# String graph with cop number 4

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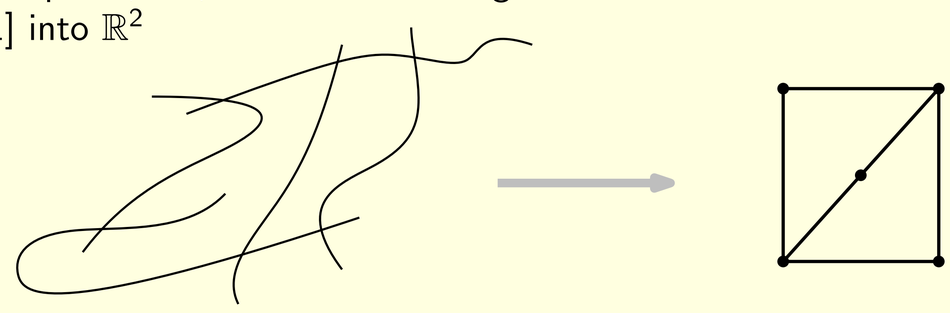
## String graphs

The class of *string graphs* is the class of intersection graphs of *strings* where each string is a bounded curve in the plane, i.e., a continuous image of the interval $[0, 1]$ into $\mathbb{R}^2$



The class of *1-string graphs* consists of string graphs where every two strings cross at most once.

## The game of Cops and Robbers

- there are two players: **Cs** and **R**;
- **Cs** and **R** see the whole graph;
- **Cs** and **R** alternate moves:

  **1st round**: **Cs** choose vertices,

  **2nd round**: **R** choses a vertex,

  **ith round**: each of **Cs** may move to an adjacent vertex as well as **R** may move to an adjacent vertex;
- the game is over when:

  **Cs** win, that is, one of **Cs** is on the same vertex as **R**,

  **R** wins, that is, **R** can move indefinitely.

## Problem

*Cop number* $c(G)$ is the smallest number of cops needed to catch the robber in a graph $G$.

Let $X = \max_{G \text{ is a string graph}} c(G)$

We have improved the gap: ~~$3$~~, $4 \leq X \leq 13$,

[Aigner and Fromme, 1984] [Gavenčiak et al., 2018]

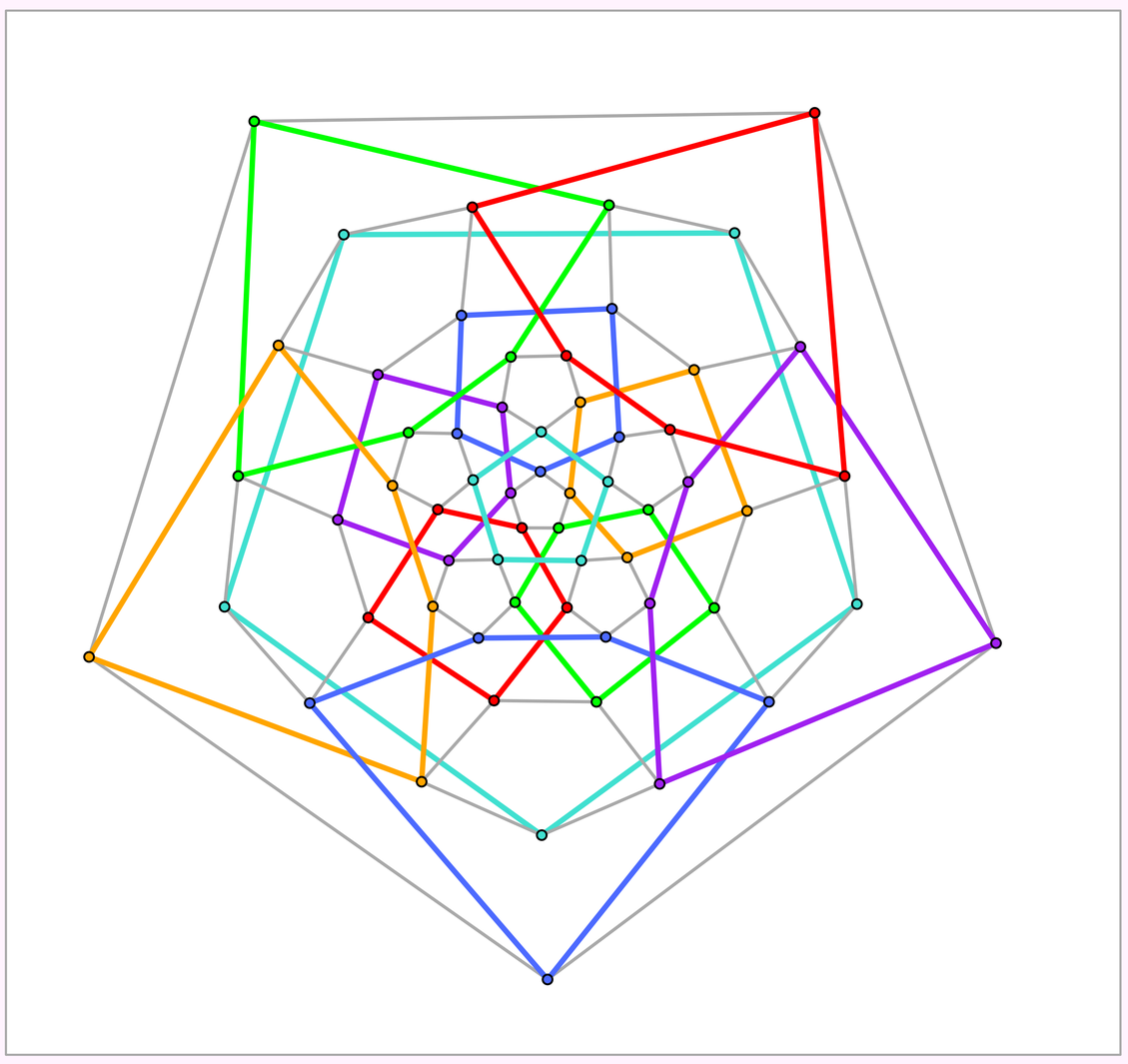
by constructing a 1-string graph $G$ with cop number 4.

## Main result: 1-string graph with cop number 4

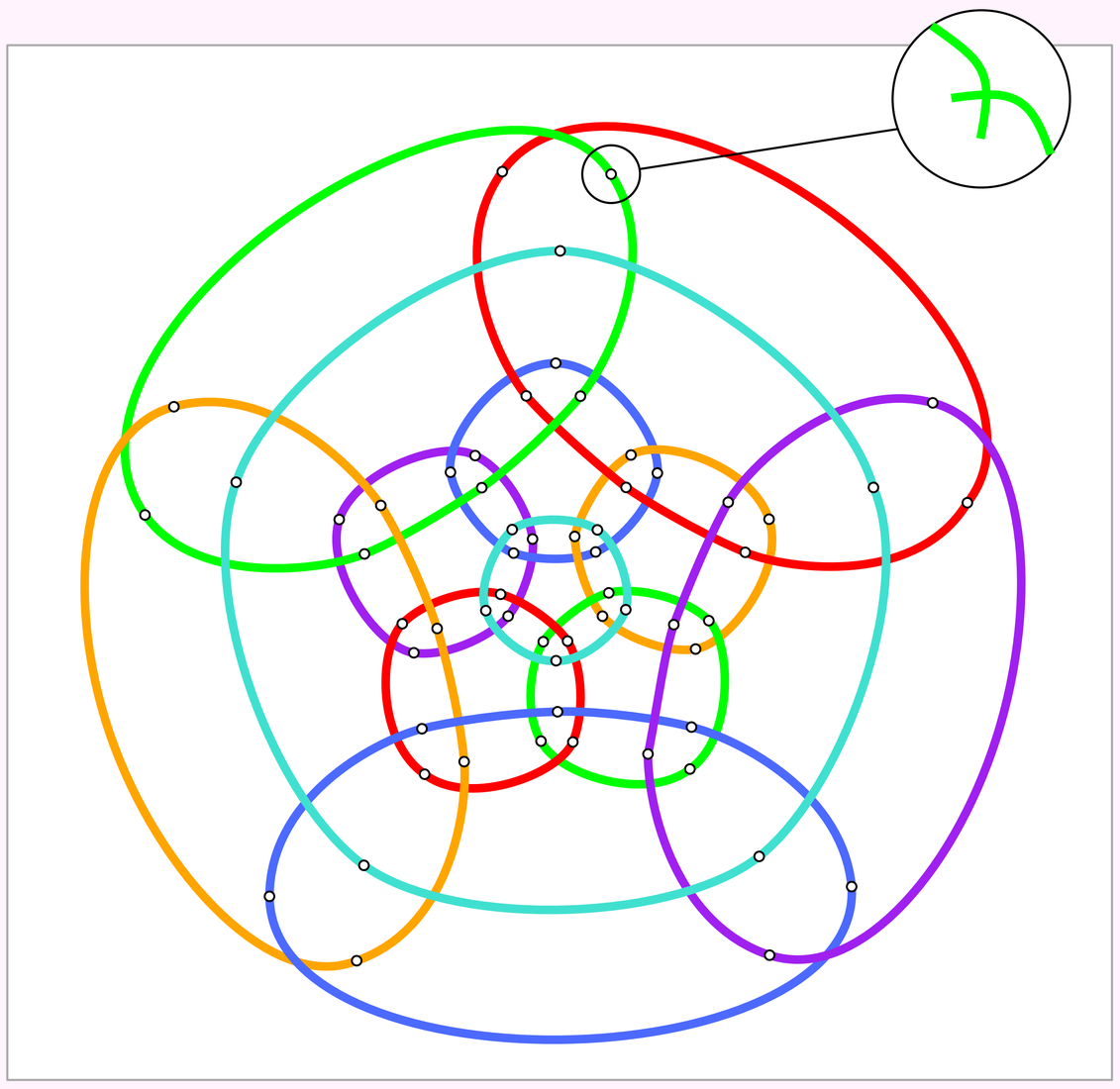
**Lemma** [Aigner and Fromme, 1984]
For a graph $G$ with min deg $\delta$ and girth at least 5, it holds that $\delta \leq c(G)$.

**Theorem** [Answers an open question by Gavenčiak et al.!]
There is a 1-string graph $G$ with min deg 4 and girth at least 5.

Construction inspired by Rhombicosidodecahedron



The graph $G$ with min degree 4 and girth 5. Does the node-link diagram above look too messy to check? Look at the string representation!



1-string representation of $G$. $G$ has

- min deg 4: each string crosses exactly 4 others (one at each of its endpoints and two in between the endpoints),
- girth $\geq 5$: check every region formed by at most four colored circles.

## Future work

Narrow the gap: $4 \leq X \leq 13$.

Generalize the lower bound construction for string graphs with string genus larger than zero. Graphs with *string genus* $k$ have string representations on a surface of genus $k$.