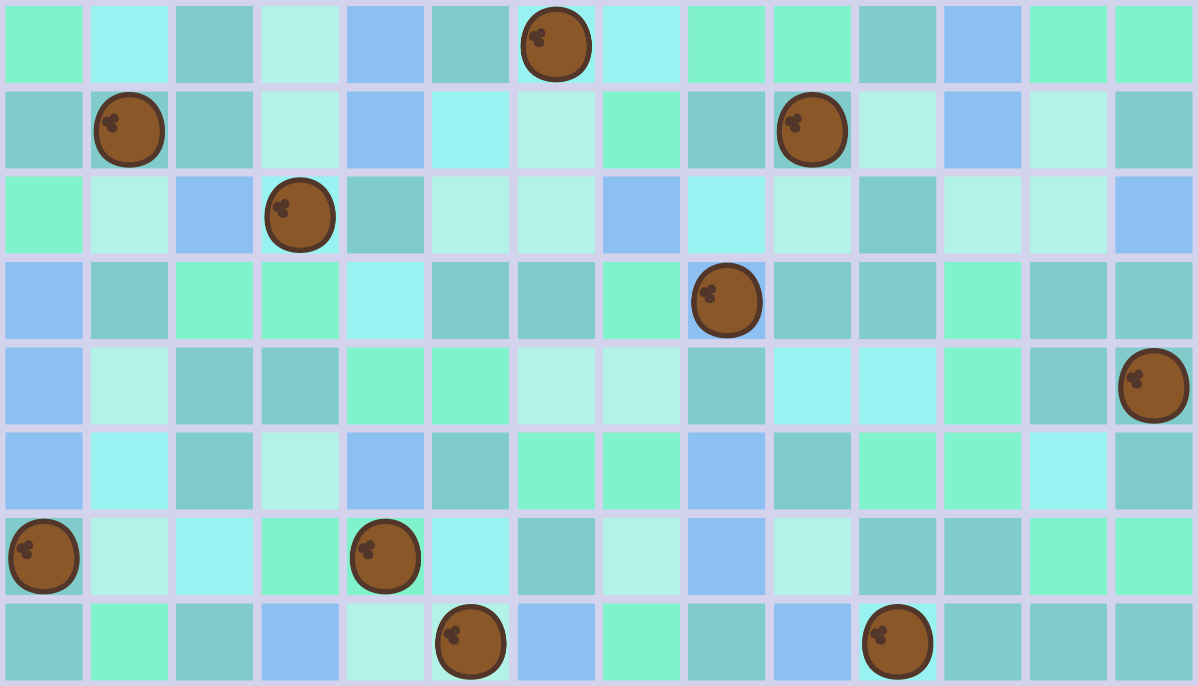
TRASH COMPACTION

Hugo Akitaya ○ Greg Aloupis ○ Maarten Löffler ○ Anika Rounds

Coconut Compaction



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COCONUT BASICS

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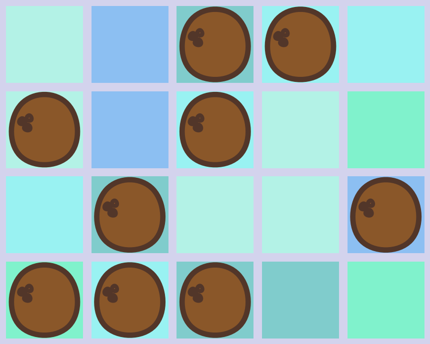

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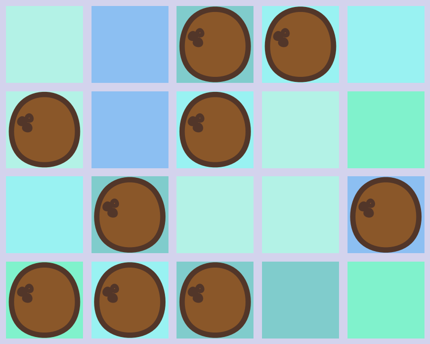

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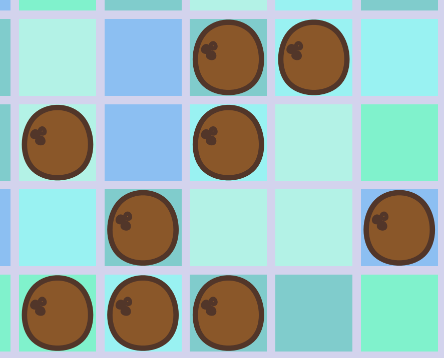

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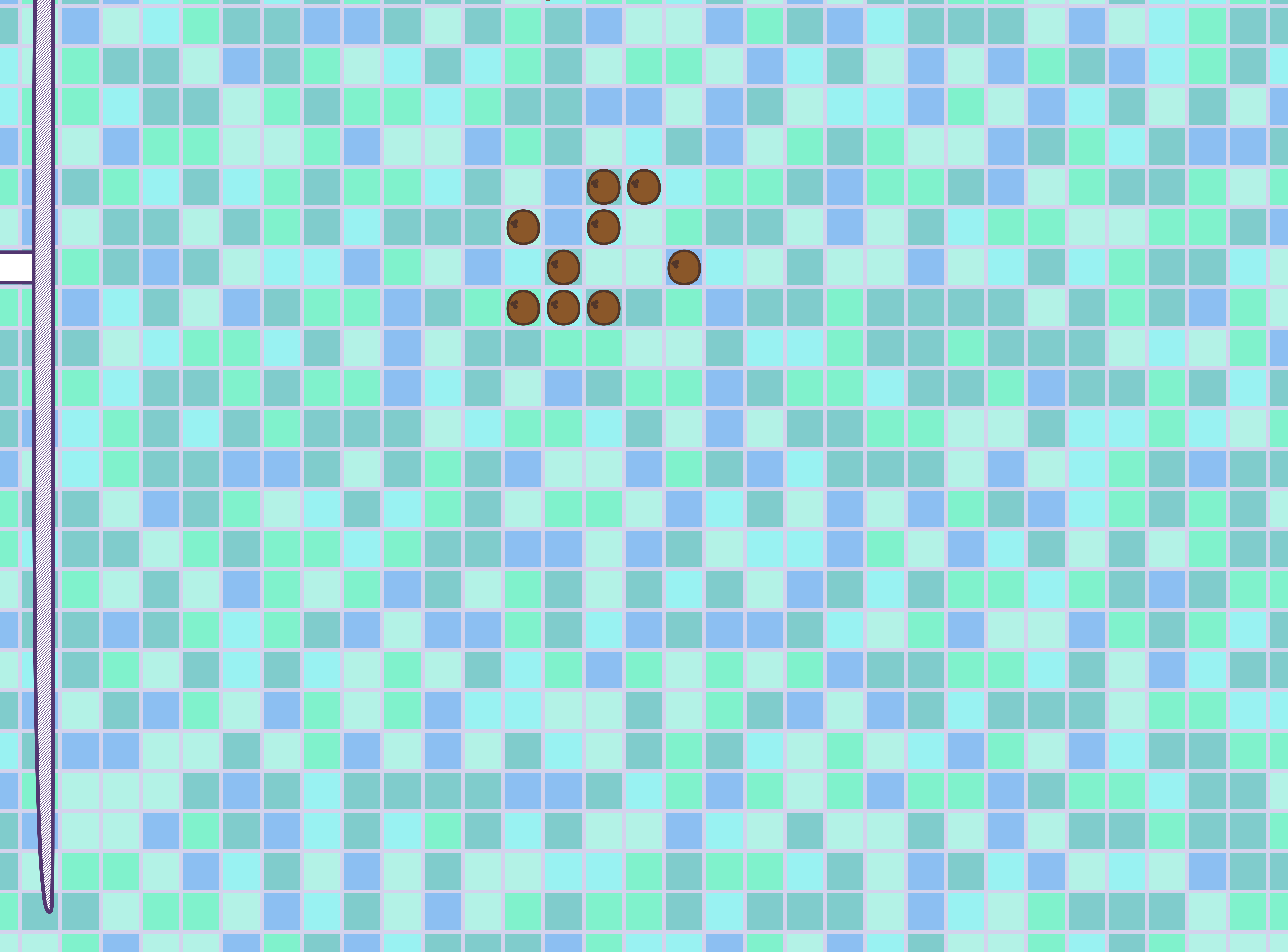

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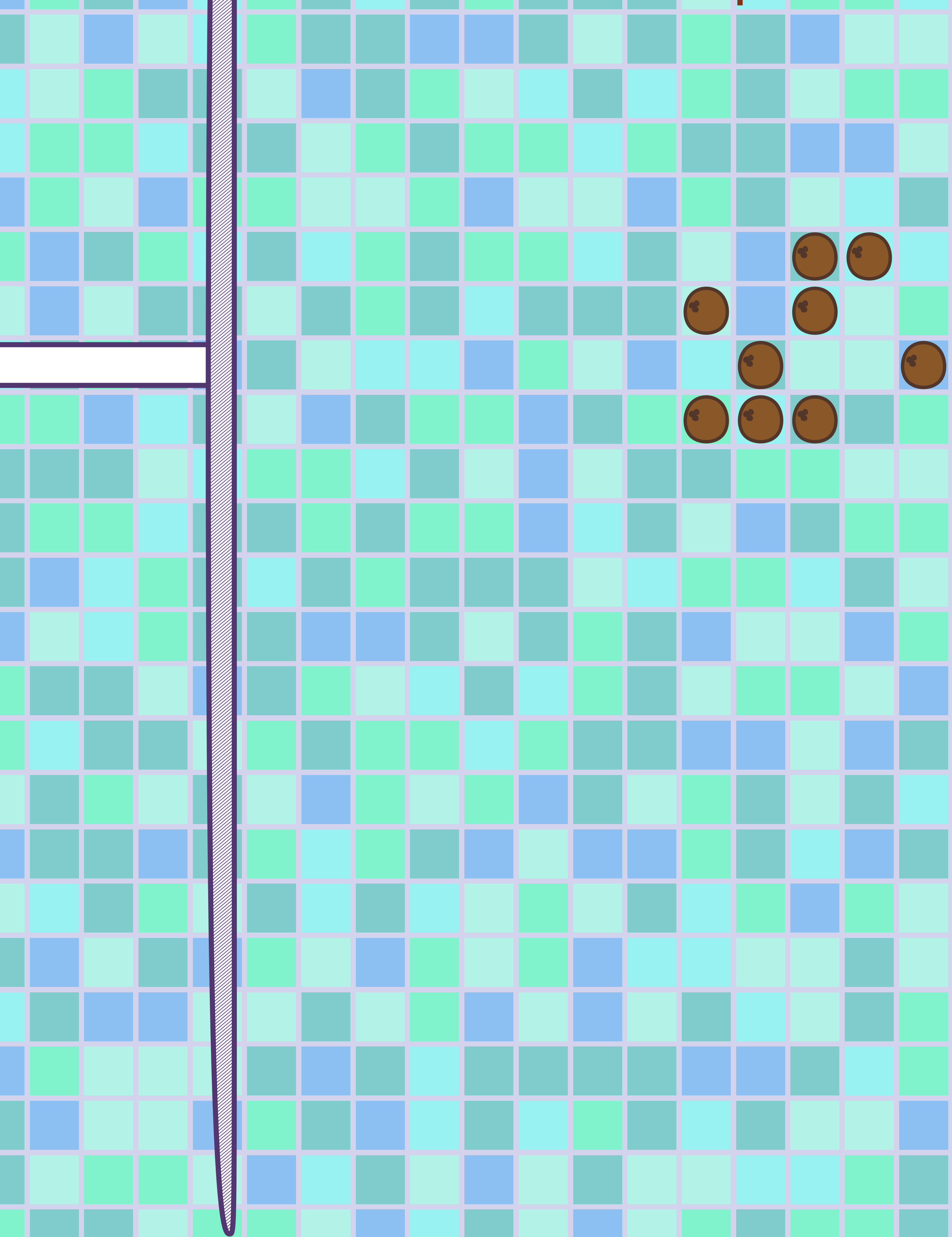

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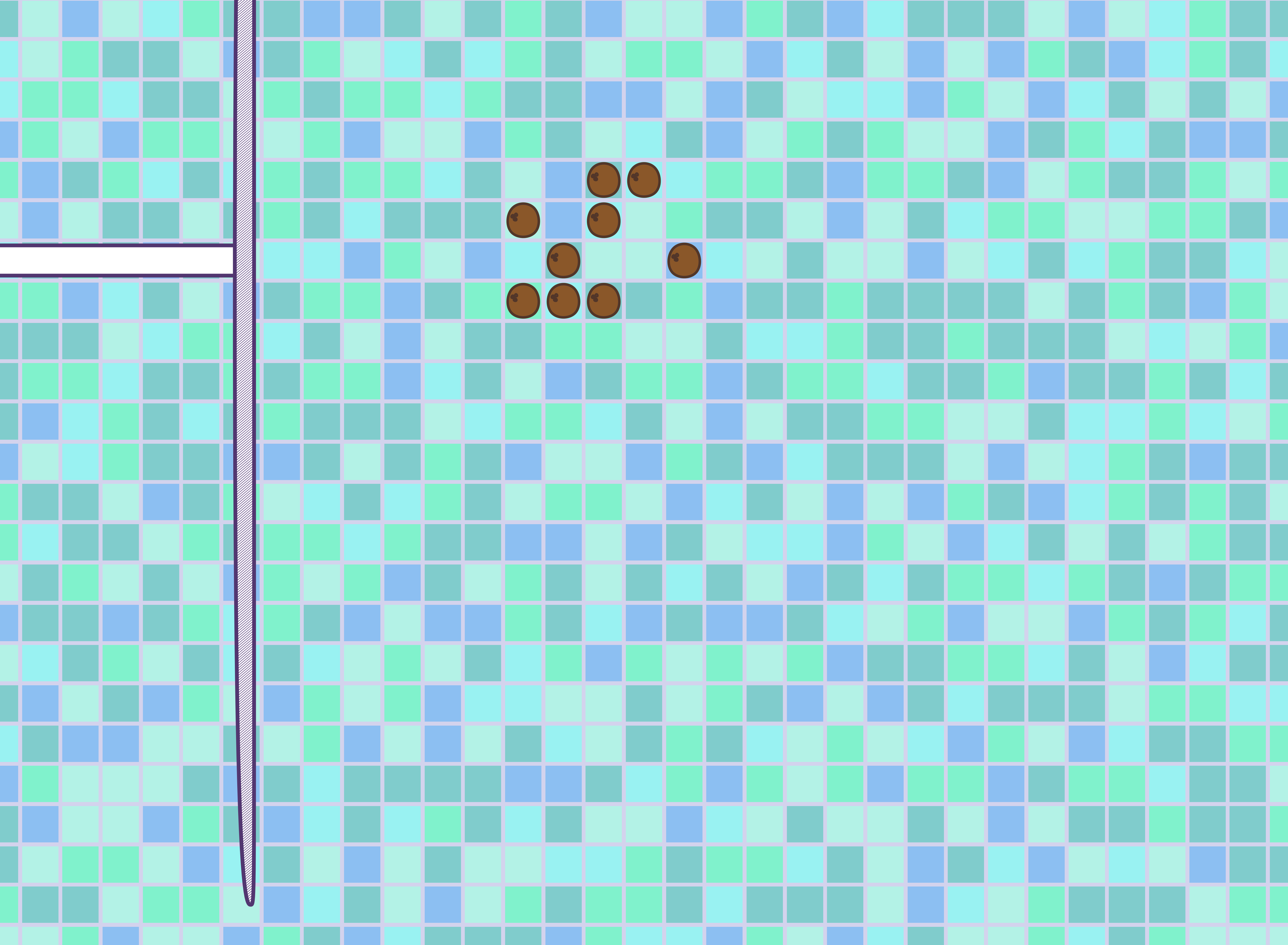

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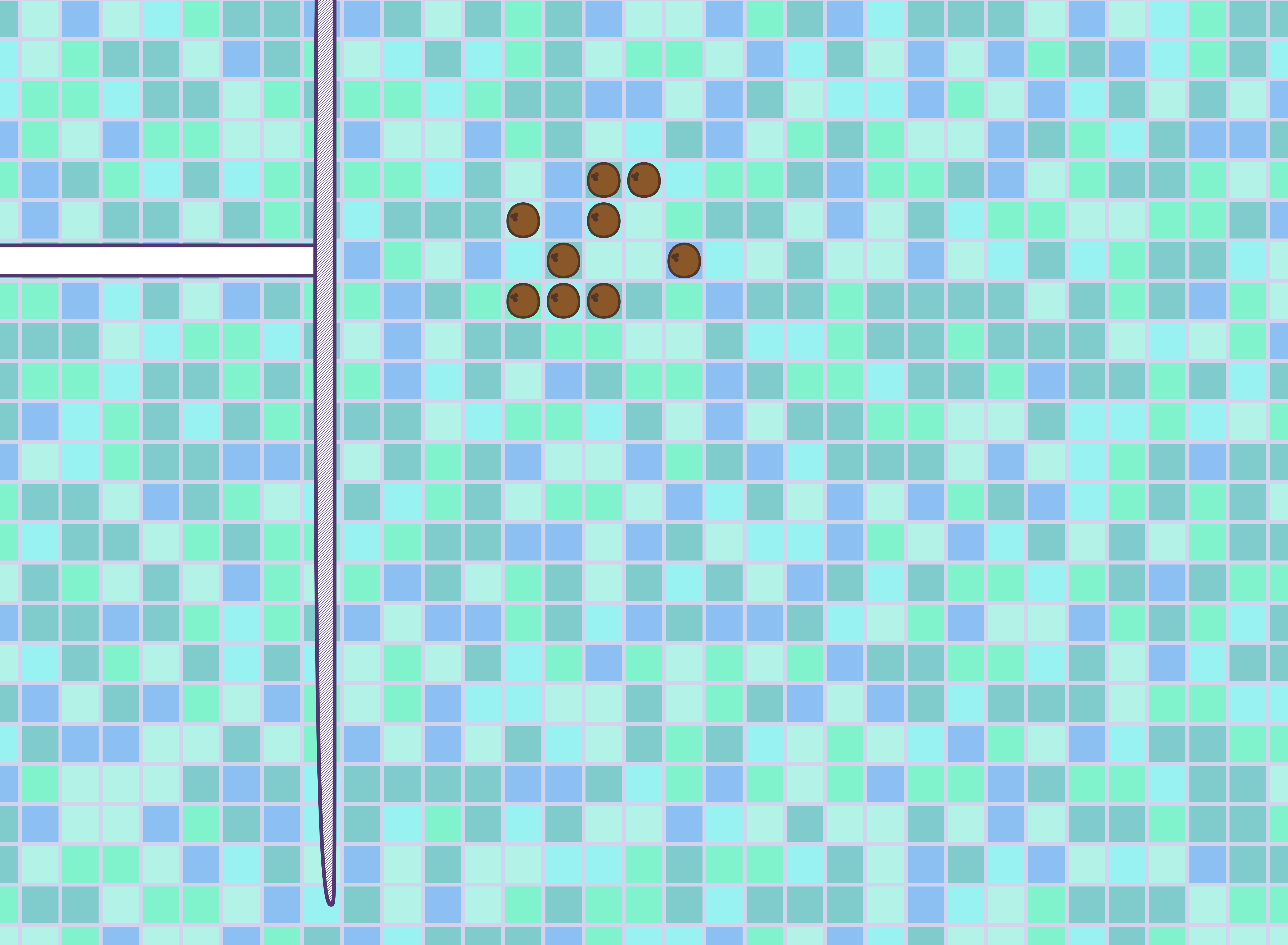

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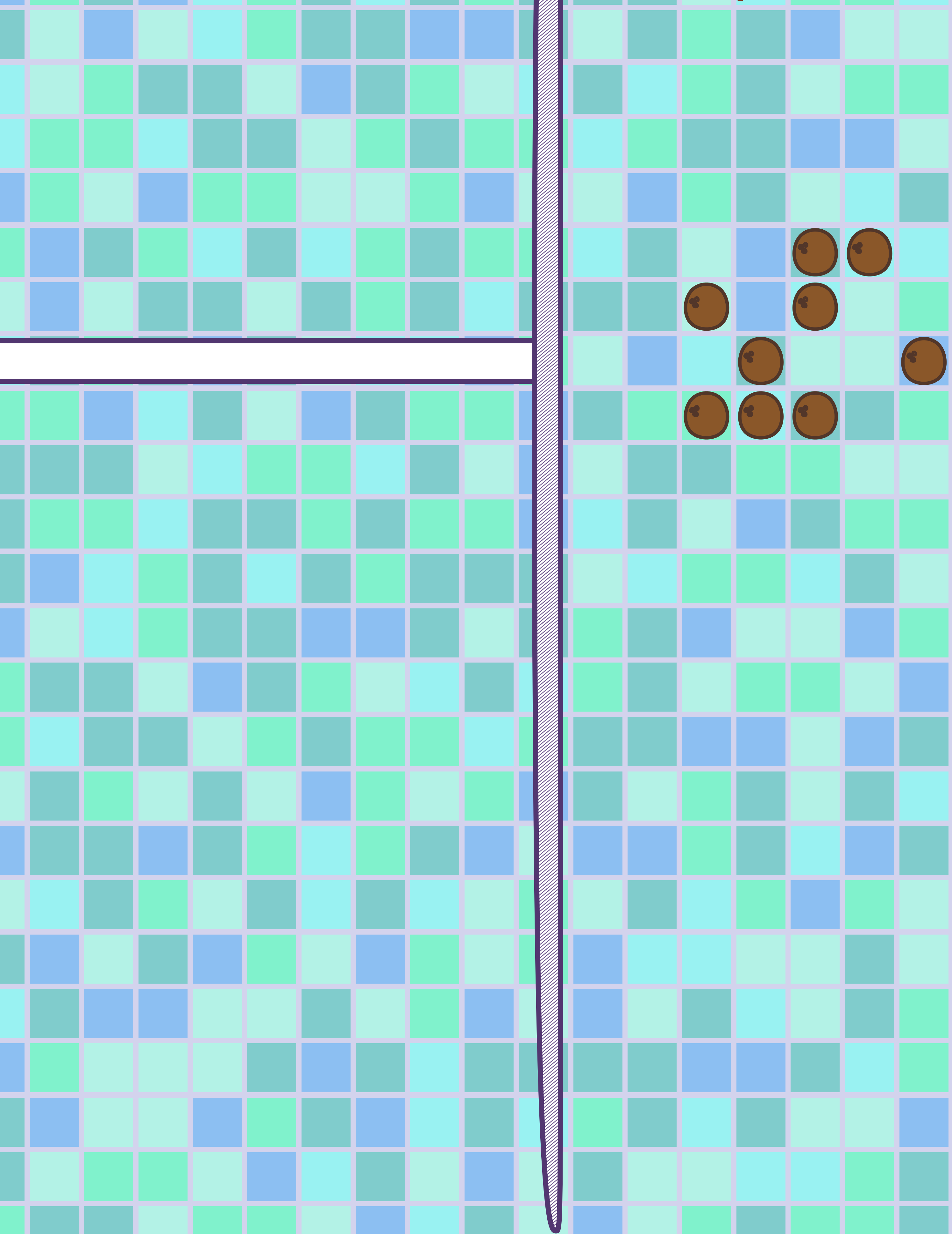

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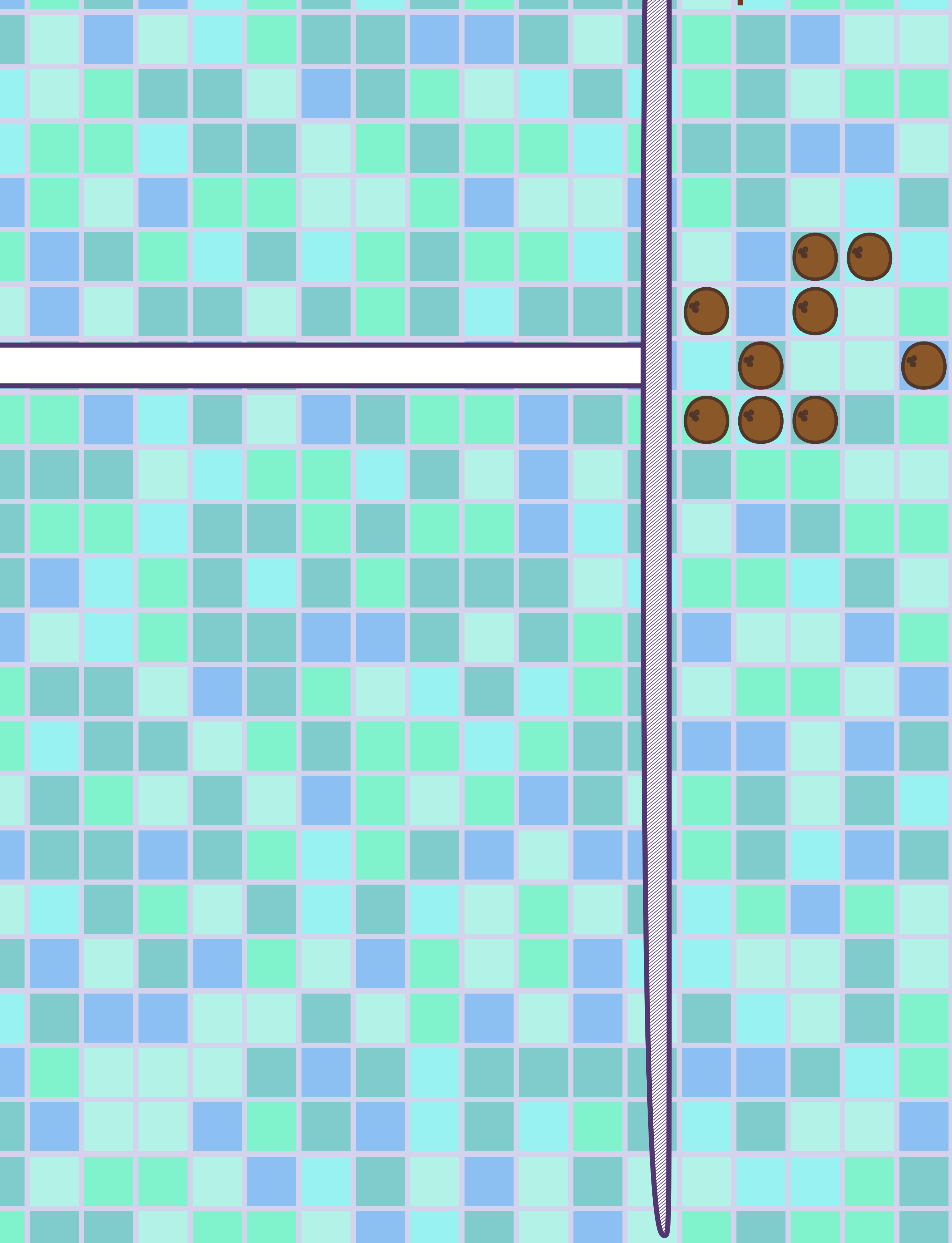

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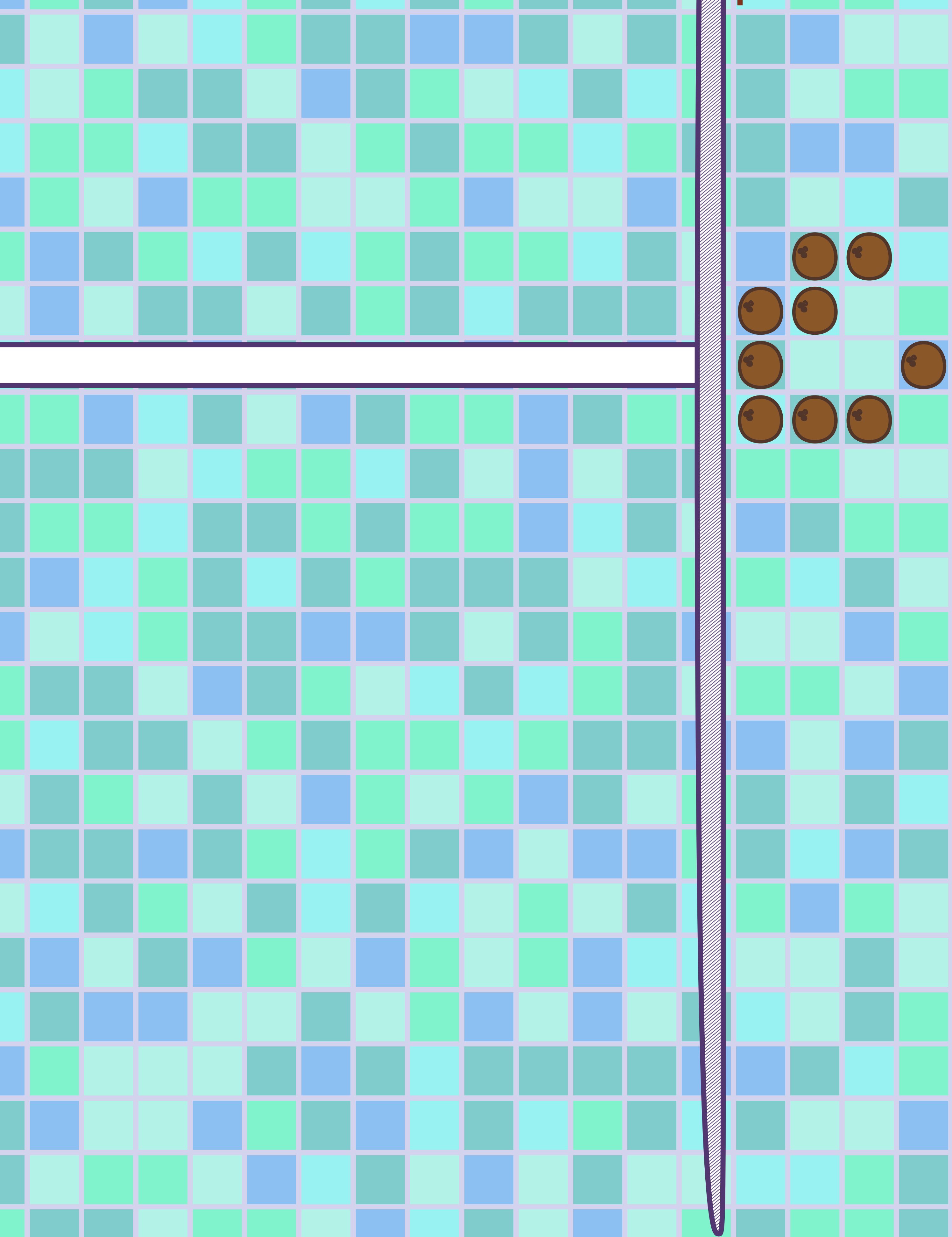

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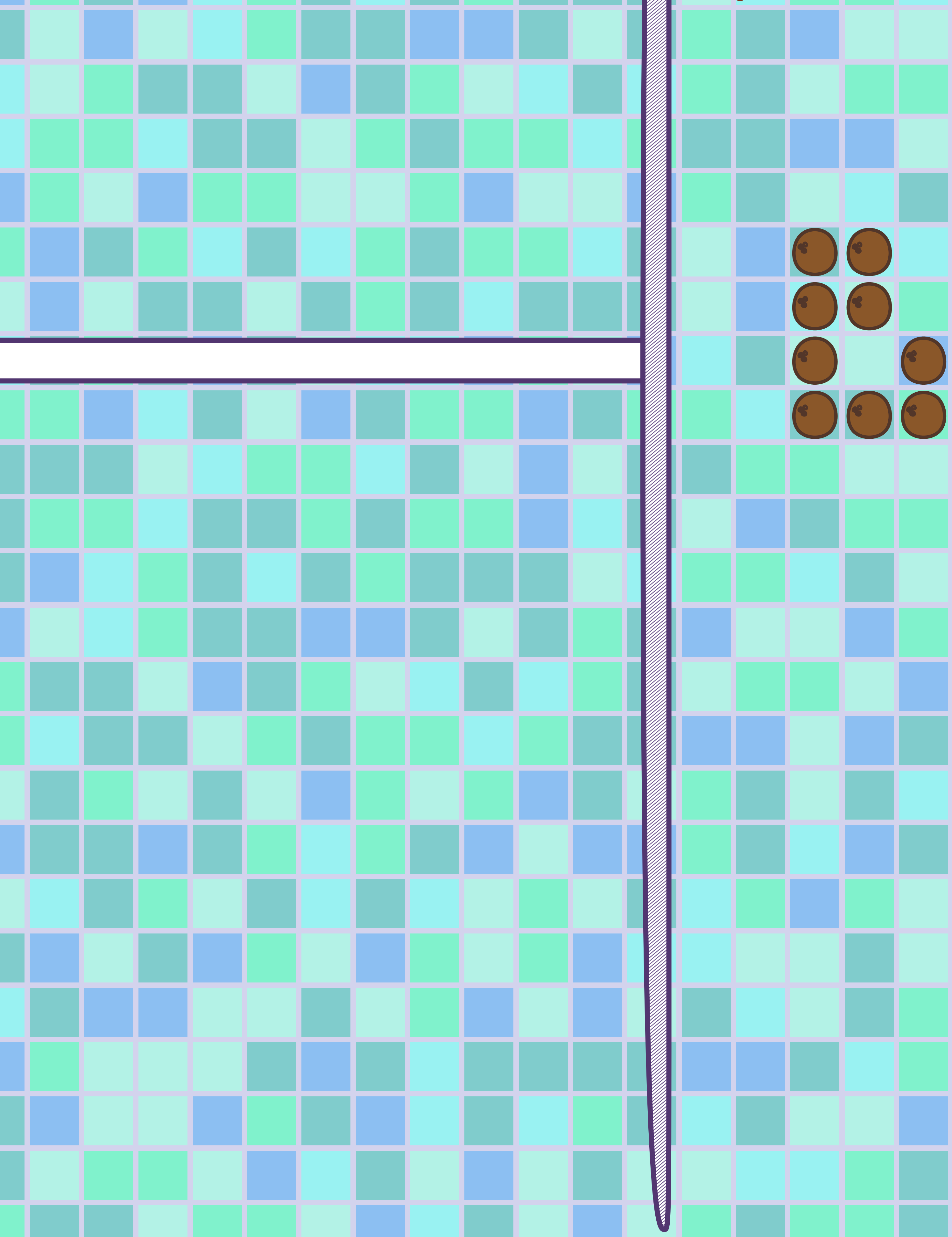

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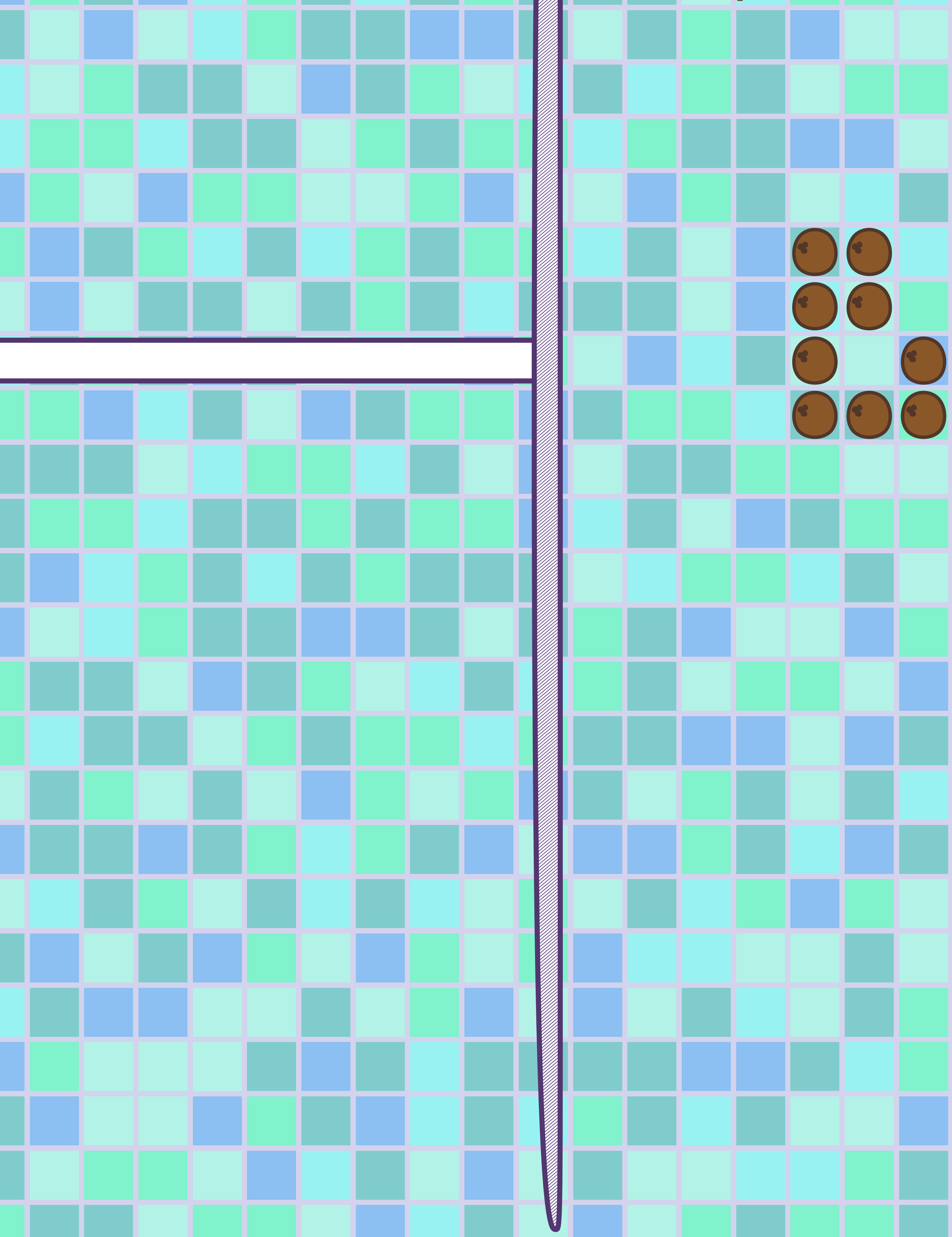

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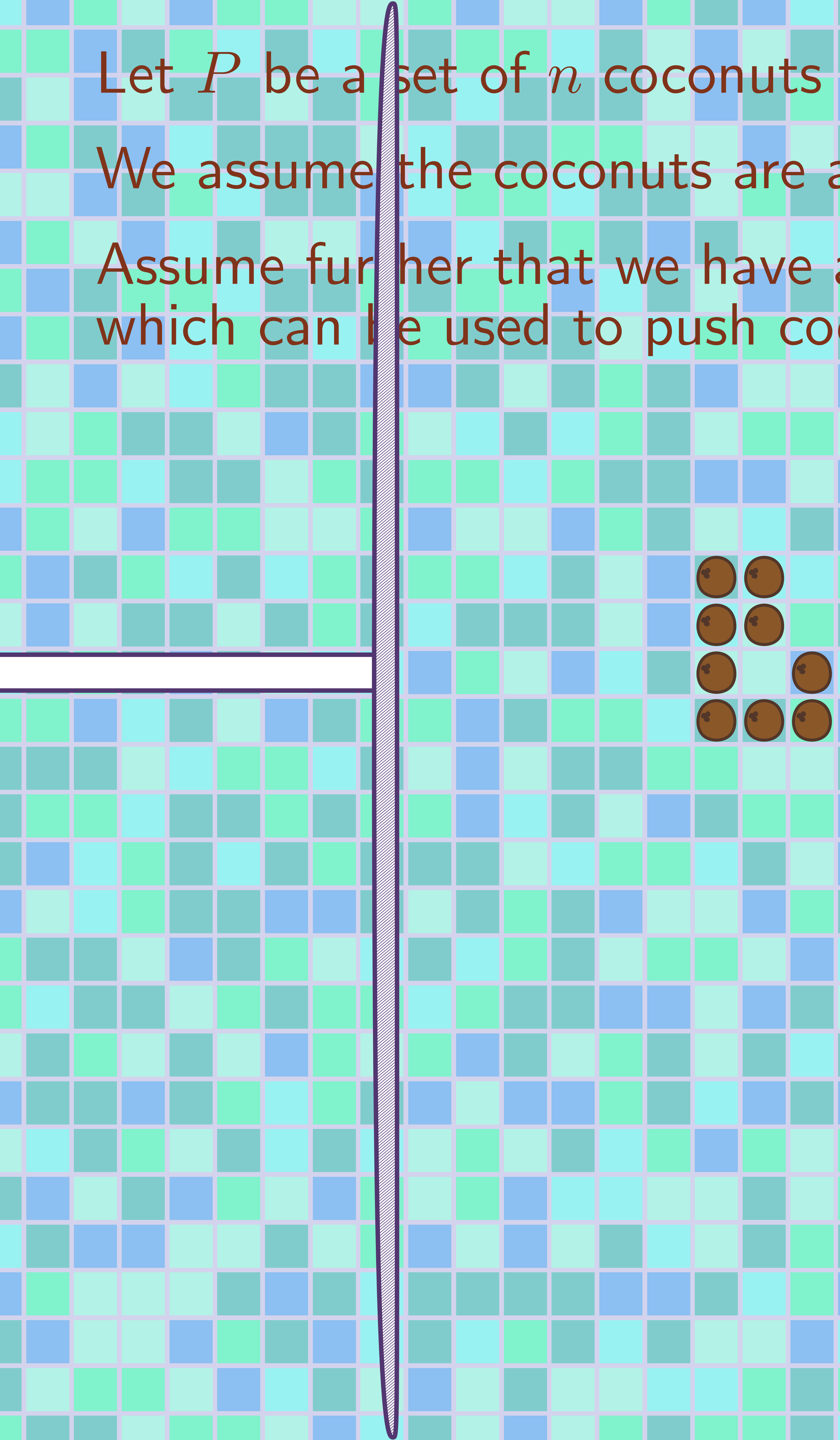

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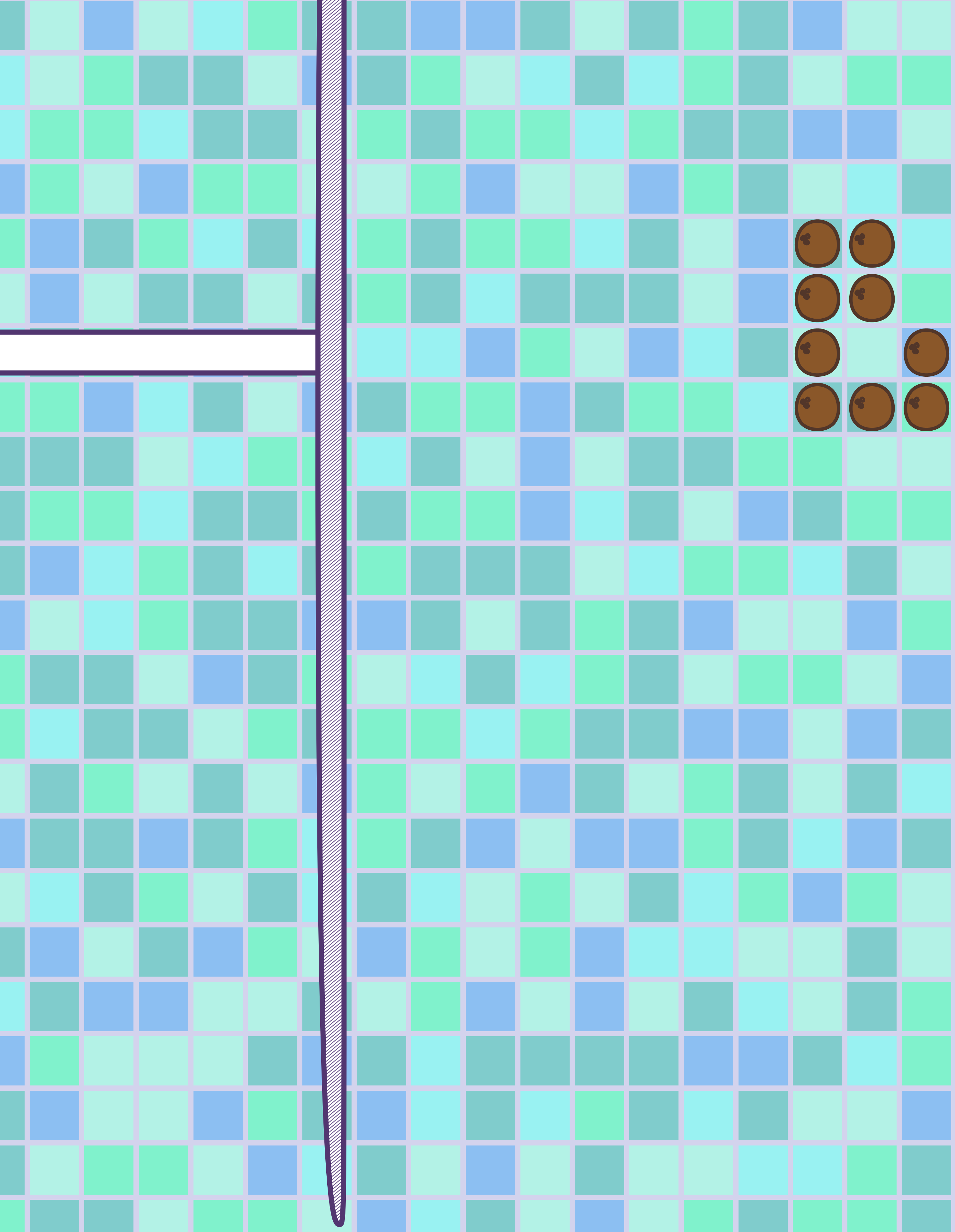

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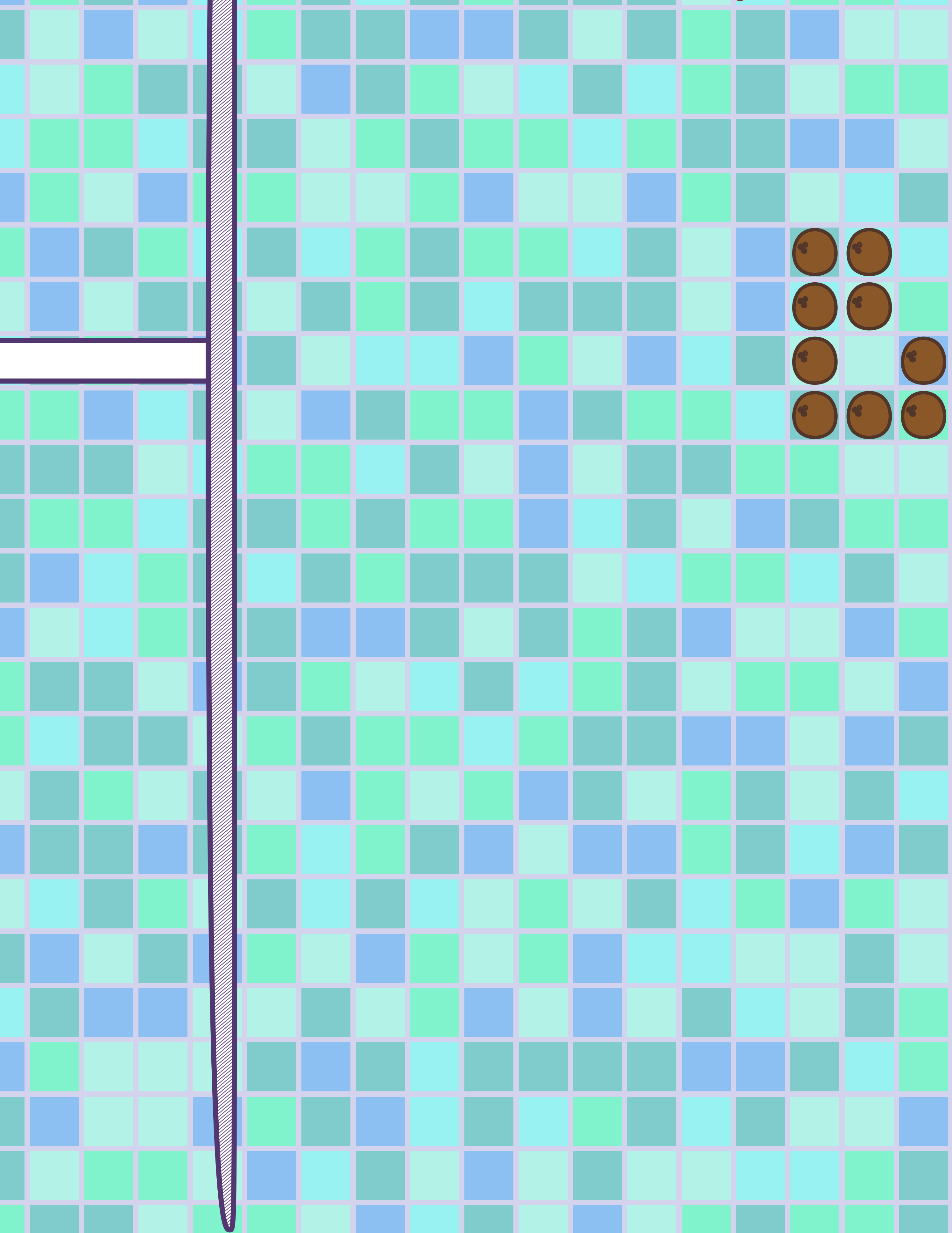

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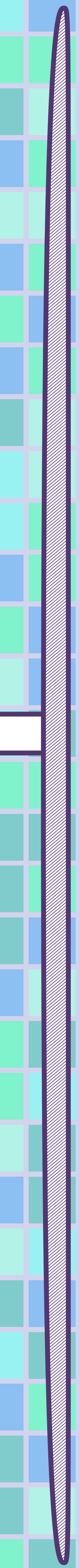

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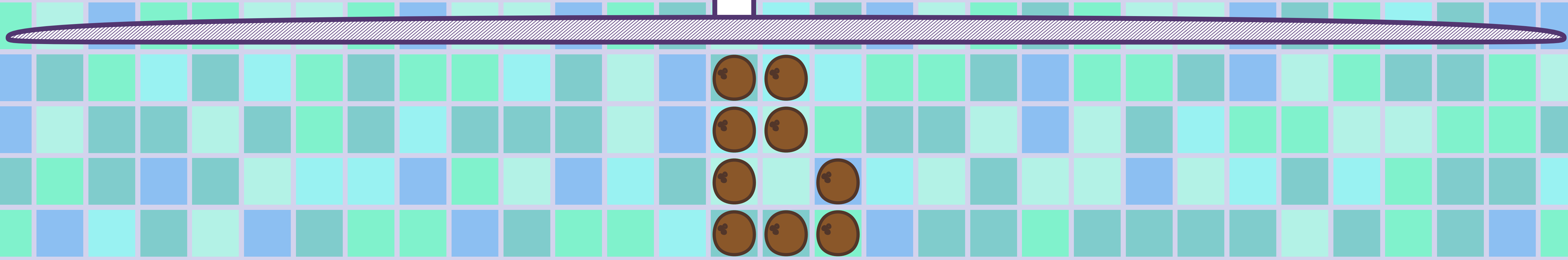

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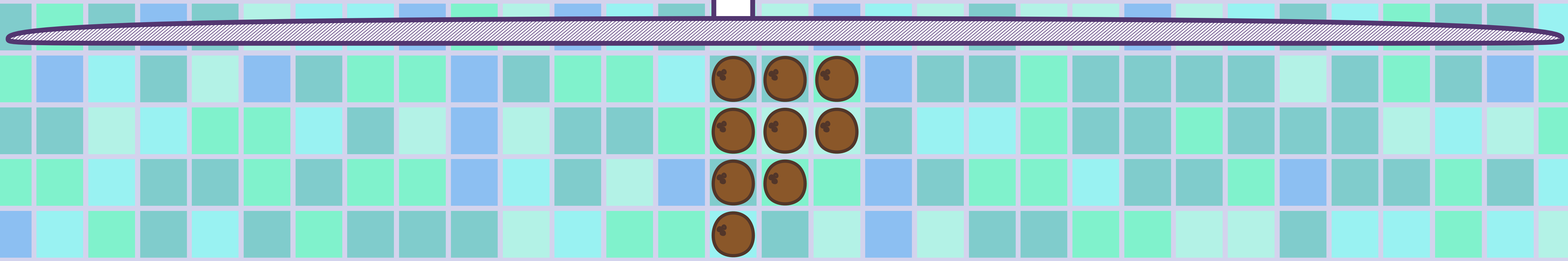

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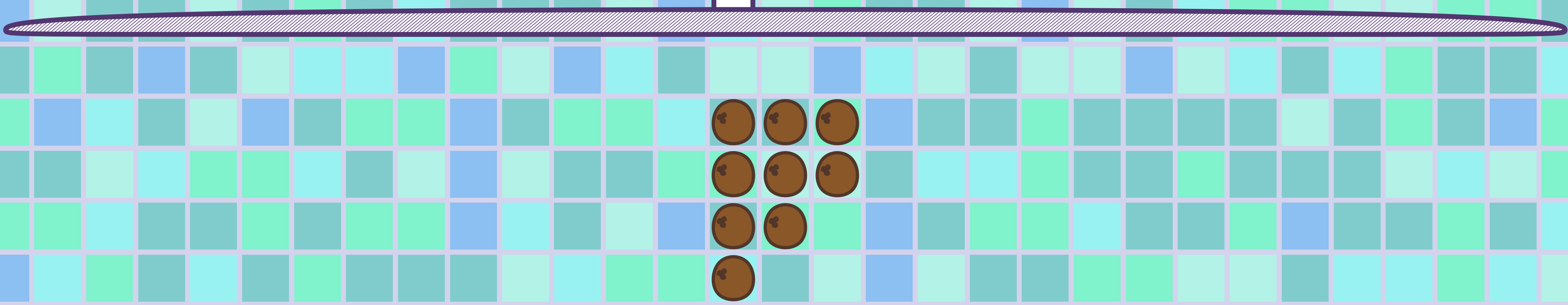

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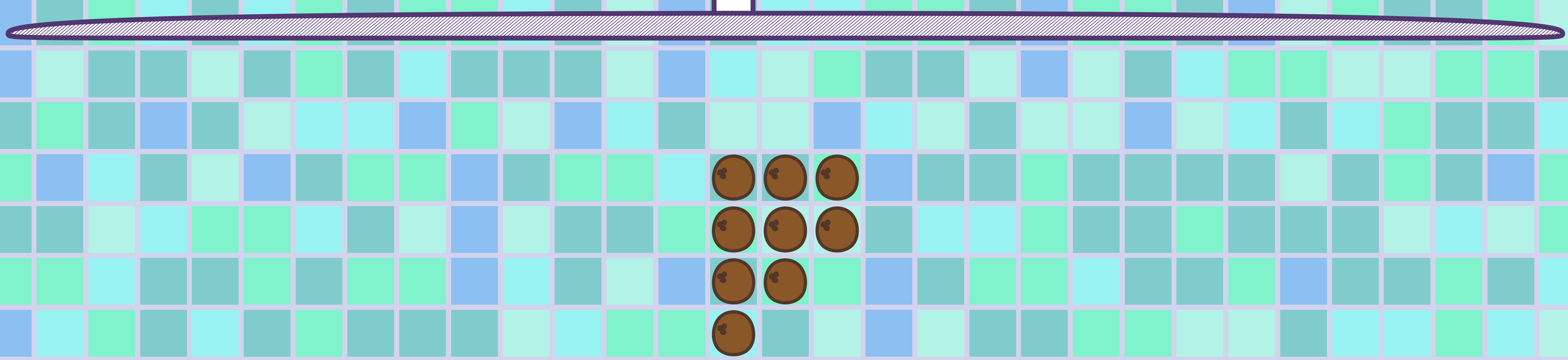

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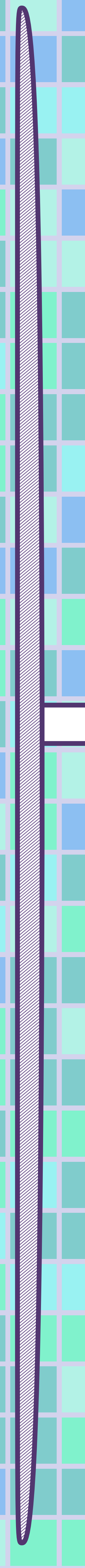

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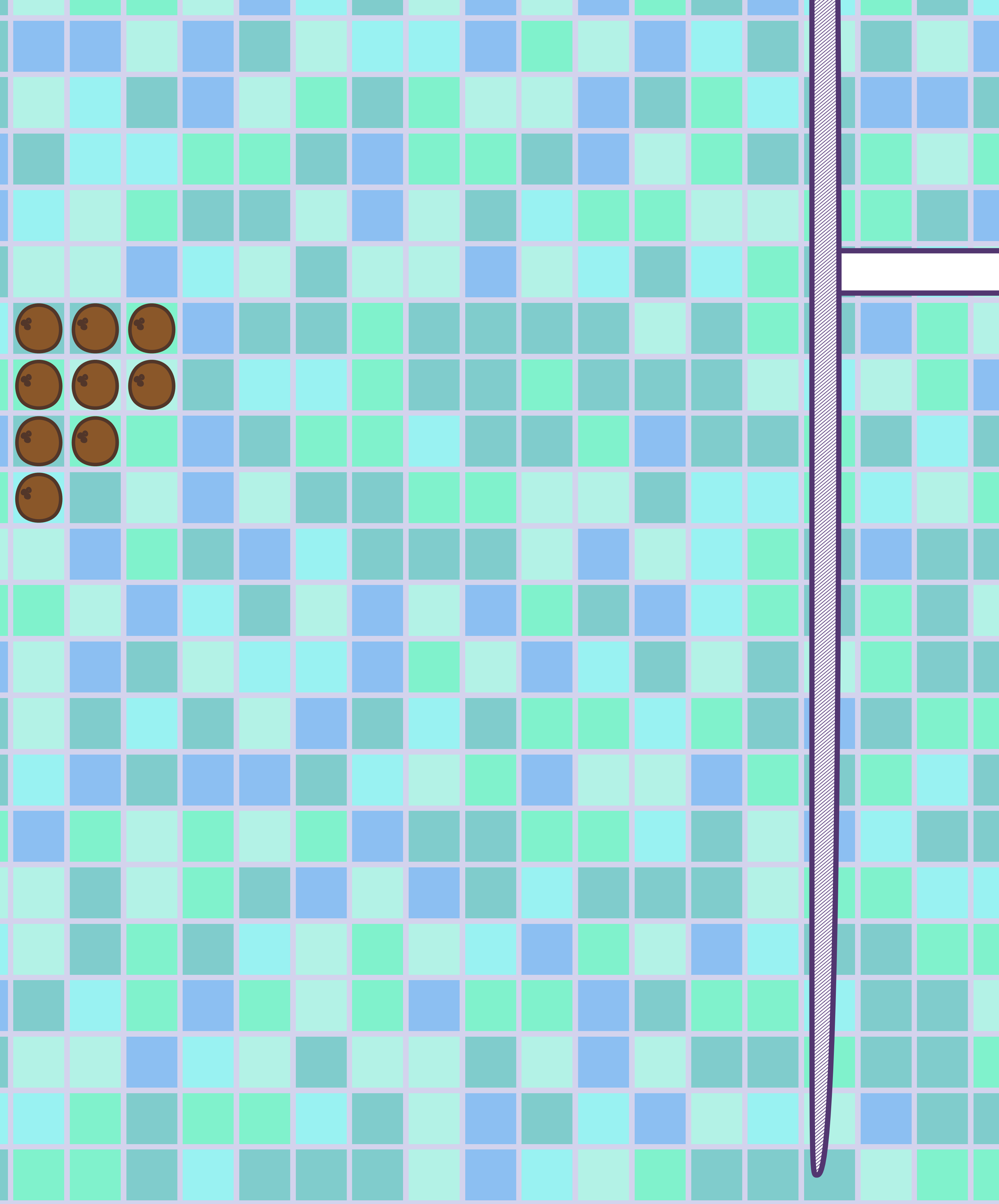

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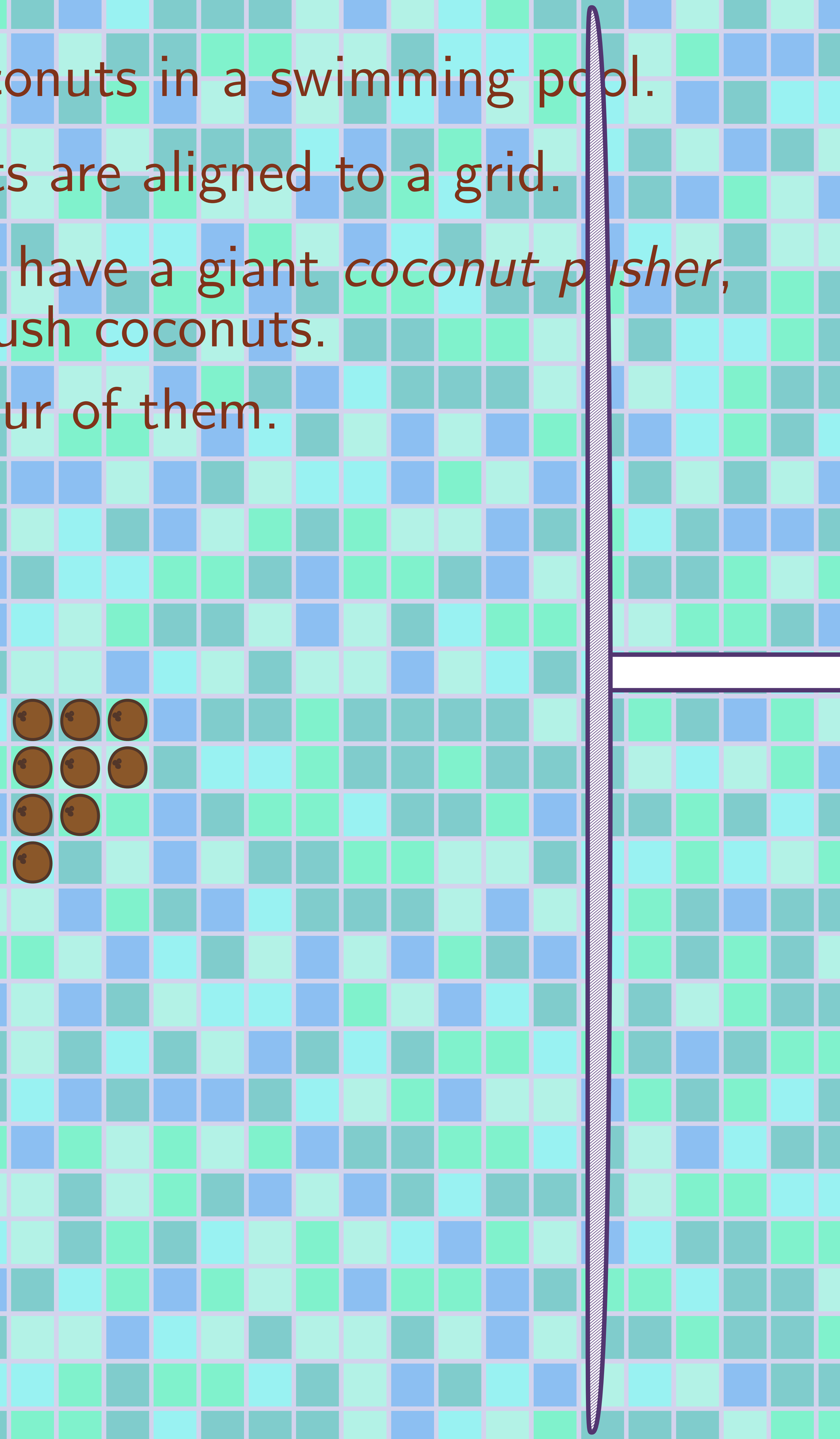

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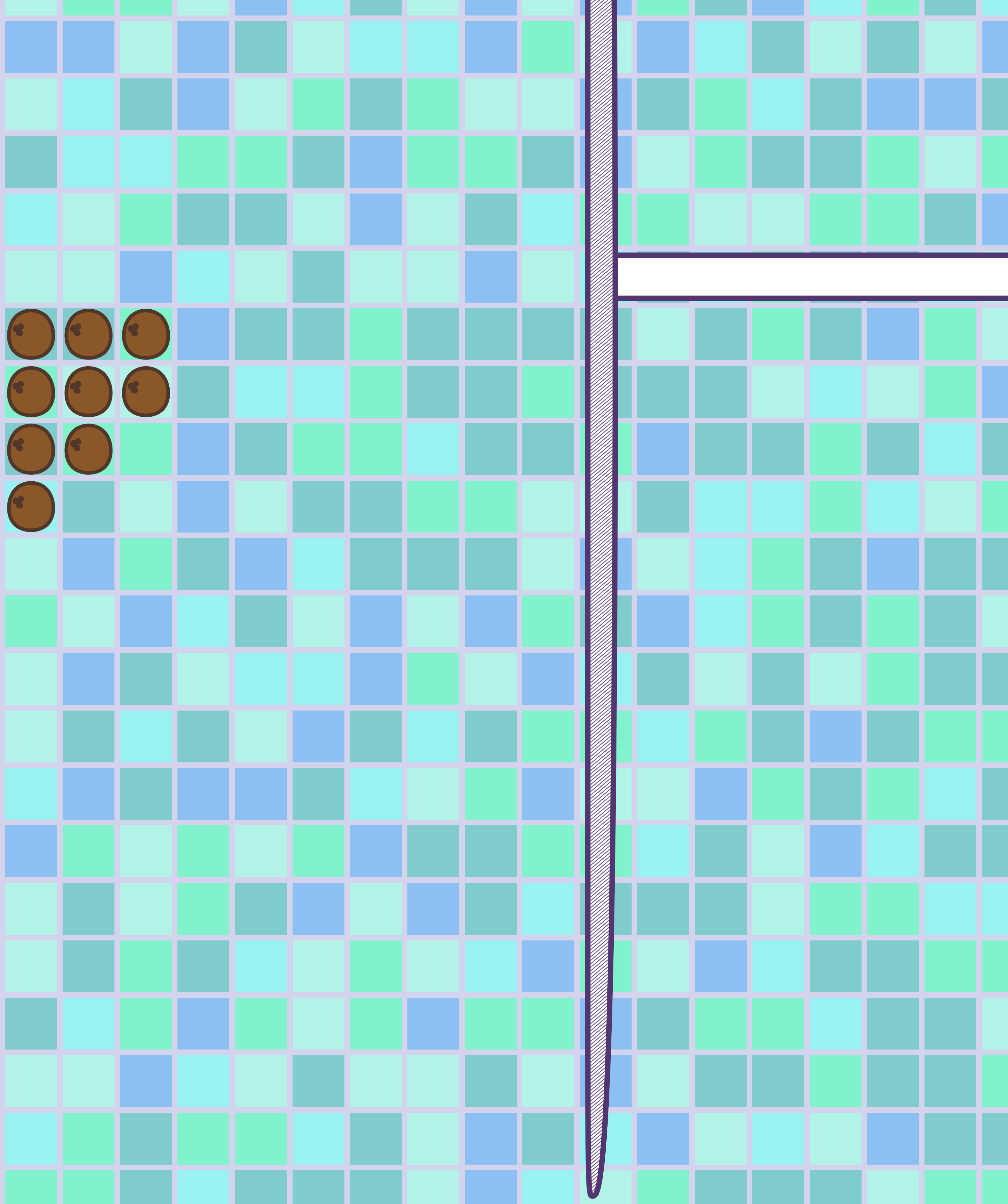

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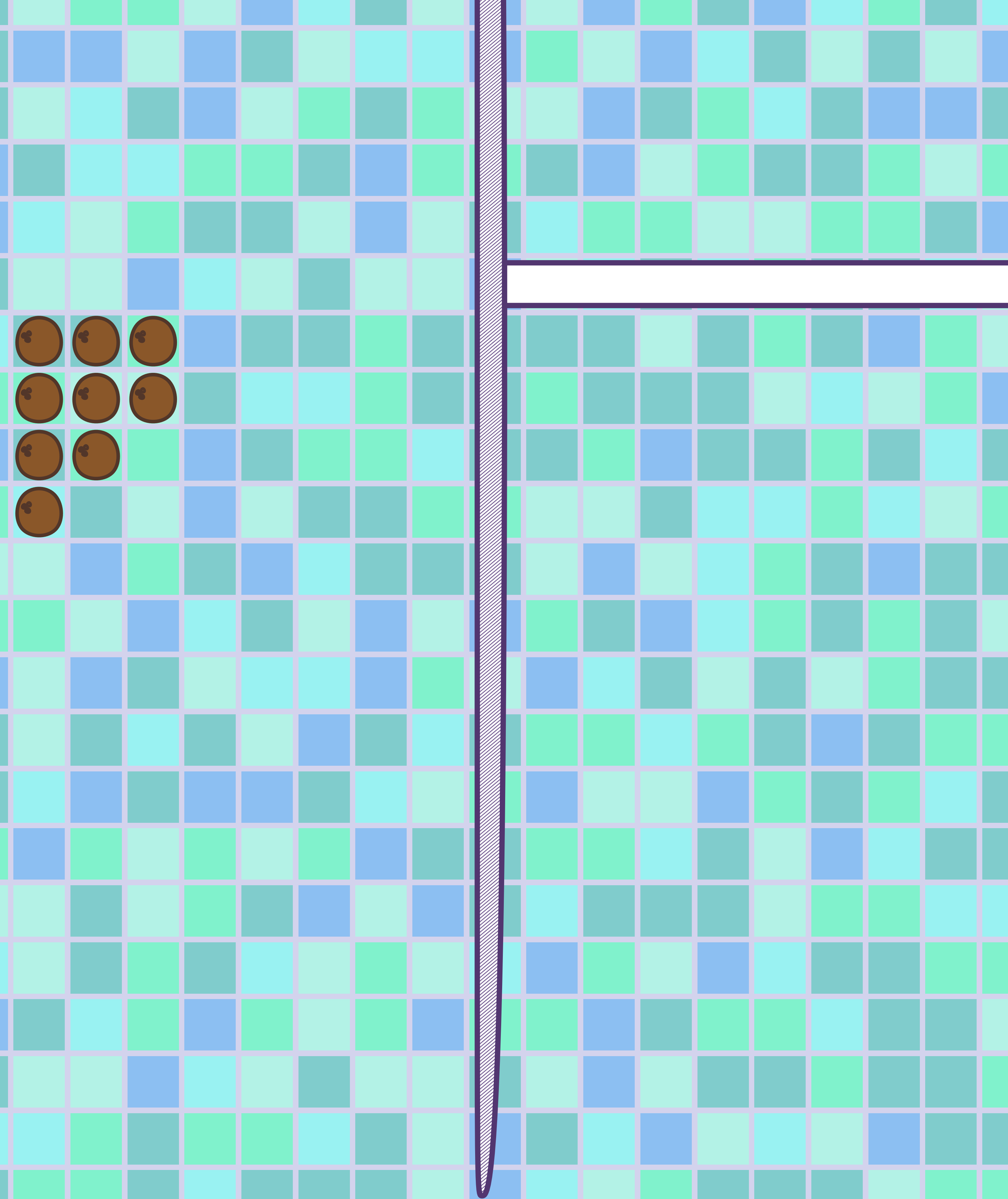

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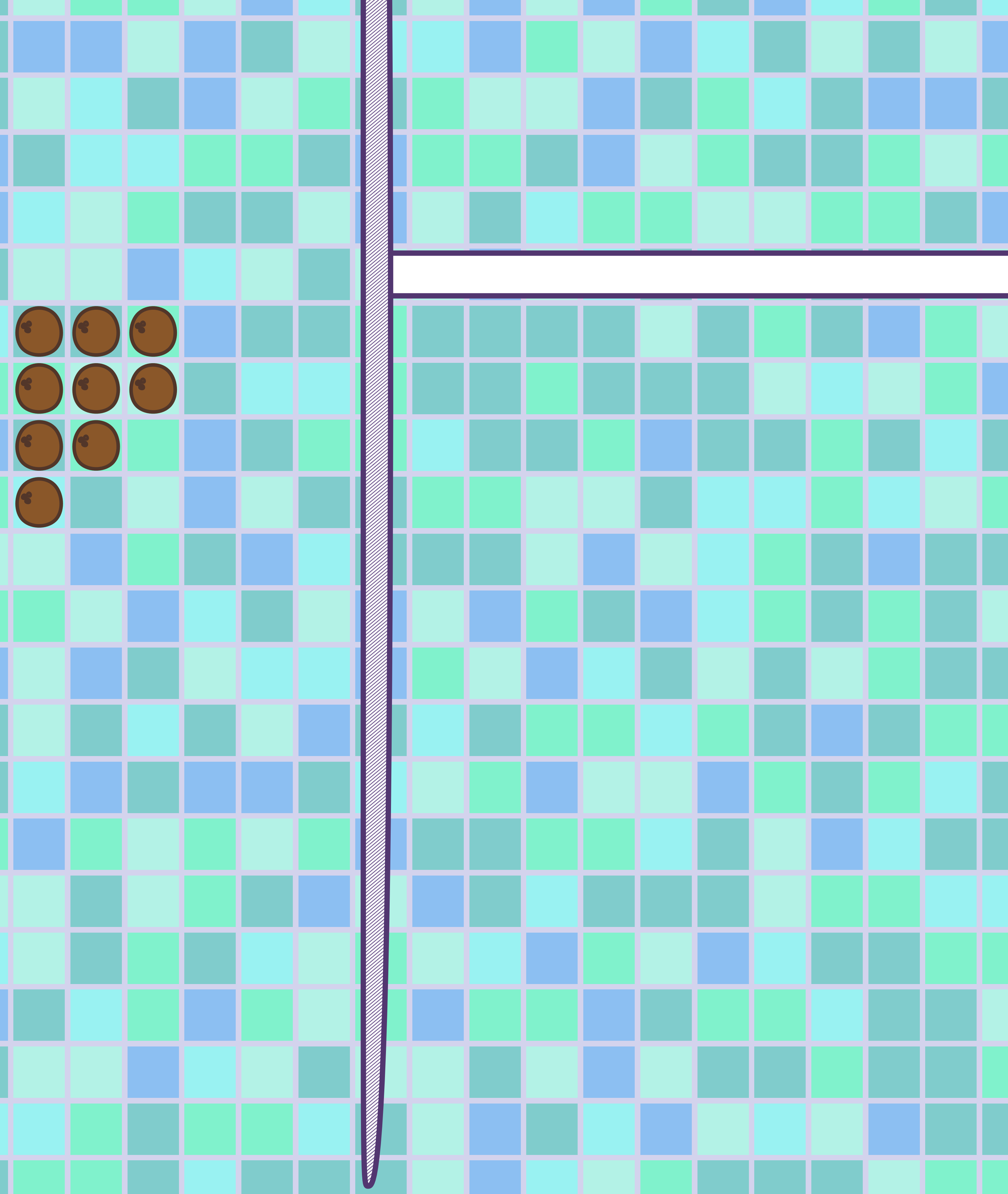

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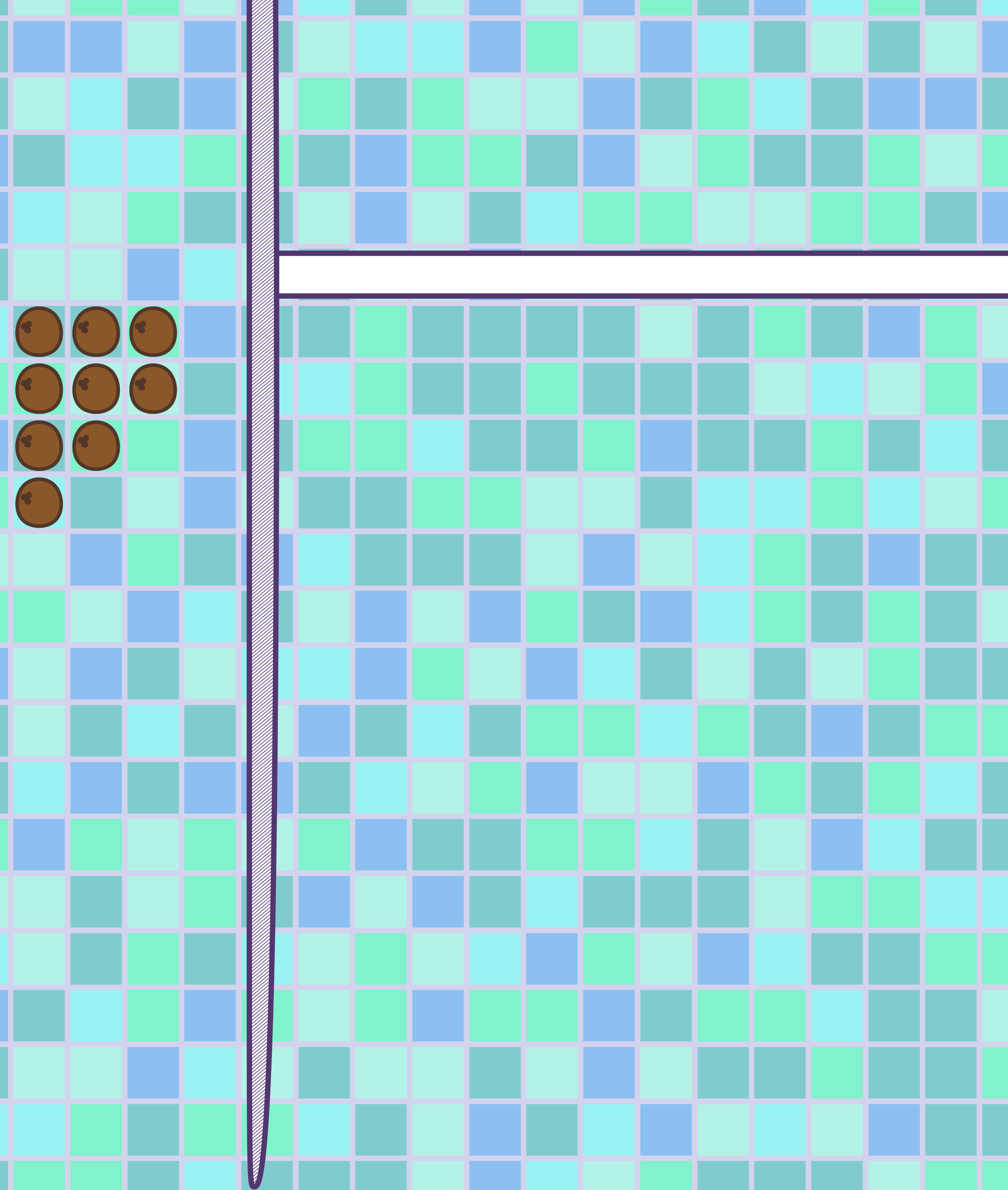

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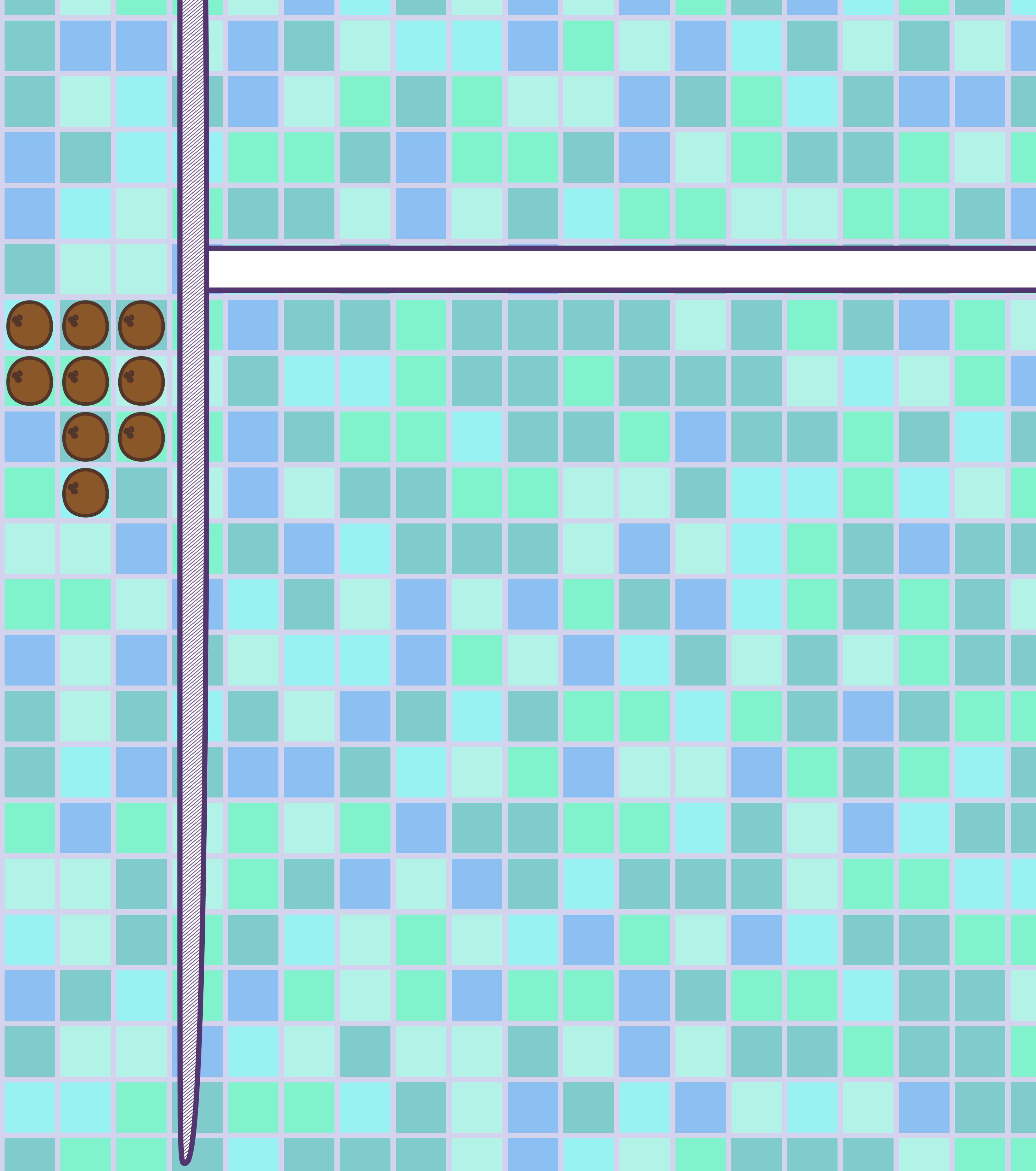

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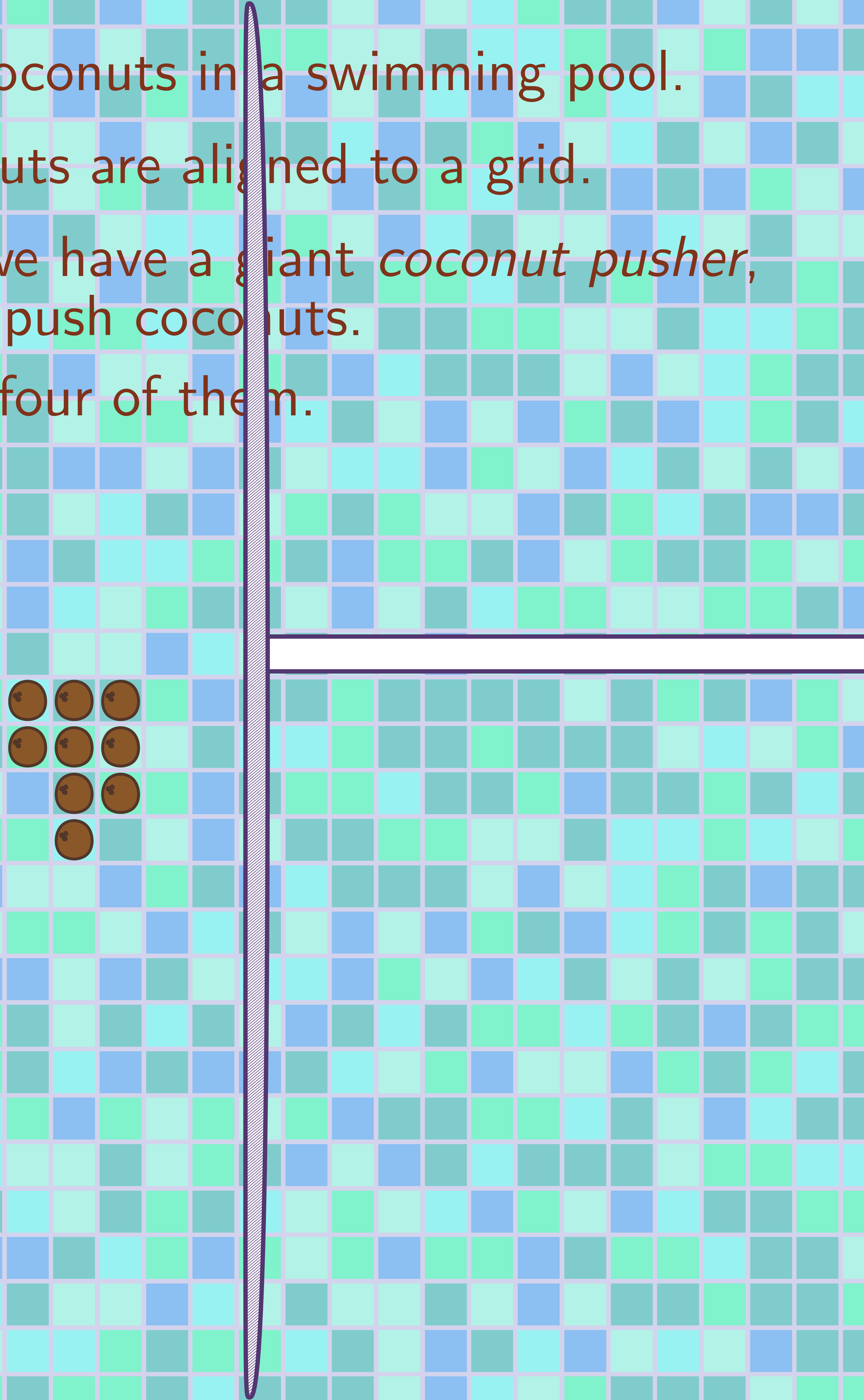

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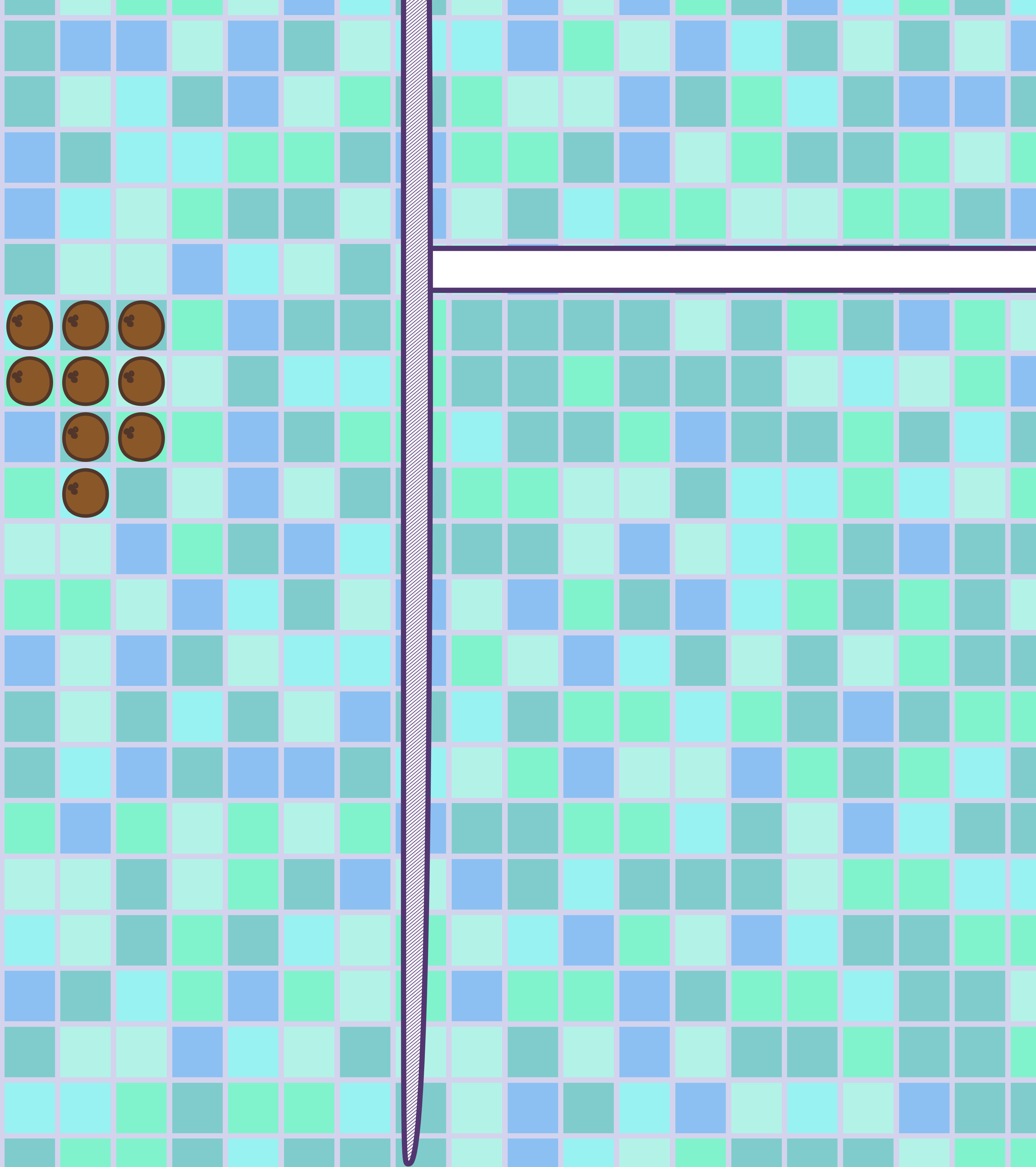

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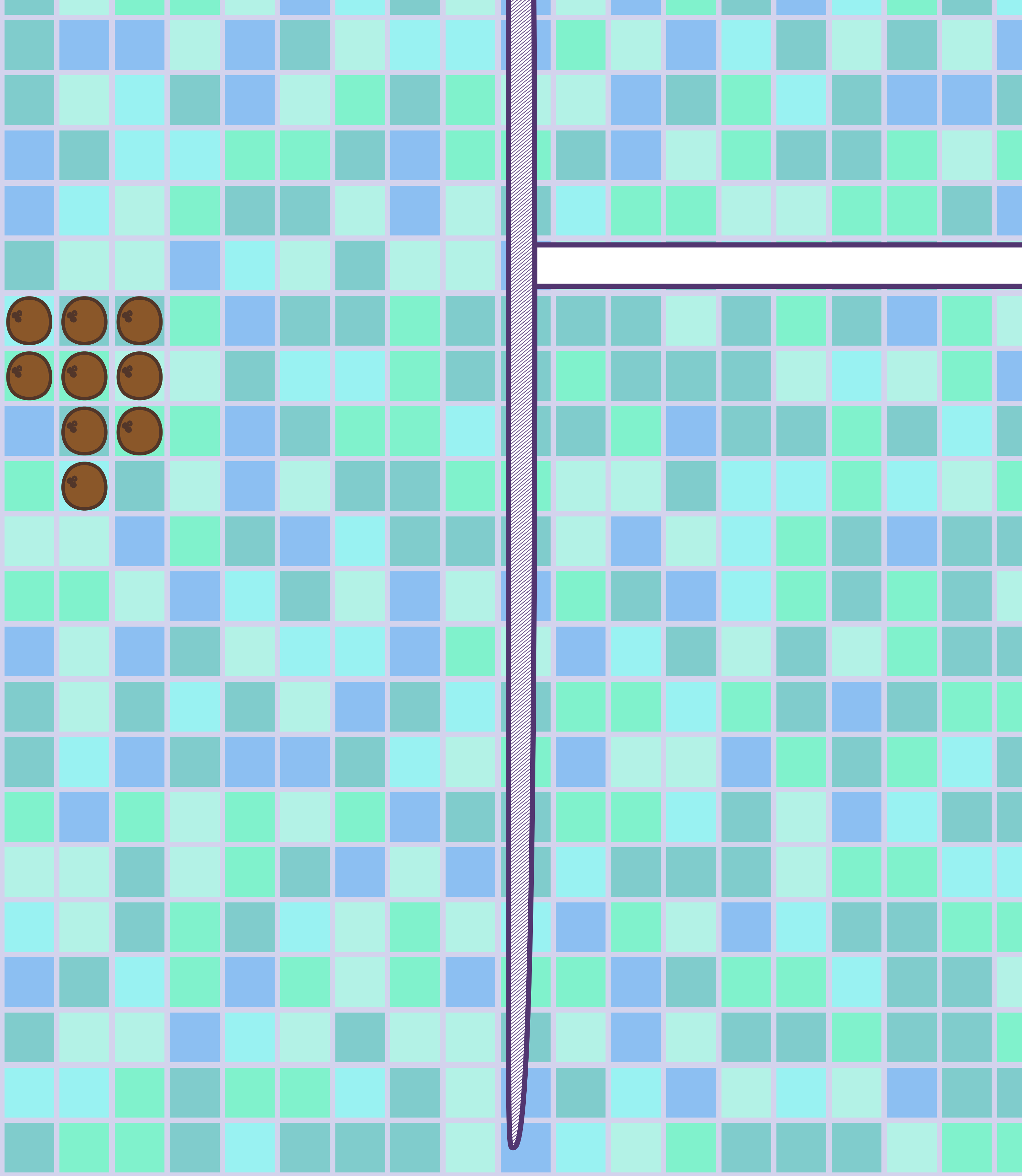

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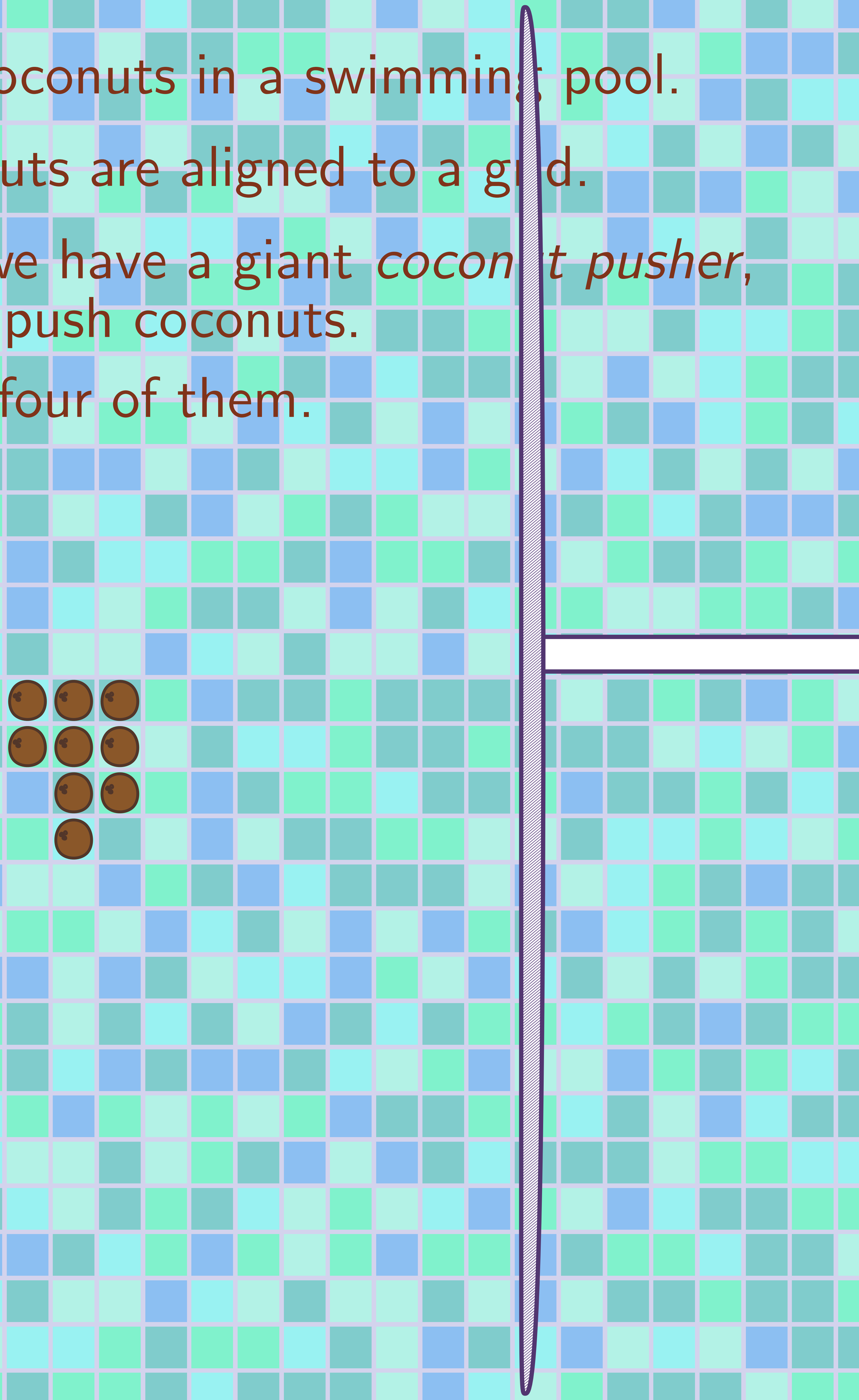

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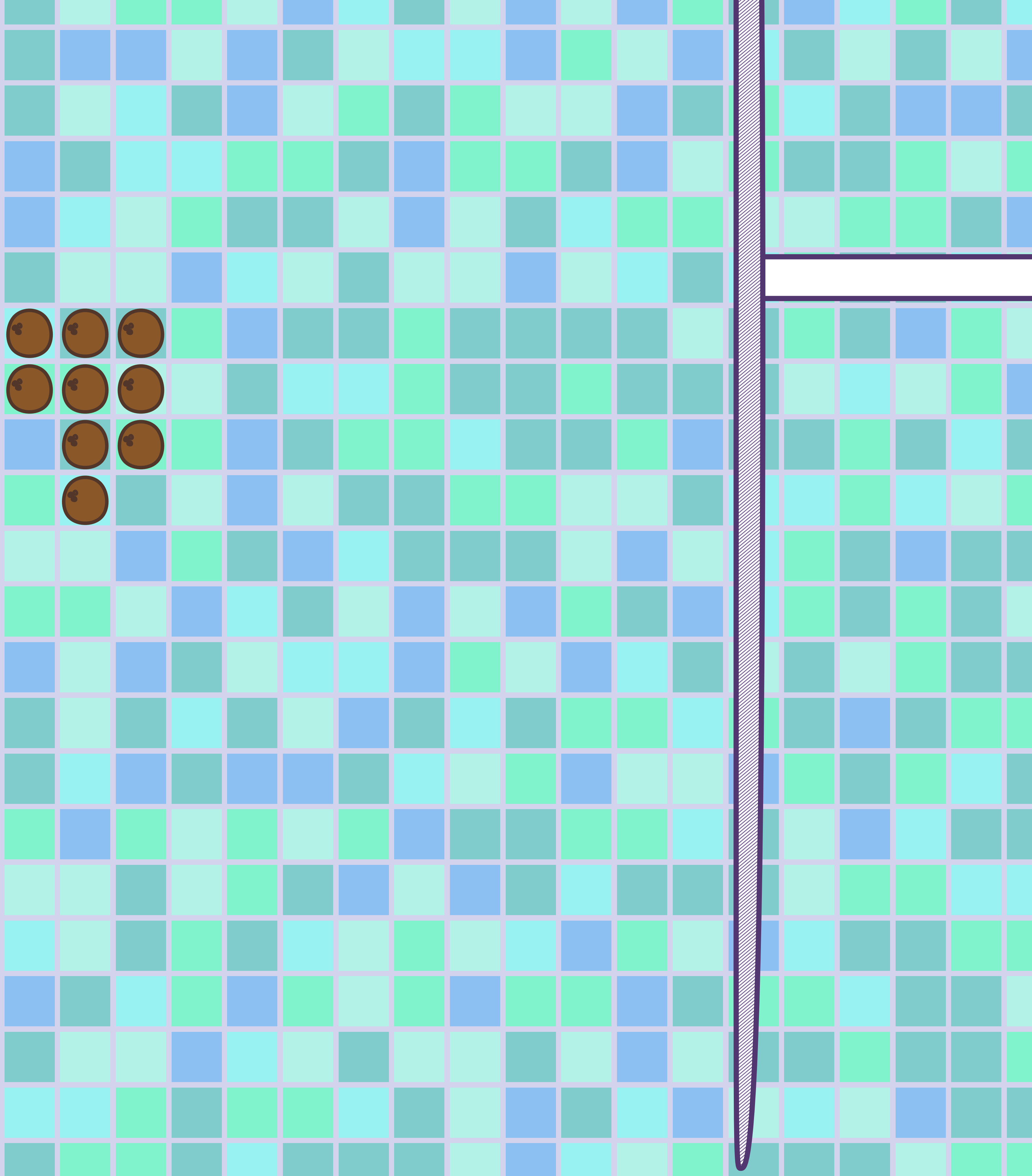

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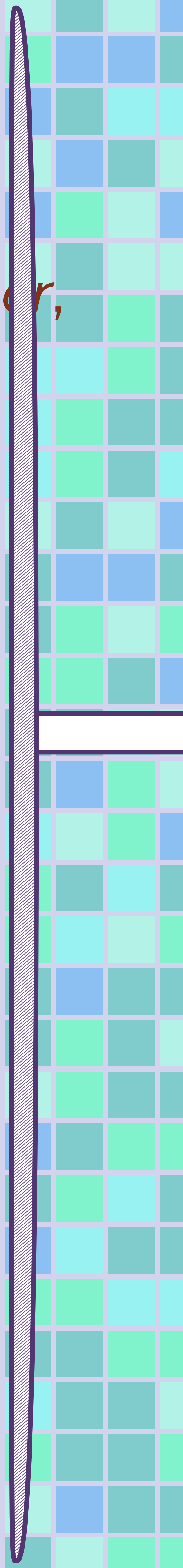

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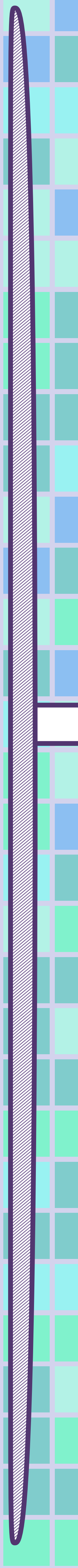

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We wish to study the behaviour of sequences of such coconut pushes.

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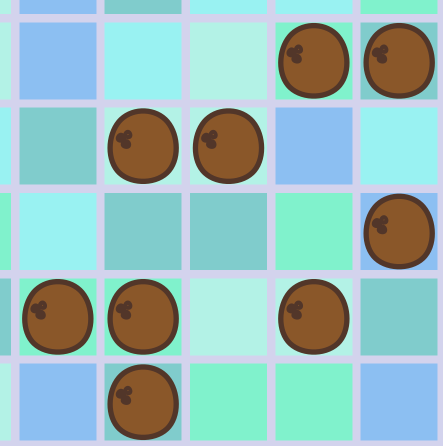

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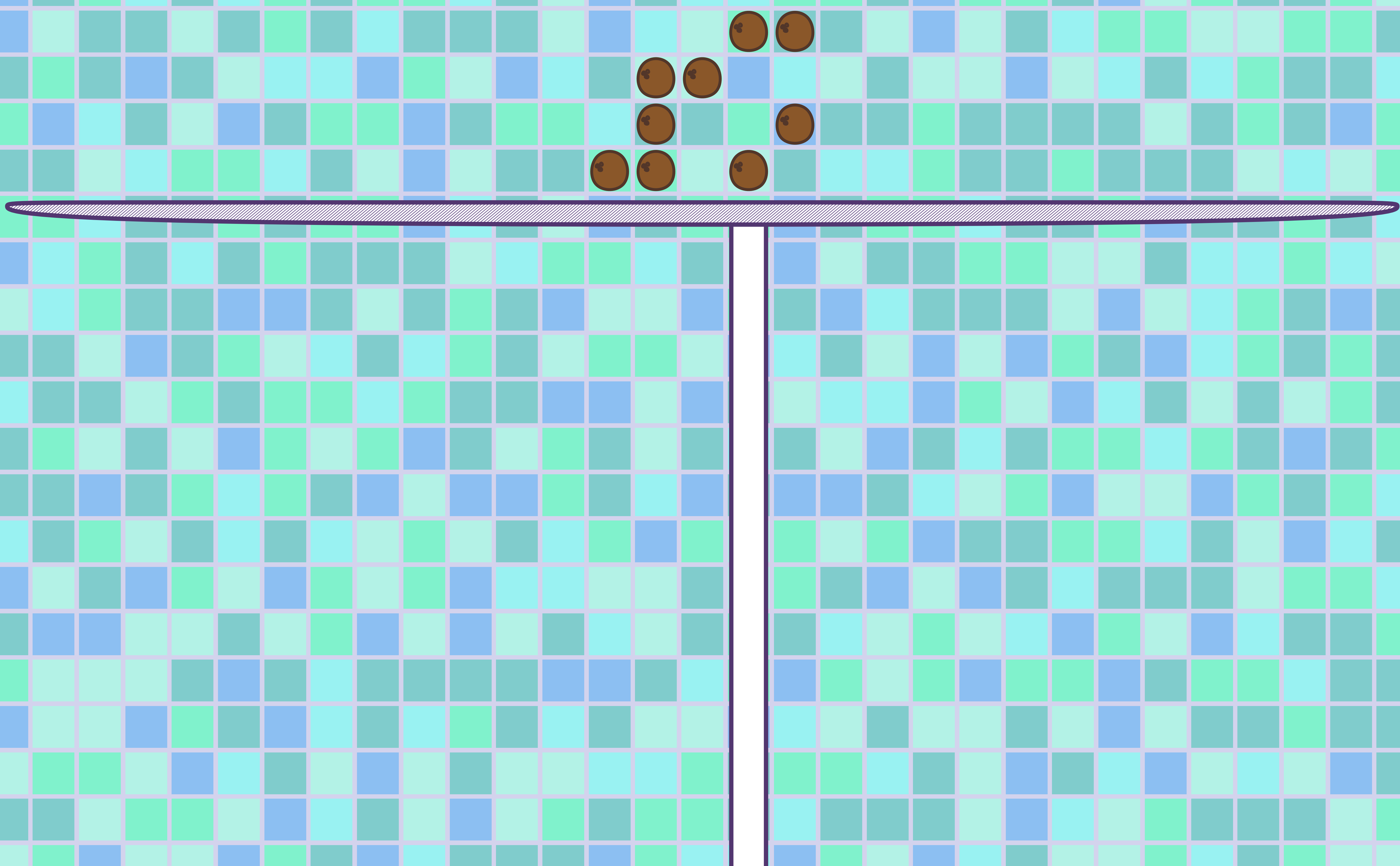

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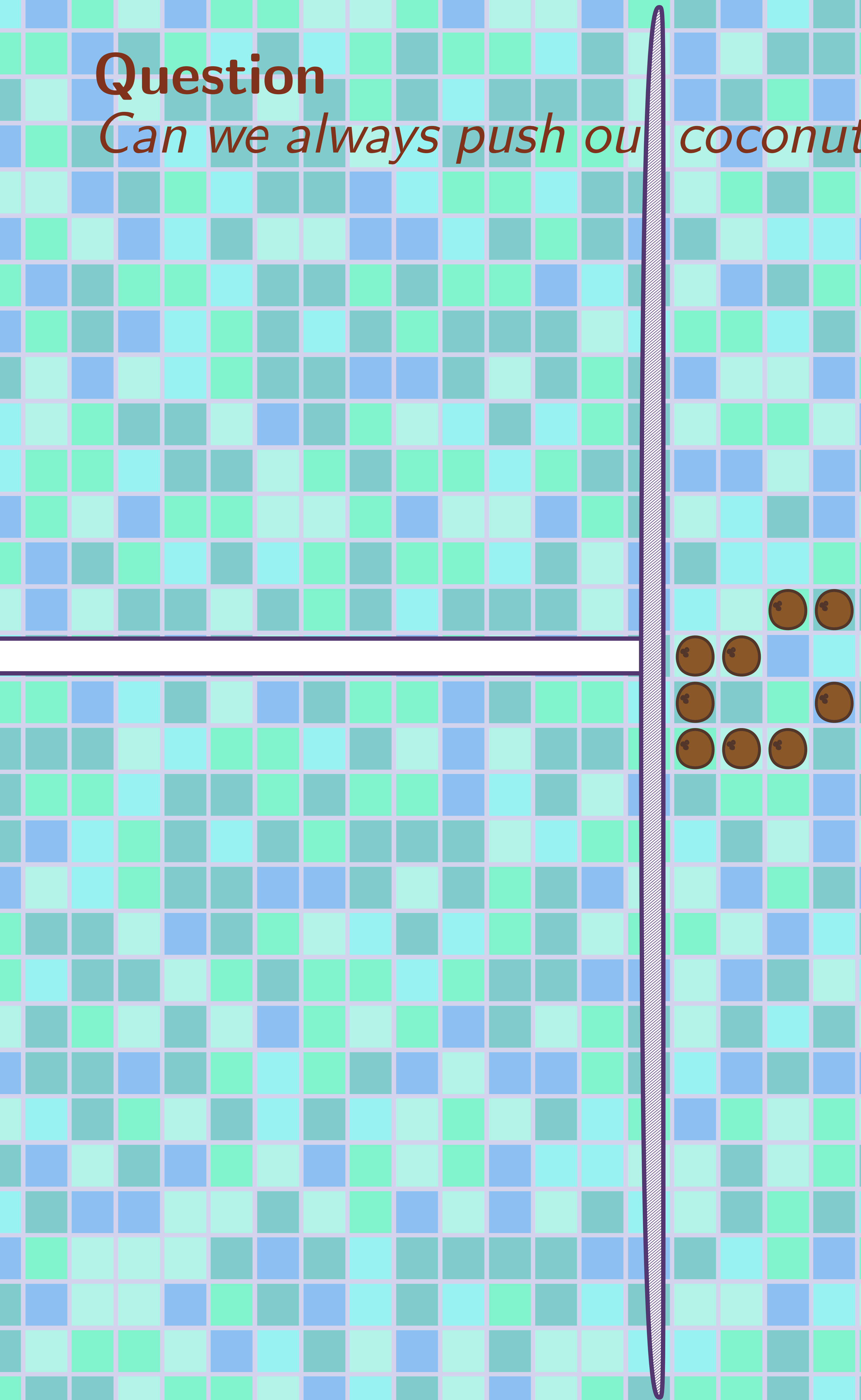

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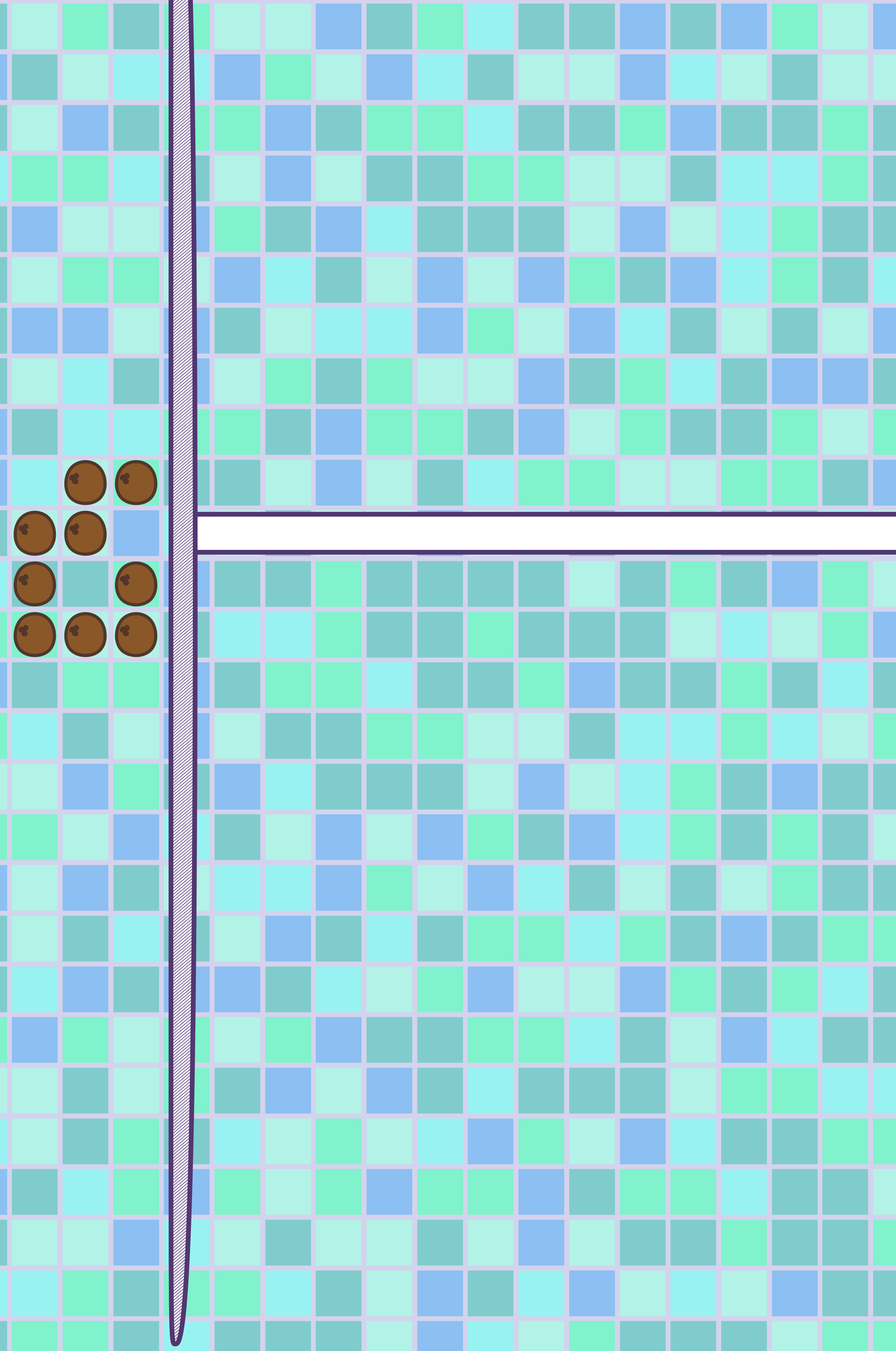

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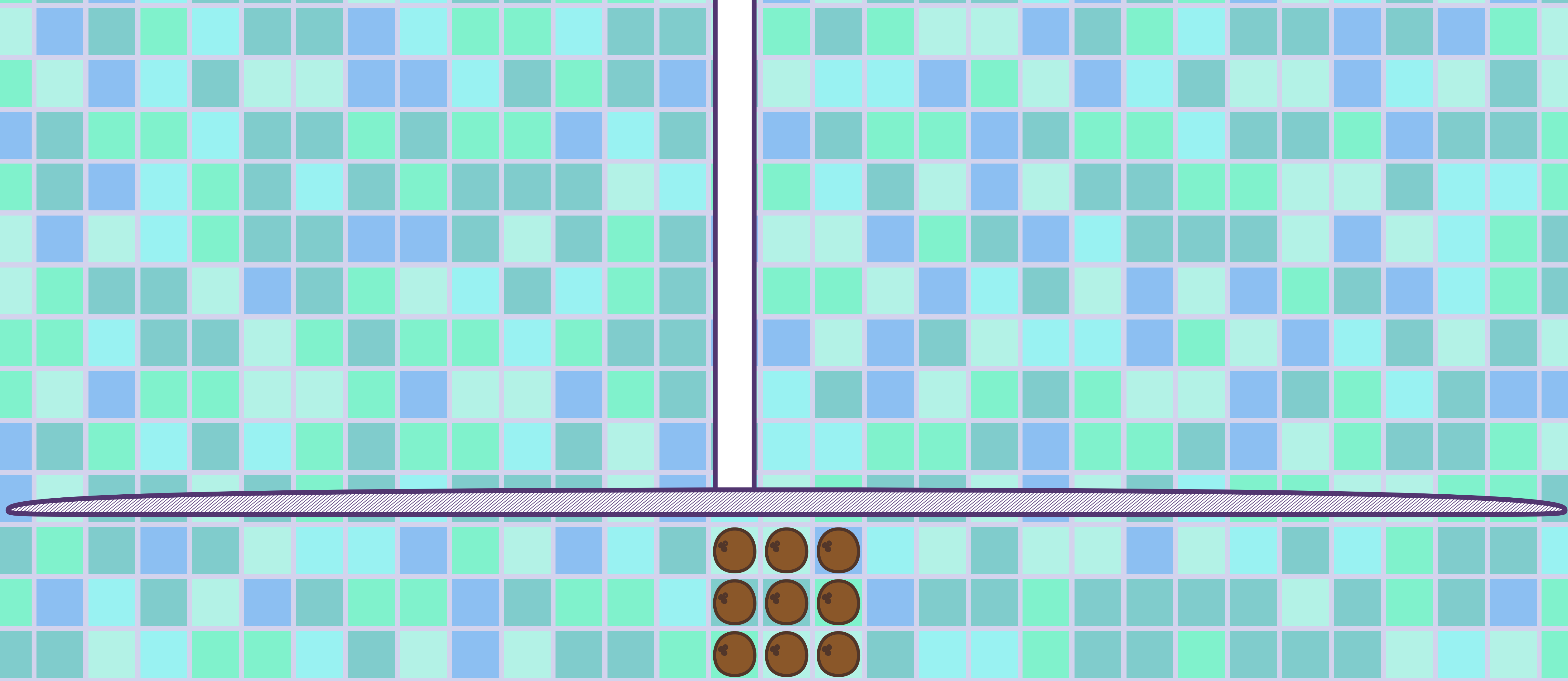

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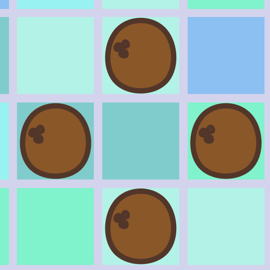

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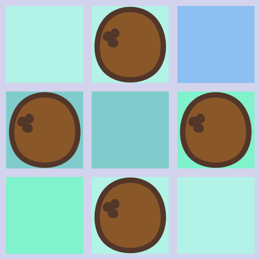

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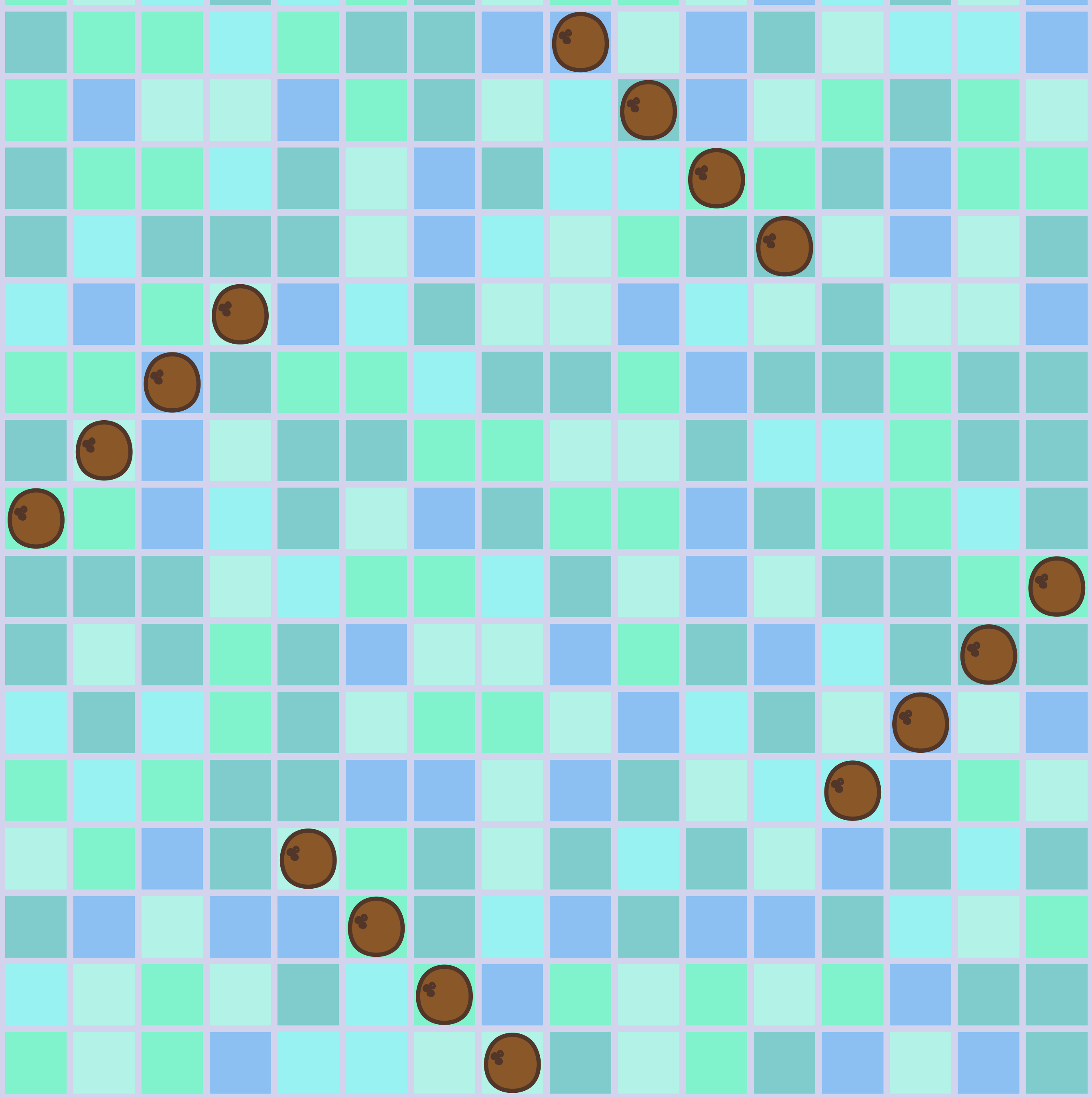

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But is it always possible?

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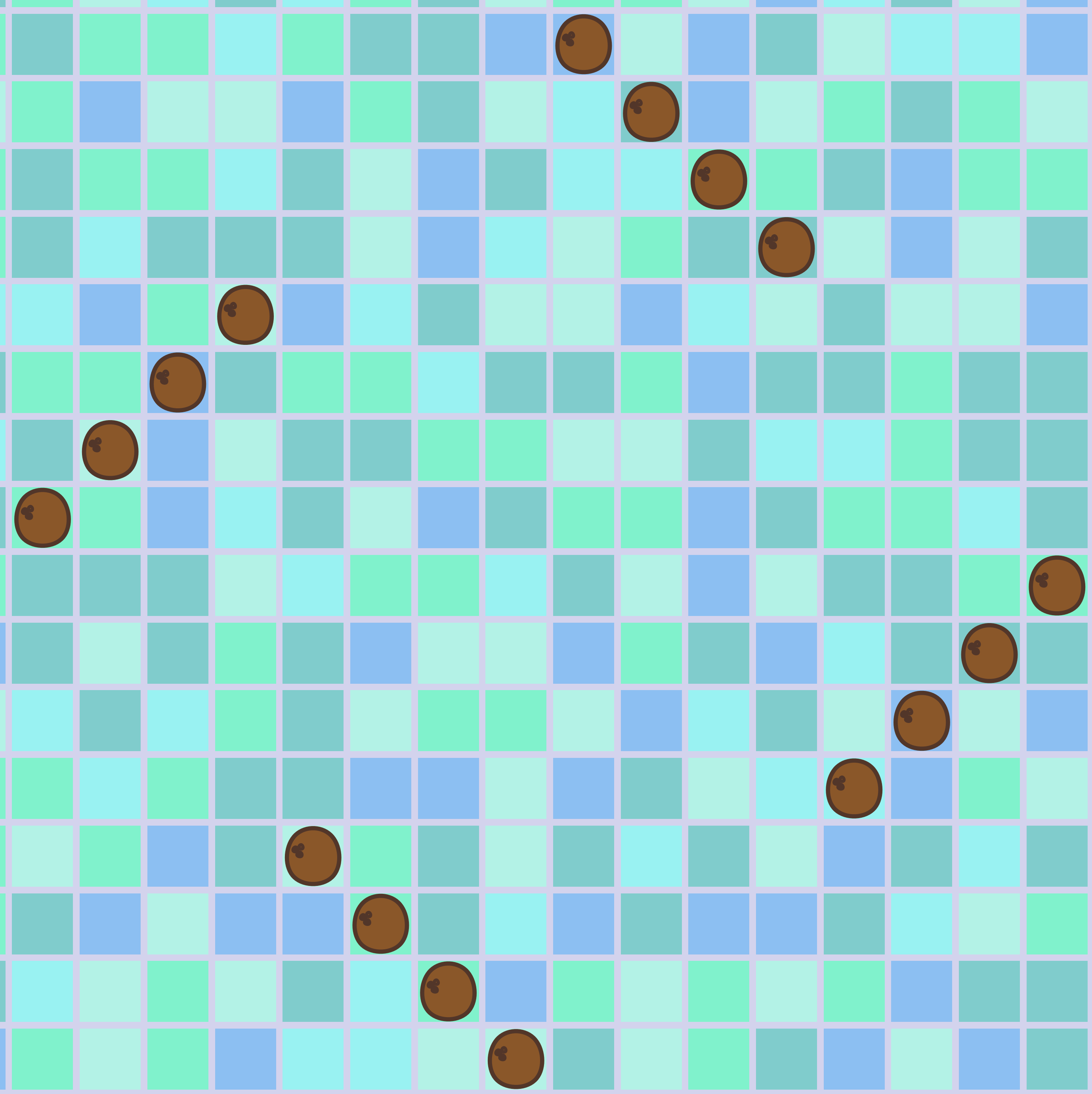

...and *some* aspect ratios are not even possible if we start with at most one coconut per row and column.

Results

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Everything inbetween is still open!

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Coconuts have no identity.

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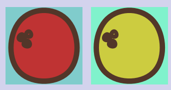

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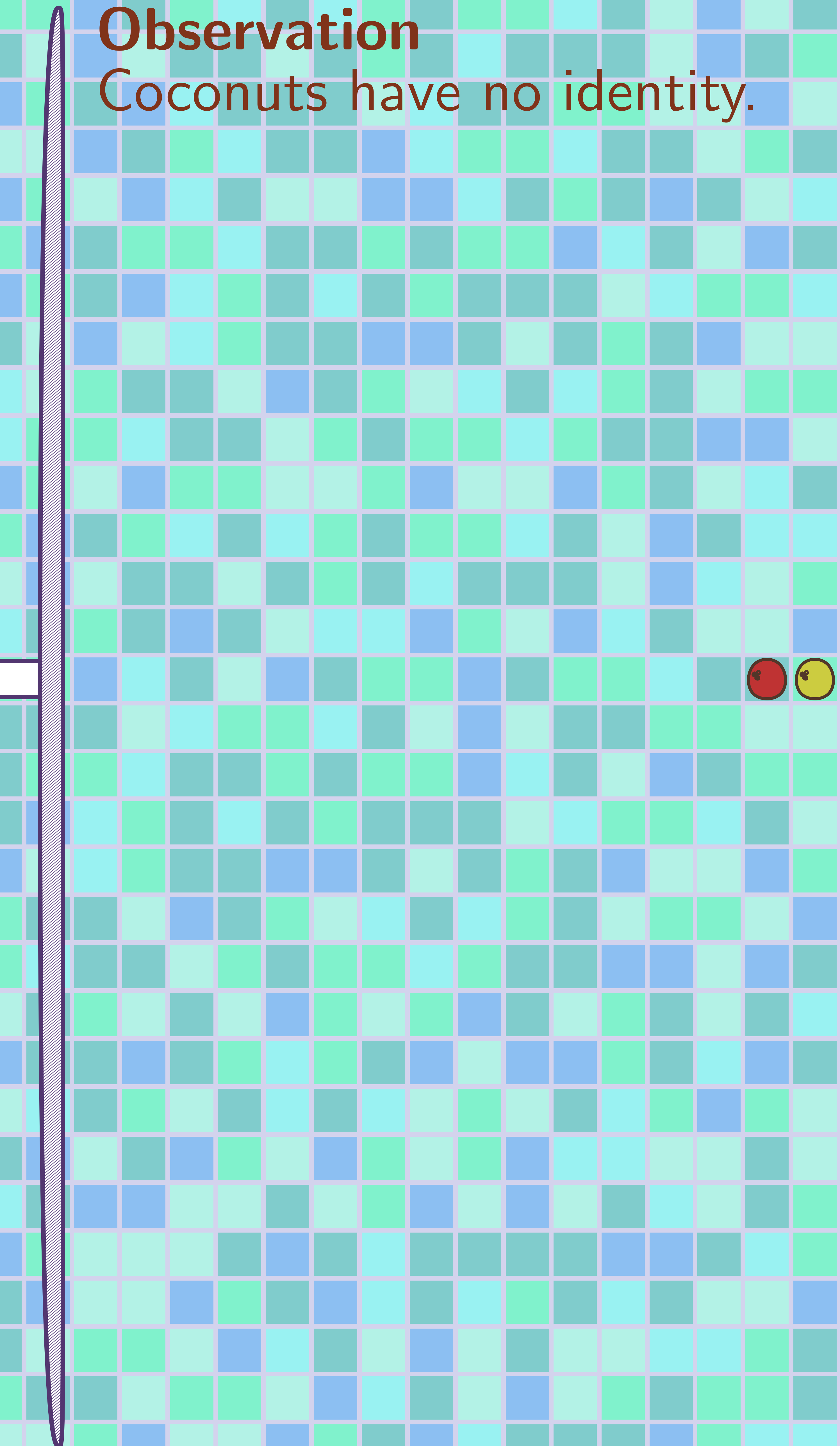

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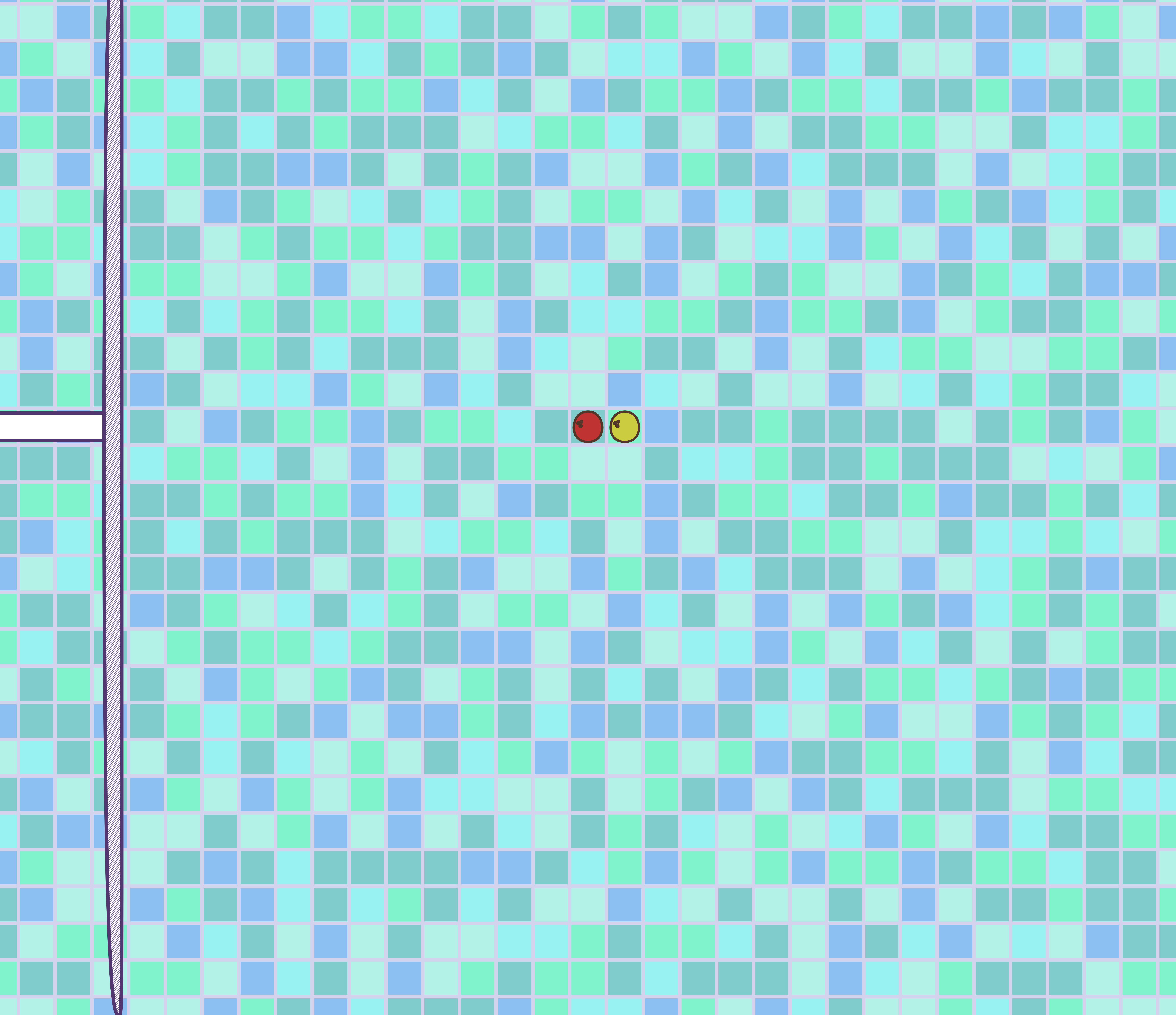

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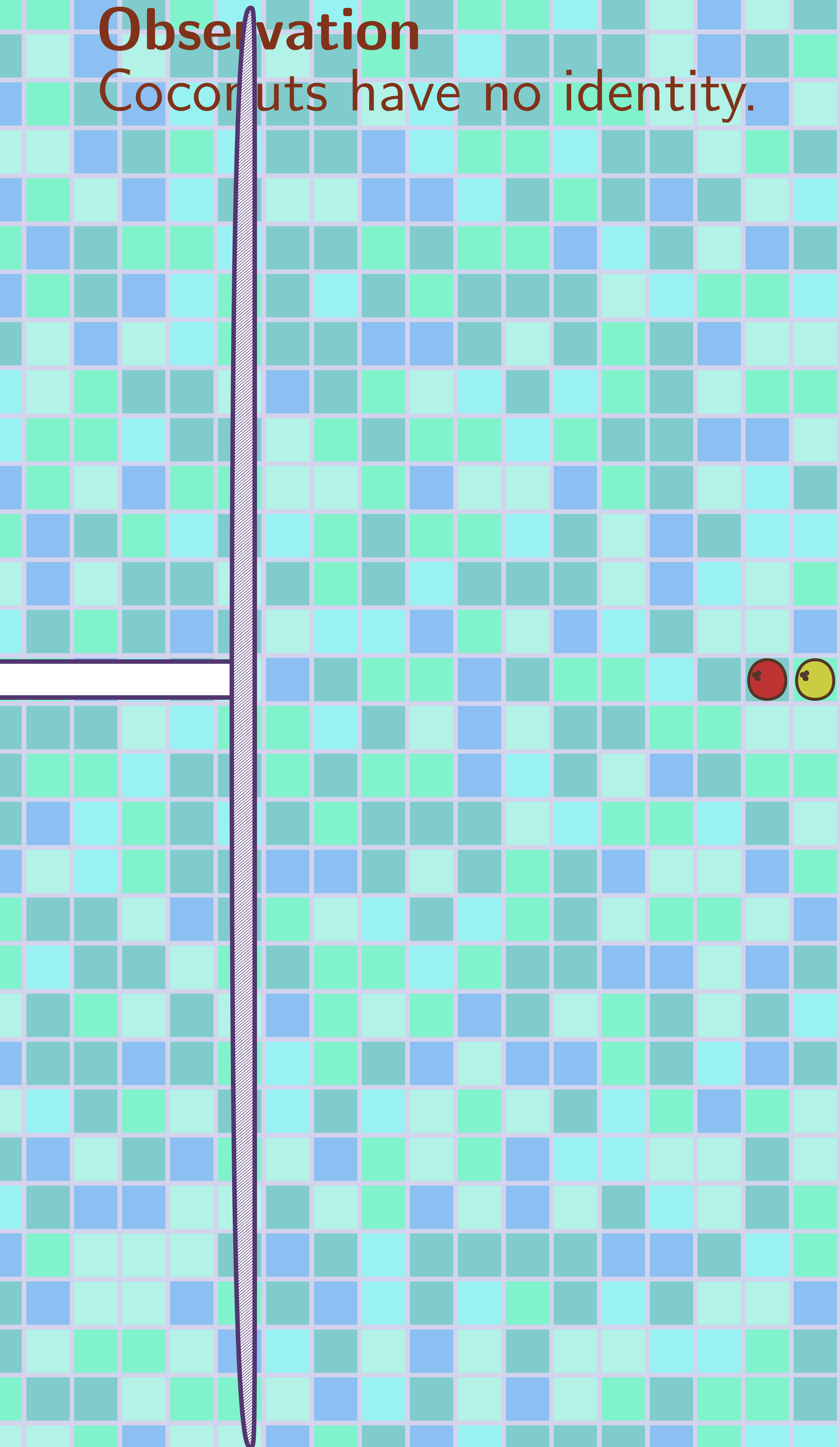

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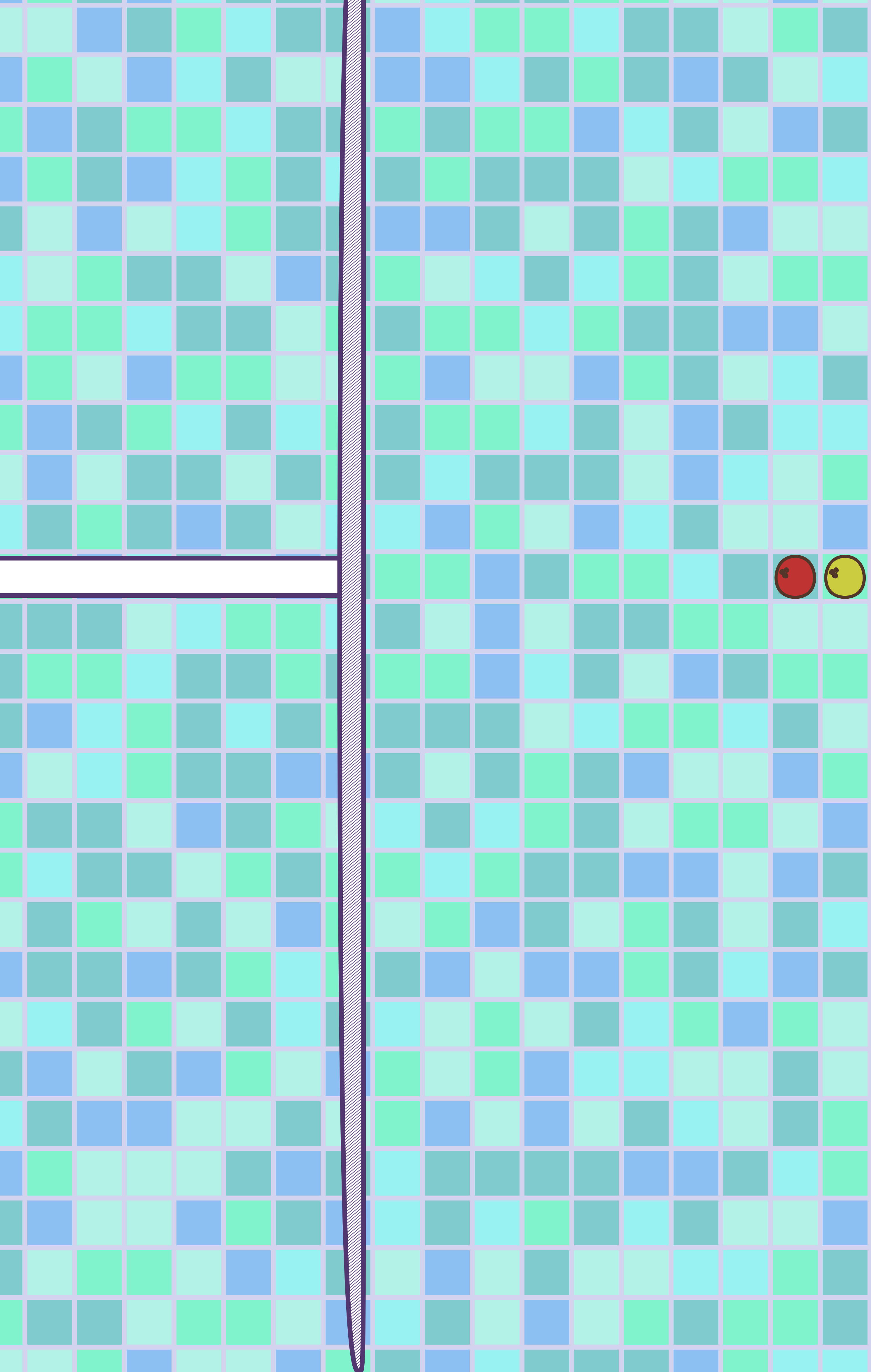

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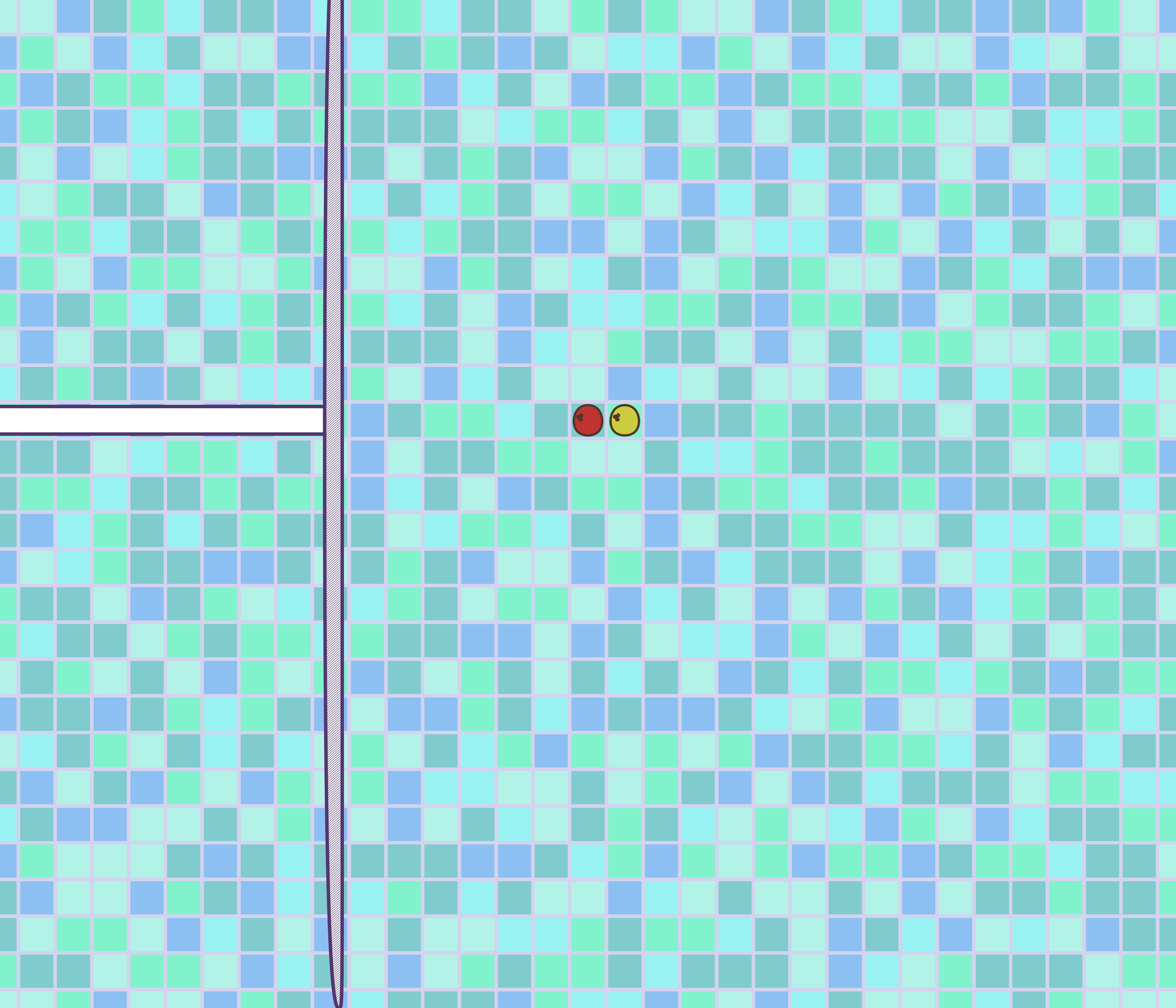

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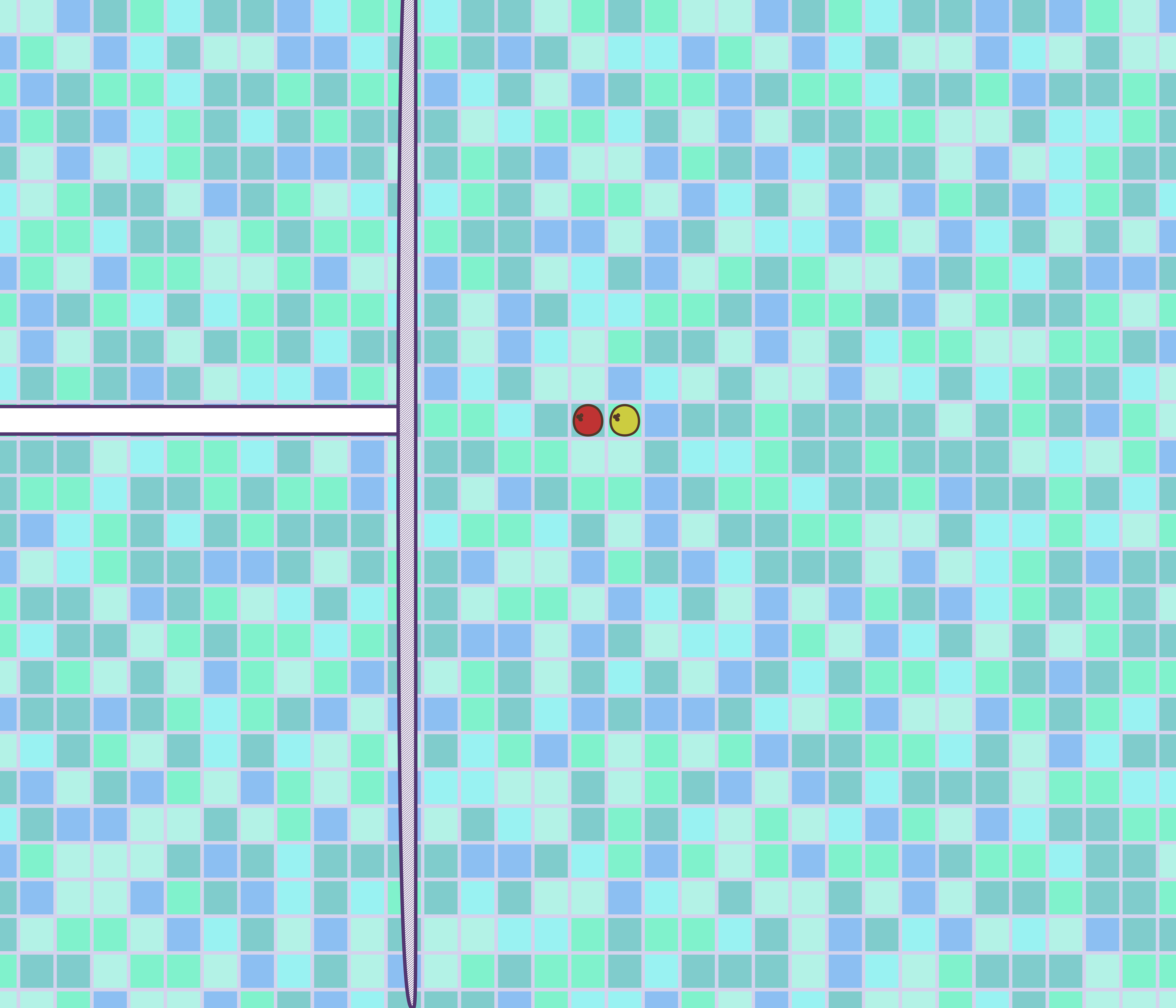

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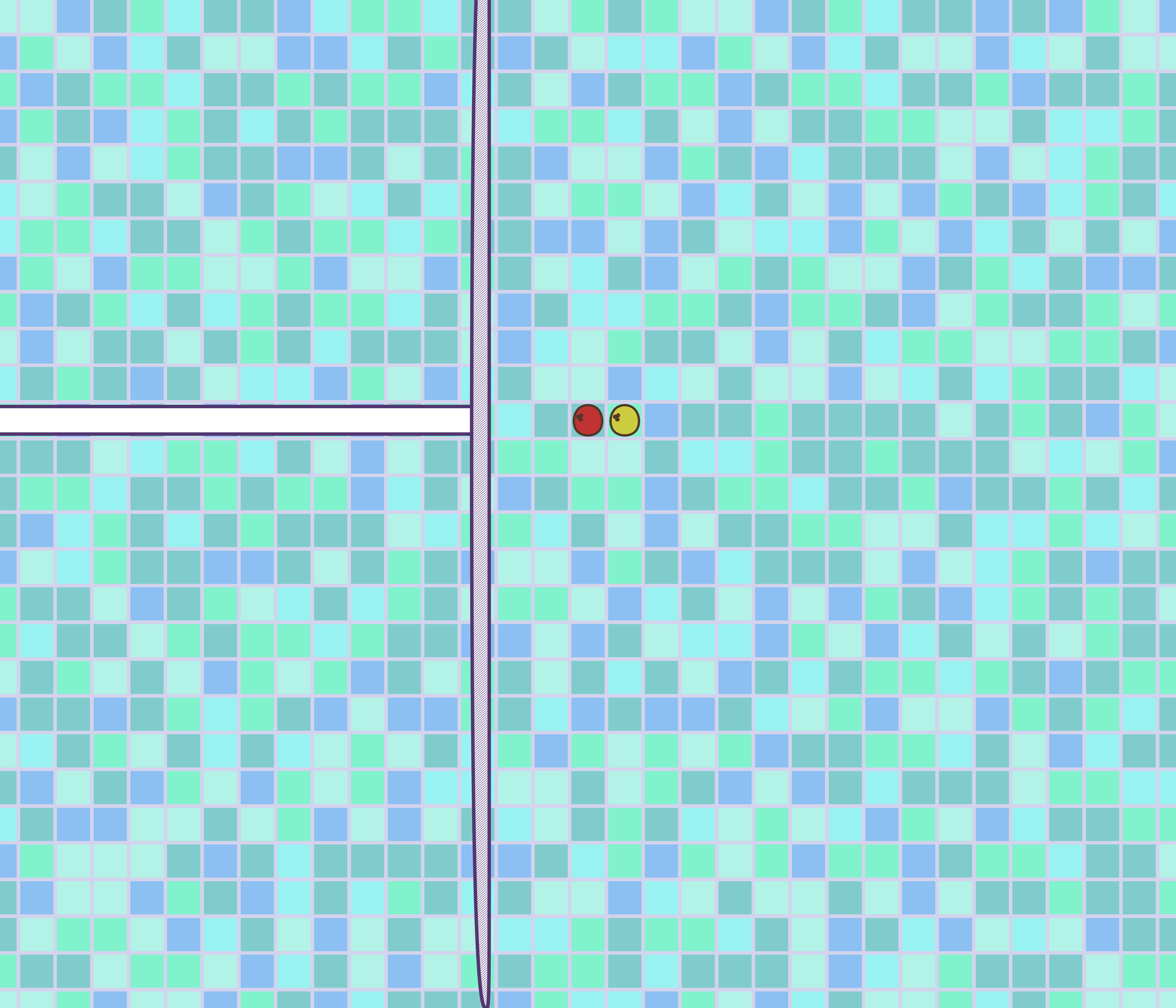

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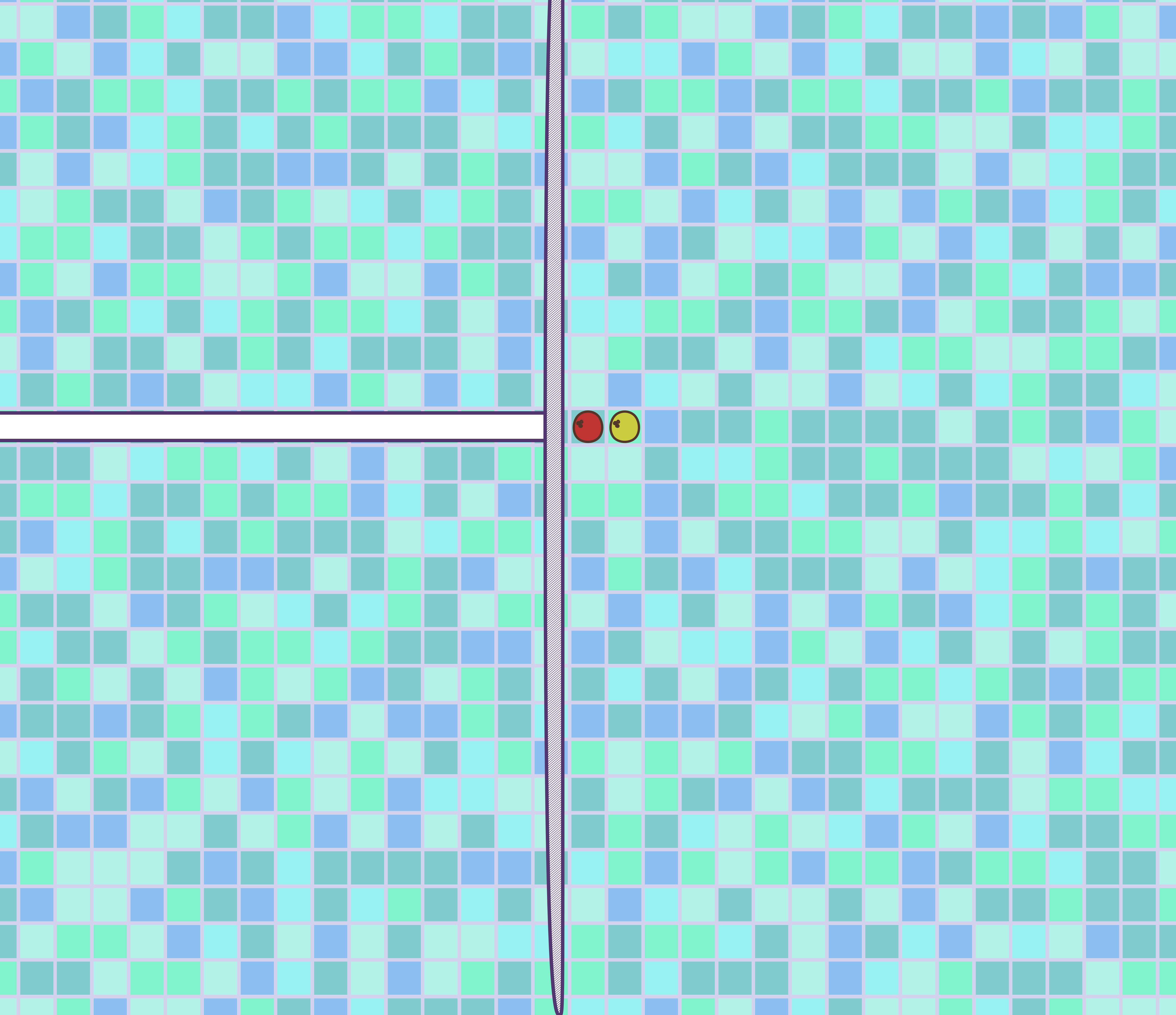

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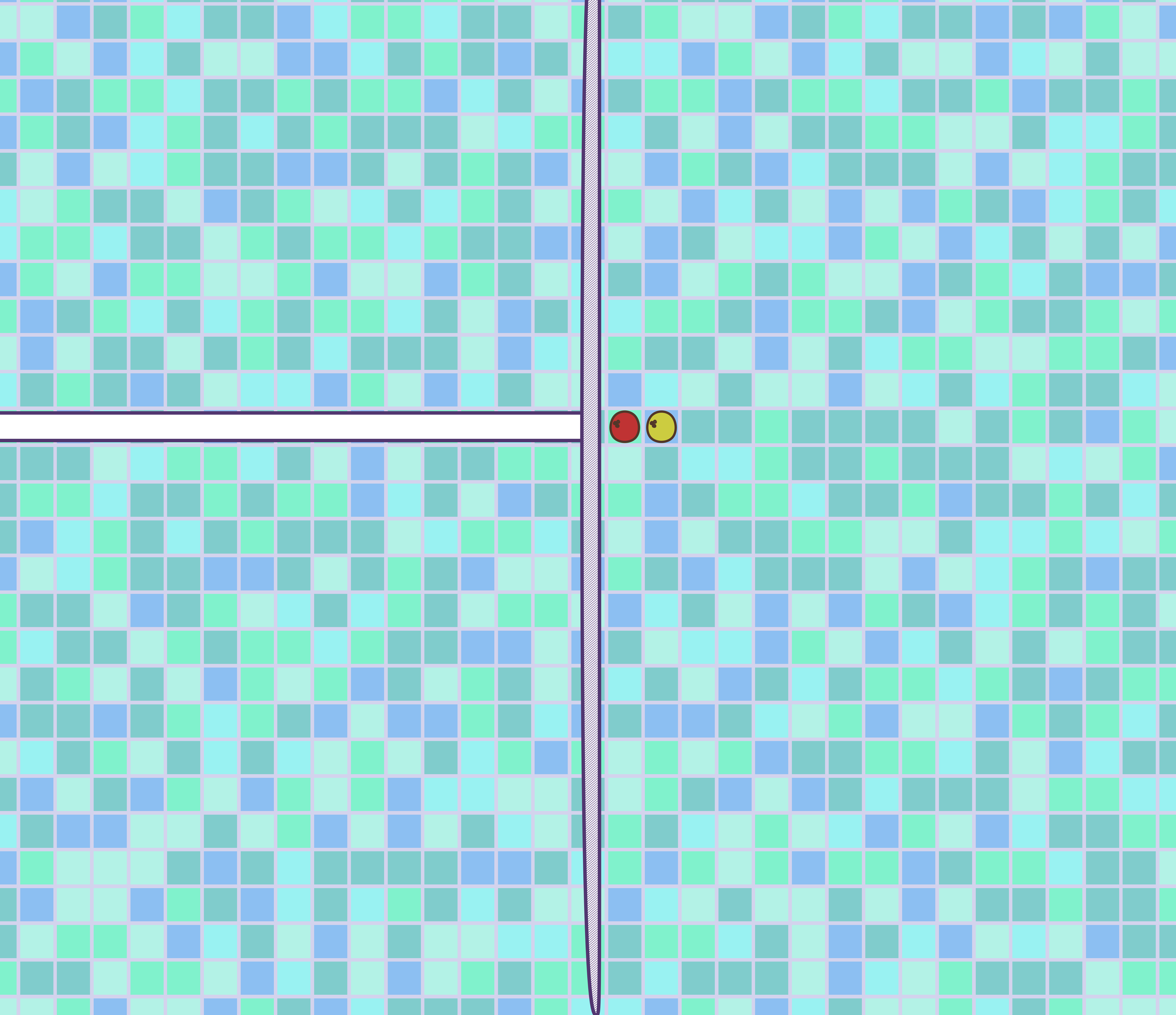

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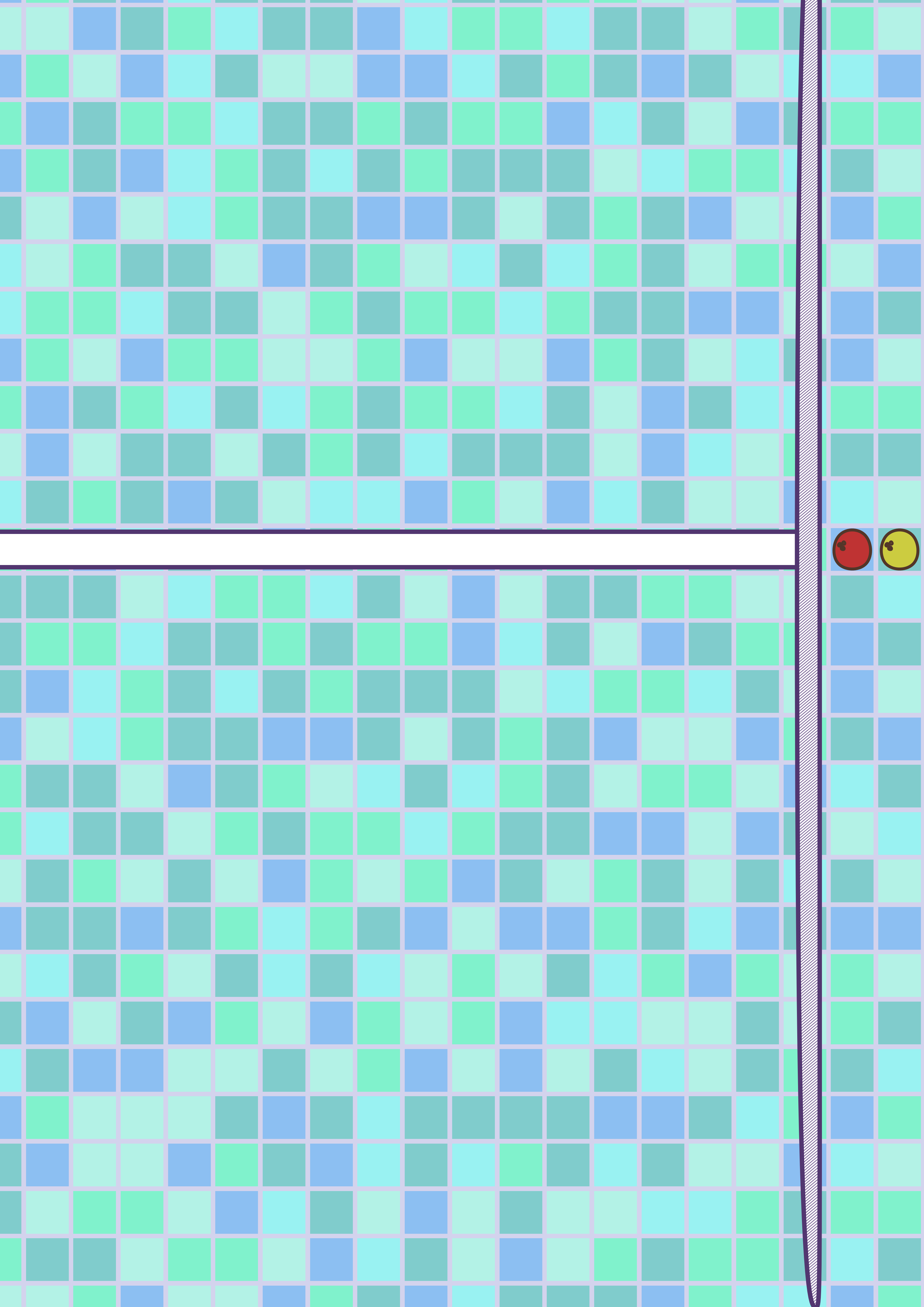

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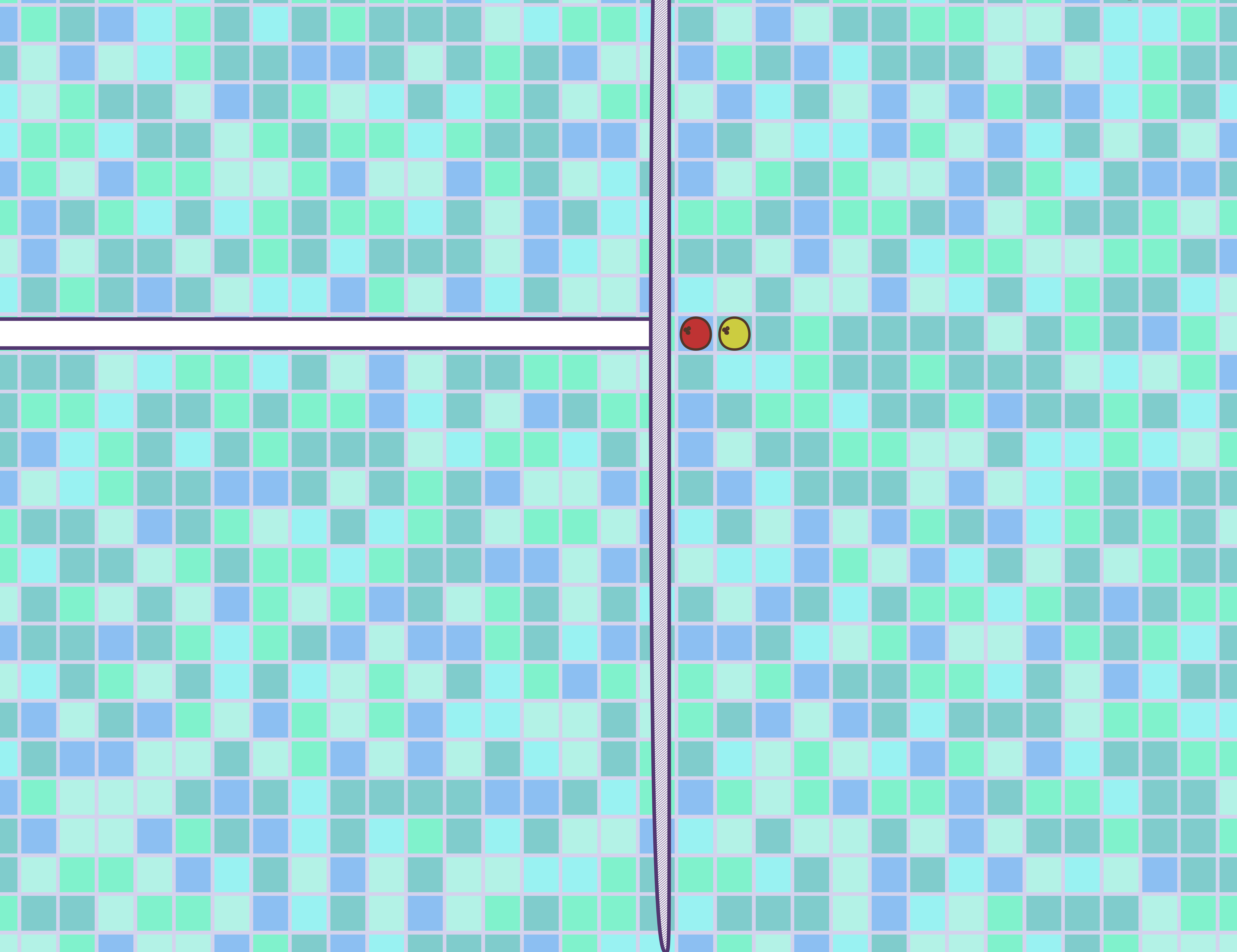

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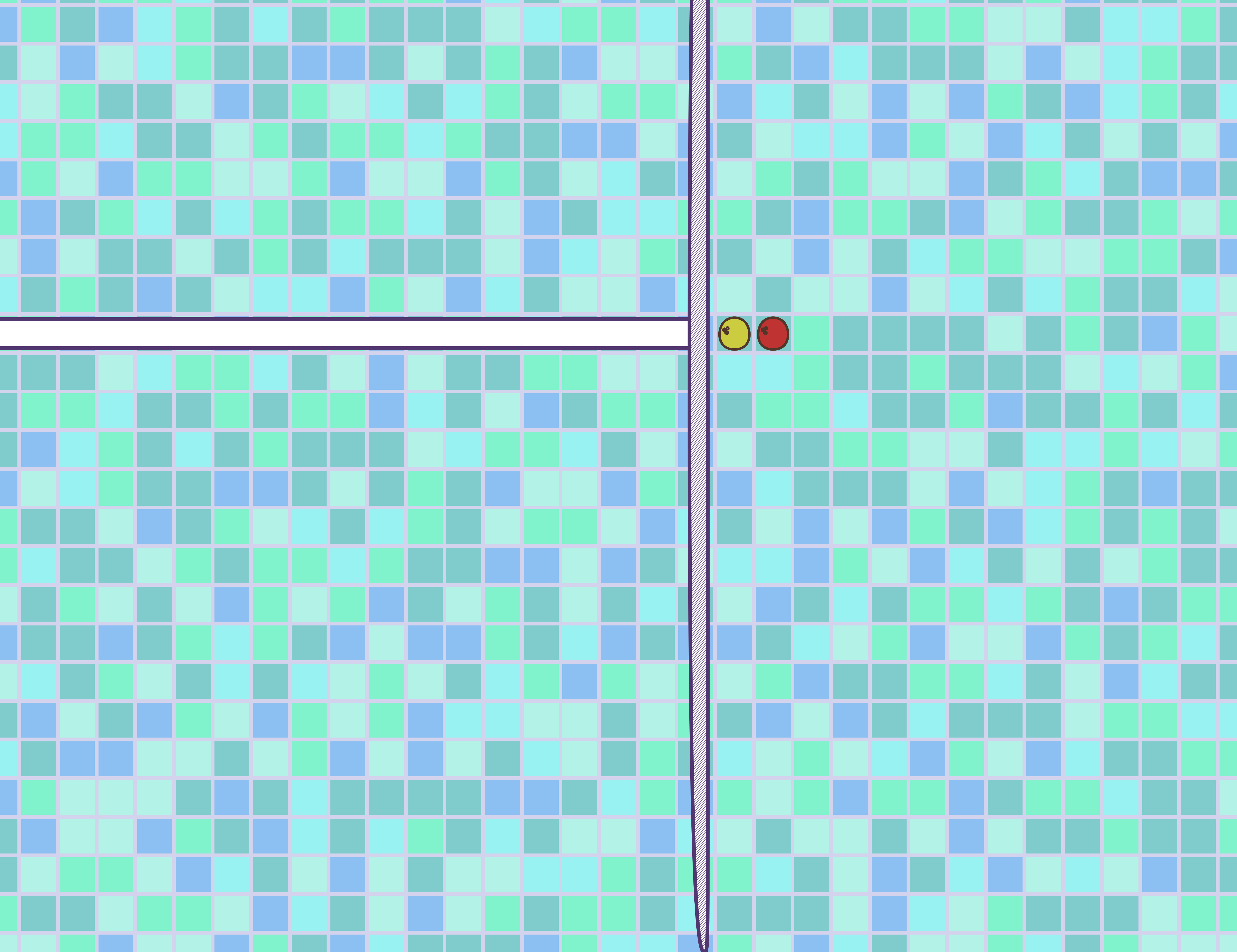

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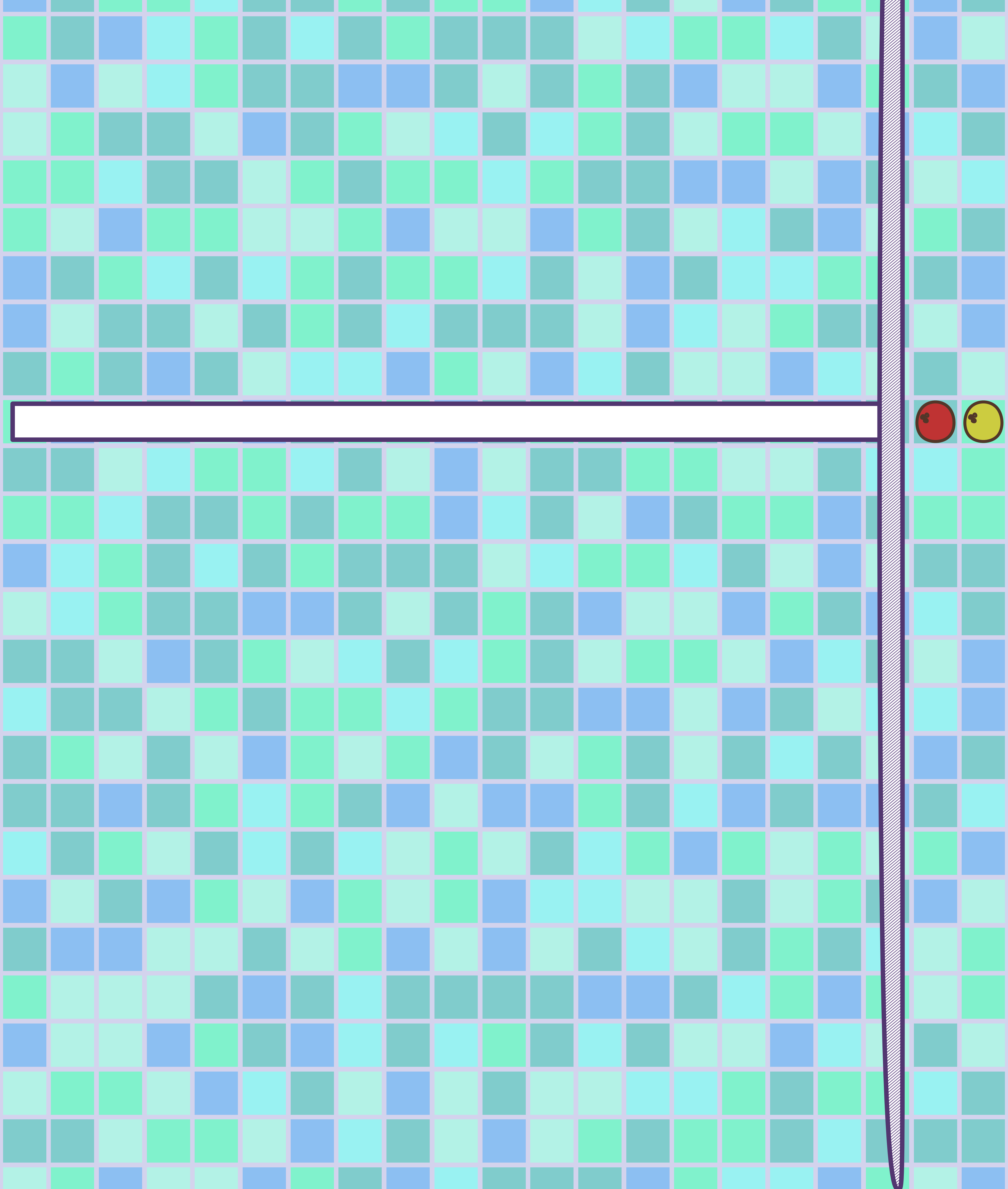

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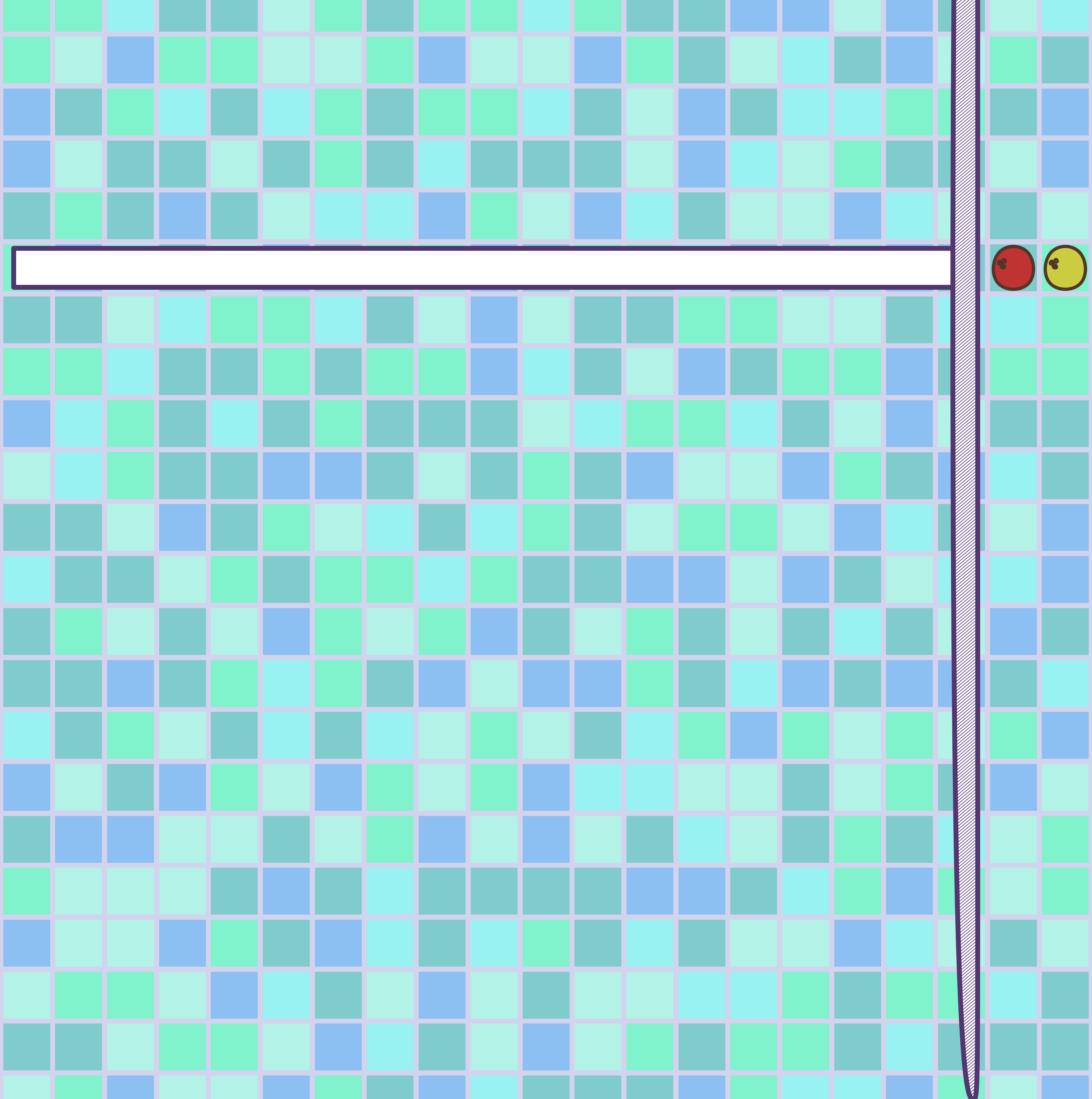

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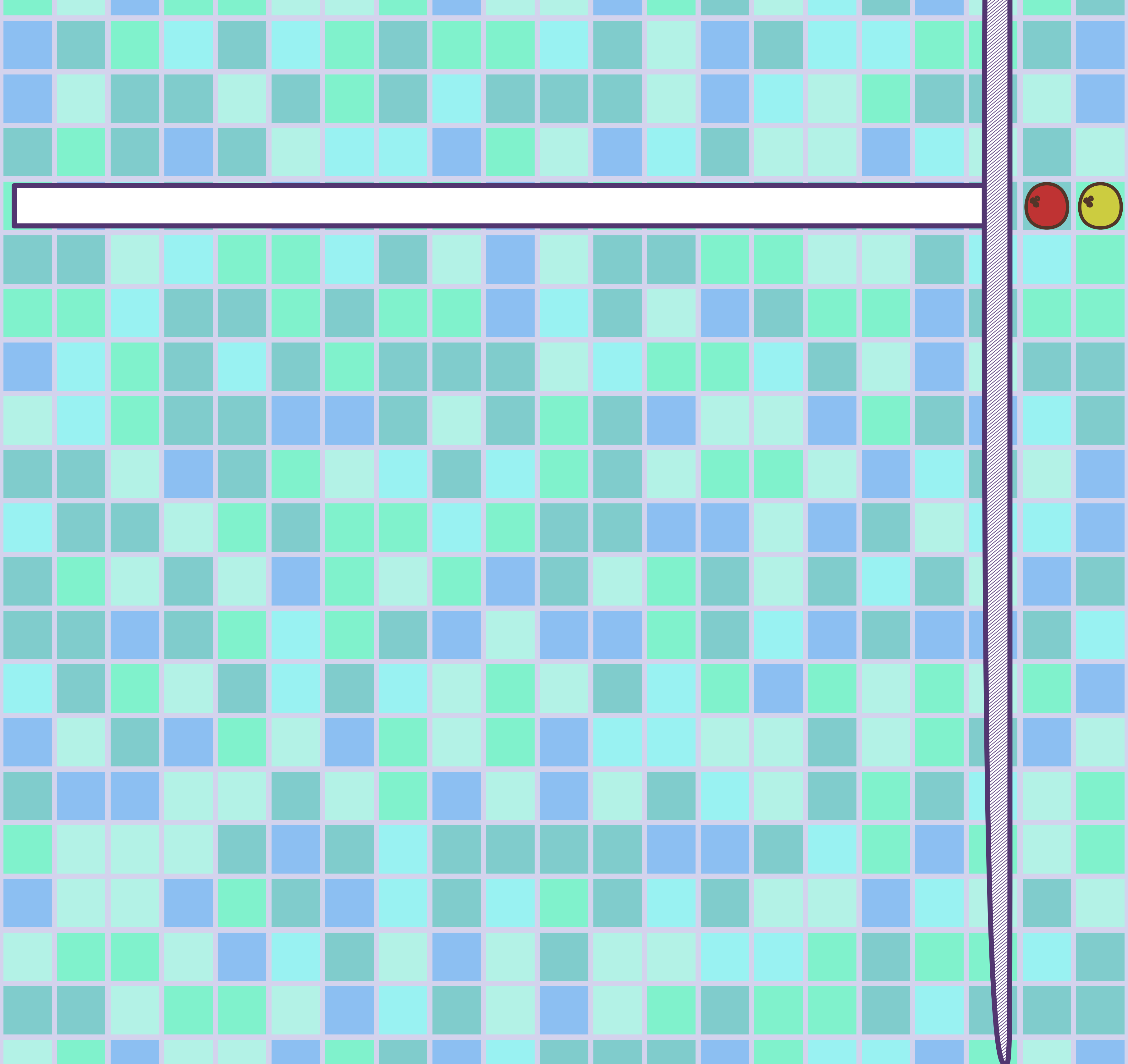

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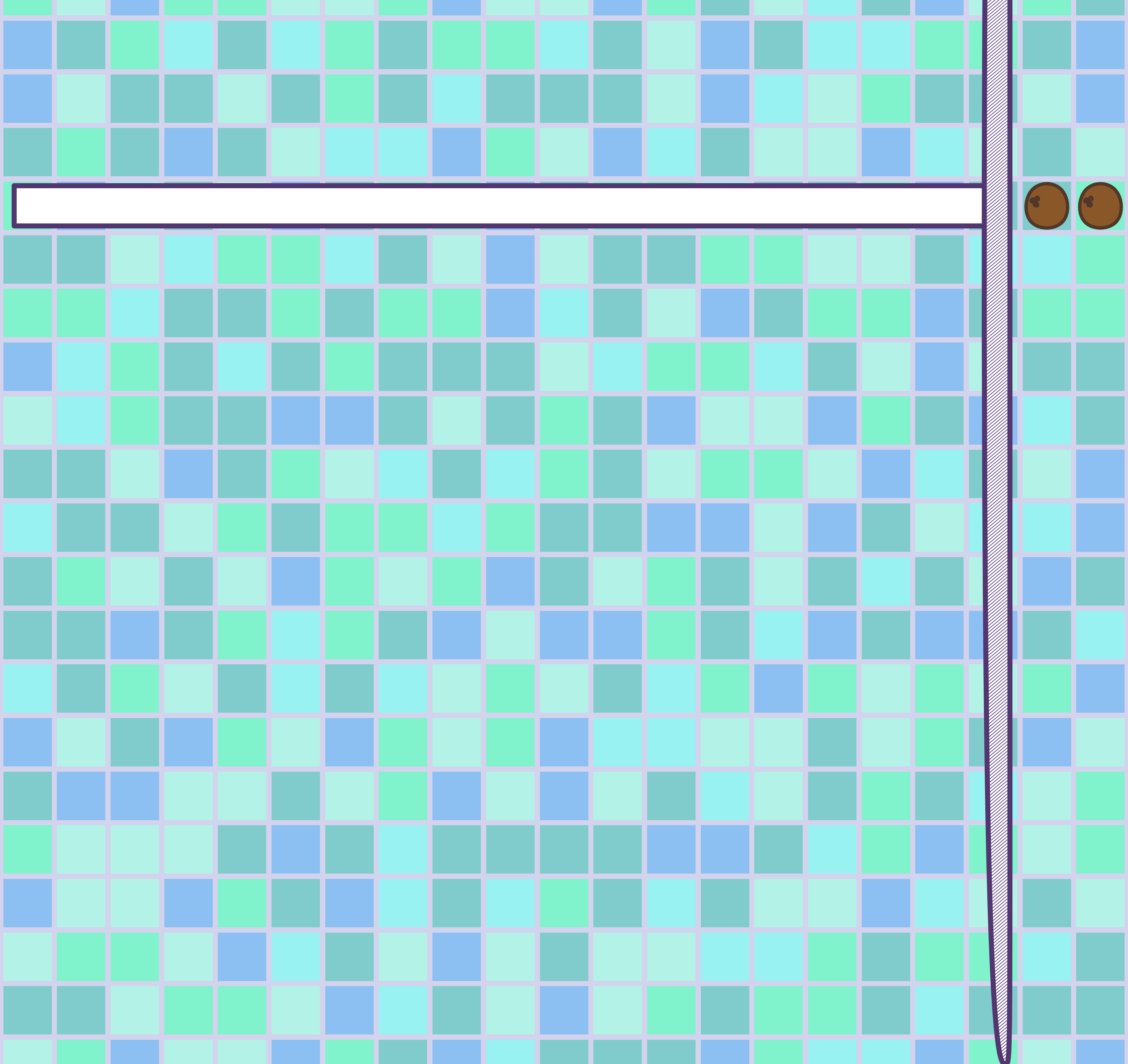

HARD COCONUTS

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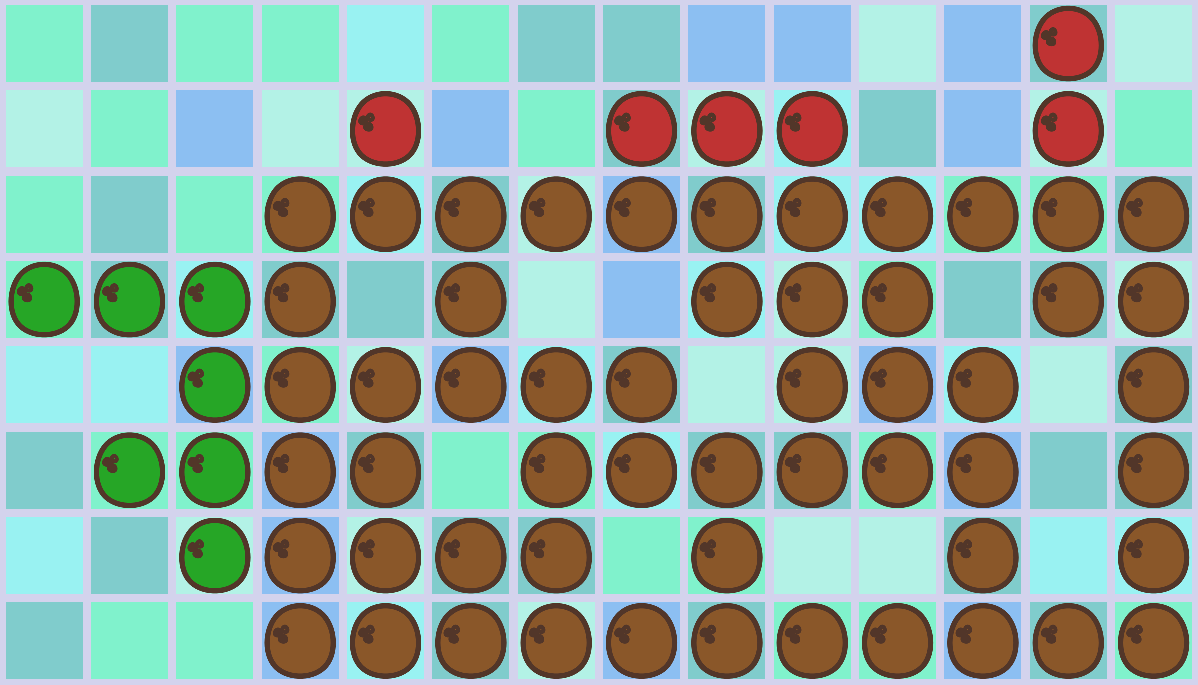

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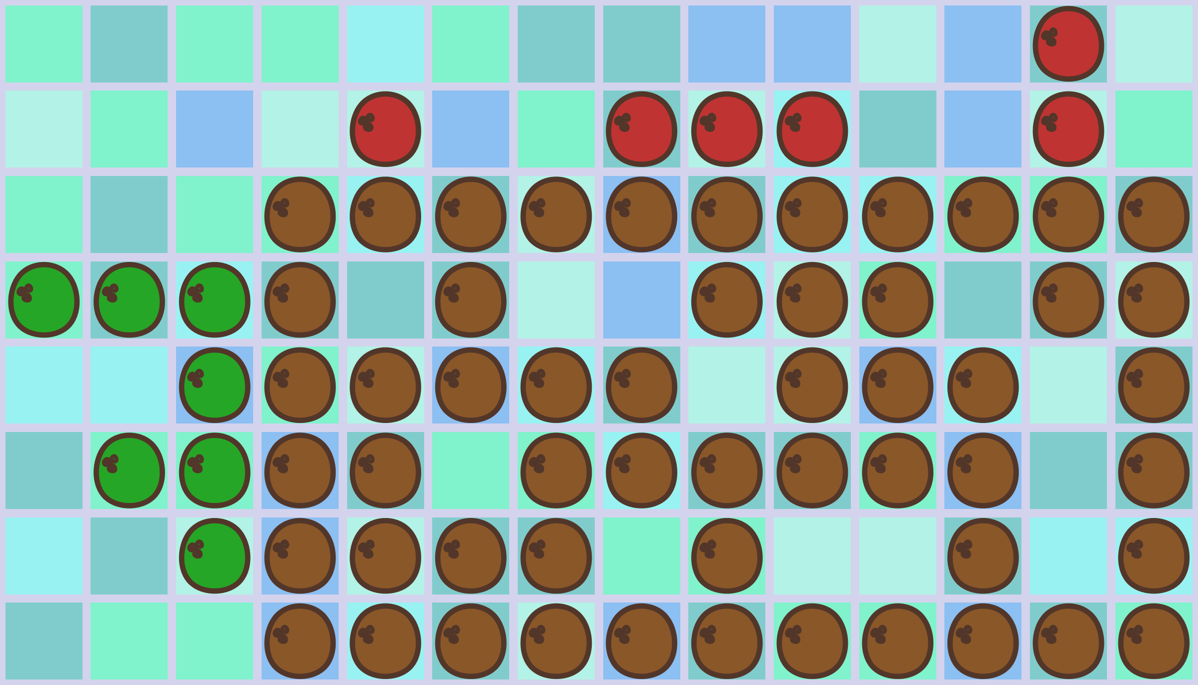

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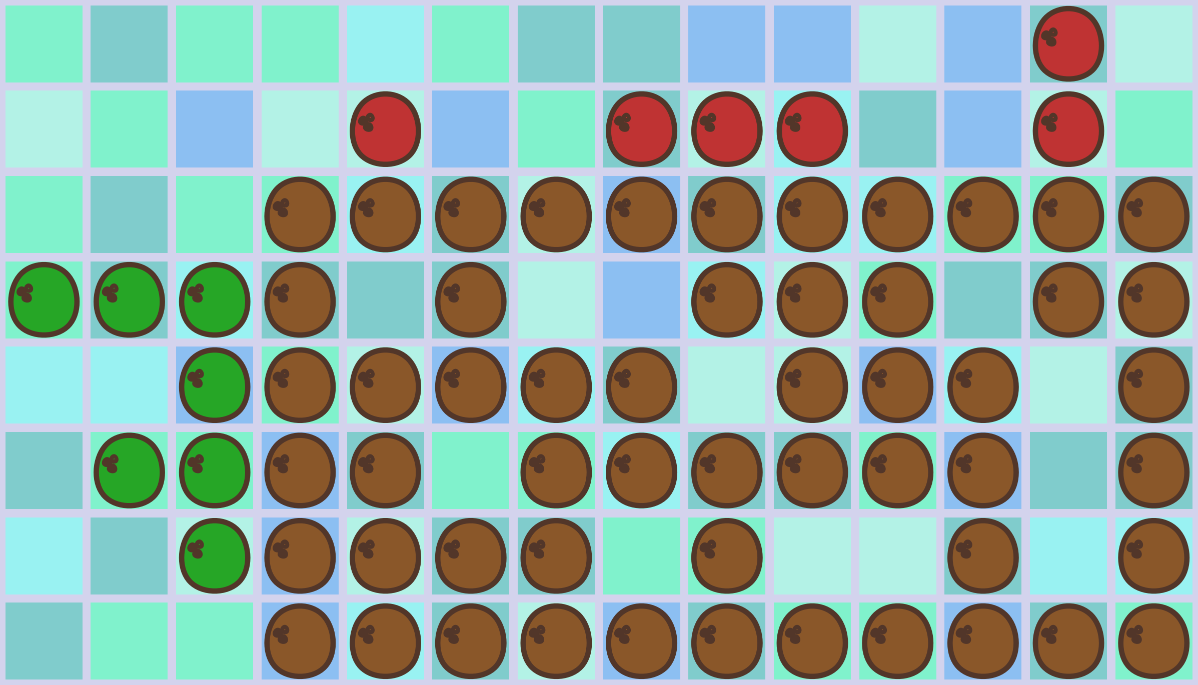

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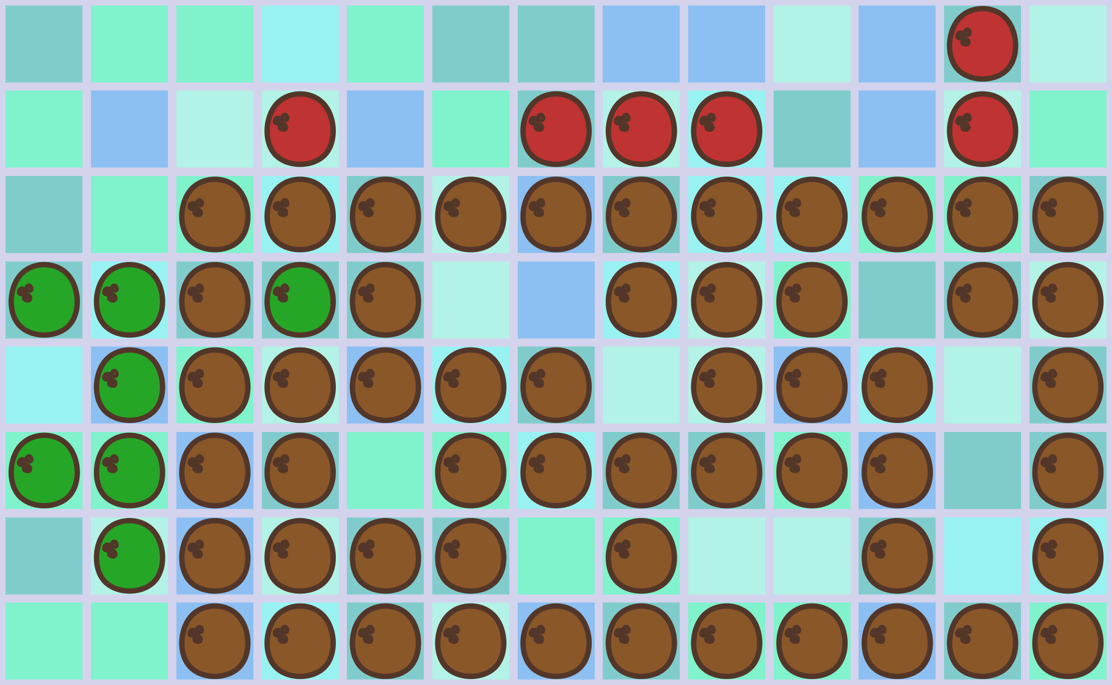

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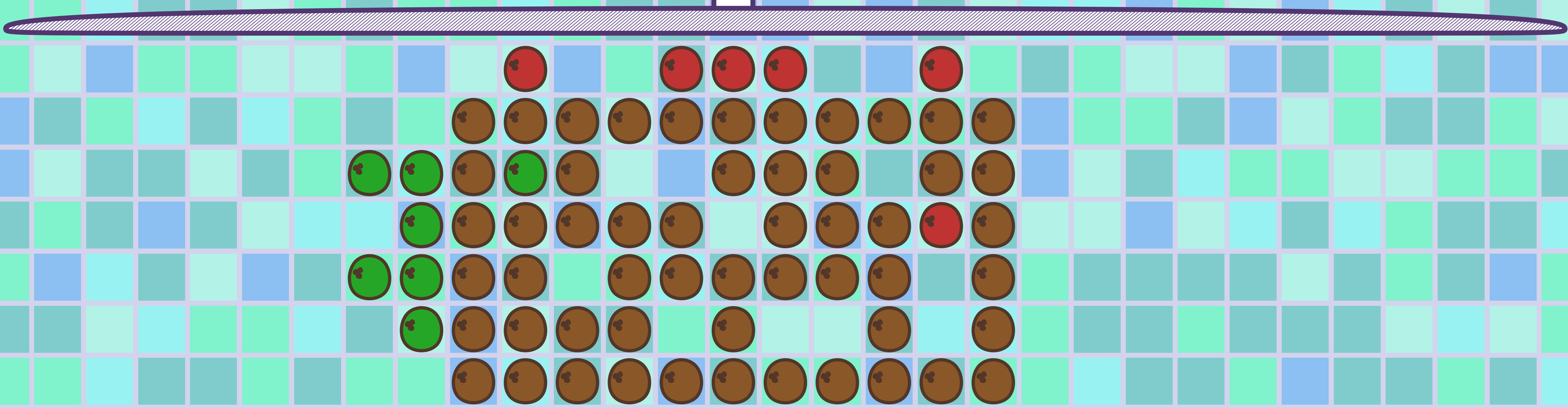

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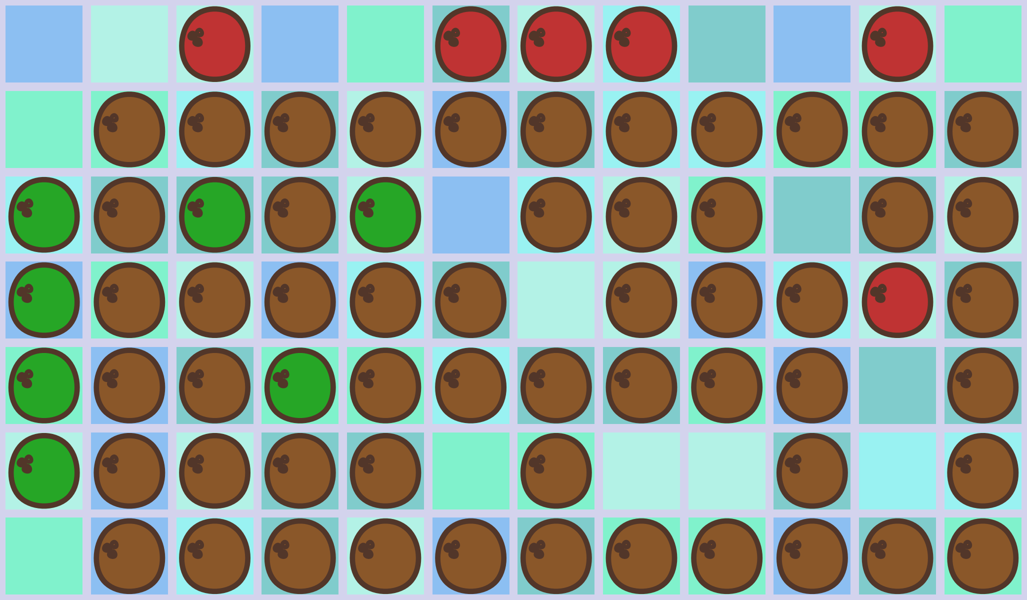

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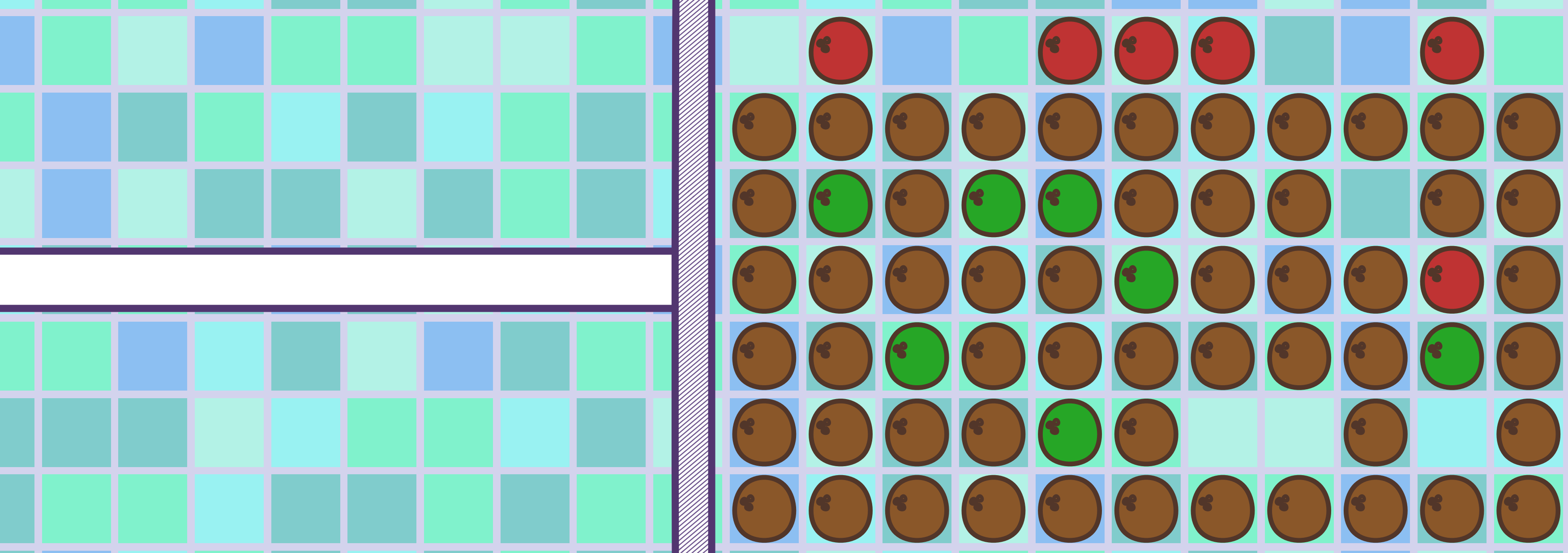

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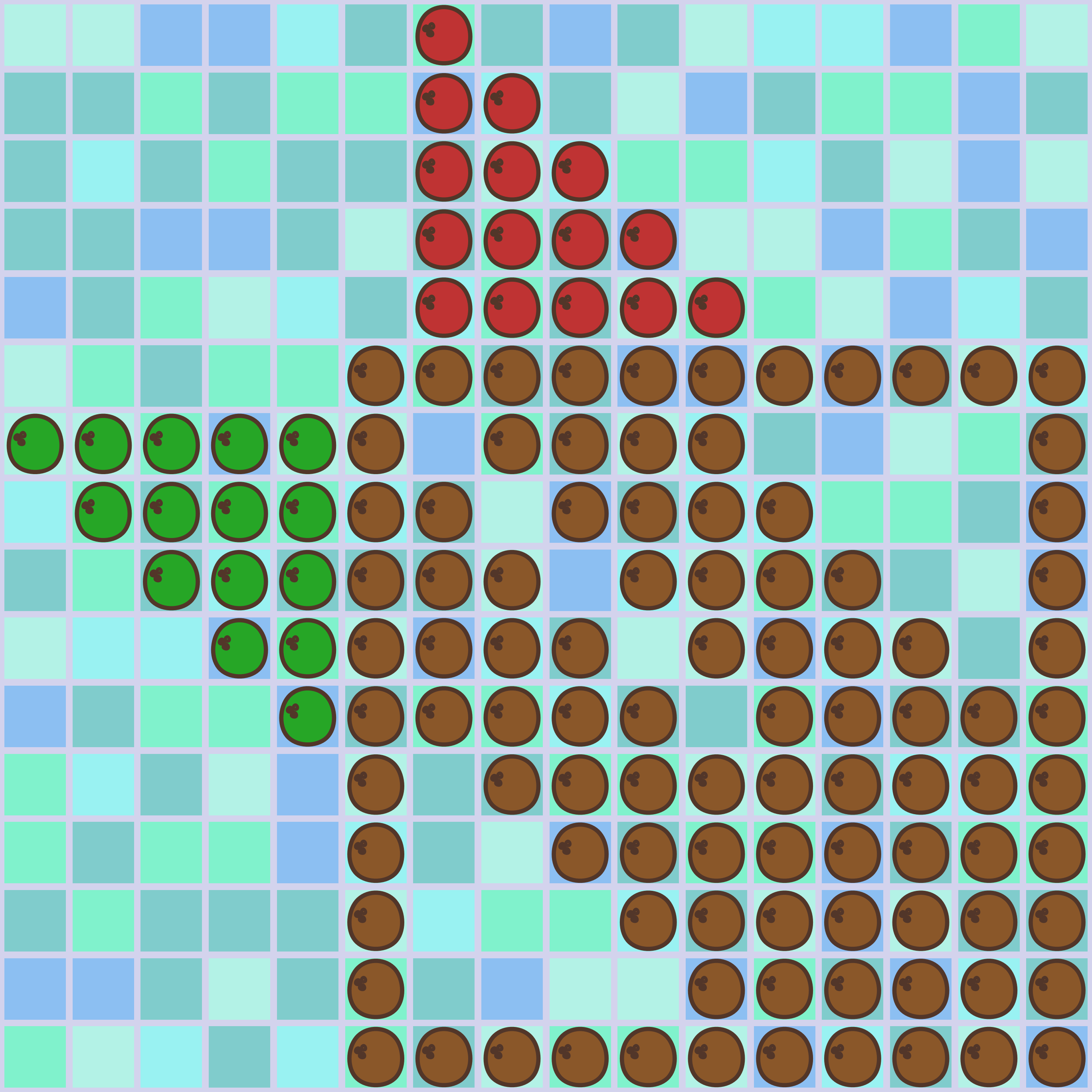

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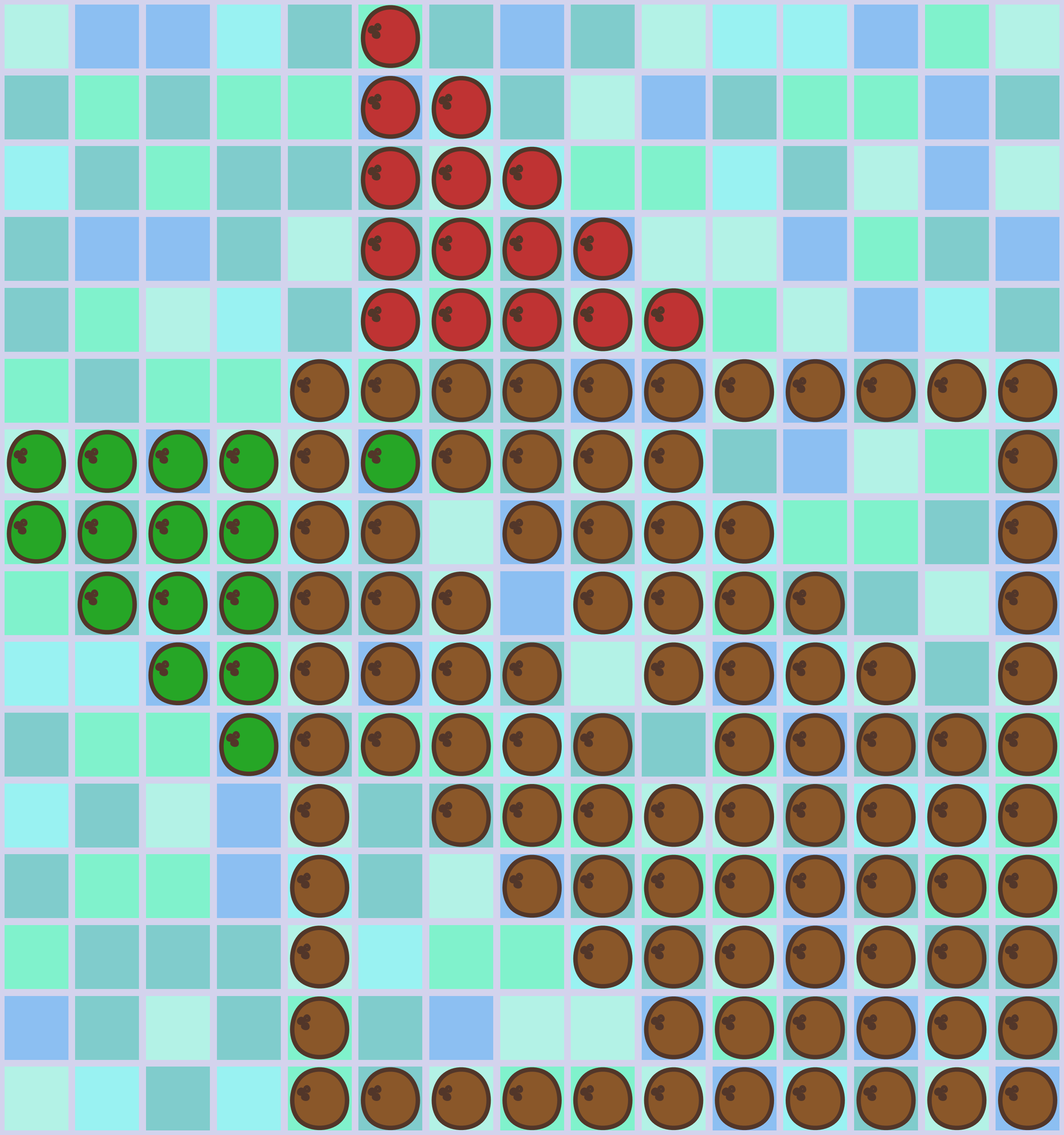

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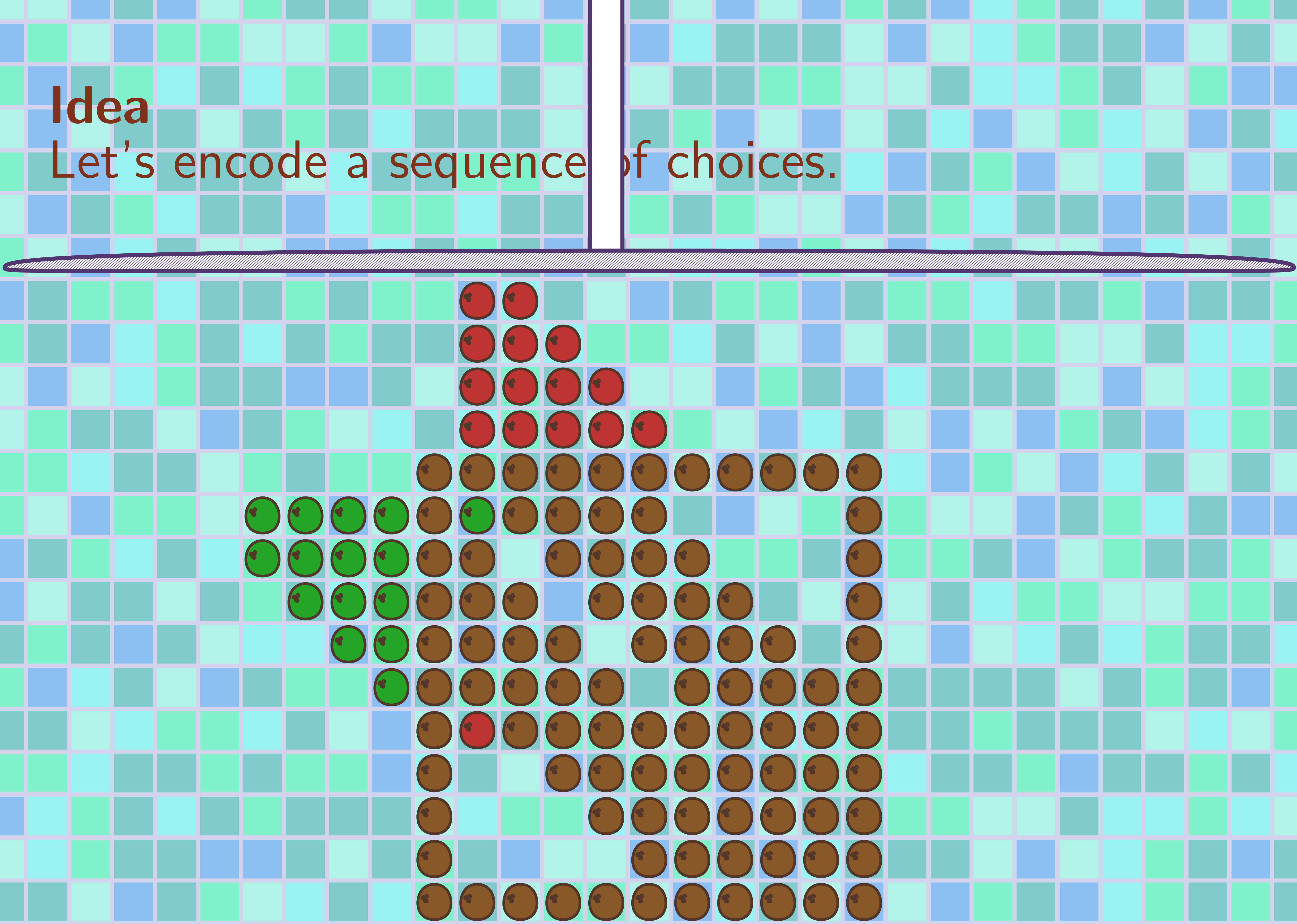

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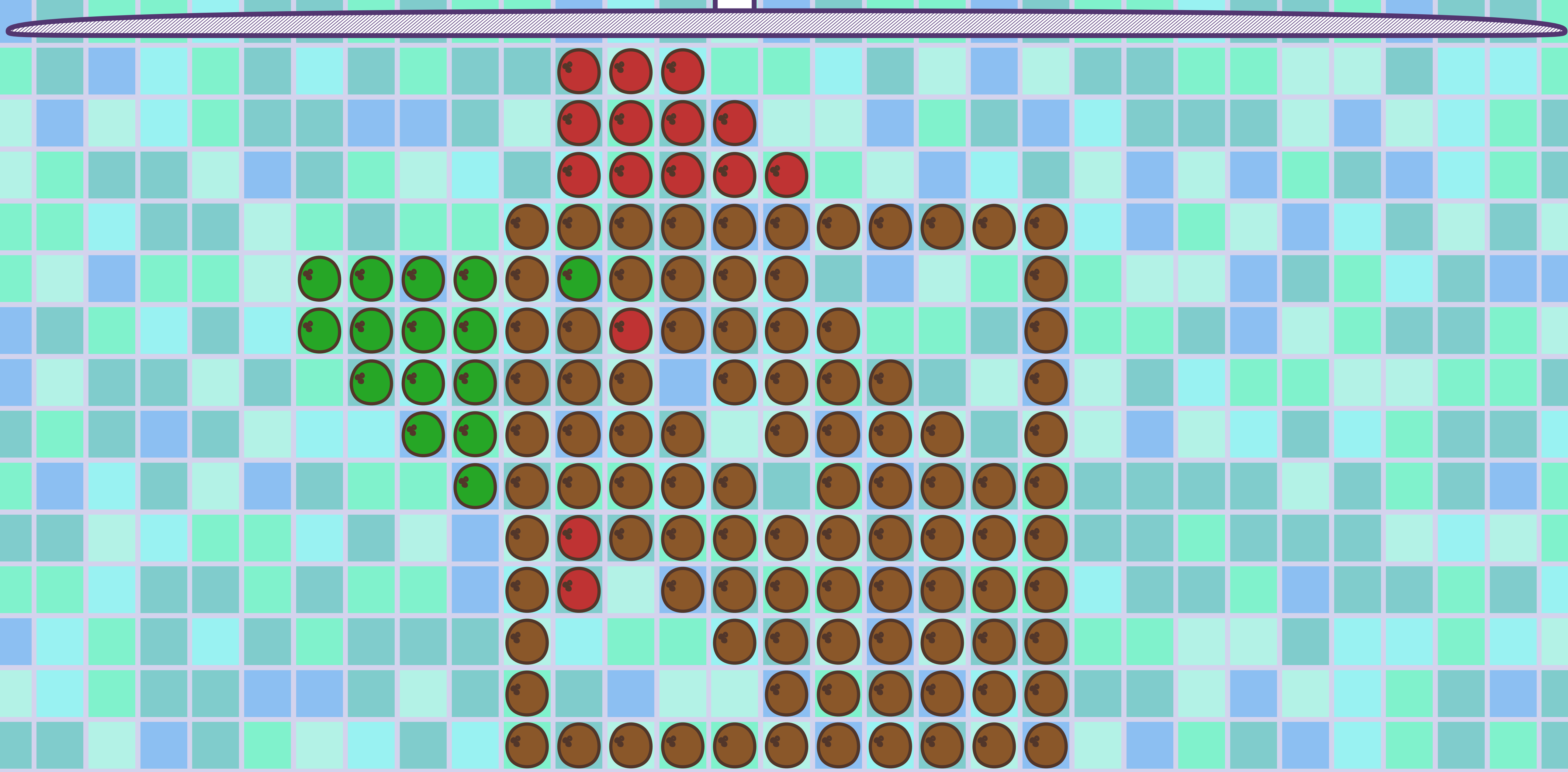

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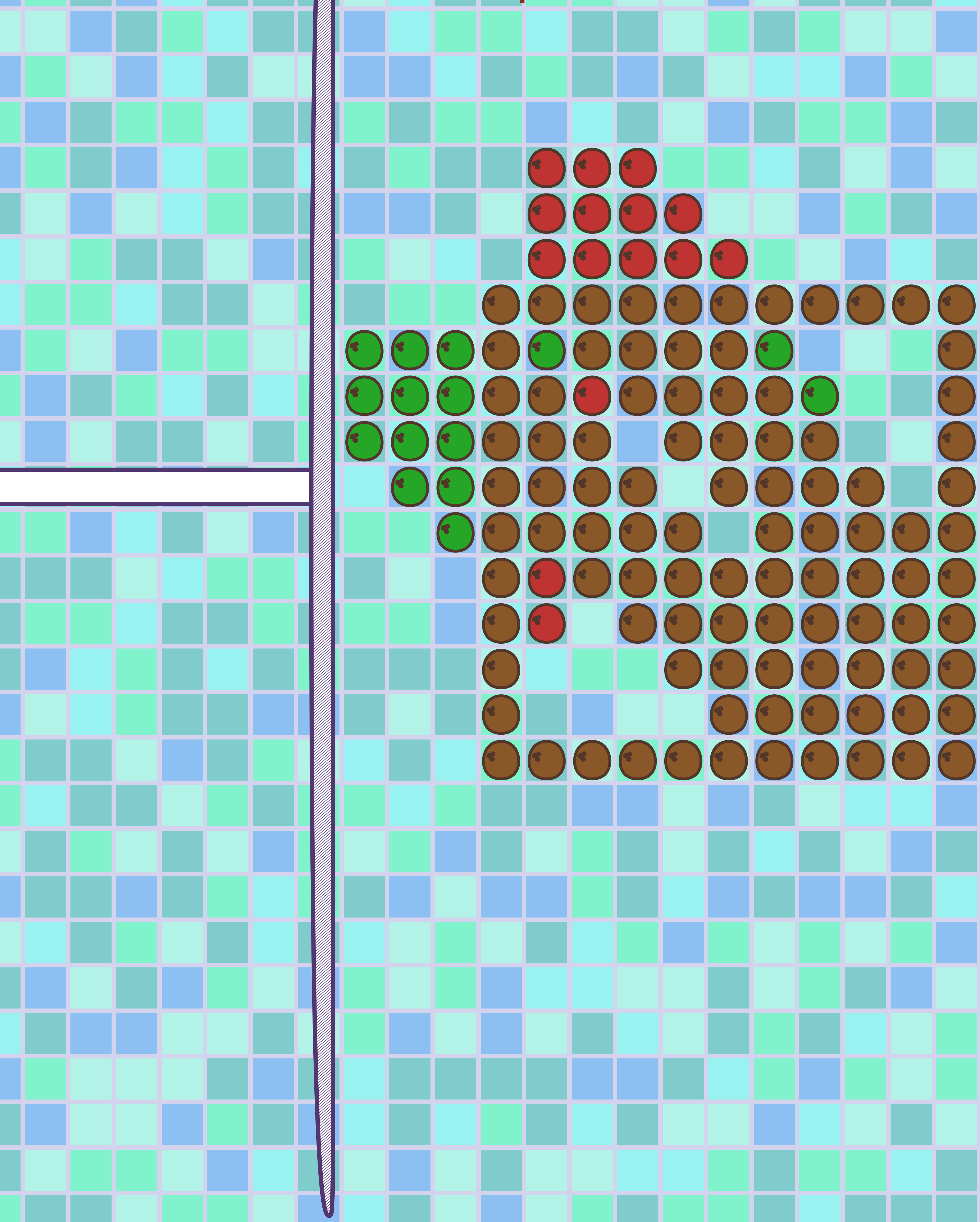

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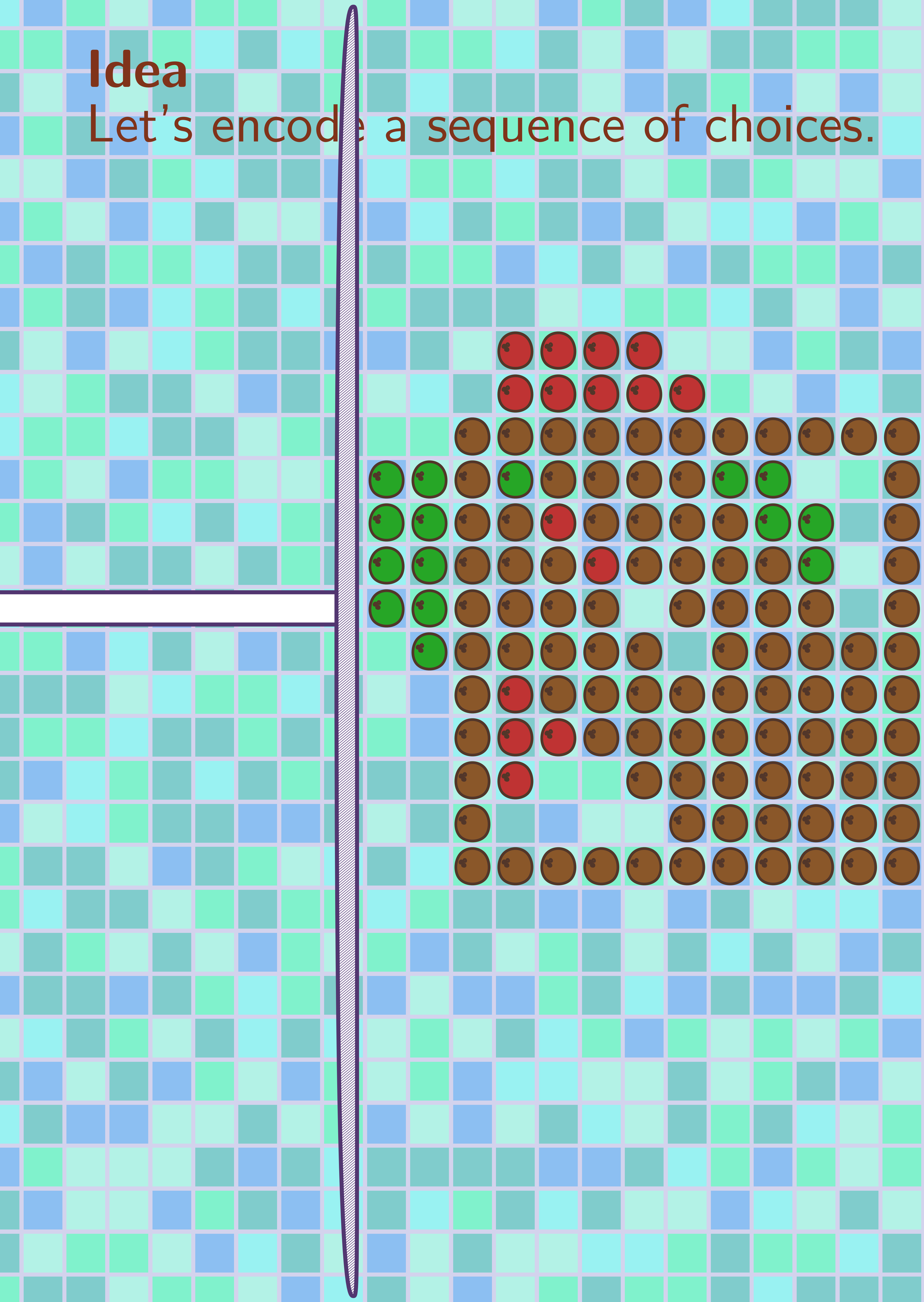

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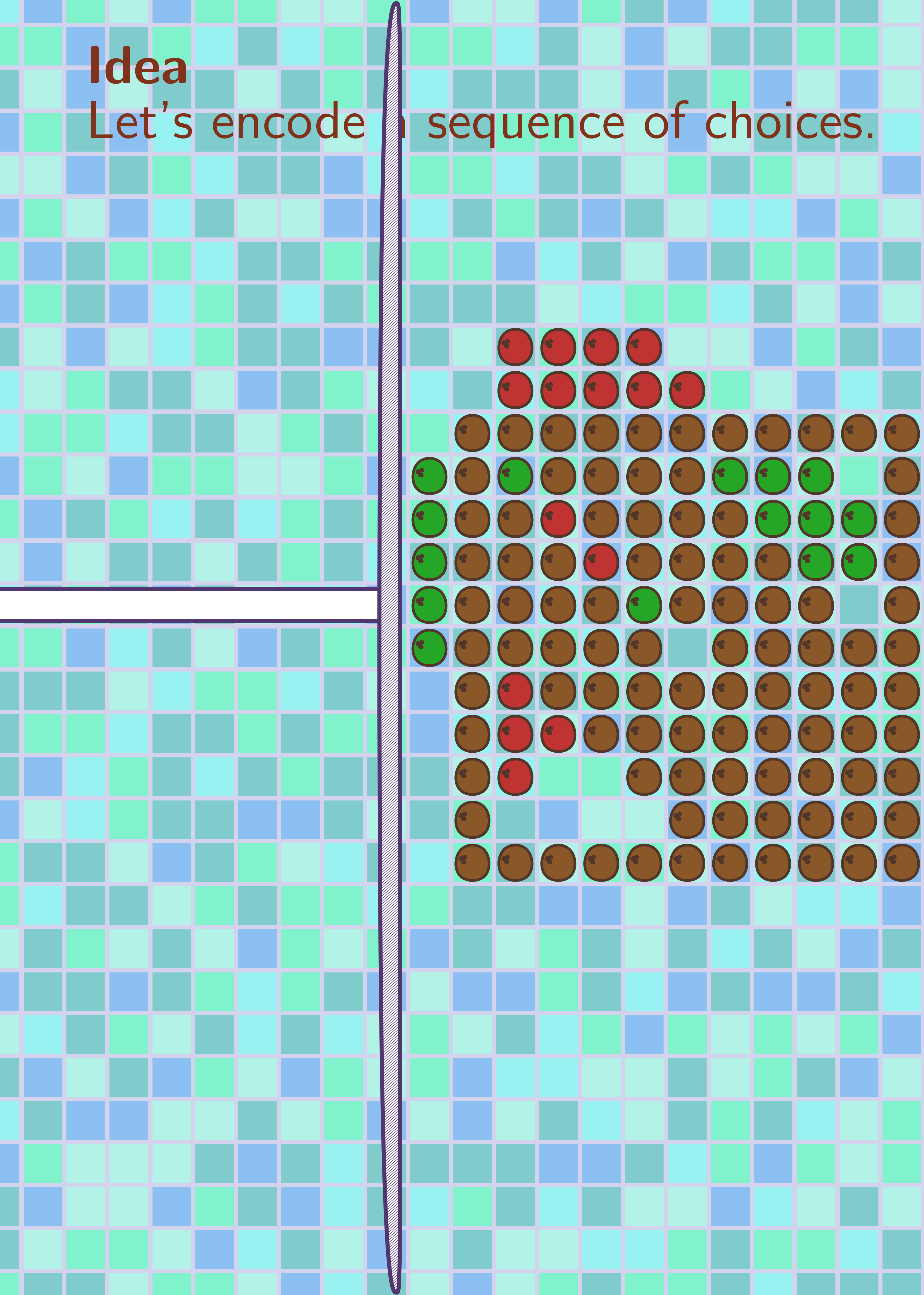

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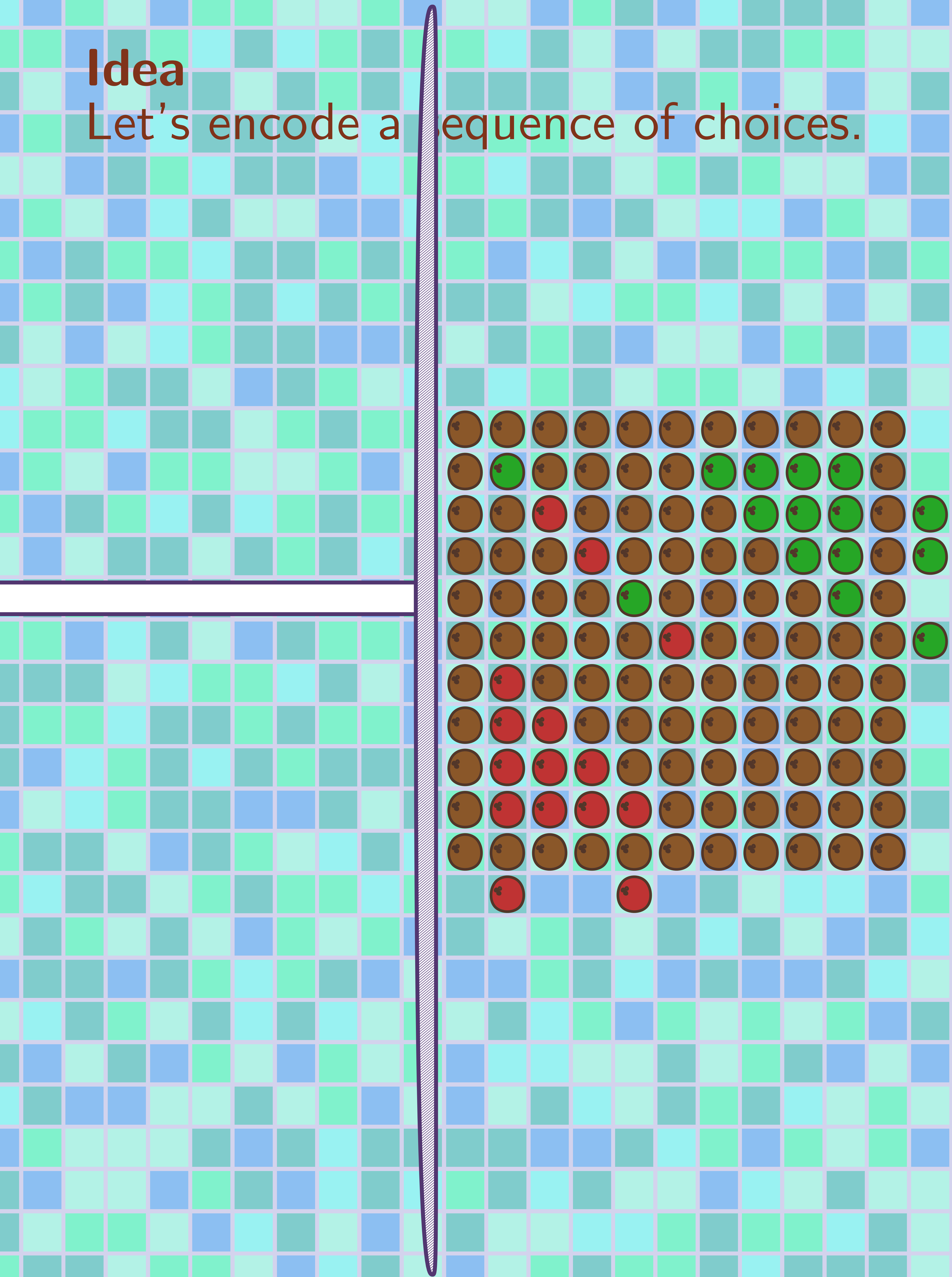

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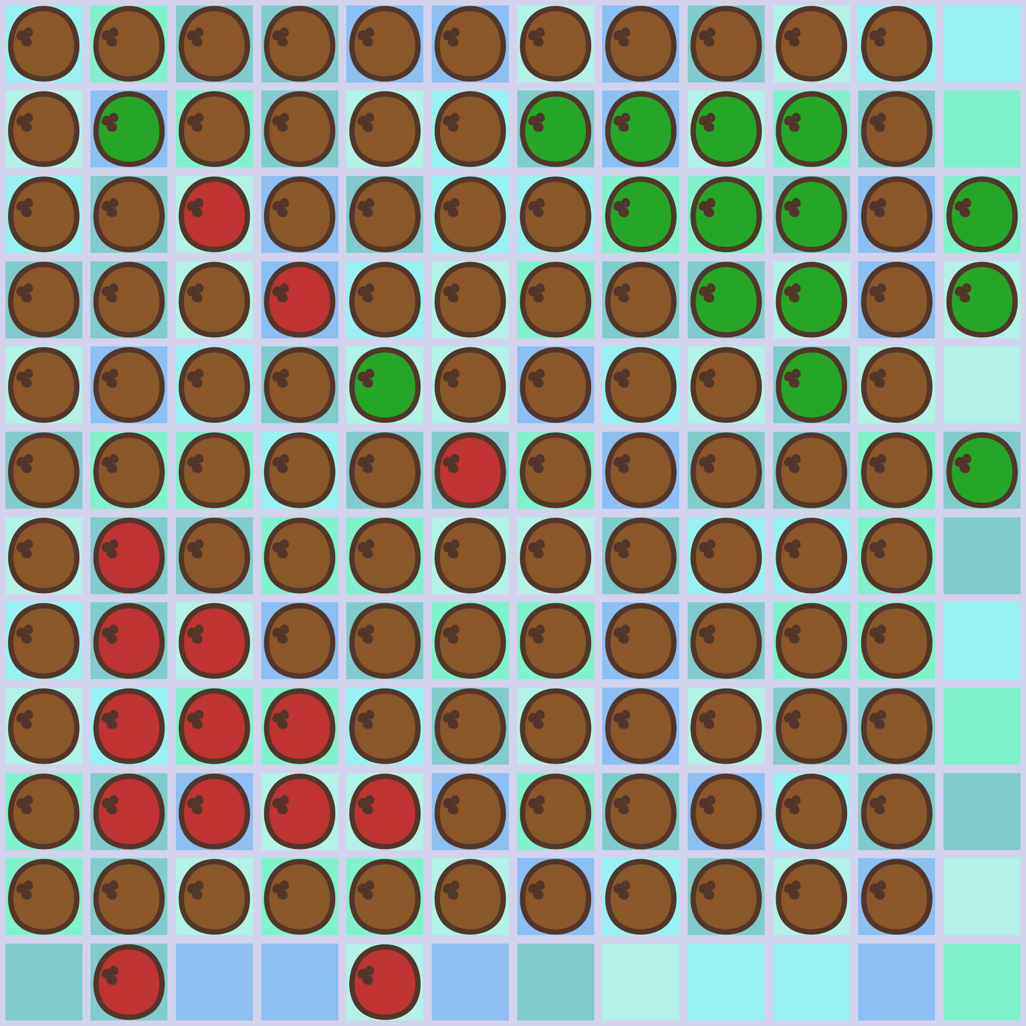
Let's encode a sequence of choices.

By choosing the order of pushes, we can make any subset spill over to the right (and bottom).



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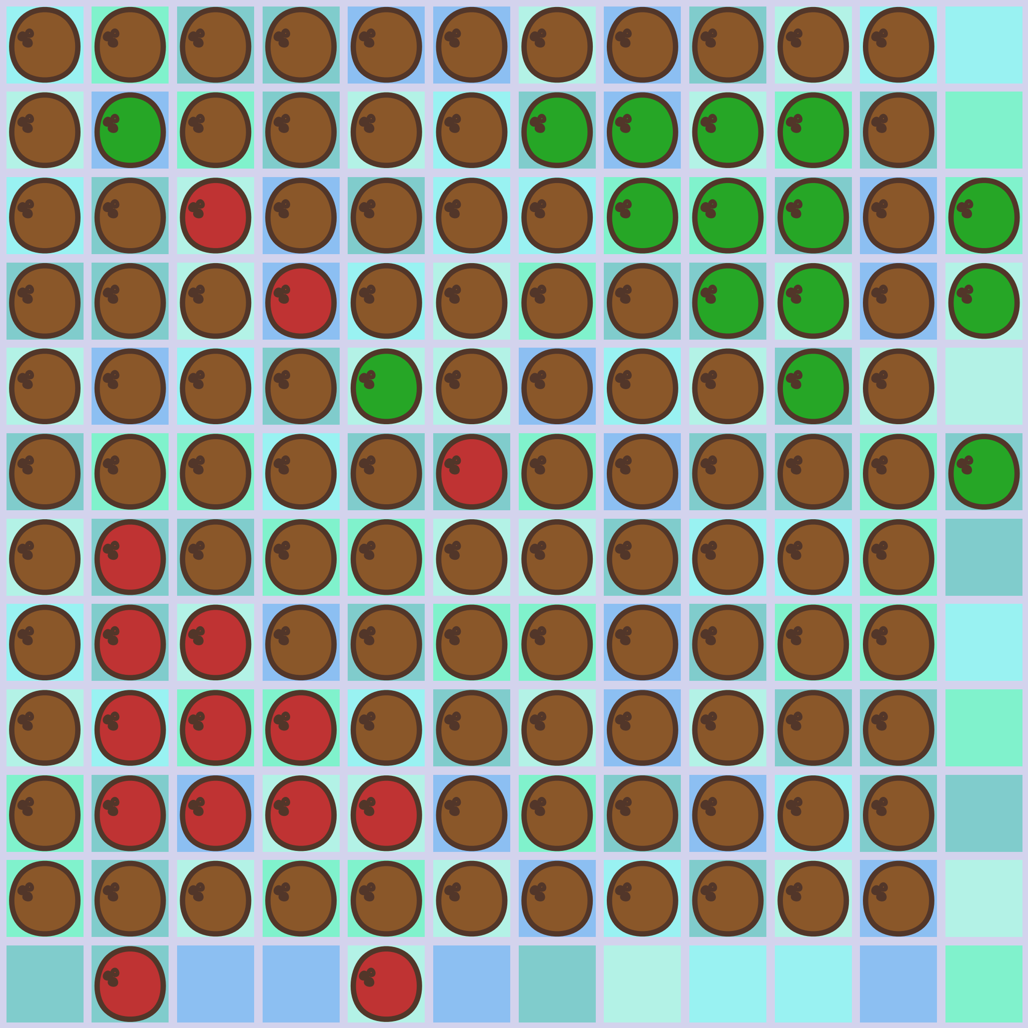

We can also do the same for multiple coconuts per row.

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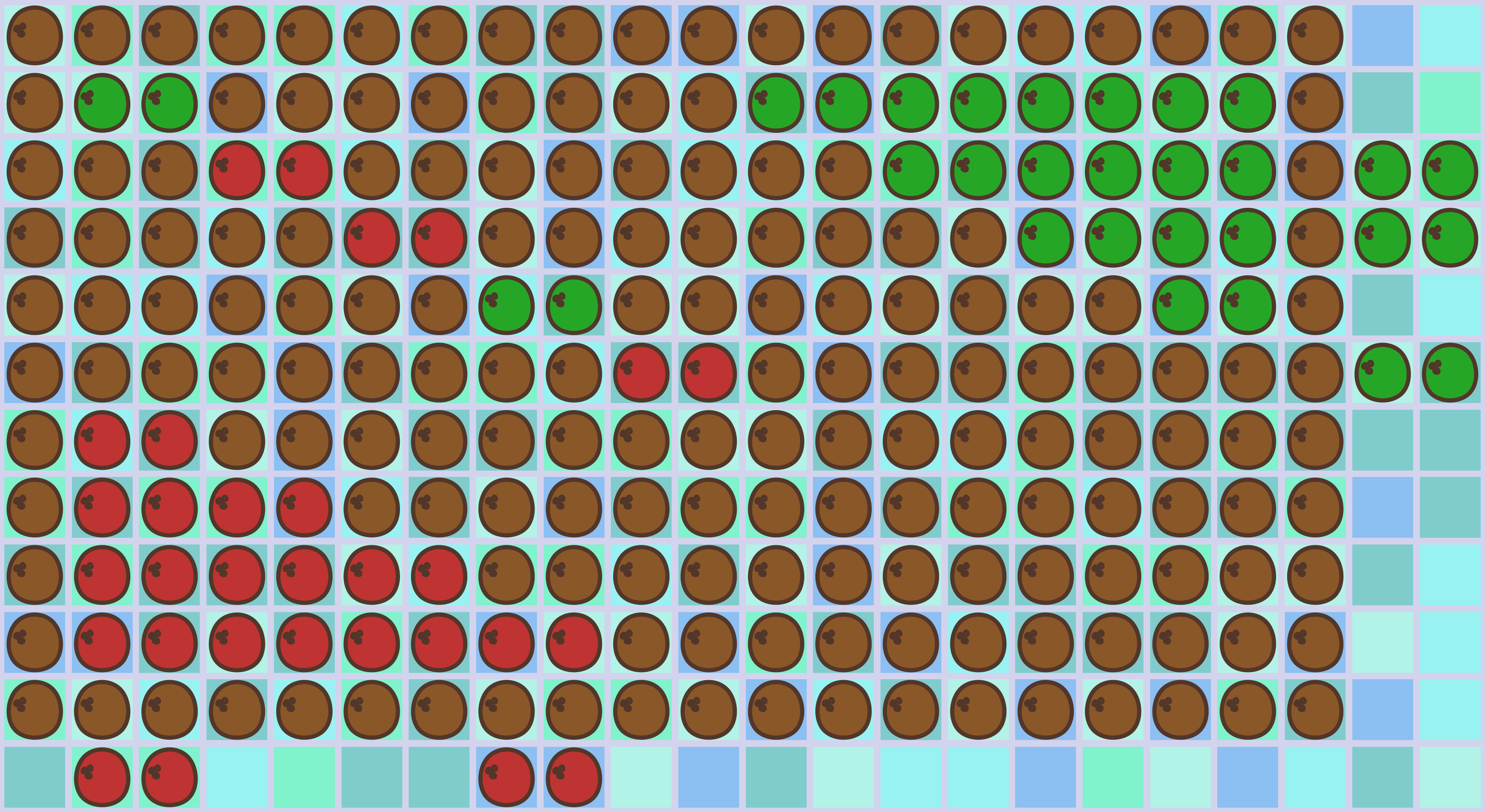

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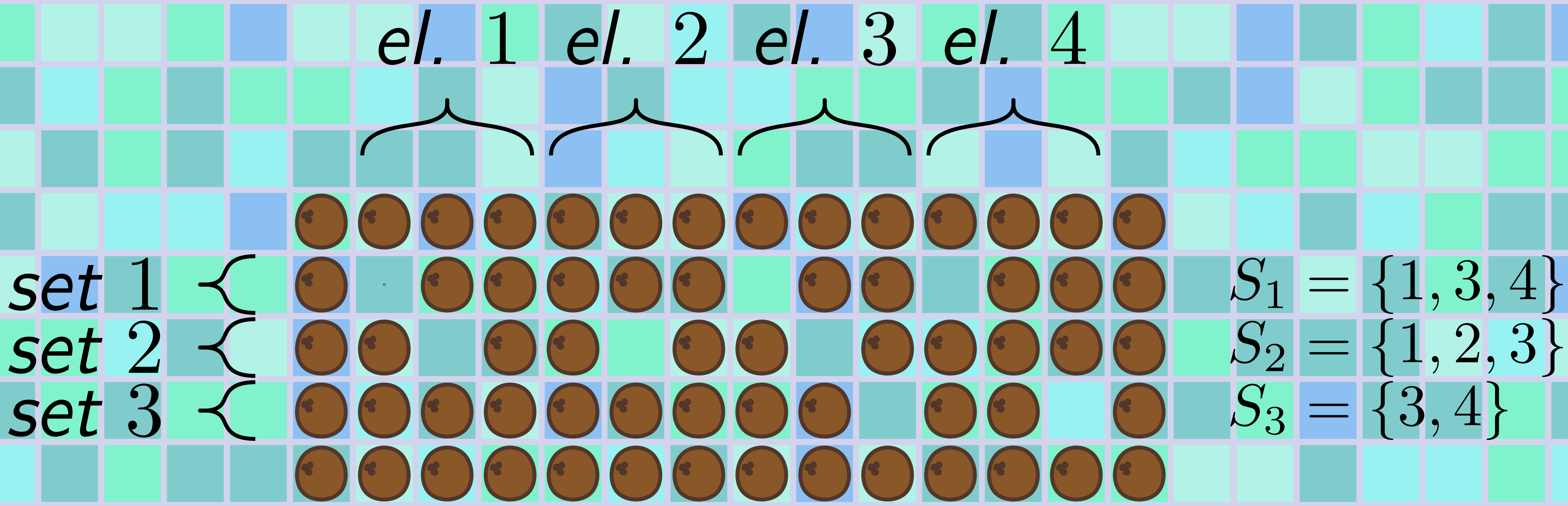

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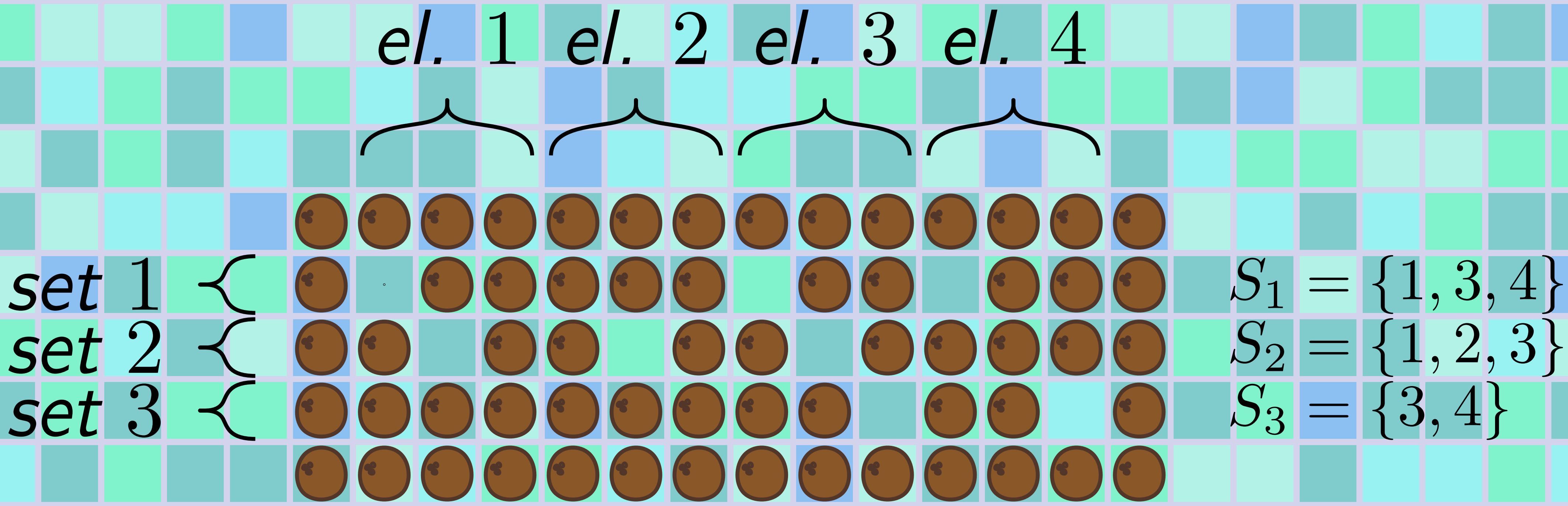

Now suppose we have coconuts coming from above in the columns of k of those elements.

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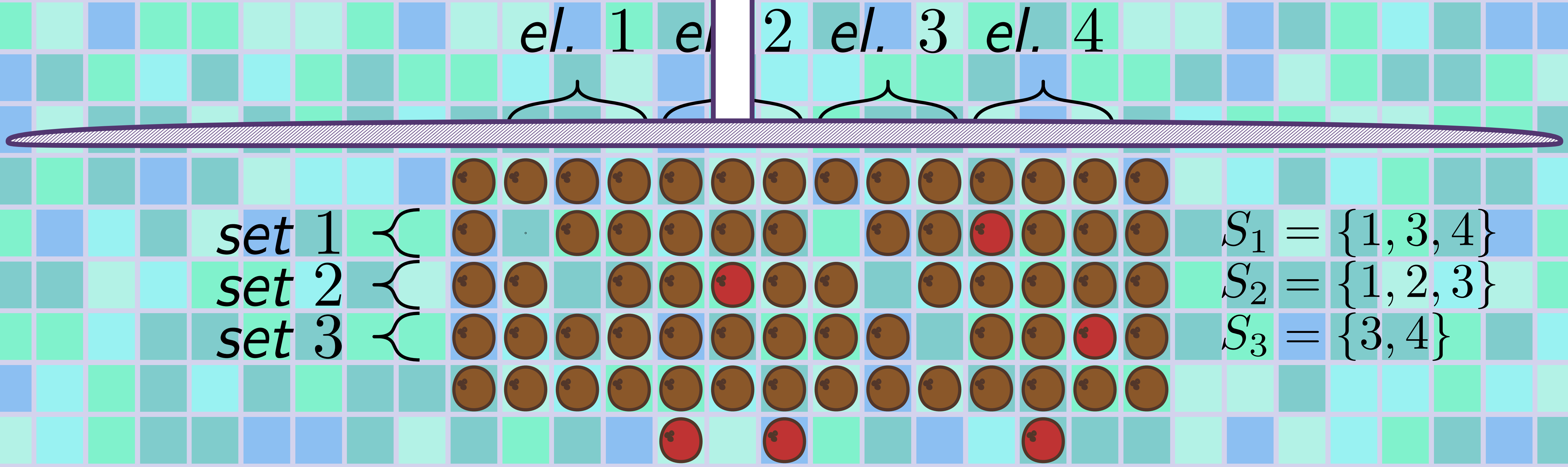

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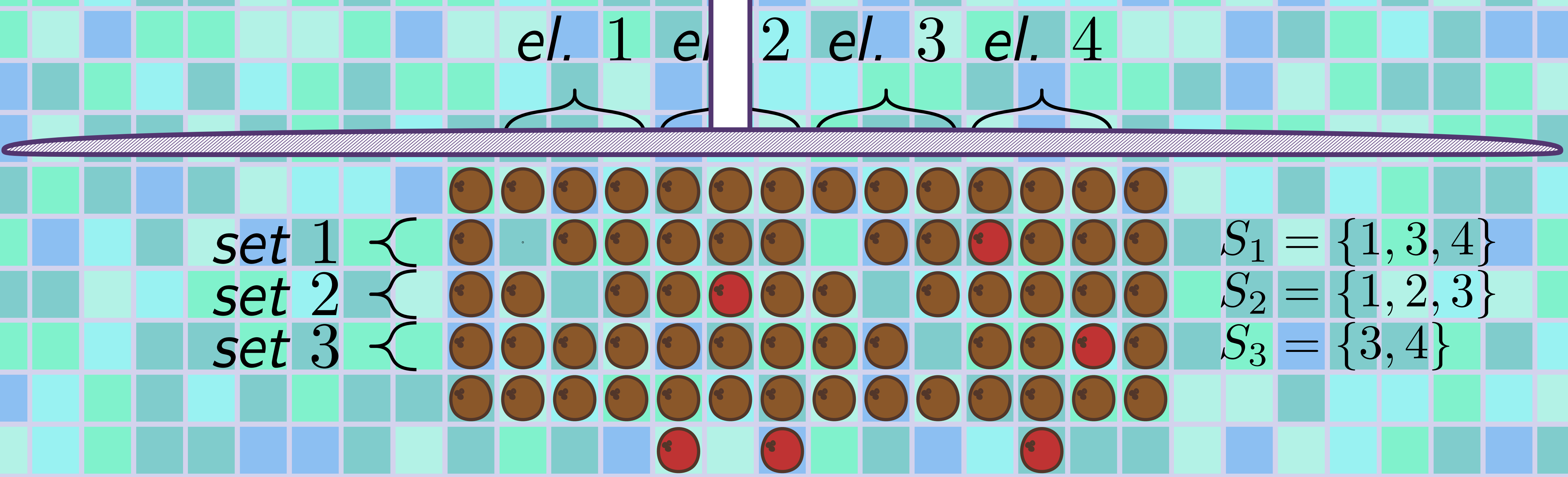

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If the elements we chose were an exact hitting set, we now have exactly one red coconut per row.

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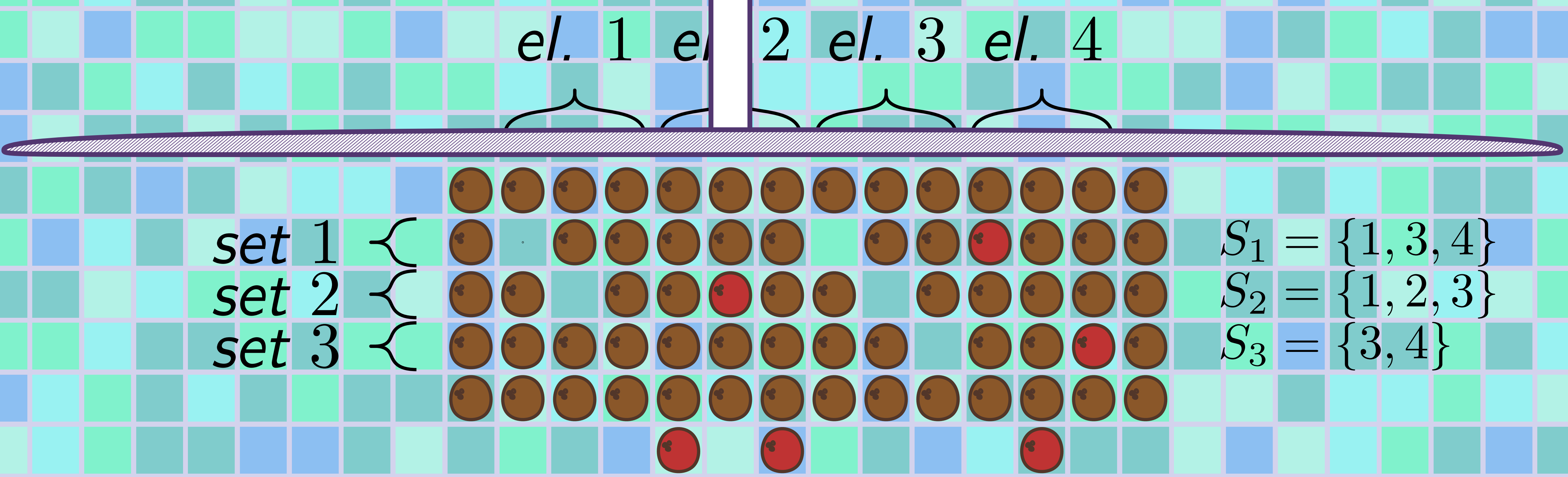

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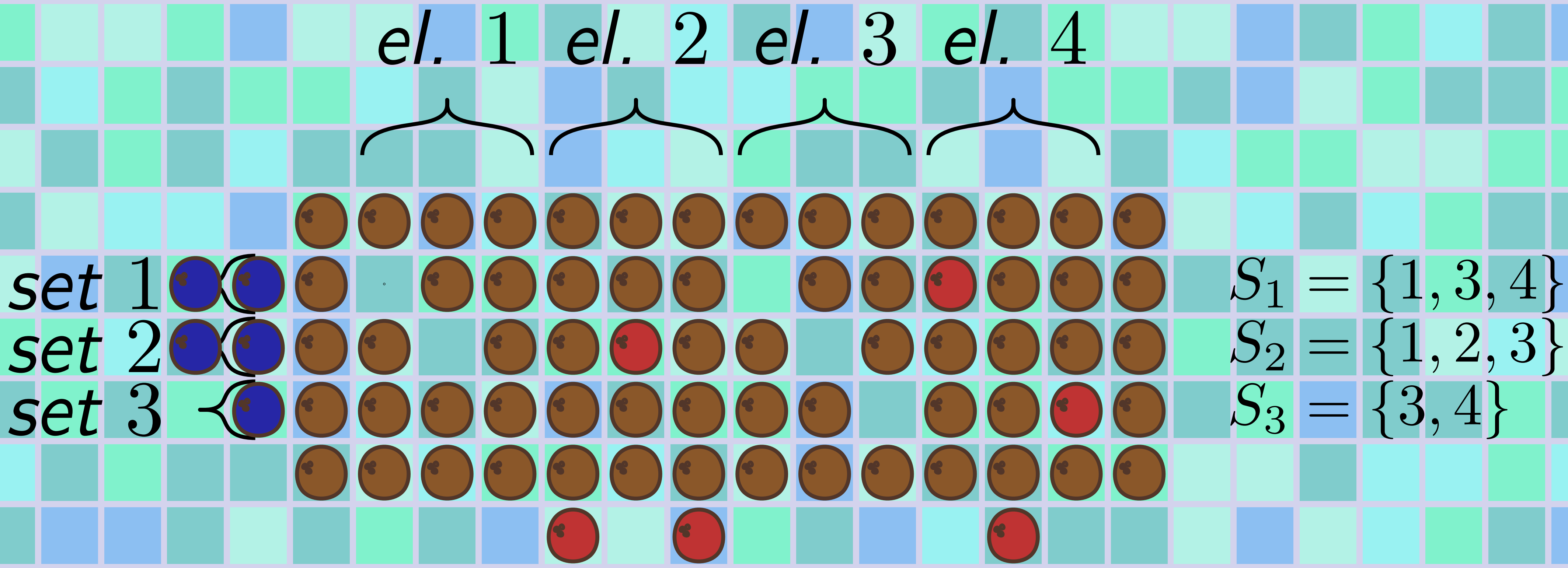

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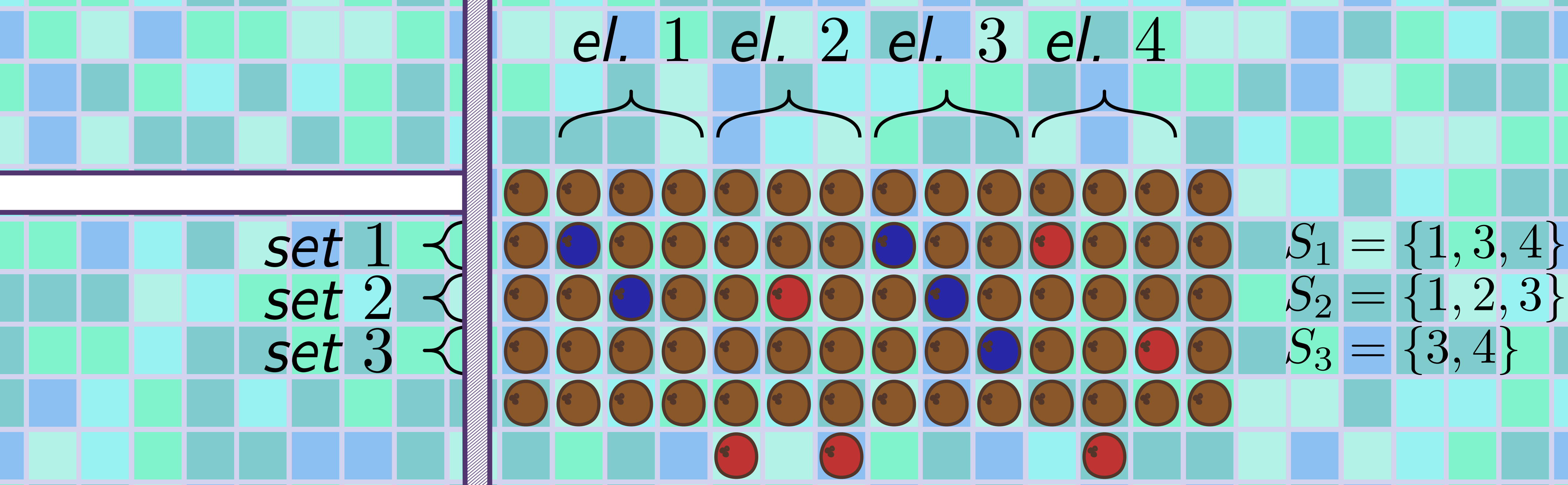

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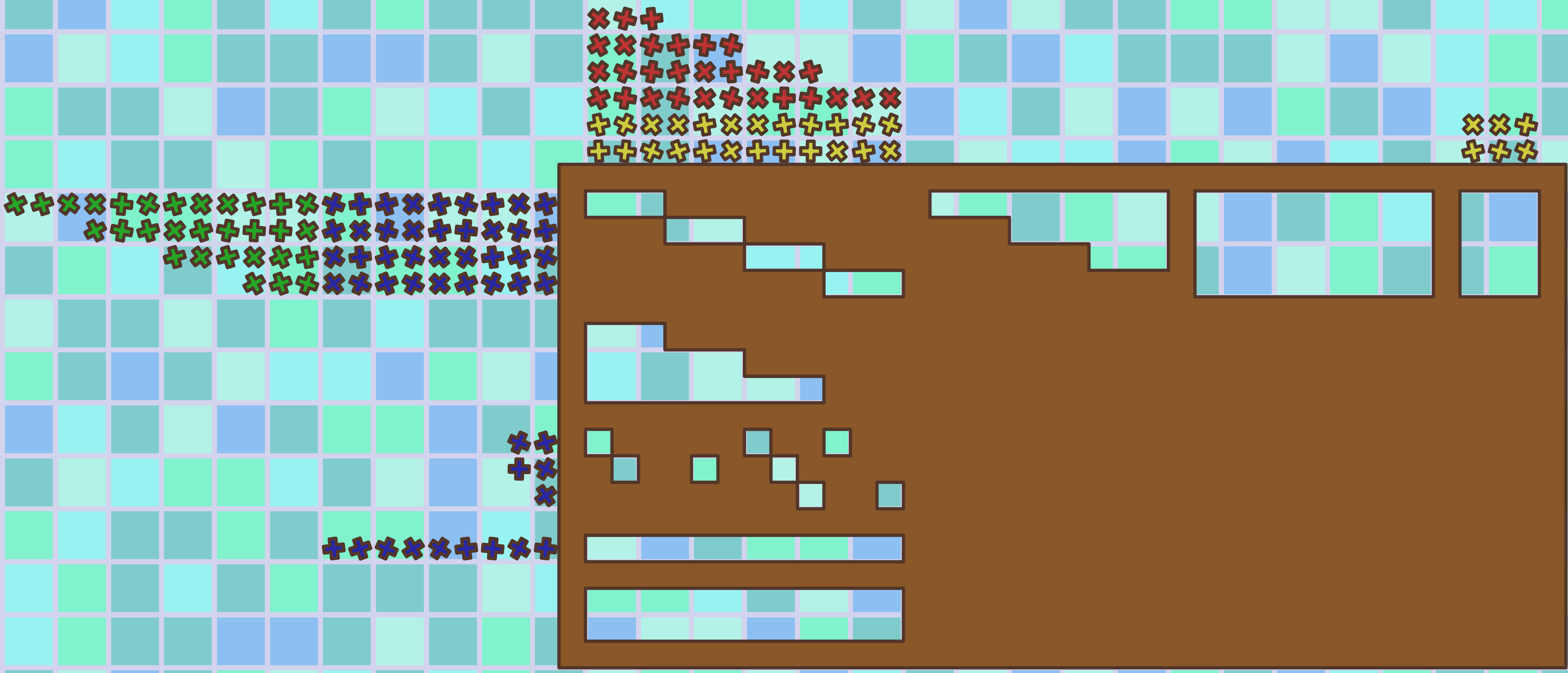

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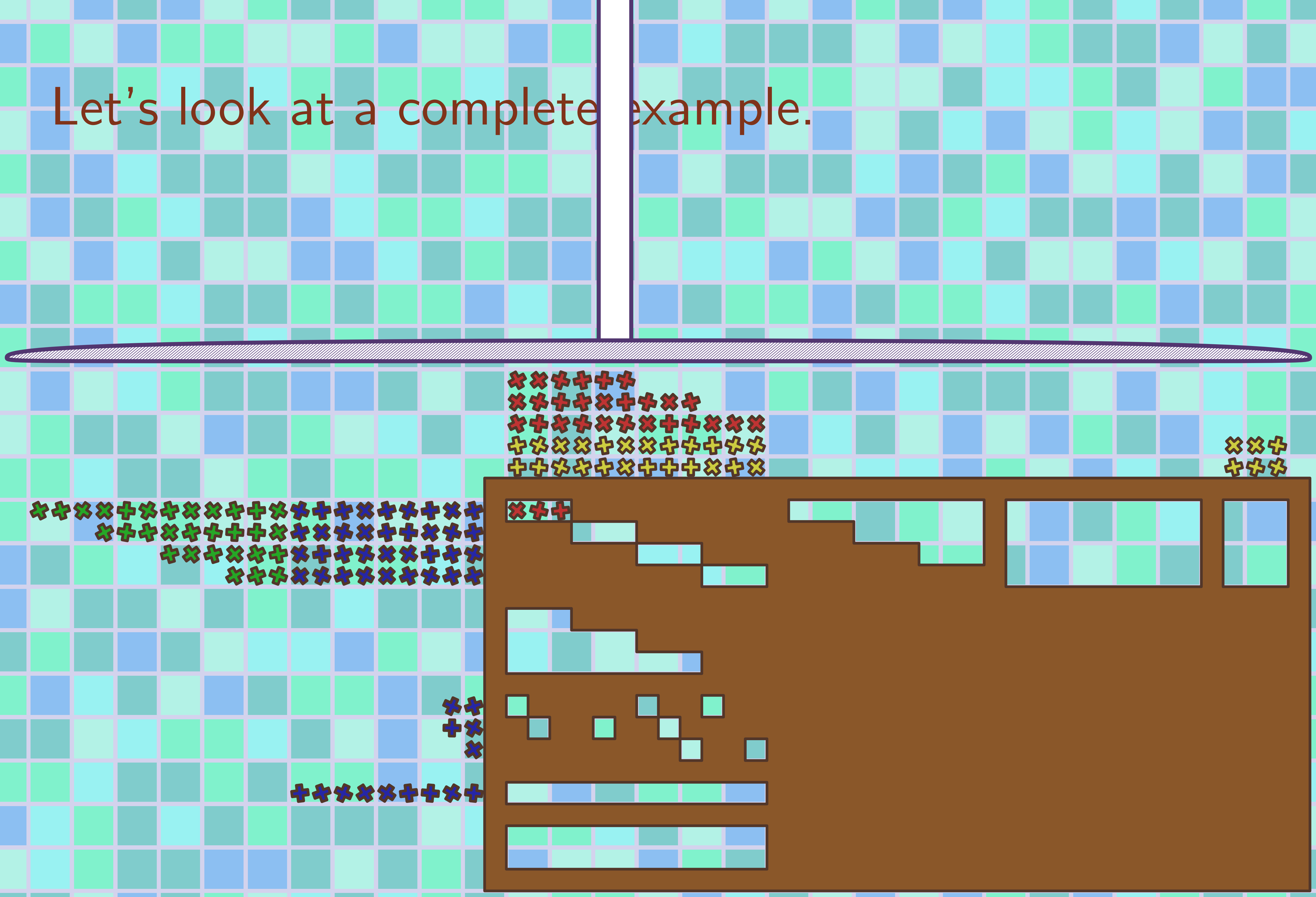

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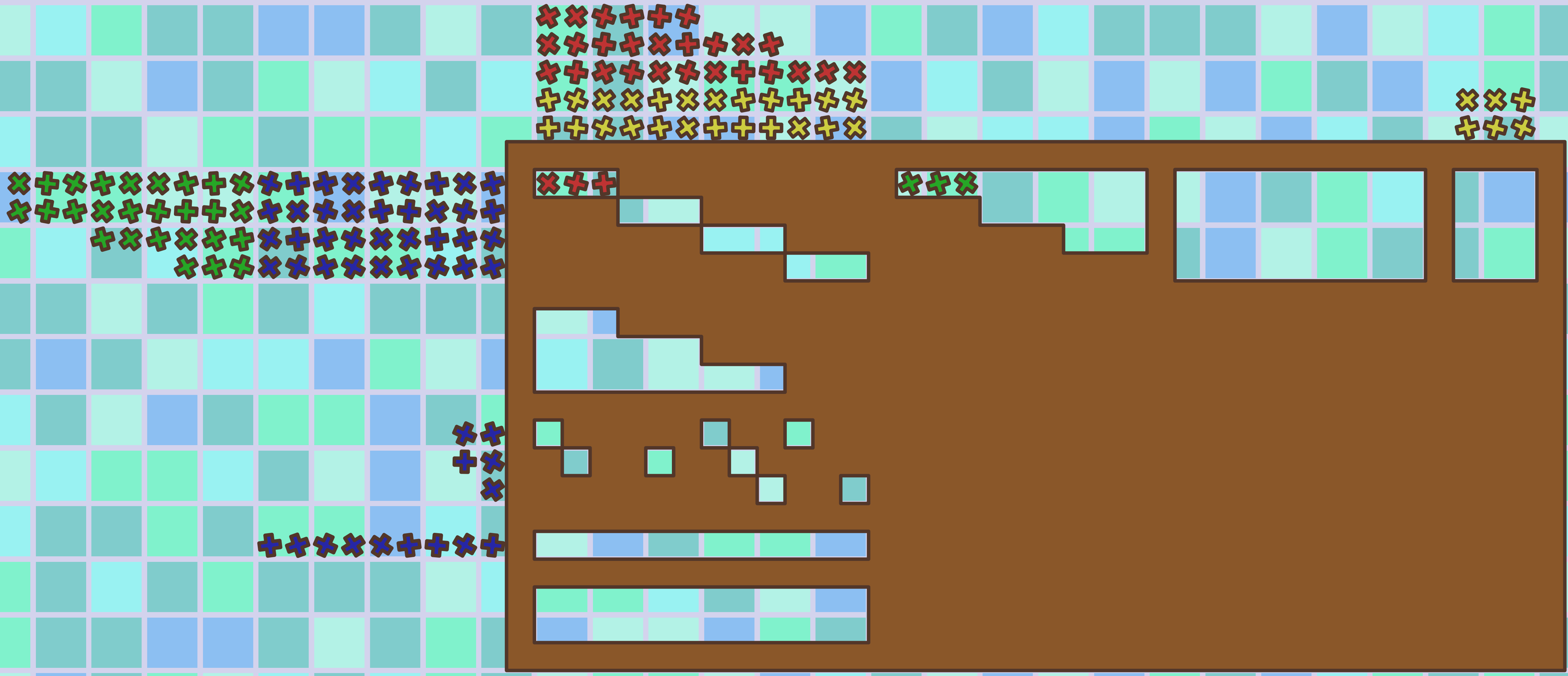

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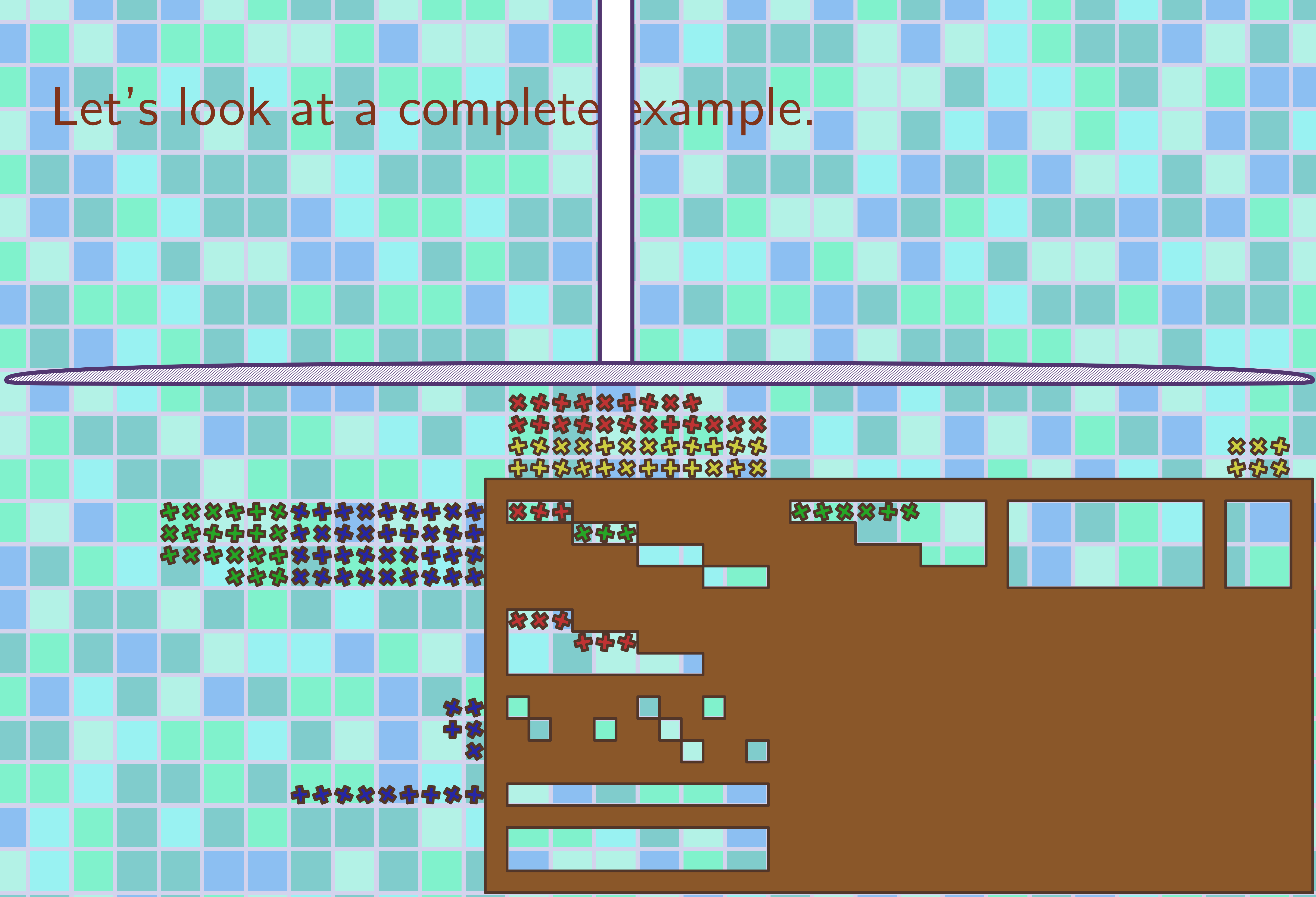

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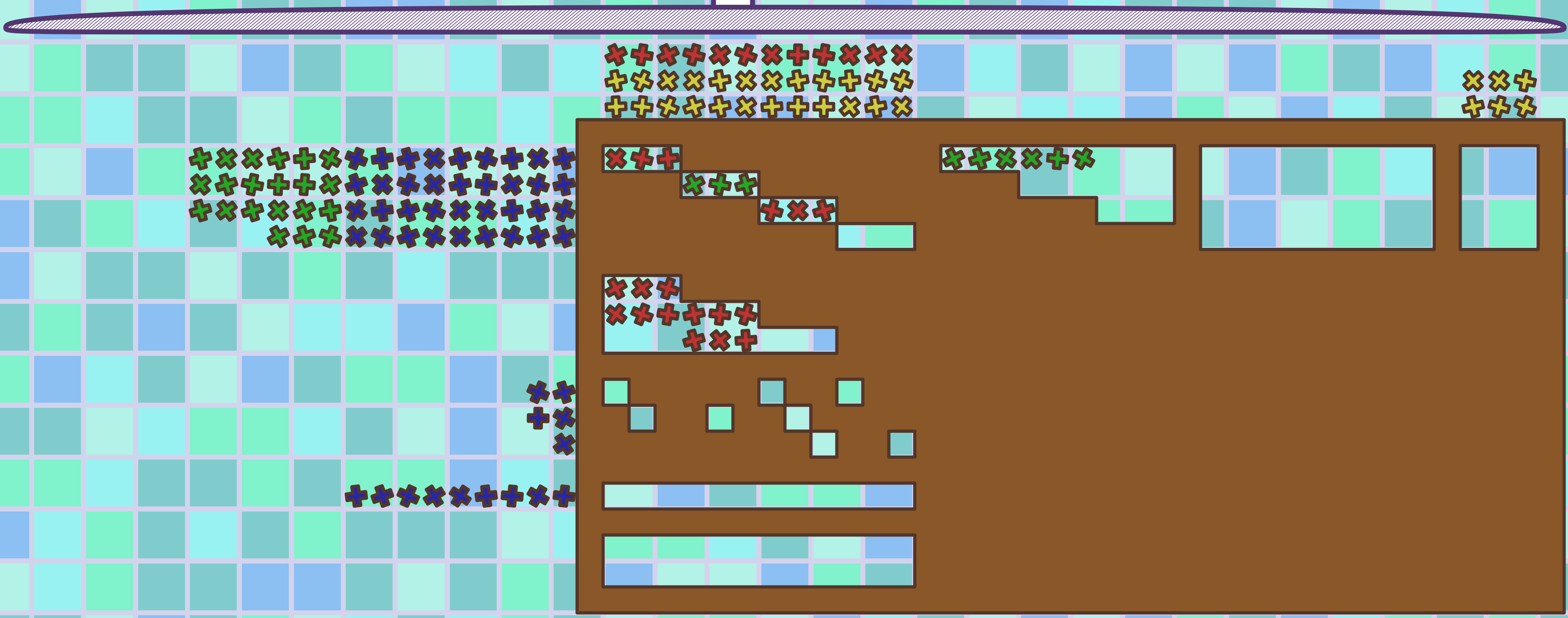

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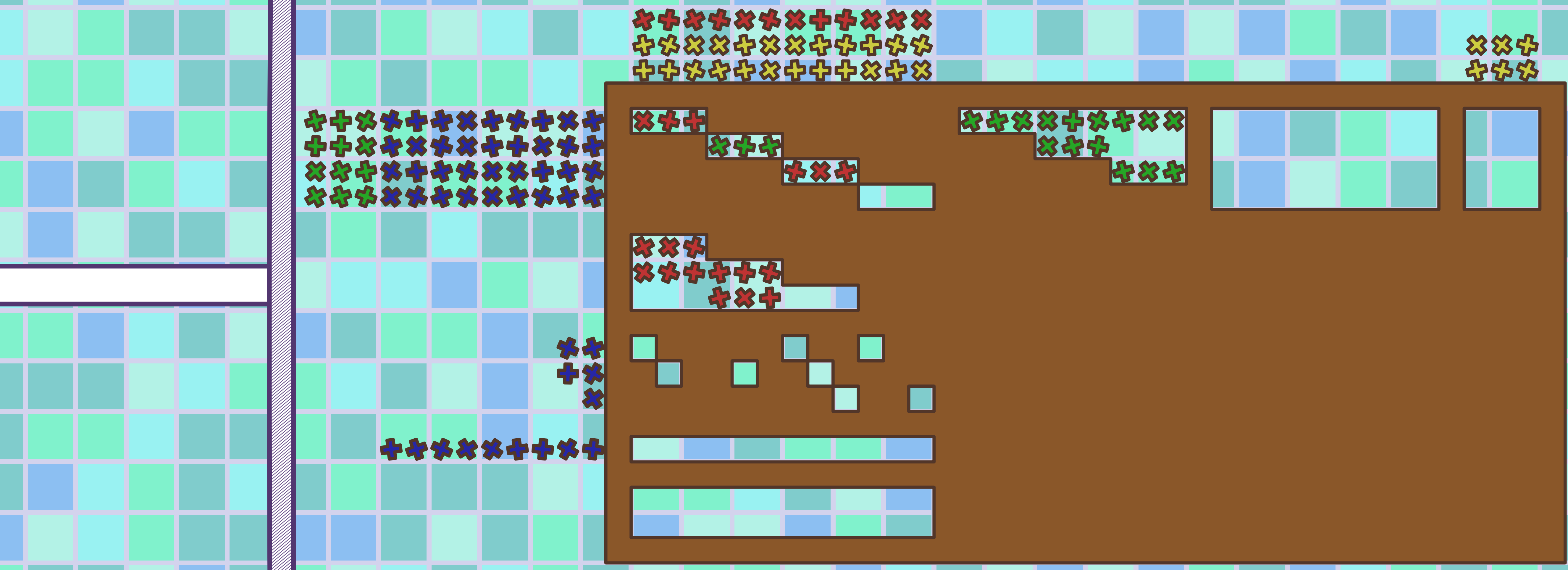

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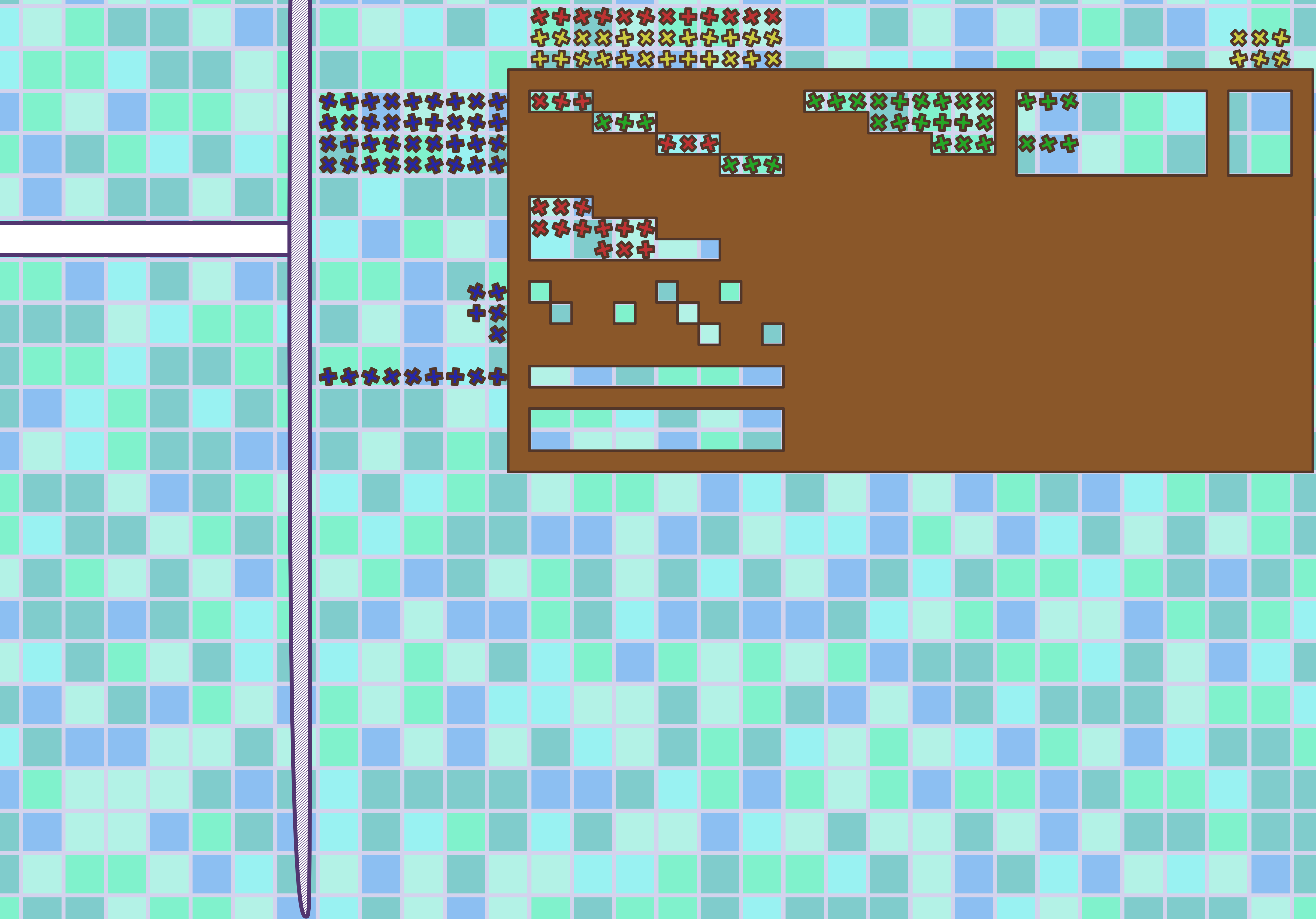

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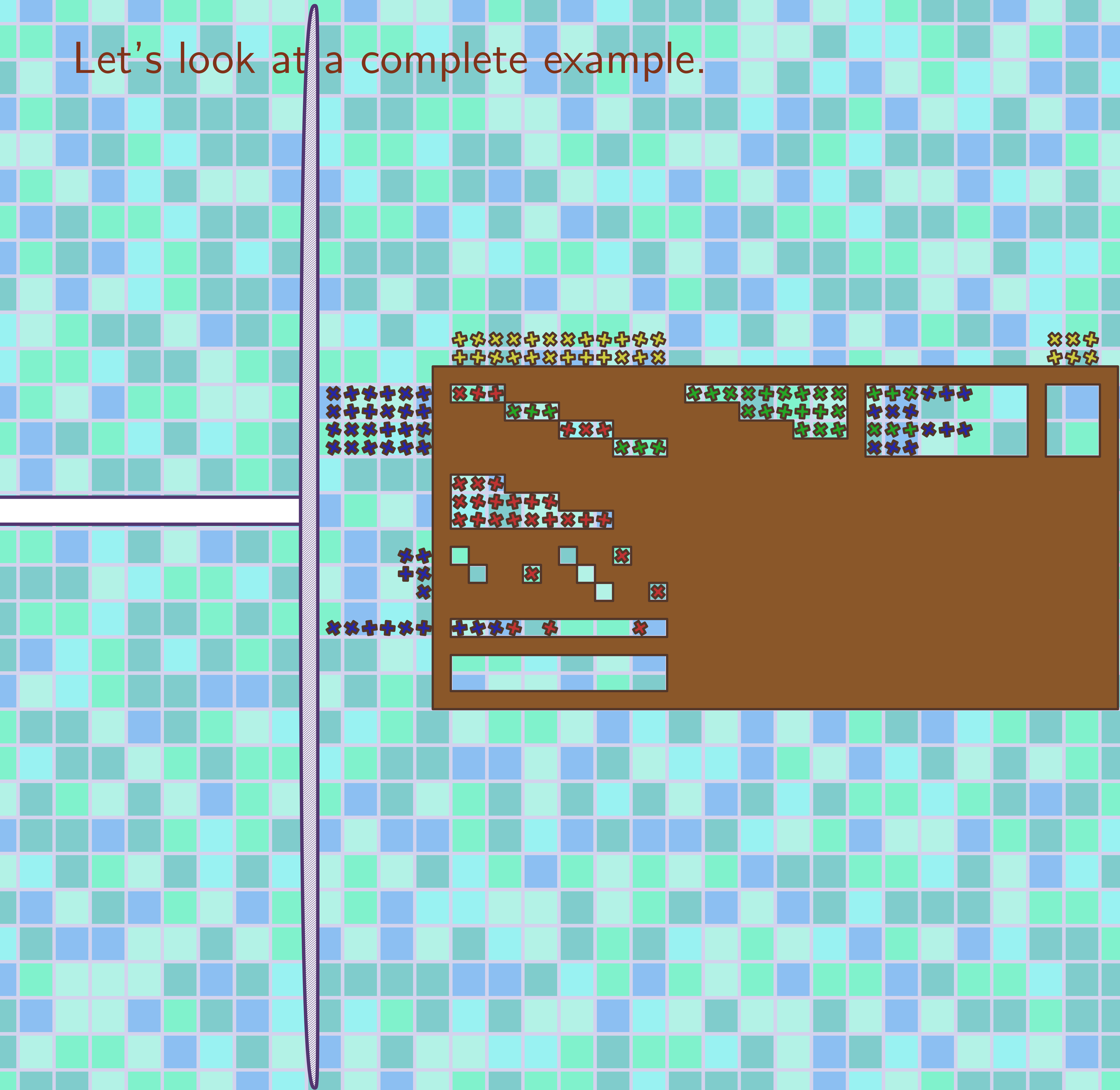

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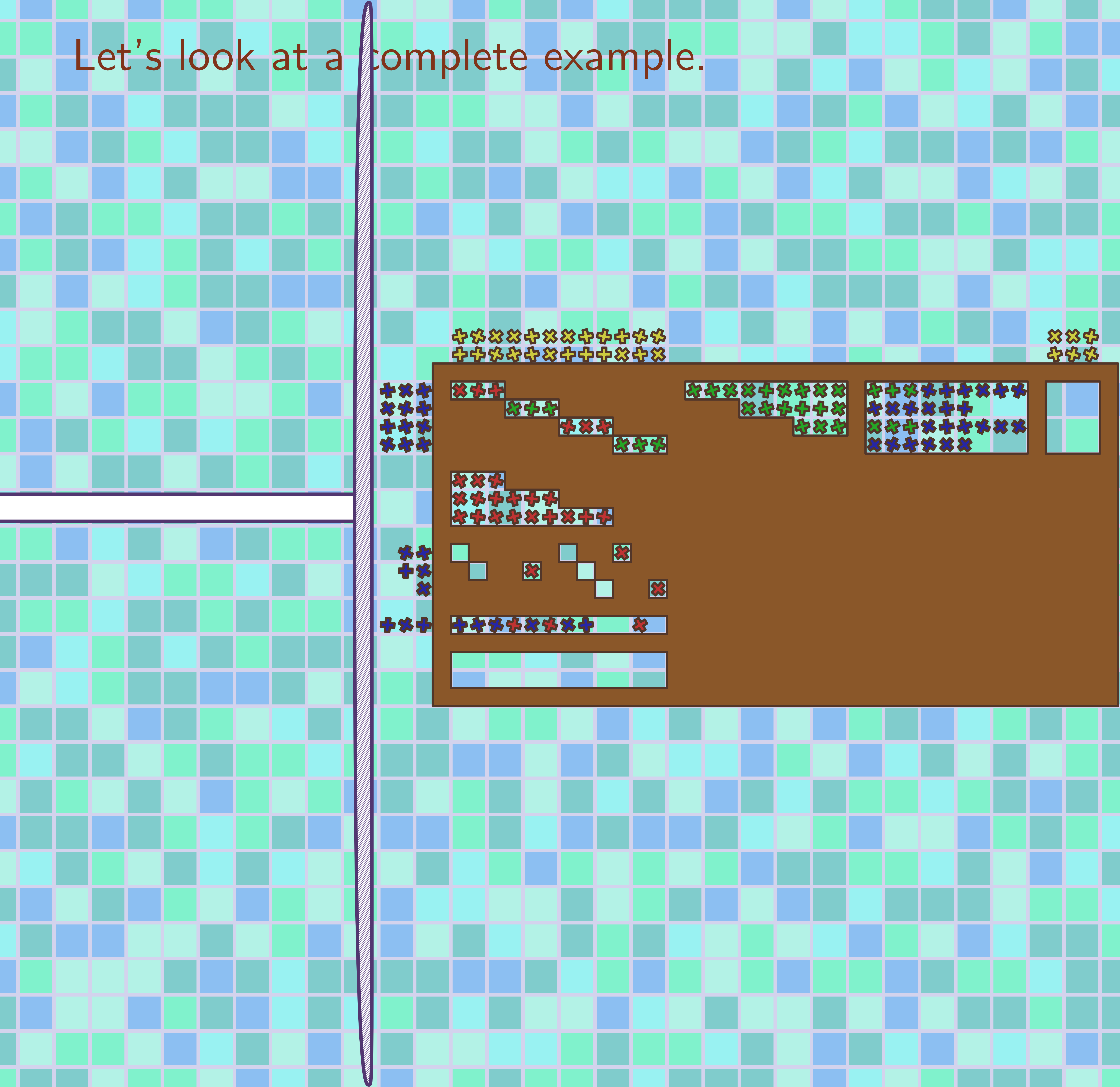

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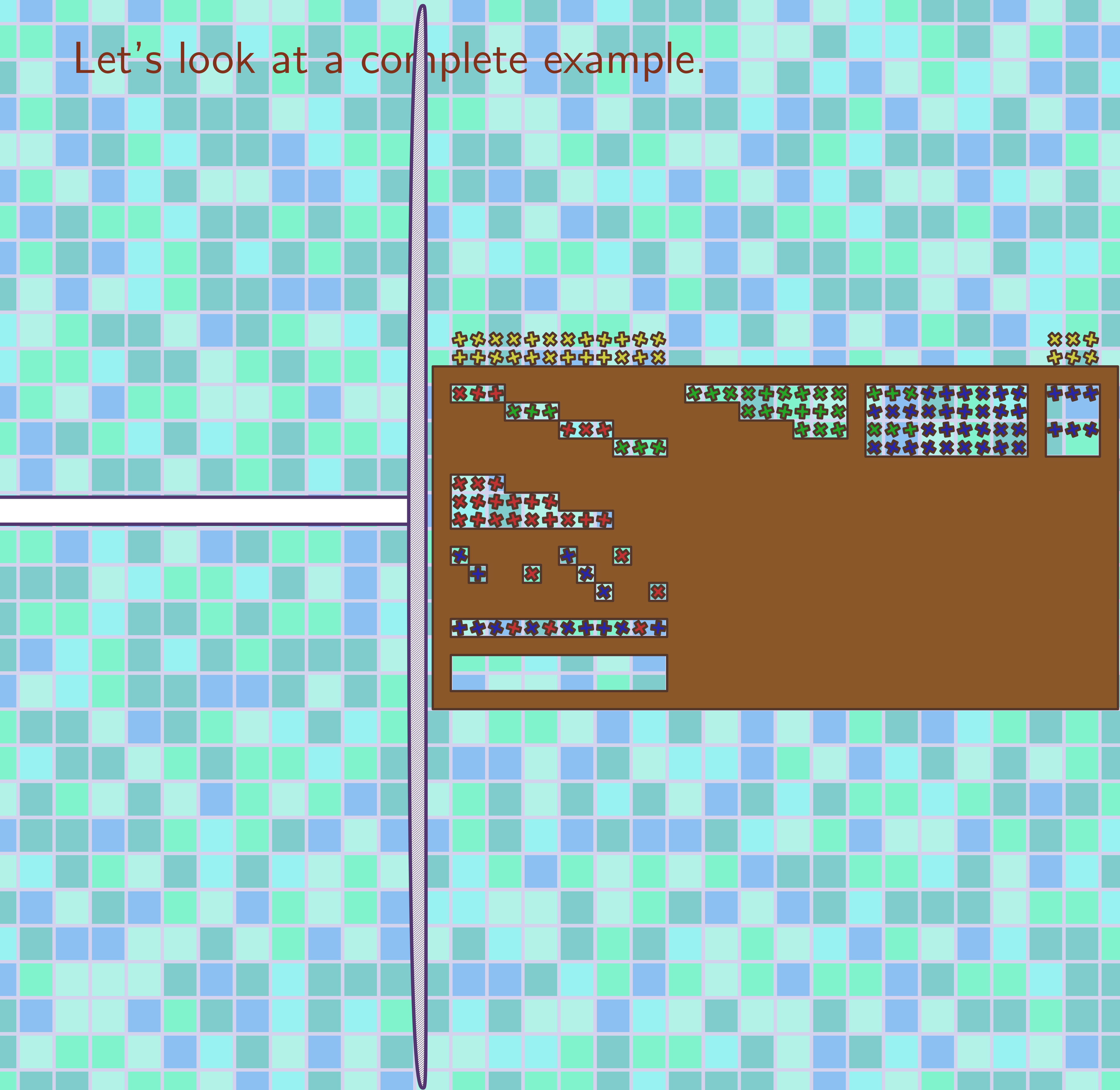

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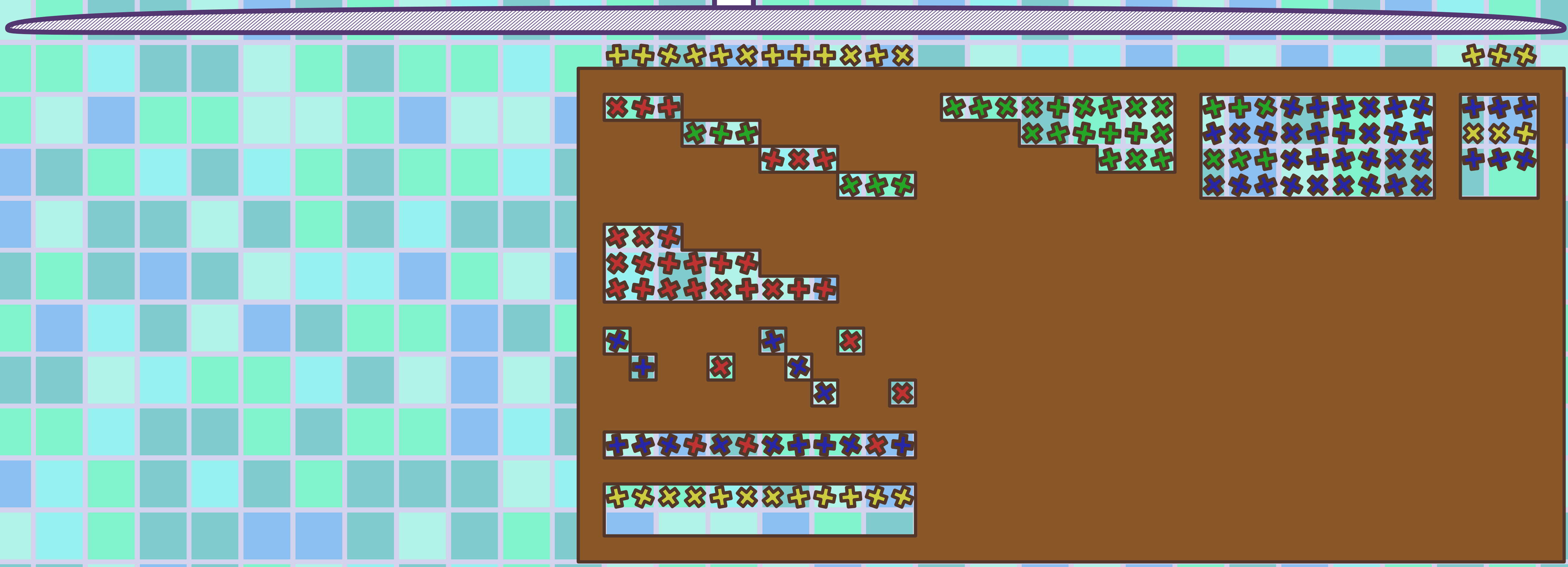

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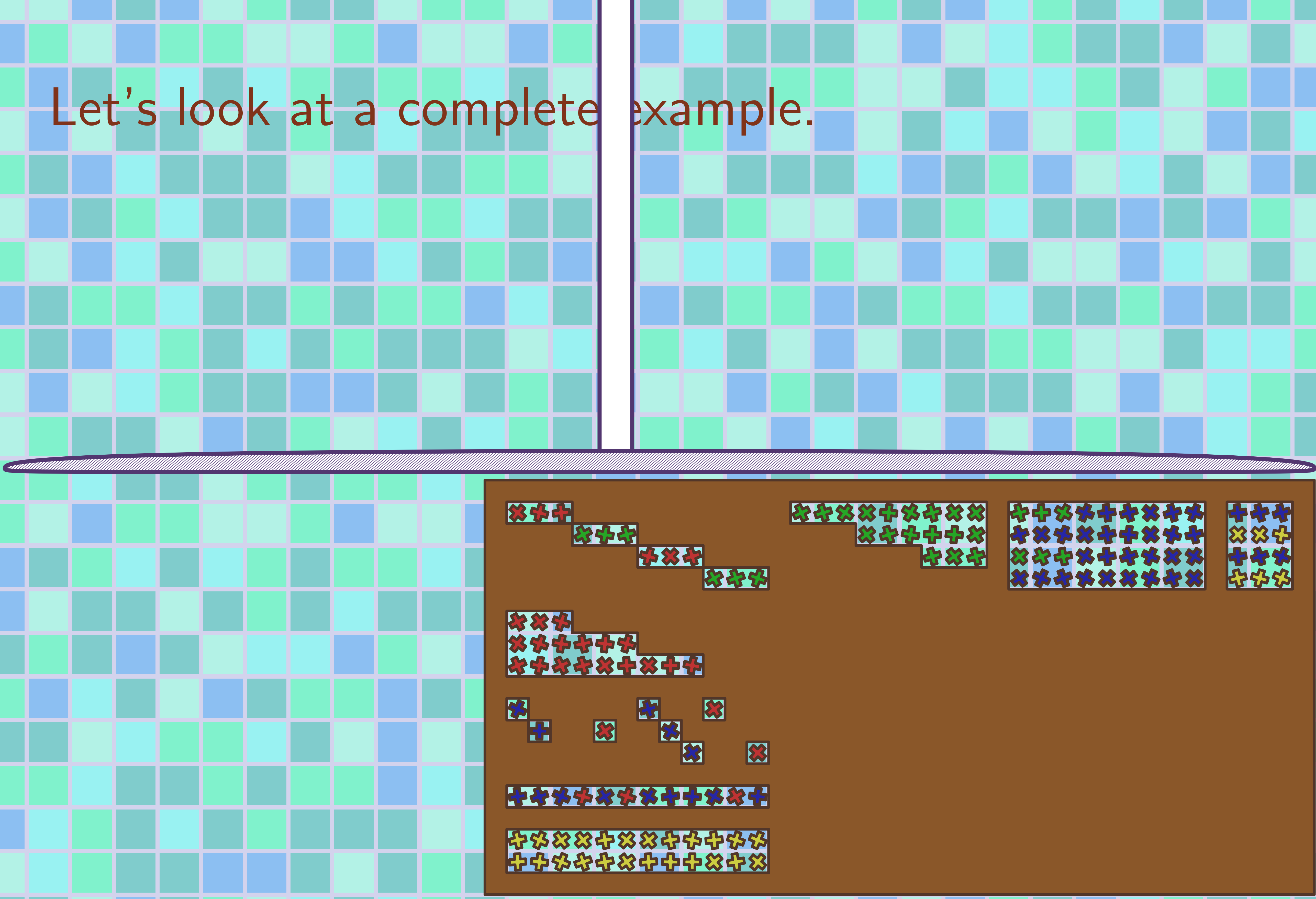

Quod erat demonstrandum.

How about four-sided coconut compaction?

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We can use the same reduction, and add one extra gadget.

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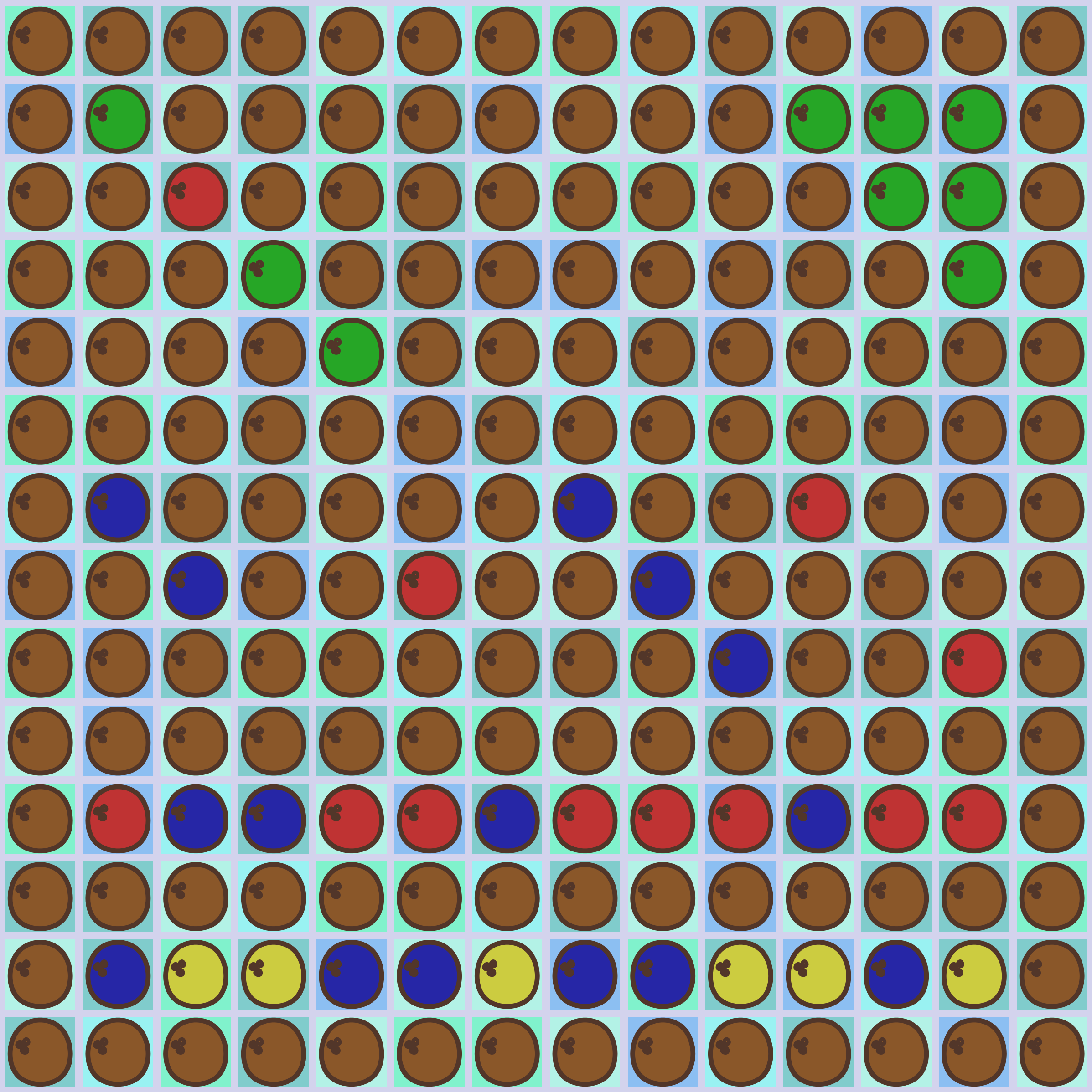

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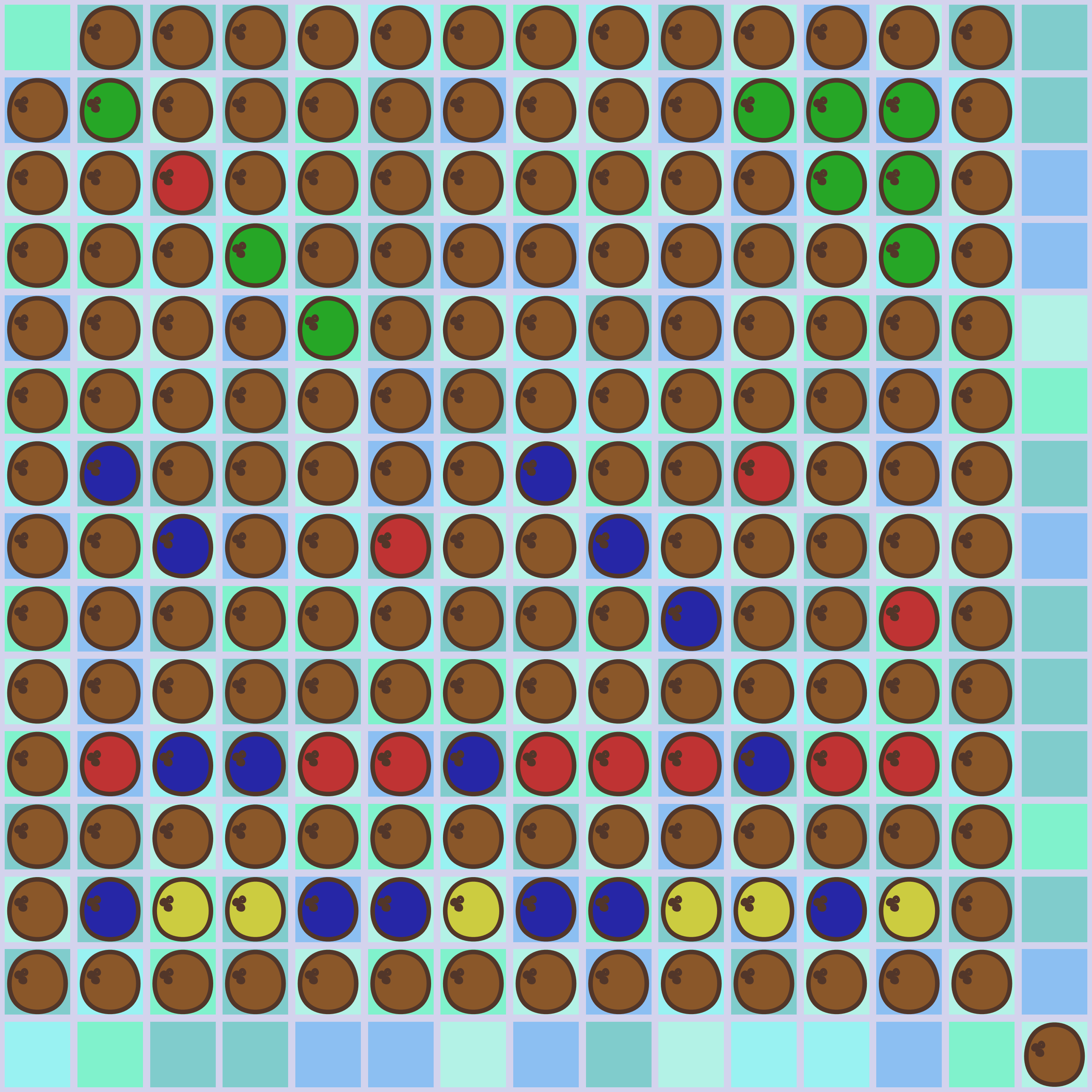

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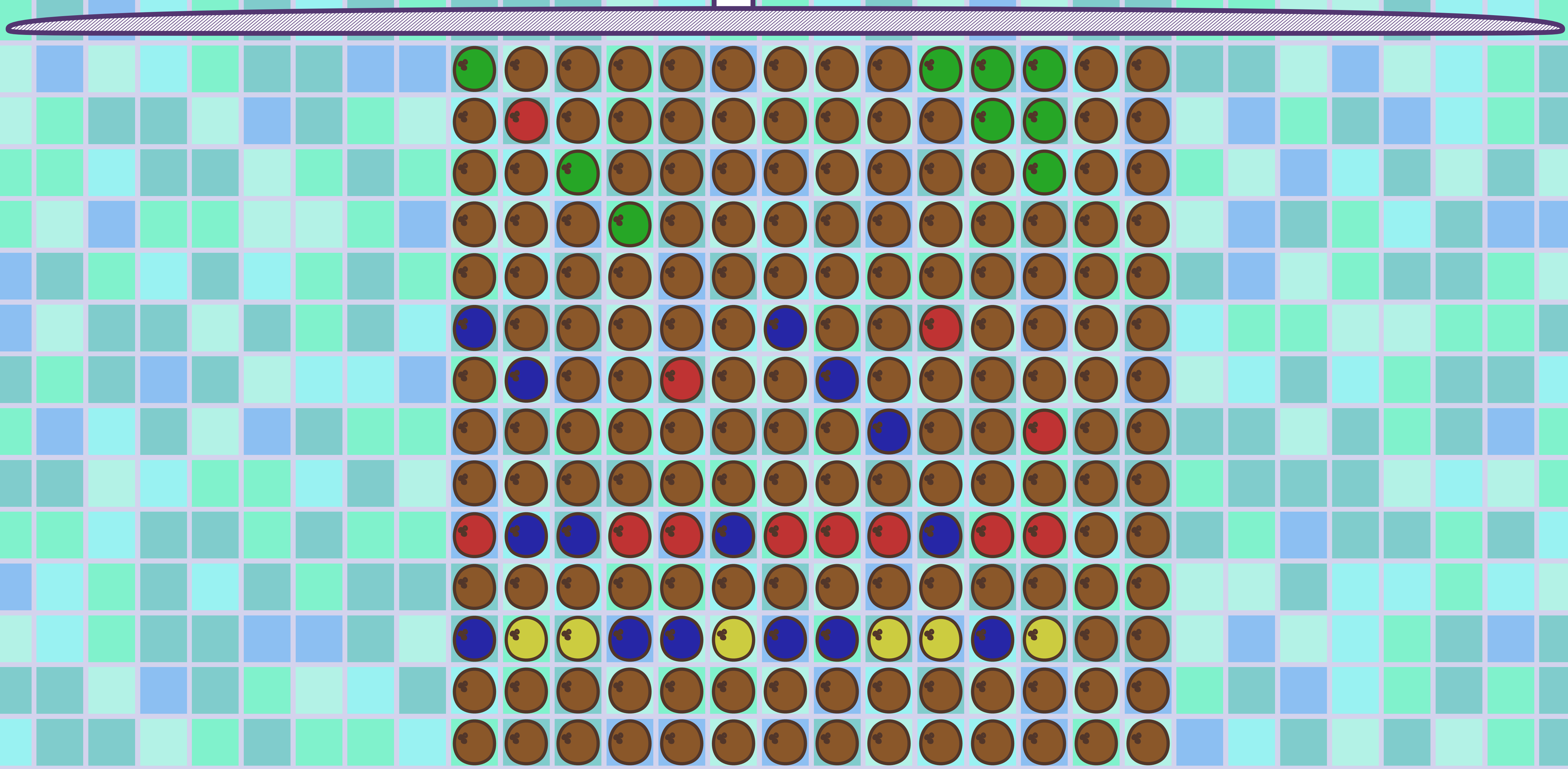

We just move one brown coconut to the bottom left.

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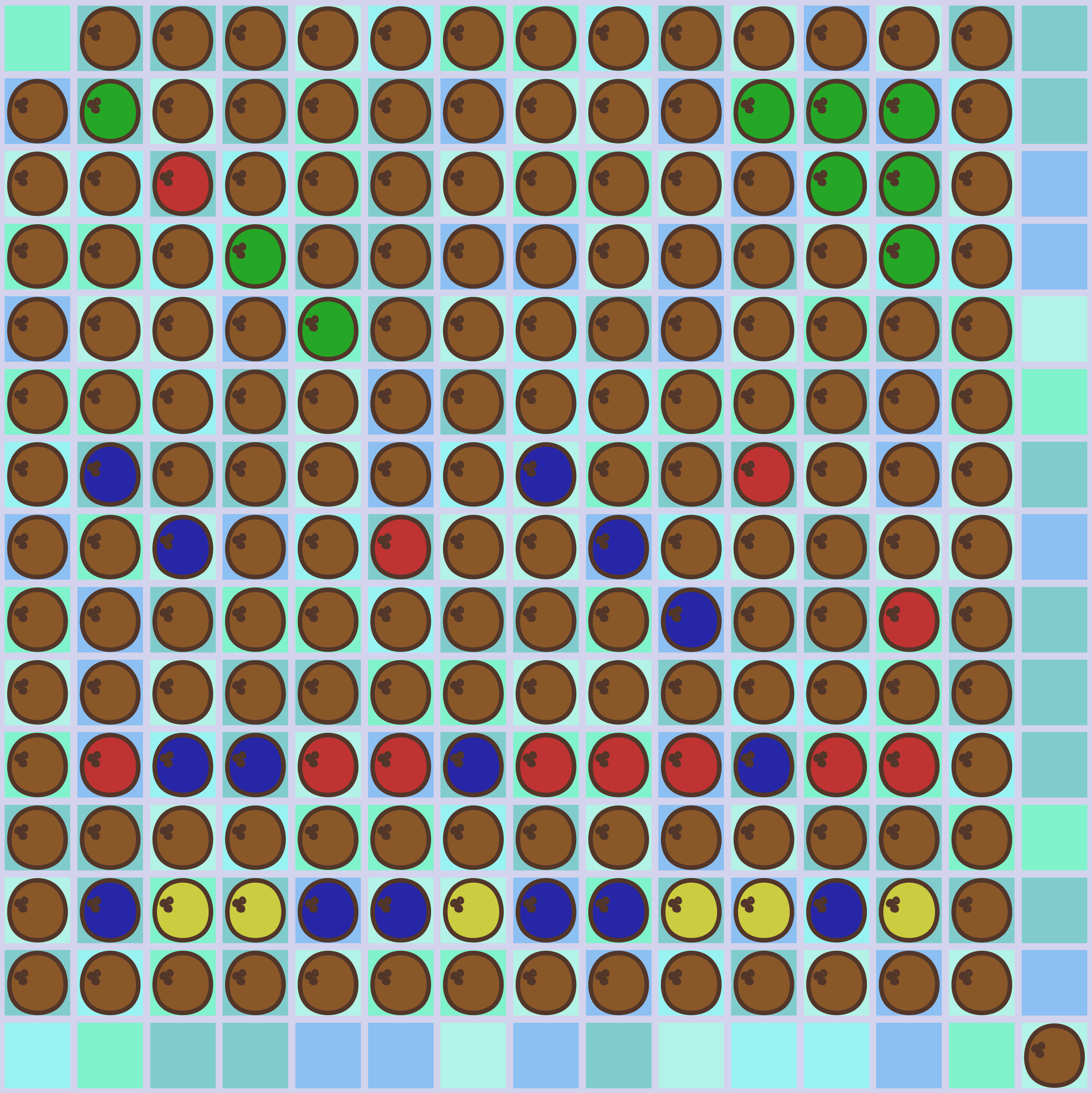

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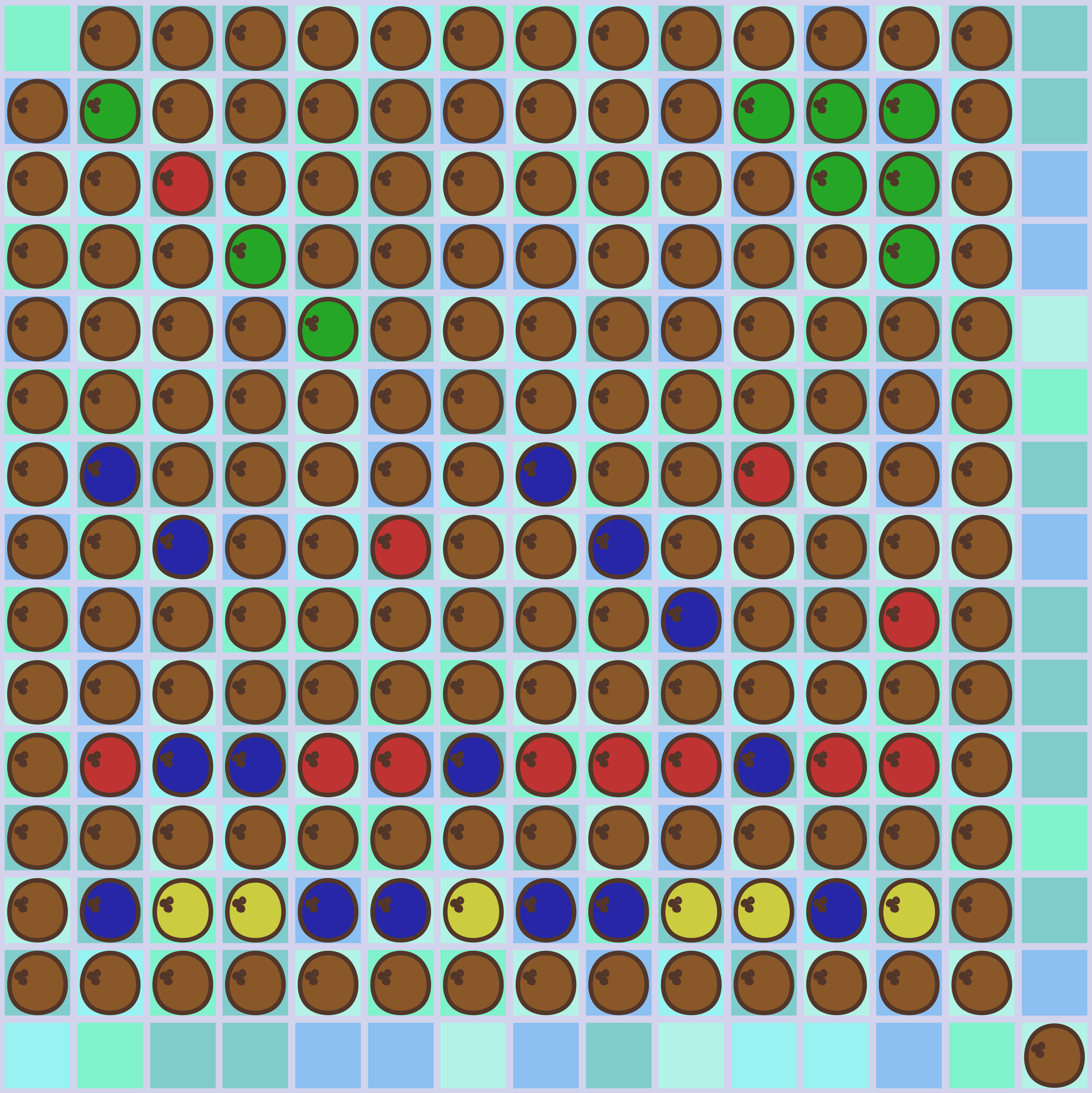

We just move one brown coconut to the bottom left.

Observation
Pushing from the bottom or right at any time creates a row or column with too many coconuts.

COCONUT QOUESTIONS?