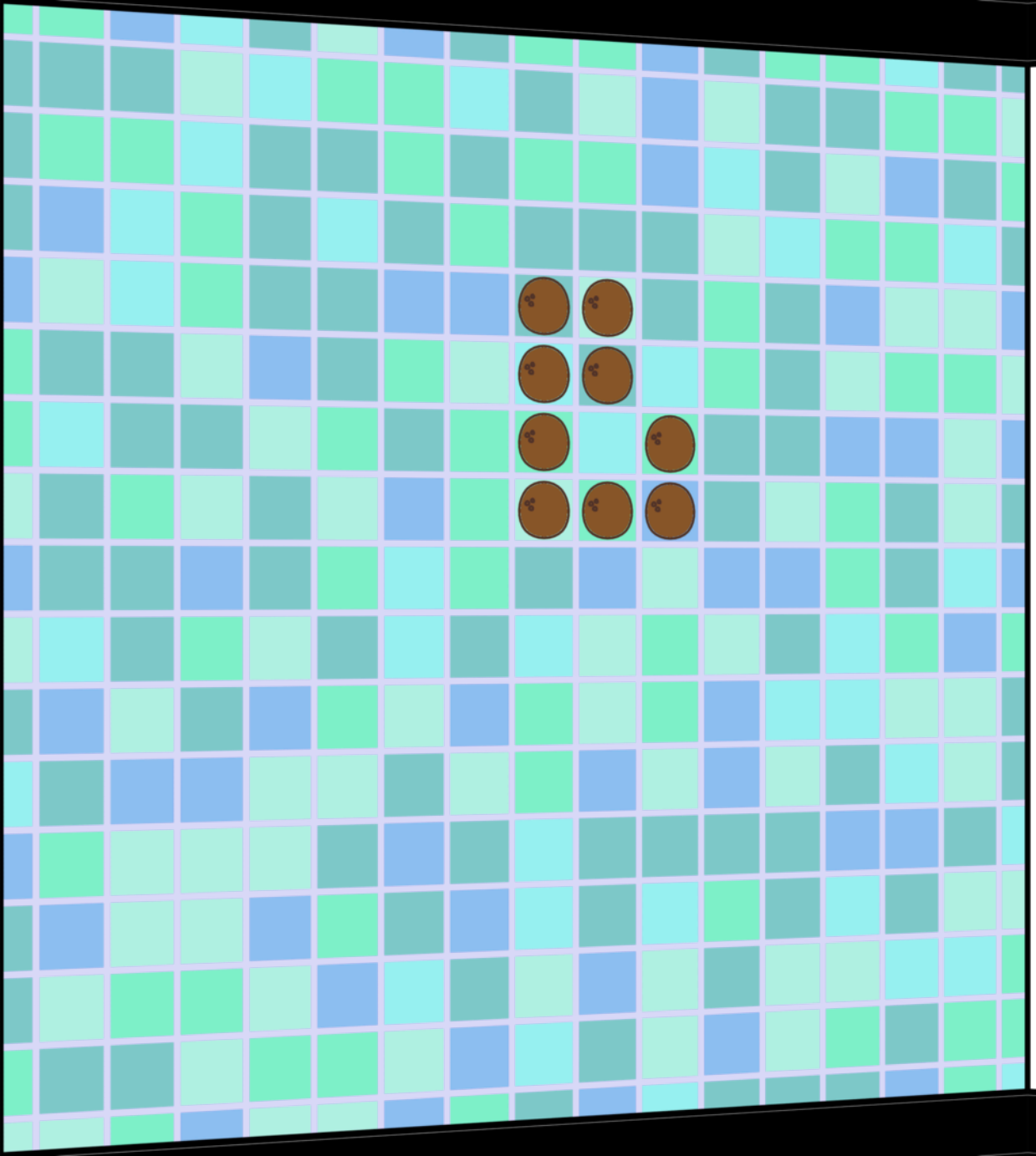

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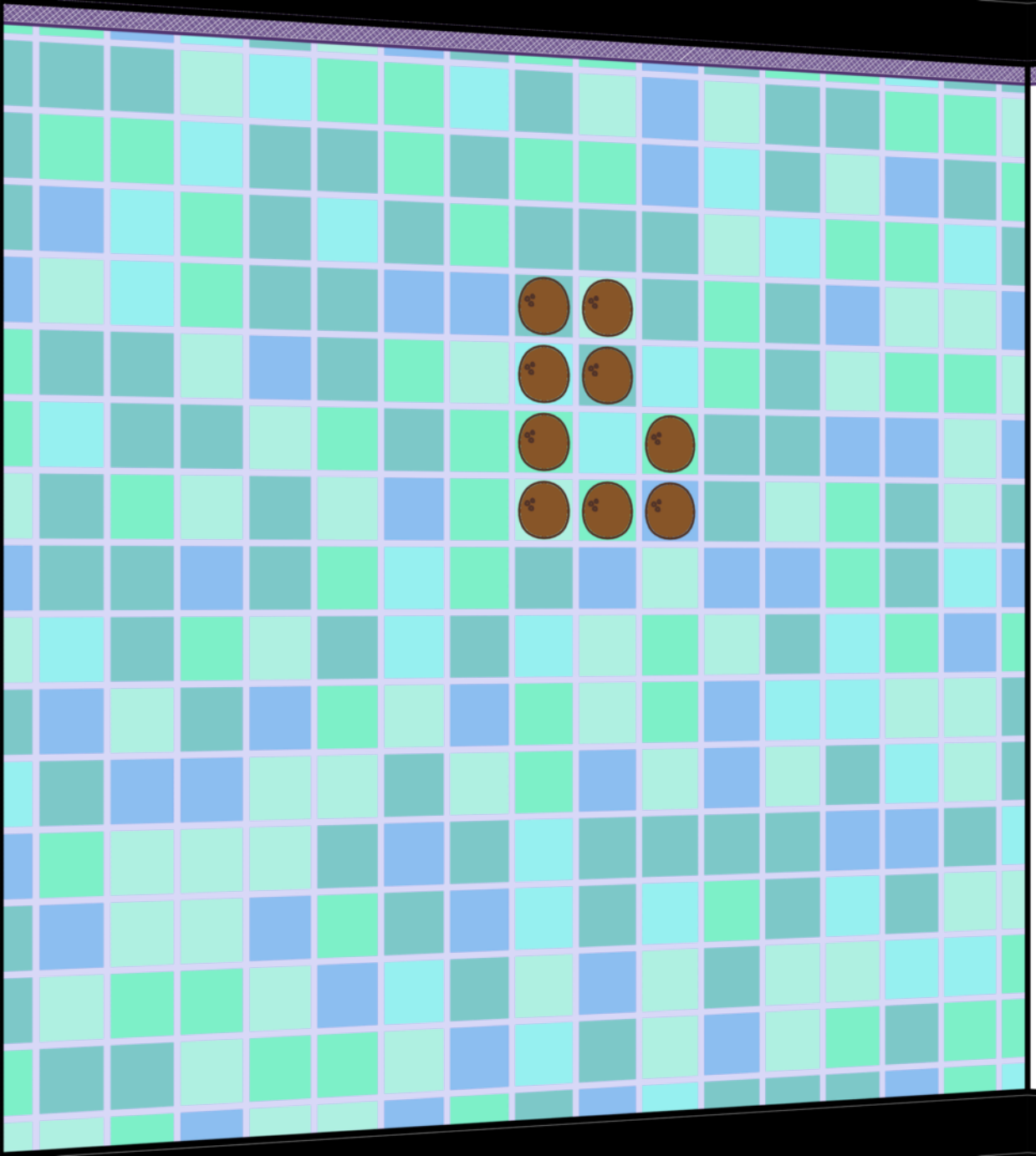

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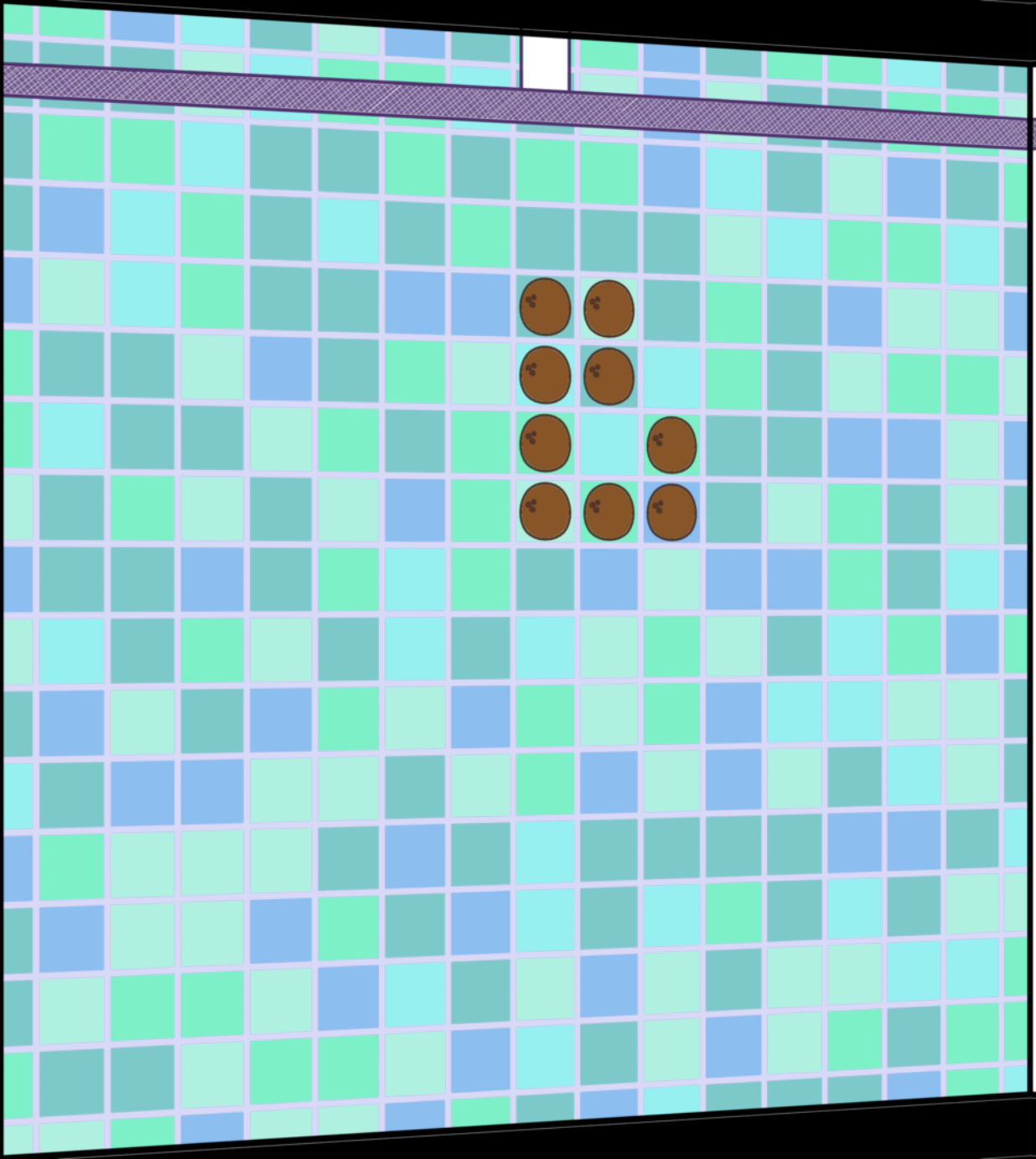

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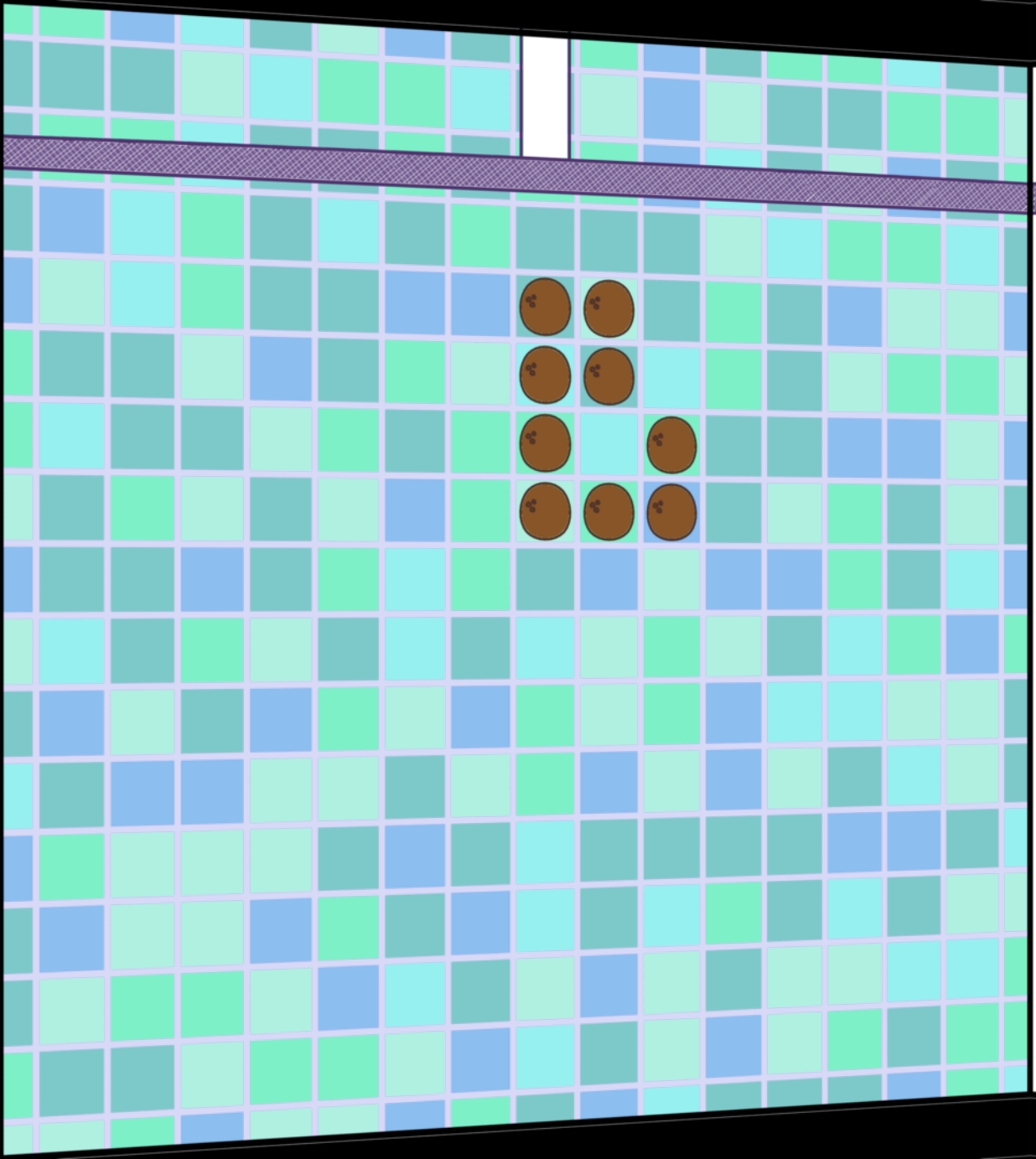

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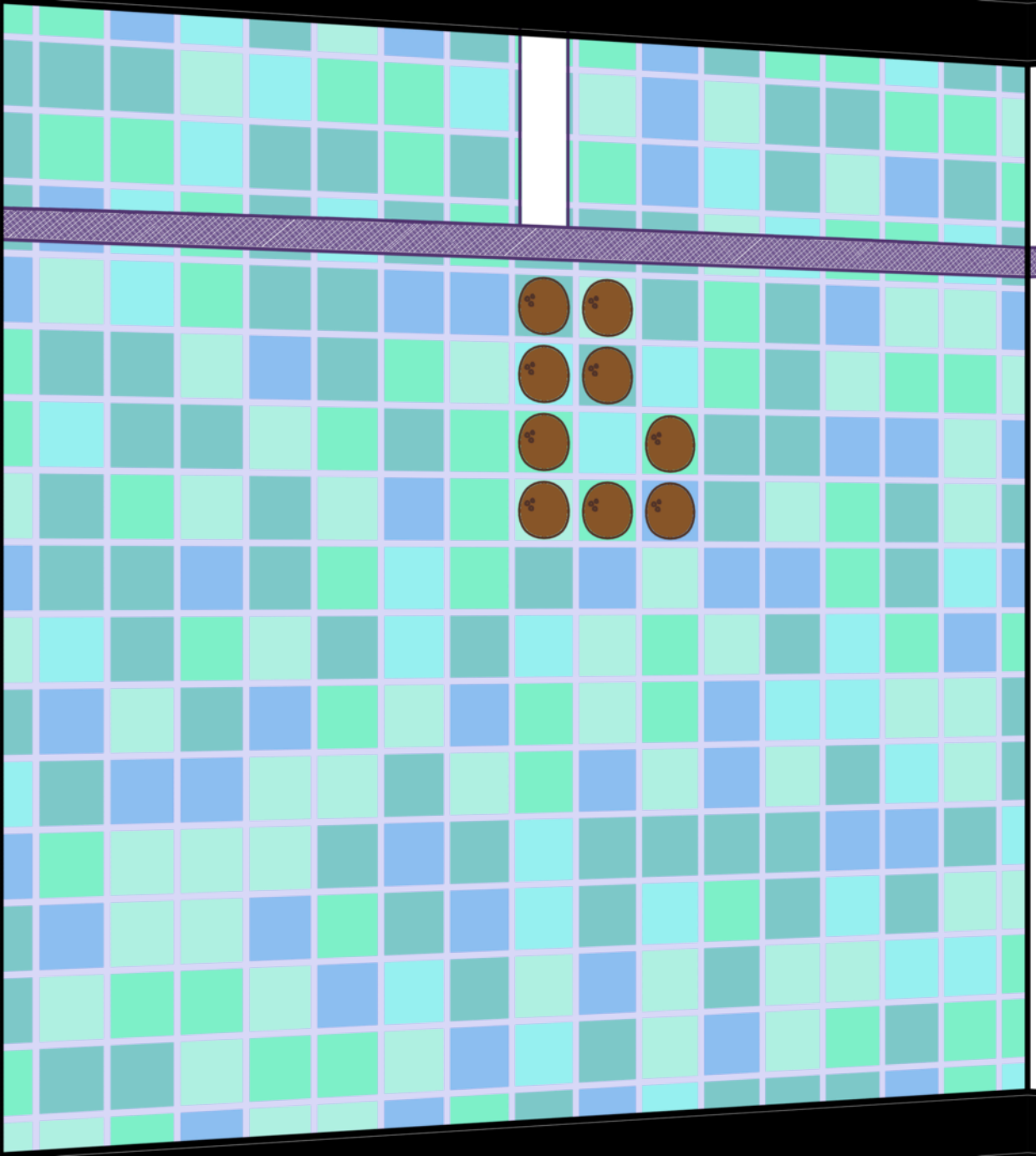

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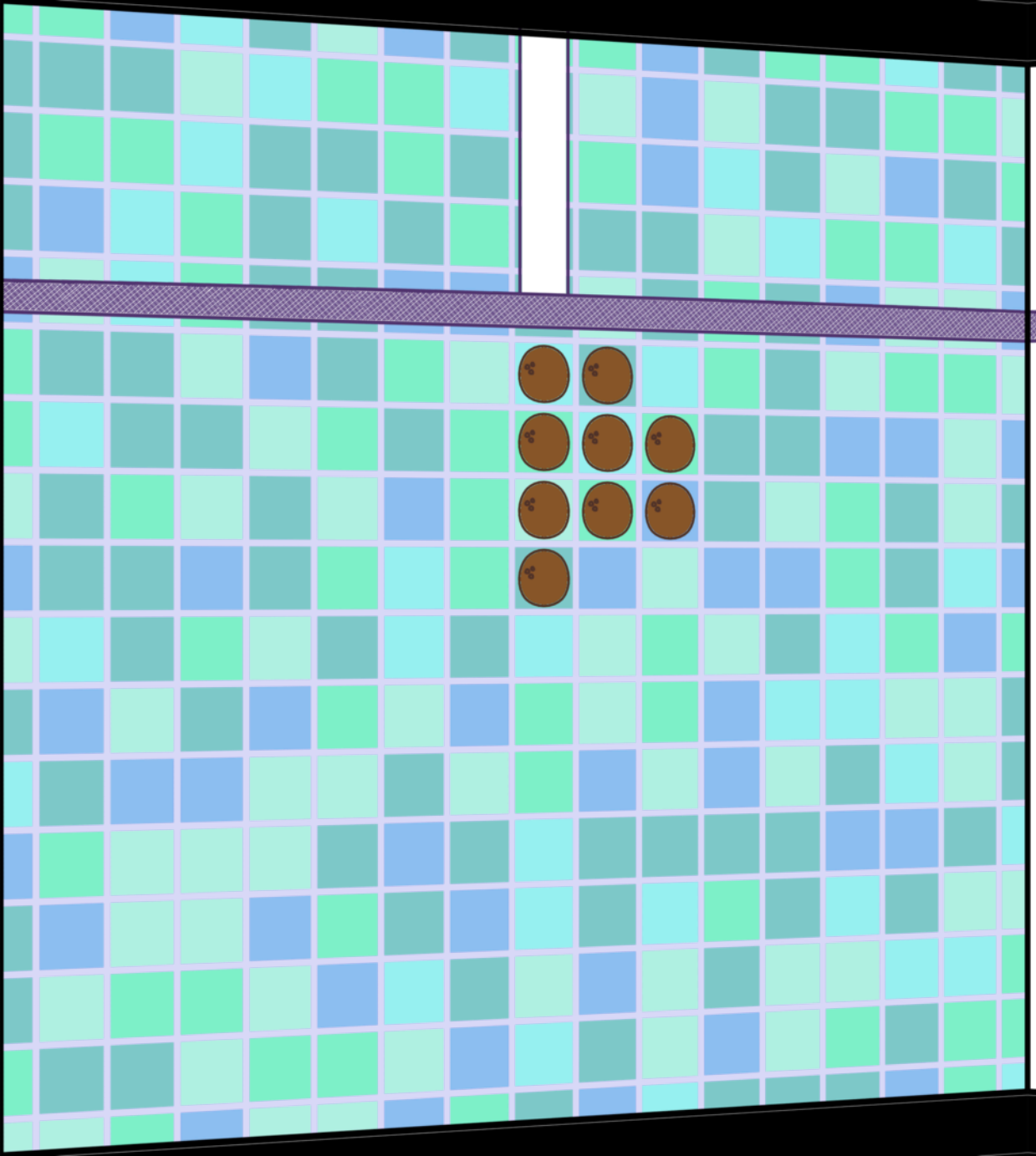

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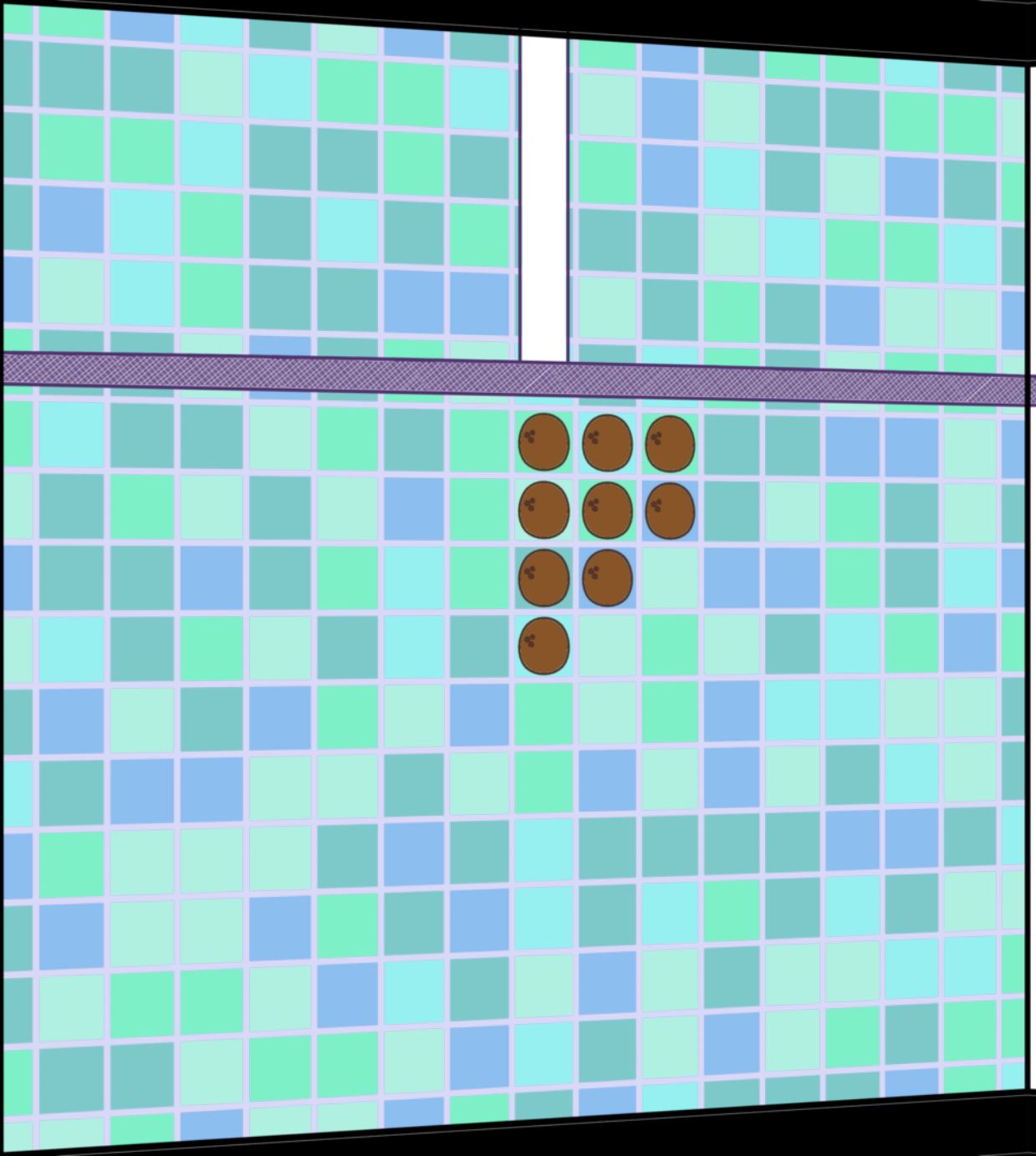

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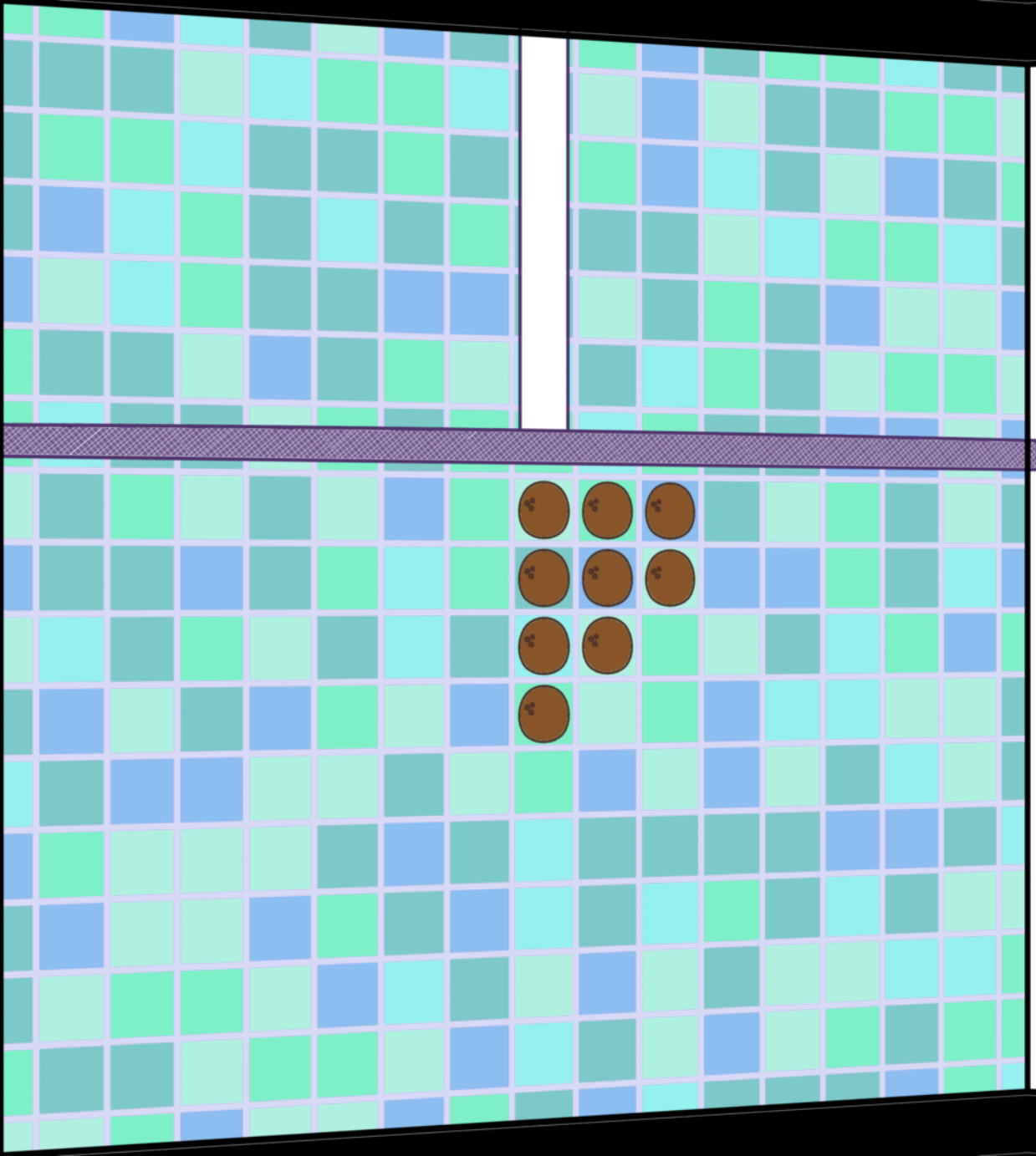

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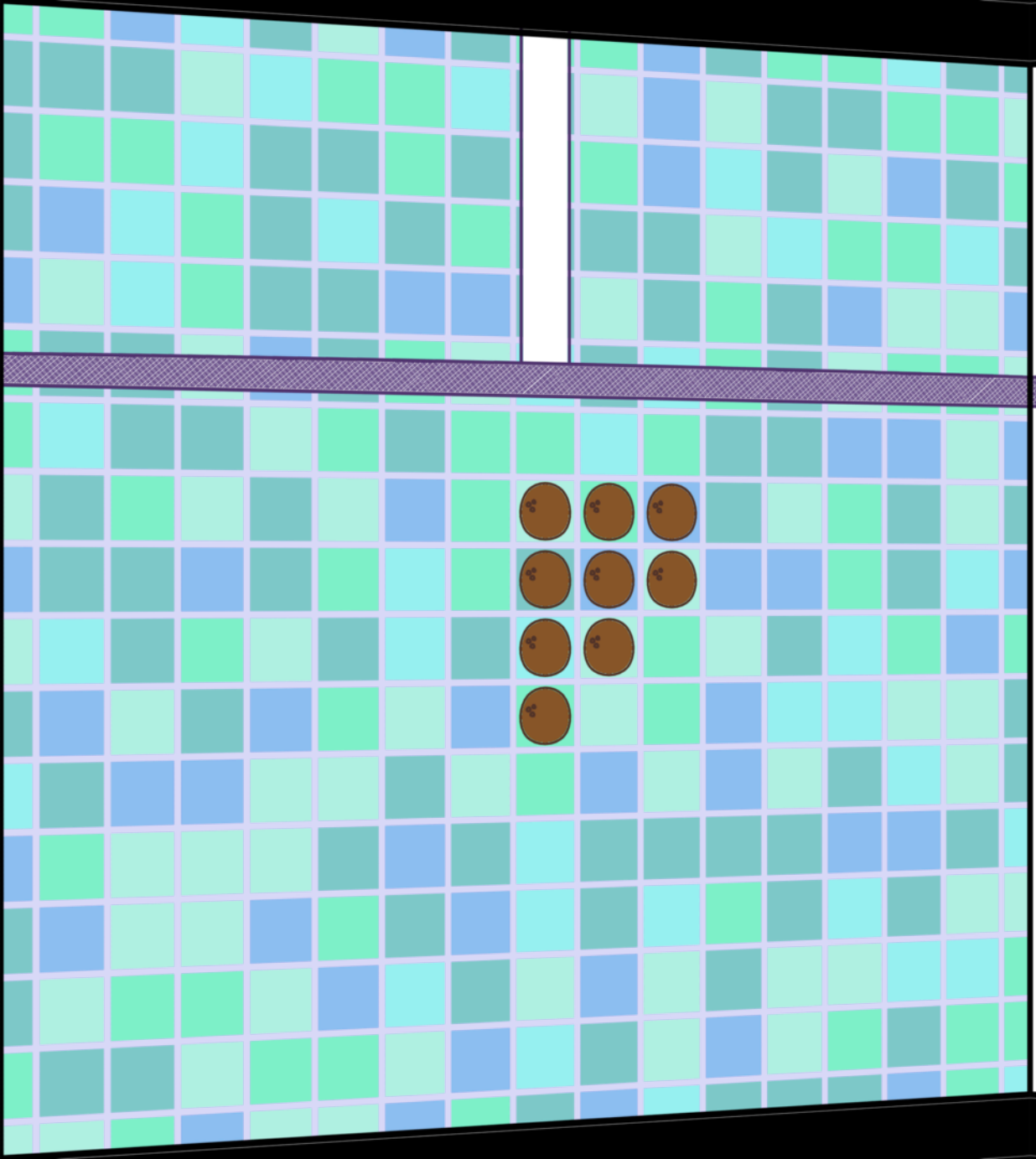

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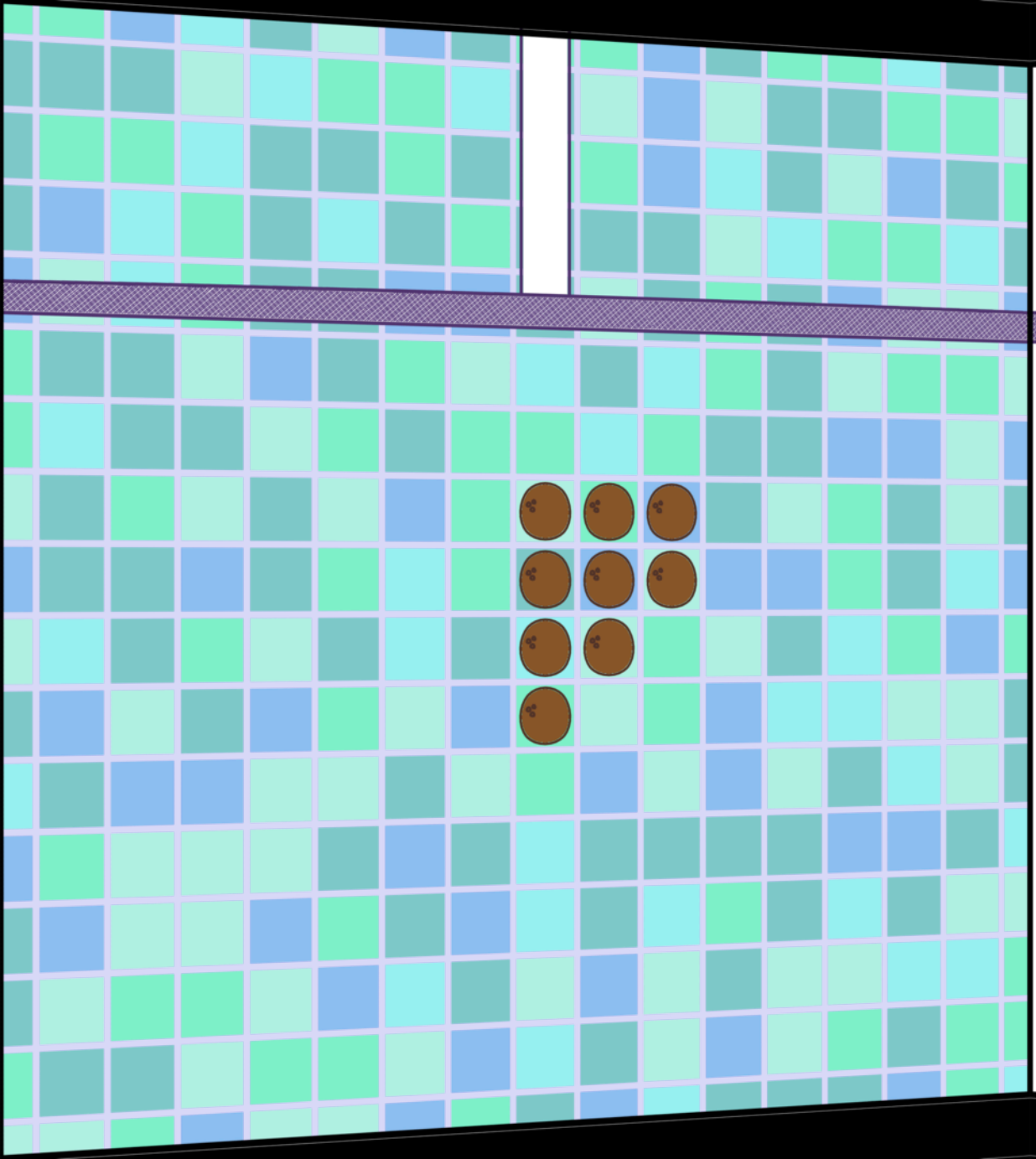

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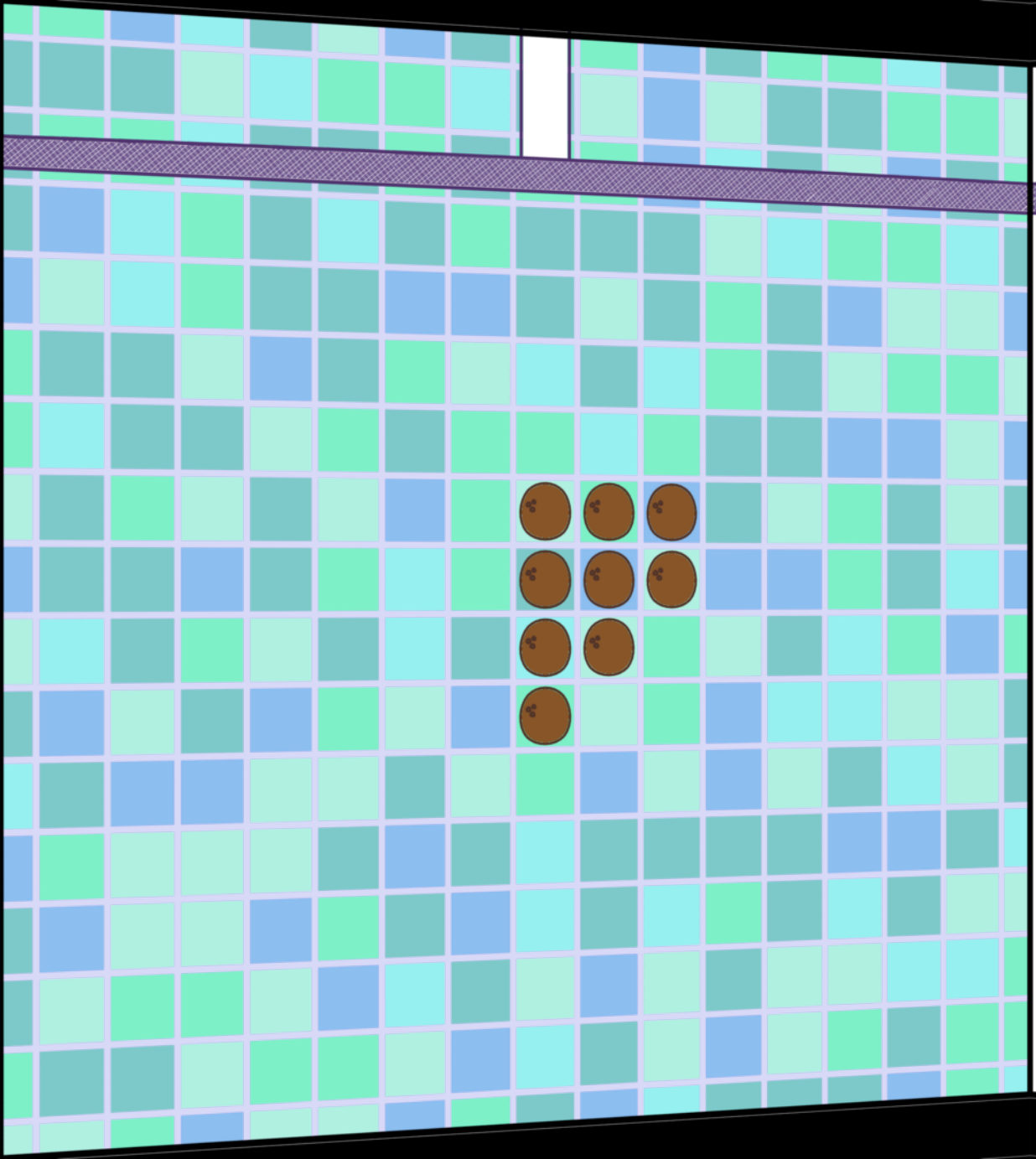

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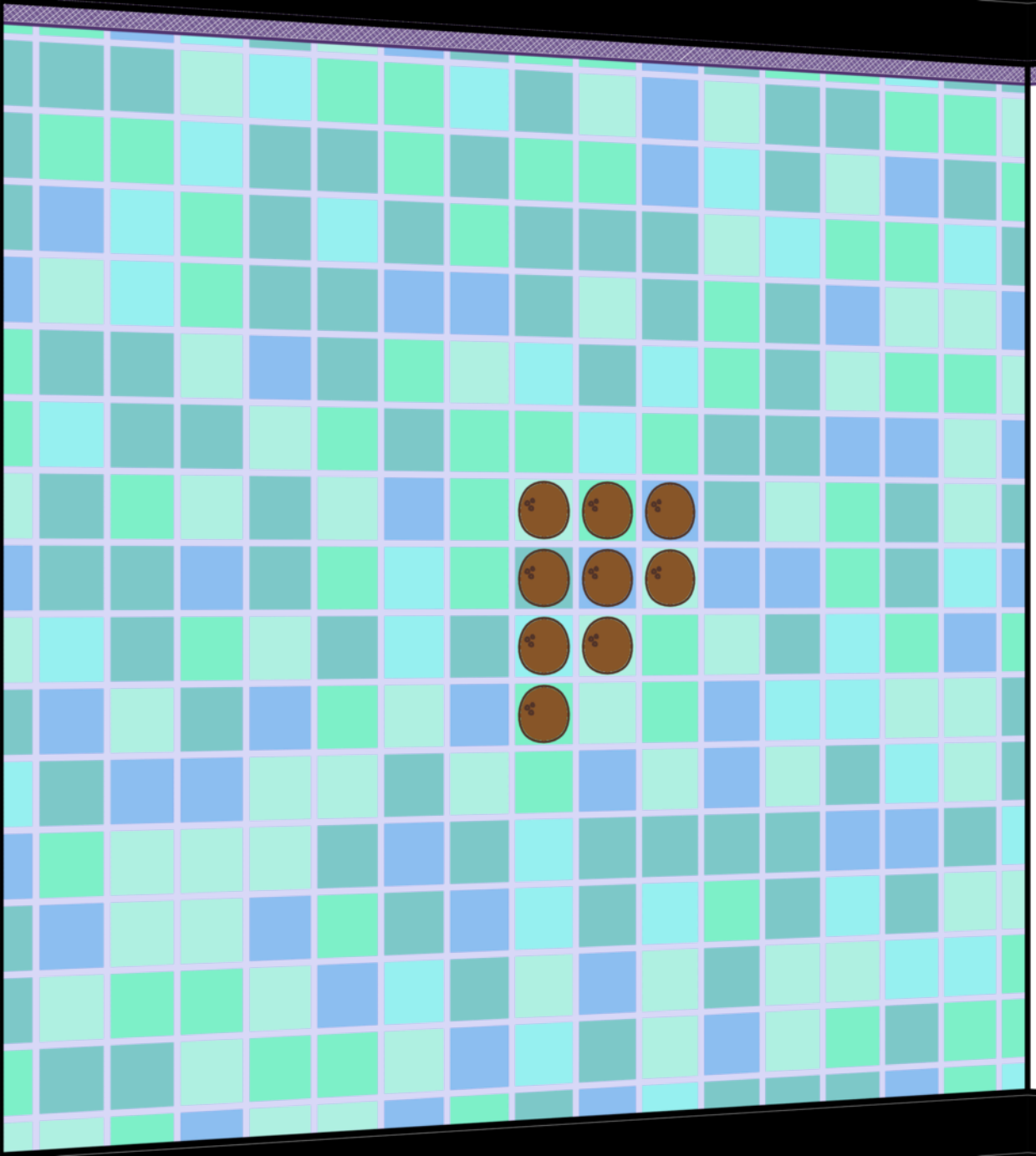

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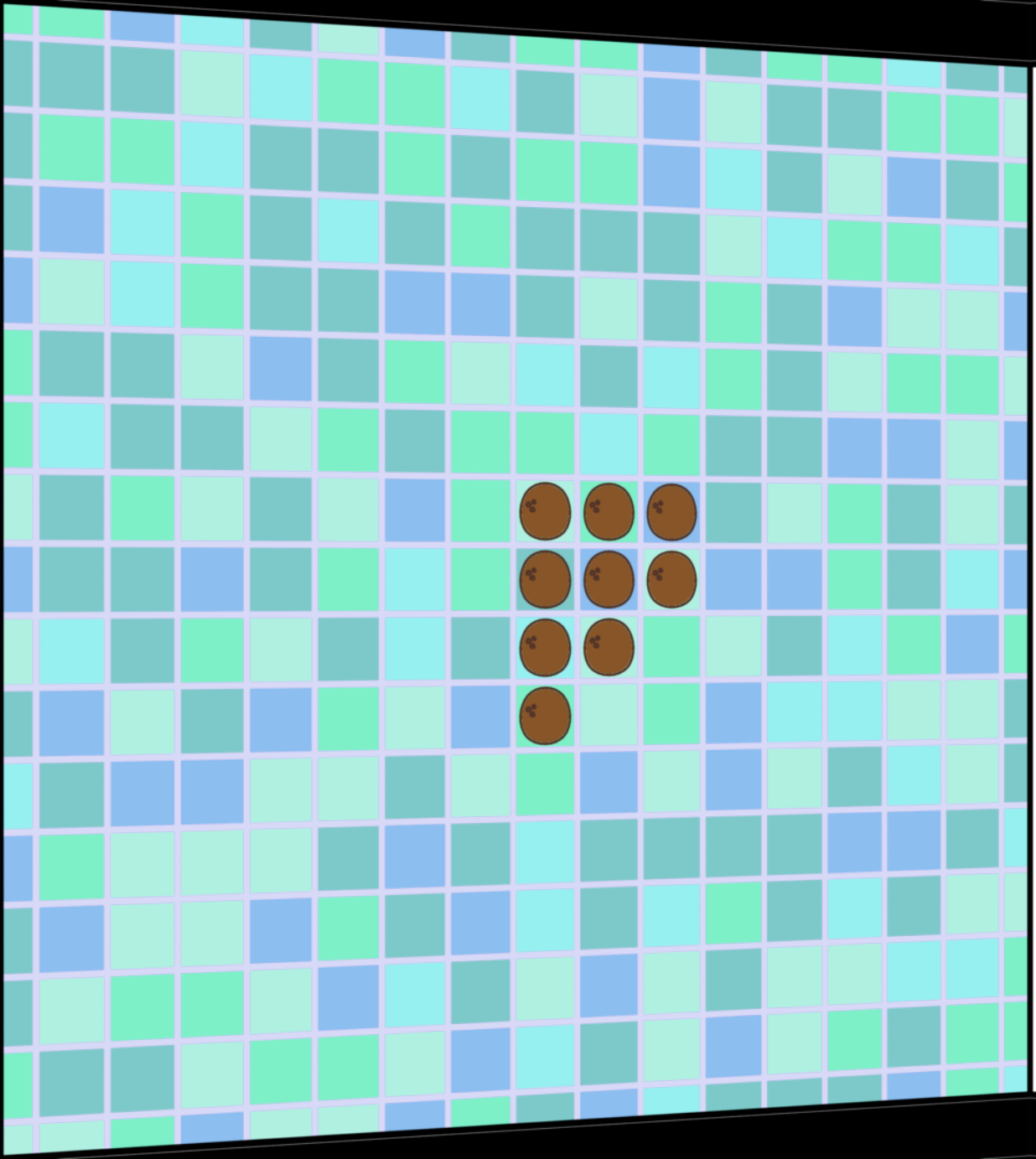

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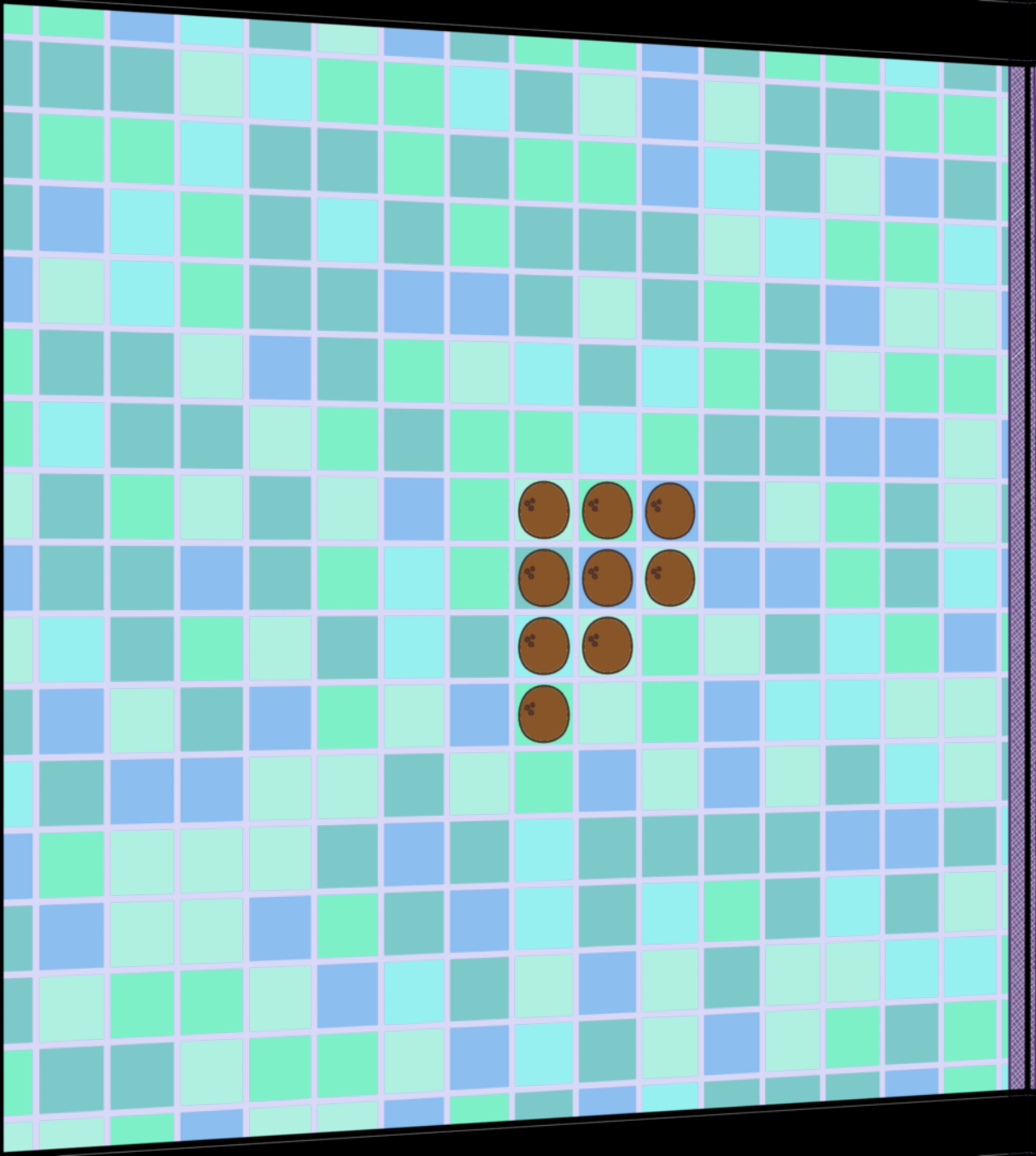

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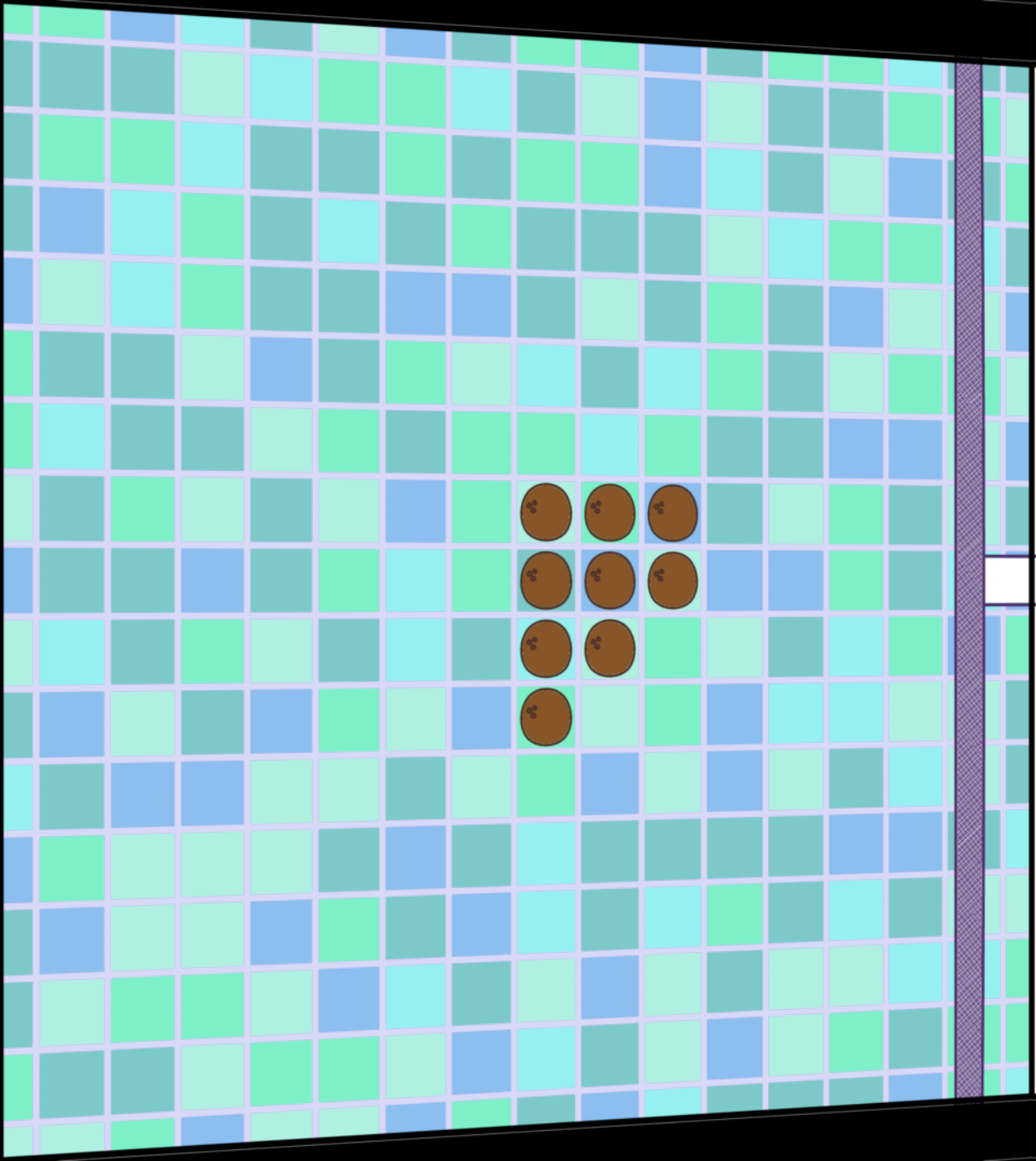

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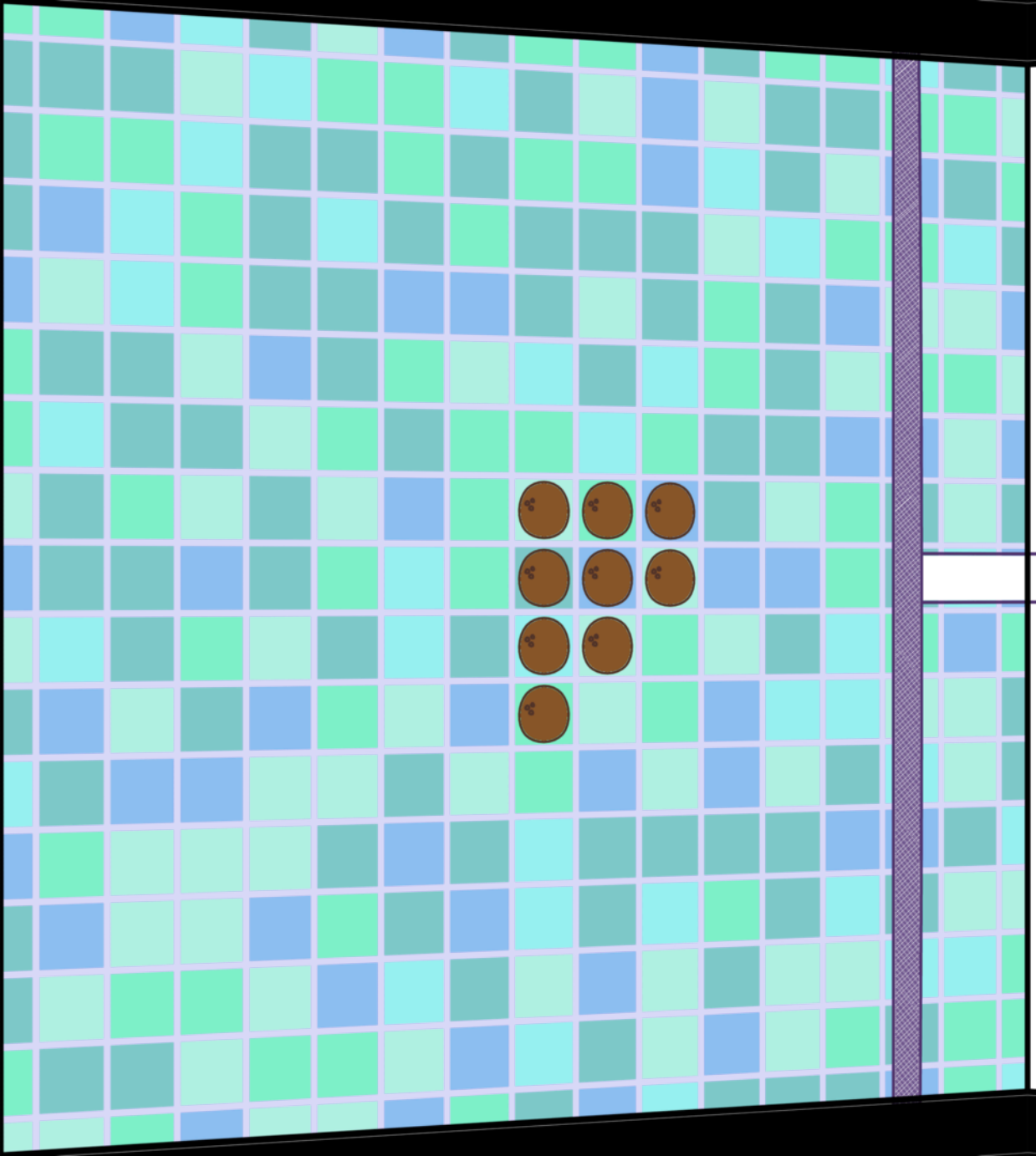

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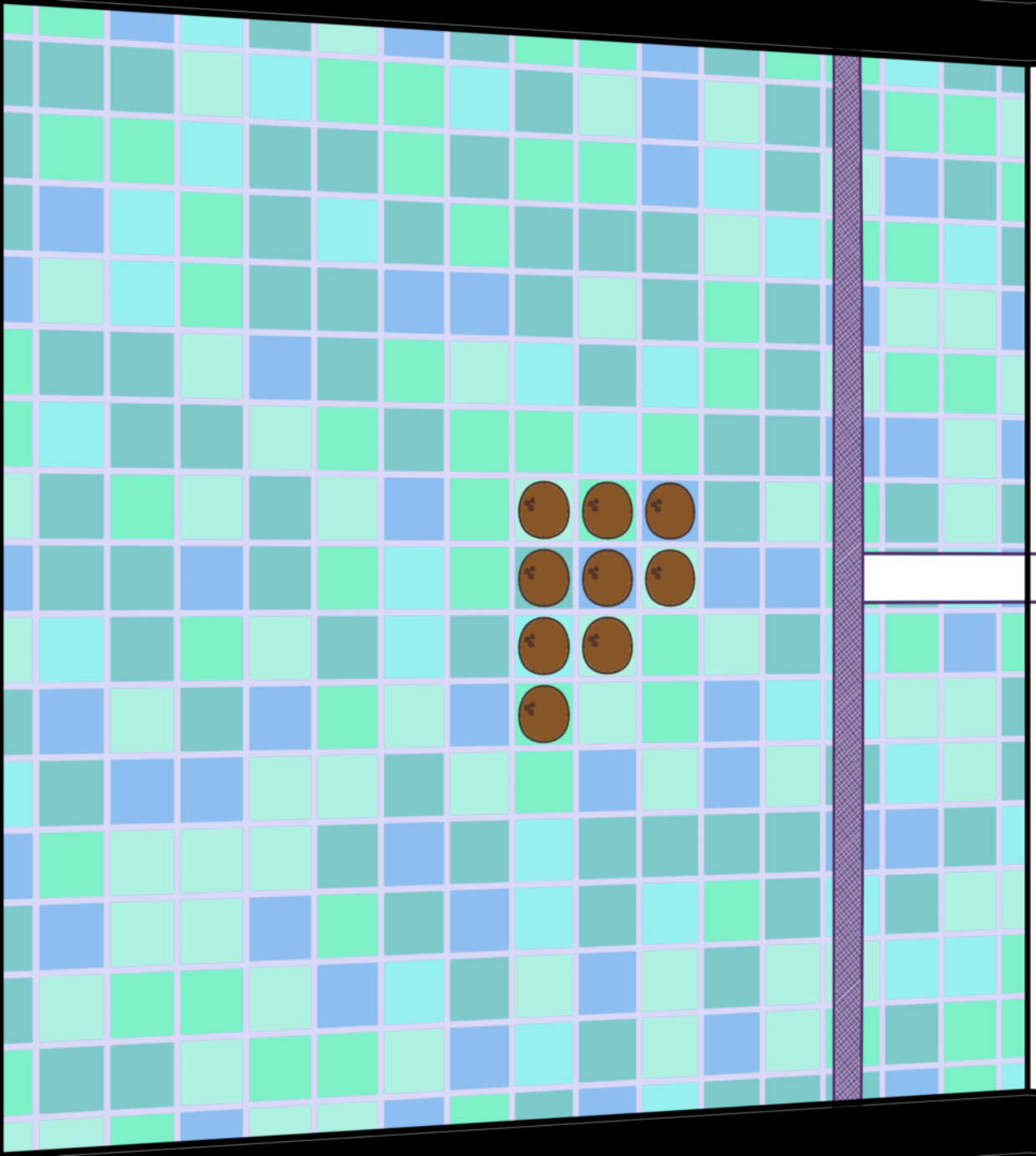

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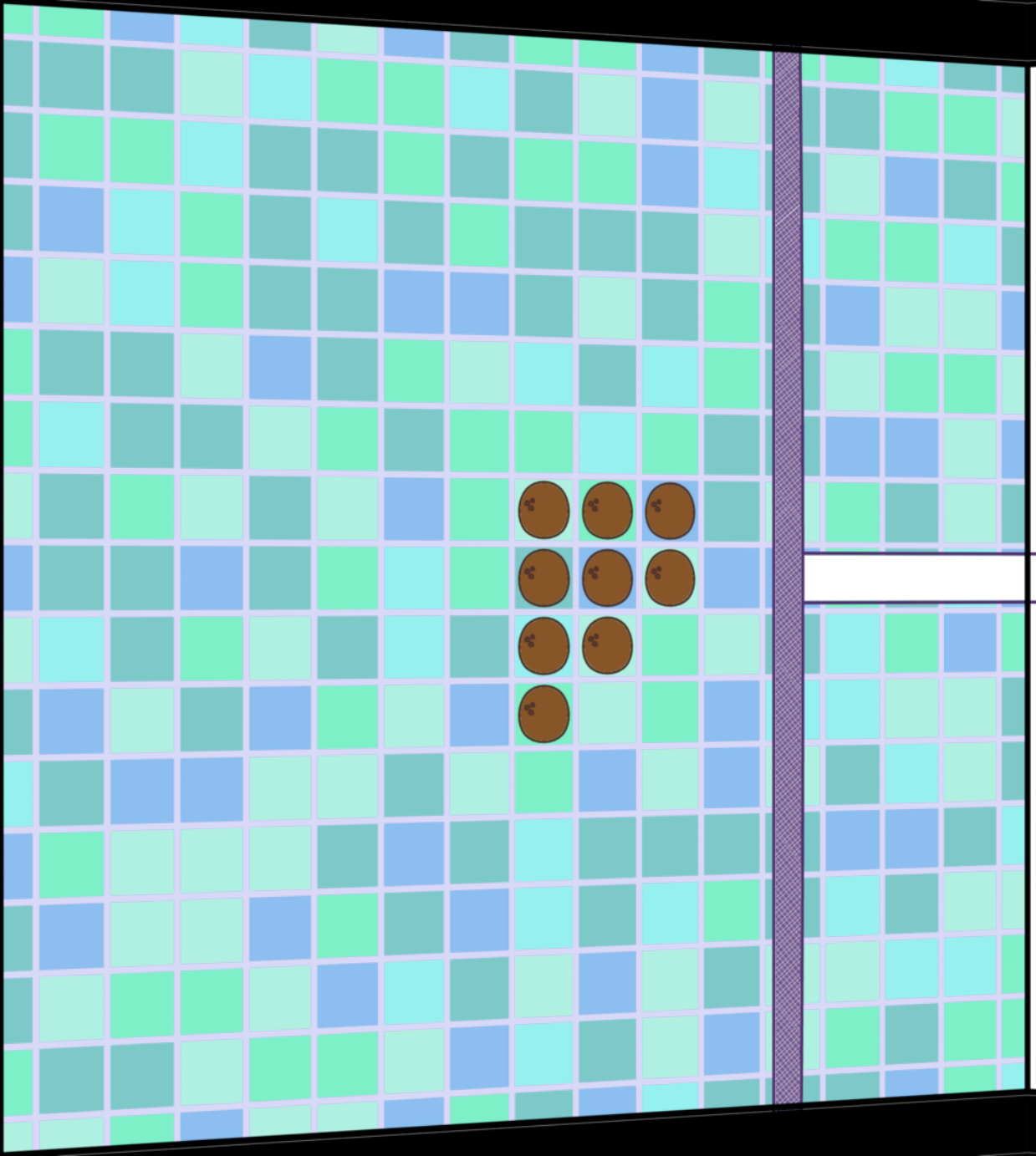

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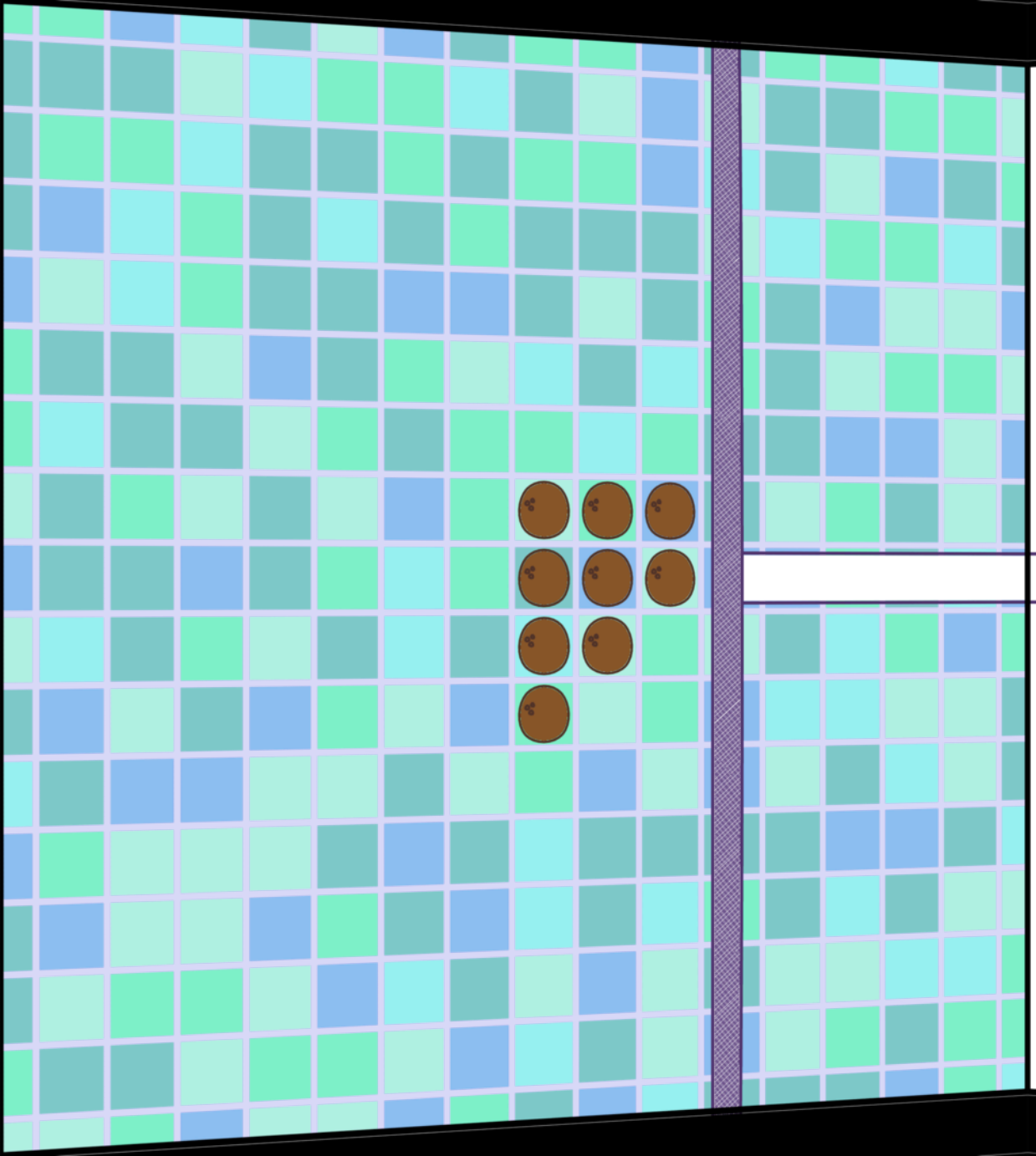

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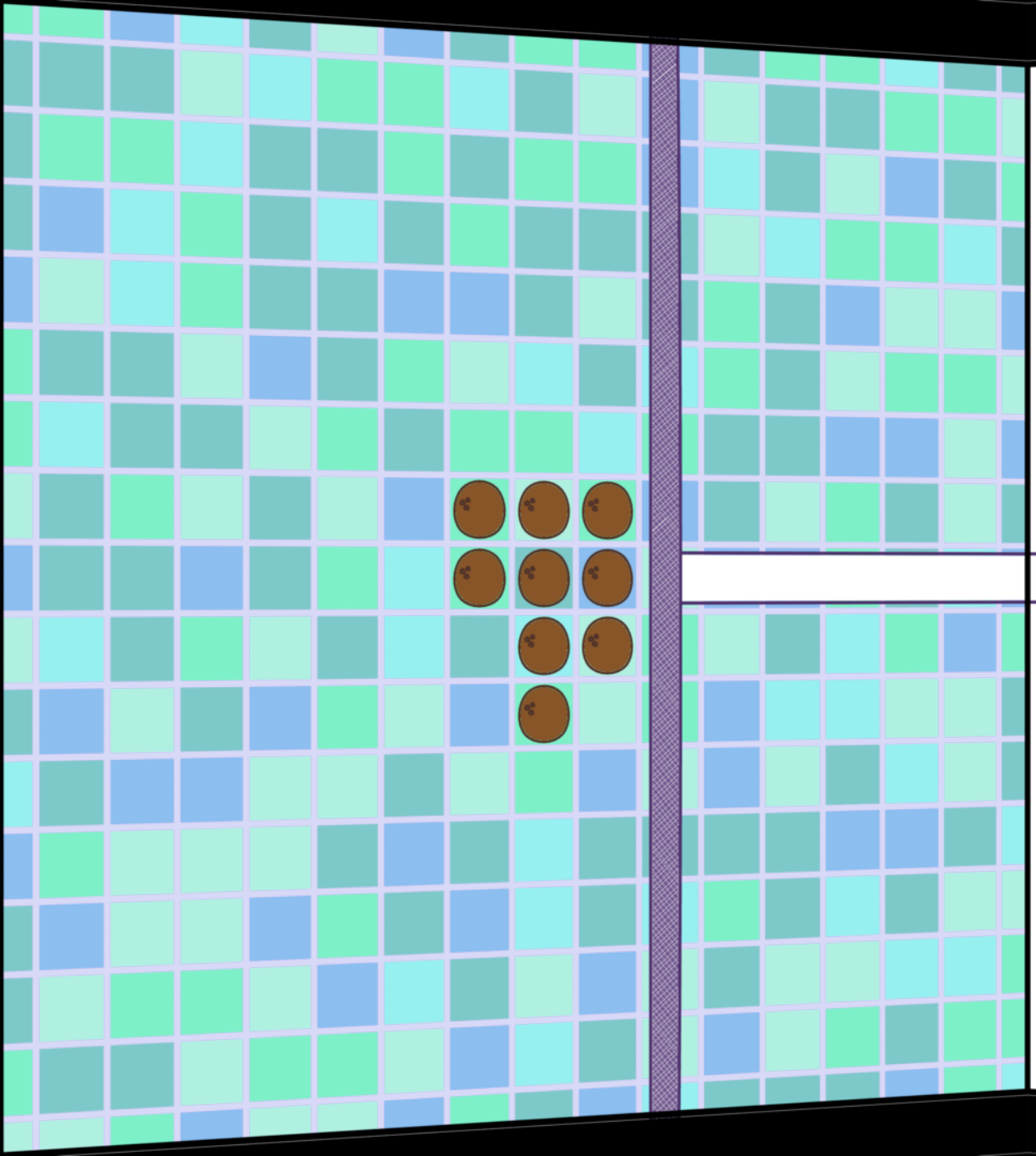

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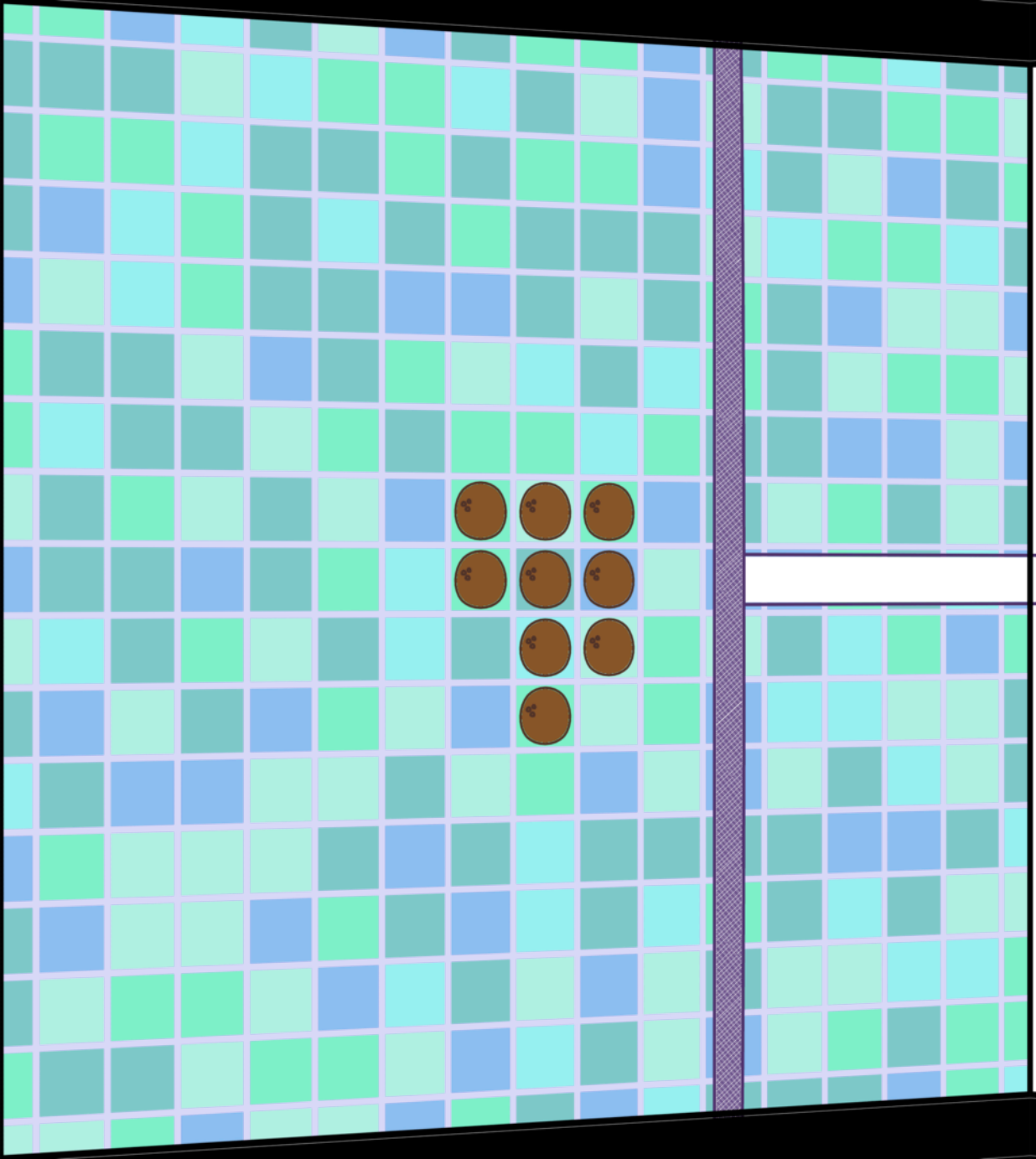

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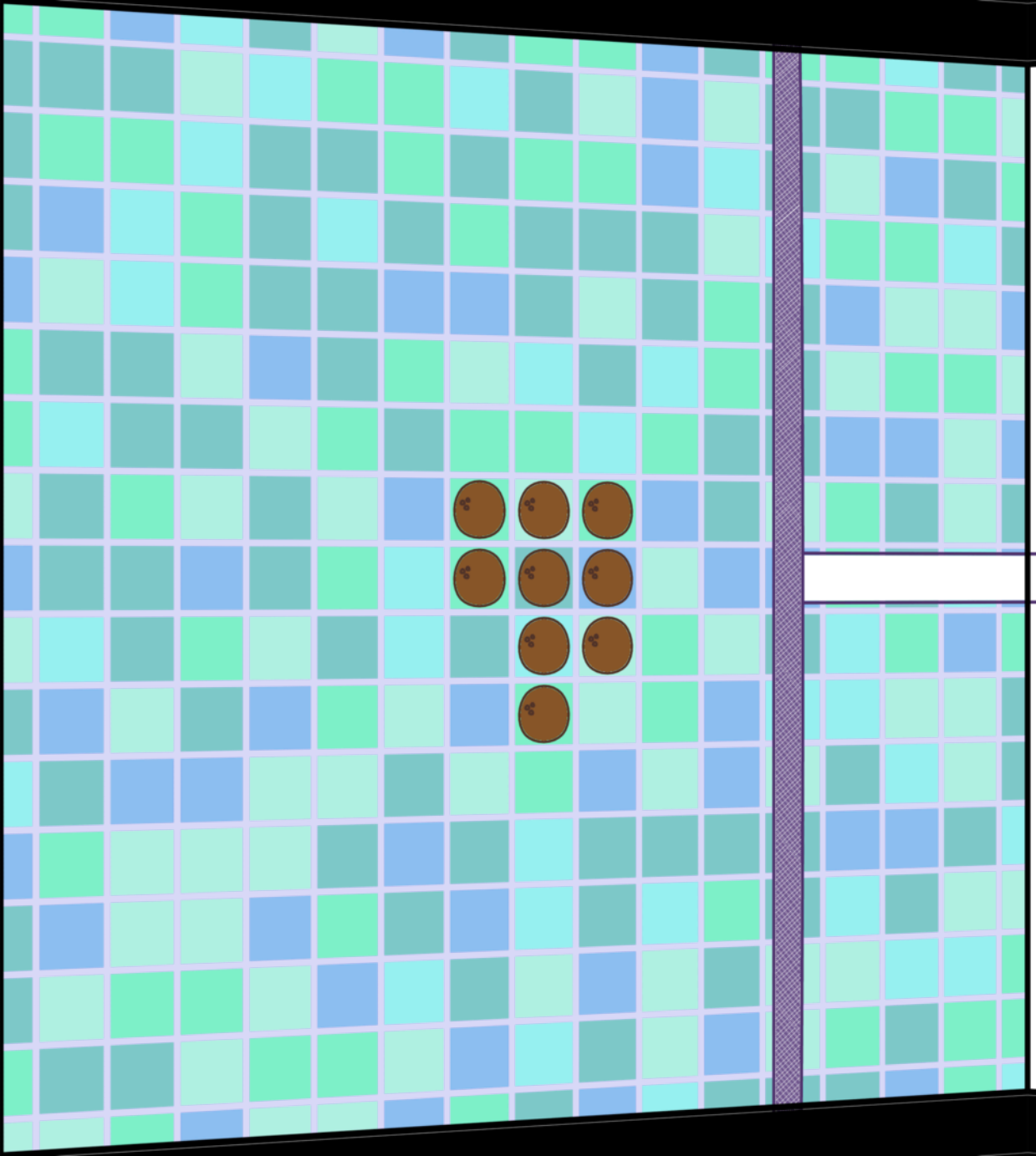

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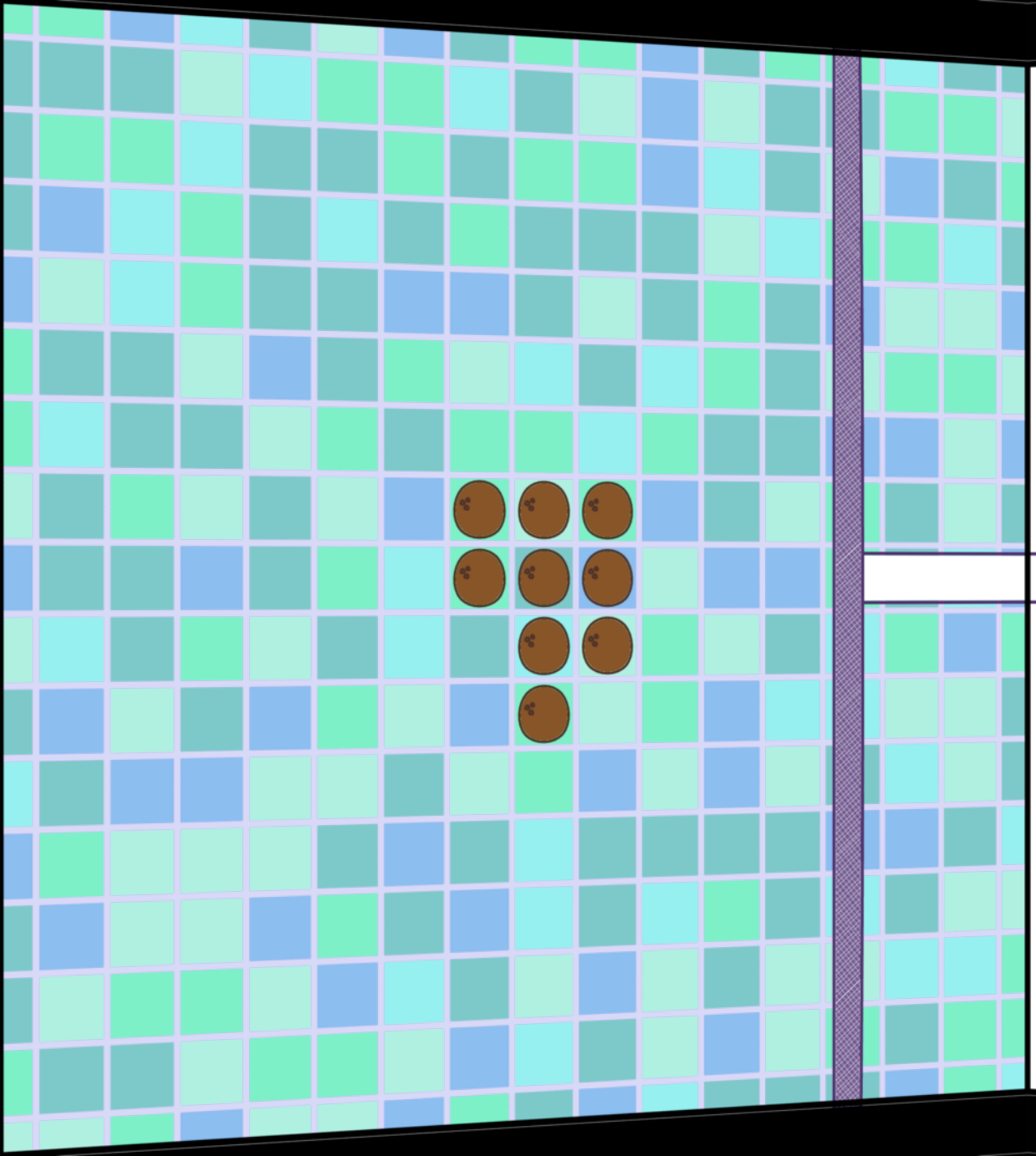

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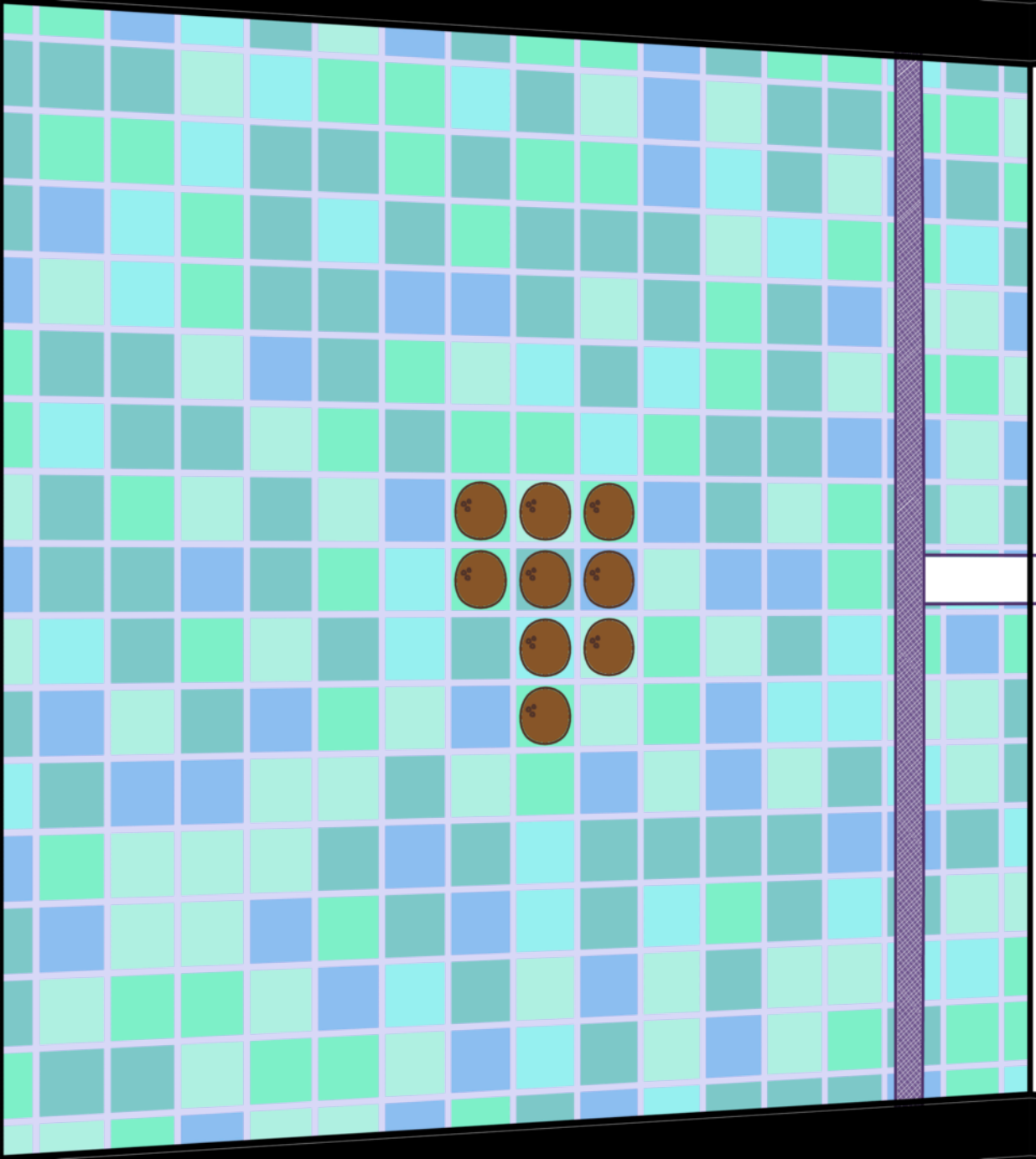

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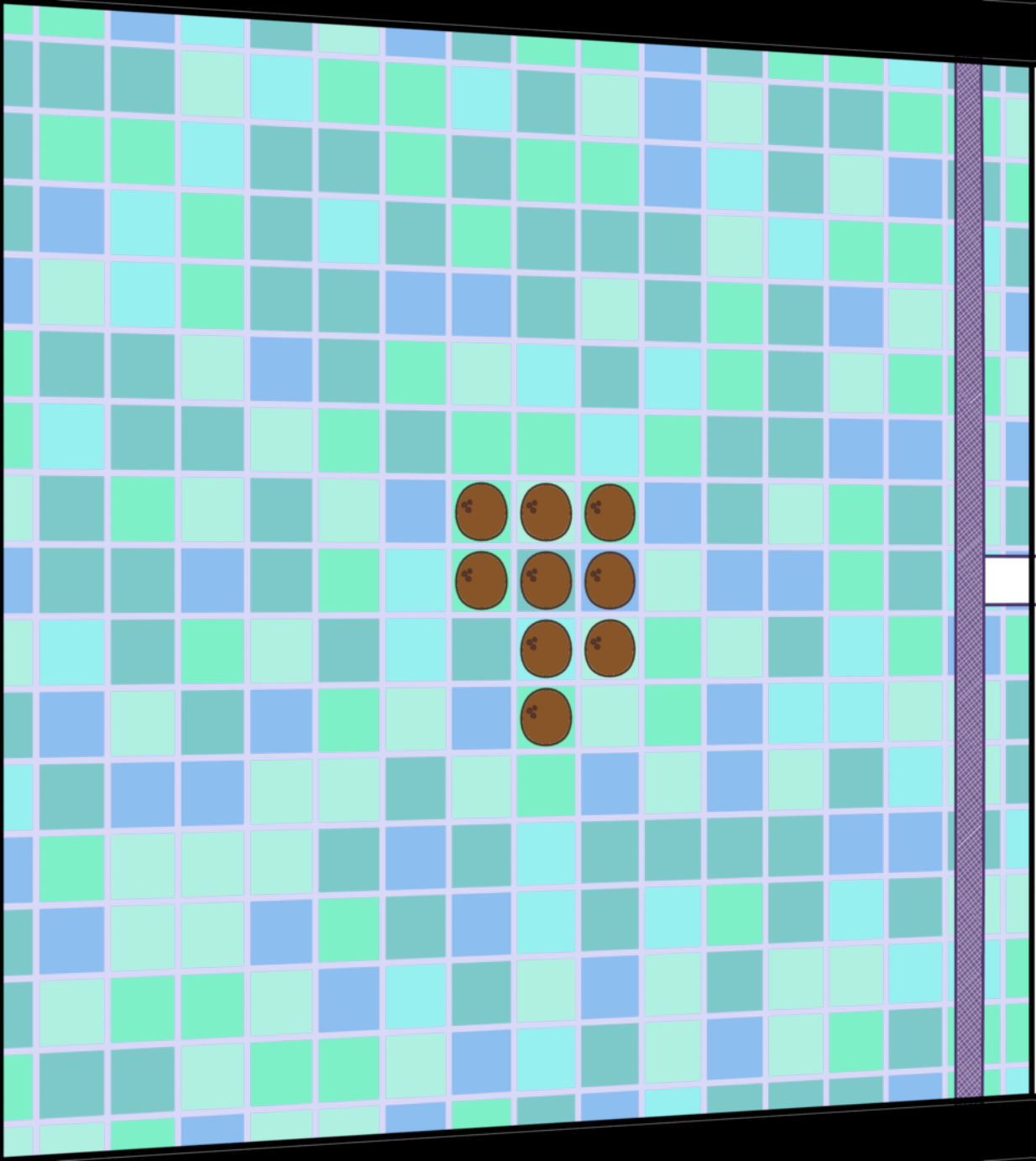

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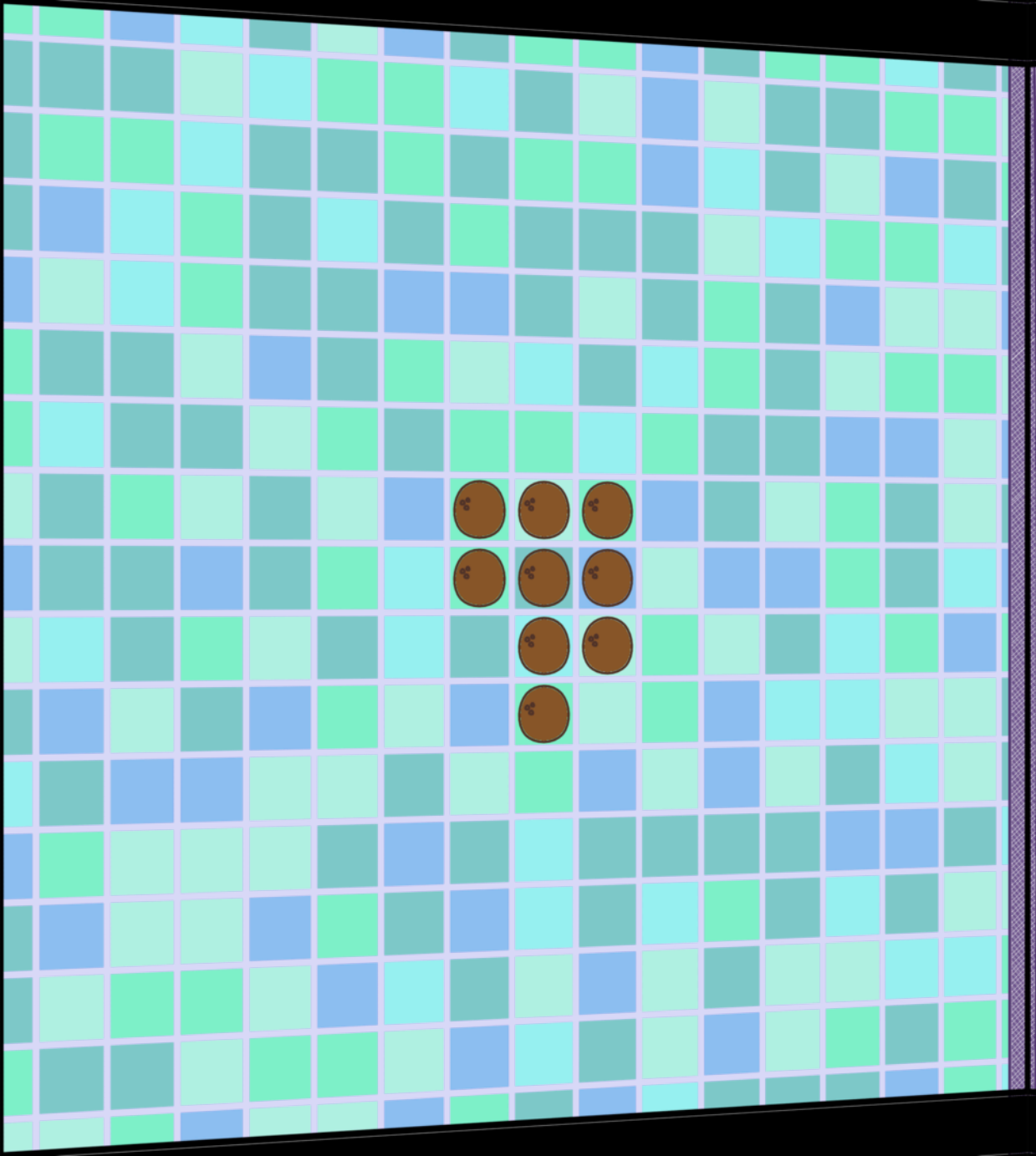

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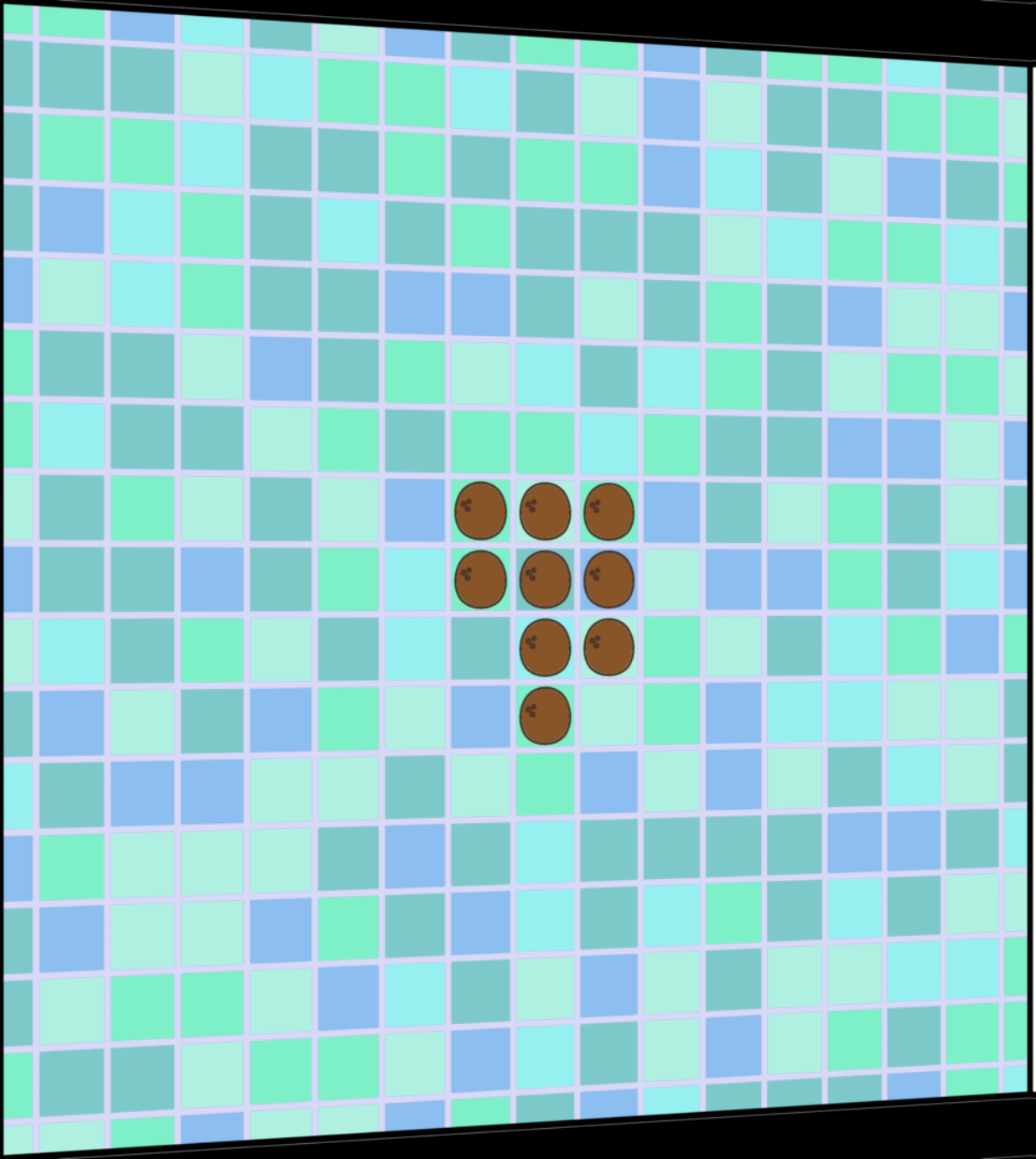

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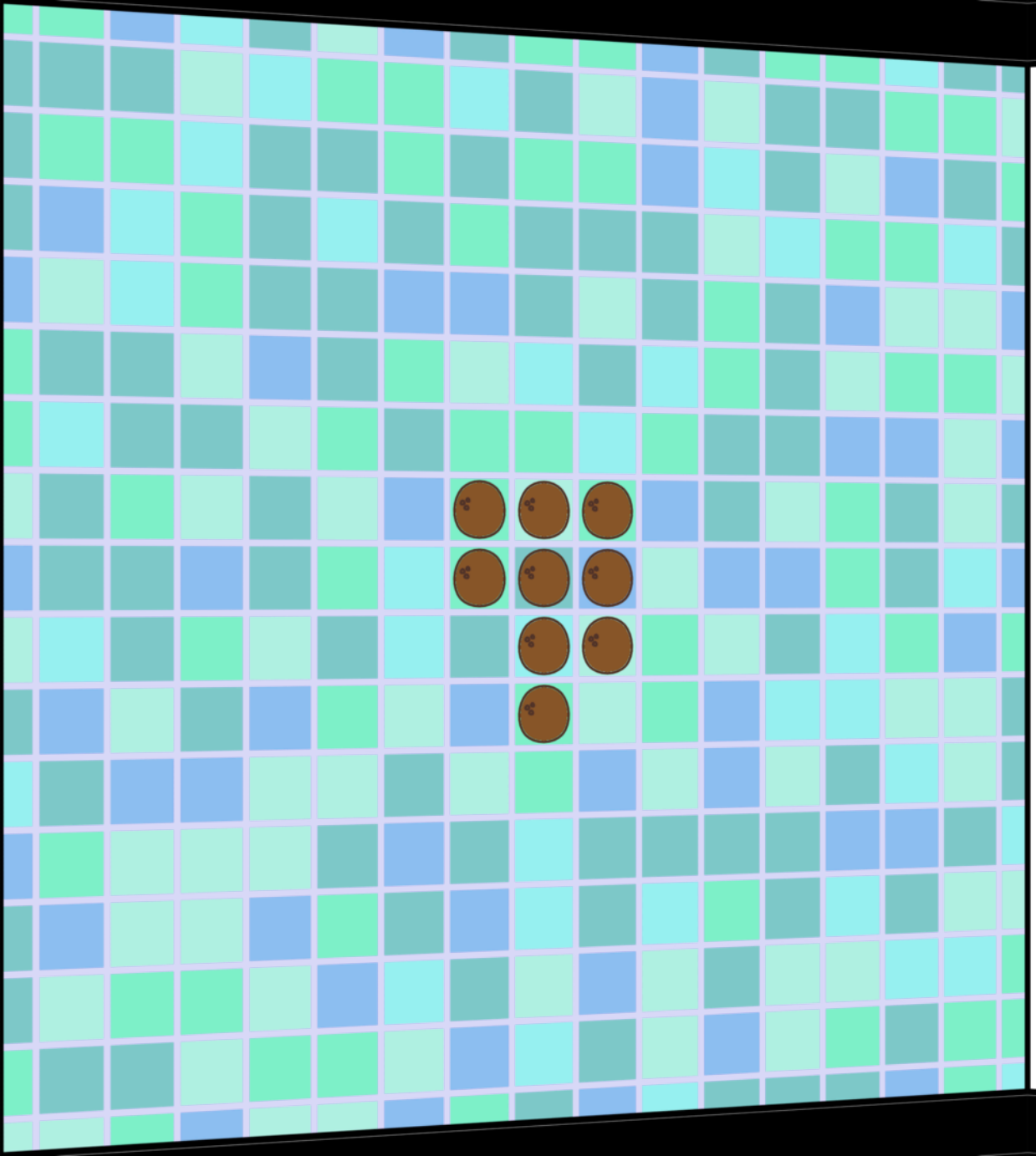

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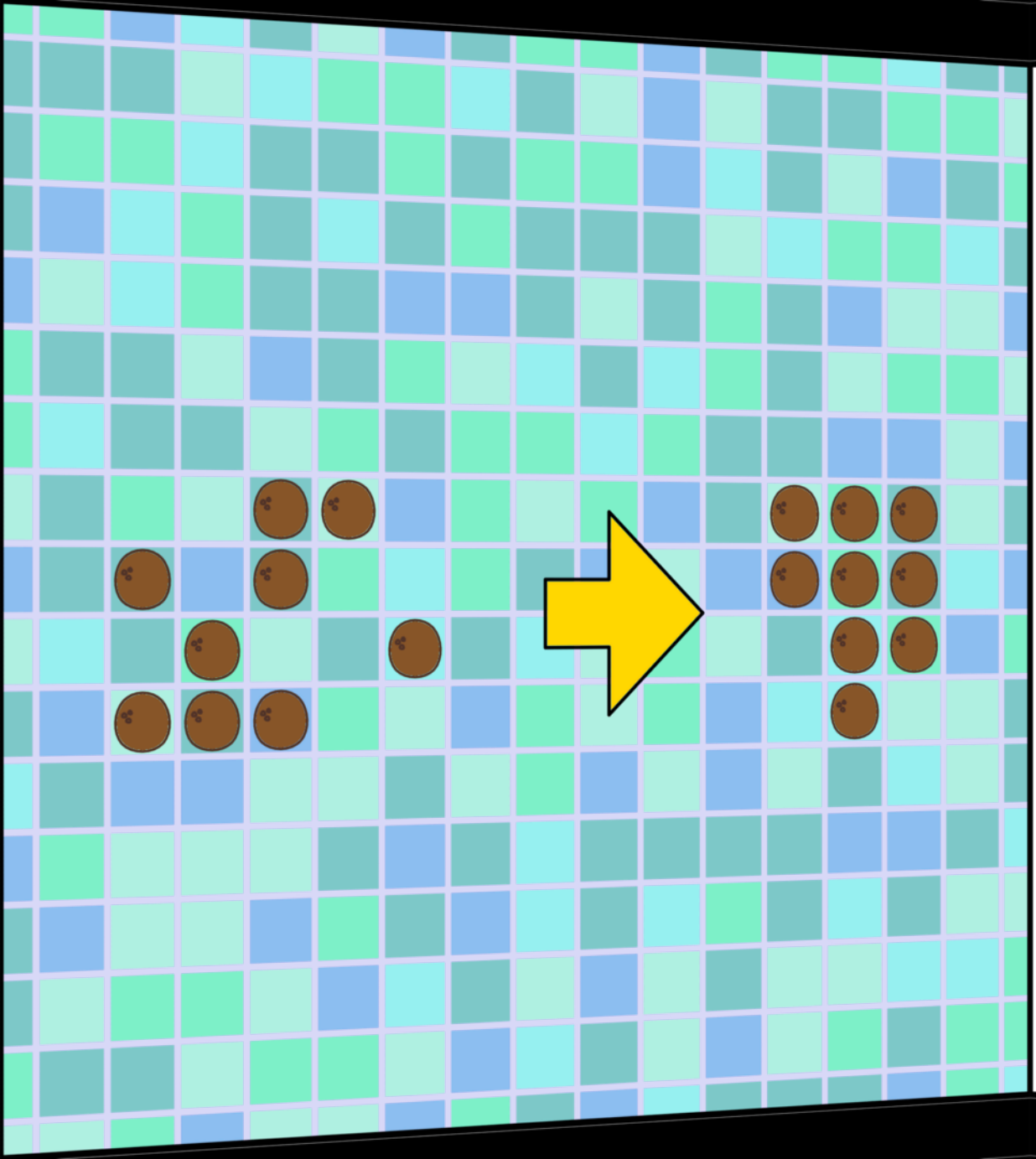
Any sequence of such coconut pushes leads to a *reconfiguration* of the coconuts.



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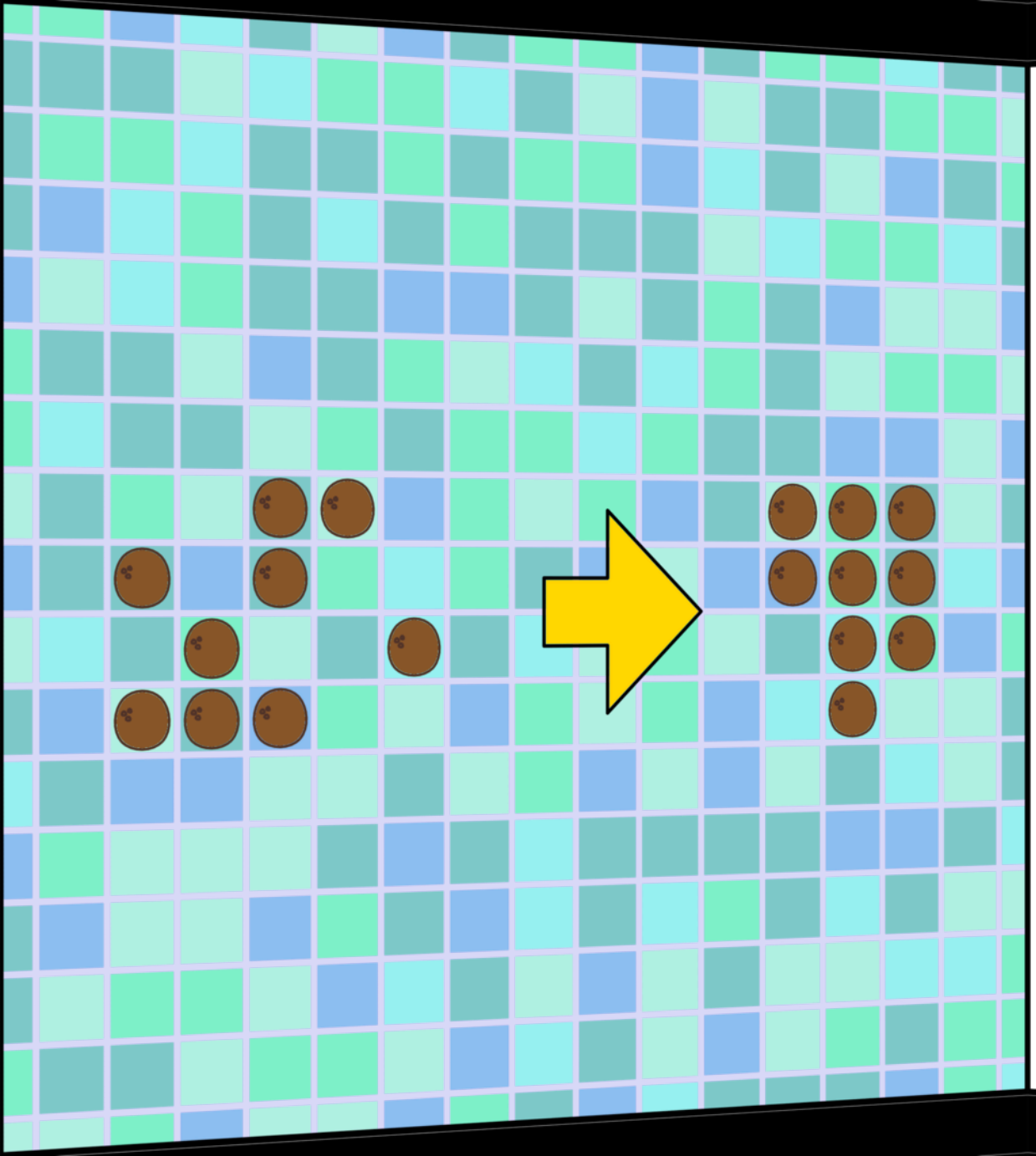
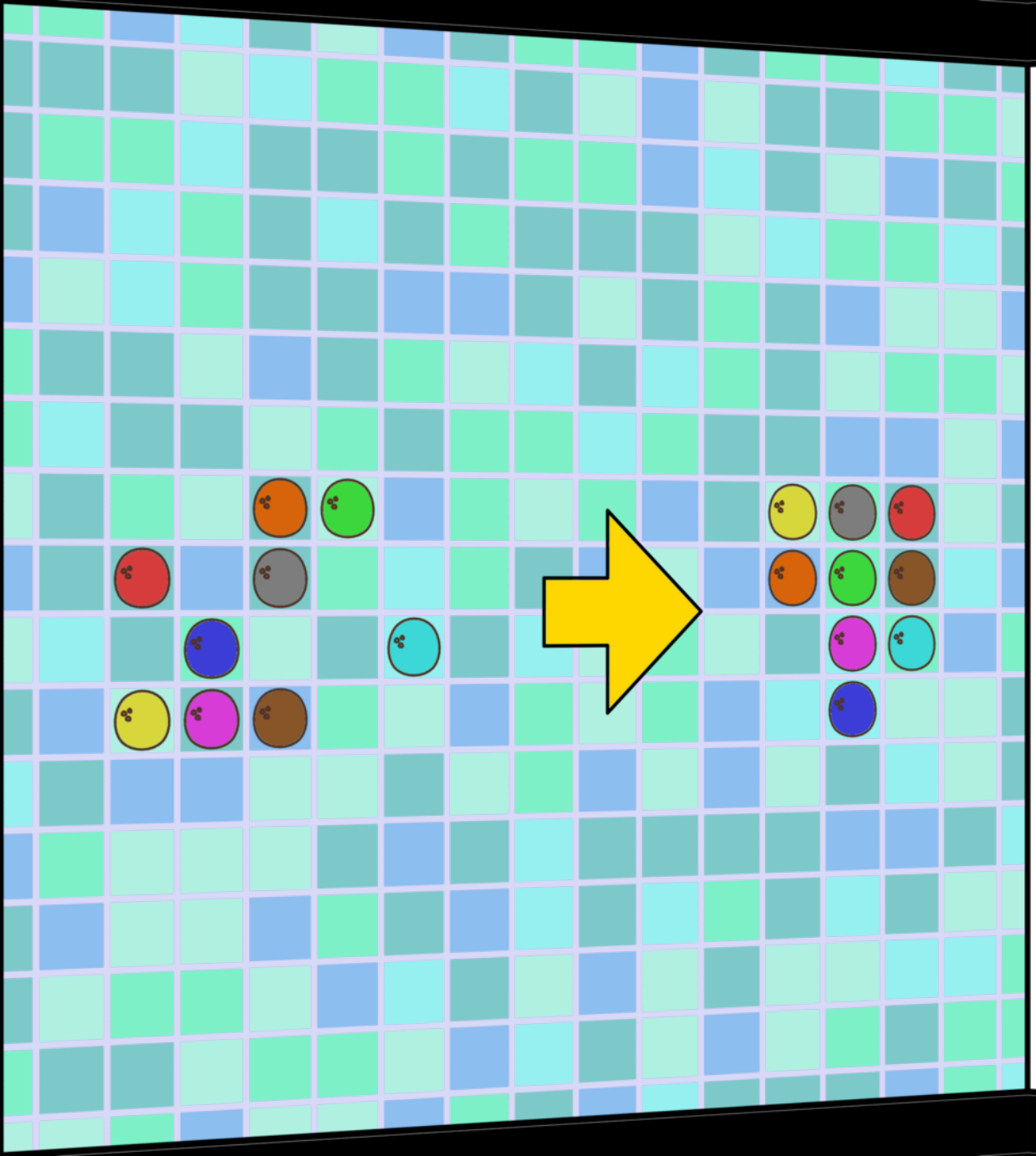


Problem 1. *(unlabeled coconut reconfiguration)*

Given a starting configuration P and a goal configuration P', is there a sequence of pushes that transforms P into P'?

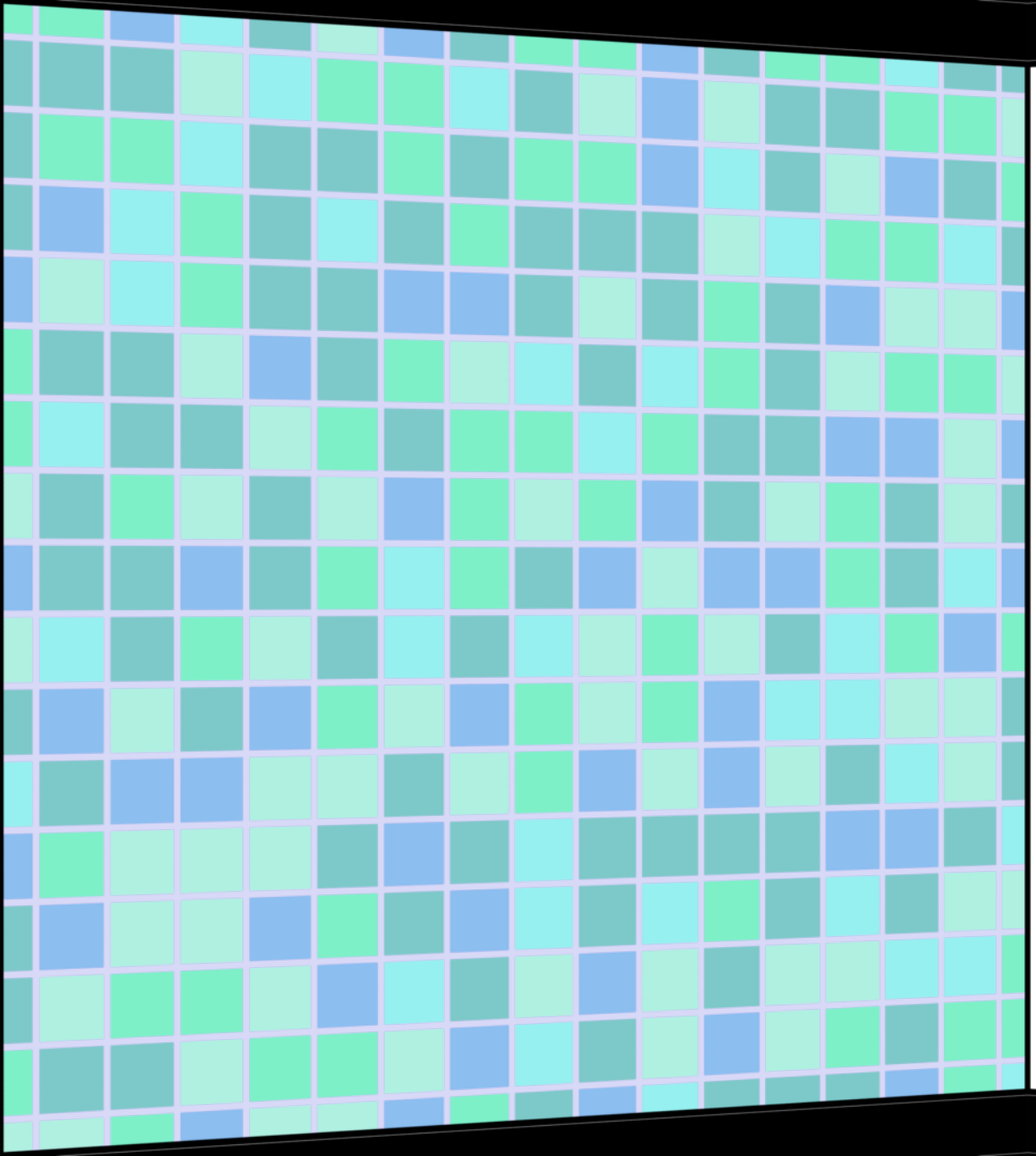


Problem 1. (*unlabeled coconut reconfiguration*)

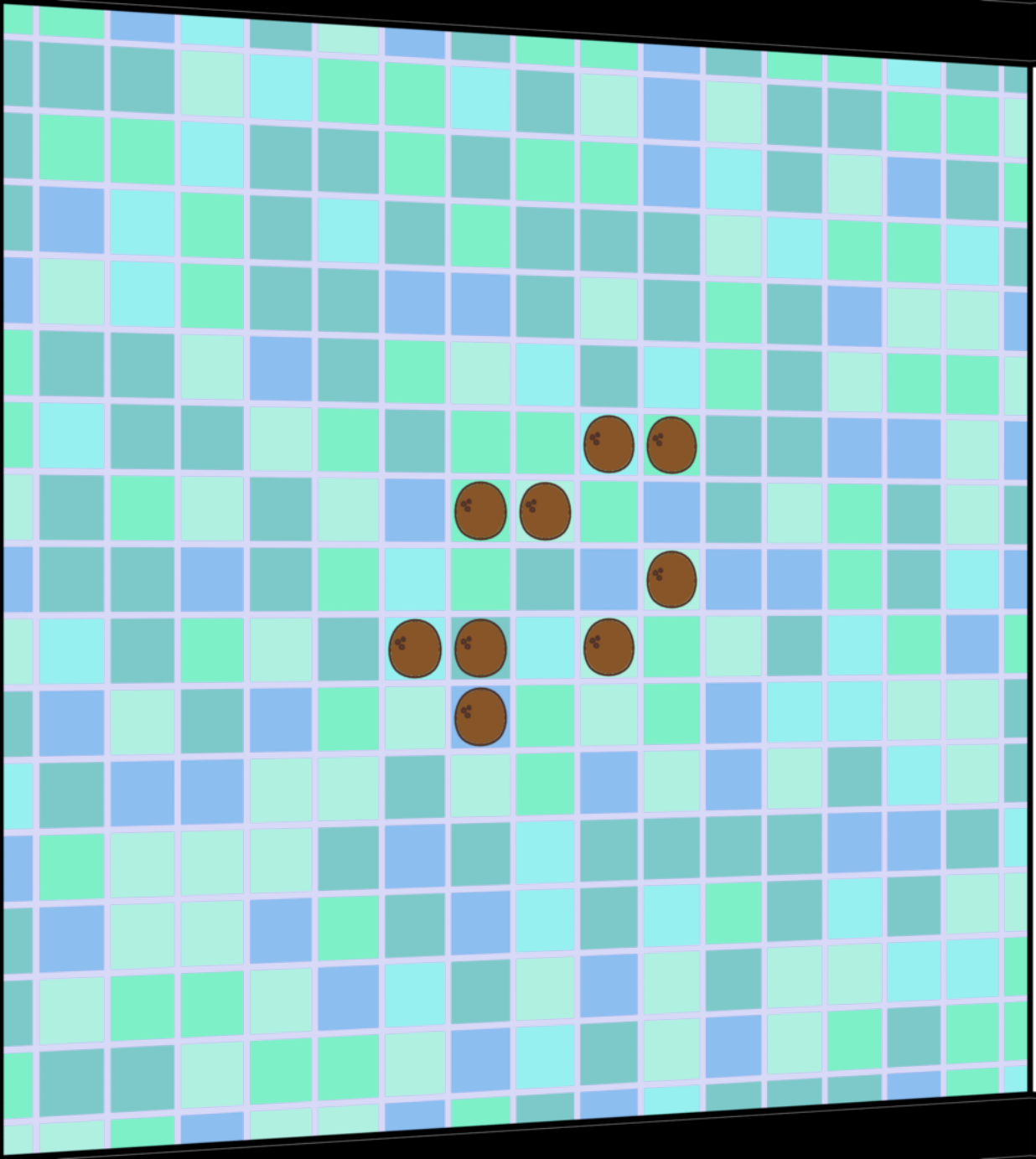
Given a starting configuration P and a goal configuration P', is there a sequence of pushes that transforms P into P'?

Problem 2. (*labeled coconut reconfiguration*)

Same thing, with labels.



Can we always push our coconuts into tidy rectangles?



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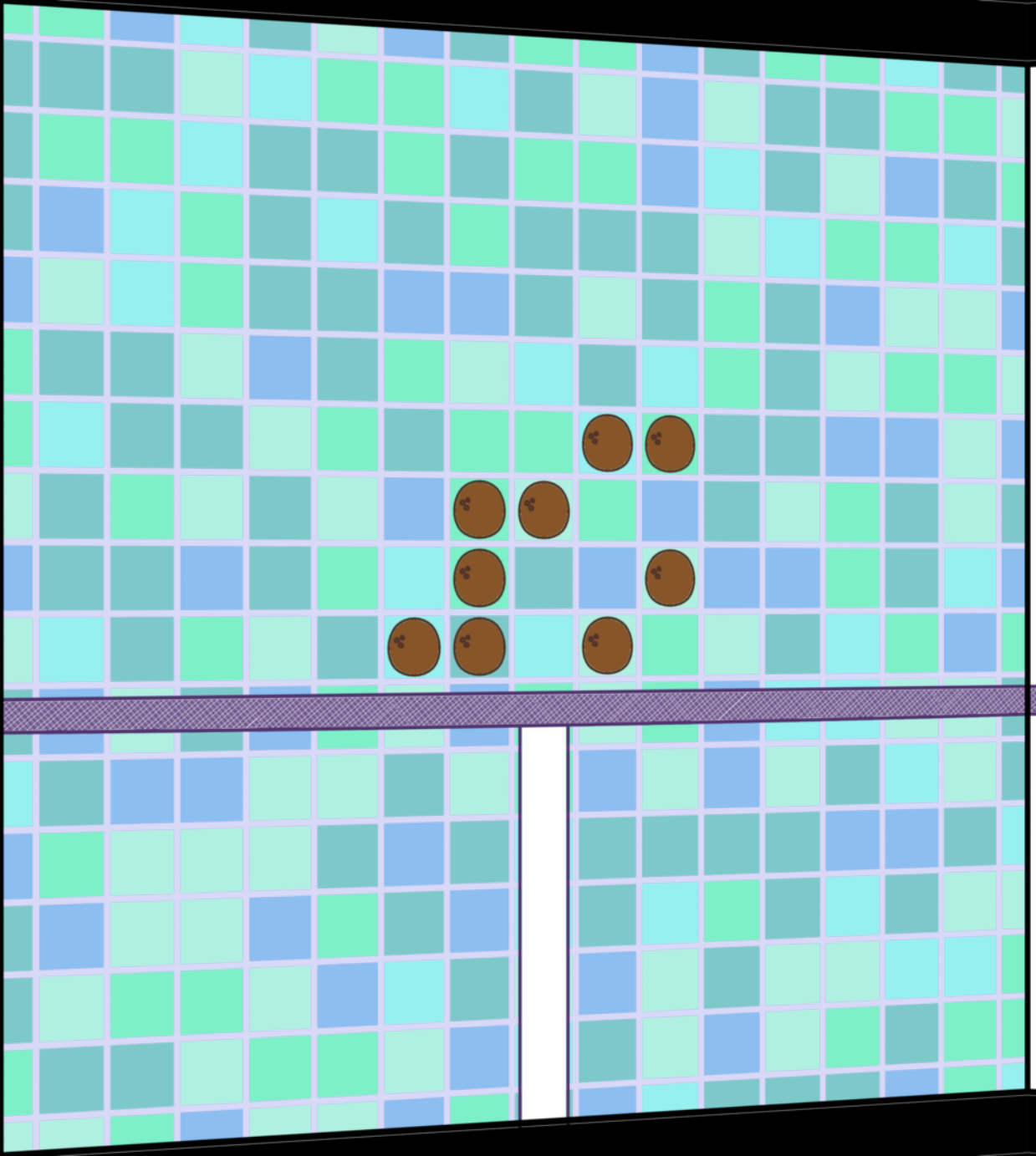

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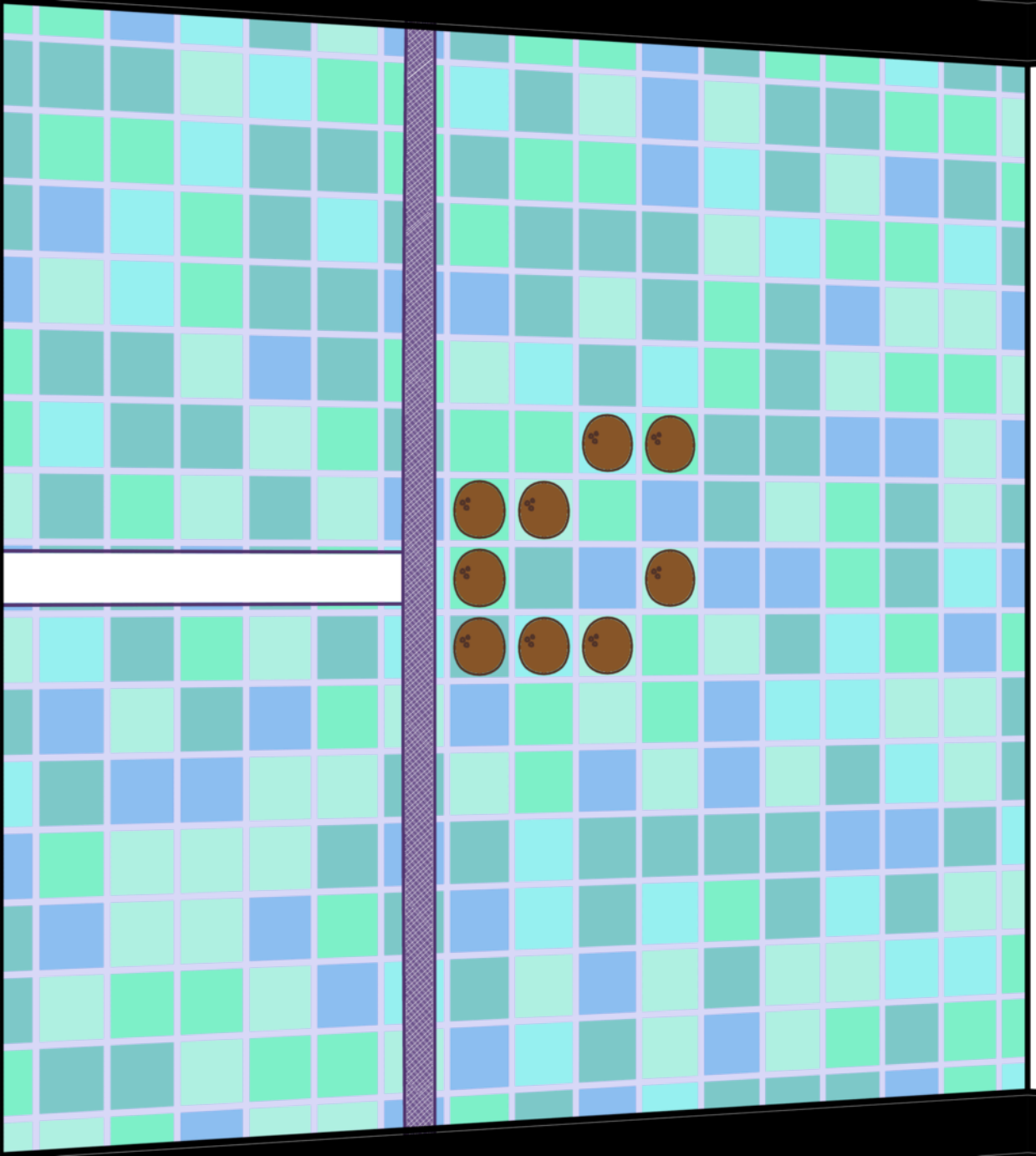

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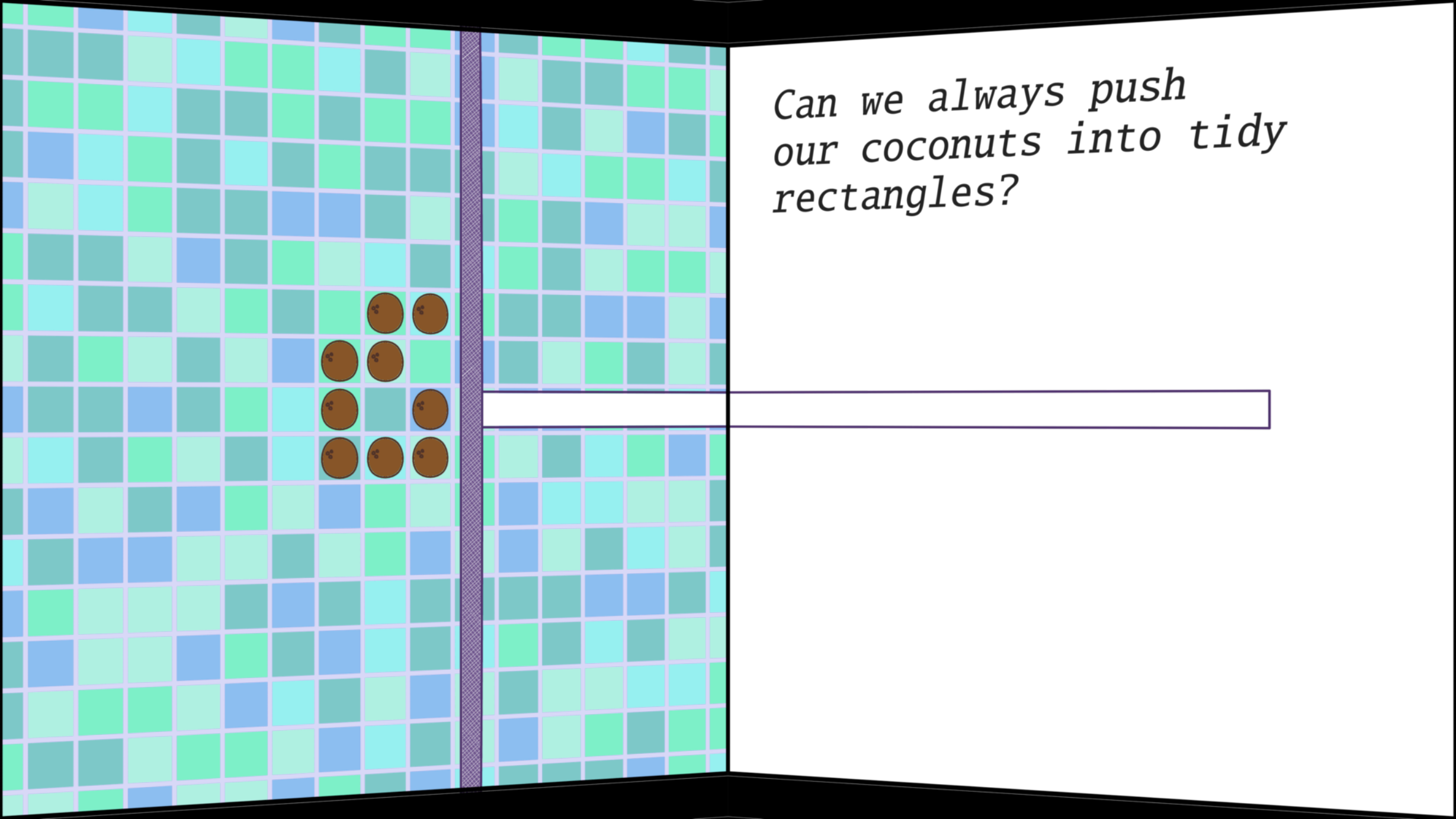
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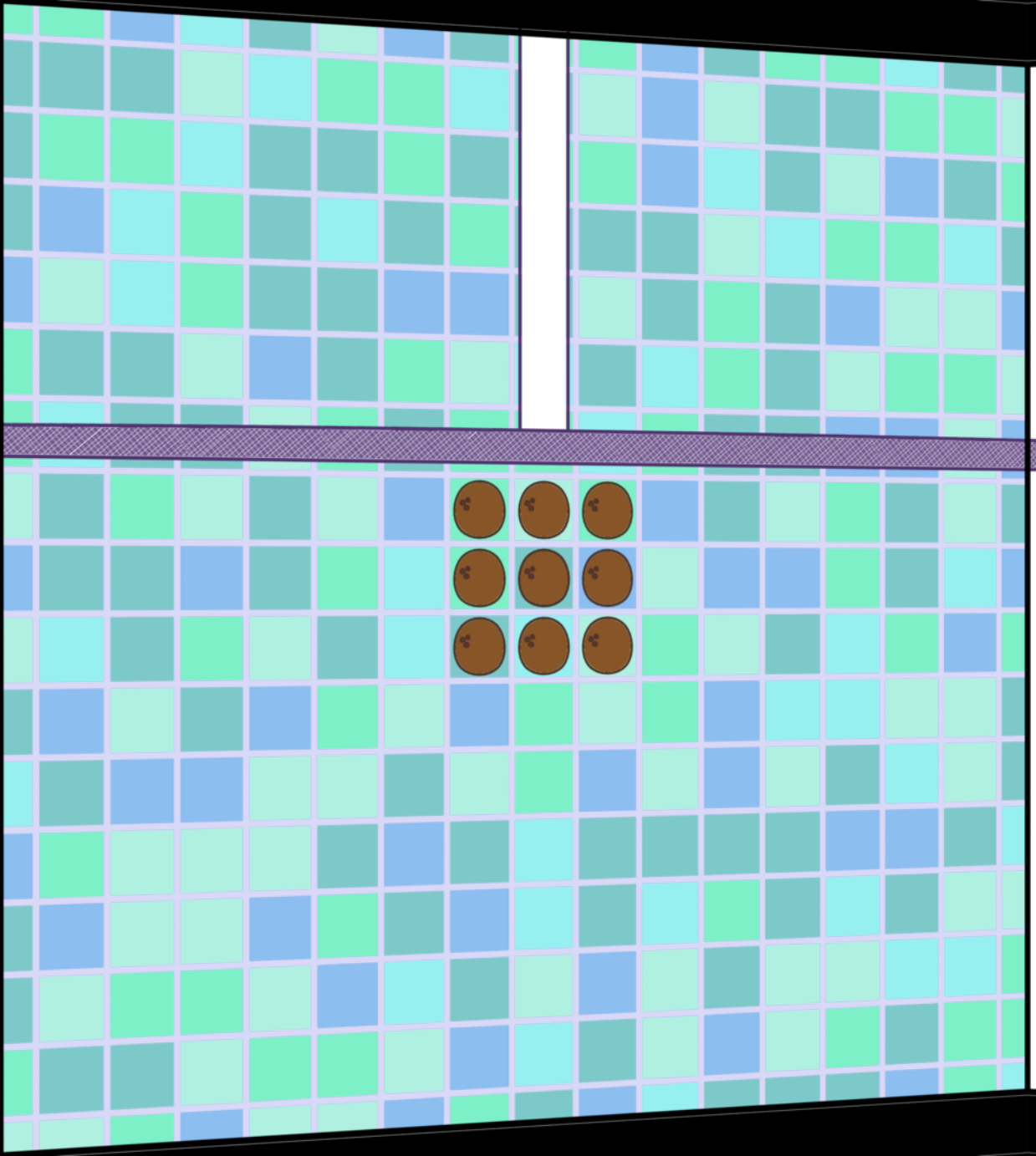

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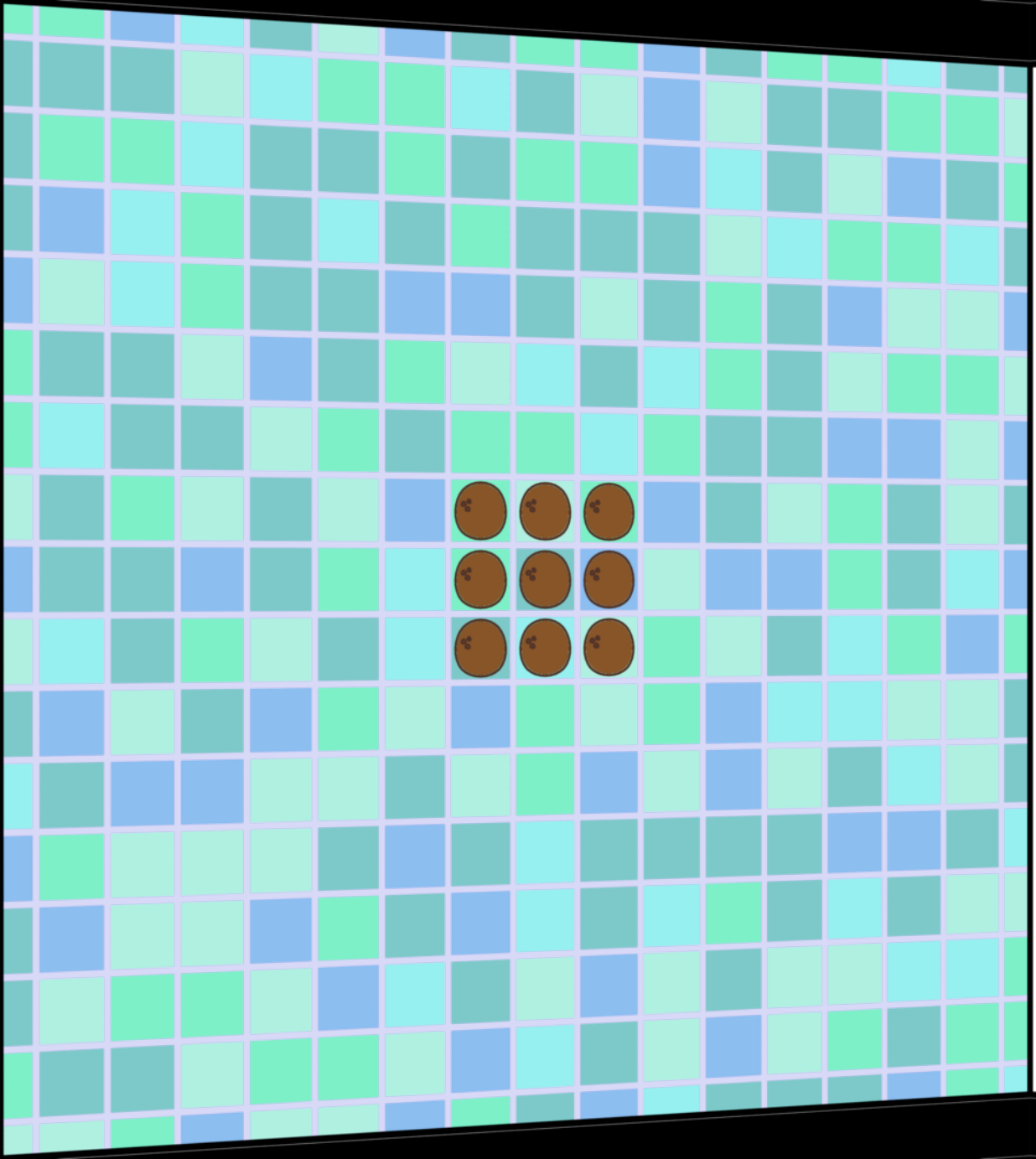

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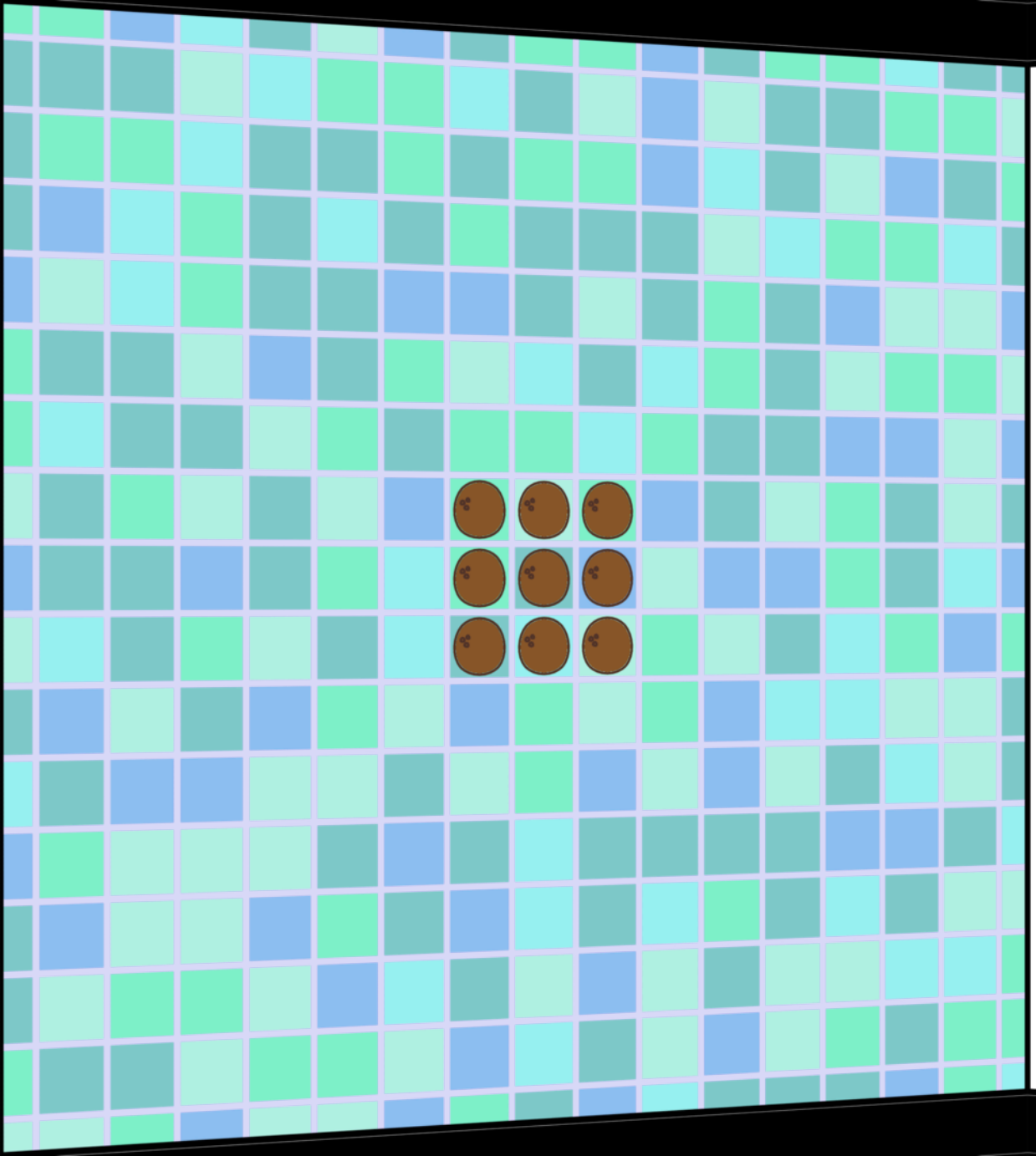

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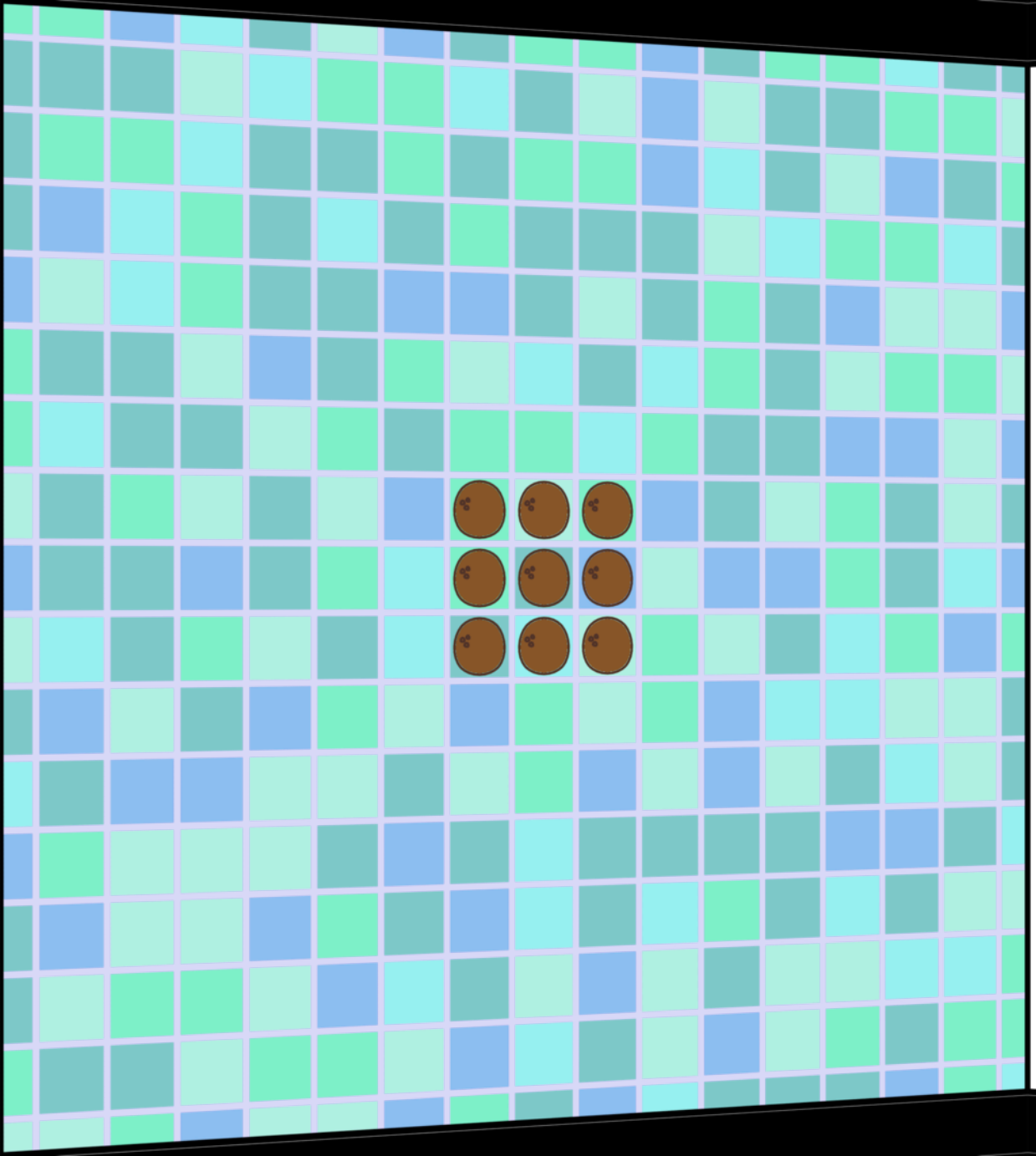

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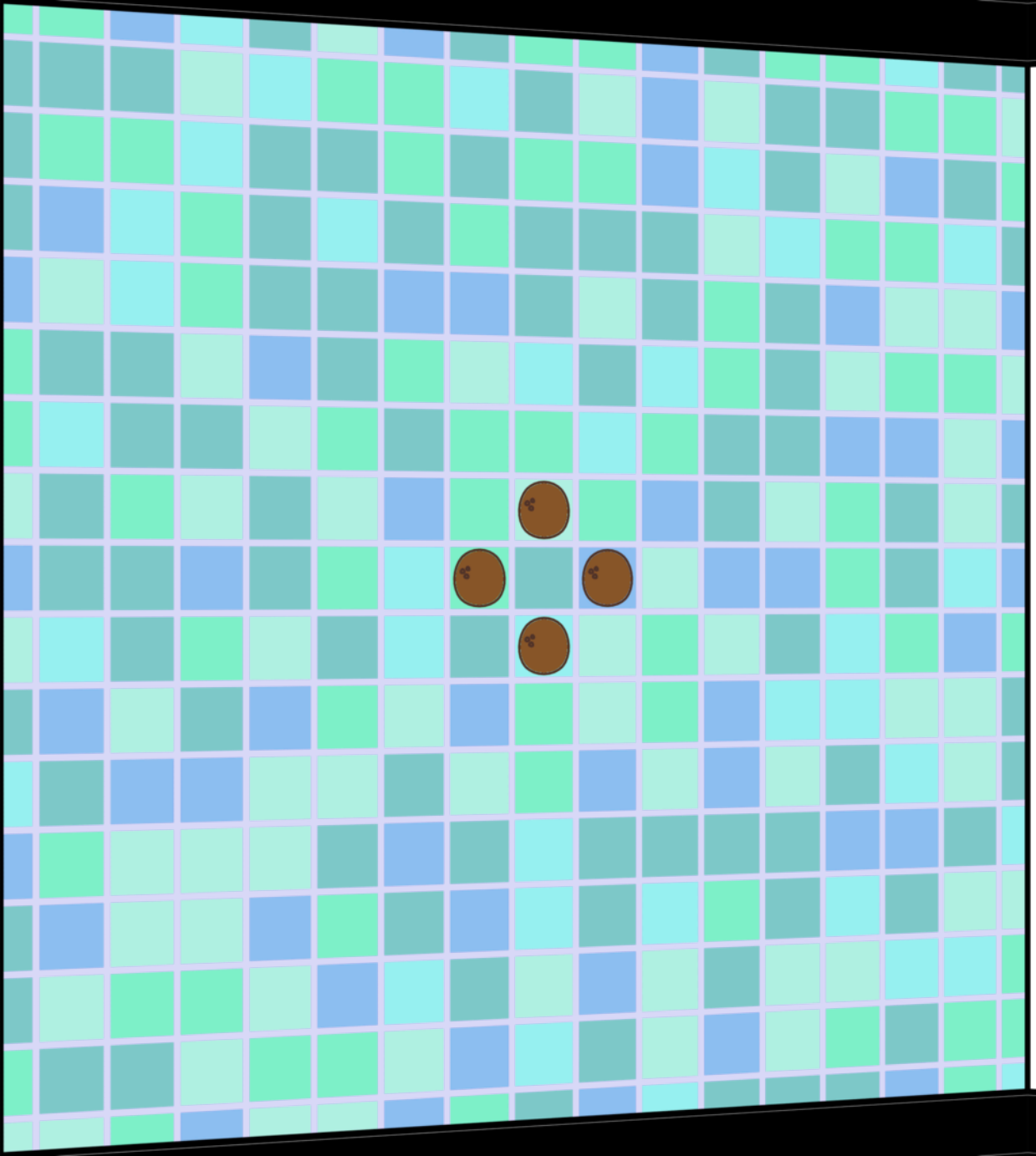

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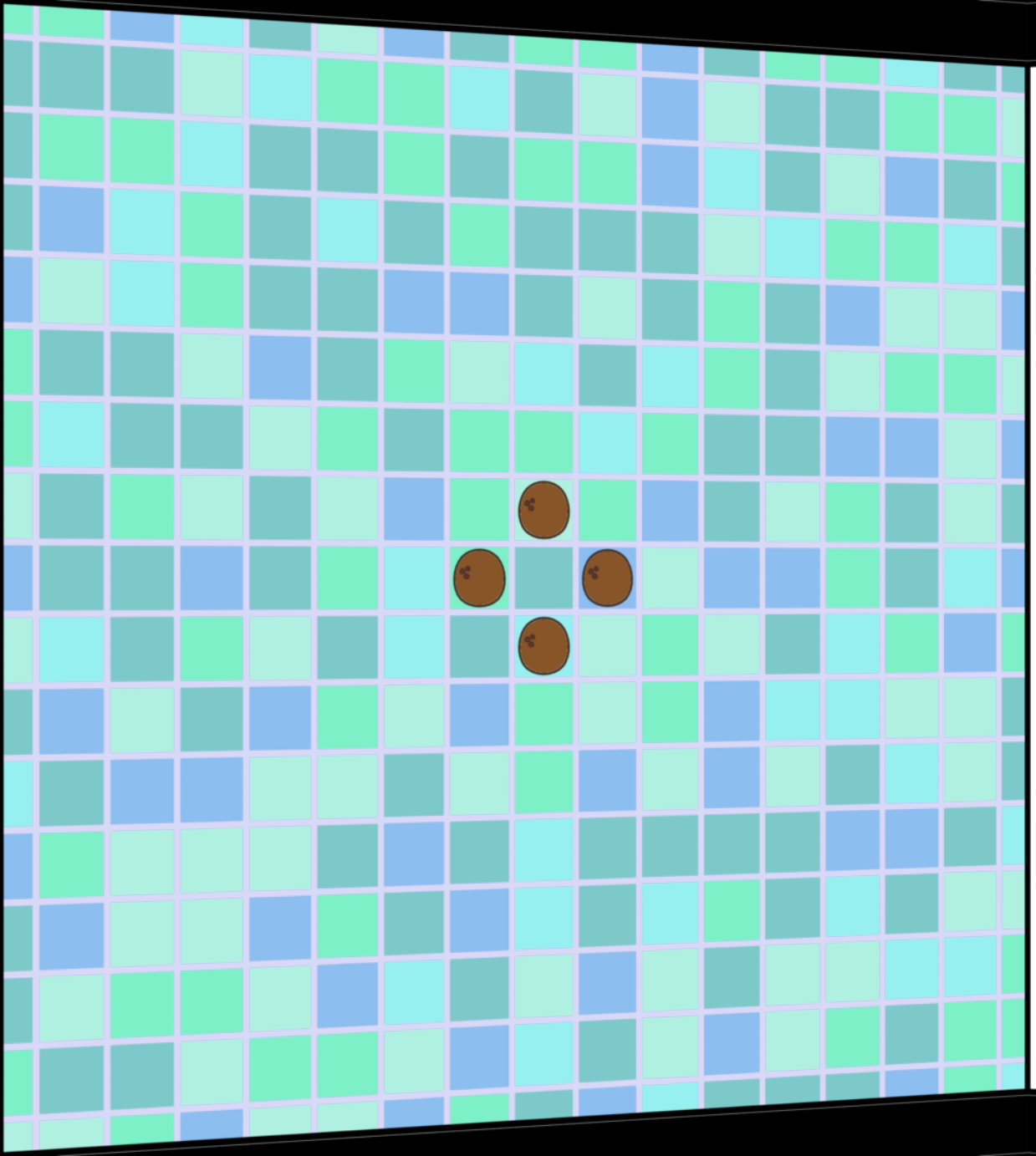

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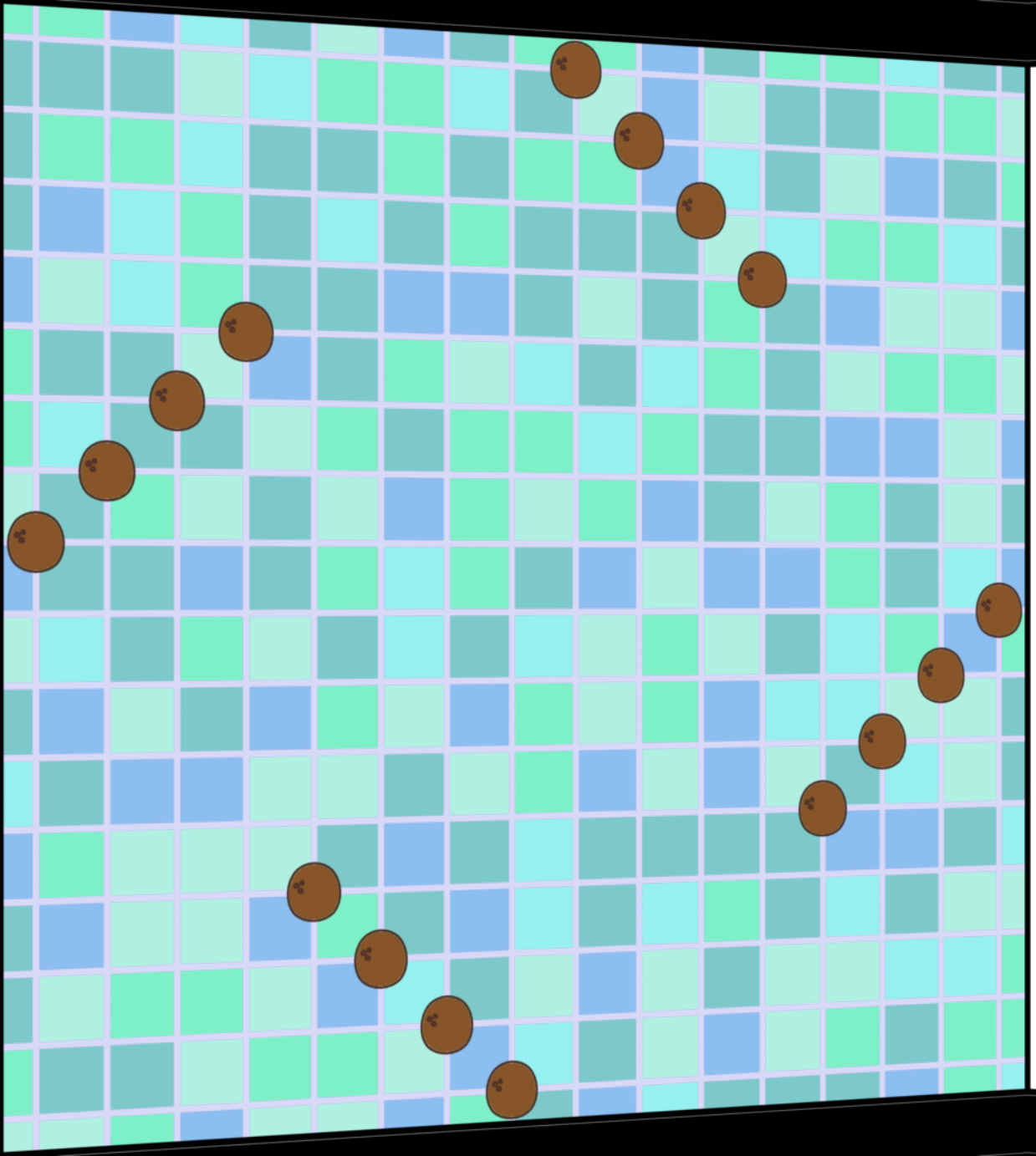

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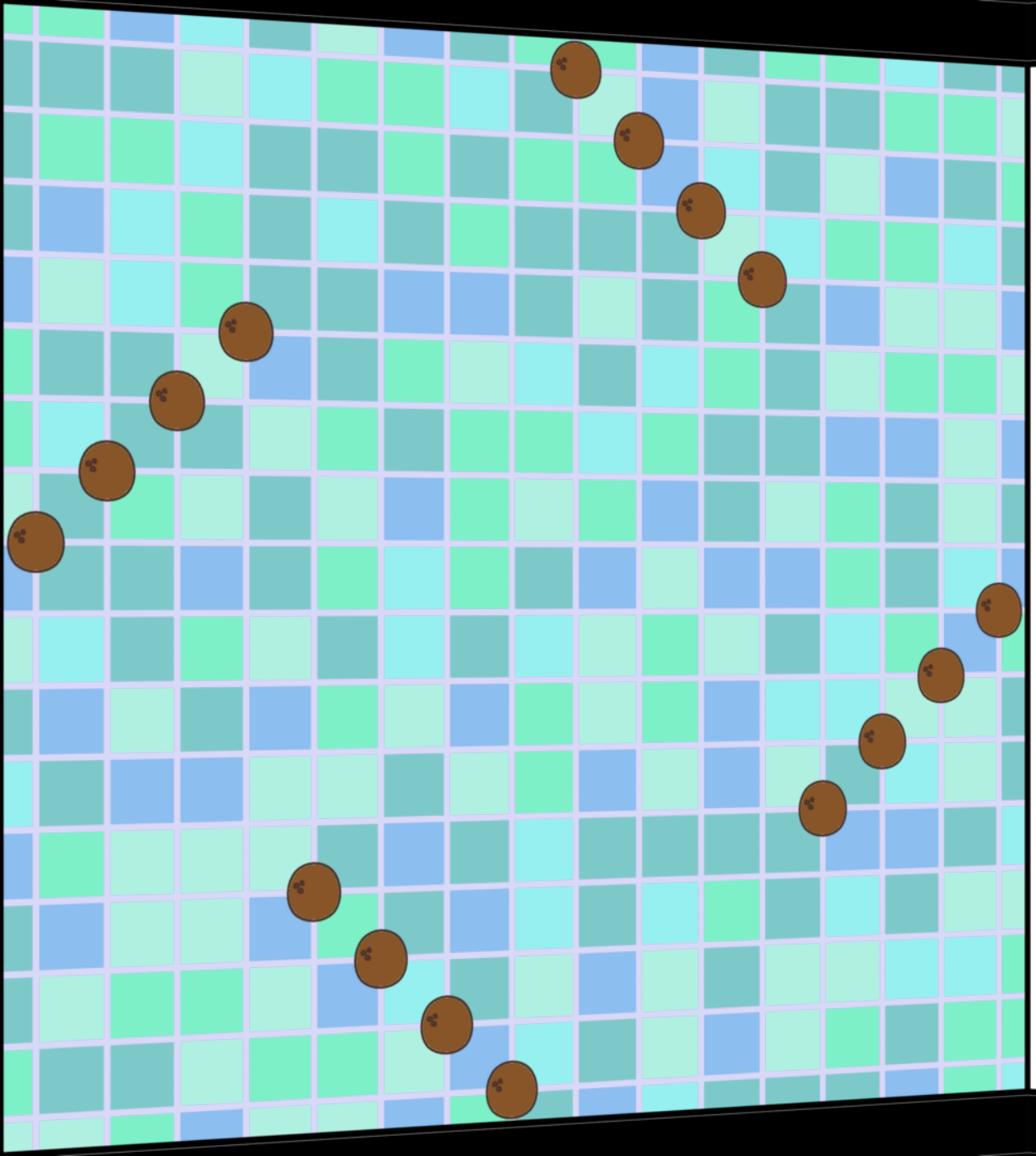

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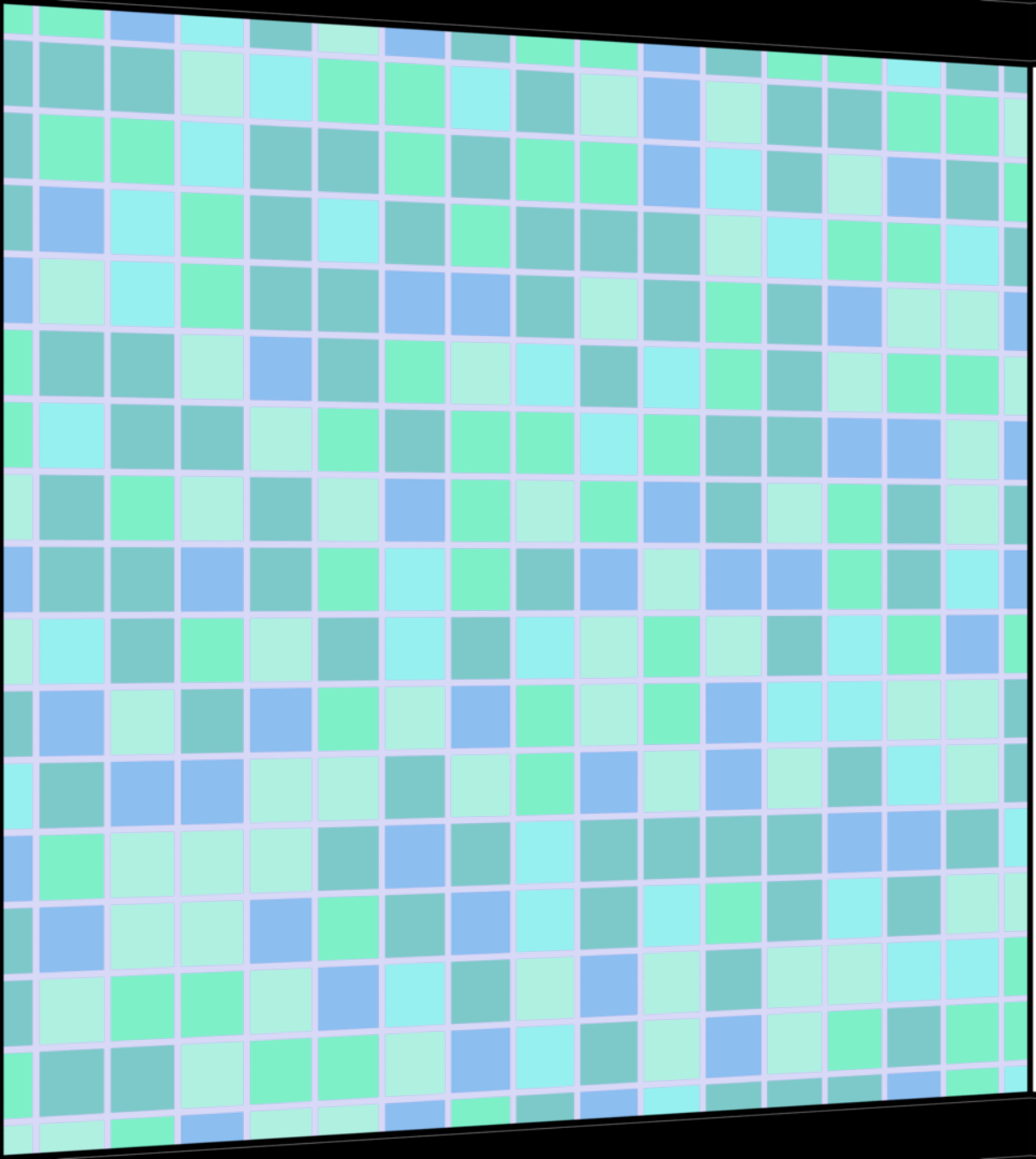
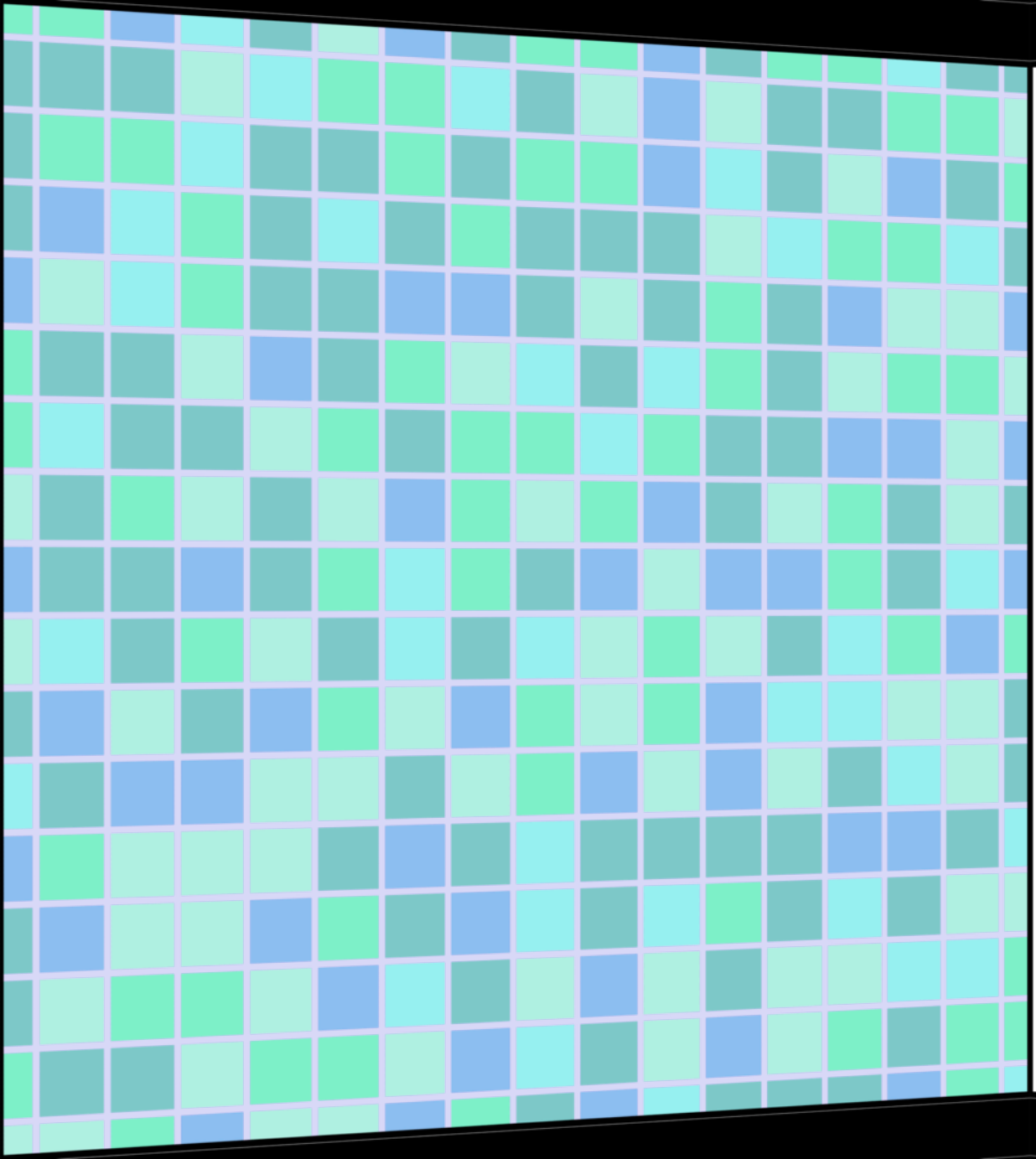


Deciding whether n coconuts can be pushed into an $a \times b$ rectangle is NP-complete.



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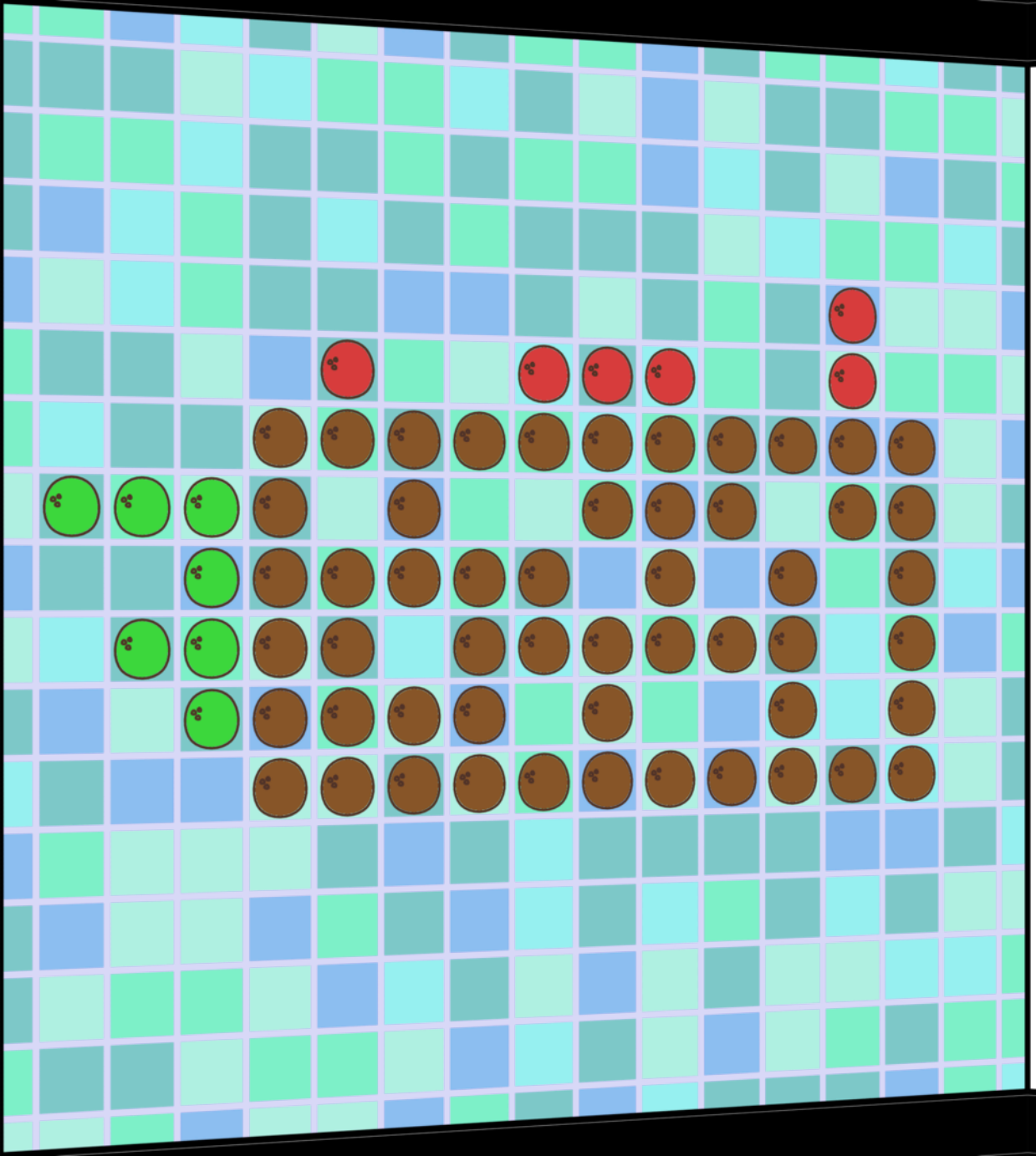

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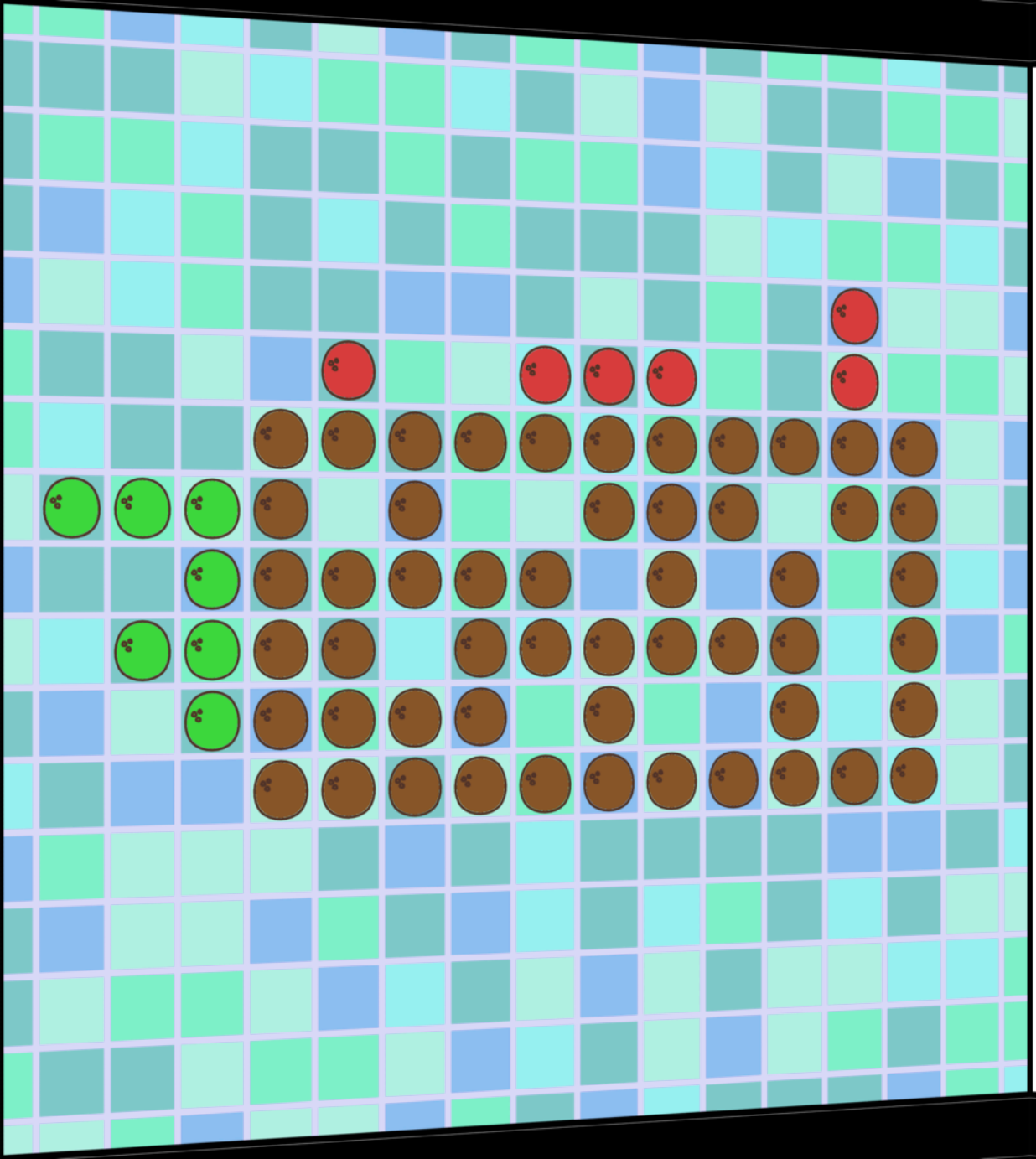

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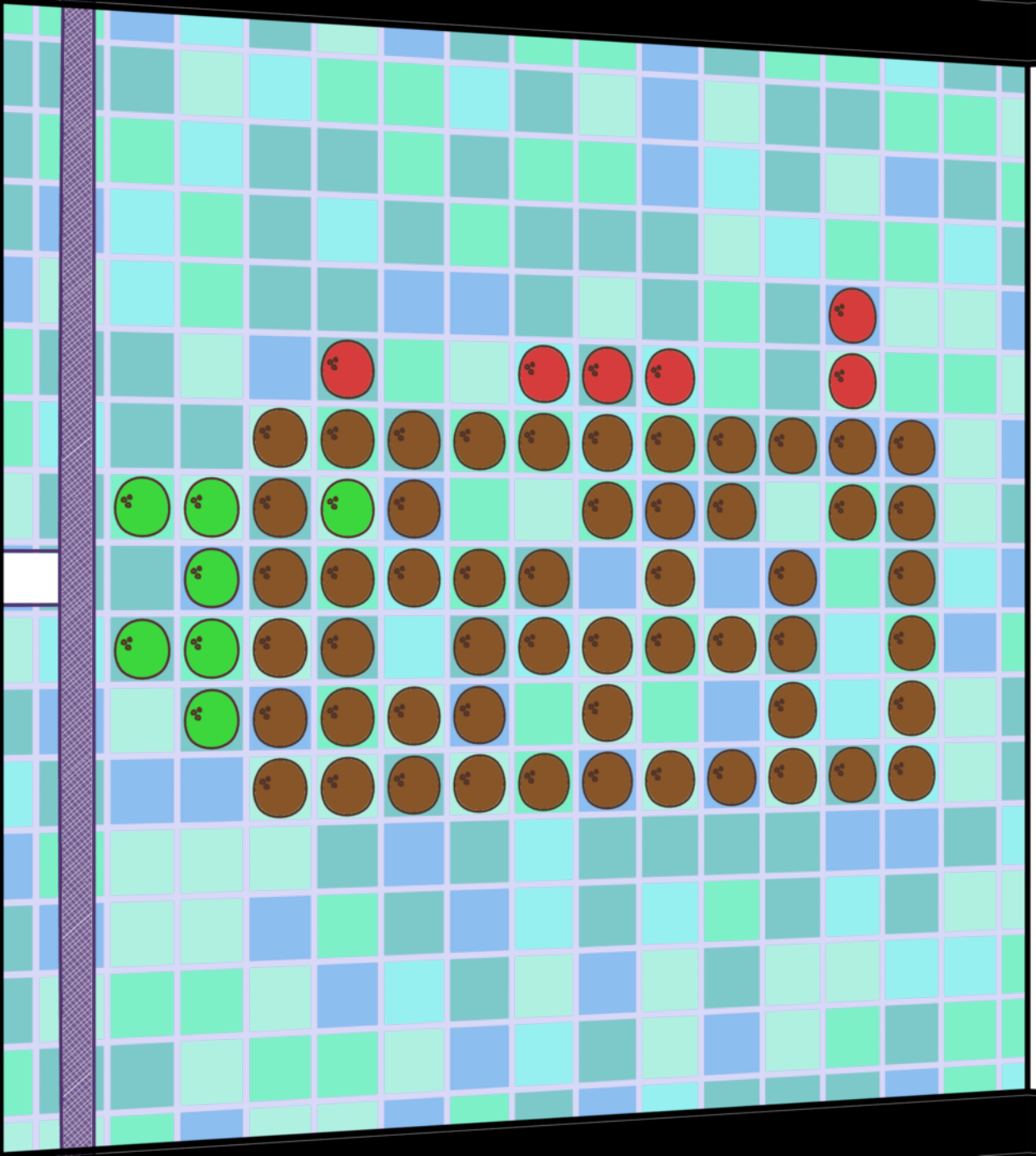

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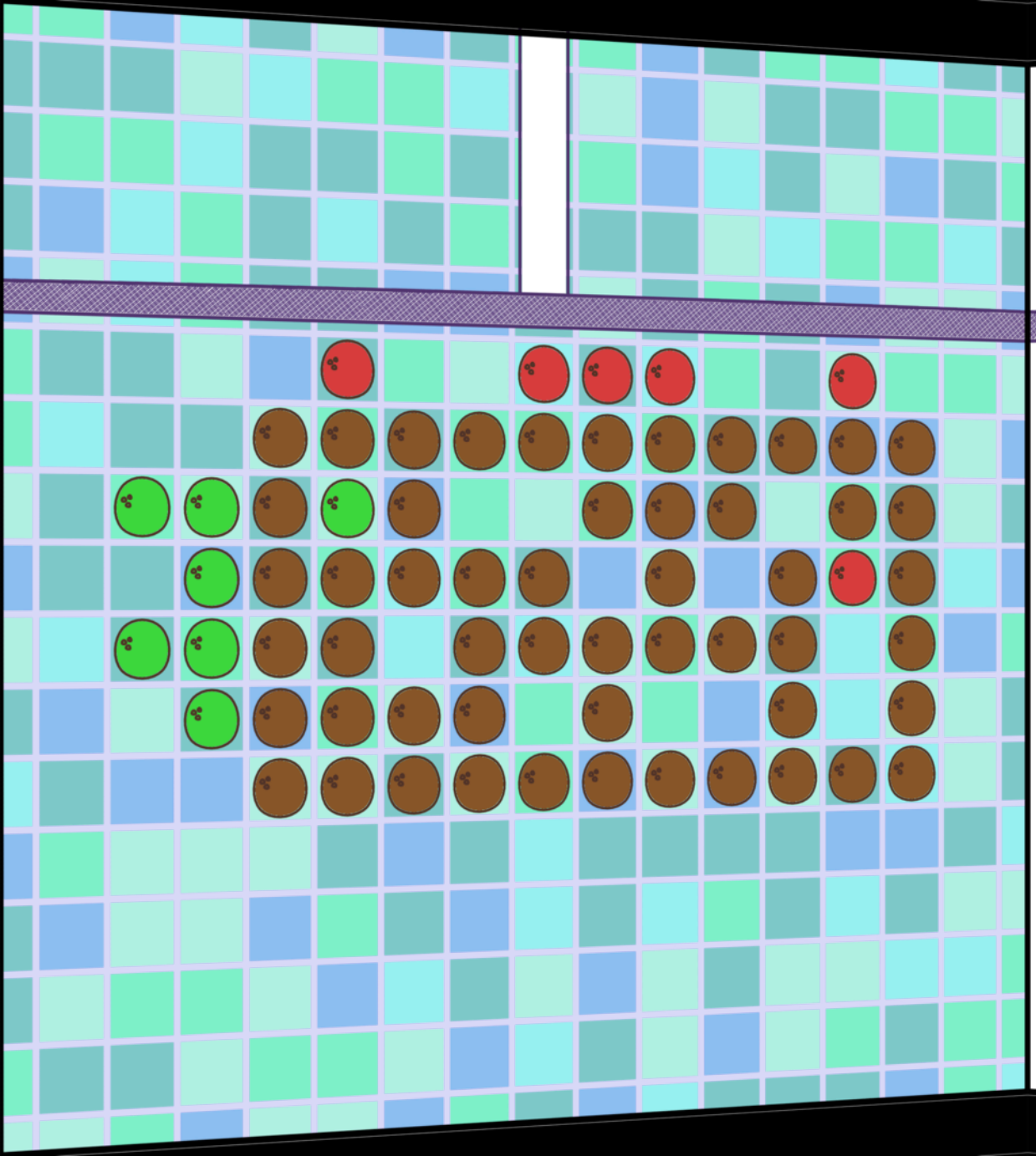

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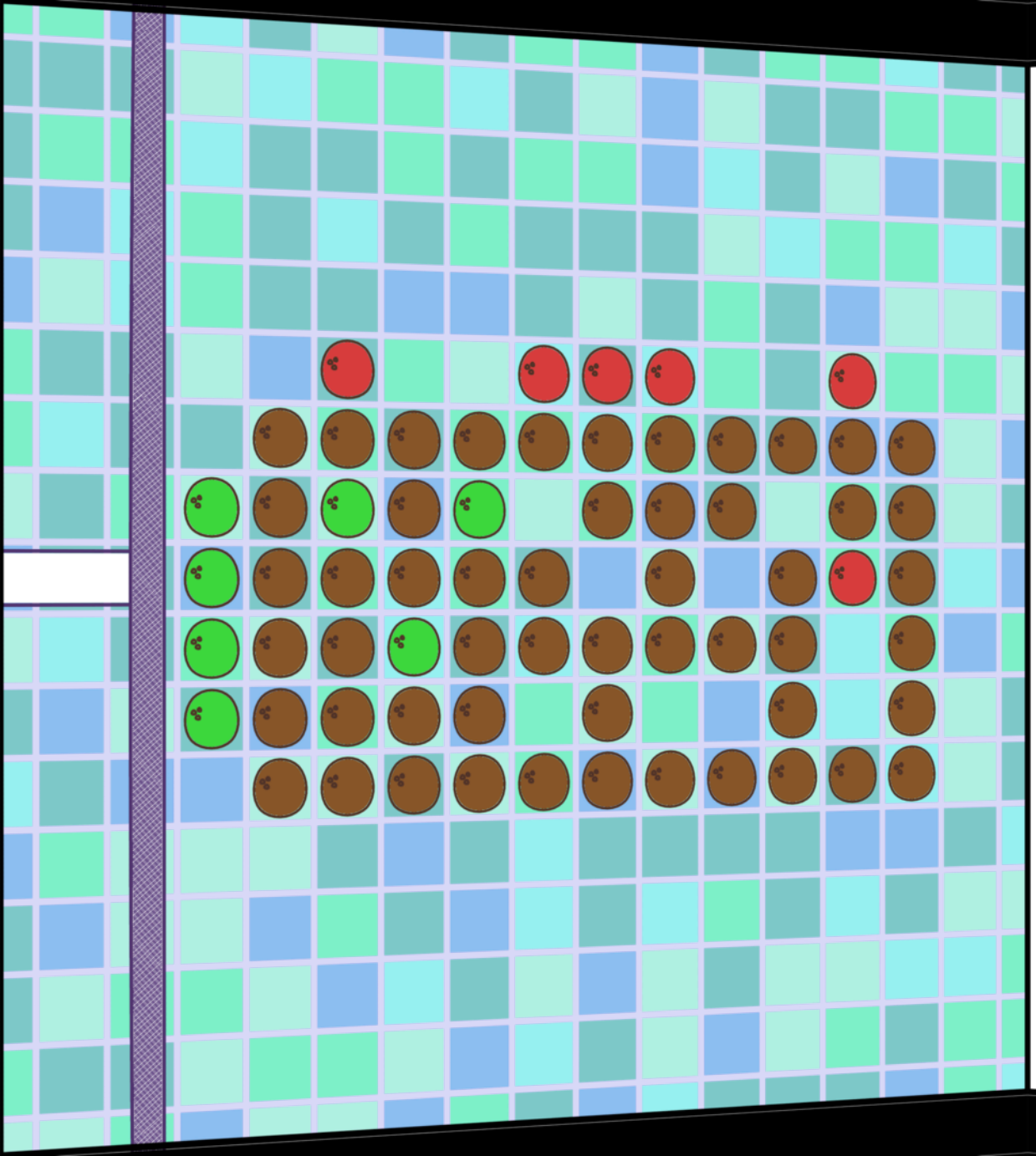

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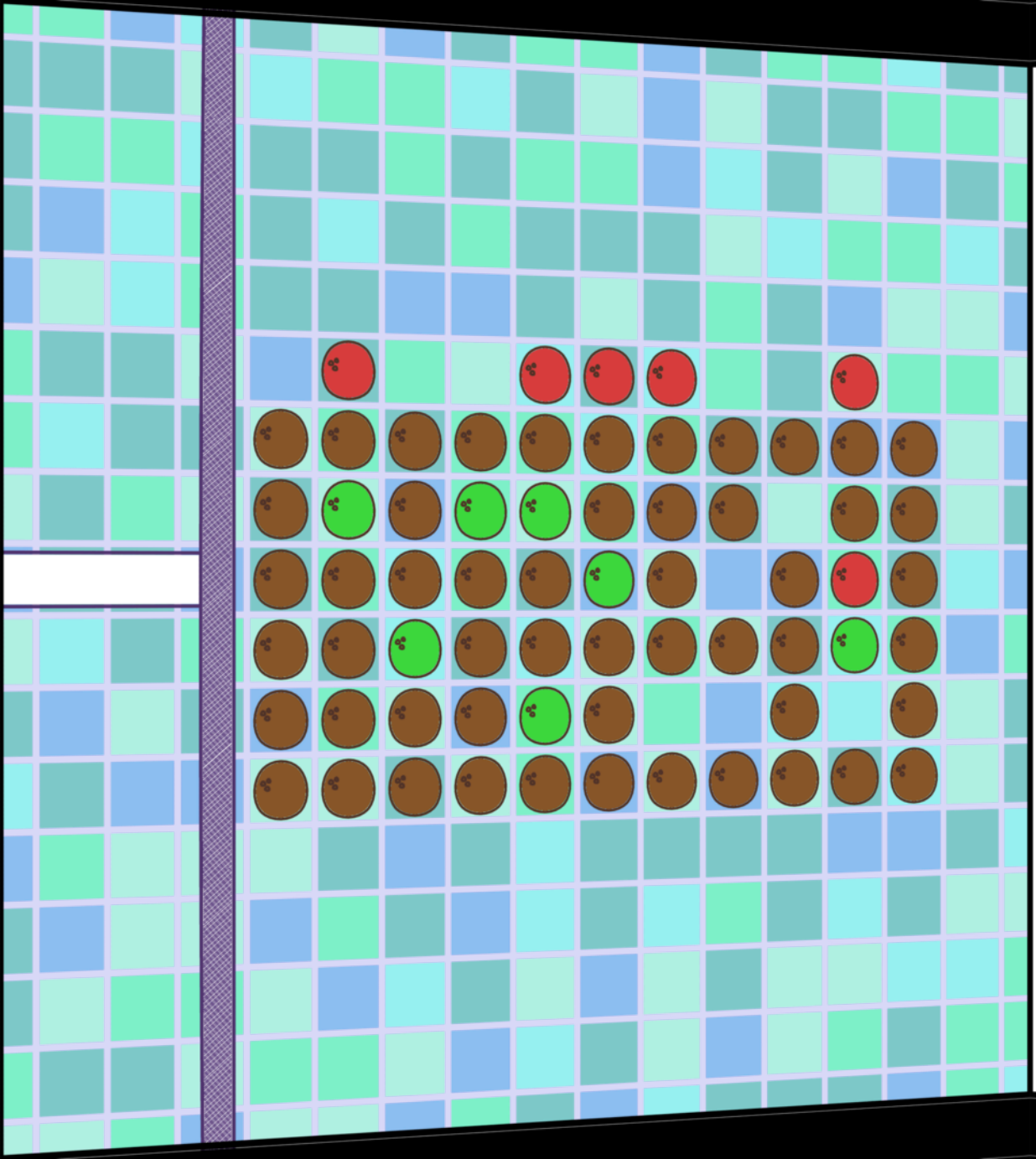

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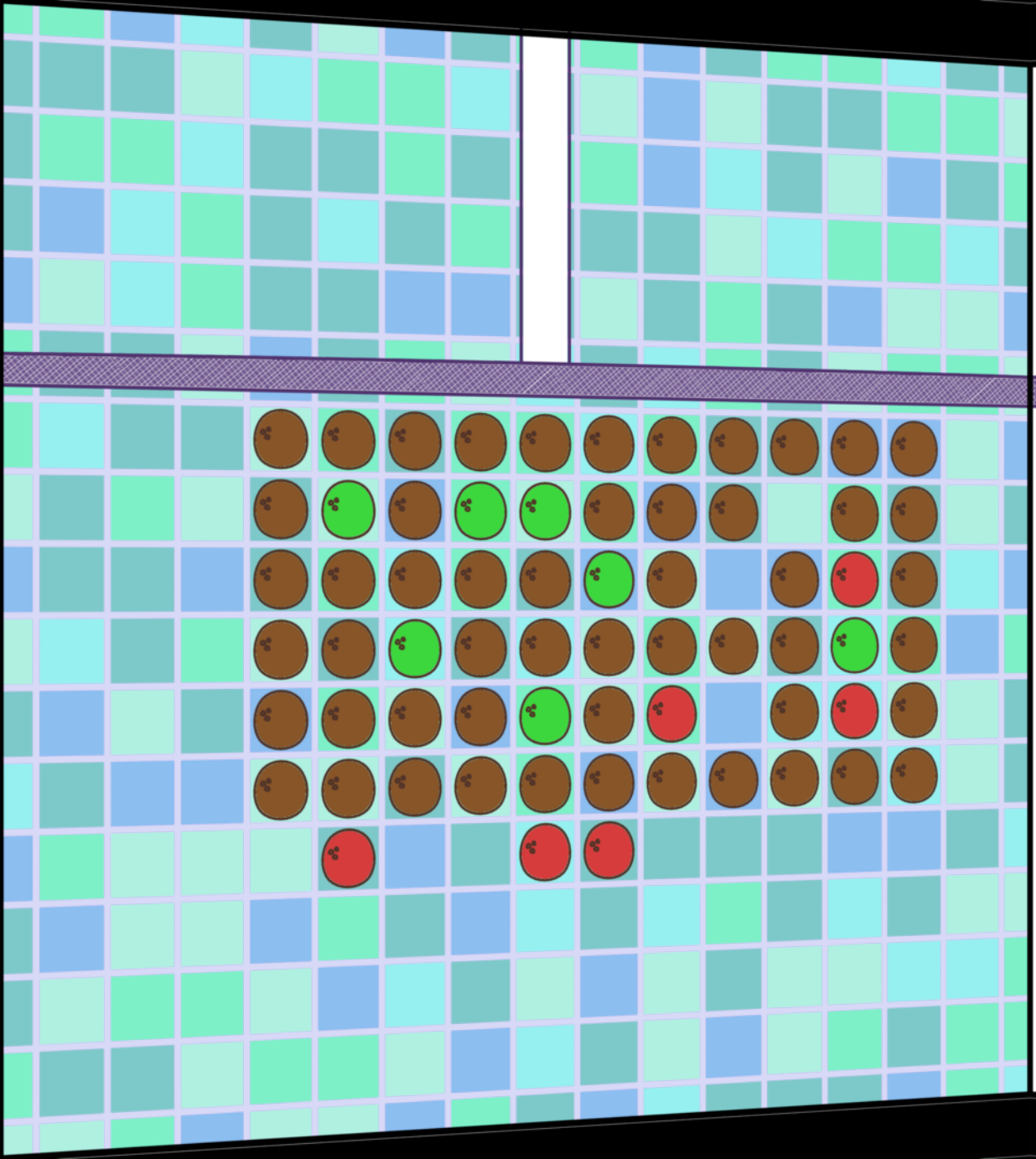

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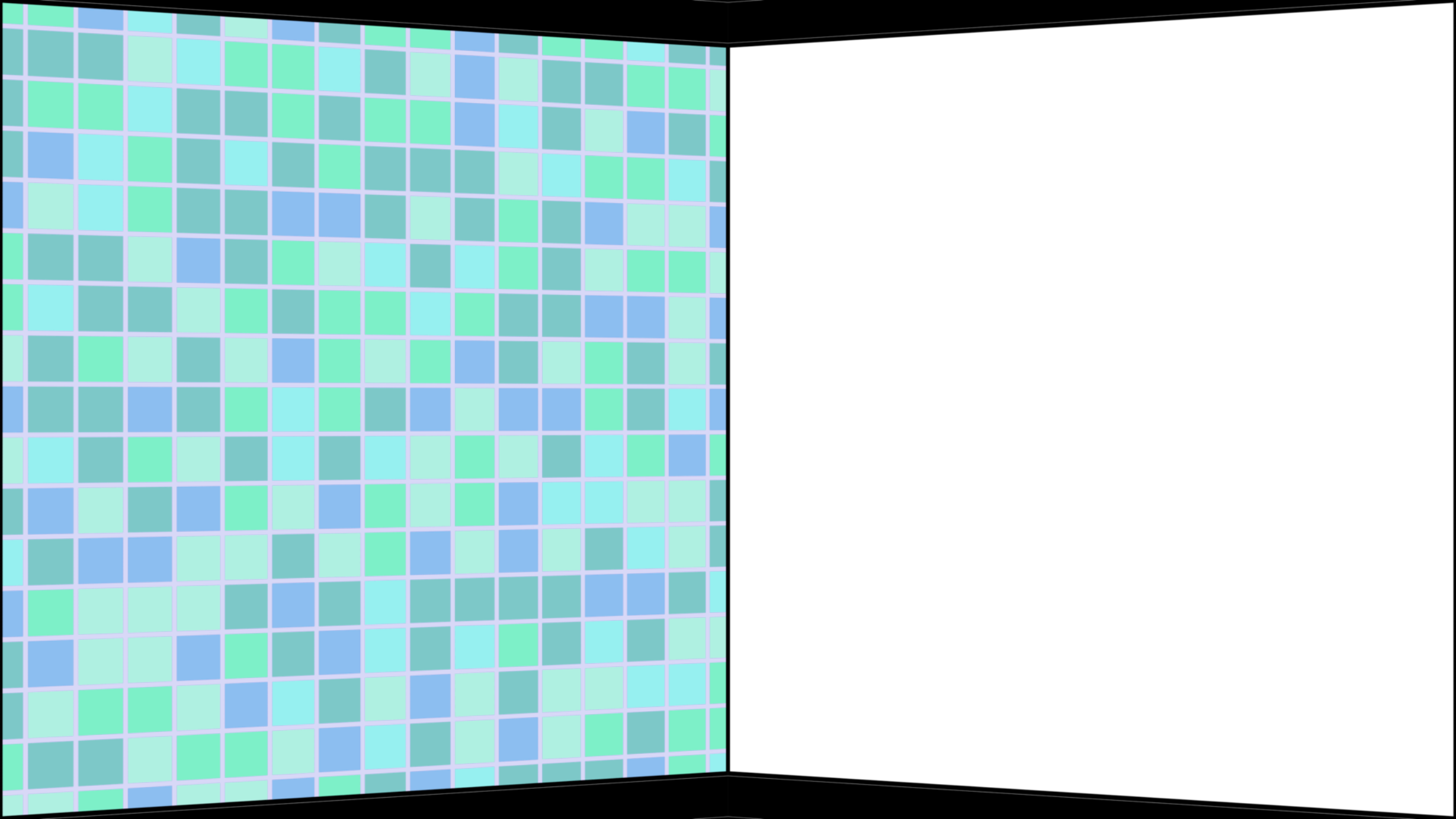

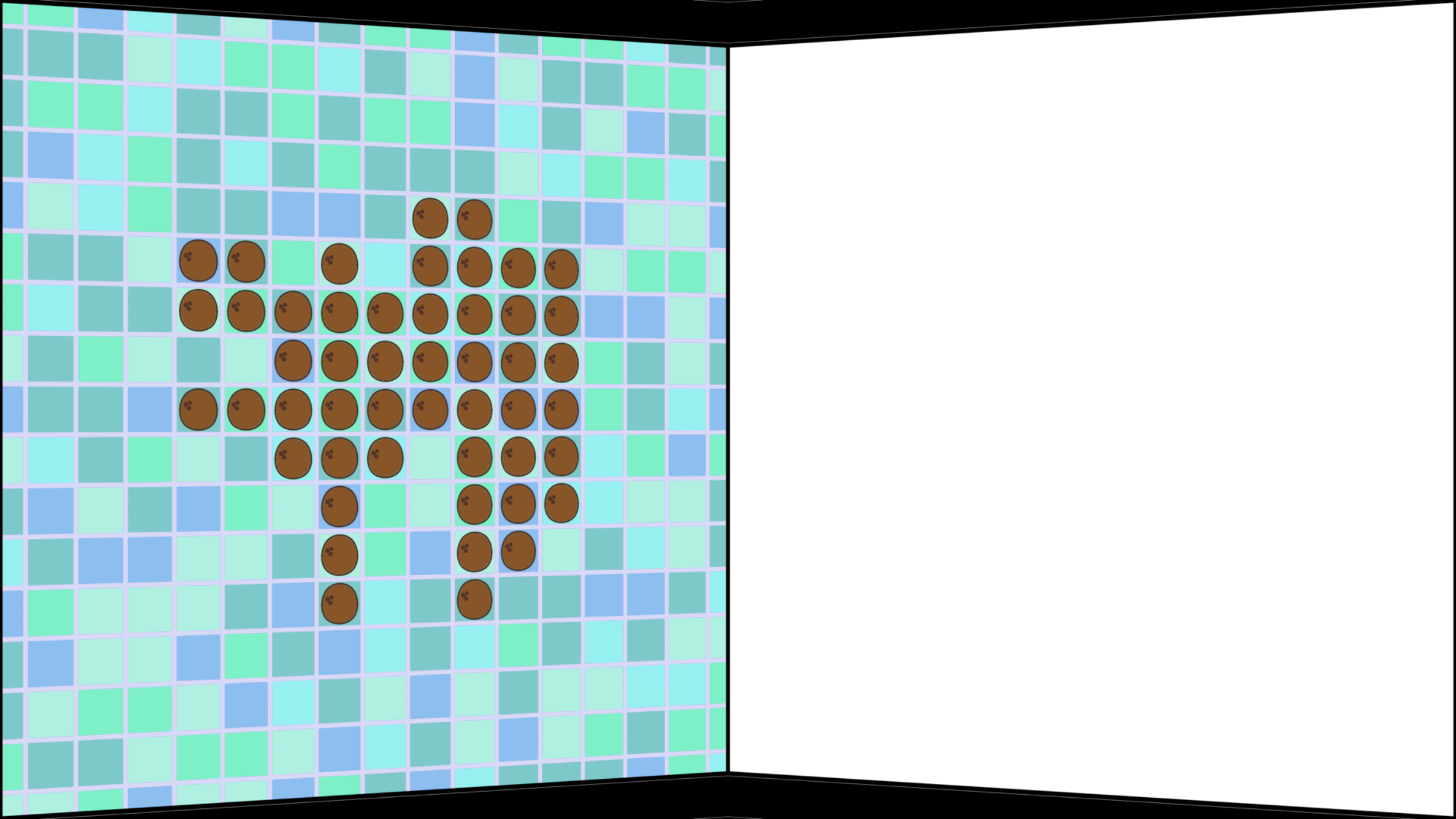

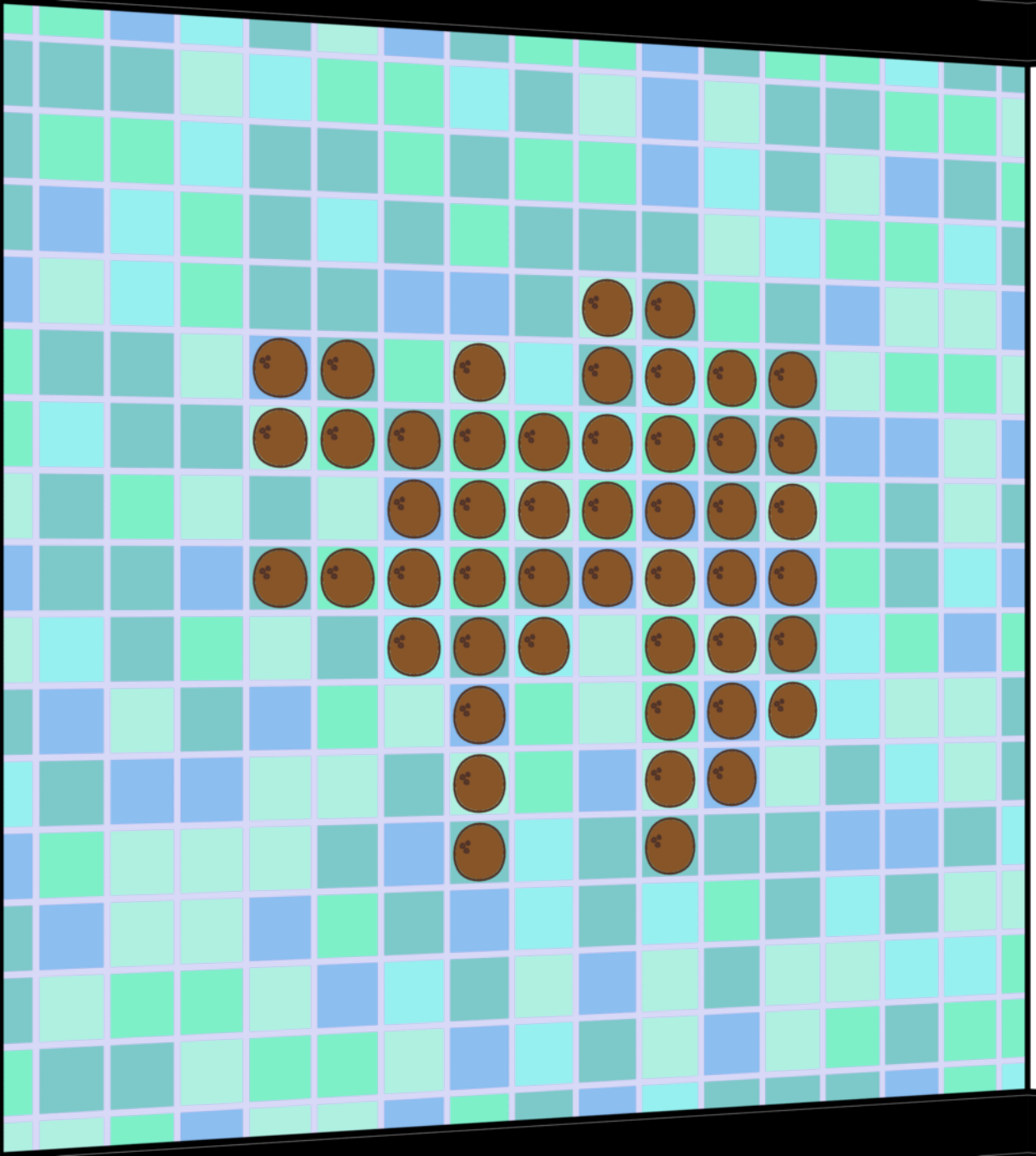

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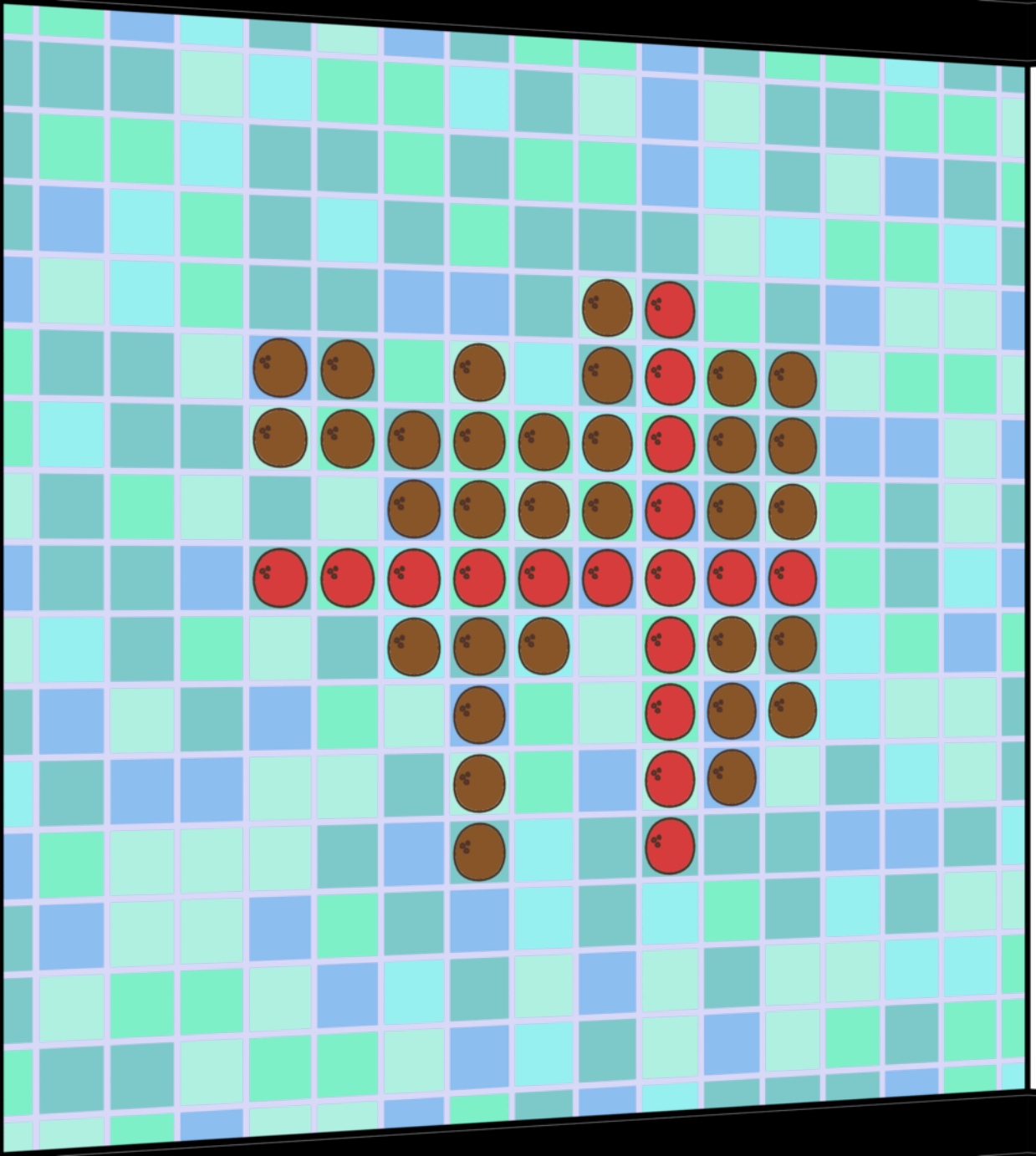

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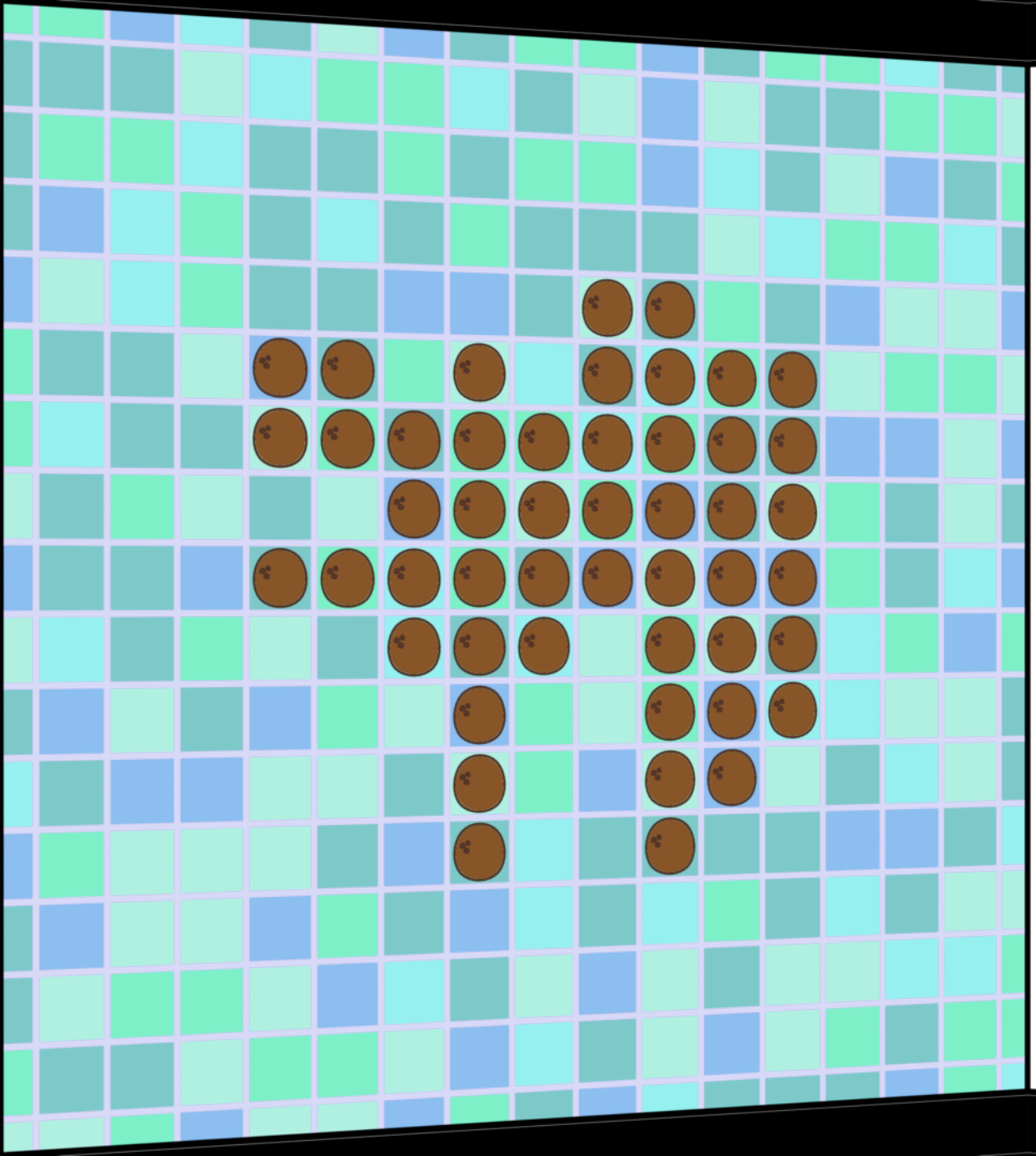

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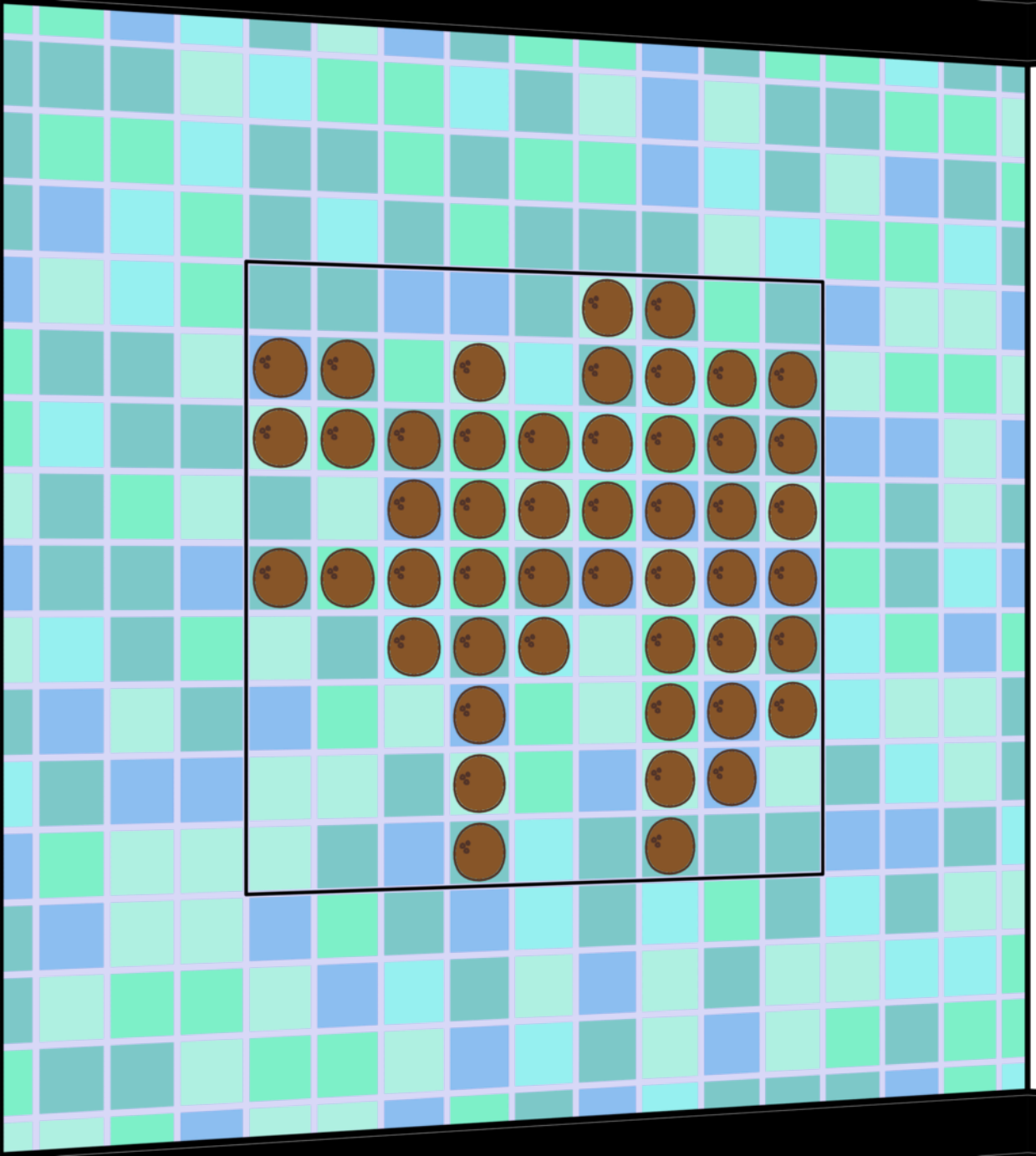

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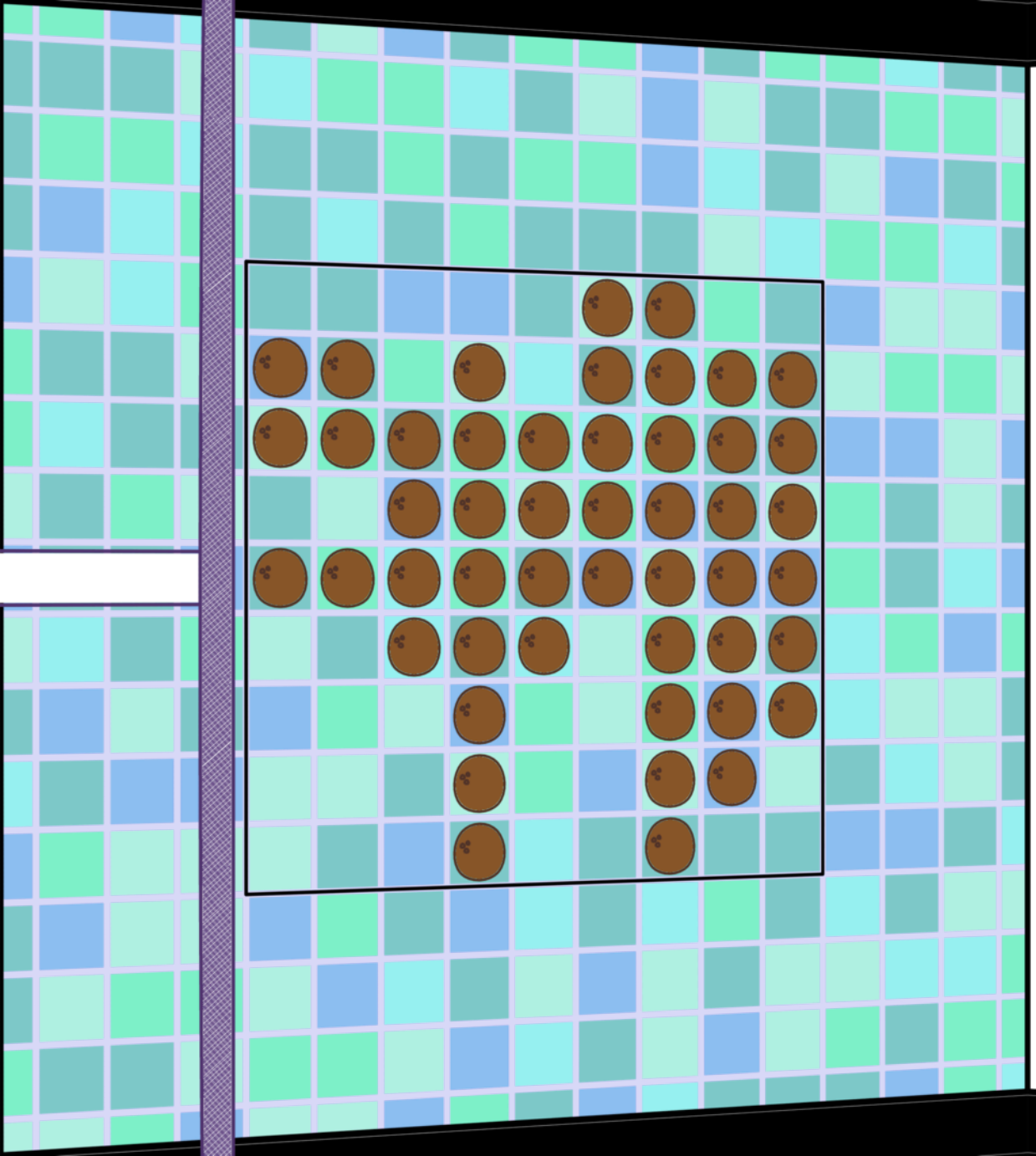

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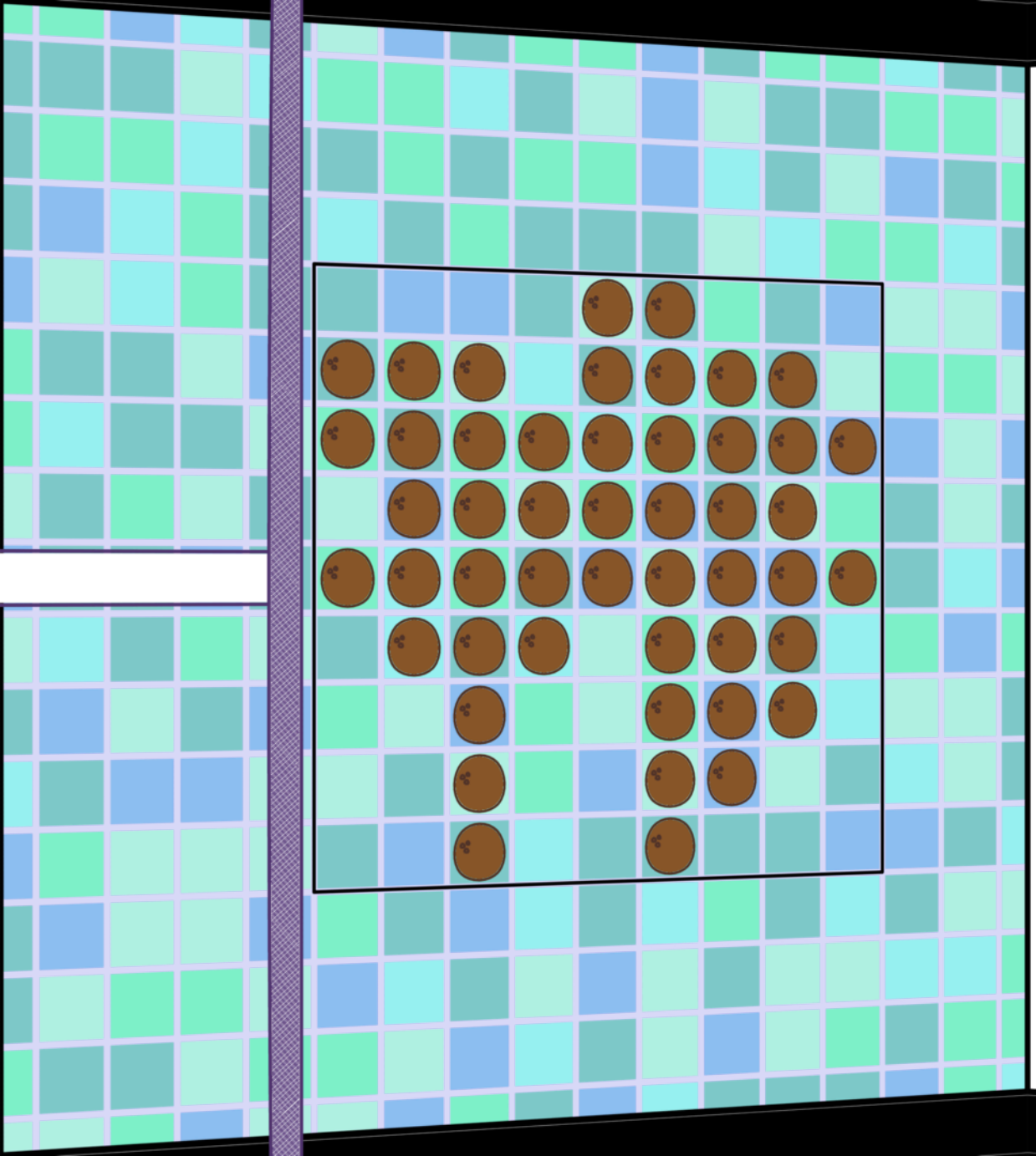

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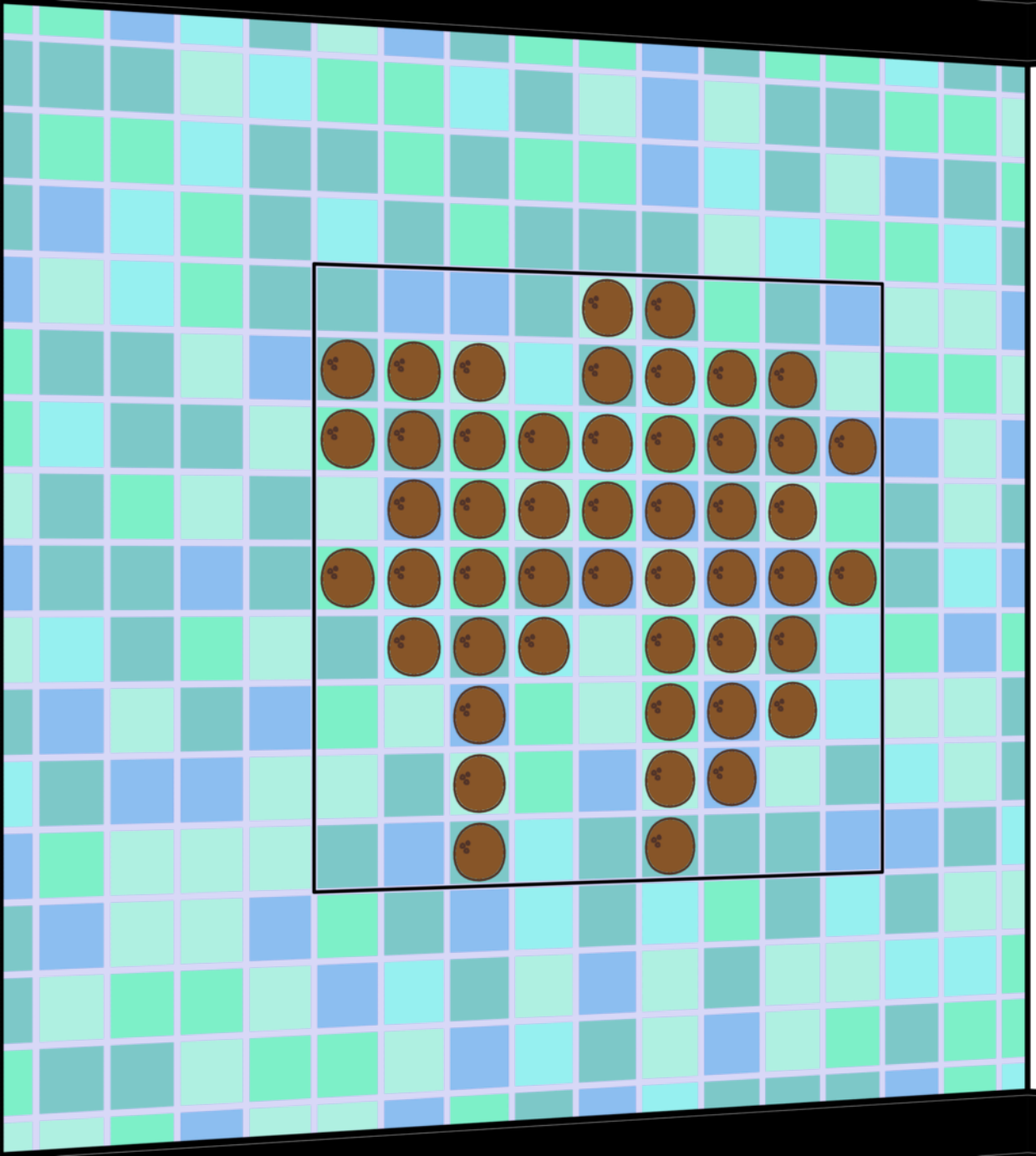

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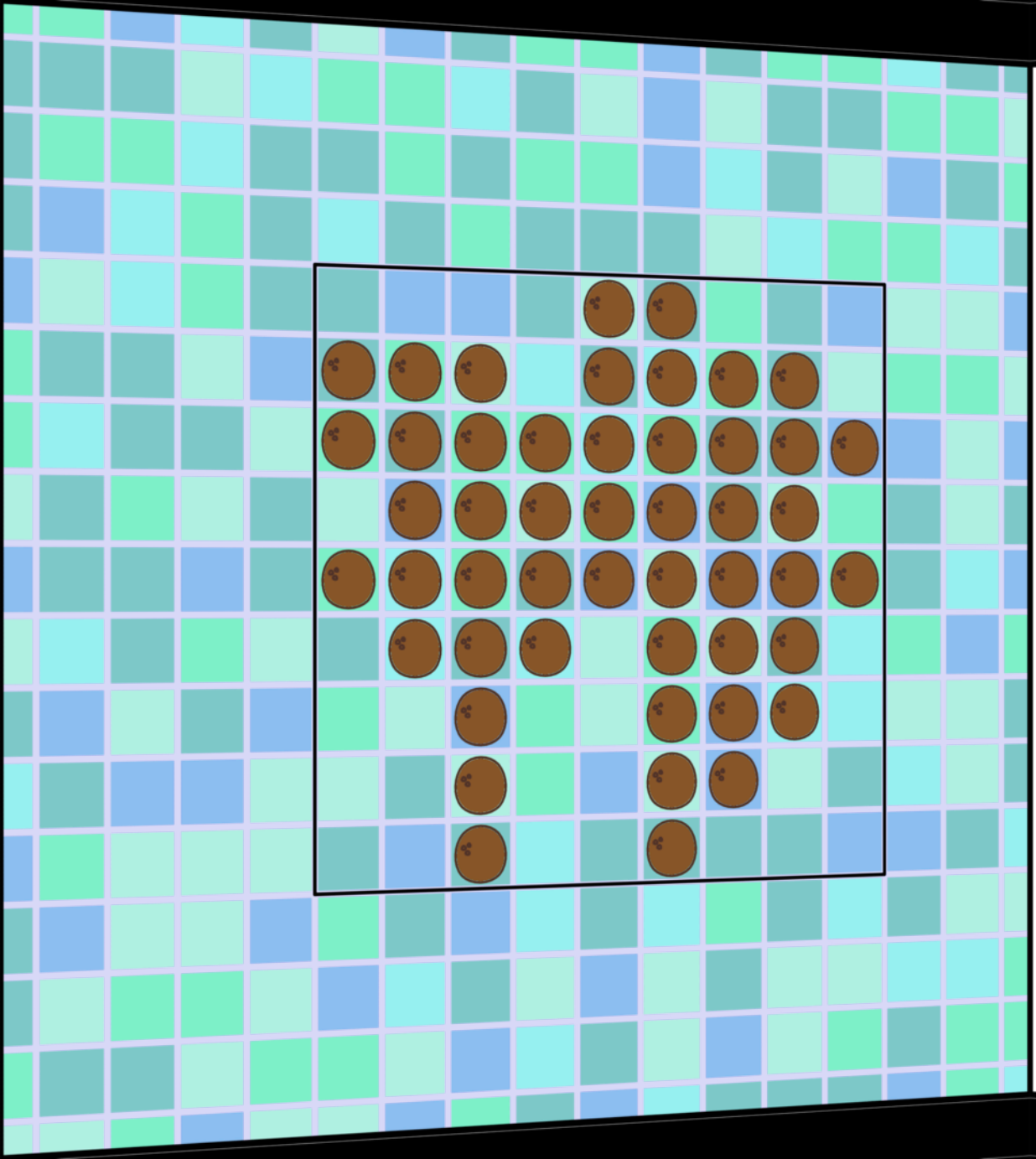

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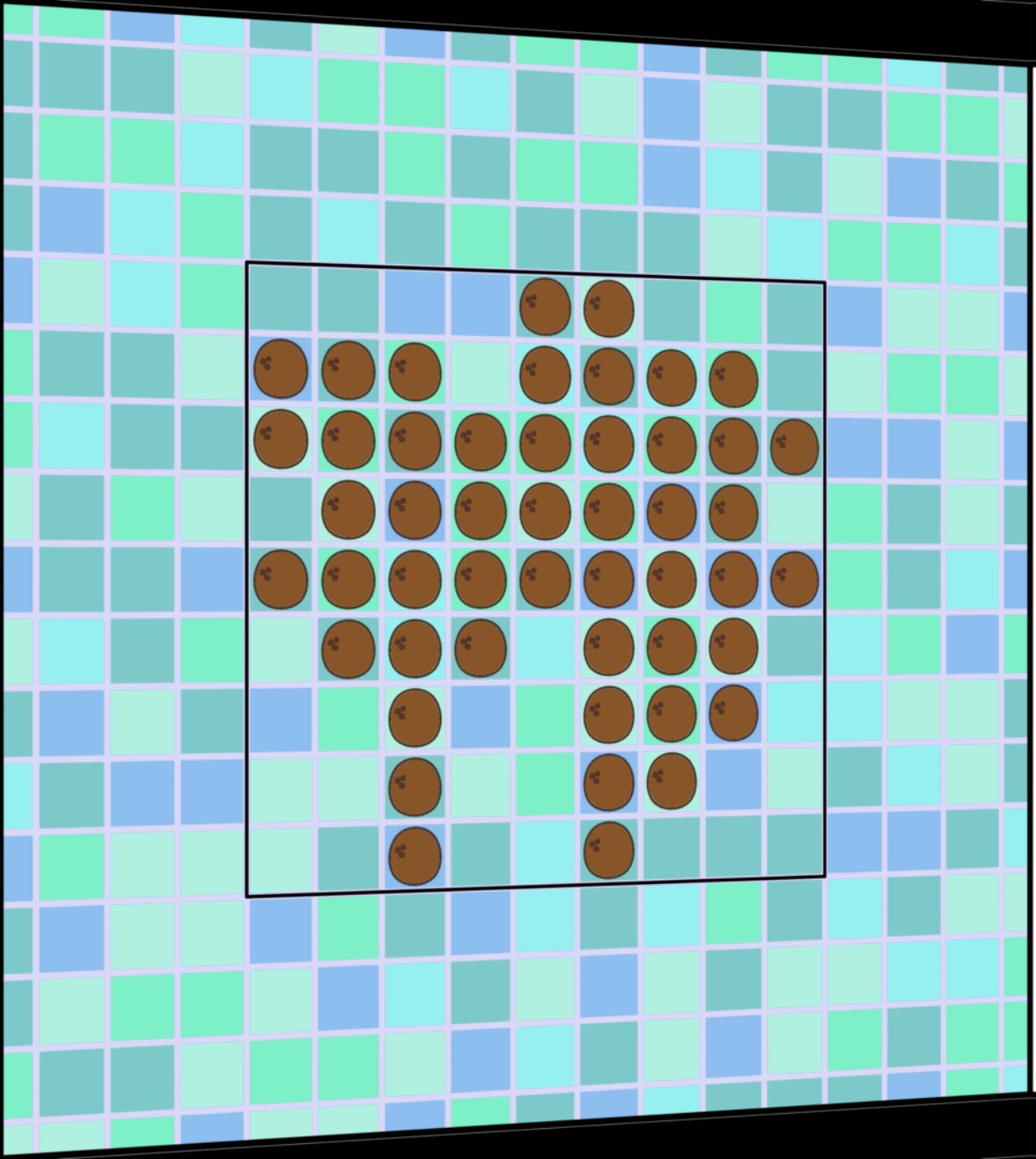

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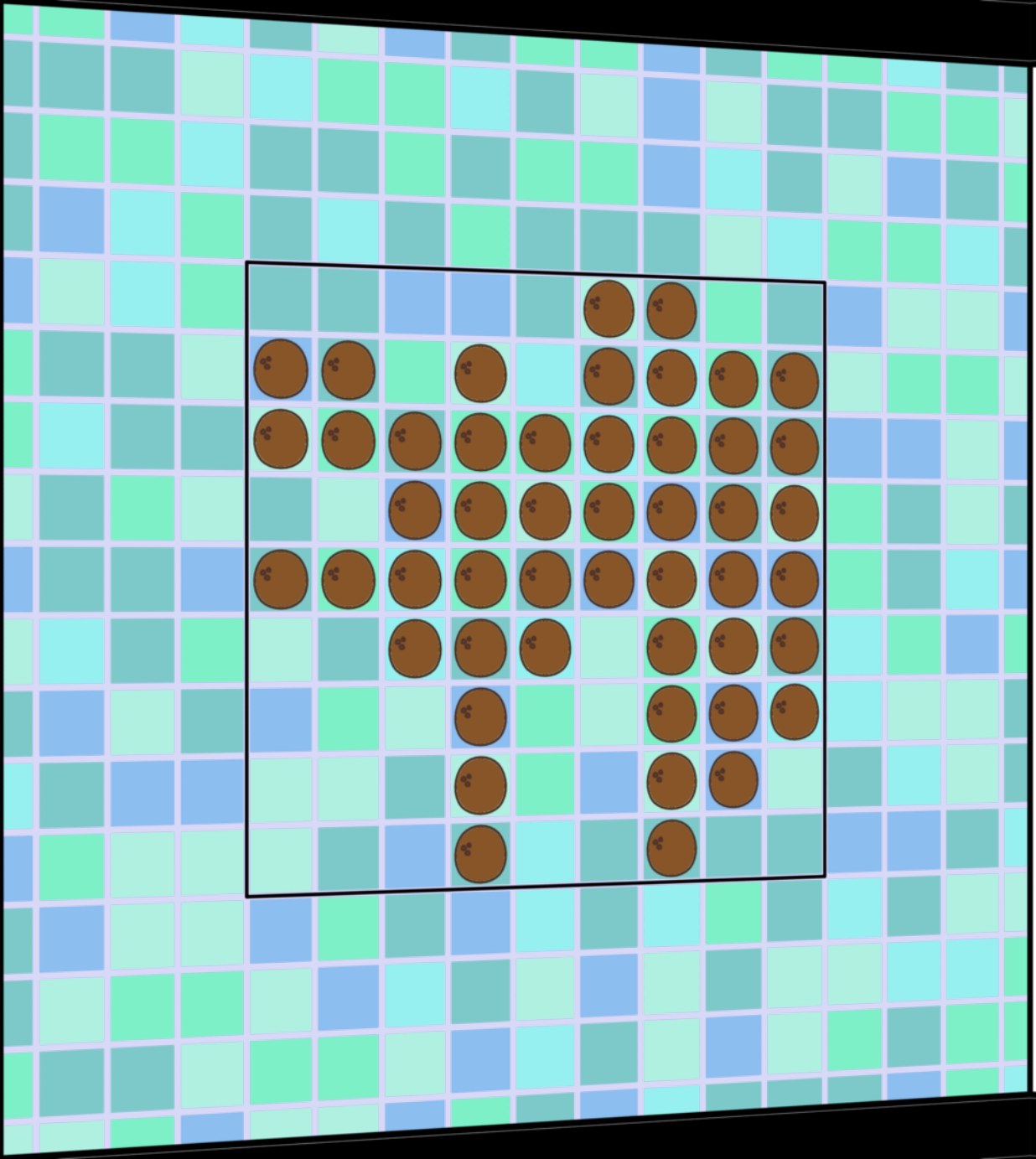

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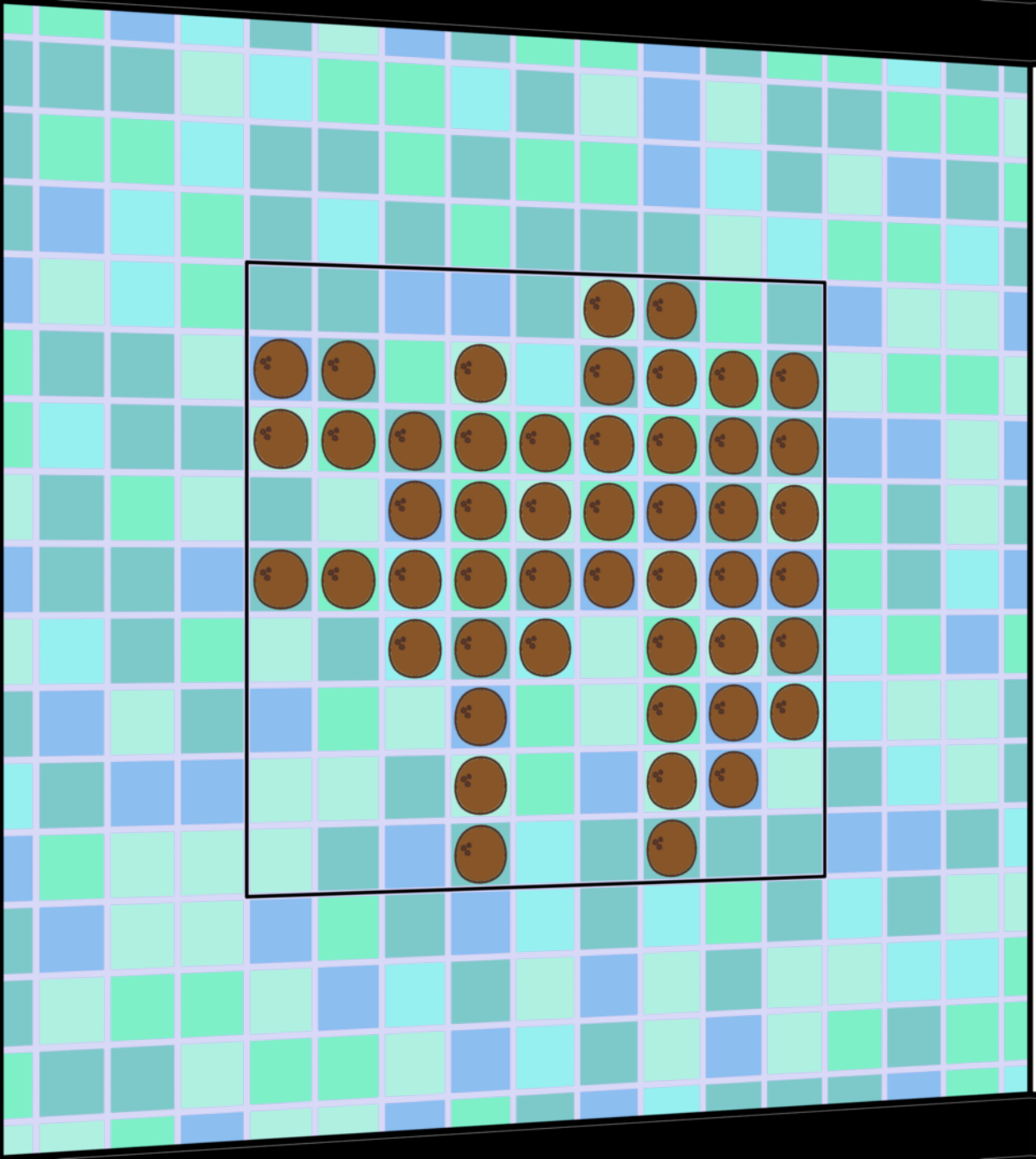

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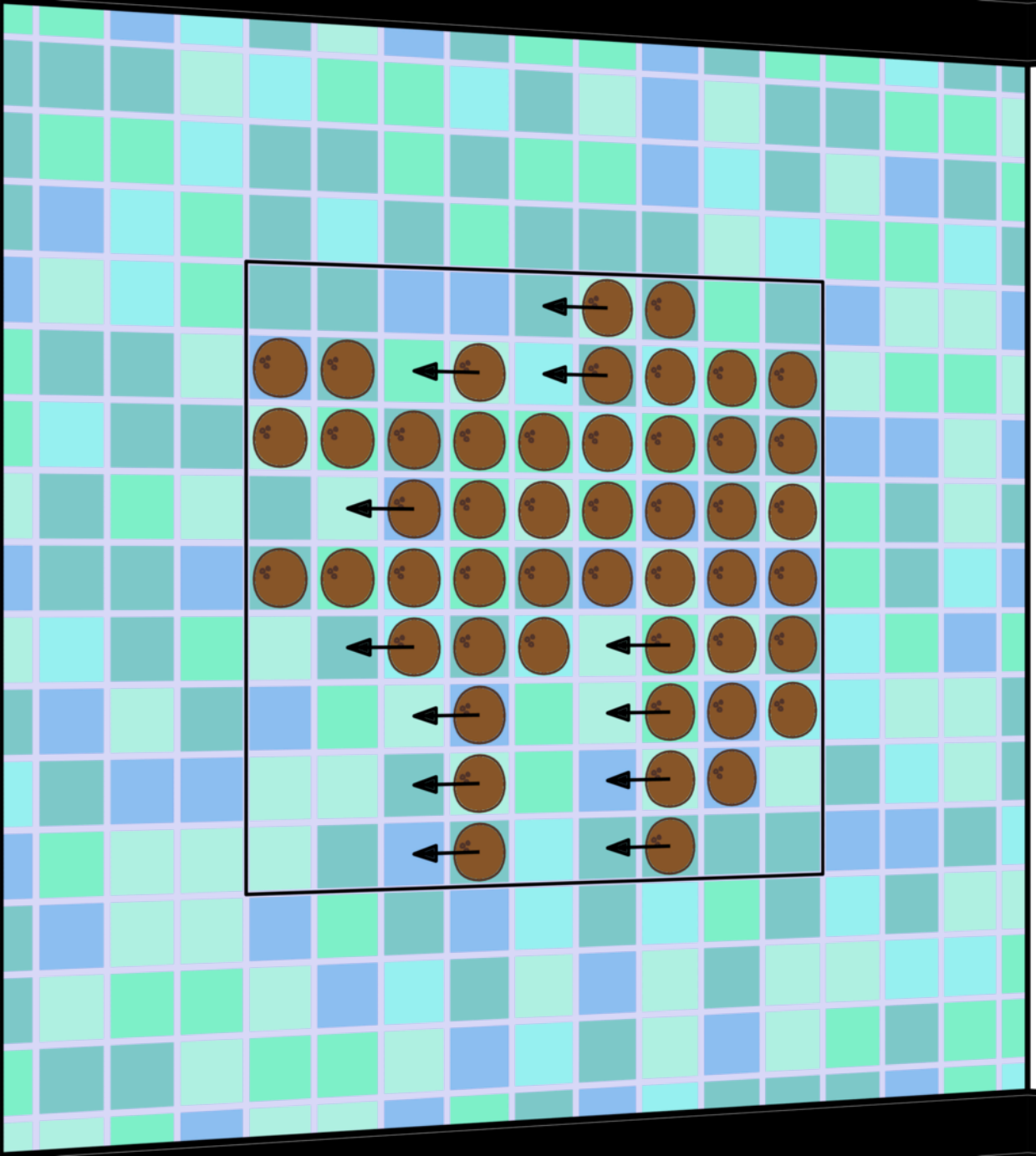

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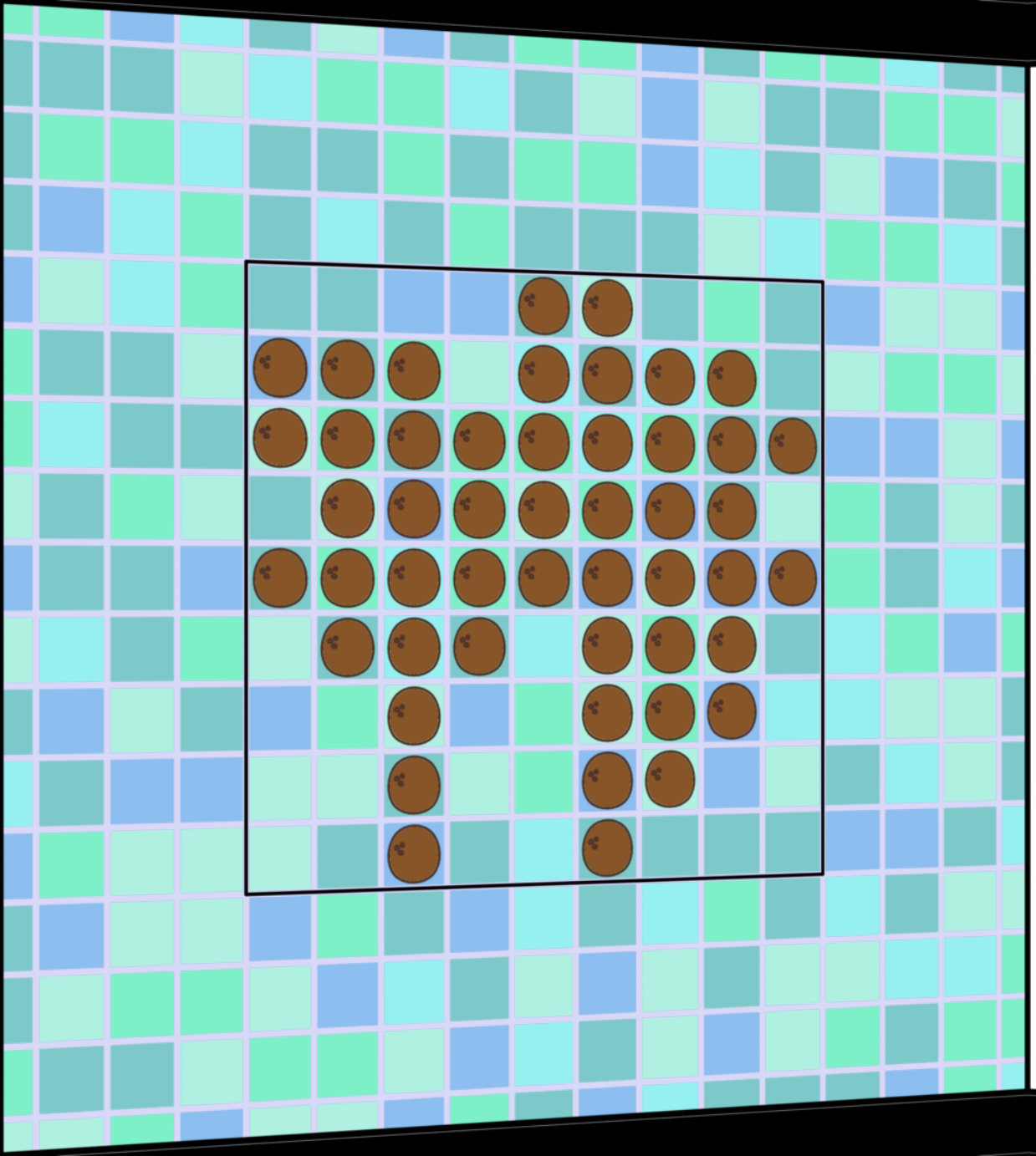

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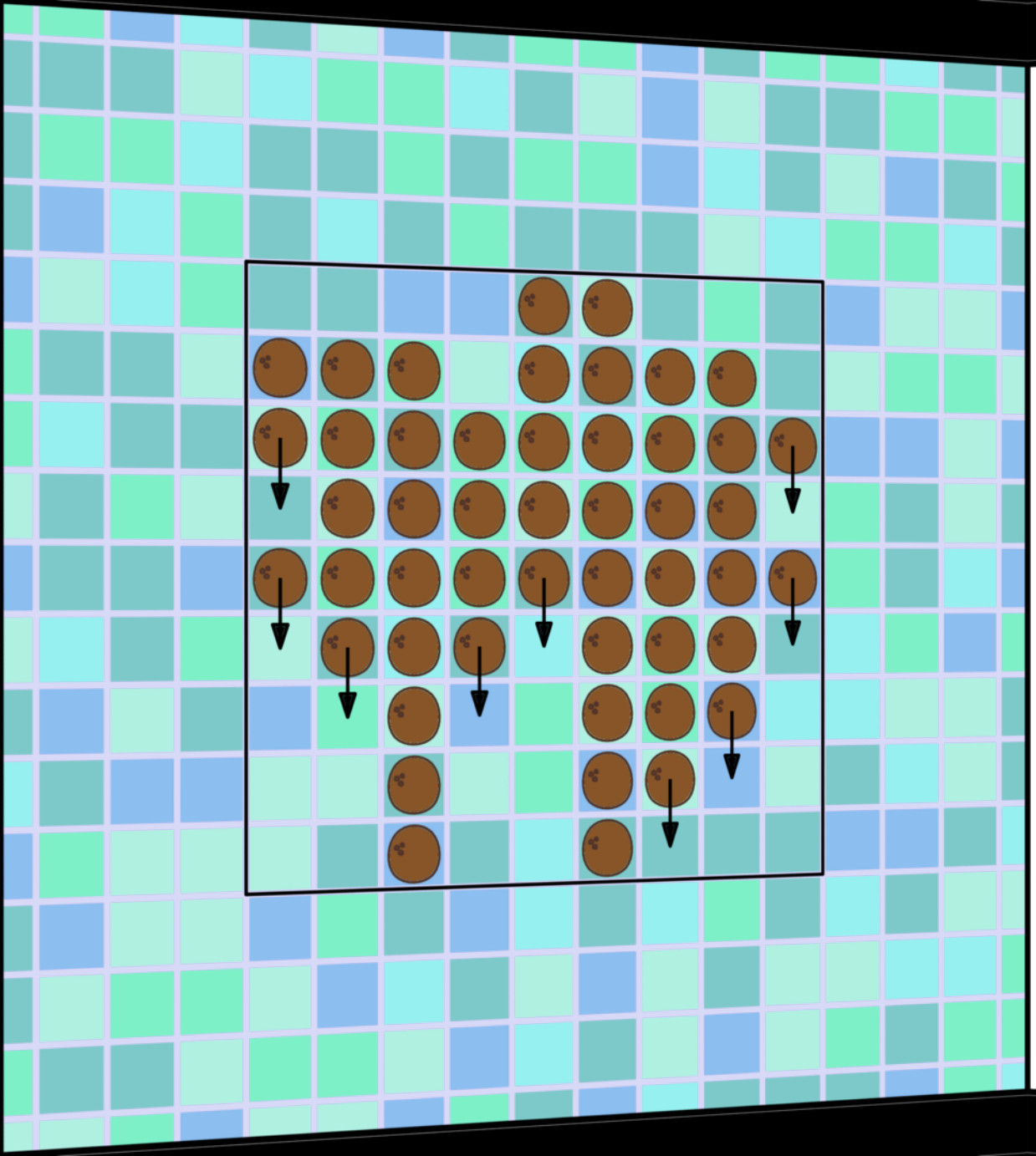

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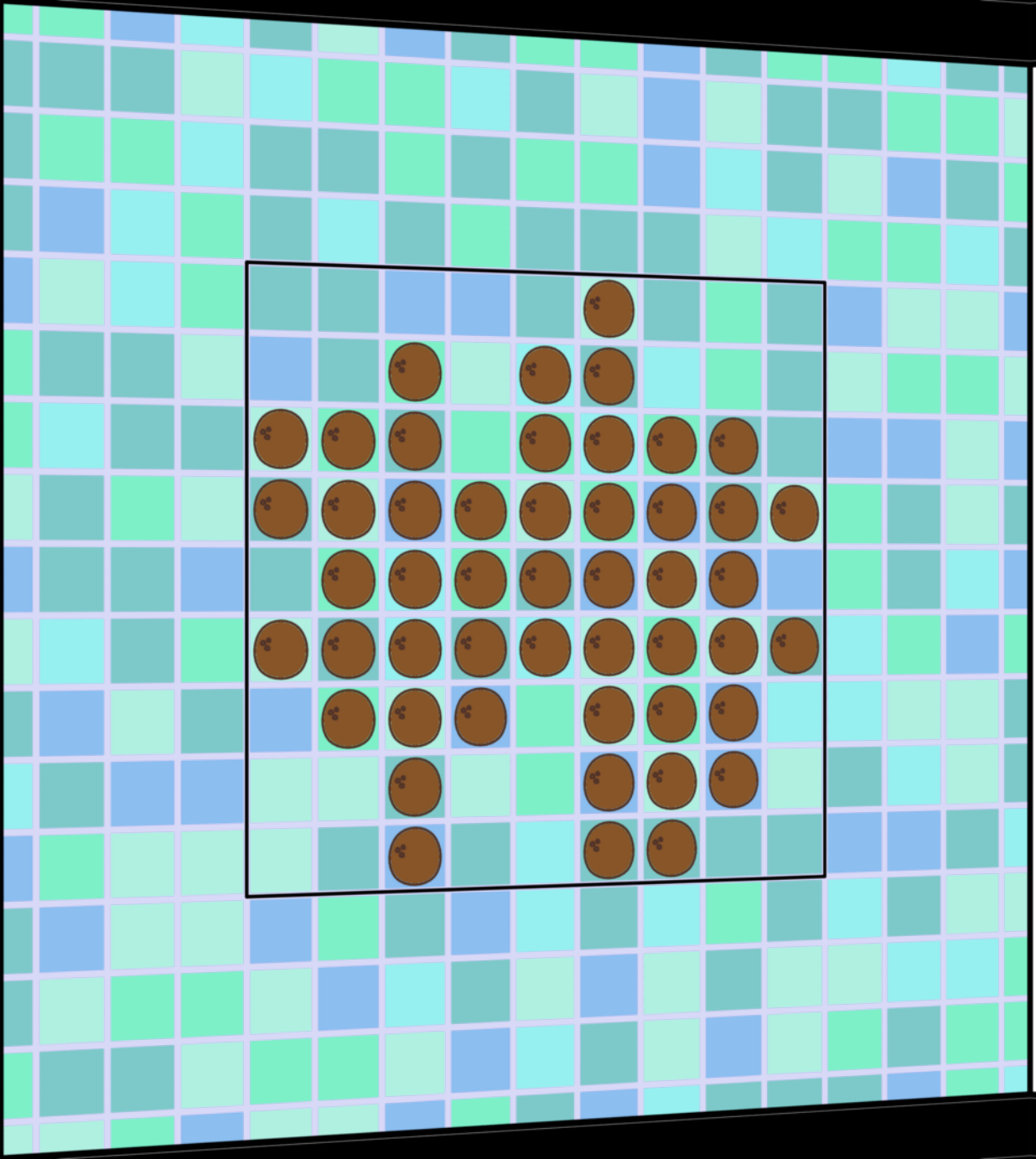

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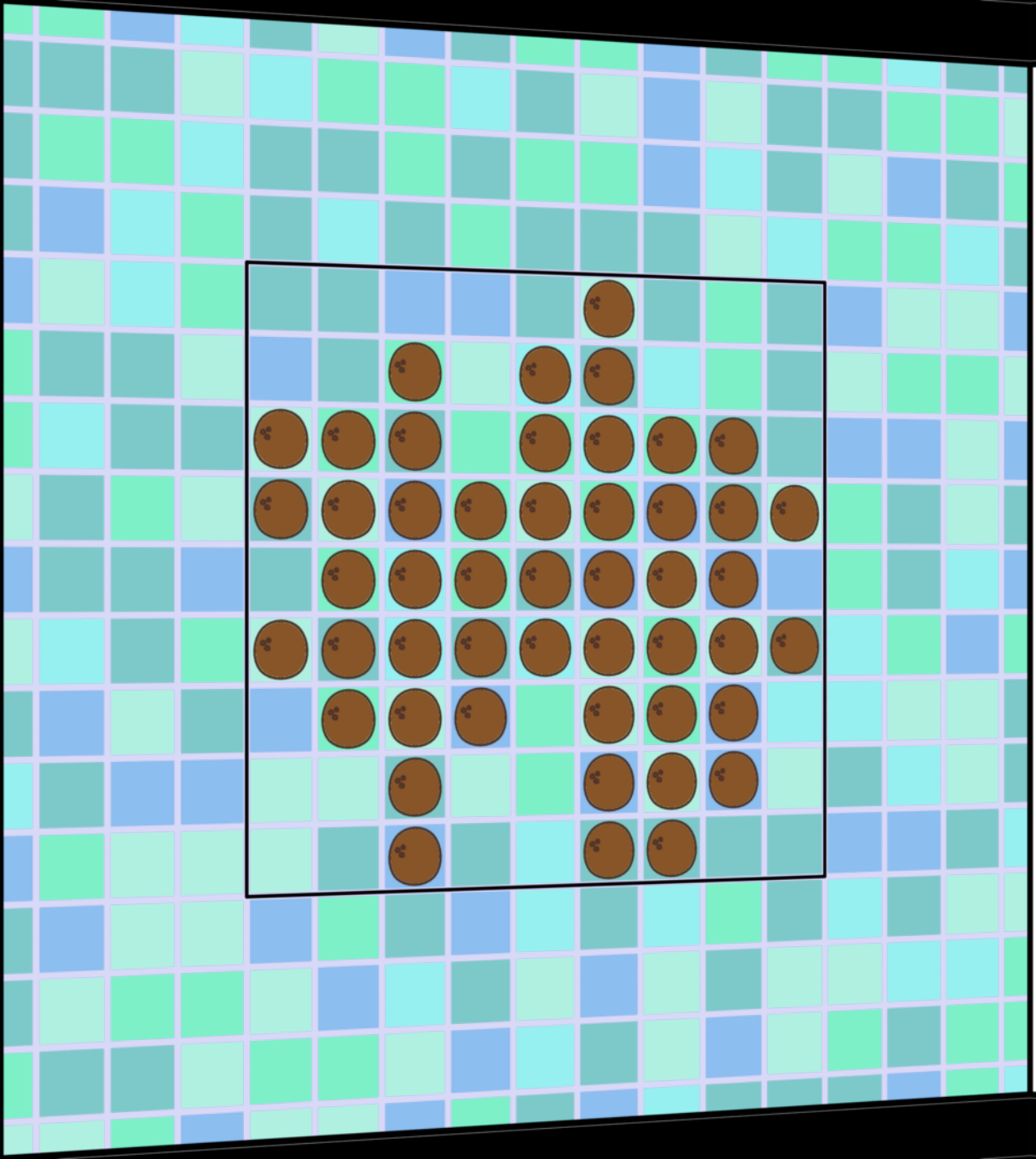

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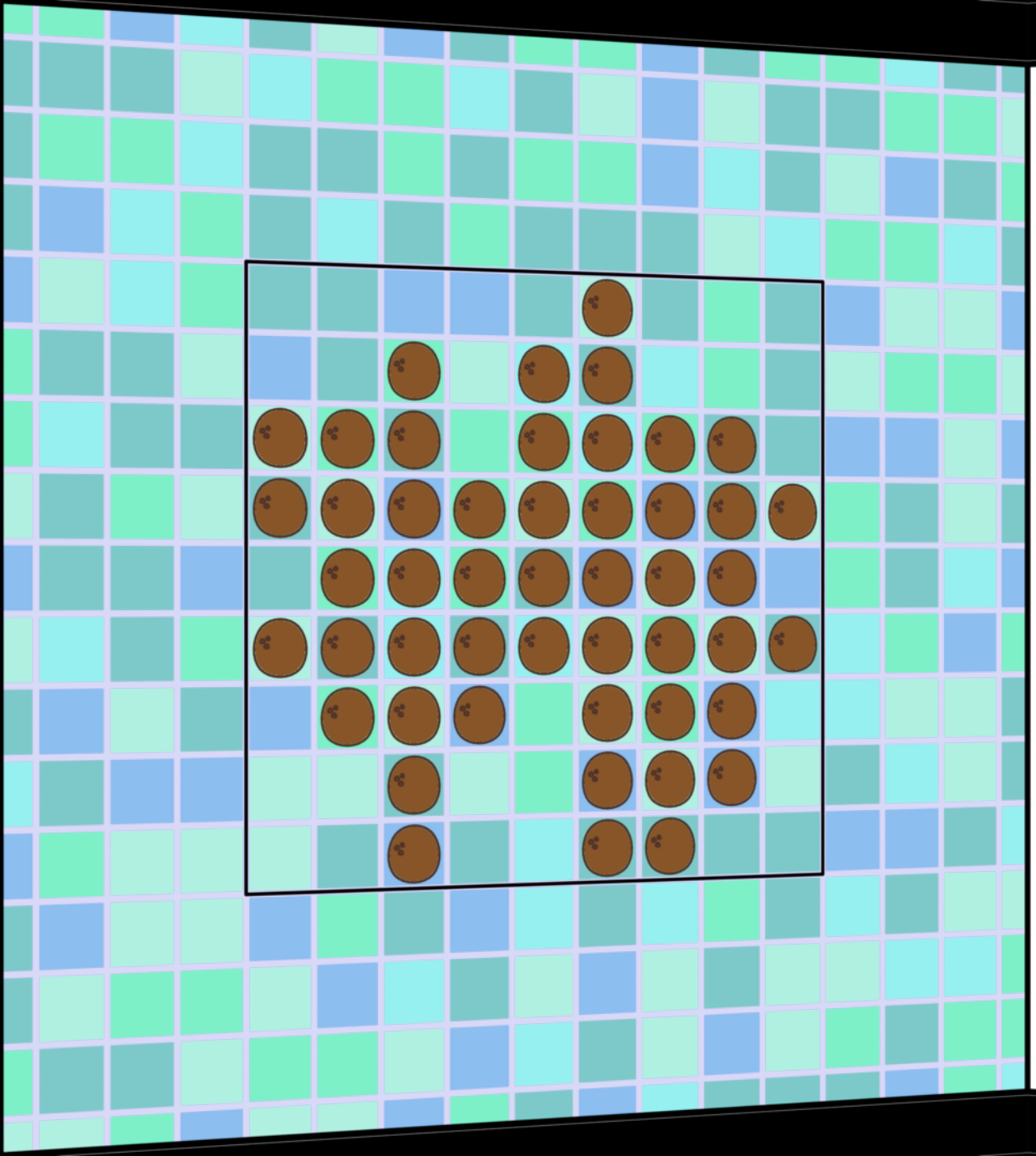
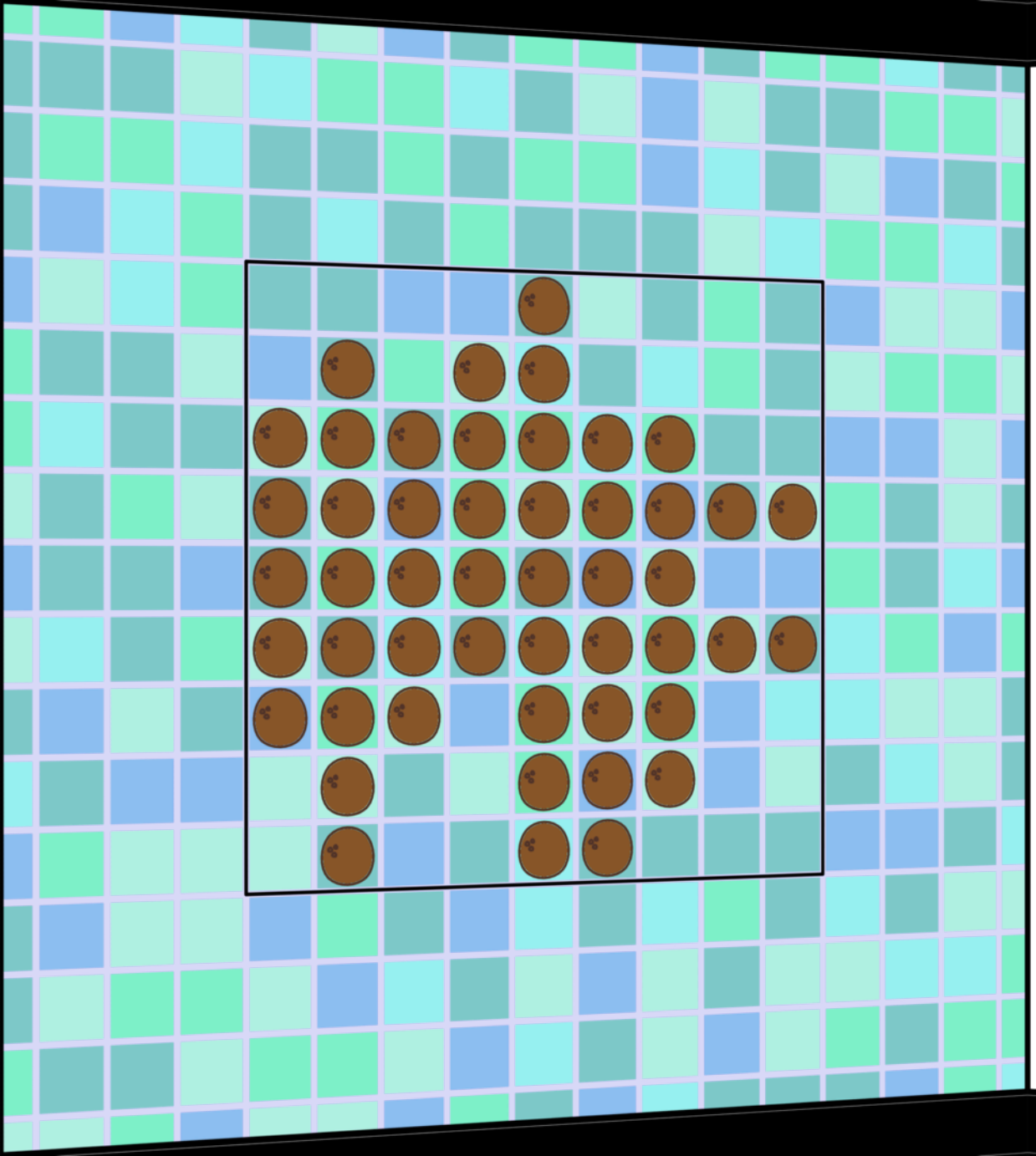
Same game, different viewpoint.



After pushing from the left and from below sufficiently often, we get a *canonical* (staircase) configuration.



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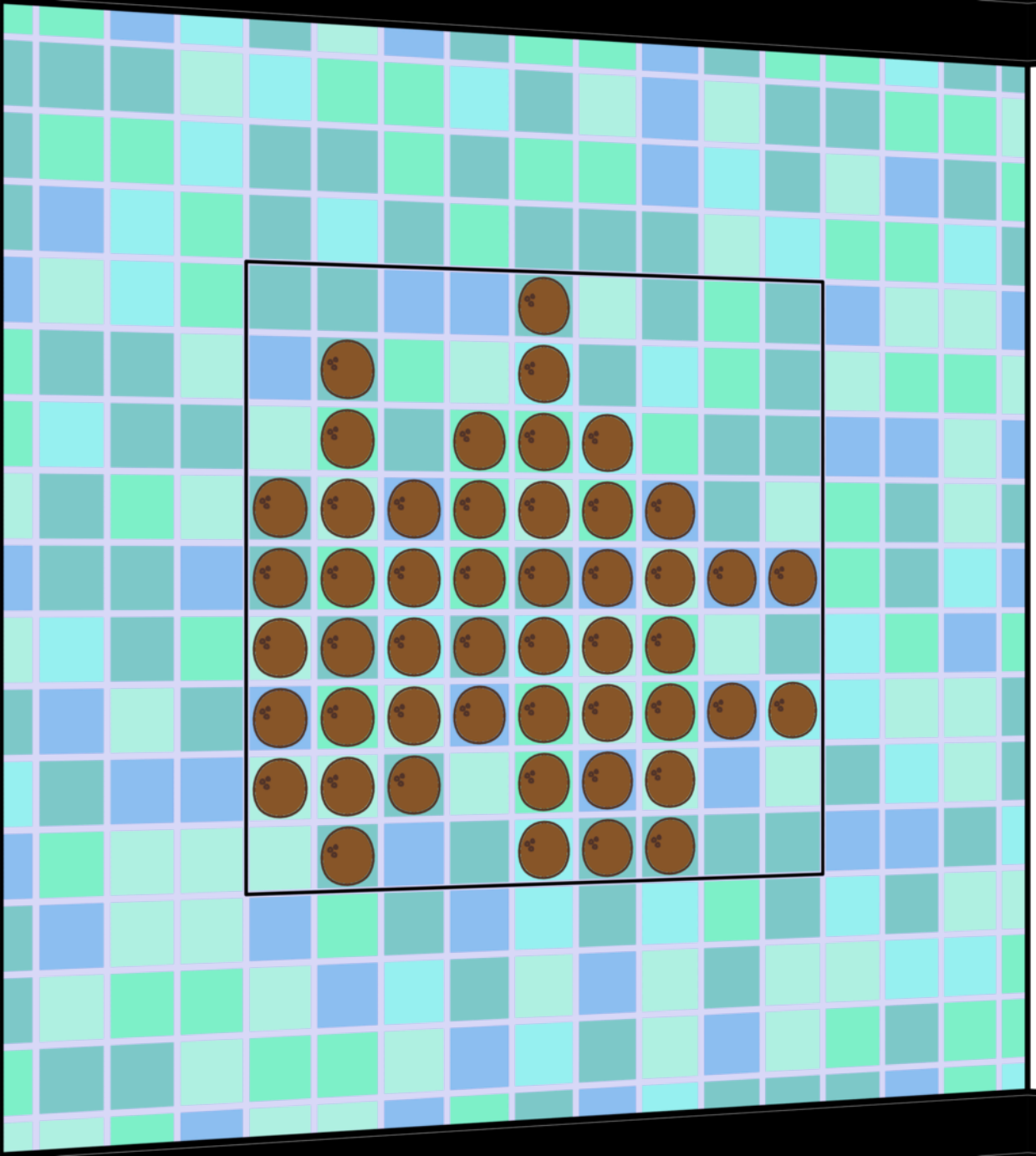

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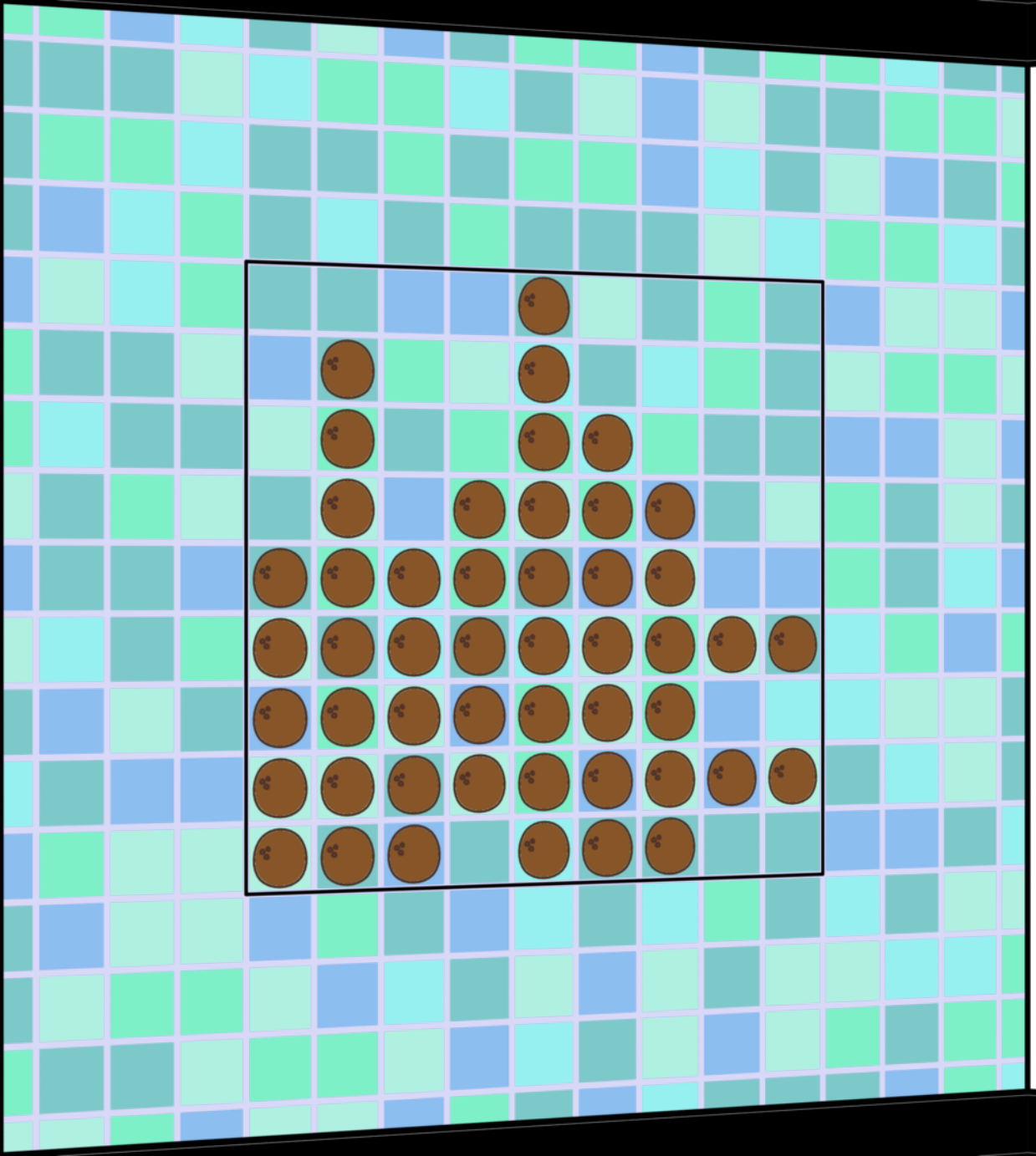

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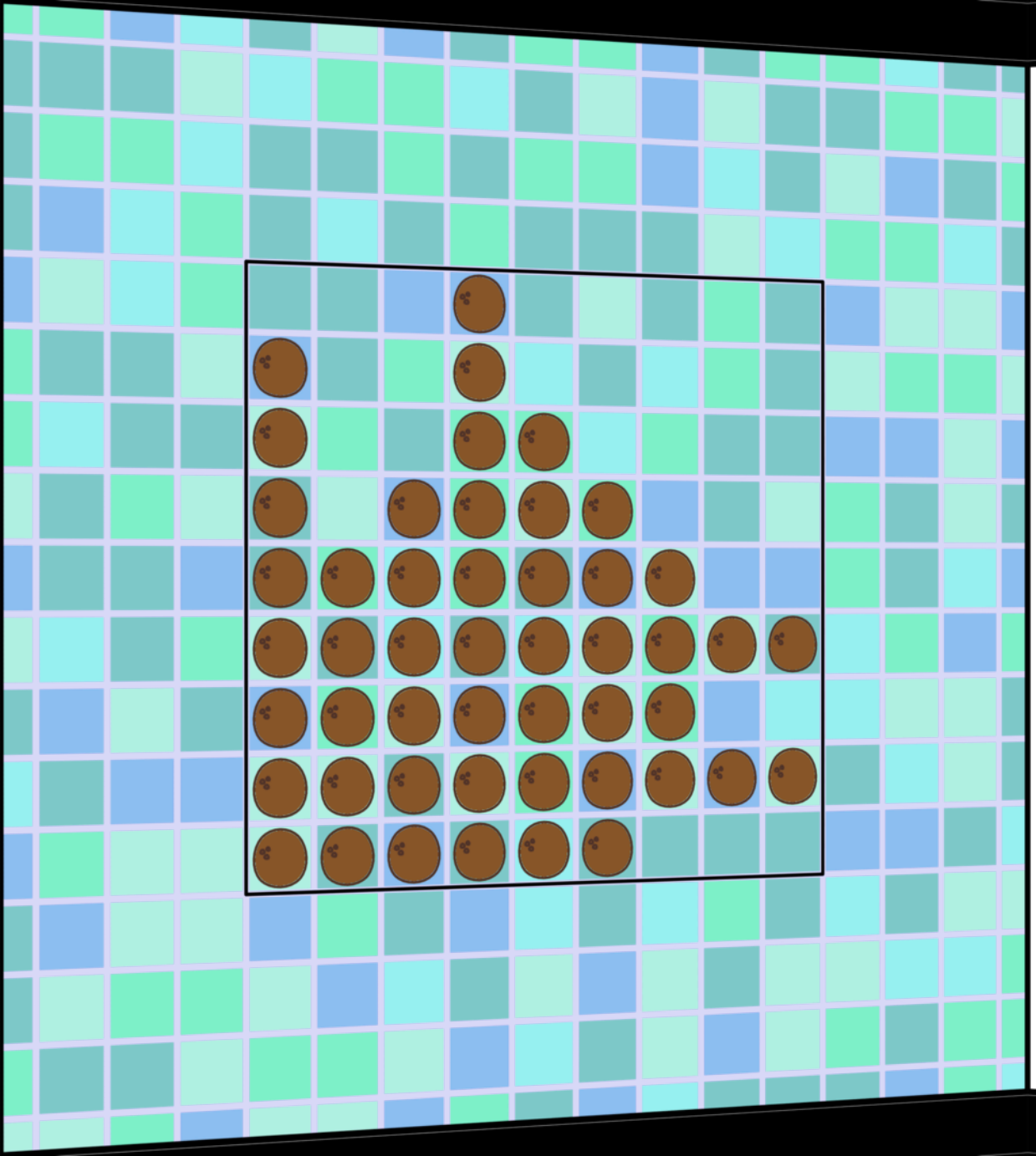

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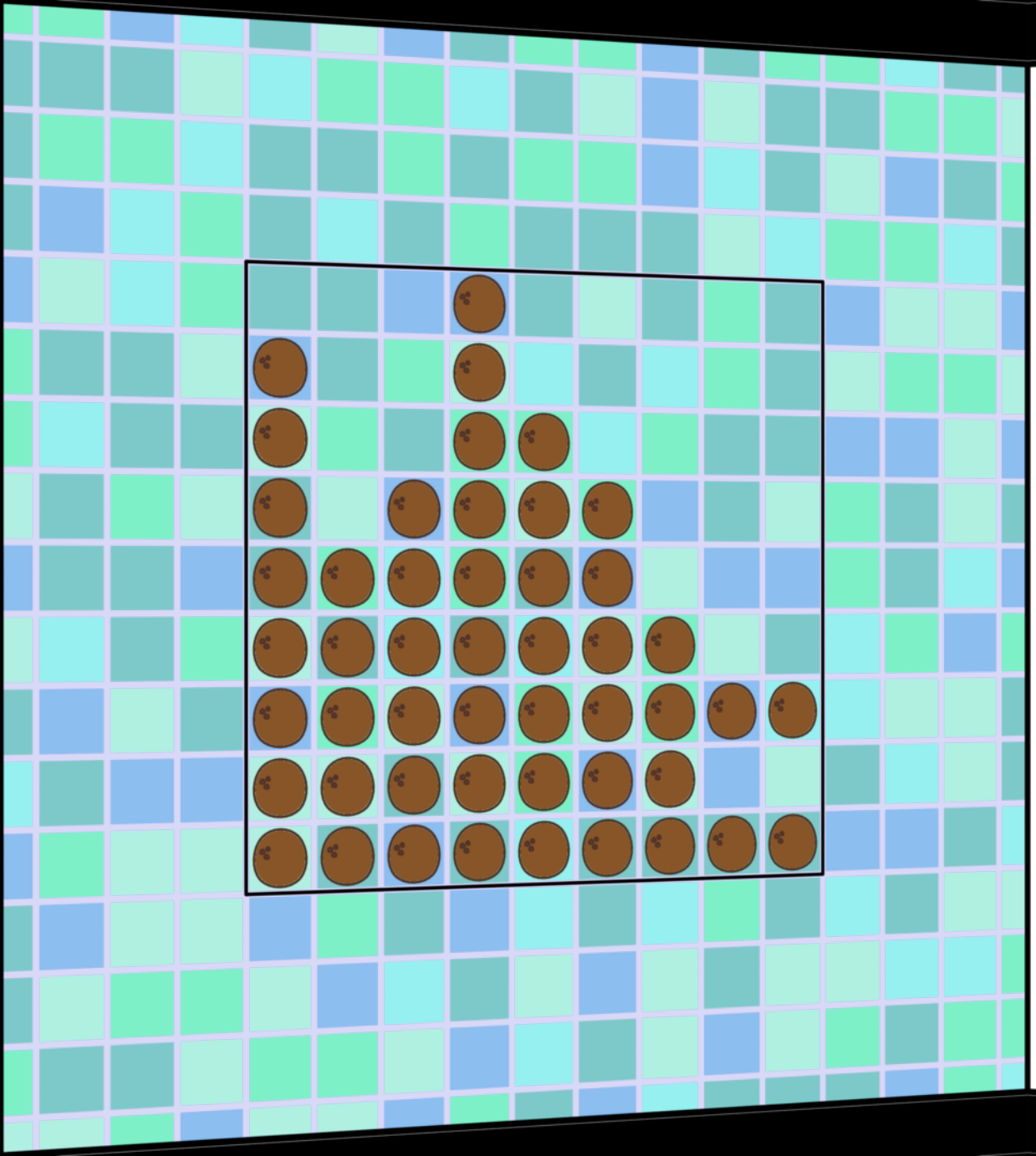

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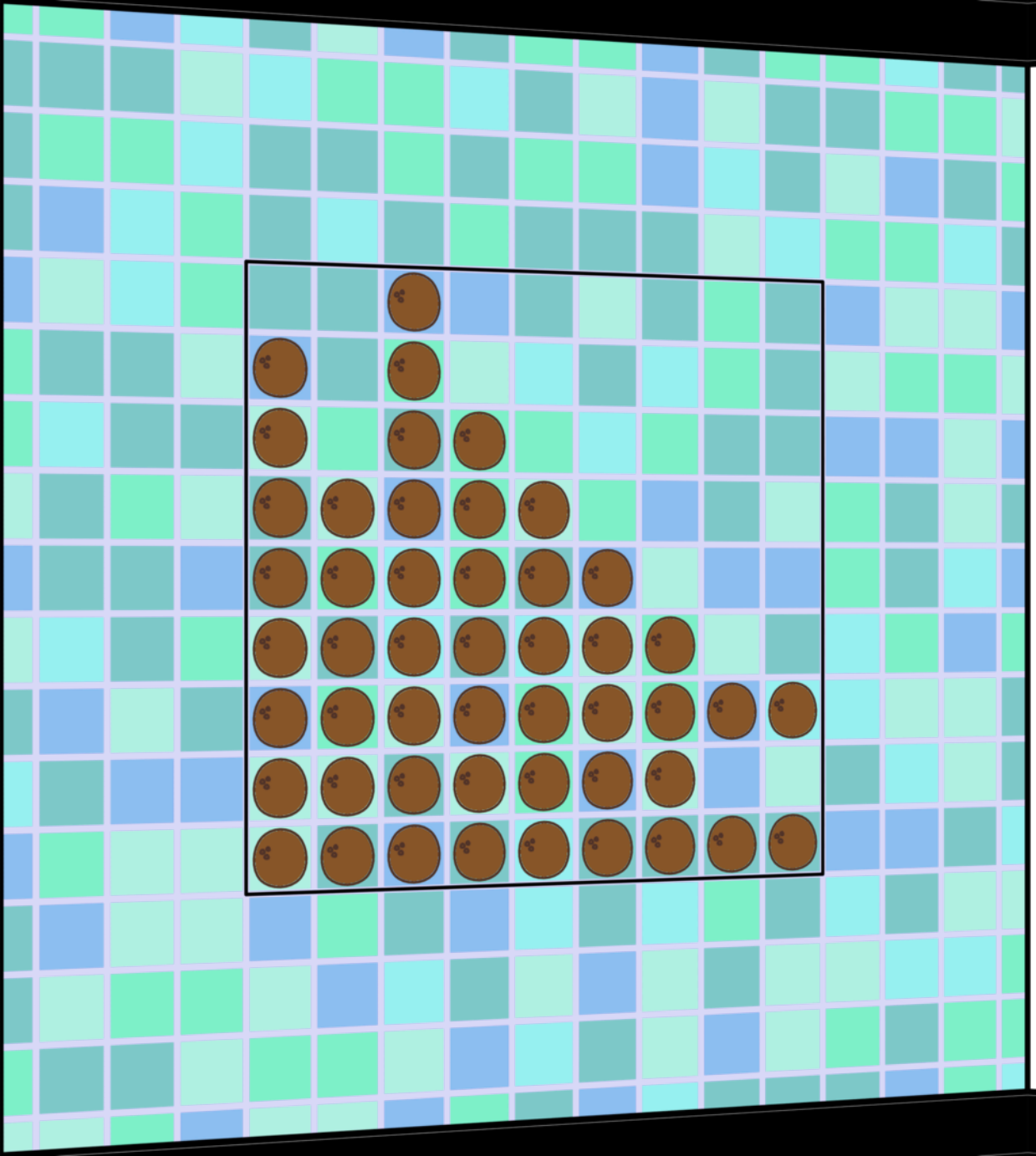

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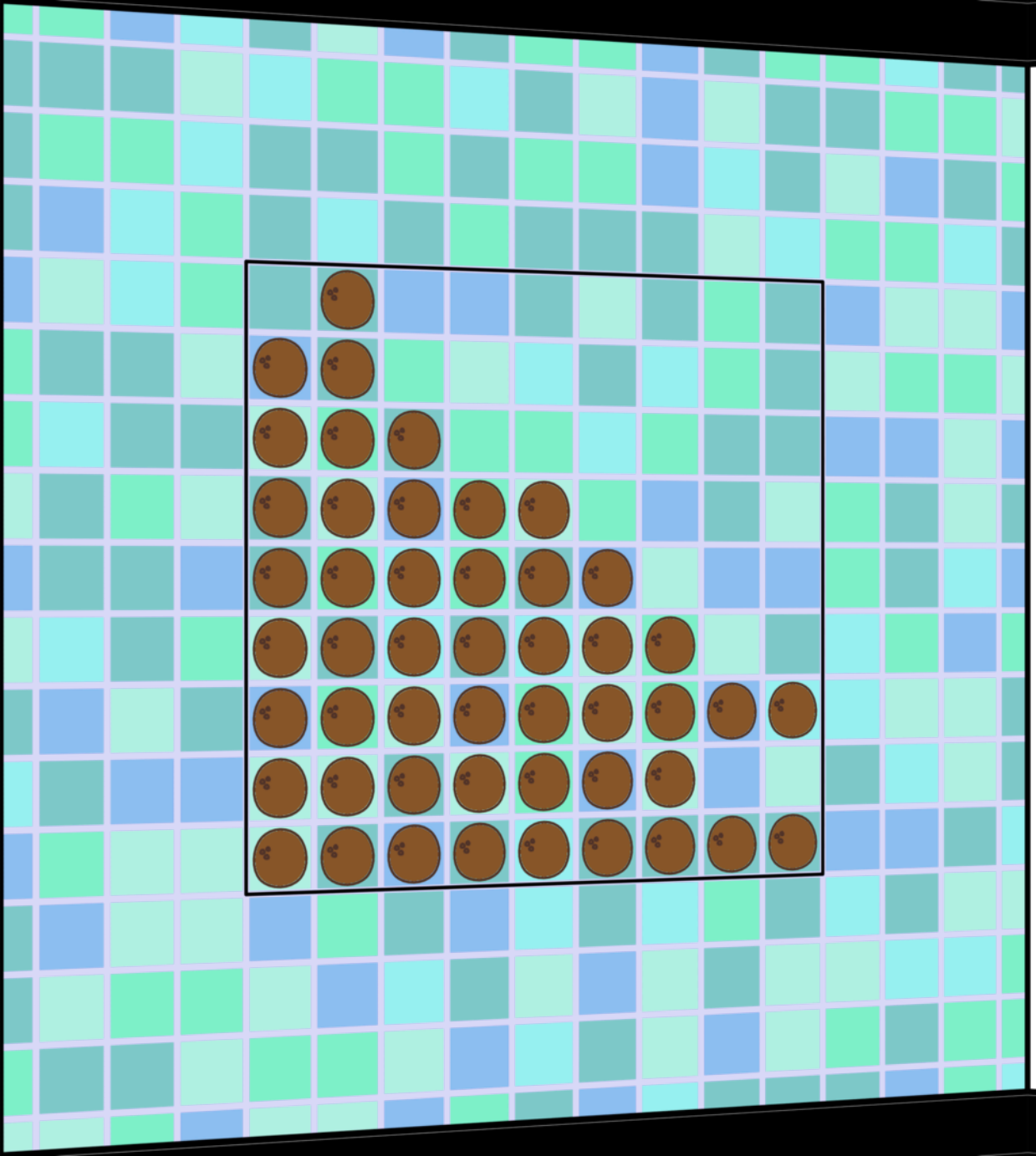

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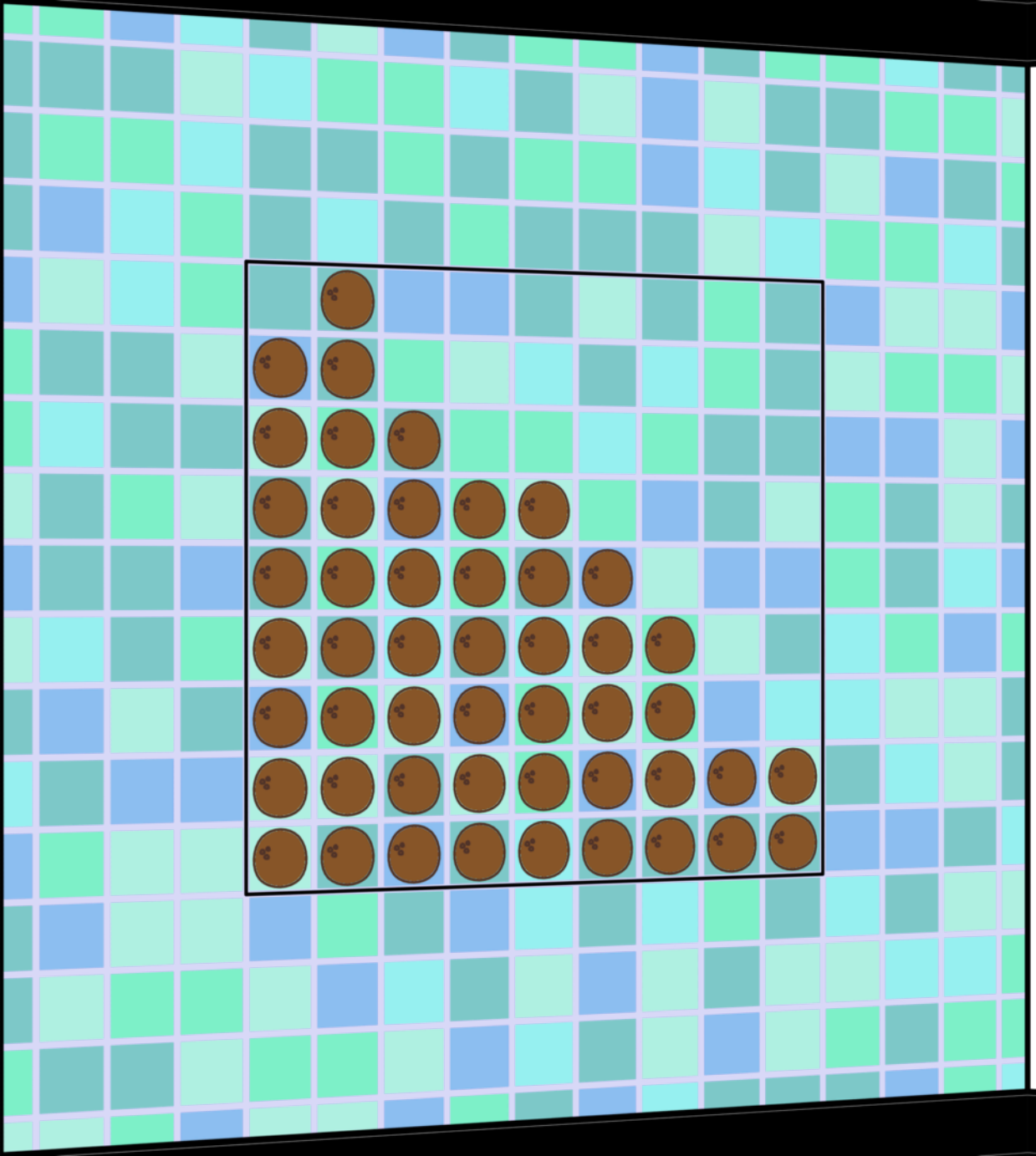

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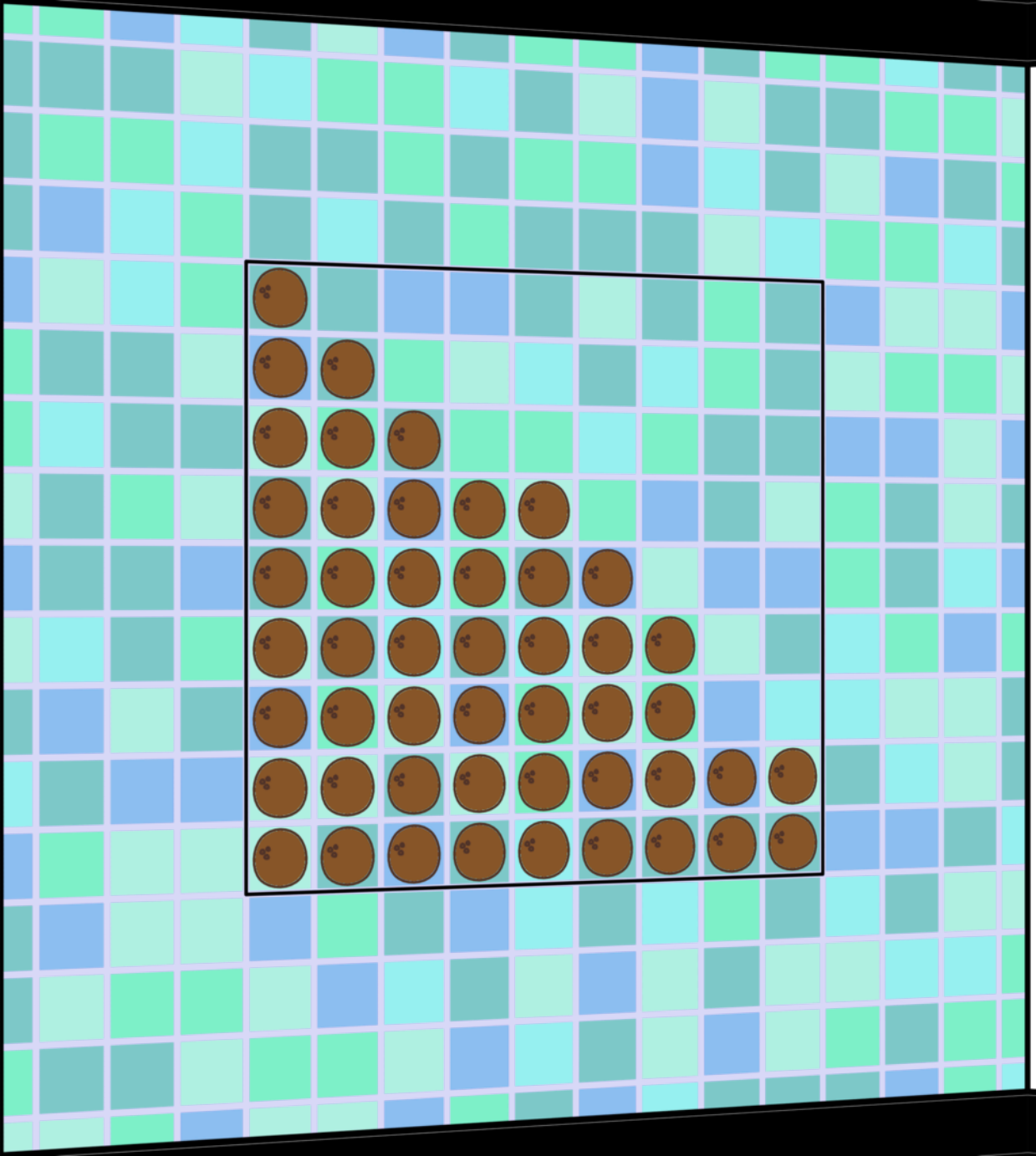

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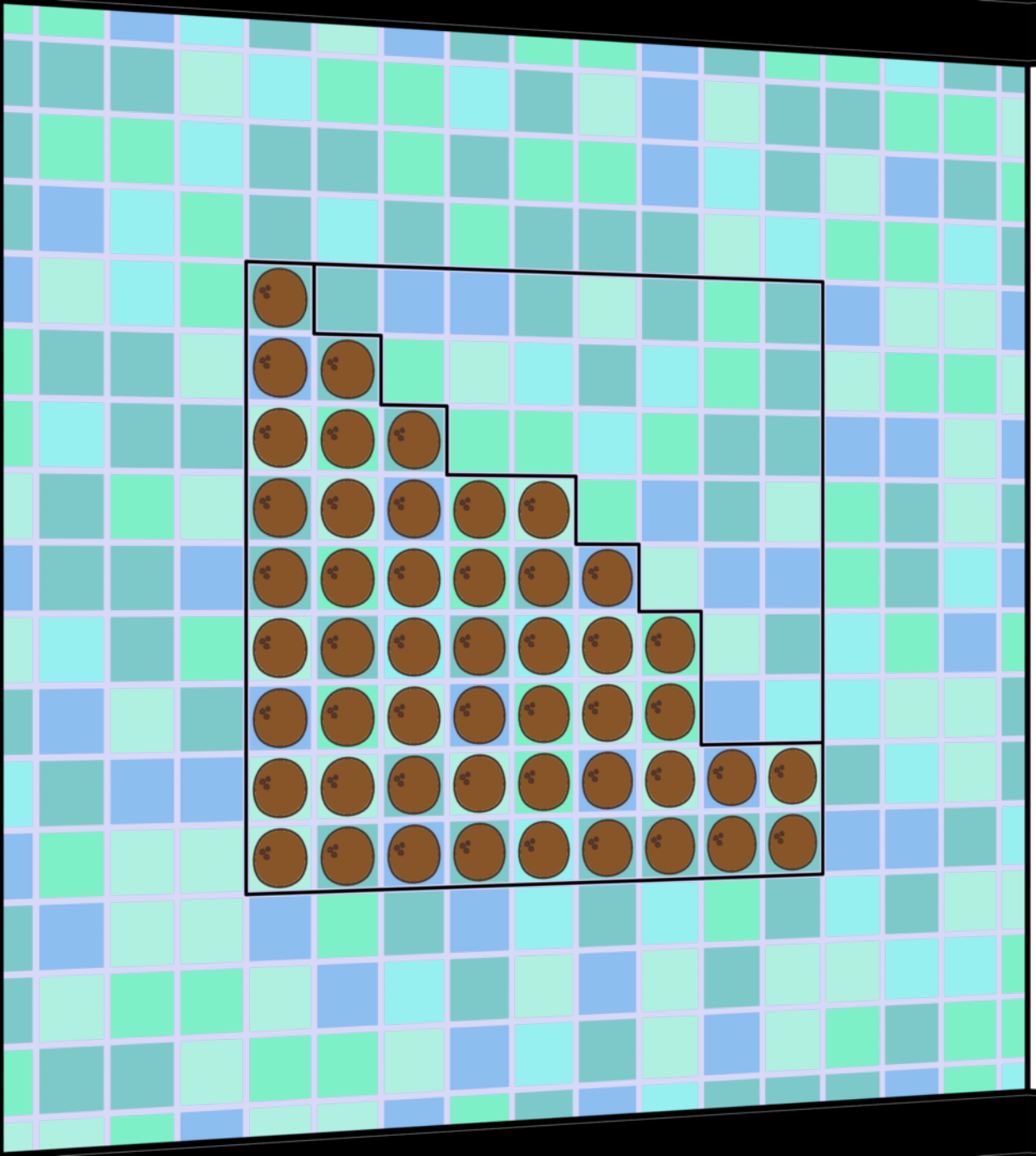

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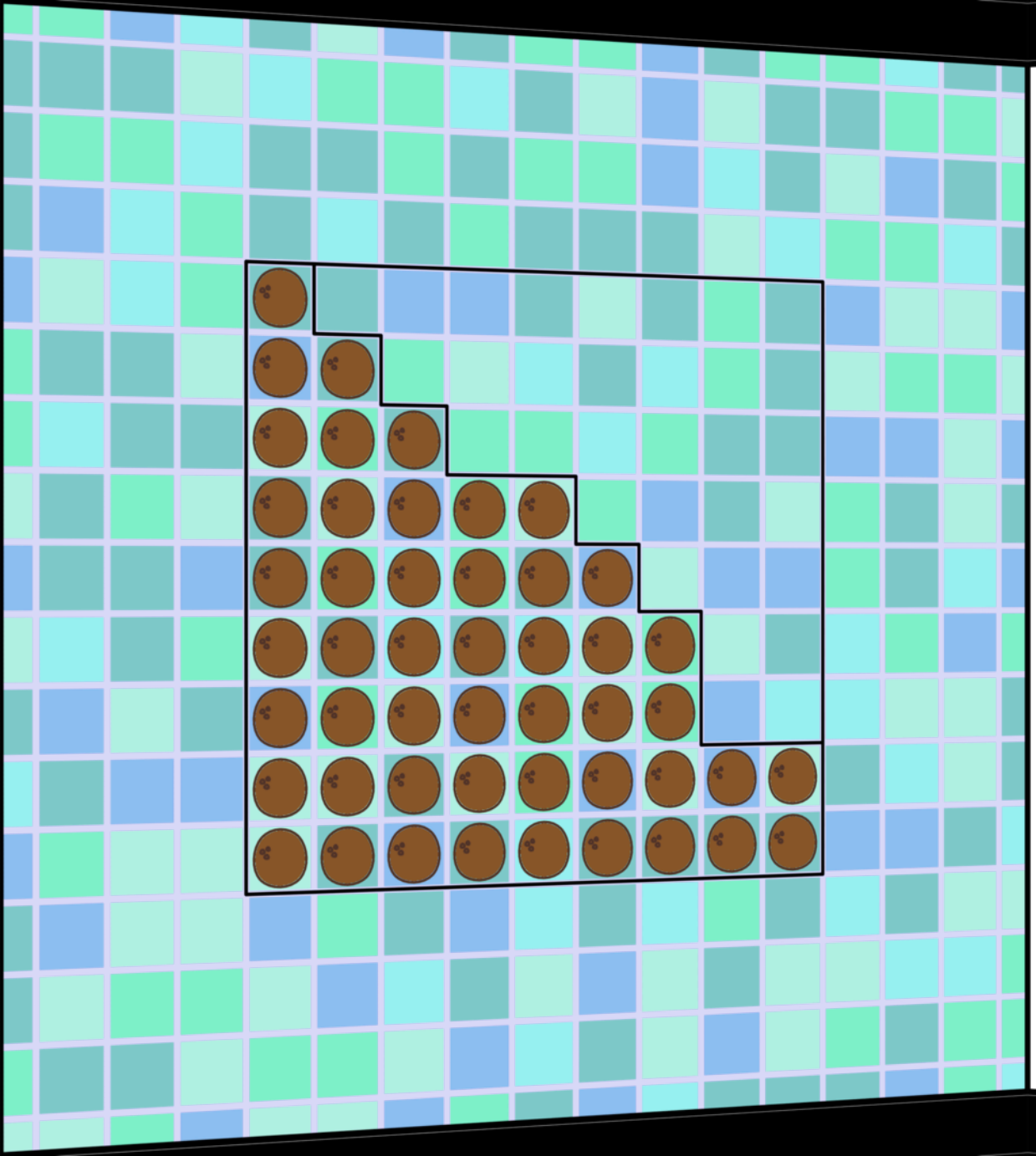

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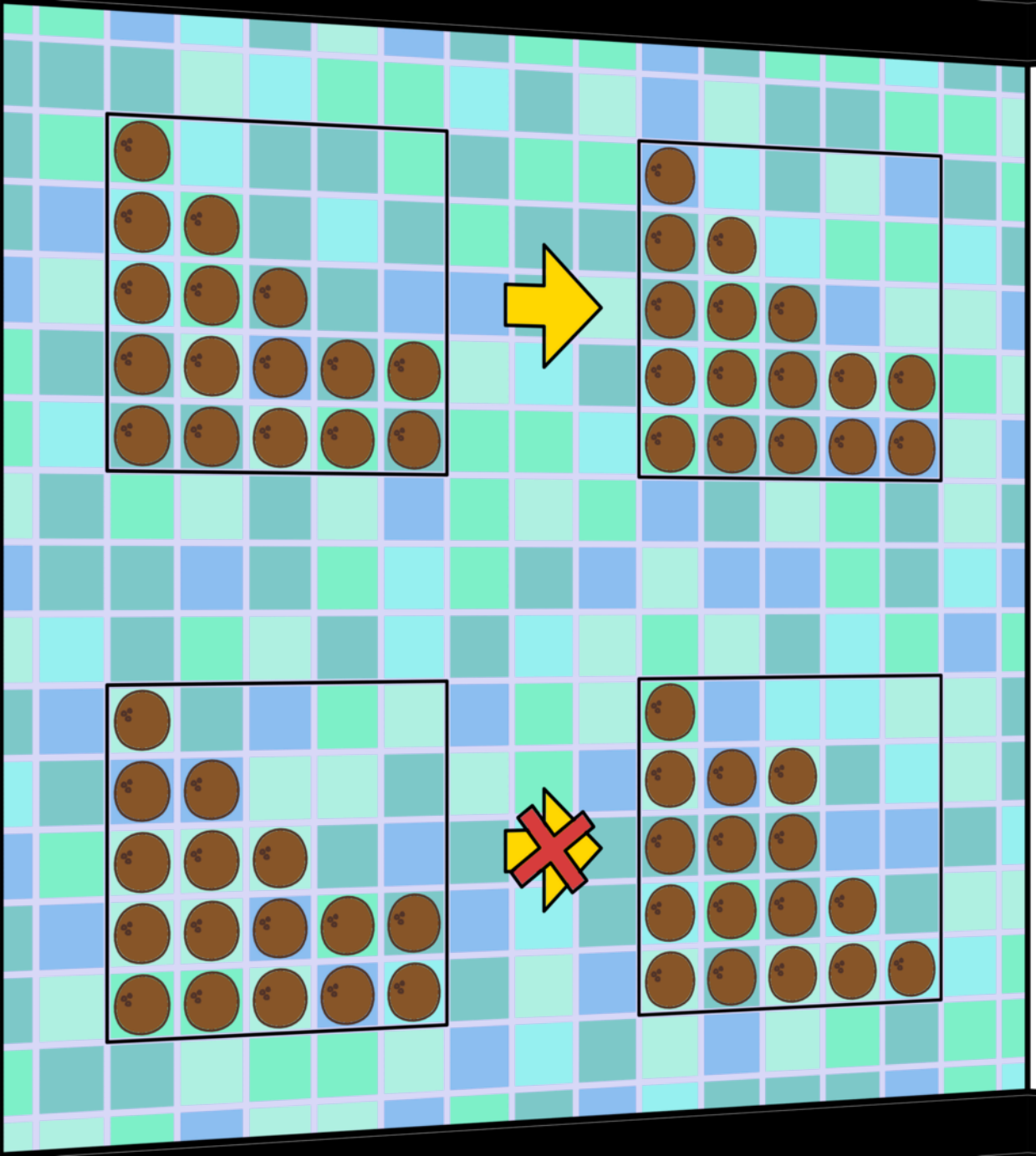
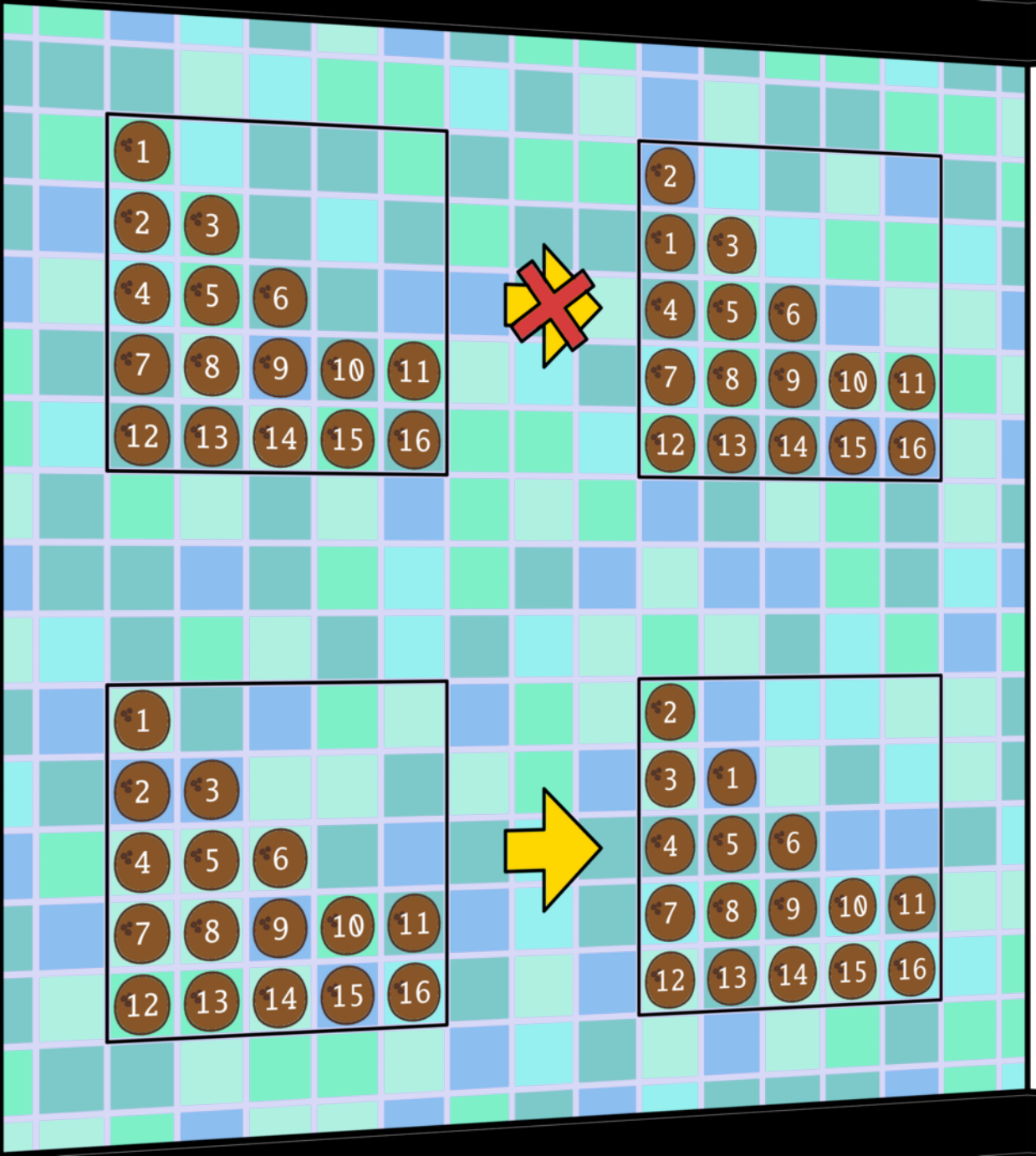
What can we say about reconfiguring canonical configurations?



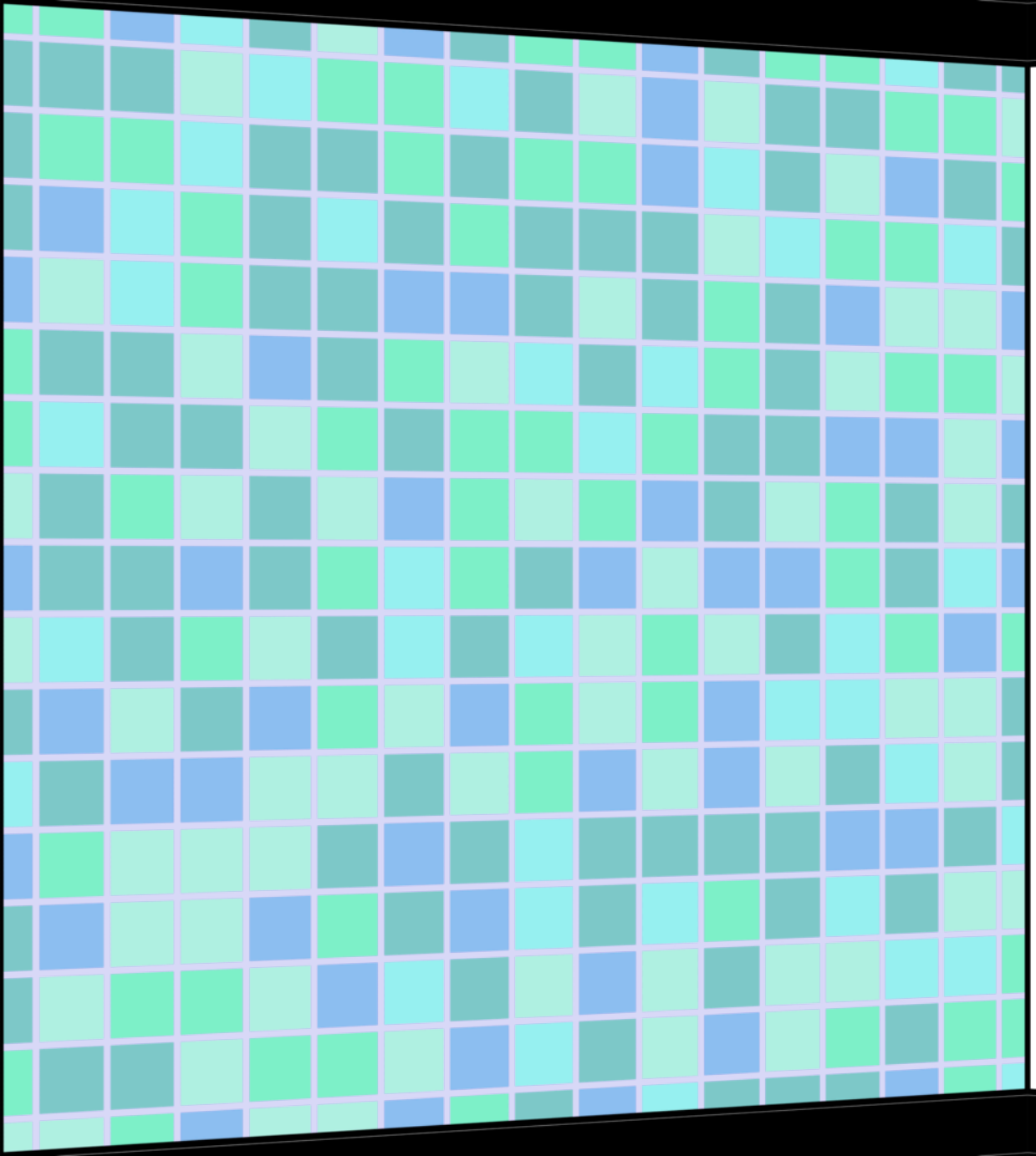
Theorem 1. Two unlabeled canonical configurations P and P' can be reached if and only if $P = P'$.



Theorem 2. Every realizable permutation of a canonical configuration is *even*.

Theorem 3. Every even permutation can be realized unless:

- there is a *core*; or
- it has this shape: 



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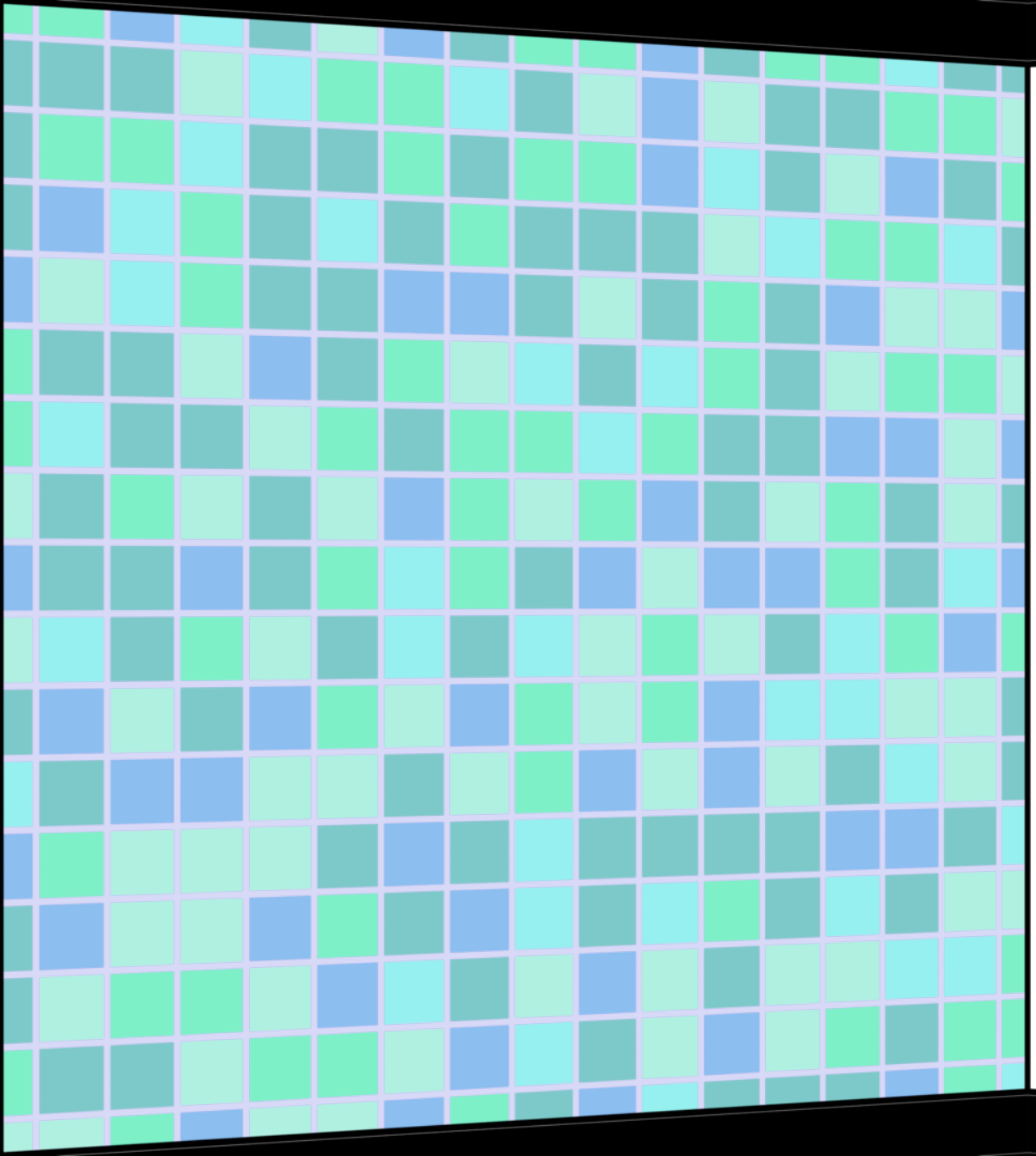

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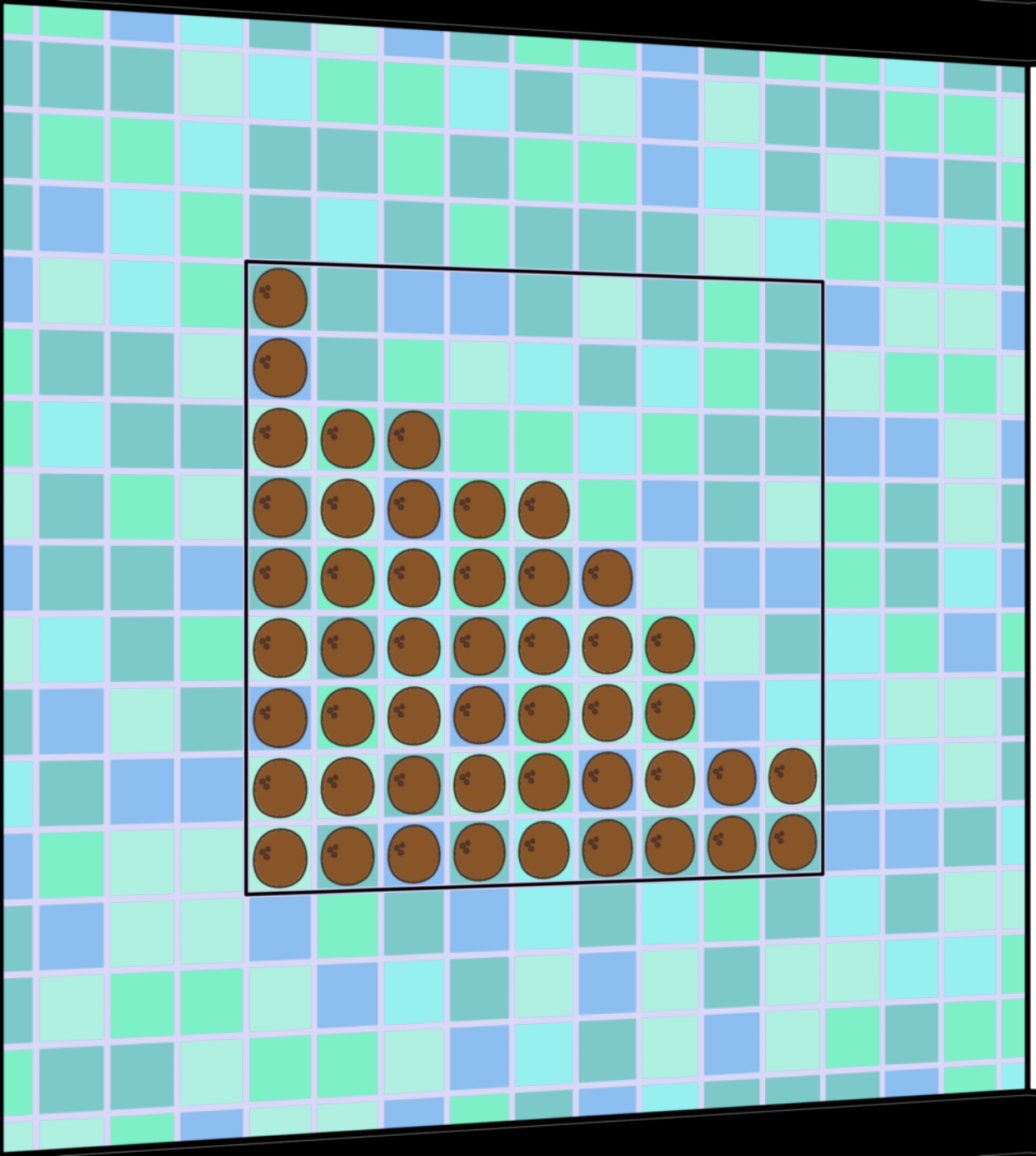

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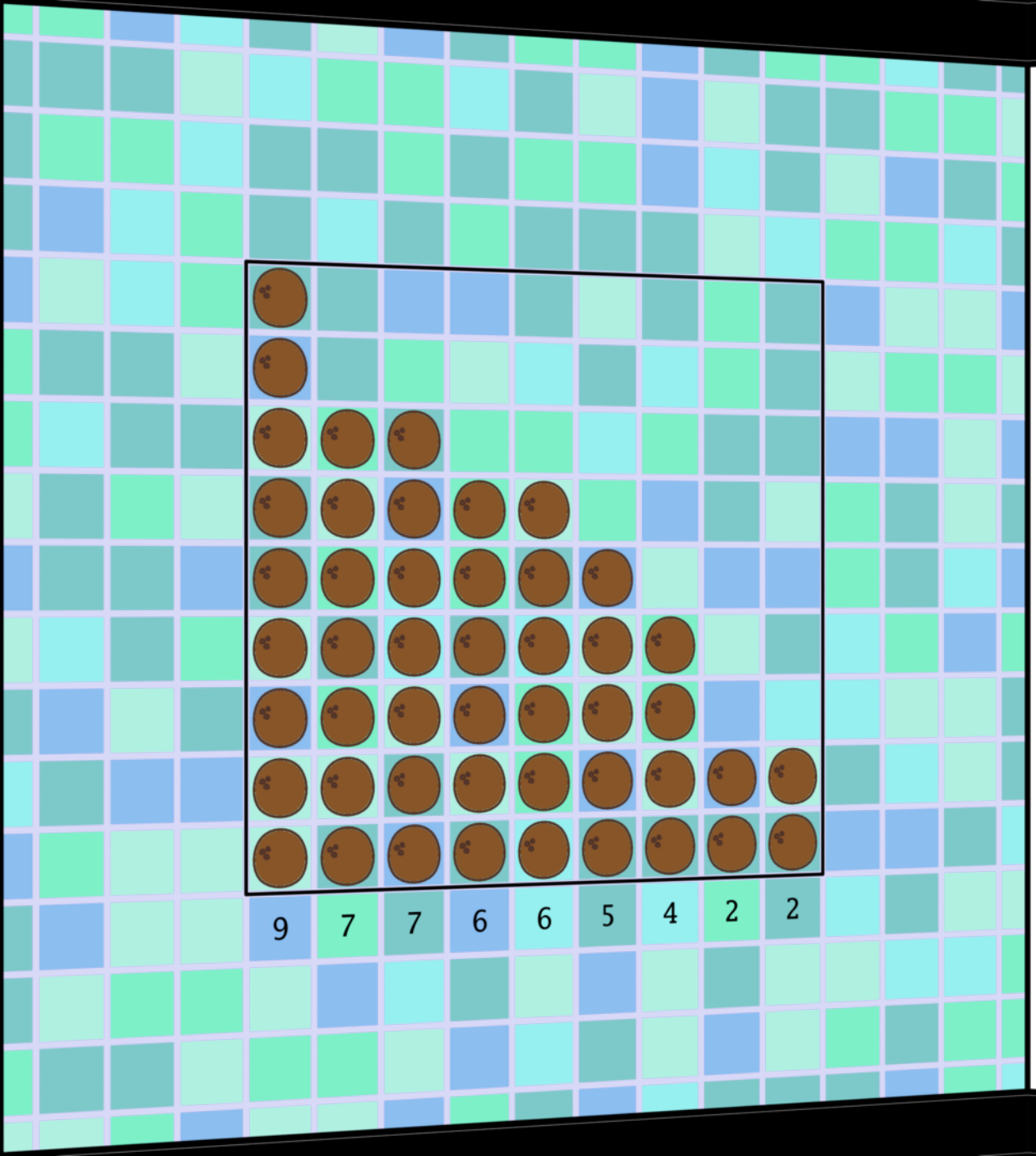

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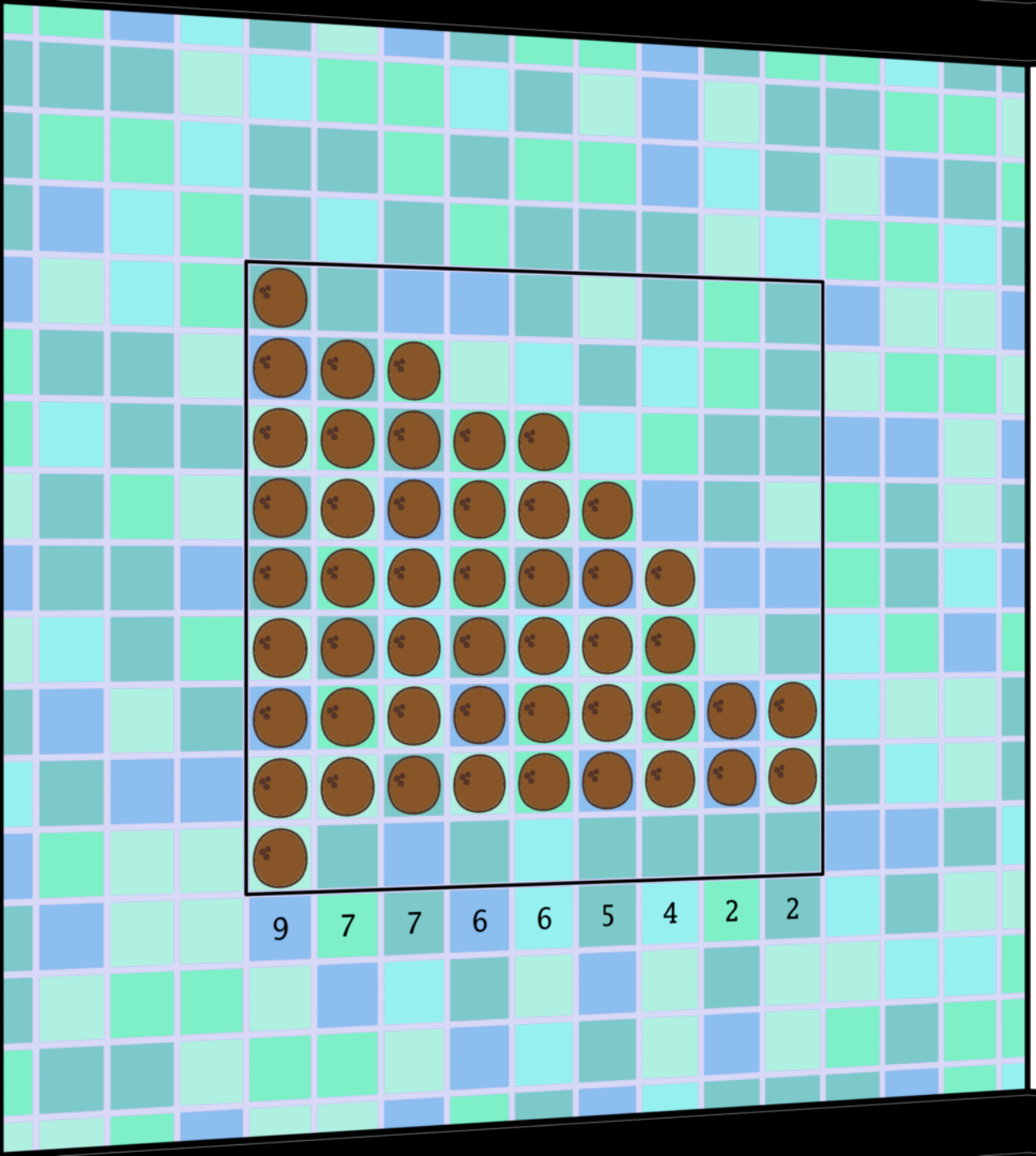

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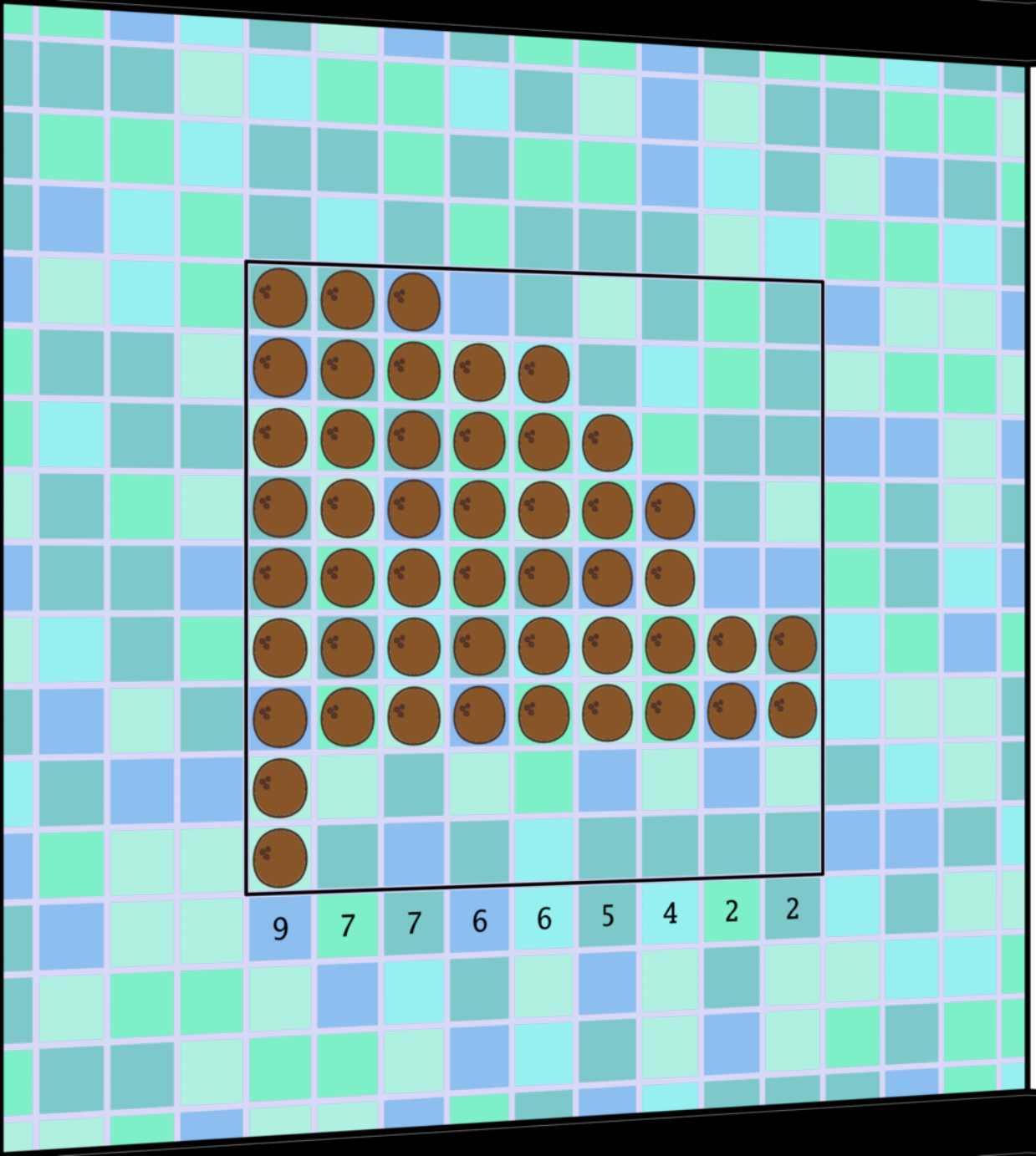

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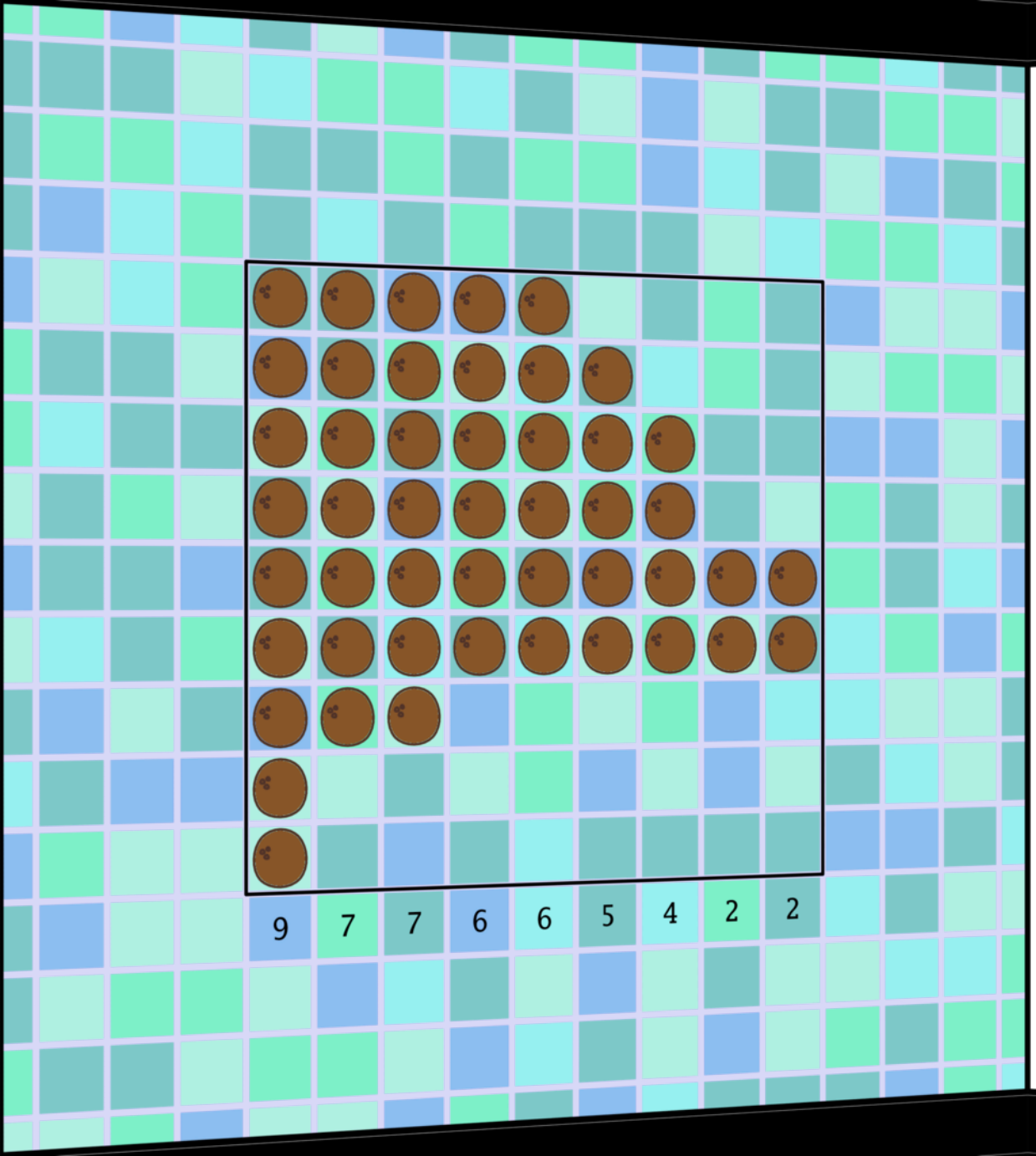

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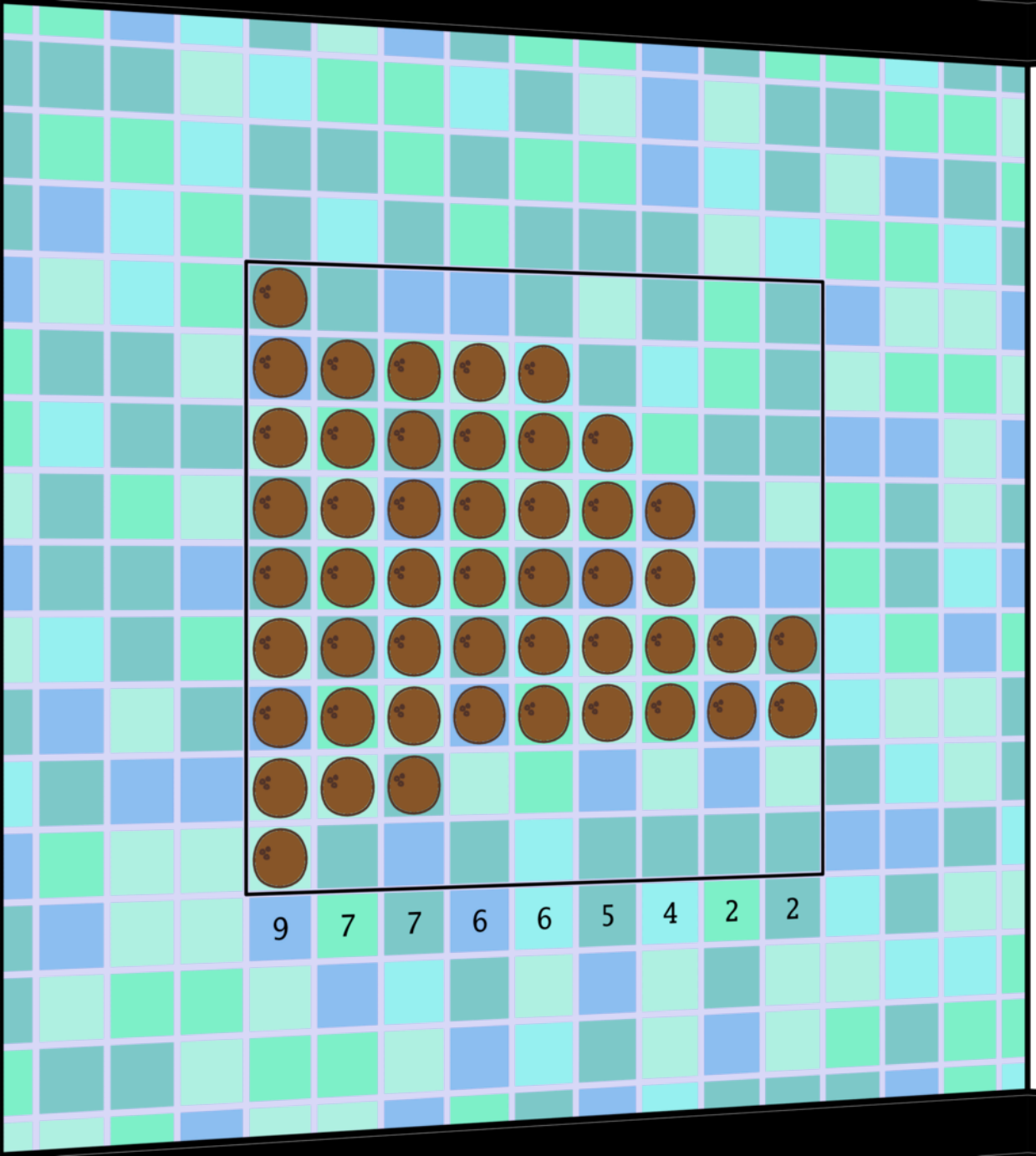

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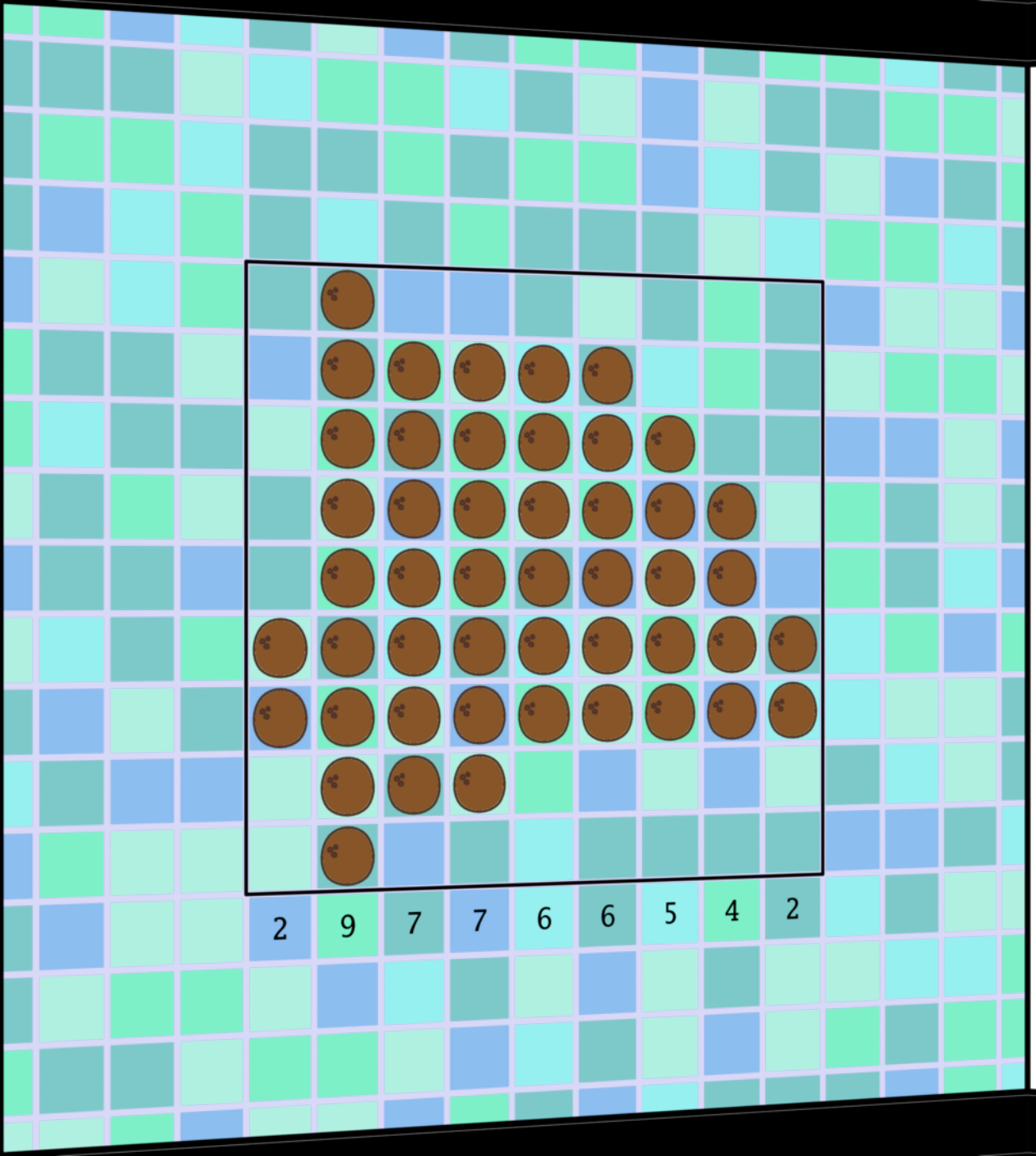

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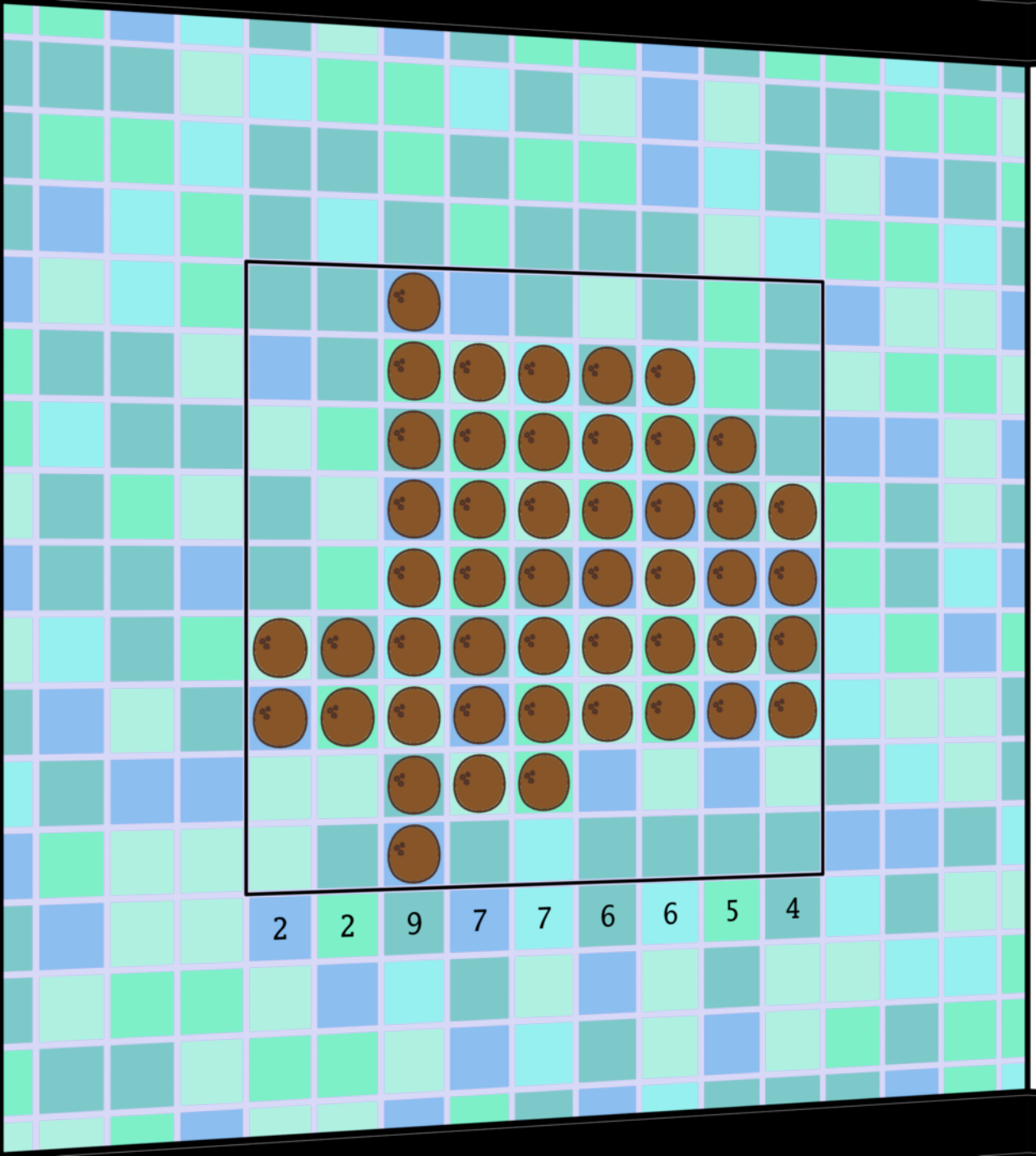

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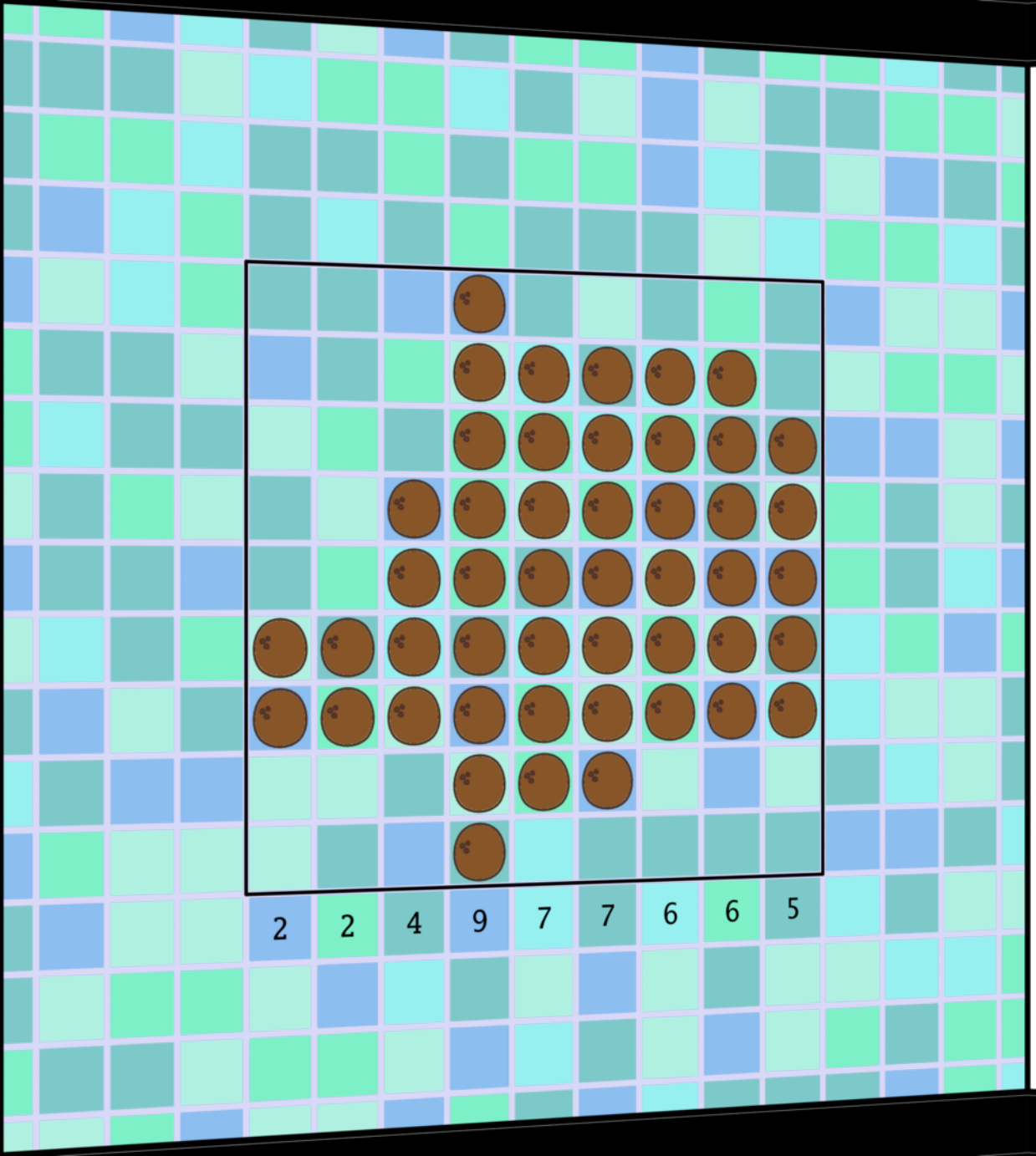

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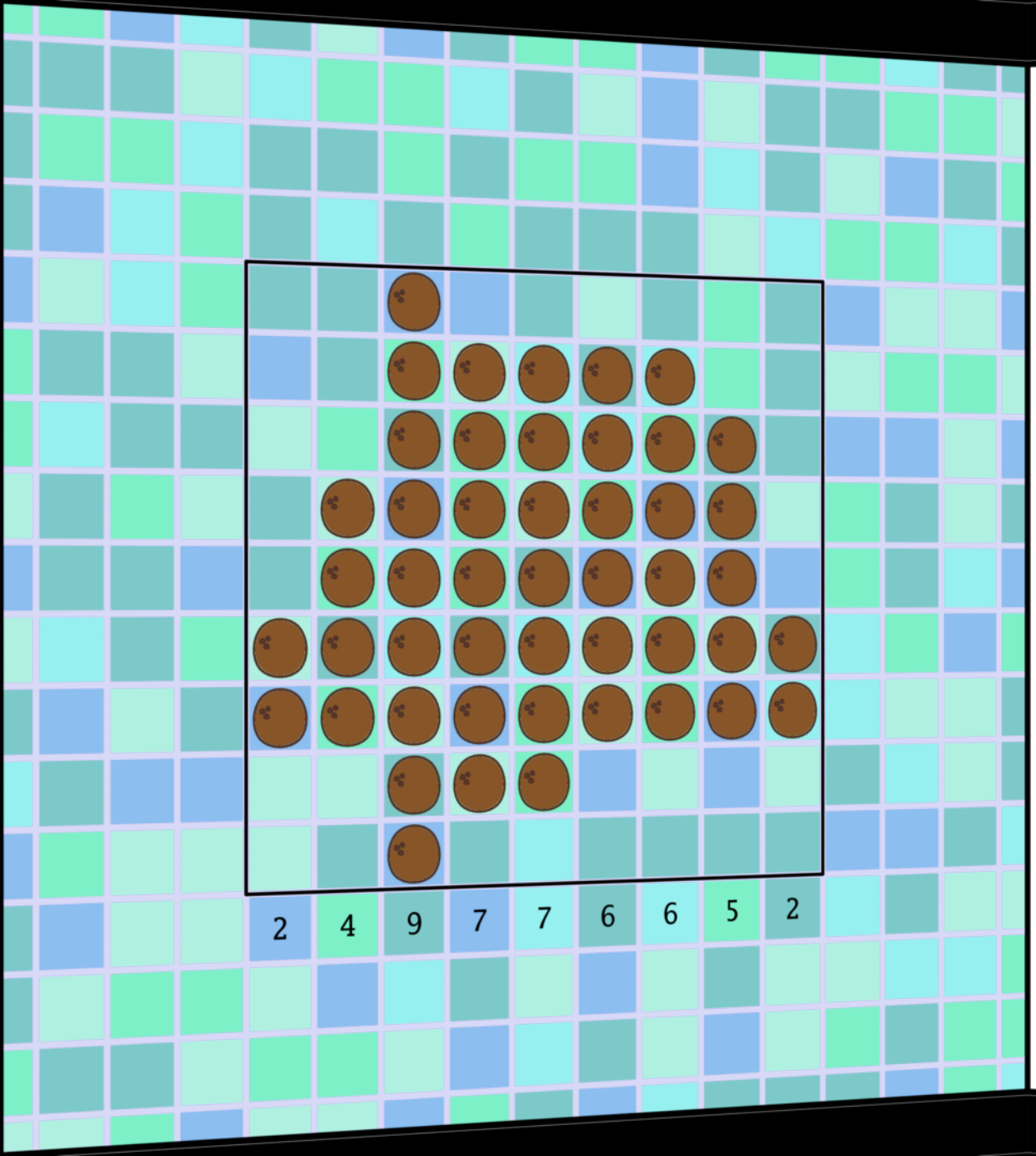

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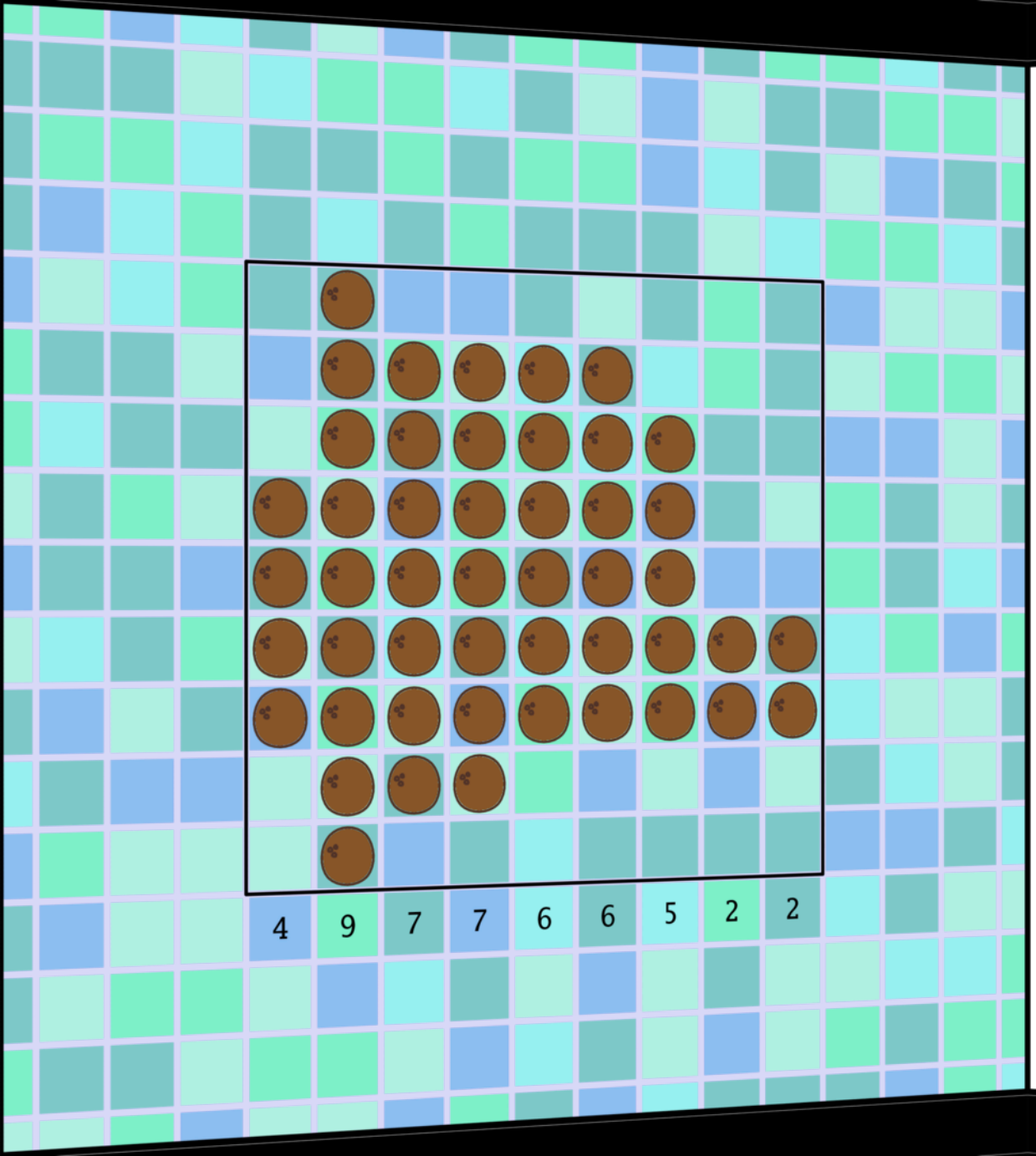

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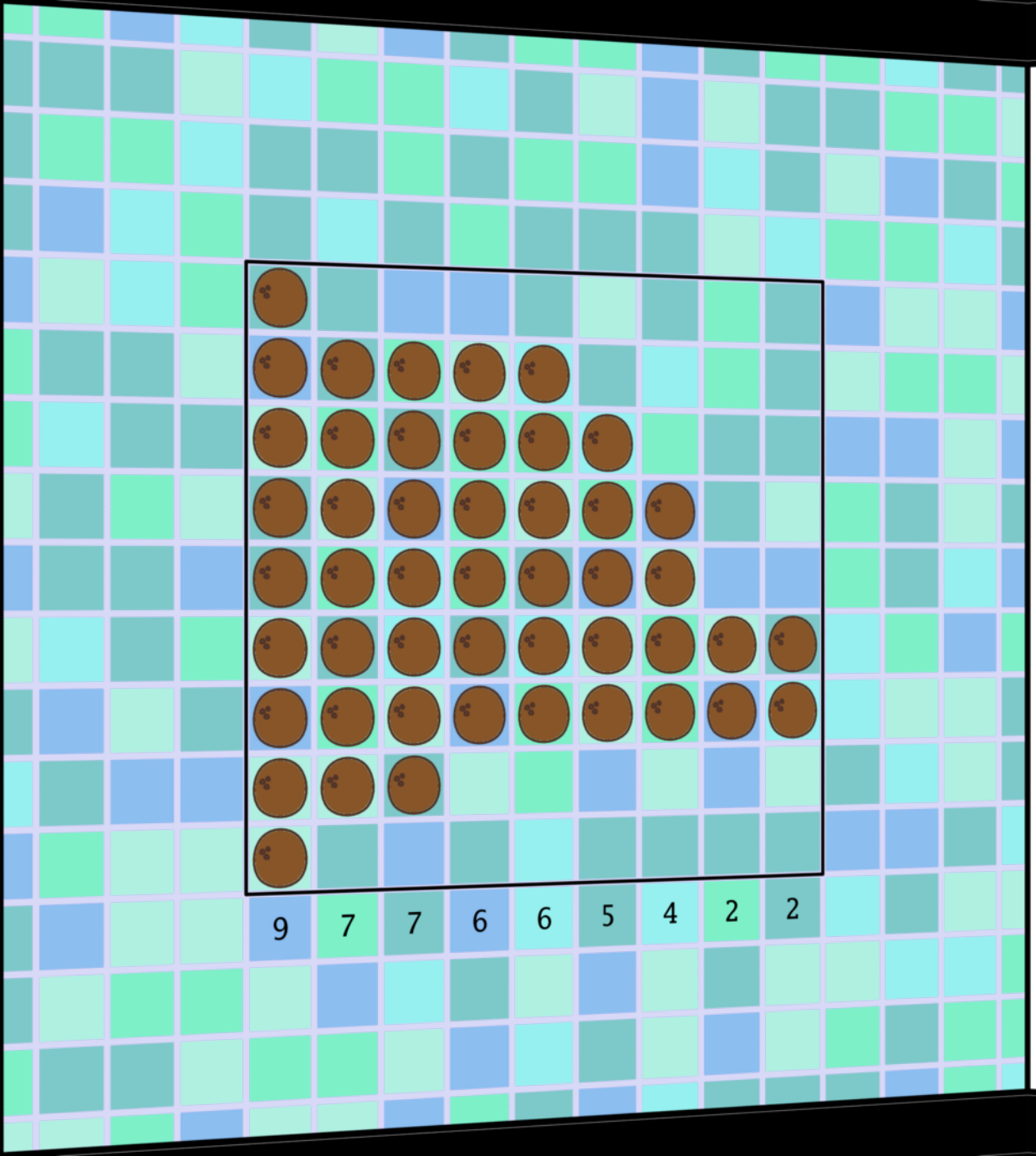

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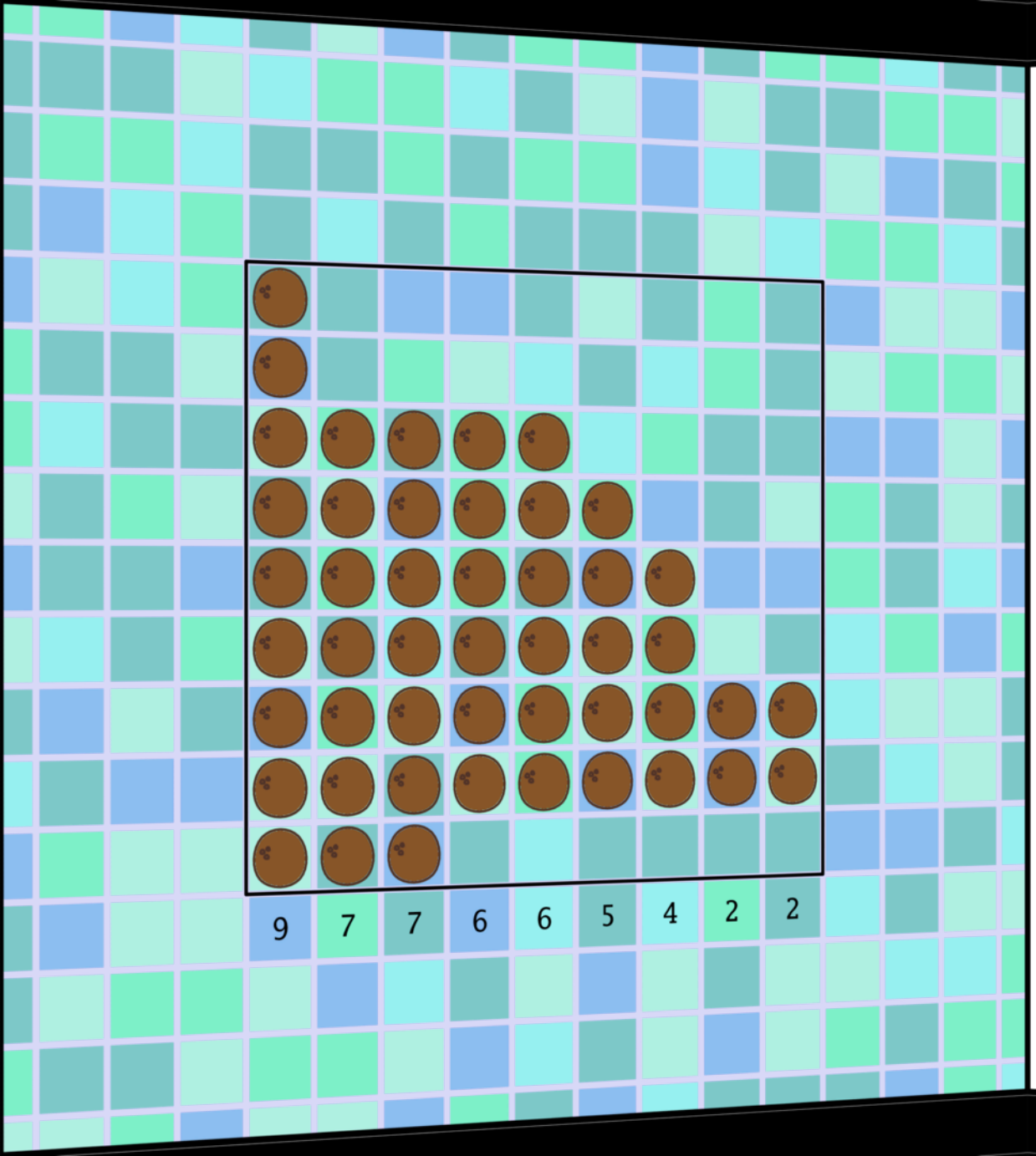

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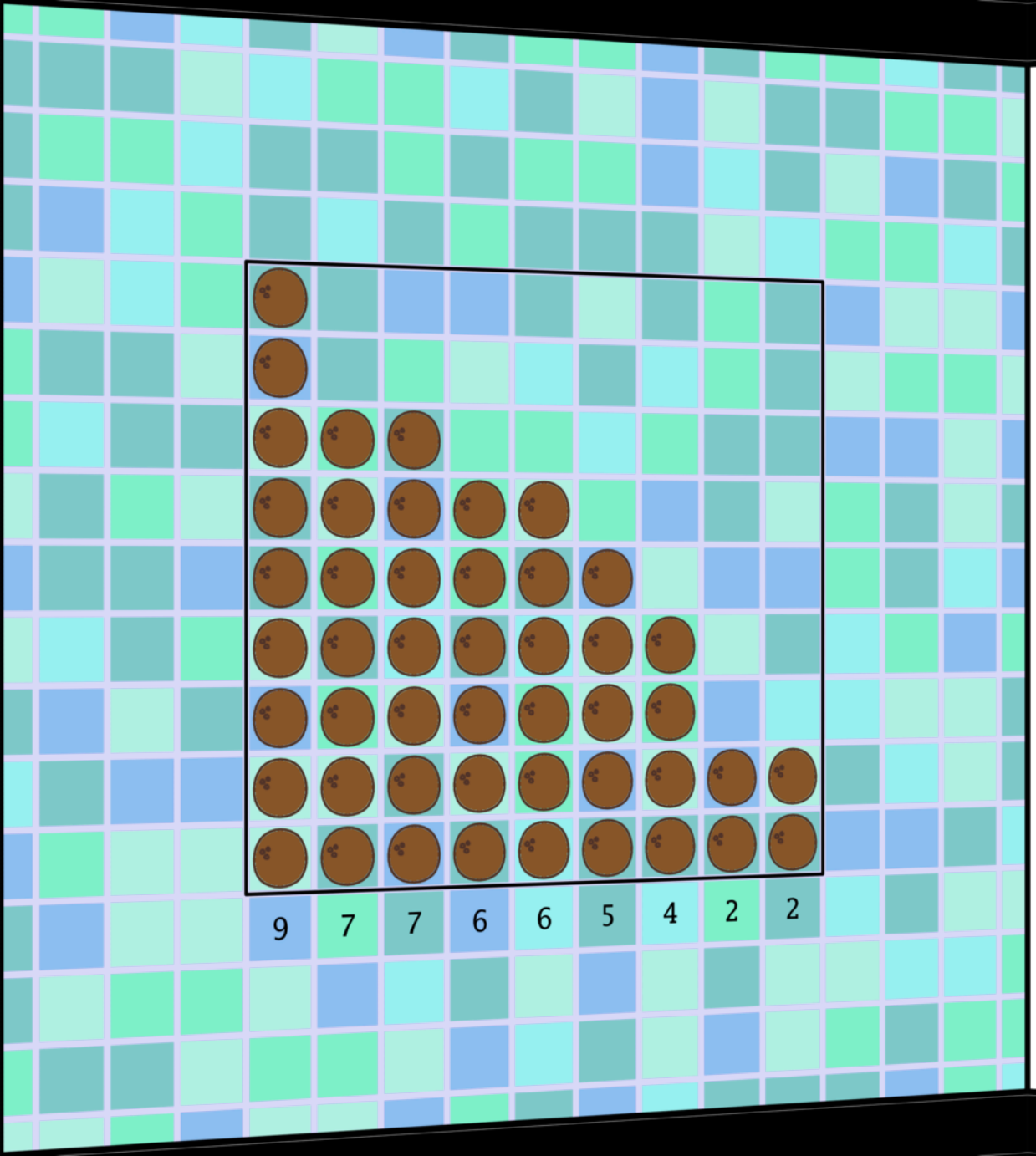

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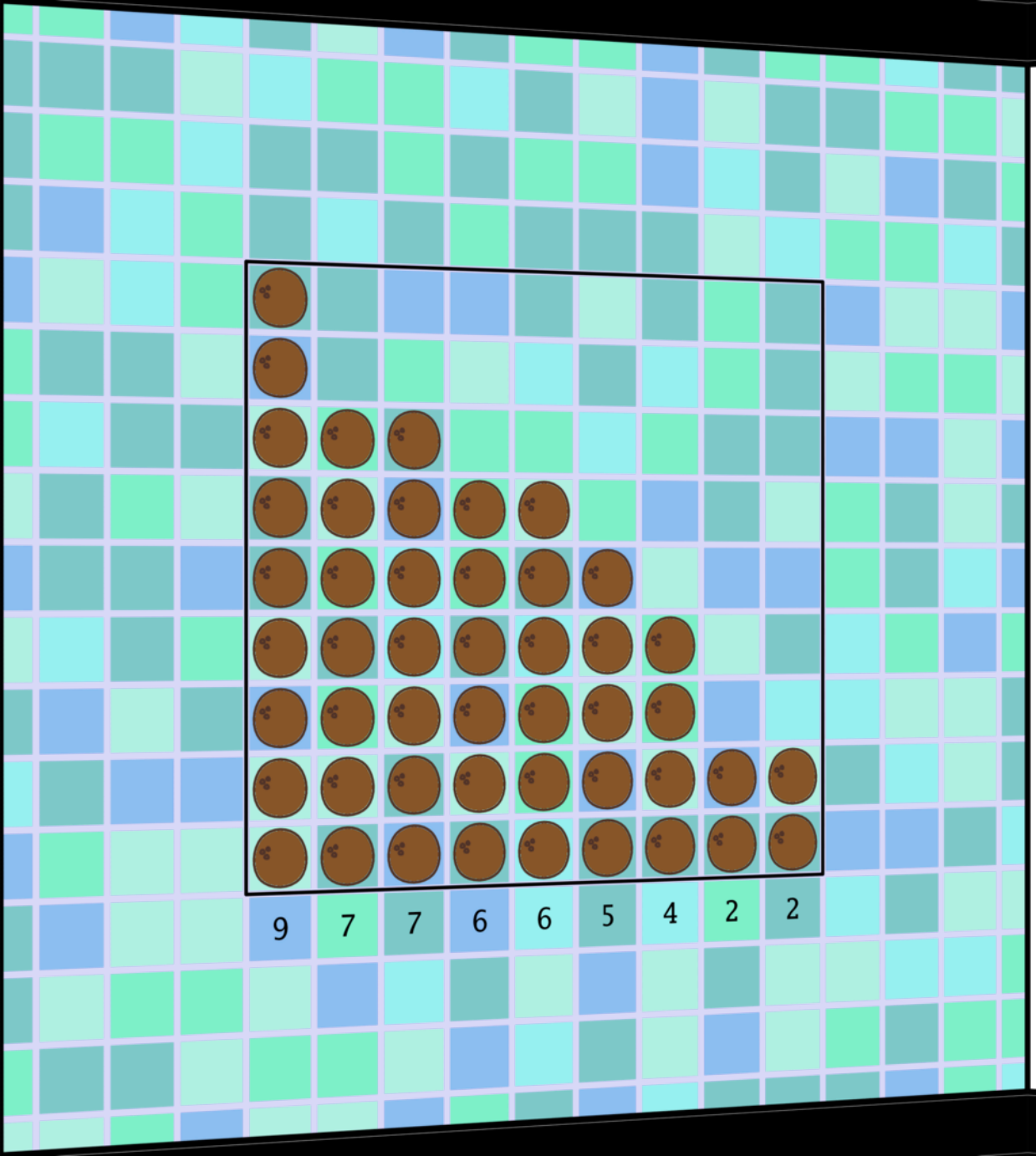

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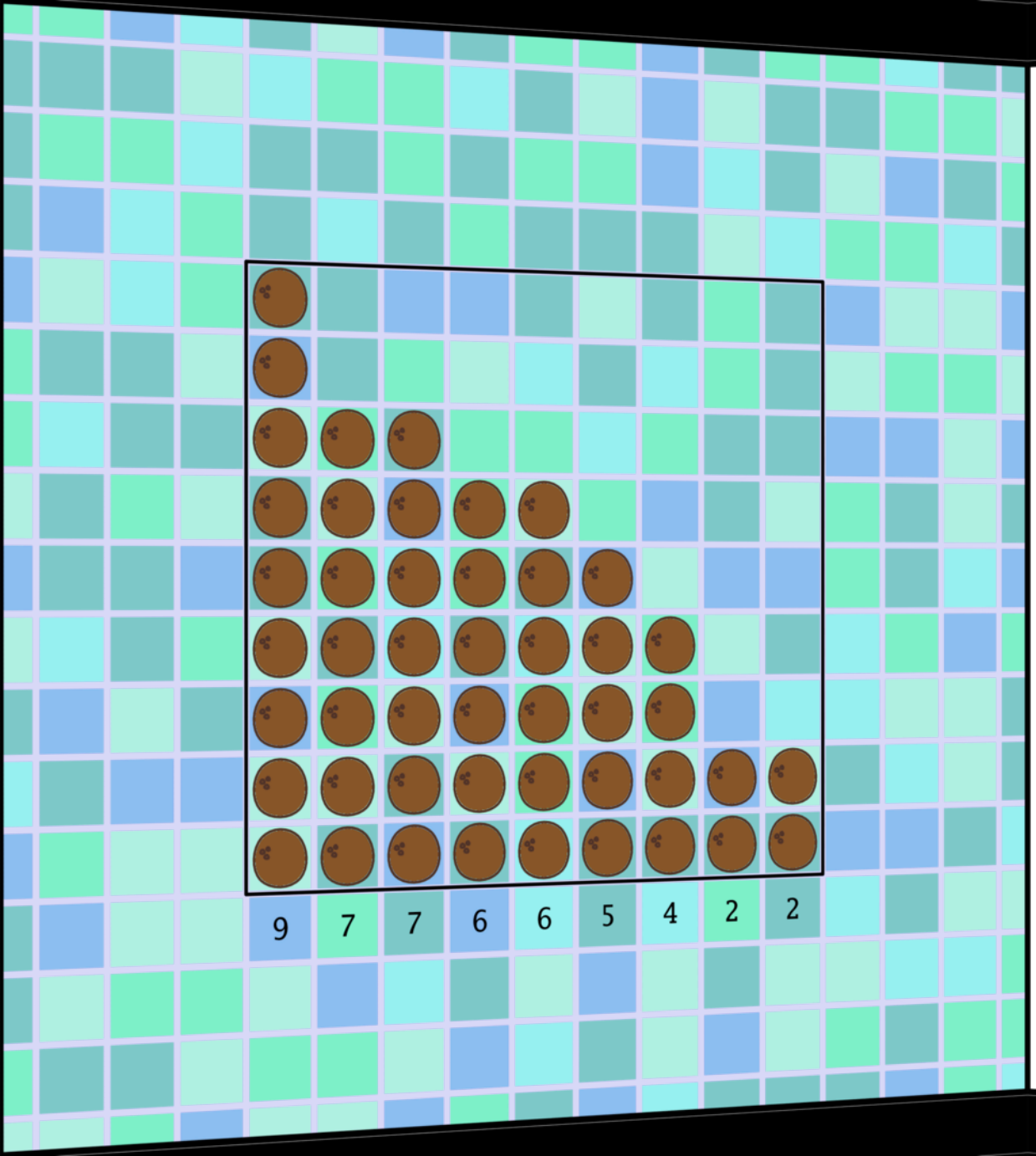
Proof idea. The number of rows and columns of given lengths are invariant.

Lemma. The number of moves is always even.



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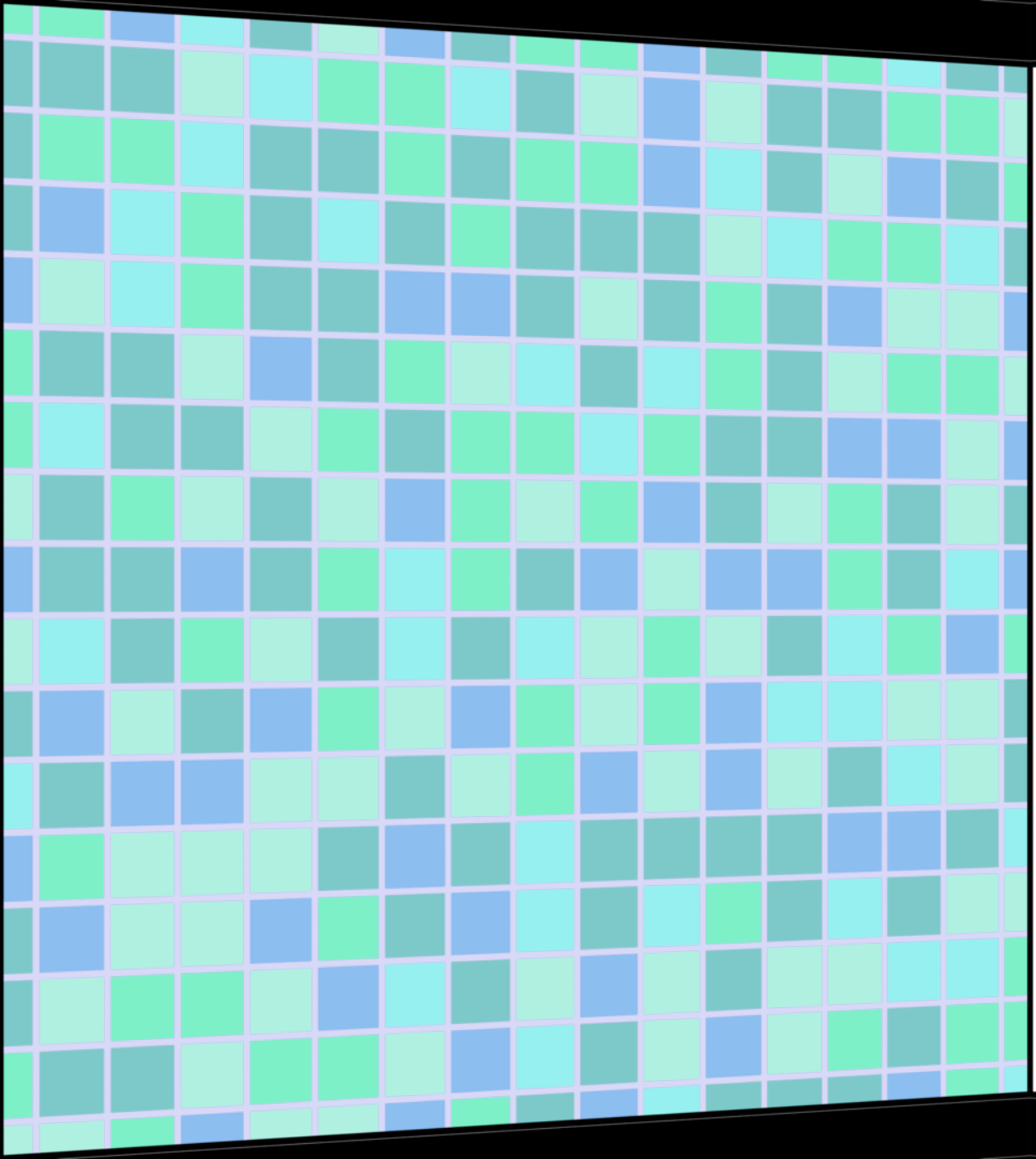
Proof. Watch the 9s.



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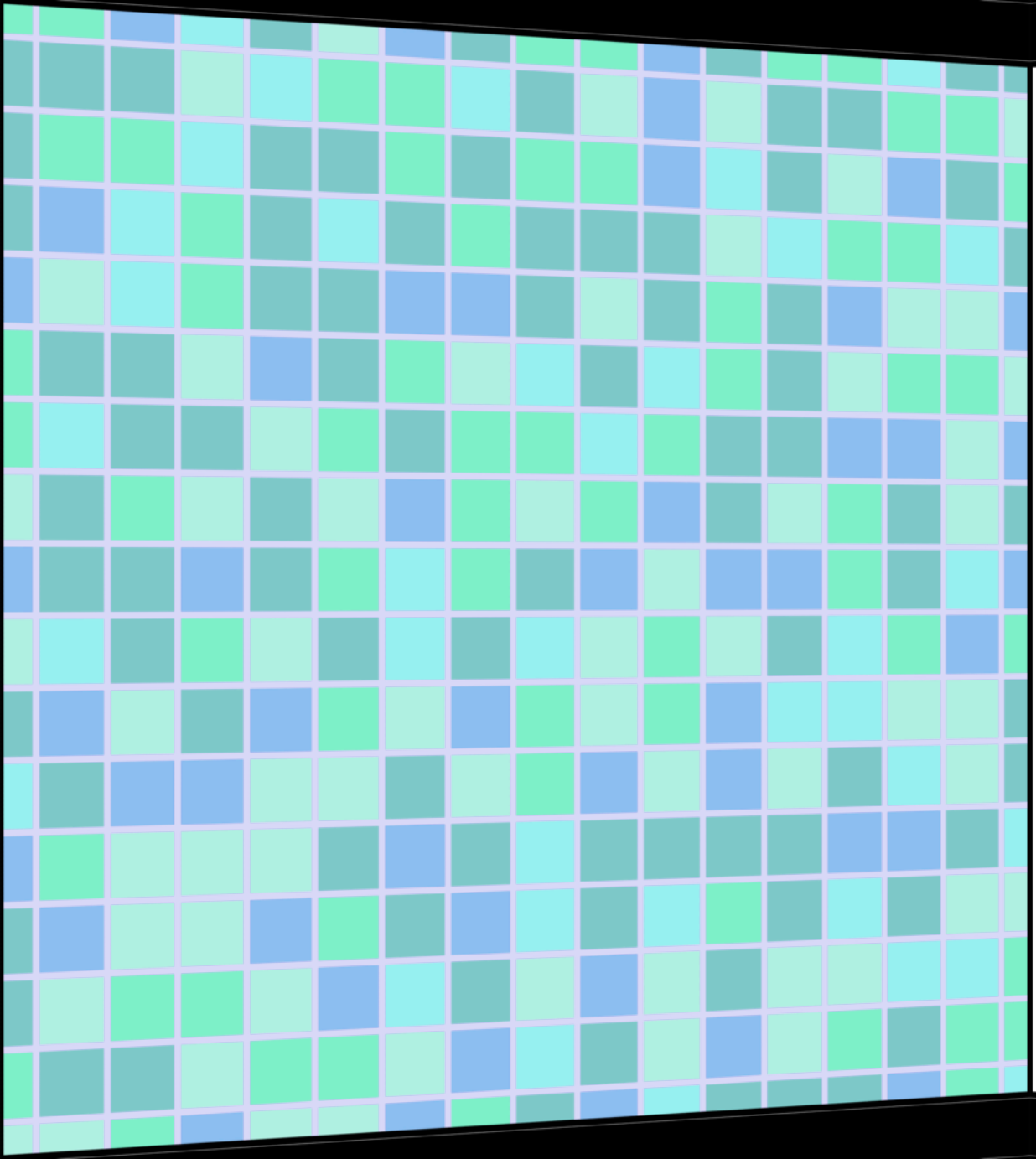

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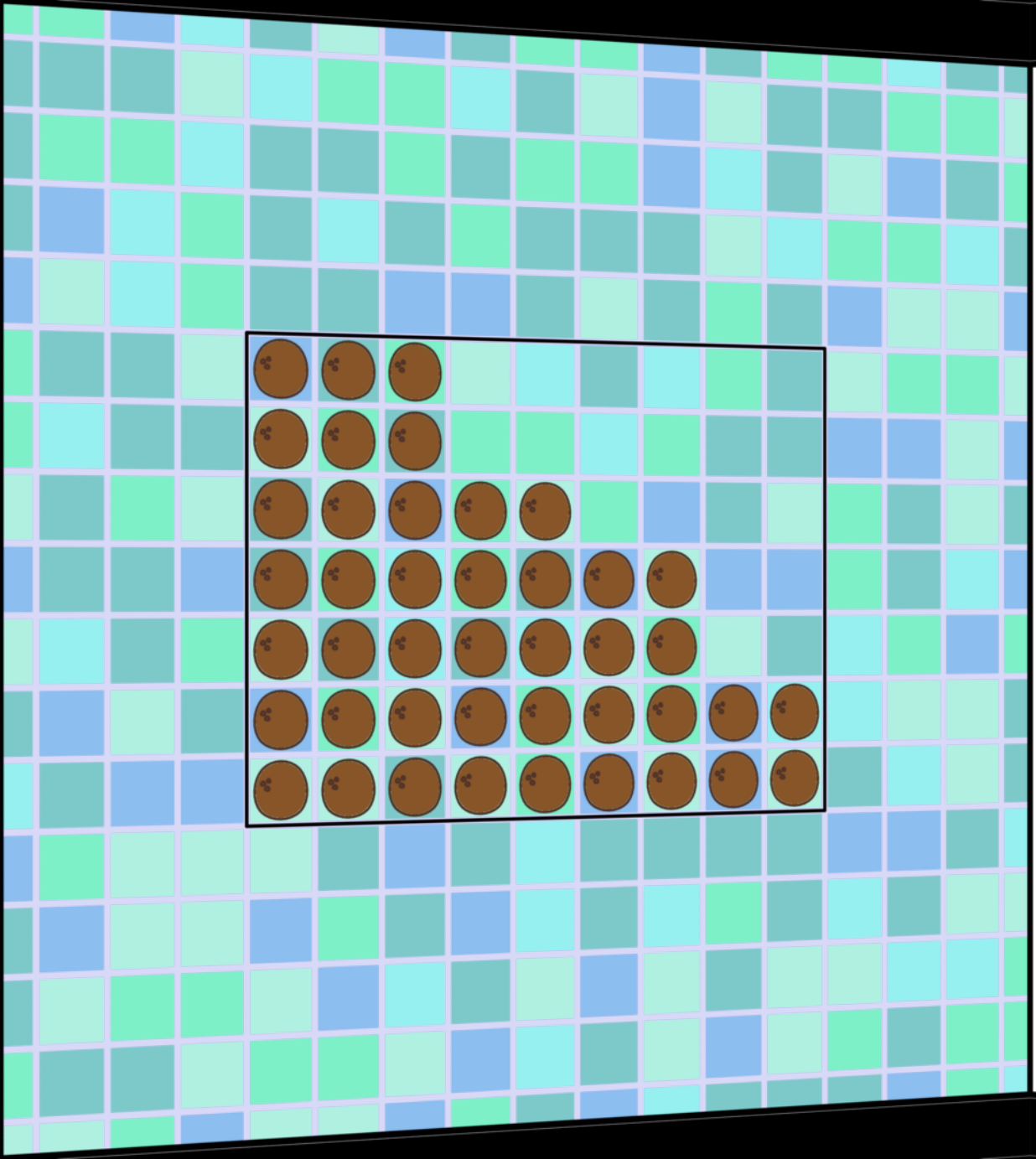

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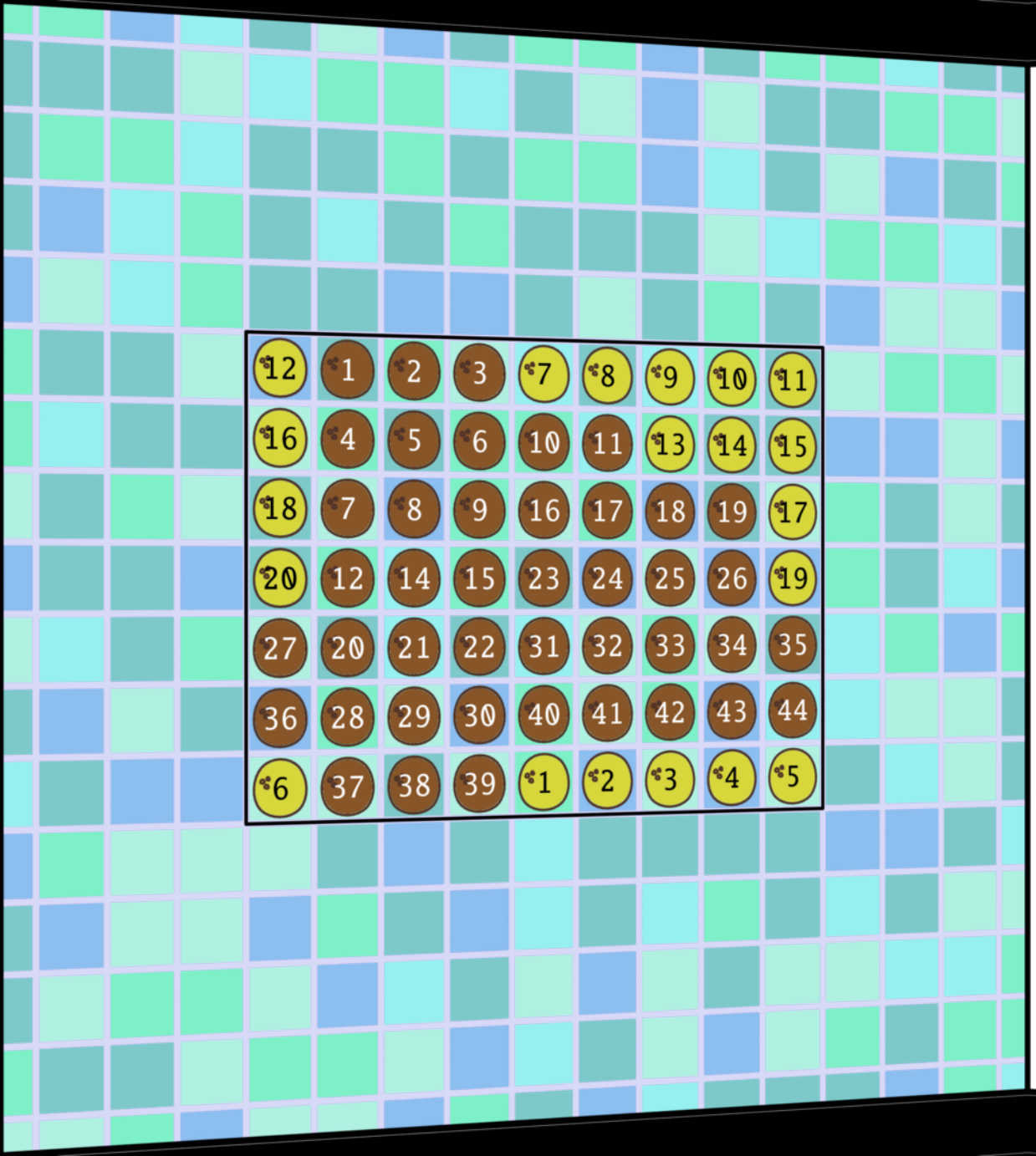

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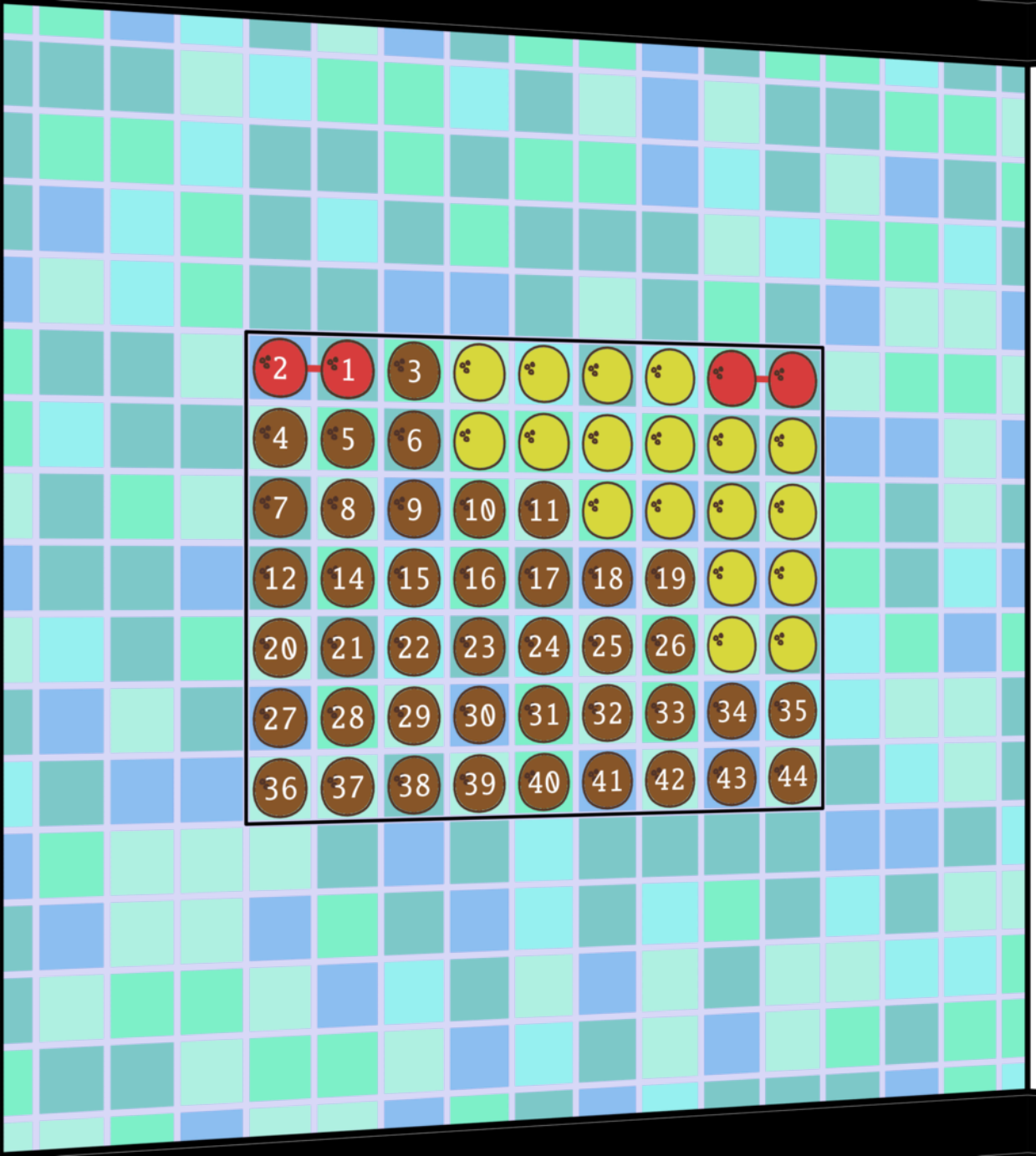

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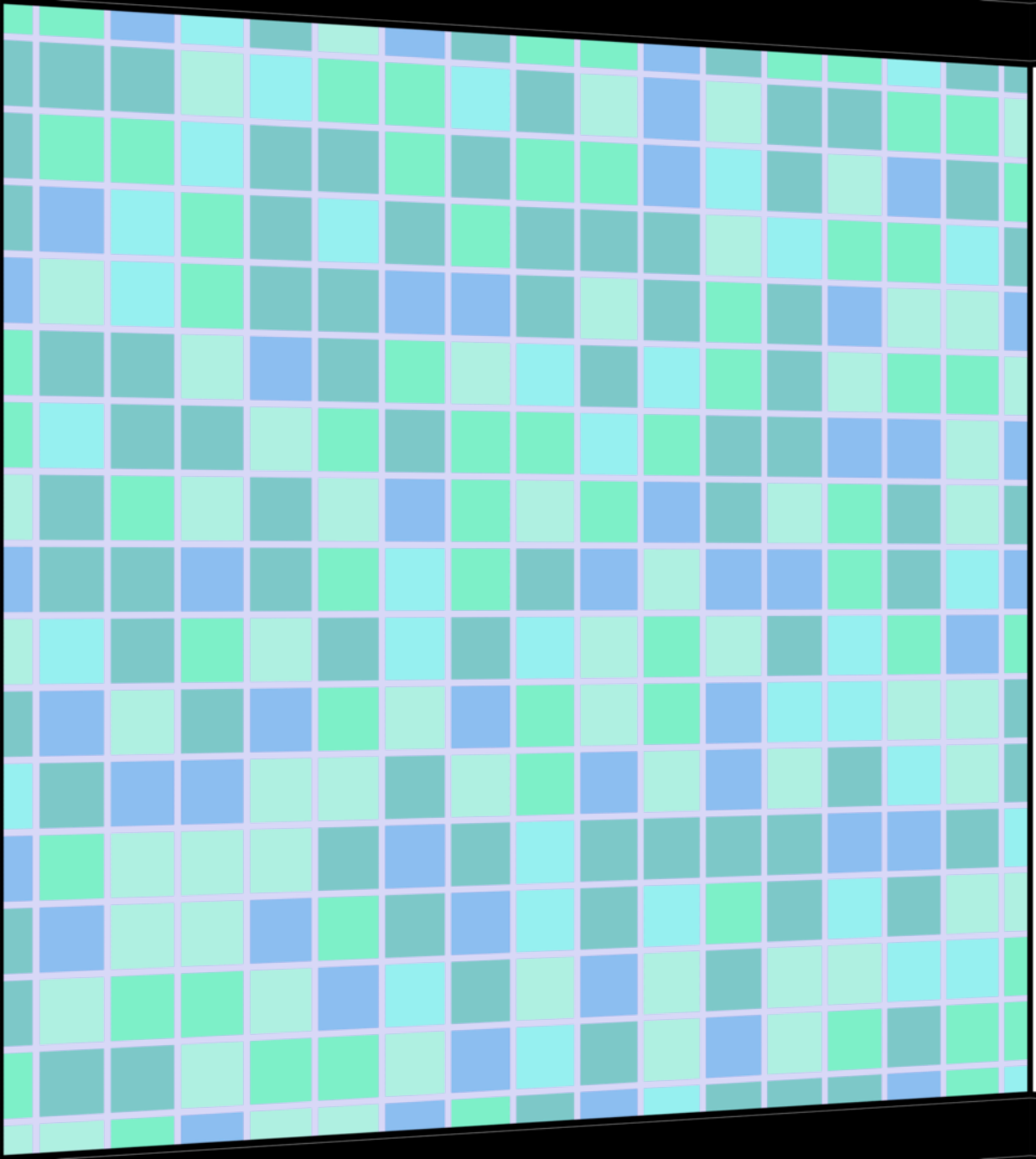
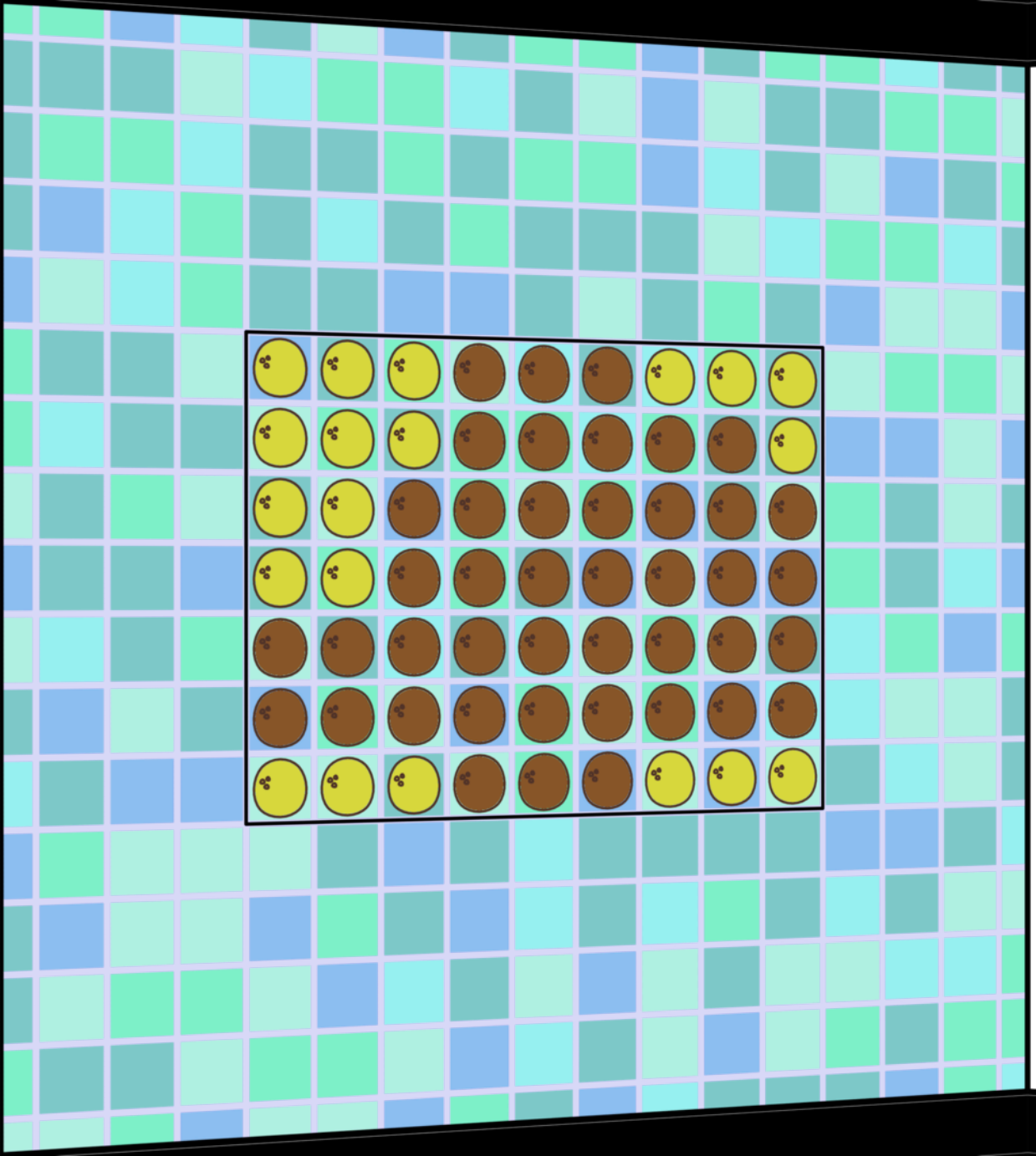


Take a closer look at the antinuts.



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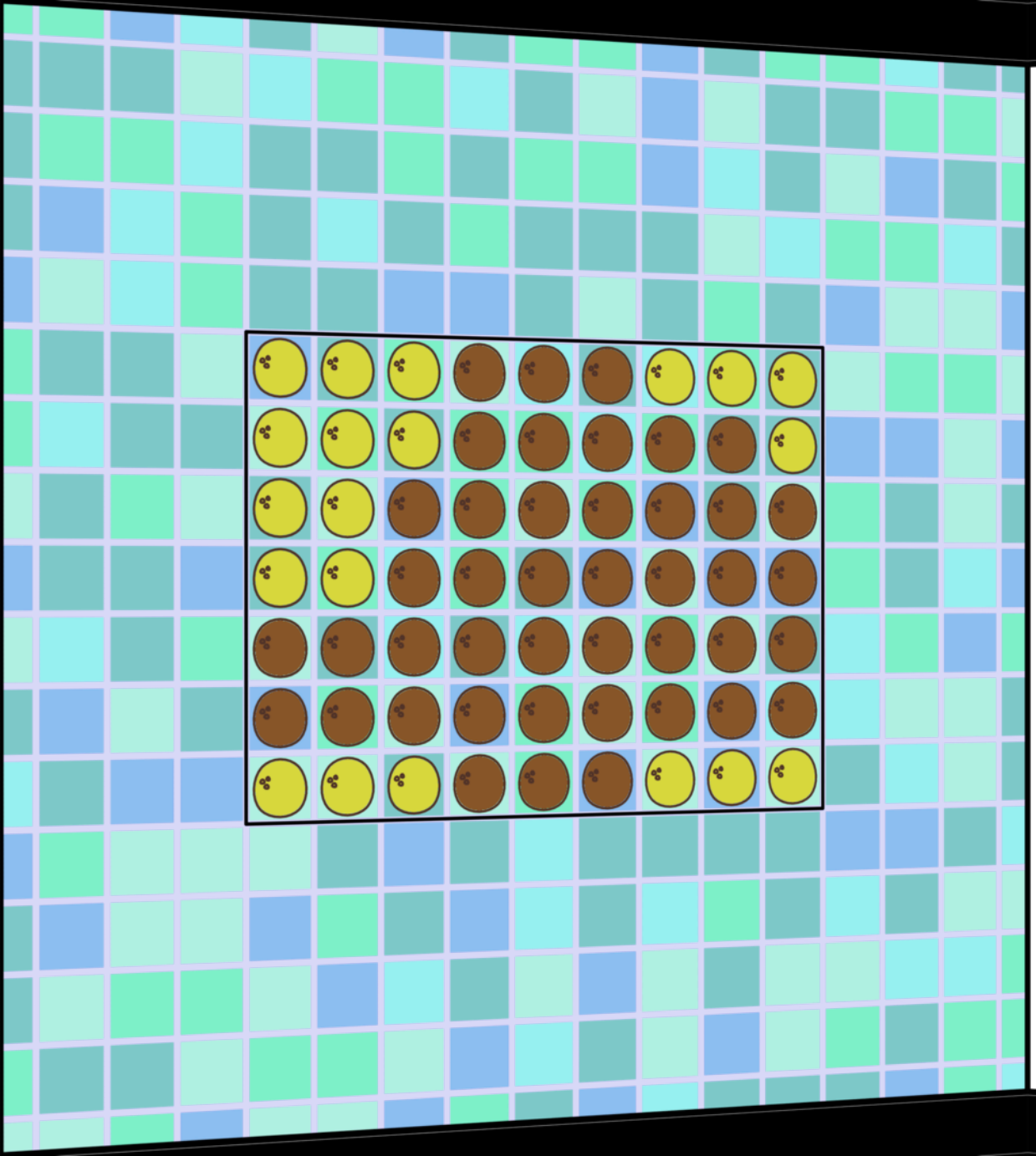
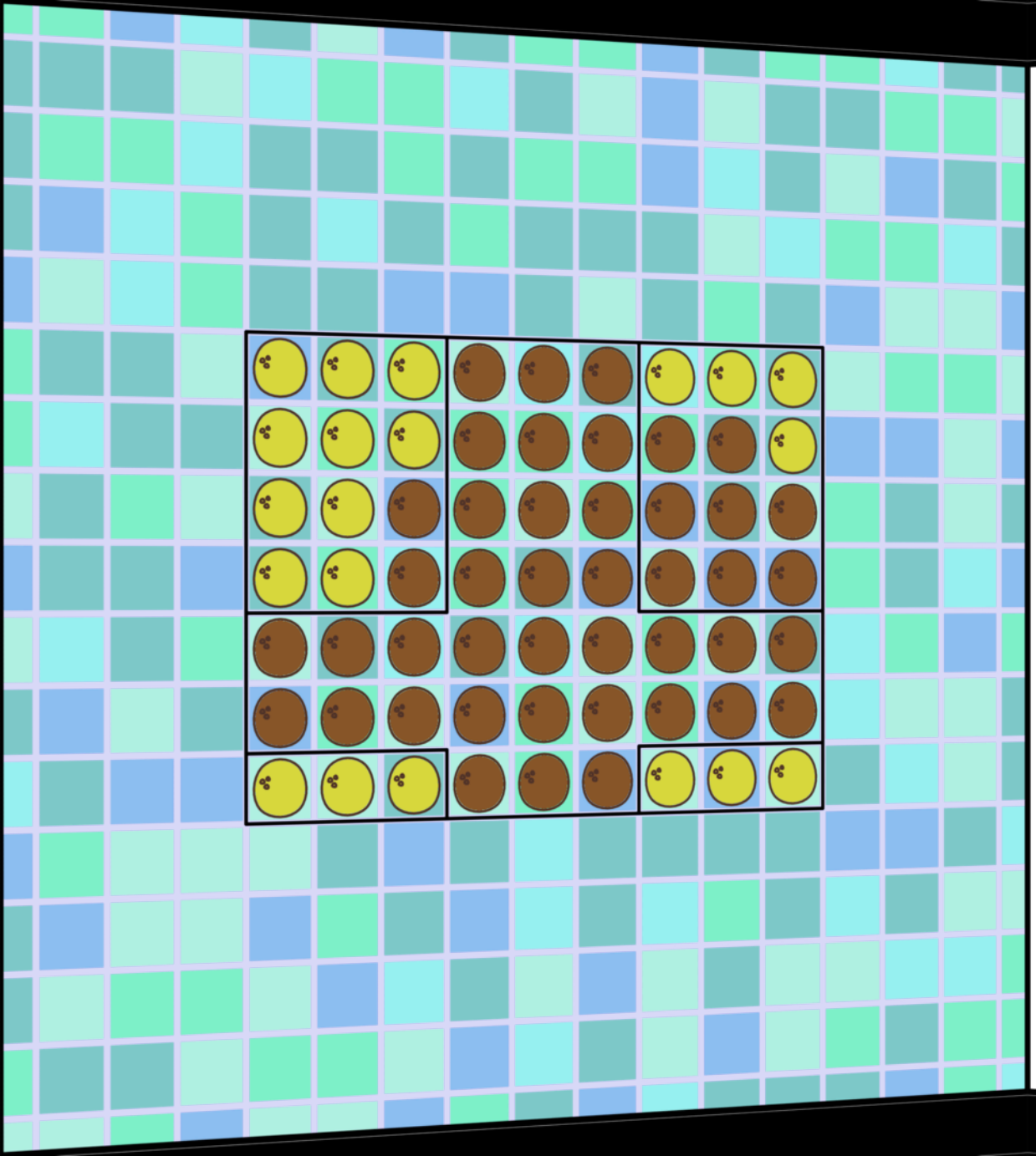


Take a closer look at the antinuts.

The full rows and columns separate them into four quadrants.



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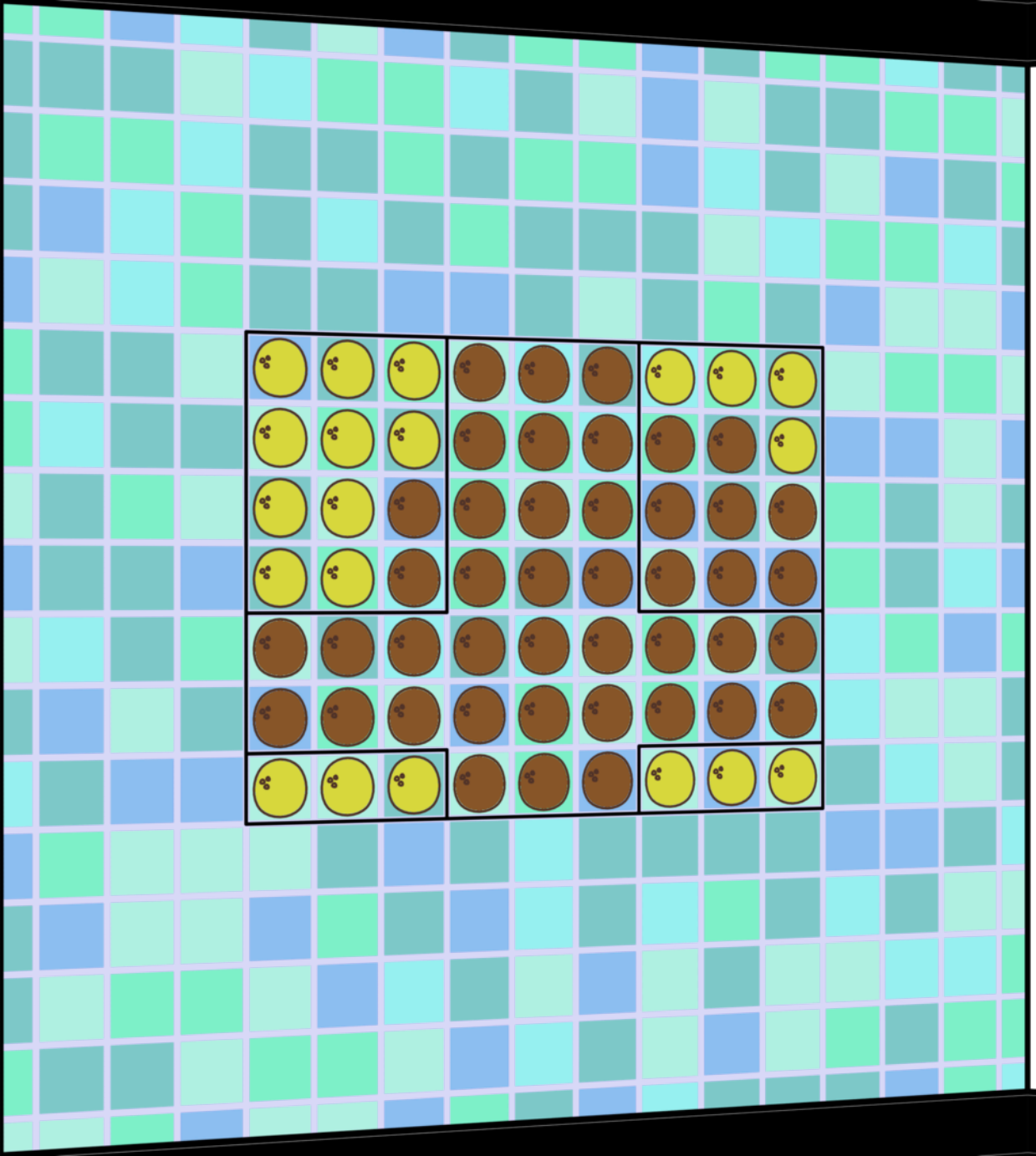

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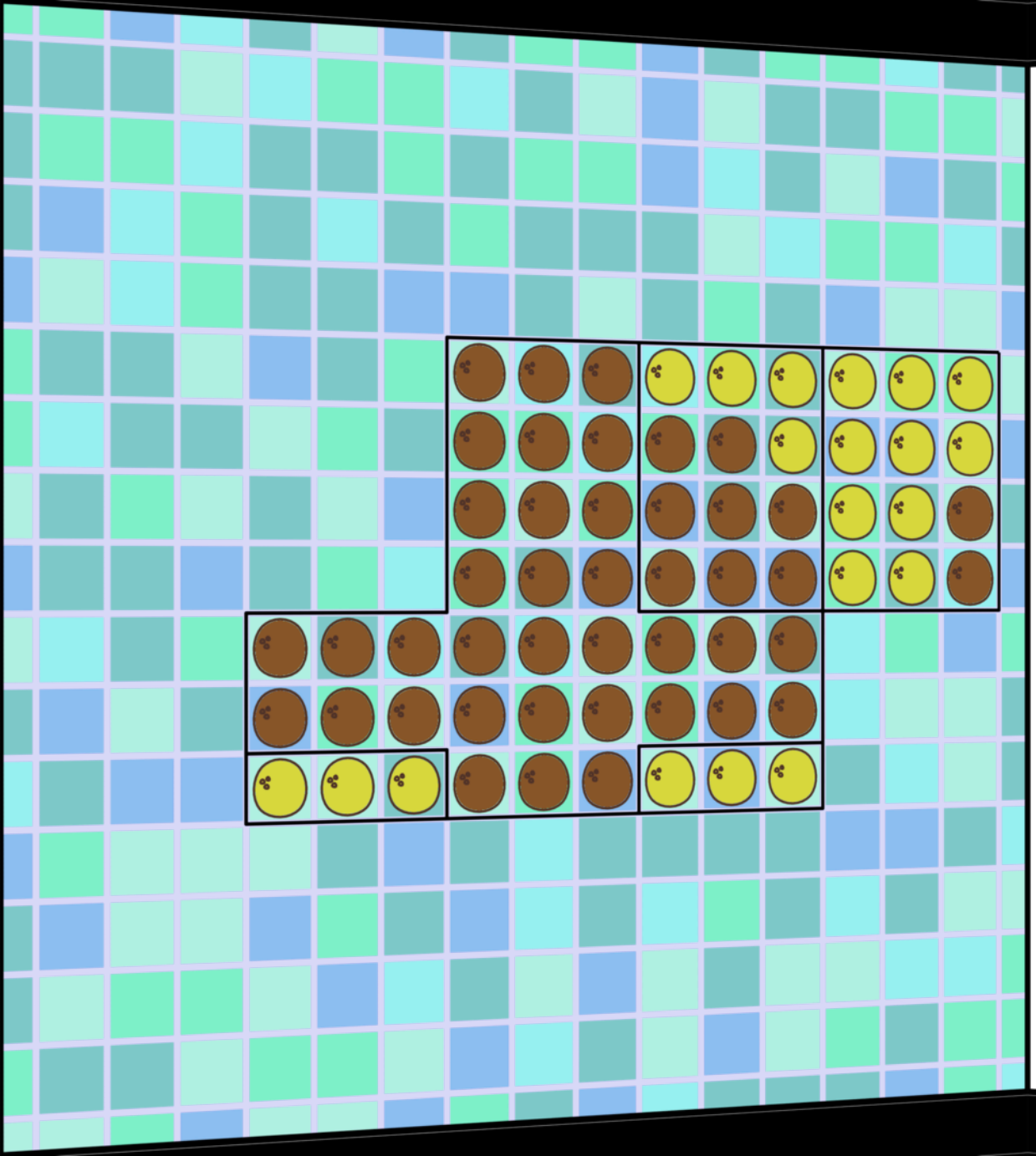

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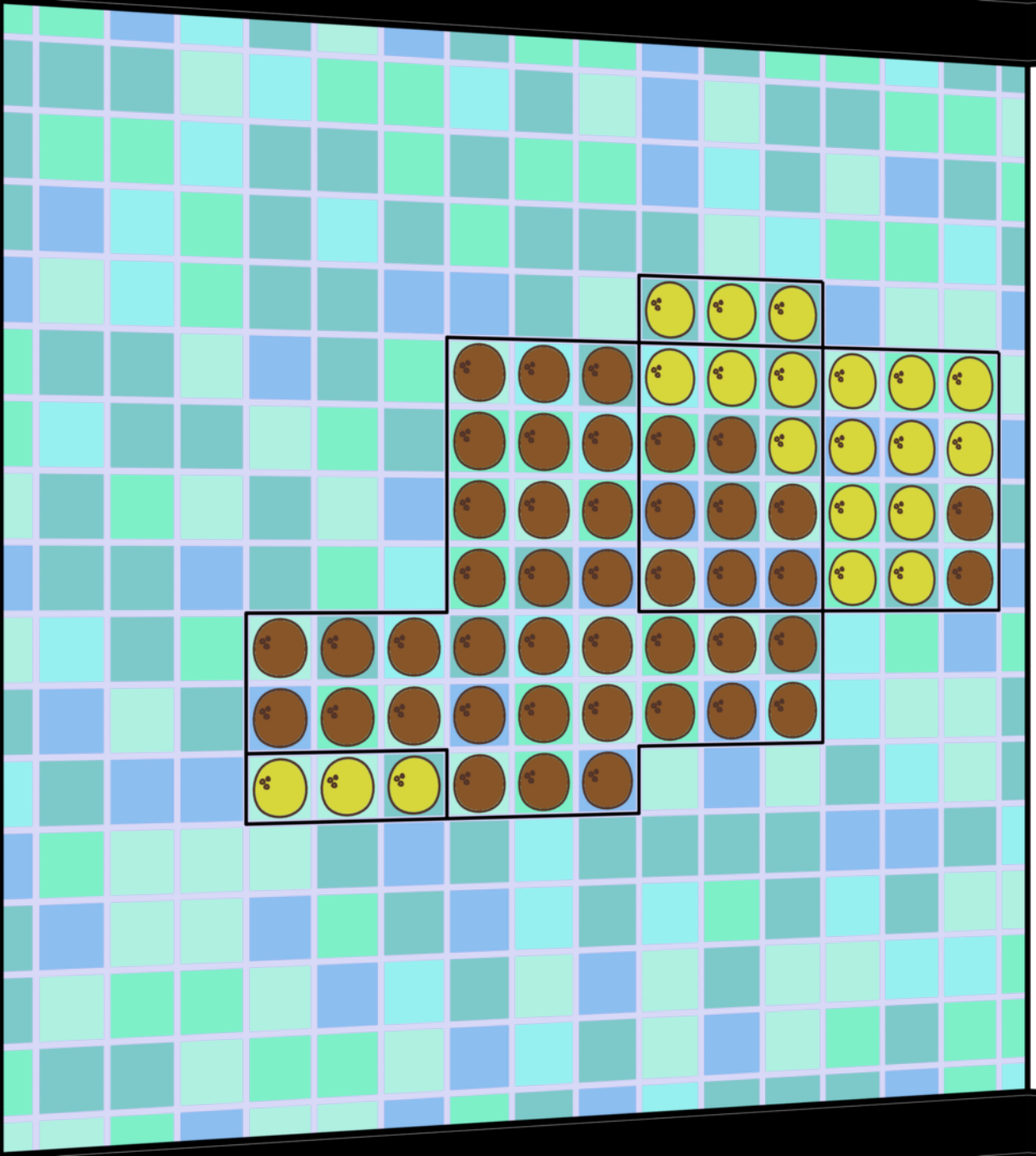

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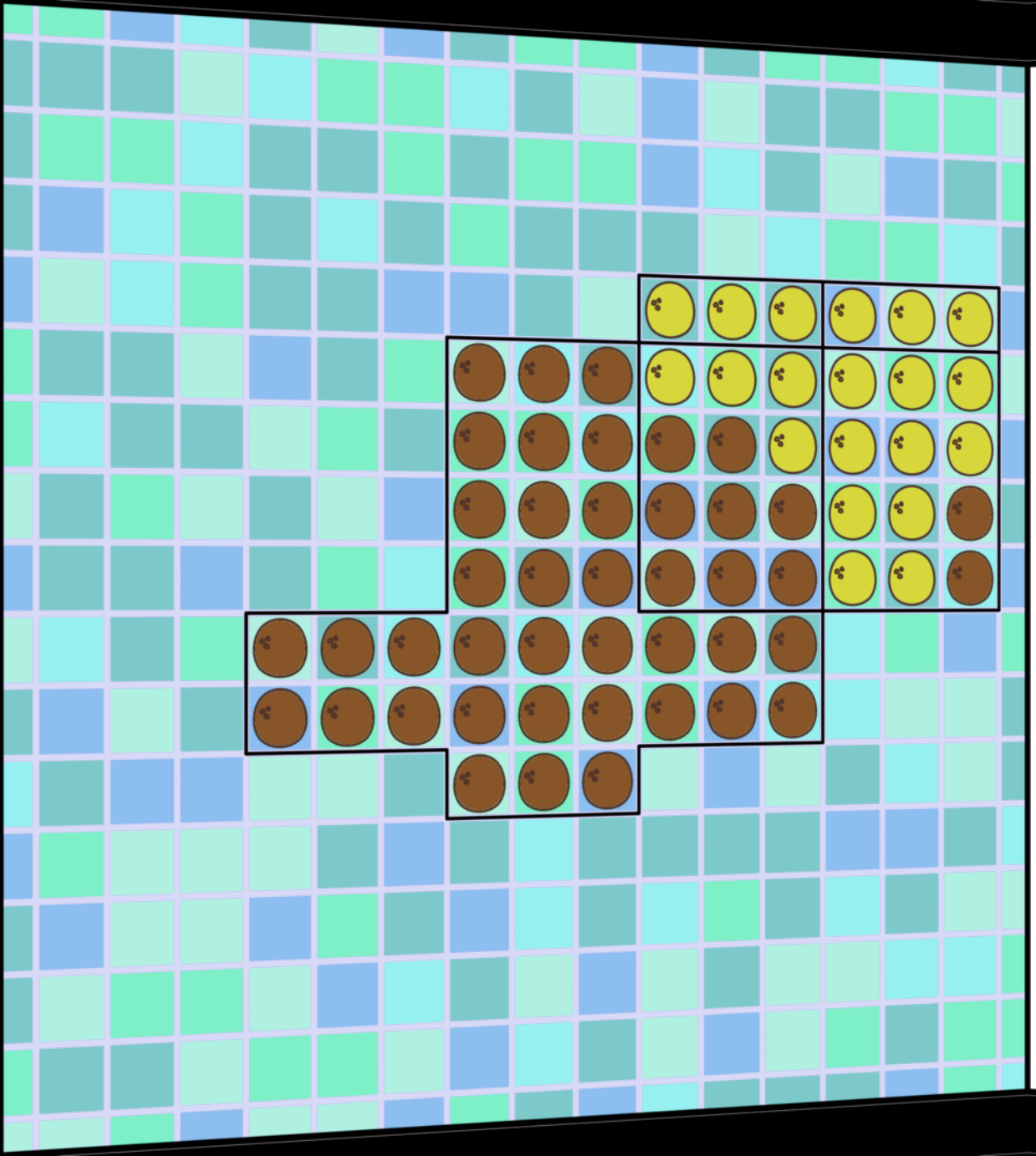

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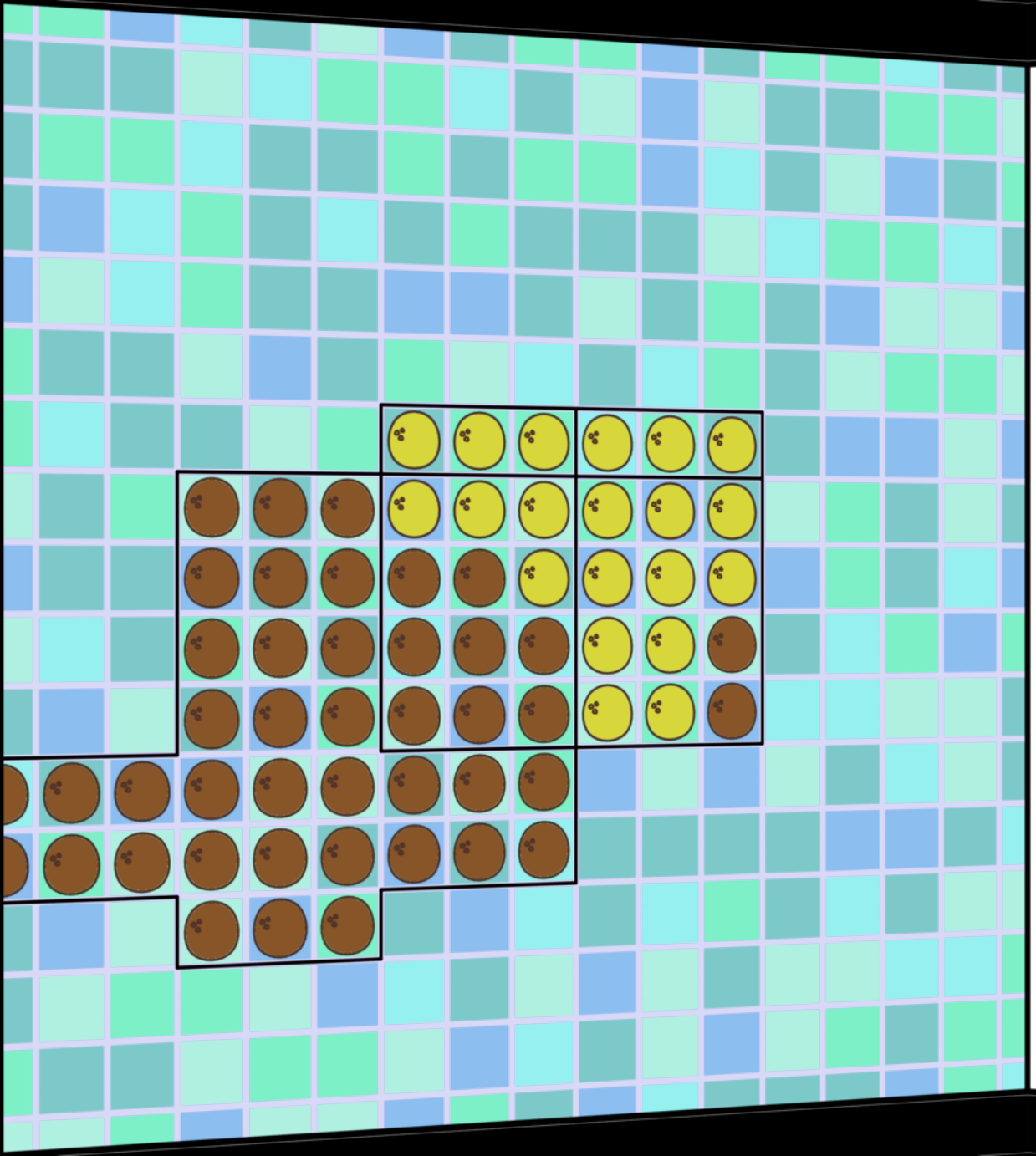
Center the view and forget about the full rows and columns.



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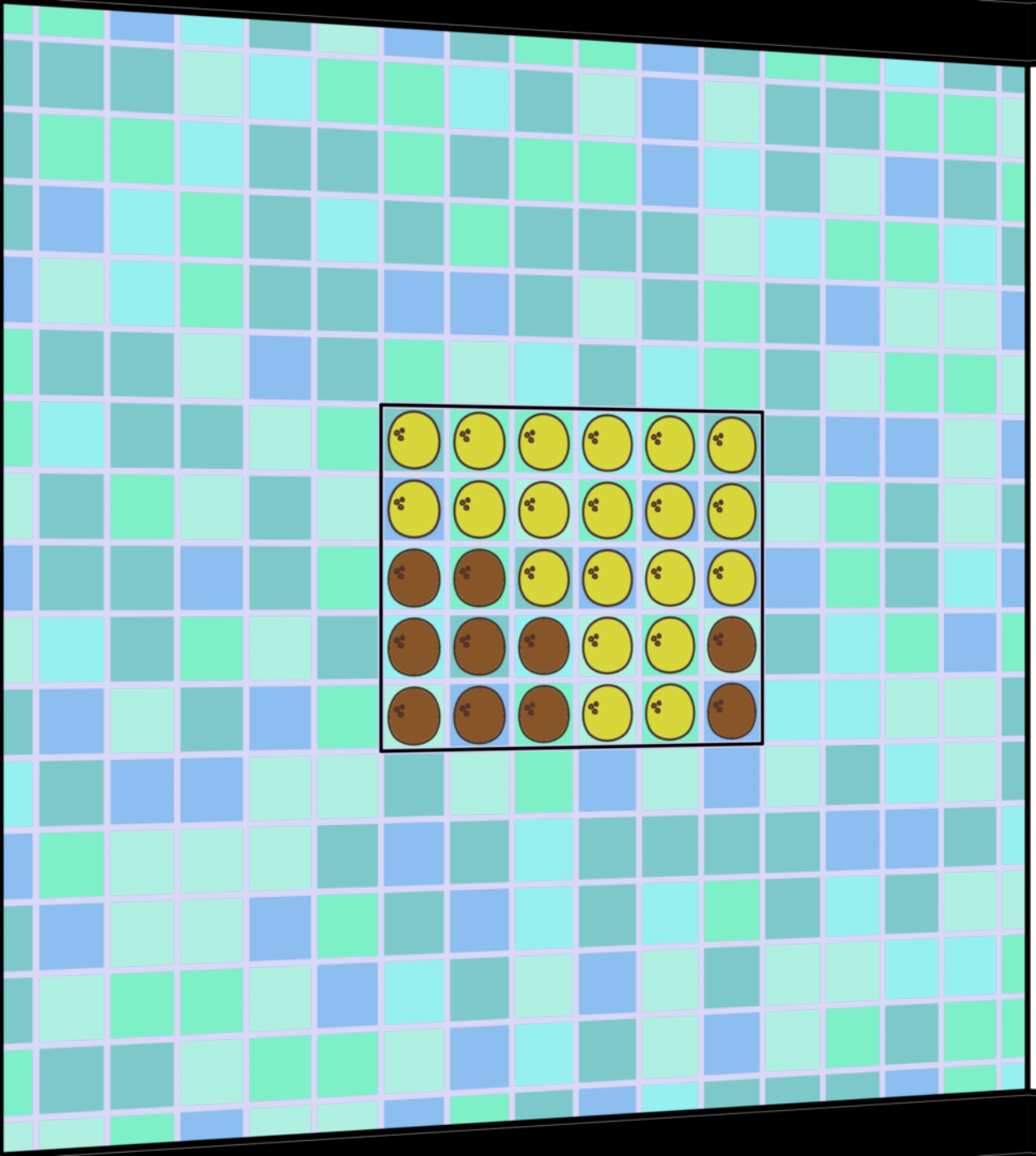

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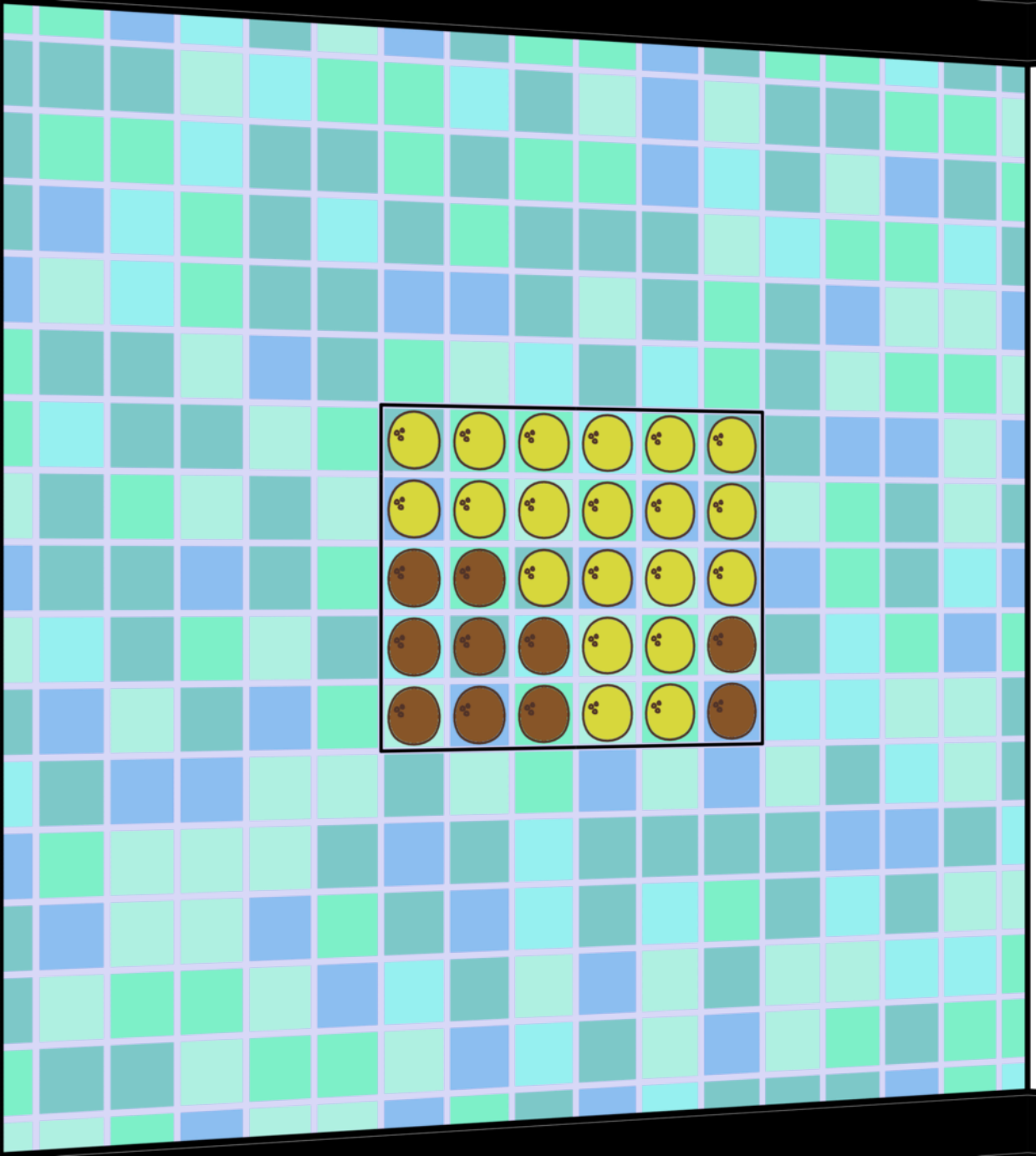

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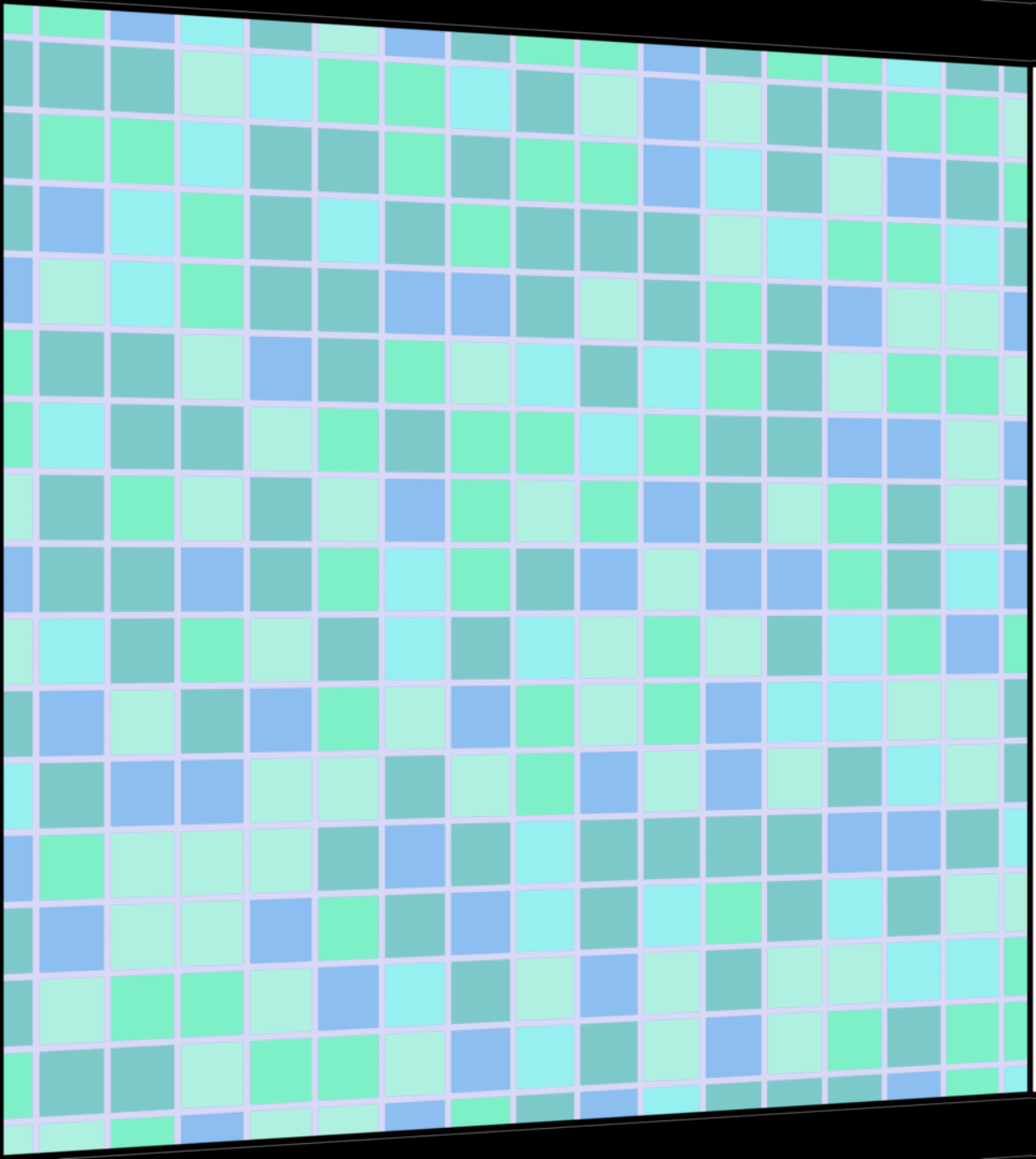
It's the *antinut-centric* view of a configuration.



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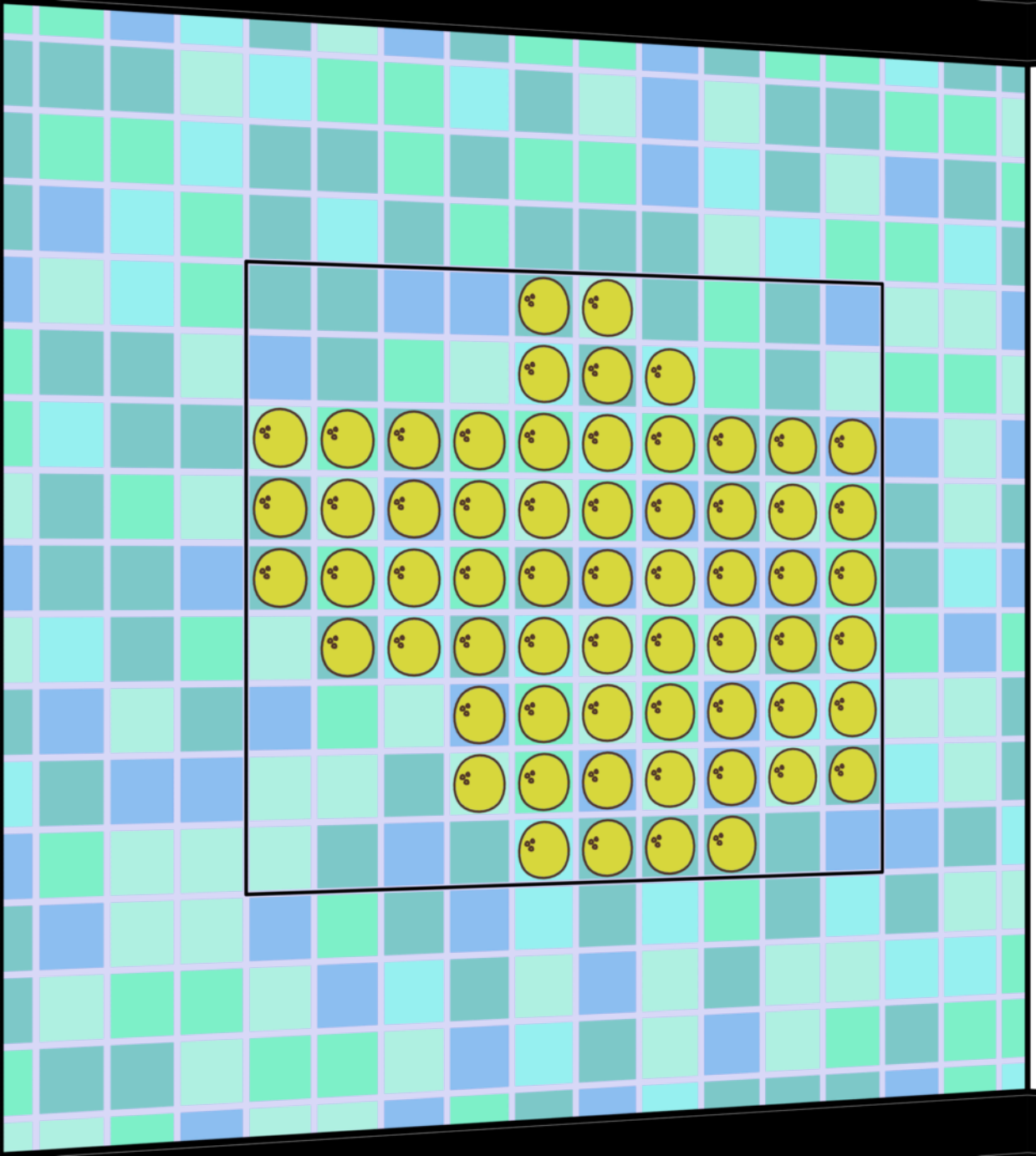

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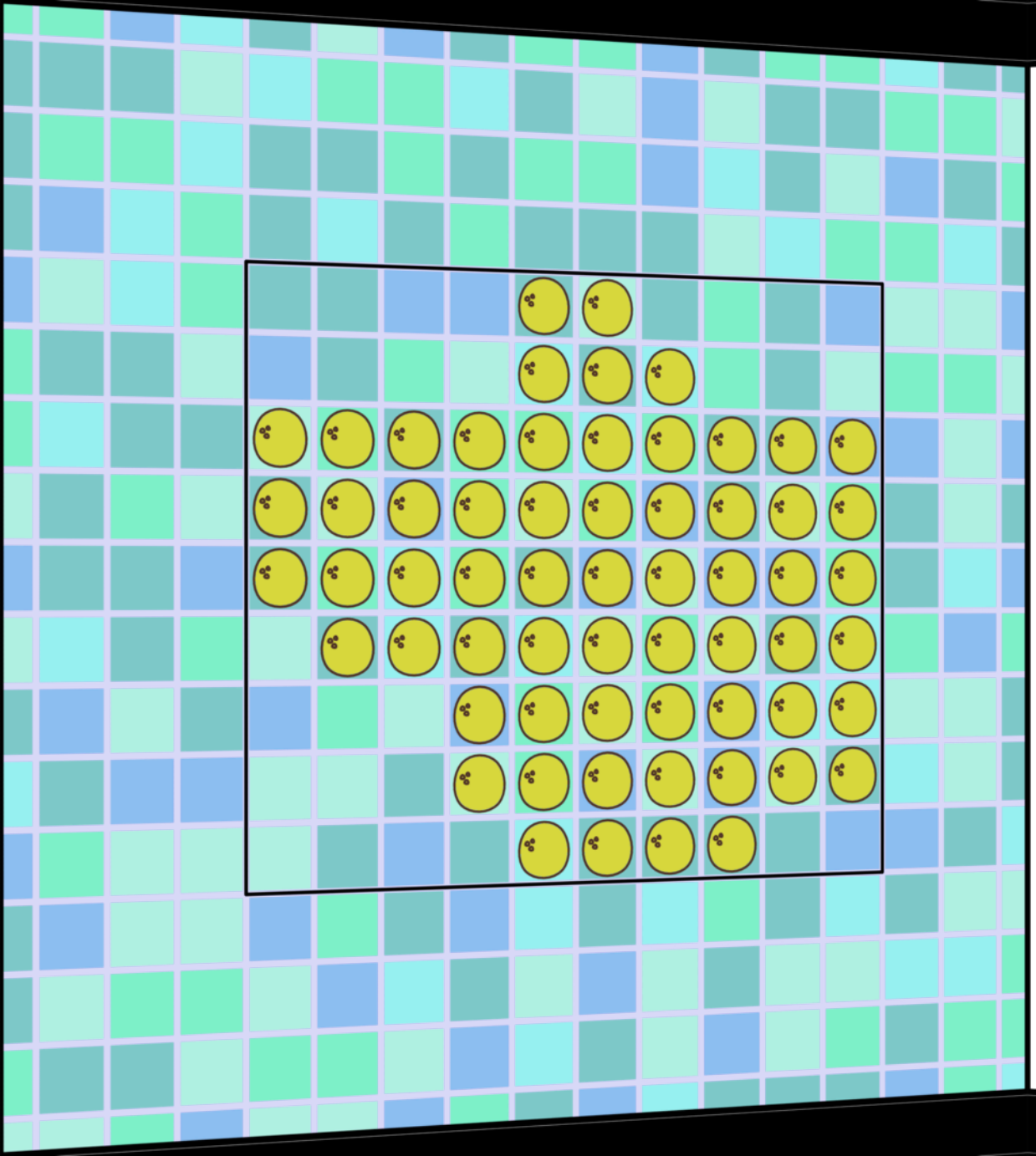

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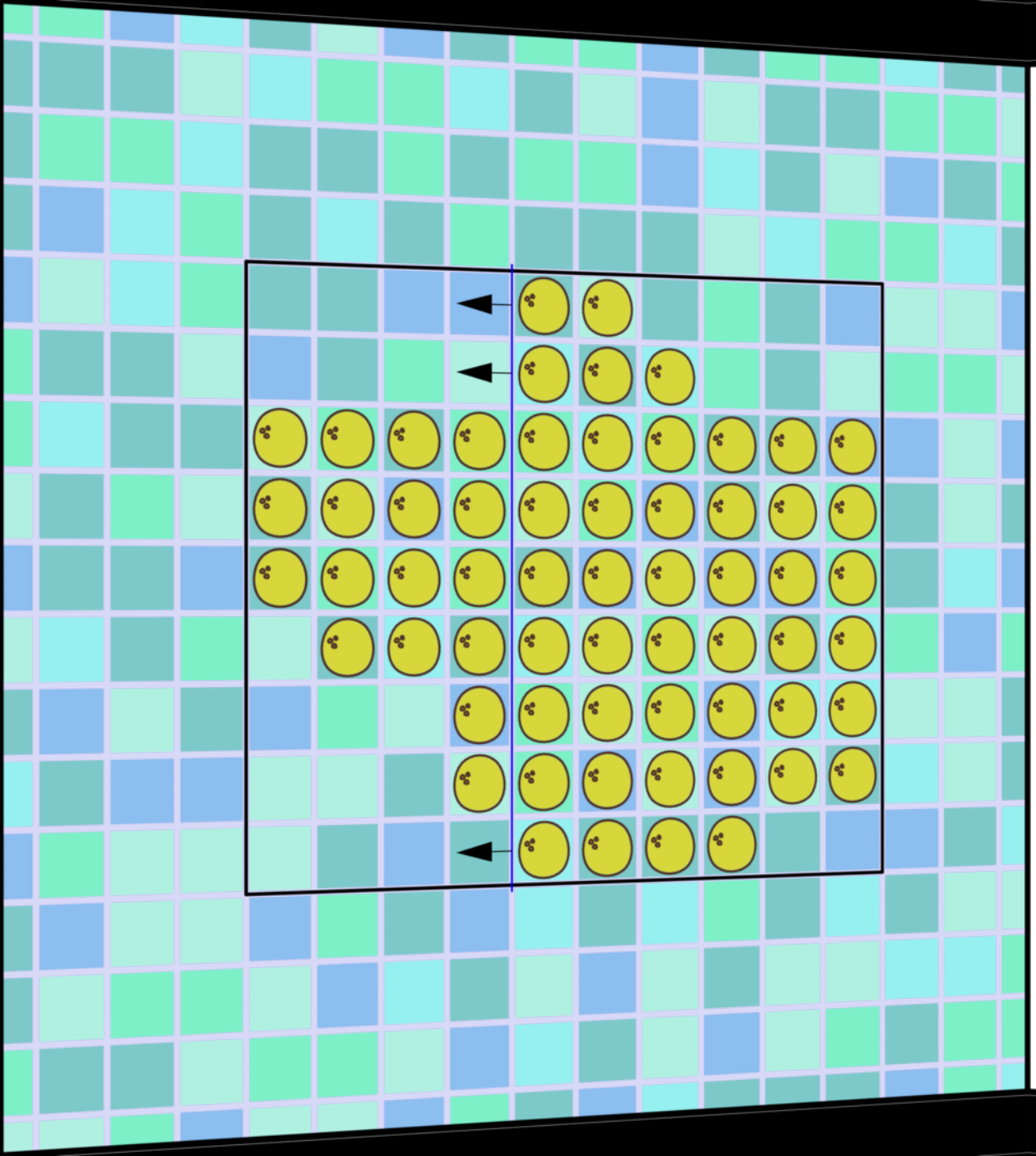

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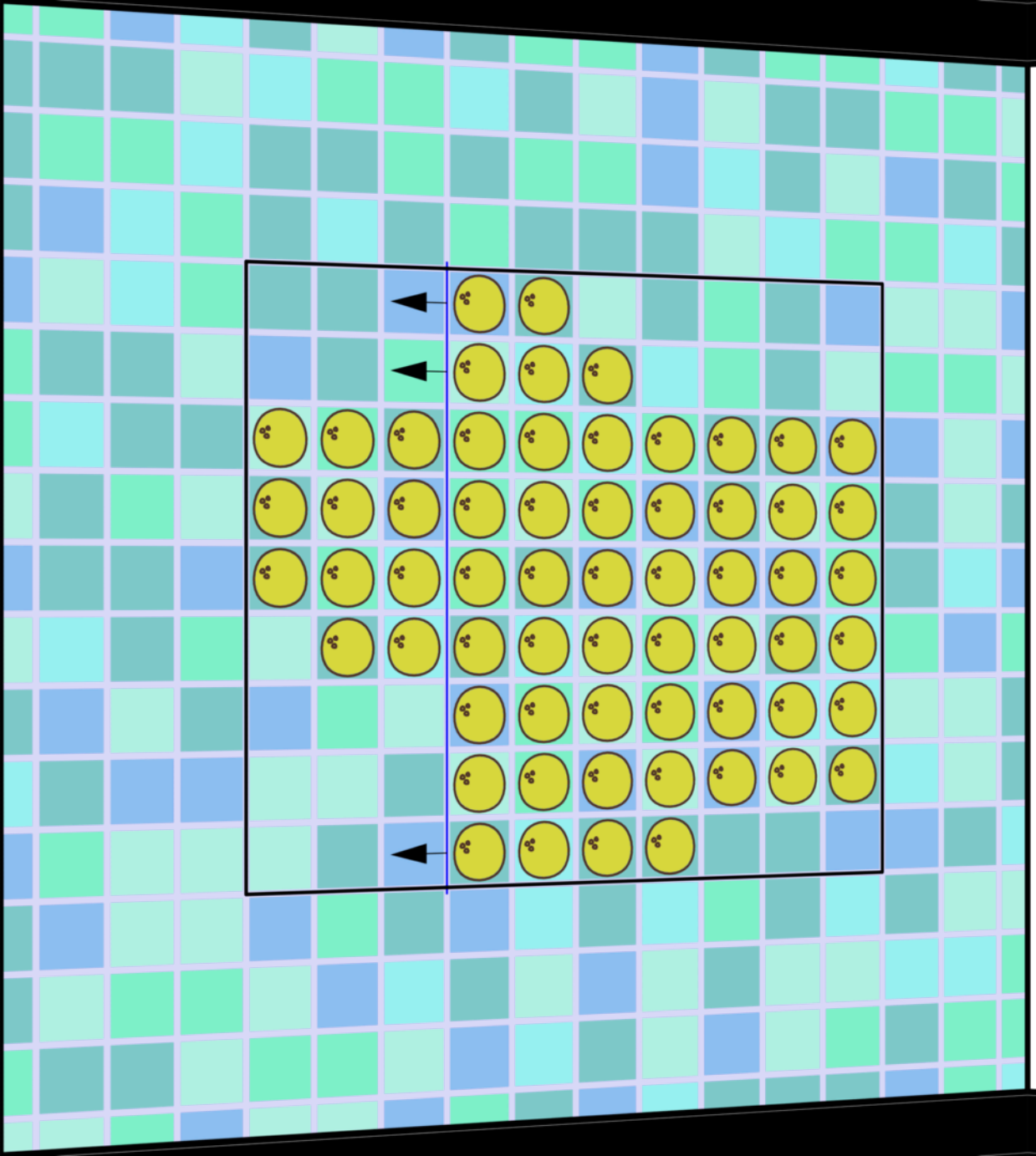

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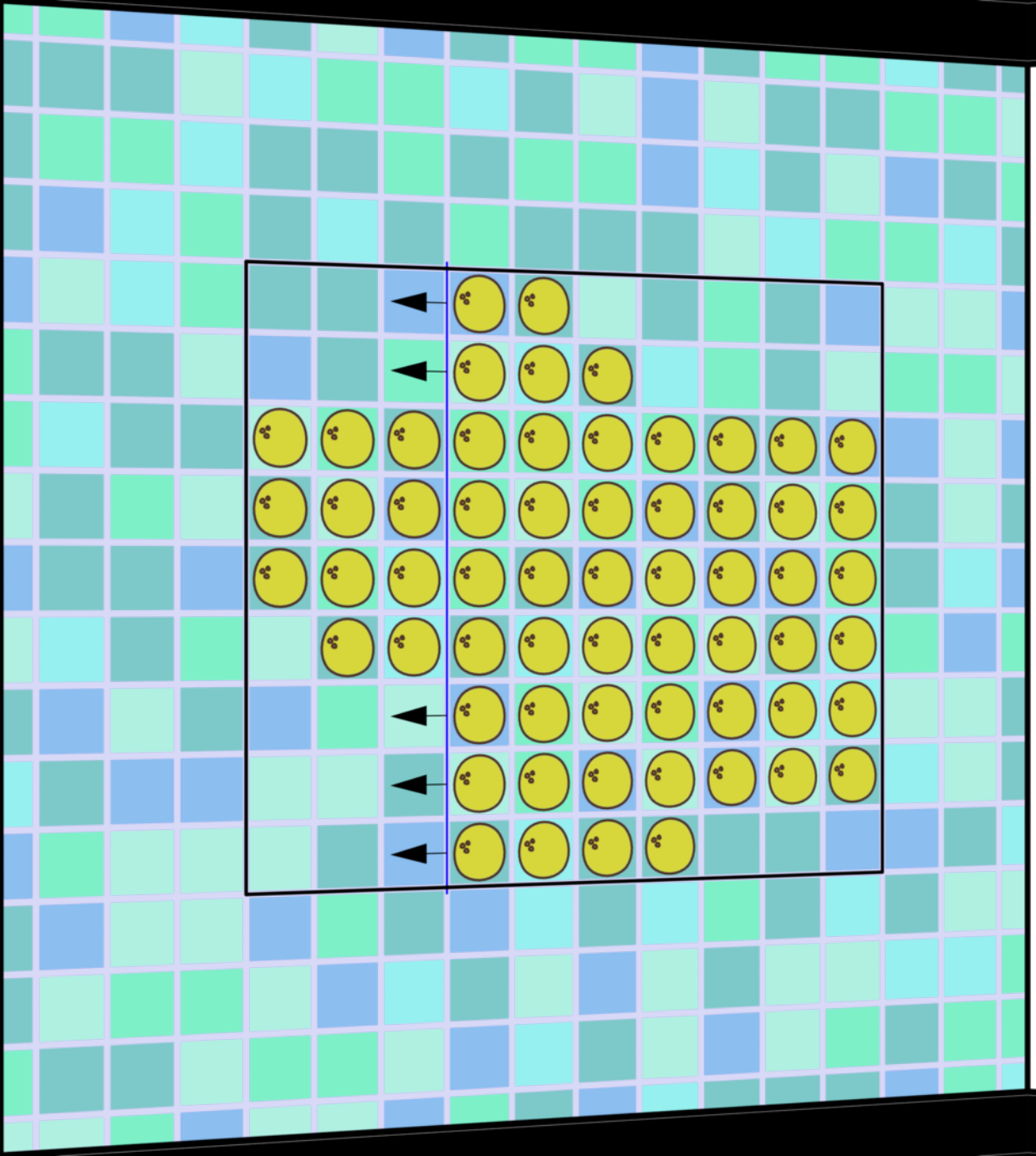

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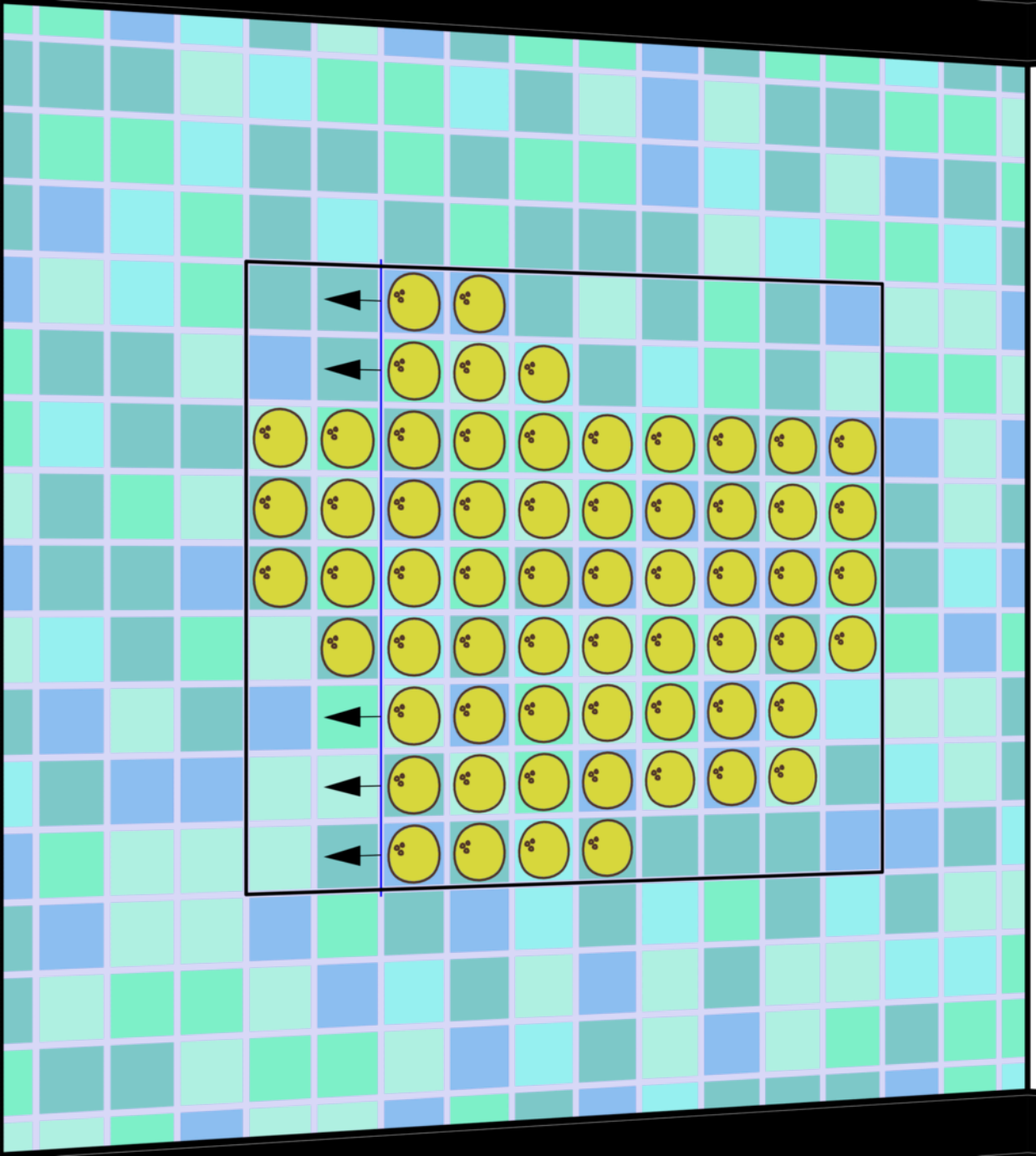

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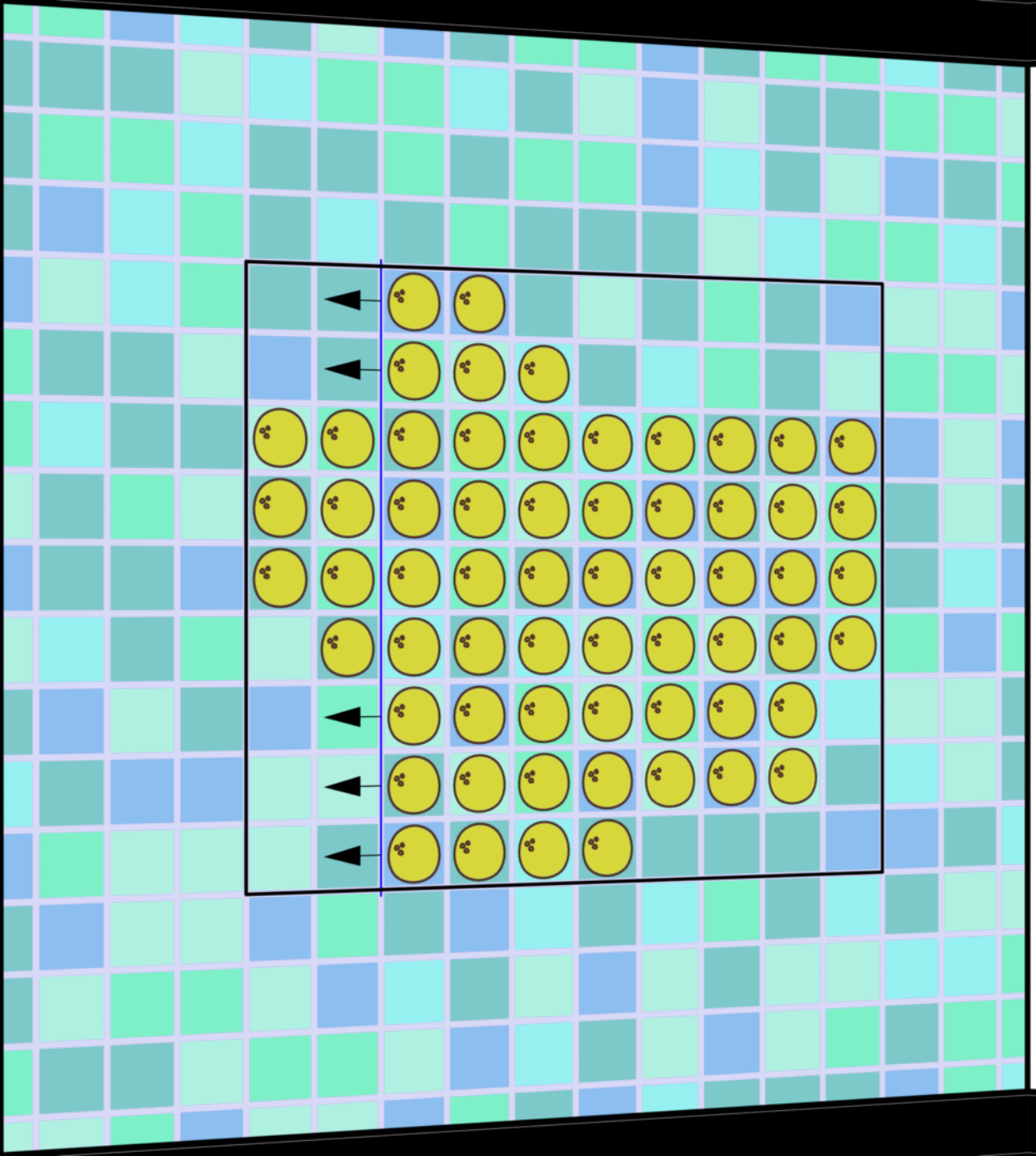

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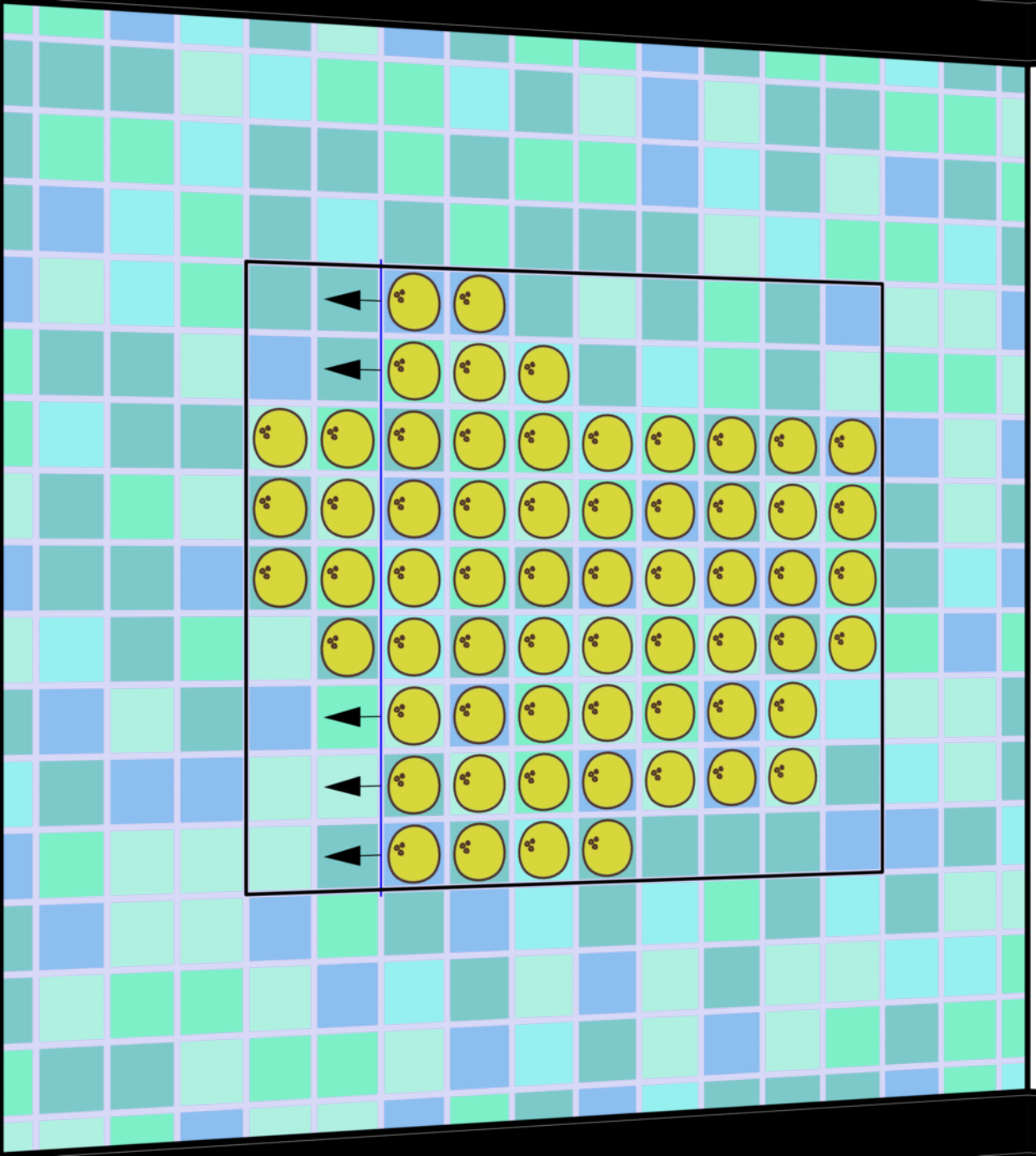

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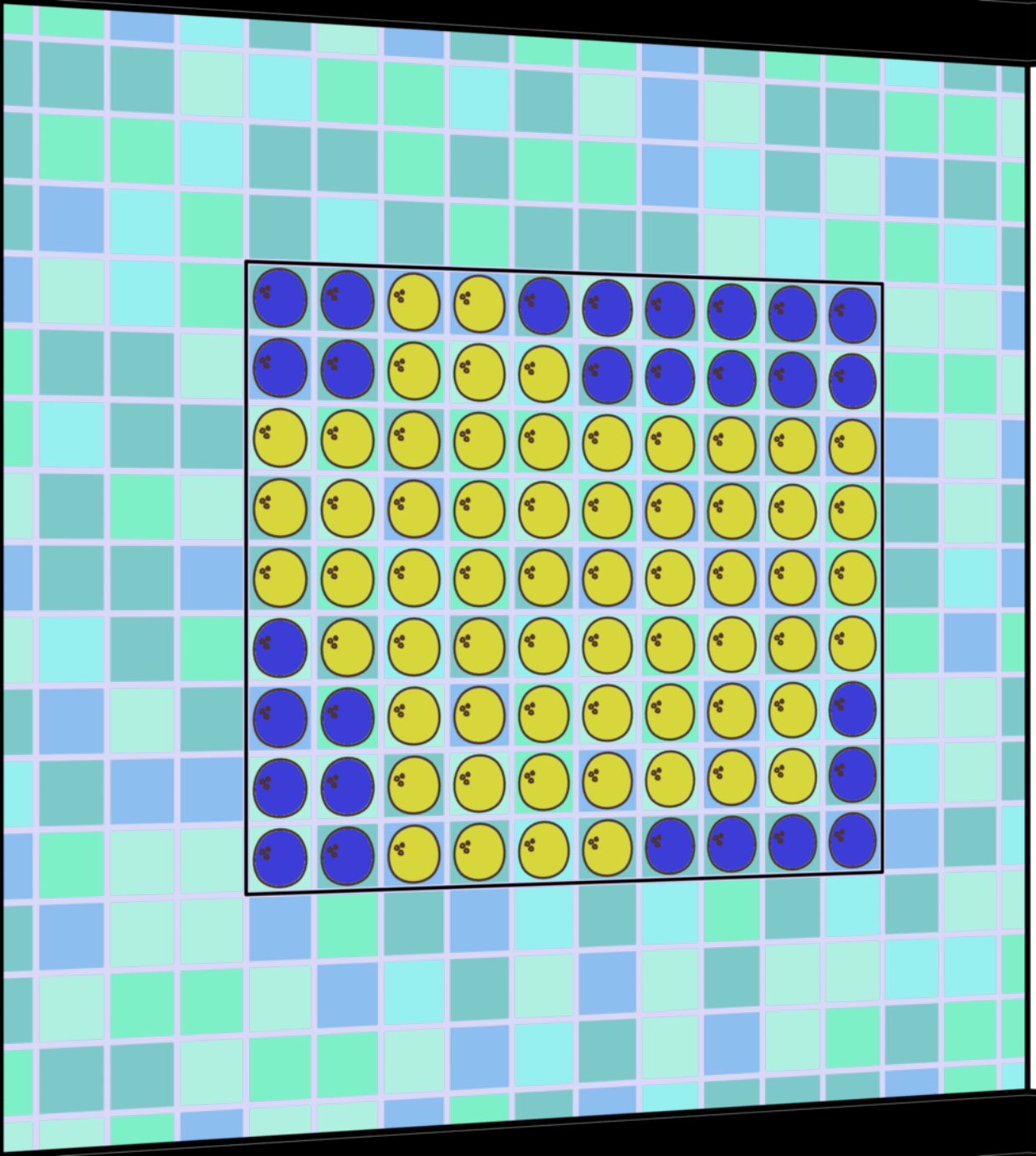

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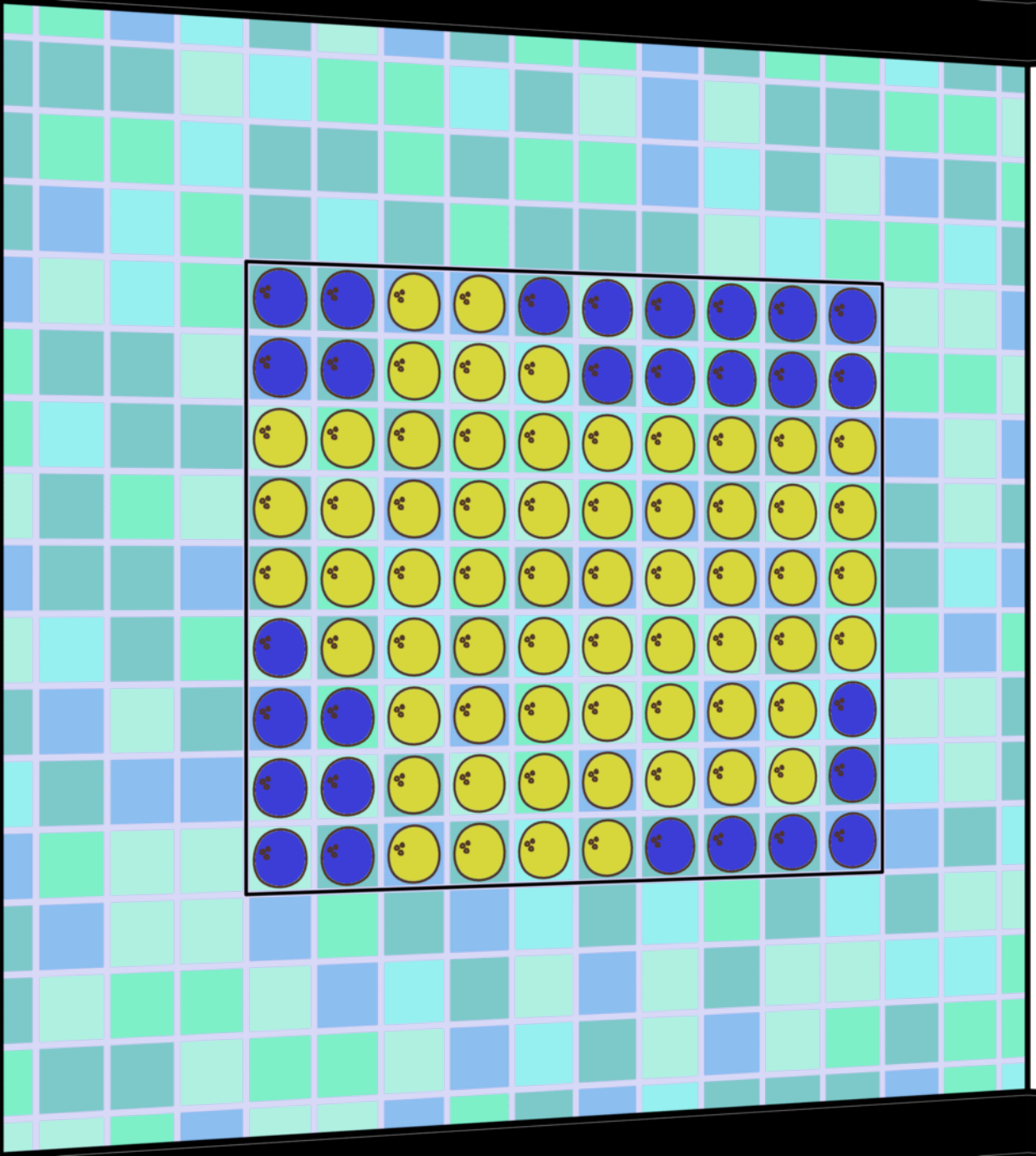

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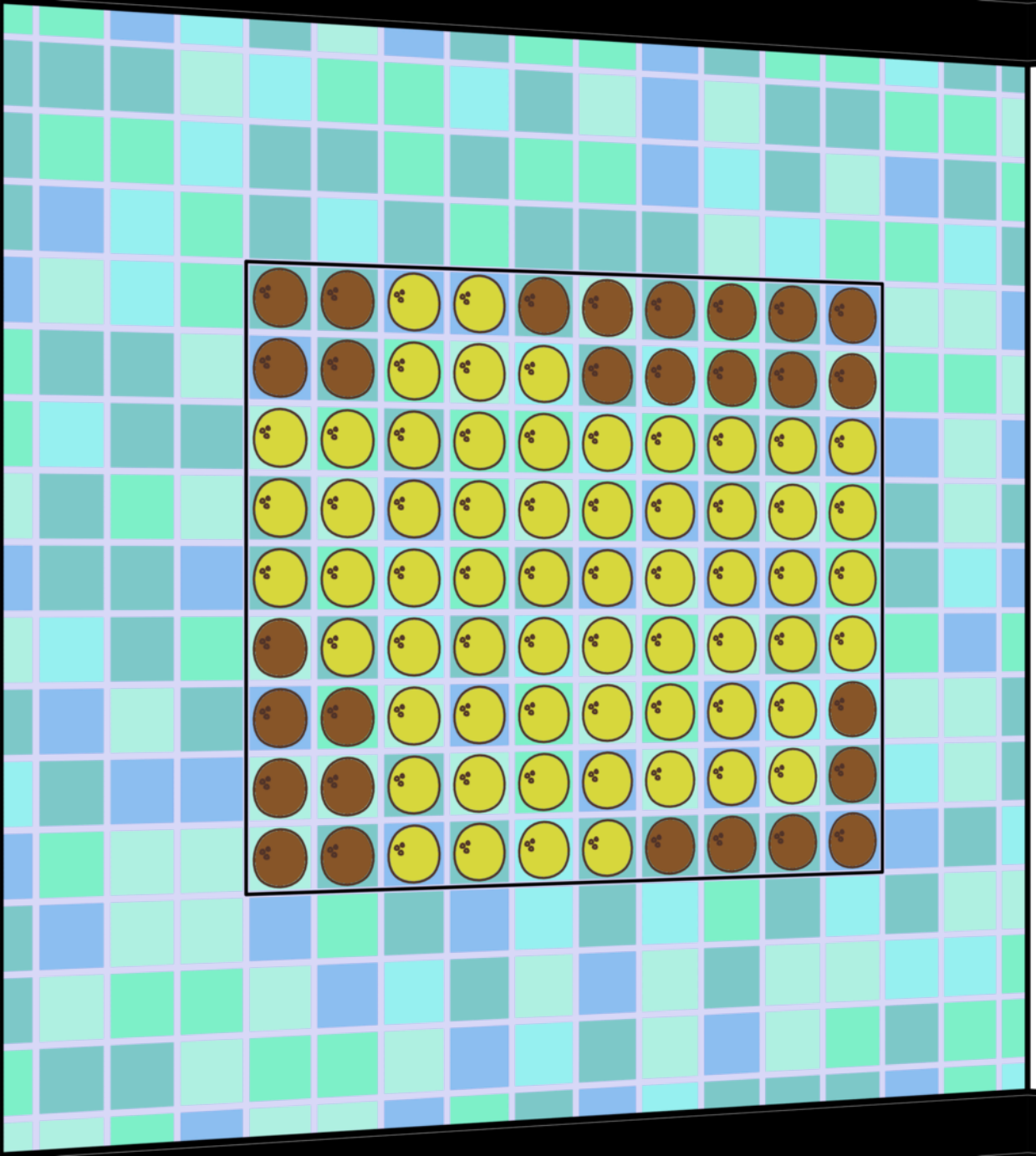

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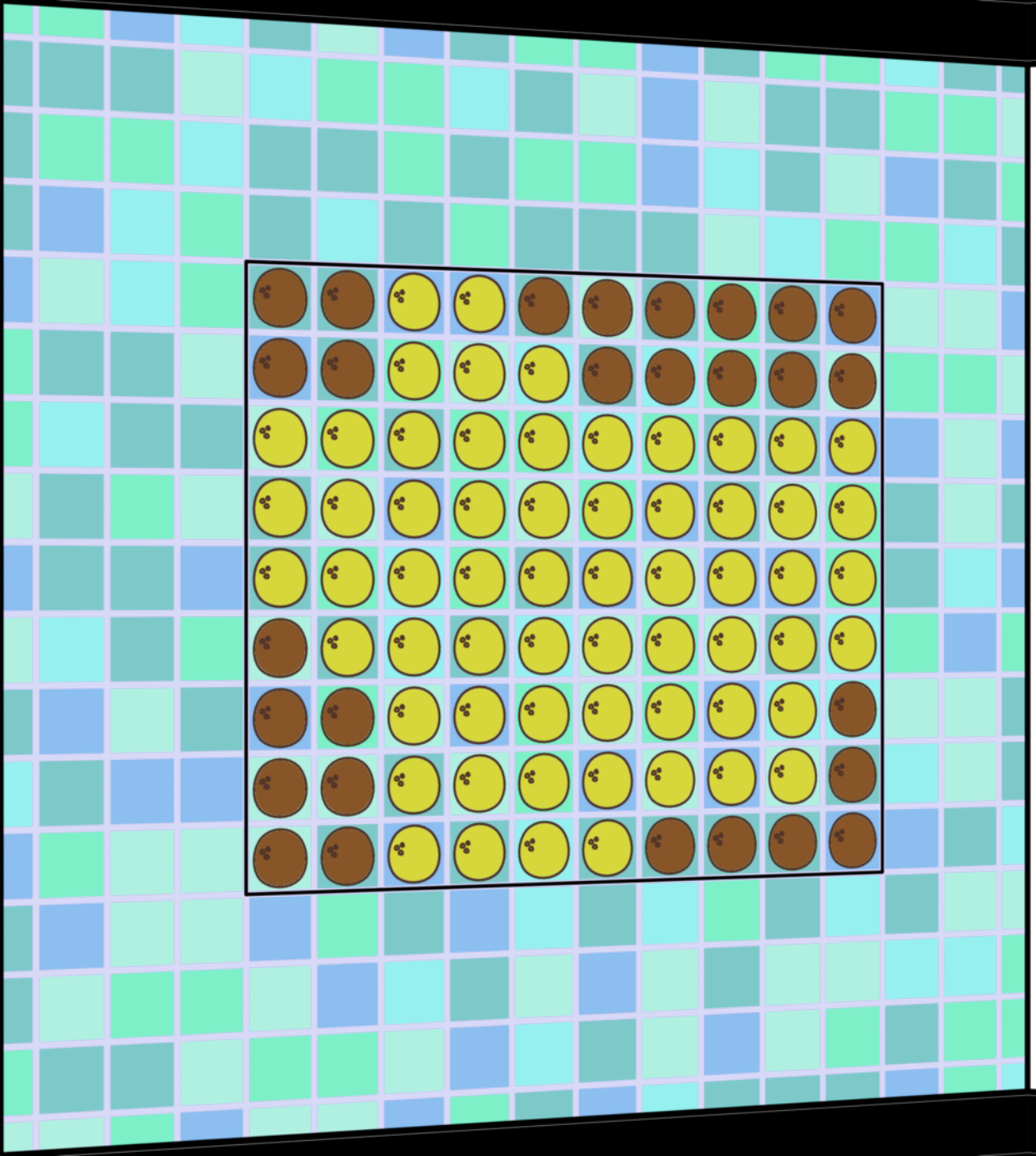

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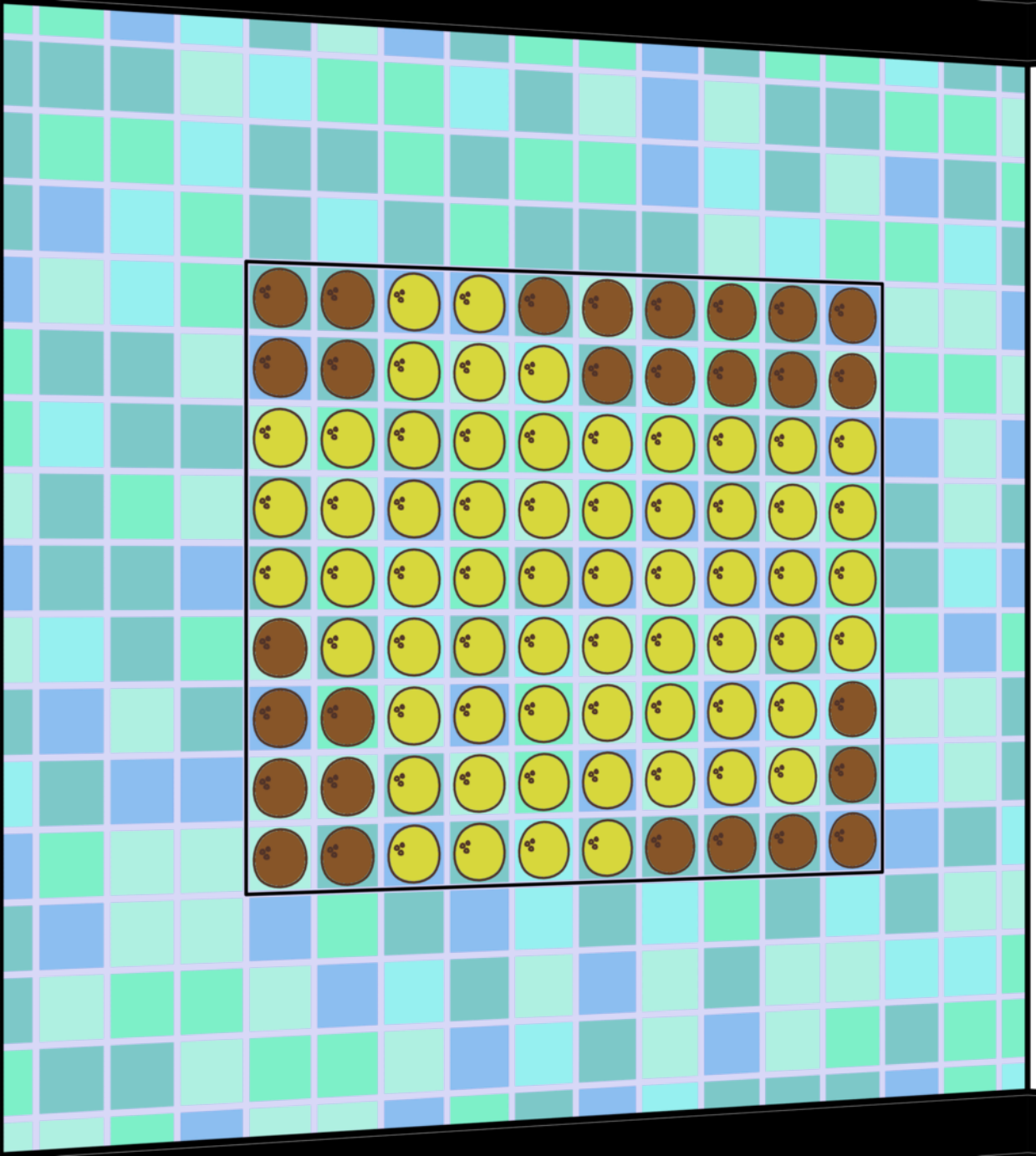

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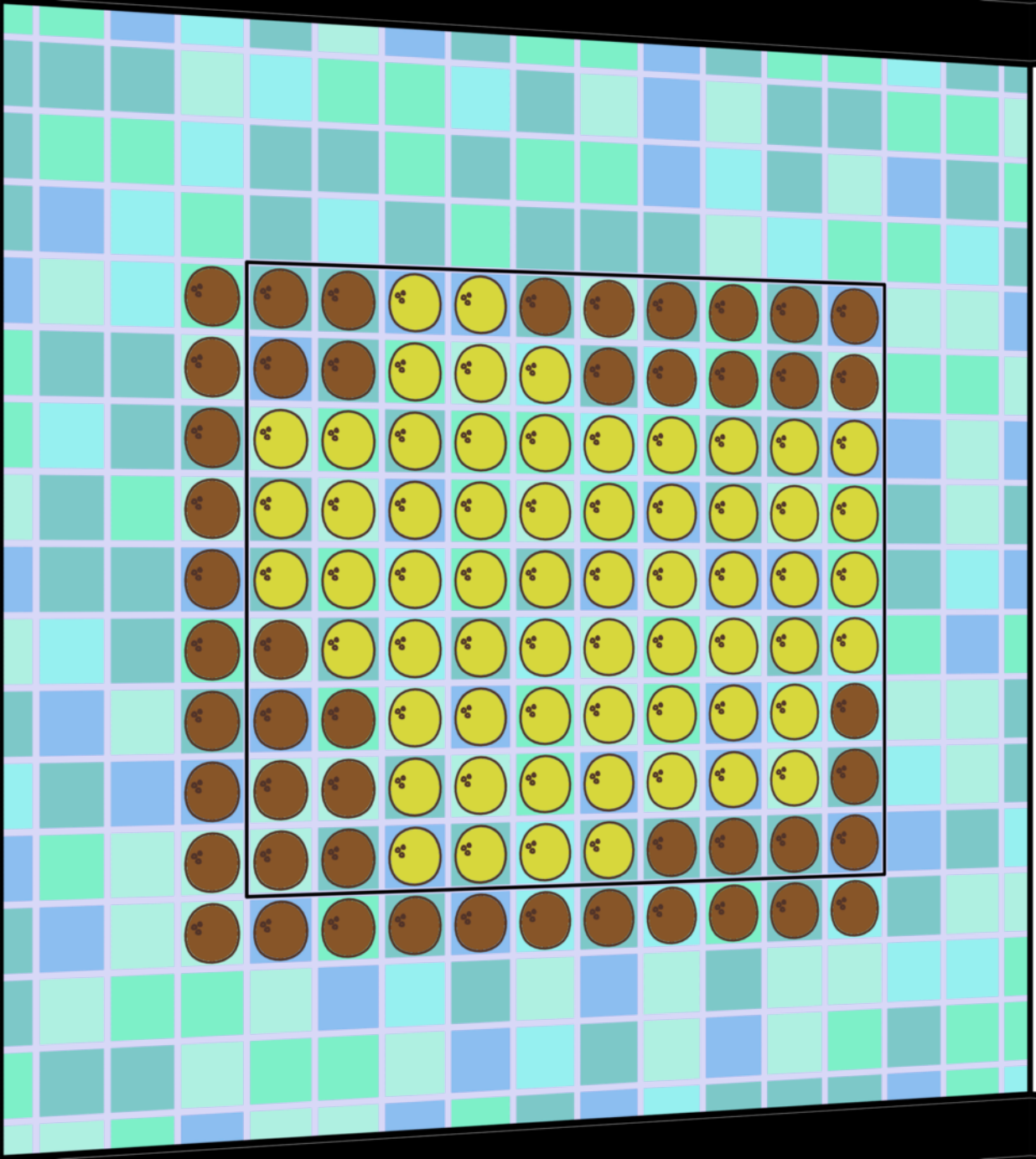

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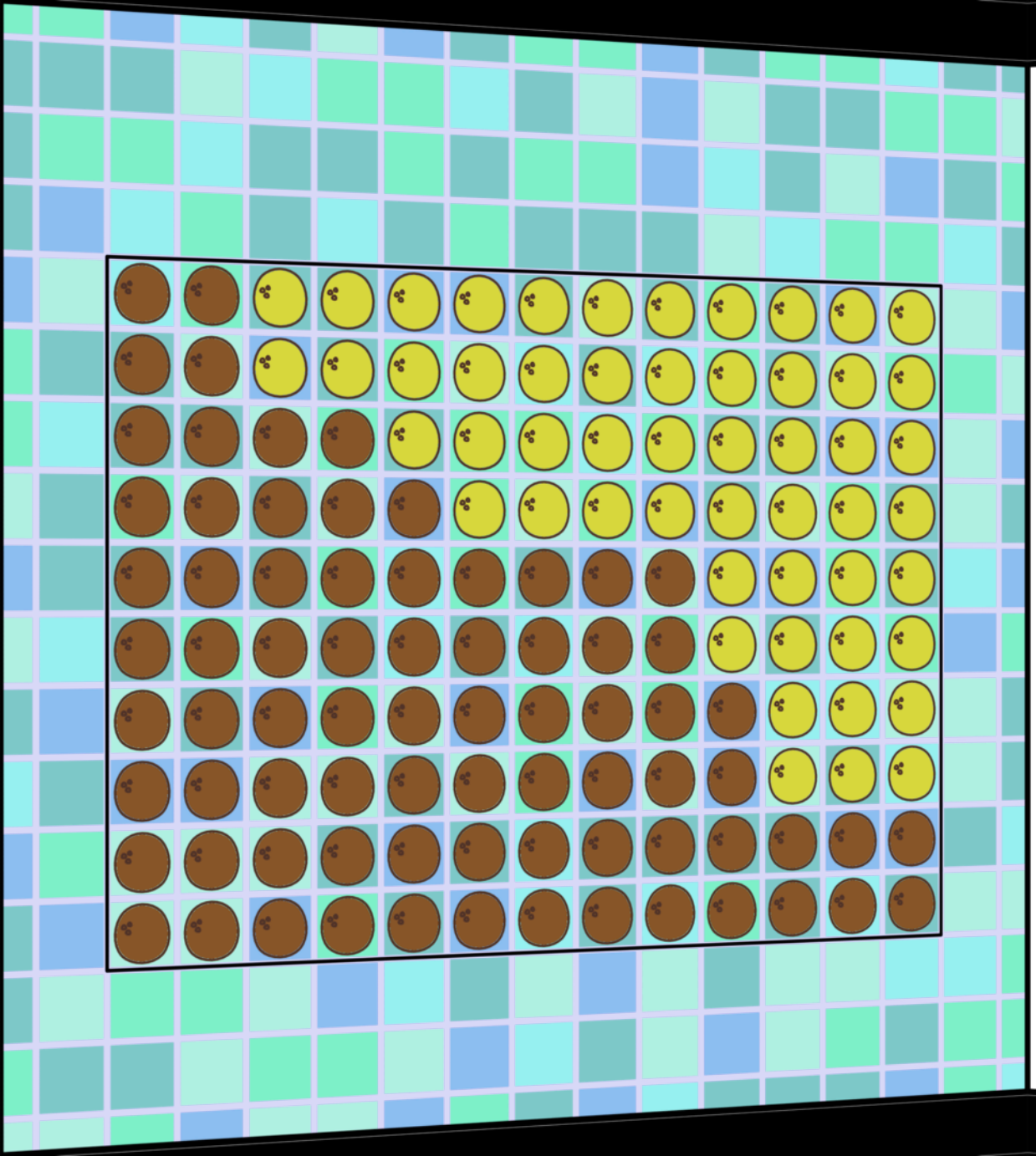

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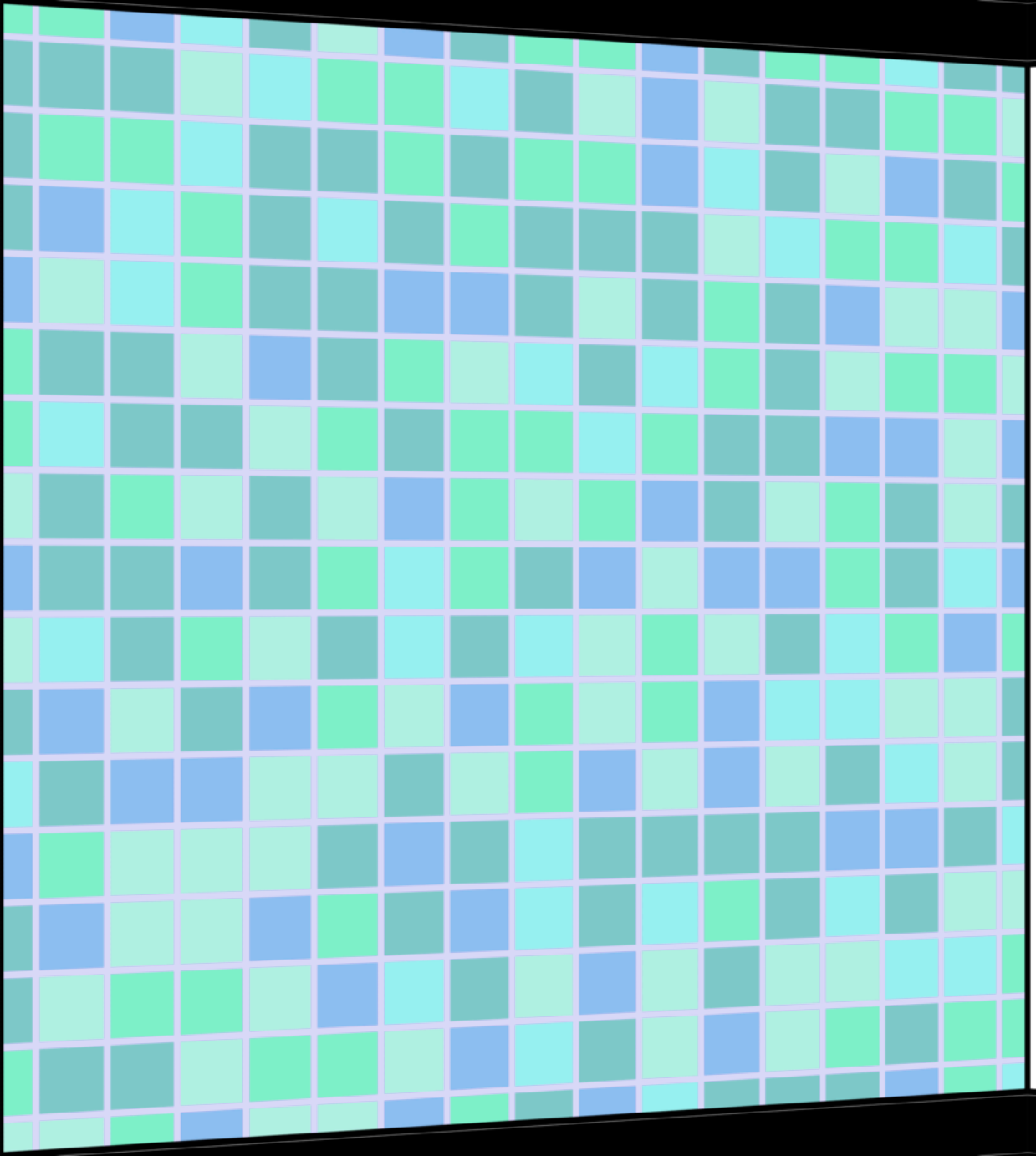

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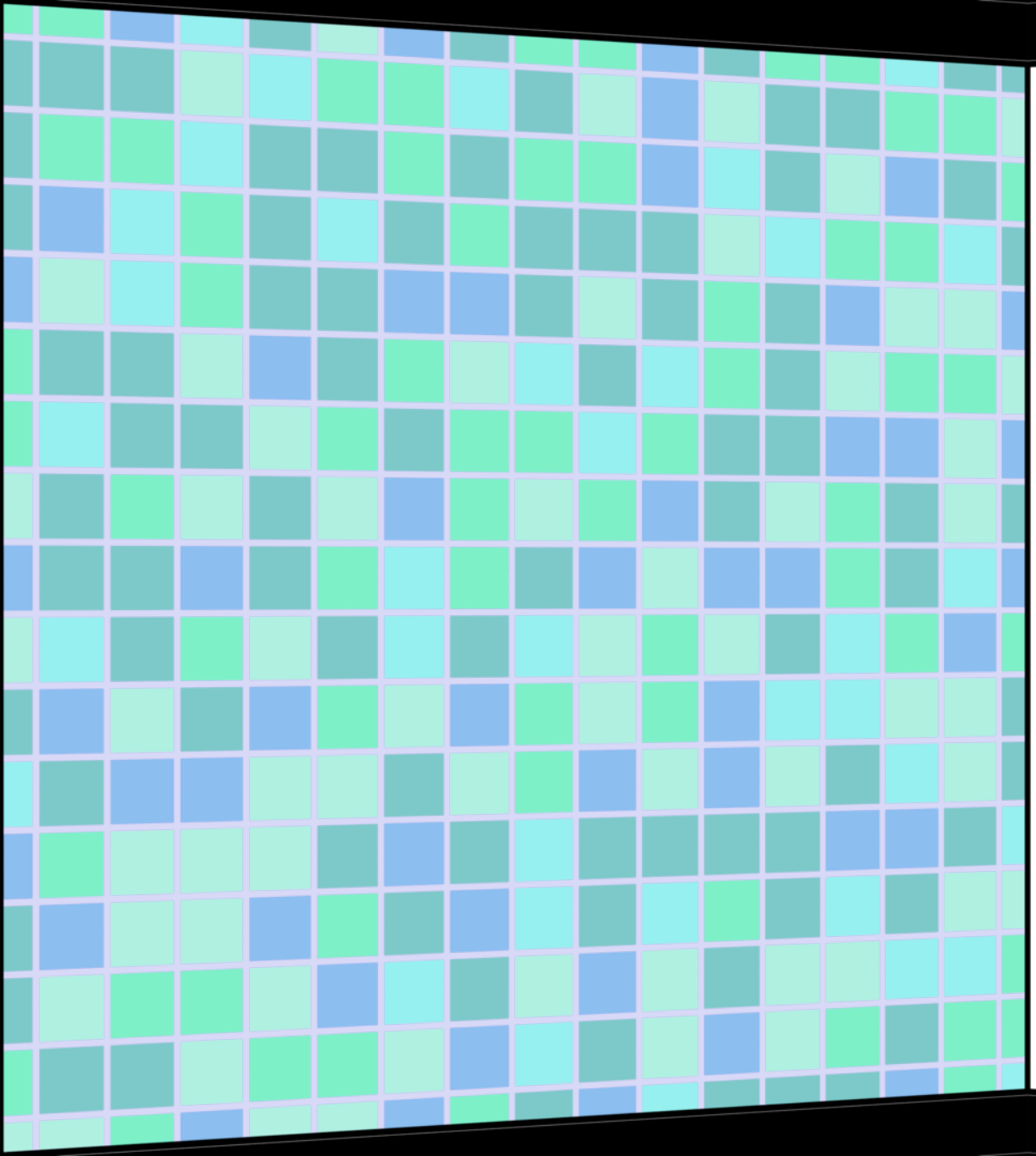

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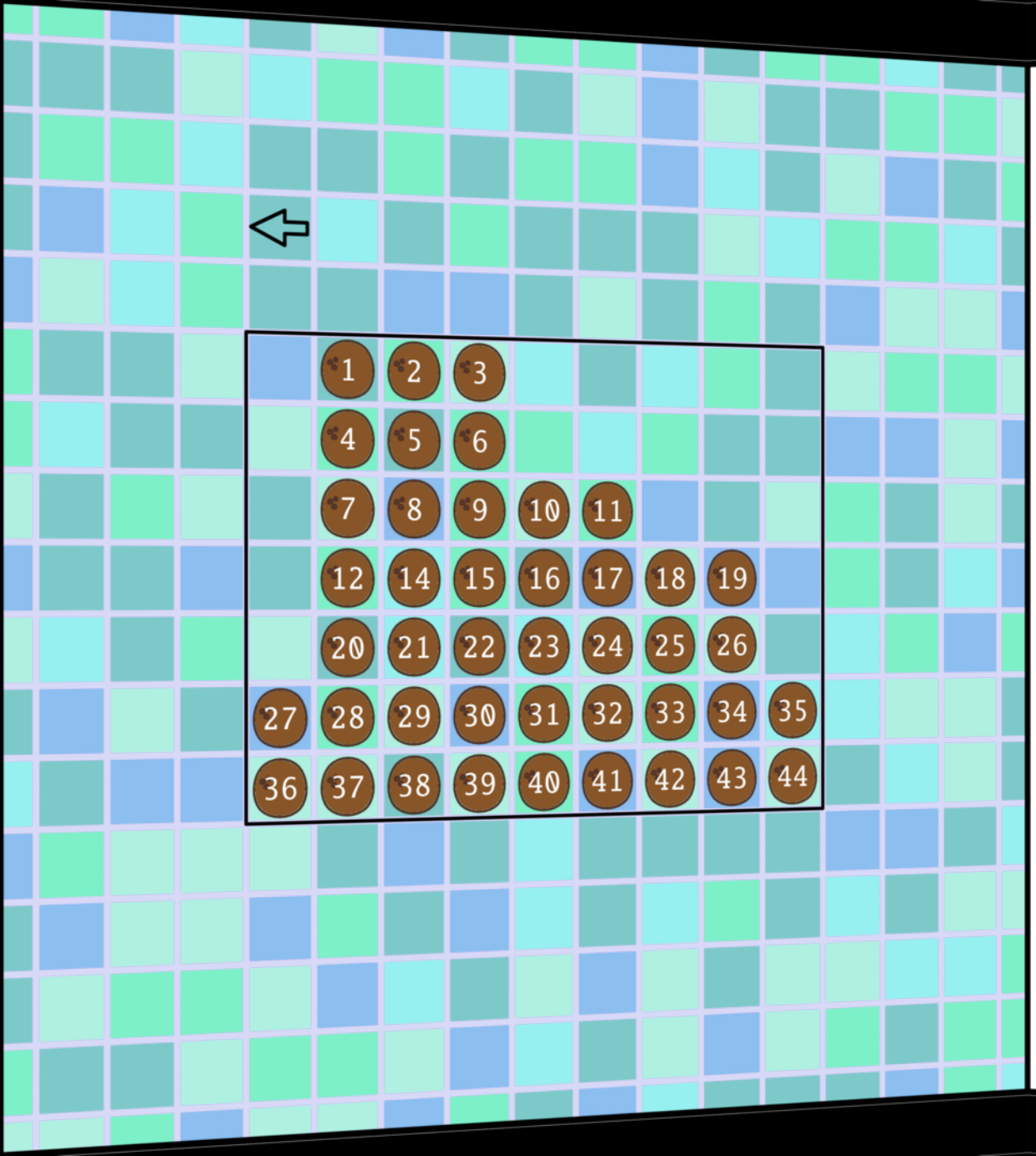

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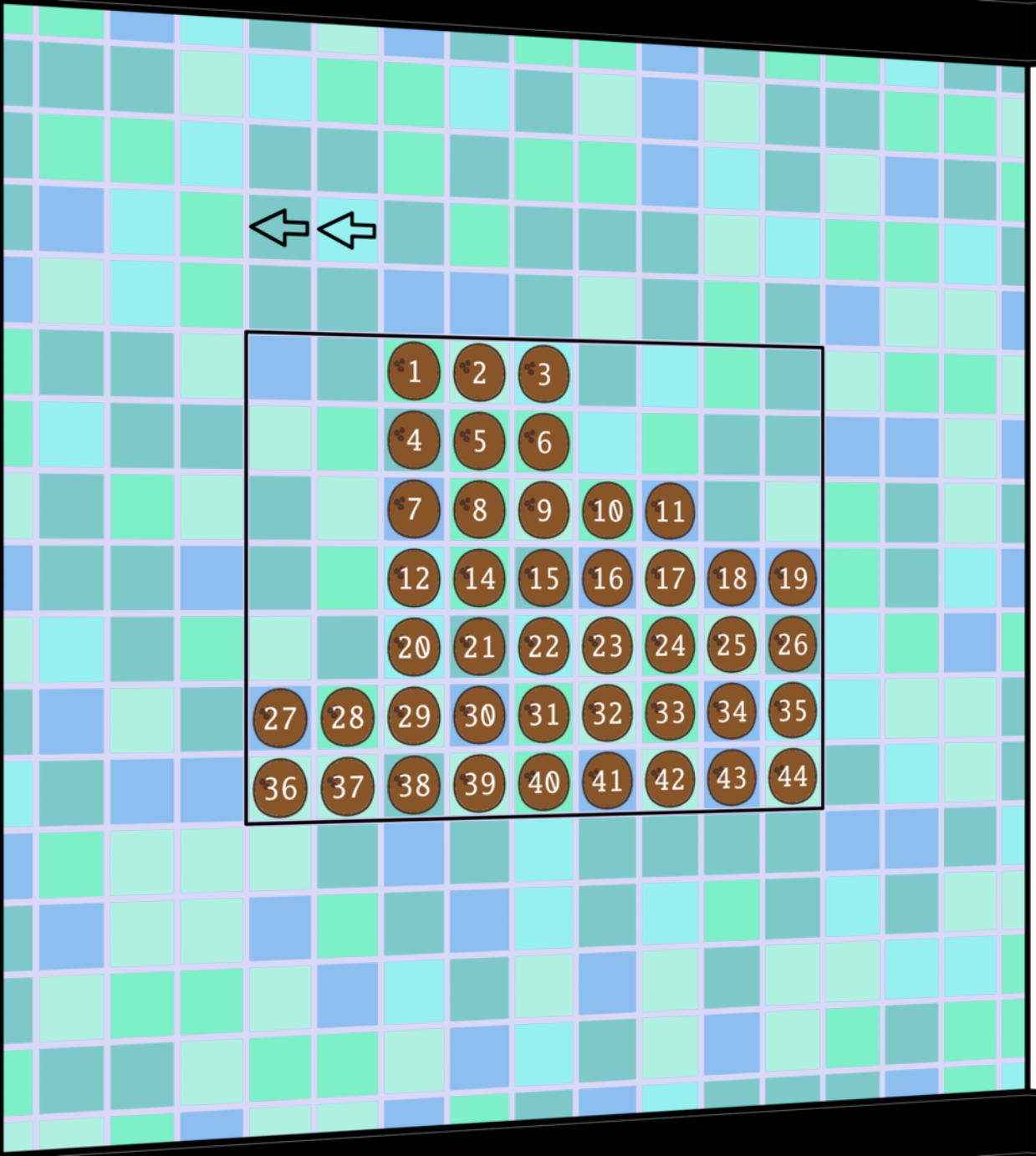

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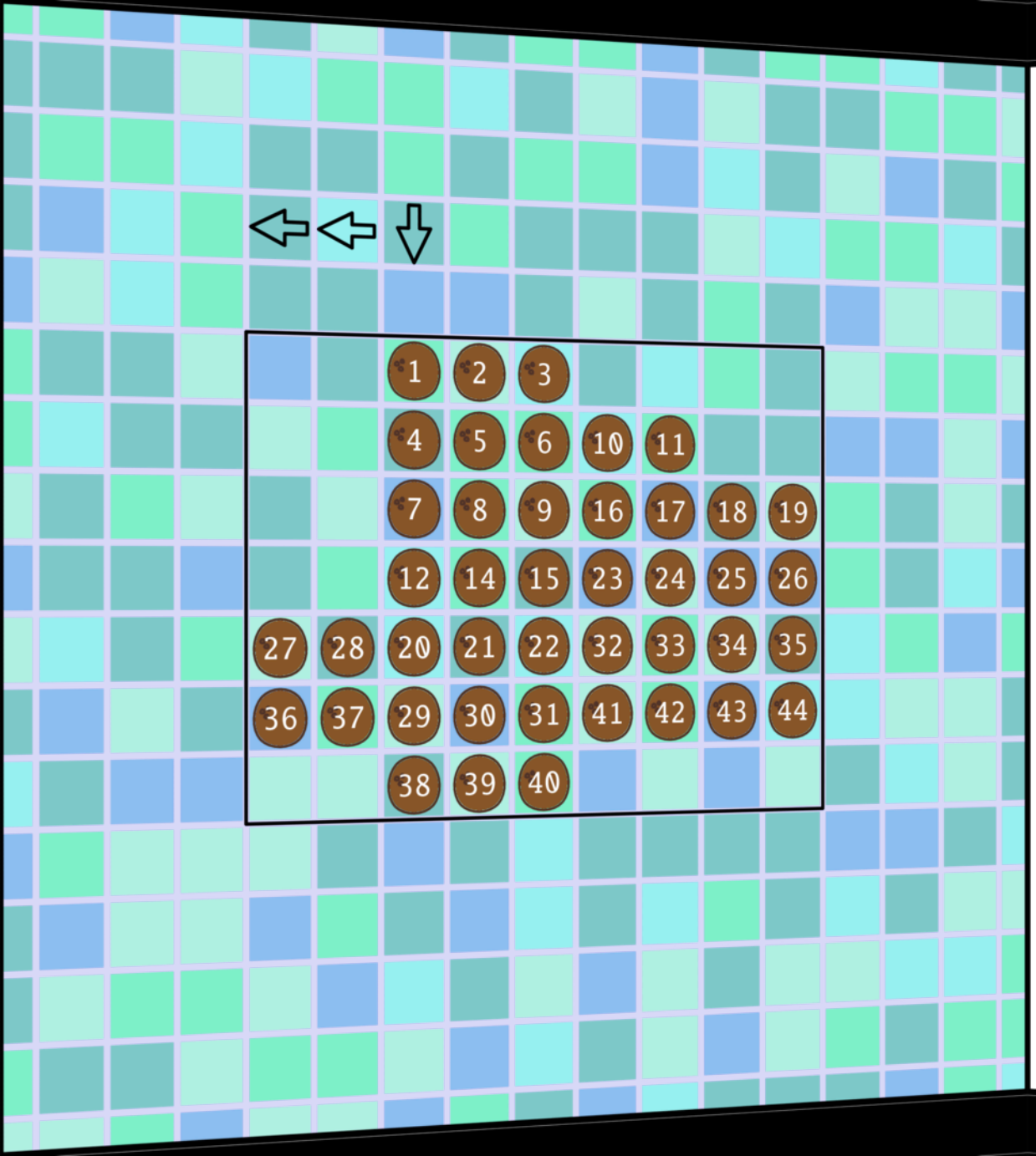

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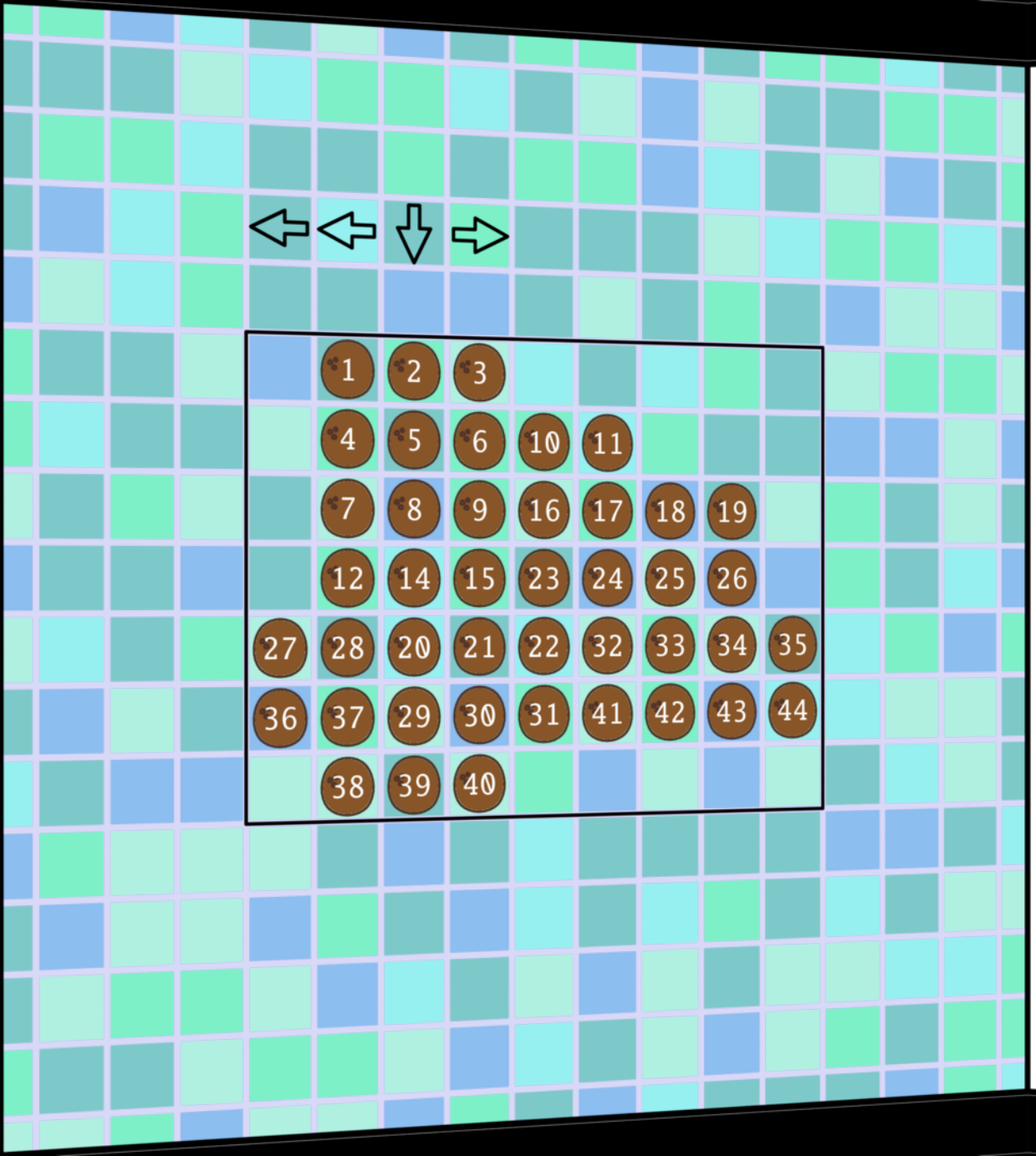

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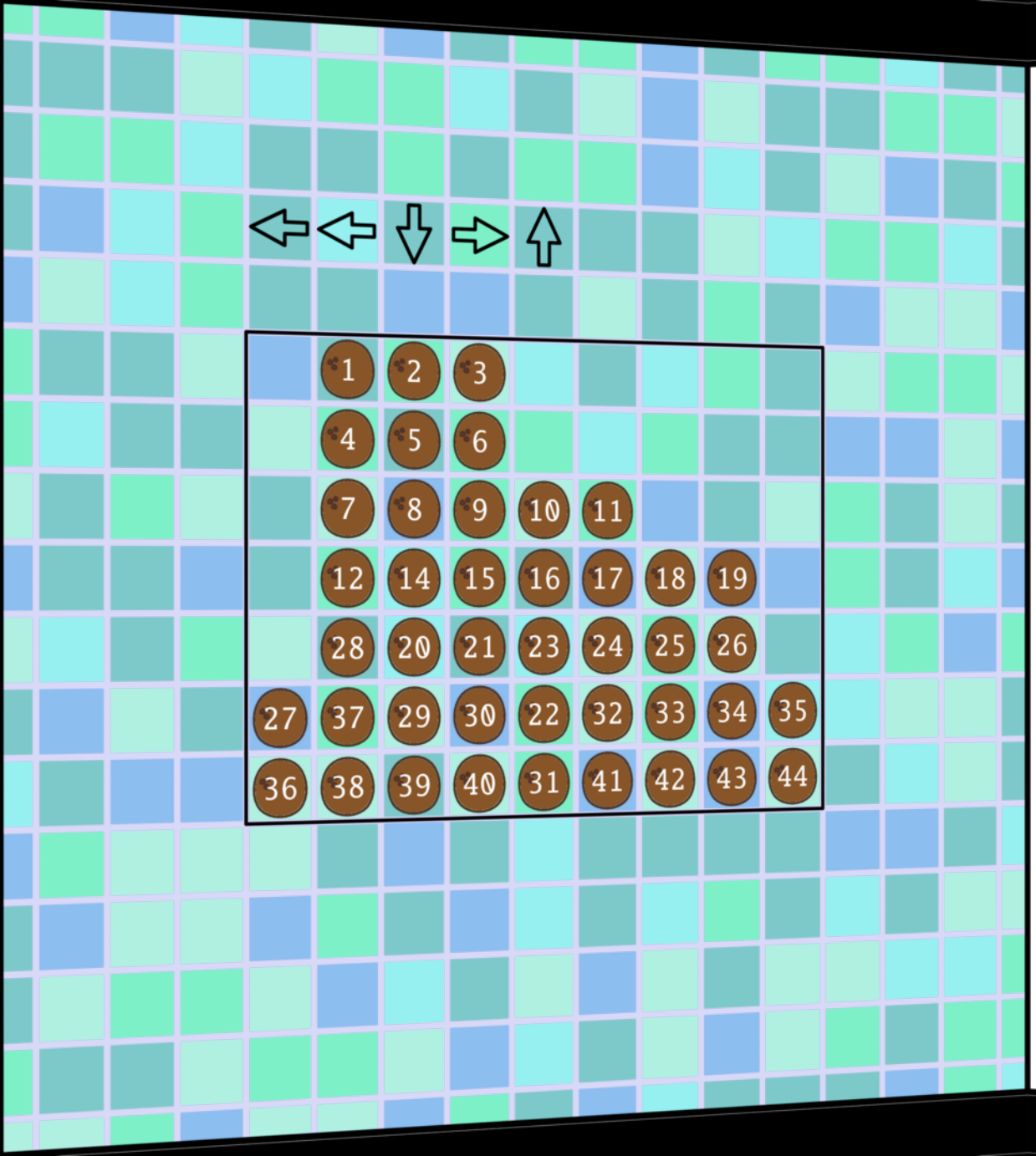

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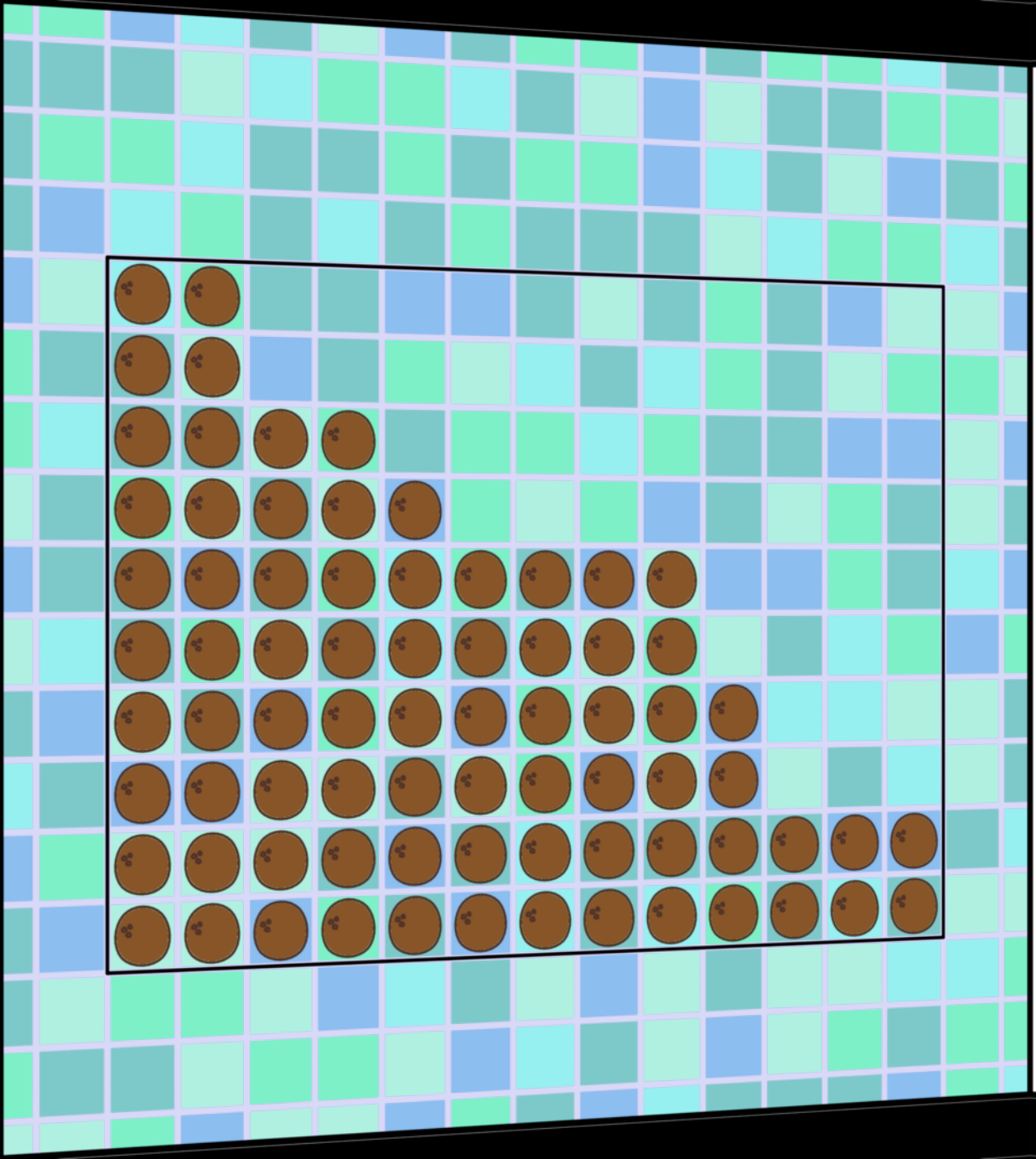

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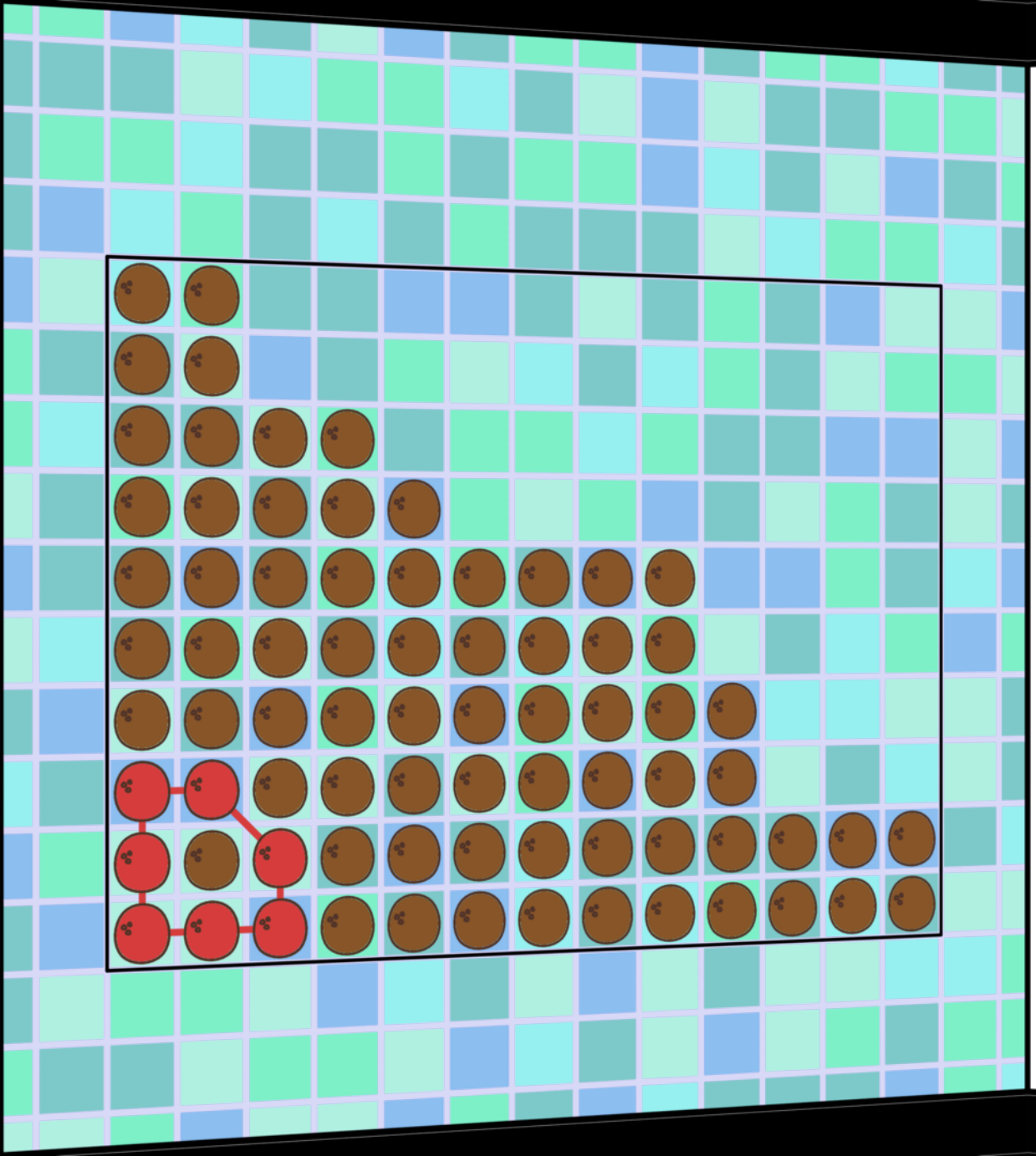

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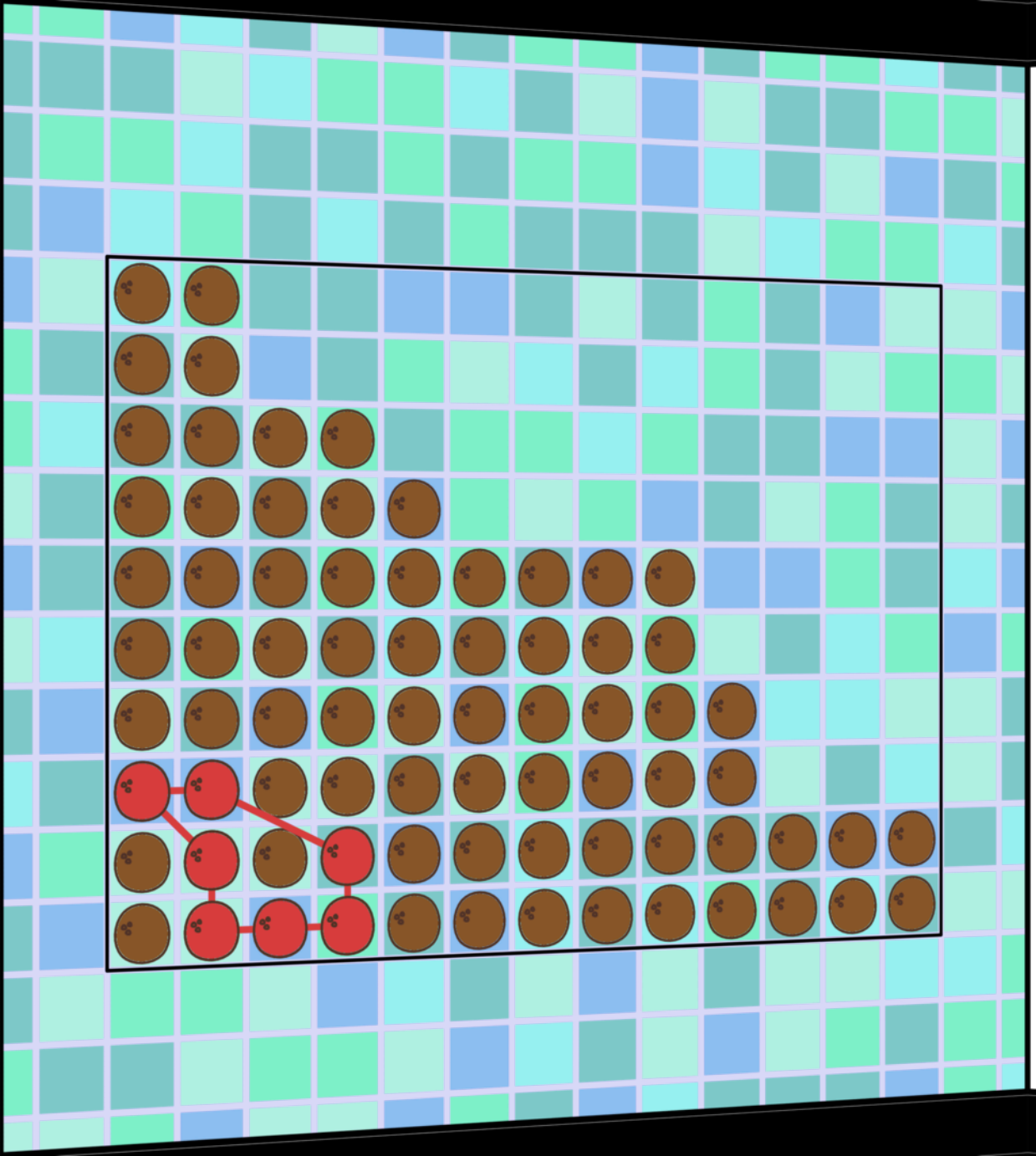

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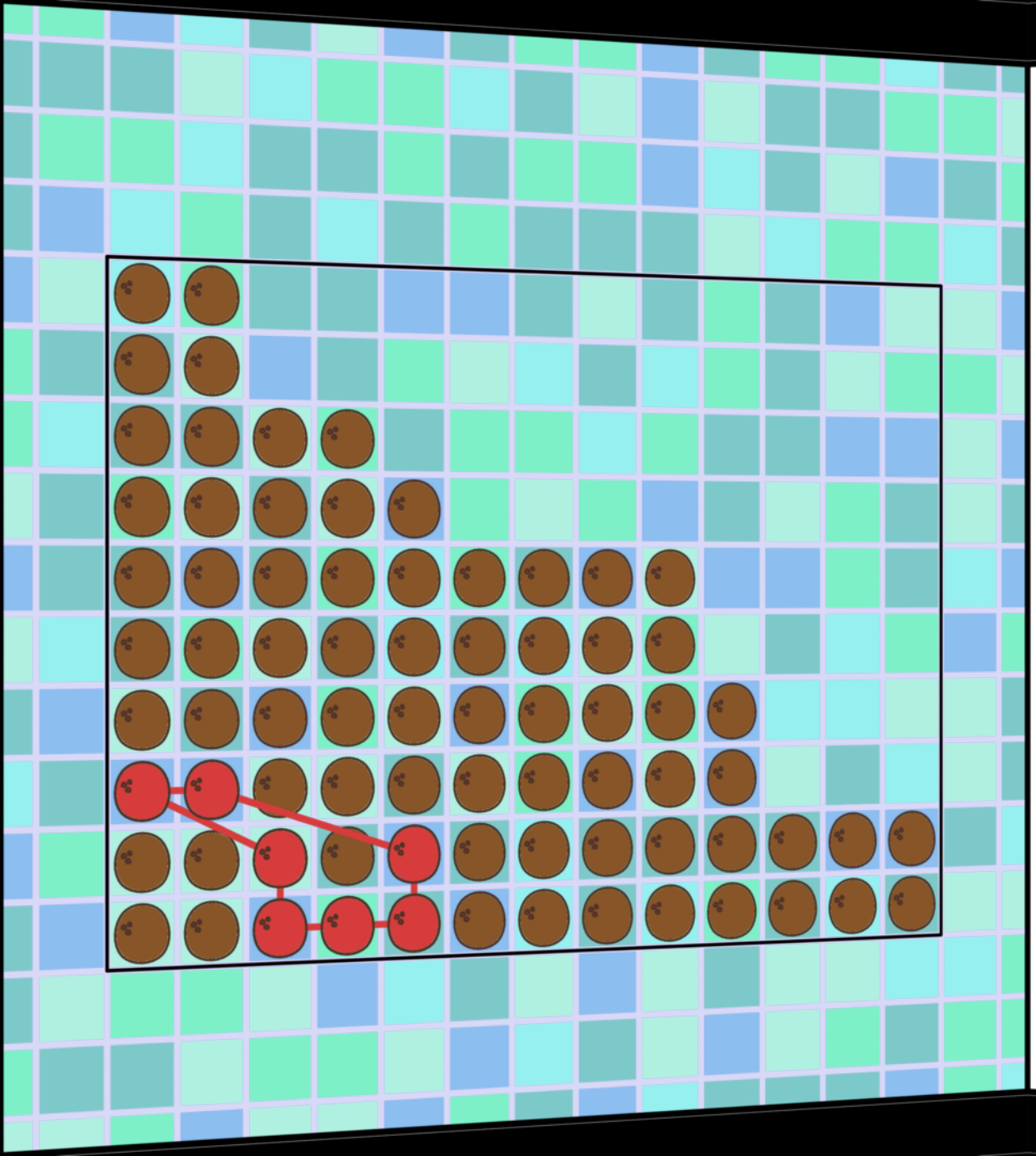

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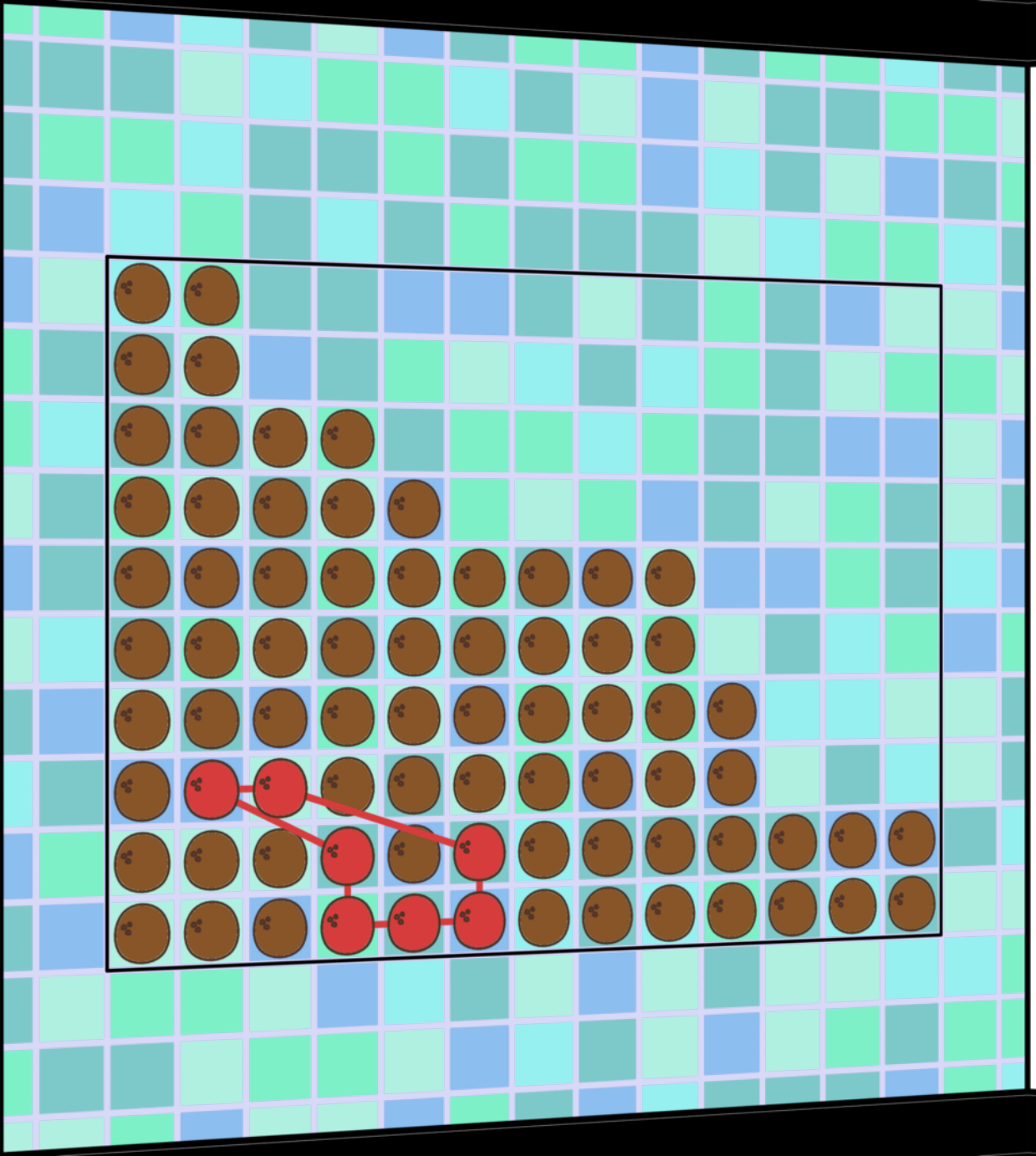

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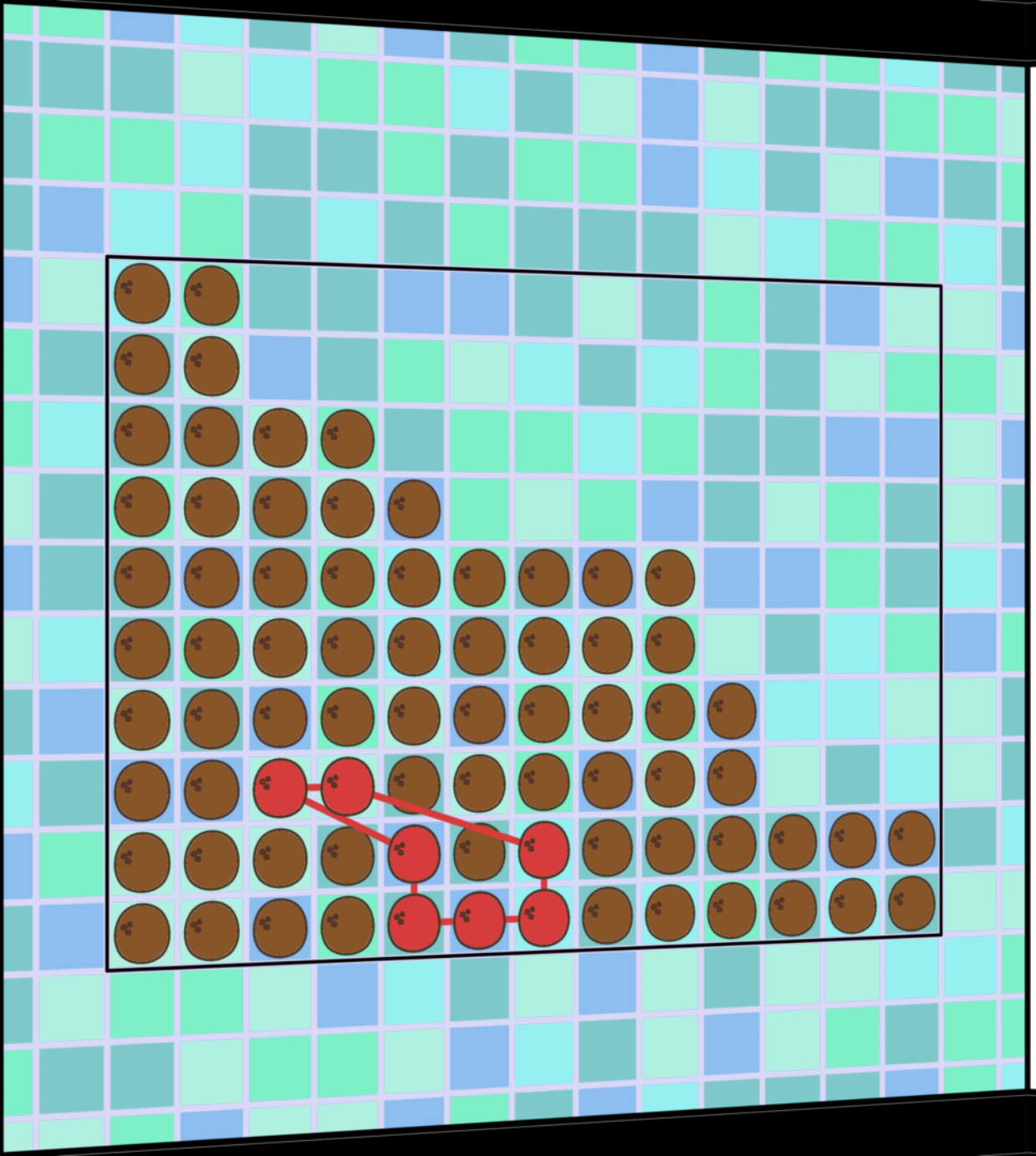

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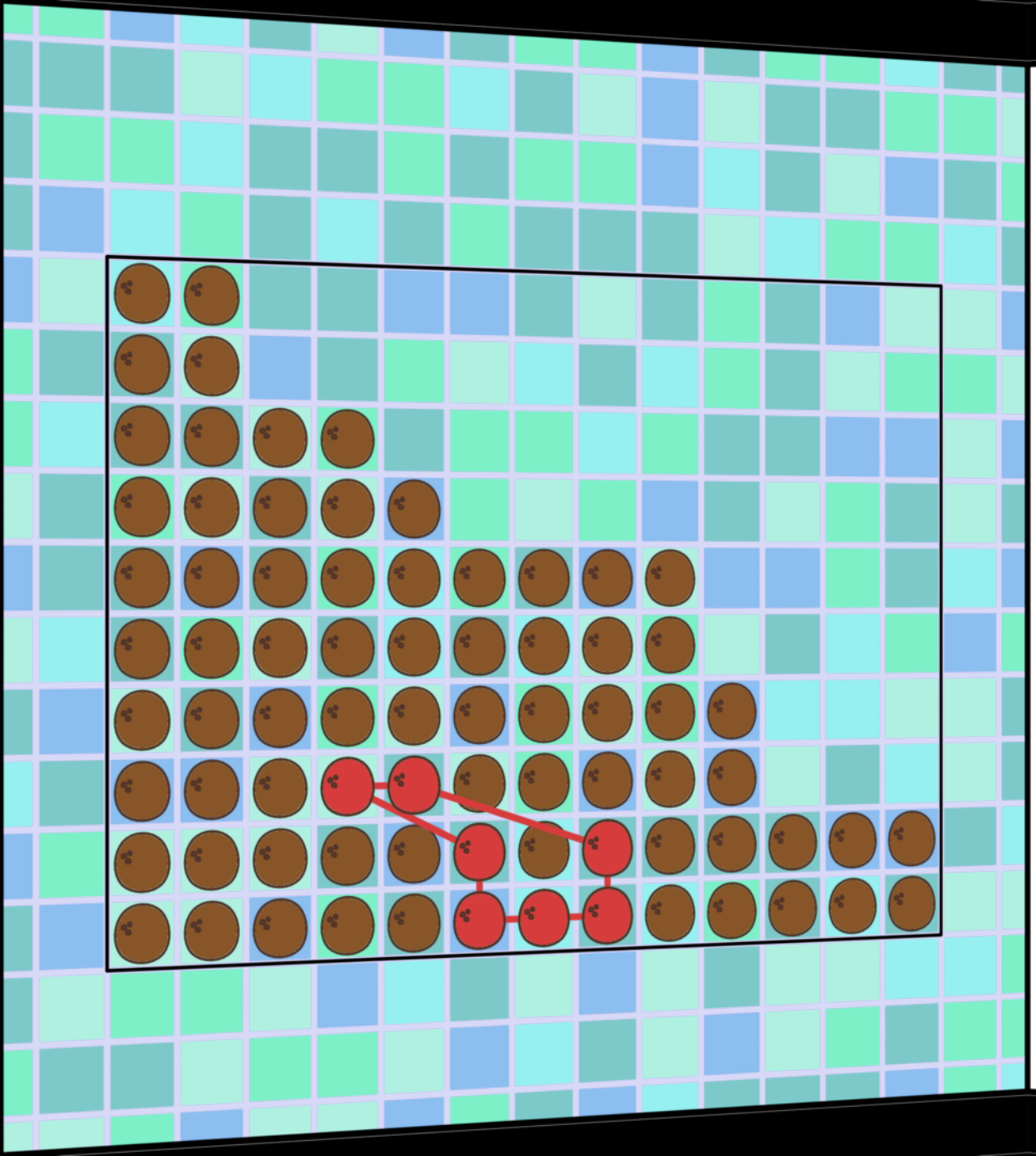

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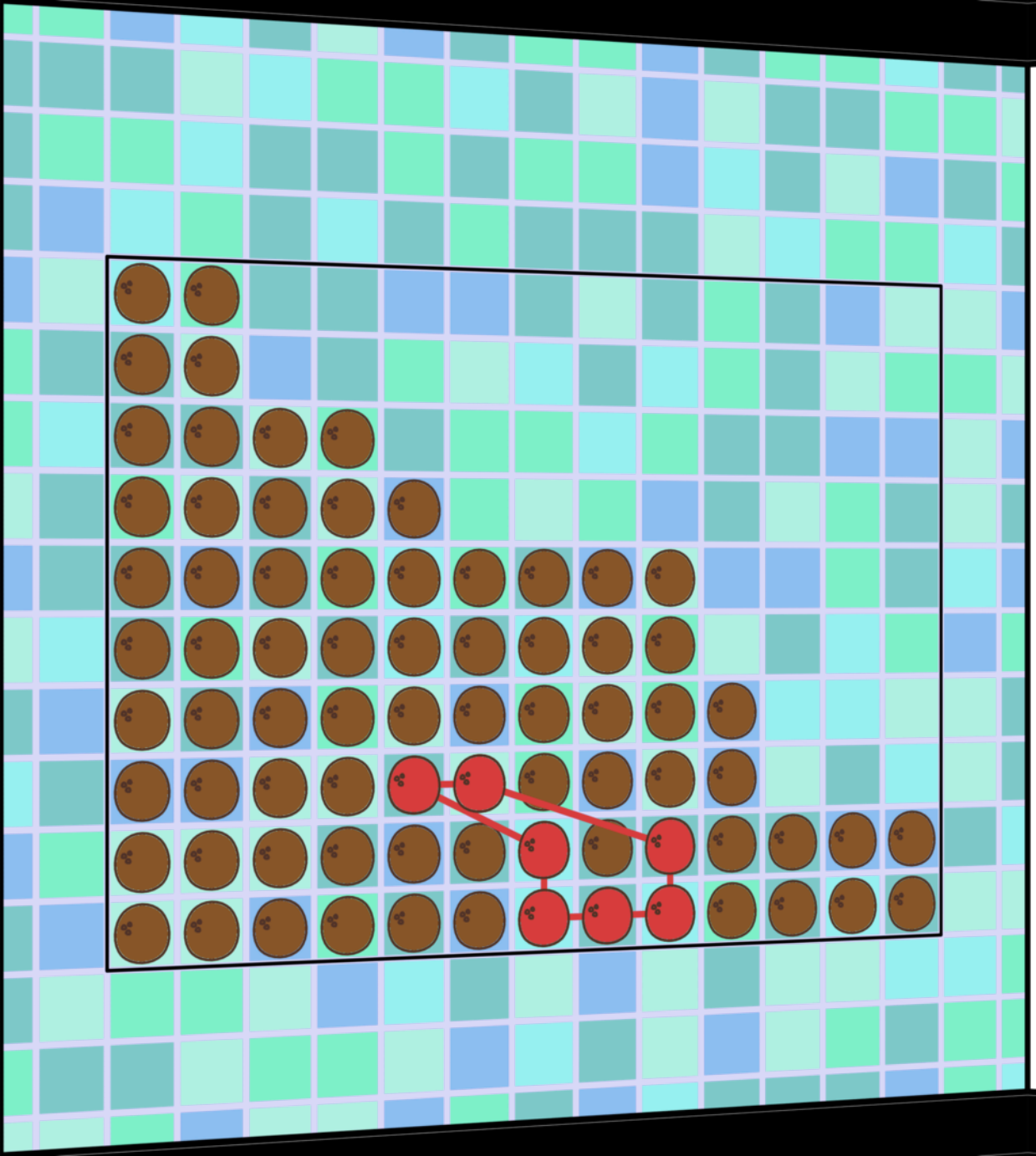

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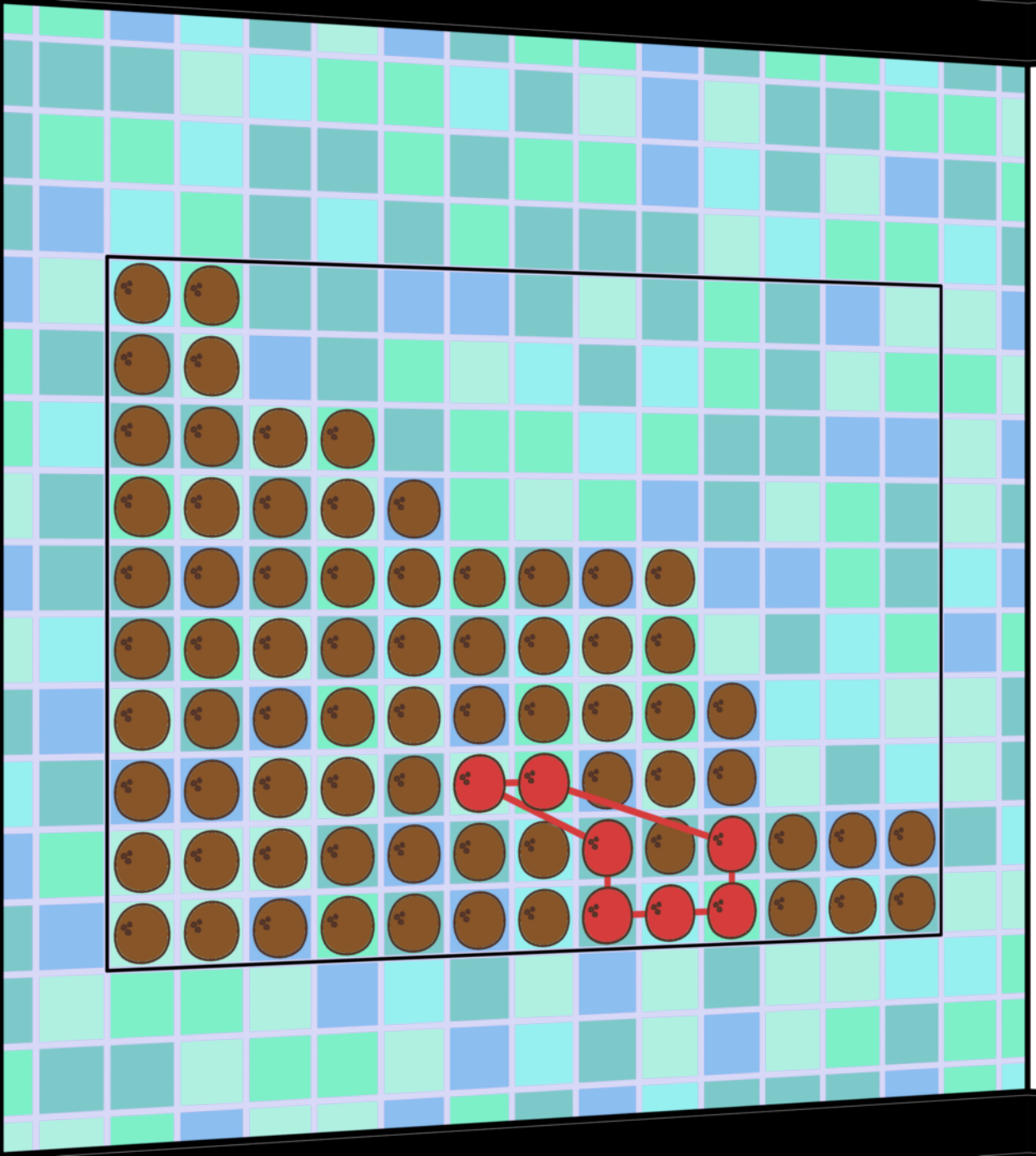

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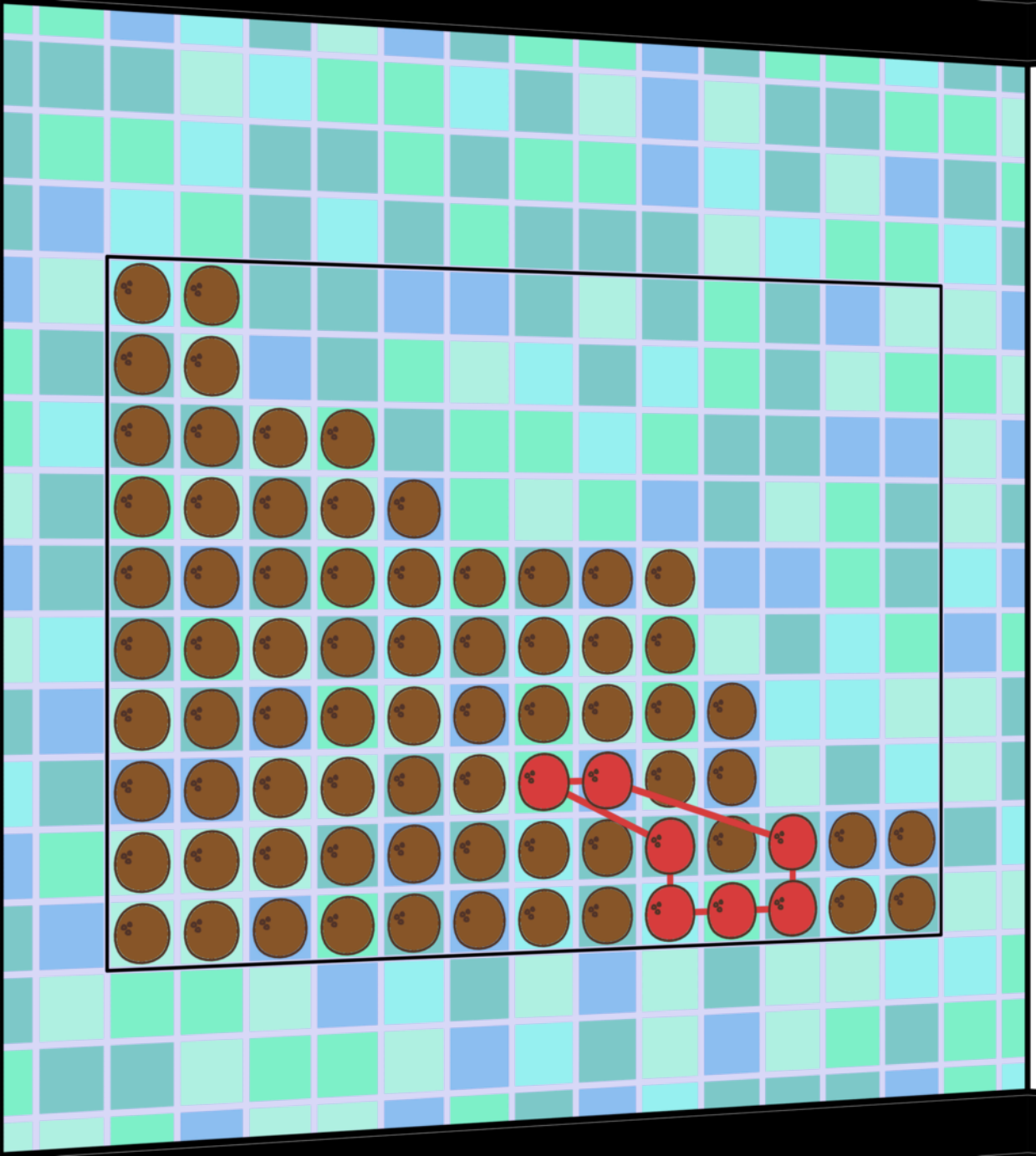

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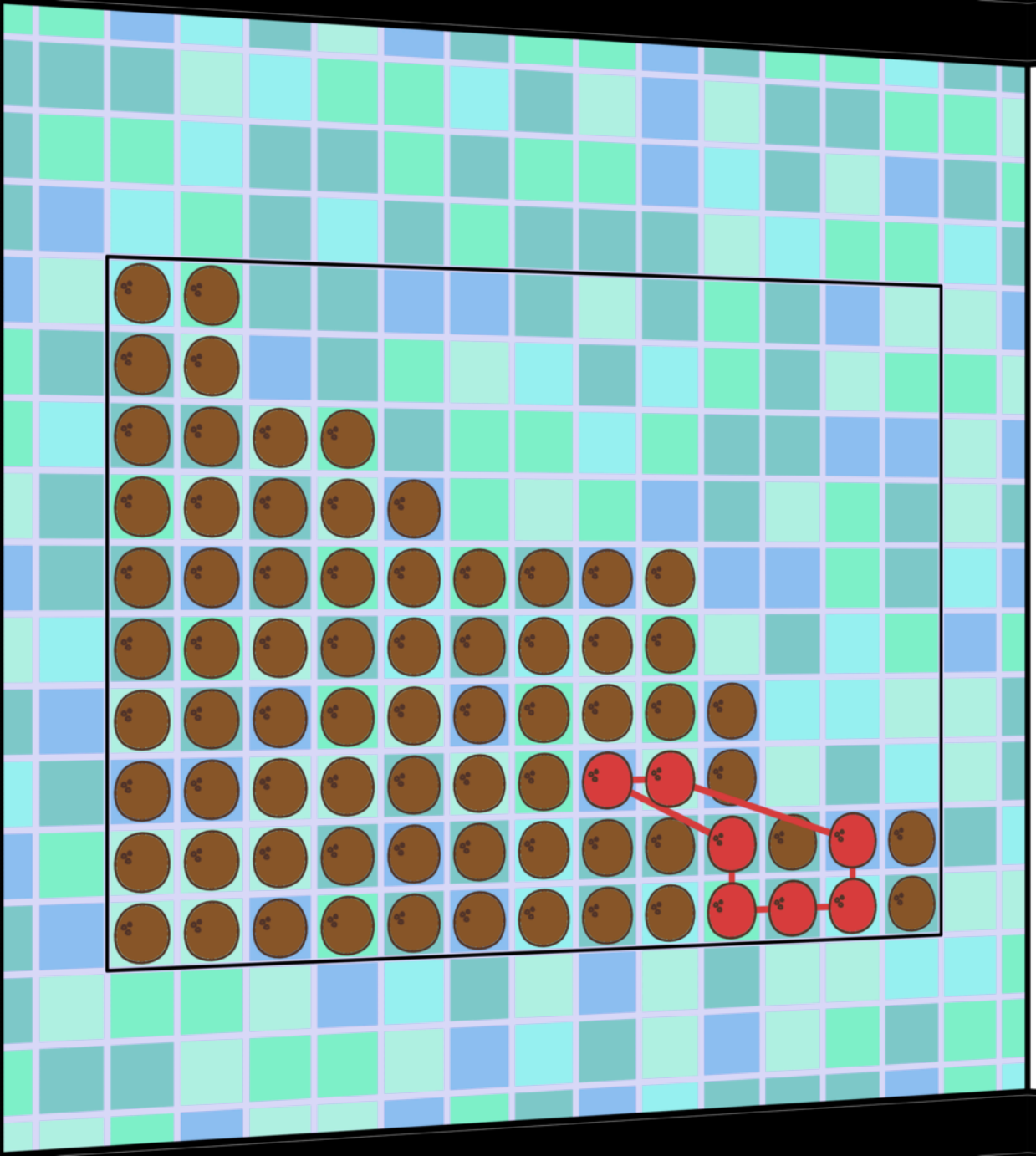

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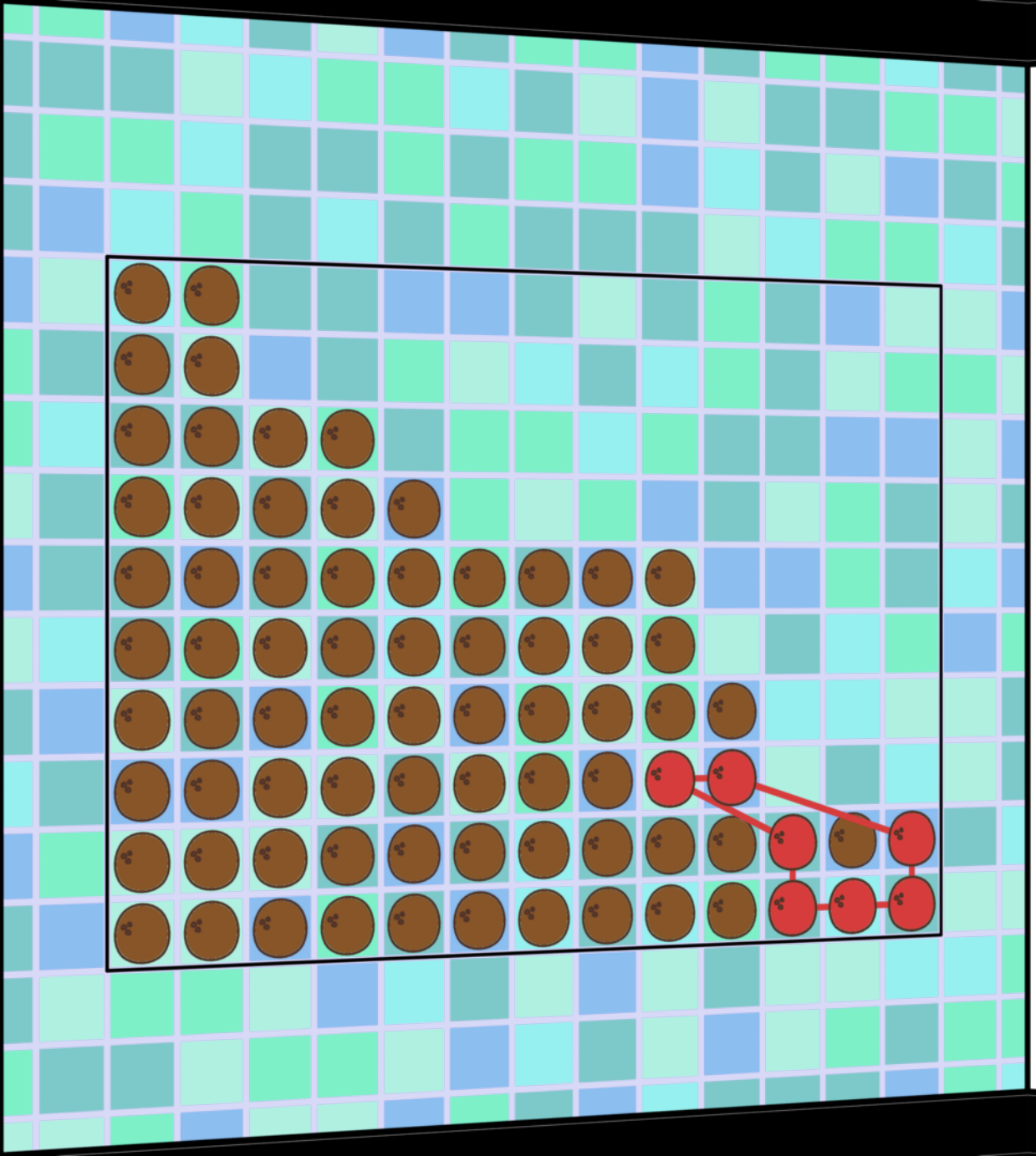

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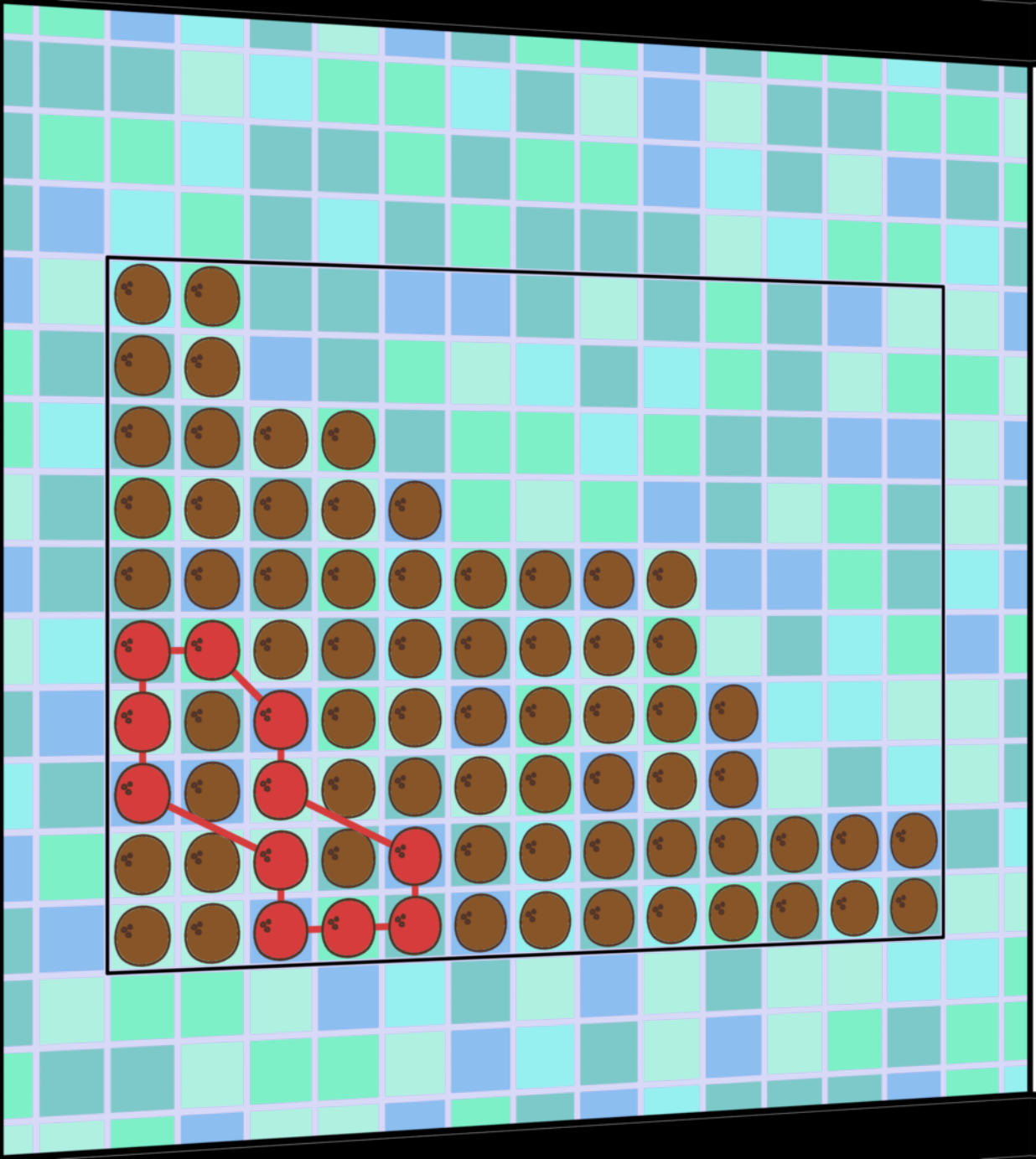

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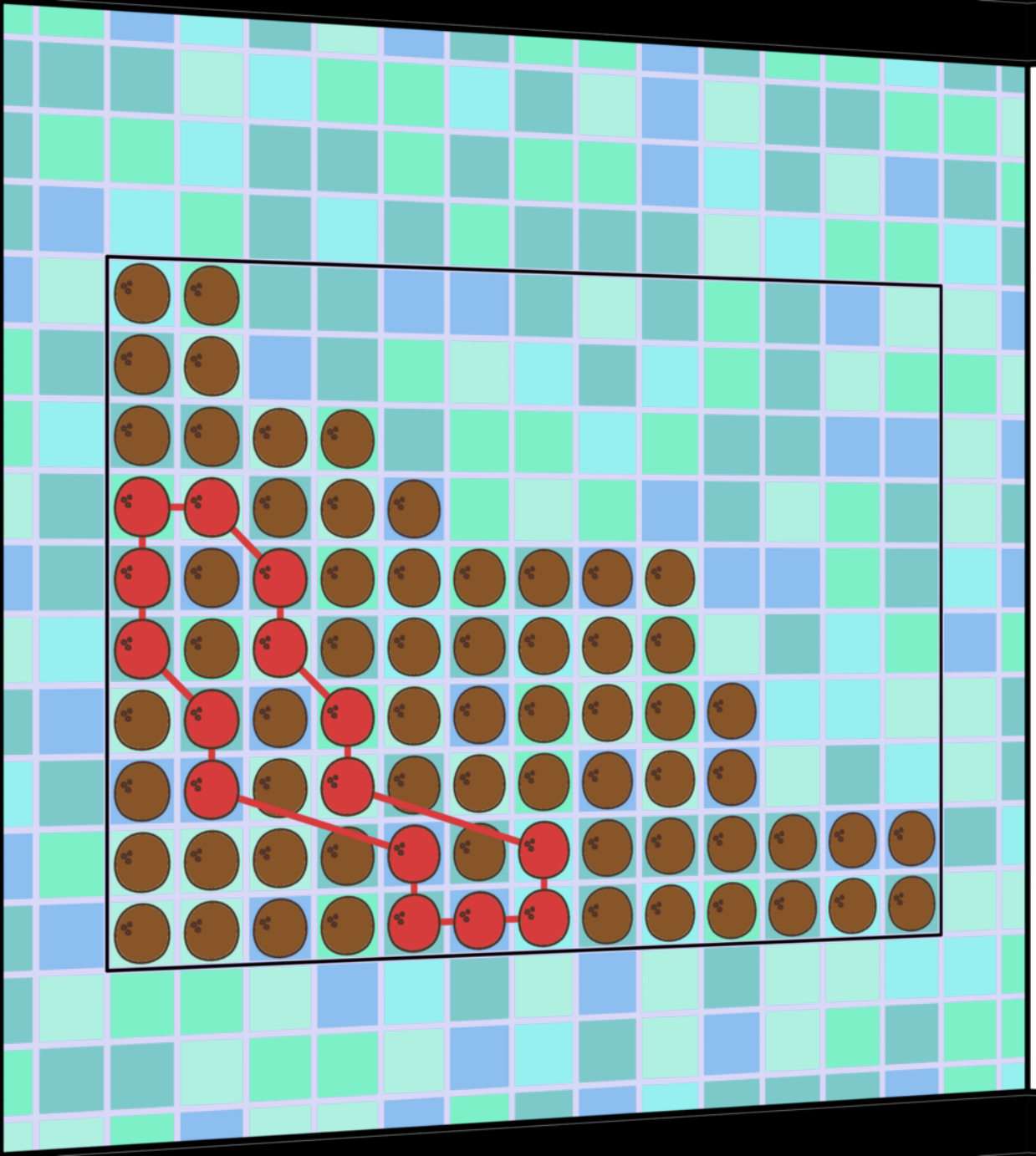

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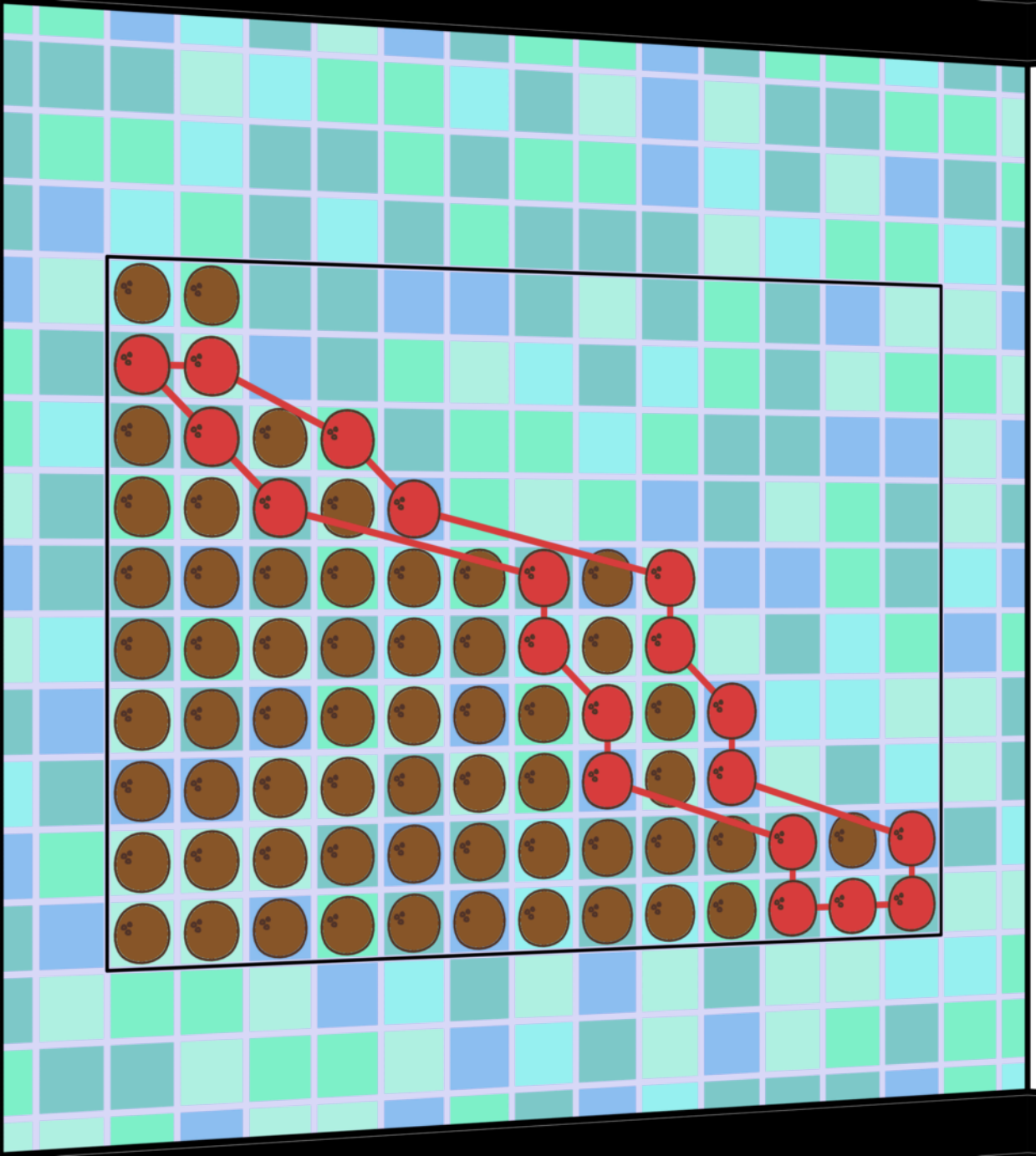

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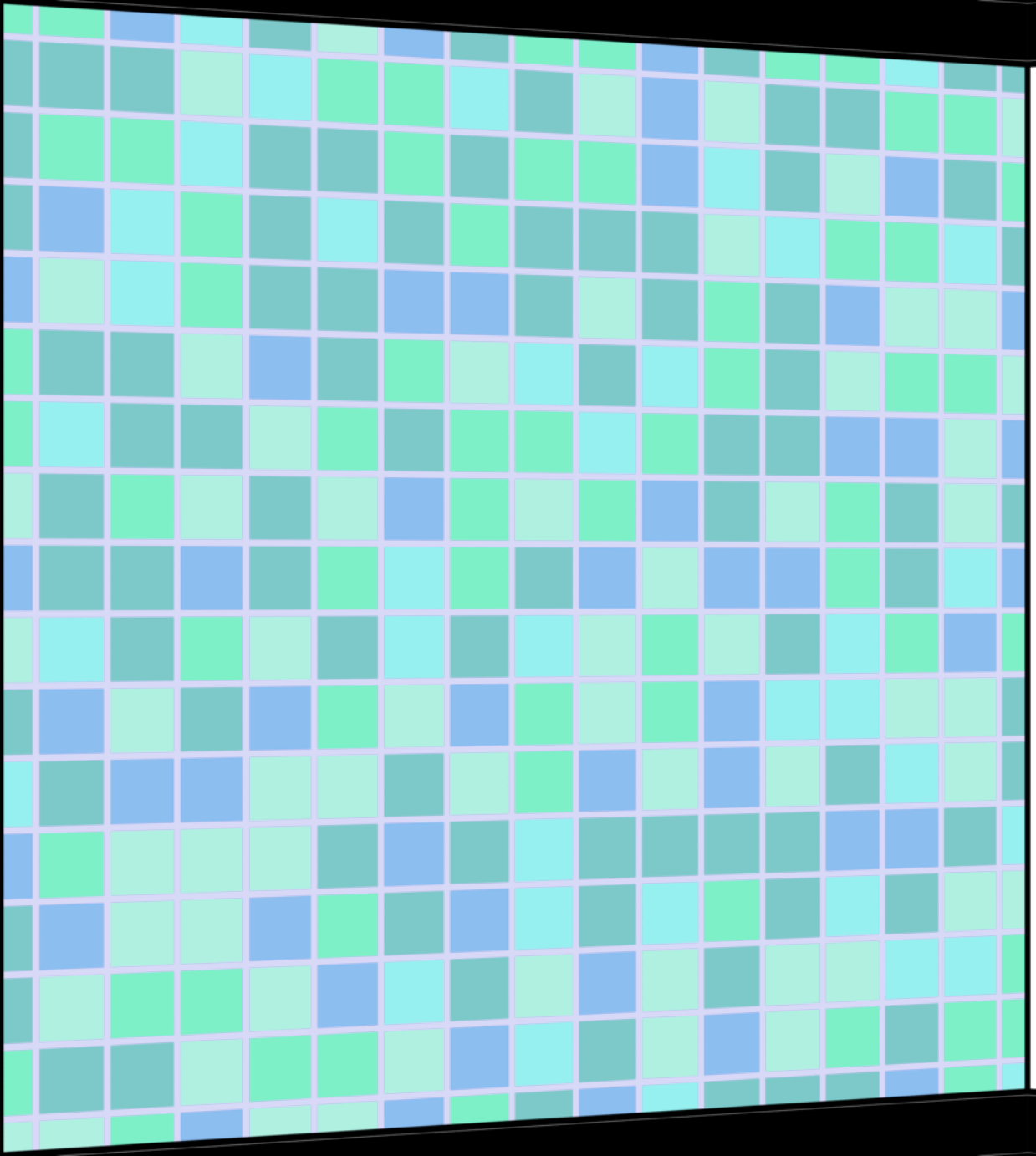

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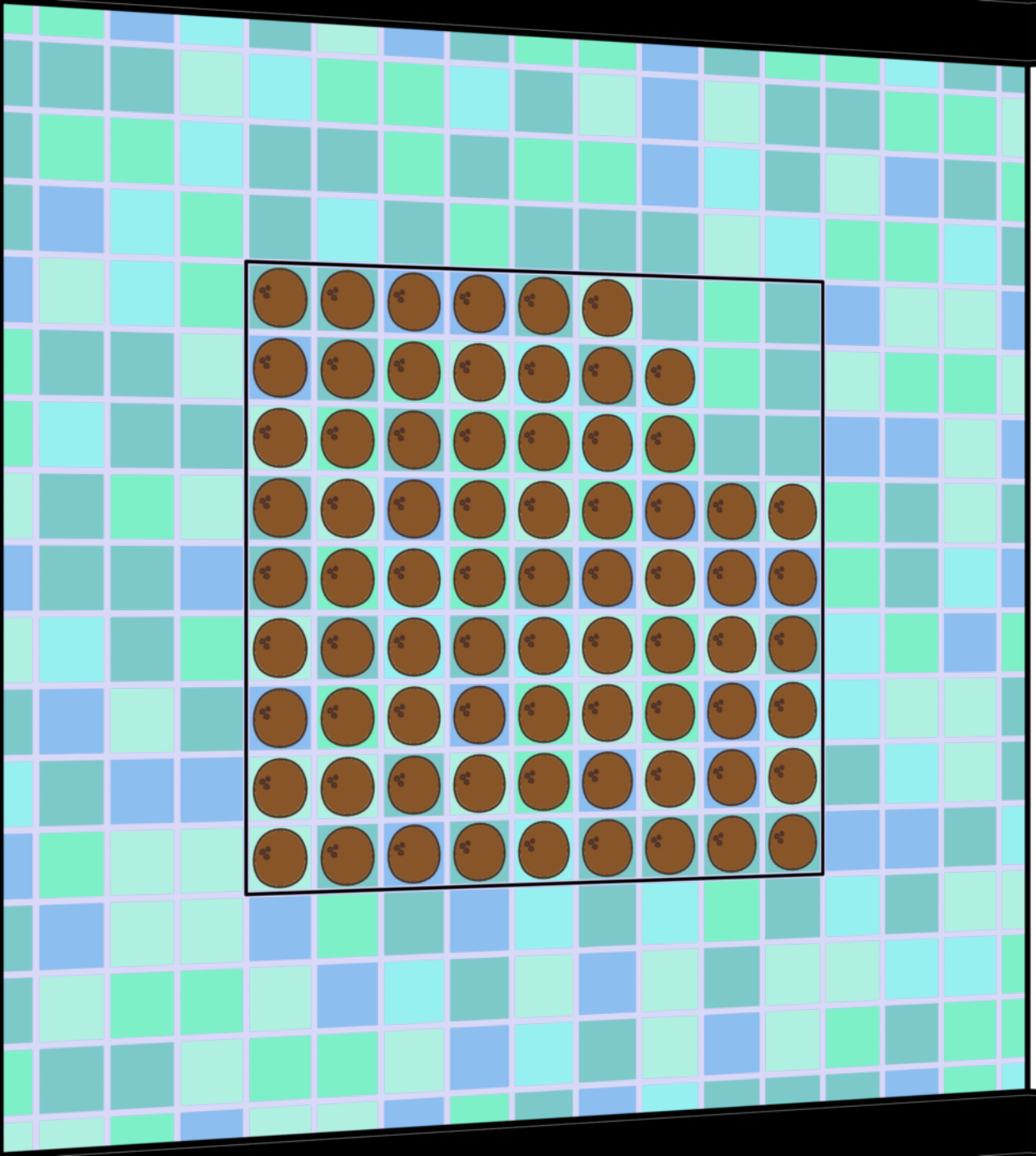

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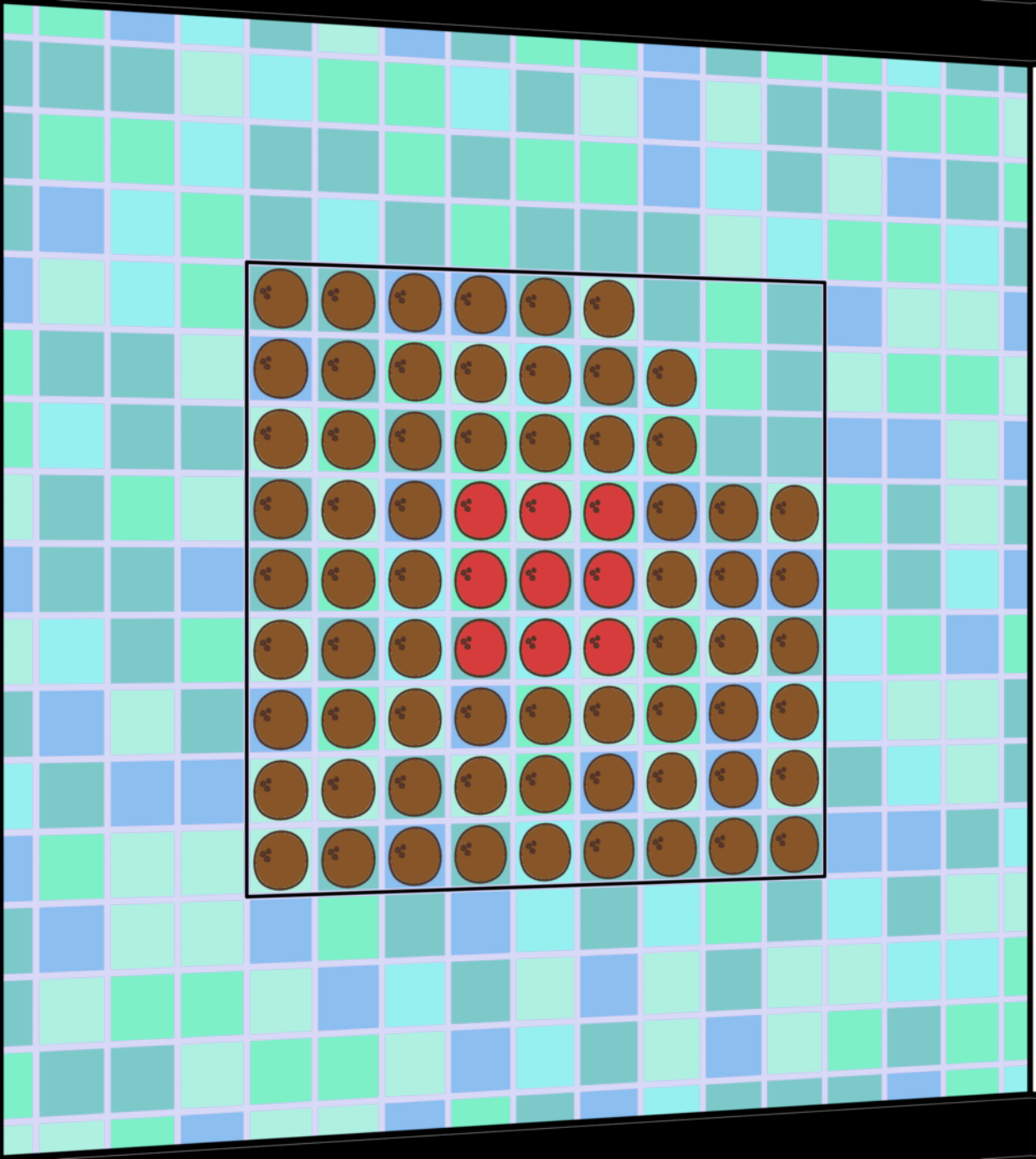

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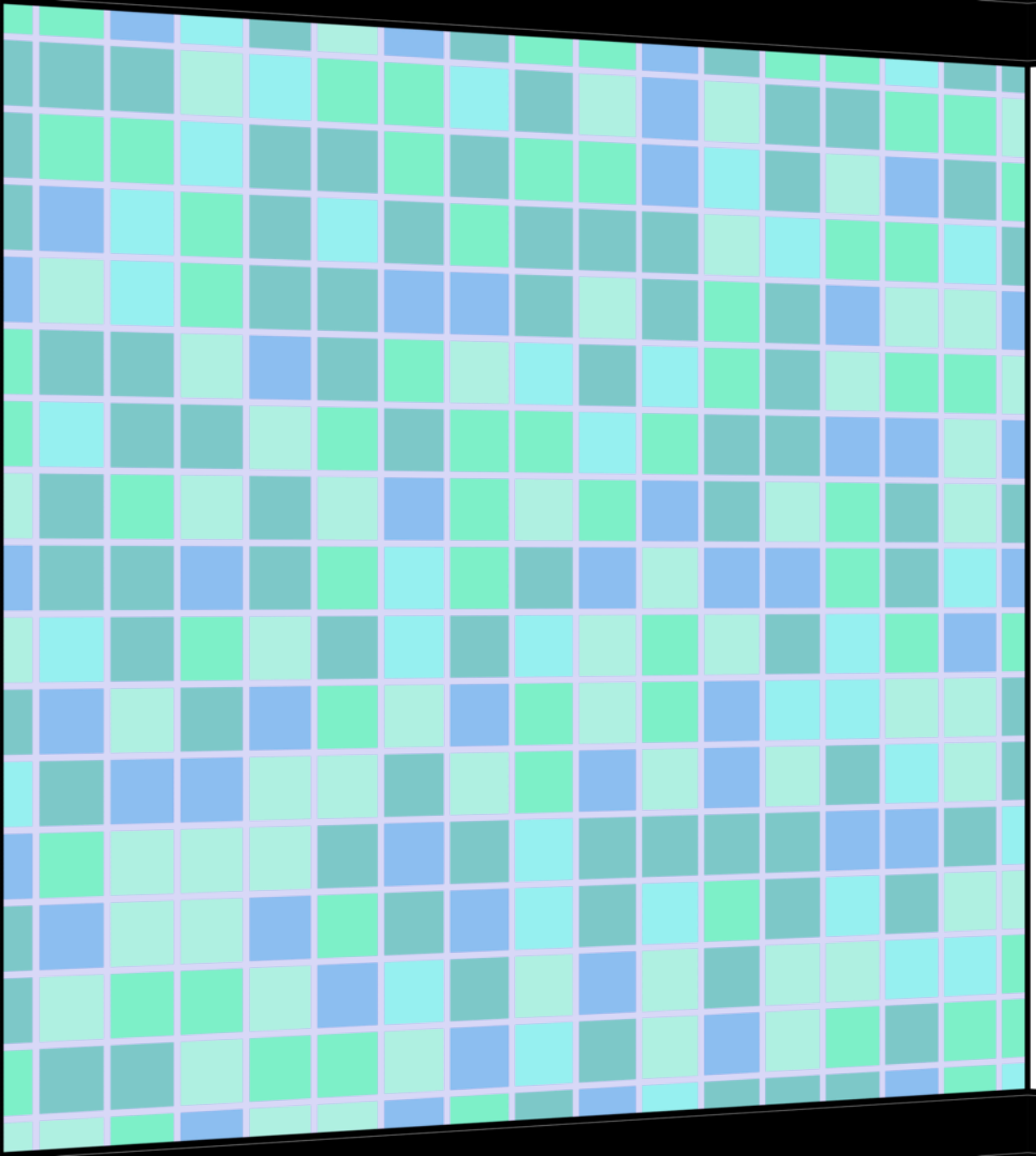
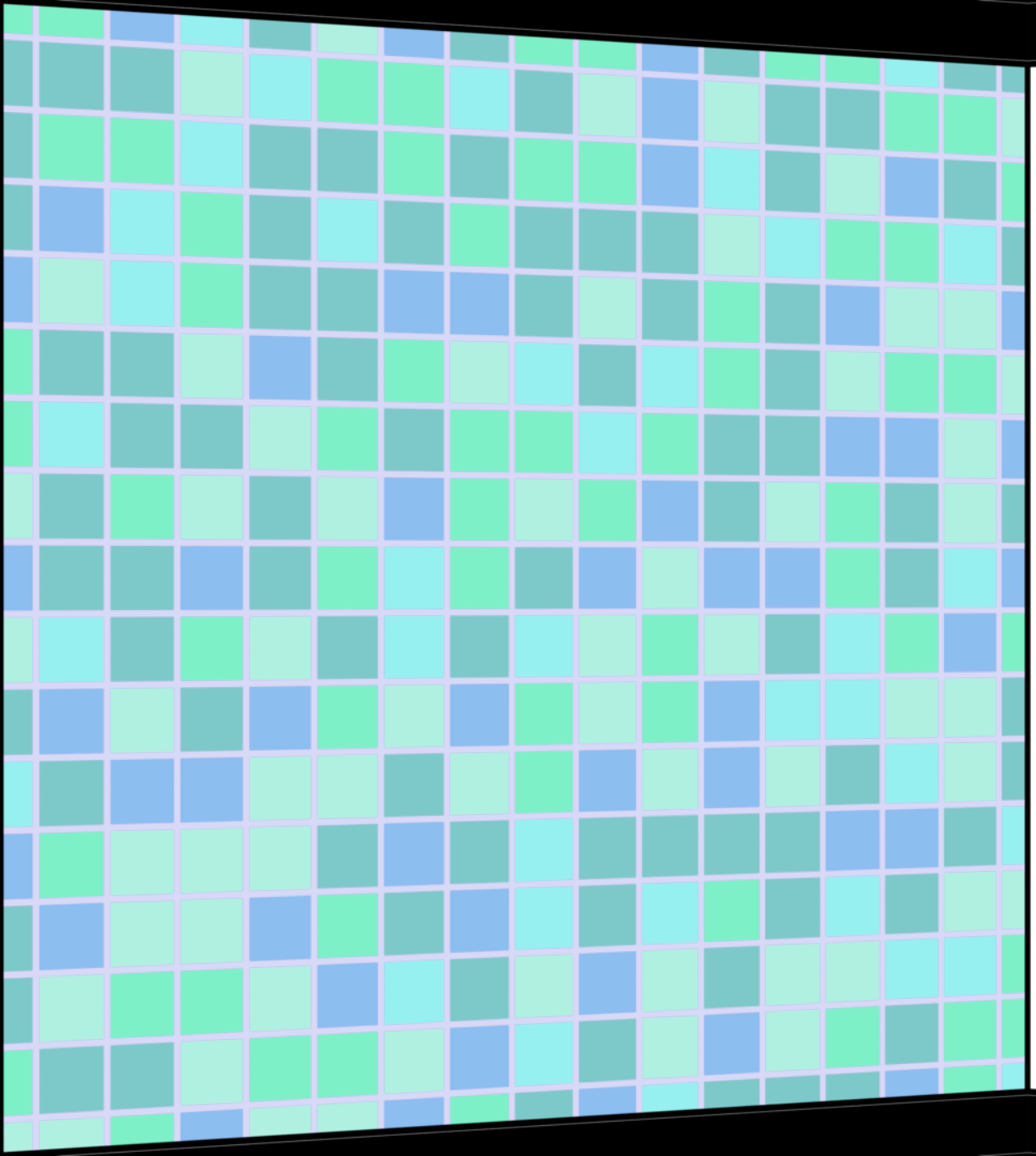
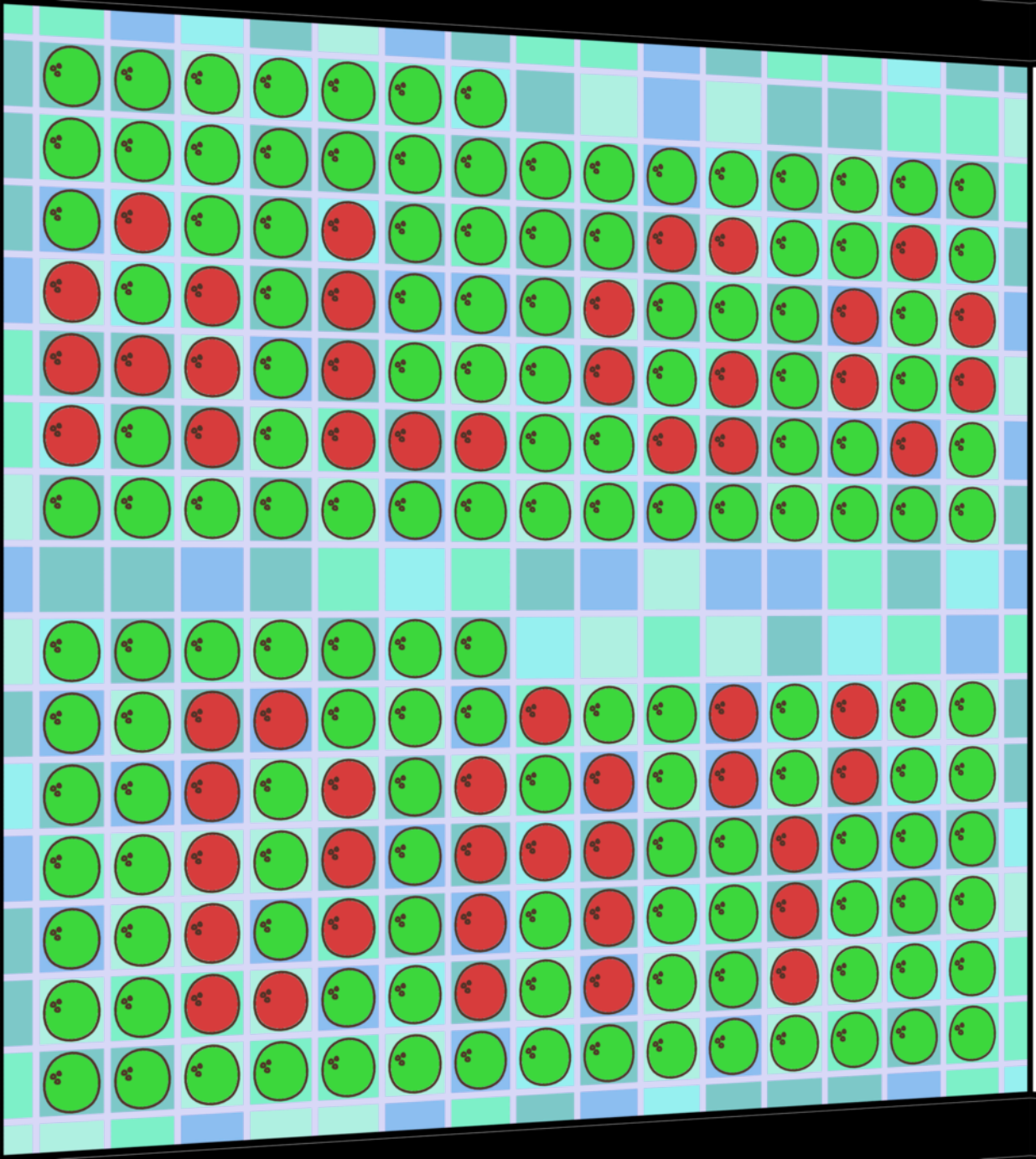


Open problem. Algorithms?



Open problem. Algorithms?

The theorem by Jones is not constructive.



Labeled Reconfiguration by Compaction

Maarten Löffler

Based on joint work with

Hugo Akitaya
Greg Aloupis
Anika Rounds
Giovanni Viglietta