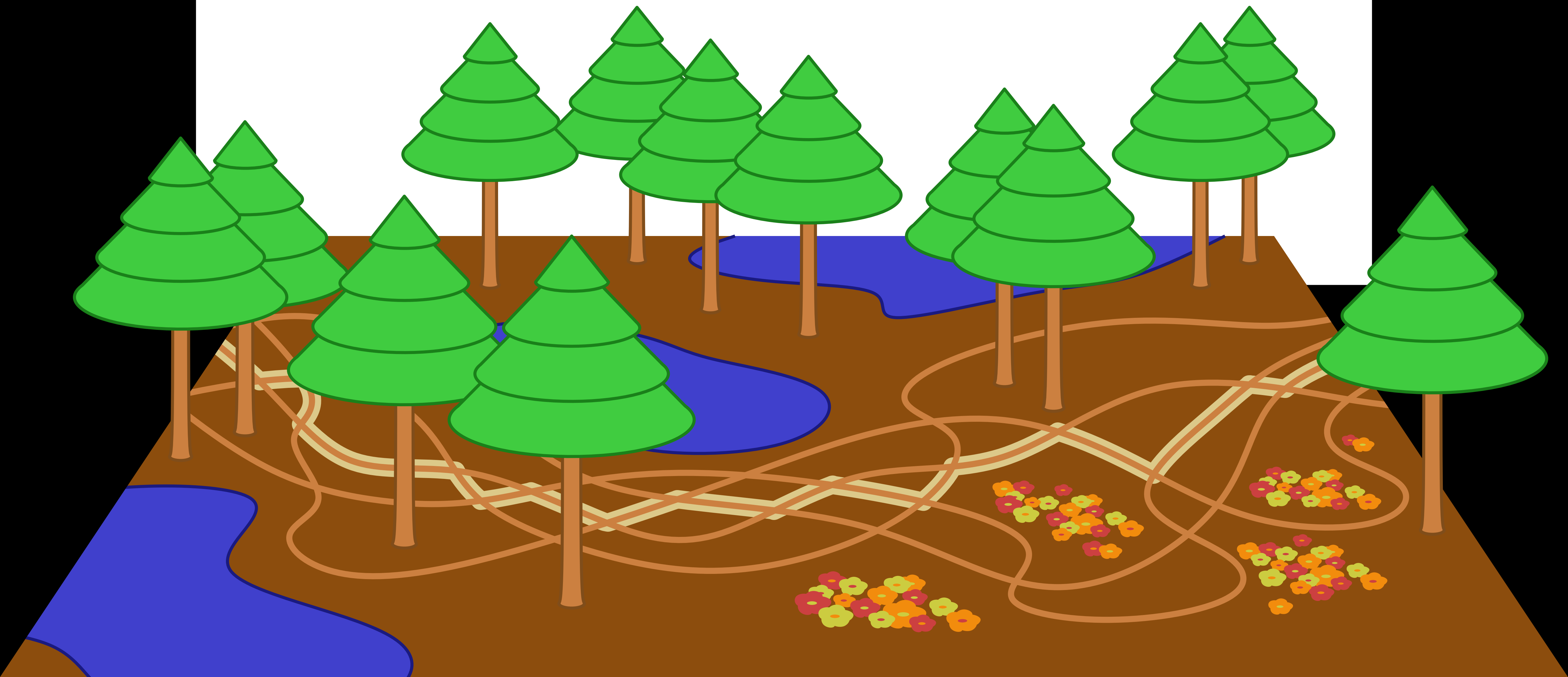
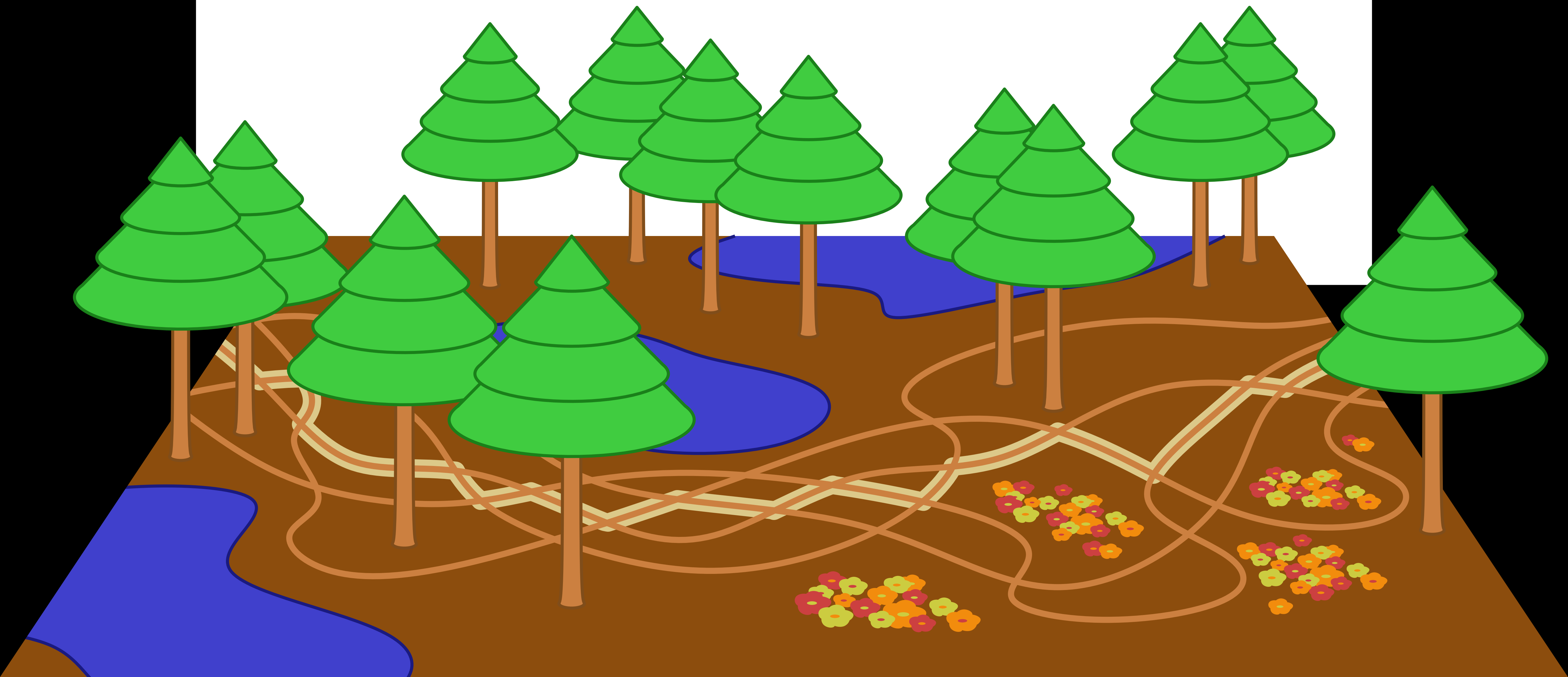
THANK YOU!



NEW APPROACHES FOR REPRESENTATIVE TRAJECTORIES

~~THANK YOU!~~



Homotopy Measures for Representative Trajectories

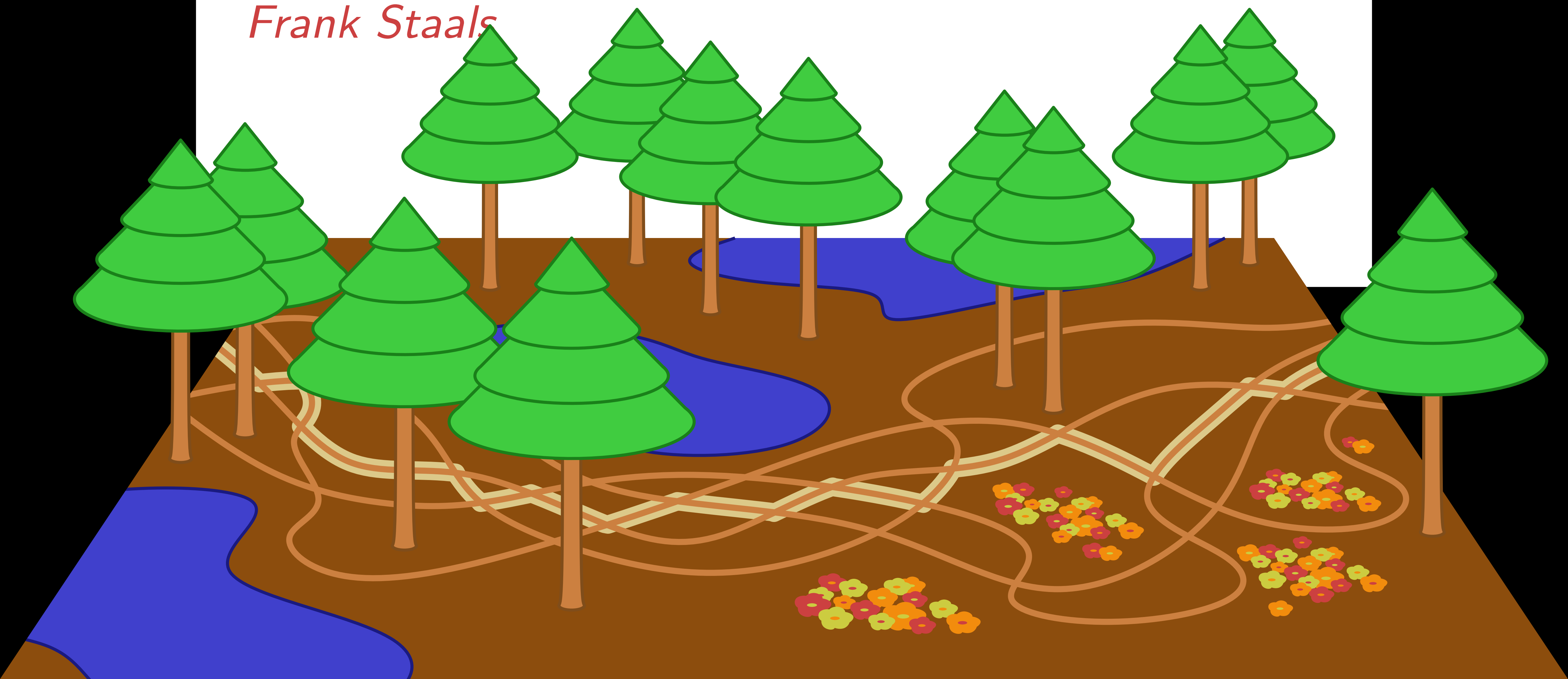
10:50 - 11:10

Erin Chambers
Irina Kostitsyna
Maarten Löffler
Frank Staals

Central Trajectories

11:10 - 11:30

Marc van Kreveld
Maarten Löffler
Frank Staals



TRAJECTORIES

TRAJECTORIES

- Let P be n points in the plane

TRAJECTORIES

- Let P be n points in the plane

TRAJECTORIES

- Let P be n points in the plane
- Now suppose your points run away

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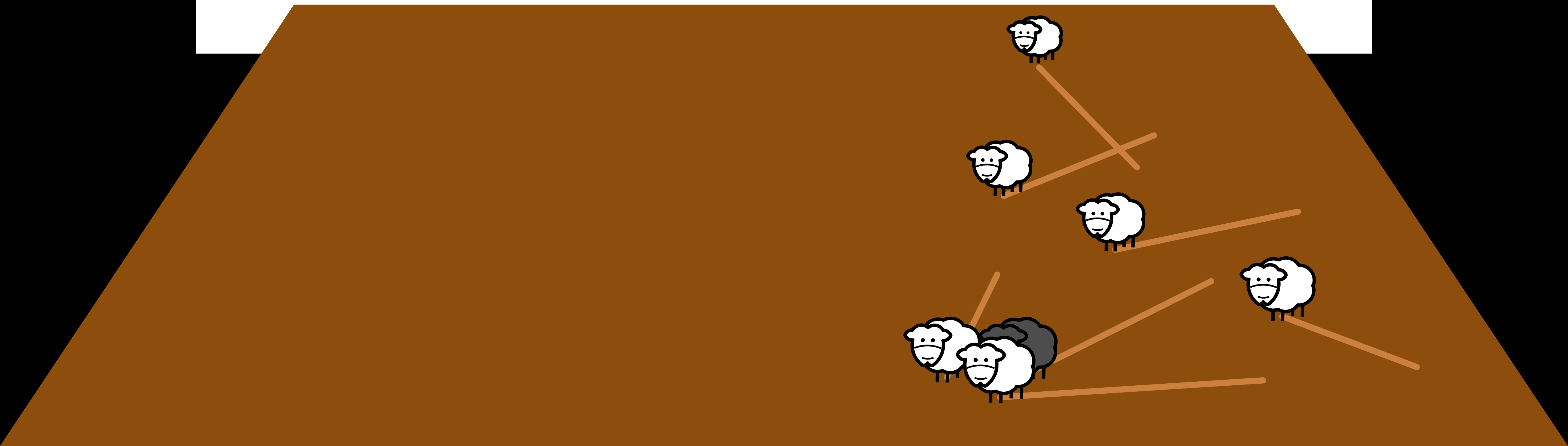

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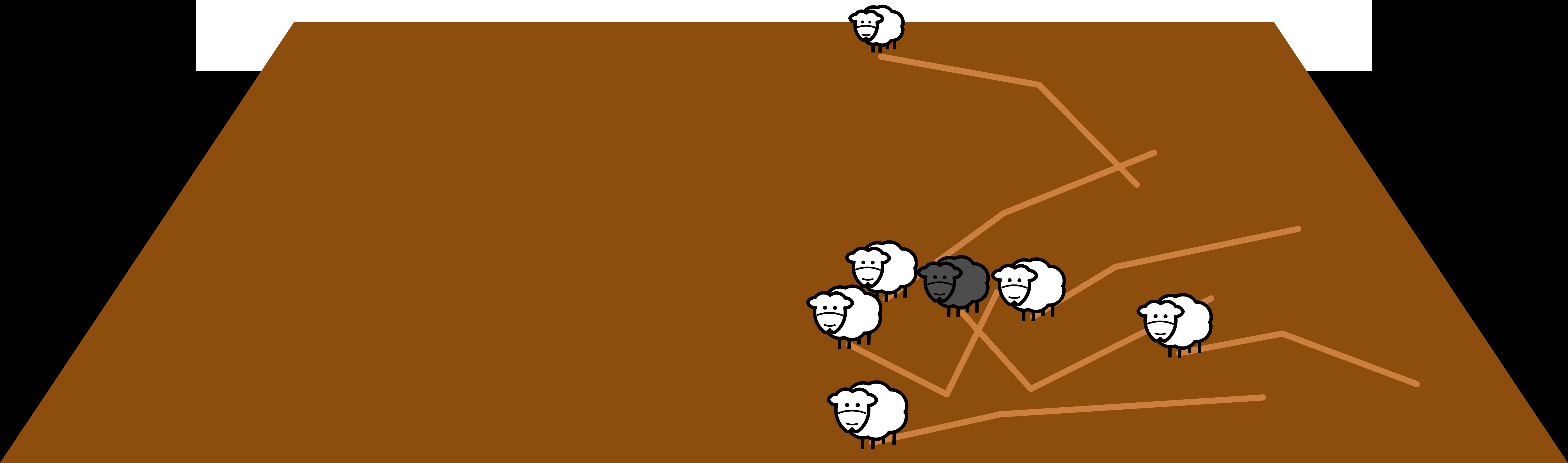

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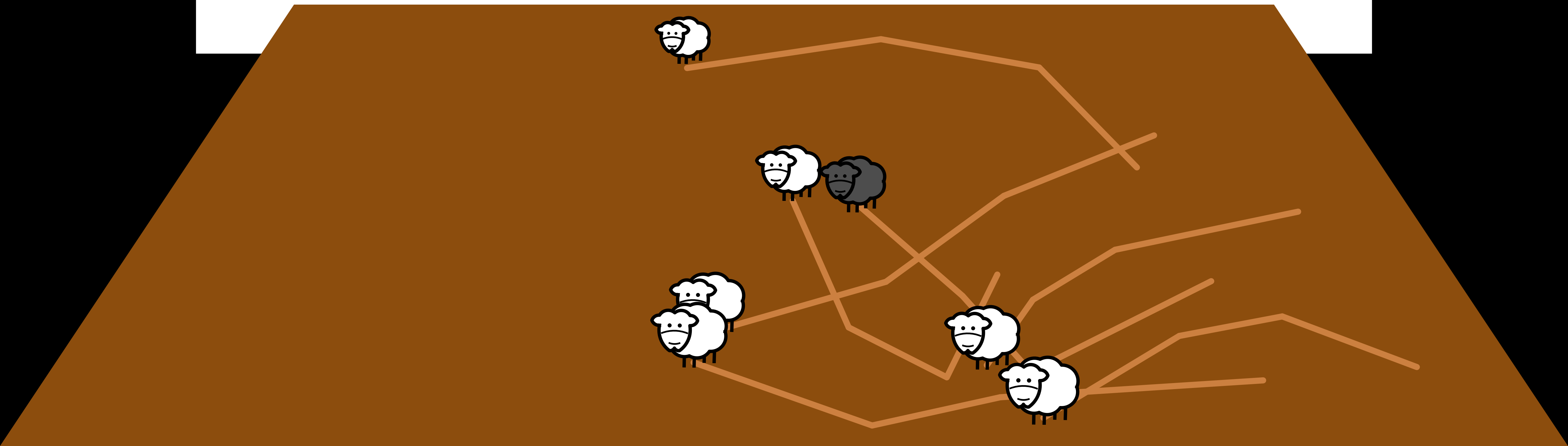

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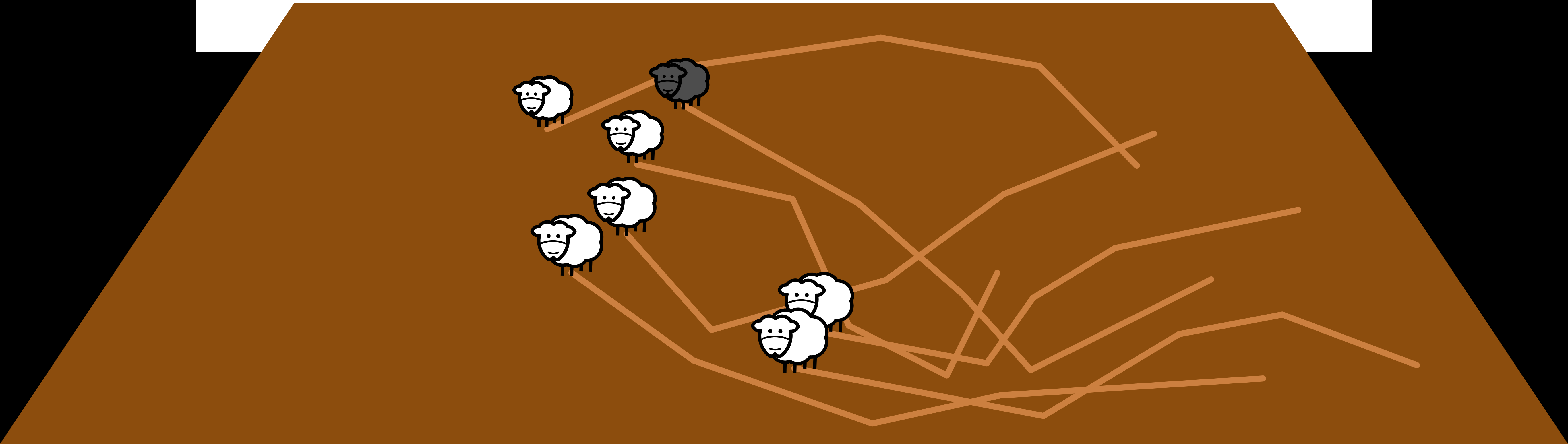

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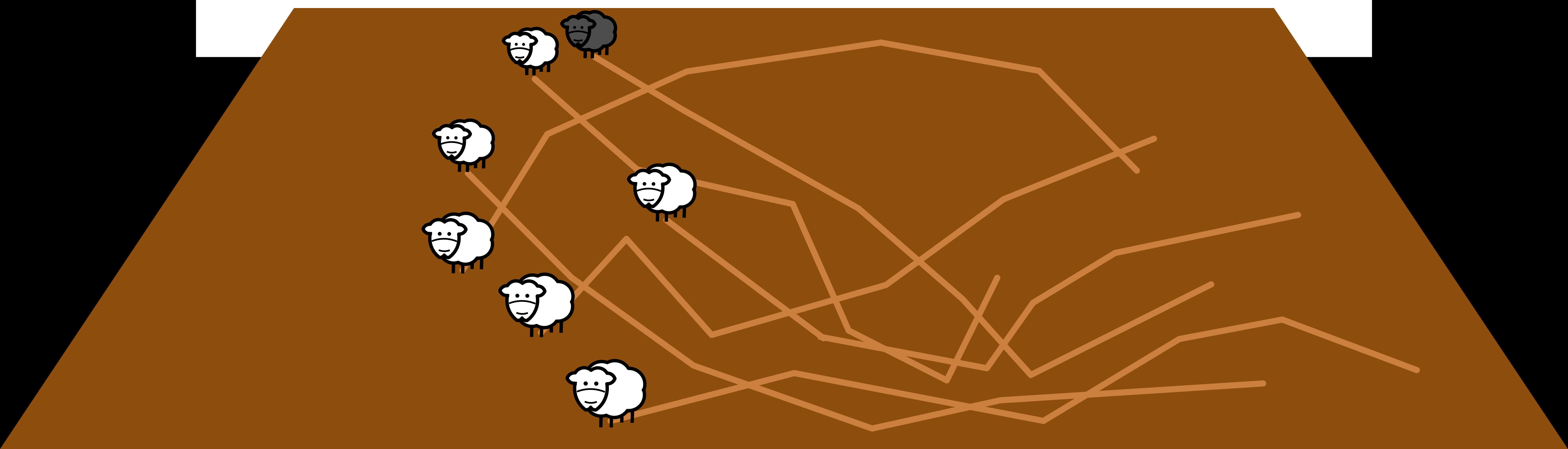
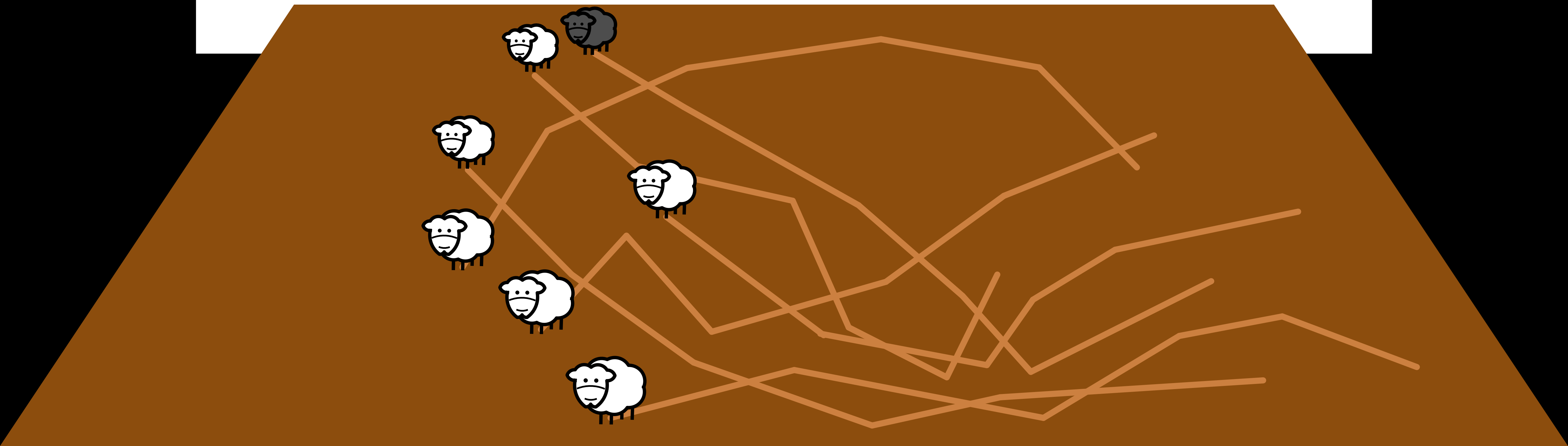


TRAJECTORIES

- Let P be n points in the plane
- Now suppose your points run away
- P traces a set of n *trajectories*



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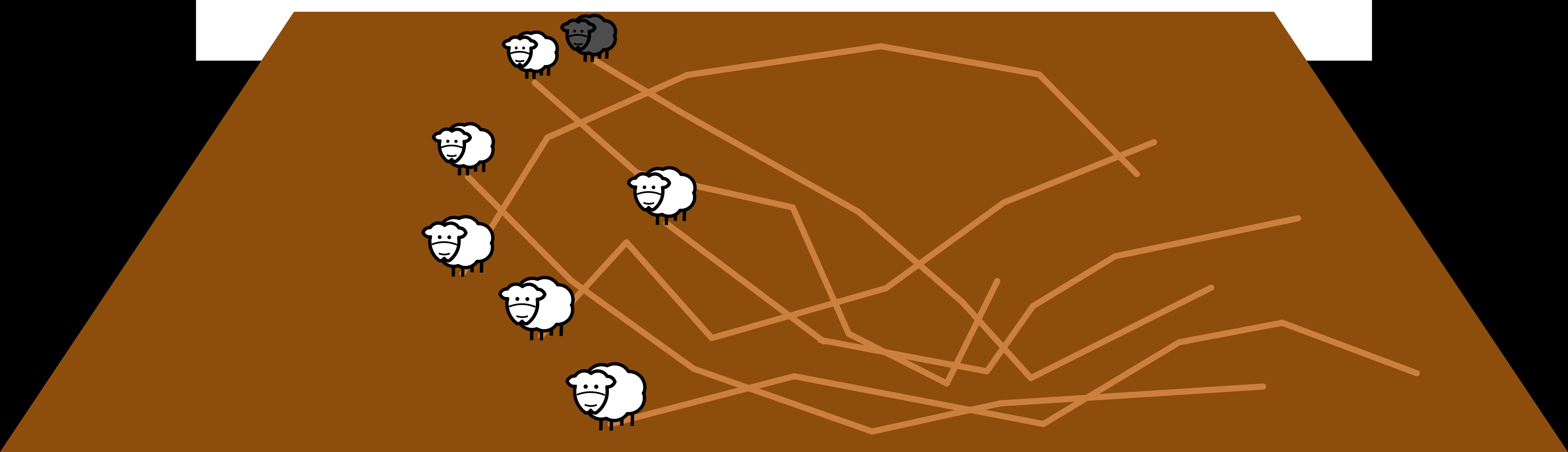
DEFINITION.

A *trajectory* is a polyline defined by a sequence of points, each with a location in $\mathbb{R}^d$ and a time stamp in $\mathbb{R}$.



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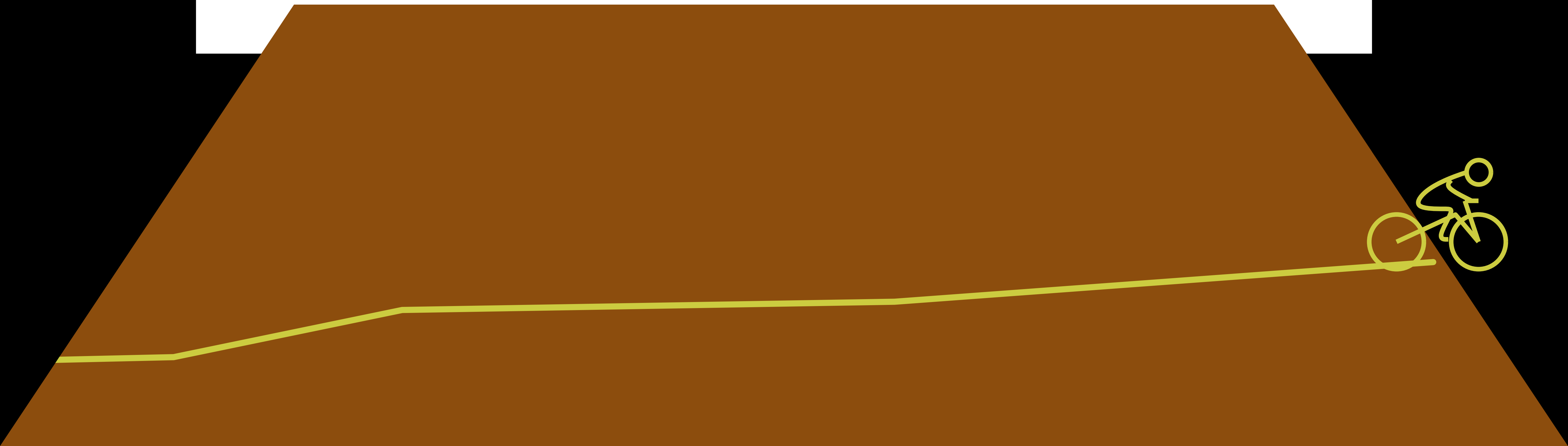

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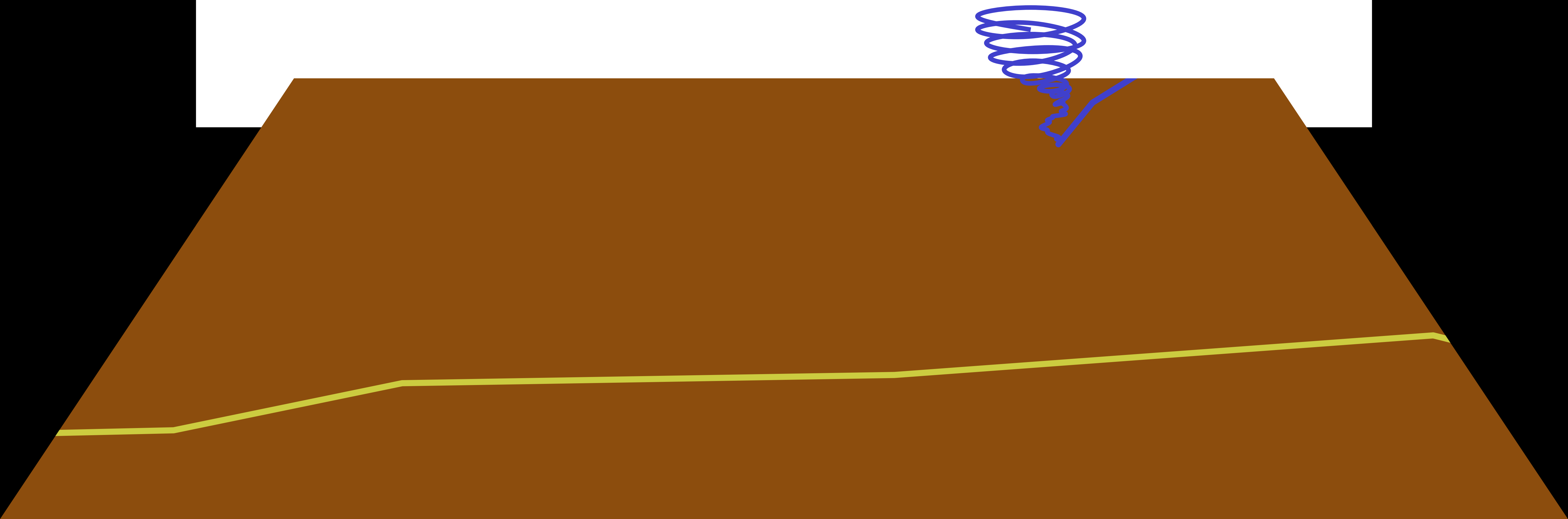

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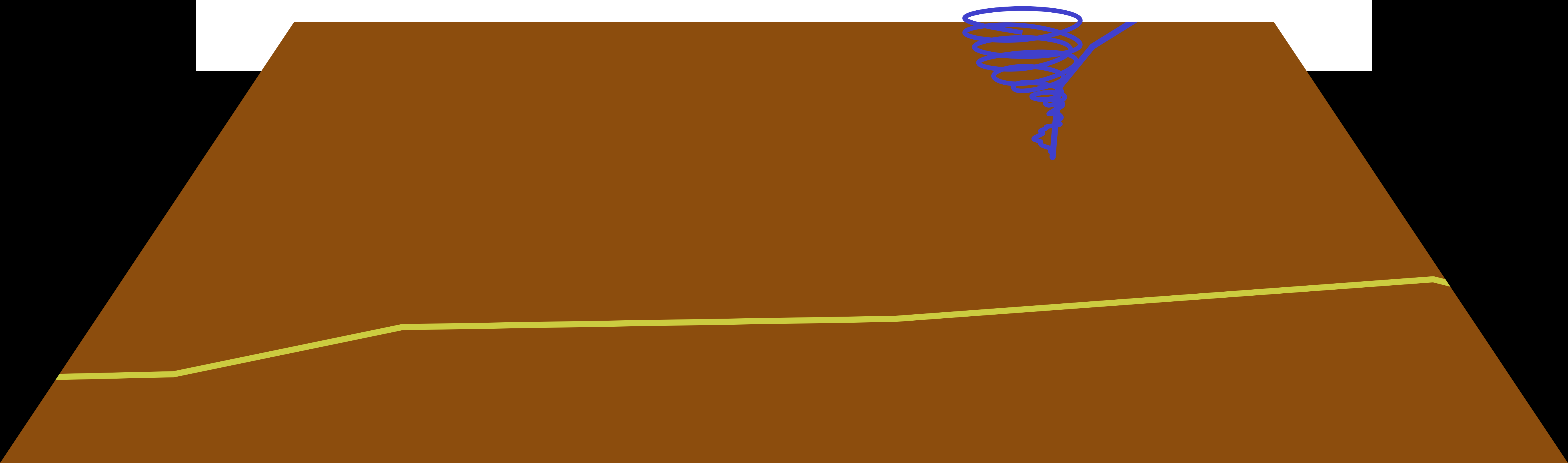

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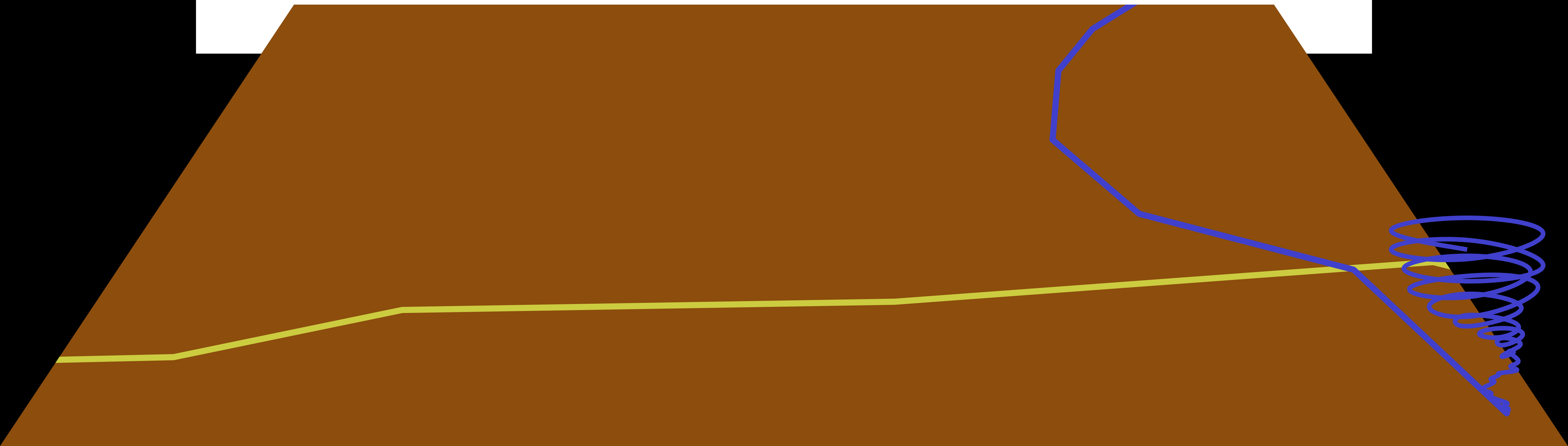

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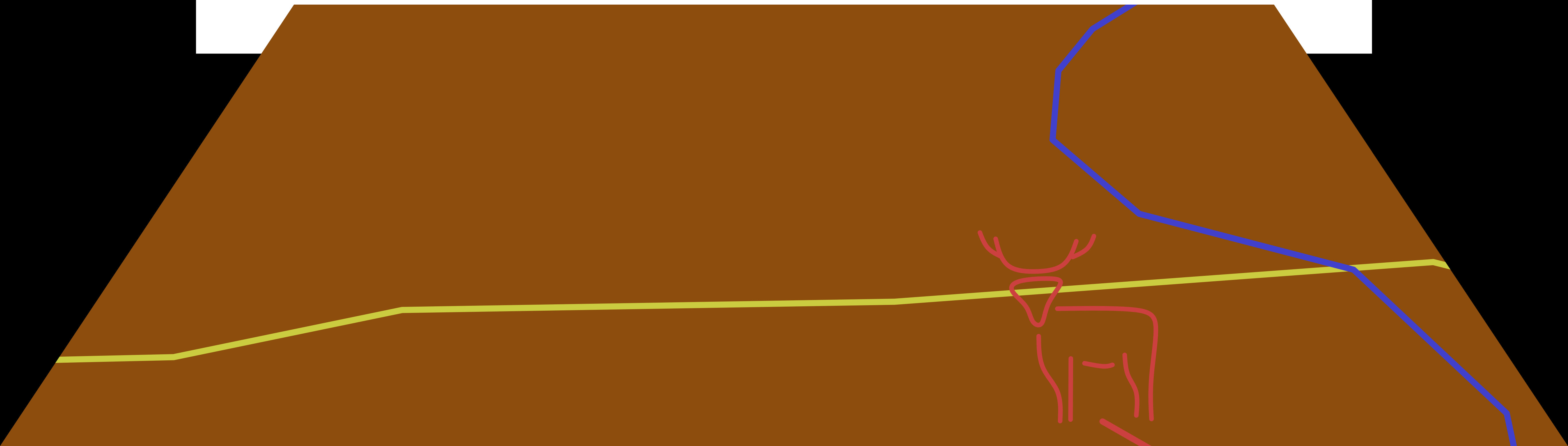

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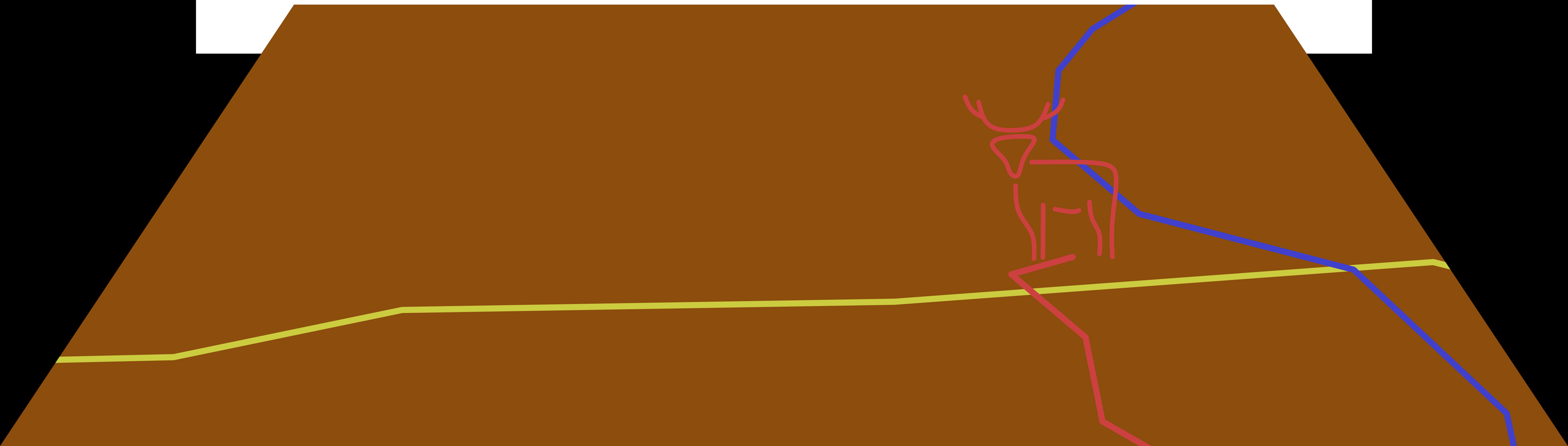

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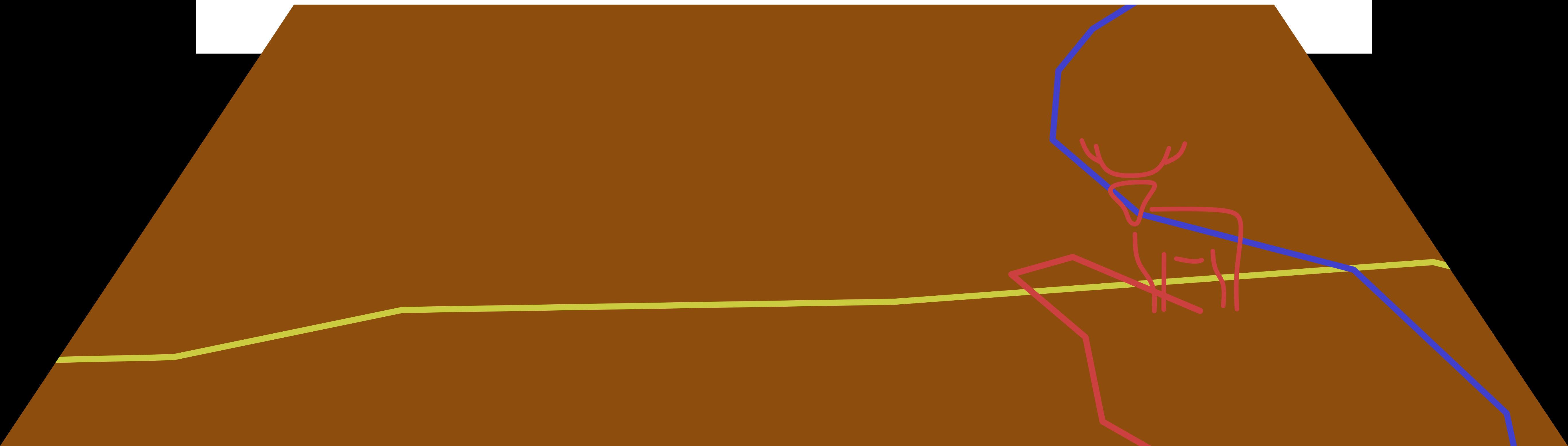

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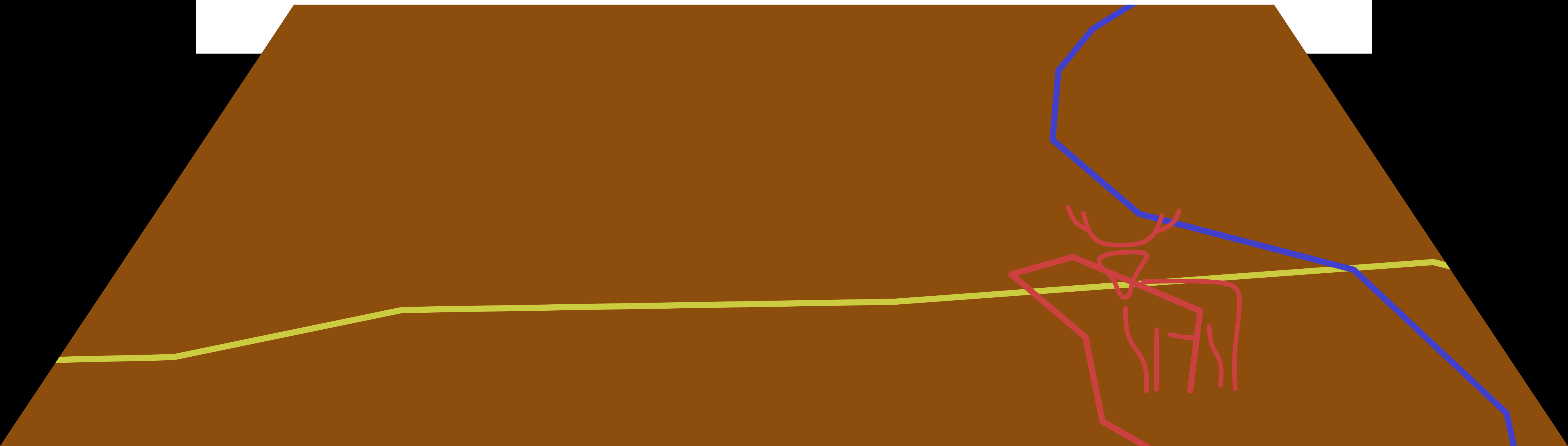

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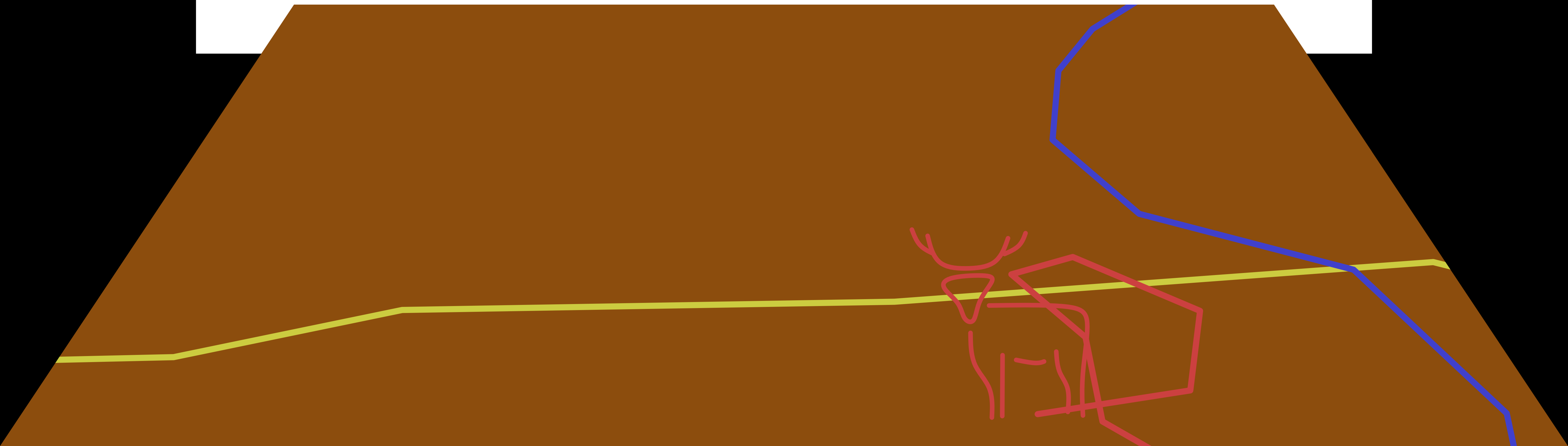

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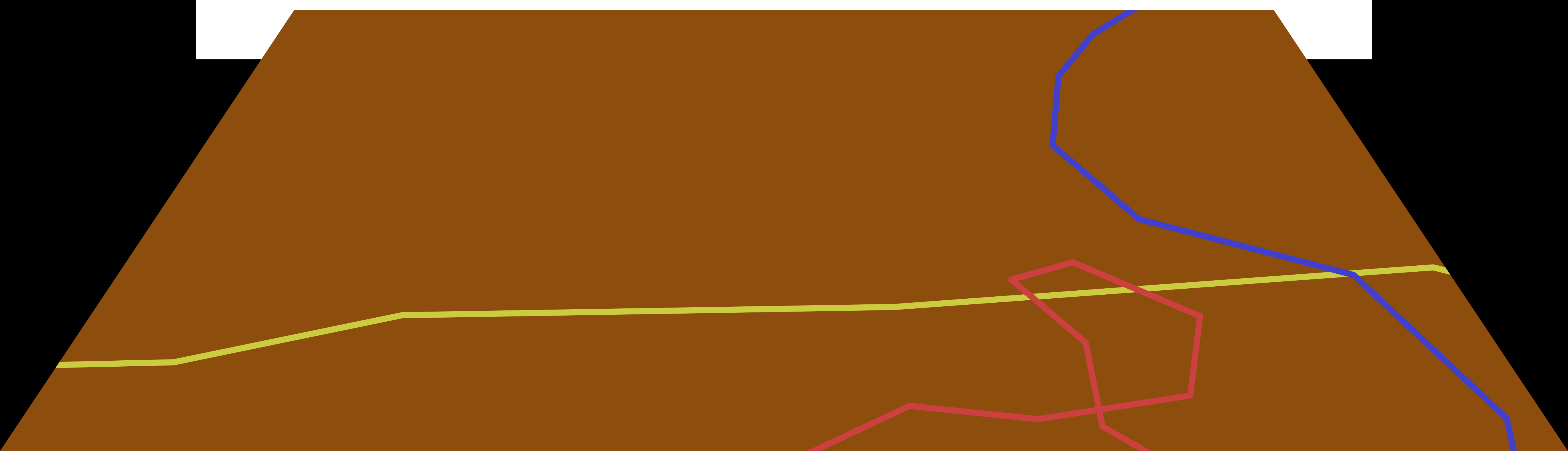
- Trajectories are interesting



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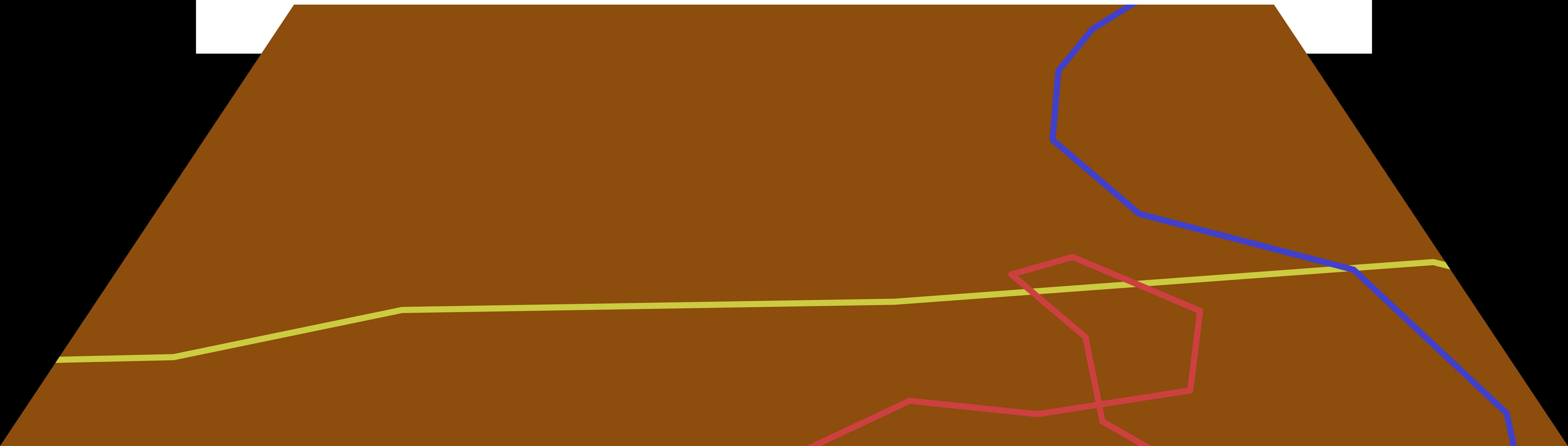
  - Many different analysis tasks



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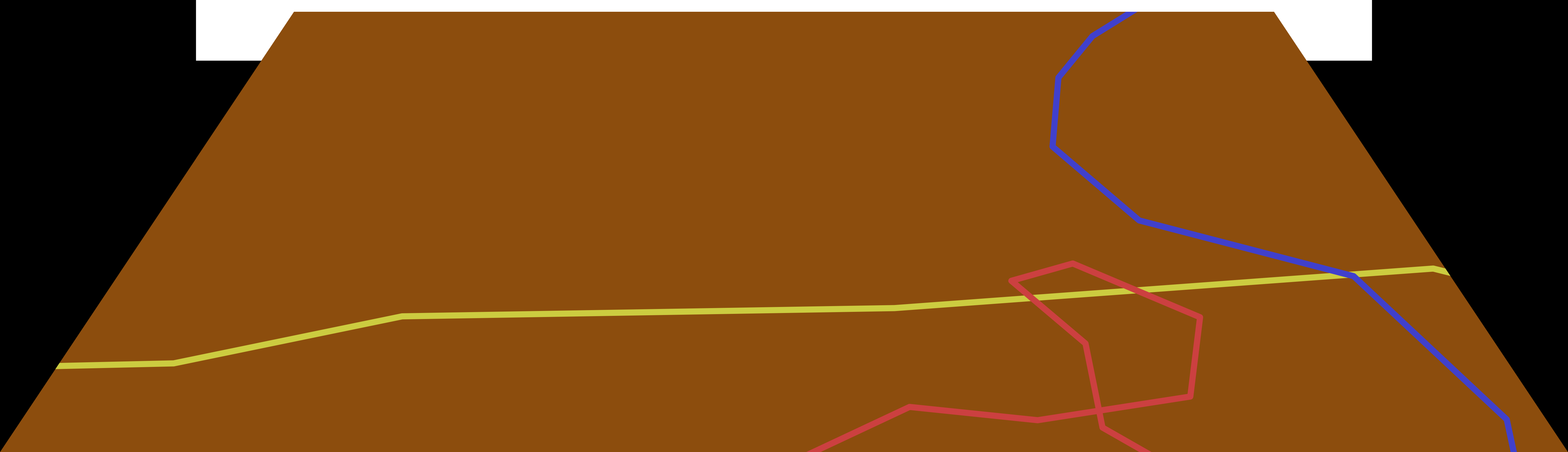

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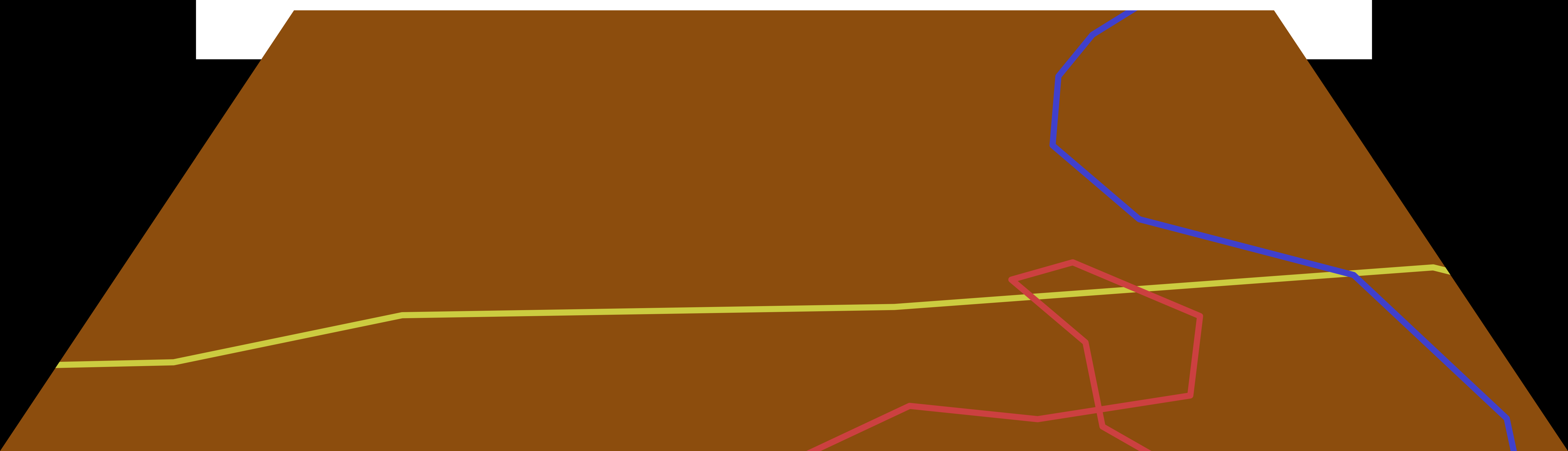

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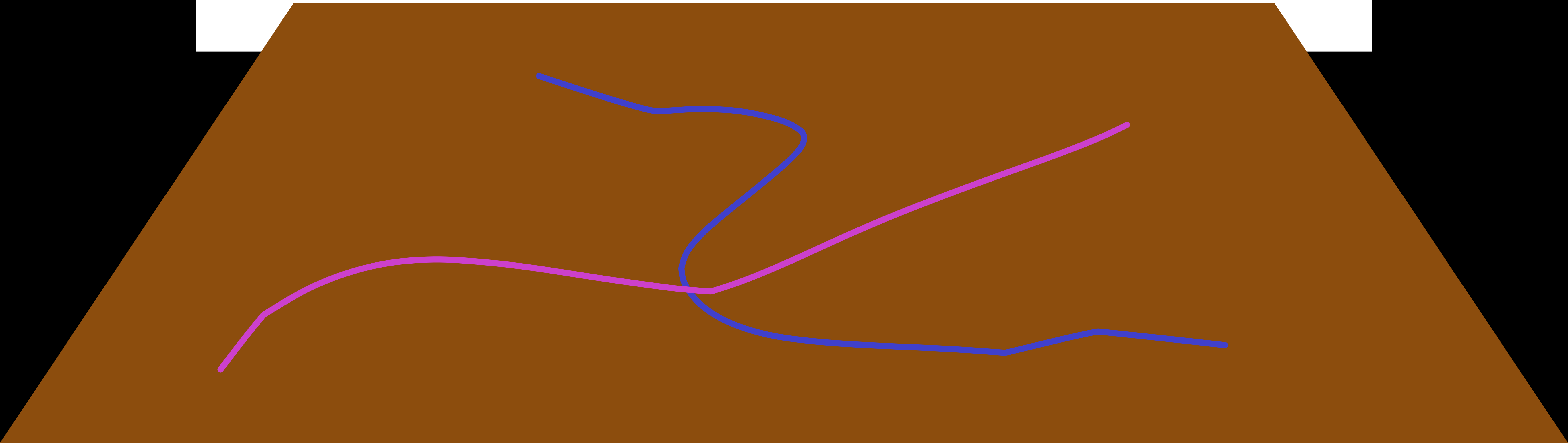

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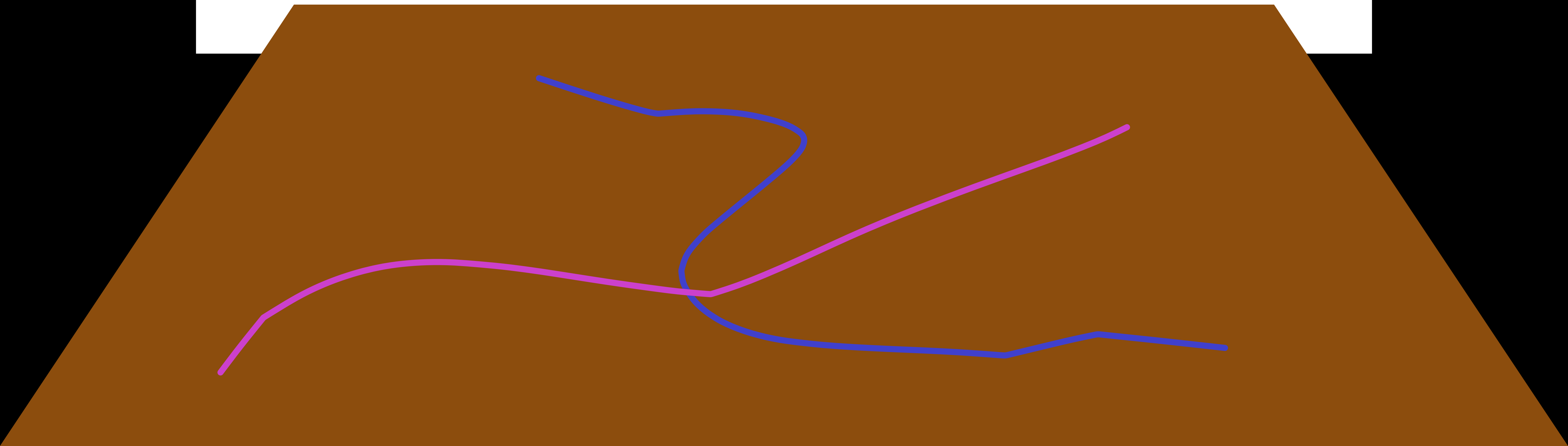

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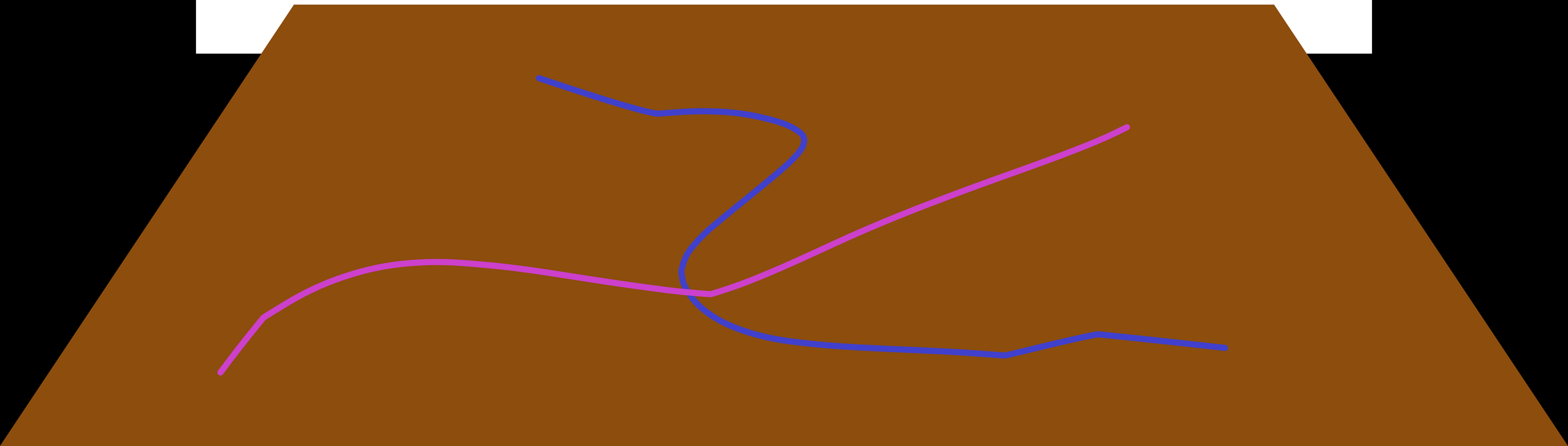
  - Keep only the representatives
- But what is a good representative?



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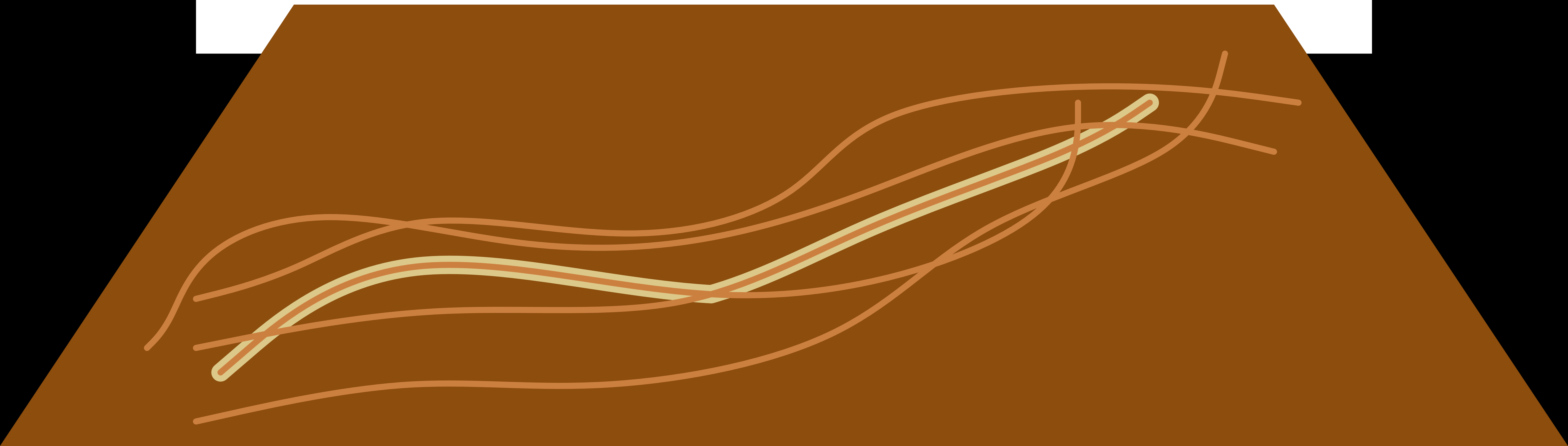

USE INPUT TRAJECTORY?

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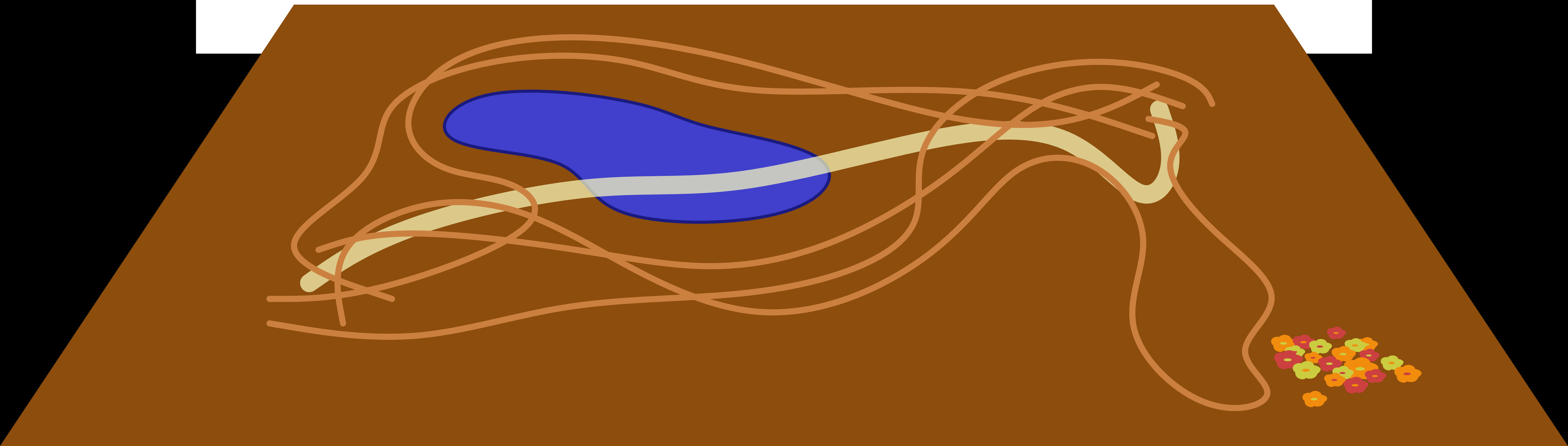

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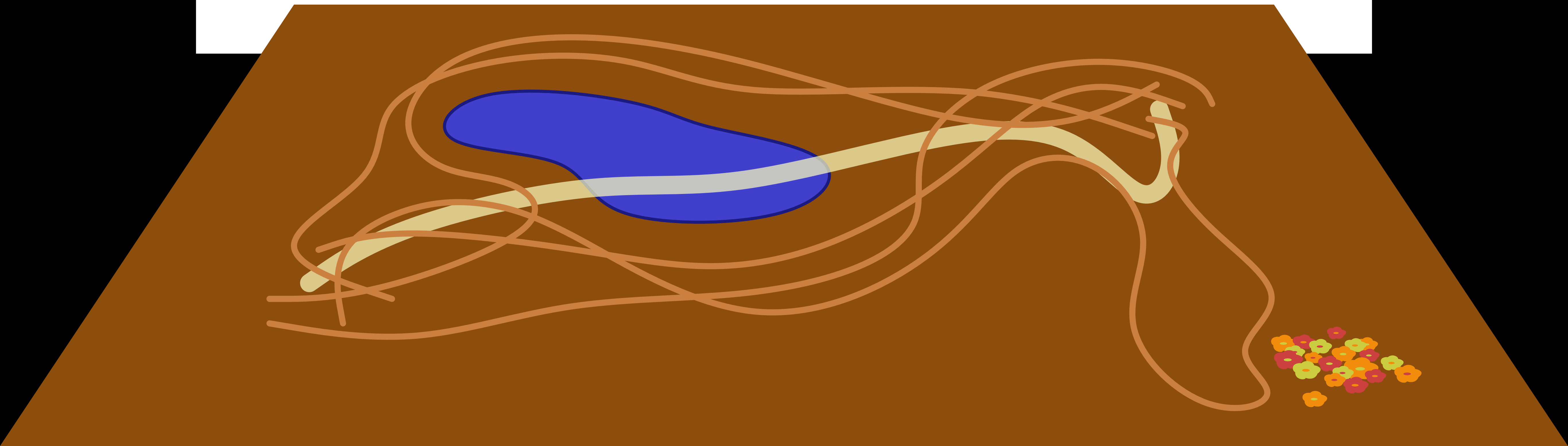

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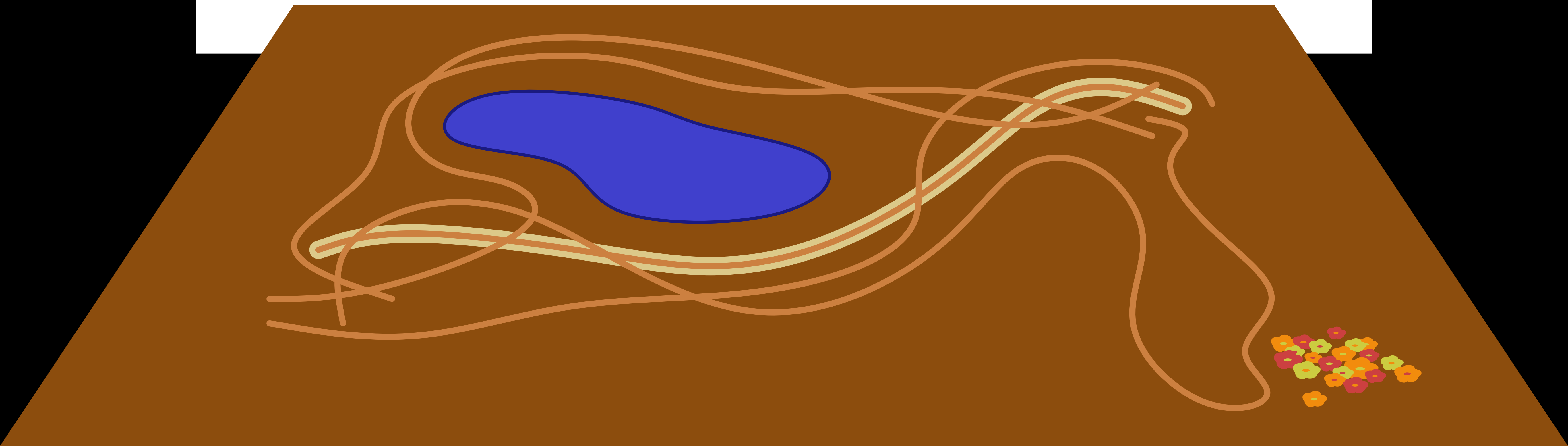

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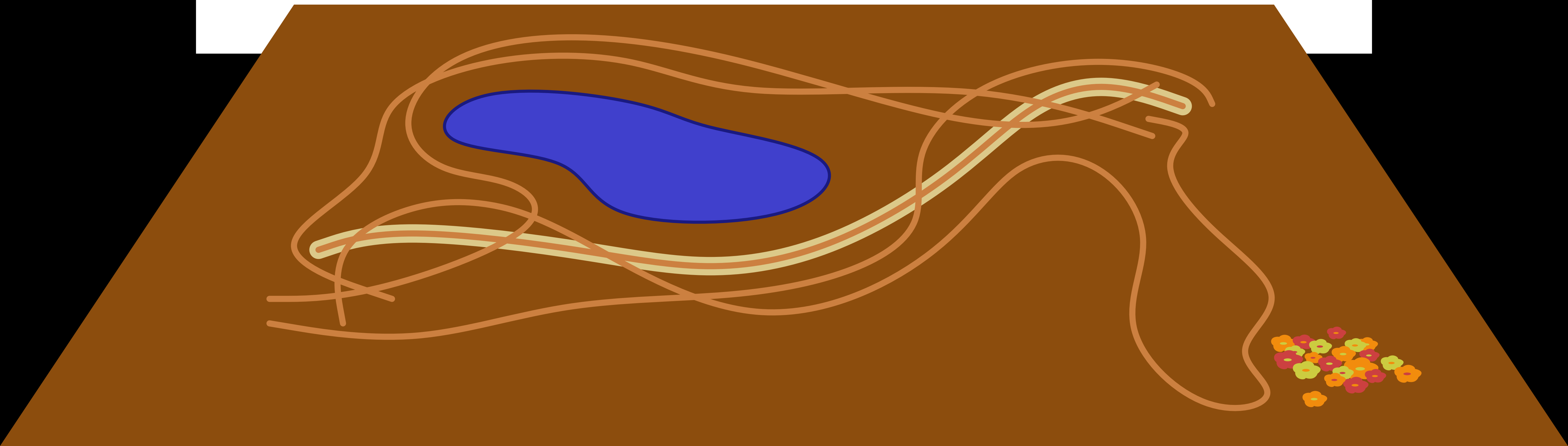

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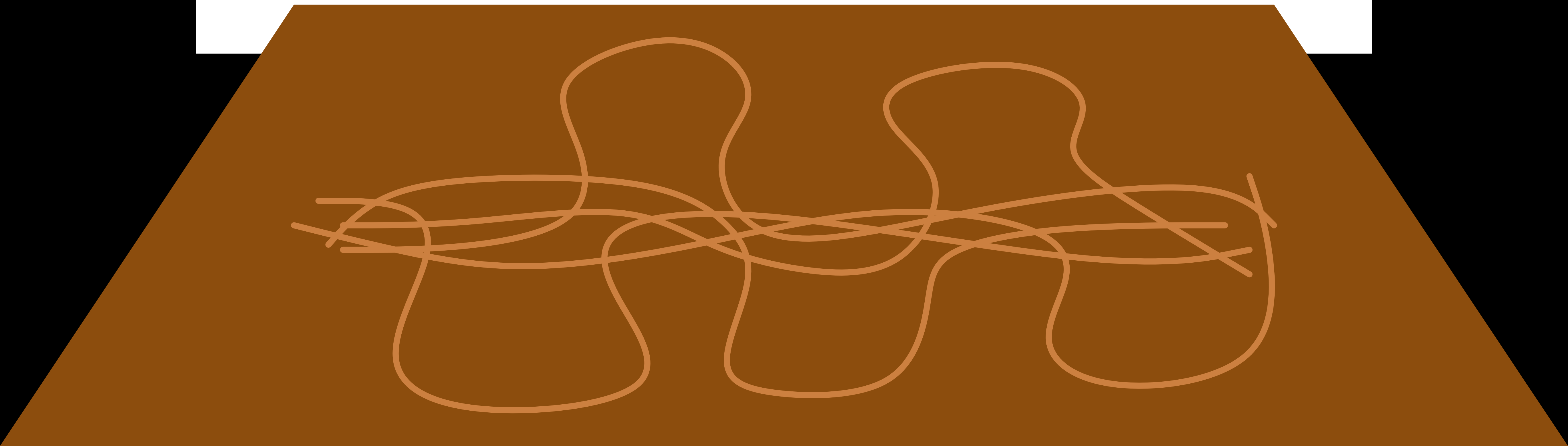

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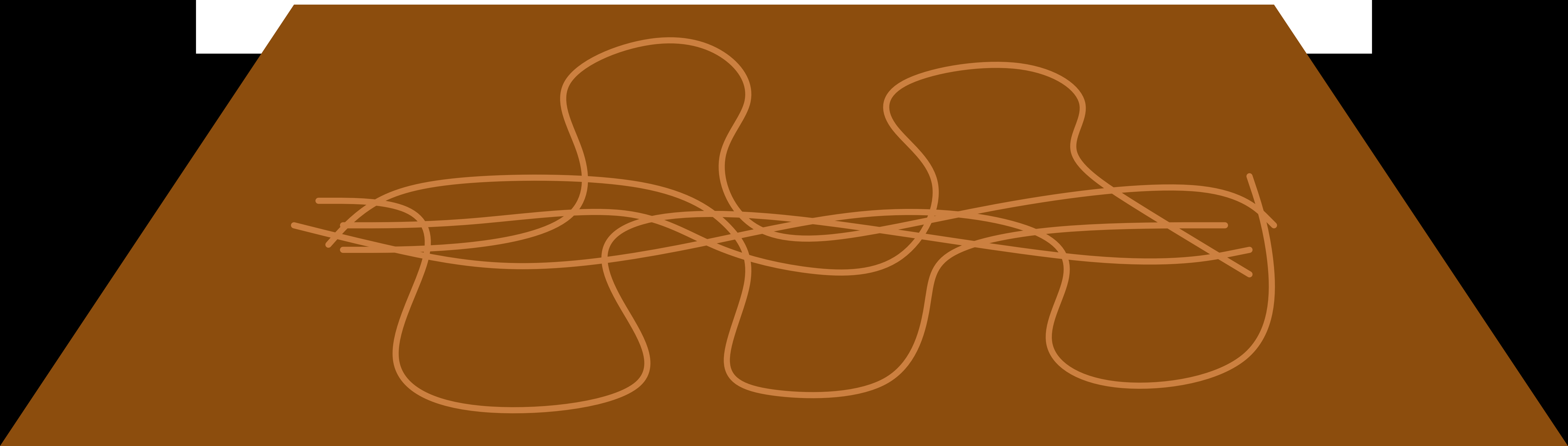

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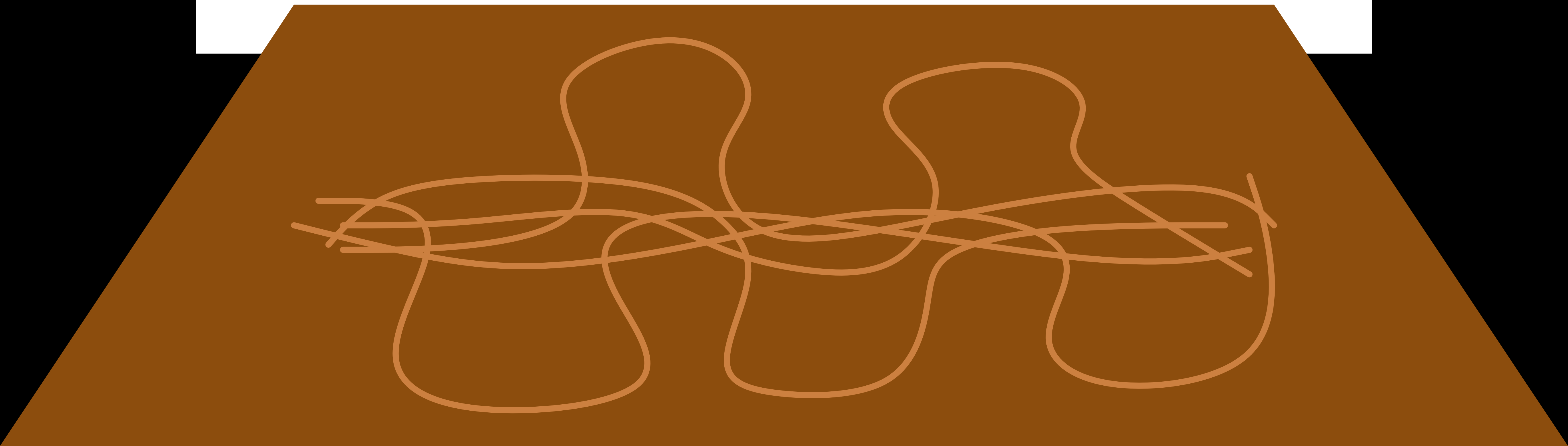

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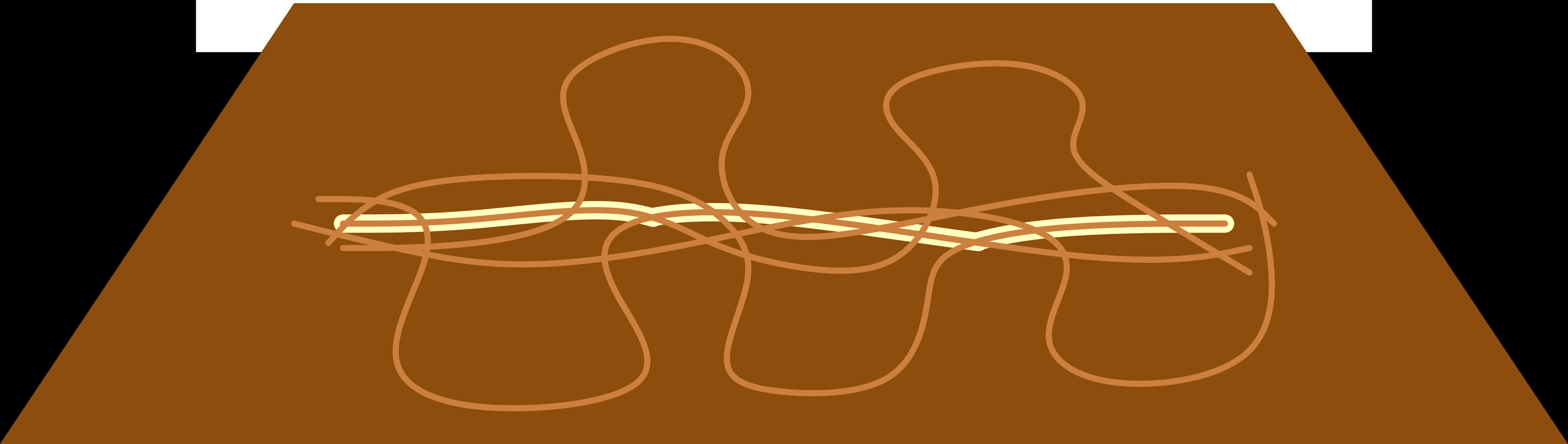

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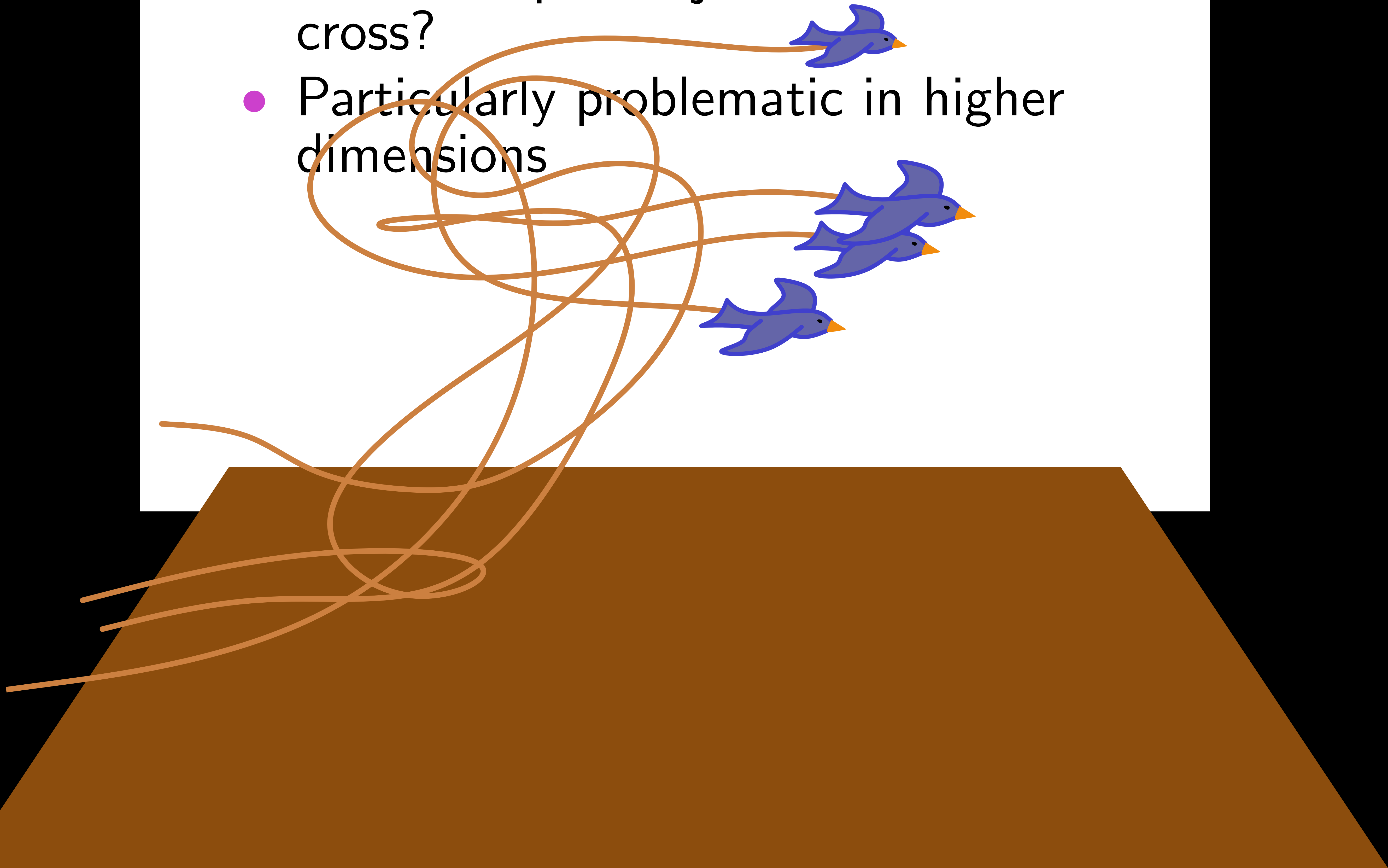

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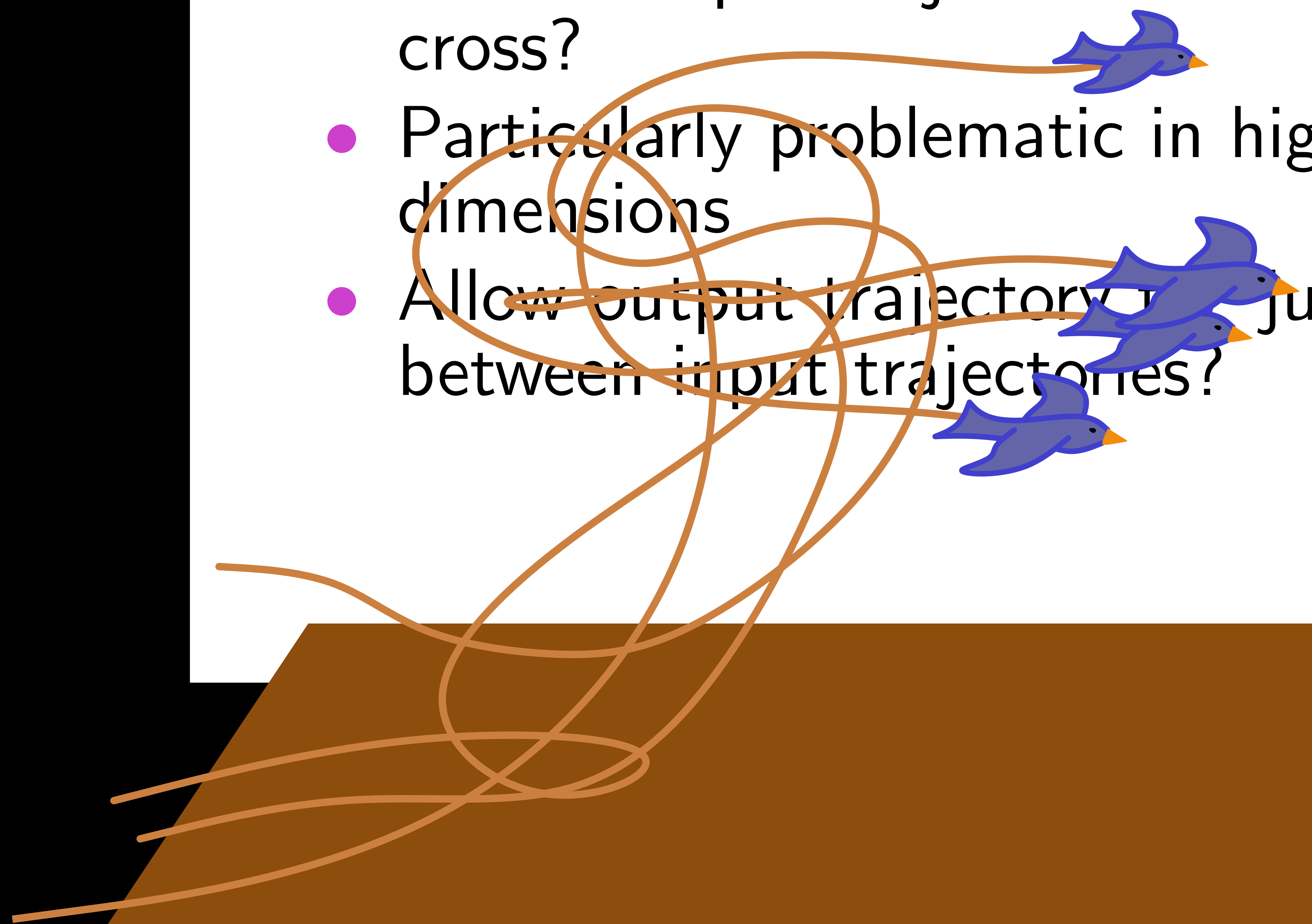
- Allow output trajectory to "jump" between input trajectories?



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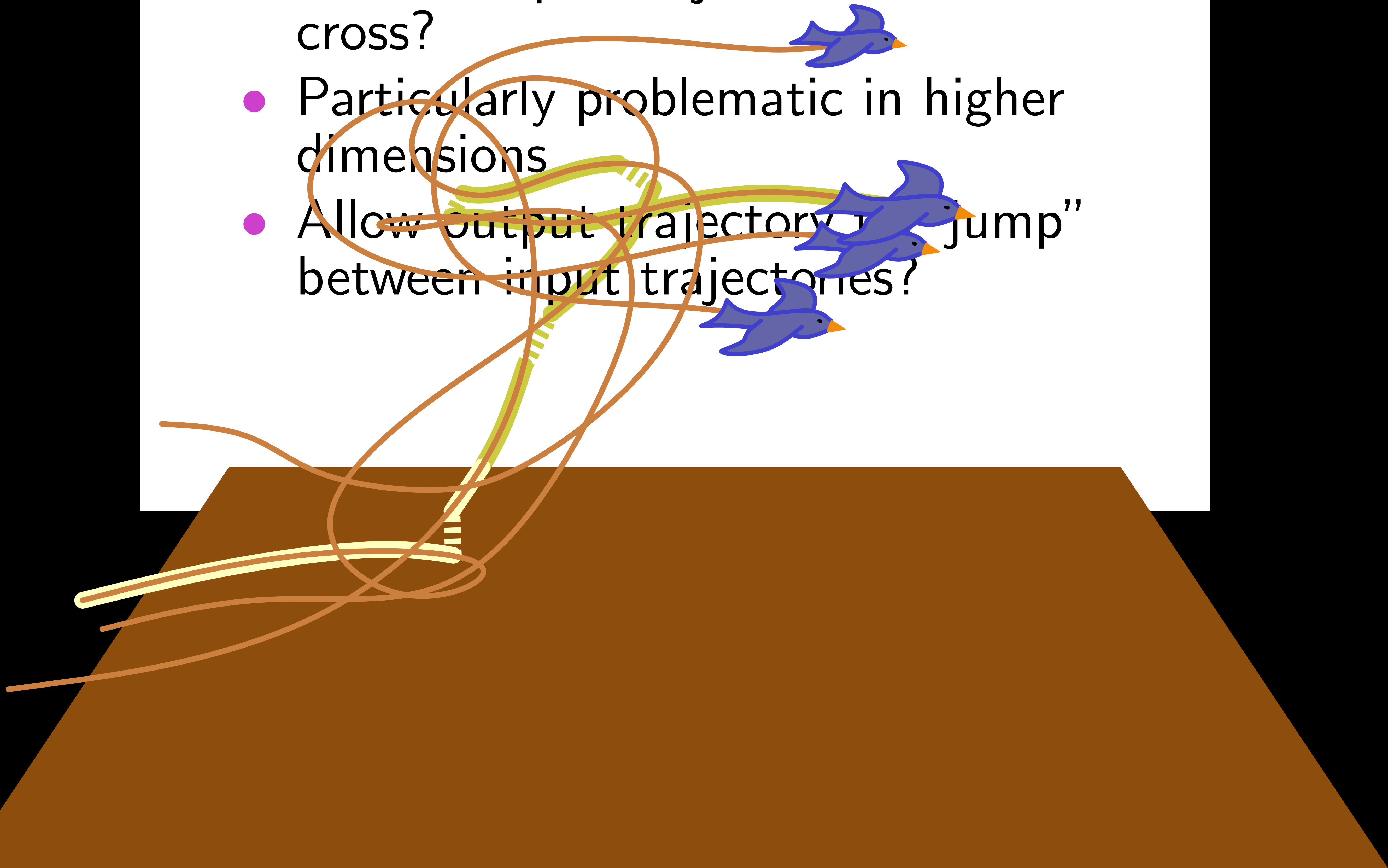

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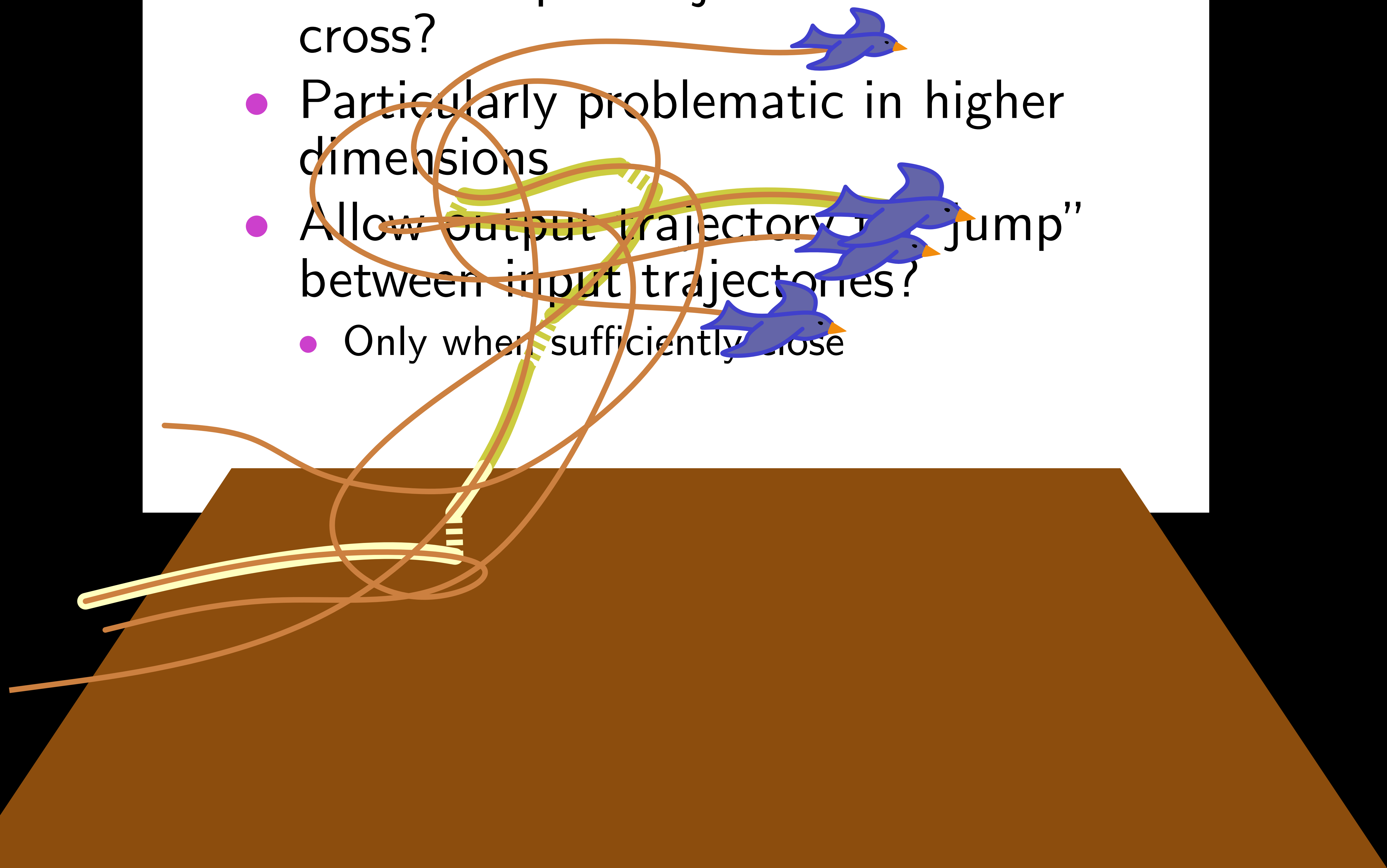
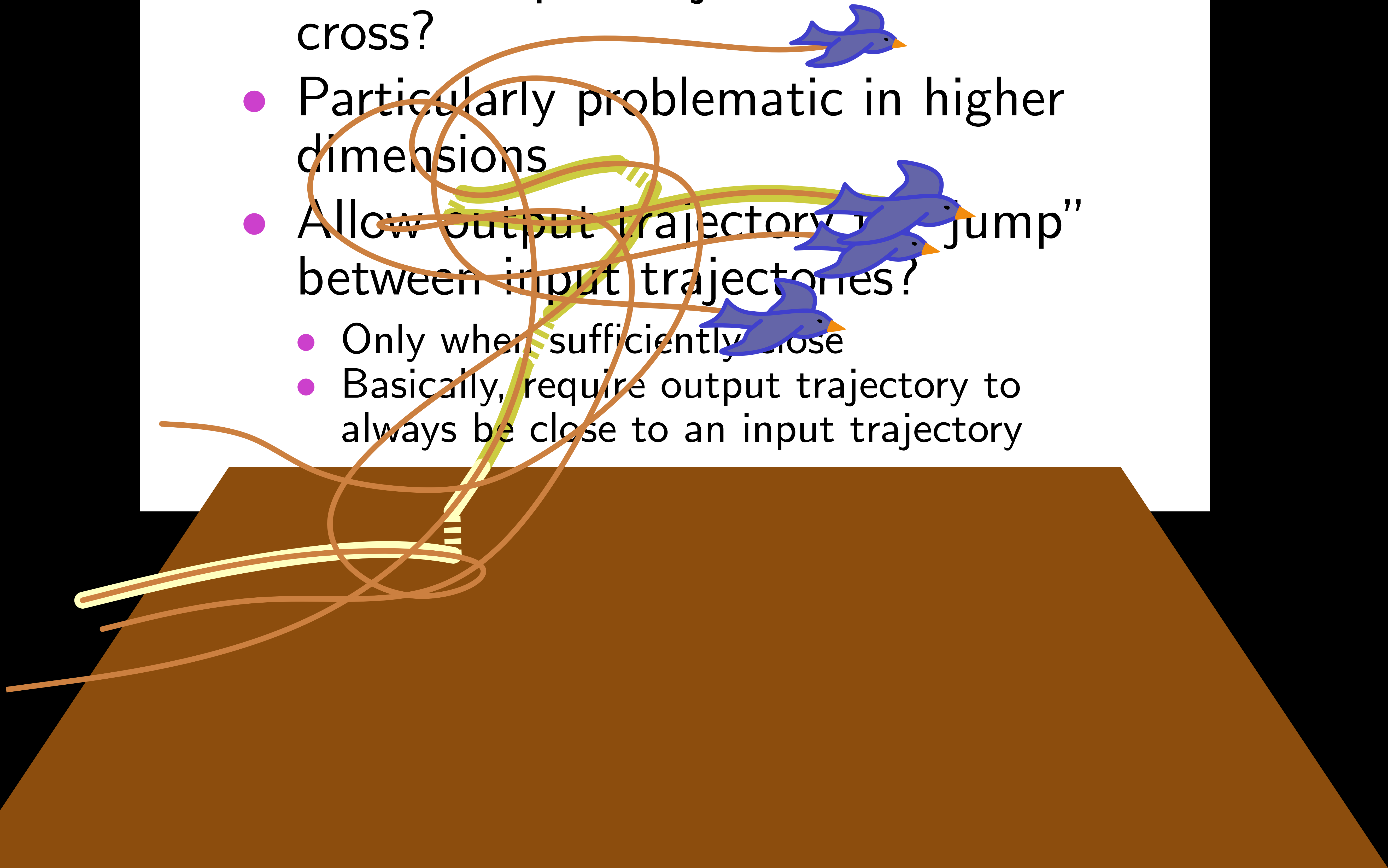


USE INPUT TRAJECTORY?

- What if input trajectories do not cross?
- Particularly problematic in higher dimensions
- Allow output trajectory to "jump" between input trajectories?
  - Only when sufficiently close
  - Basically, require output trajectory to always be close to an input trajectory



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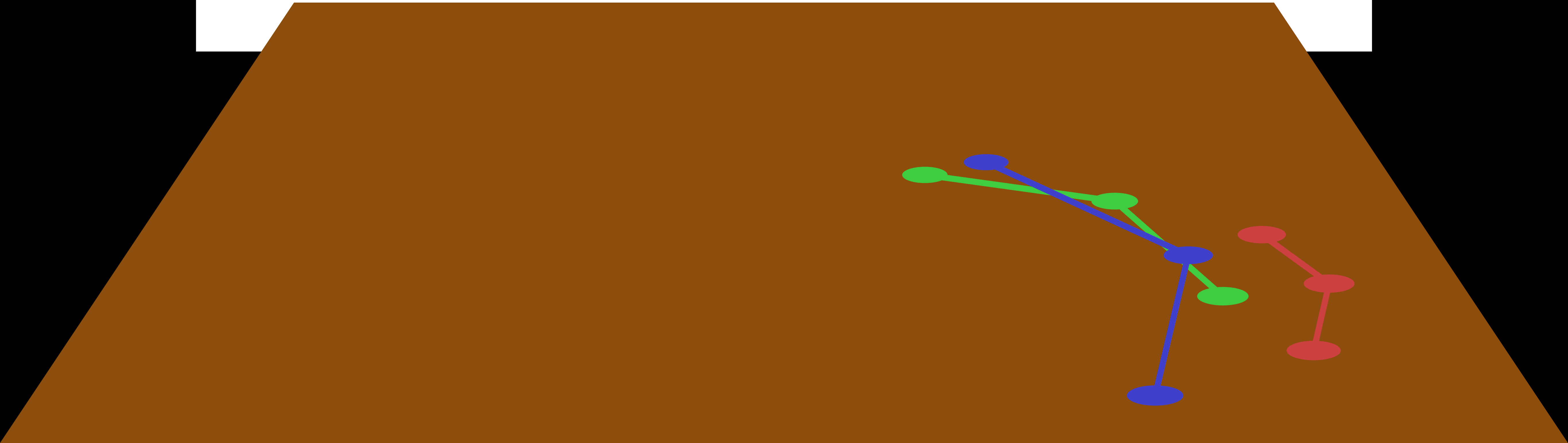

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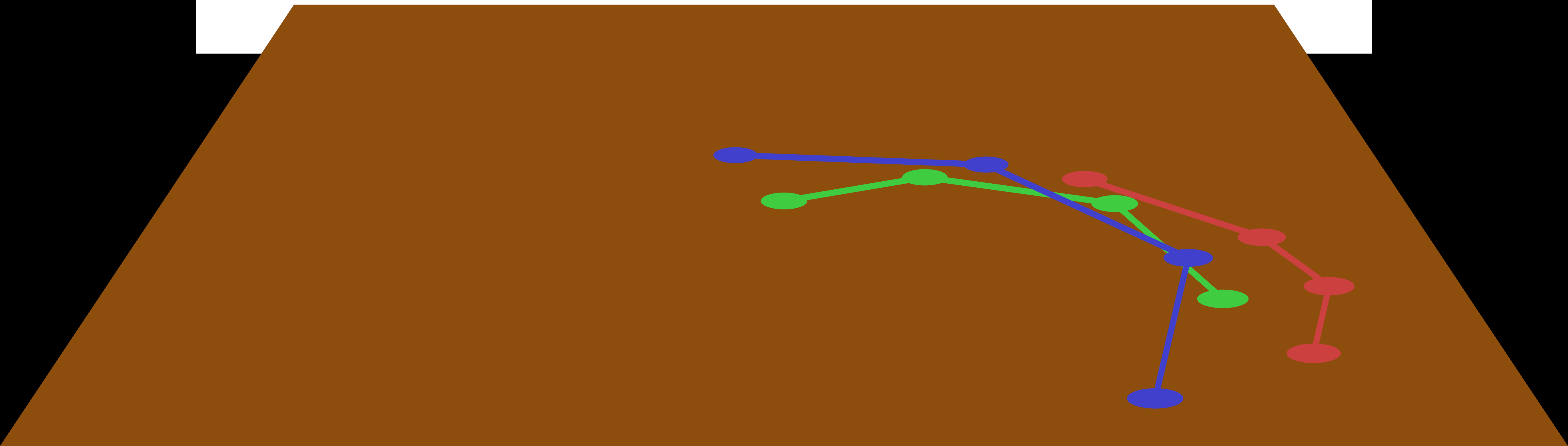

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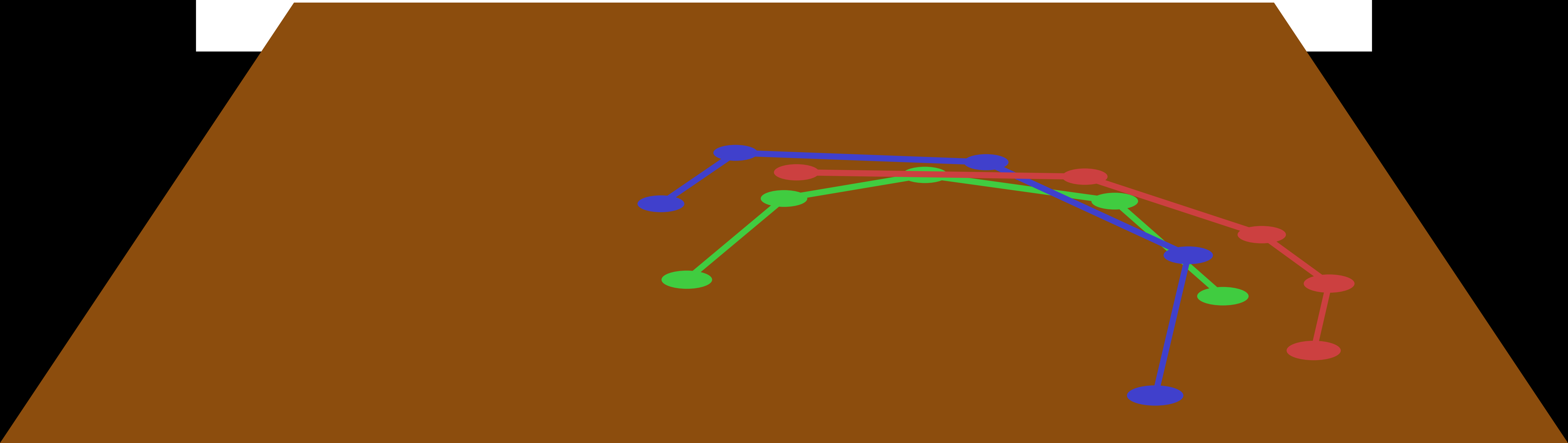

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- Depending on the application, time may be relevant or not
  - These curves are quite similar
  - But at $t = 3$, the points were spread
- See time as another dimension?

DO YOU HAVE TIME TO CARE?

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- Depending on the application, time may be relevant or not
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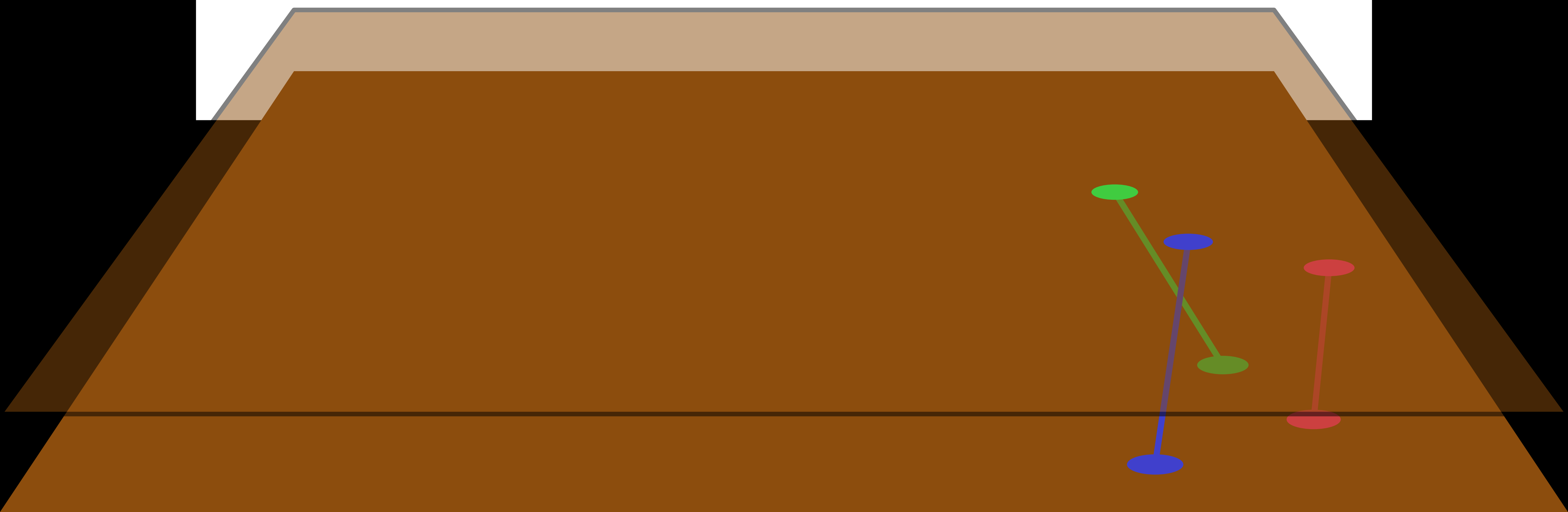

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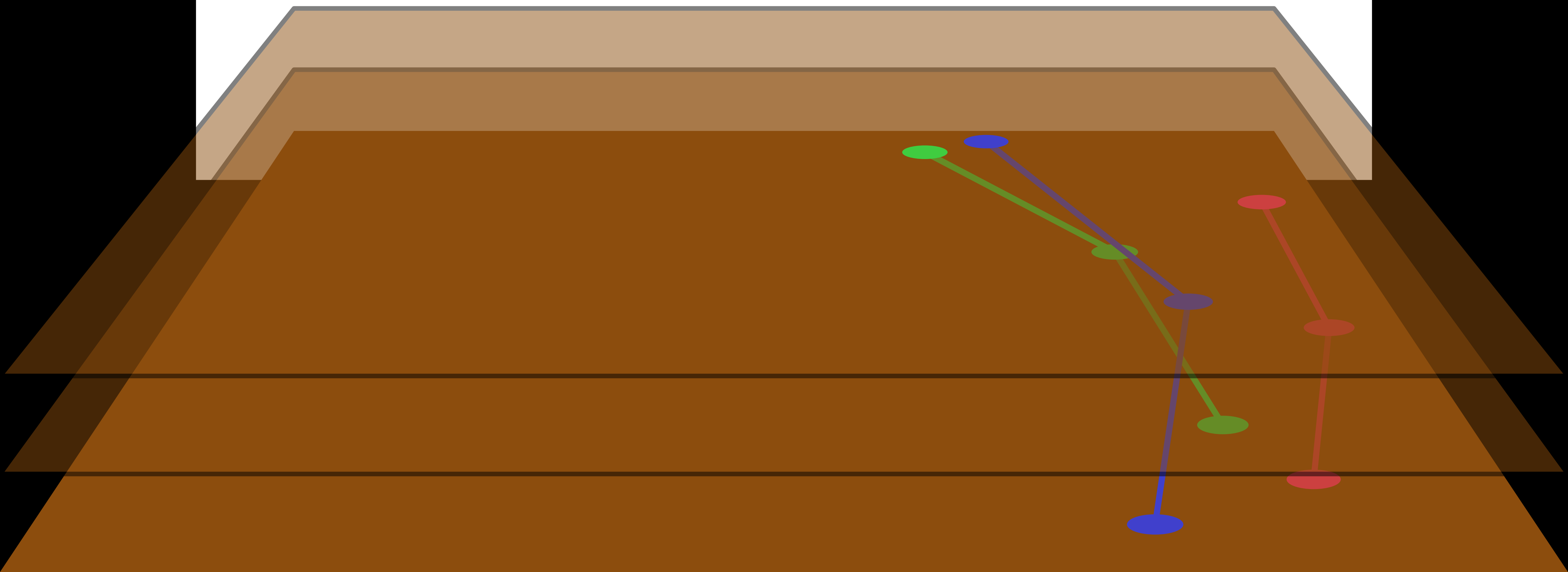

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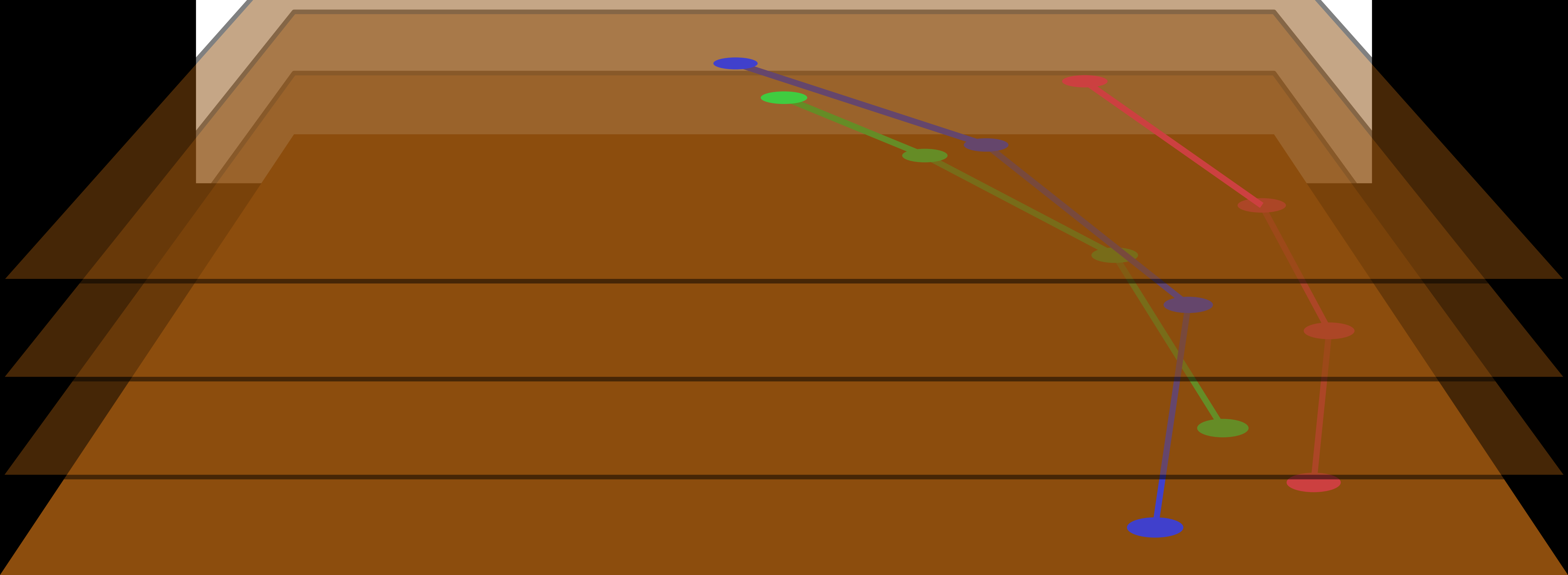

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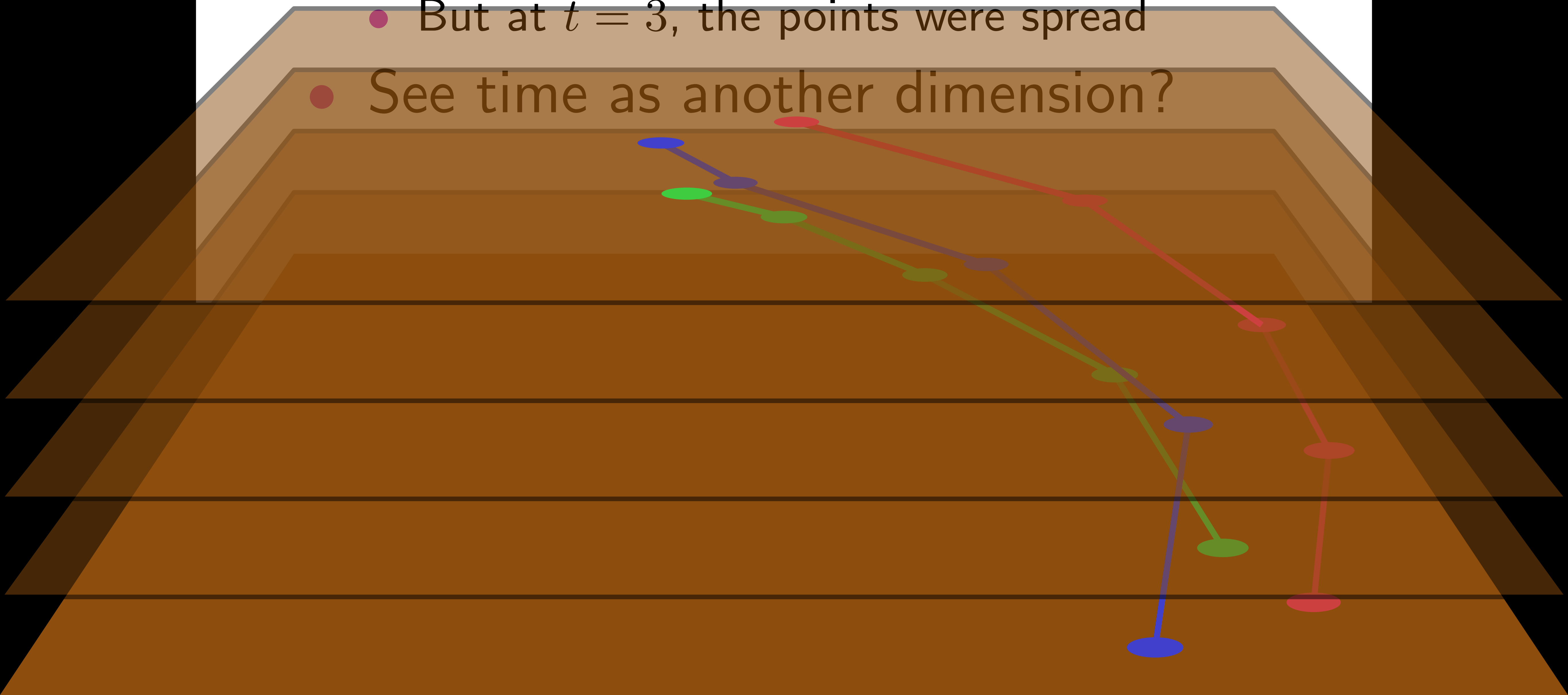

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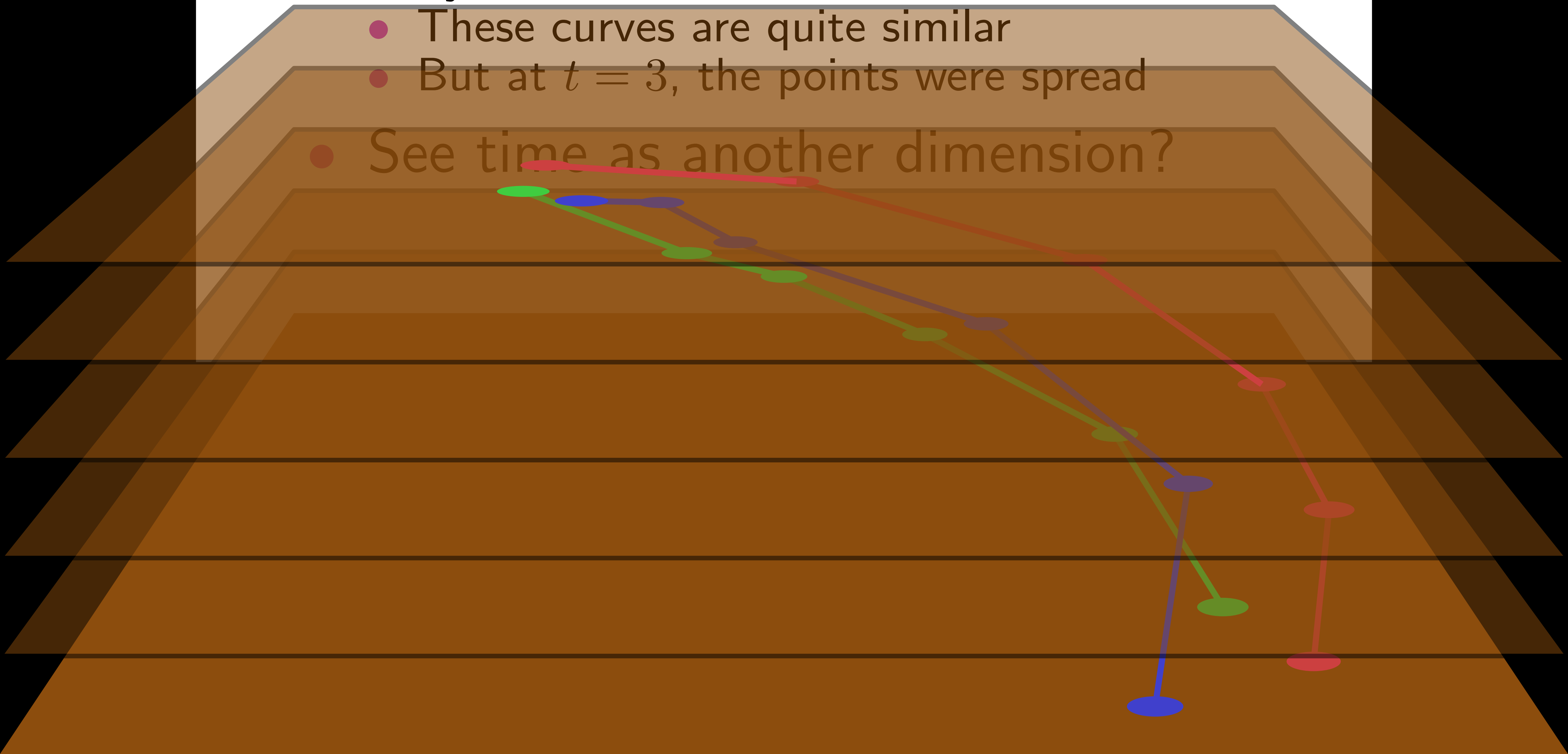

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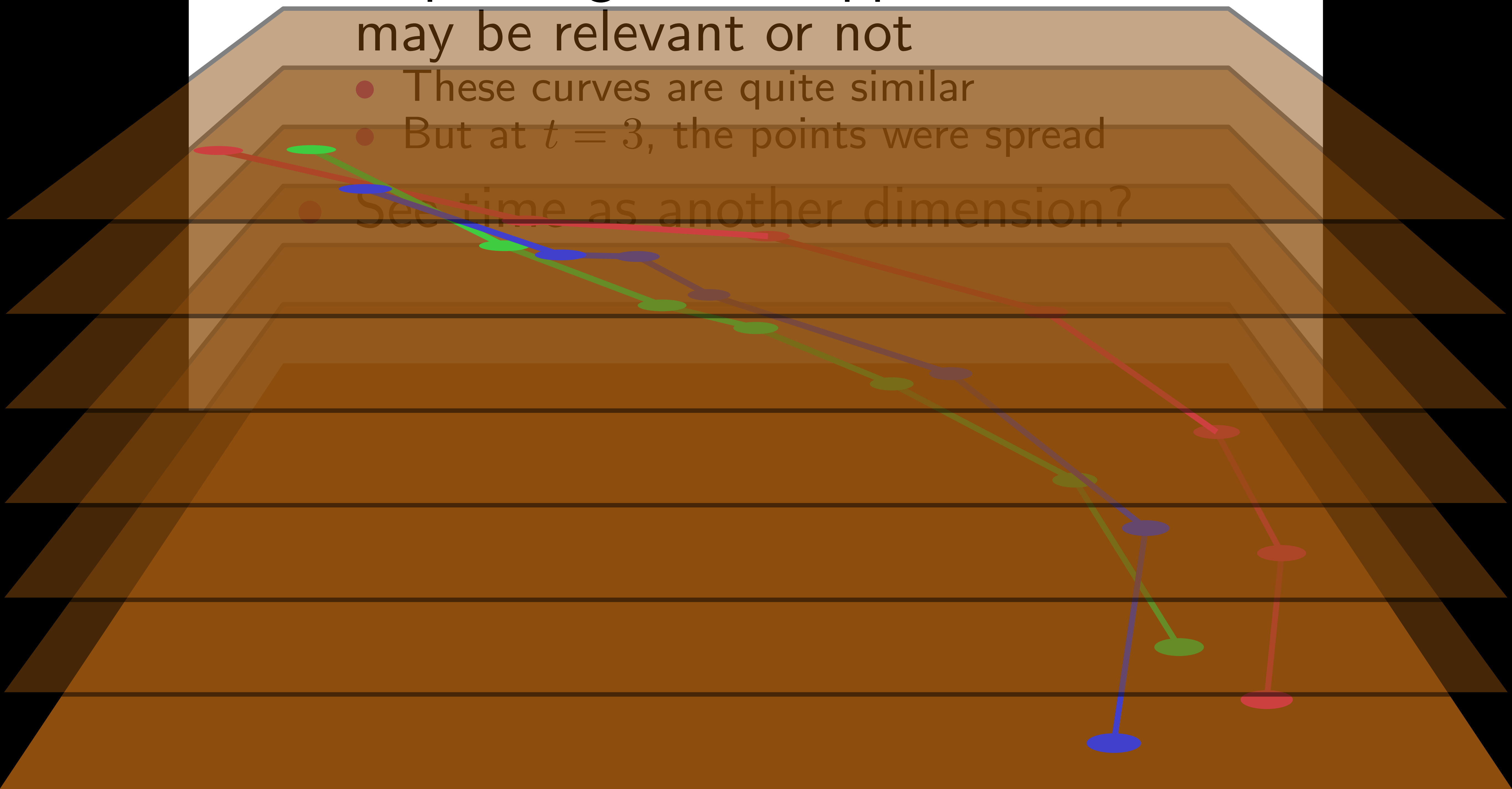

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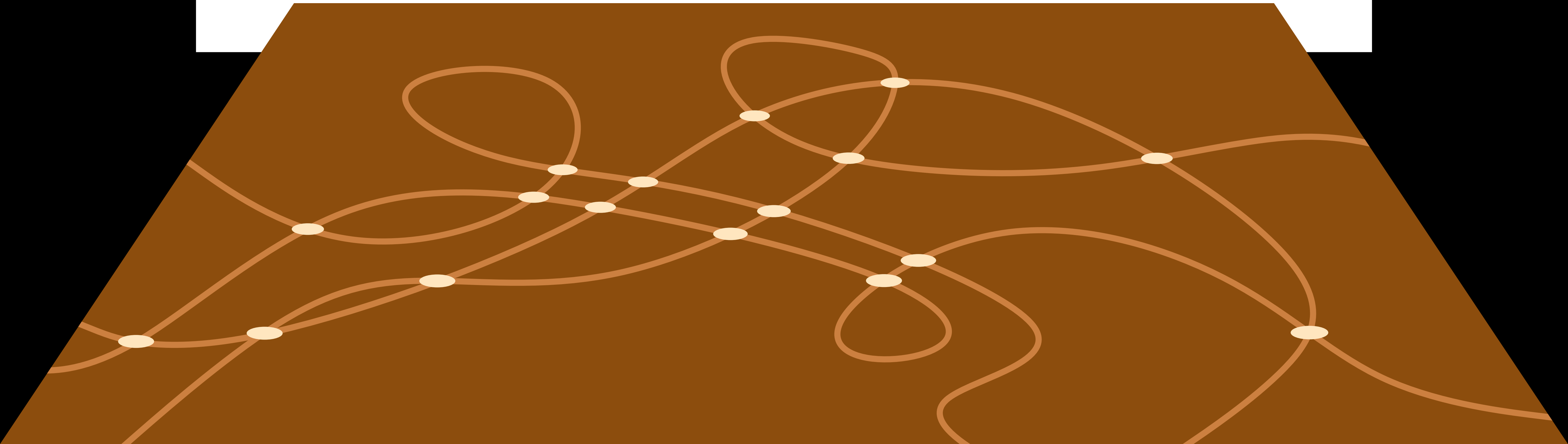

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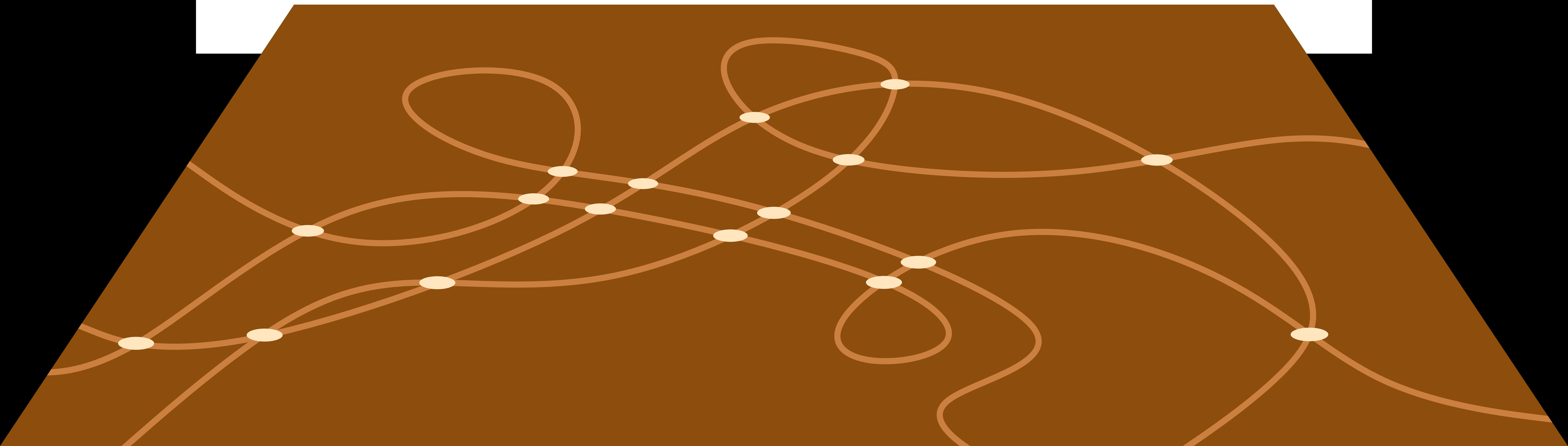

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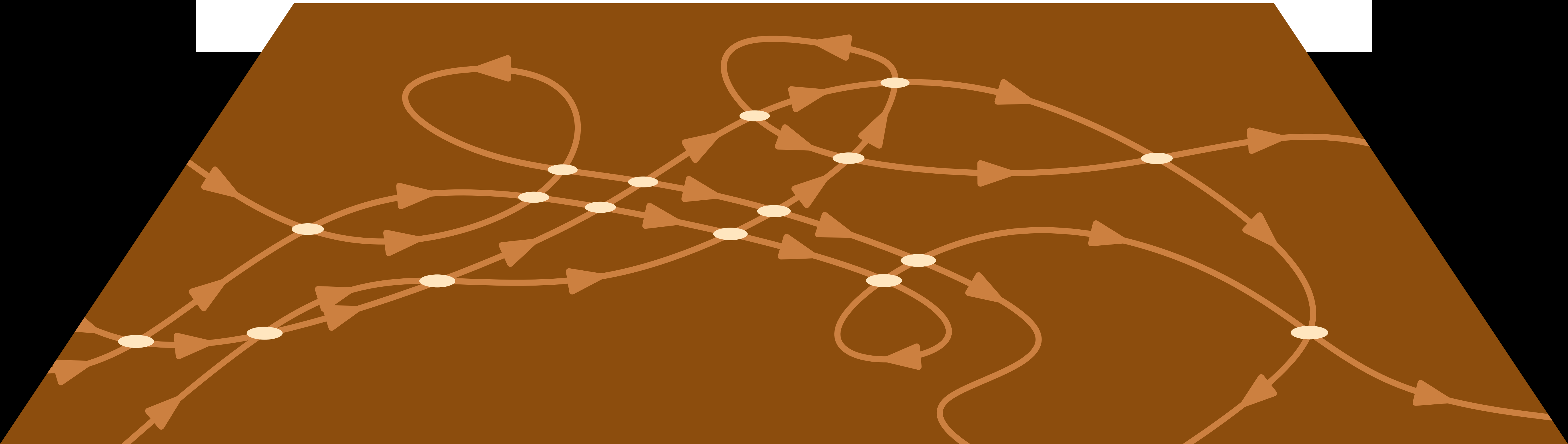

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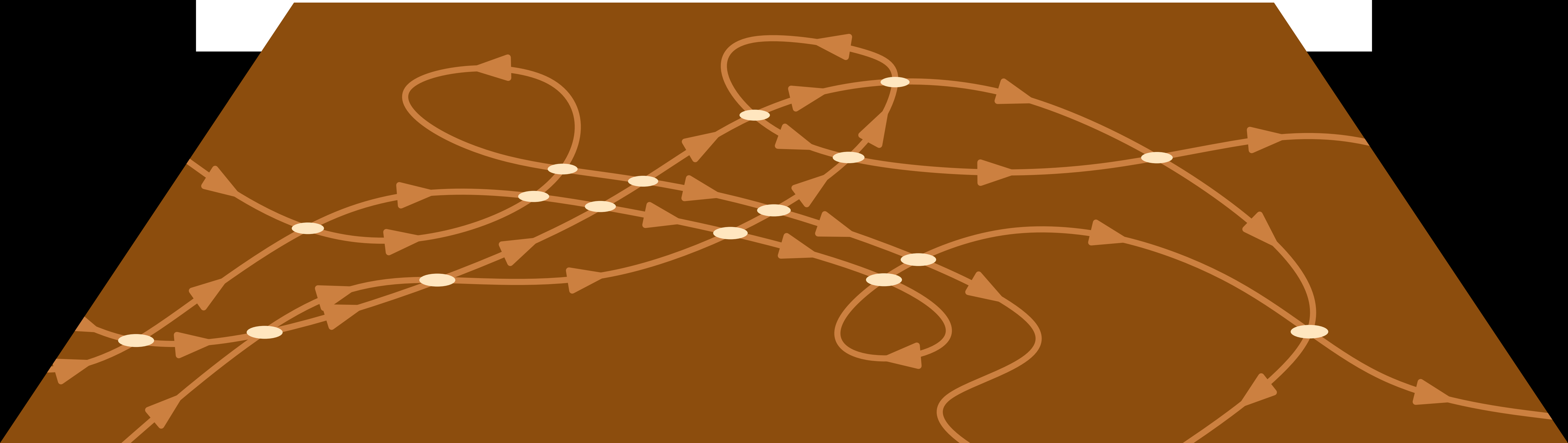

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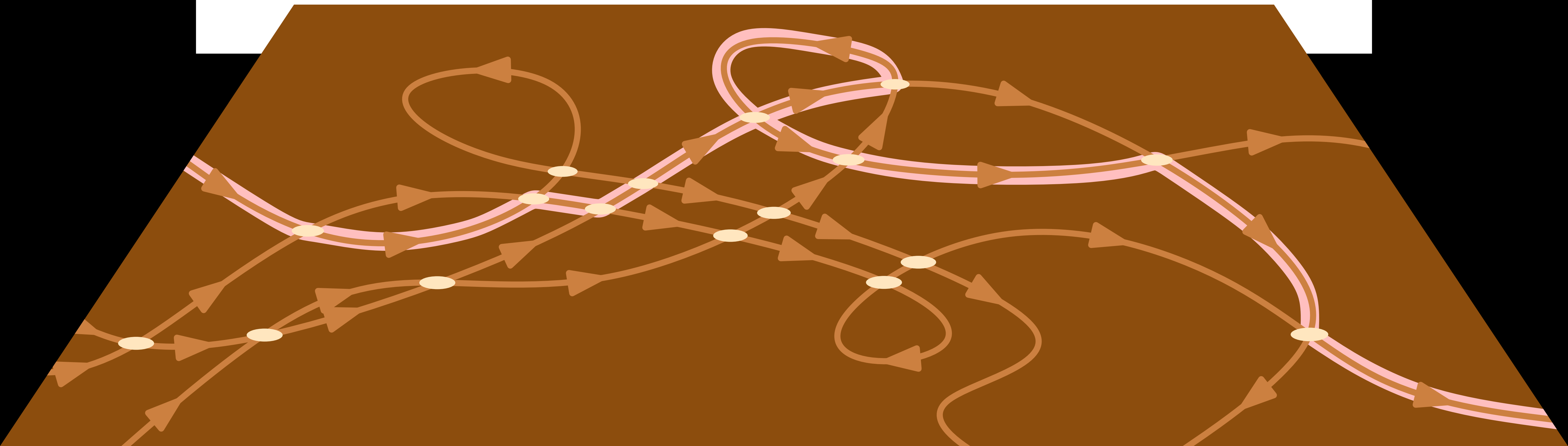

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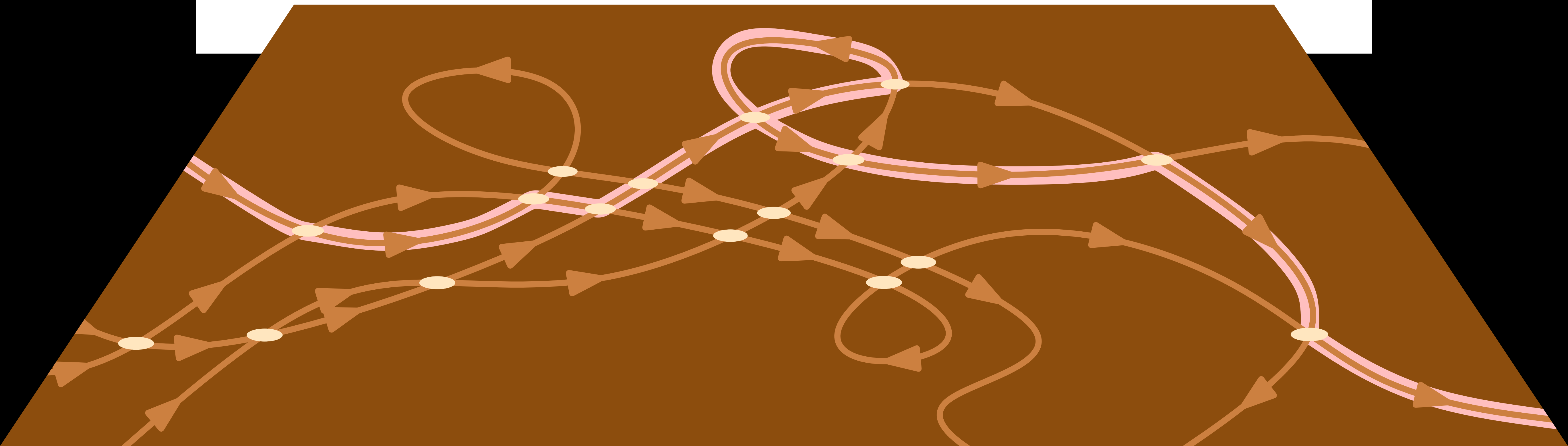
  - It should have sensible topology



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  - It should have sensible topology
  - But we cannot ignore the geometry

PROBLEM STATEMENT

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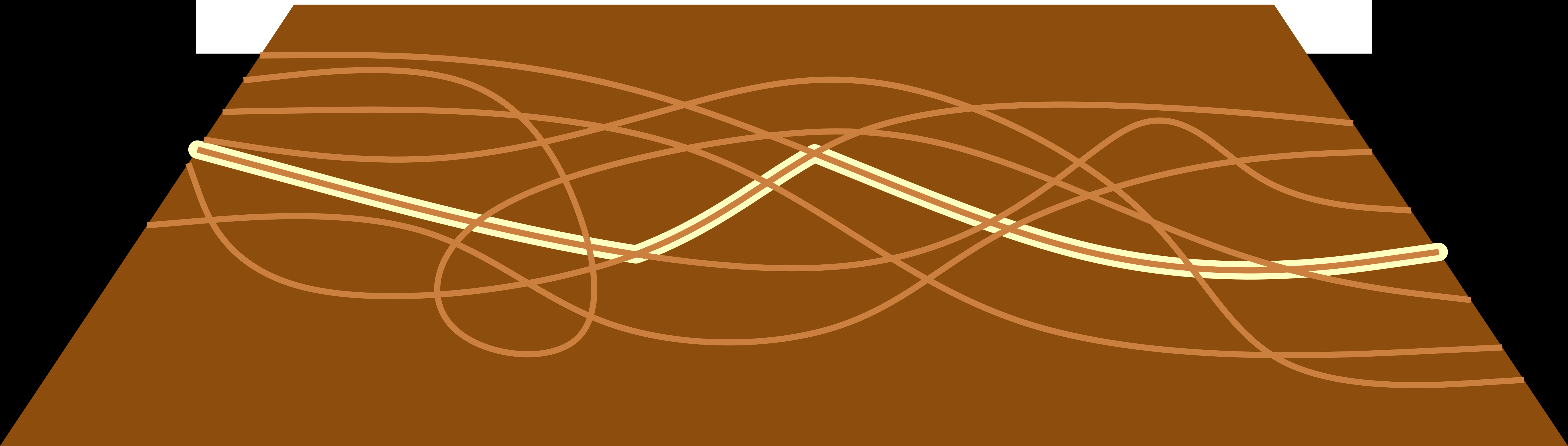
Given a set of ‘similar’ trajectories,
Find a representative trajectory:



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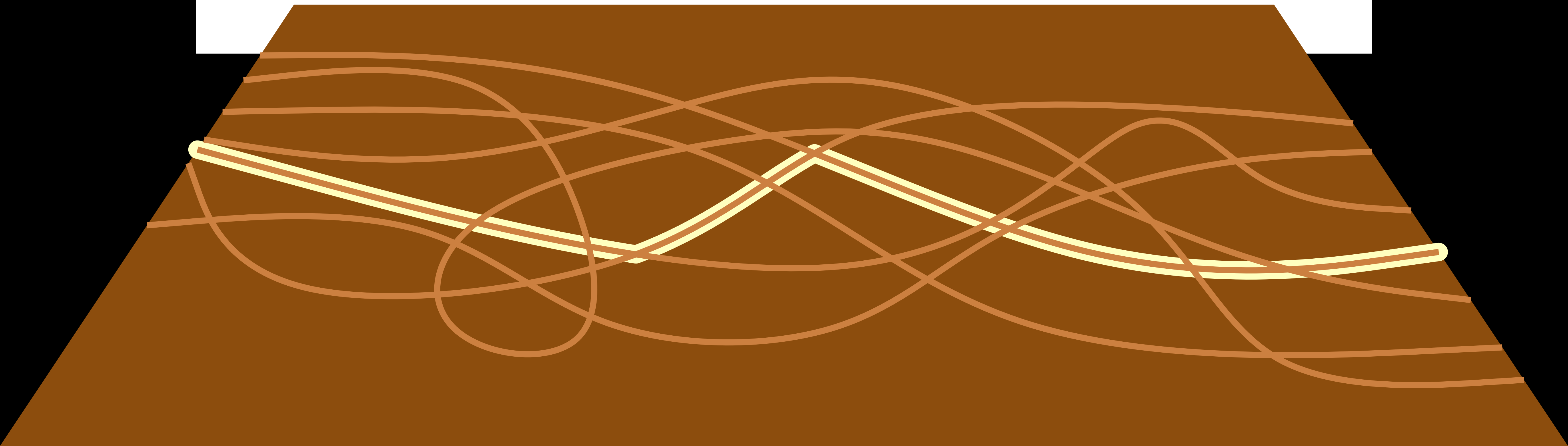
- consists of segments of input trajectories



PROBLEM STATEMENT

Given a set of ‘similar’ trajectories,
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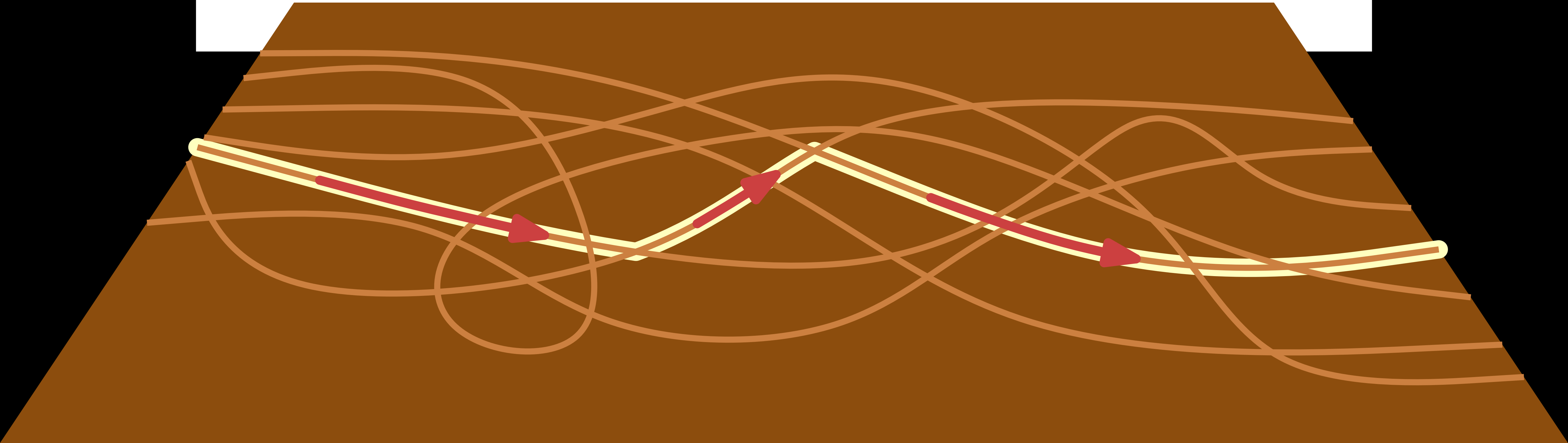

PROBLEM STATEMENT

Given a set of 'similar' trajectories,
Find a representative trajectory:

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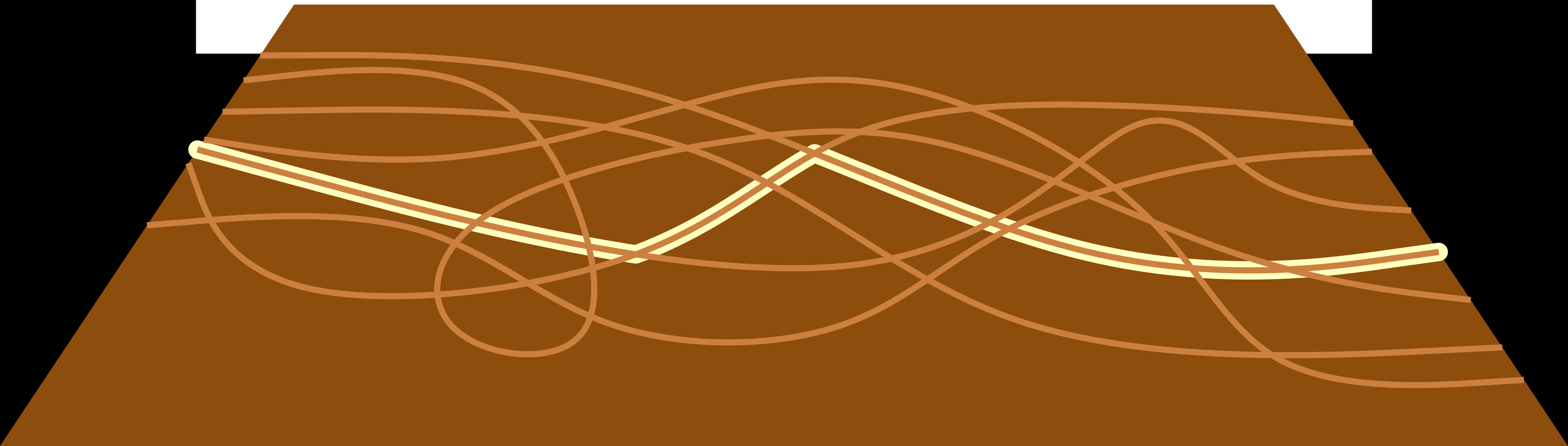
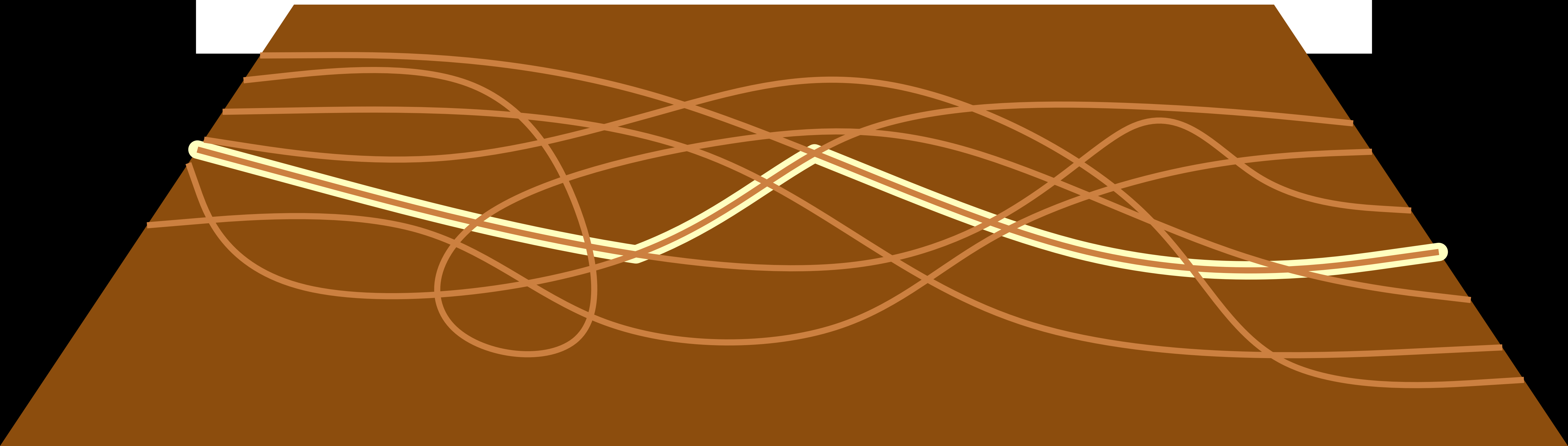


PROBLEM STATEMENT

Given a set of ‘similar’ trajectories,
Find a representative trajectory:

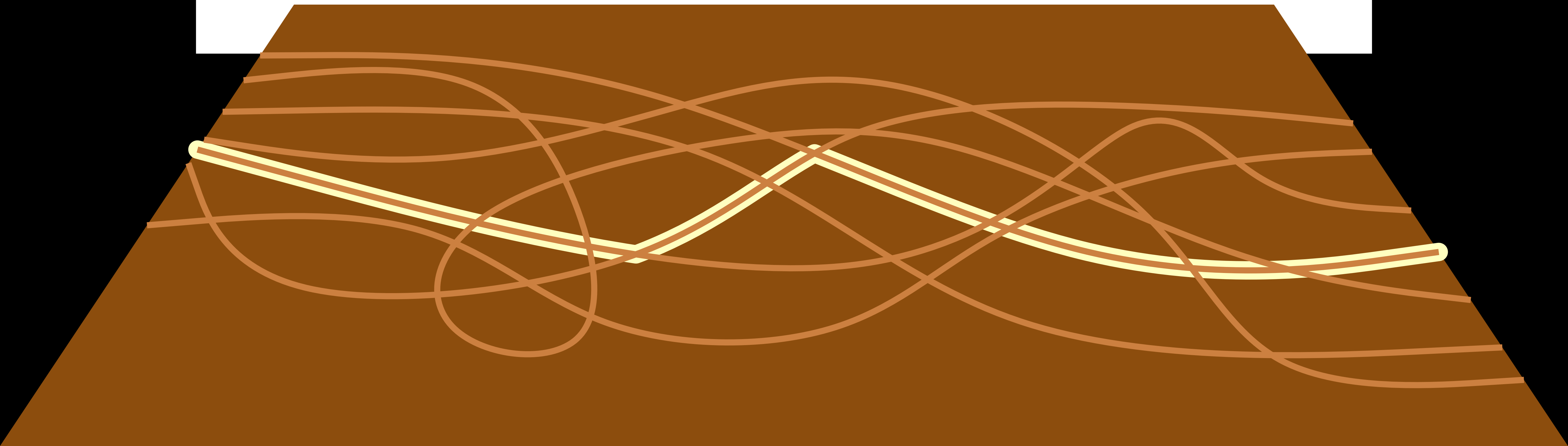
- consists of segments of input trajectories
- uses each segment in the correct direction
- all segments appear in the correct order
- represents trajectories ‘well’



PROBLEM STATEMENT

Given a set of 'similar' trajectories,
Find a representative trajectory:

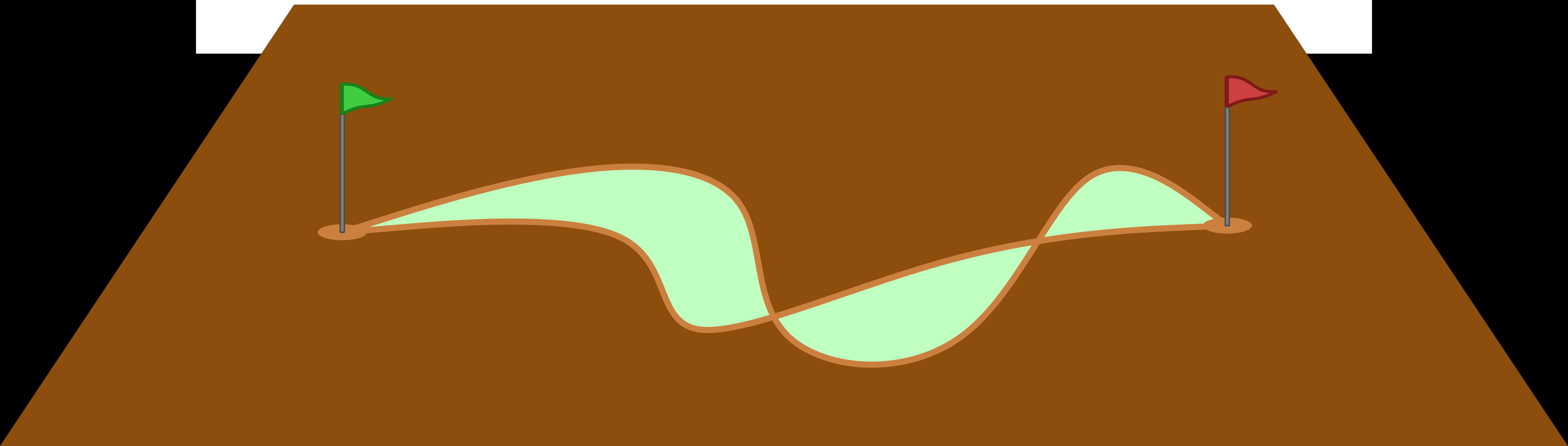
- consists of segments of input trajectories
- uses each segment in the correct direction
- all segments appear in the correct order
- minimizes homotopy area



HOMOTOPY AREA*

$$\int_{p \in \mathbb{R}^2} |\omega(p, \gamma)| \; \mathrm{d}s(p)$$

* As defined in [Chambers, Wang '13]



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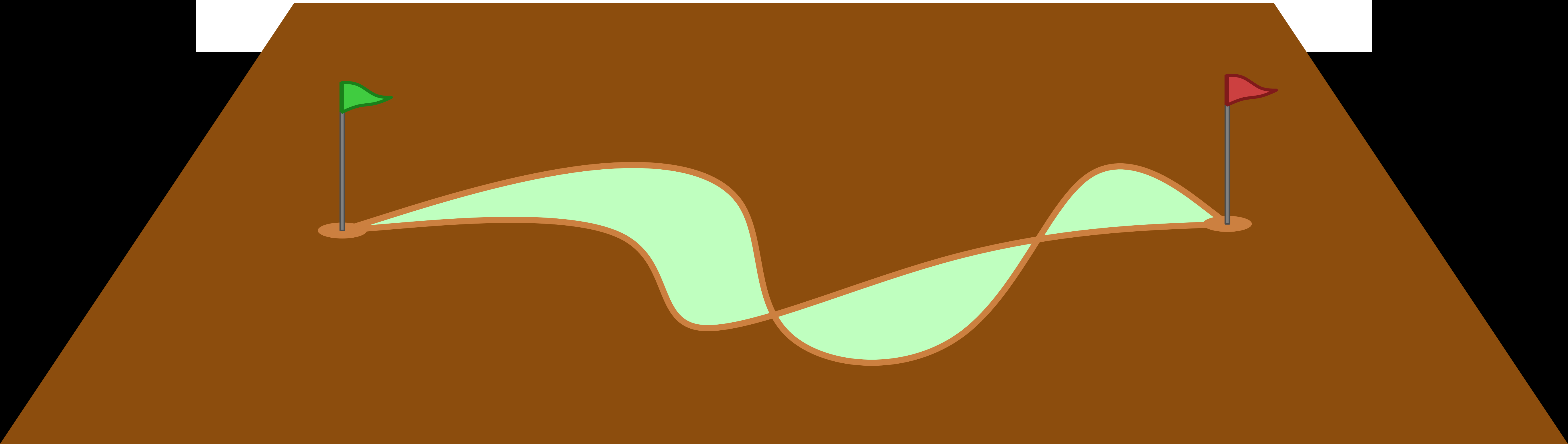

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Trajectories:

- start in s and end in t

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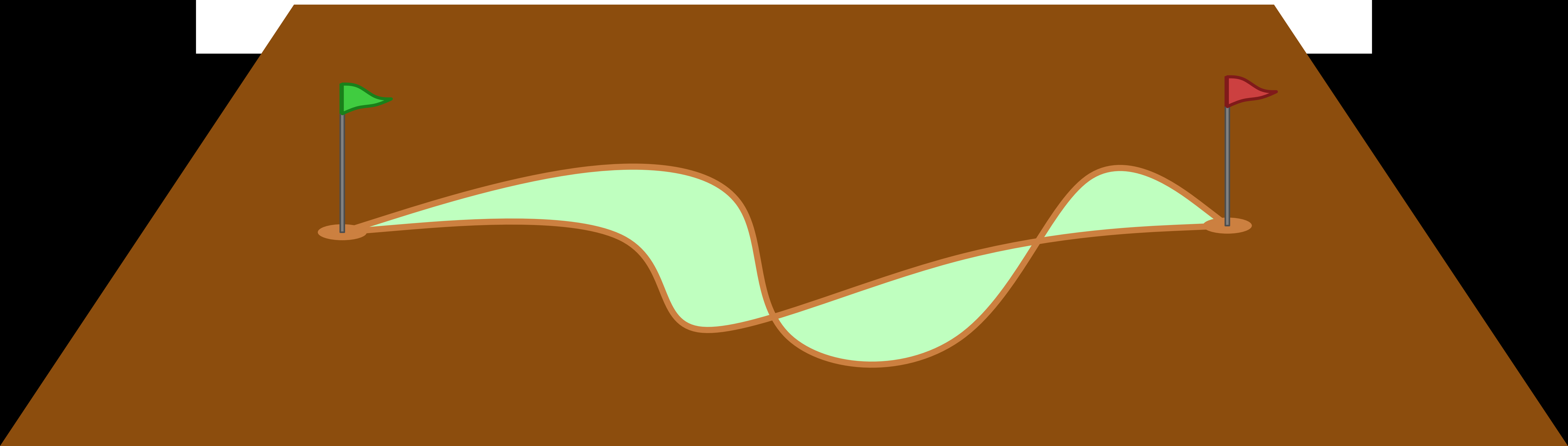

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- start in s and end in t
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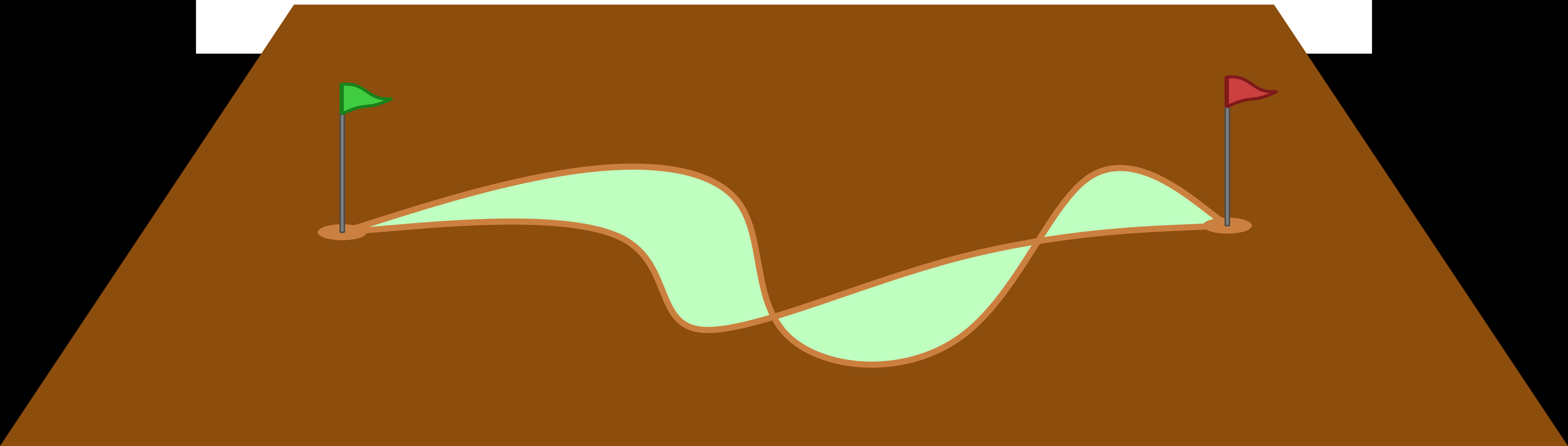

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Trajectories:

- start in s and end in t
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- are homotopic

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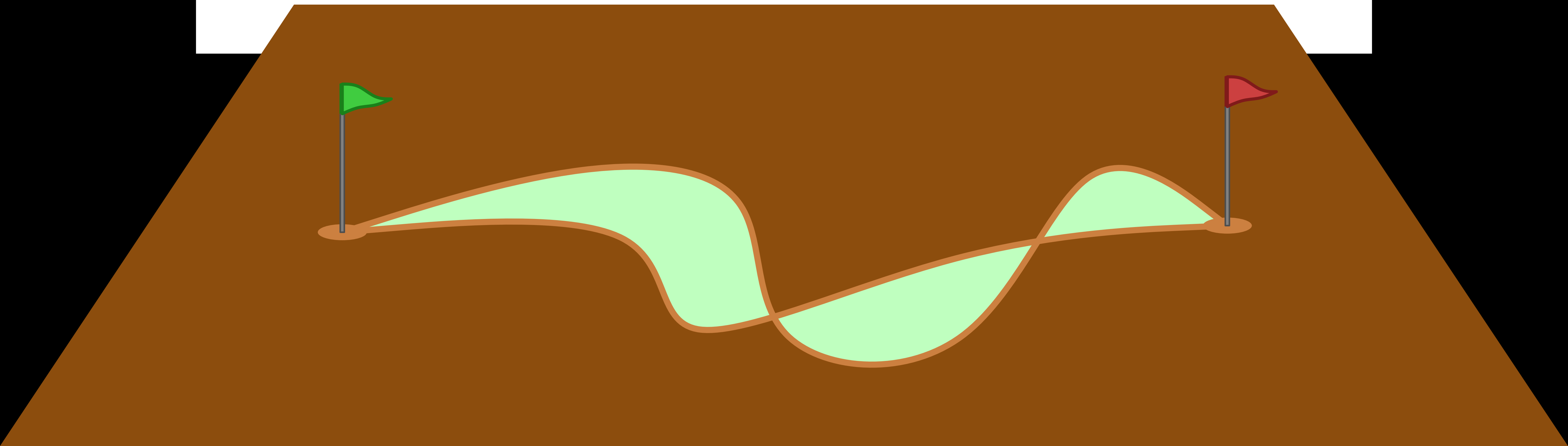

PROBLEM STATEMENT

Given a set of simple 'similar' trajectories, that all start in the same point and all end in the same point,
Find a representative trajectory:

- consists of segments of input trajectories
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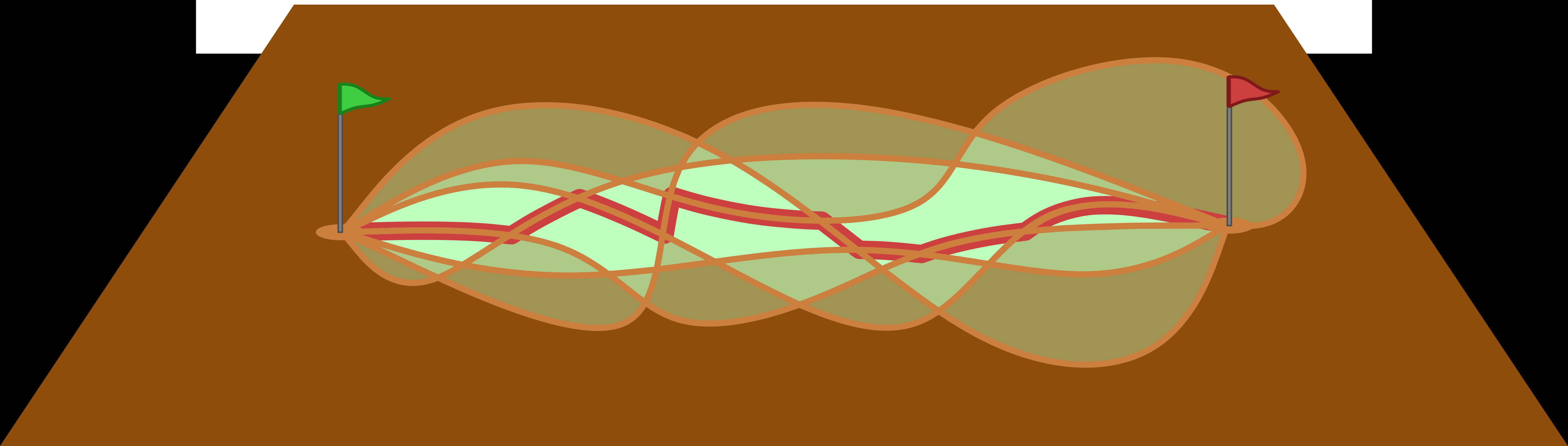
- is simple
- minimizes max/avg homotopy area

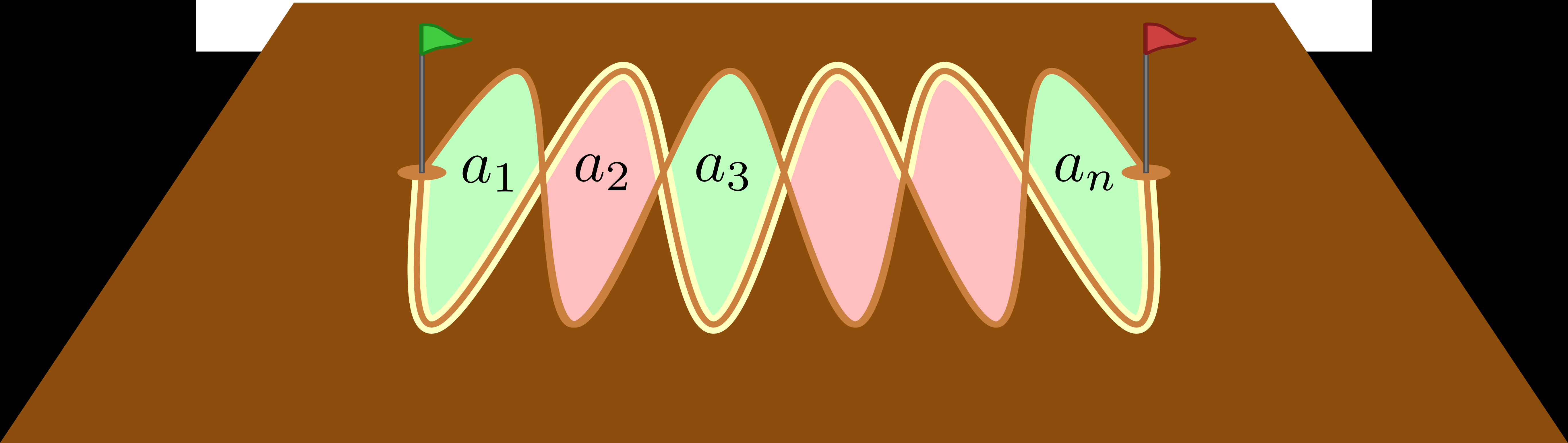


MIN MAX IS NP-HARD

Reduction from PARTITION:

Partition a set of integers $S = \{a_1, a_2, \ldots, a_n\}$ into two subsets S_1 and S_2 with equal total sums:

$$\sum_{a \in S_1} a = \sum_{a \in S_2} a$$



MIN AVG IS EQUIVALENT TO MEDIAN*

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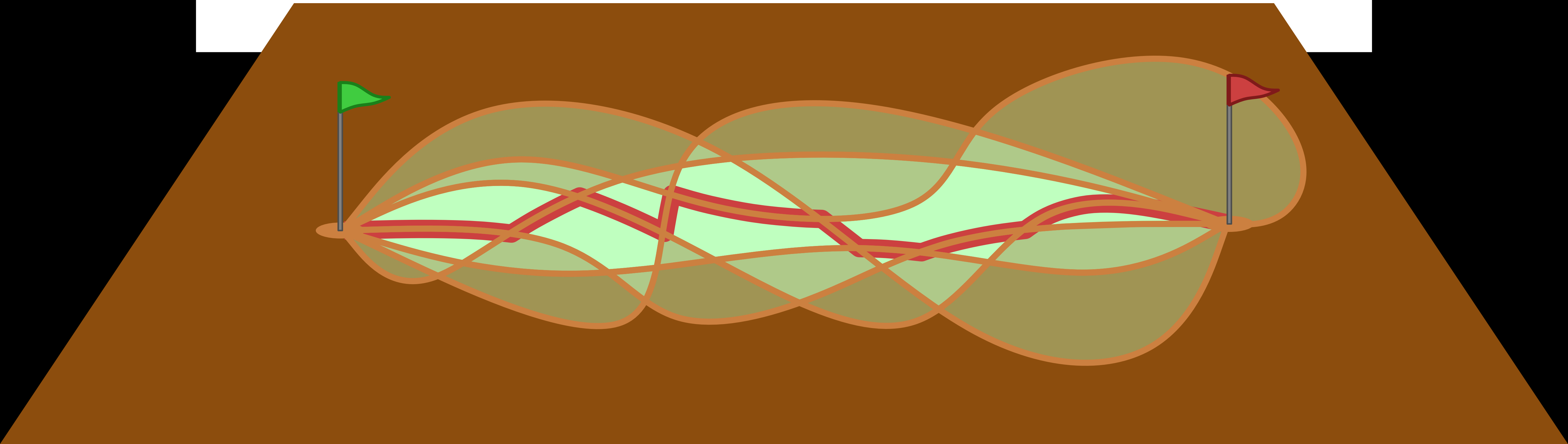

under certain conditions

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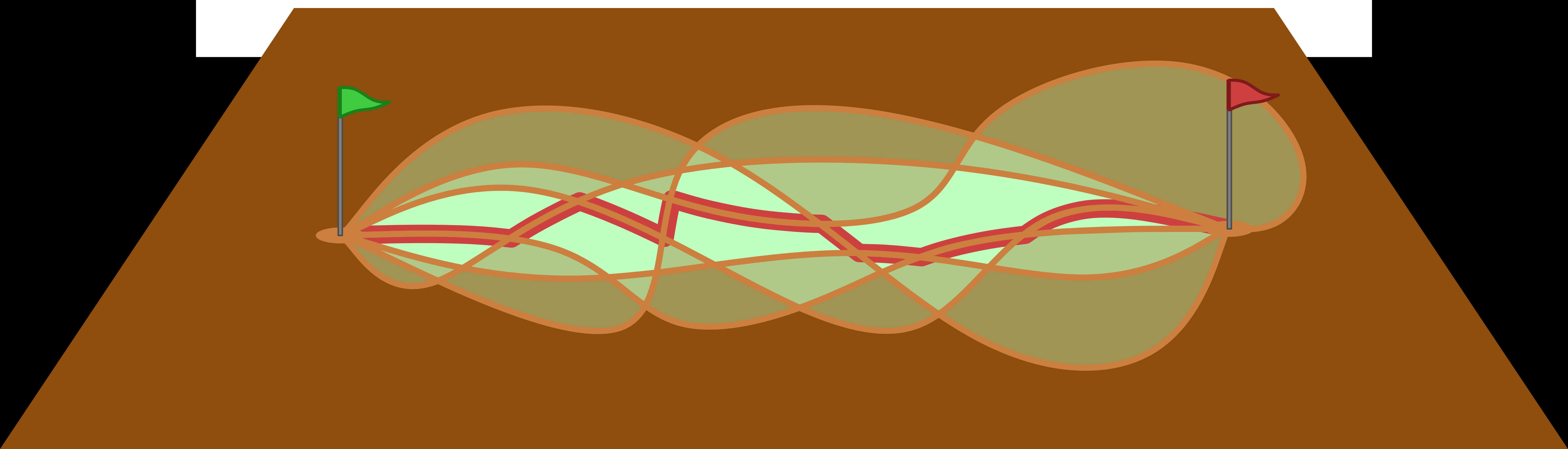

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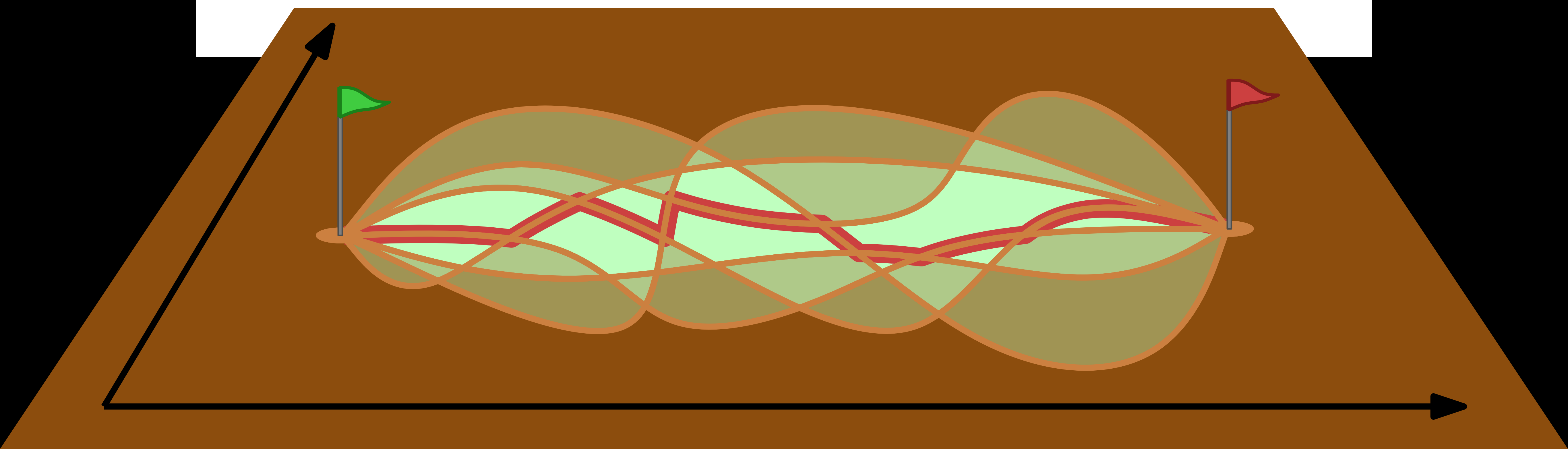

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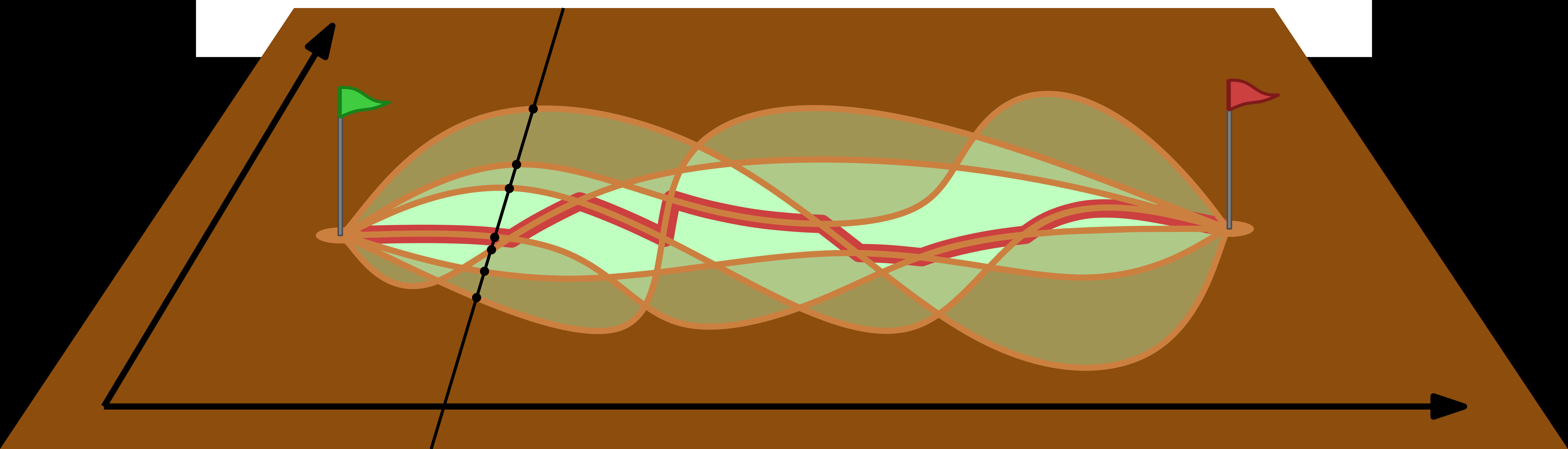

MIN AVG IS EQUIVALENT TO MEDIAN*

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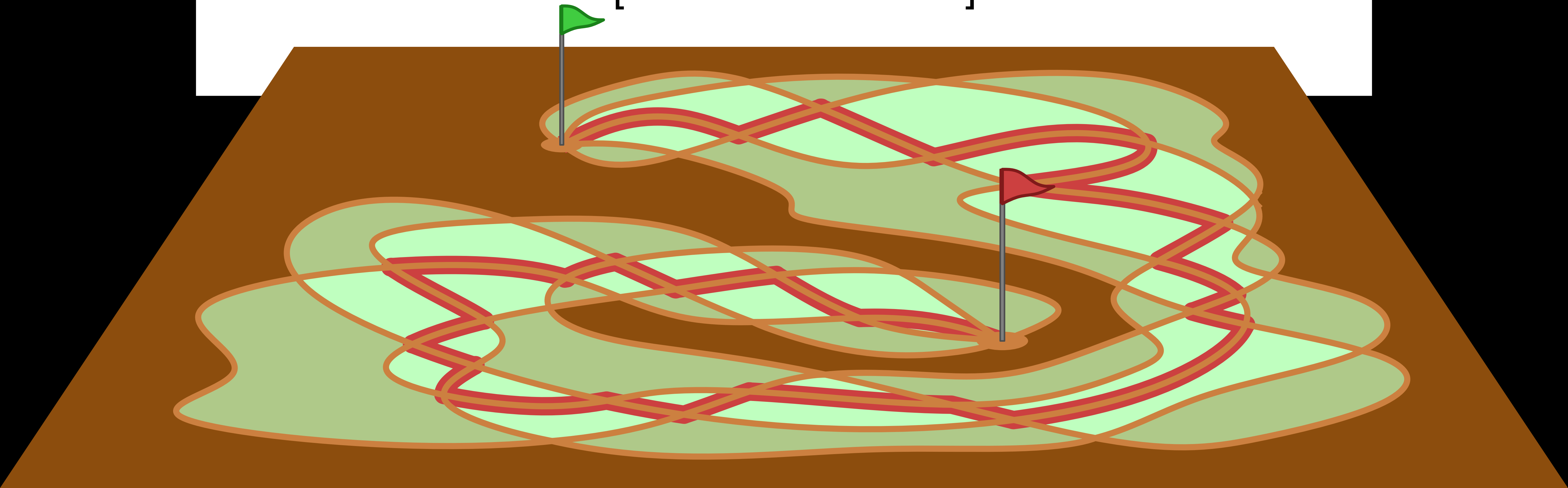

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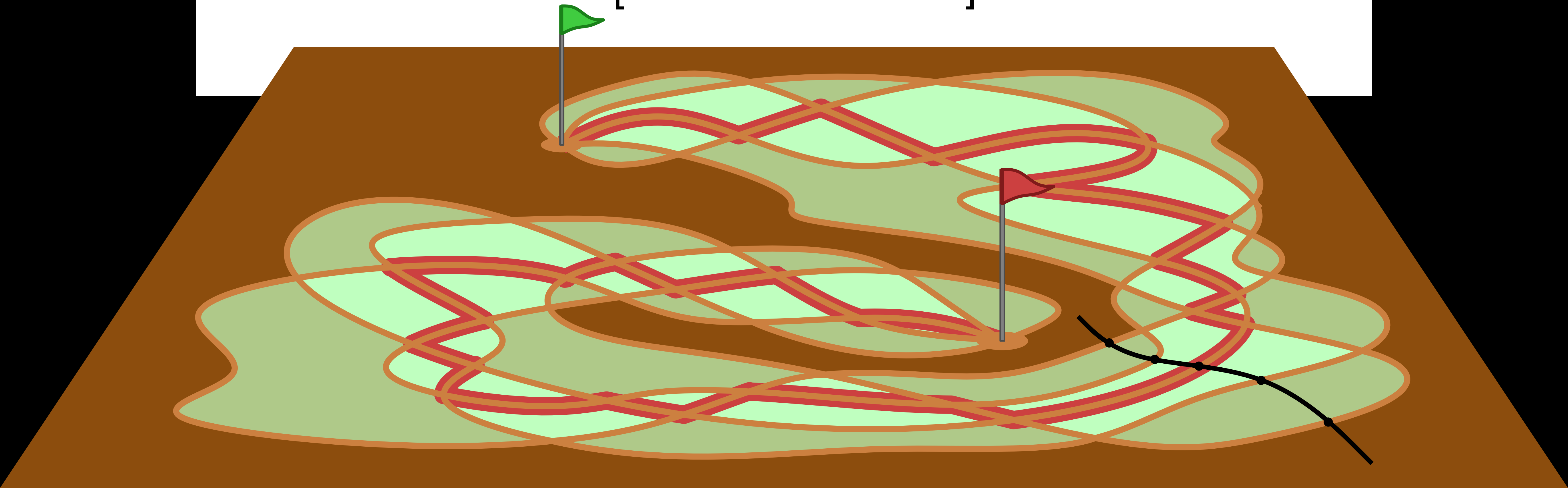
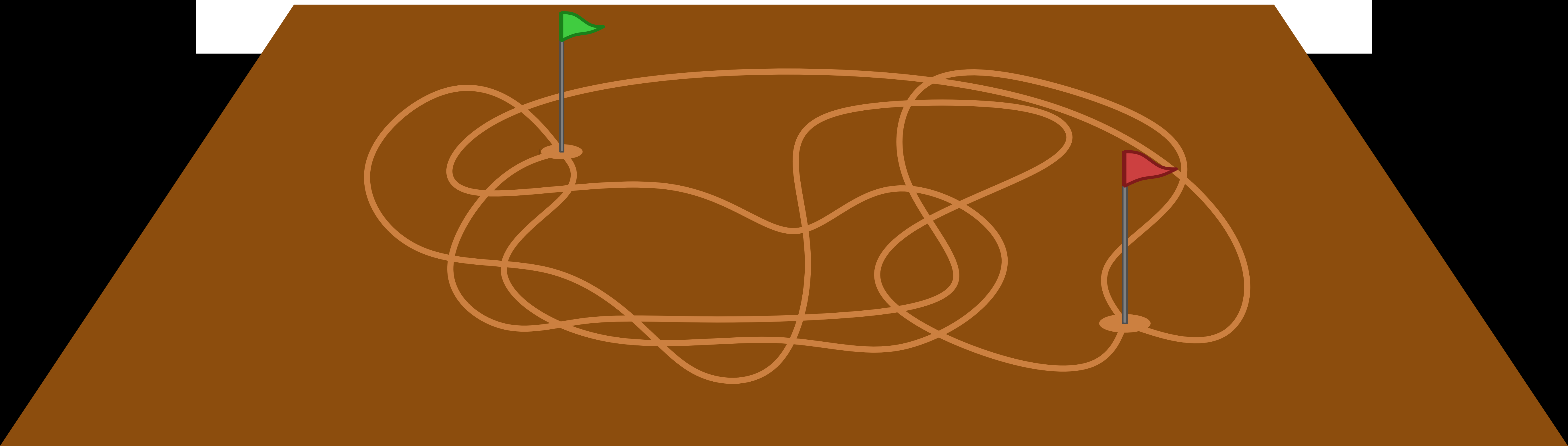


FUTURE WORK

Non-DAG:



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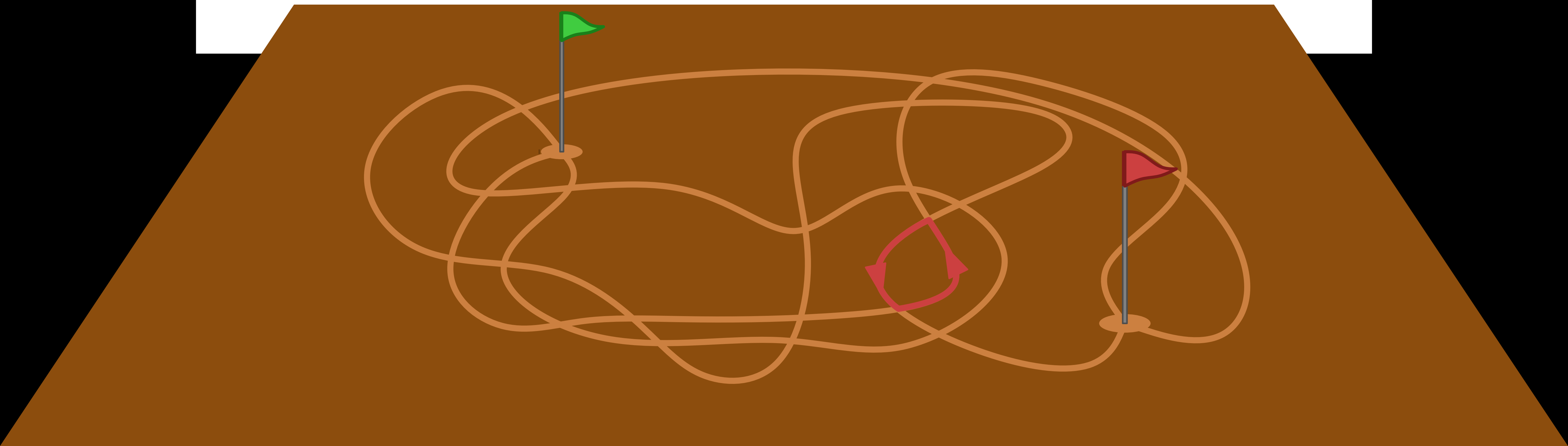
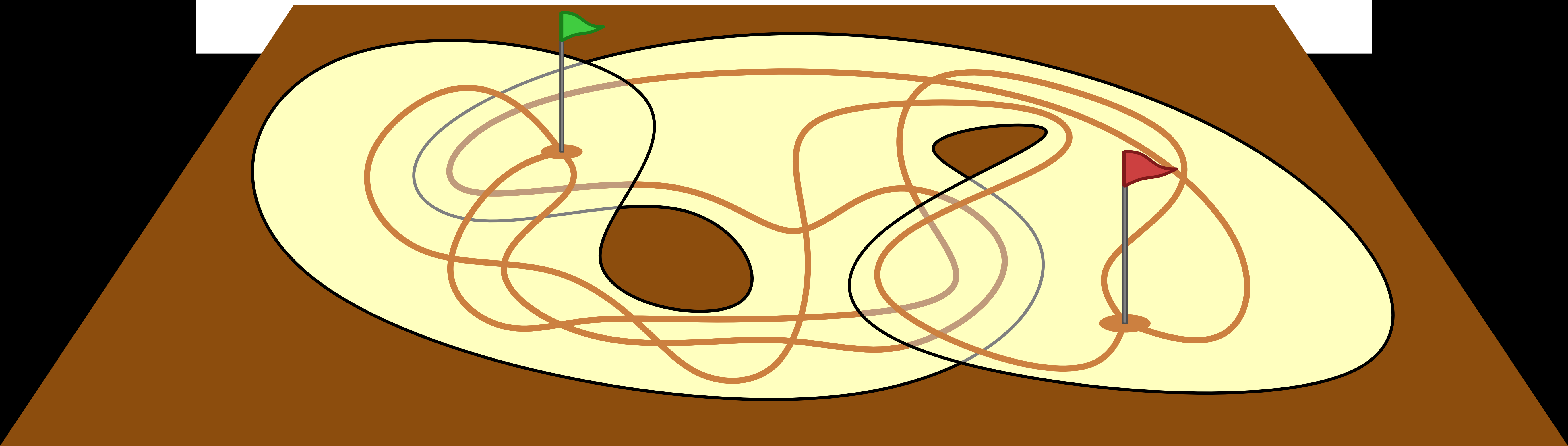


FUTURE WORK

Non-DAG:

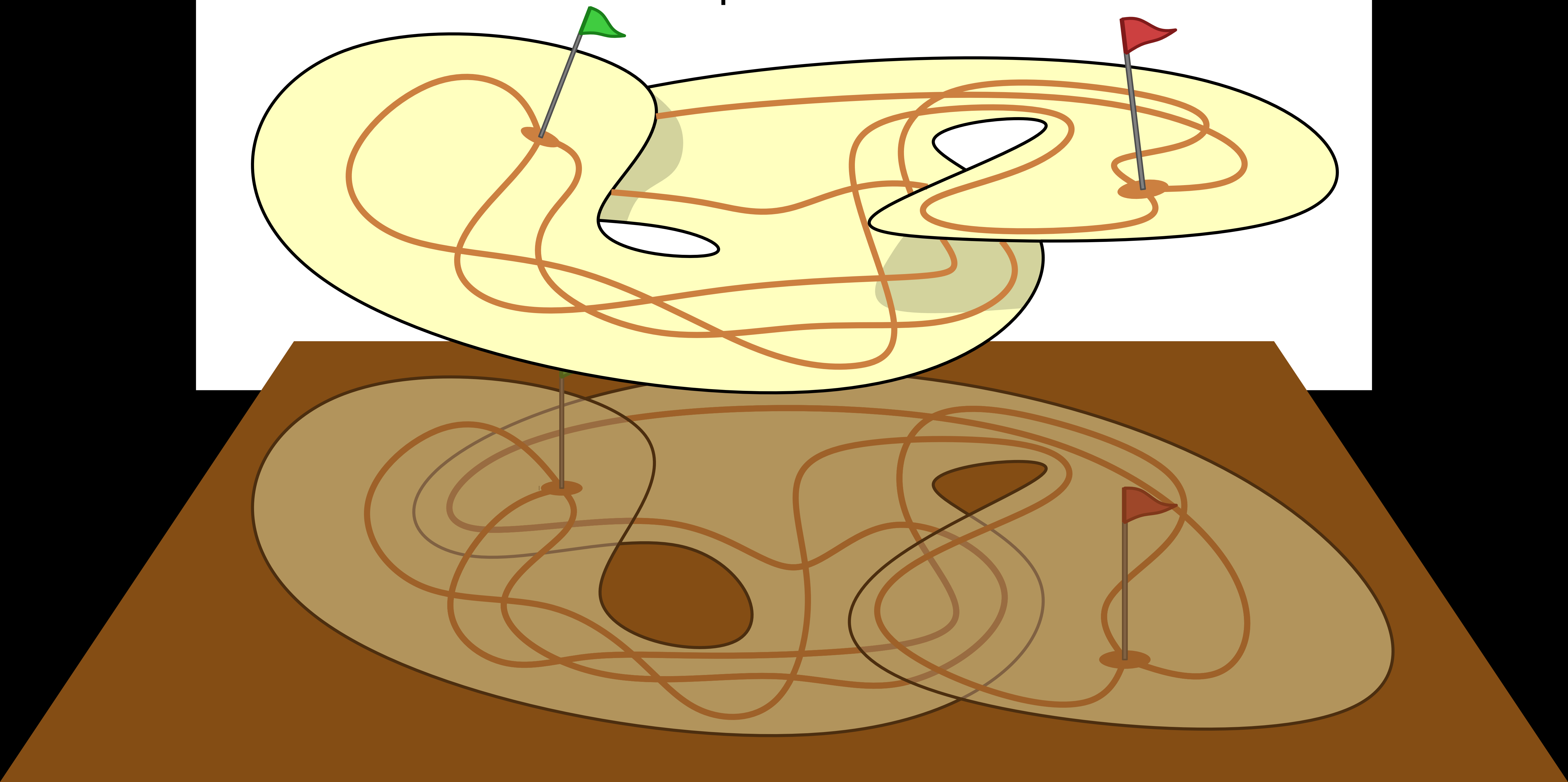
- define ‘corridor’



FUTURE WORK

Non-DAG:

- define ‘corridor’
- ‘lift’ to non-DAG space



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  - “simple median” optimises homotopy area
  - acyclic trajectory intersection graph
  - open problem: find appropriate space

INTERMISSION

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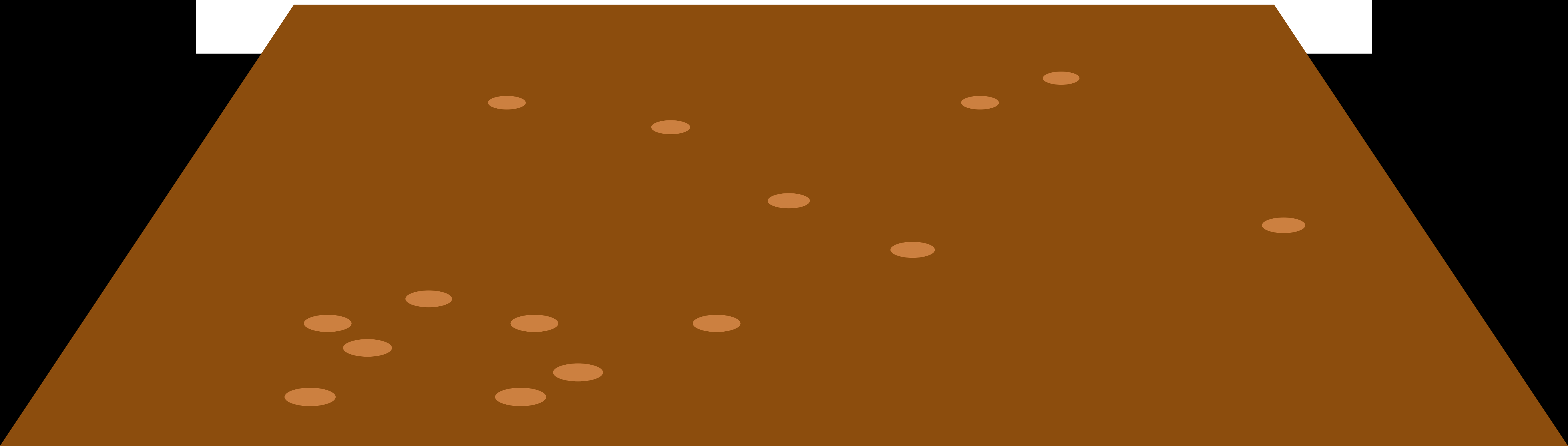

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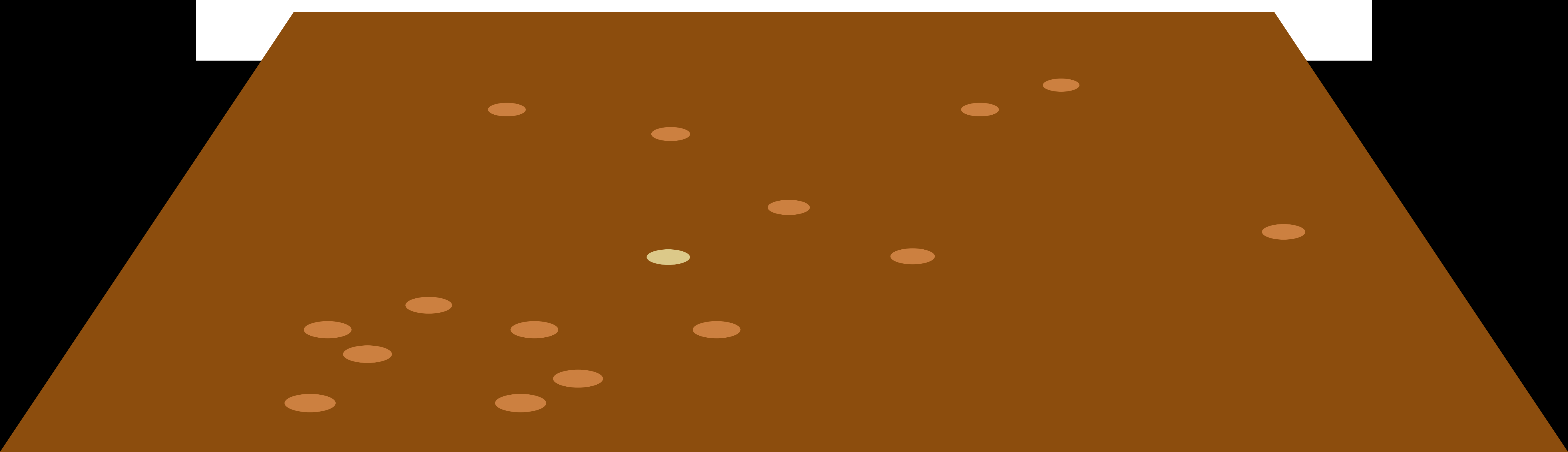

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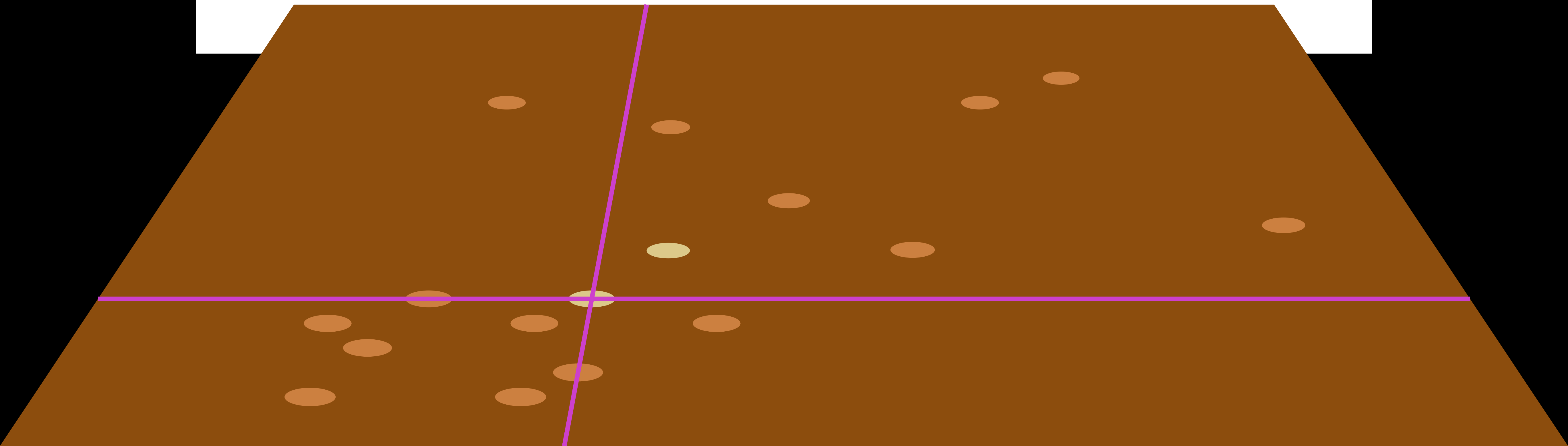

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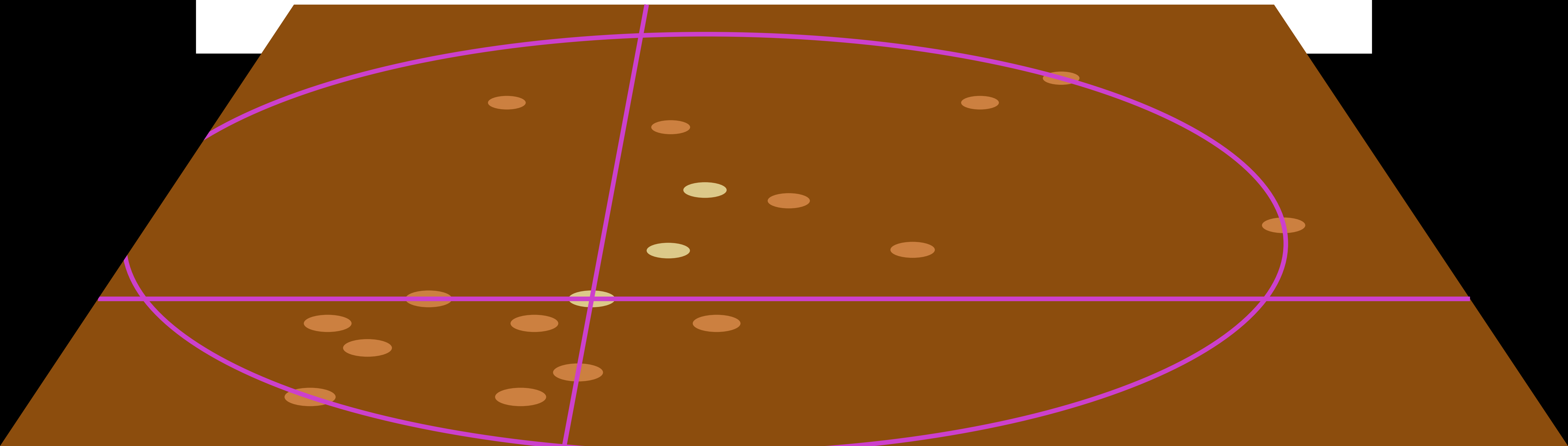
  - Centre of smallest enclosing circle



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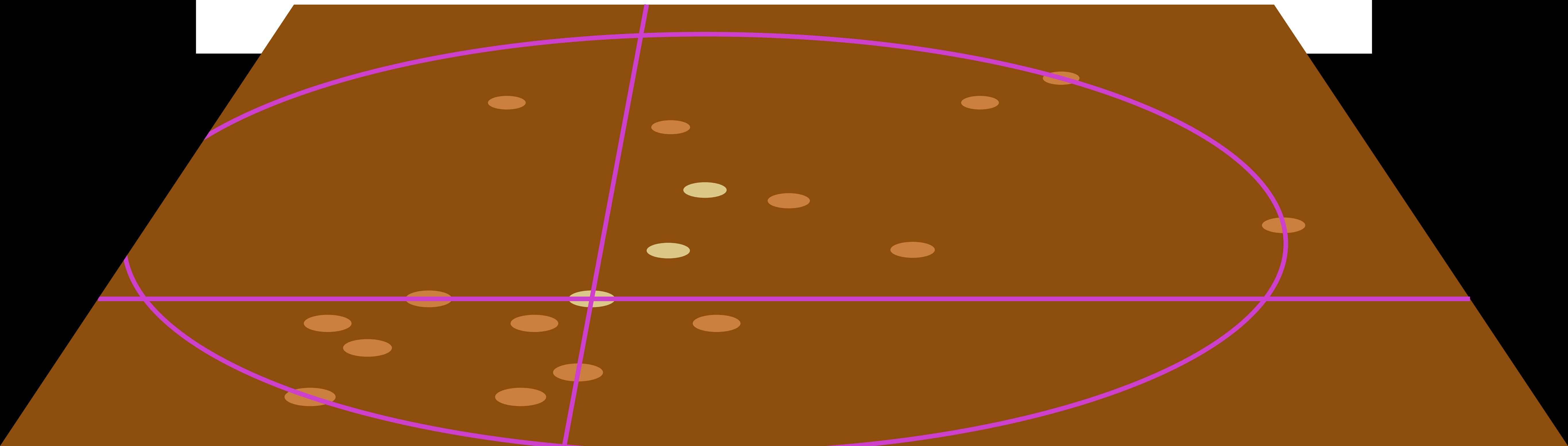
- We want to be close all the time!



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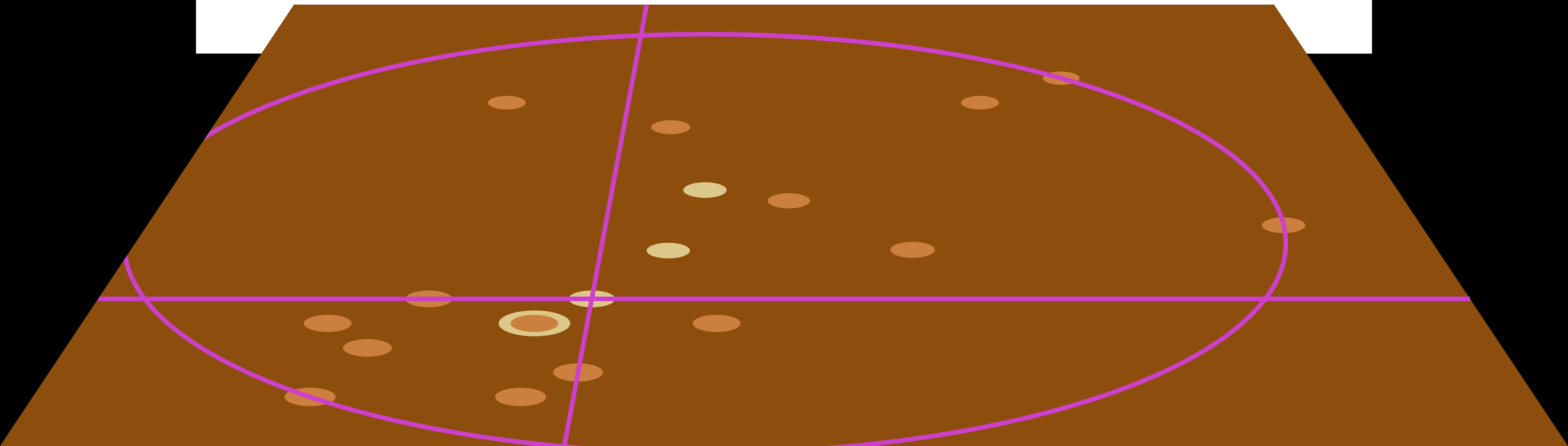
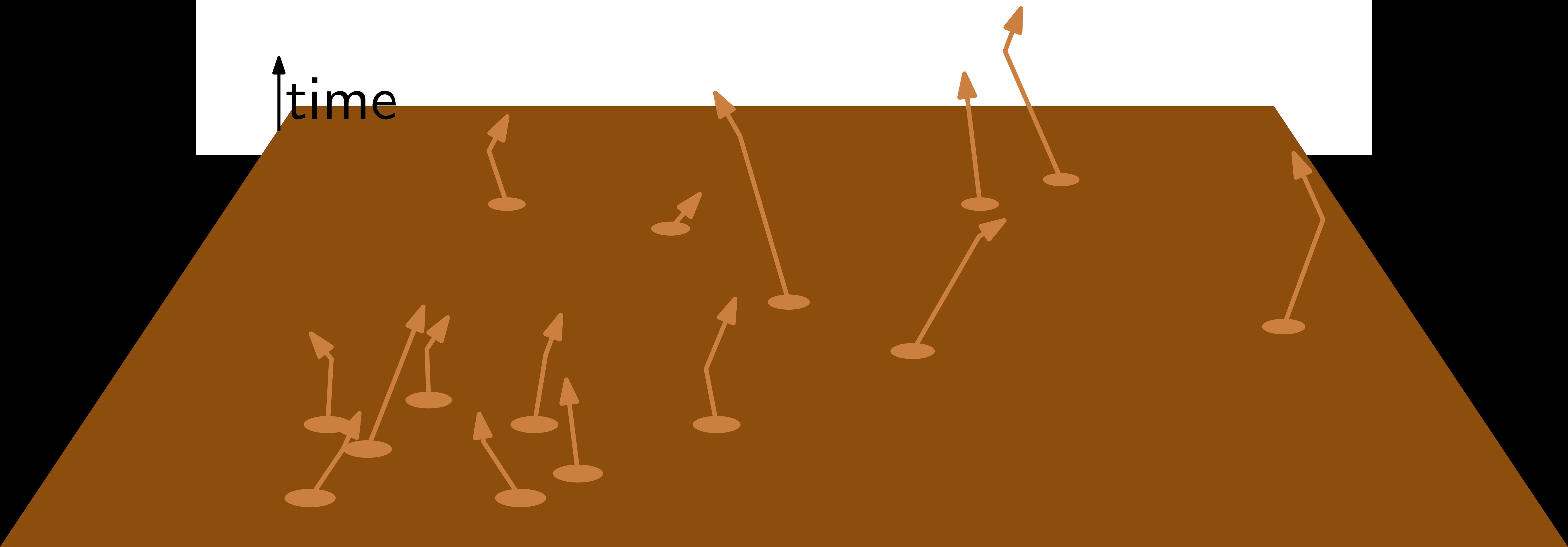


CENTRAL TRAJECTORIES



CENTRAL TRAJECTORIES

- At any time t, a central trajectory $\mathcal{C}$ should
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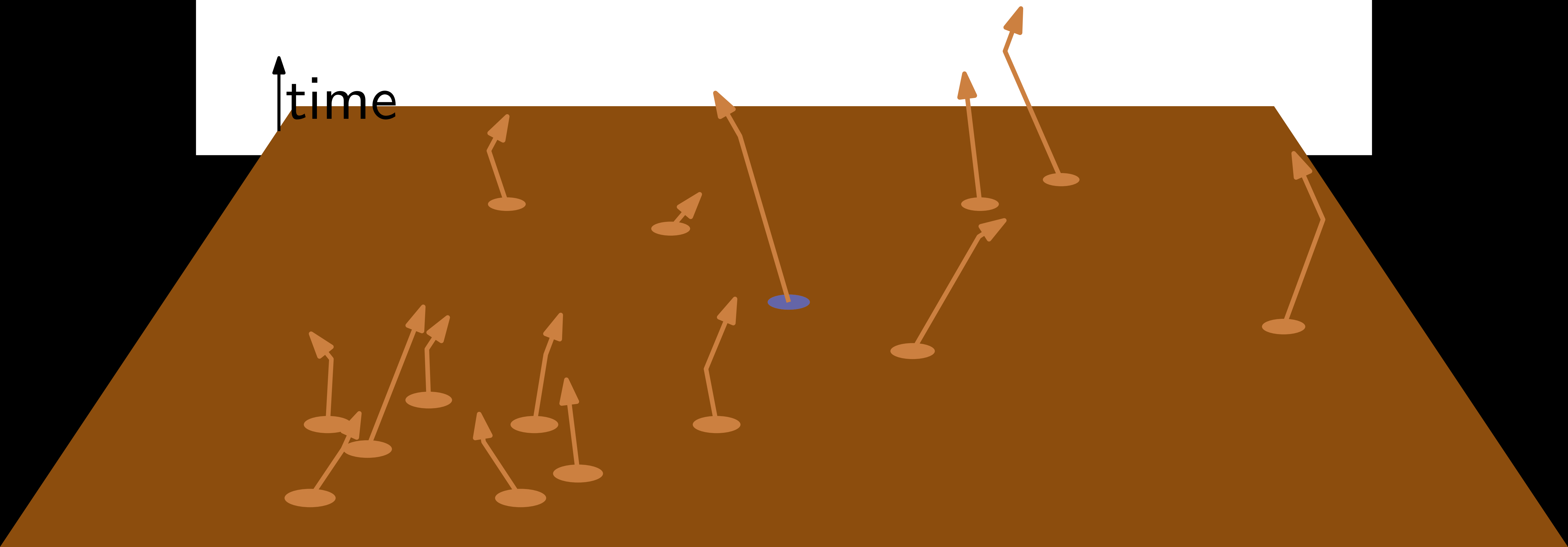
  - be as central as possible.



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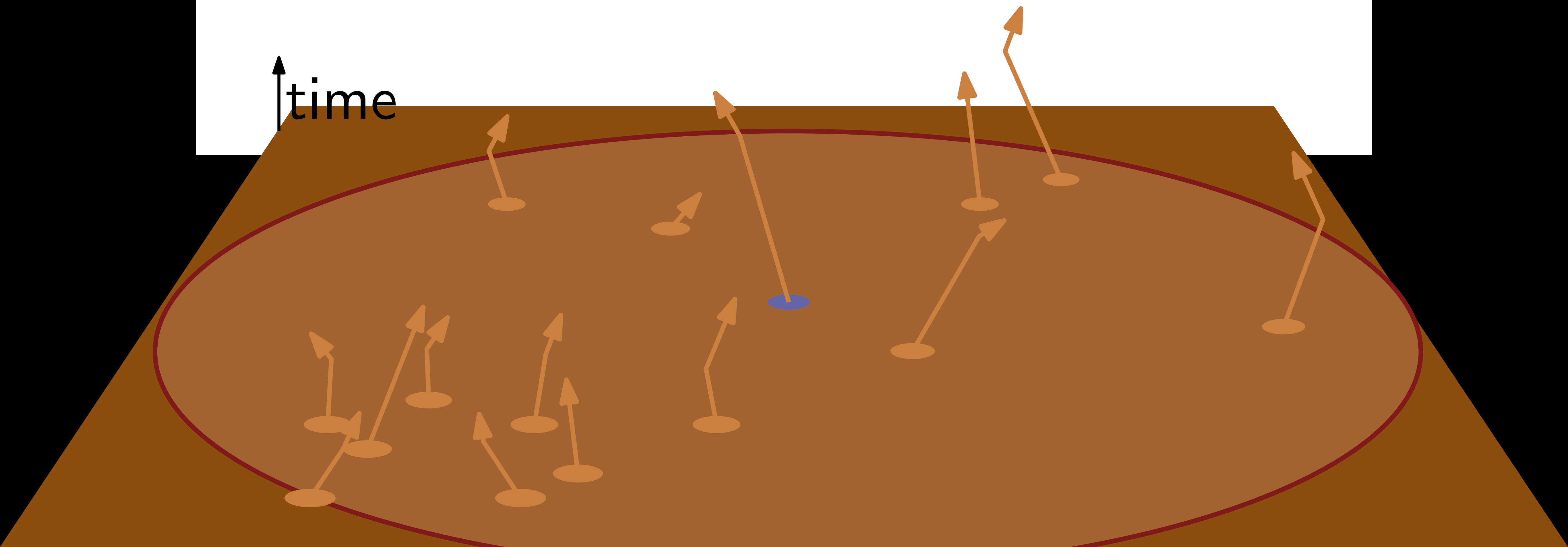

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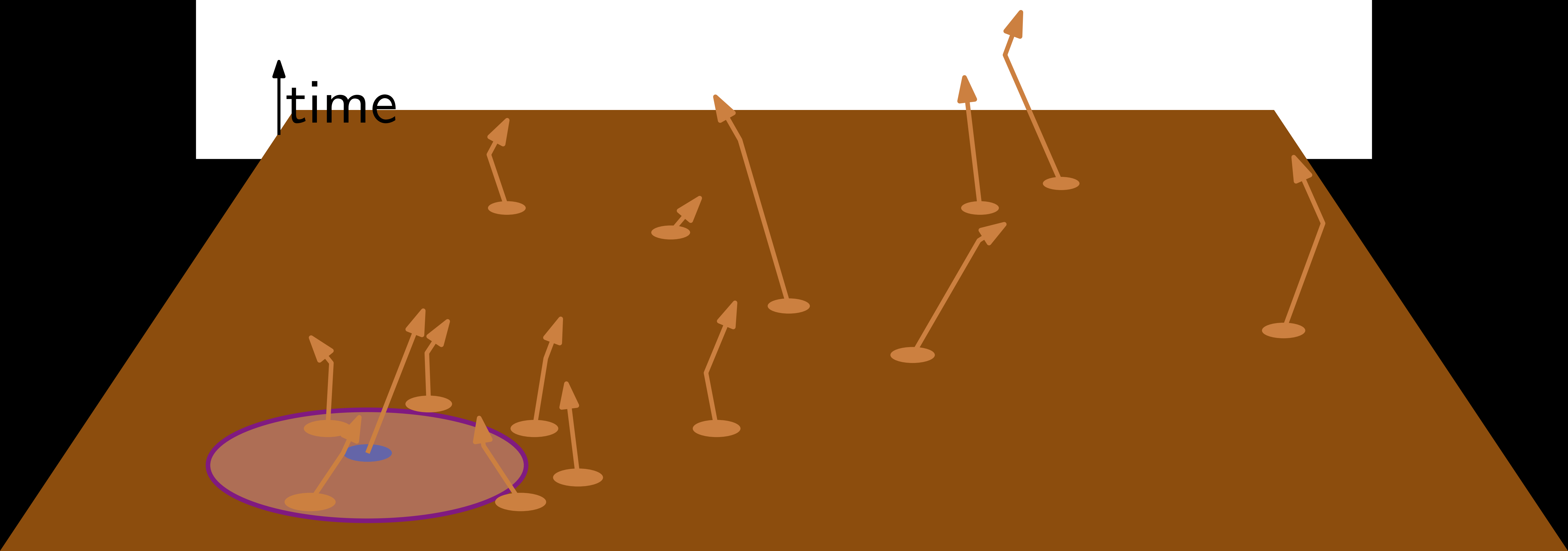
    $$D(\sigma, t) = \max_{\psi \in P} \|\sigma(t)\psi(t)\|$$
  - jump over a distance at most ε.



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  - jump over a distance at most ε.
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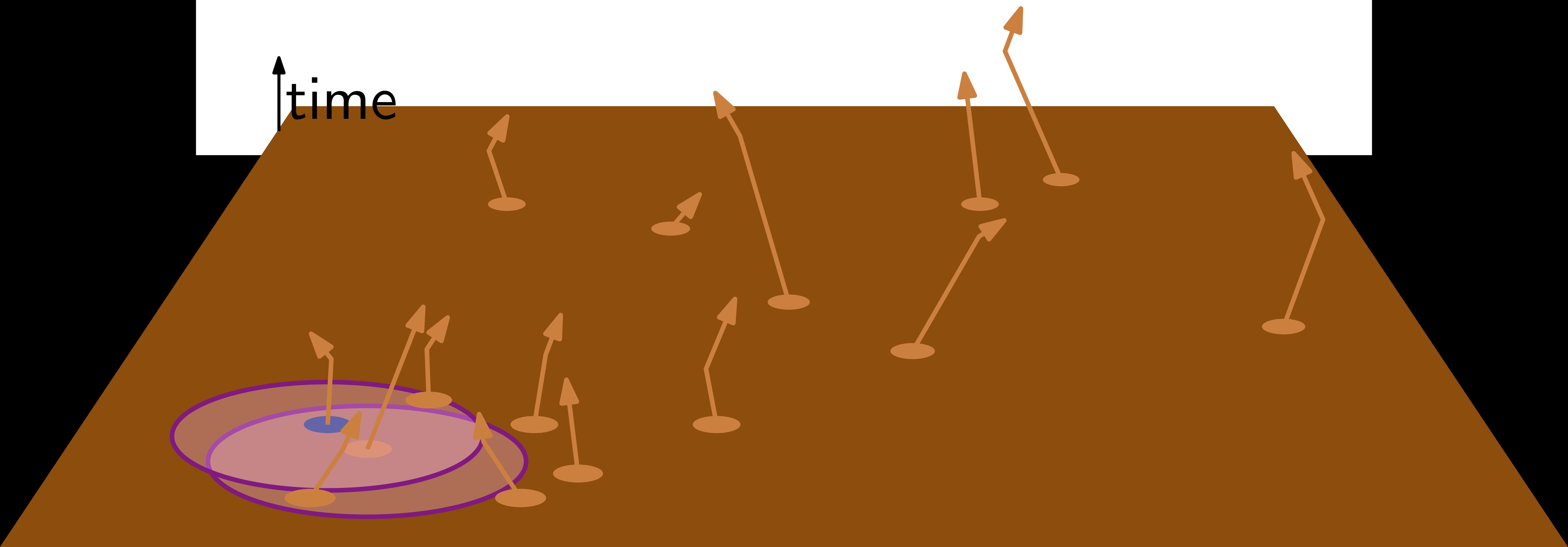

CENTRAL TRAJECTORIES

- At any time t, a central trajectory $\mathcal{C}$ should
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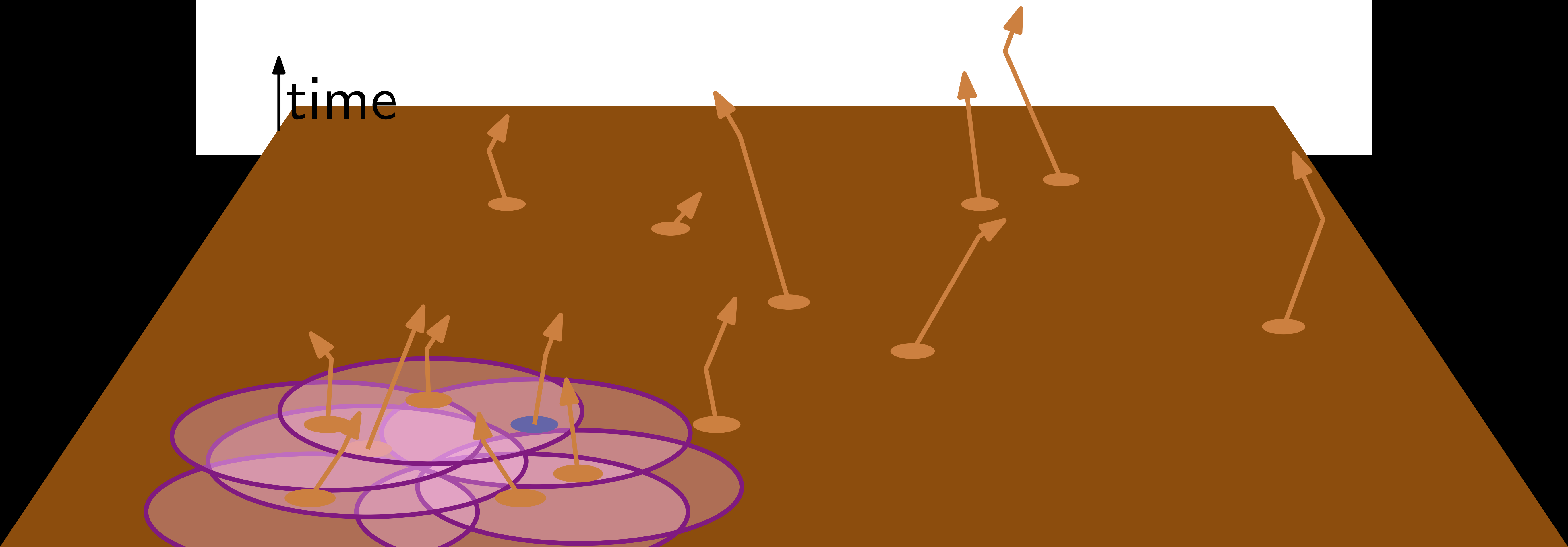
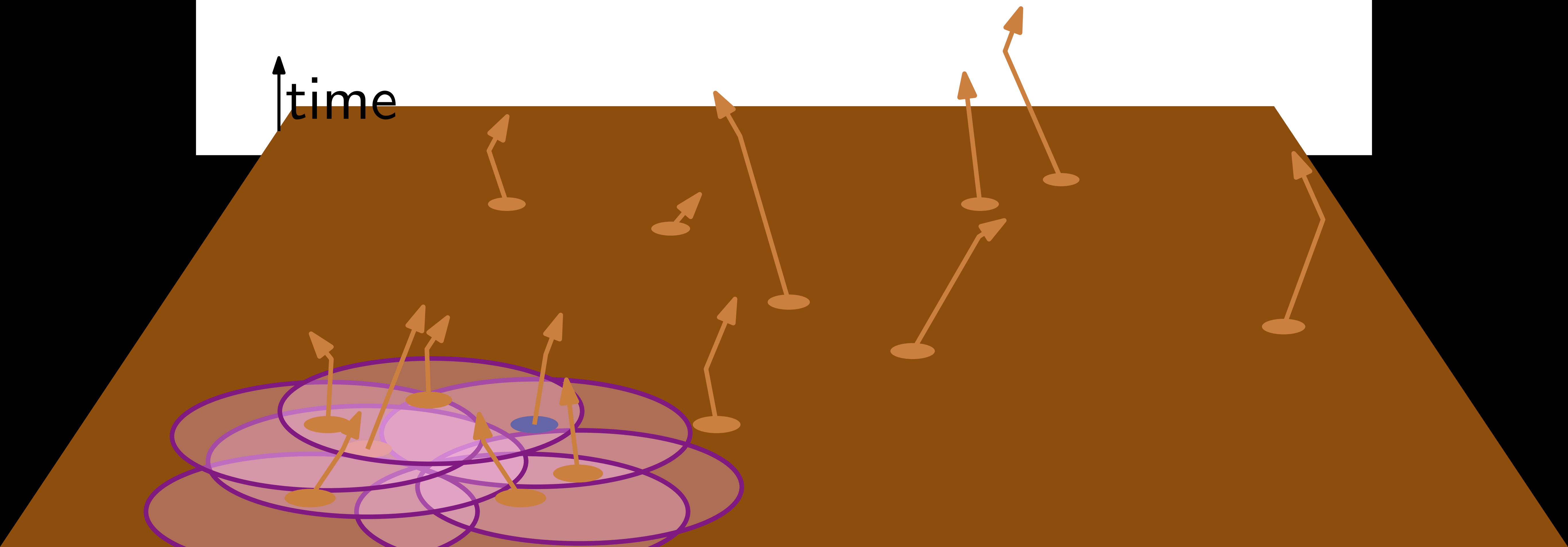


CENTRAL TRAJECTORIES

- Globally, $\mathcal{C}$ should minimize

$$\mathcal{D}(\mathcal{T}) = \int_{t_0}^{t_\tau} D(\mathcal{T}, t)\,\mathrm{d}t.$$



RESULTS

	Complexity $\mathcal{C}$		Algorithm
	Lower bound	Upper bound	
$\mathbb{R}^1$	$\Omega(\tau n^2)$	$O(\tau n^2)$	$O(\tau n^2 \log n)$
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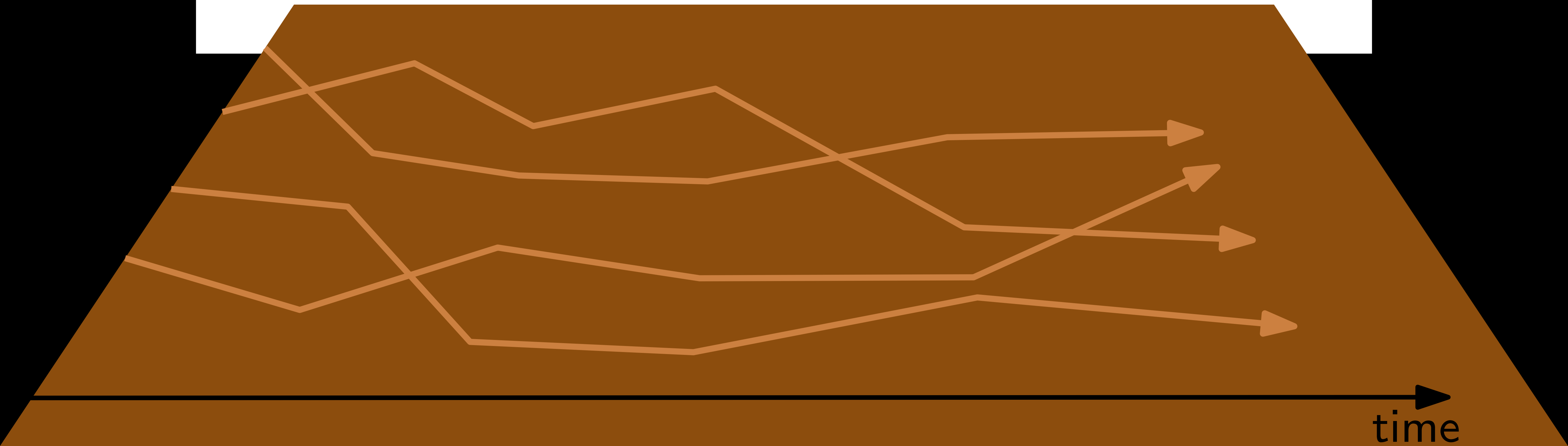

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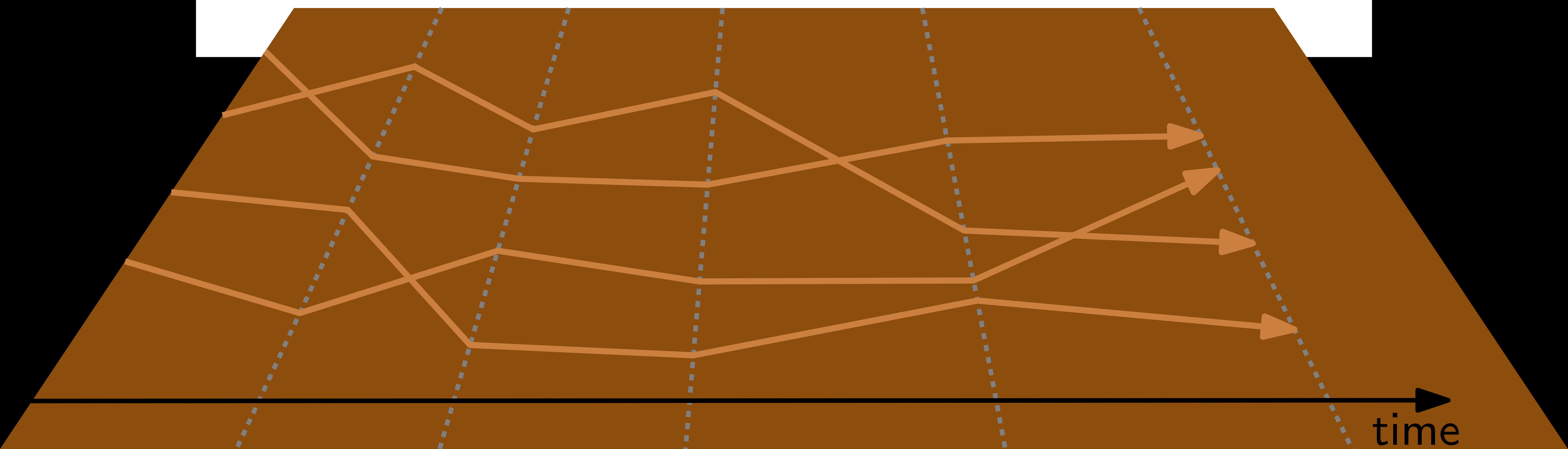

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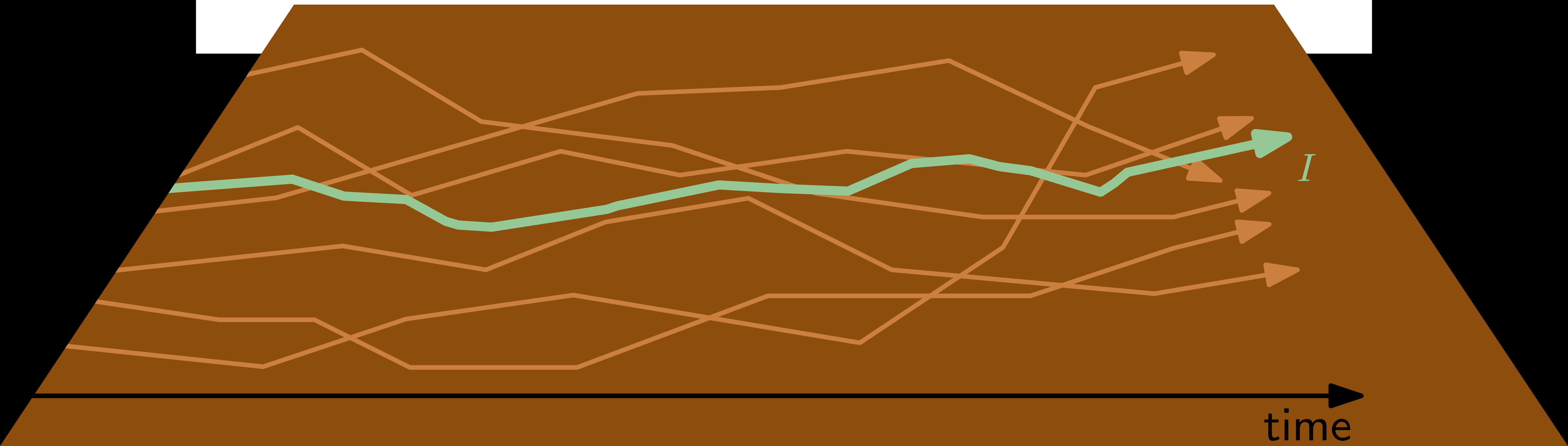
- Let I be the *ideal trajectory*: the trajectory that minimizes $\mathcal{D}$.



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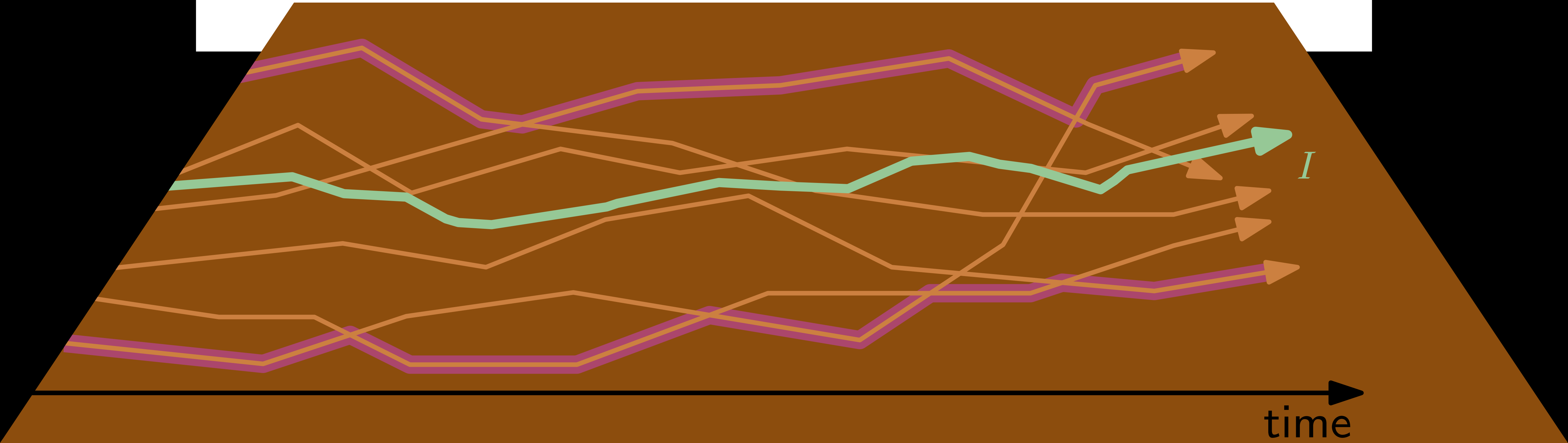

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A central trajectory $\mathcal{C}$ minimizes

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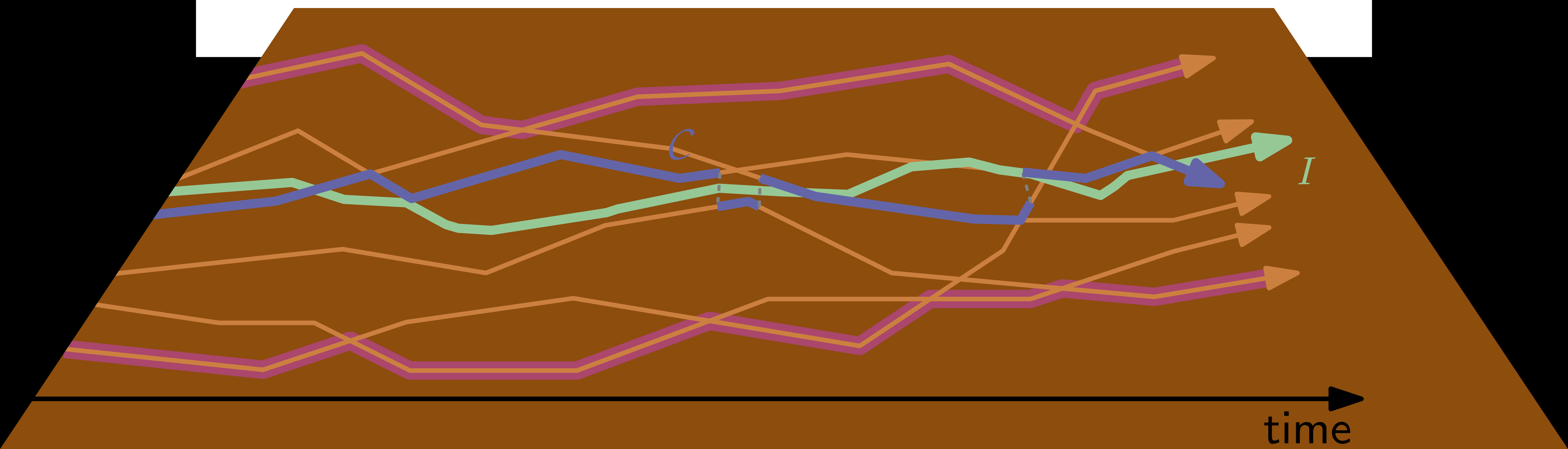

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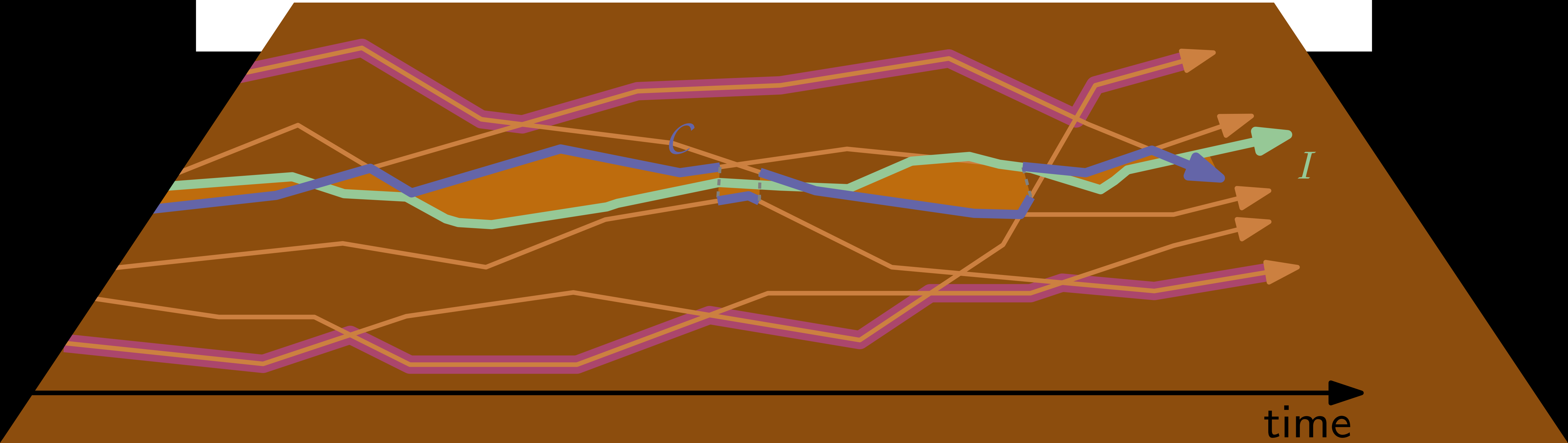

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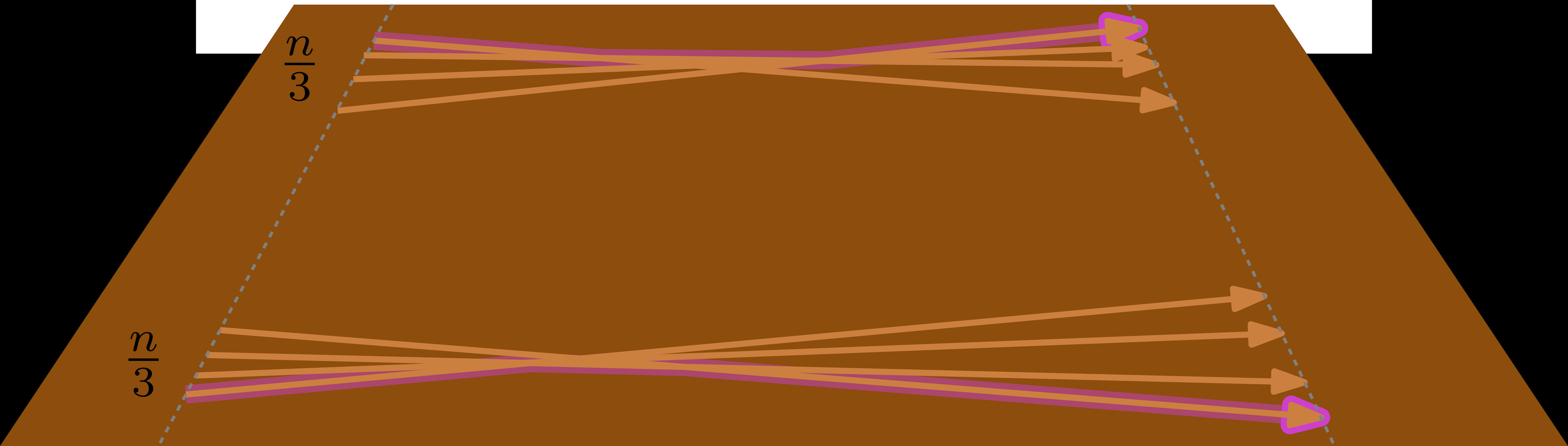

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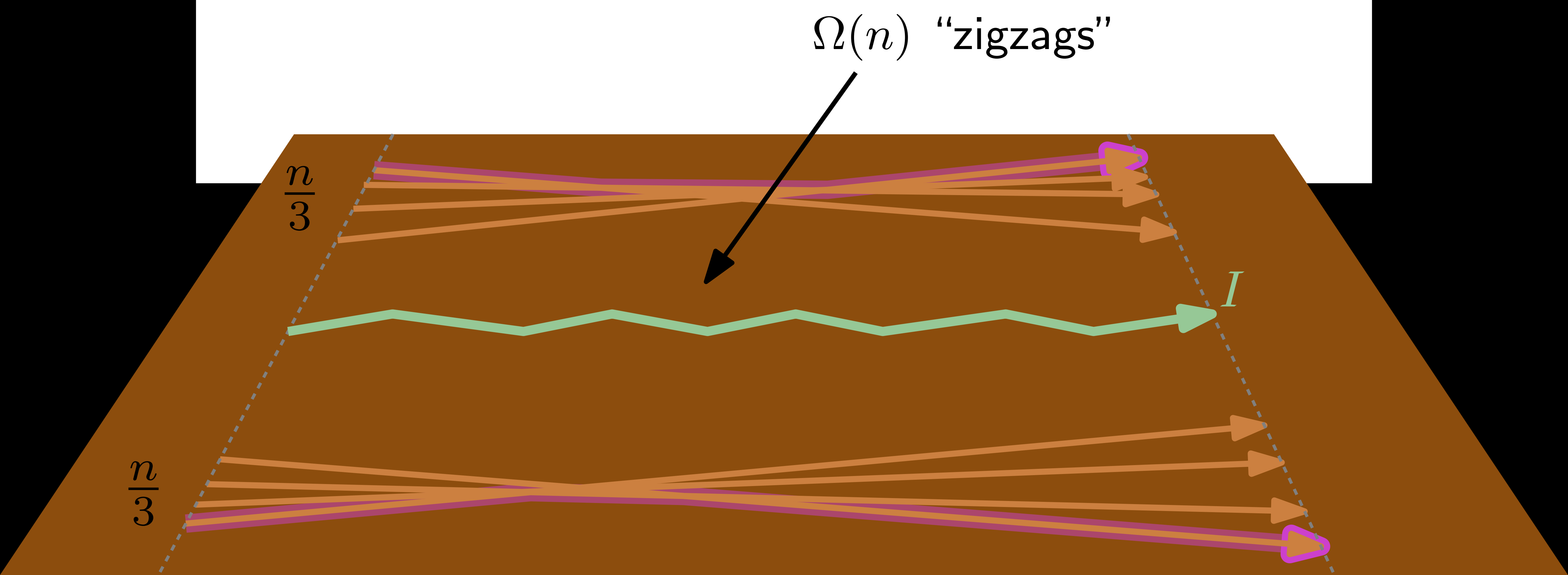

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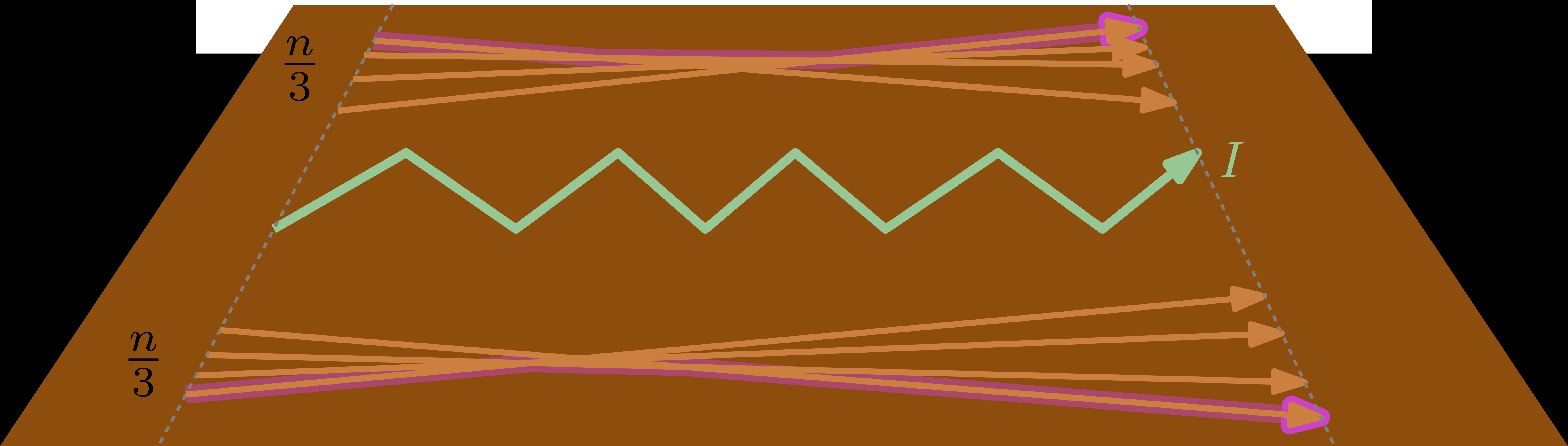

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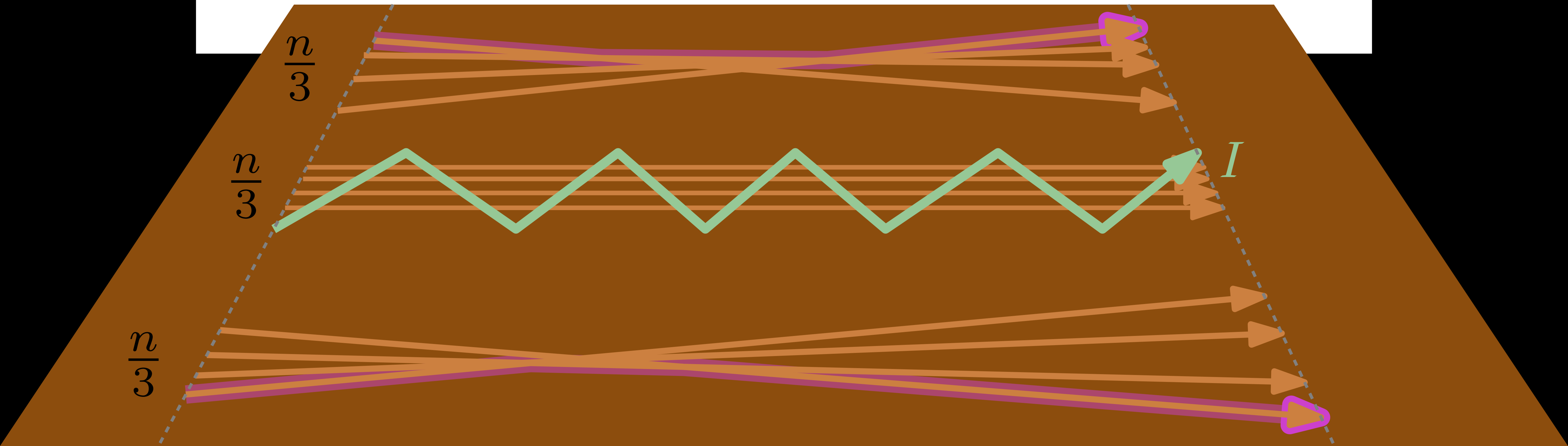

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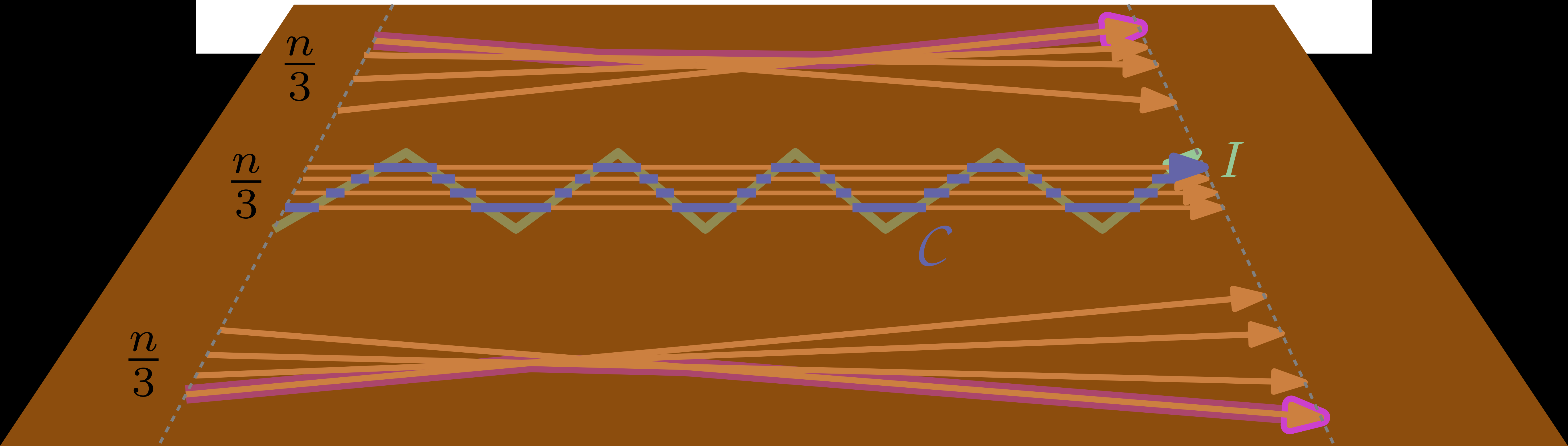

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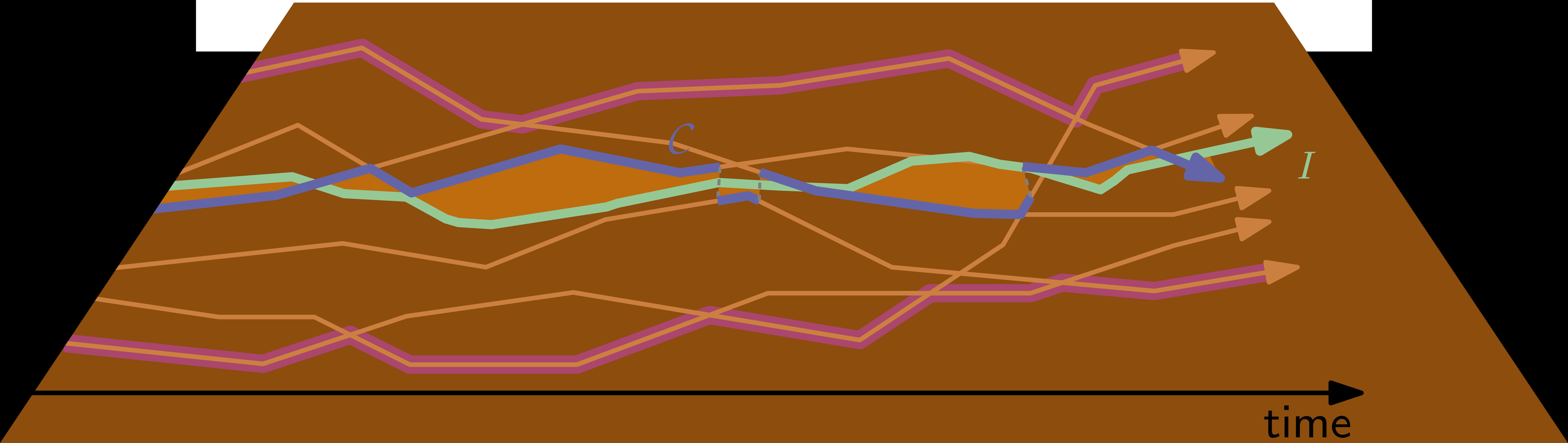
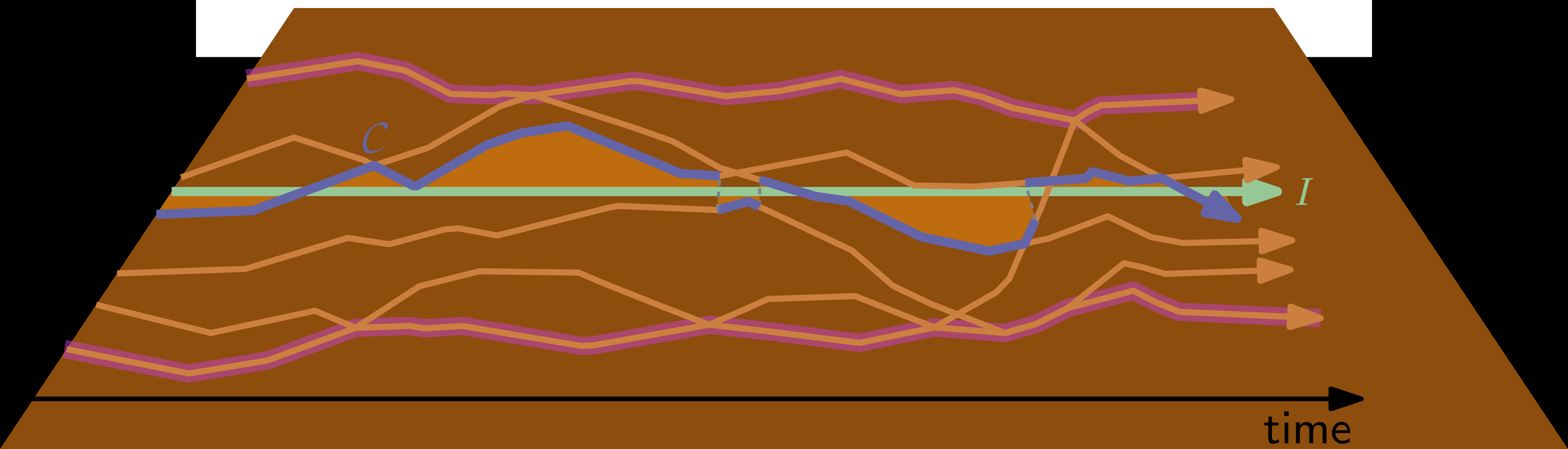
- "Straighten" I



UPPER BOUND

- "Straighten" I
  - We now have n trajectories, with $O(\tau n)$ vertices each



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OBSERVATION.
$\mathcal{C}$ can jump from σ to ψ at time t
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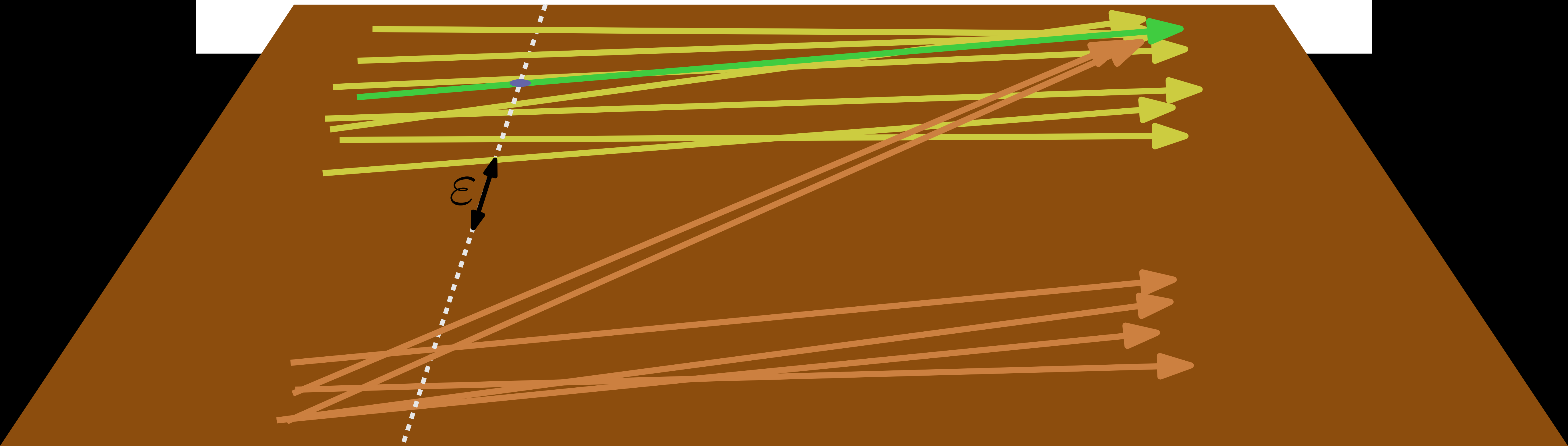
σ and ψ are ε-connected at time t.



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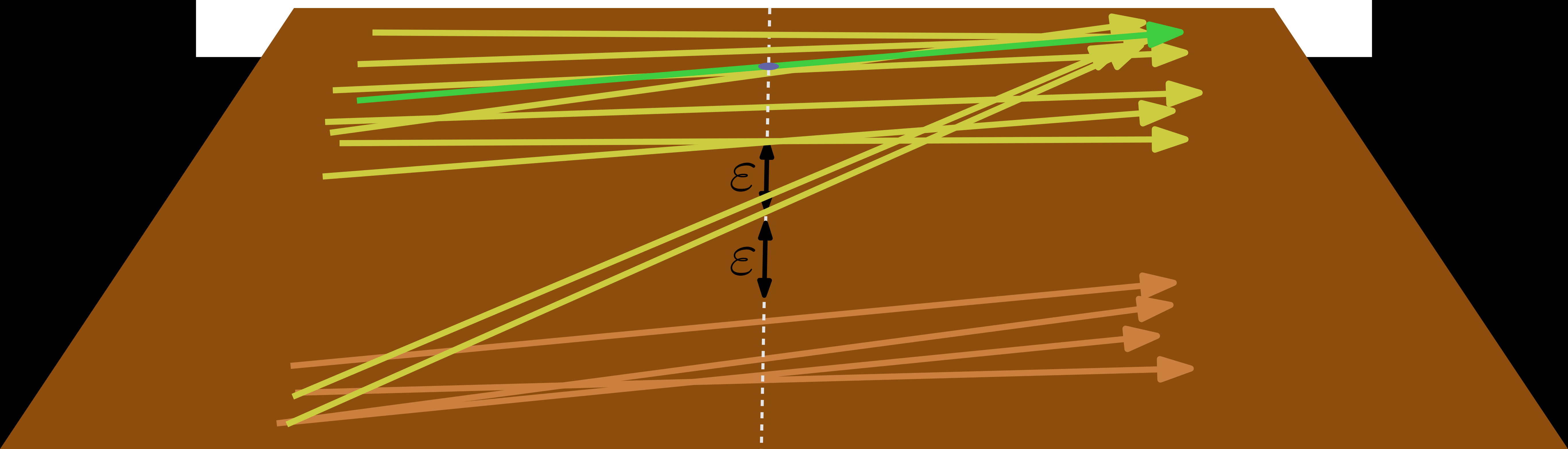

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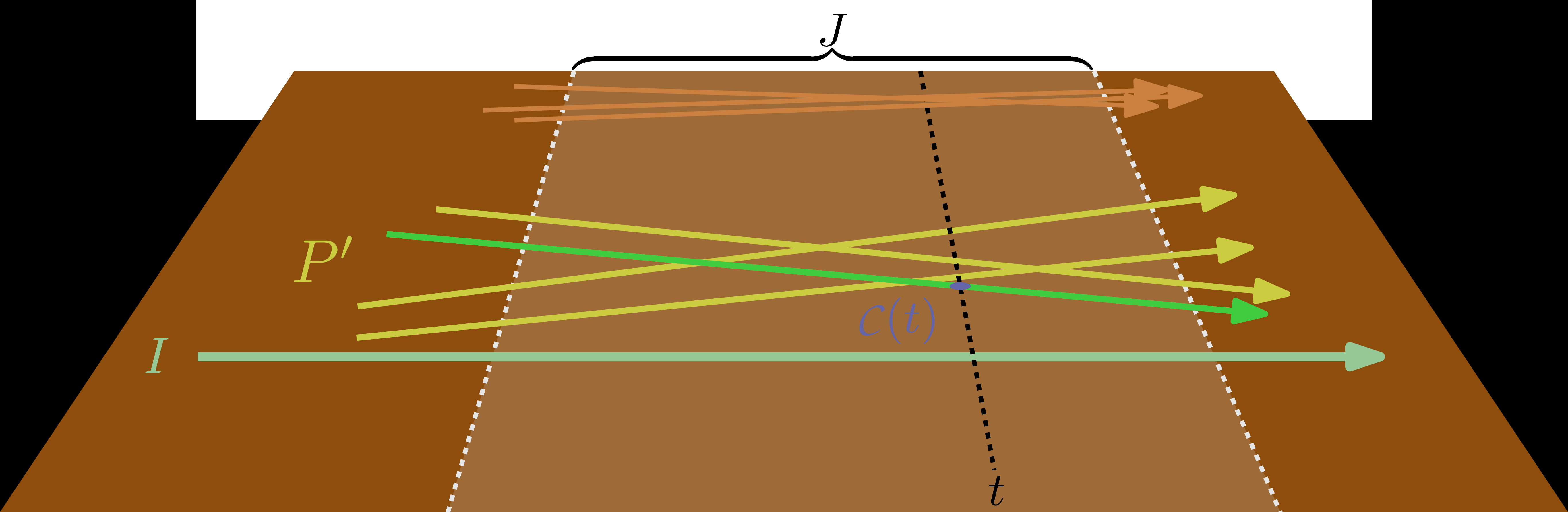
- Let $P' \ni \sigma$ be a maximal set of entities that is ε-connected during interval J, and s.t. $\mathcal{C} \in P'$ during J.



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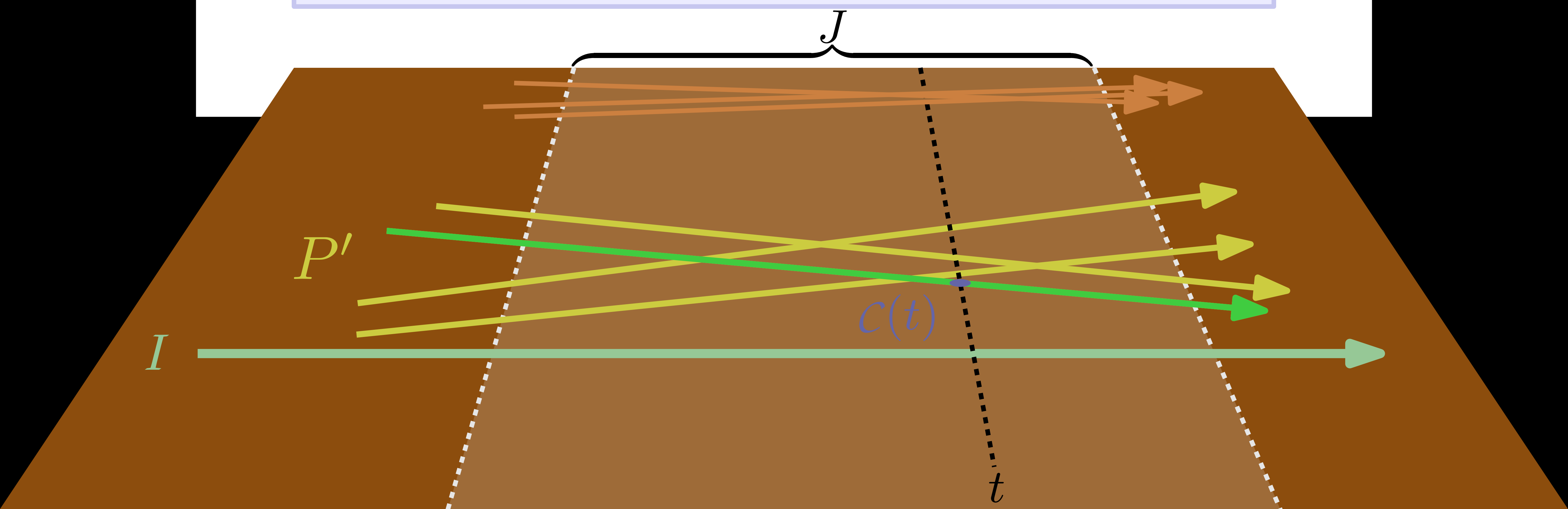

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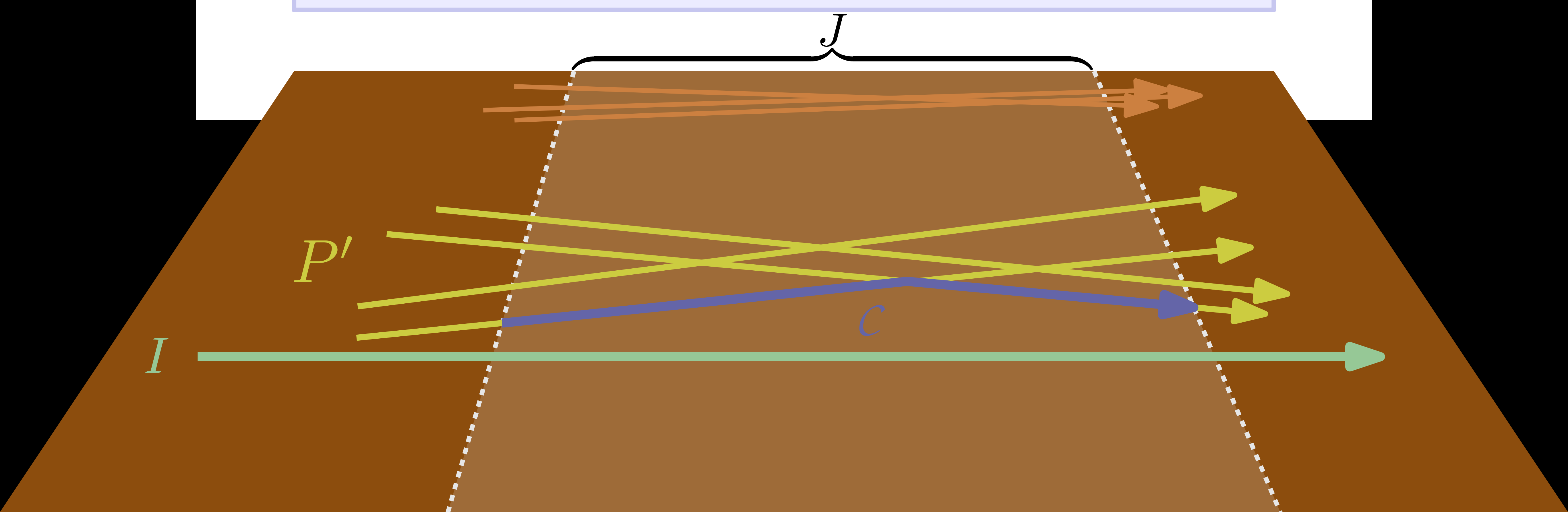

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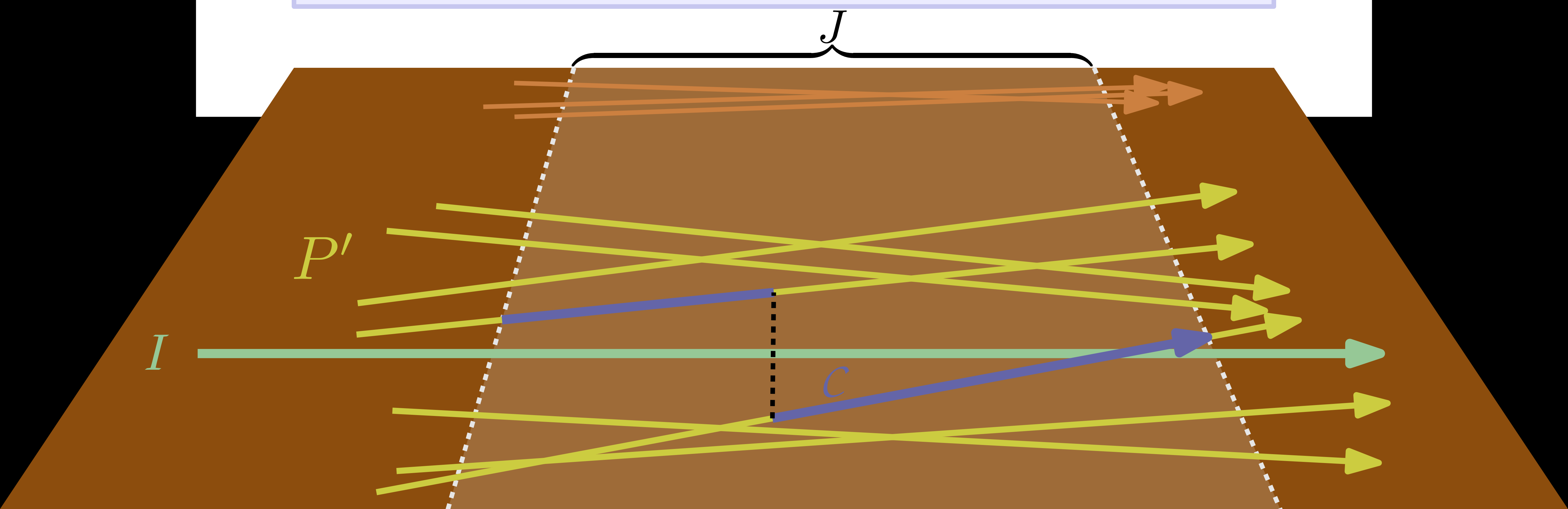

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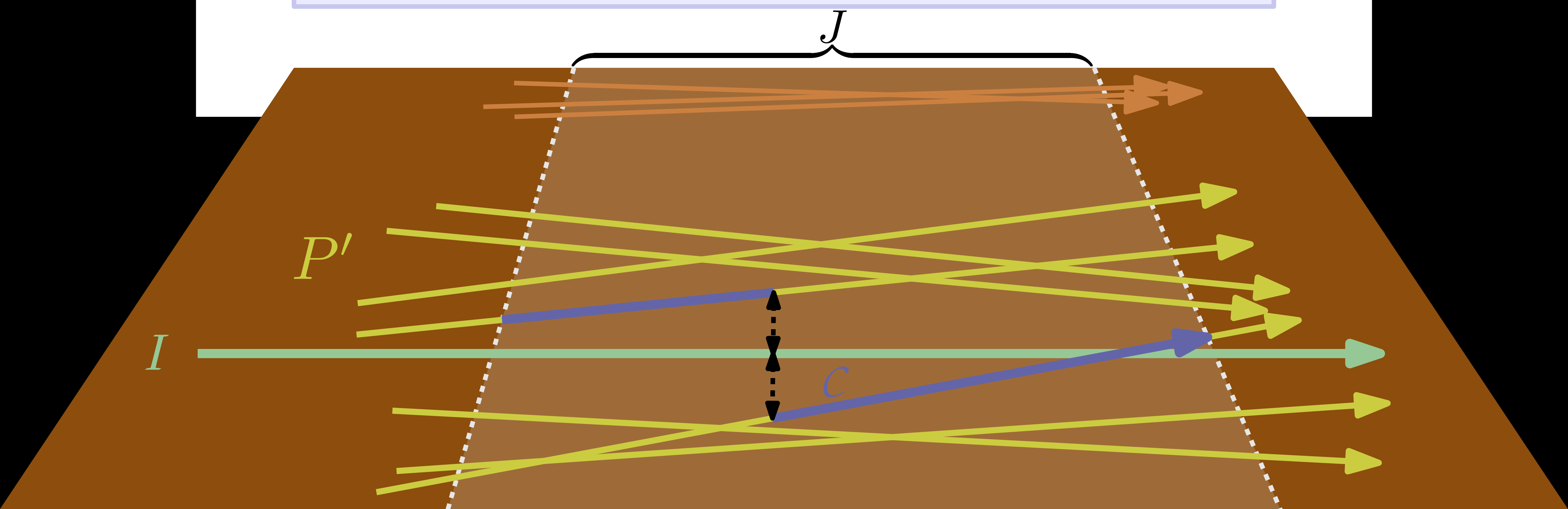

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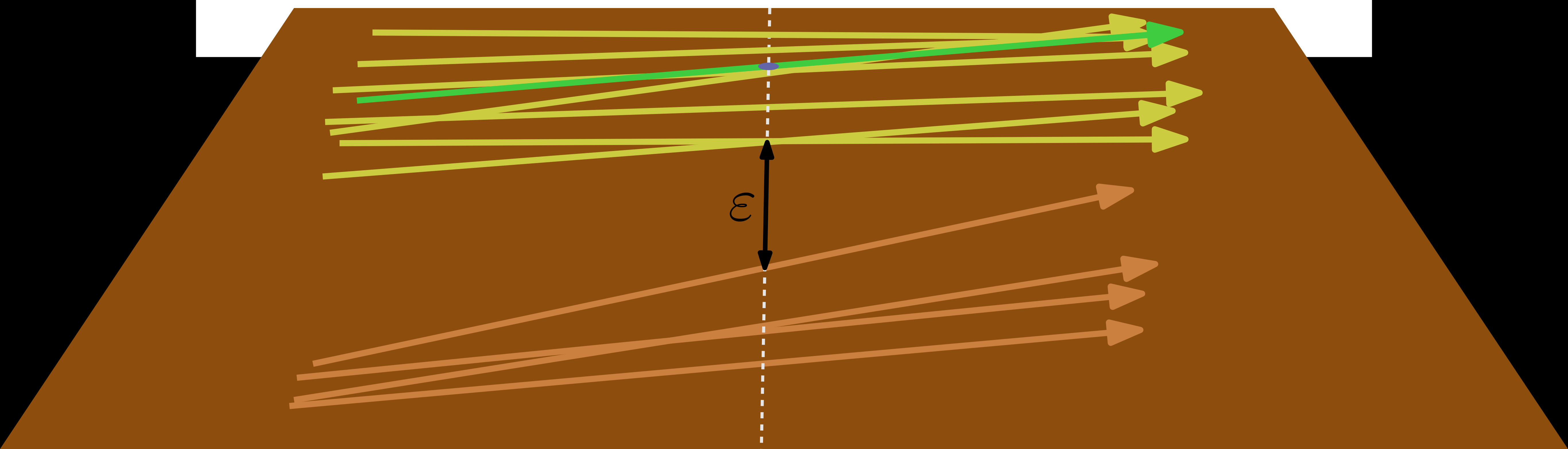
- $\mathcal{C}$ may have a vertex at time t if:
  - Partition into ε-connected sets changes at time t, or



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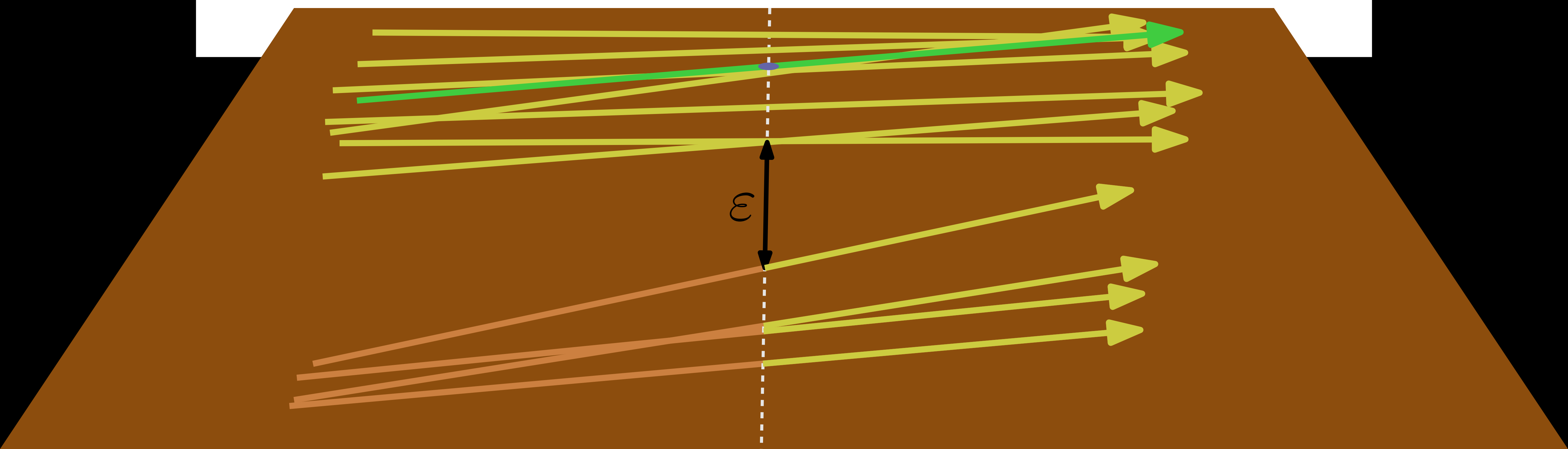

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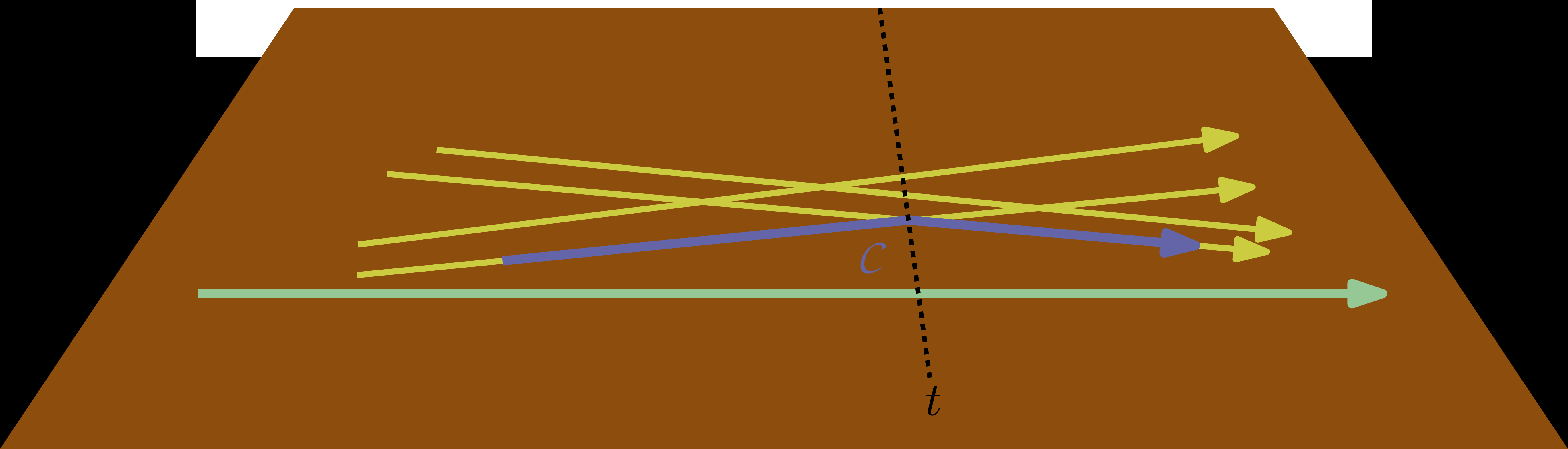

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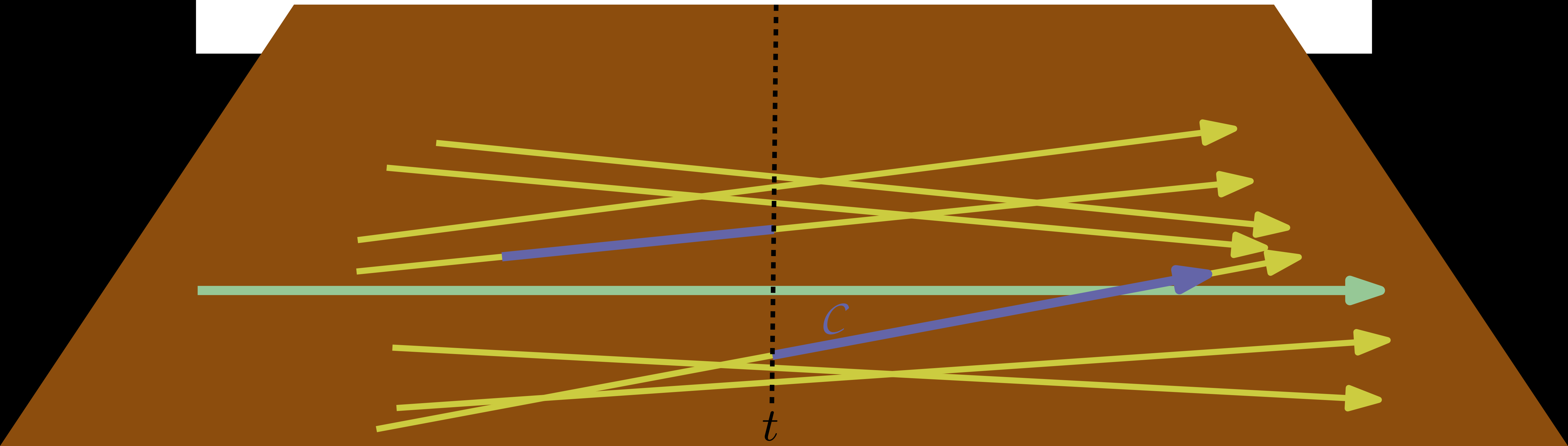

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- $\mathcal{C}$ may have a vertex at time t if:
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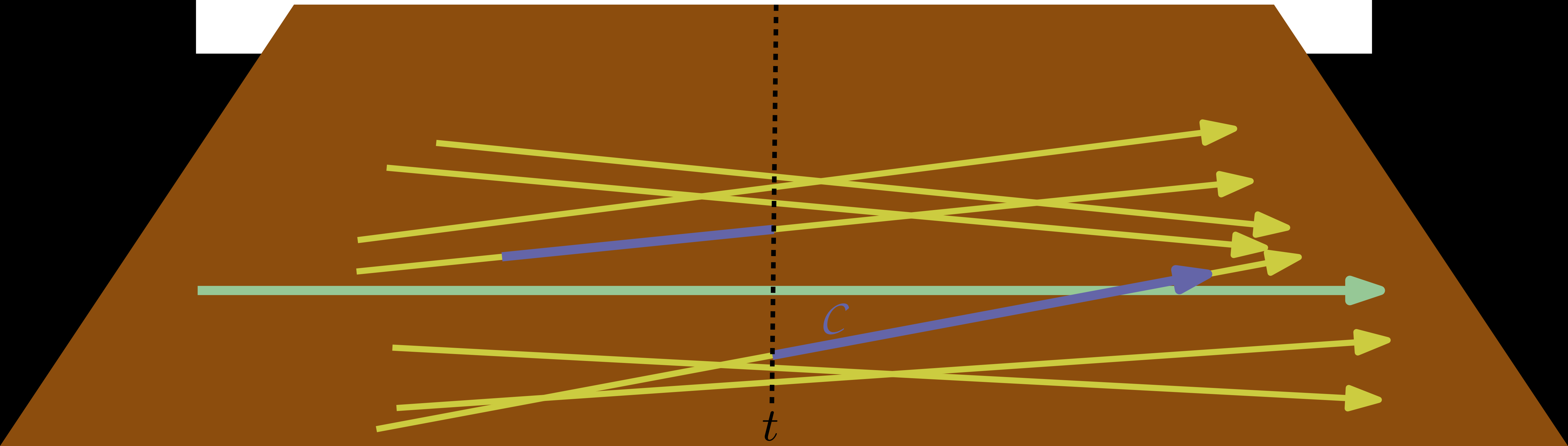
    - Intersection of the trajectories $O(\tau n^2)$
    - Jump crossing I $O(\tau n^2)$



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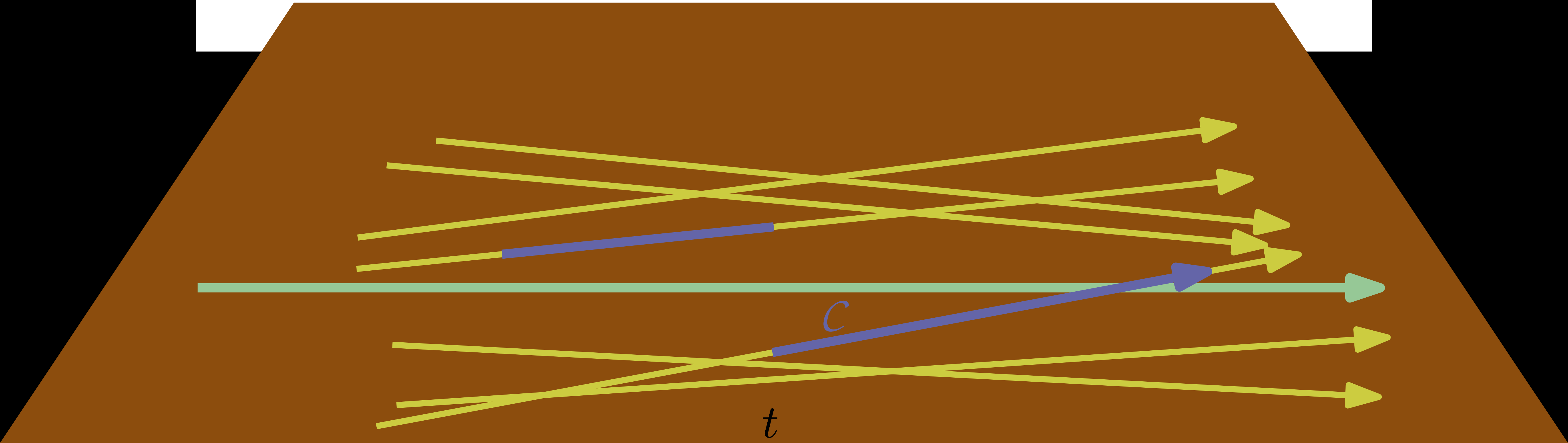
THEOREM.
Given a set of n trajectories in $\mathbb{R}^1$, each with vertices at times $t_0, .., t_\tau$, a central trajectory $\mathcal{C}$ has worst case complexity $O(\tau n^2)$.



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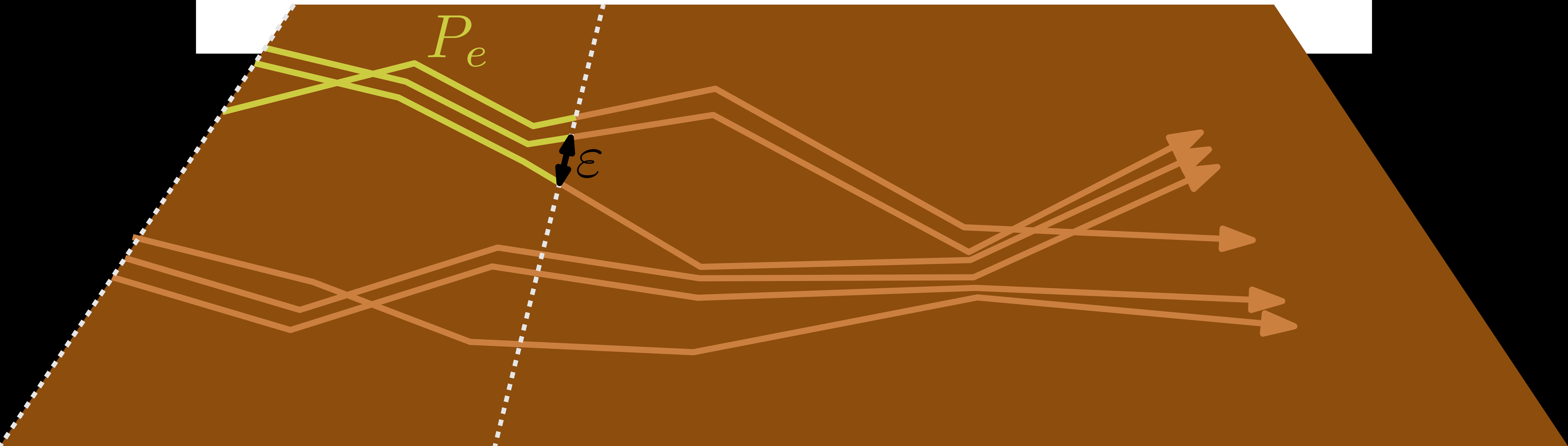

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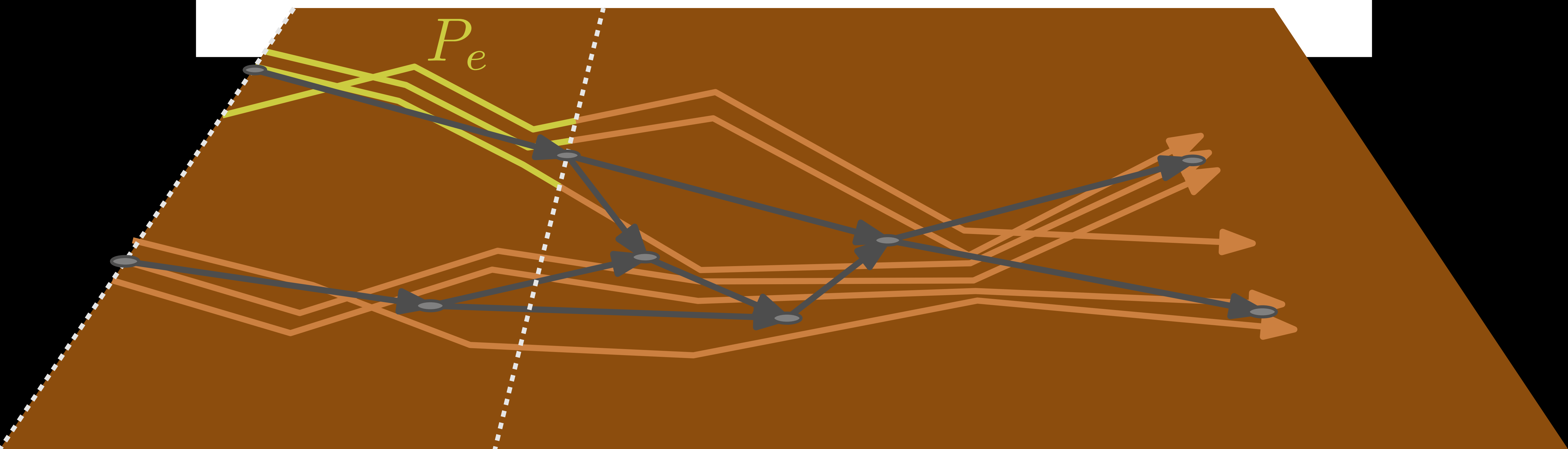

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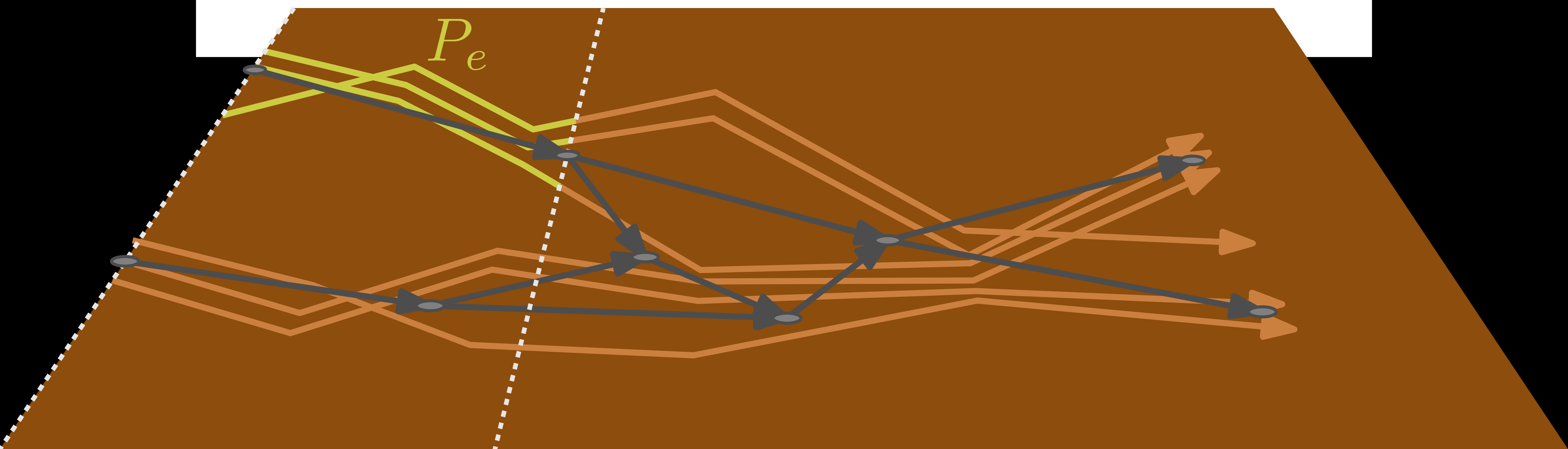

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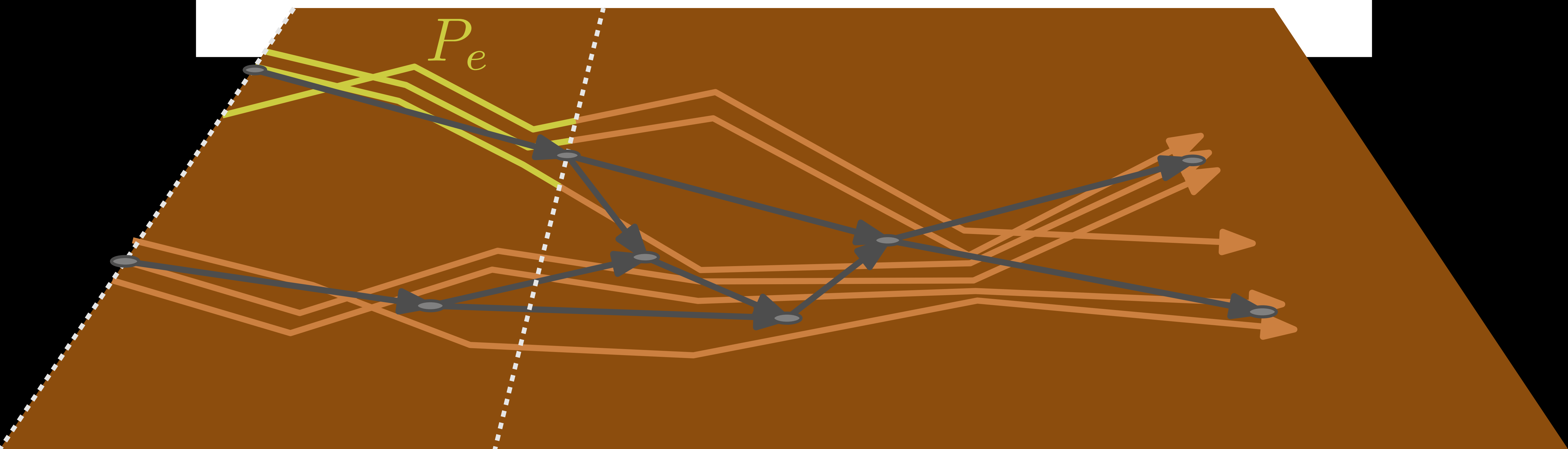
    $$\int \mathcal{L}(\{f_\sigma \mid \sigma \in P_e\})(t)\,\mathrm{d}t$$
  - Constructing $\mathcal{R}$ takes $O(\tau n^2 \log n)$ time.



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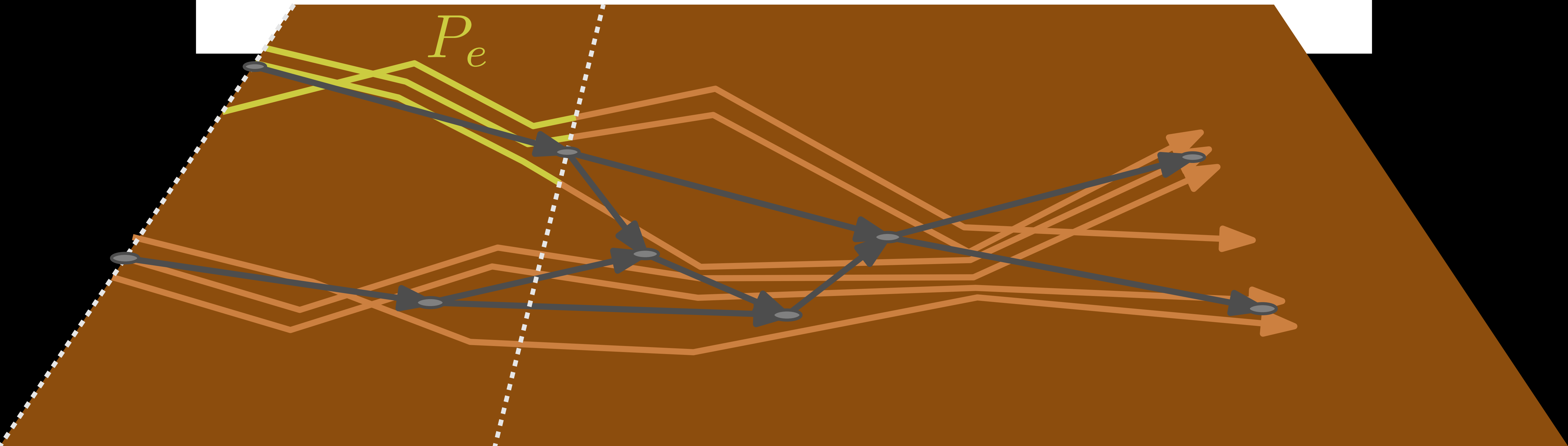

ENTITIES MOVING IN $\mathbb{R}^d$

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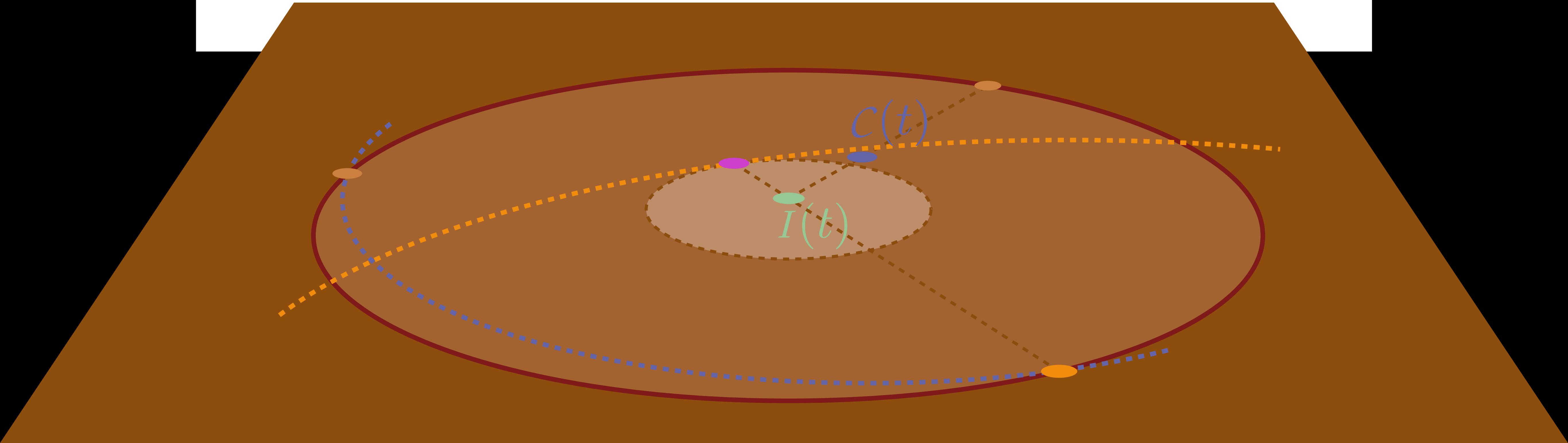

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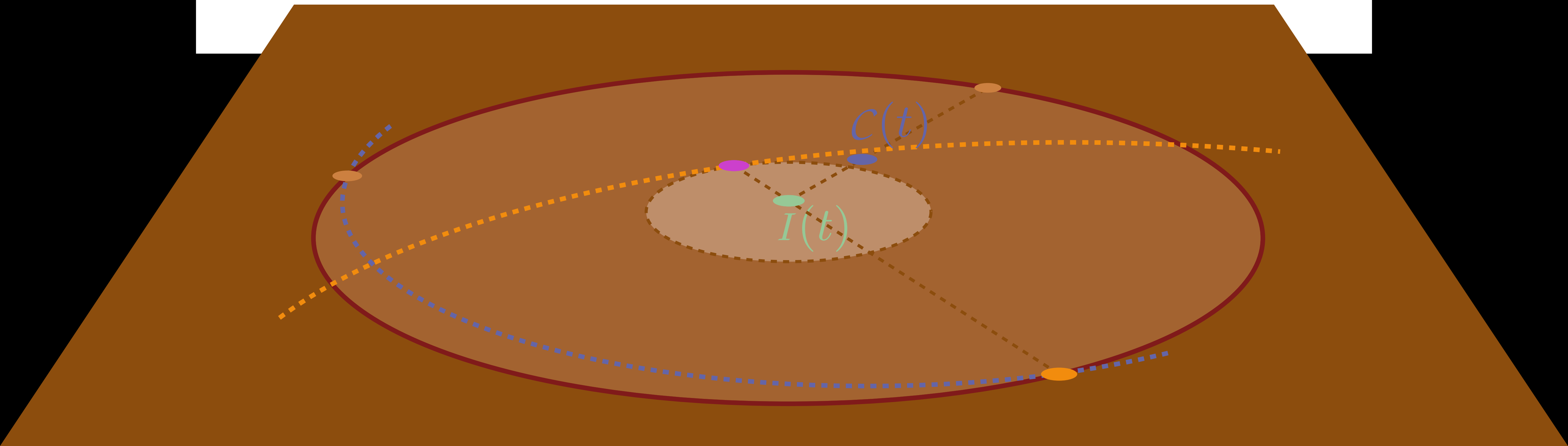

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  - Same observations hold, and allow to show that $\mathcal{C}$ has complexity $O(\tau n^2 \sqrt{n})$.
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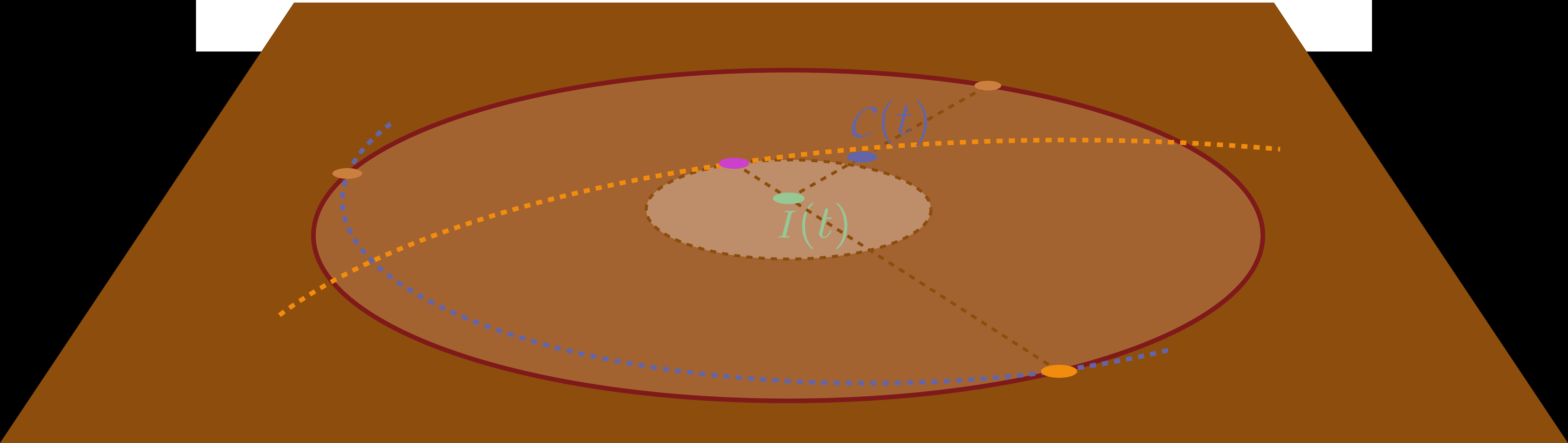

ENTITIES MOVING IN $\mathbb{R}^d$

OPEN PROBLEM.
How to compute lower envelopes for all edges of $\mathcal{R}$ efficiently?

- The weight of edge e is

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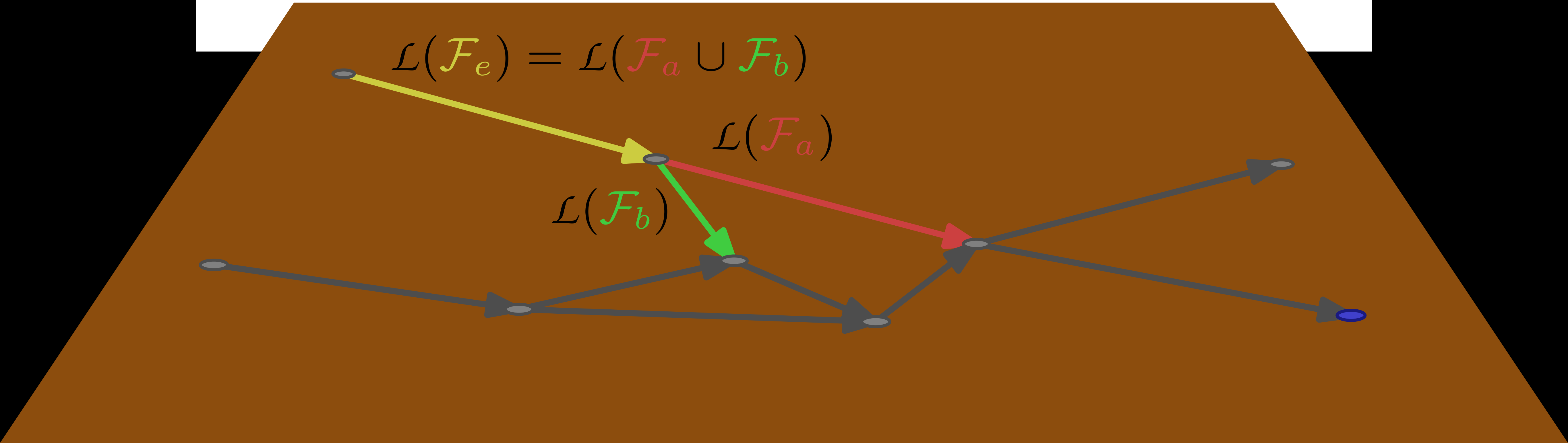

CONCLUSIONS

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  - “simple median” optimises homotopy area
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THANK YOU!