ON THE COMPLEXITY OF BARRIER RESILIENCE FOR FAT REGIONS

Matias Korman
Maarten Löffler
Rodrigo Silveira
Darren Strash

INTRODUCTION

Let t be a desirable point in the plane.

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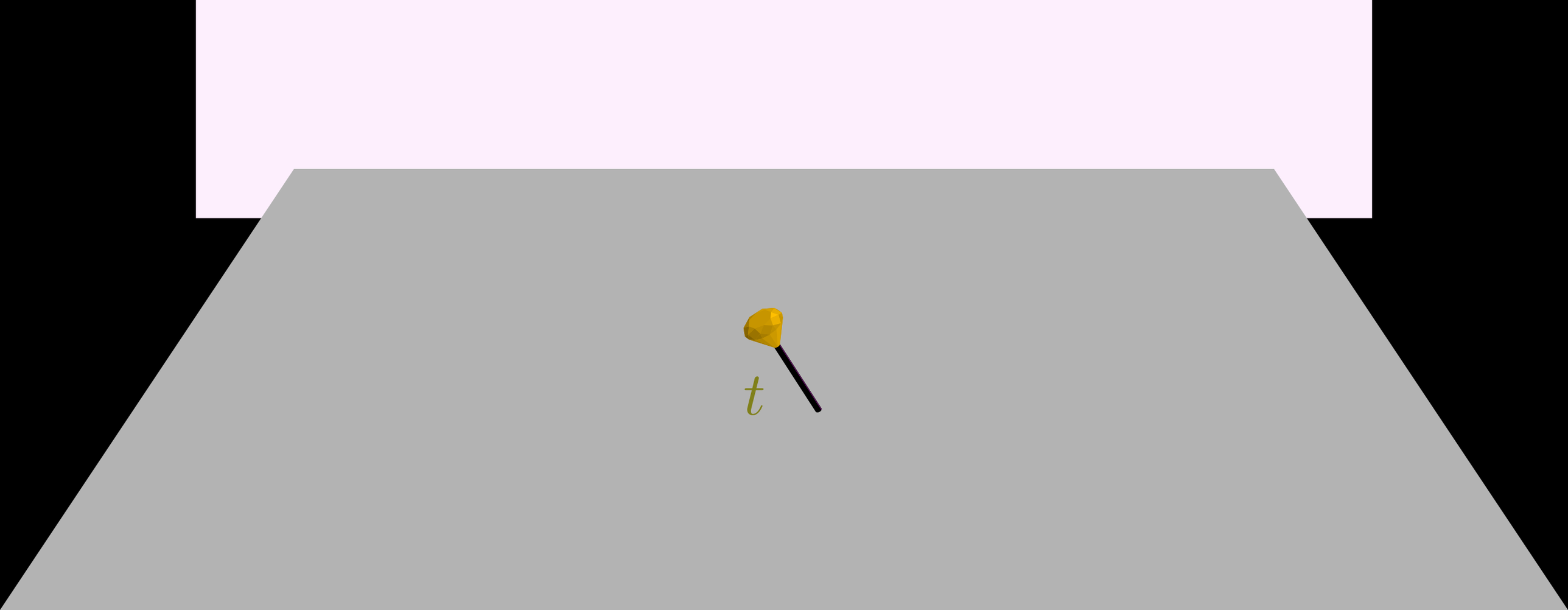

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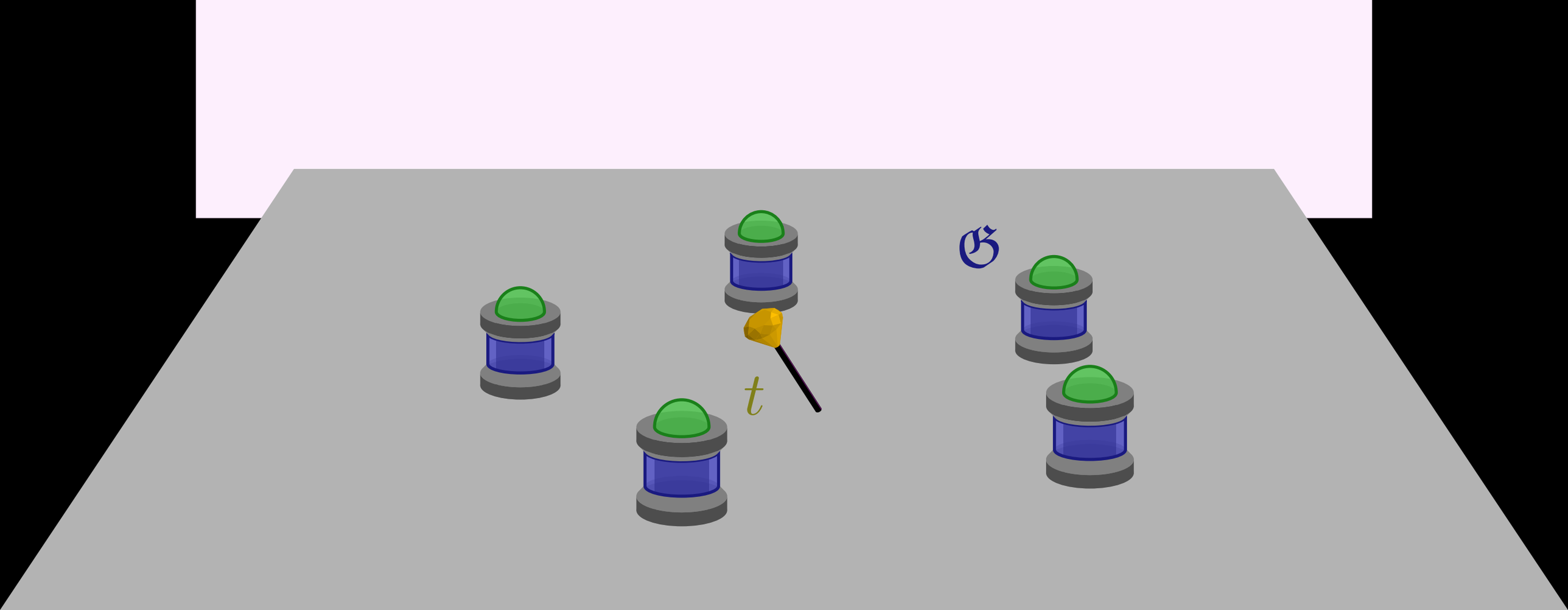

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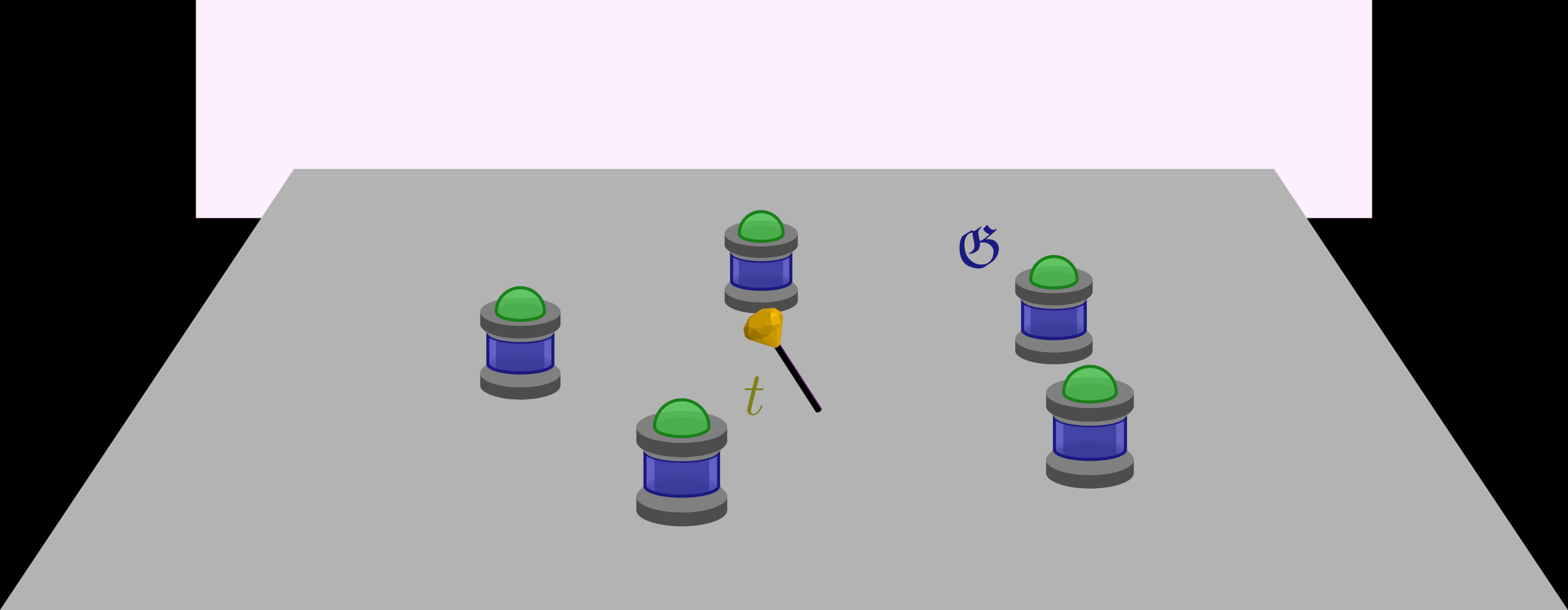

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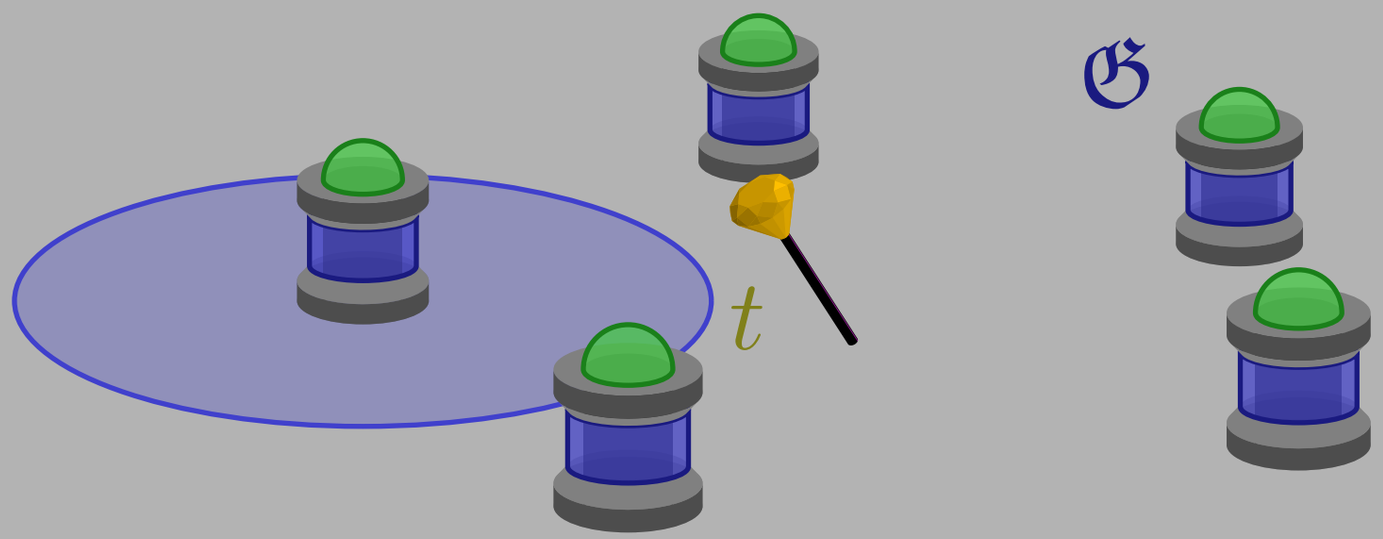

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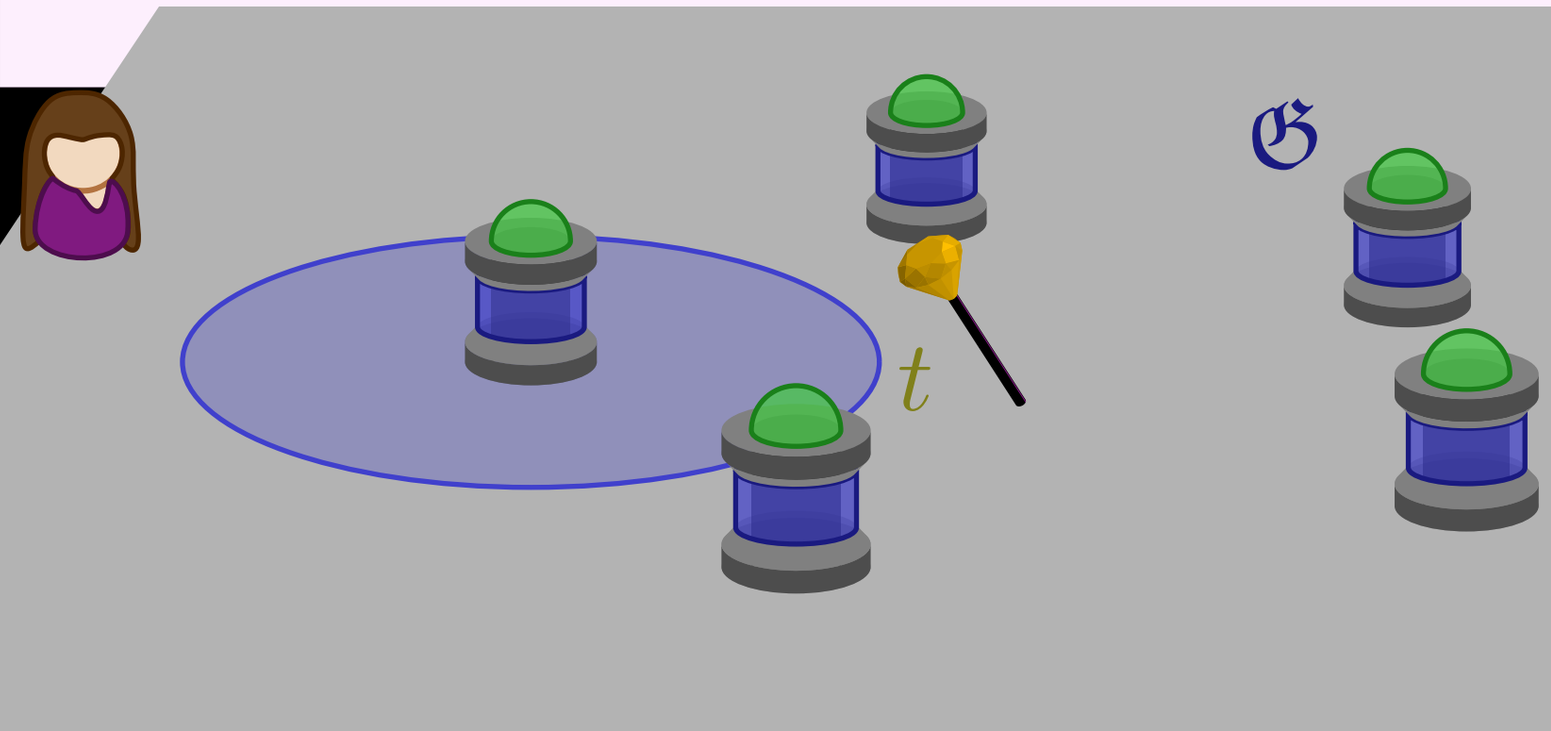

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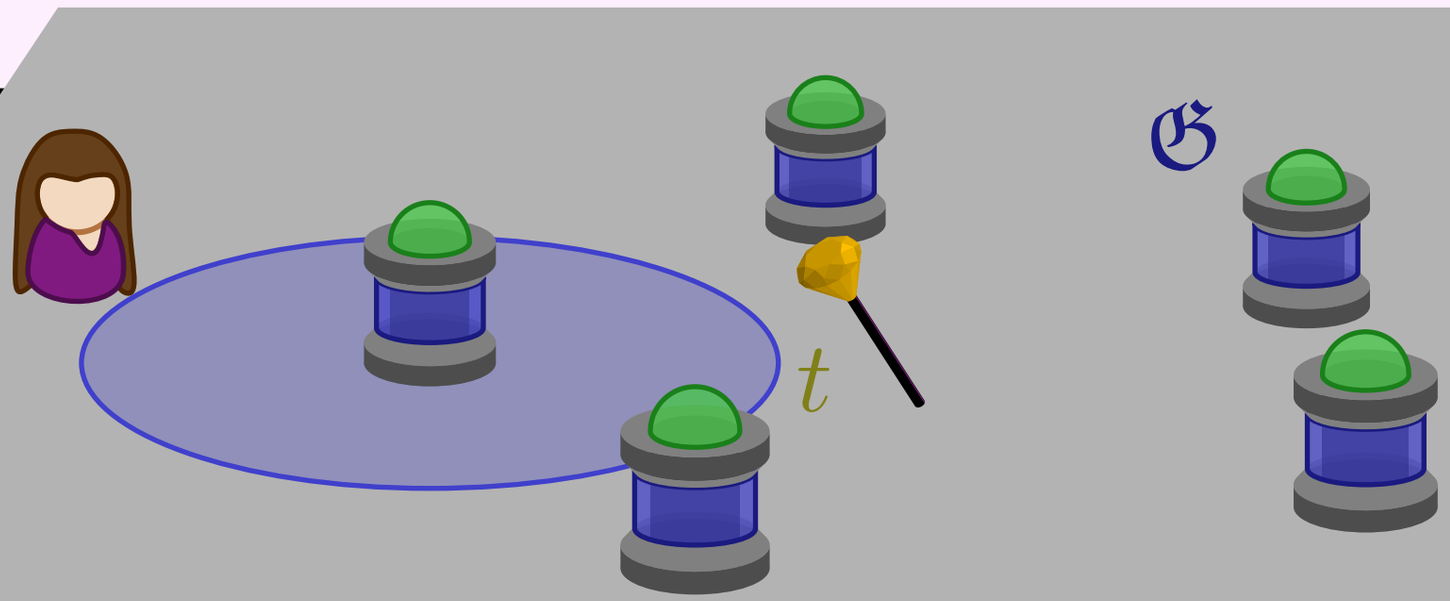

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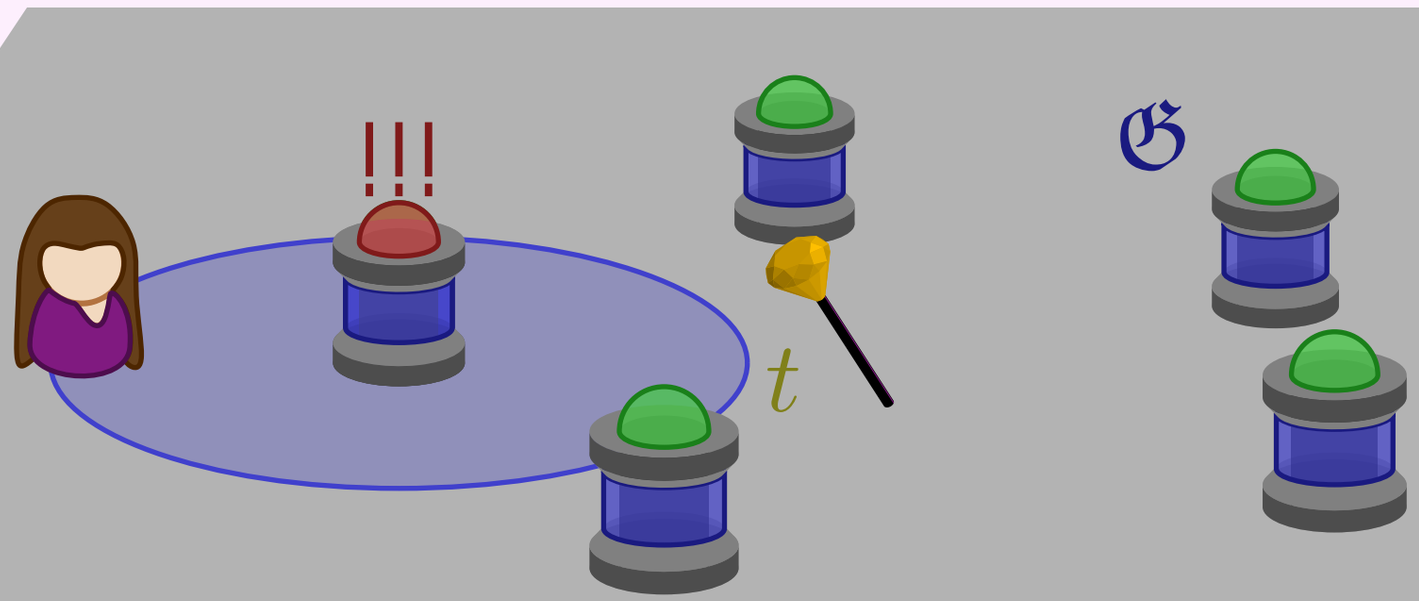

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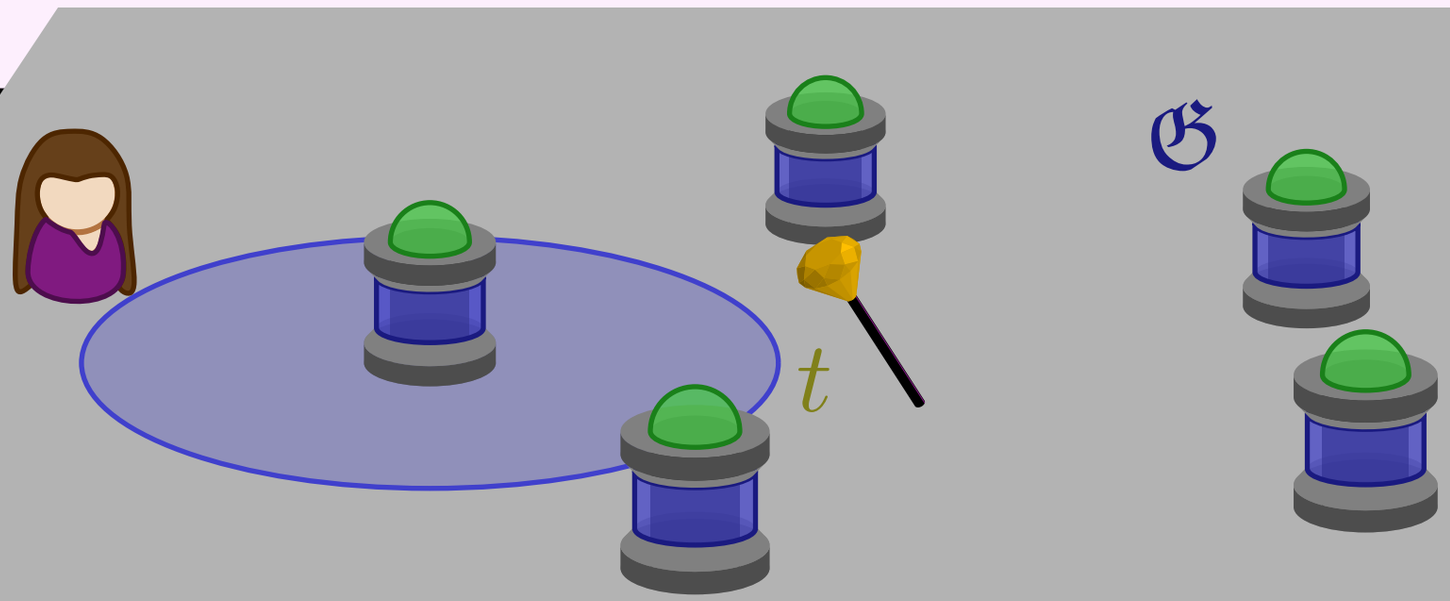

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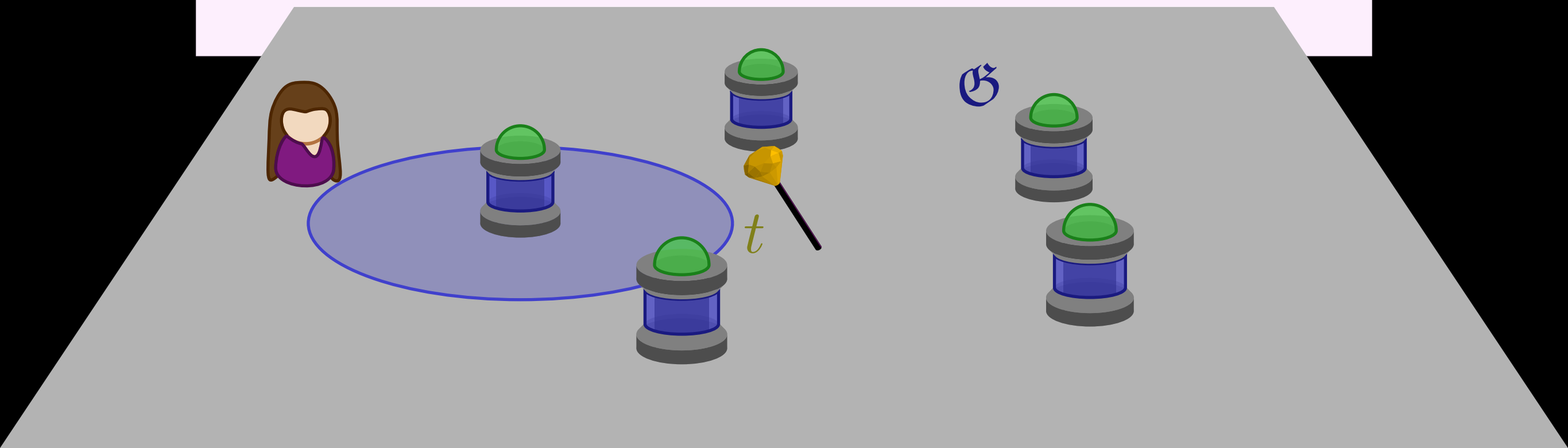

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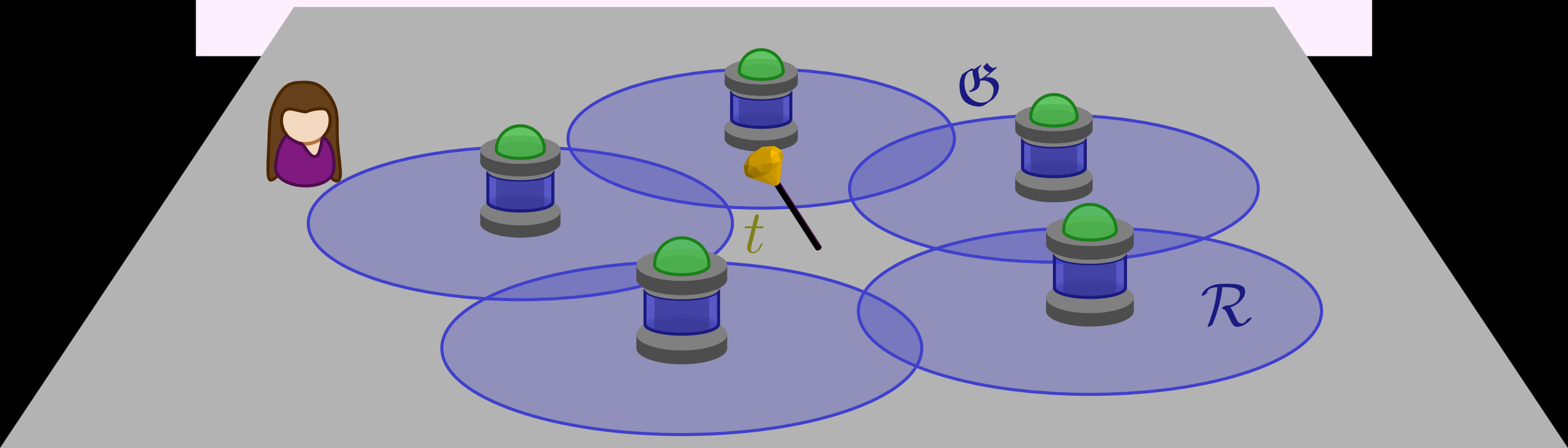

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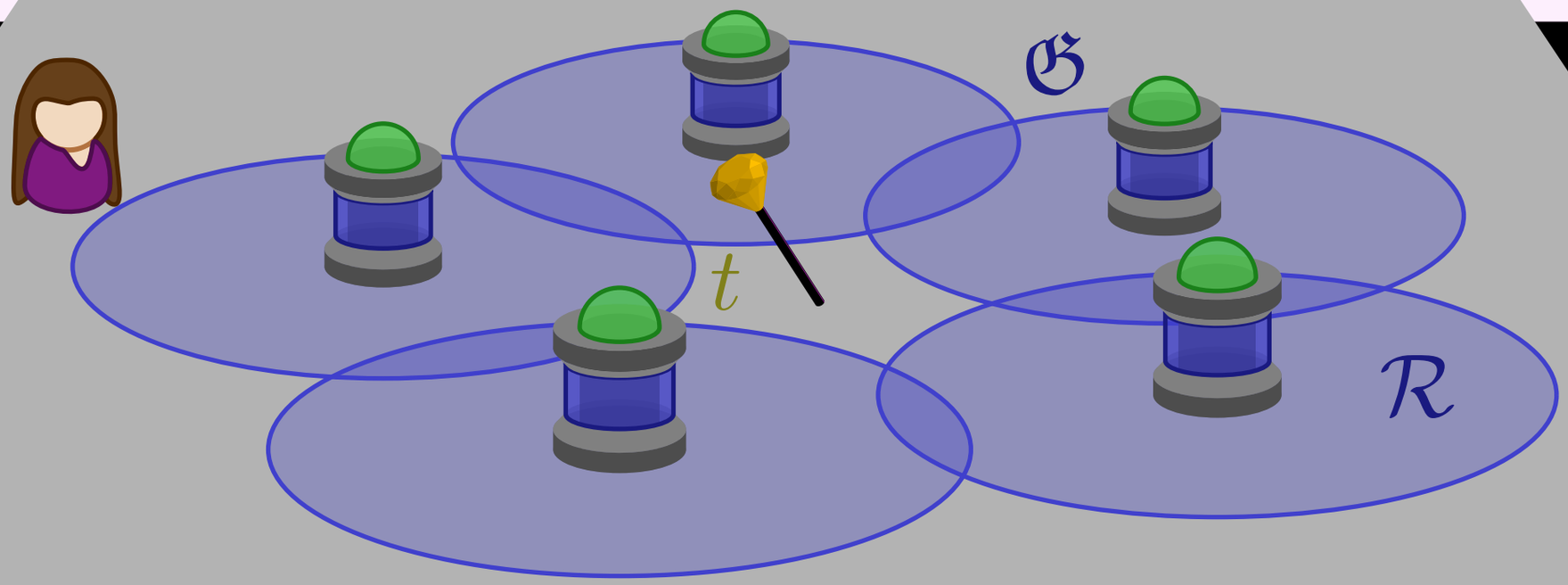
We say $\mathcal{R}$ *guards* t if every path to t passes through one of the regions.



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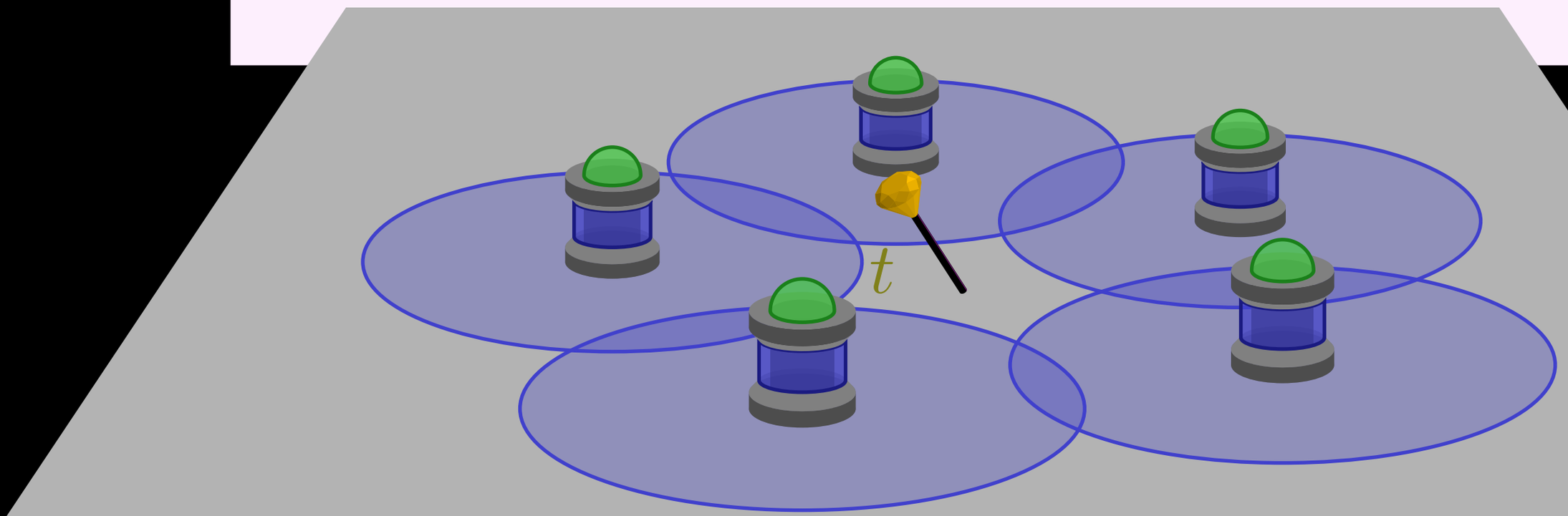

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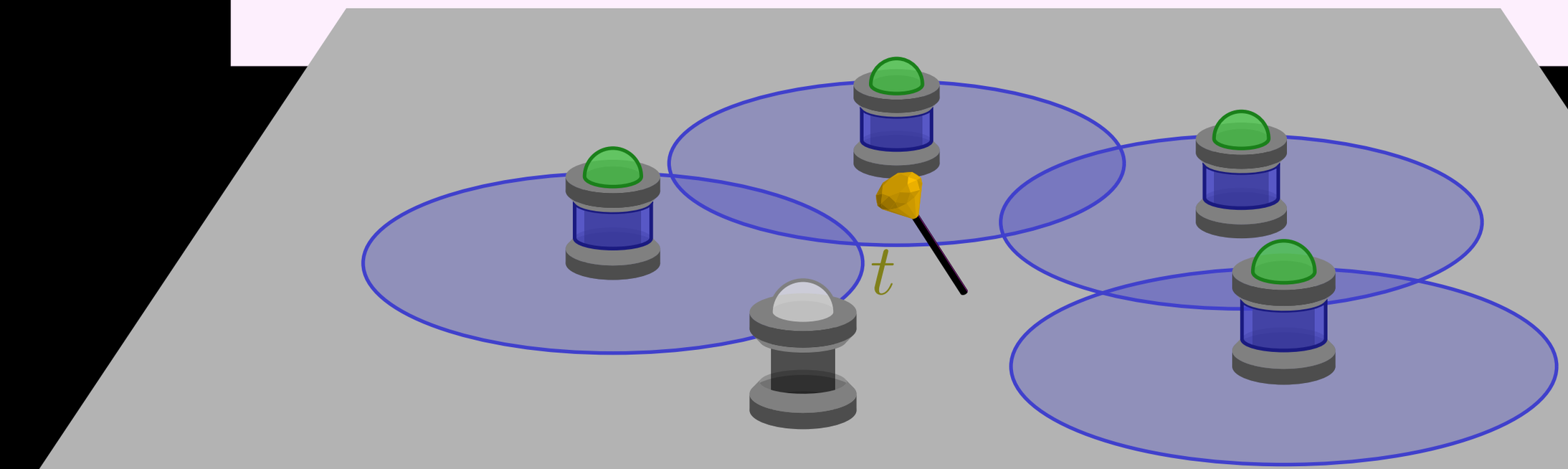

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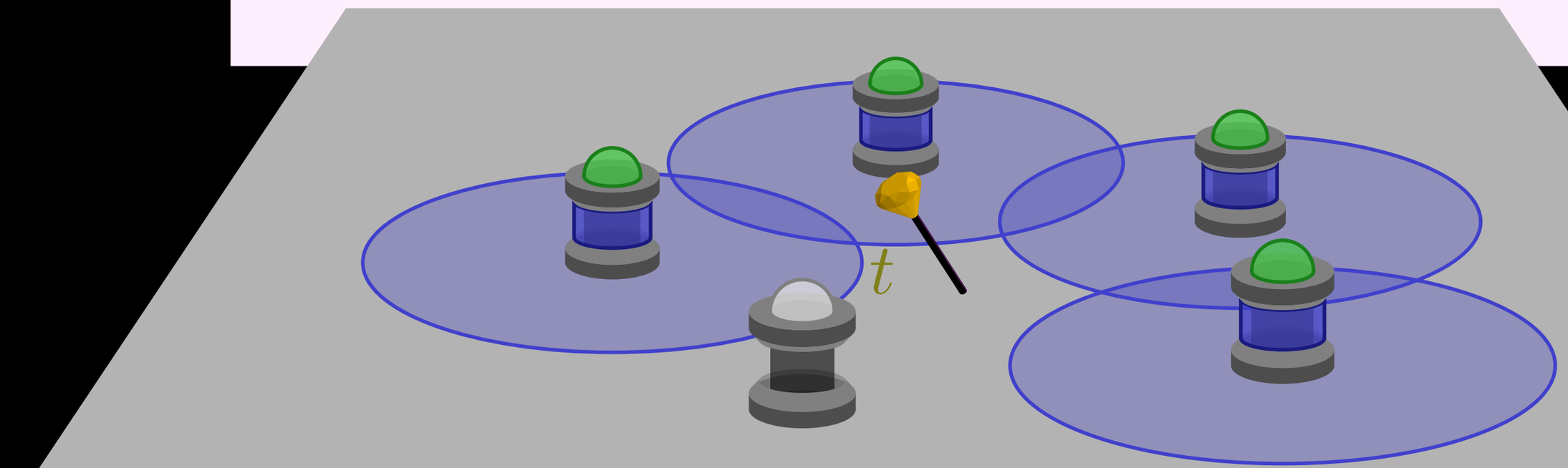
When they do, t may no longer be guarded!



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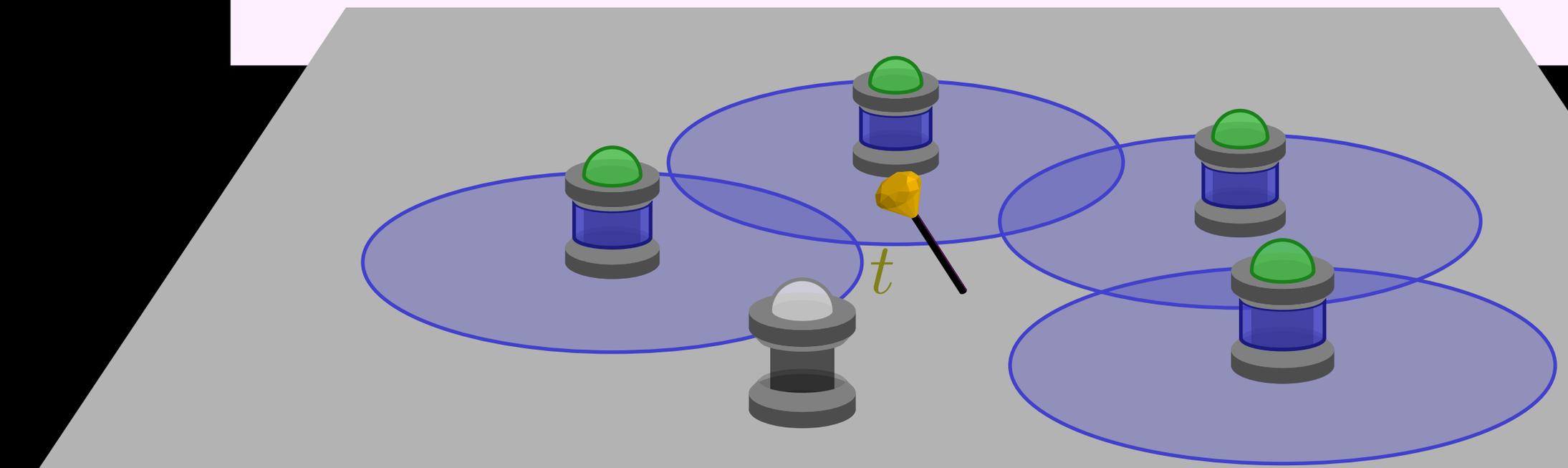

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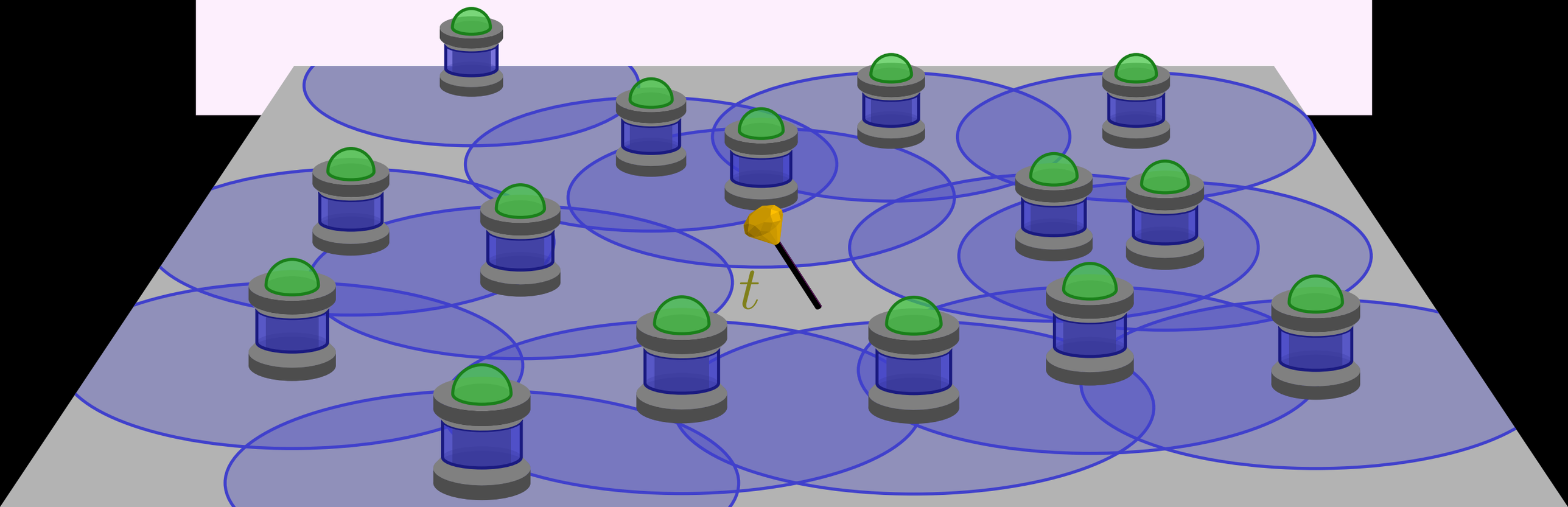

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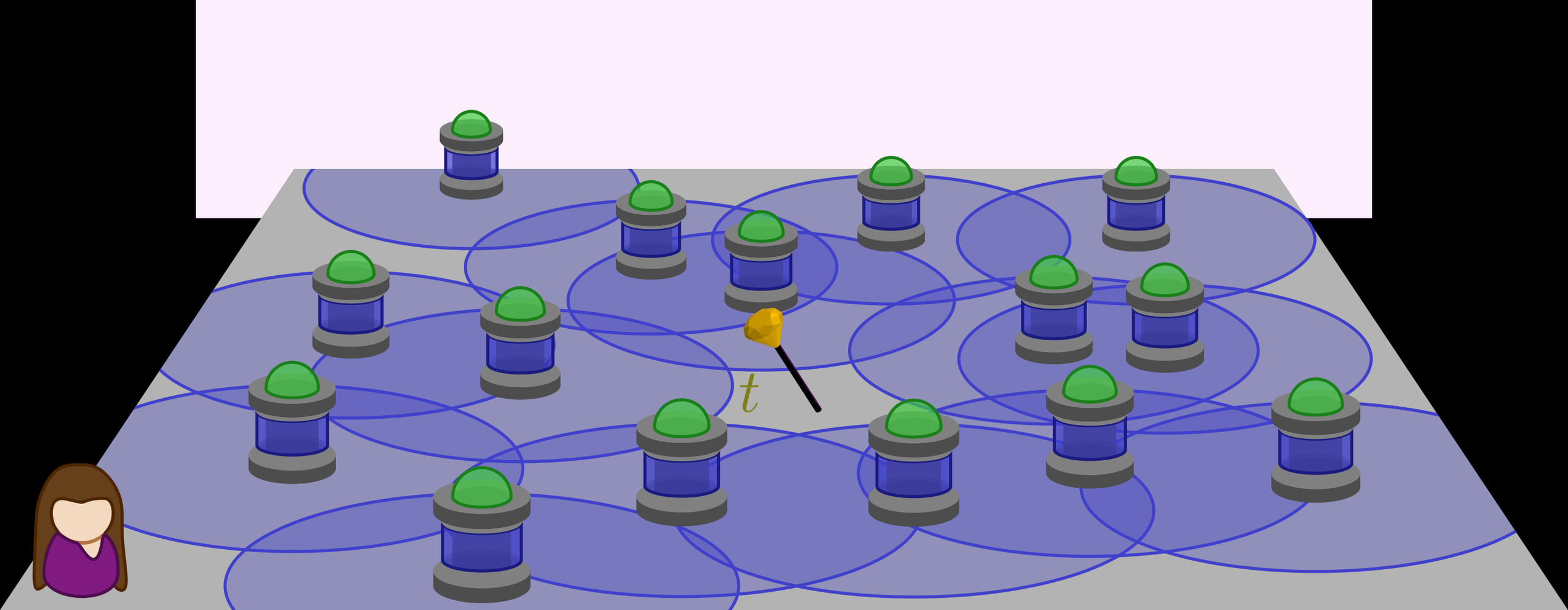

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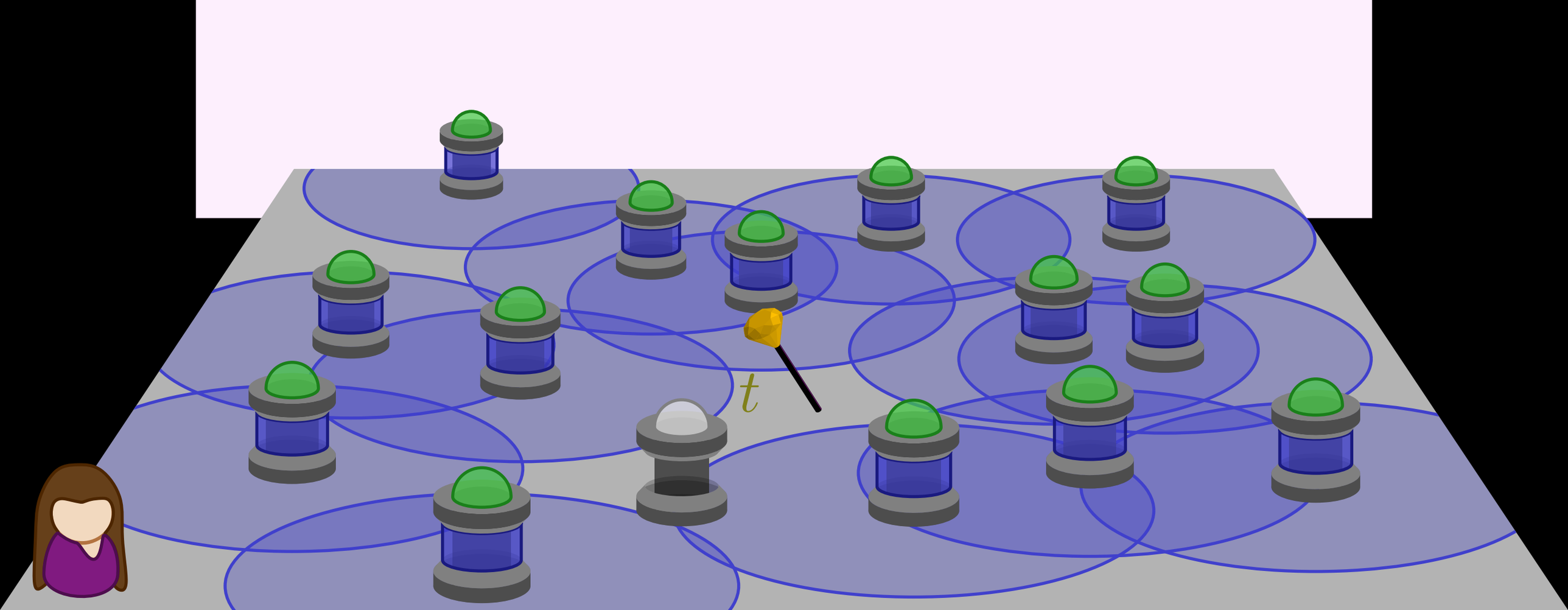

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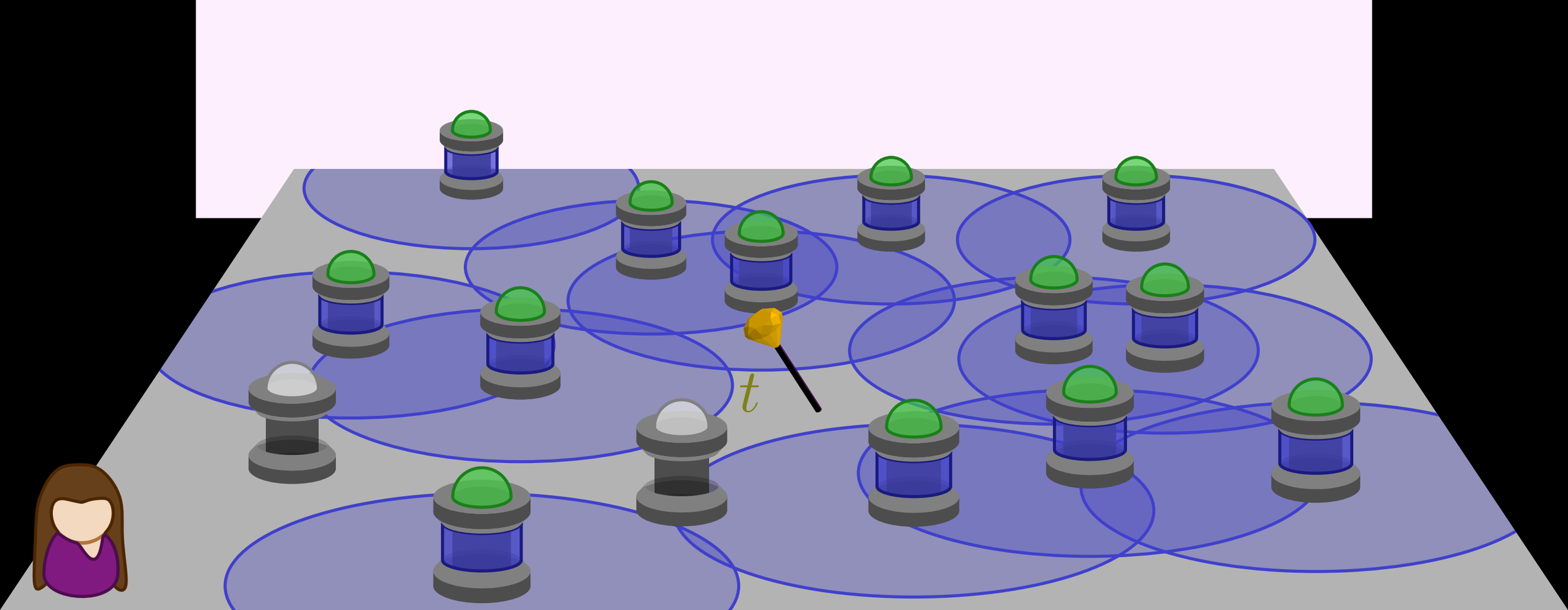

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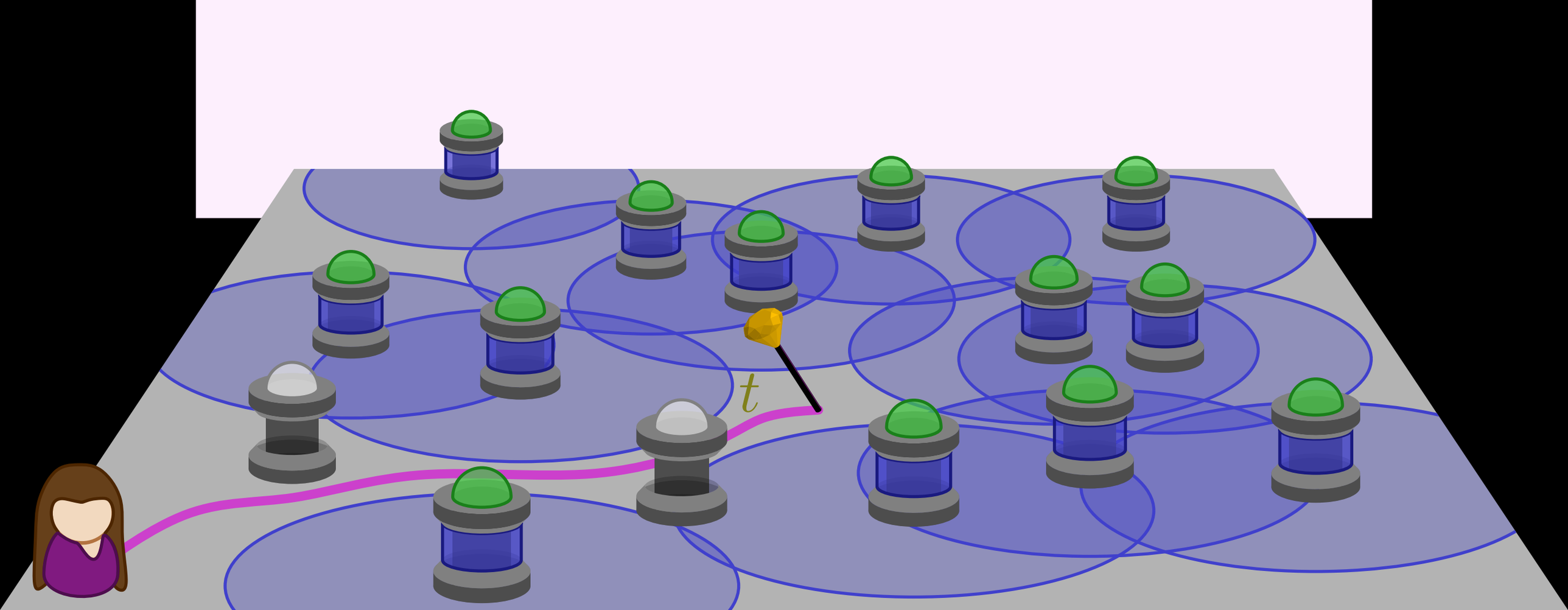

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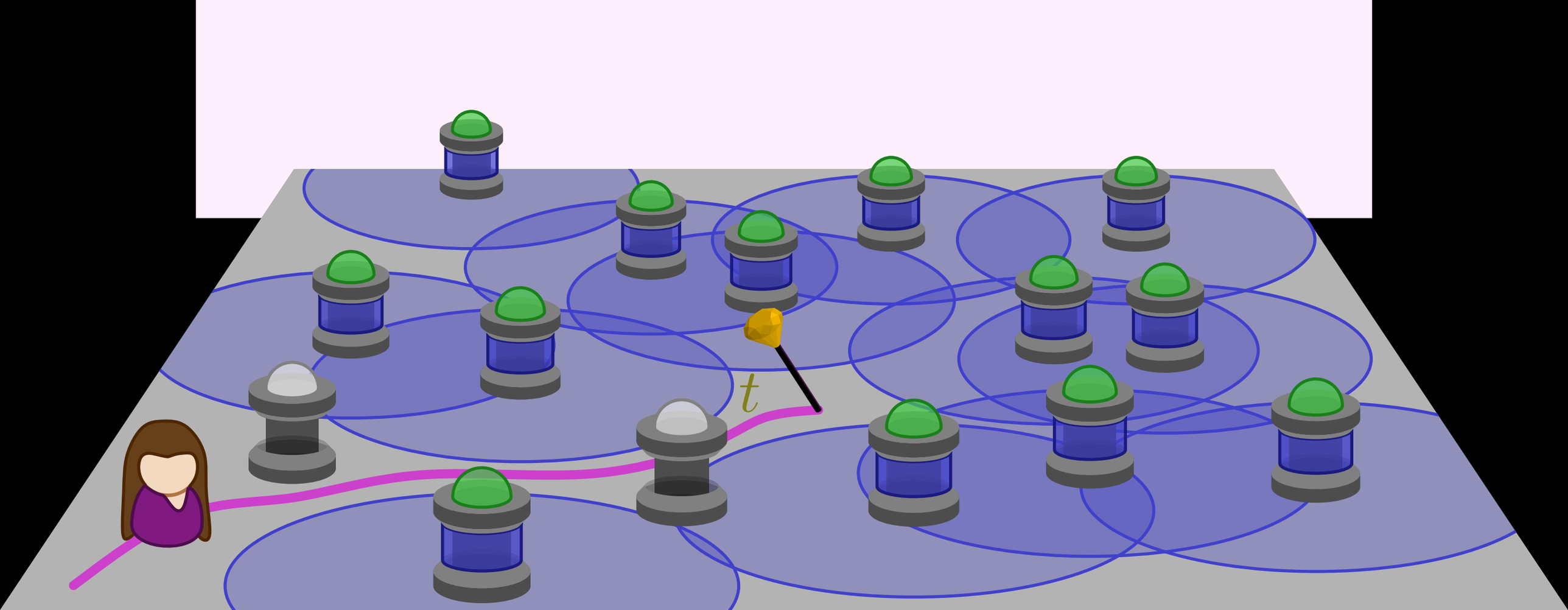

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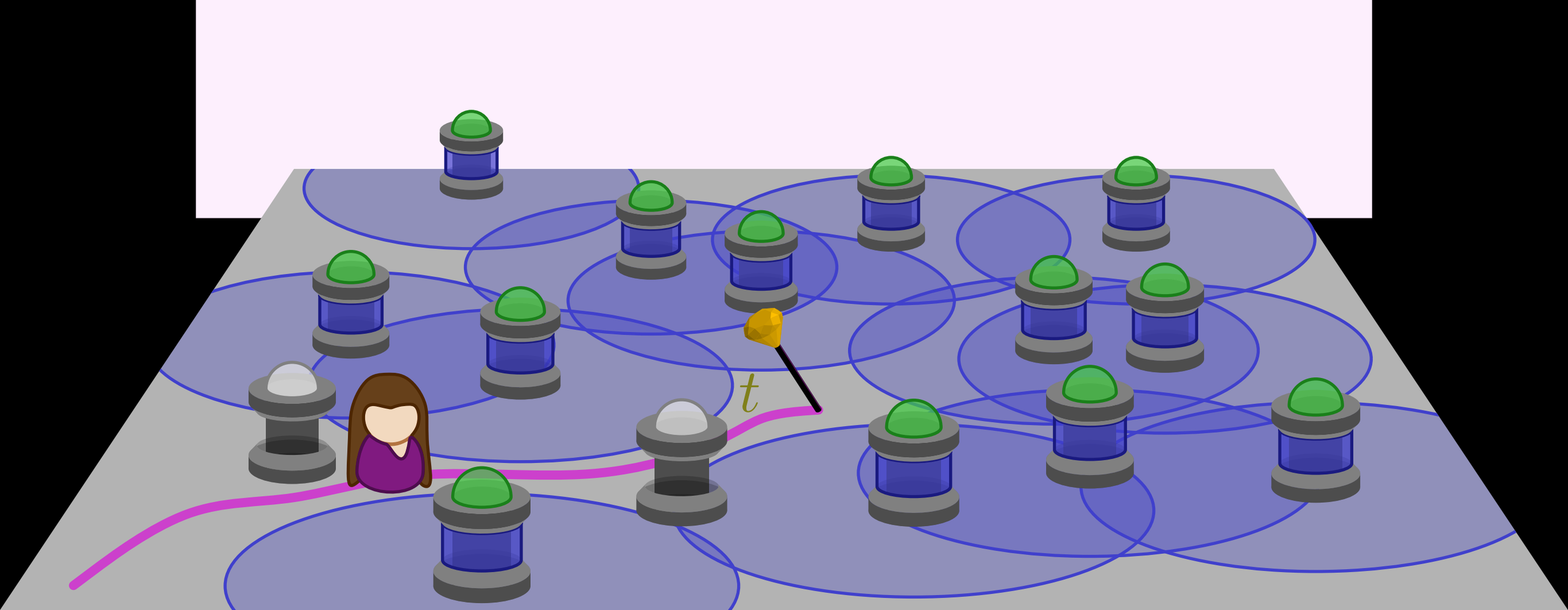

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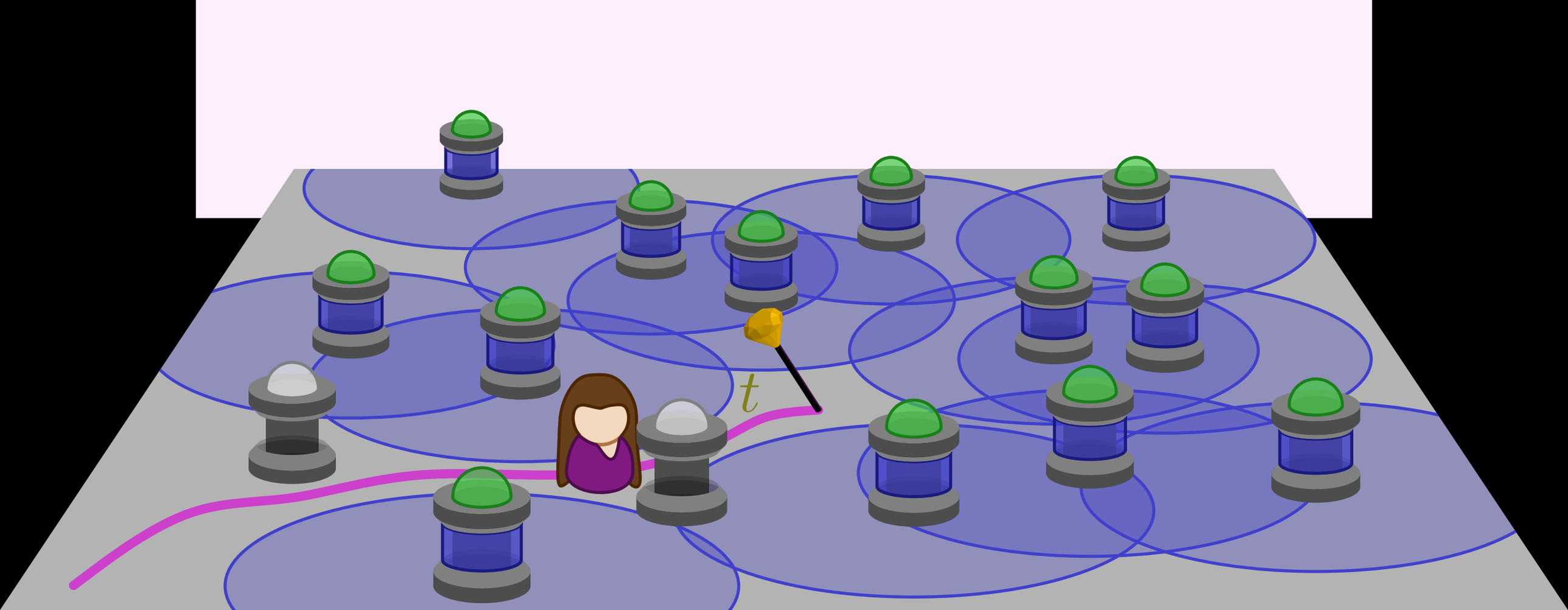

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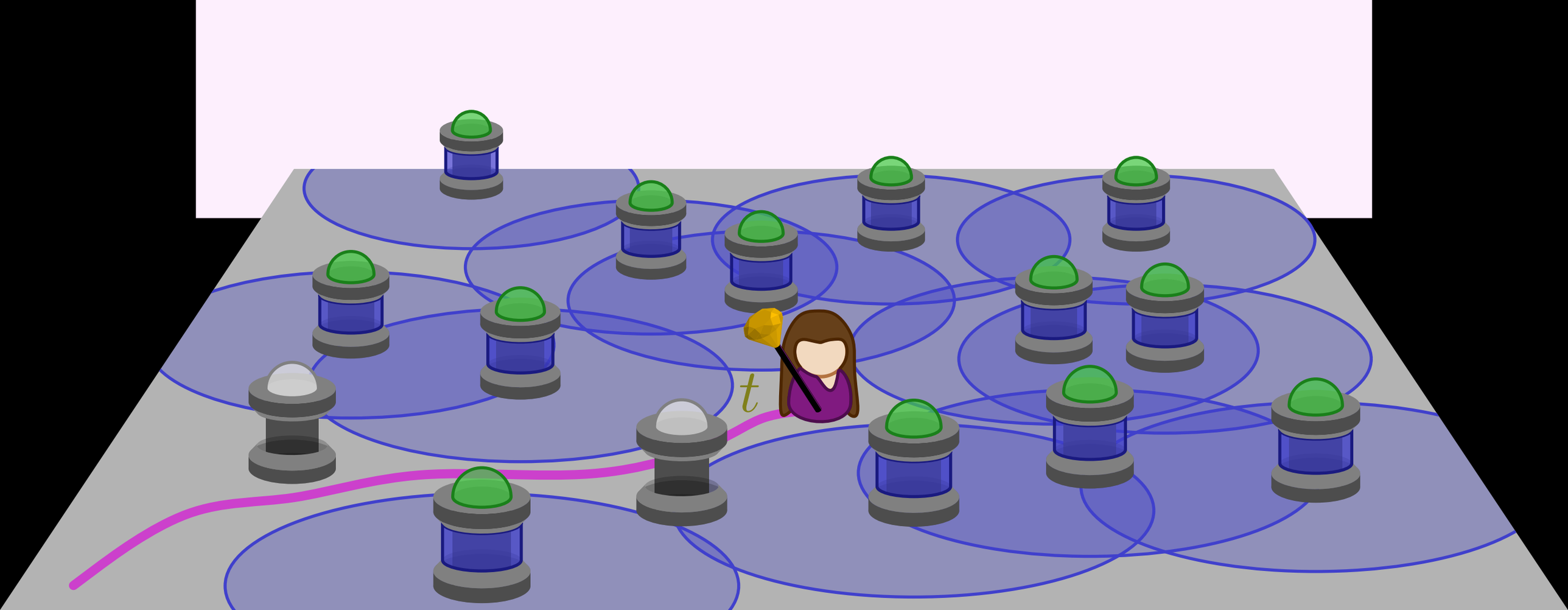

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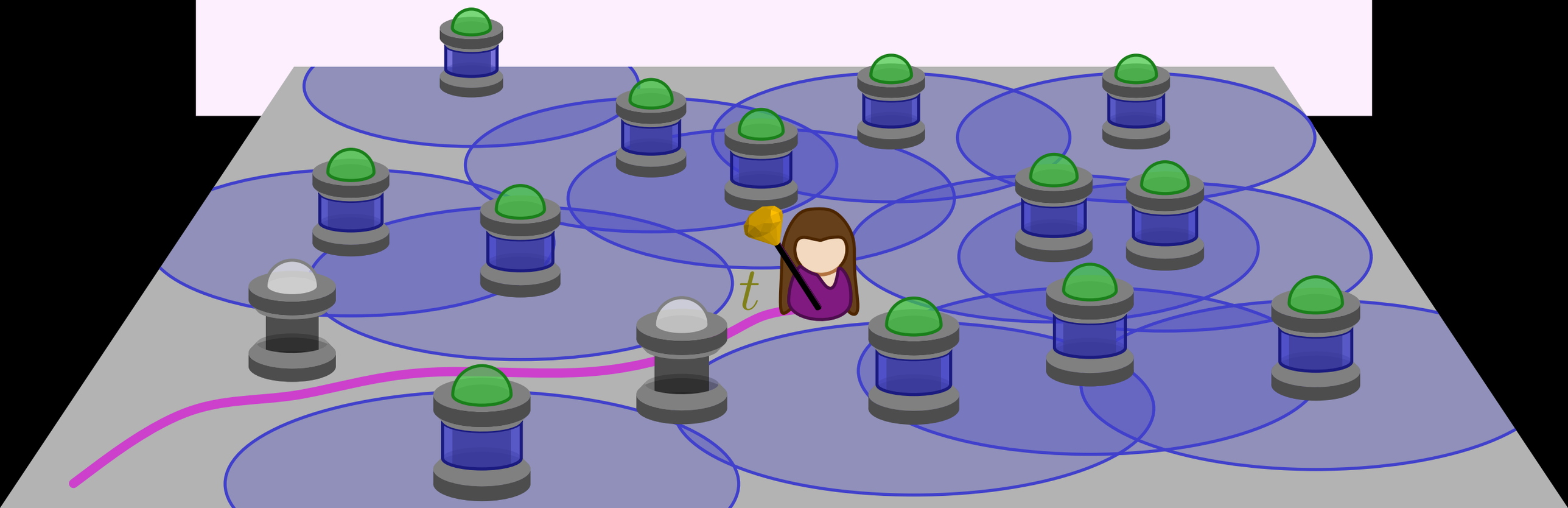

KNOWN & NEW RESULTS

When guarding a *strip*, the resilience can be computed in polynomial time.

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The resilience of unit disks in the plane can be $\frac{5}{6}$-approximated.

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The resilience of unit disks in the plane can be $\frac{5}{6}$-approximated.

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Computing the resilience of line segments in the plane is NP-hard.

(Cabello, Giannopoulos & Knauer, 2011)

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Computing the resilience of β-fat regions is NP-hard.

FIXED PARAMETER ALGORITHM

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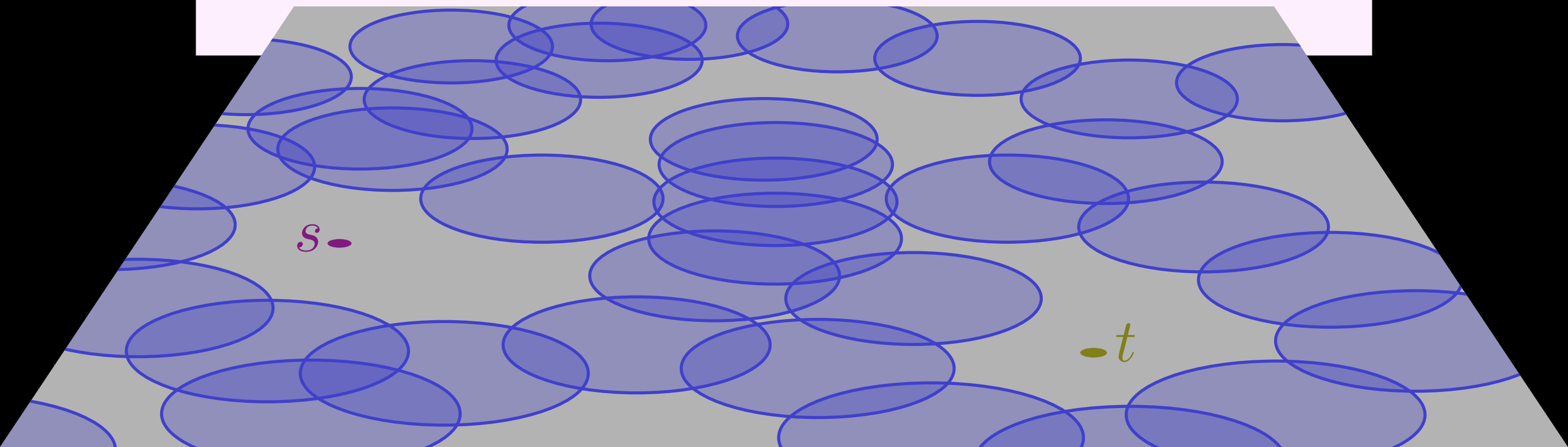

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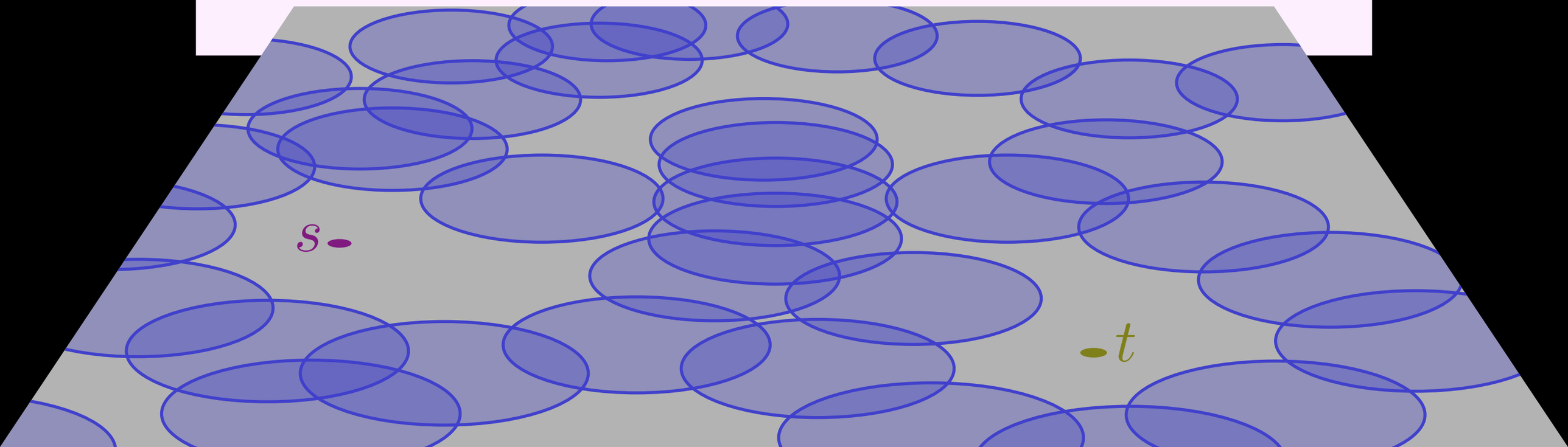

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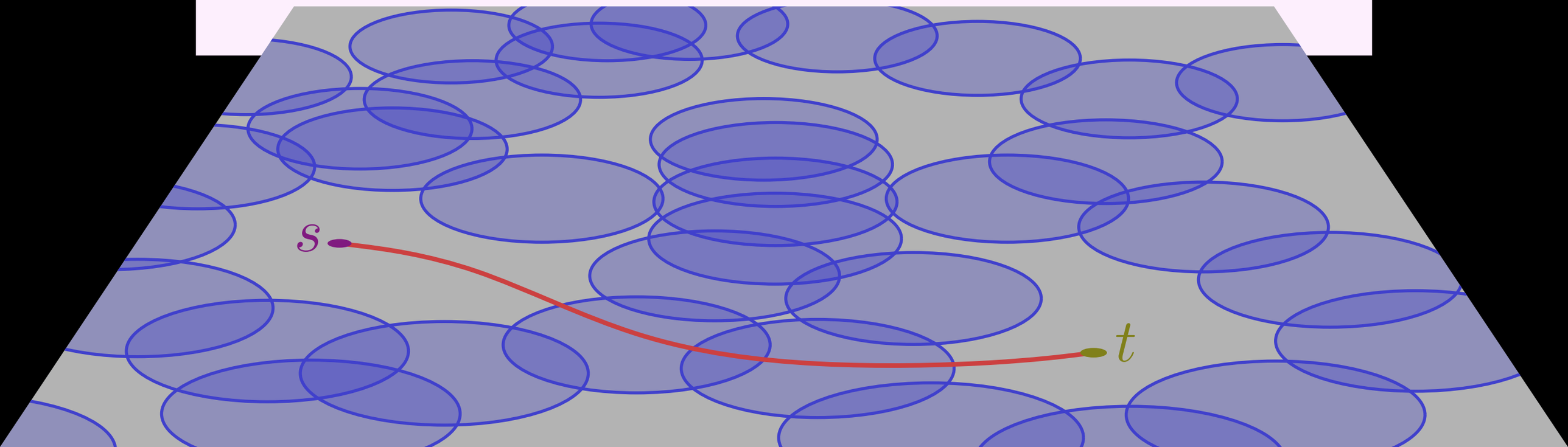

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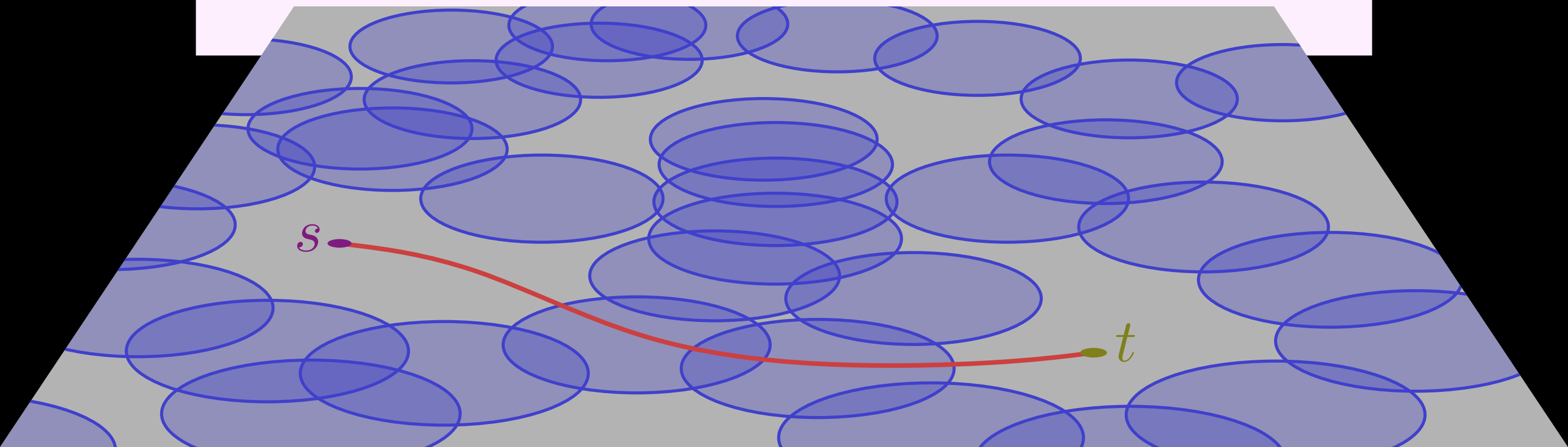
This path gives an upper bound on the resilience.



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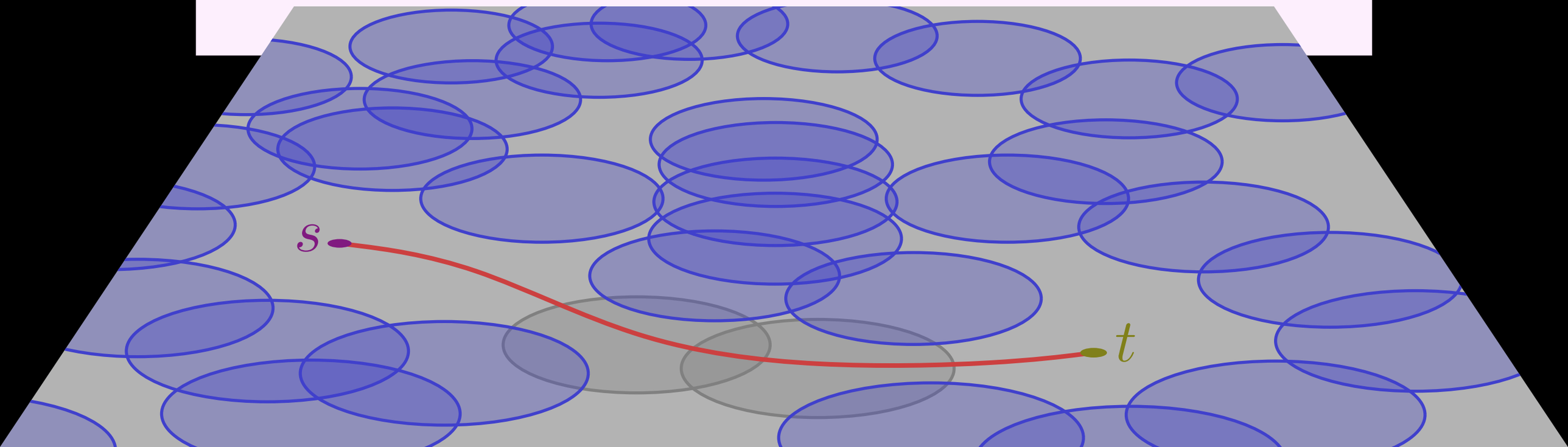

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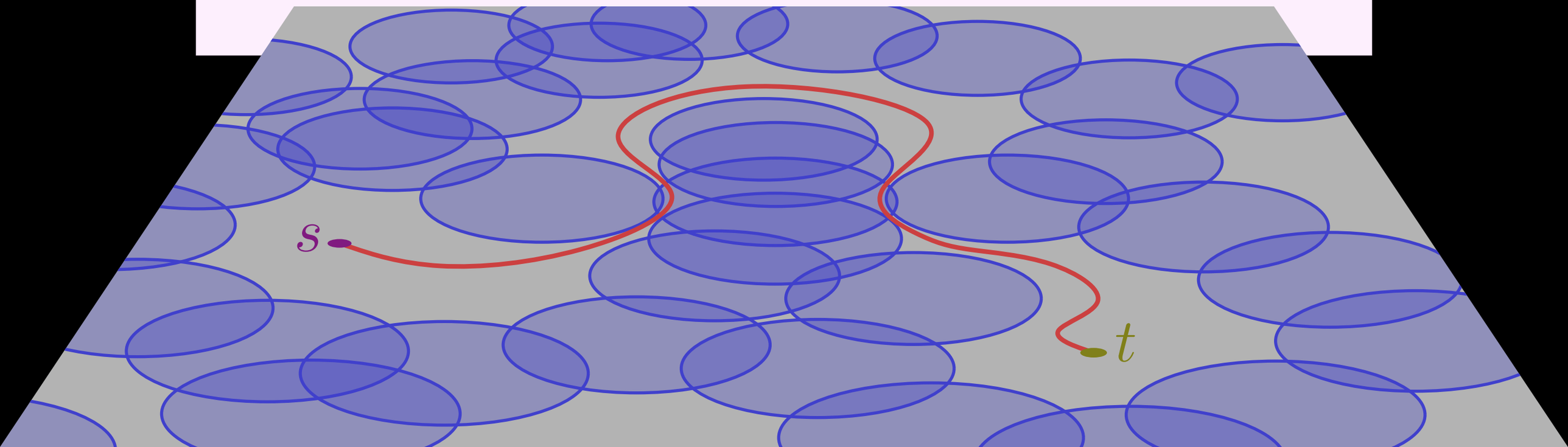

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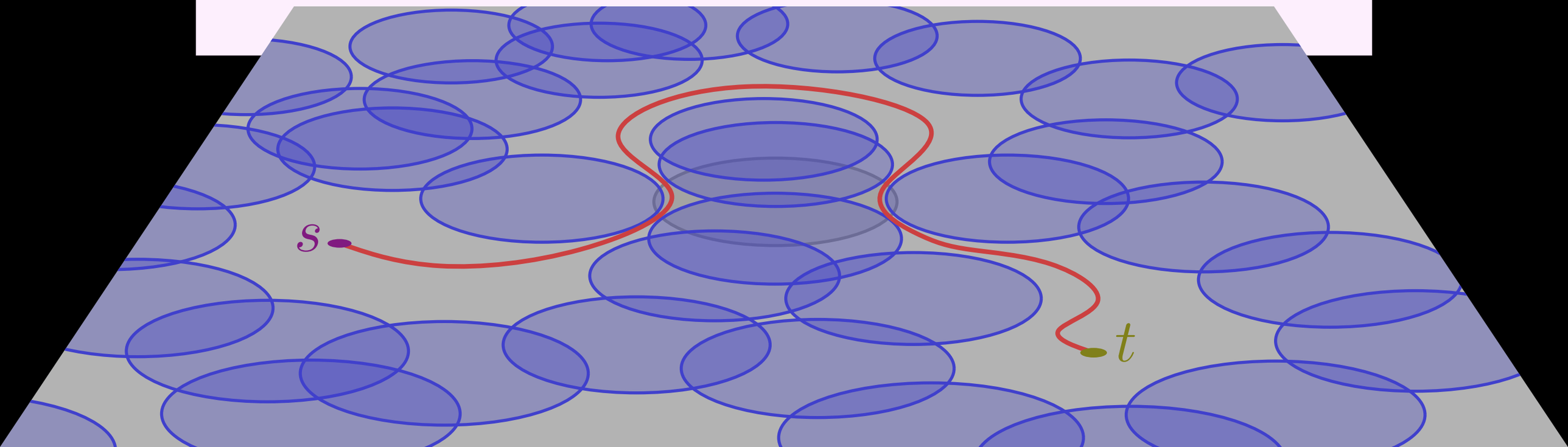

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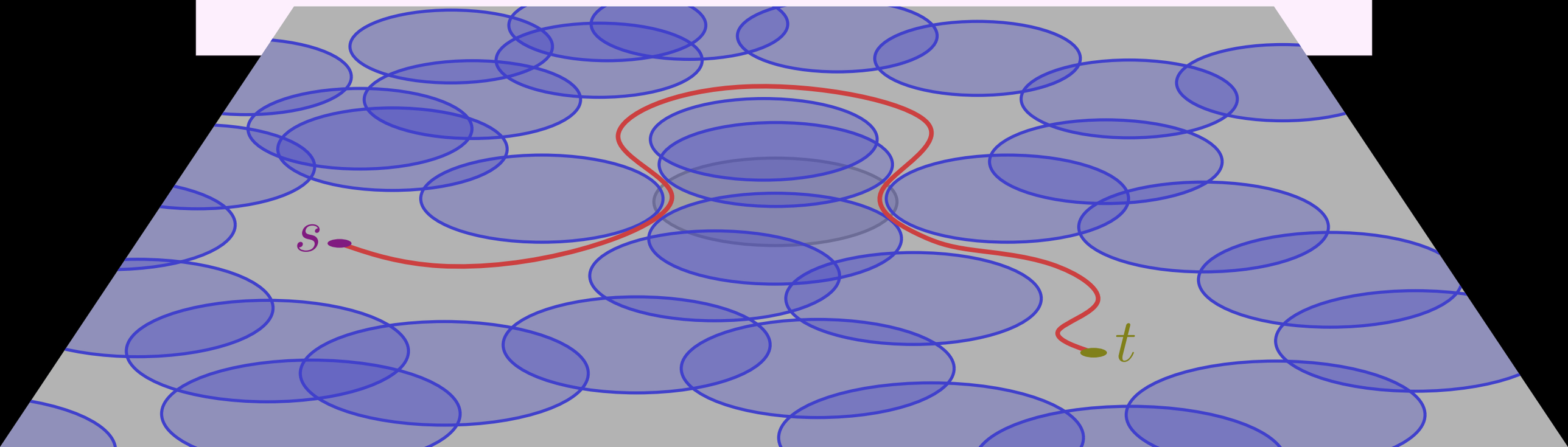
LEMMA: The thickness is at most twice the resilience. (Bereg & Kirkpatrick, 2009)



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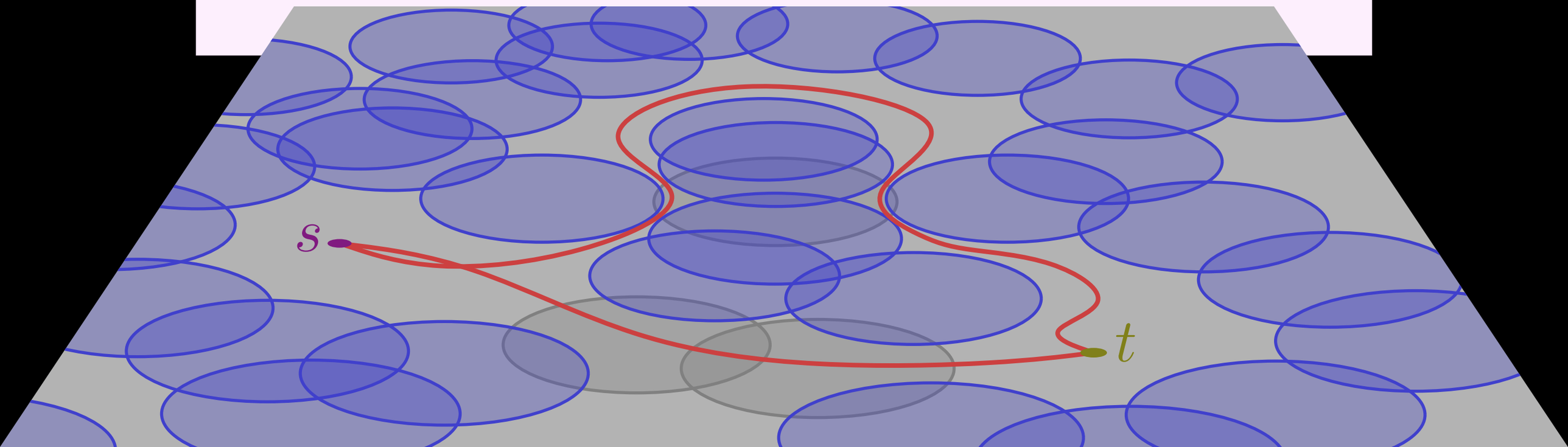

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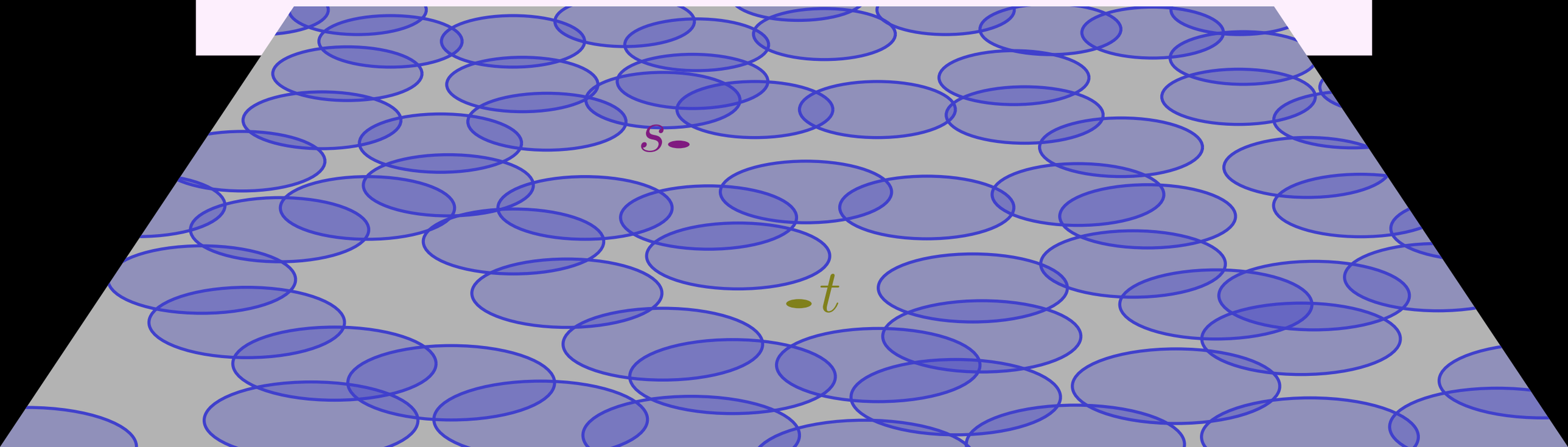

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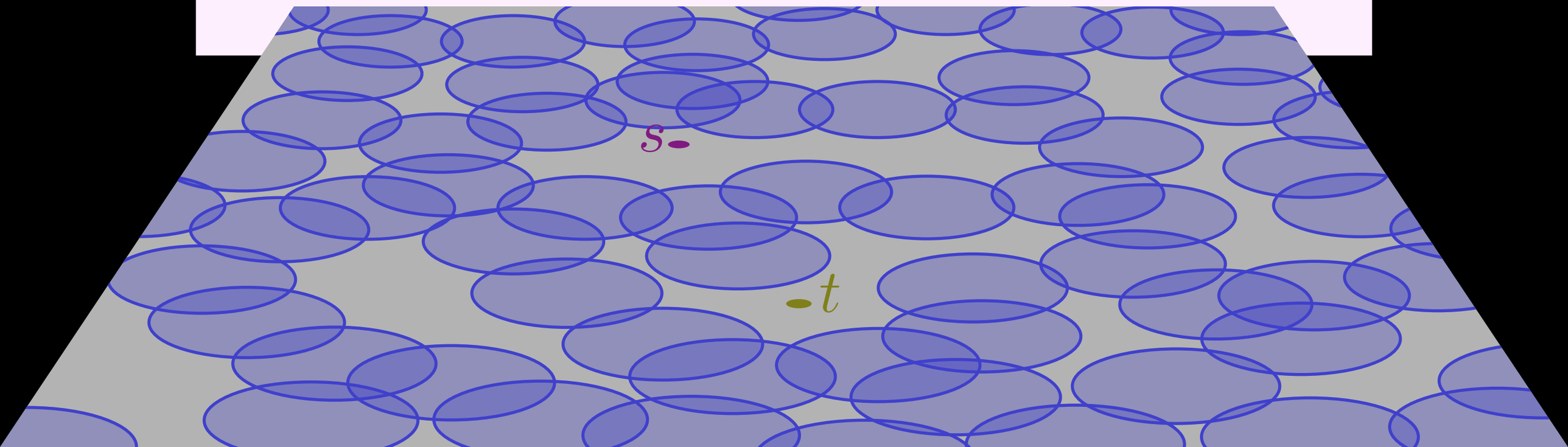

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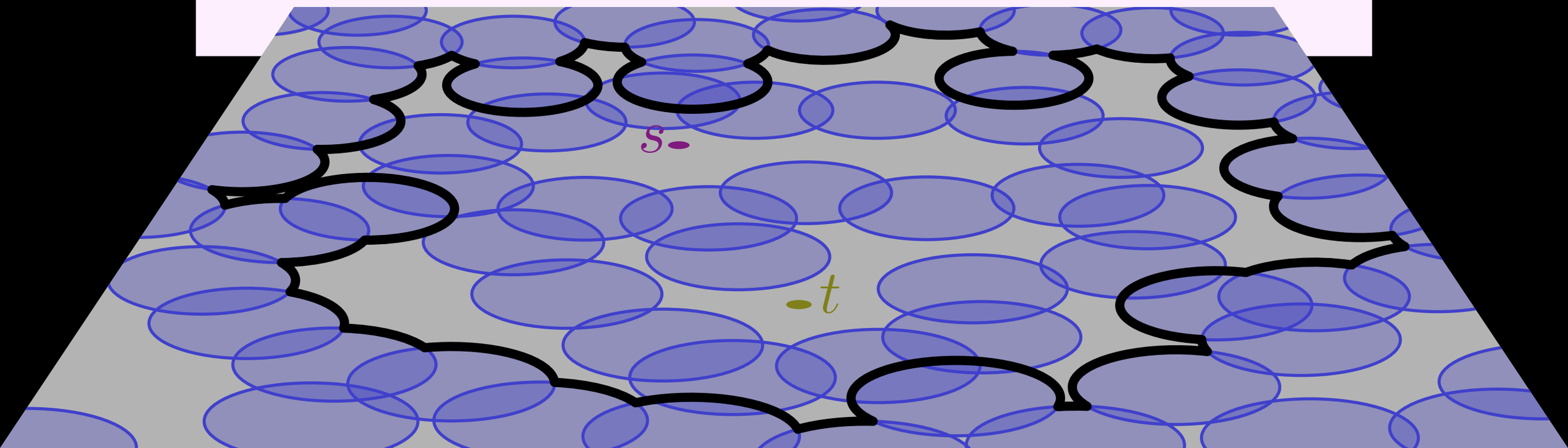

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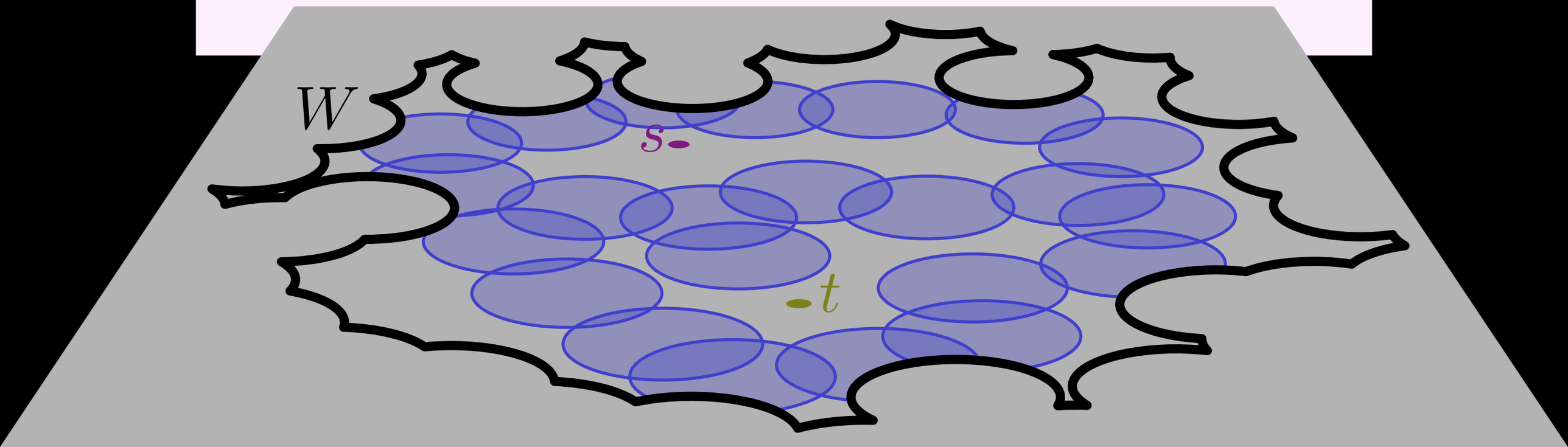

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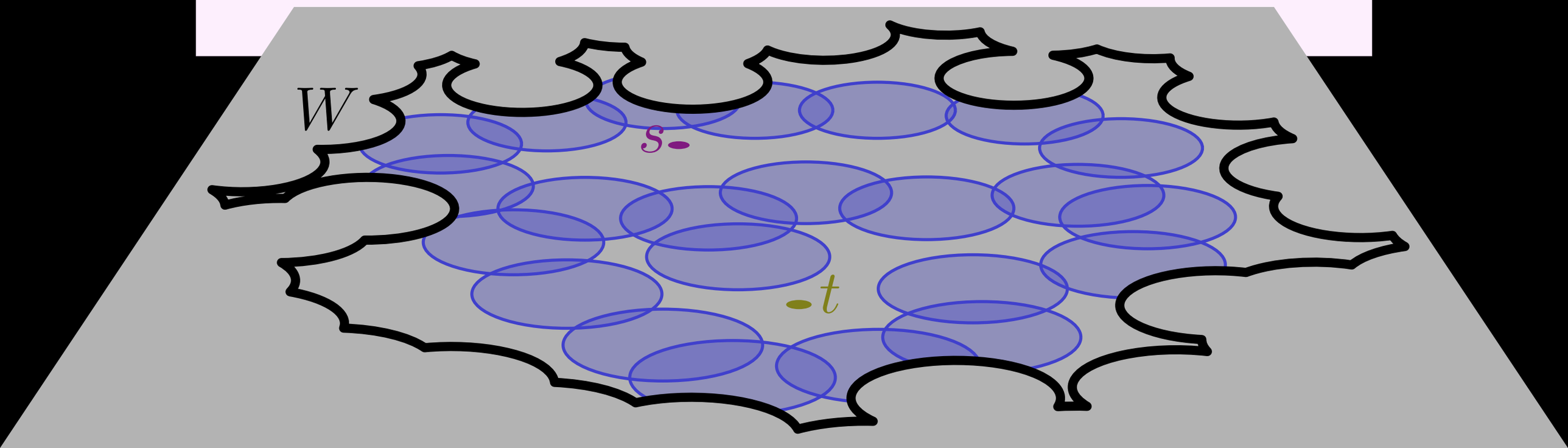

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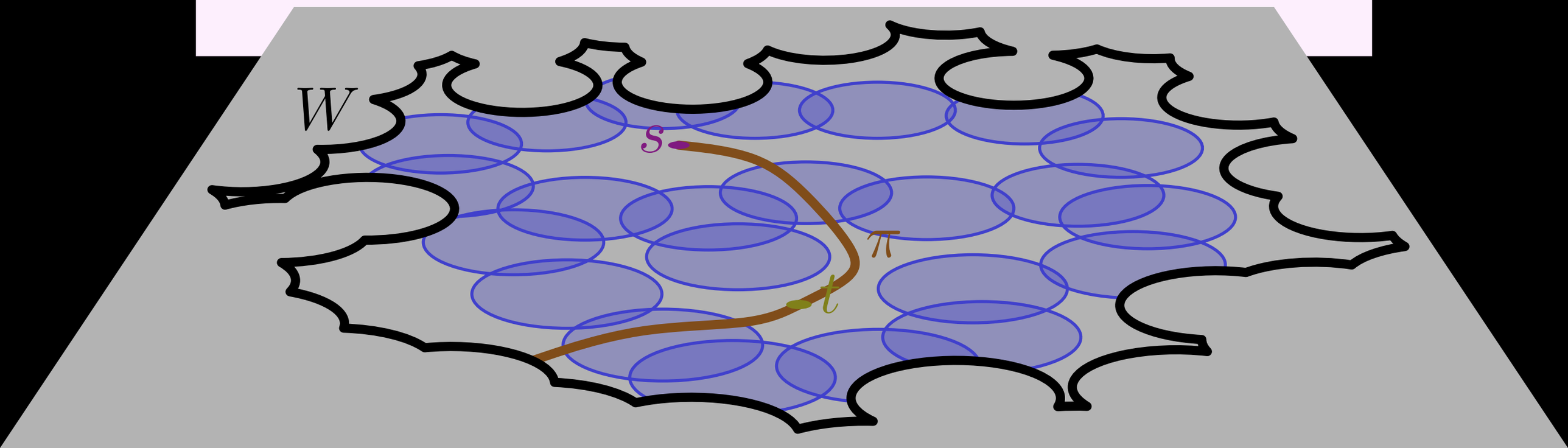

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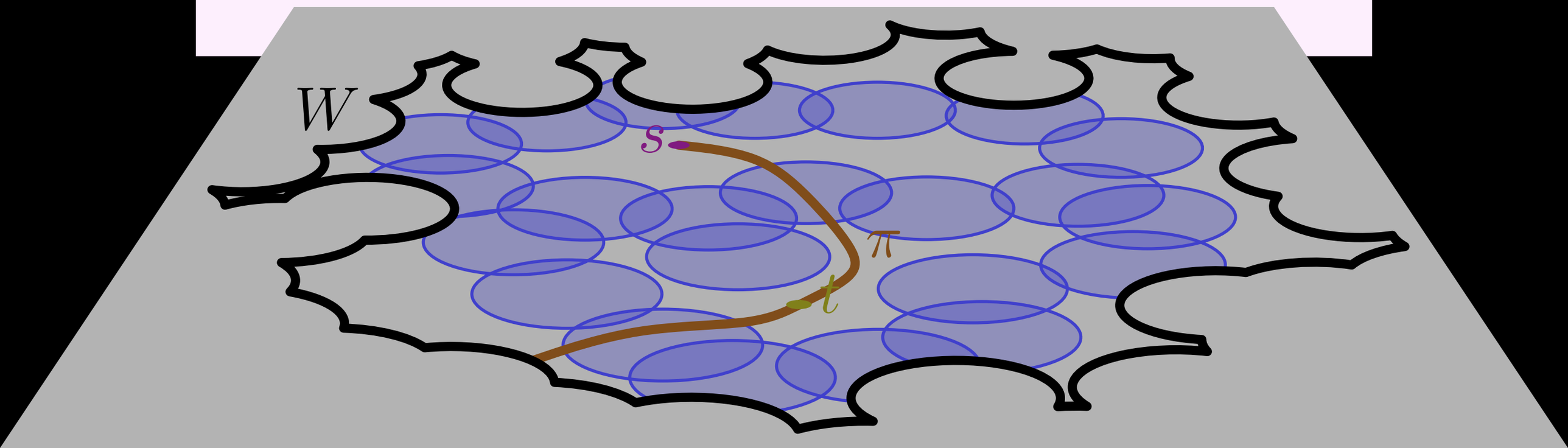

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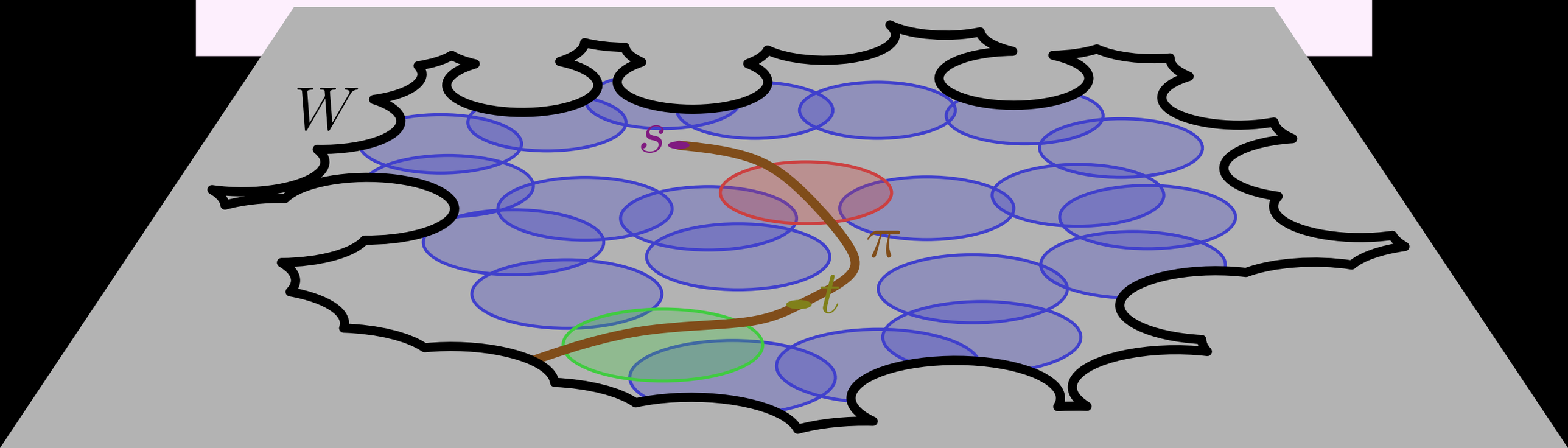
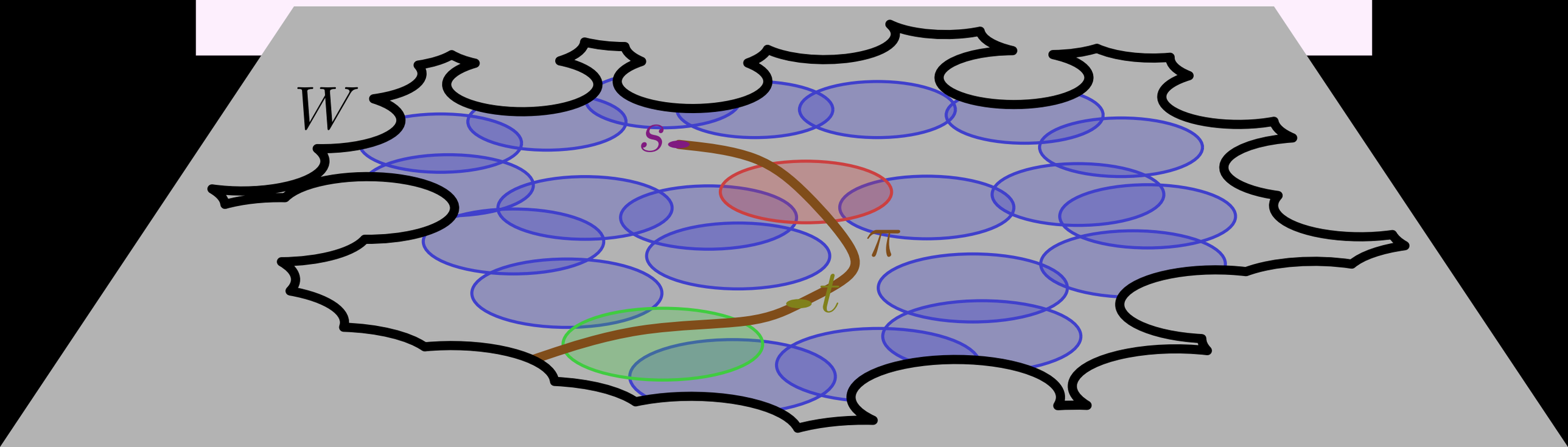


IDEA: Cut open the domain along π.



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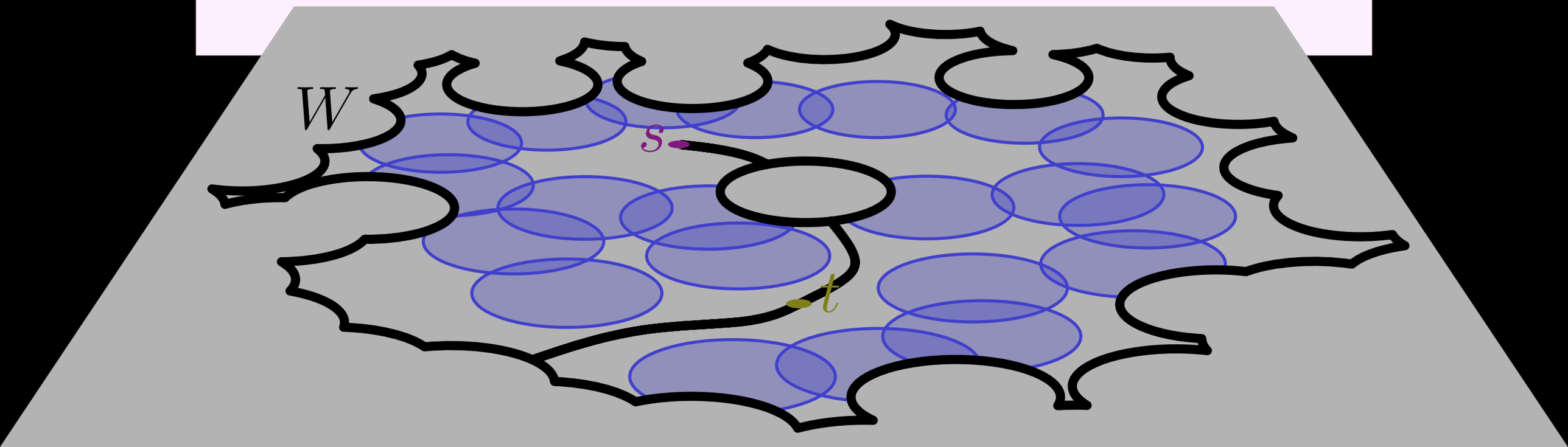

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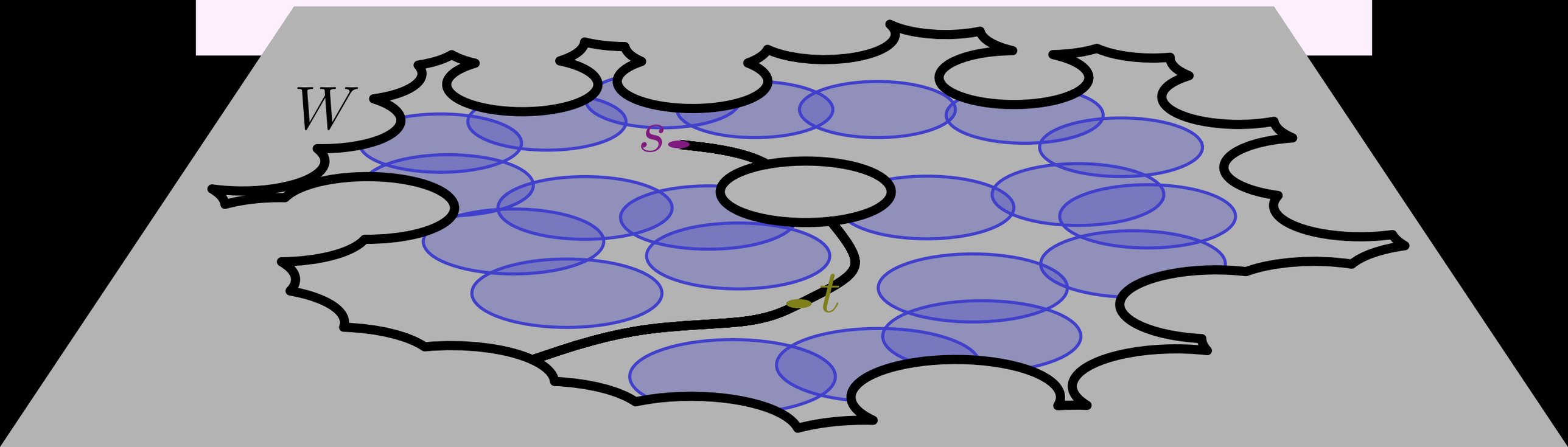

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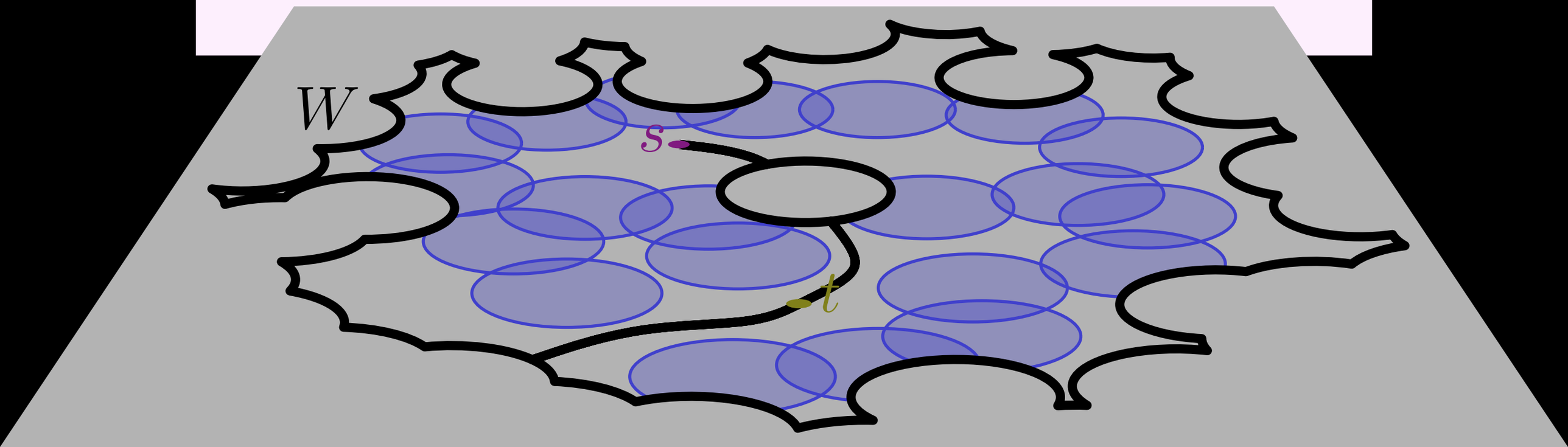

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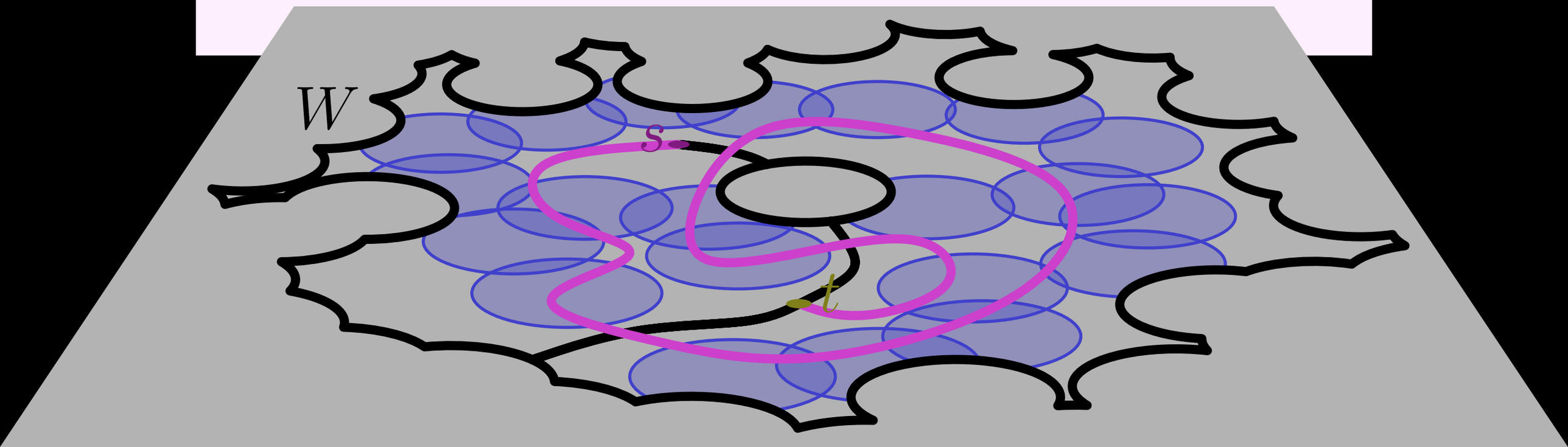

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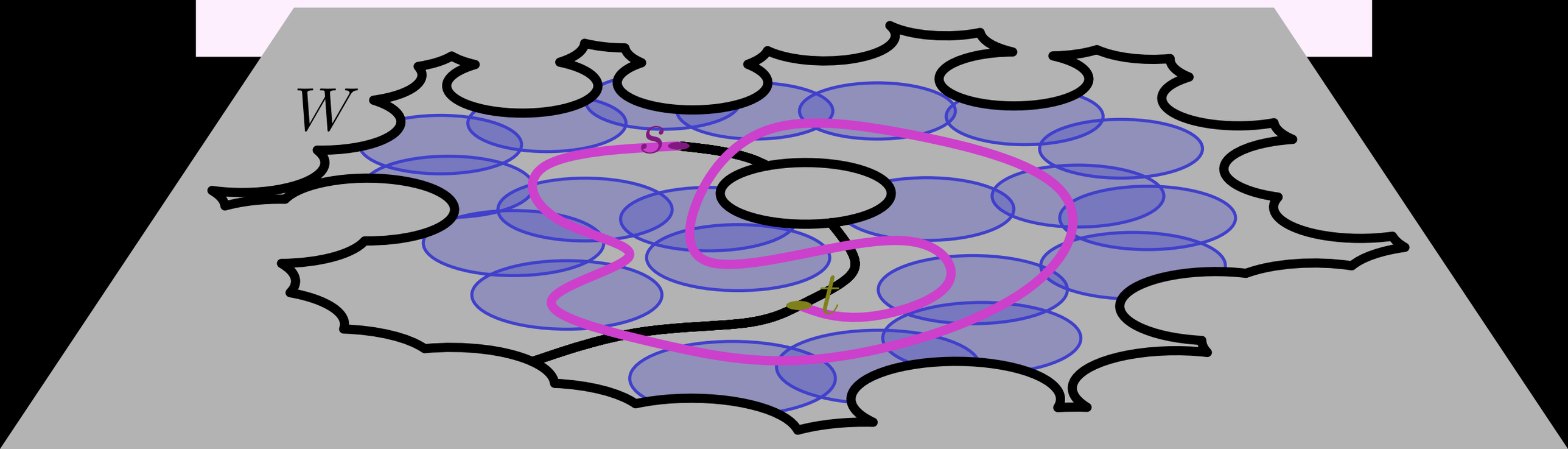

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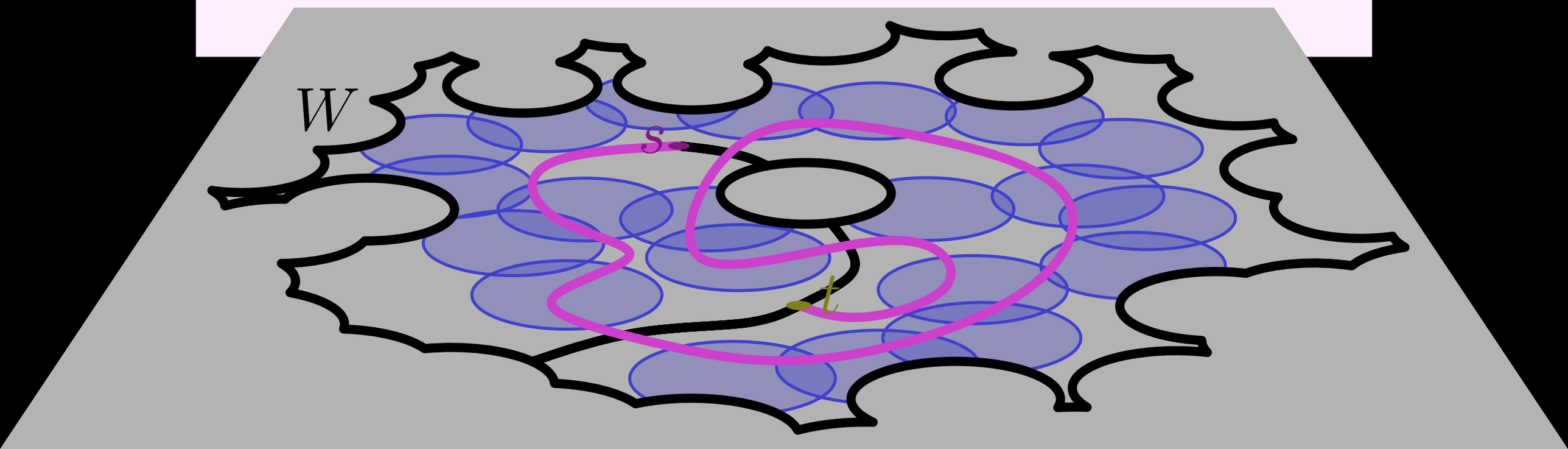

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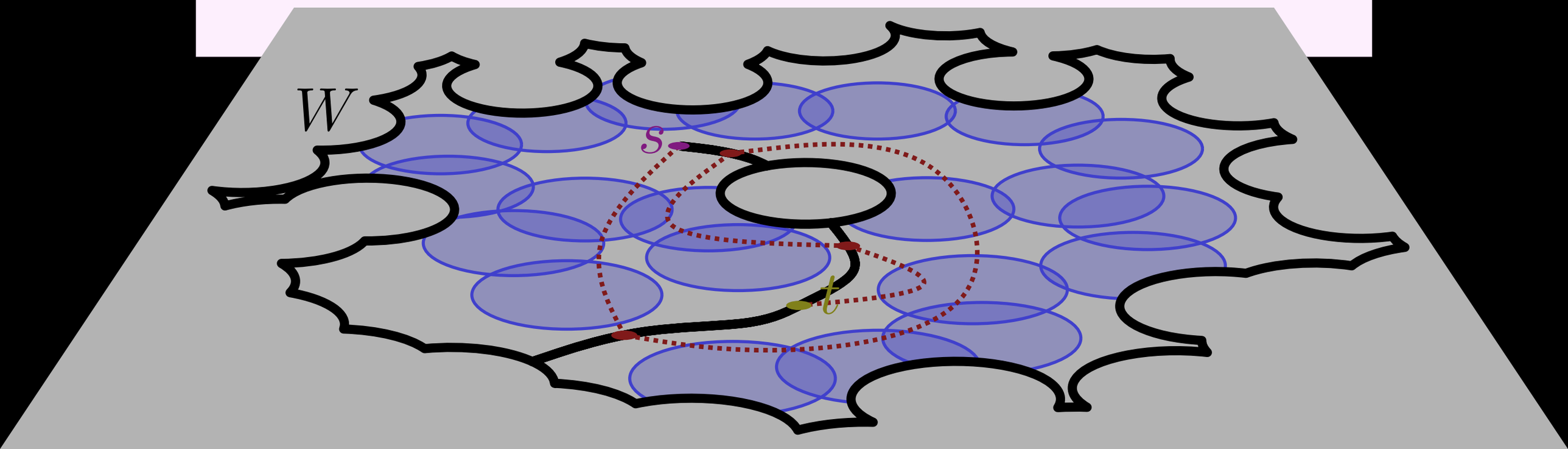

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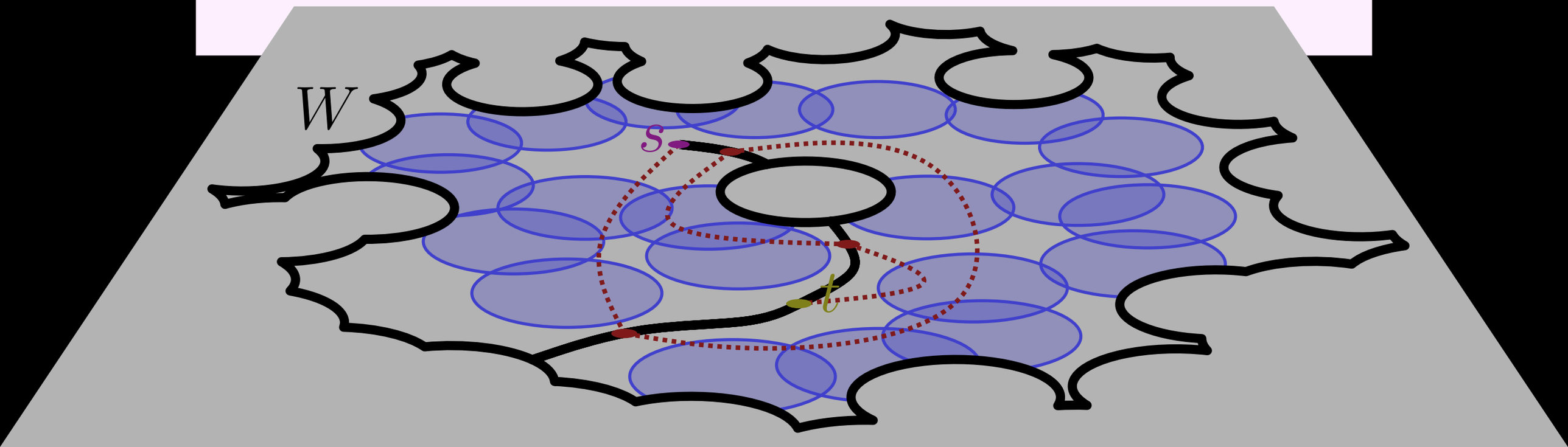

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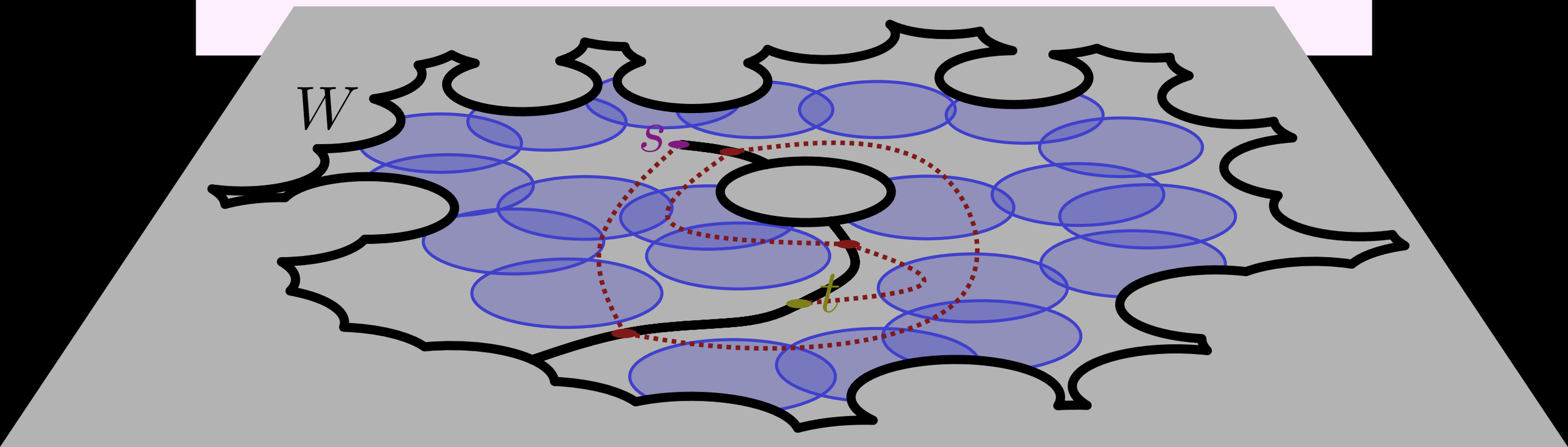

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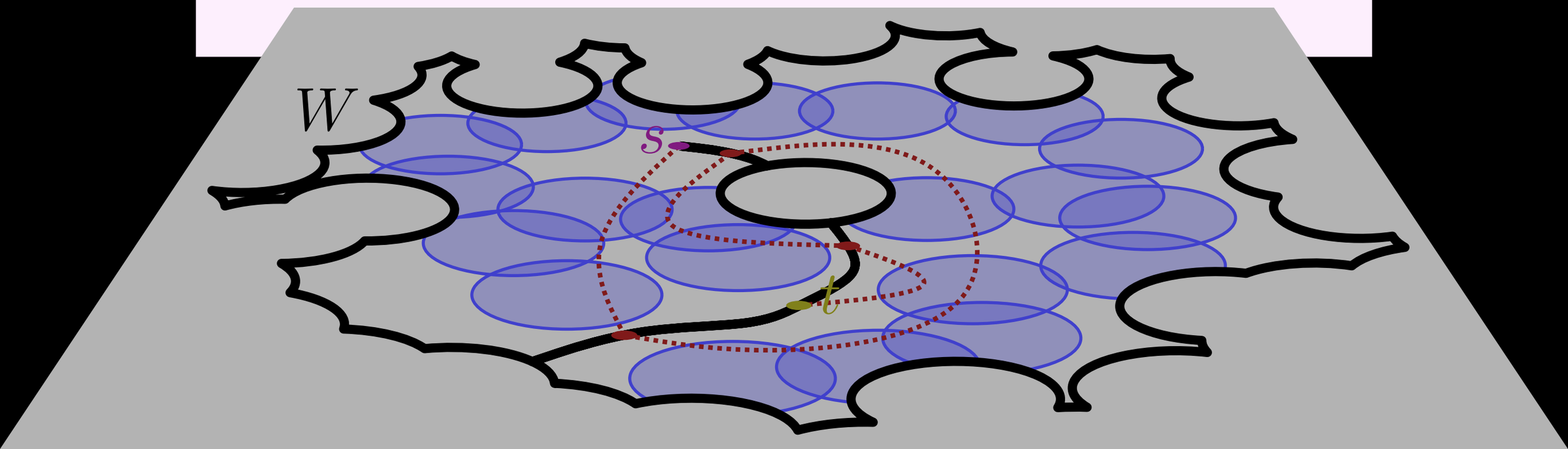

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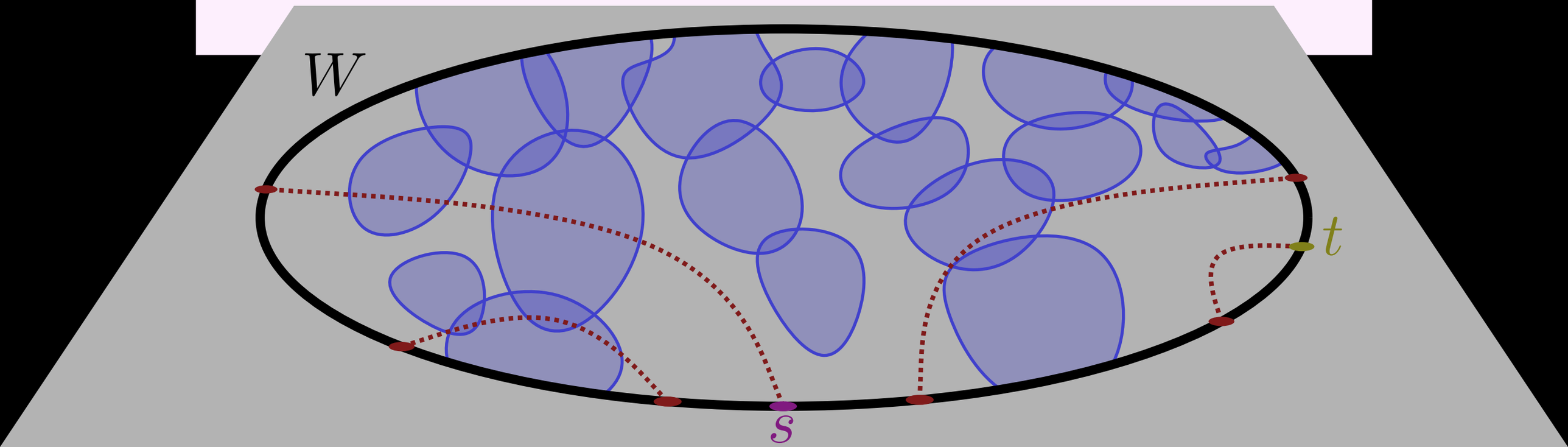

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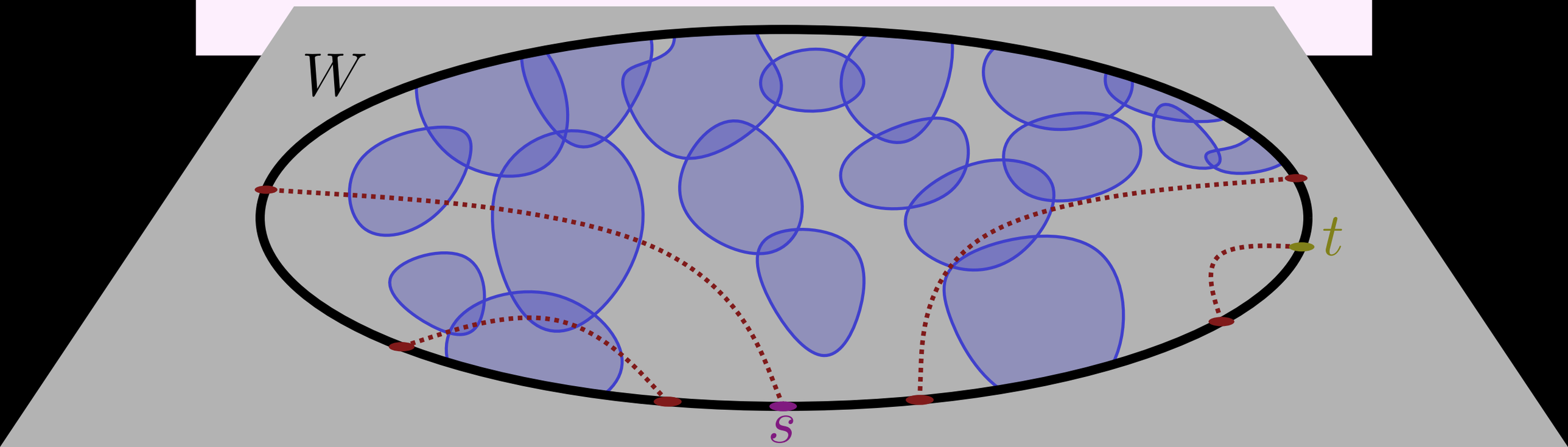

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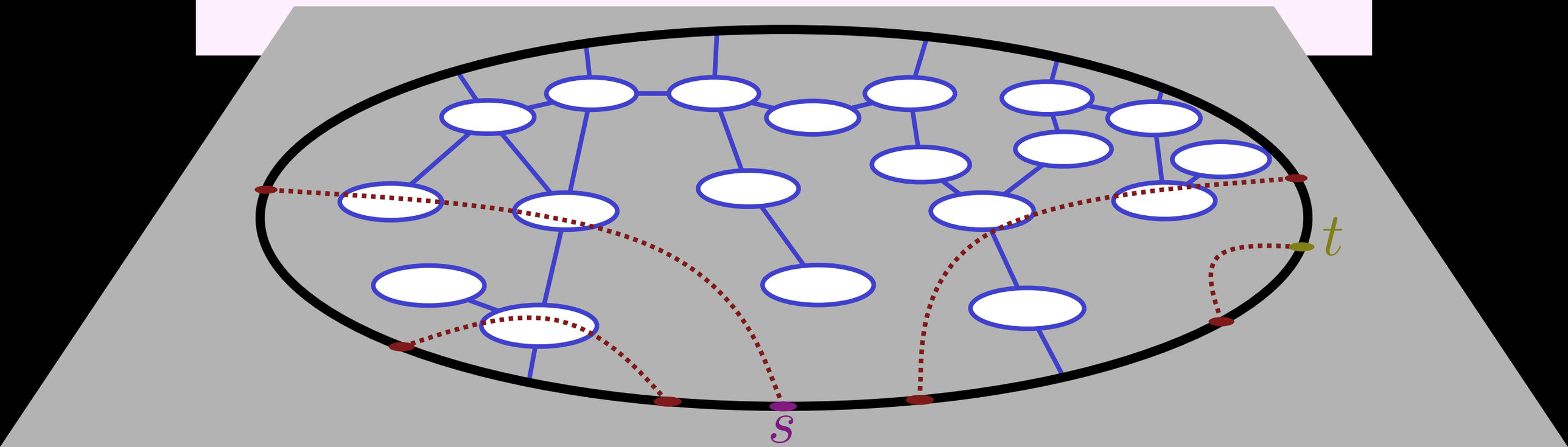

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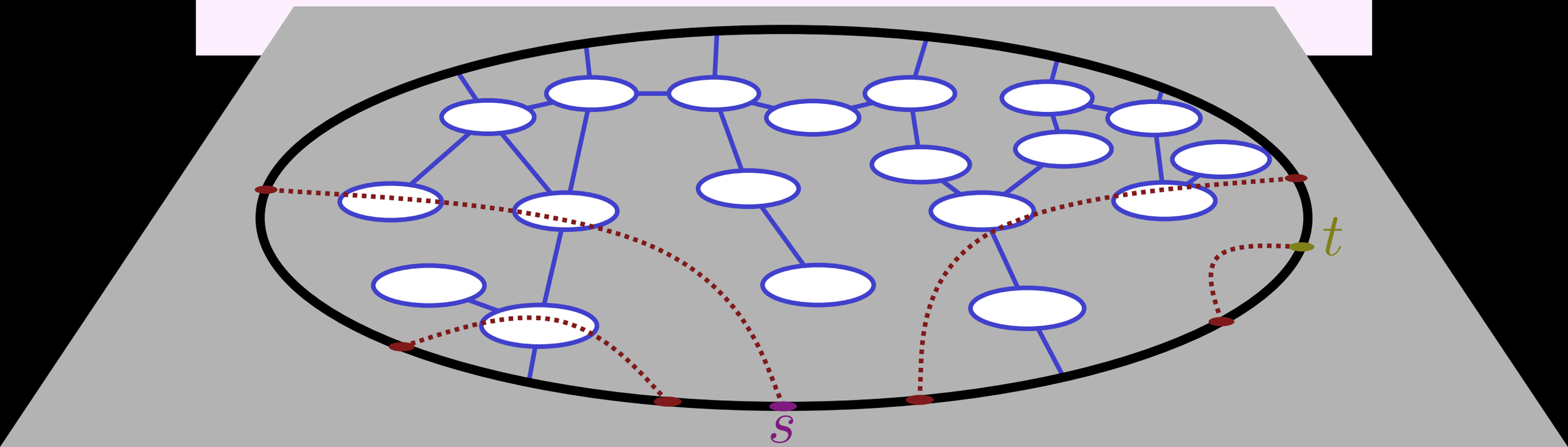

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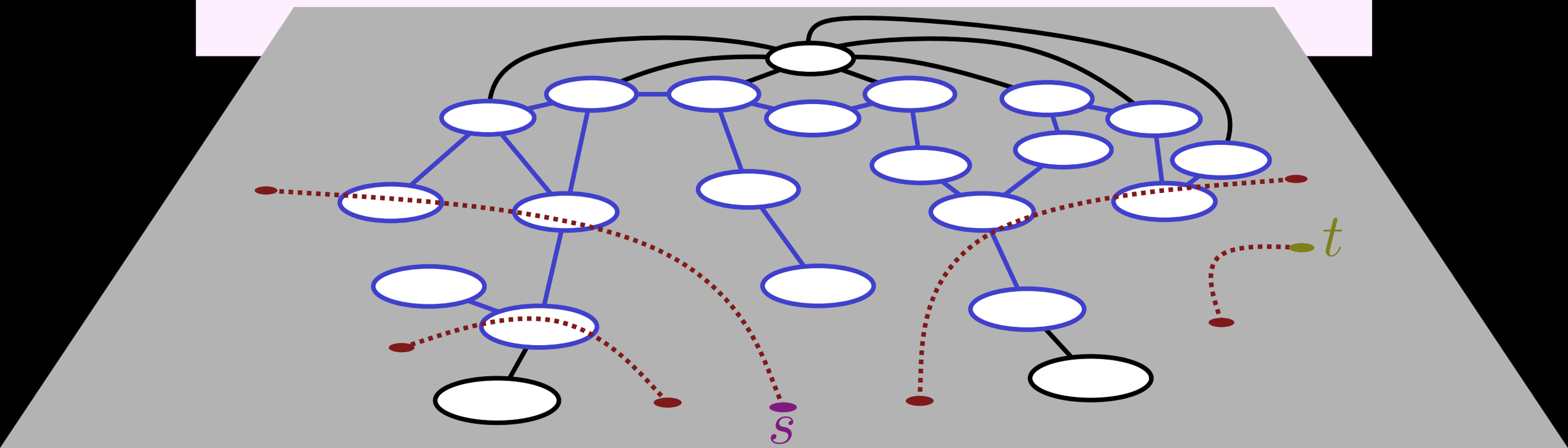

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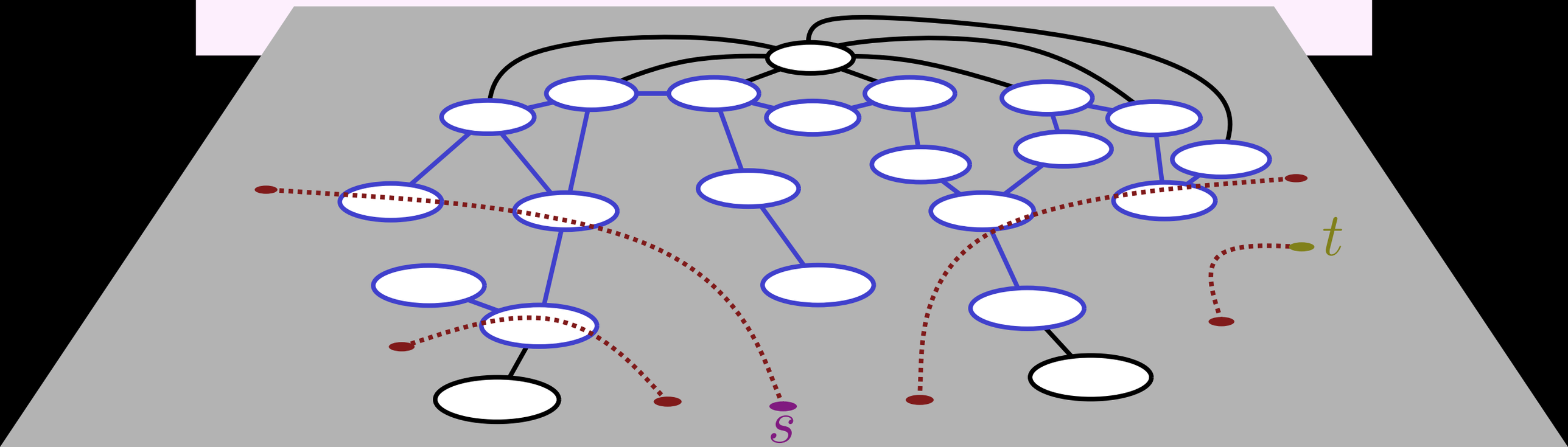

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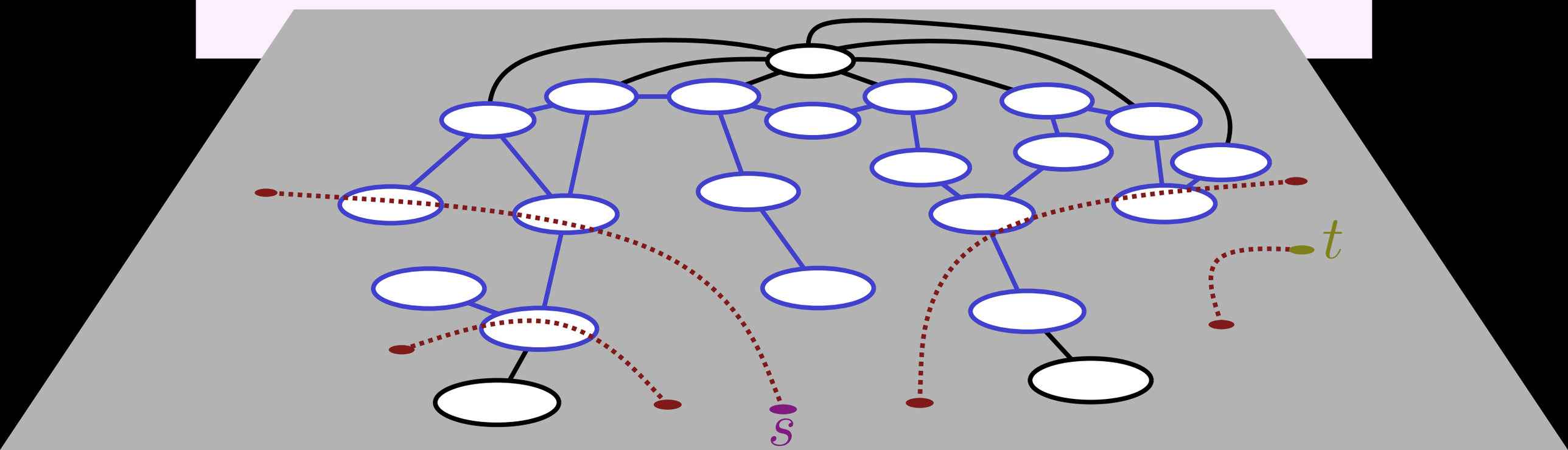

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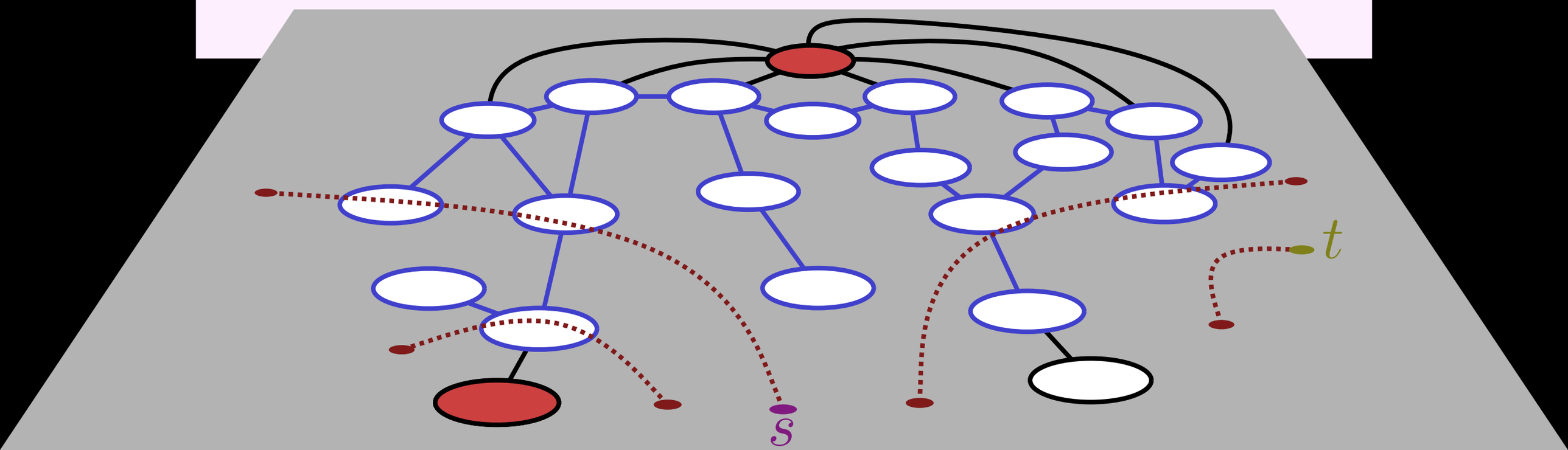

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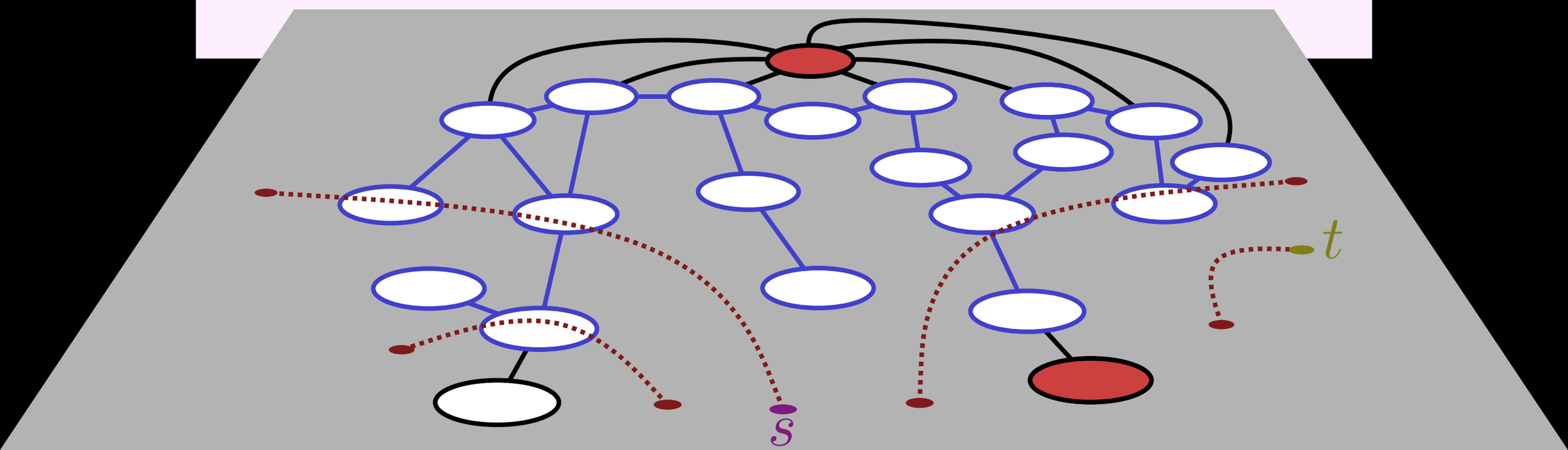

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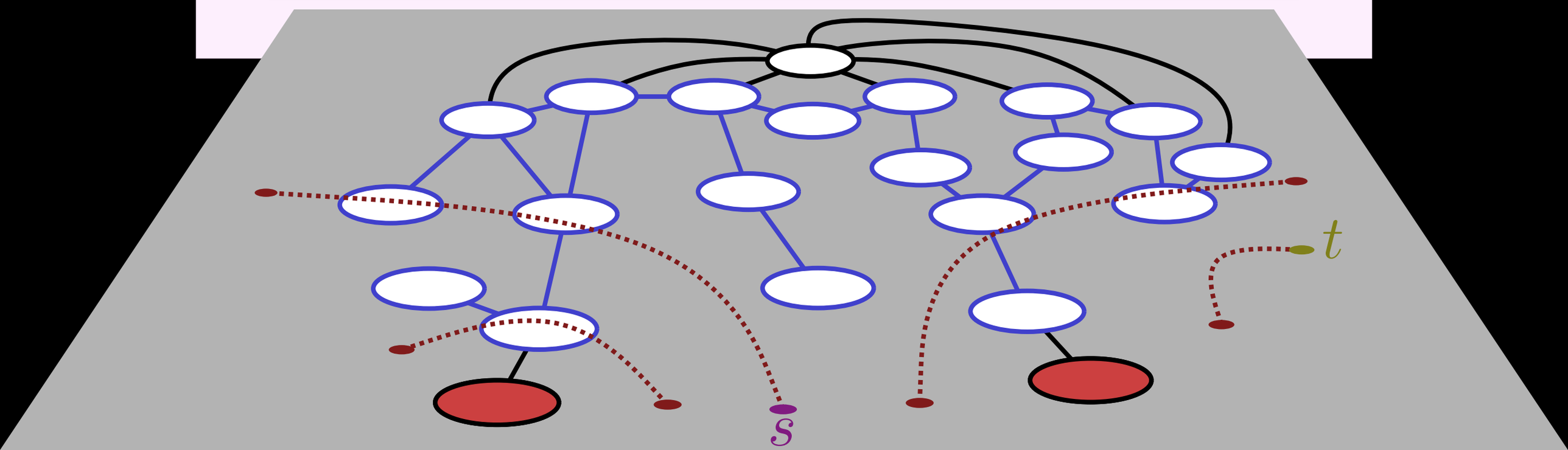

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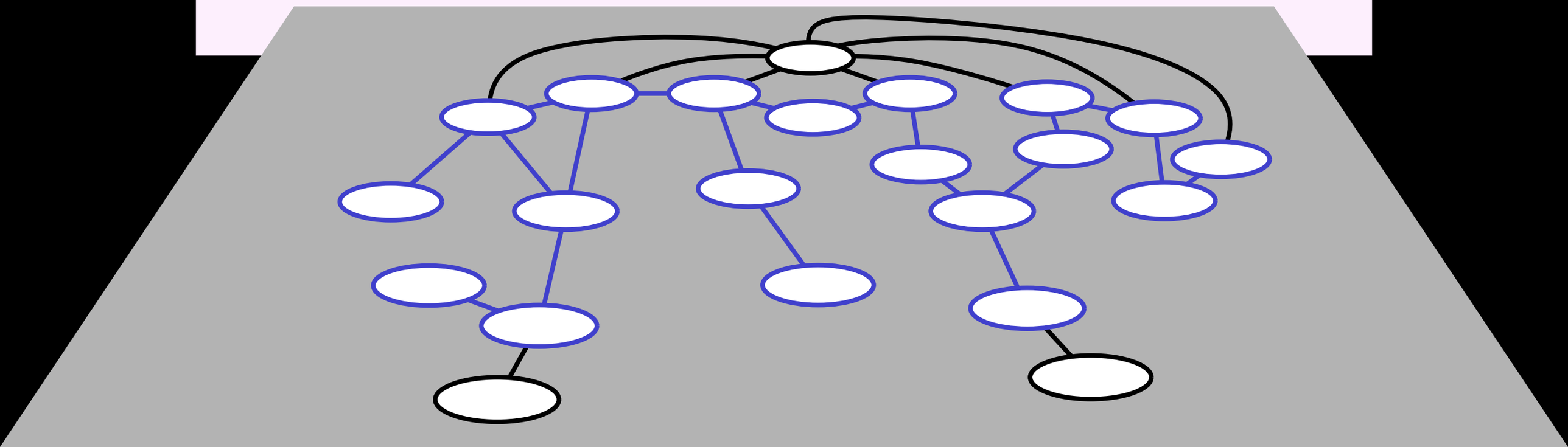

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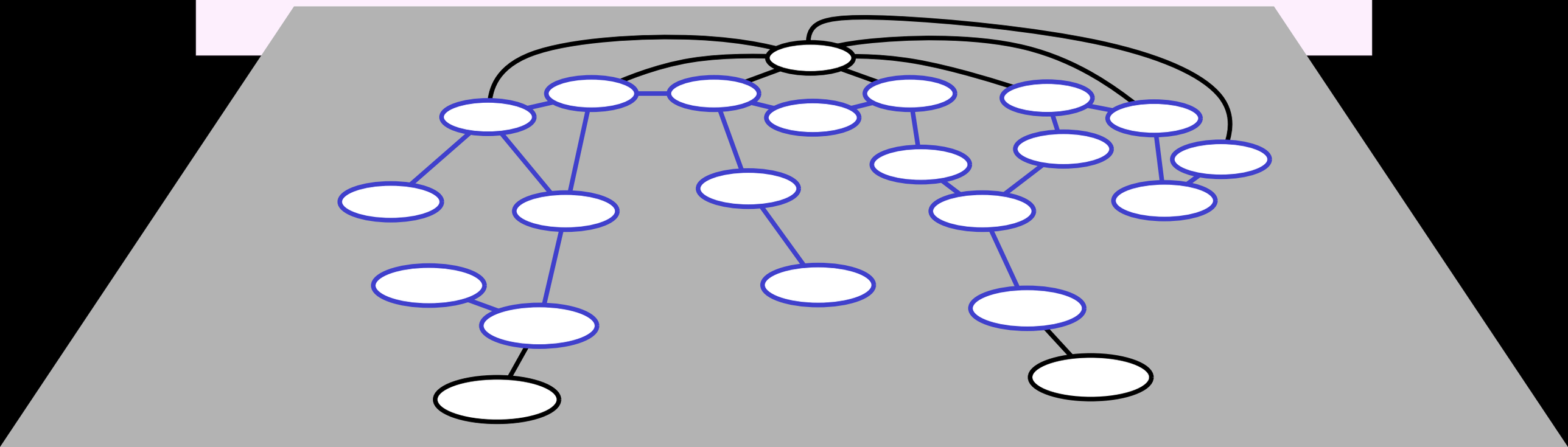

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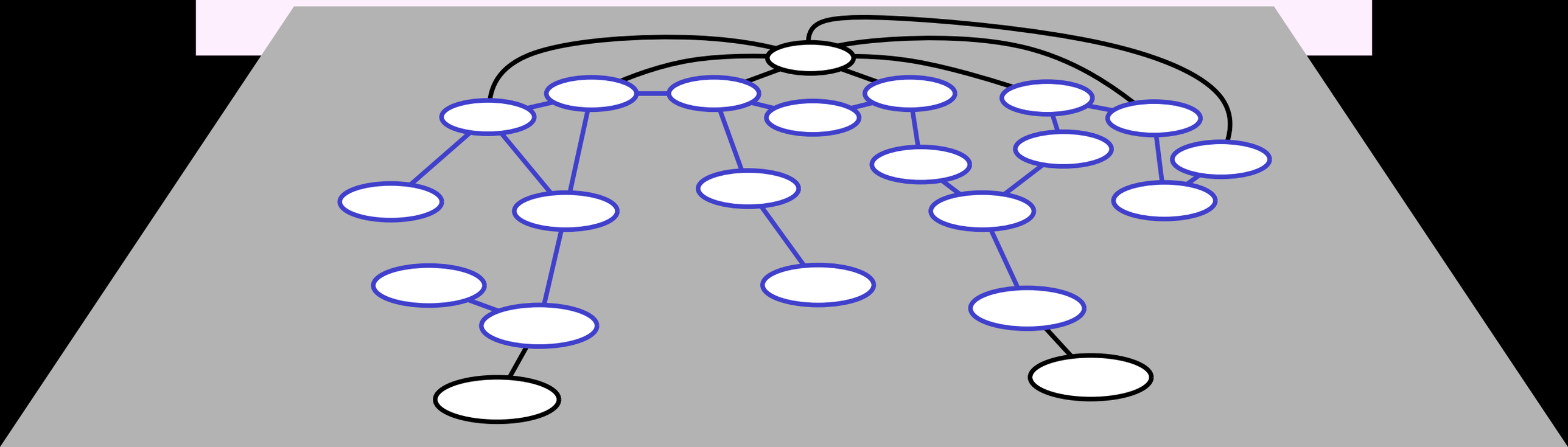

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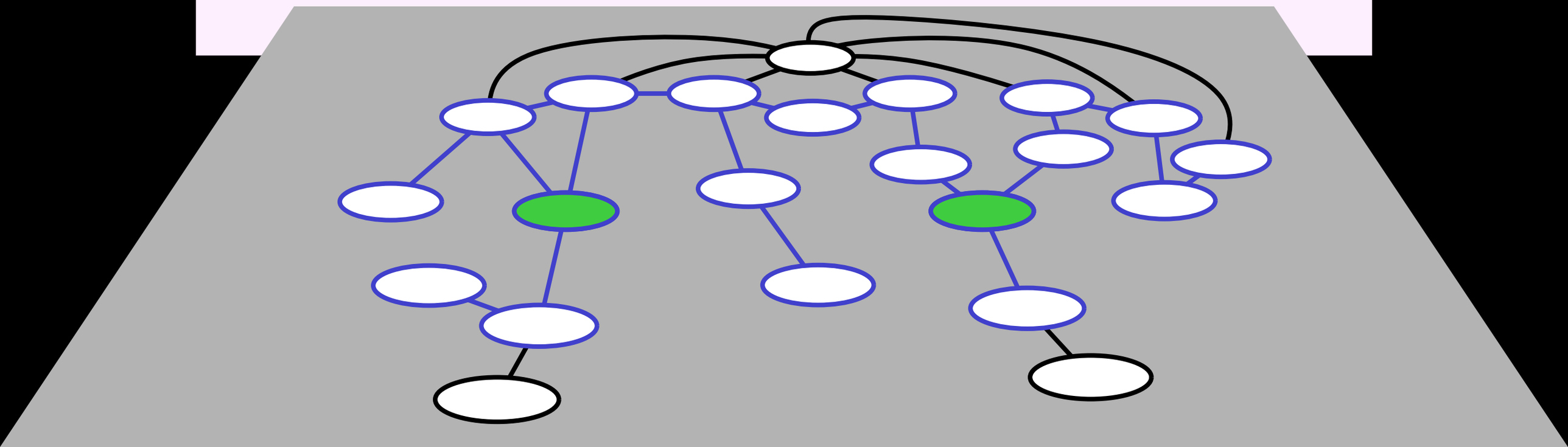

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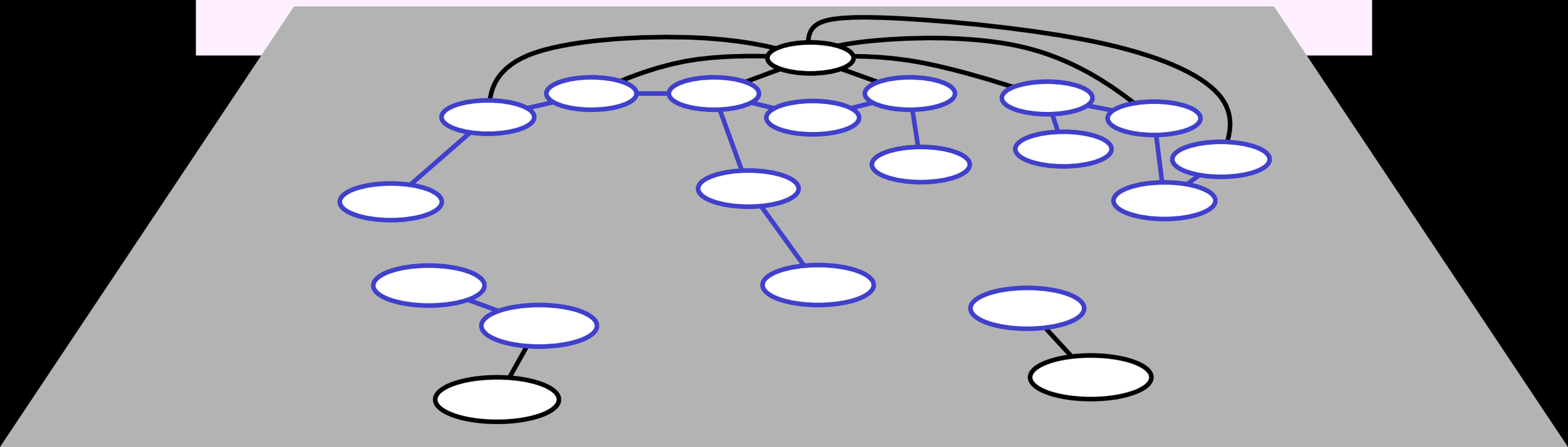

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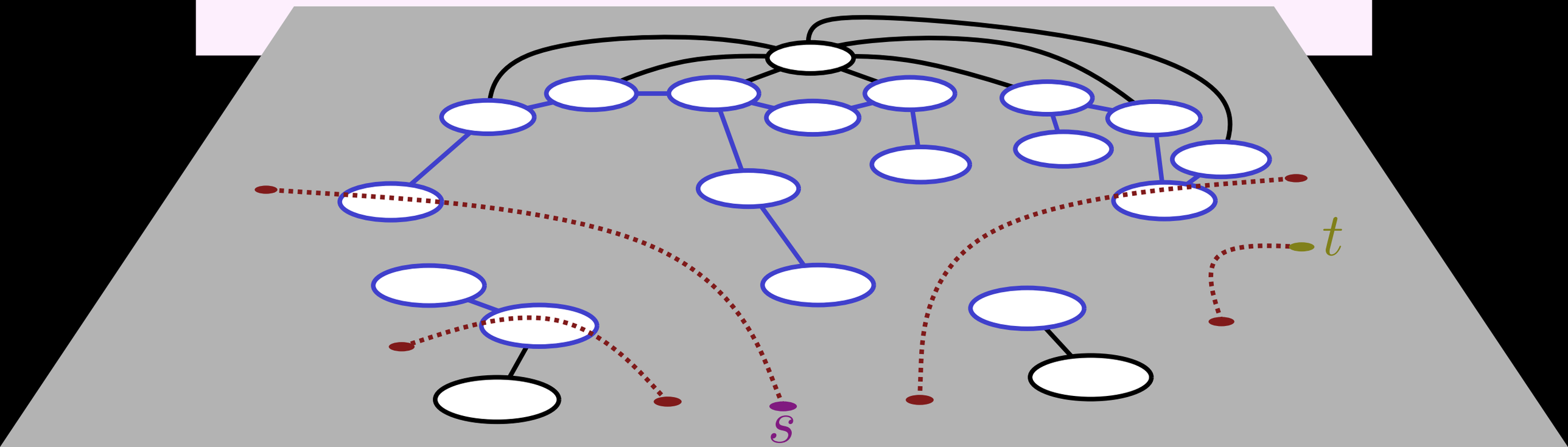

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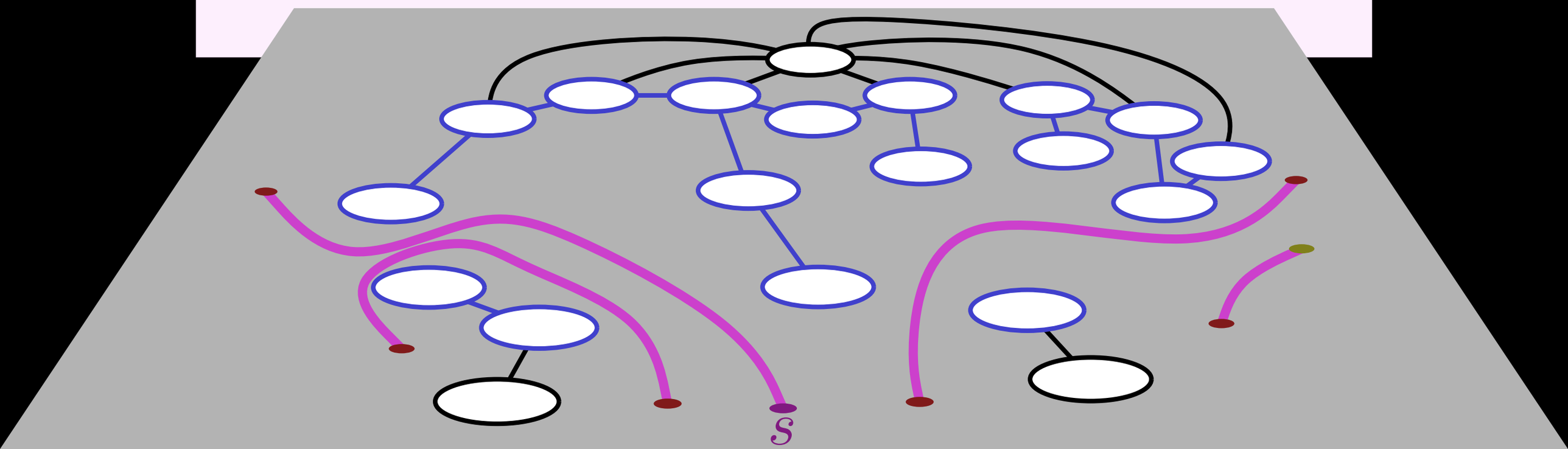

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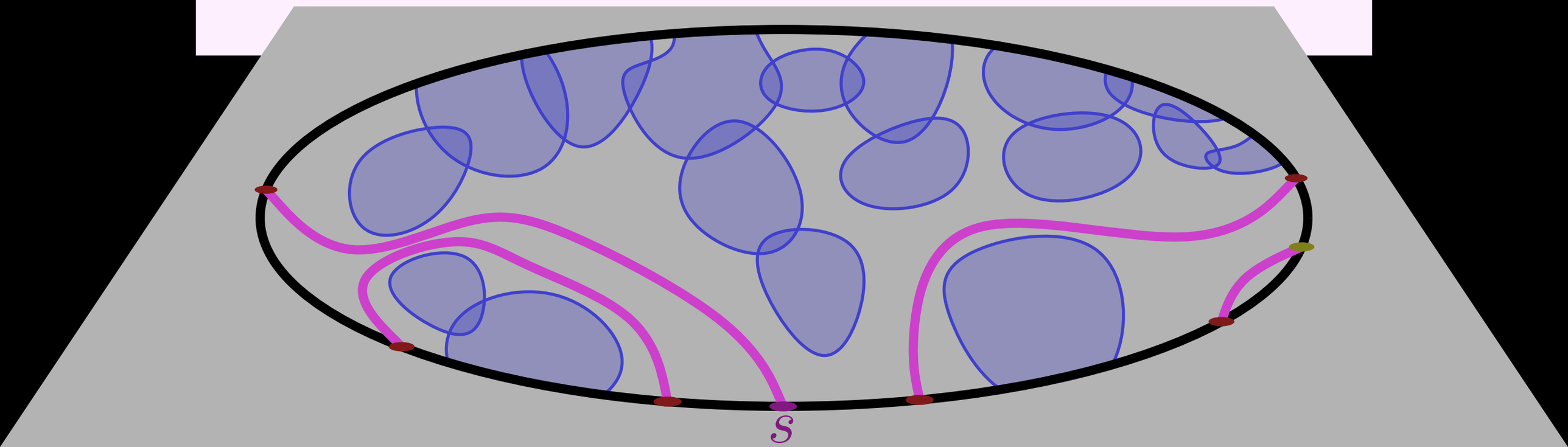

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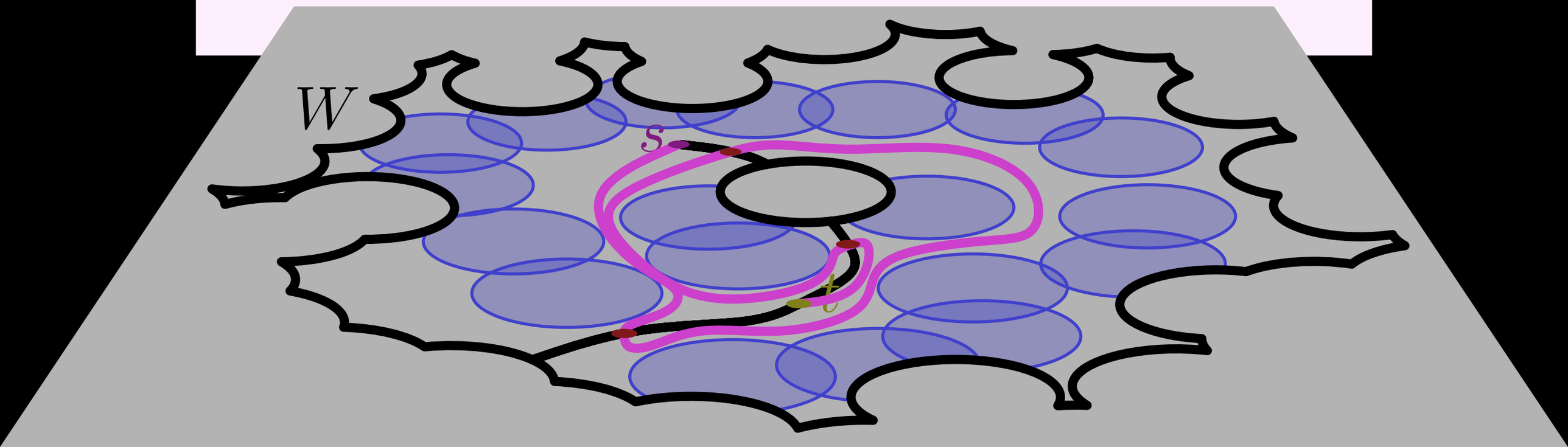

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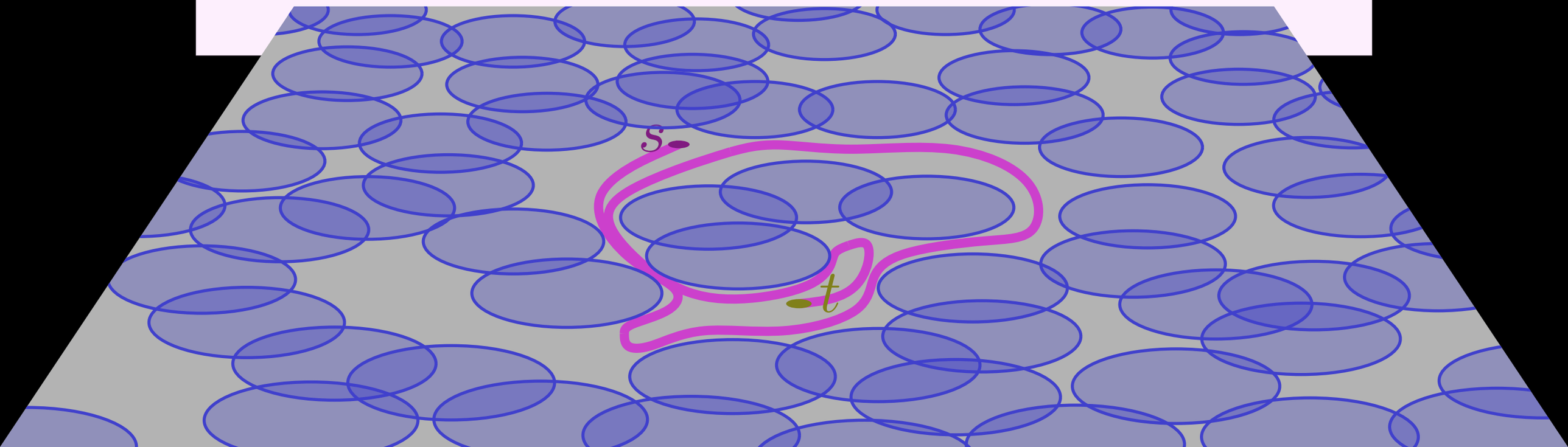

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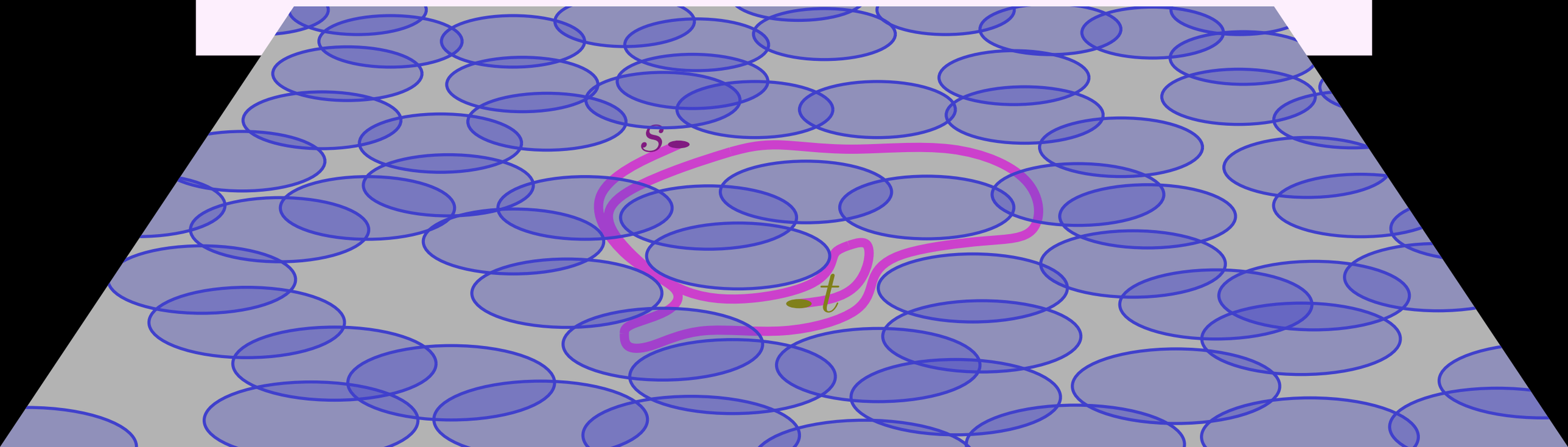

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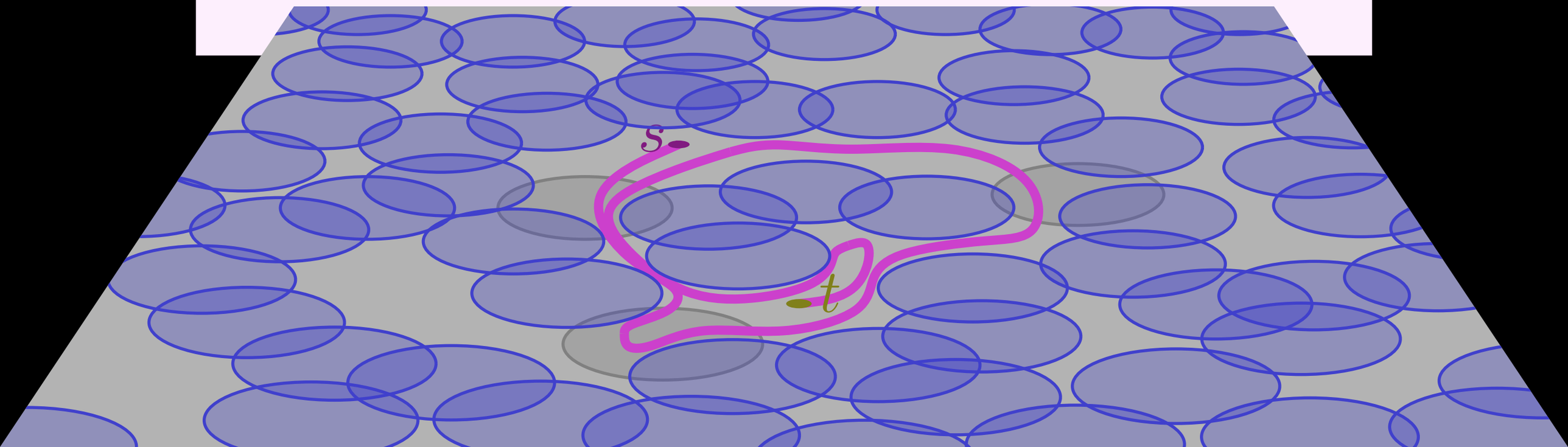

ε-APPROXIMATION ALGORITHM

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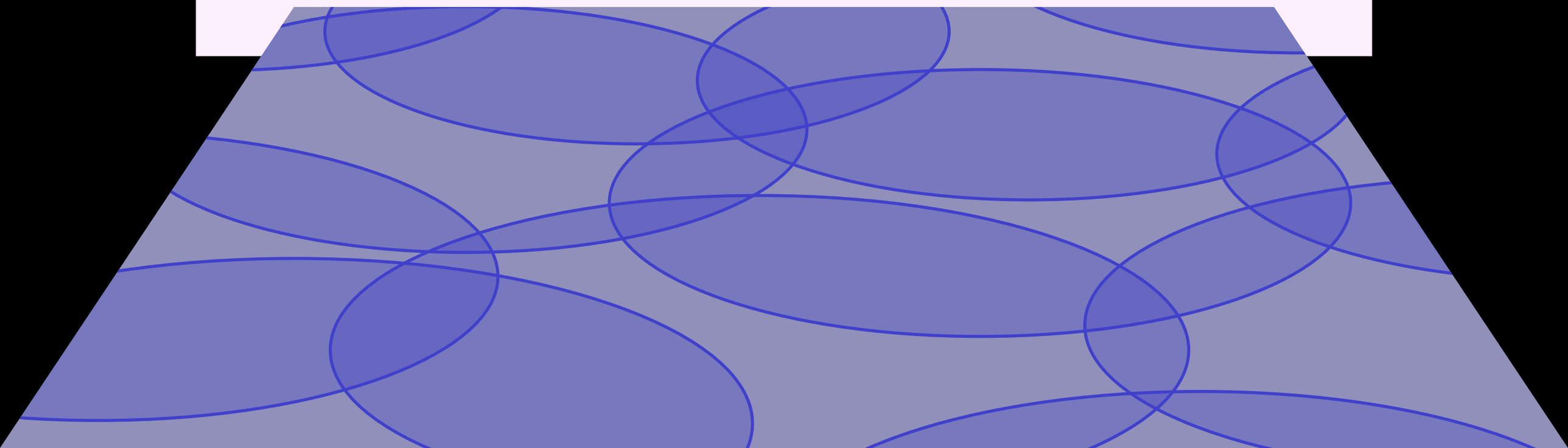

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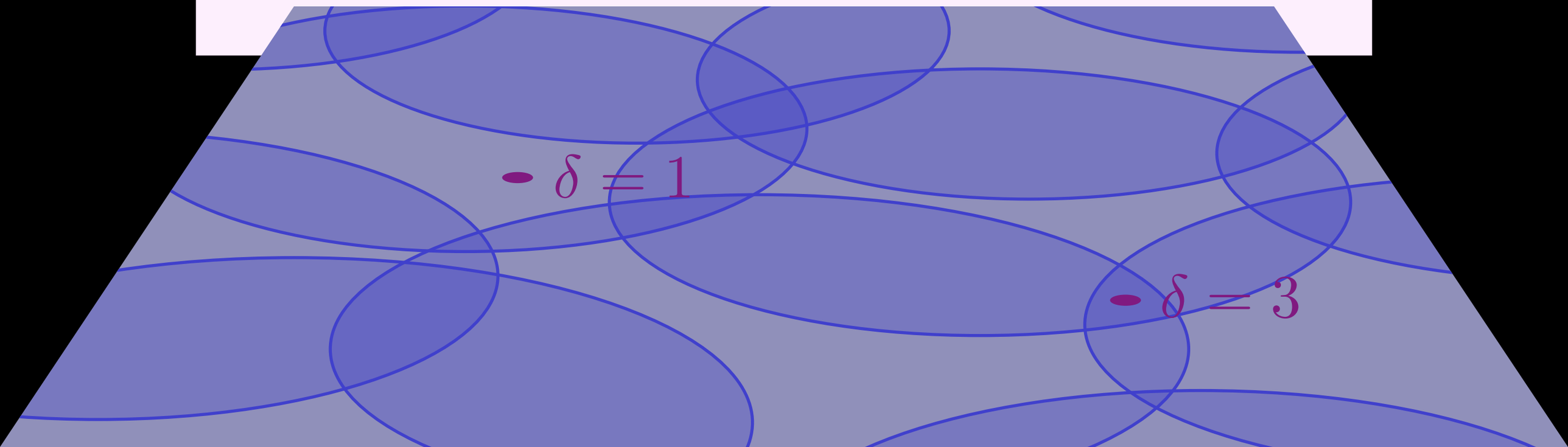

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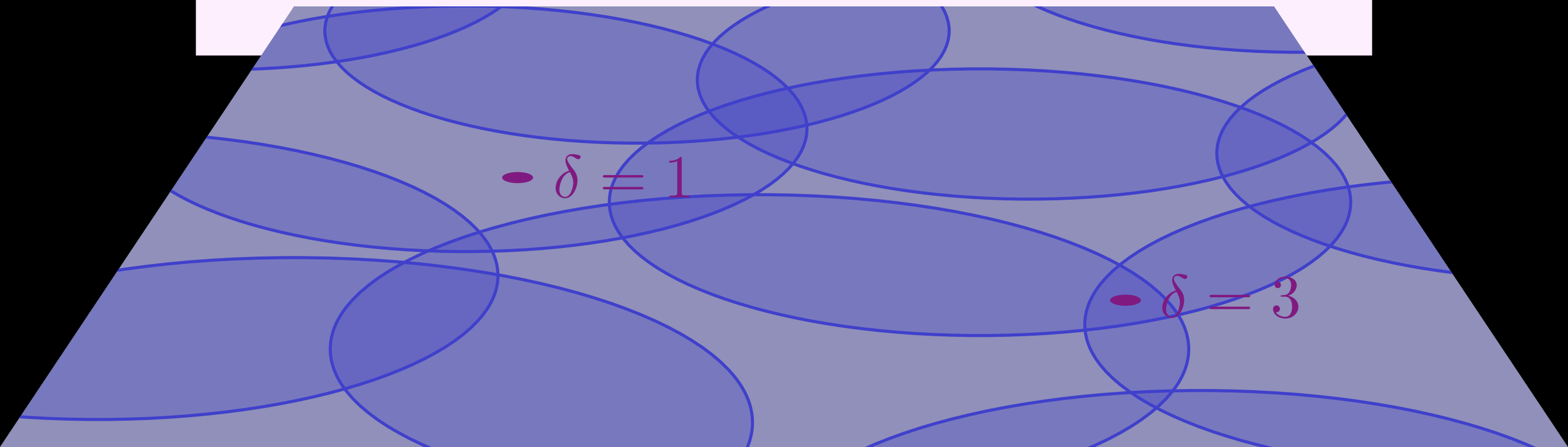

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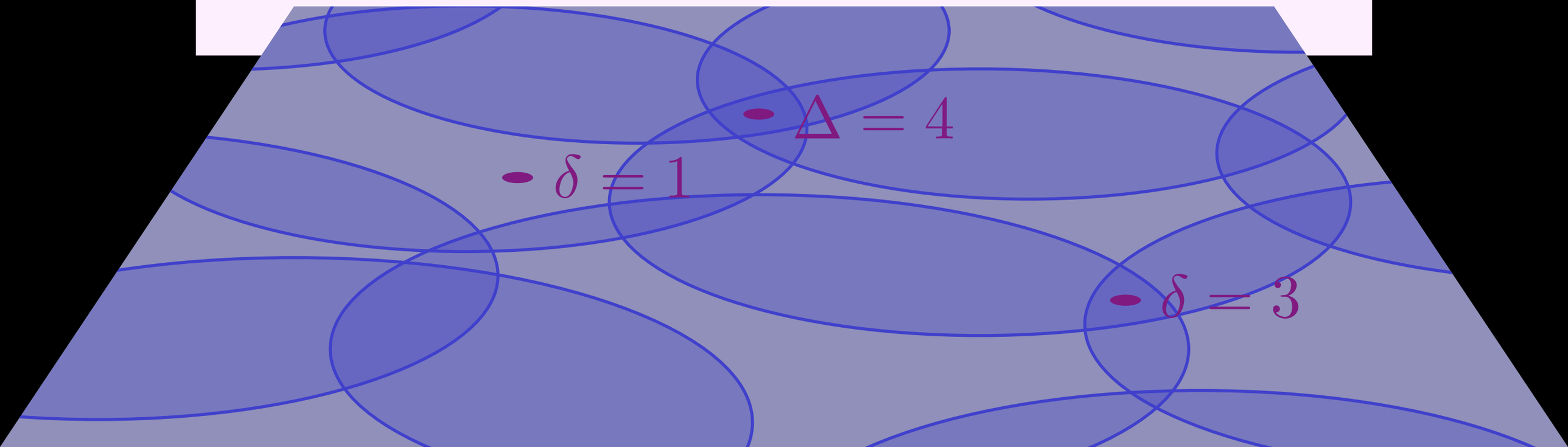

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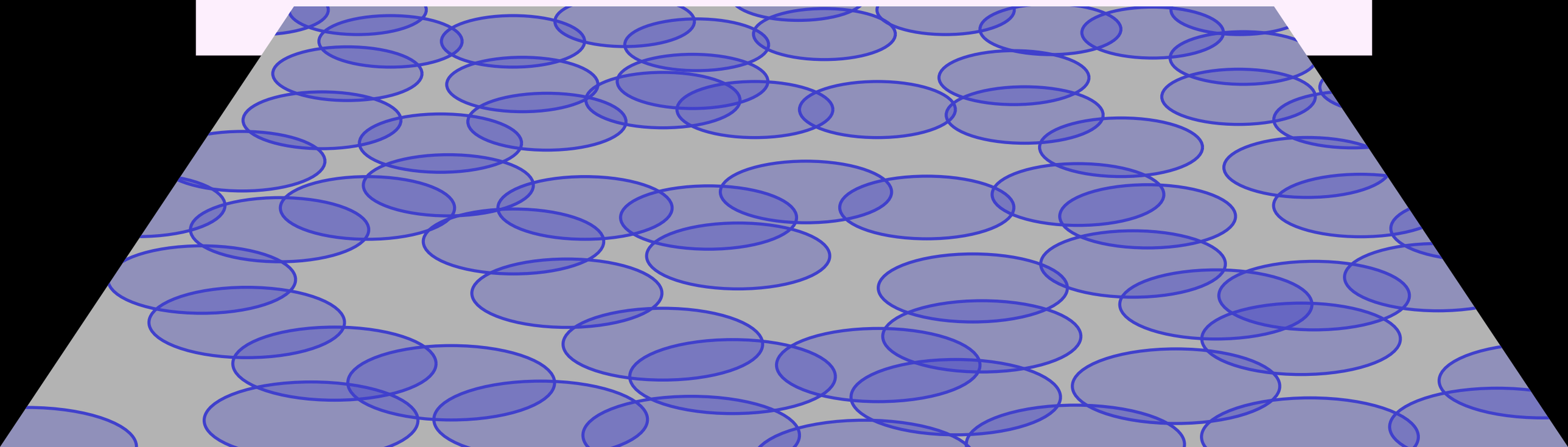

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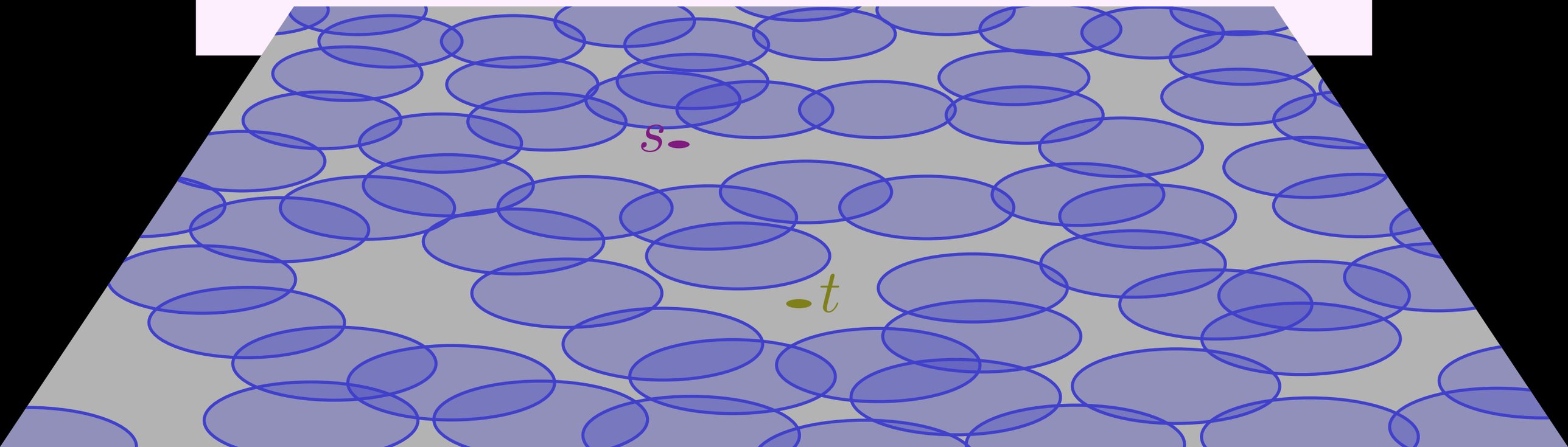

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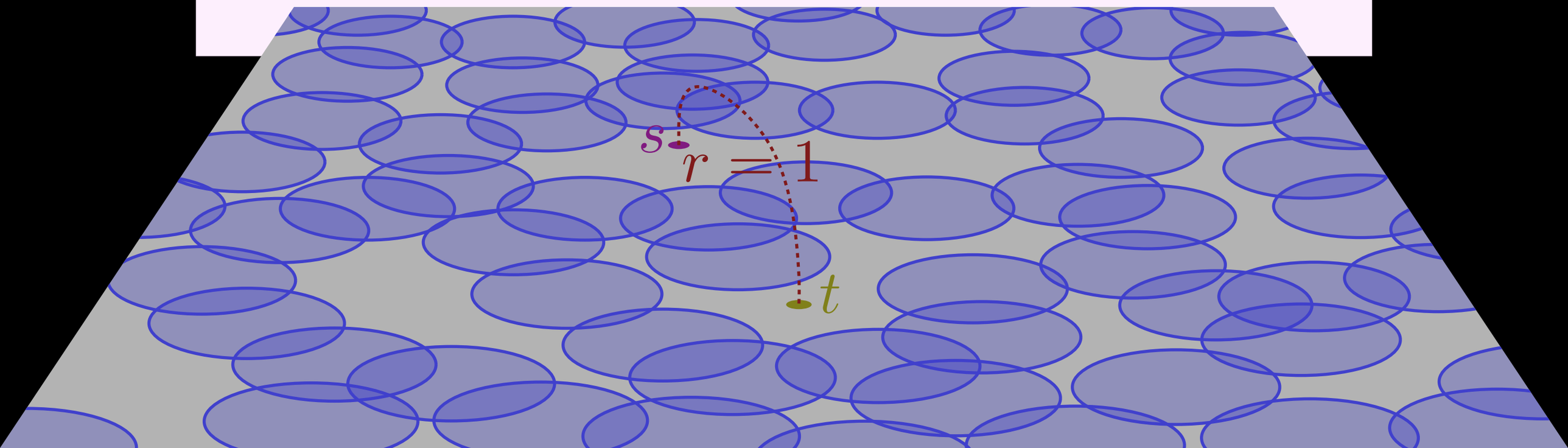

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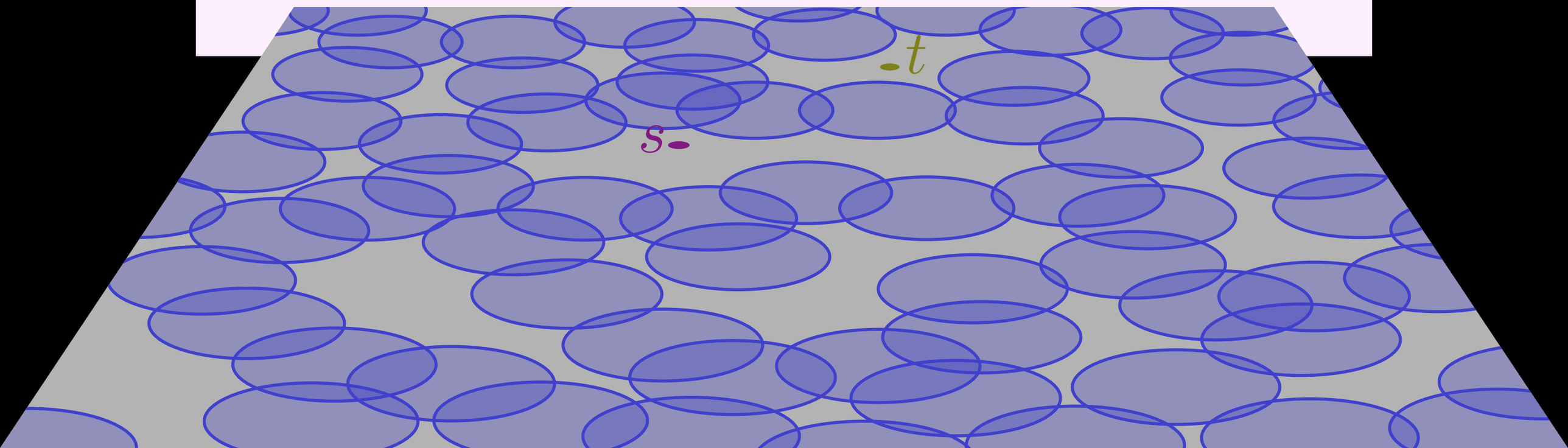

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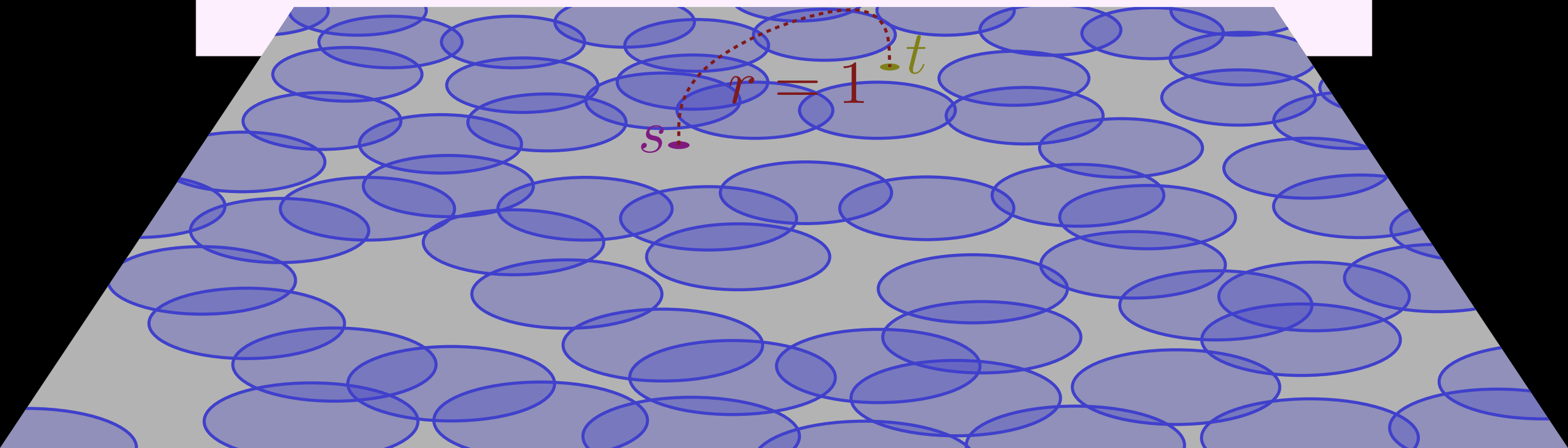

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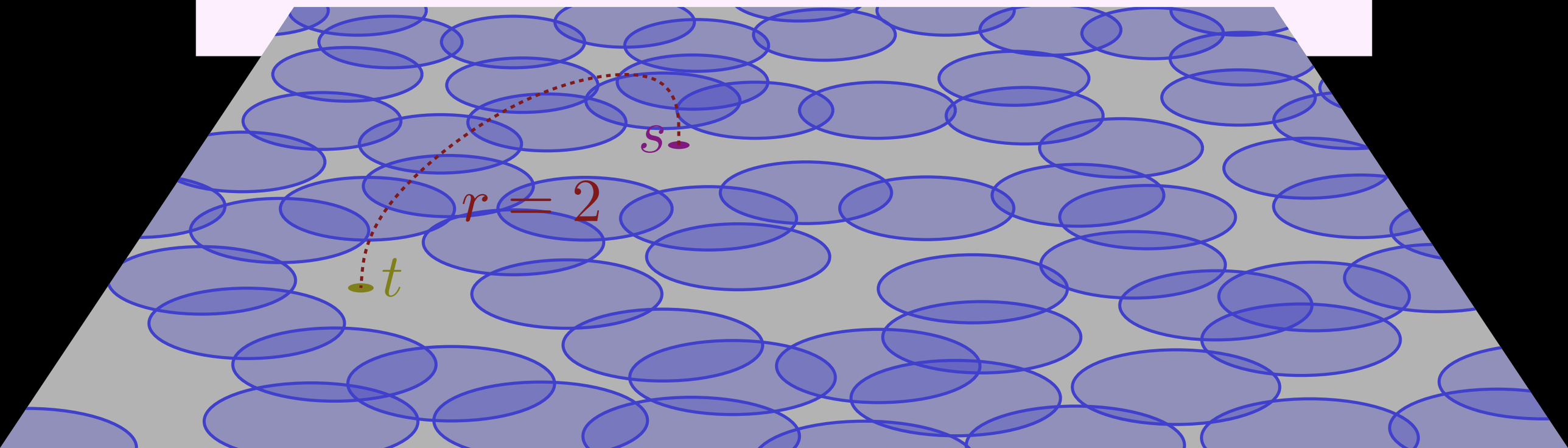

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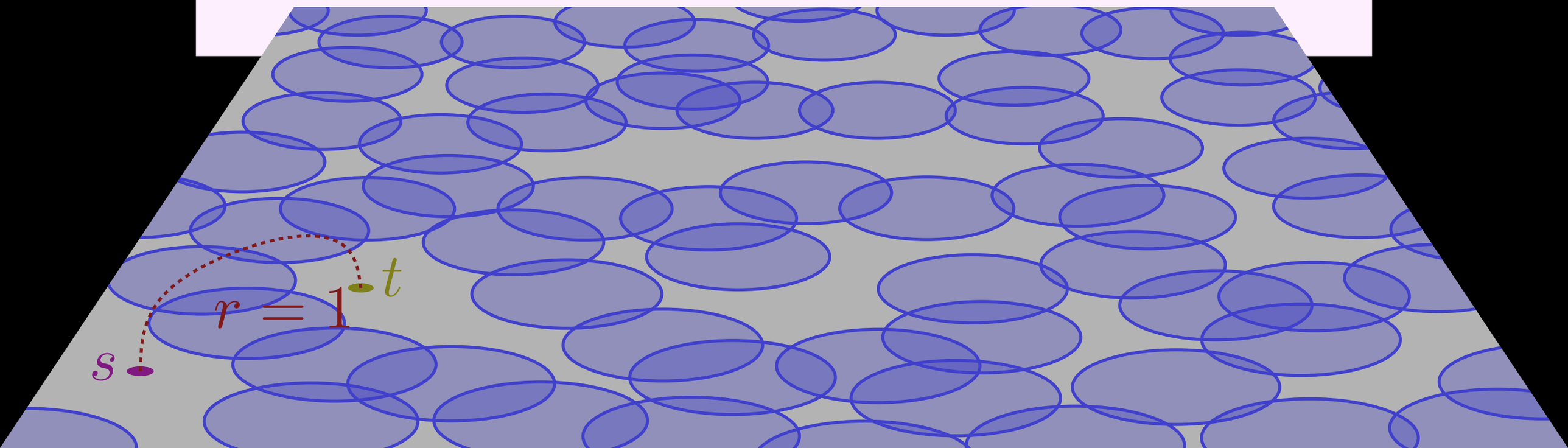

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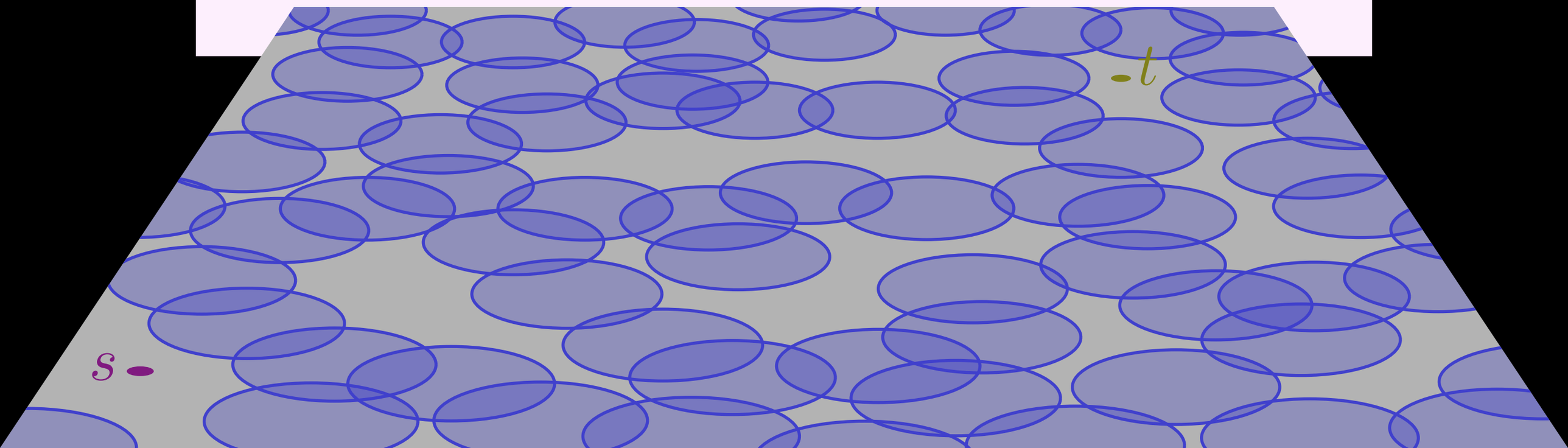

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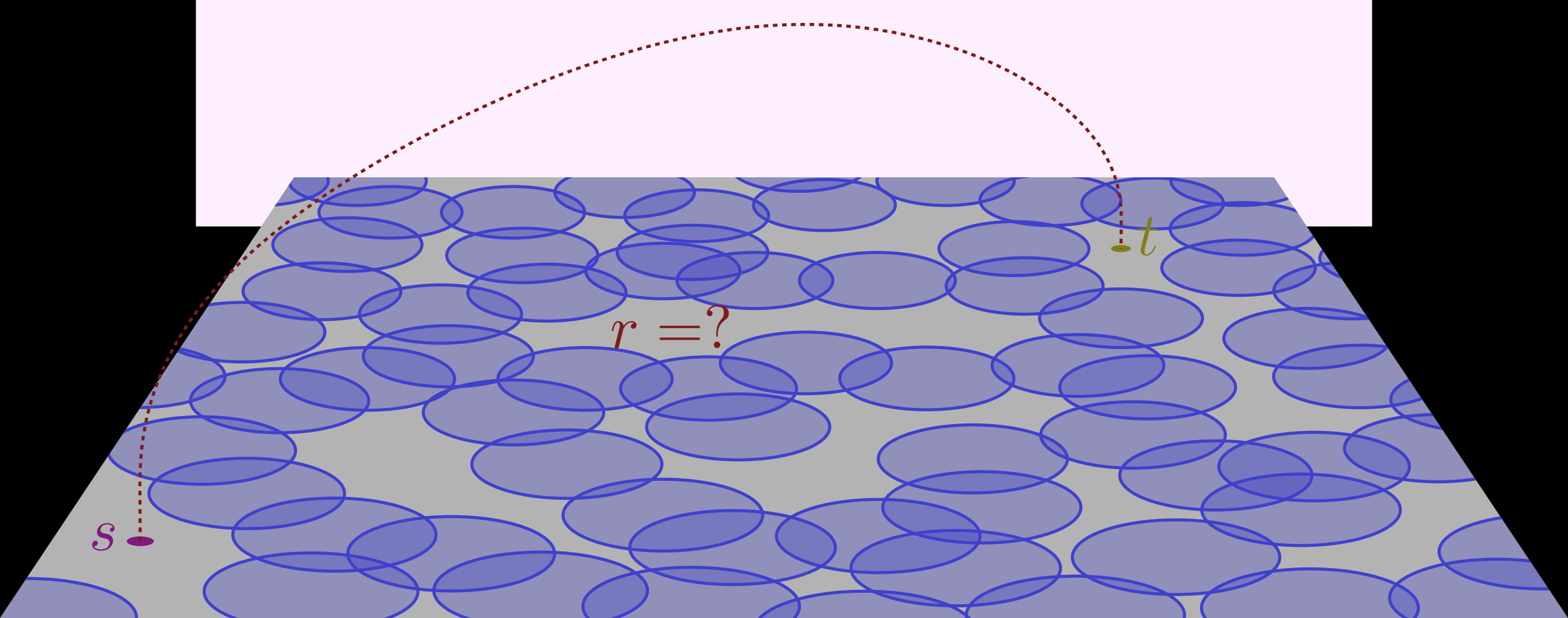

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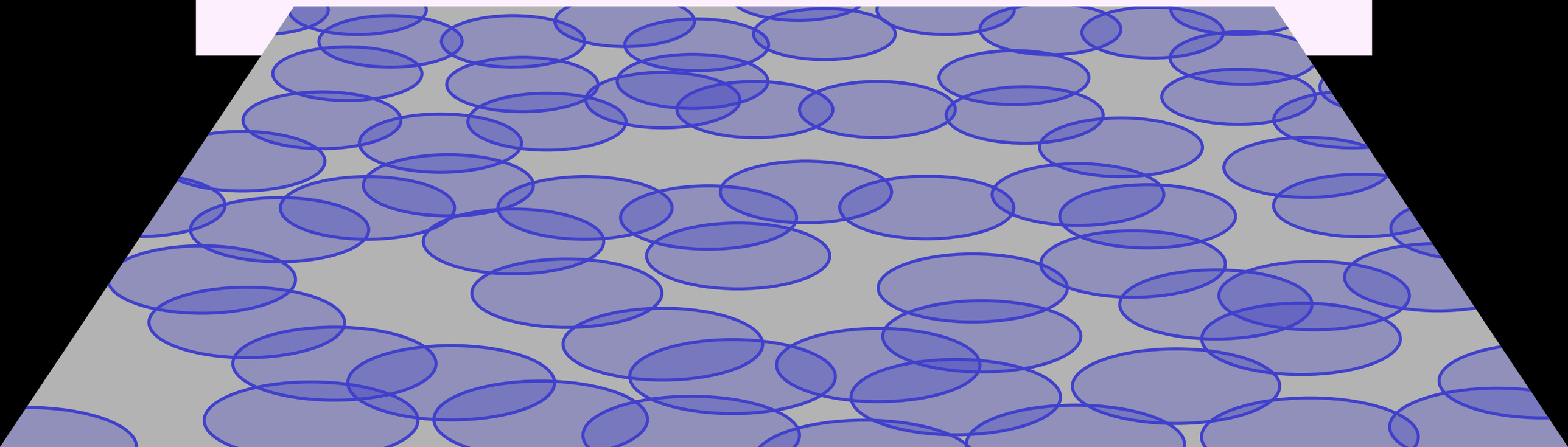
Augment the dual graph of $\mathcal{R}$ with extra edges.



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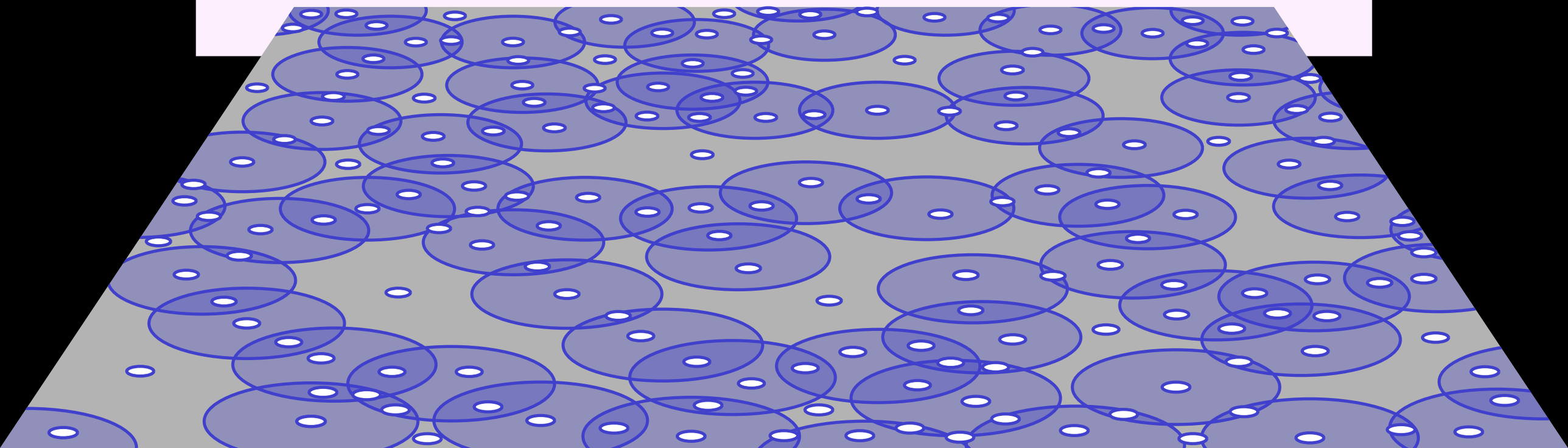

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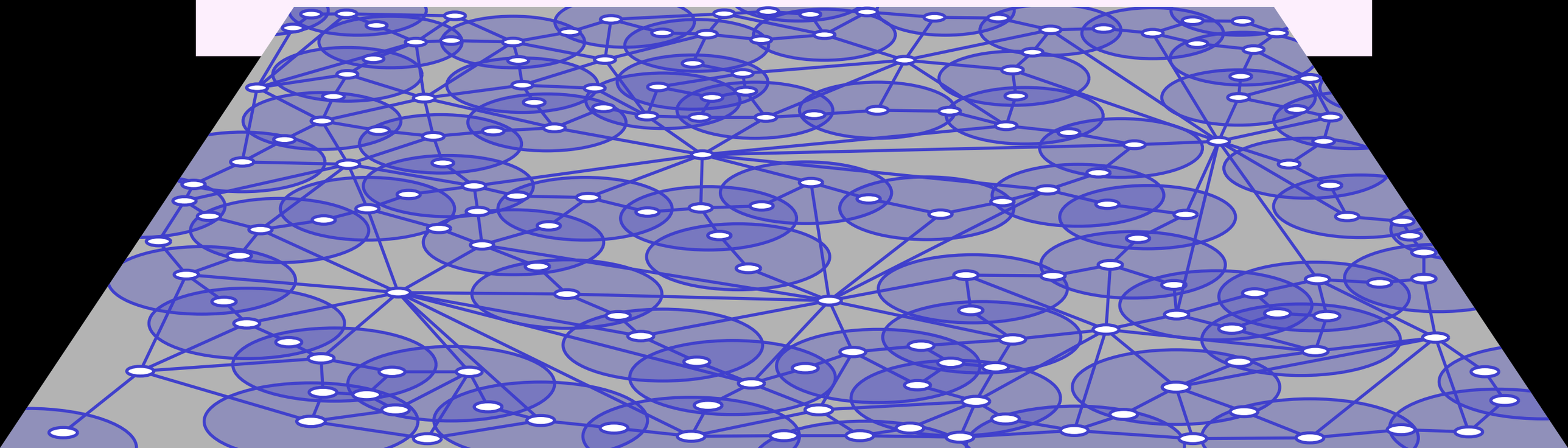

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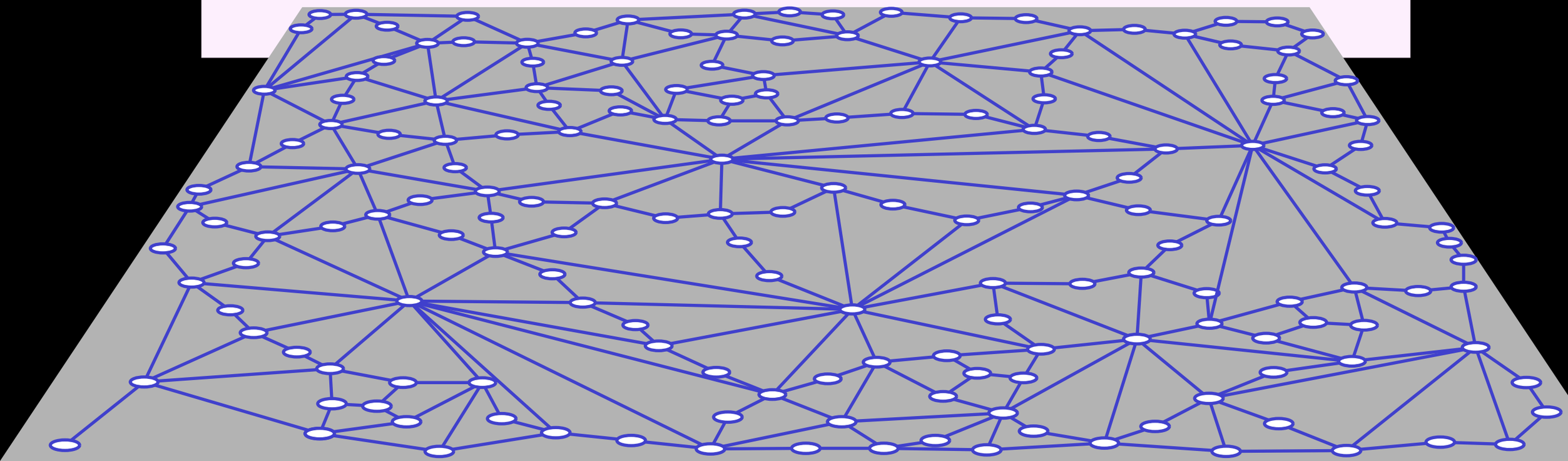

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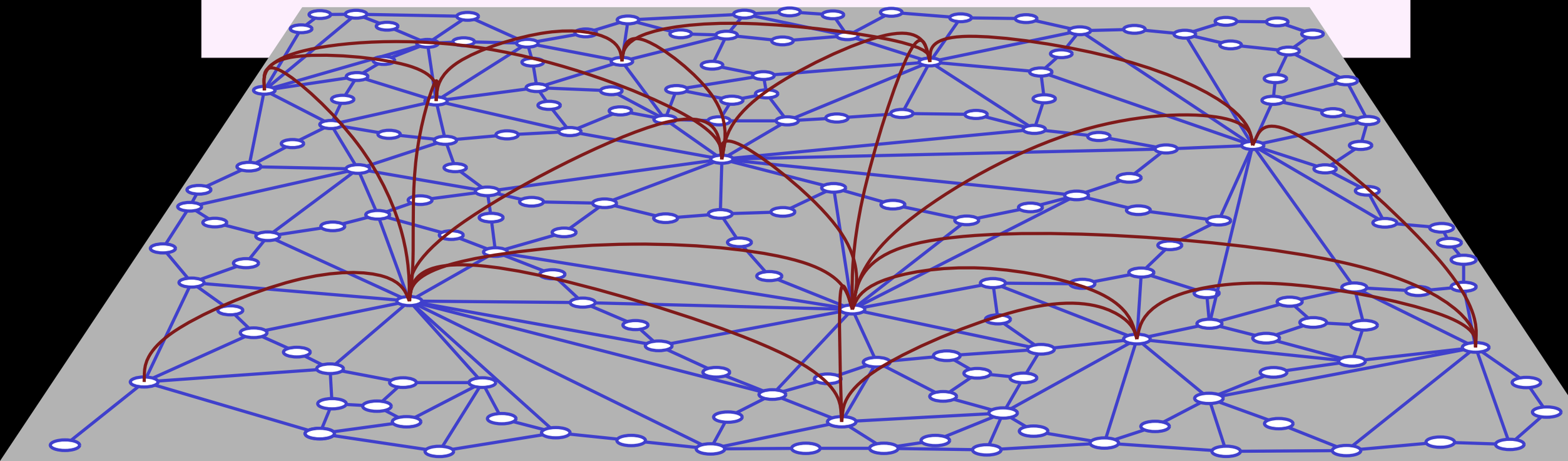

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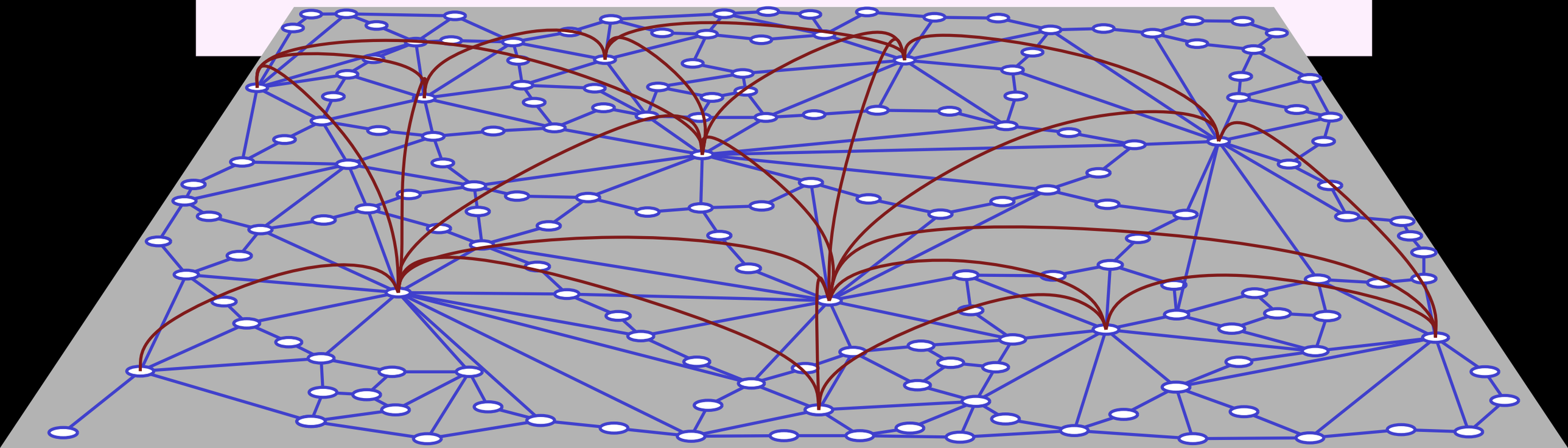
Compute the shortest path in the resulting graph.



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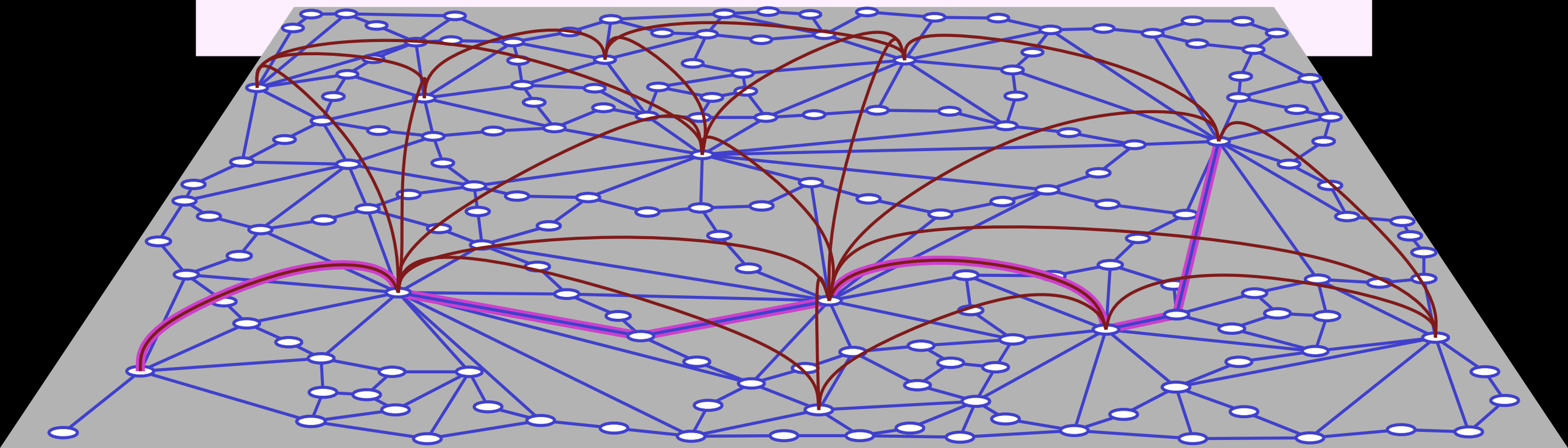

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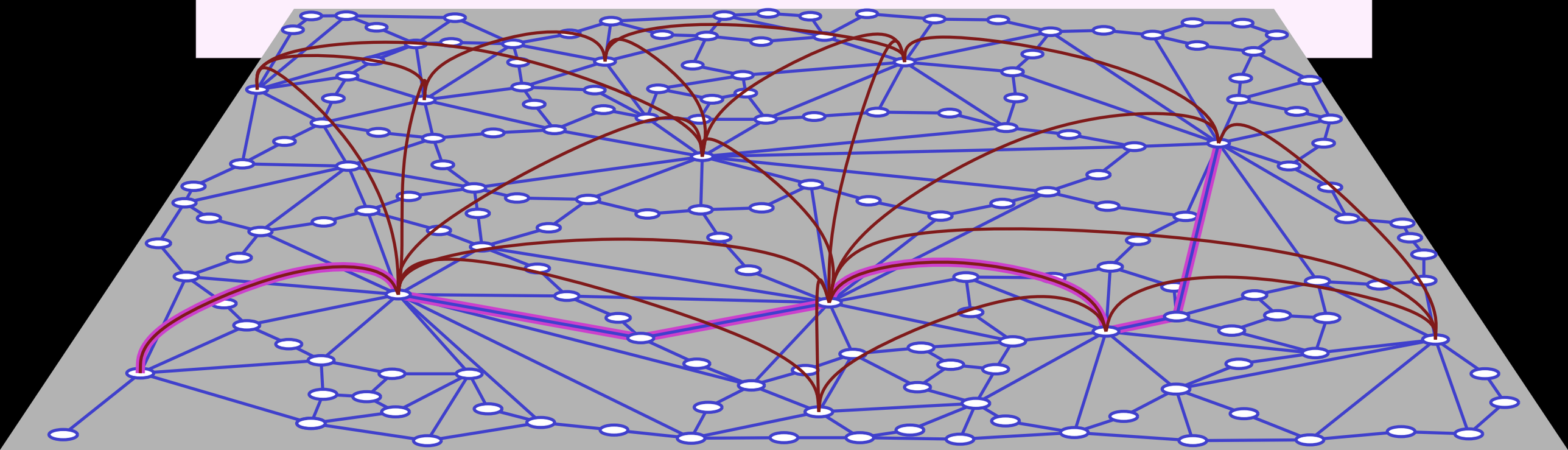
CLAIM: This is a $1 + \varepsilon$ approximation.



CONCLUSIONS

We show that resilience for unit disks is FPT, and leads this to an EPTAS.

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OPEN QUESTION: Can the resilience of unit disks be computed in polynomial time?

THANKS! ;)