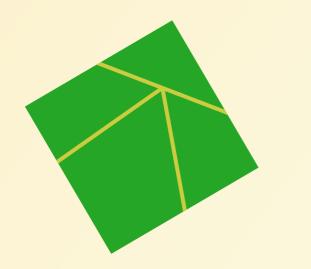
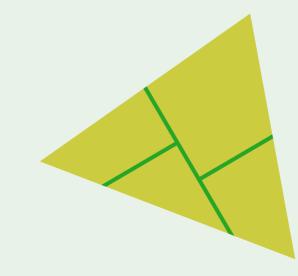
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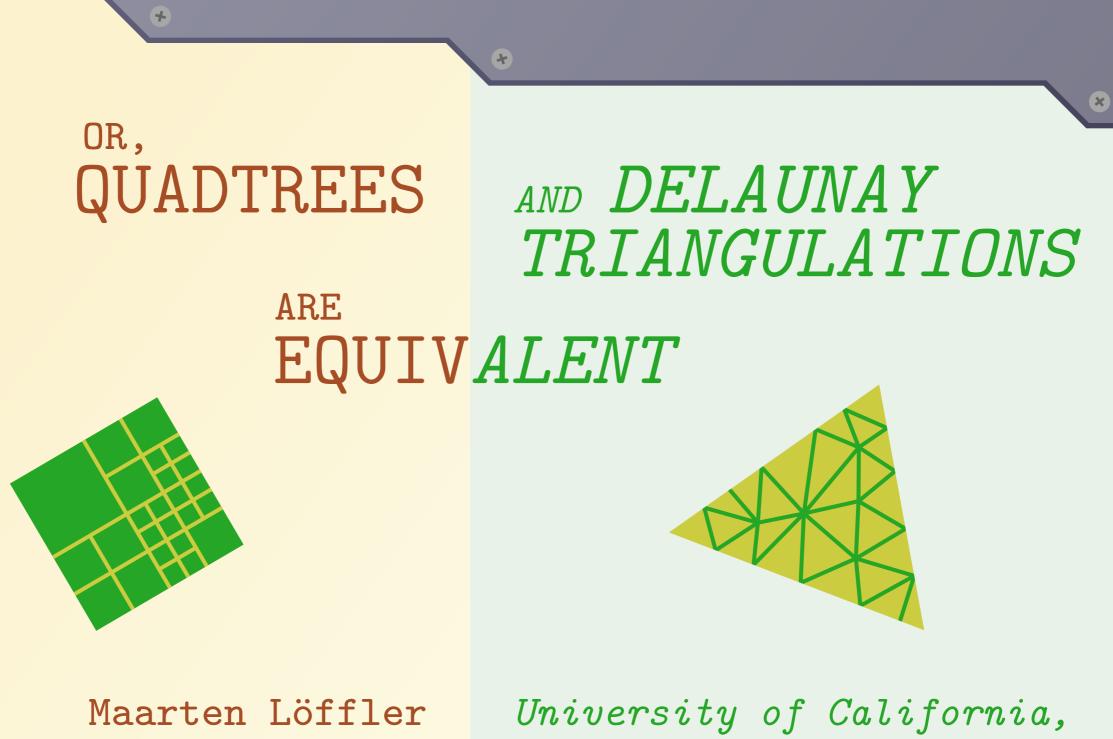
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Maarten Löffler Wolfgang Mulzer University of California, Irvine Freie Universität Berlin



Wolfgang Mulzer

University of California, Irvine Freie Universität Berlin

PART I INTRODUCTION

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Let P be a set of n points in the plane.

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Let P be a set of n points in the plane.

A proximity structure on P is 'a structure that stores some kind of useful local information'.

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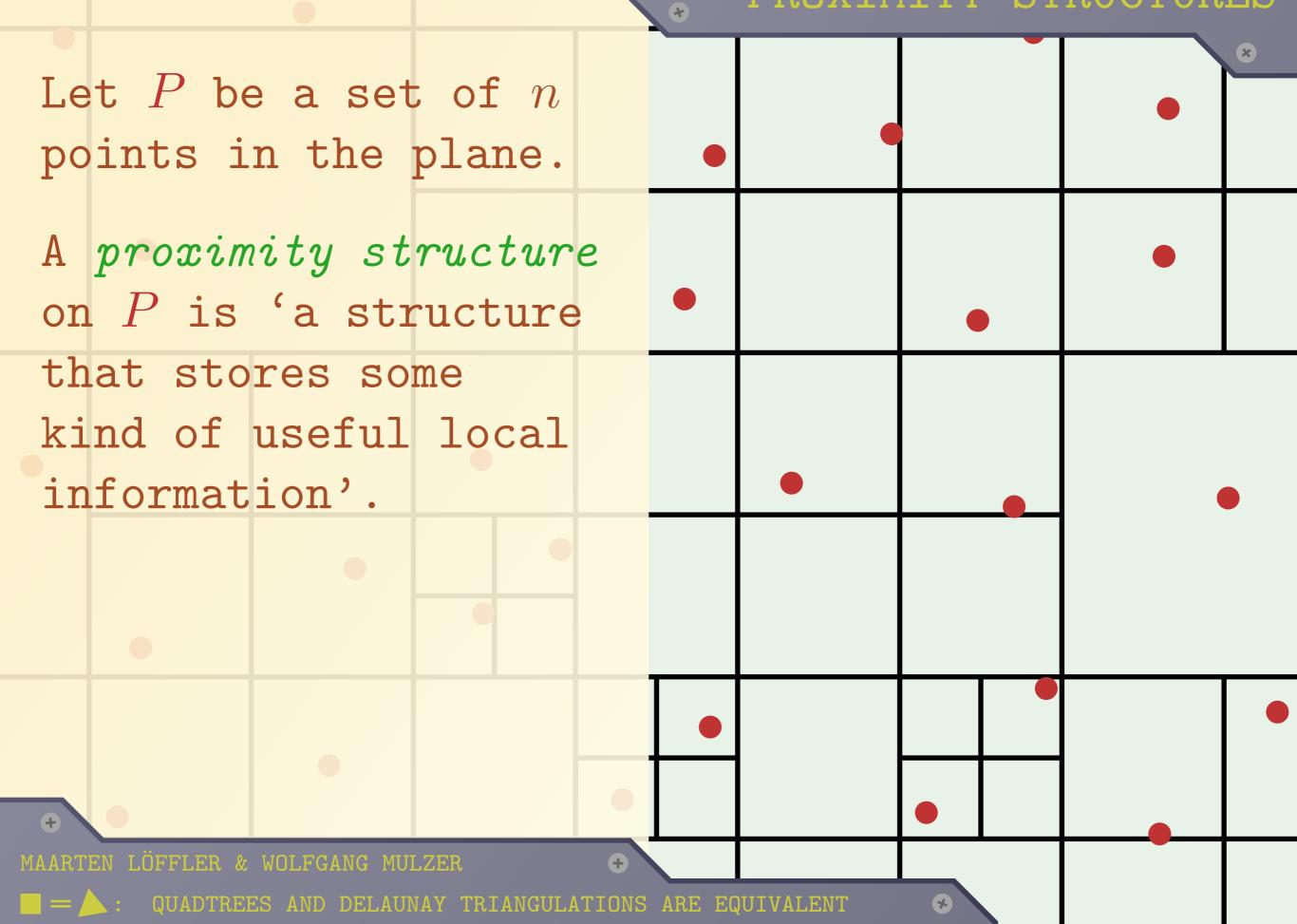
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All these structures take $\Omega(n\log n)$ time to build.

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Intuitively, though, once you built one, you should be able to derive the others.

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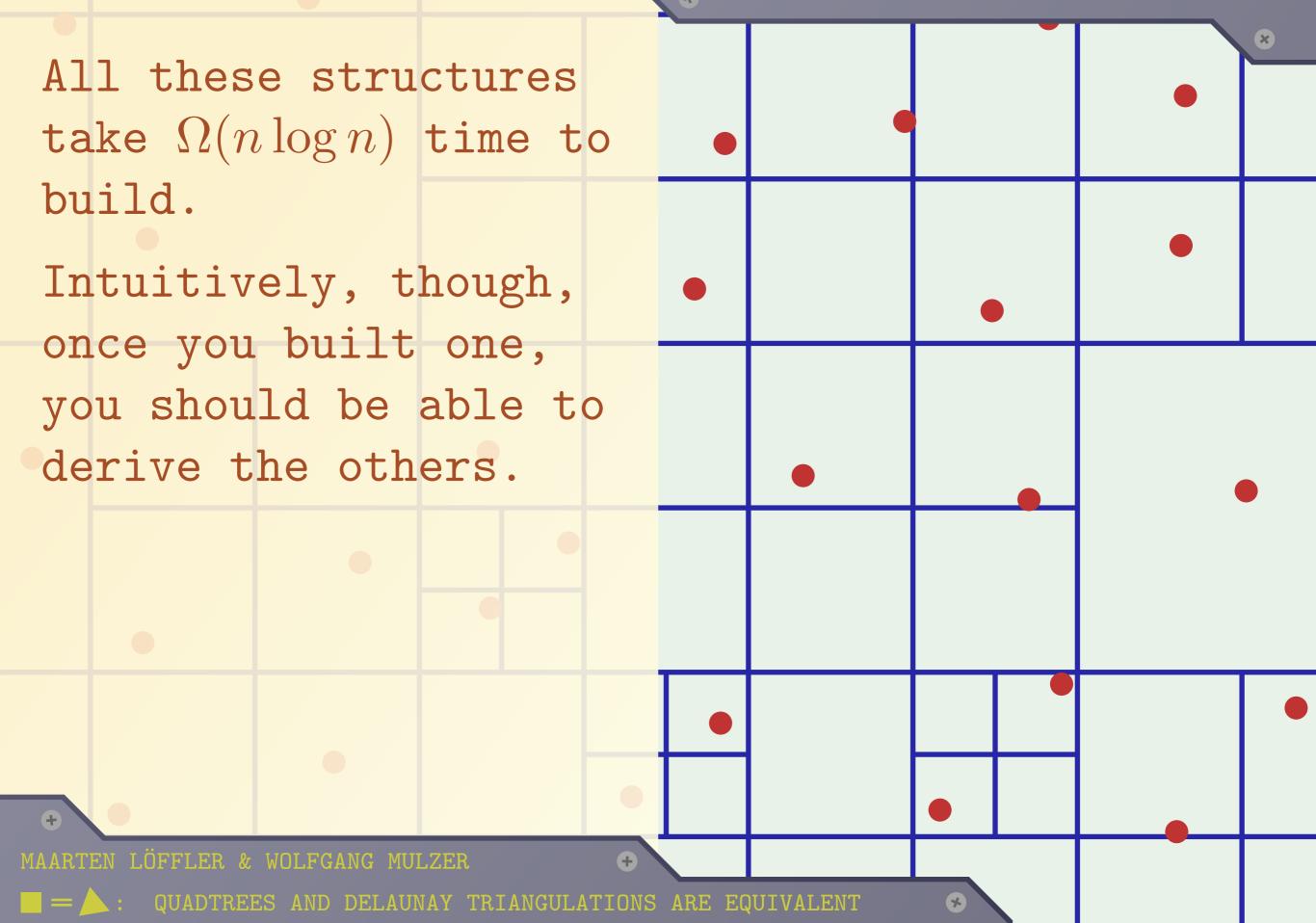
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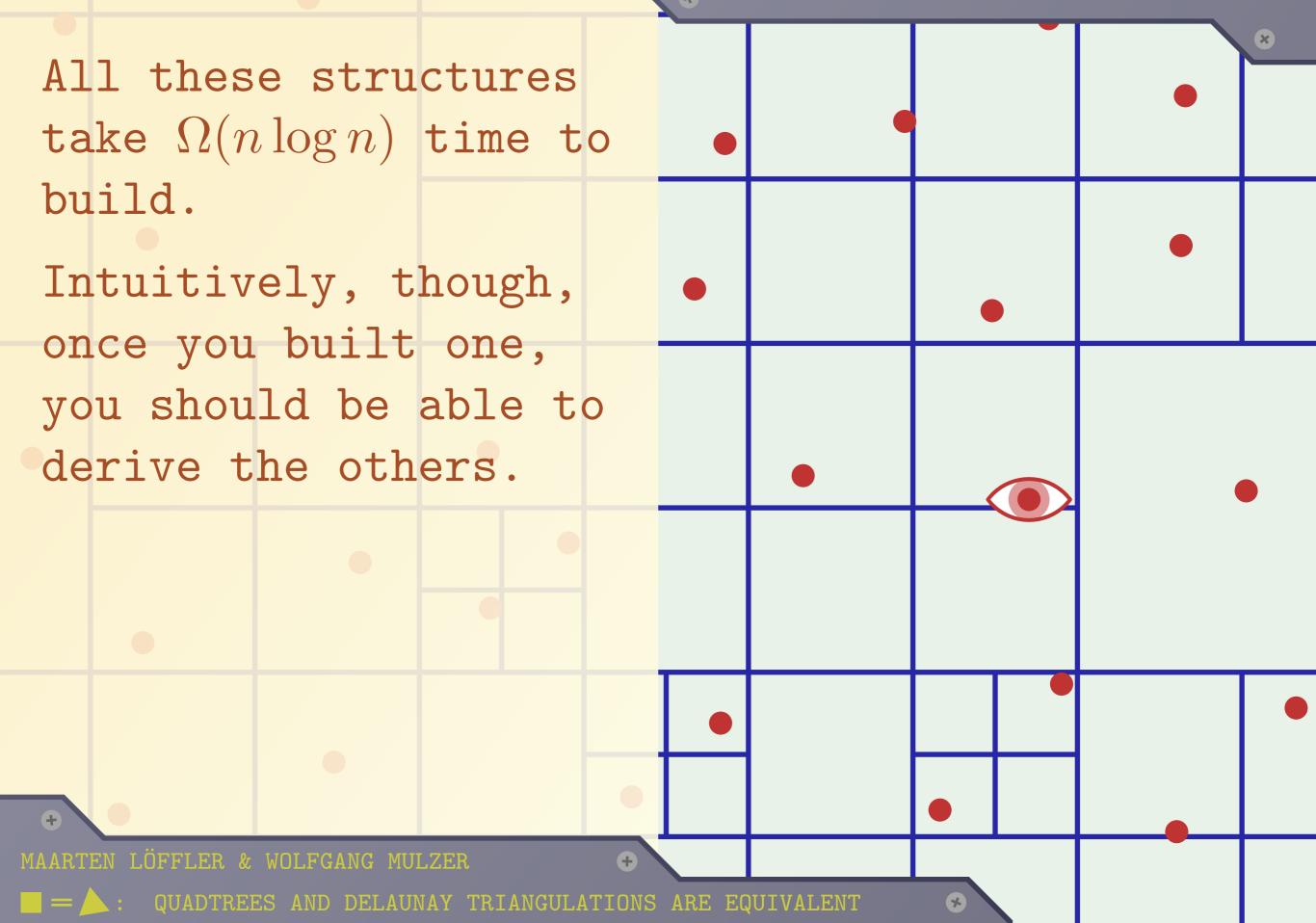
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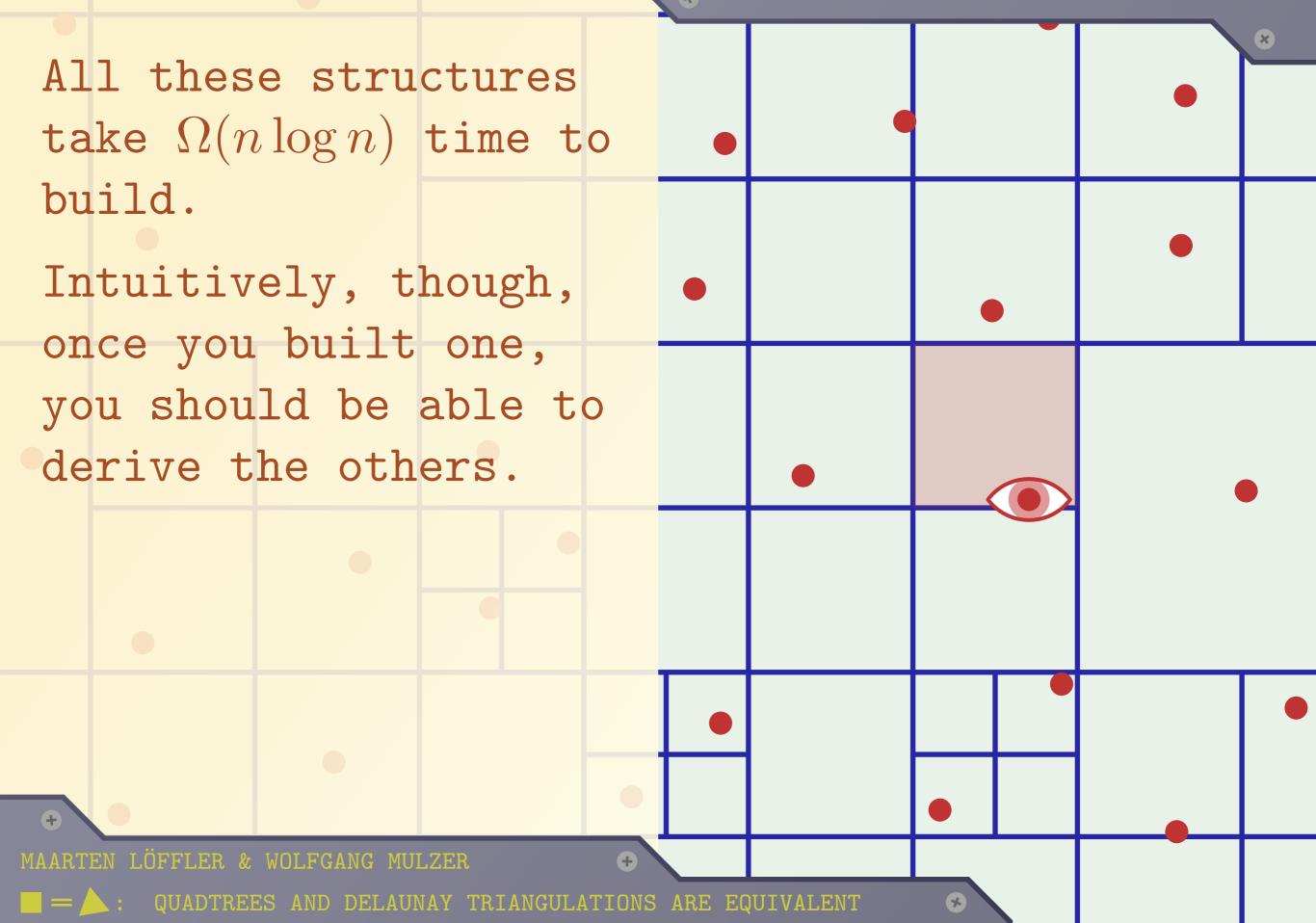
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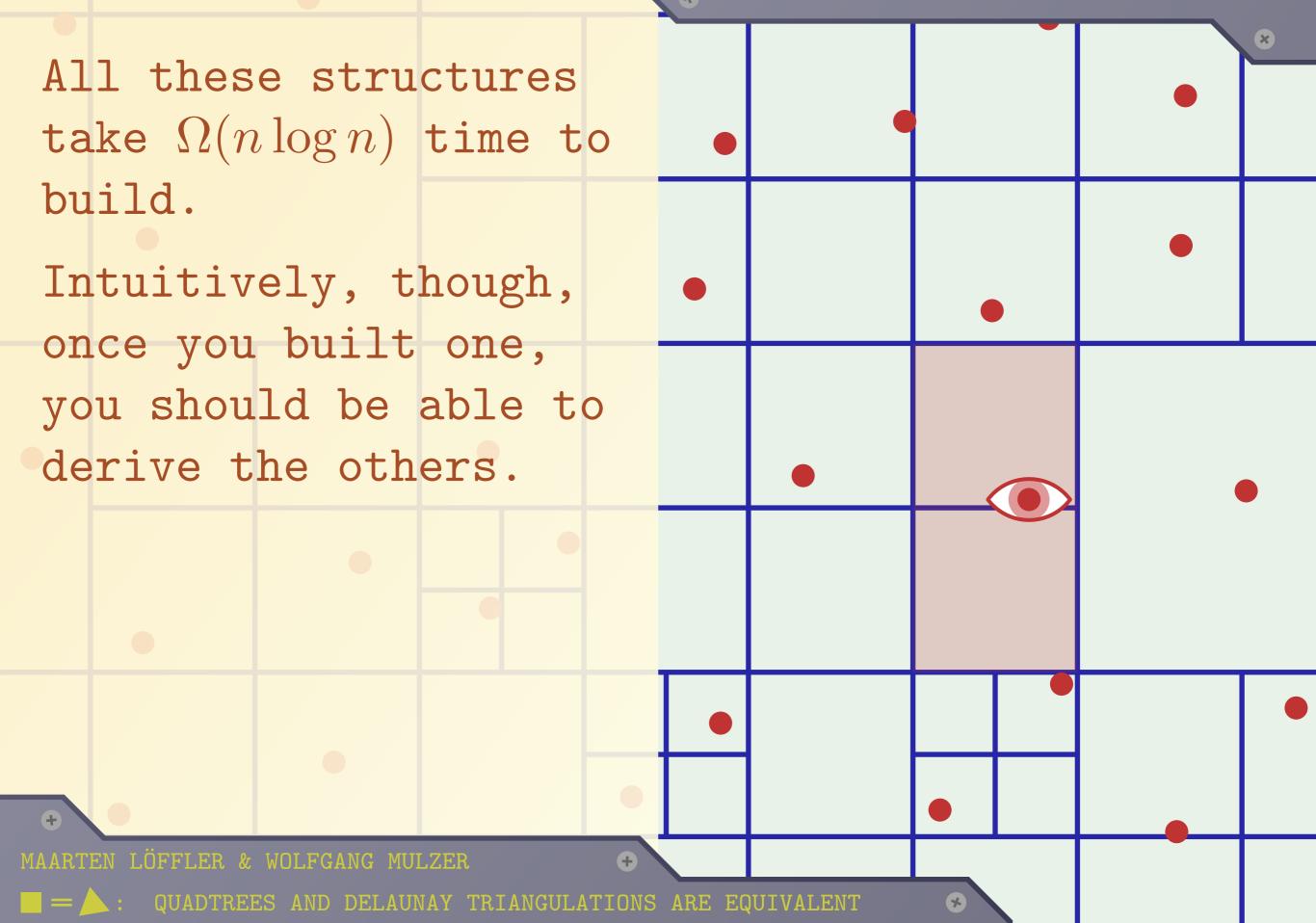
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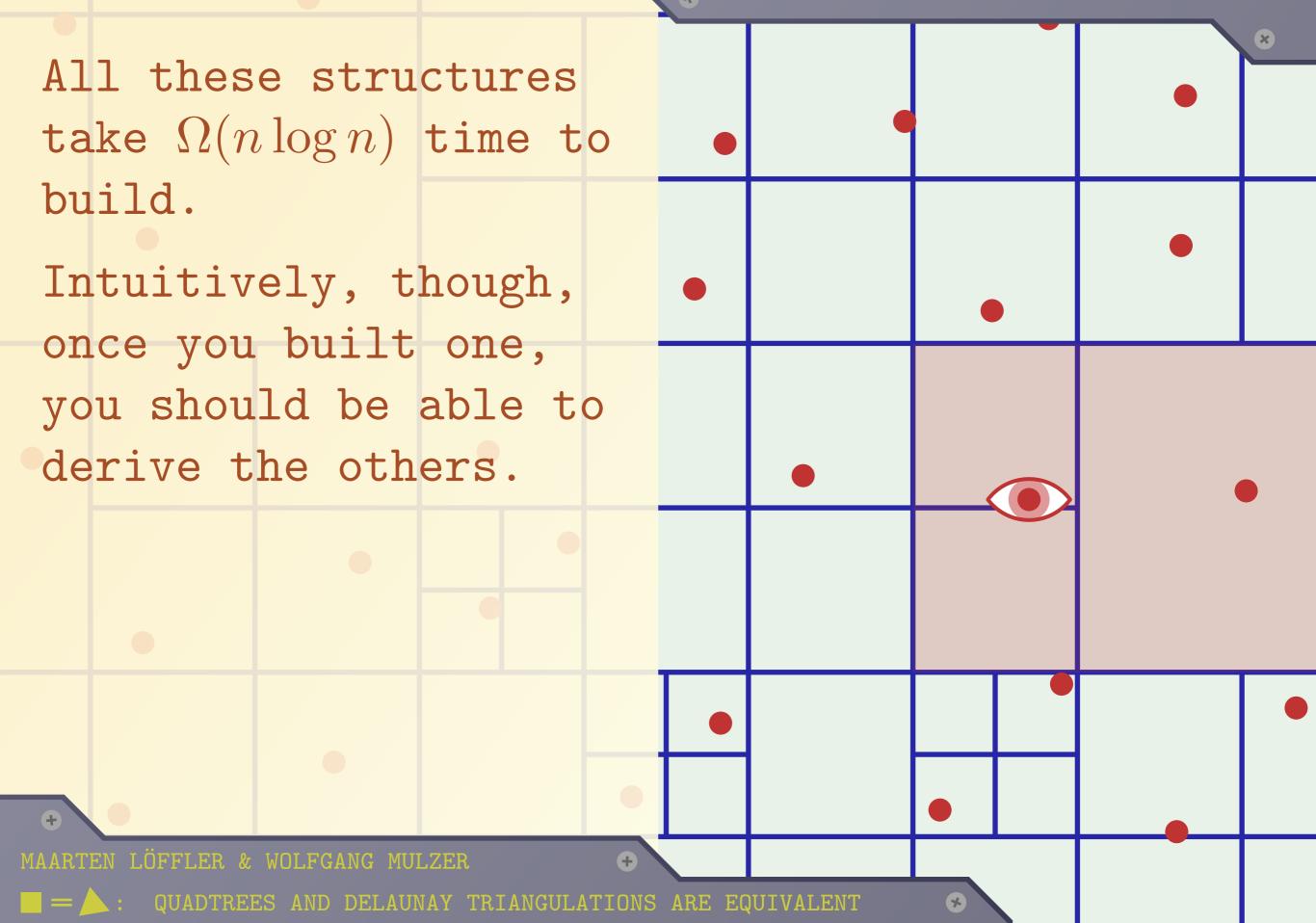
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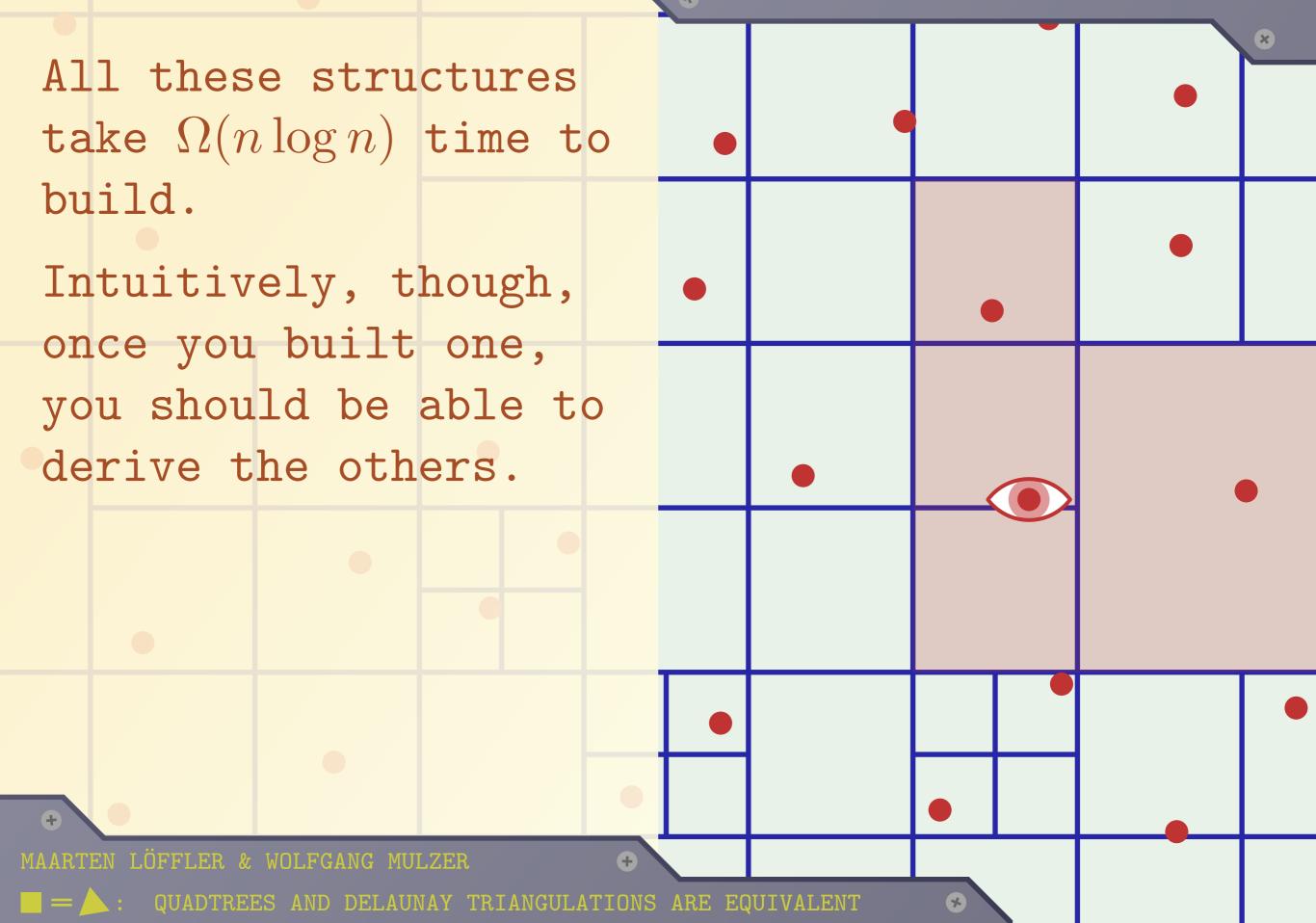


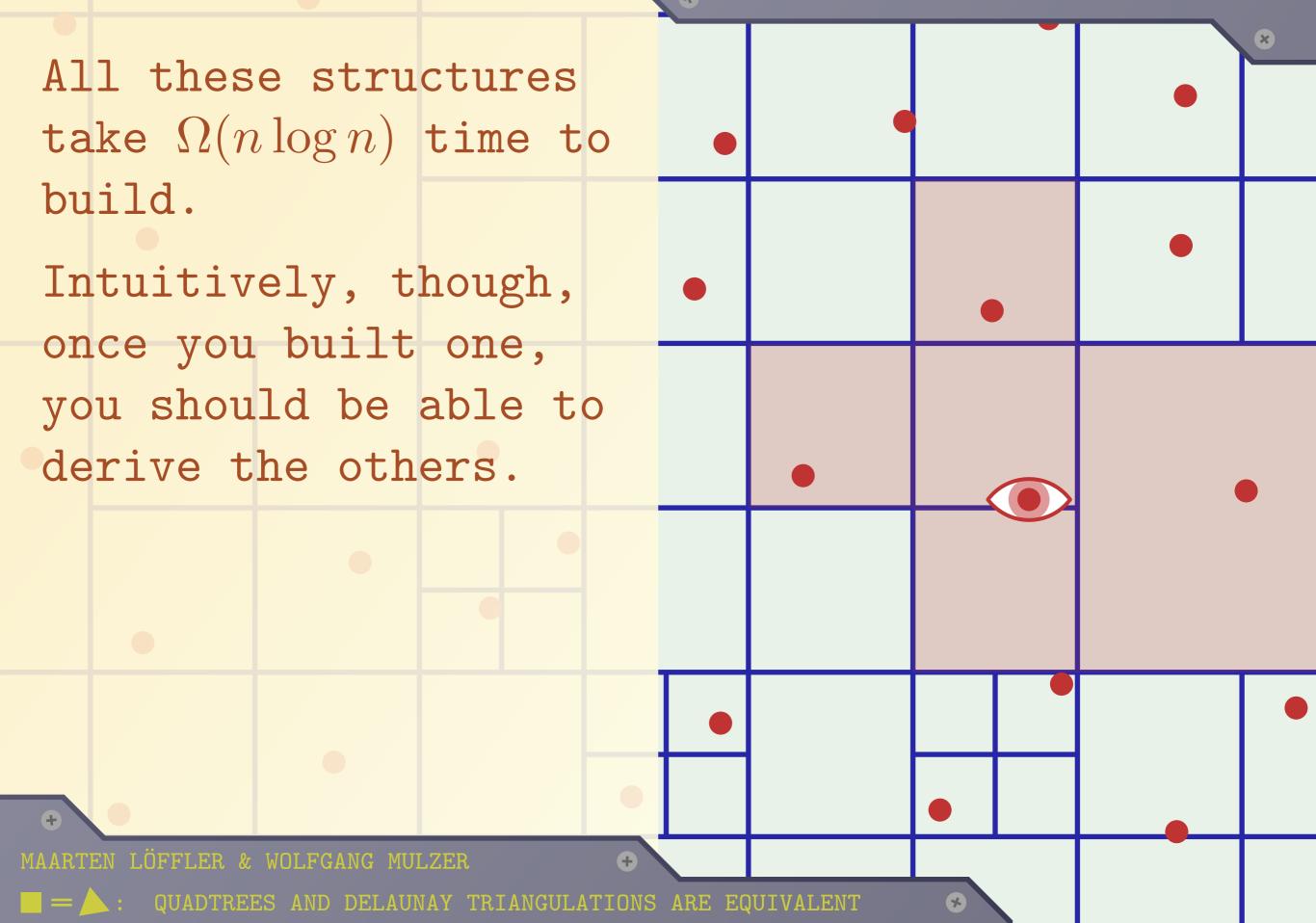


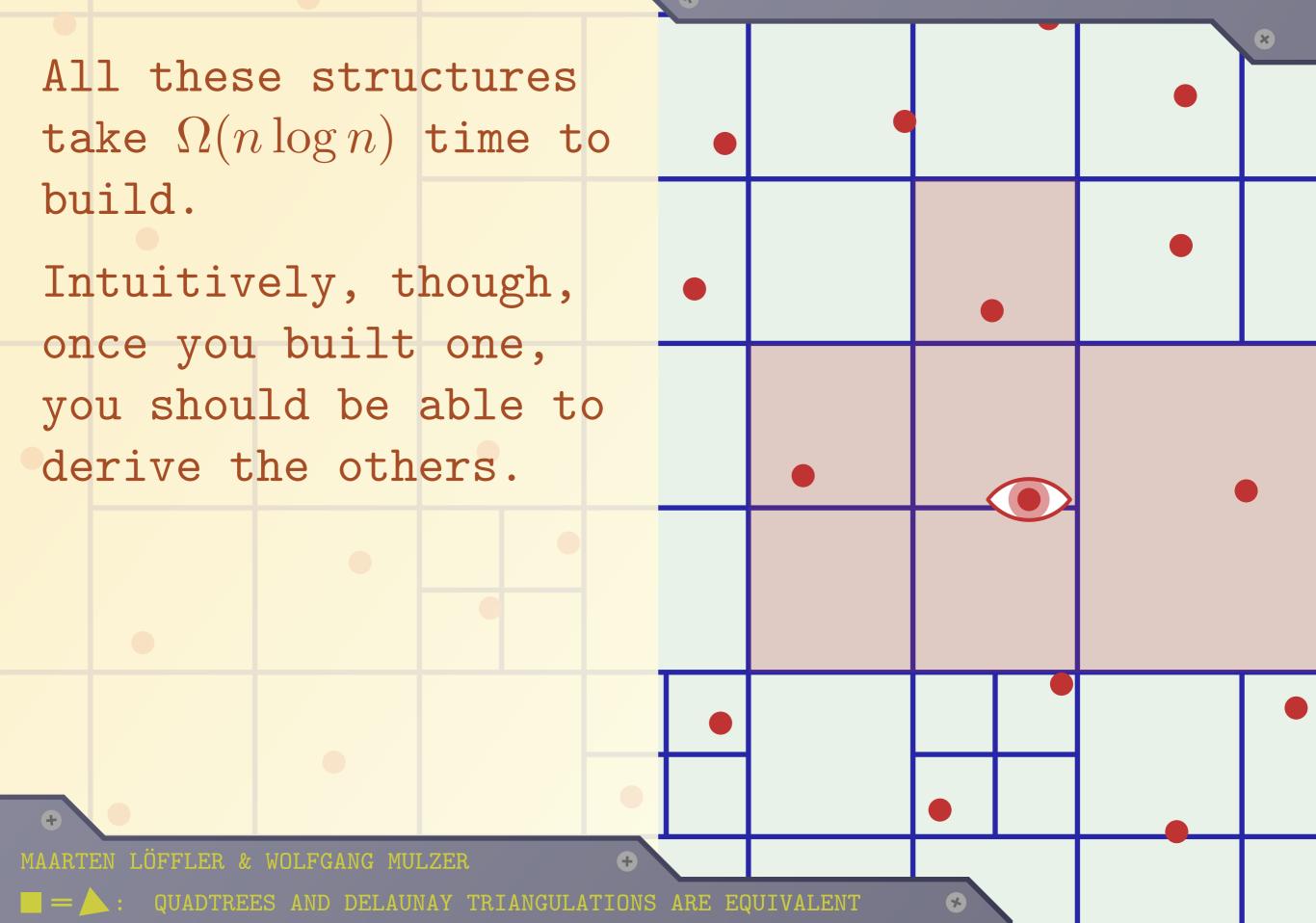


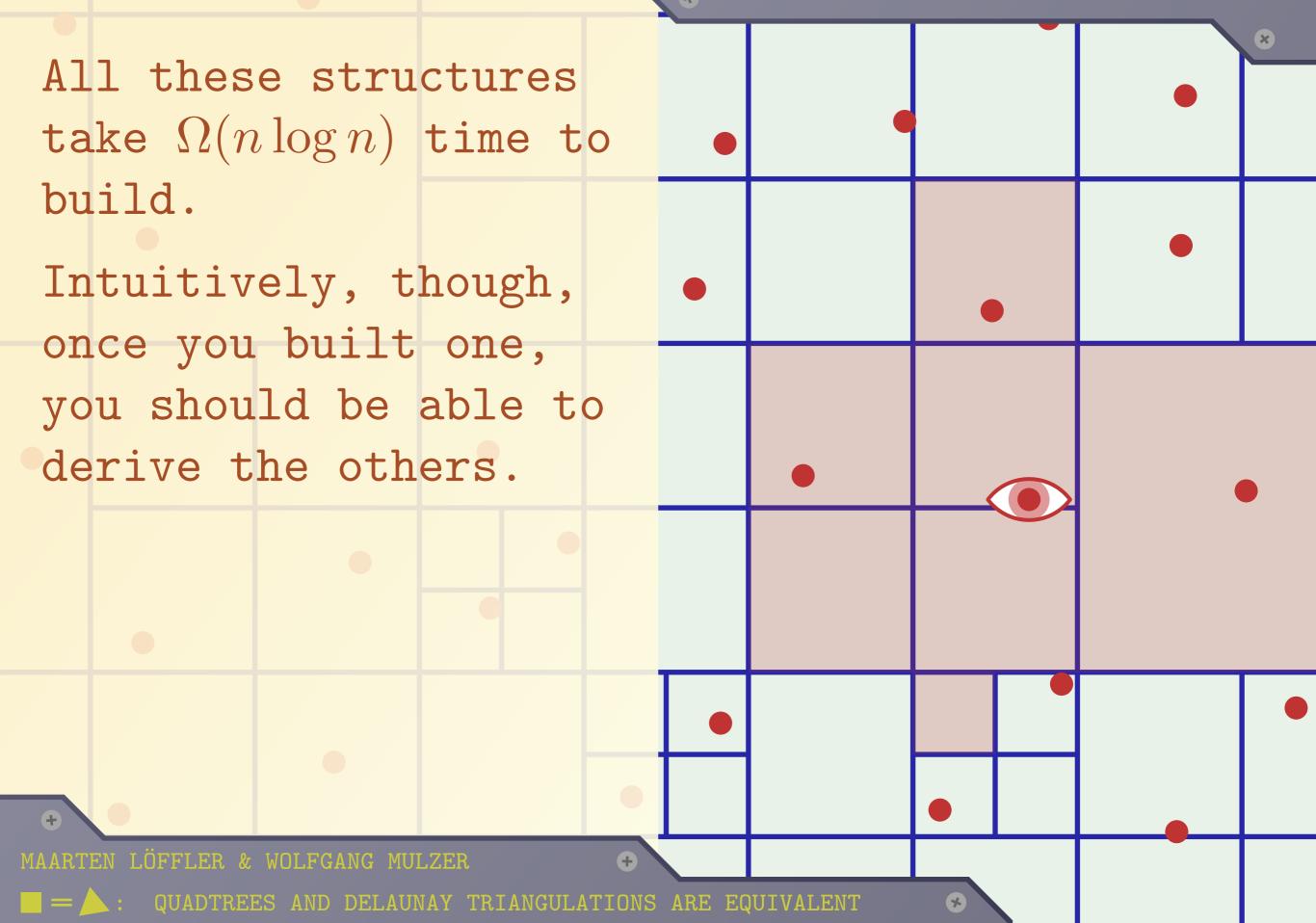


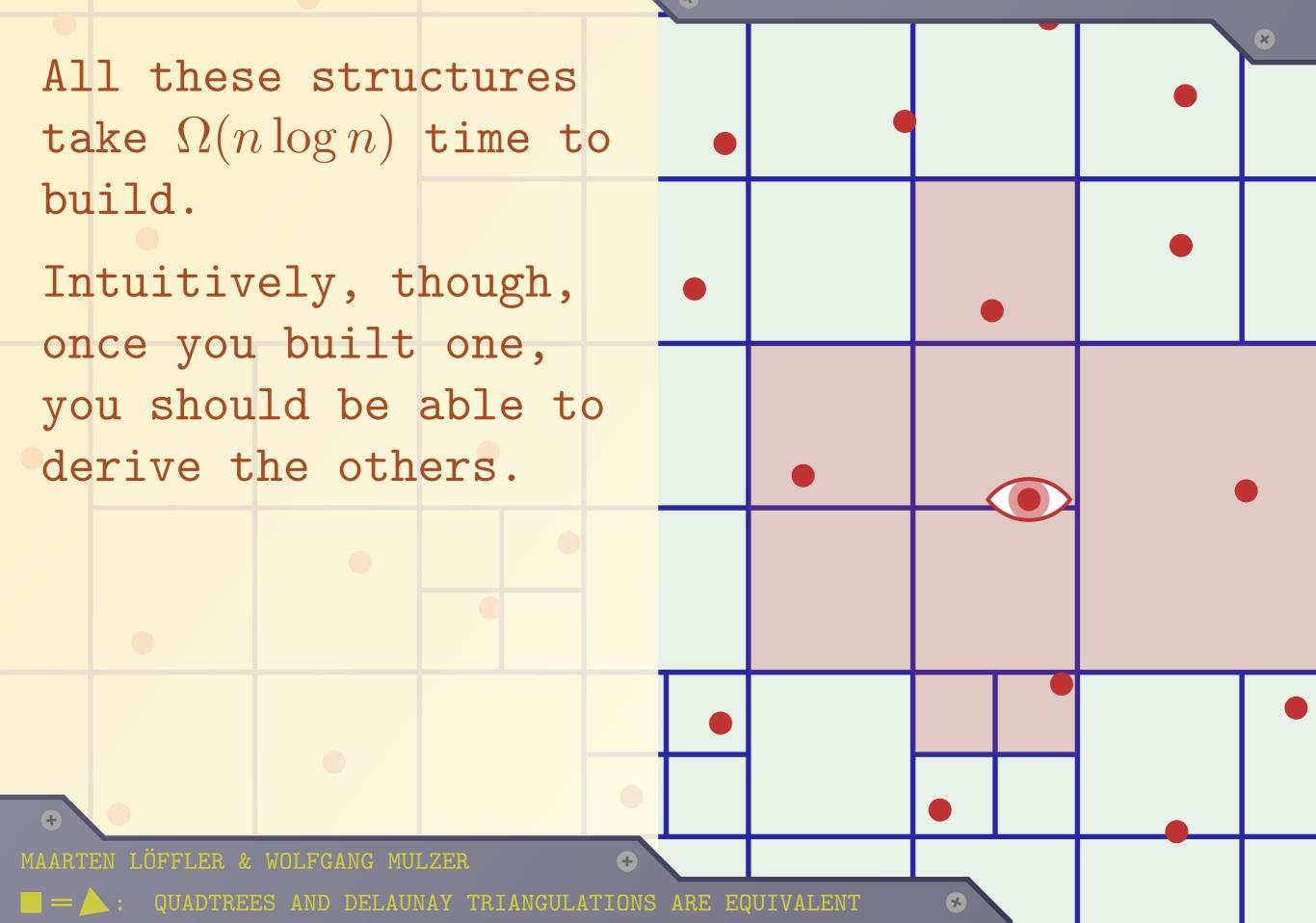


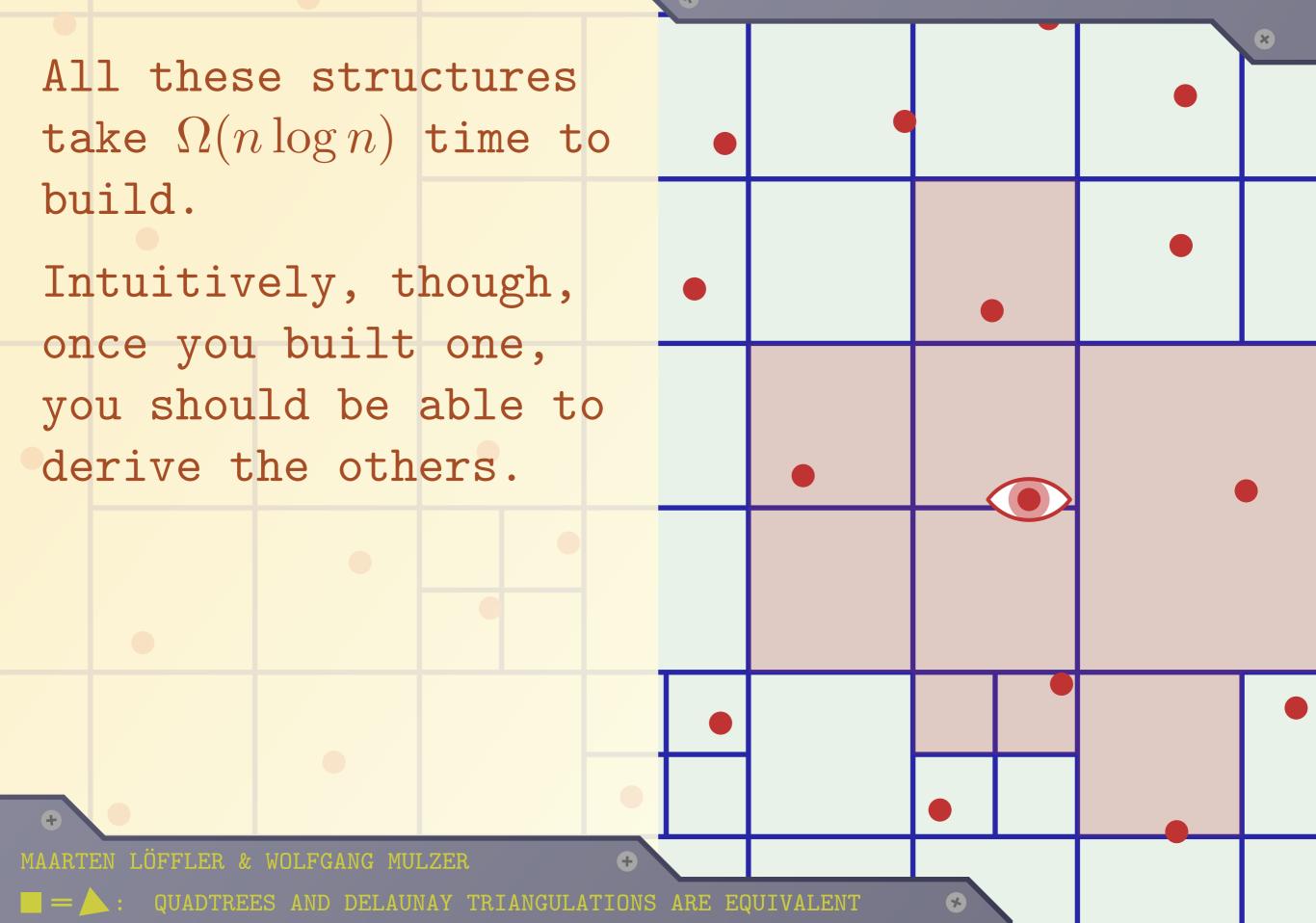


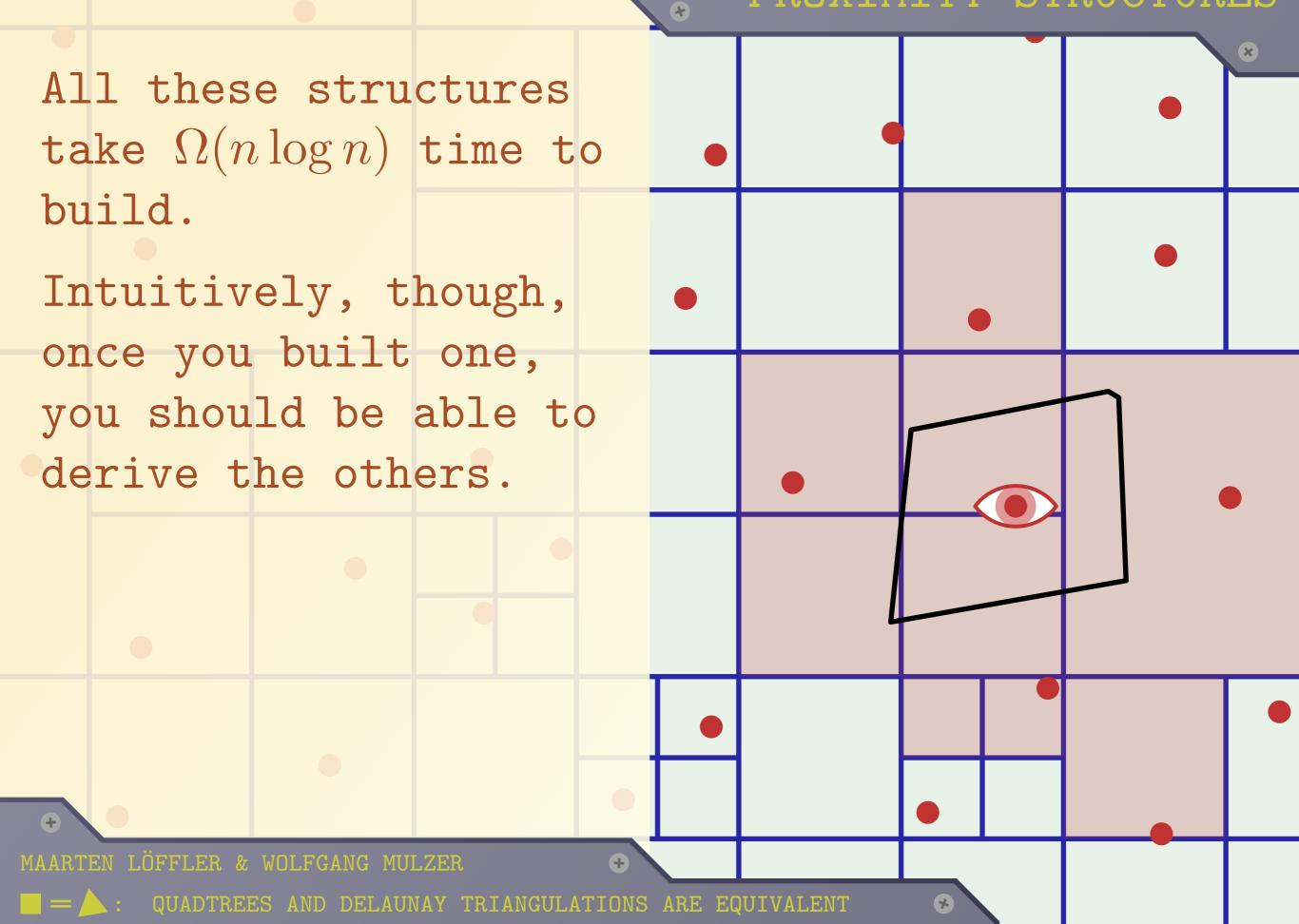


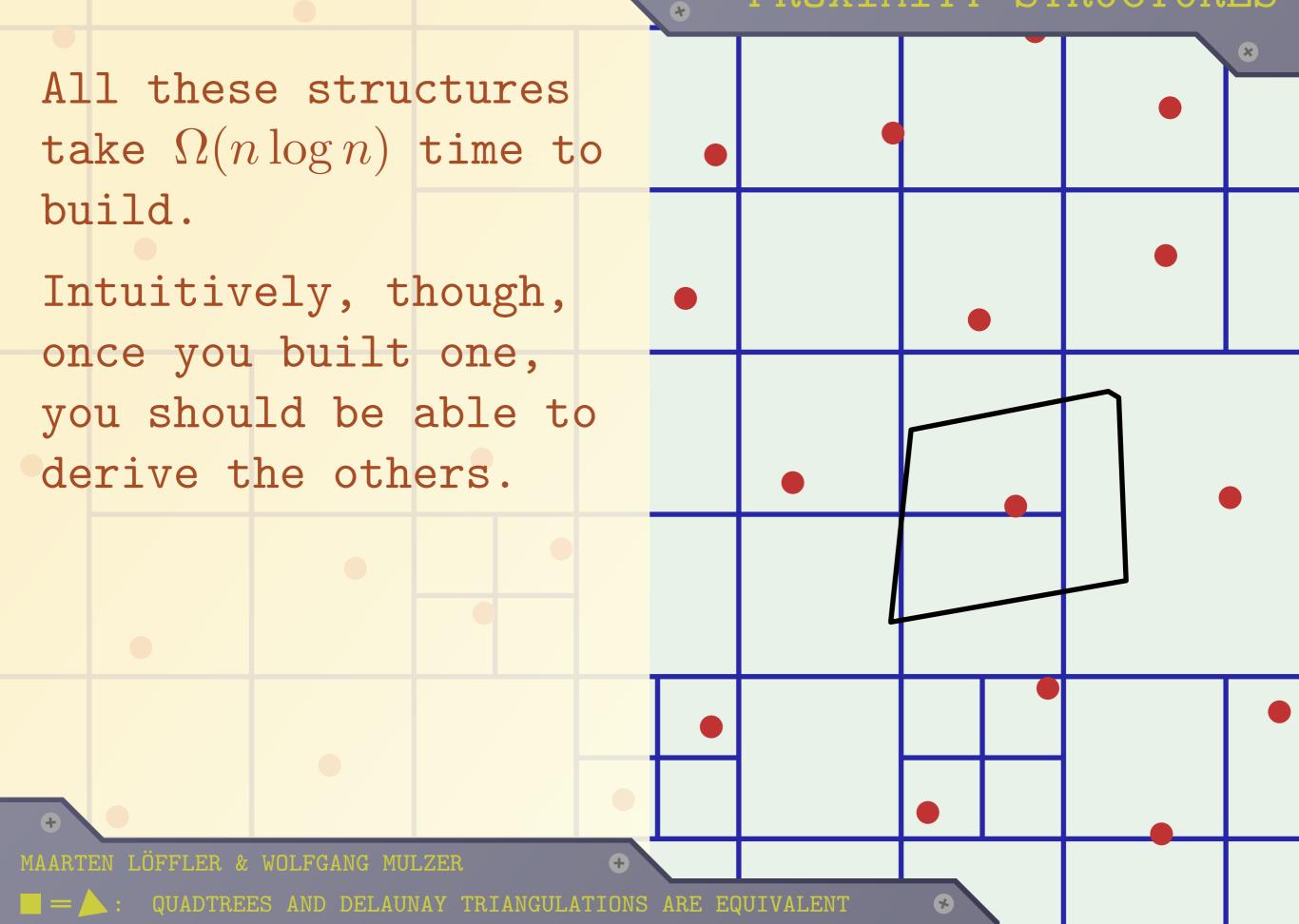


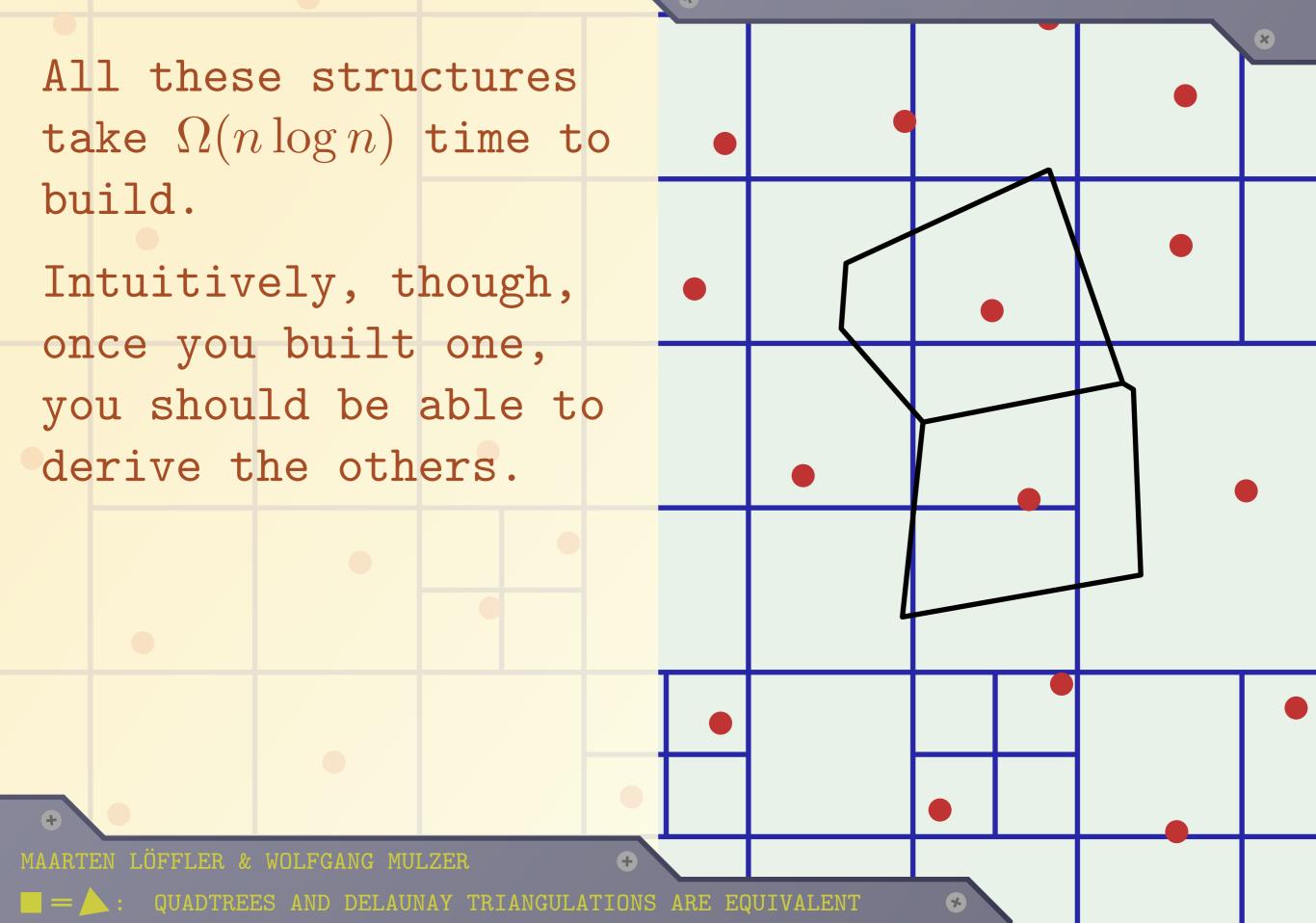












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All these structures take $\Omega(n \log n)$ time to build.

Intuitively, though, once you built one, you should be able to derive the others.

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All these structures take $\Omega(n \log n)$ time to build.

Intuitively, though, once you built one, you should be able to derive the others.

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All these structures take $\Omega(n \log n)$ time to build.

Intuitively, though, once you built one, you should be able to derive the others.

Since information is local, such a conversion should take O(n) time.

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Unfortunately, not all point sets behave themselves.

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Unfortunately, not all point sets behave themselves.

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A single point may have more than $\Omega(1)$ neighbours.

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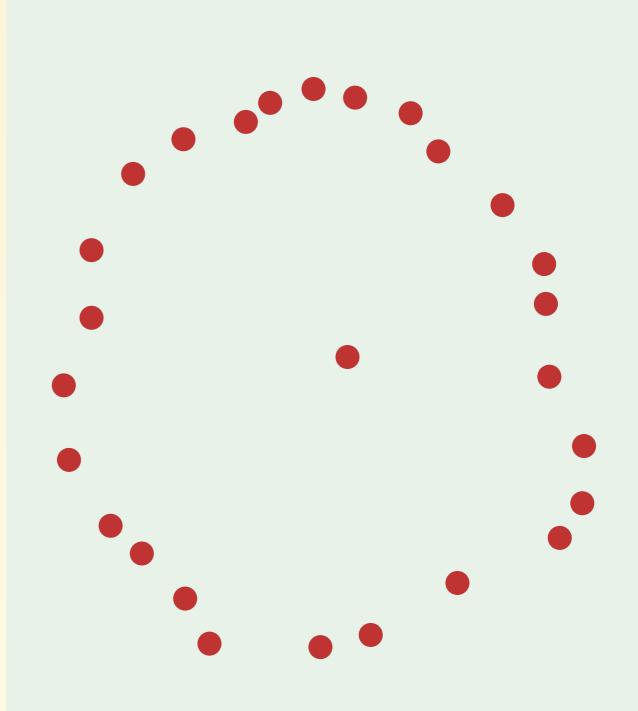
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PROXIMITY STRUCTURES

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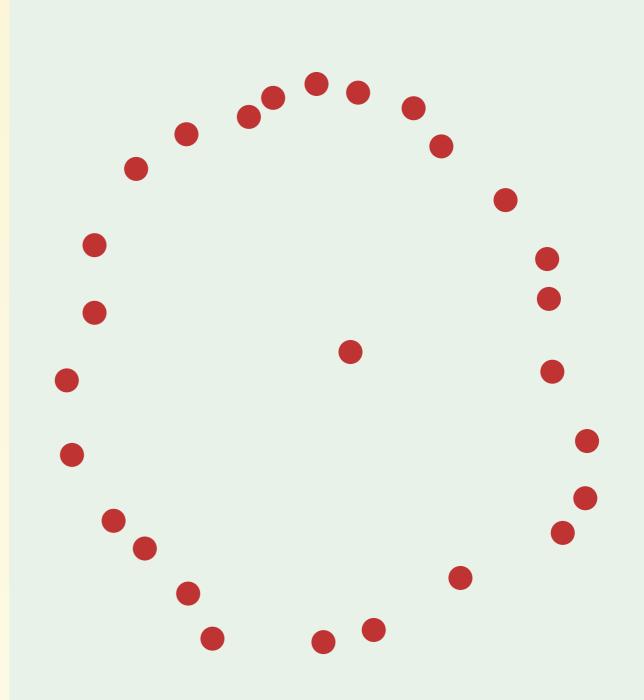
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Unfortunately, not all point sets behave themselves.

A single point may have more than $\Omega(1)$ neighbours.

The scale of a point set need not be uniform.



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PROXIMITY STRUCTURES

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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

Unfortunately, not all point sets behave themselves.

A single point may have more than $\Omega(1)$ neighbours.

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PROXIMITY STRUCTURES

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Unfortunately, not all point sets behave themselves.

A single point may have more than $\Omega(1)$ neighbours.

The scale of a point set need not be uniform.

What can we do?

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Do we...

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Do we...

assume that in practice, point sets are well-behaved, and design useable algorithms that only work (or are only efficient) on certain classes of point sets?

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Do we...

assume that in practice, point sets are well-behaved, -0Rand design useable algorithms that only work (or are only efficient) on certain classes of point sets?

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Do we...

assume that in practice, point sets are well-behaved, -0Rand design useable algorithms that only work (or are only efficient) on certain classes of point sets?

insist on being general, and design devilishly complicated algorithms that nobody will ever use, but that are able to handle arbitrary point sets?

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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

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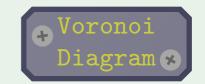
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[Dirichlet, 1850]

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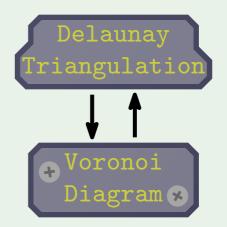
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[Dirichlet, 1850] [Delaunay, 1934]

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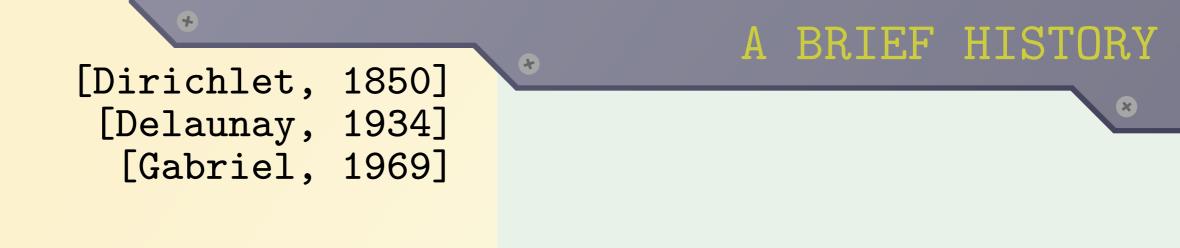


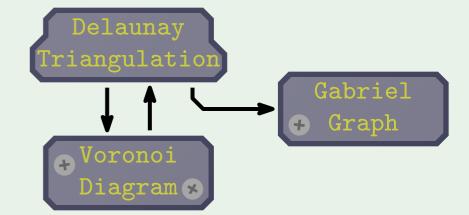
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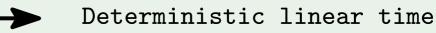
Deterministic linear time

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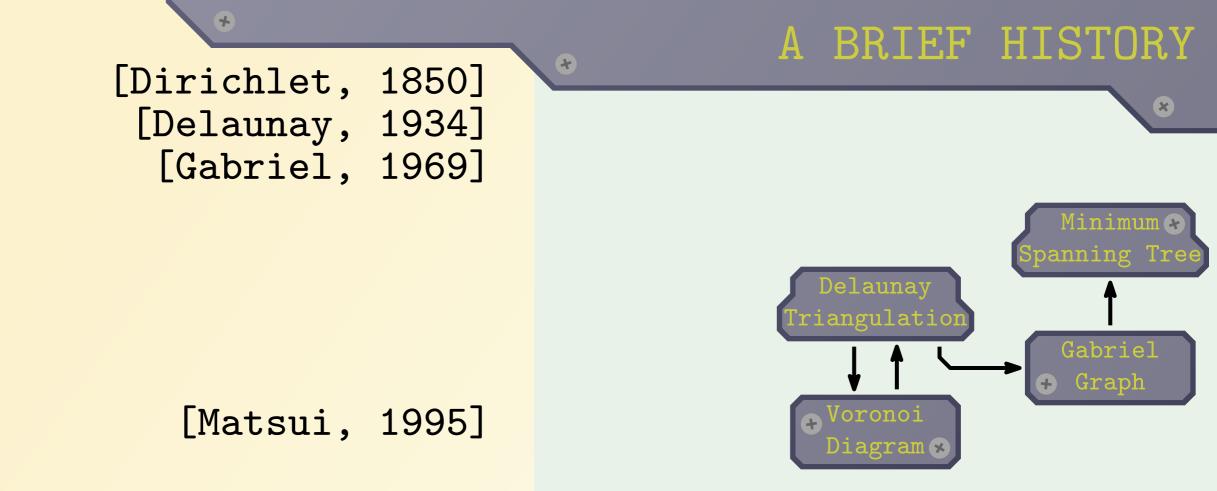


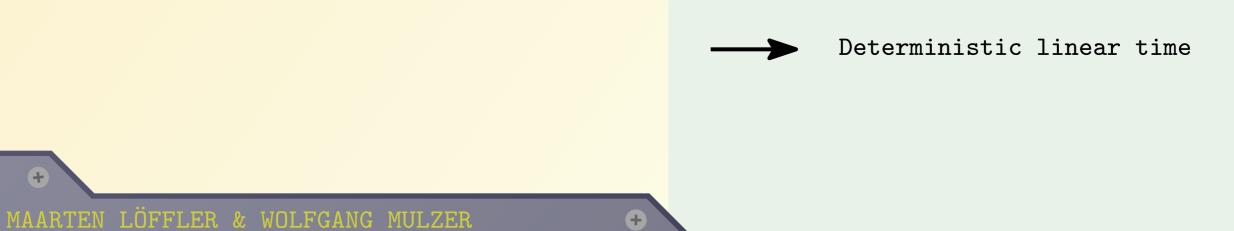


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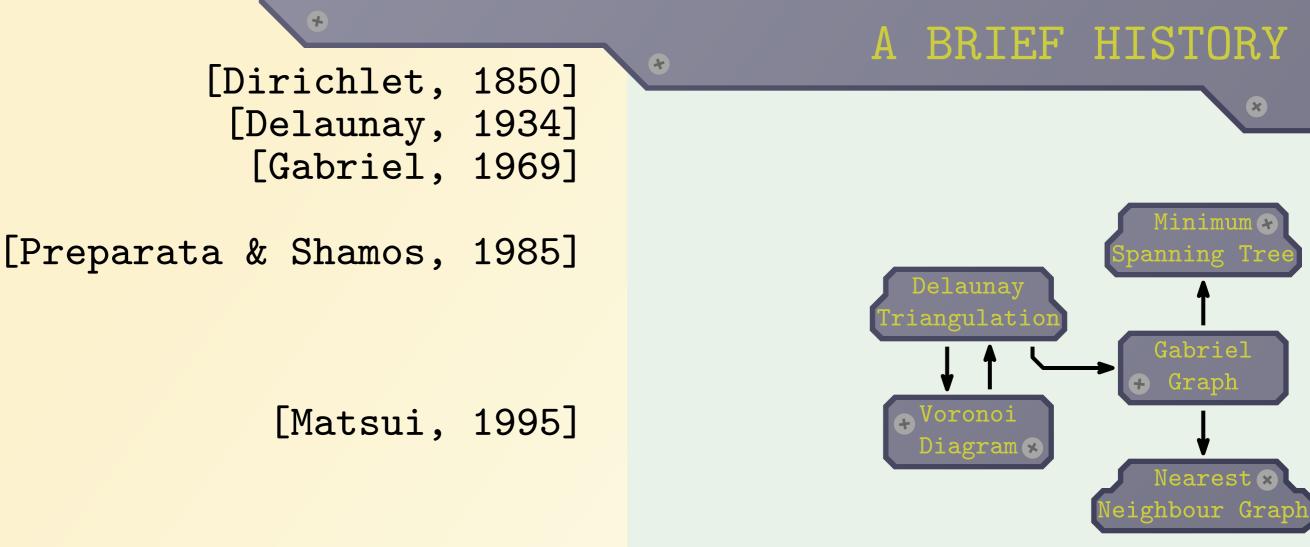


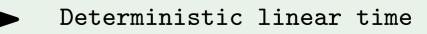


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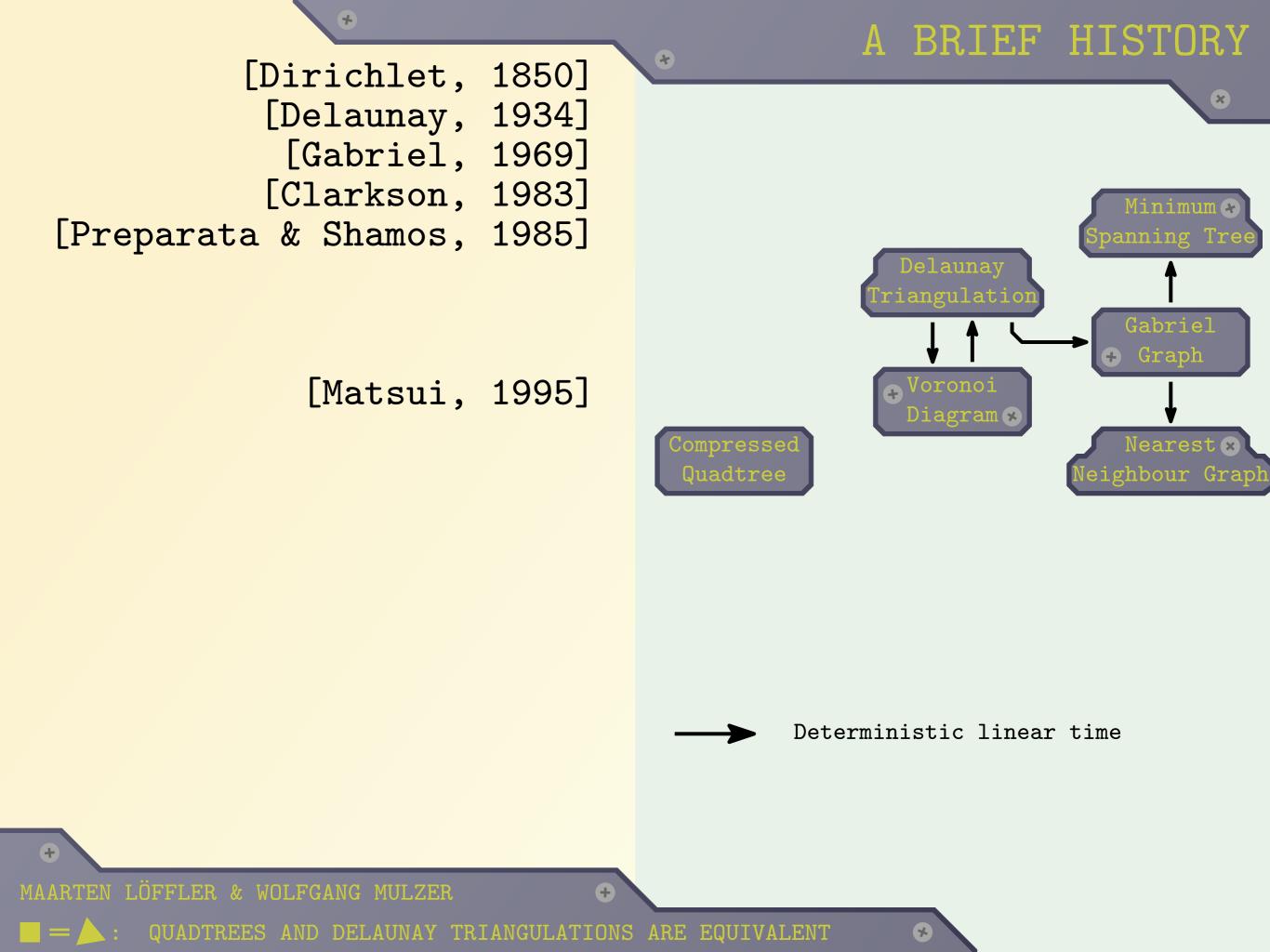


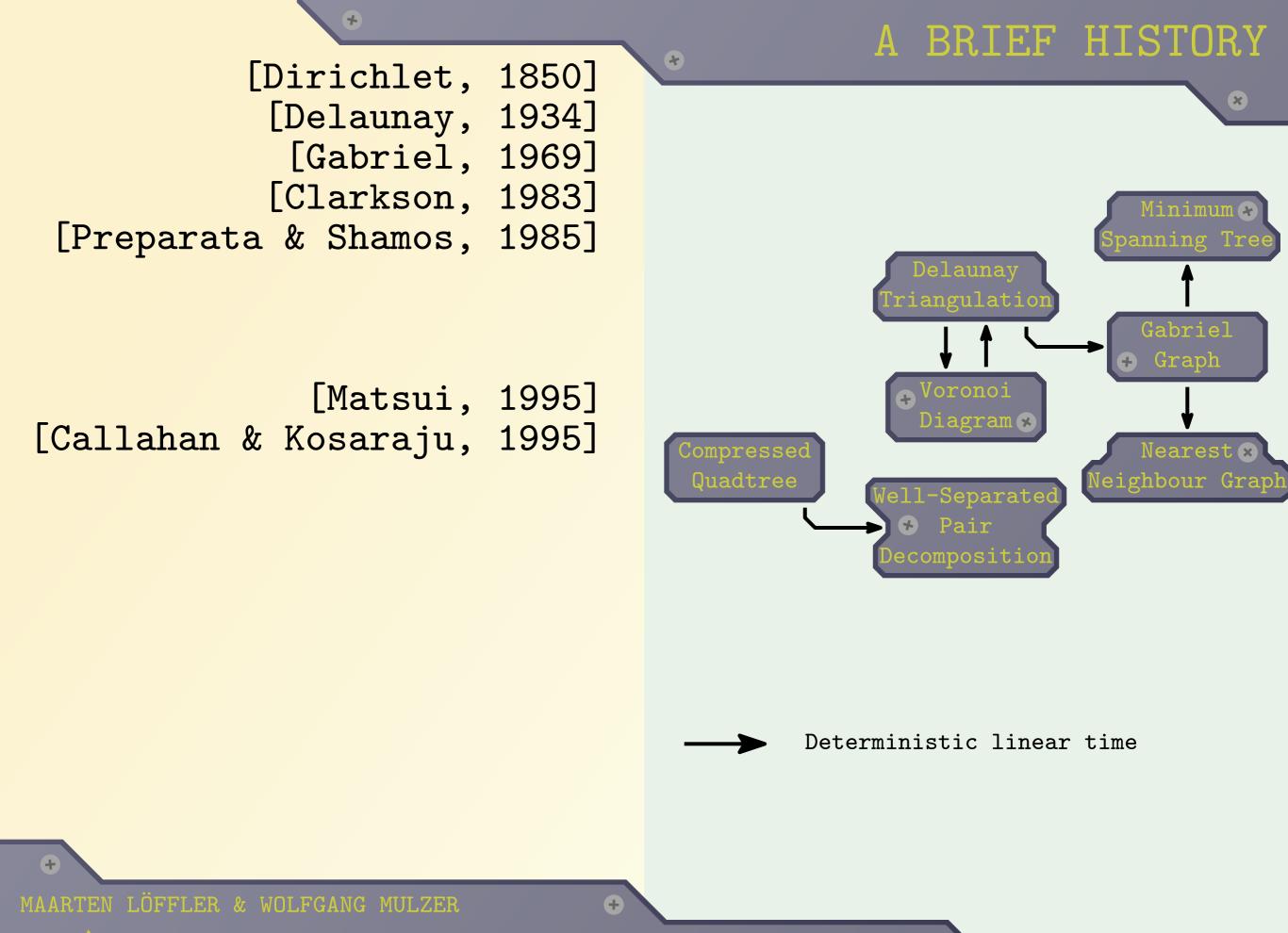
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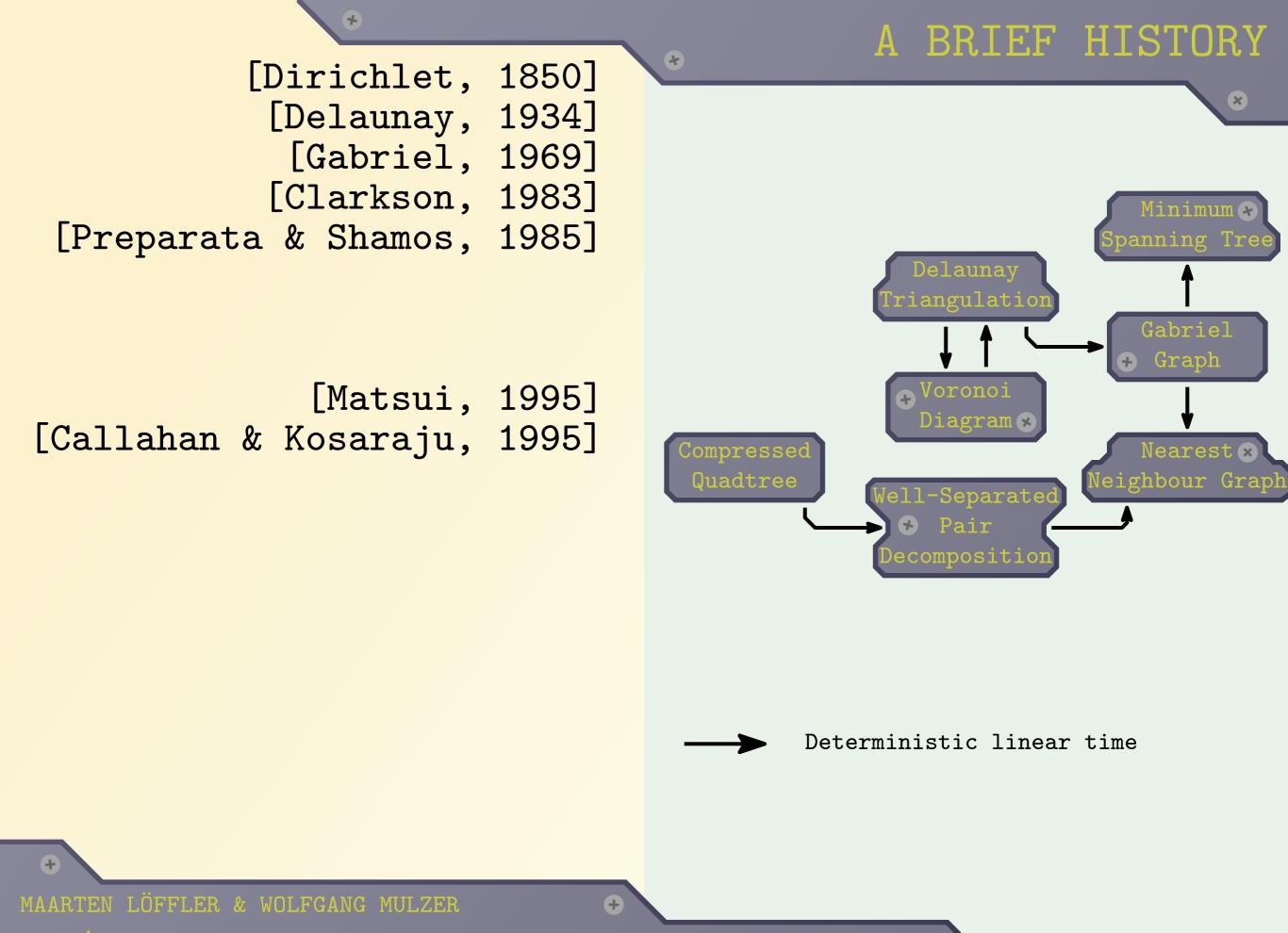
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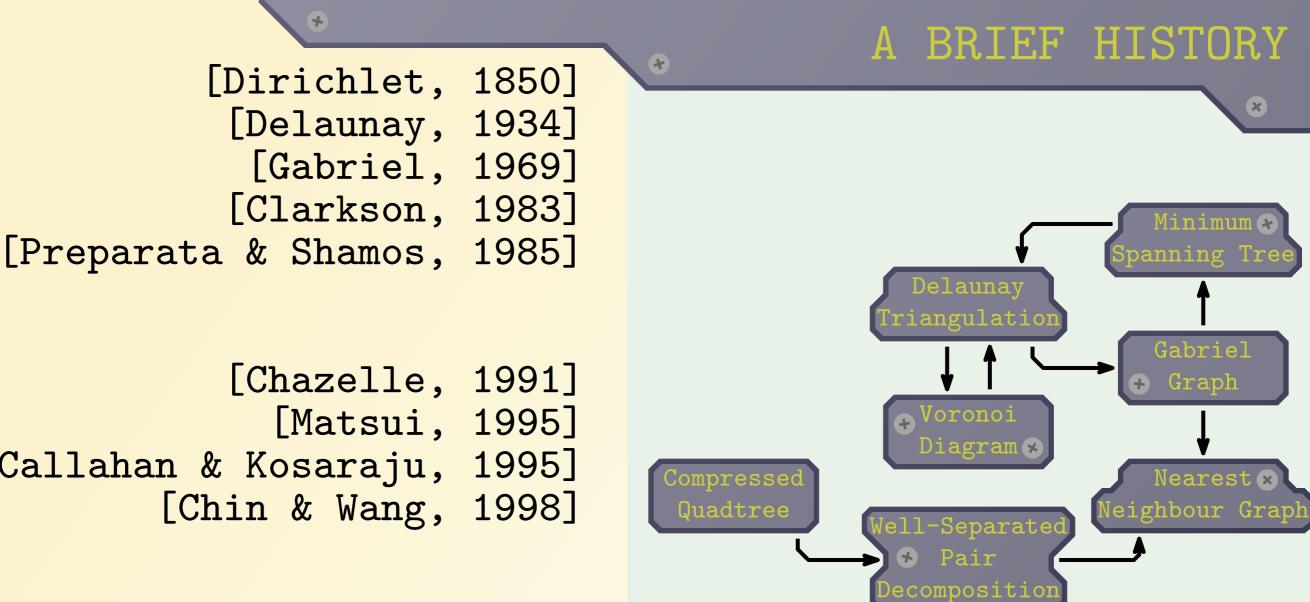
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[Callahan & Kosaraju,

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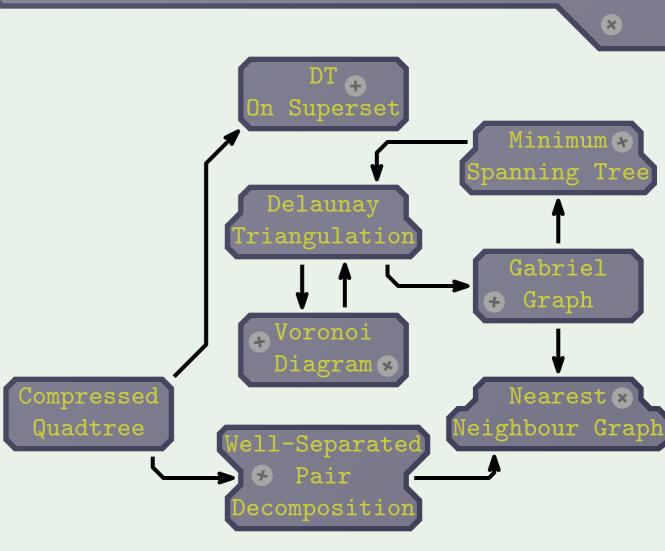
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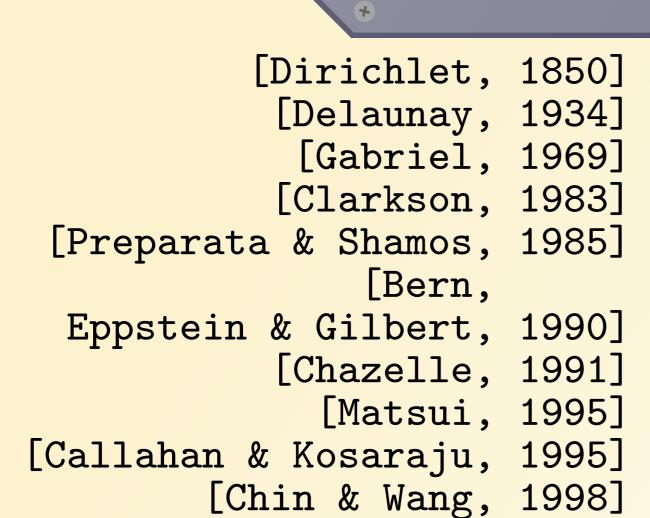
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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT





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Deterministic linear time

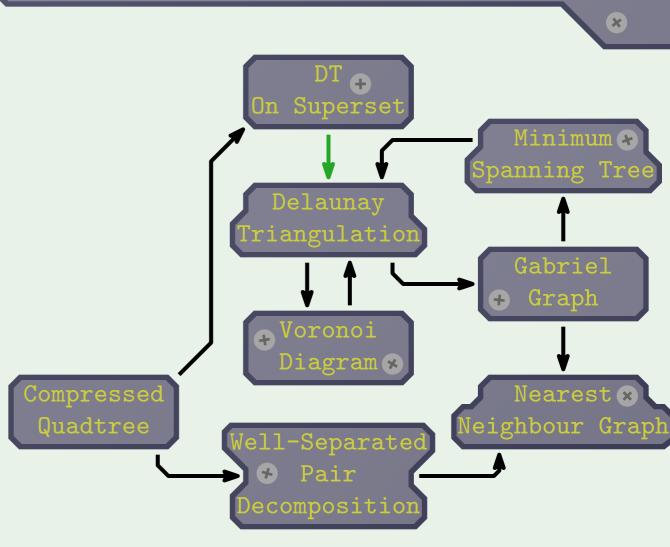
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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT





[Dirichlet, 1850] [Delaunay, 1934] [Gabriel, 1969] 1983] [Clarkson, [Preparata & Shamos, 1985] [Bern, Eppstein & Gilbert, 1990] [Chazelle, 1991] [Matsui, 1995] [Callahan & Kosaraju, 1995] 1998] [Chin & Wang,

[Chazelle, Devillers, Hurtado, Mora, Sacristán & Teillaud, 2001]

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Randomised linear time

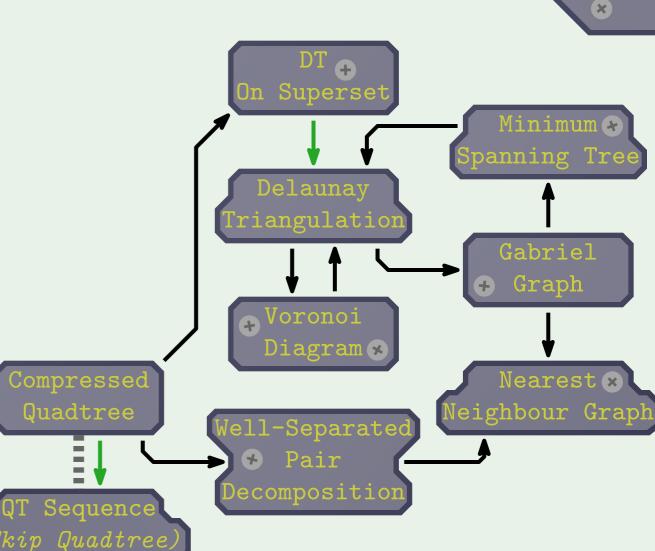
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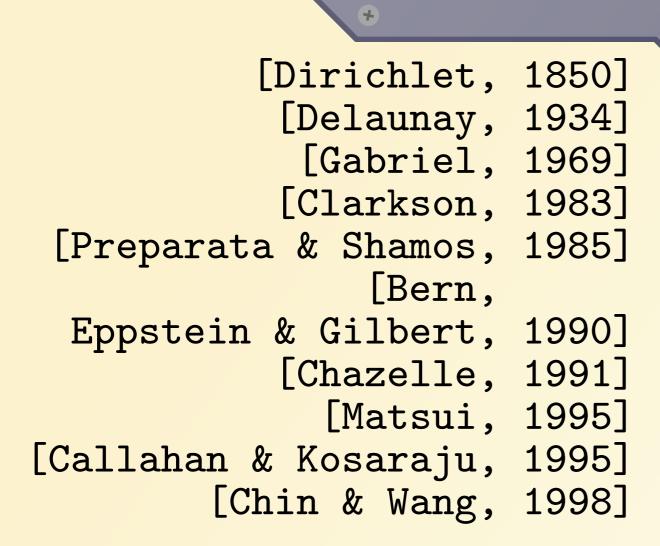
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Deterministic linear time
Randomised linear time

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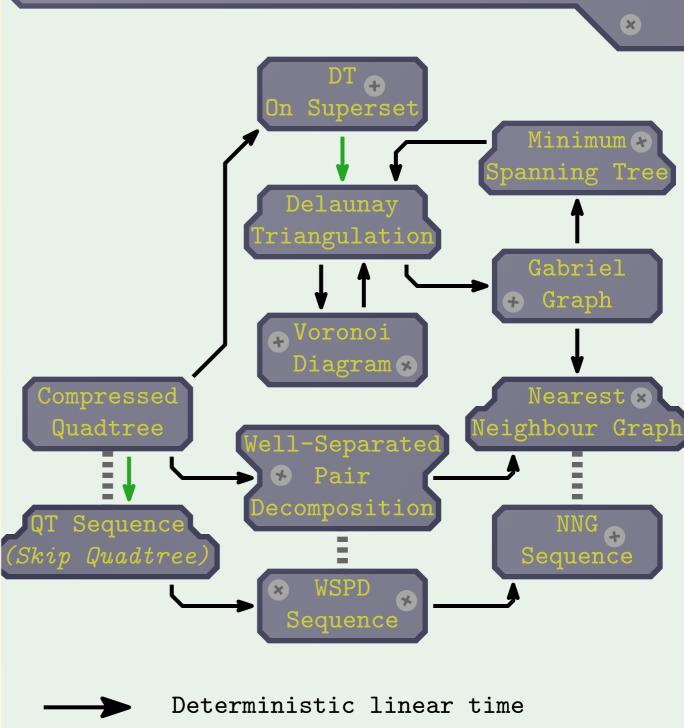
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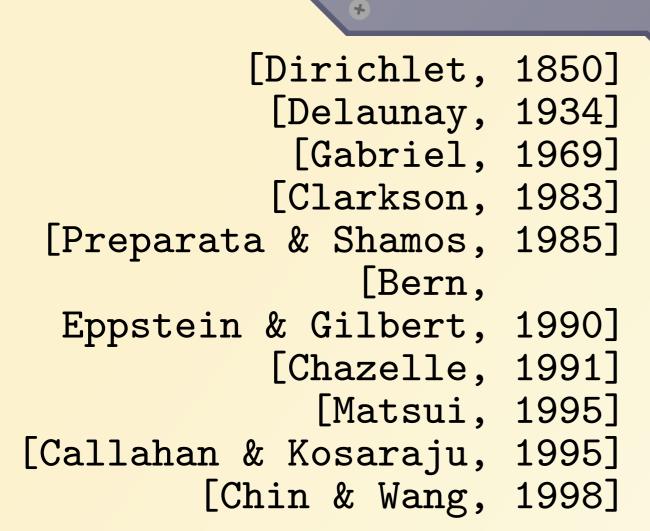
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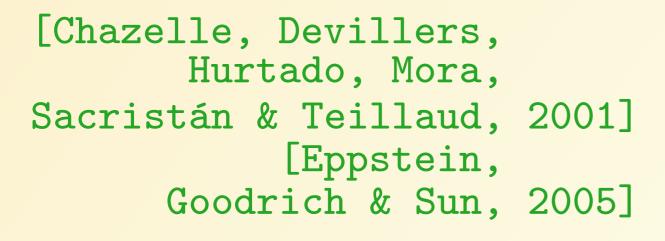
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Randomised linear time

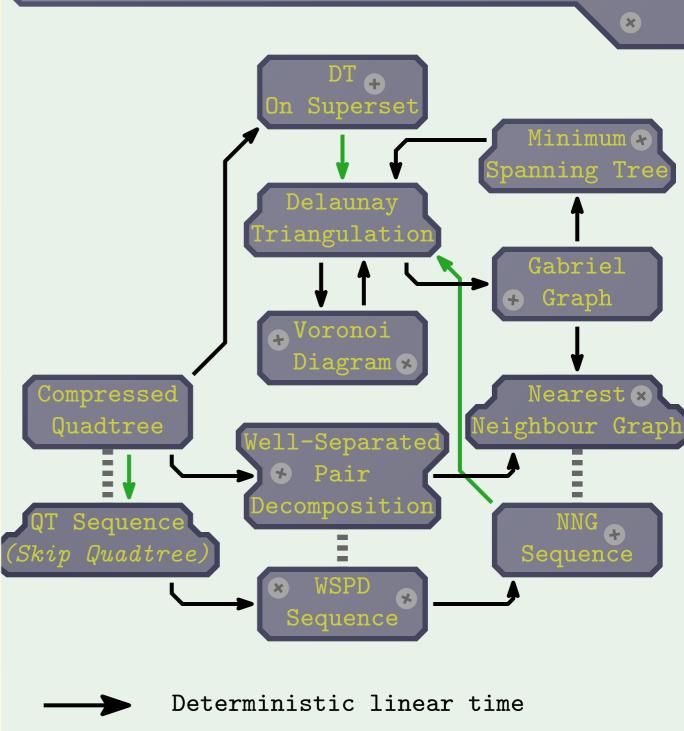
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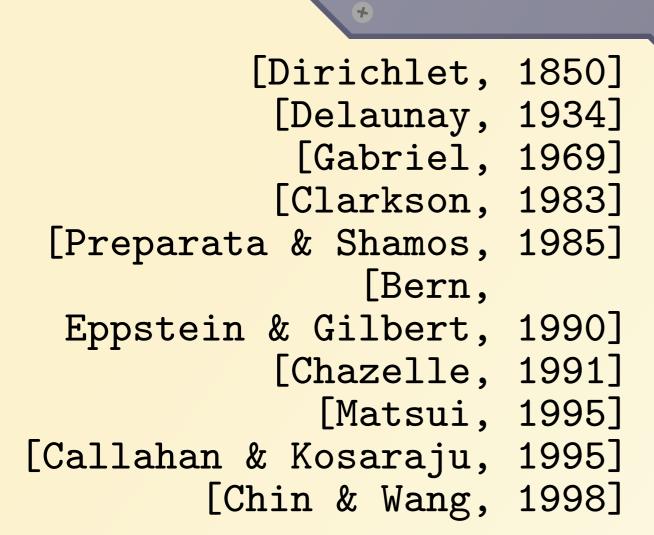
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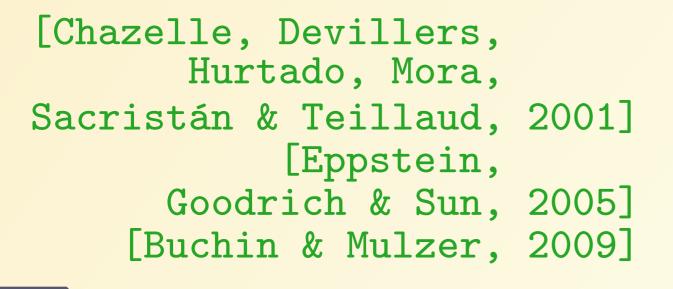
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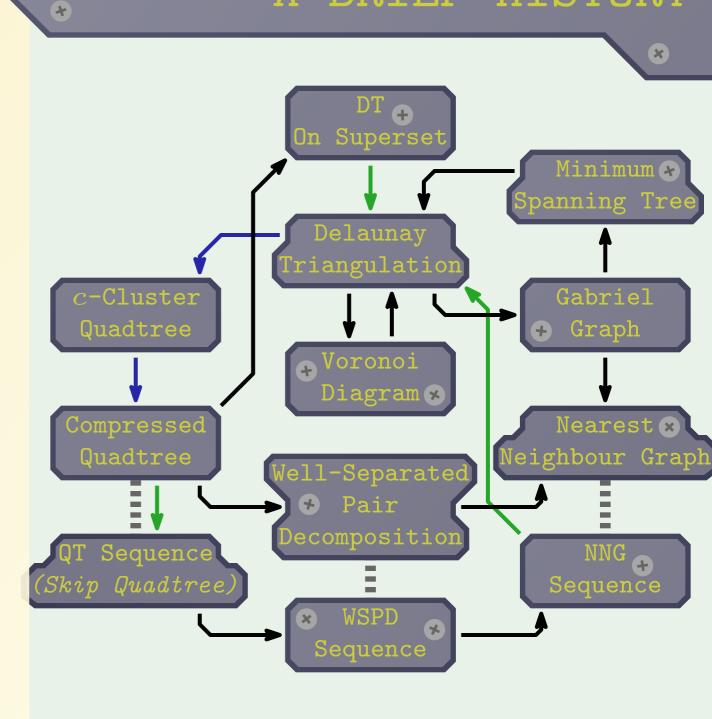
Randomised linear time

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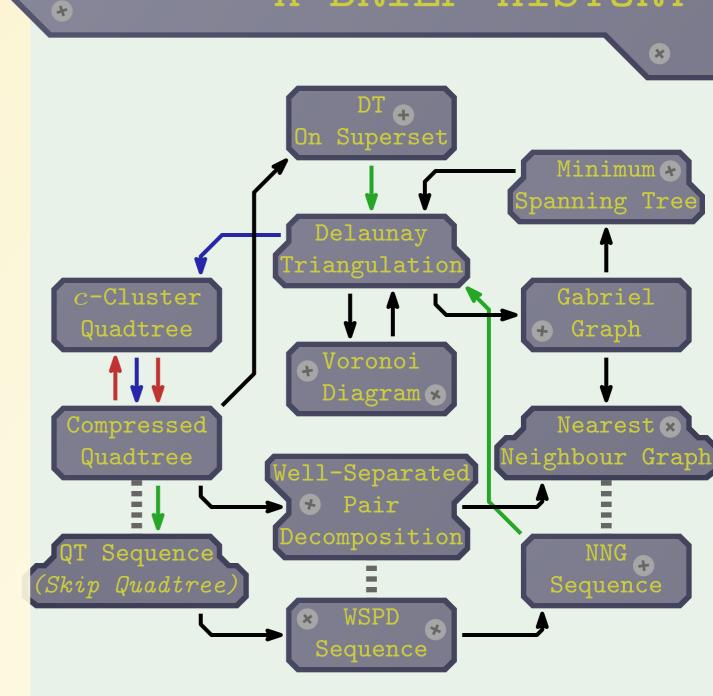
Linear time with floor operation



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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT



- Deterministic linear time
 - Randomised linear time
 - Linear time with floor operation
 - Deterministic linear time

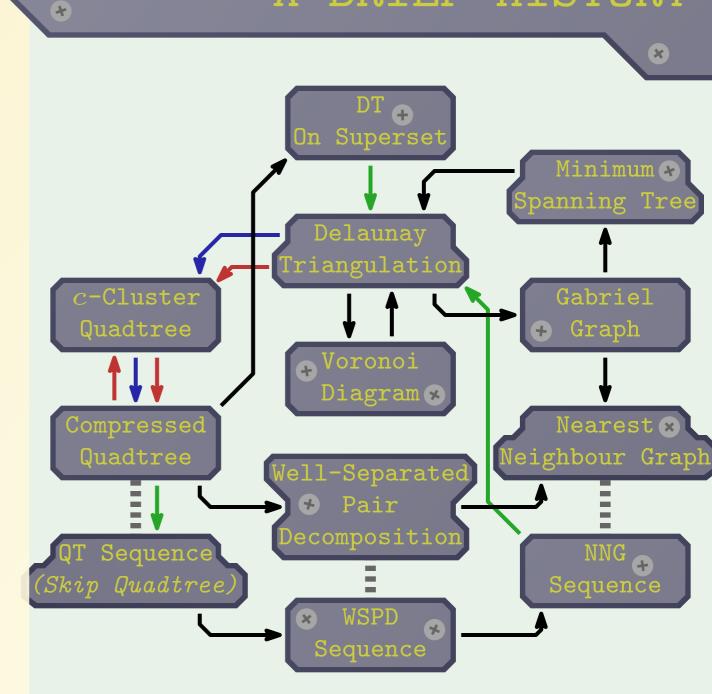
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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

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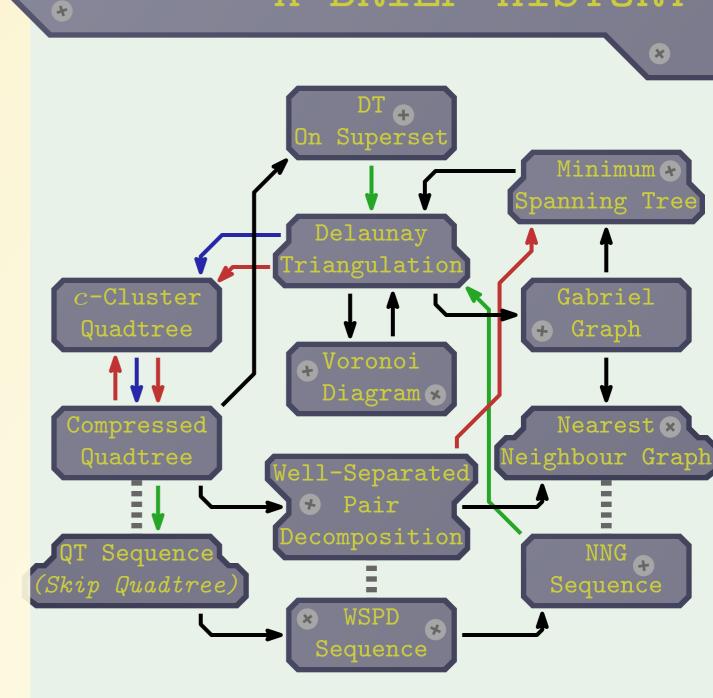
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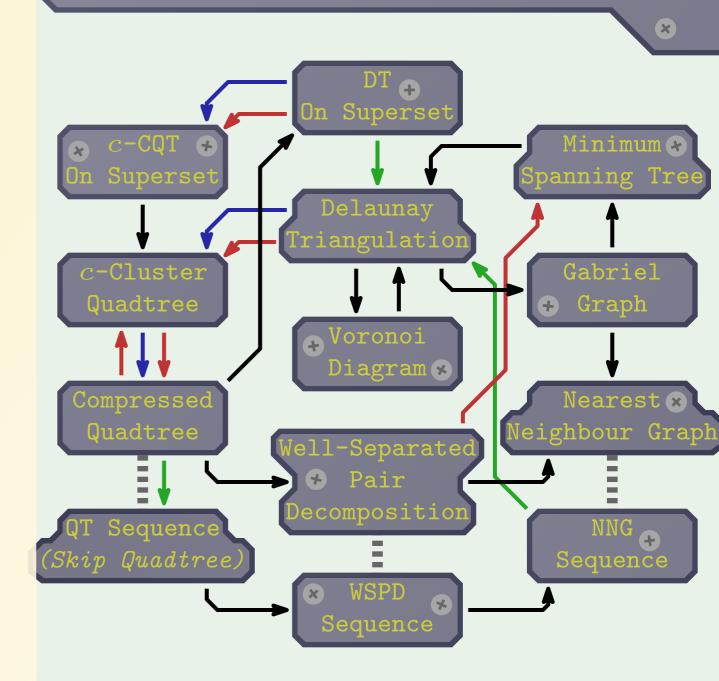
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PART II PRELIMINARIES

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The Delaunay triangulation (DT) is a triangulation D of a point set P that has an edge between two points if there is an empty circle through these points.

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DELAUNAY TRIANGULATION

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The Delaunay triangulation (DT) is a triangulation D of a point set P that has an edge between two points if there is an empty circle through these points.

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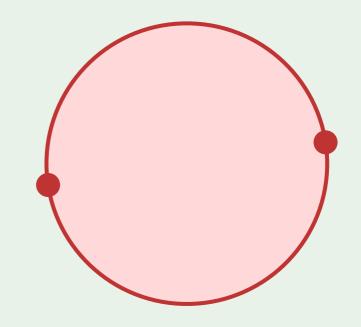
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The Delaunay triangulation (DT) is a triangulation D of a point set P that has an edge between two points if there is an empty circle through these points.



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DELAUNAY TRIANGULATION

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The Euclidean minimum spanning tree (EMST) is the minimum spanning tree of the complete graph on a point set P.

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The EMST is a subgraph of the DT.

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A quadtree T for a set of points Pis a hierarchical subdivision of a square into smaller squares that separates P.

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COMPRESSED QUADTREE

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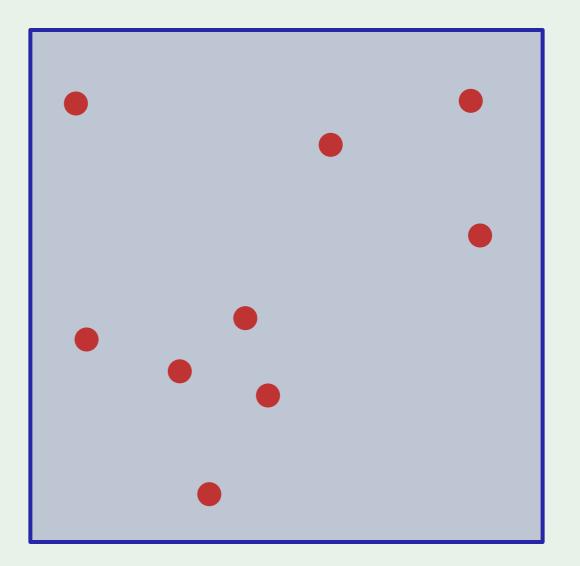
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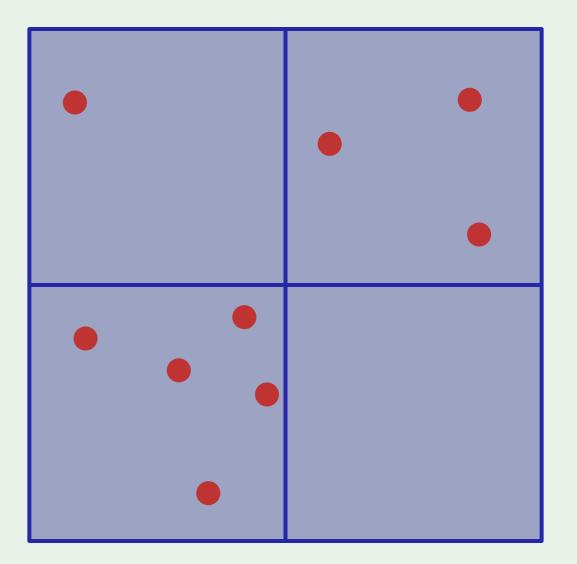
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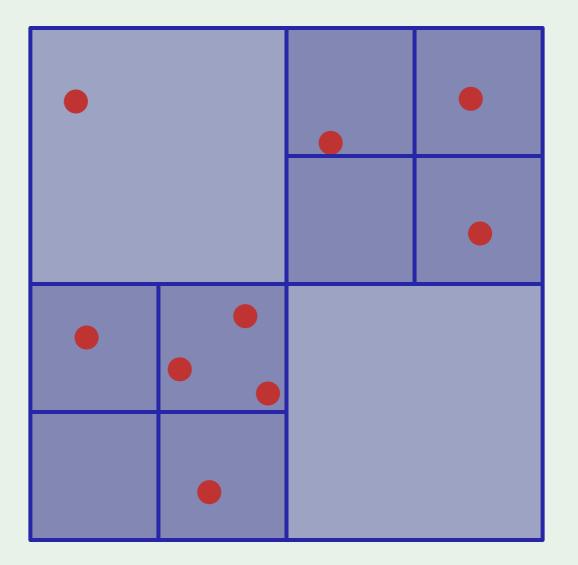
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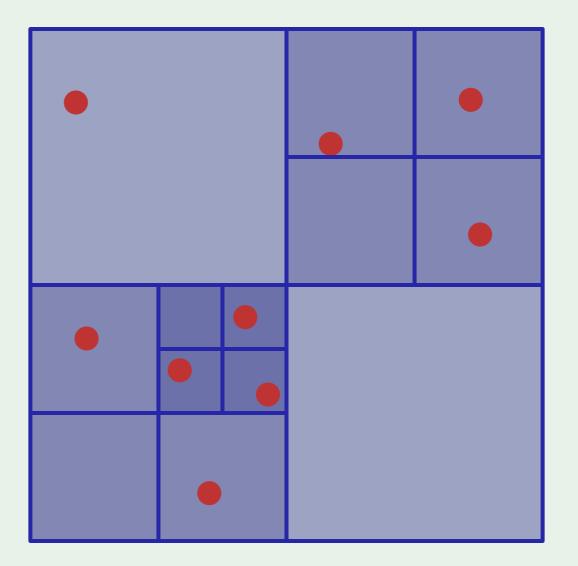
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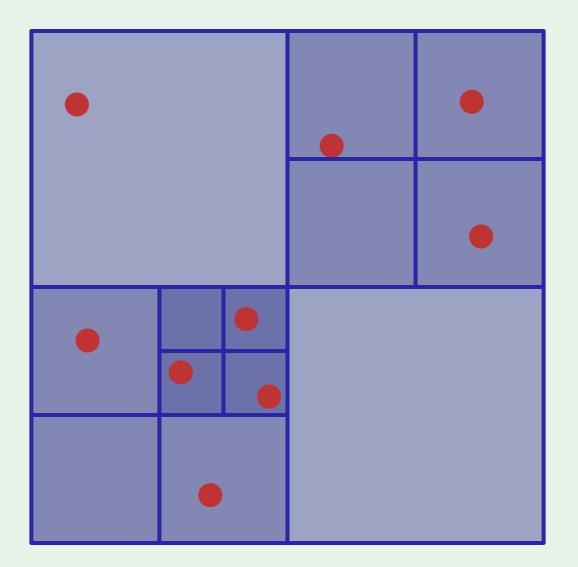
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A quadtree T for a set of points Pis a hierarchical subdivision of a square into smaller squares that separates P.

For point sets with large spread, a quadtree can be compressed.

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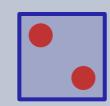
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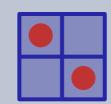
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A c-cluster in a point set P is a subset $C \subset P$ whose distance to the rest of P is at least cits diameter.

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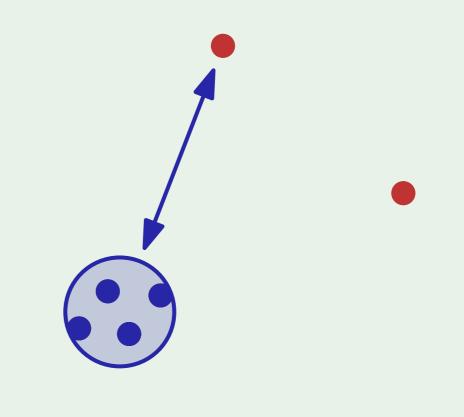
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A *c*-cluster in a point set P is a subset $C \subset P$ whose distance to the rest of P is at least cits diameter.

The c-clusters on P form a hierarchy.

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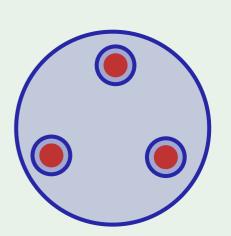
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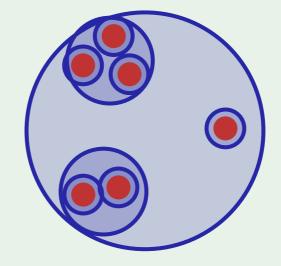
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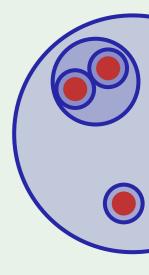
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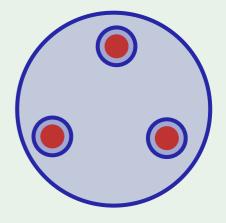
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A *c-cluster* in a point set P is a subset $C \subset P$ whose distance to the rest of P is at least cits diameter.

The c-clusters on P form a hierarchy.

This *c*-cluster tree can have linear degree.

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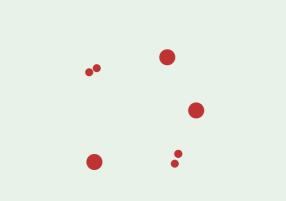
A c-cluster quadtree is a c-cluster tree augmented with a quadtree on its high-degree nodes.

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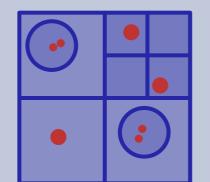
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c-CLUSTER QUADTREE

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A c-well-separated pair is a pair of point sets U, Vwhose distance to each other is at least c times their diameters.

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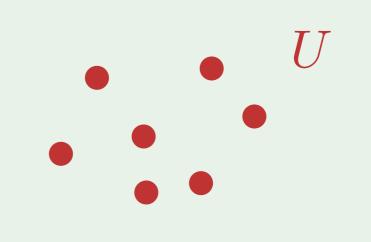
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A c-well-separated pair decomposition (WSPD) of a set of points P is a set of well-separated pairs of subsets of P such that every pair of points $p,q \in P$ appear in exactly one pair of subsets.

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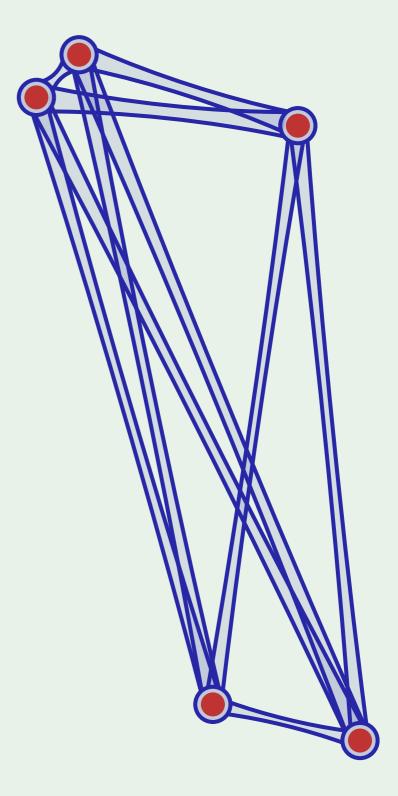
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A WSPD with O(|P|) pairs always exists.

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A WSPD with O(|P|) pairs always exists.

One way to construct one is from a hierarchical subdivision (such as a *c*-cluster quadtree).

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Suppose we have a WSPD \mathcal{P} on P.

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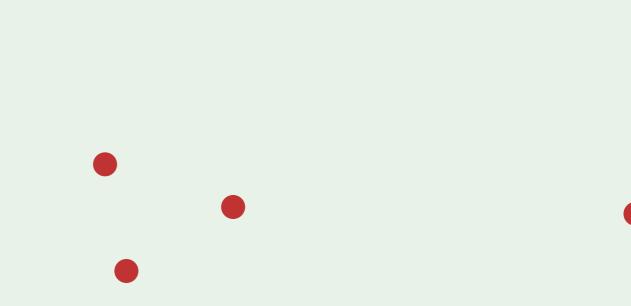
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Suppose we have a WSPD \mathcal{P} on P. Let G be the graph

that contains the shortest edge between two points in any pair of \mathcal{P} .

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Suppose we have a WSPD \mathcal{P} on P.

Let G be the graph that contains the shortest edge between two points in any pair of \mathcal{P} .

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Suppose we have a WSPD \mathcal{P} on P.

Let G be the graph that contains the shortest edge between two points in any pair of \mathcal{P} .

Claim: G has linear size and contains the MST of P.

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Problem: \mathcal{P} has quadratic weight.

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Problem: \mathcal{P} has quadratic weight.

Idea: most heavy
pairs are far away
and don't contribute
to the MST anyway.

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Problem: \mathcal{P} has quadratic weight.

Idea: most heavy
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to the MST anyway.

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Let \mathcal{P}_{ϕ} be the pairs in \mathcal{P} with general direction ϕ .

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Problem: \mathcal{P} has quadratic weight.

Idea: most heavy pairs are far away and don't contribute to the MST anyway.

Let \mathcal{P}_{ϕ} be the pairs in \mathcal{P} with general direction ϕ .

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Problem: \mathcal{P} has quadratic weight.

Idea: most heavy pairs are far away and don't contribute to the MST anyway.

Let \mathcal{P}_{ϕ} be the pairs in \mathcal{P} with general direction ϕ .

We construct a G_{ϕ} separately from \mathcal{P}_{ϕ} .

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For each $p \in P$, find the k closest pairs in \mathcal{P}_{ϕ} , and remove pfrom all other pairs.

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 \mathcal{P}

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For each $p \in P$, find the k closest pairs in \mathcal{P}_{ϕ} , and remove pfrom all other pairs.

Let \mathcal{P}'_{ϕ} be the resulting pair set.

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For each $p \in P$, find the k closest pairs in \mathcal{P}_{ϕ} , and remove pfrom all other pairs.

Let \mathcal{P}'_{ϕ} be the resulting pair set.

 \mathcal{P}' , the union of \mathcal{P}'_{ϕ} over all ϕ , is no longer a pair decomposition of P. But...

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Claim: \mathcal{P}' has linear weight.

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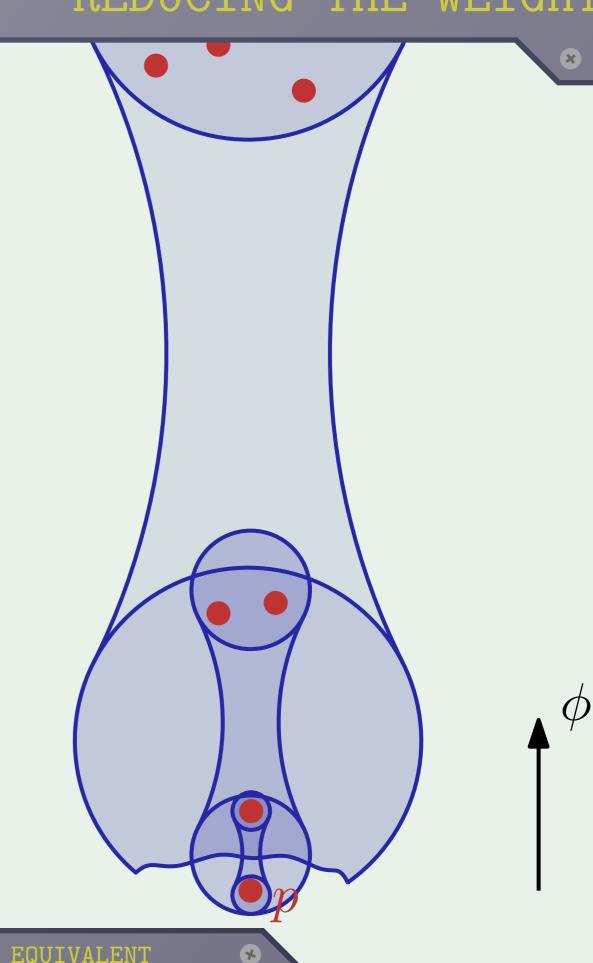
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Claim: \mathcal{P}' has linear weight.

Claim: \mathcal{P}' still contains all edges of the MST of P.



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Claim: \mathcal{P}' has linear weight.

Claim: \mathcal{P}' still contains all edges of the MST of P.

Claim: and as if that wasn't cool enough, \mathcal{P}' can even be computed in linear time.

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ÇOMPUTING CLOSEST PAIRS

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Now, for each pair $(U,V) \in \mathcal{P}'_{\phi}$, we need to find the closest pair of points.

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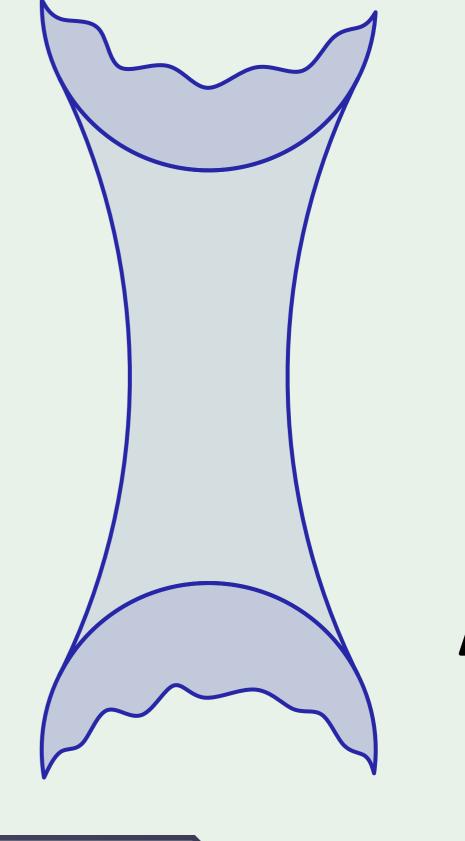
MAARTEN LÖFFLER & WOLFGANG MULZER

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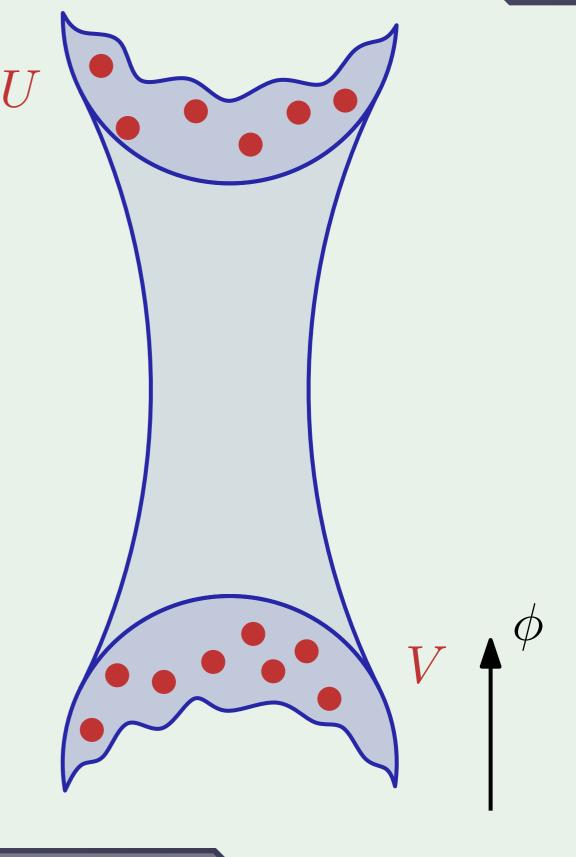
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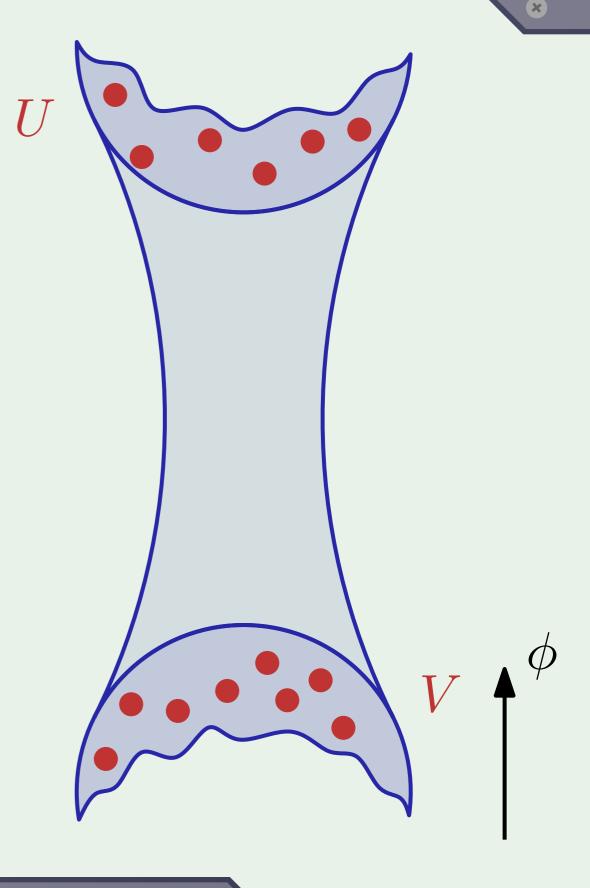
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Now, for each pair $(U,V) \in \mathcal{P}'_{\phi}$, we need to find the closest pair of points.

Claim: if the points would be sorted on x-coordinate, we could find it in linear time.



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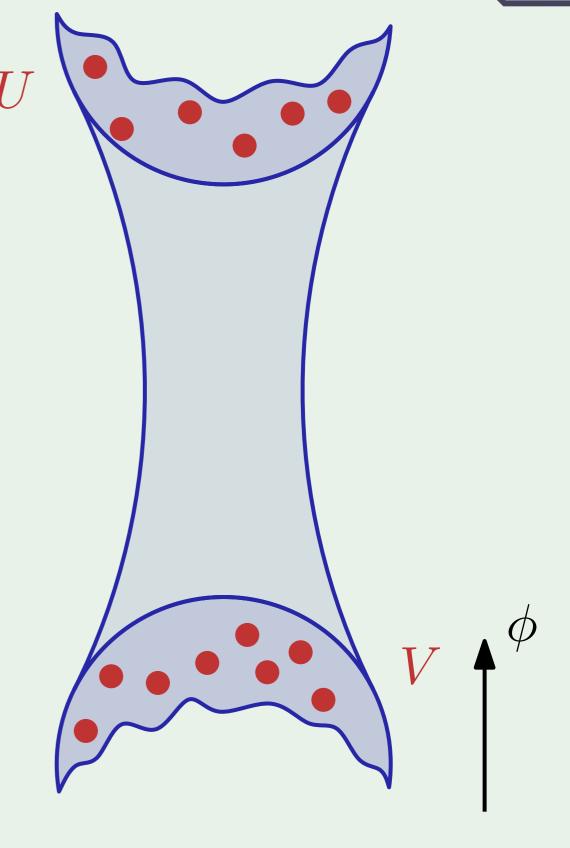
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Now, for each pair $(U,V) \in \mathcal{P}'_{\phi}$, we need to find the closest pair of points.

Claim: if the points would be sorted on x-coordinate, we could find it in linear time.

Problem: sorting takes $\Theta(n \log n)$ time.



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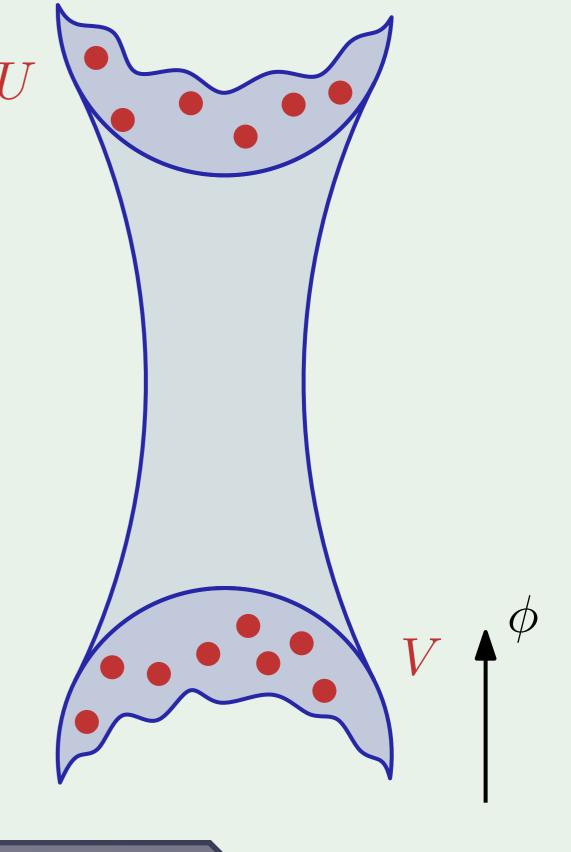
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Claim: we only need to sort the points whose 'upward cone' is empty.



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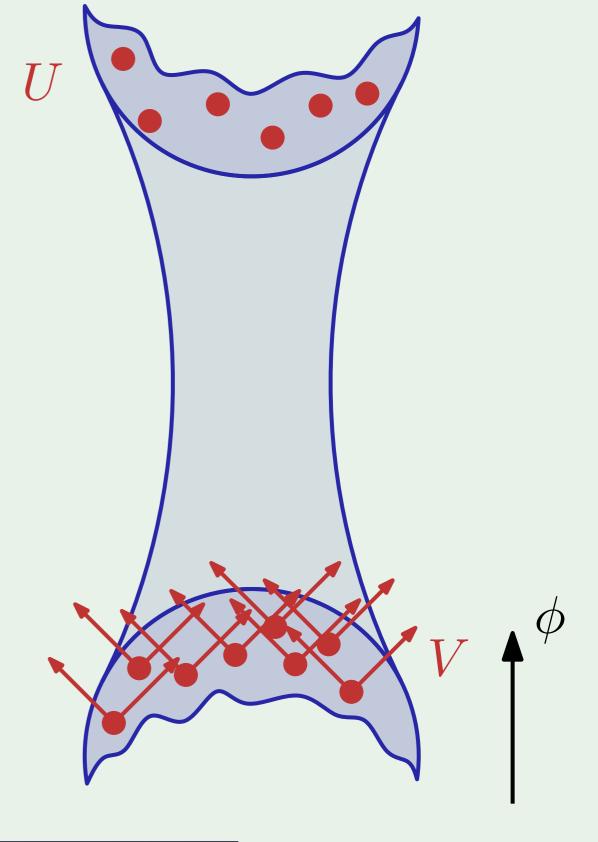
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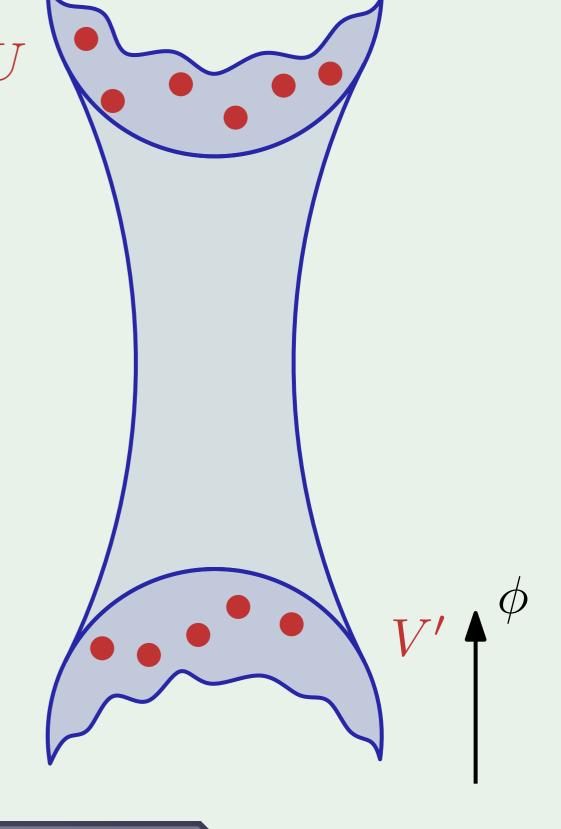
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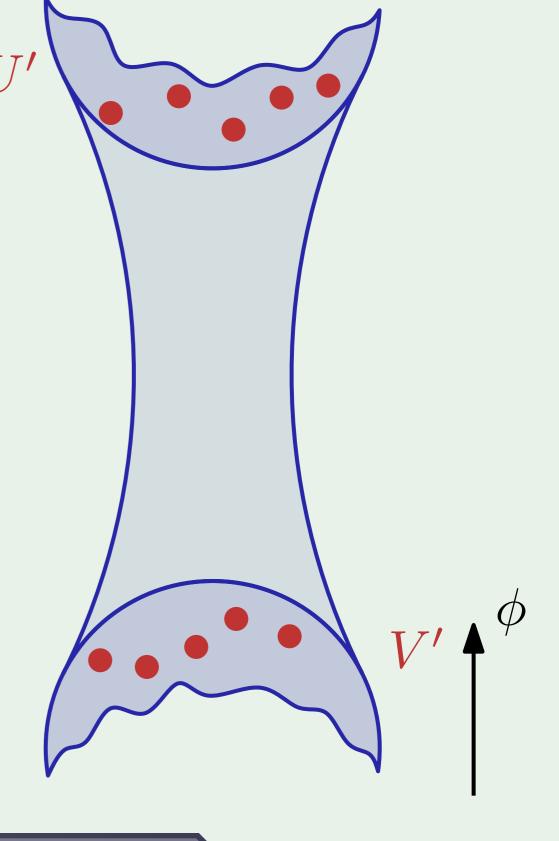
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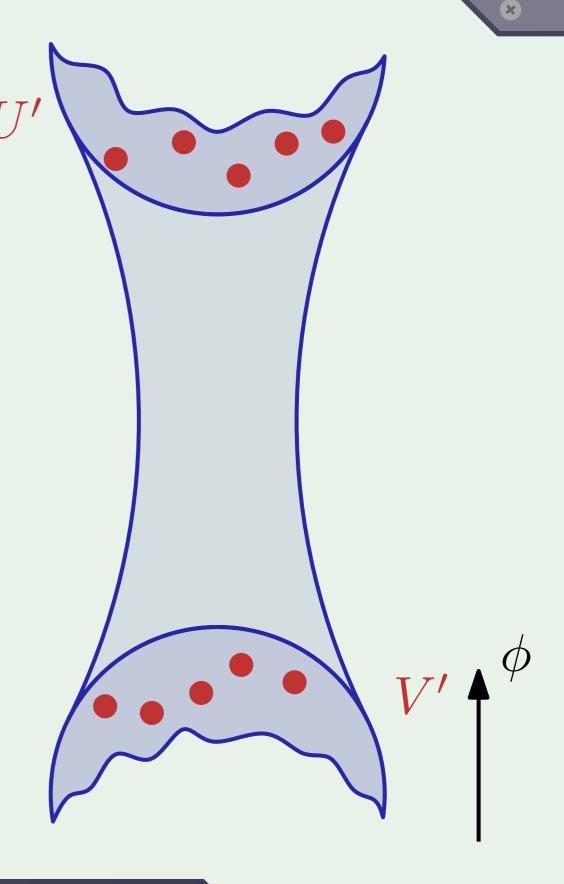
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So, we only *locally* need the correct *x*-order.



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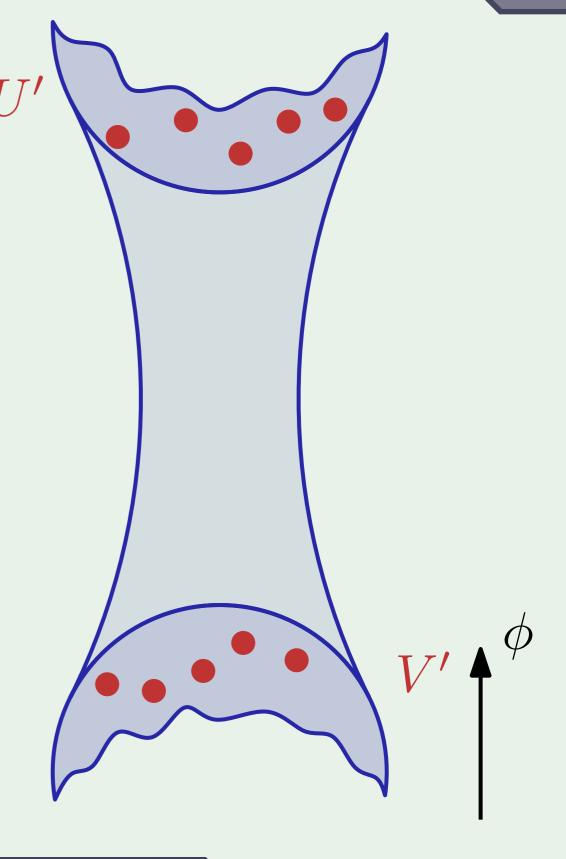
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Claim: we only need to sort the points whose 'upward cone' is empty.

So, we only *locally* need the correct *x*-order.

But we have that information: we started with a WSPD of *P*!



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Consider $\mathcal{P}_{\phi+\frac{1}{2}\pi}$, the pairs perpendicular to those in \mathcal{P}_{ϕ} .

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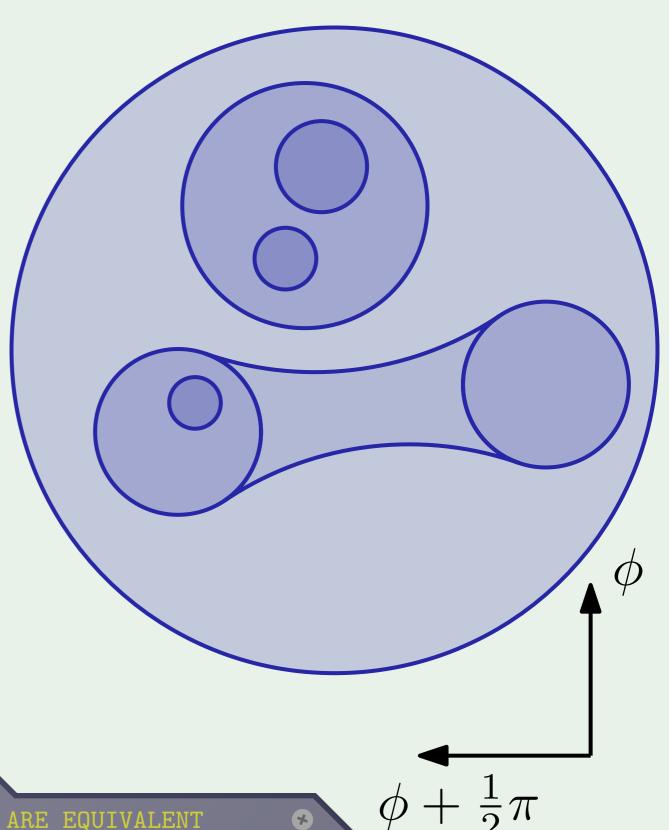
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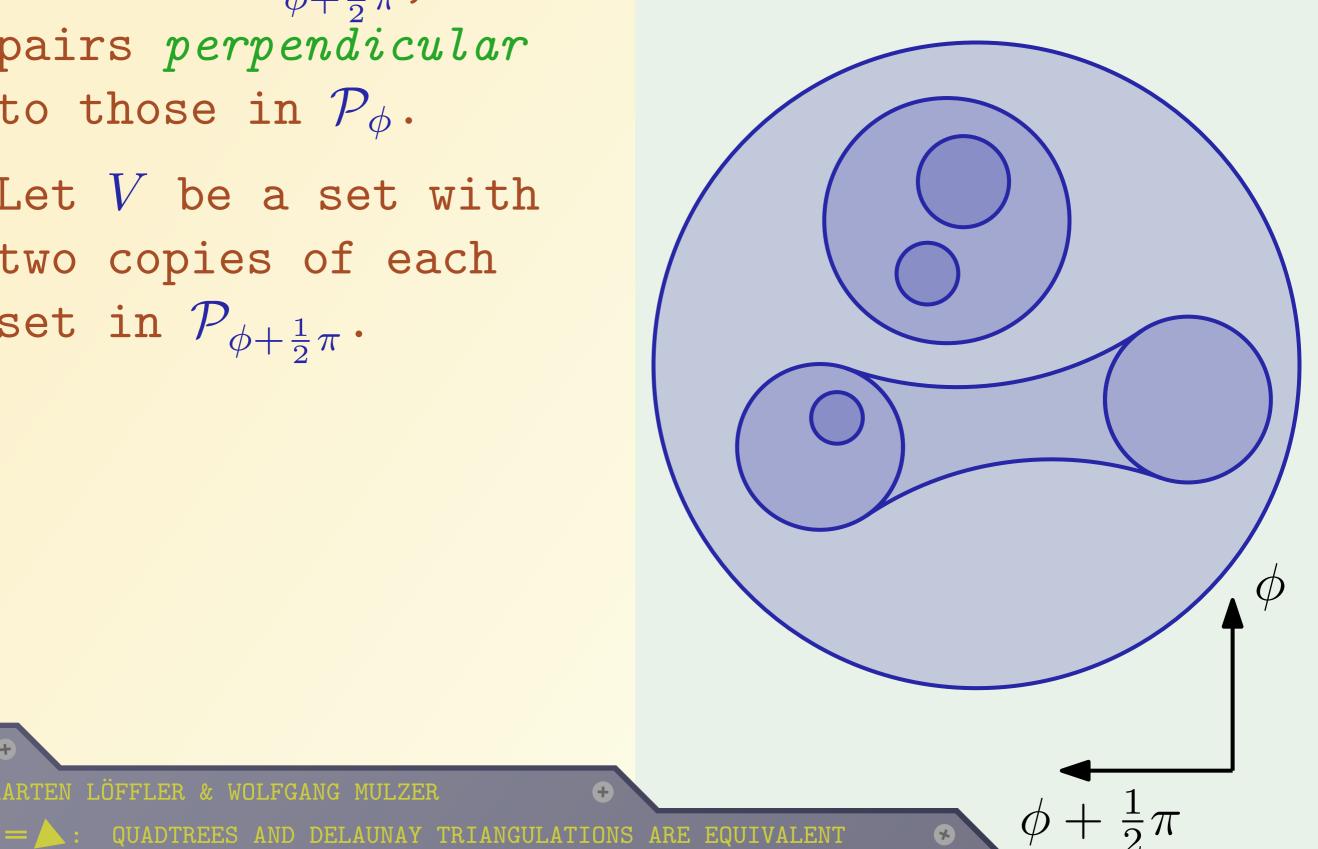
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Consider $\mathcal{P}_{\phi+\frac{1}{2}\pi}$, the pairs perpendicular to those in \mathcal{P}_{ϕ} .

Let V be a set with two copies of each set in $\mathcal{P}_{\phi+rac{1}{2}\pi}$.

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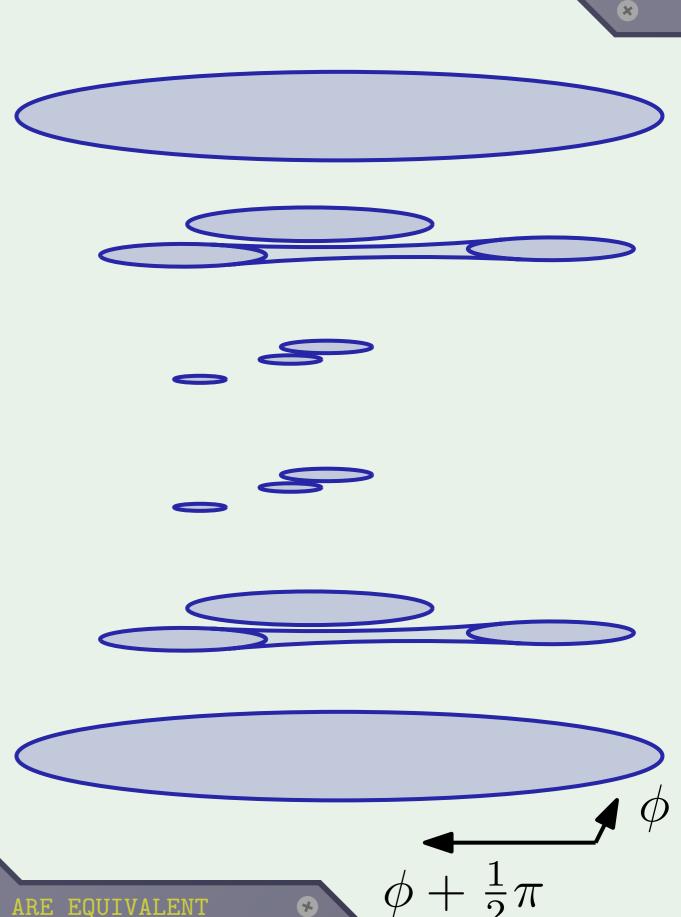
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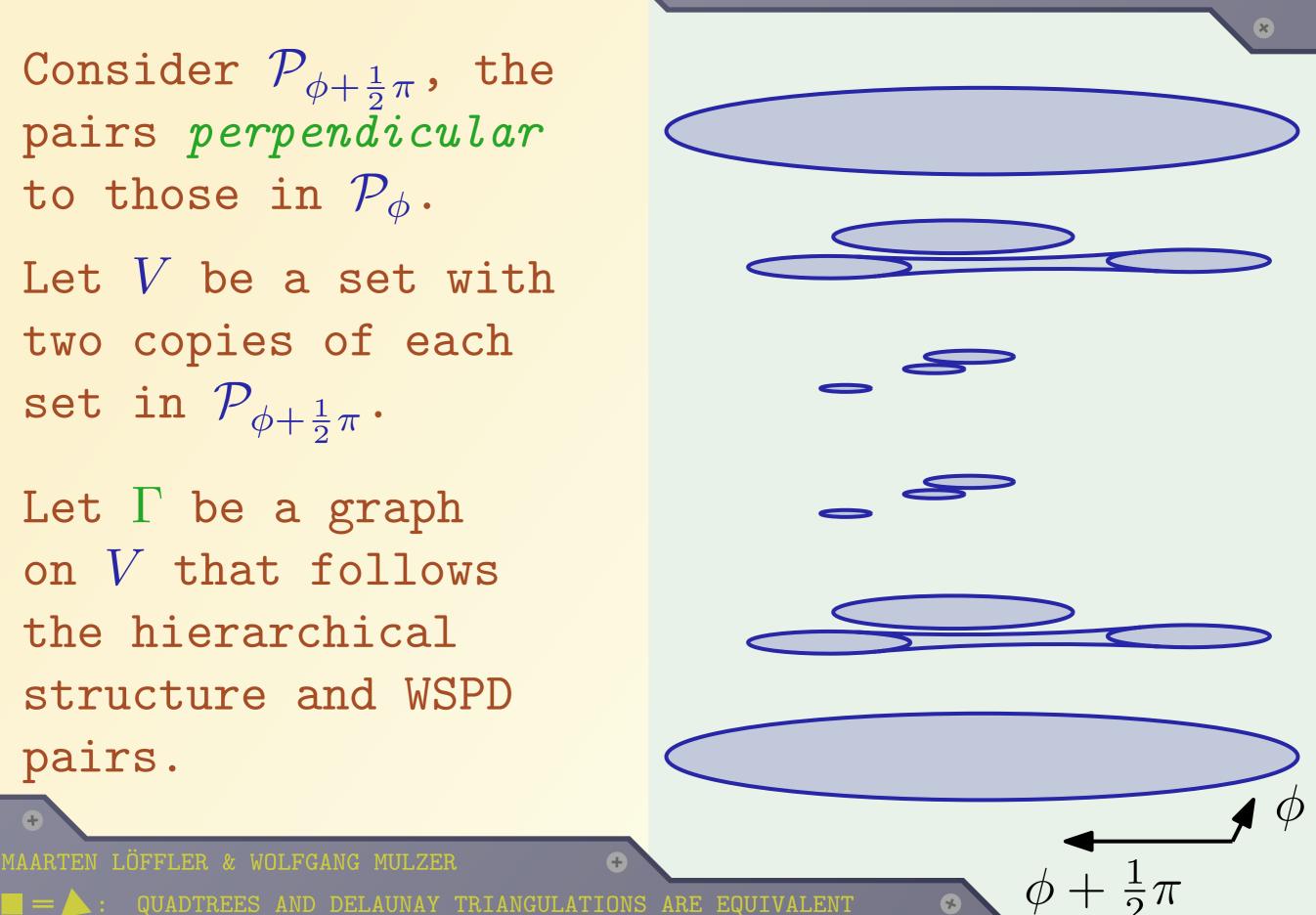
Consider $\mathcal{P}_{\phi+\frac{1}{2}\pi}$, the pairs perpendicular to those in \mathcal{P}_{ϕ} .

Let V be a set with two copies of each set in $\mathcal{P}_{\phi+\frac{1}{2}\pi}$.

Let Γ be a graph on V that follows the hierarchical structure and WSPD pairs.

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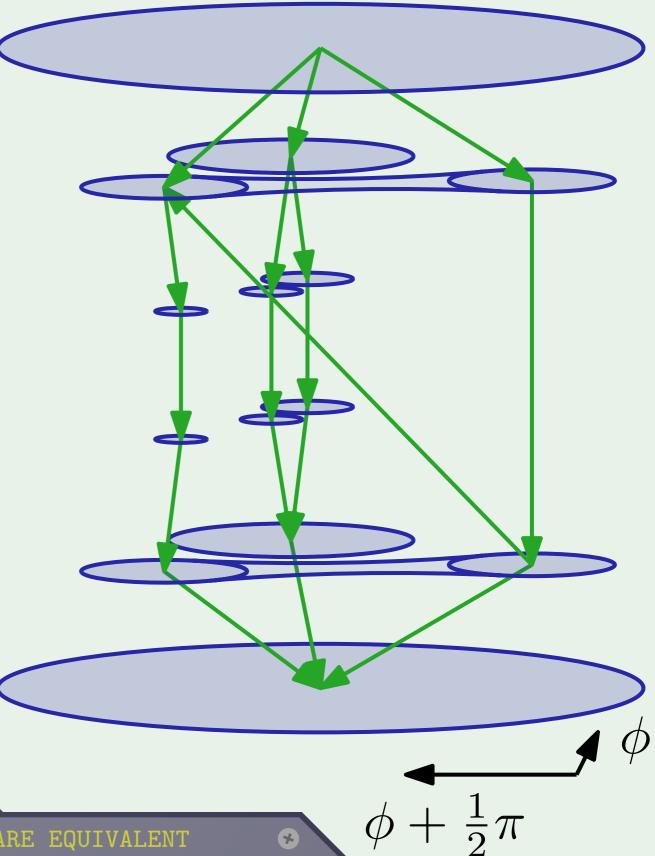
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Claim: Γ is acyclic.

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Claim: Γ is acyclic. Claim: If a point p is roughly in direction ϕ + $\frac{1}{2}\pi$ as seen from a point q, then q comes before pin Γ .

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 ϕ $\frac{1}{2}\pi$ = 📐 : QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

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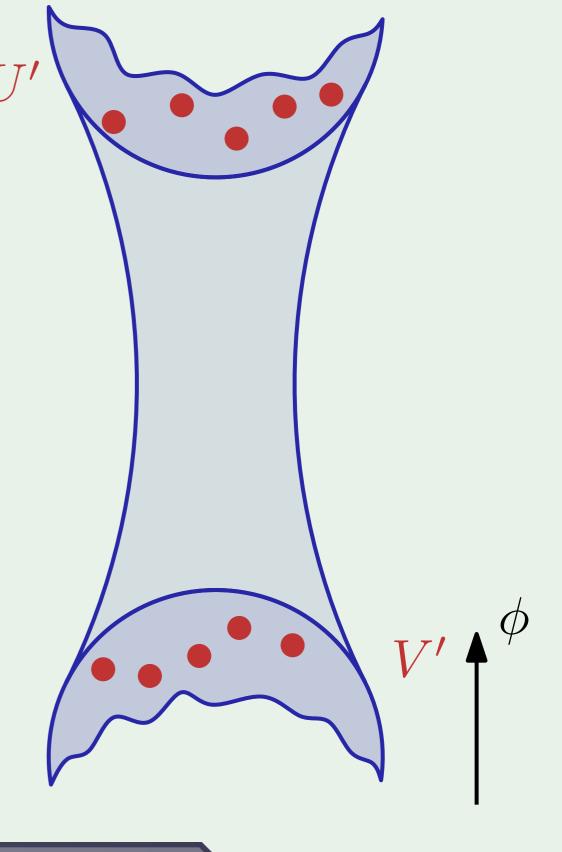
Claim: We can sort P conforming Γ in linear time.

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Now, we can find the closest edge e between U' and V' in O(|U| + |V|) time.



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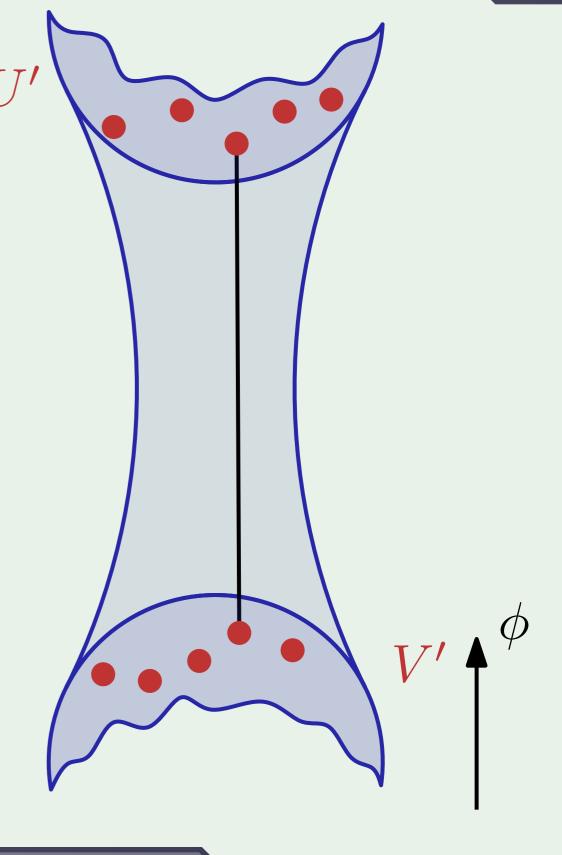
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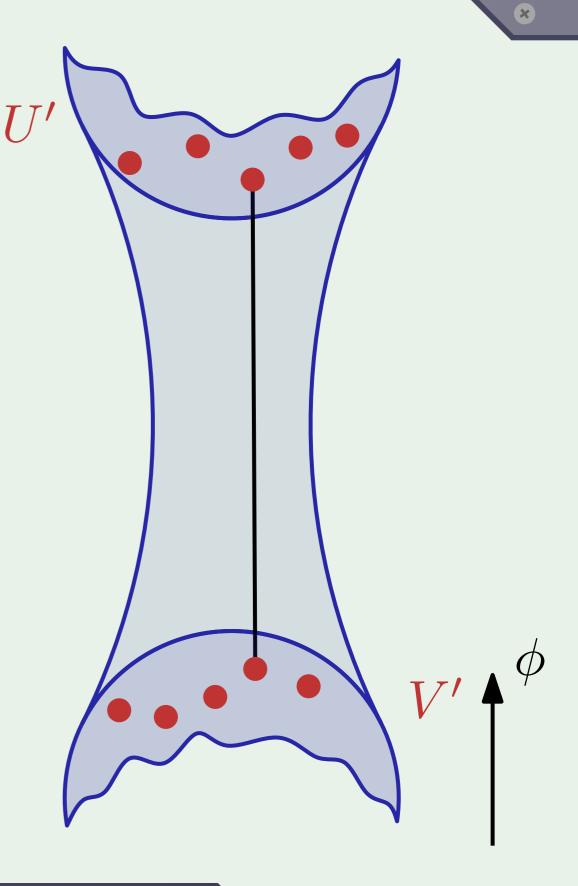
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Now, we can find the closest edge e between U' and V' in O(|U| + |V|) time.

So, we can find G_ϕ in O(n) time.



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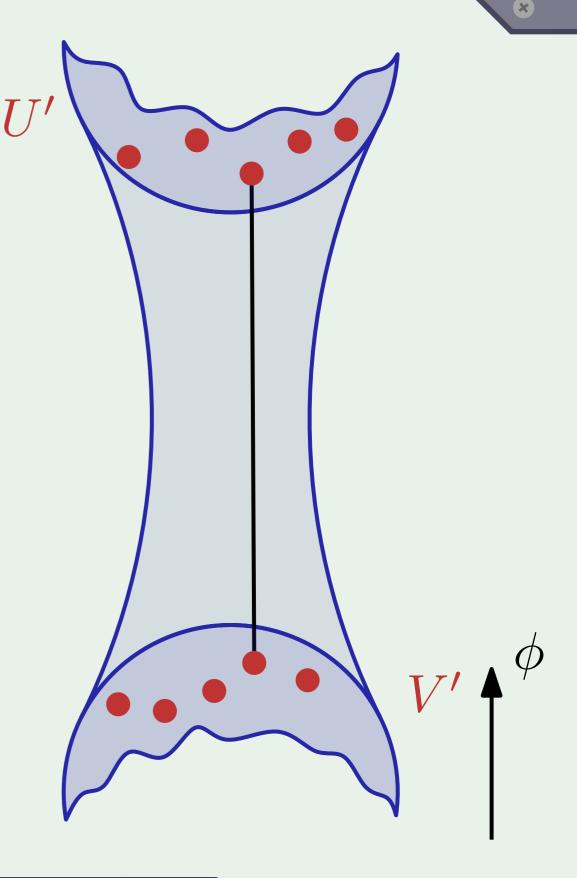
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Now, we can find the closest edge e between U' and V' in O(|U| + |V|) time.

So, we can find G_{ϕ} in O(n) time.

Yay! We found G, a linear size supergraph of the MST of P, in linear time!



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Problem: computing a MST may take up to $O(n\alpha(n))$ time.

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Problem: computing a MST may take up to $O(n\alpha(n))$ time.

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For *planar* graphs, this is only O(n).

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Problem: computing a MST may take up to $O(n\alpha(n))$ time.

For *planar* graphs, this is only O(n).

Unfortunately, G need not be planar.

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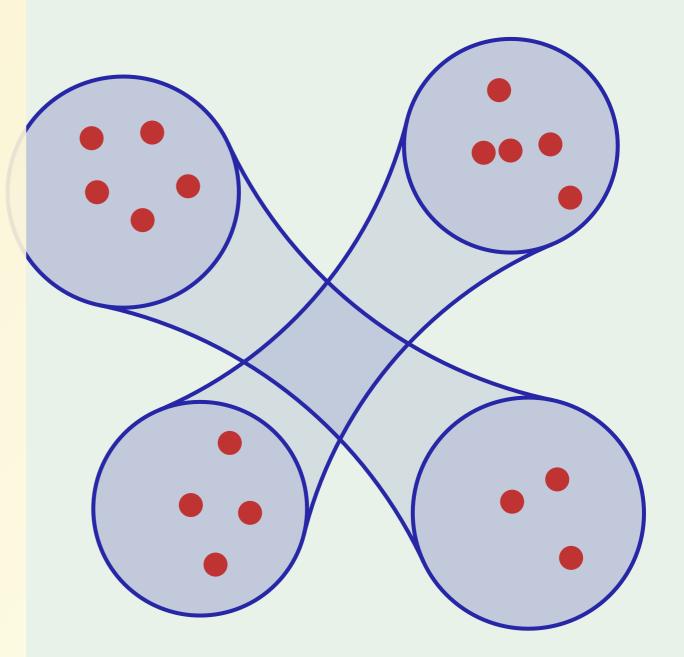
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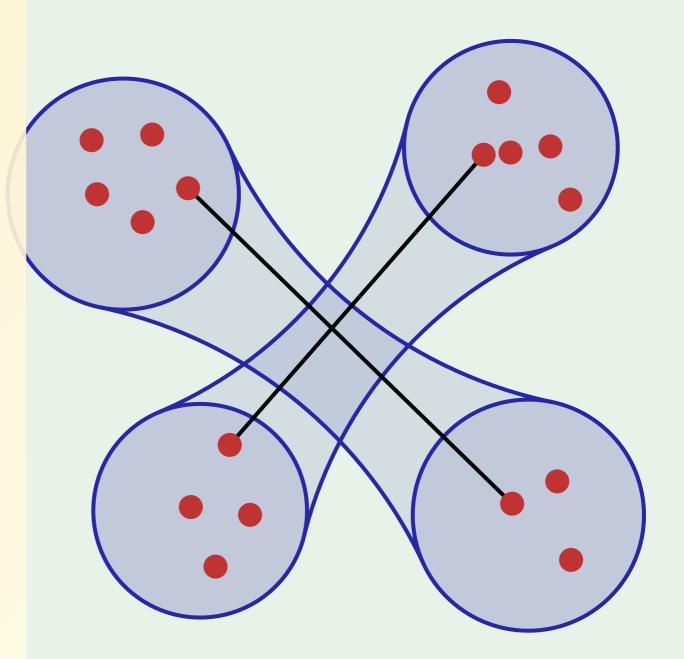
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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

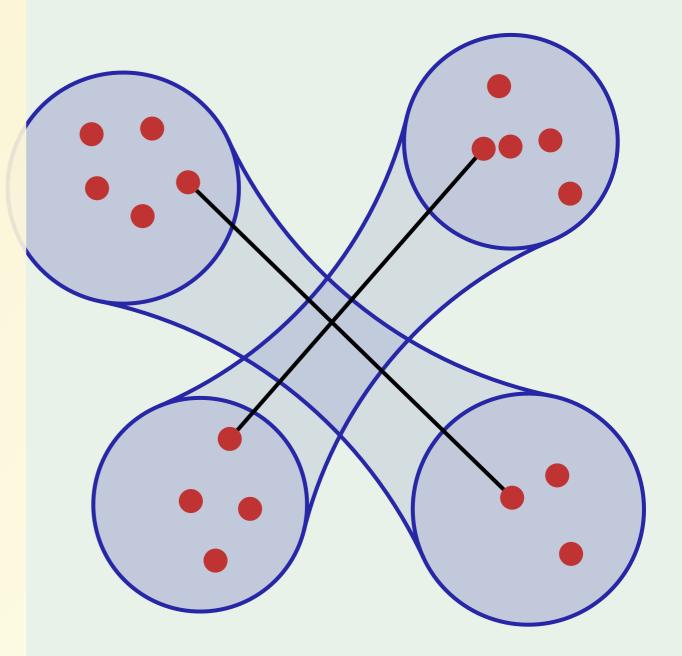
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Problem: computing a MST may take up to $O(n\alpha(n))$ time.

For *planar* graphs, this is only O(n).

Unfortunately, G need not be planar.

Claim: any edge eof G crosses at most O(1) edges of length $\Omega(|e|)$.



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Start with P and G.

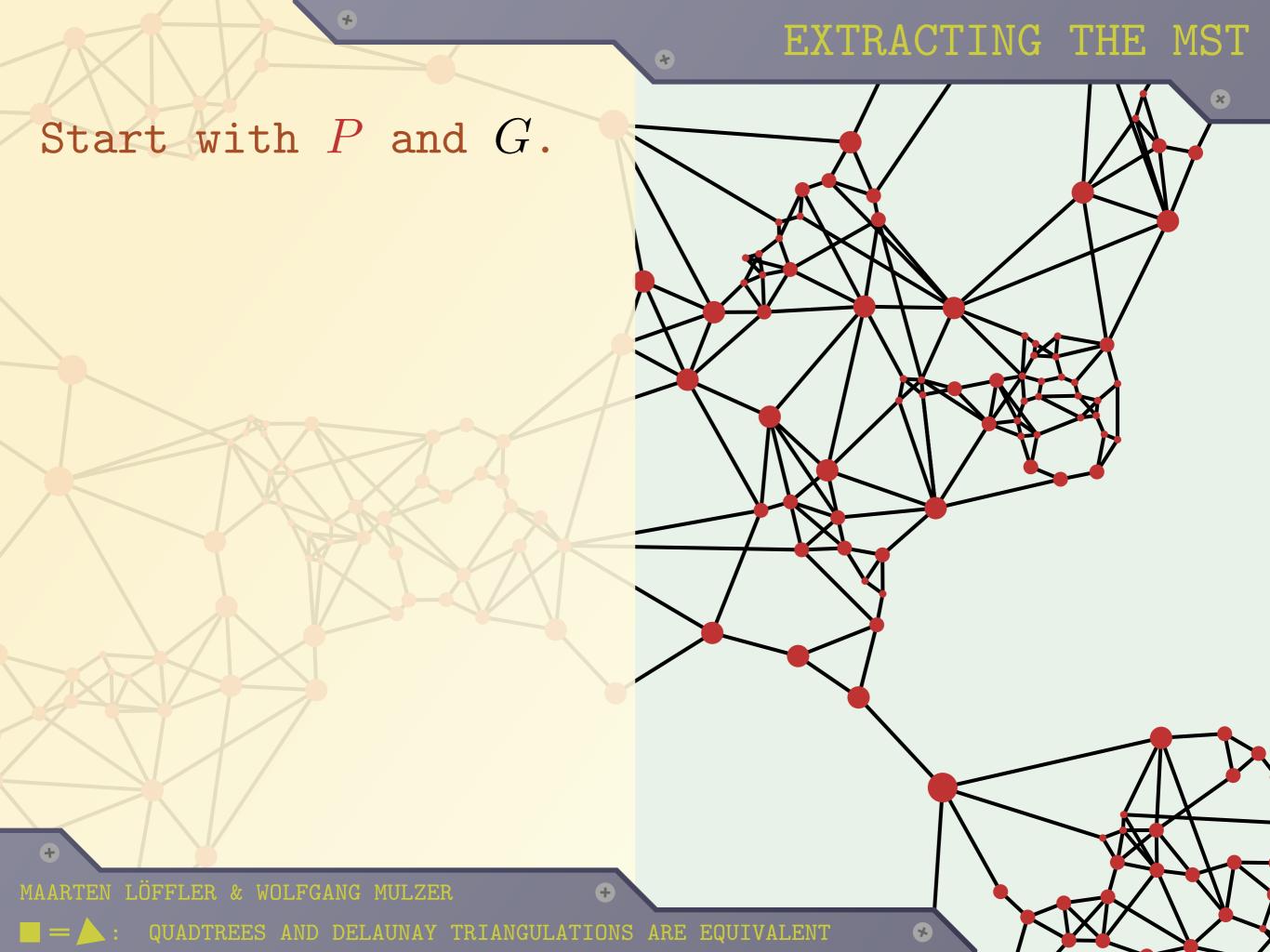
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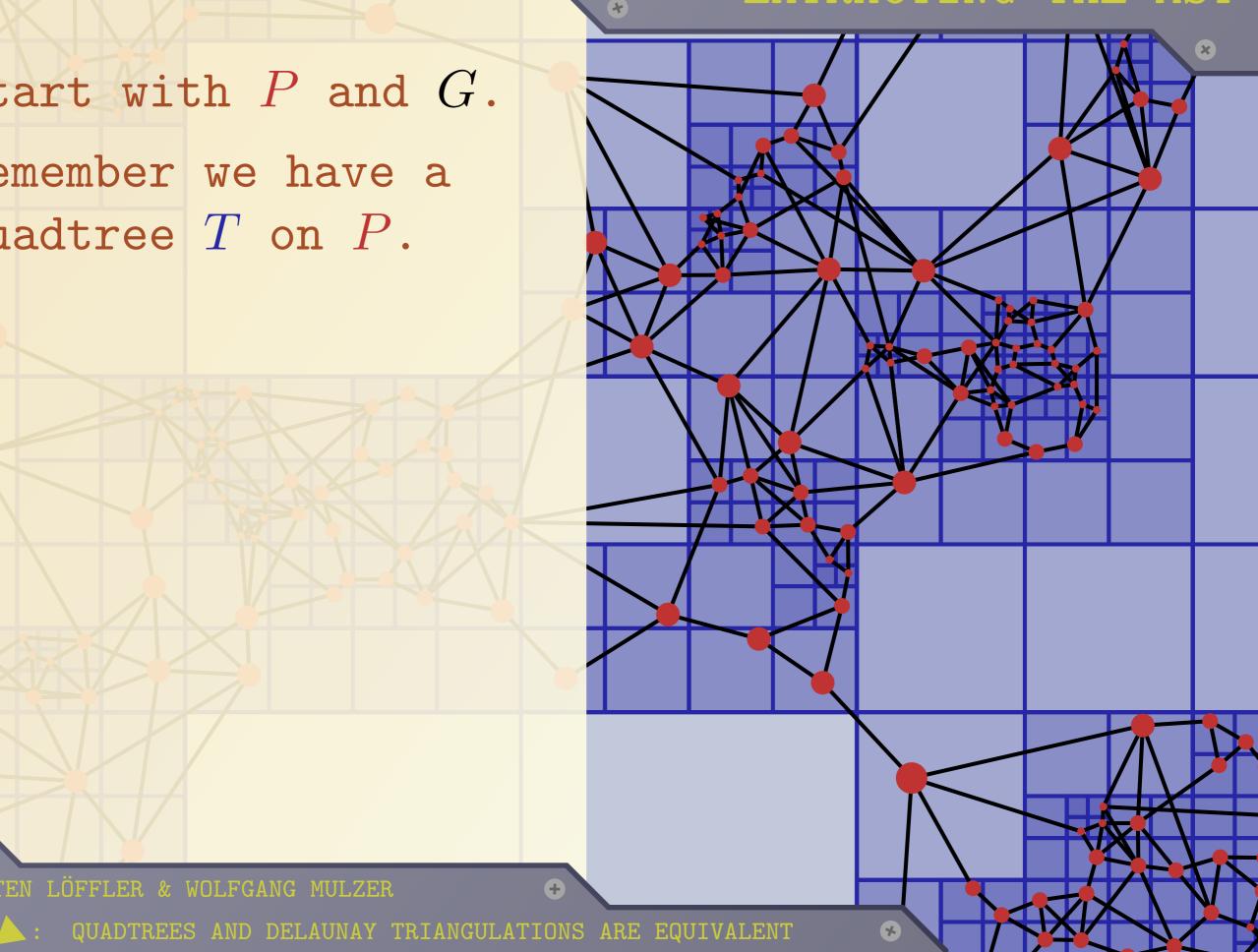
Start with P and G. Remember we have a quadtree T on P.

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Start with P and G.

Remember we have a quadtree T on P.

We can process the edges of G by increasing length, Borůvka-style.



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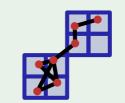
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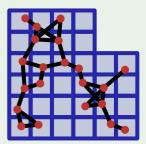
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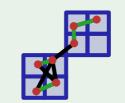


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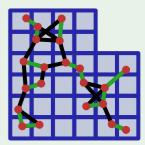
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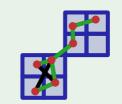


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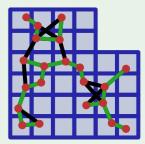
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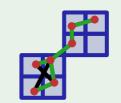


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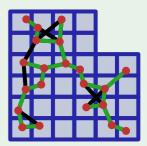
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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

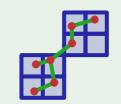


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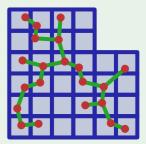
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Start with P and G.

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Remember we have a quadtree T on P.

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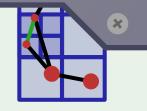
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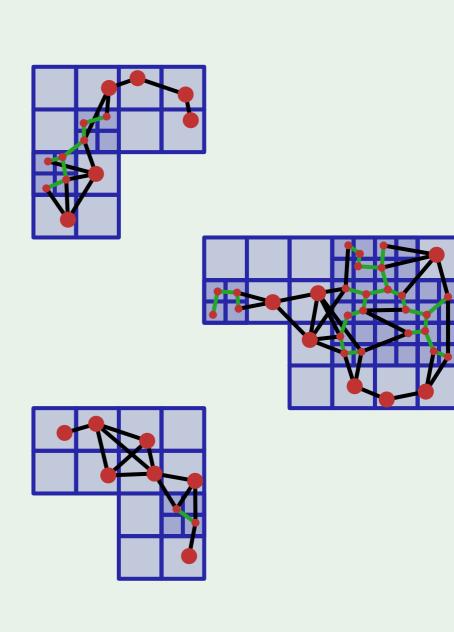
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Start with P and G.

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Ignore edges within components.



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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

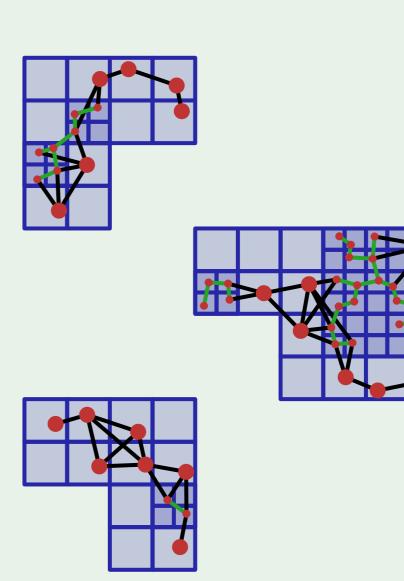
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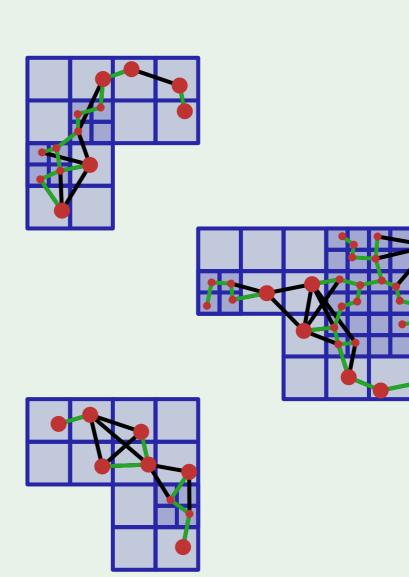
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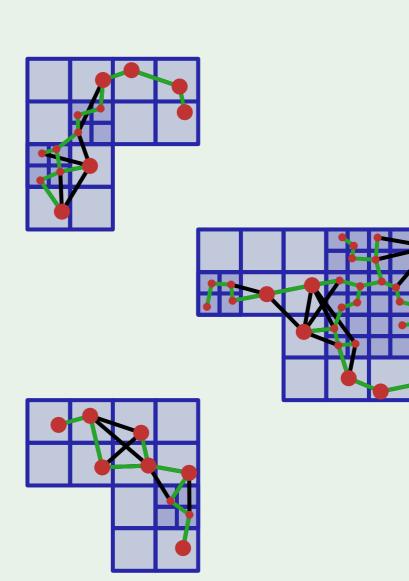
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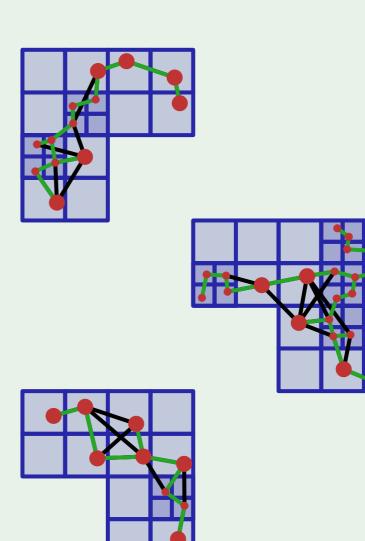
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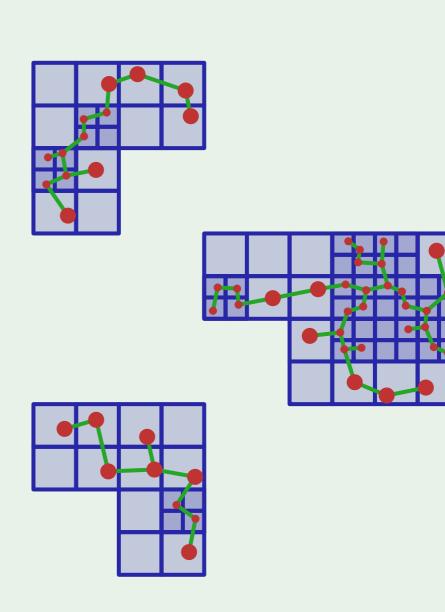
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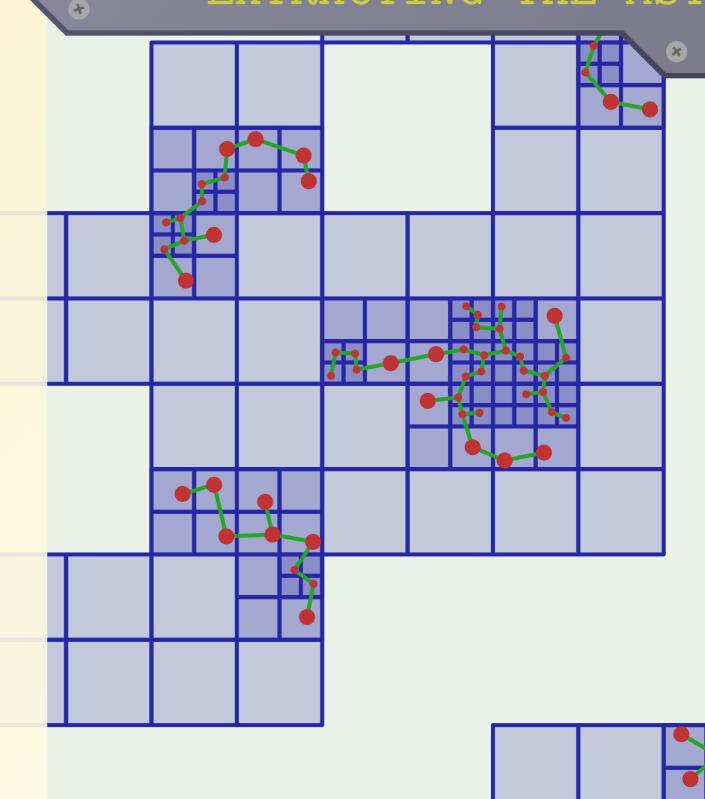
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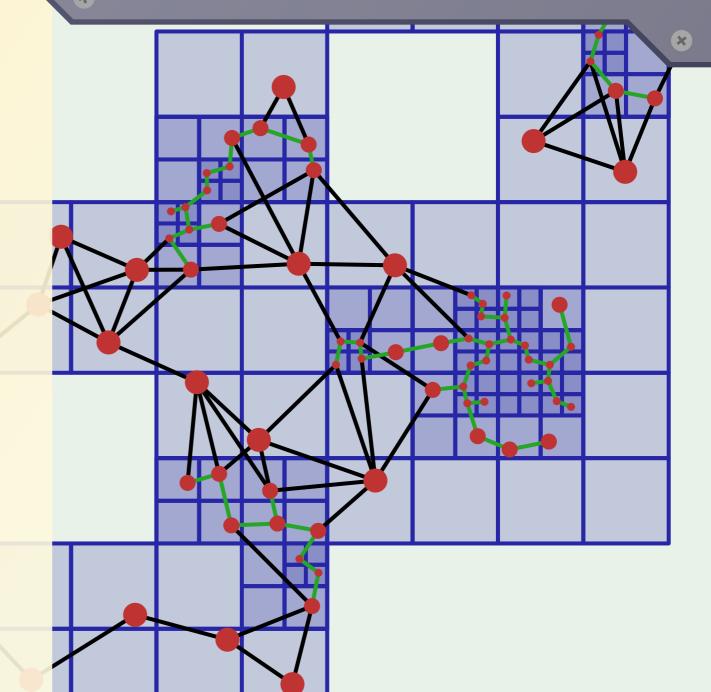
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QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

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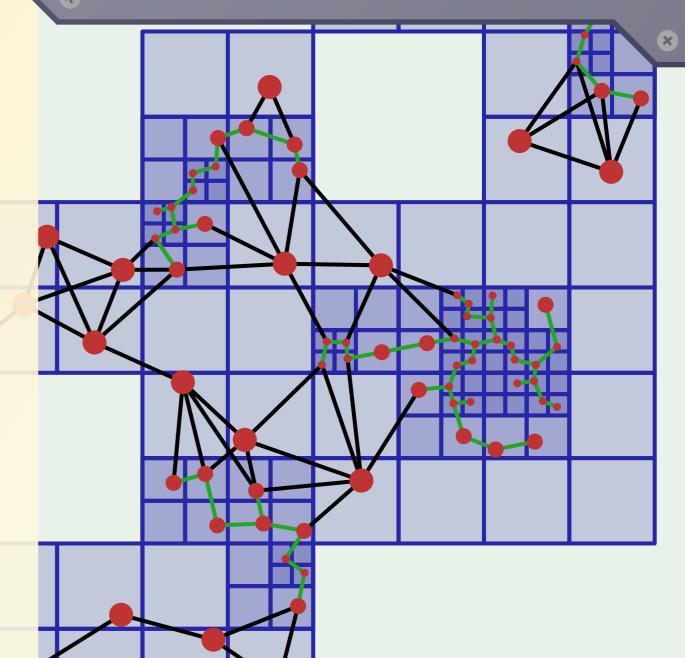
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= 📐 : 🛛 QUADTREES AND DELAUNAY TRIANGULATIONS ARE EQUIVALENT

Start with P and G. Remember we have a quadtree T on P.

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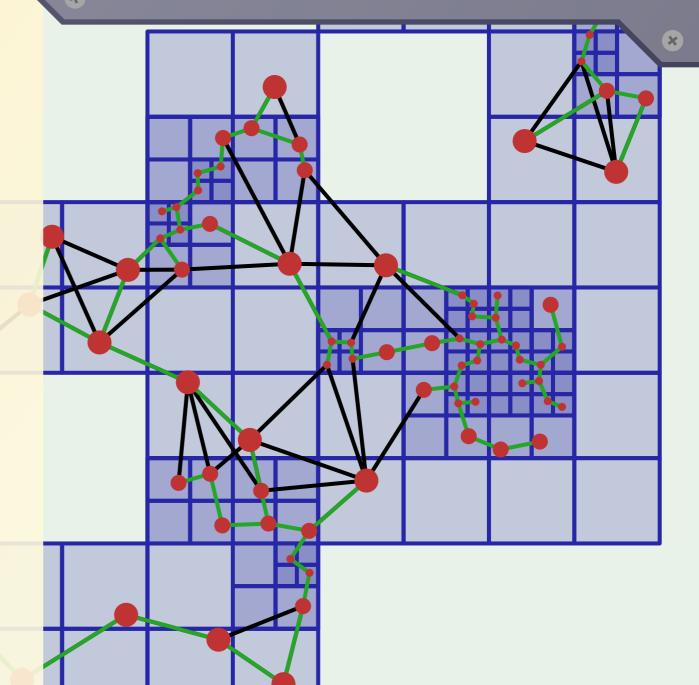
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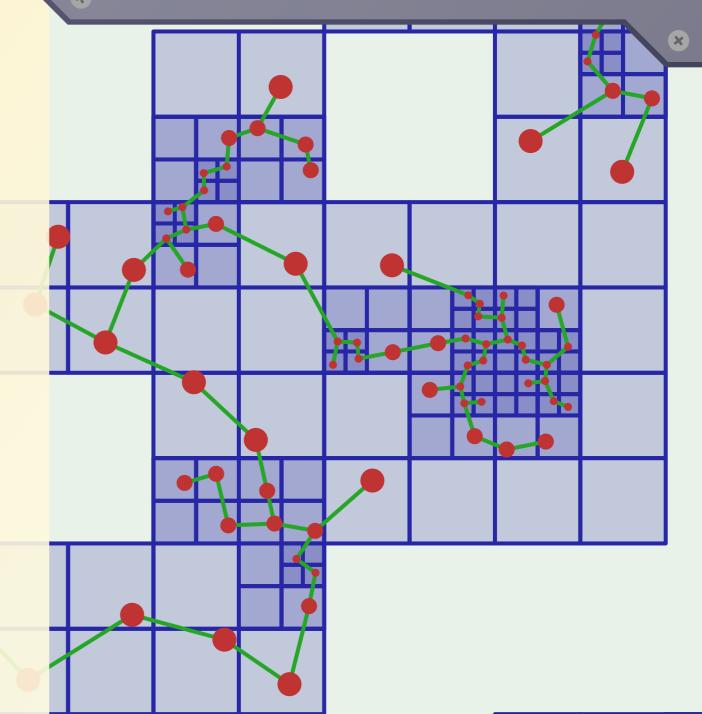
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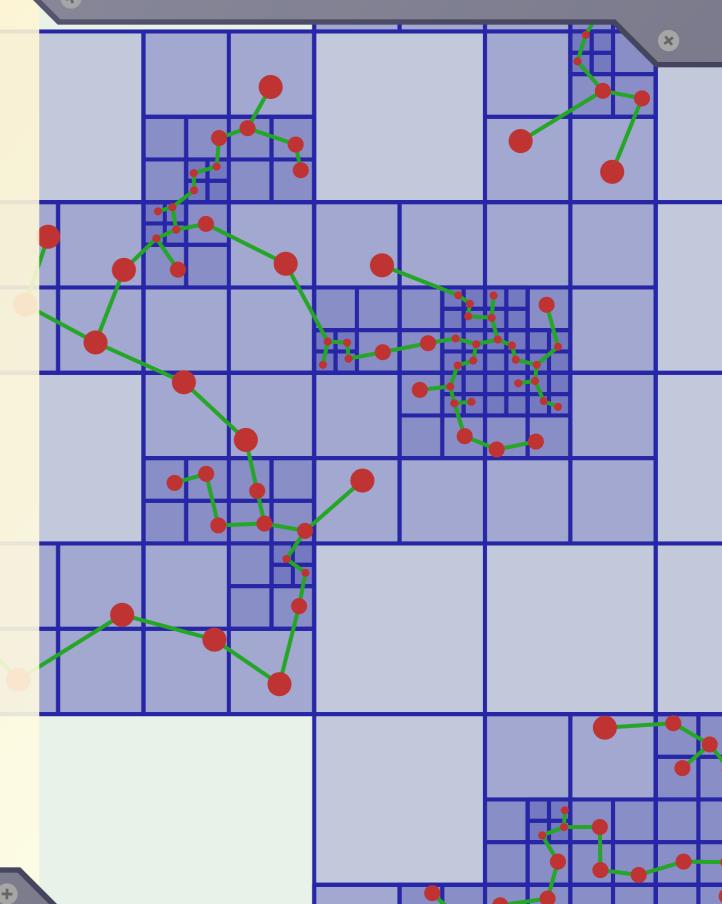
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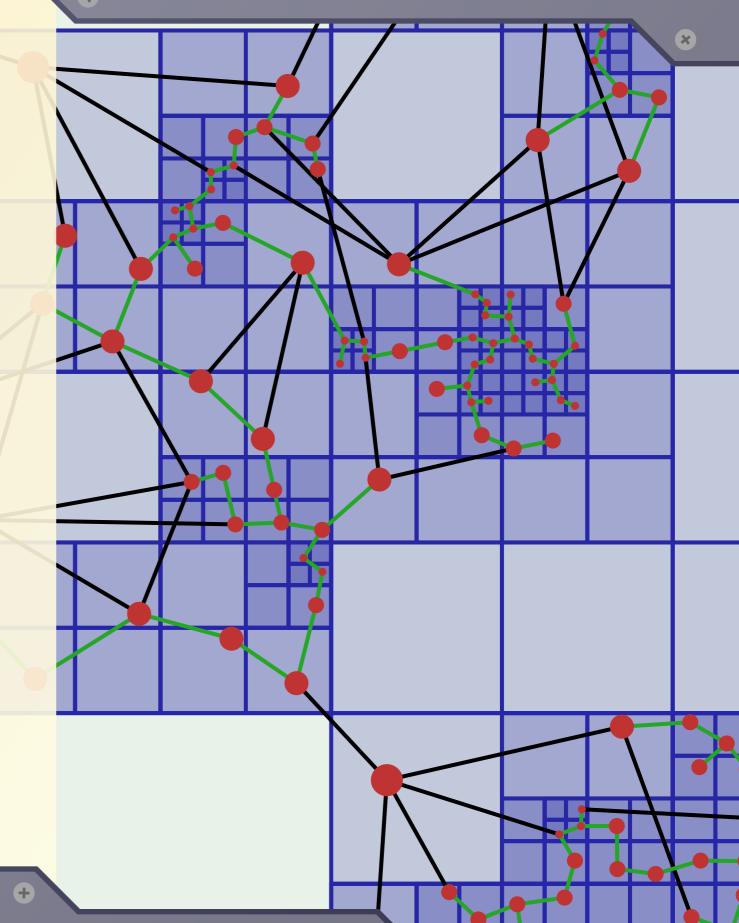
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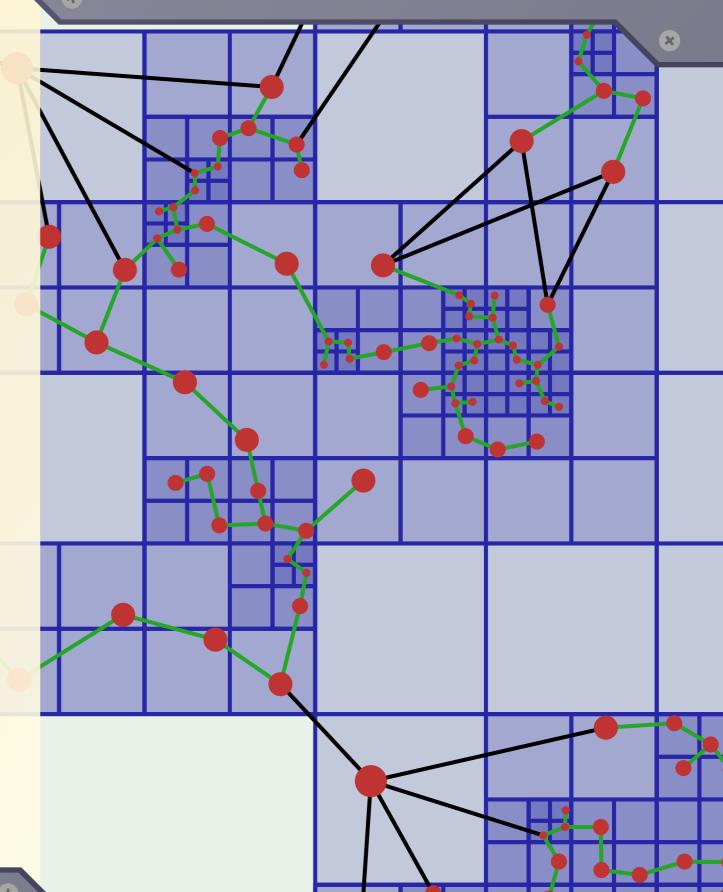
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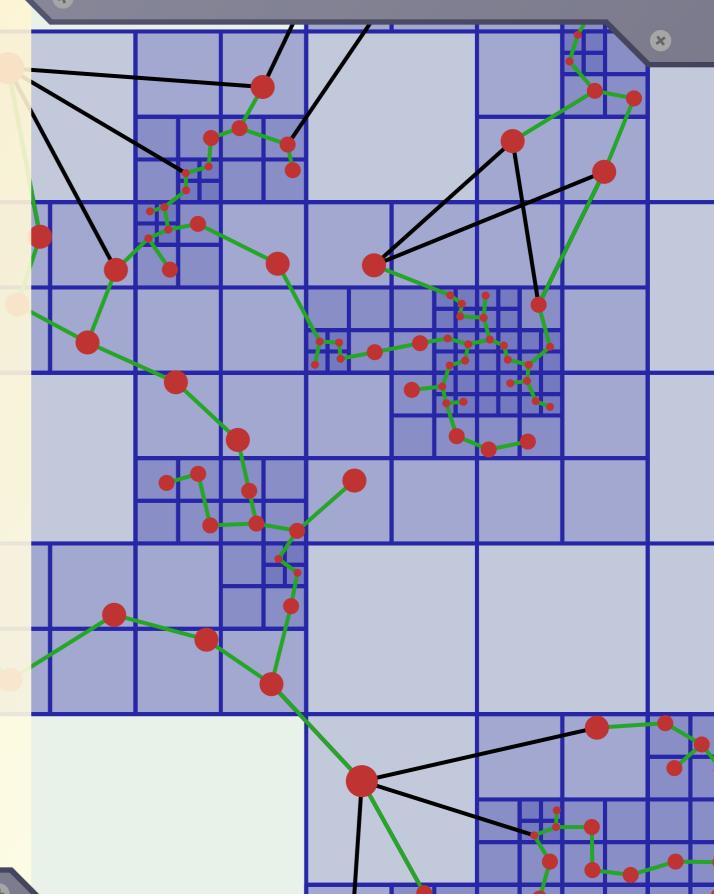
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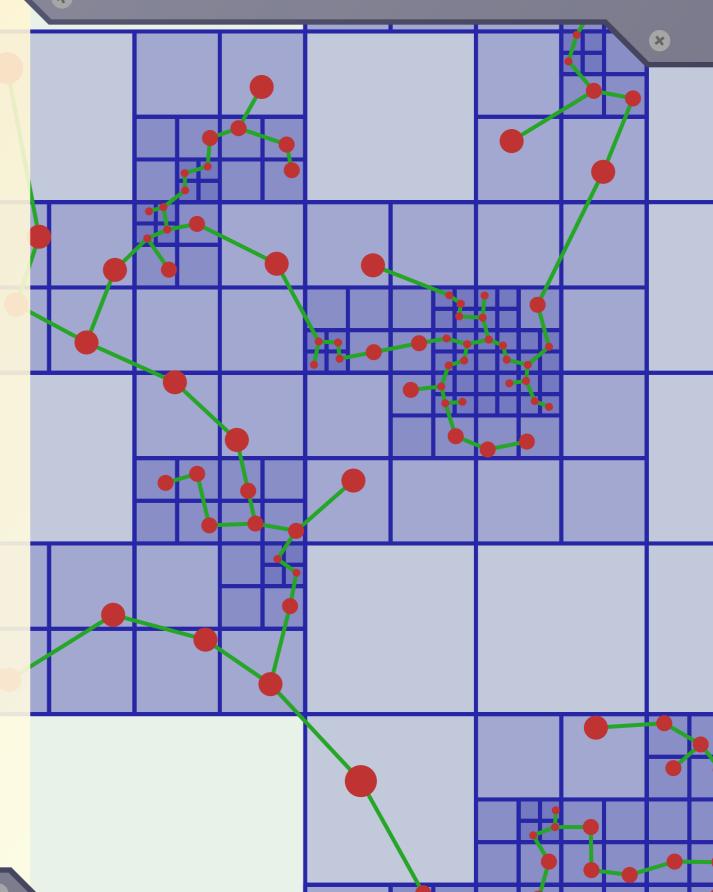
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Done!

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CONCLUSION



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CONCLUSION

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Given a quadtree on a set of points P, we can compute the MST of P in linear time.

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CONCLUSION

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Given a quadtree on a set of points P, we can compute the MST of P in linear time.

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Many major proximity structures on planar point sets can be derived from each other in linear deterministic time.

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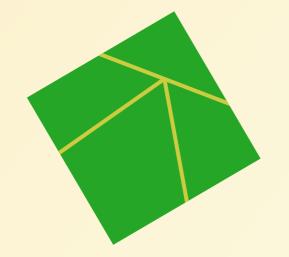
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SQUARES ARE TRIANGLES

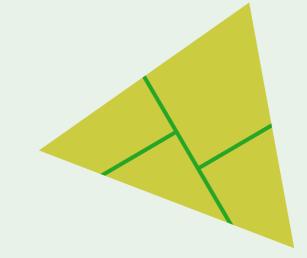
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THANK YOU!

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ANY QUESTIONS?

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