REGION-BASED APPROXIMATION ALGORITHMS FOR VISIBILITY BETWEEN IMPRECISE LOCATIONS

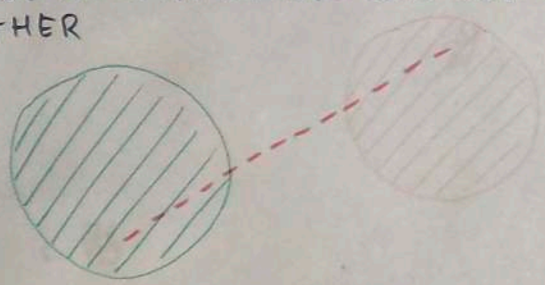
KEVIN BUCHIN • IRINA KOSTITSYNA • MAARTEN LÖFFLER • RODRIGO S.

ACT I

REGION-BASED APPROXIMATION WHAT?

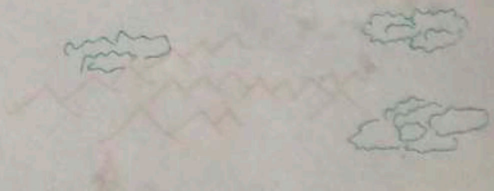
PROBLEM STATEMENT

COMPUTE THE PROBABILITY THAT TWO RANDOM POINTS CAN SEE EACH OTHER



MONKEY BEHAVIOUR

DO WANDERING MONKEY GROUPS ALTER THEIR COURSE WHEN THEY SEE EACH OTHER?



[ERIK WILLEMS, 2013]

CONVEXITY MEASURE

CAN WE PARAMETERISE THE EXTENT TO WHICH A POLYGON IS CONVEX?

NA KOSTITSYNA • MAARTEN LÖF

ACT I

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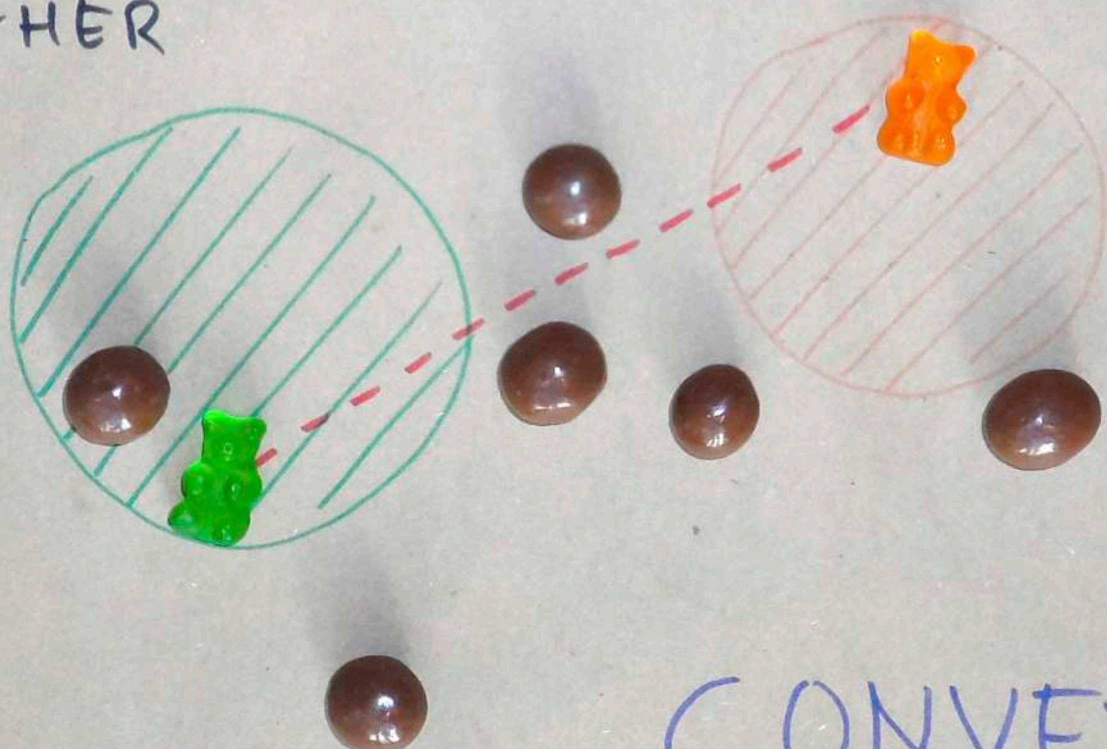

CONVEXITY M

CAN WE PARAMETERIS

MONKEY BEHAVIOUR

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ASURE

THE EXTENT
CONVEX?

[ERIK WILLEMS, 2013]

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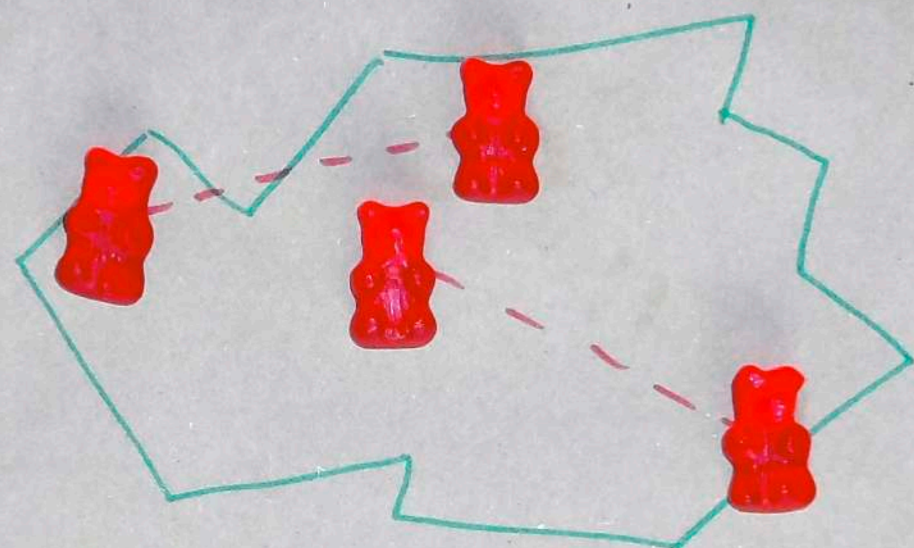


[GÜNTER ROTE, 2012]

[GÜNTER ROTE, 2012]

ACT II

WAIT, ISN'T THAT A SOLVED PROBLEM?

RATION

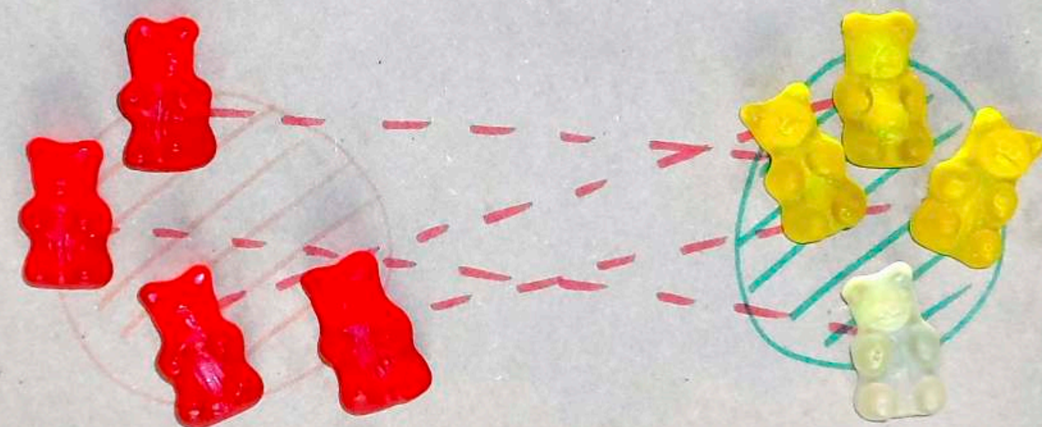
CK YOUR SEAT
NDOM HATS

POINT SA

JUST TRY TW
AND TEST THEI

POINT SAMPLING

JUST TRY TWO RANDOM POINTS,
AND TEST THEIR VISIBILITY!



(AND THEN DO IT AGAN. AND AGAIN.)

CKS

ILLUSTRATION

PLEASE CHECK YOUR SEAT
FOR ANY RANDOM HATS

(AND THEN D

DRAWBACKS

- YOU NEED A RELIABLE SOURCE OF RANDOMNESS
- YOU NEED TO TAKE A LOT OF SAMPLES
- MOST LIKELY YOU STILL WON'T GET THE RIGHT ANSWER
- YOU JUST NEVER KNOW FOR SURE!

- YOU NEED TO TAKE A LOT OF SAMPLES
- MOST LIKELY YOU STILL WON'T GET THE RIGHT ANSWER
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ACT III

A FUNDAMENTAL APRROACH, WITH INTEGRALS AND STUFF

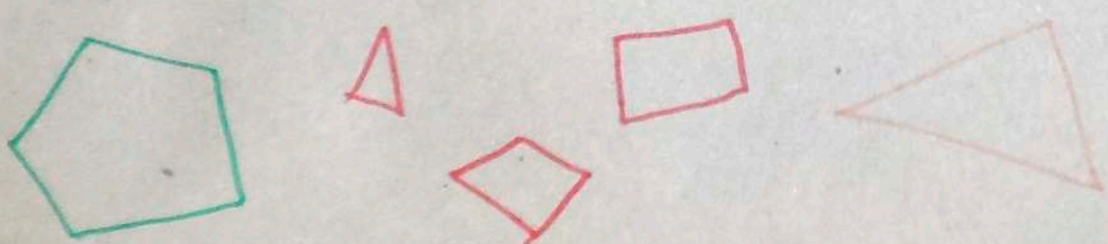
BLEM

OLYGONS OF

OLLECTION

FORMAL PROBLEM

GIVEN TWO CONVEX POLYGONS OF COMPLEXITY N AND A COLLECTION OF OBSTACLES OF COMPLEXITY M, COMPUTE THE PROBABILITY THAT TWO POINTS, TAKEN FROM BOTH POLYGONS, ARE MUTUALLY VISIBLE



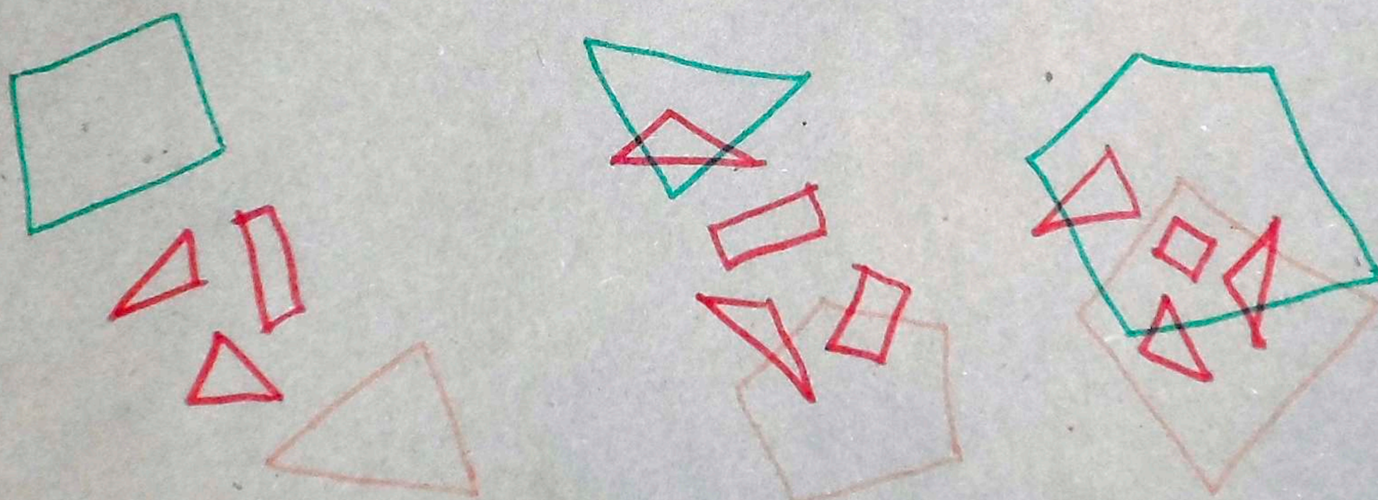
SO

OT

THAT
OTH
SIBLE

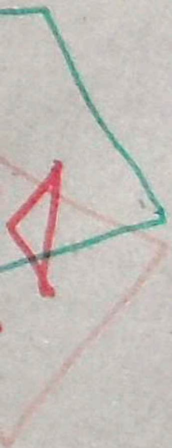
VARIANTS

SOME POLYGONS OVERLAP.
OTHERS DON'T.



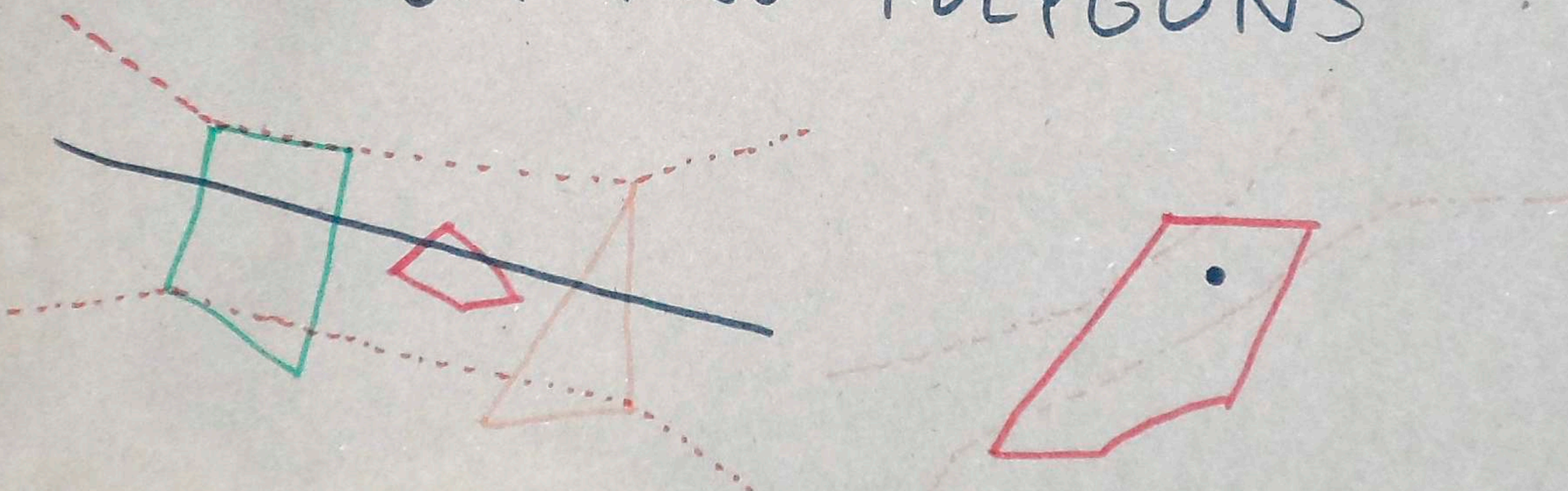
POI

WE W
THE



POINT-LINE DUALITY

WE WANT TO ARGUE ABOUT THE SPACE OF ALL LINES THROUGH TWO POLYGONS



THE CALCULATION

$$I = \iint\limits_{C} \left(\int_{x_1(\alpha,\beta)}^{x_2(\alpha,\beta)} \int_{x_3(\alpha,\beta)}^{x_4(\alpha,\beta)} (x_2 - x_1)\, dx_2\, dx_1 \right) d\alpha\, d\beta = \iint\limits_{C} \frac{(x\ldots}{\ldots}$$

$$I_{ij} = \iint\limits_{C} x_i x_j^2 \, d\alpha\, d\beta = \iint\limits_{C} \frac{(b_i\beta - c_i)(b_j\beta - c_j)^2}{(b_i\alpha + a_i)(b_j\alpha + a_j)^2}\, d\alpha\, d\beta = \sum_{C_{\ldots}}$$

$$F_{ij}(\alpha) = \int \left(\int_{A_1\alpha + B_1}^{A_2\alpha + B_2} \frac{(b_i\beta - c_i)(b_j\beta - c_j)^2}{(b_i\alpha + a_i)(b_j\alpha + a_j)^2}\, d\alpha = \frac{\log(a_i + \alpha b_i)}{12 b_i^2 b_j^2} \left[4(A \ldots \right.\right.$$

$$+ \frac{1}{24 b_i^2 b_j^2} \left[3\alpha^2 (\ldots \right.$$

$$= \iint_C \frac{(x_2-x_1)(x_4-x_3)(x_3+x_4-x_1-x_2)}{2} d\alpha d\beta = \frac{1}{2} \iint_C (-x_1^2 \ldots$$

$$+ x_1^2 x \ldots$$

$$\ldots d\alpha d\beta = \sum_{C_v \subset C} \int_{\alpha_1}^{\alpha_2} \left(\int_{A_1\alpha+B_1}^{A_2\alpha+B_2} \frac{(b_i\beta-c_i)(b_j\beta-c_j)^2}{(b_i\alpha+a_i)(b_j\alpha+a_j)^2} \right.$$

$$\frac{\ldots+\alpha b_i)}{b_i^2} \left[4(A_2^3-A_1^3)(a_jb_i-a_ib_j)(b_ic_j-b_jc_i) + 3(A_2^4-A_1^4) \ldots \right.$$

$$\frac{\ldots}{\ldots^2 b_j^2} \left[3\alpha^2(A_2^4-A_1^4)b_ib_j + \alpha\left((A_2^4-A_1^4)(12a_jb_i-6a_ib_j) \ldots \right.\right.$$

...CULATION

$$\iint_C \left(\int_{x_1(\alpha,\beta)}^{x_2(\alpha,\beta)} \int_{x_3(\alpha,\beta)}^{x_4(\alpha,\beta)} (x_2 - x_1)\, dx_2\, dx_1 \right) d\alpha\, d\beta = \iint_C \ldots$$

$$I_{ij} = \iint_C X_i X_j^2 \, d\alpha\, d\beta = \iint_C \frac{(b_i\beta - c_i)(b_j\beta - c_j)^2}{(b_i\alpha + a_i)(b_j\alpha + a_j)^2}\, d\alpha\, d\beta = \sum \ldots$$

$$F_{ij}(\alpha) = \int \left(\int_{A_1\alpha + B_1}^{A_2\alpha + B_2} \frac{(b_i\beta - c_i)(b_j\beta - c_j)^2}{(b_i\alpha + a_i)(b_j\alpha + a_j)^2} \right) d\alpha = \frac{\log(a_i + \alpha b_i)}{12 b_i^2 b_j^2}\left[4(A_2^3 - \ldots \right.$$

$$+ \frac{1}{24 b_i^2 b_j^2}\left[3\alpha^2 (A_2^4 - \ldots \right.$$

(OTHER CASES ARE SIMILAR BUT SLIGHTLY MORE COMPLICATED. PLEASE ...

THEOREM

THE DESIRED PROBABILI...
CAN BE COMPUTED IN TI...
12)

MORE COMPLICATED.

THEOREM

THE DESIRED PROBABILITY CAN BE COMPUTED IN TIME

$$O(M \cdot (M+N)^2)$$

ACT IV

LIGHTLY MORE COMP...

THEOREM

THE DESIRED PROBABILITY CAN BE COMPUTED IN TIME

$$O(M \cdot (M+N)^2)$$

ACT IV

SURELY WE CAN DO BETTER?

ACE

OF

E END

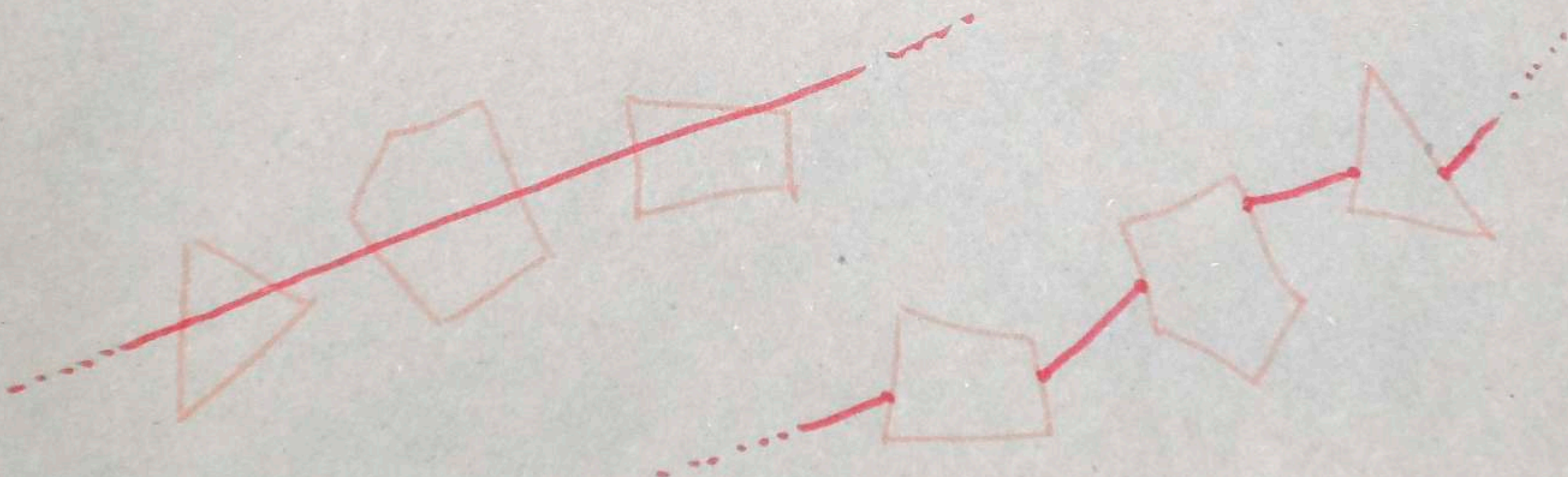
LAYER

EACH LINE

MAXIMAL SEG

SEGMENT SPACE

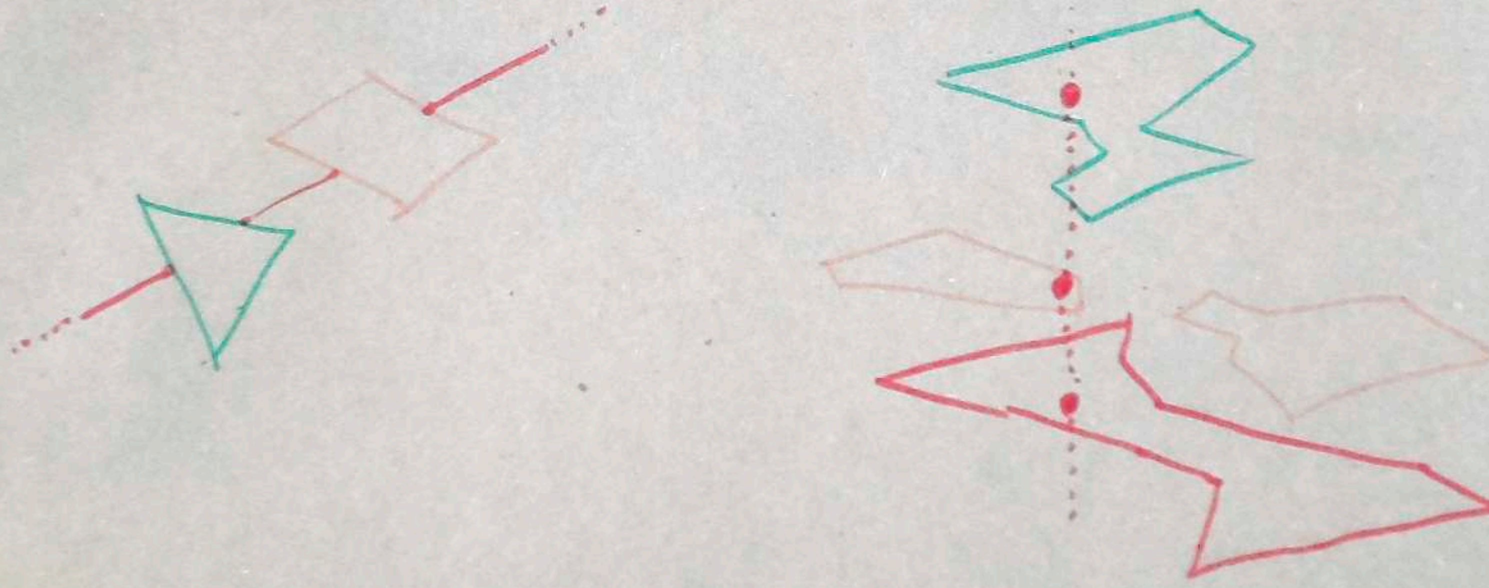
CONSIDER THE SPACE OF LINE SEGMENTS WHOSE END POINTS TOUCH AN OBSTACLE



TH

LAYERS OF SEGMENTS

EACH LINE MAY CONSIST OF M MAXIMAL SEGMENTS, ADDING A DISCRETE DIMENSION TO THE SEGMENT SPACE



THEOREM

THE DESIRED PROBABILITY CAN ALSO BE COMPUTED IN JUST $O((M+N)^2)$ TIME

THE DESIRED PROBABILITY CAN ALSO BE COMPUTED IN JUST $O((M+N)^2)$ TIME

ACT V

BUT WHAT ABOUT REAL DISTRIBUTIONS?

S DON'T

GAUSSIANS

REAL RANDOM POINTS DON'T HAVE UNIFORM DISTRIBUTIONS

FOR NORMAL DISTRIBUTIONS, OUR INTEGRAL HAS NO CLOSED-FORM SOLUTION!

AP

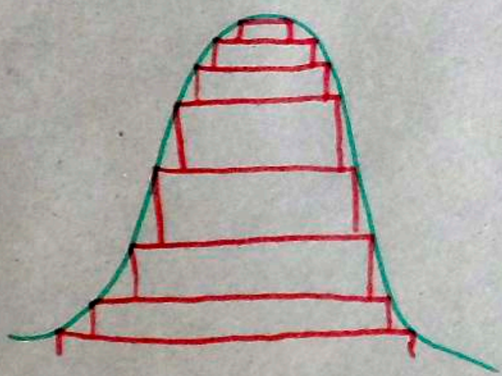
WE

DIS

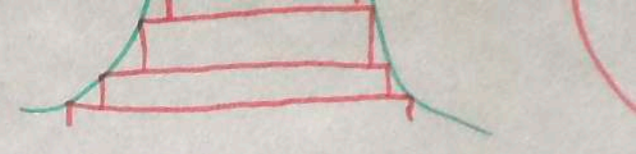
DIS

APPROXIMATION BY DISKS

WE CAN APRROXIMATE A GAUSSIAN DISTRIBUTION WITH A BUNCH OF UNIFORM DISTRIBUTIONS ON CONCENTRIC DISKS

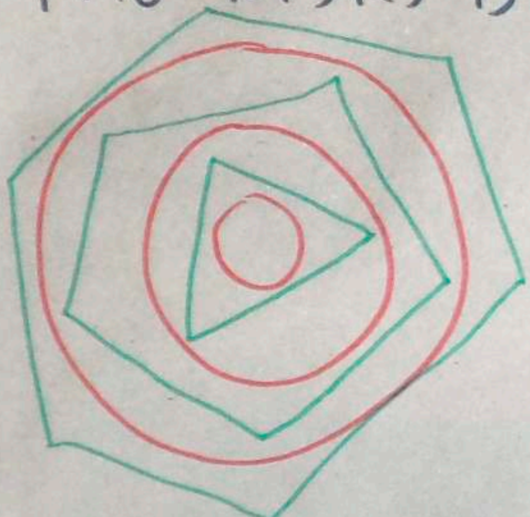


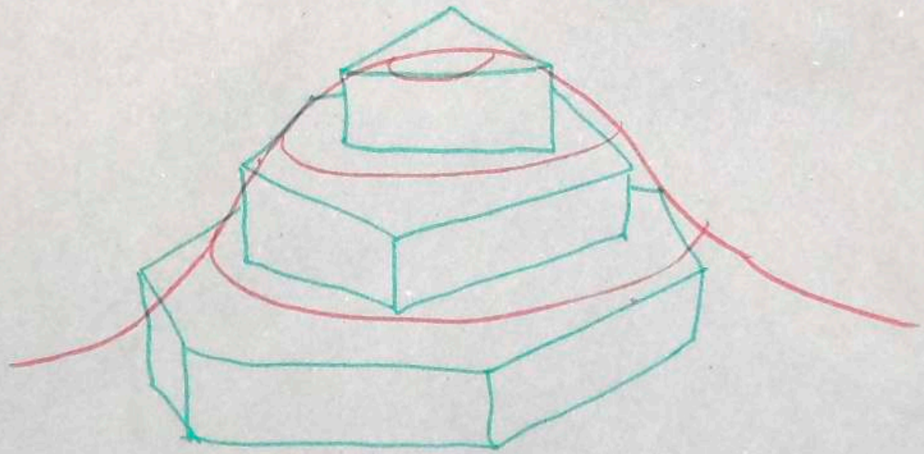
ITH POLYGONS



APPROXIMATION WITH POLYGONS

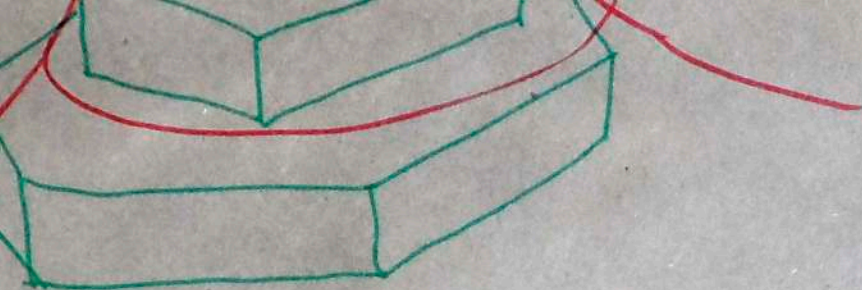
AND THEN, WE CAN IN TURN APPROXIMATE THE DISKS BY REGULAR POLYGONS





THEOREM

A NORMAL DISTRIBUTION CAN BE ε-APPROXIMATED WITH POLYGONS OF TOTAL COMPLEXITY $O(\sqrt{\varepsilon})$



WITH POLYGON
COMPLEXITY

ACT VI

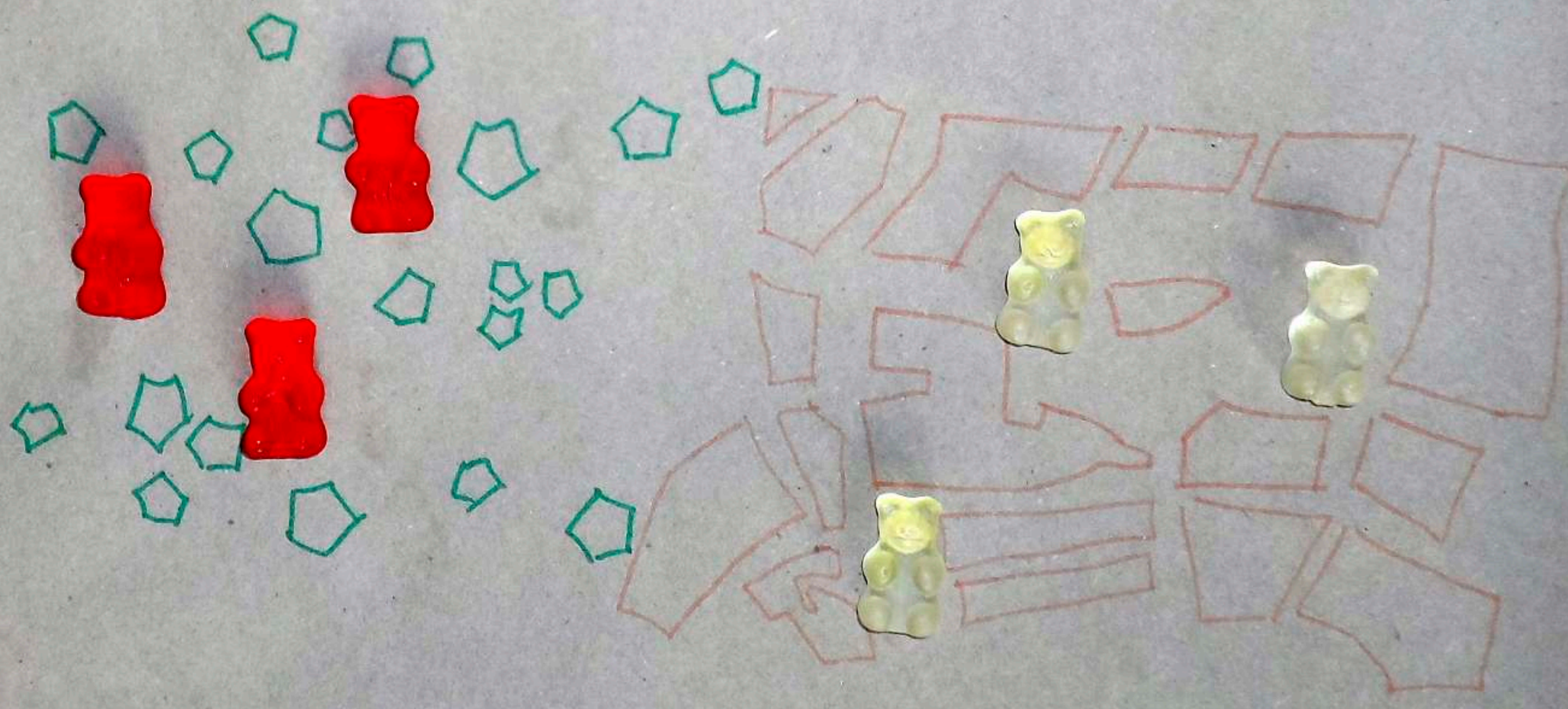
ISN'T THIS SUPPOSED TO BE AN EXPERIMENTAL PAPER?

SET

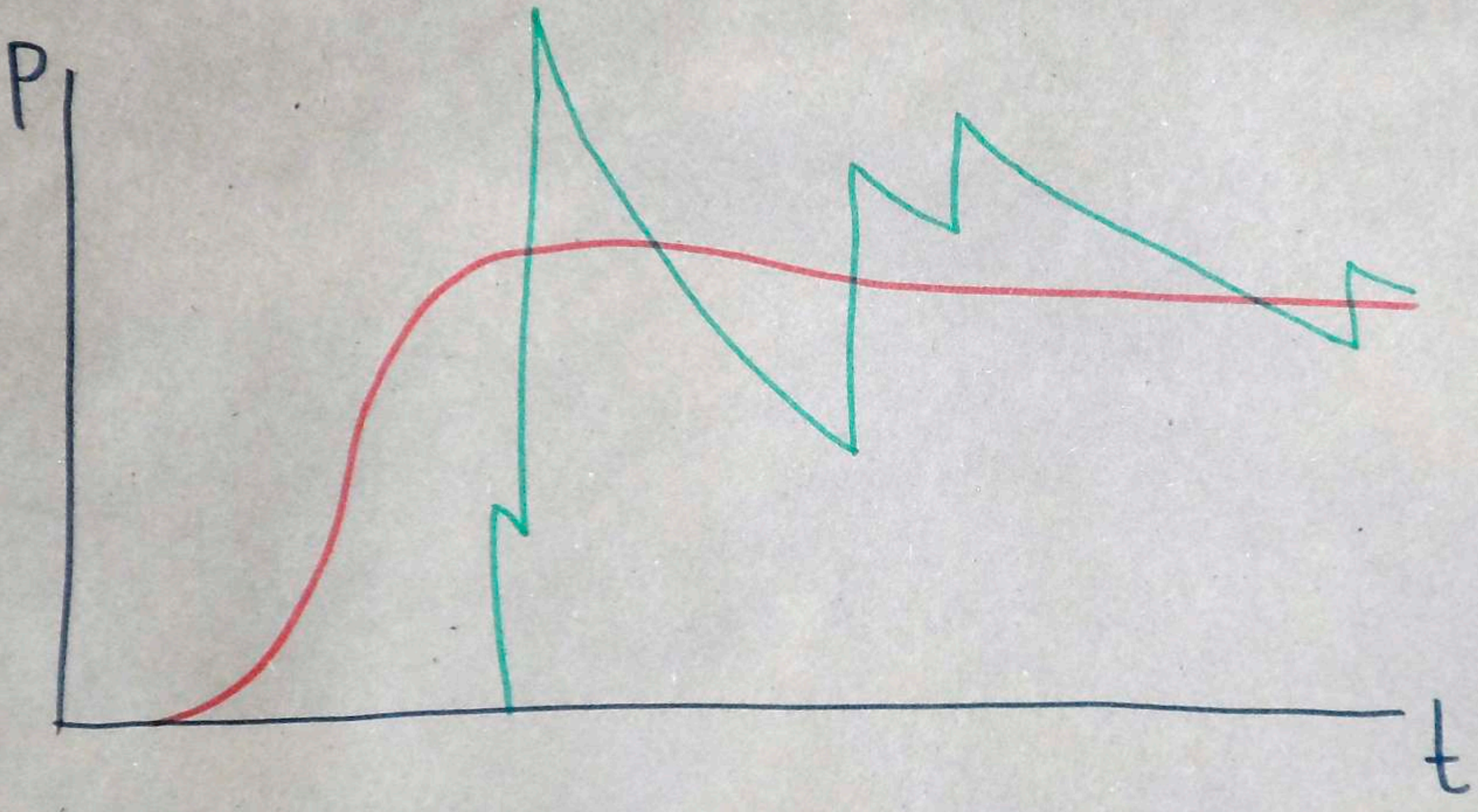
WE PER
A FORE

SETTINGS

WE PERFORMED TESTS IN
A FOREST AN URBAN ENVIRONMENT



RESULTS



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THE END