
COMPUTING WAVE IMPACT IN SELF-ORGANISED MUSSEL BEDS
APRIL
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11:30
EuroCG17
Malmö
JOHAN VAN DE KOPPEL
MAARTEN LÖFFLER
TIM OPHELDERS

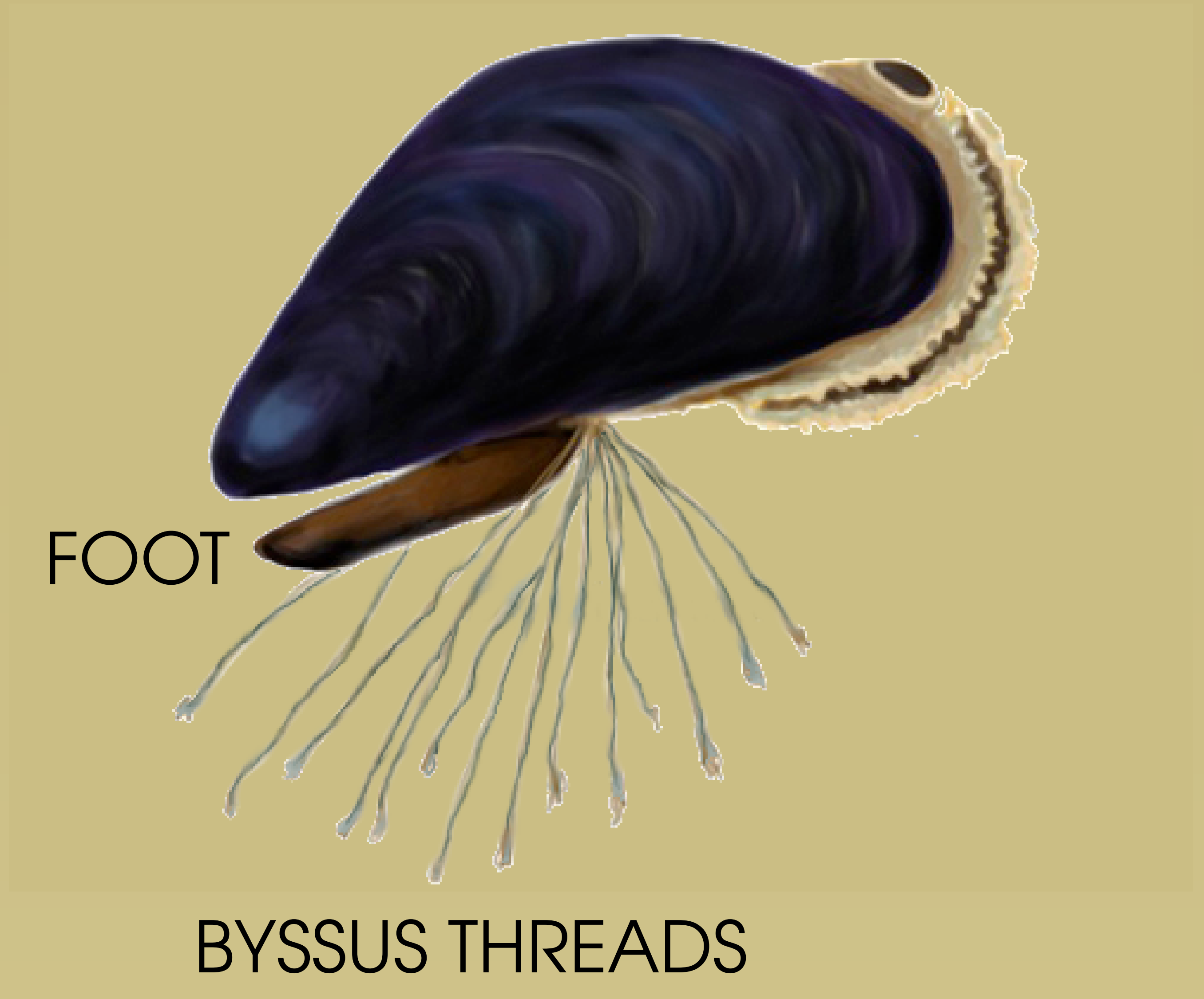
A CRASH COURSE IN MUSSLEOLOGY

DEFINITION

Mussel is the common name used for members of several families of bivalve molluscs, from saltwater and freshwater habitats. These groups have in common a shell whose outline is elongated and asymmetrical compared with other edible clams, which are often more or less rounded or oval.

A CRASH COURSE IN MUSSLEOLOGY

Mussels have a foot that they use to walk around.



A CRASH COURSE IN MUSSLEOLOGY

Mussels have a foot that they use to walk around.

Mussels have byssus threads to attach themselves.

MUSSEL PATTERNS AND SELF-ORGANISATION

You can find mussels in shallow water with strong tides, for instance attached to rocks or piers.

MUSSEL PATTERNS AND SELF-ORGANISATION

You can find mussels in shallow water with strong tides, for instance attached to rocks or piers.

You can also find them in large sandy areas, attached to each other in interesting patterns.

WHY?

HYPOTHESIS

Mussels need to be close to each other for stability against waves.
Mussels also need to be close to empty space for their food supply.

WHY?

Access to food is easy to model, but wave impact is much harder to understand.

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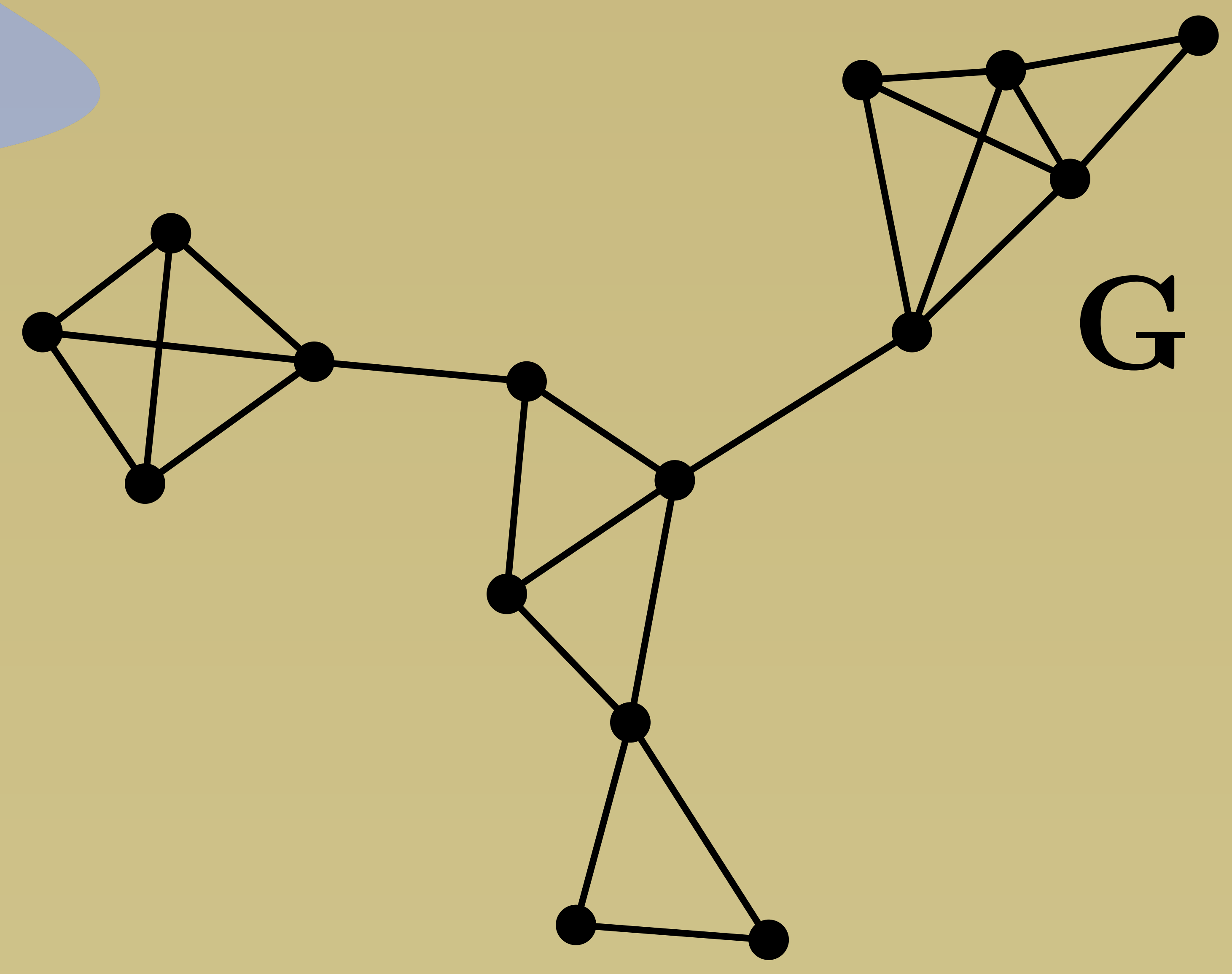
Therefore, marine biologists use computers to understand which mussel patterns are stable.

A WAVE-RESISTANT MUSSEL MODEL



We model the mussels and their byssus connections as a plane geometric graph $\mathbf{G}$.

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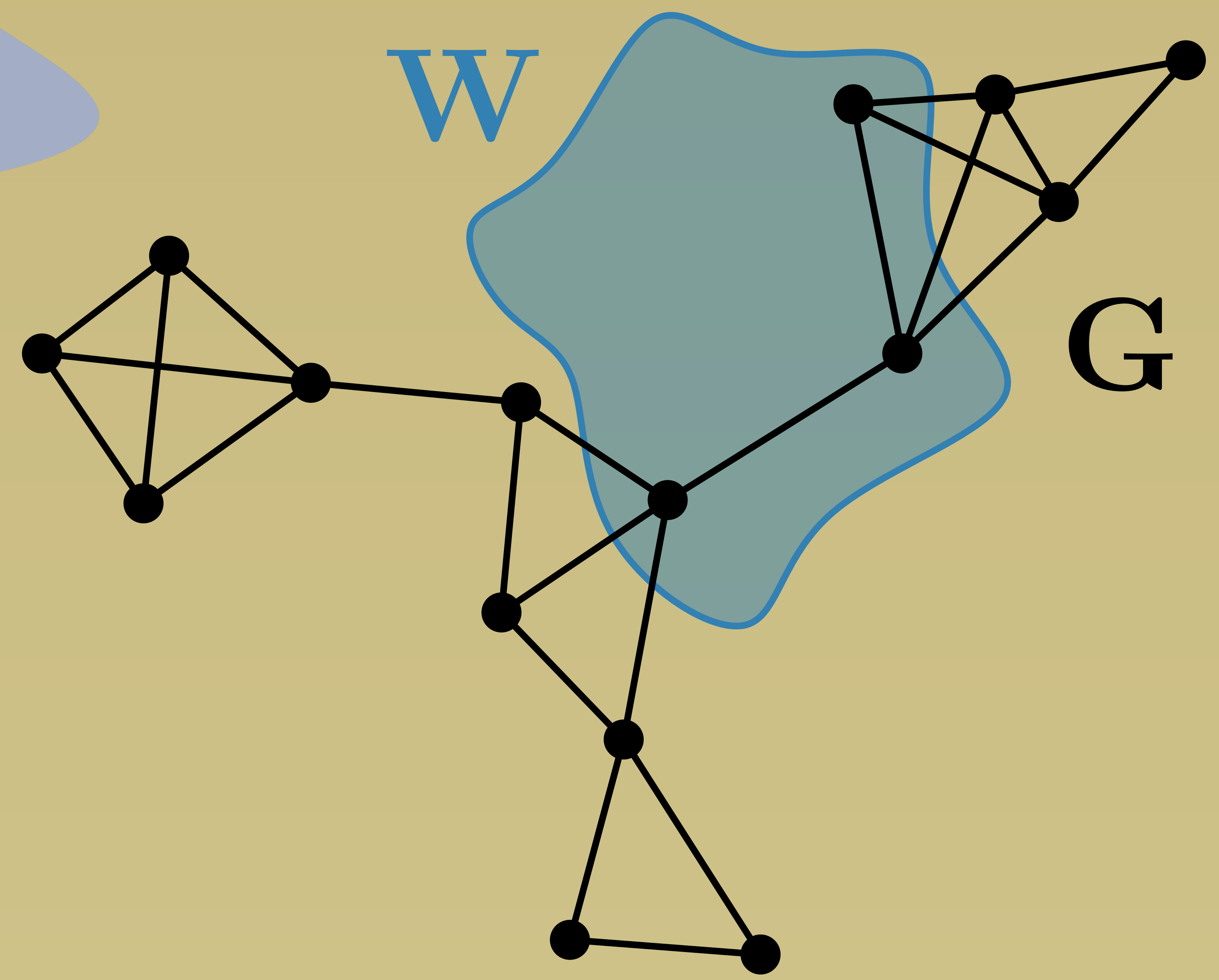
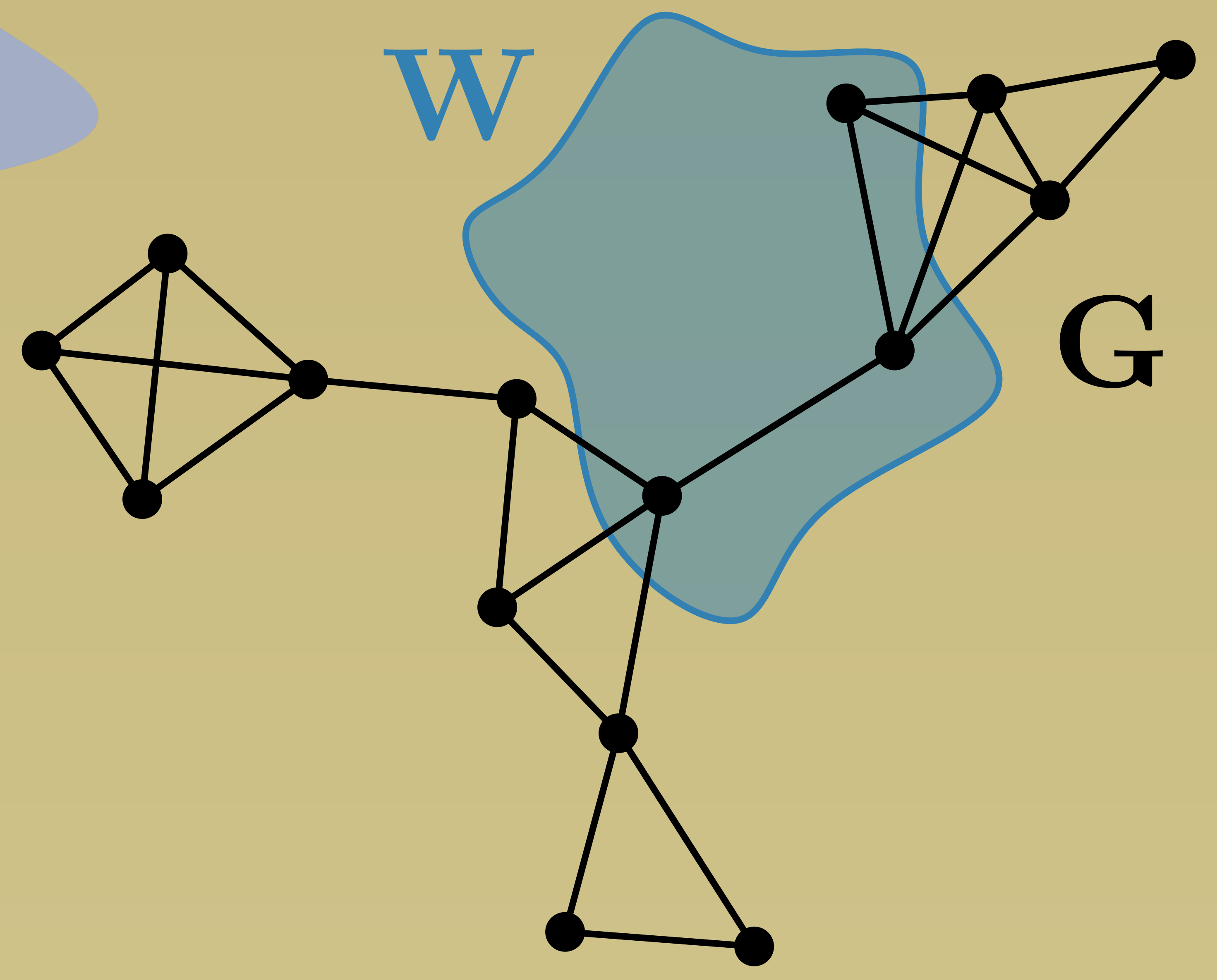


We model the mussels and their byssus connections as a plane geometric graph **G**.

We model a wave as a geometric region **W**.

A WAVE-RESISTANT MUSSEL MODEL



QUESTION

For a given G and W, do all mussels stay fixed or do some mussels dislodge?

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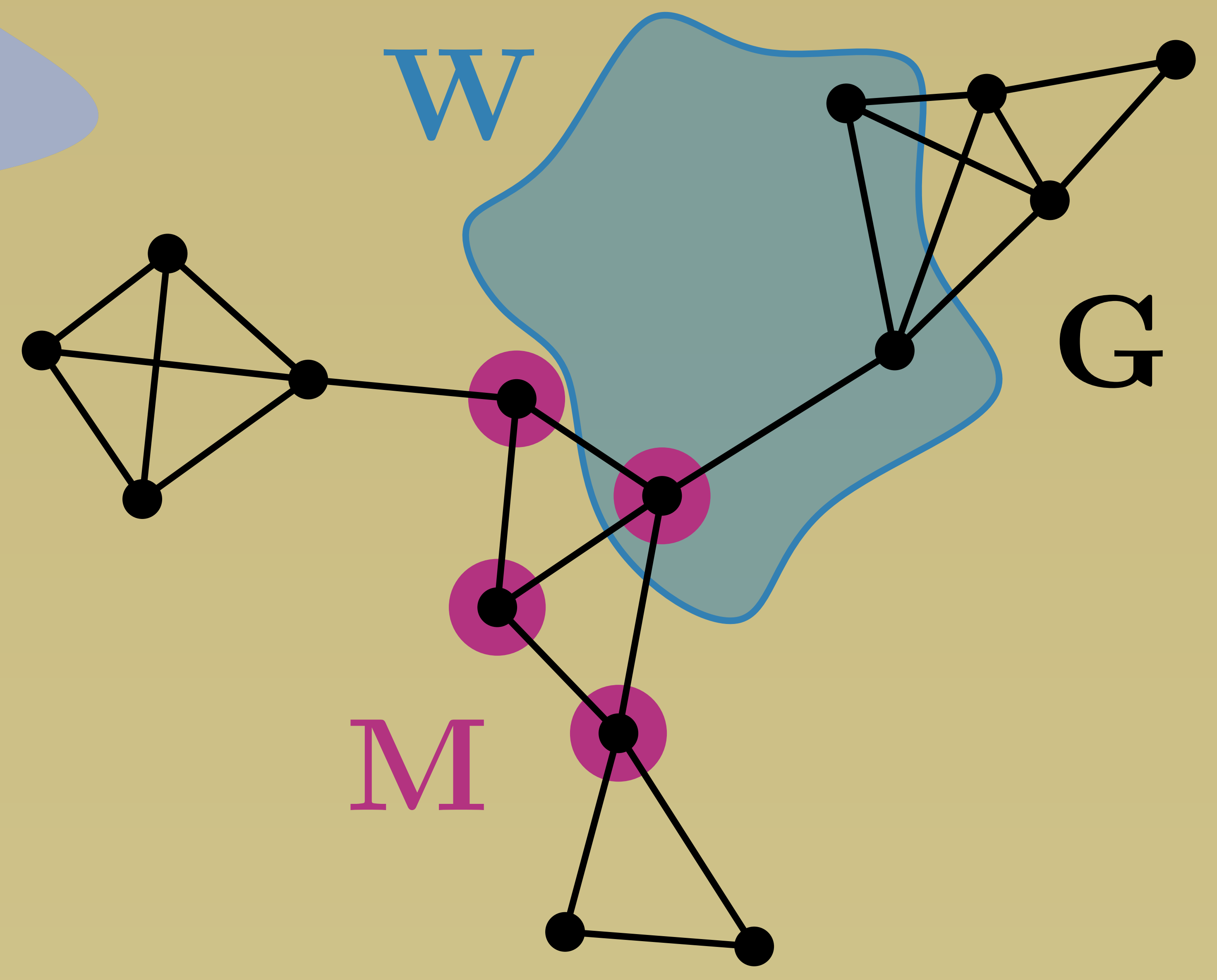

Consider a set of mussels $\mathbf{M}$.

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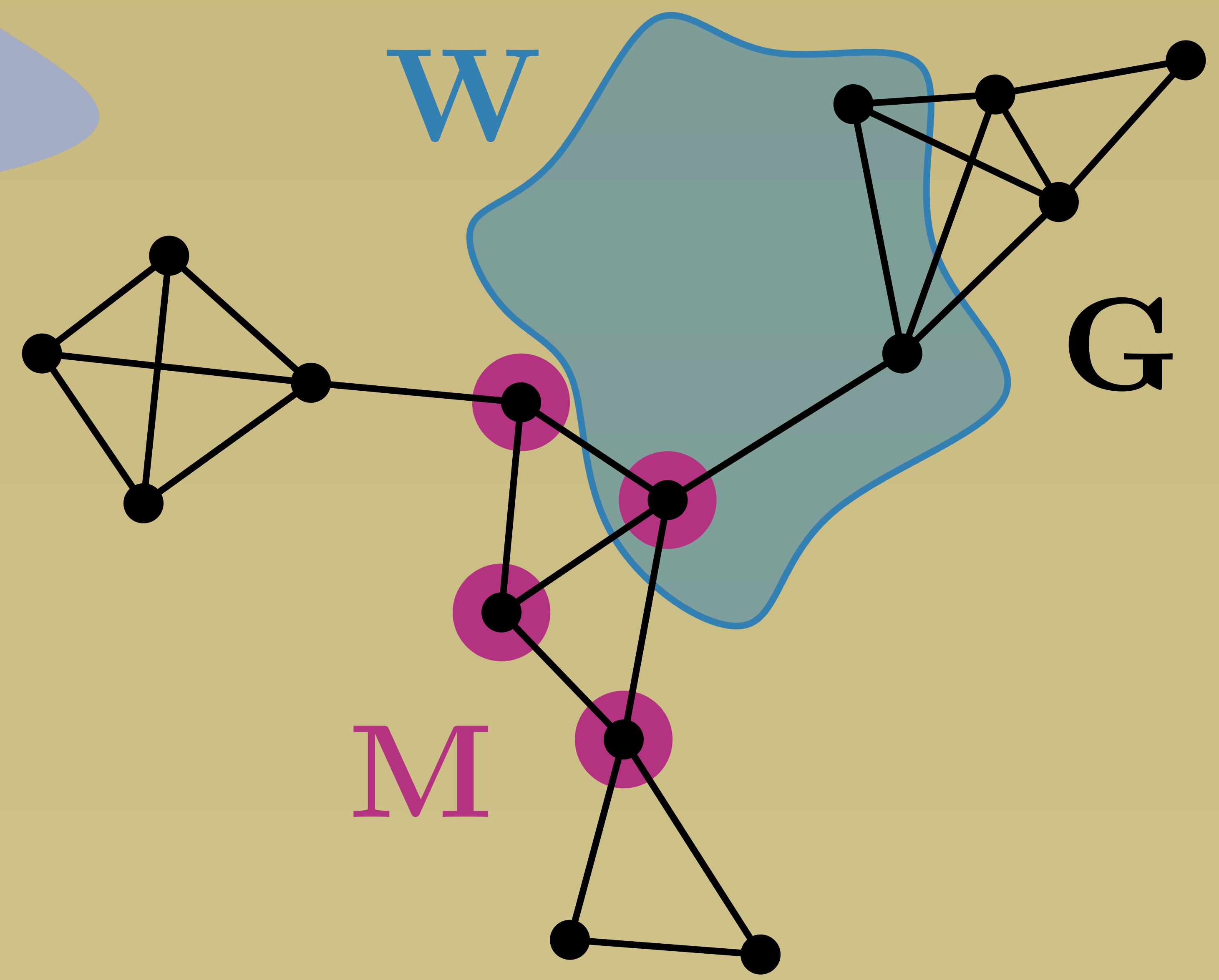
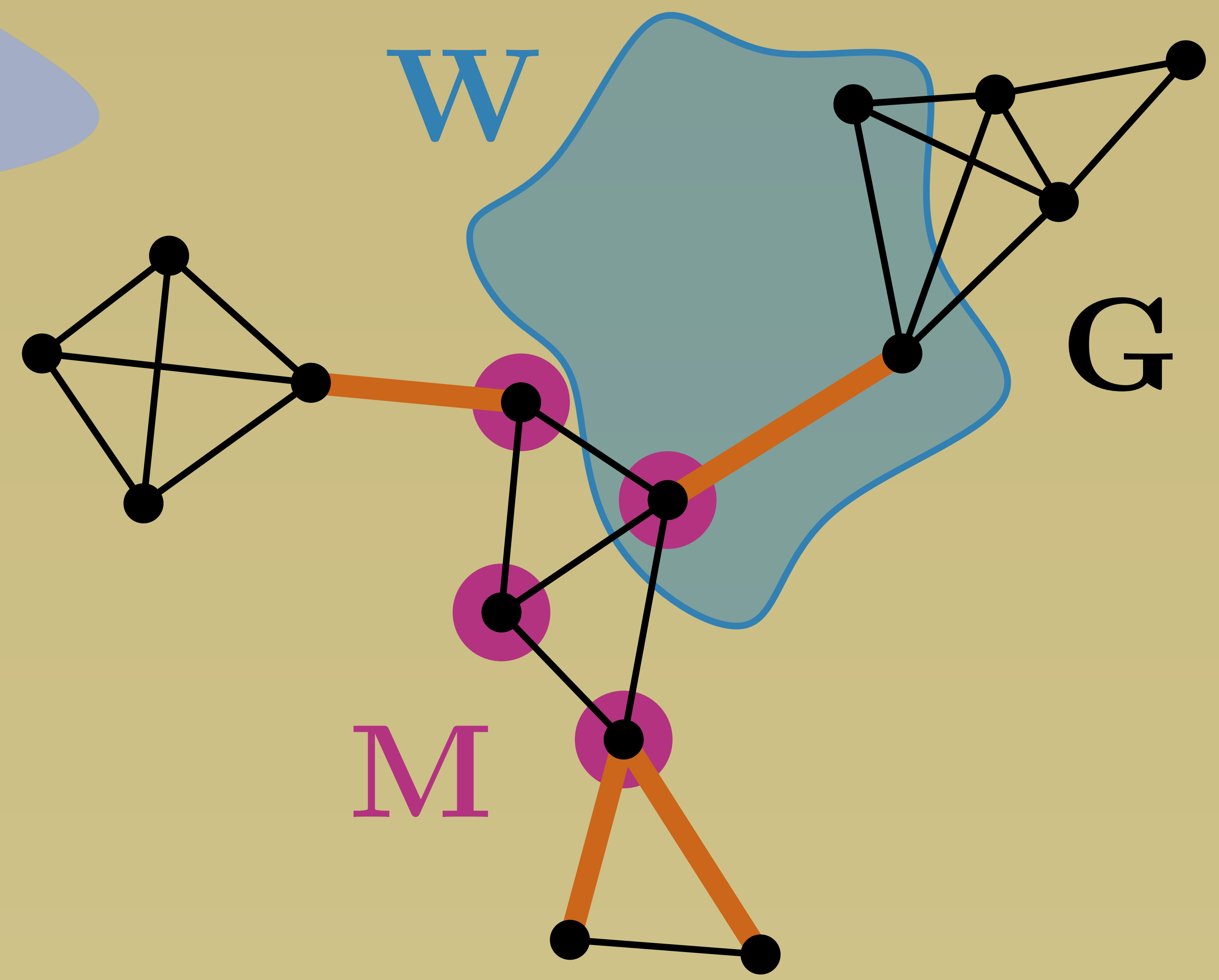


Consider a set of mussels M.

We want to know whether wave W exerts enough force on M to dislodge exactly M.

A WAVE-RESISTANT MUSSEL MODEL

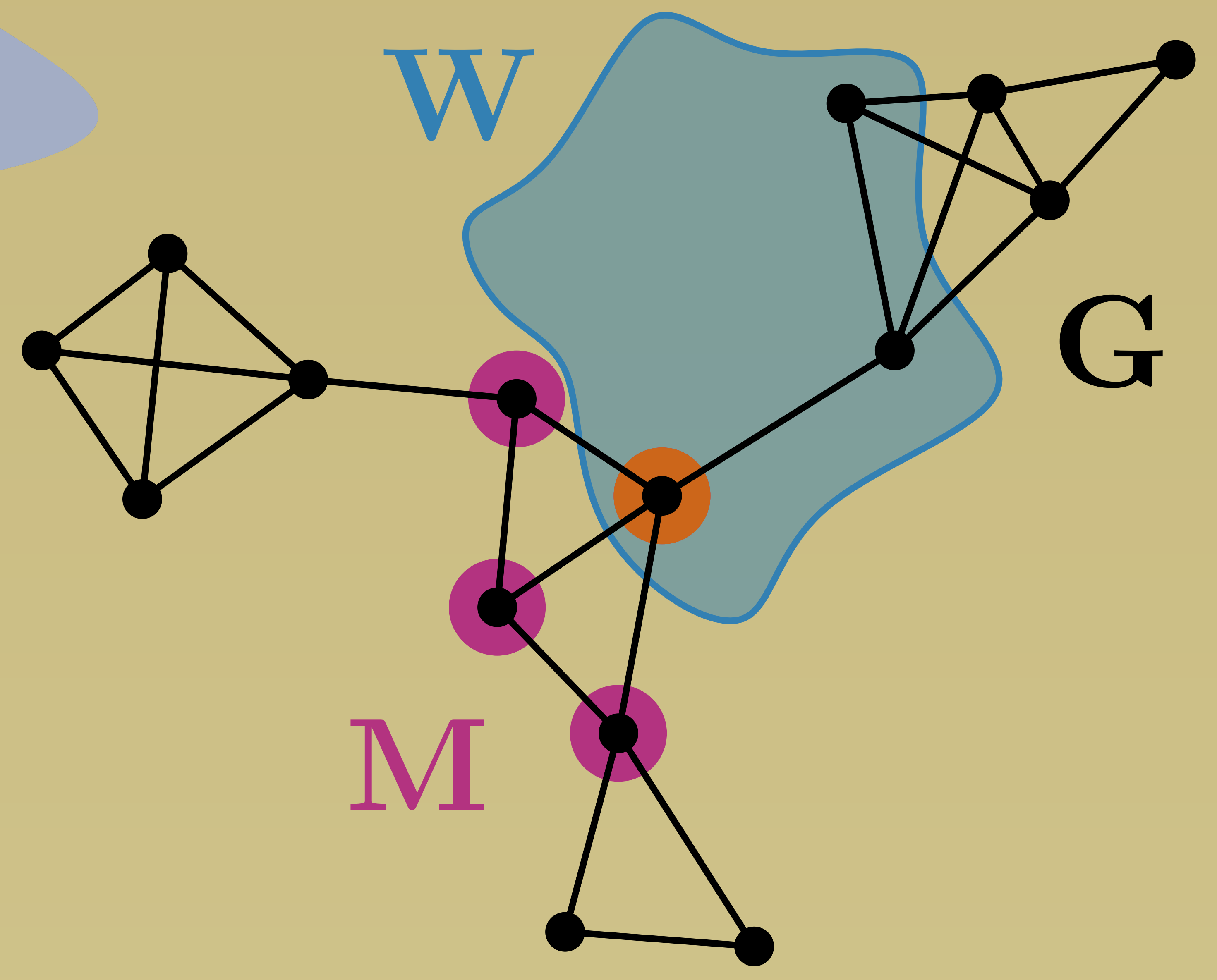


$\mathbf{C}(\mathbf{M}) = 4$

Let $\mathbf{C}(\mathbf{M})$ be the number of connections between $\mathbf{M}$ and the rest of the mussels.

$$\mathbf{C}(\mathbf{M}) = |\{(a,b) \in E : a \in \mathbf{M} \land b \notin \mathbf{M}\}|$$

A WAVE-RESISTANT MUSSEL MODEL

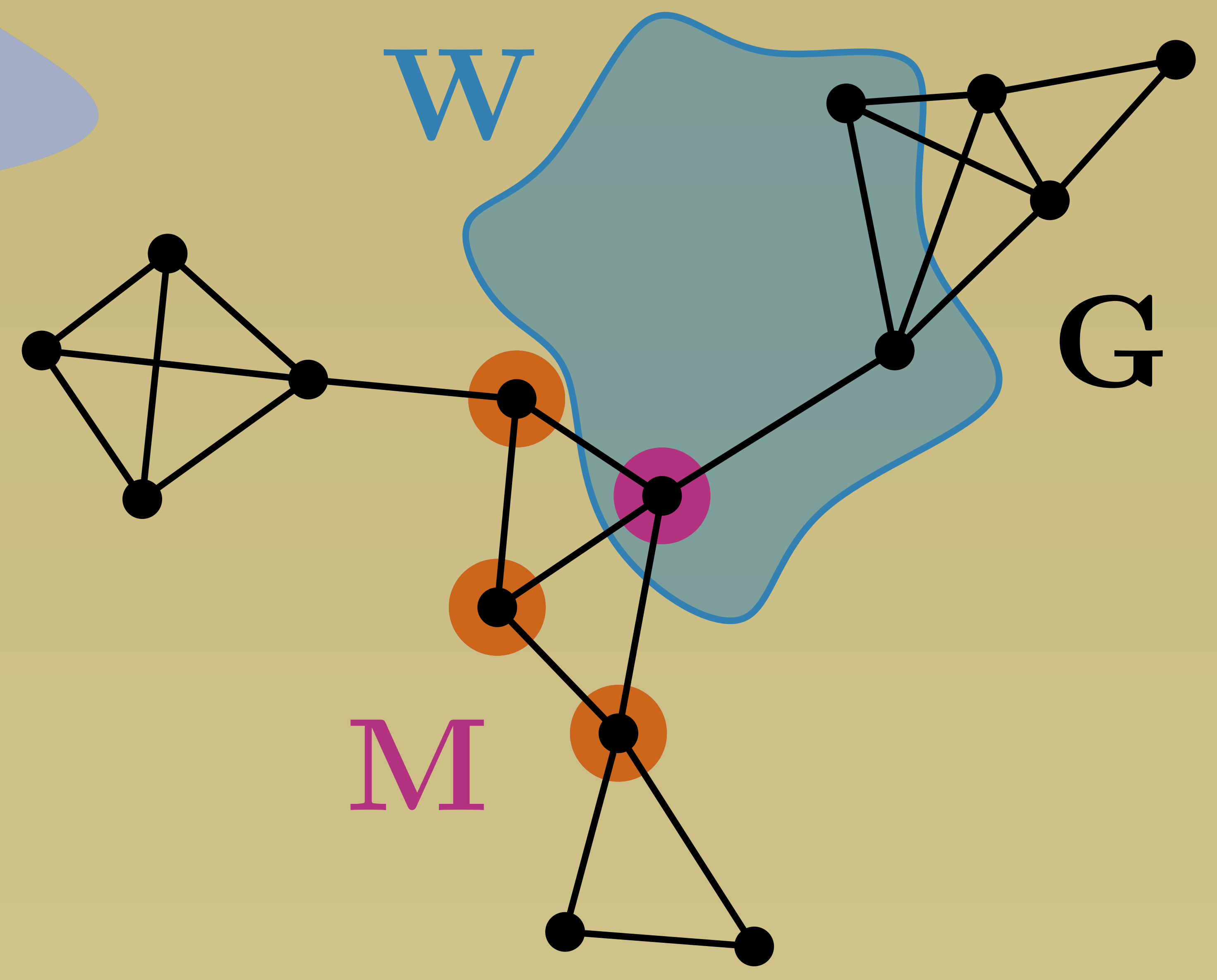


$\mathbf{I}(\mathbf{M}) = 1$

Let $\mathbf{I}(\mathbf{M})$ be the number of mussels of $\mathbf{M}$ that are inside the wave impact zone.

$$\mathbf{I}(\mathbf{M}) = |\mathbf{M} \cap \mathbf{W}|$$

A WAVE-RESISTANT MUSSEL MODEL

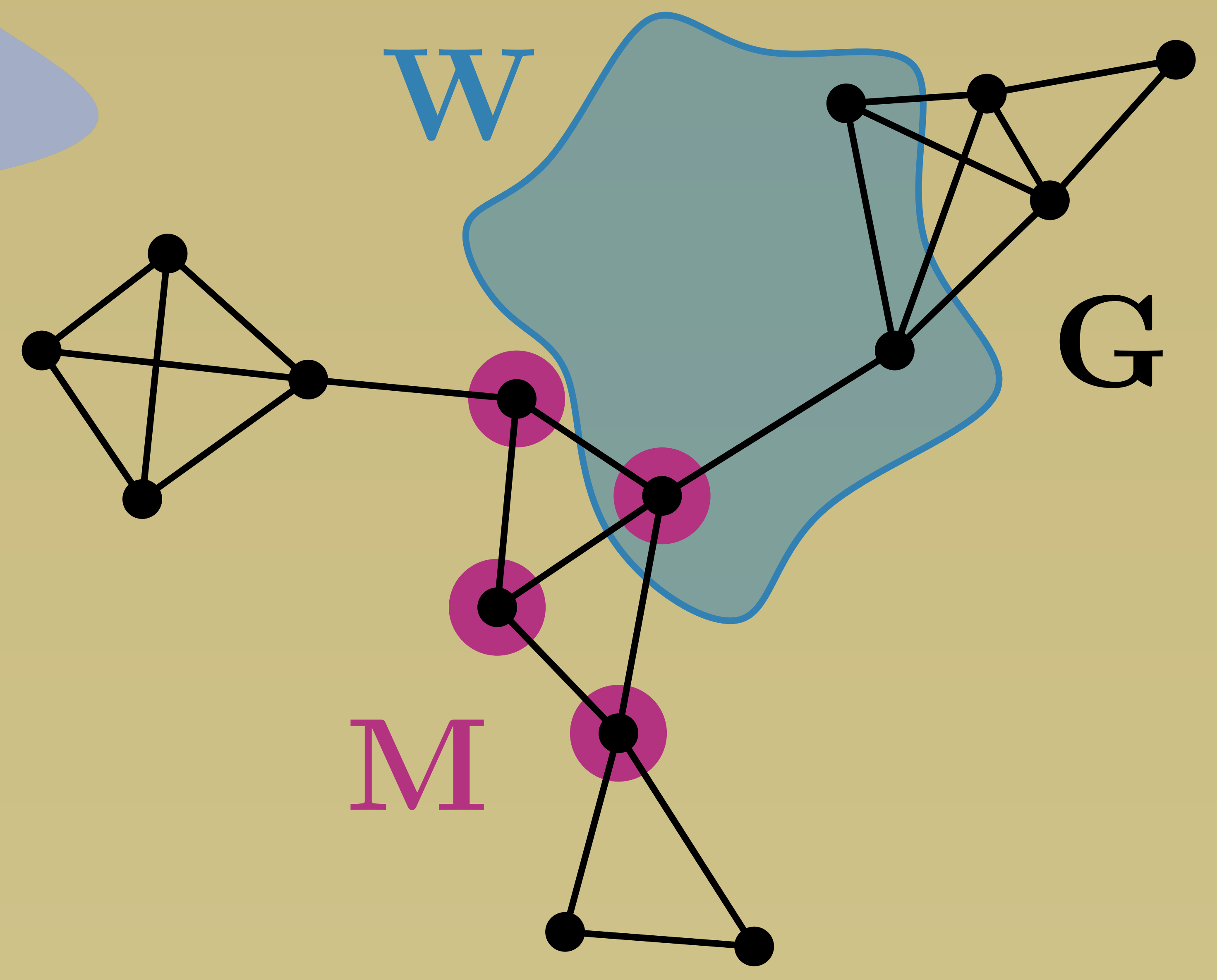


$$\mathbf{O}(\mathbf{M}) = 3$$

Let $\mathbf{O}(\mathbf{M})$ be the number of mussels of $\mathbf{M}$ that are outside the wave impact zone.

$$\mathbf{O}(\mathbf{M}) = |\mathbf{M} \setminus \mathbf{W}|$$

A WAVE-RESISTANT MUSSEL MODEL



We define $\mathbf{F}(\mathrm{M})$ to be a weighted average of $\mathbf{I}(\mathrm{M})$, $\mathbf{O}(\mathrm{M})$ and $\mathbf{C}(\mathrm{M})$.

$$\mathbf{F}(\mathrm{M}) = w_{\mathbf{I}}\mathbf{I}(\mathrm{M}) - w_{\mathbf{O}}\mathbf{O}(\mathrm{M}) - w_{\mathbf{C}}\mathbf{C}(\mathrm{M})$$

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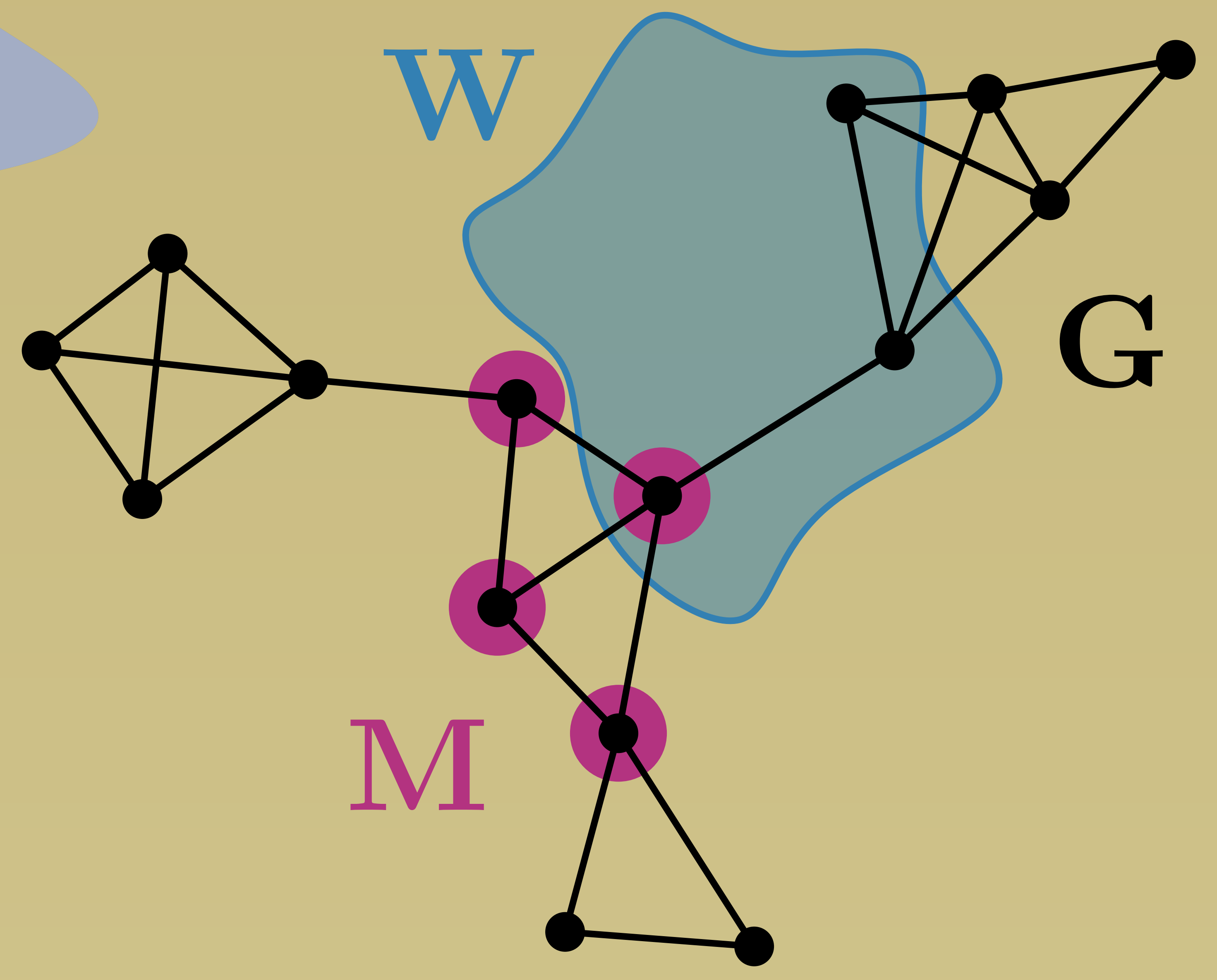

We define $\mathbf{F}(\mathbf{M})$ to be a weighted average of $\mathbf{I}(\mathbf{M})$, $\mathbf{O}(\mathbf{M})$ and $\mathbf{C}(\mathbf{M})$.

The idea is that $\mathbf{M}$ dislodges if and only if $\mathbf{F}(\mathbf{M}) > 0$.

A WAVE-RESISTANT MUSSEL MODEL

LEMMA

If we remove the set $\mathbf{M}$ that maximises $\mathbf{F}$ from $\mathbf{G}$, then all remaining sets have negative or zero potential in the resulting graph.

MUSSEL BEDS ARE REALLY JUST DISK INTERSECTION GRAPHS

Mussels connect themselves to everything solid they can reach.

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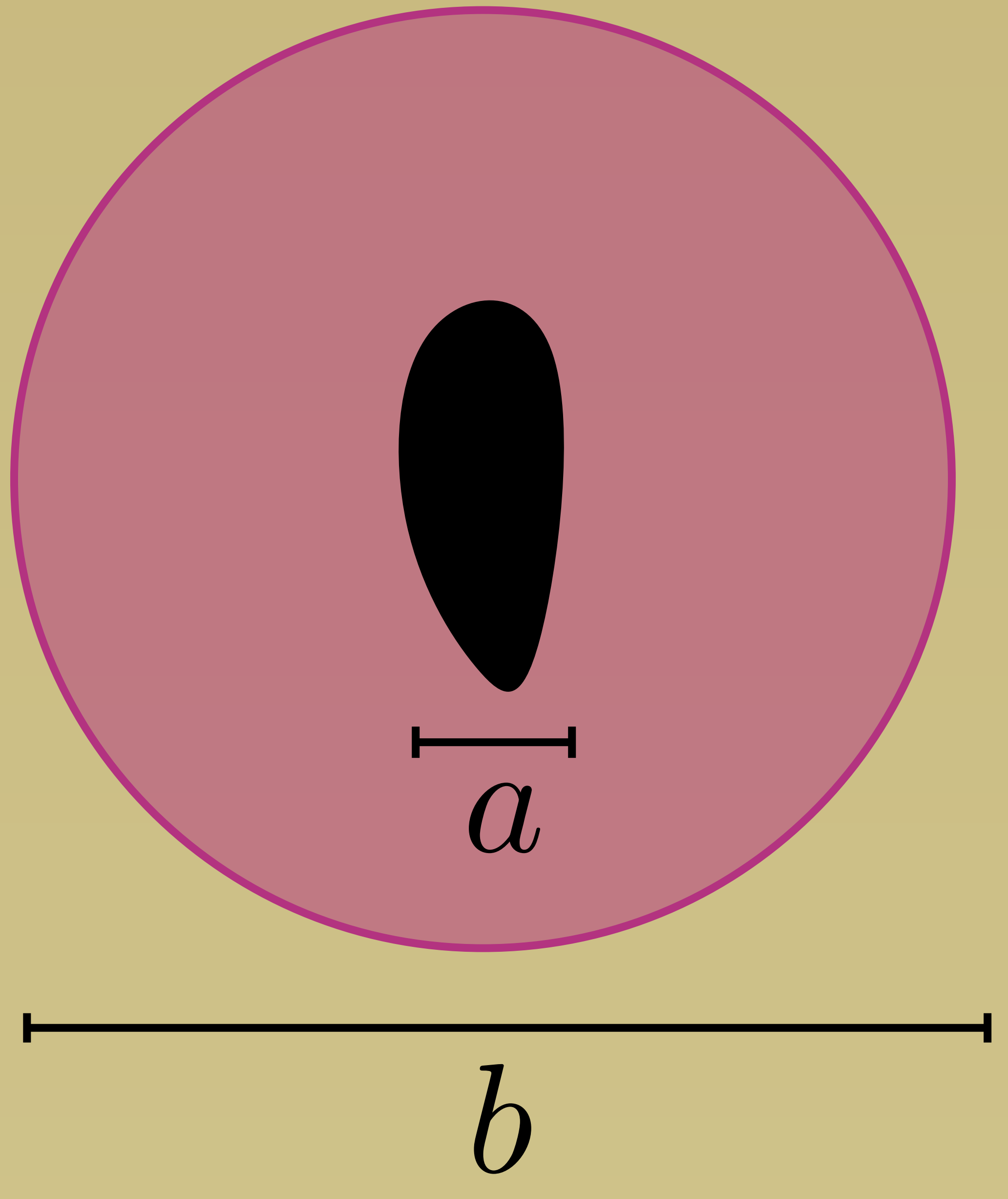
Byssus threads grow to a maximum length from a given point, the byssus pit.

MUSSEL BEDS ARE REALLY JUST DISK INTERSECTION GRAPHS

Let ϕ be the ratio between byssus range and mussel size.



$$\phi = \frac{a}{b}$$

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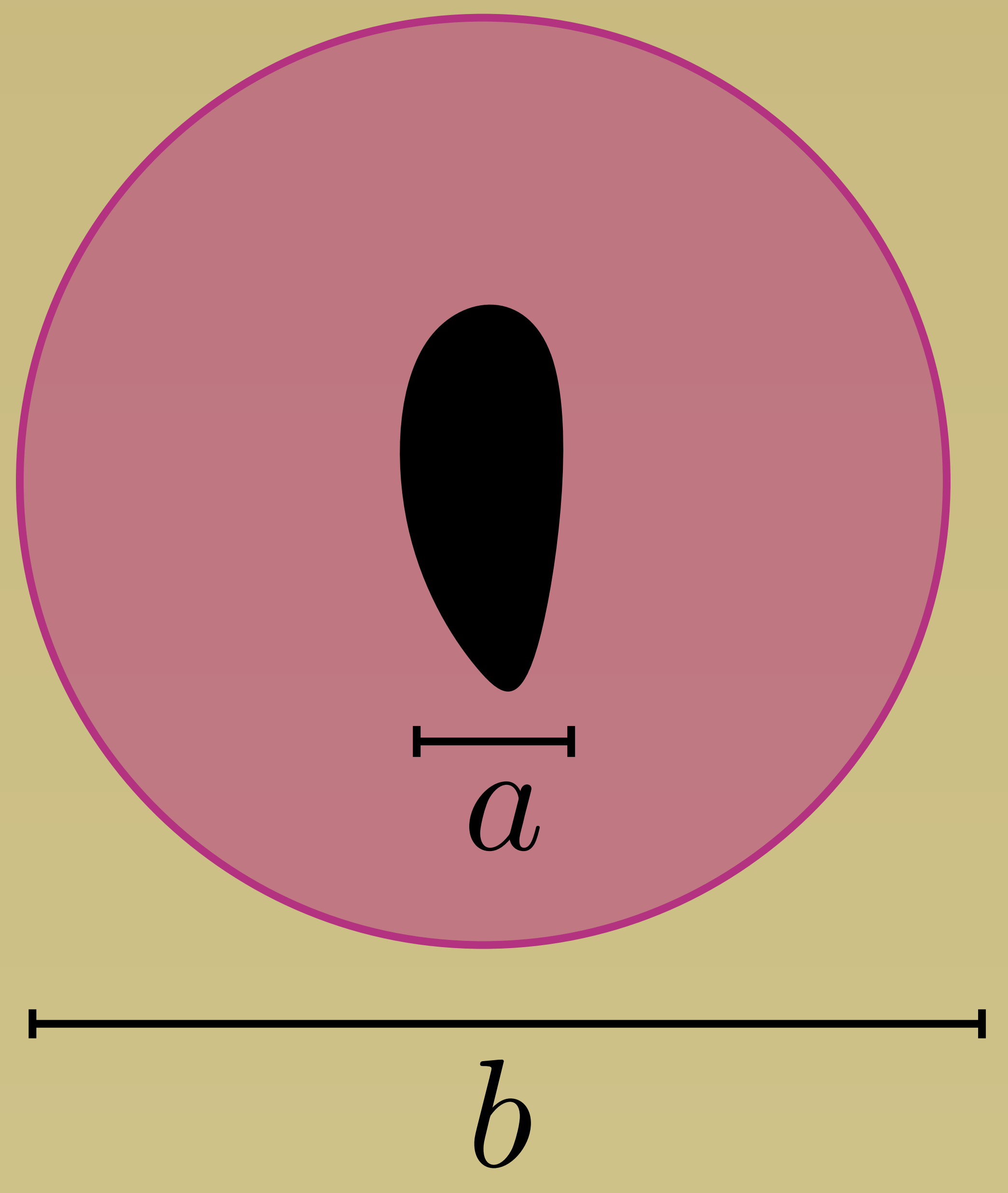
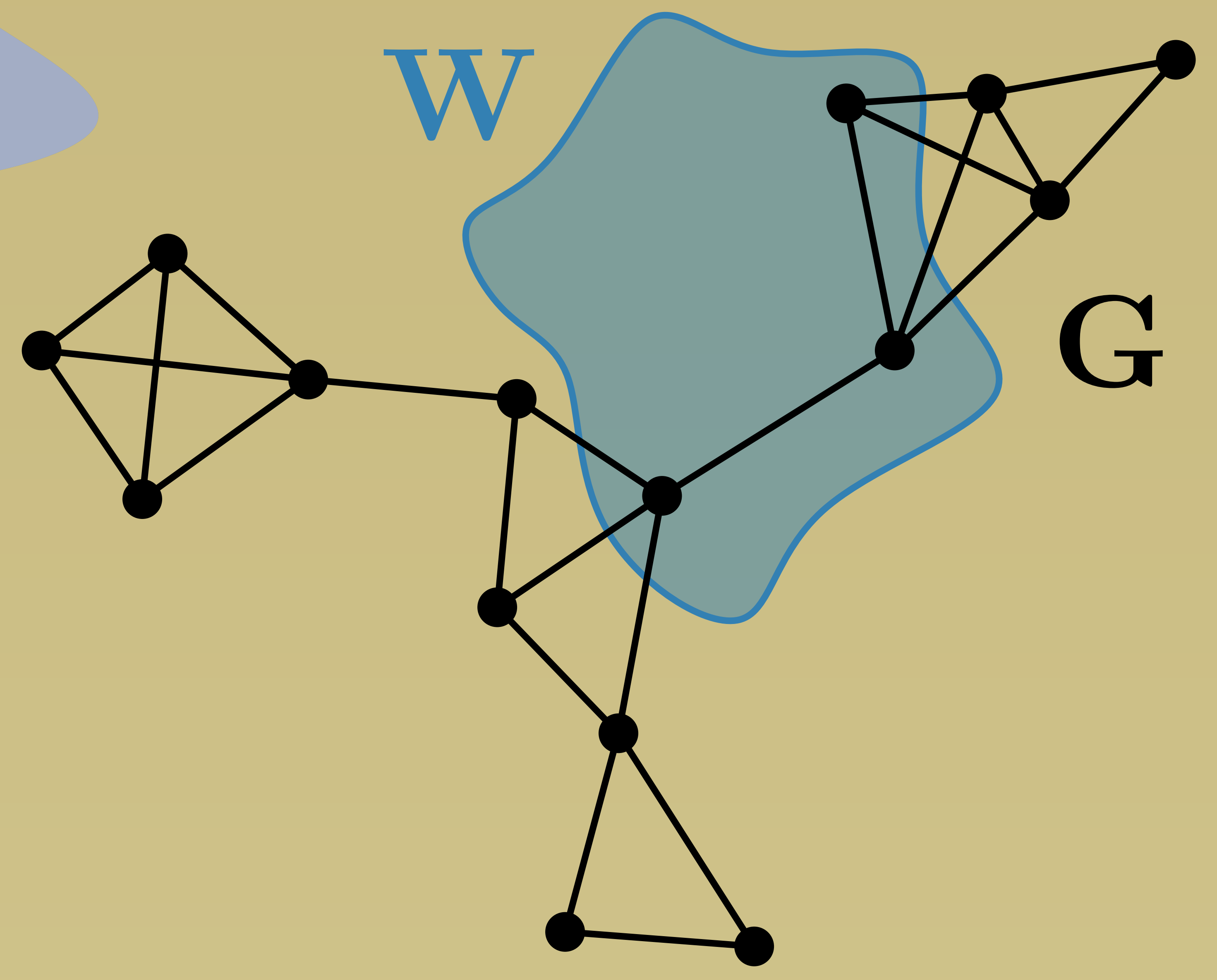


$$\phi = \frac{a}{b}$$

Let ϕ be the ratio between byssus range and mussel size.

THEOREM
Given a set of n mussels in the plane, we can construct $\mathbf{G}$ in $O(n(\log n + \phi^2))$ time.

THE FLOW OF THE WAVE



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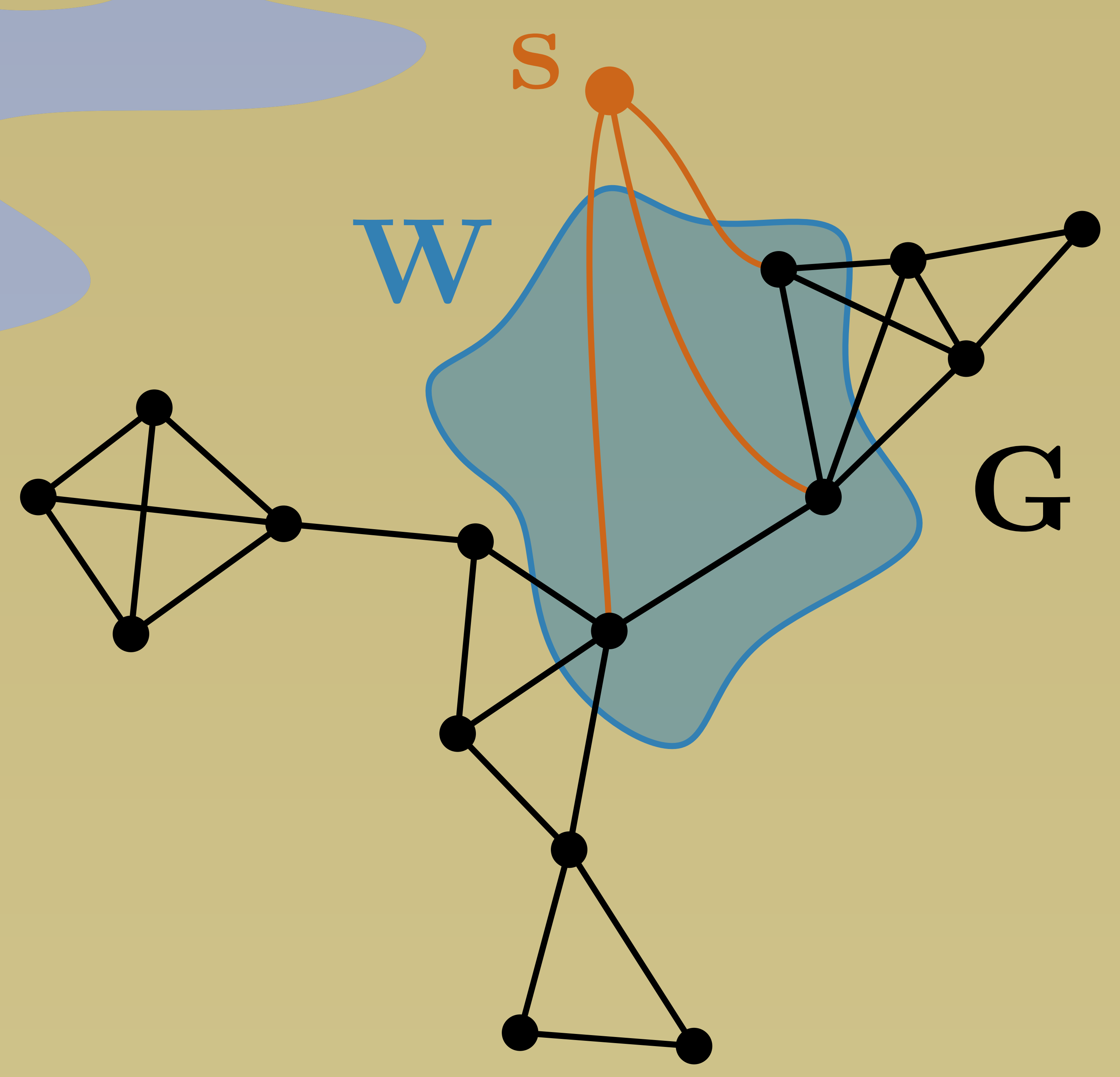

We create a new vertex s,
attached to all vertices in W.

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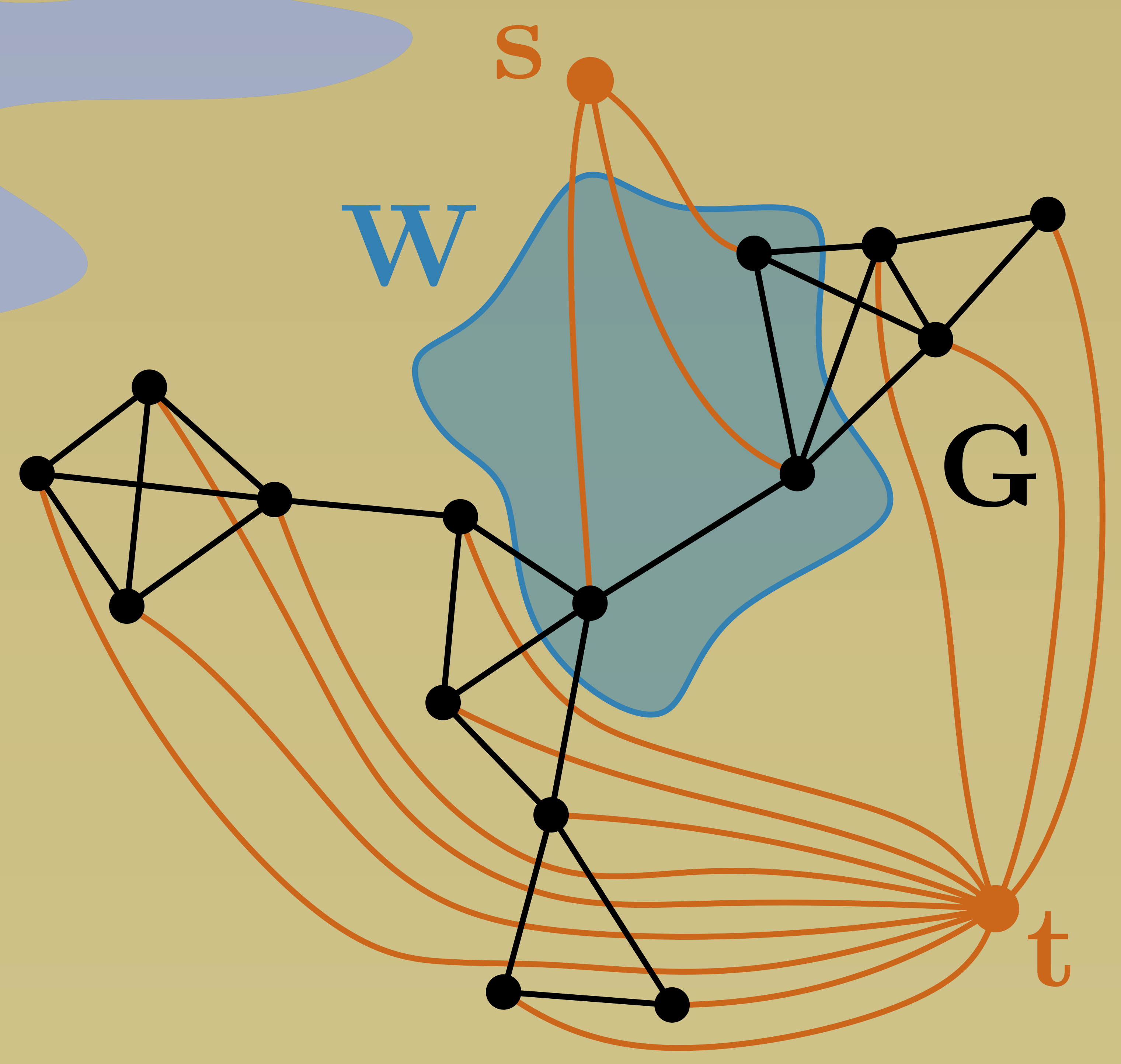
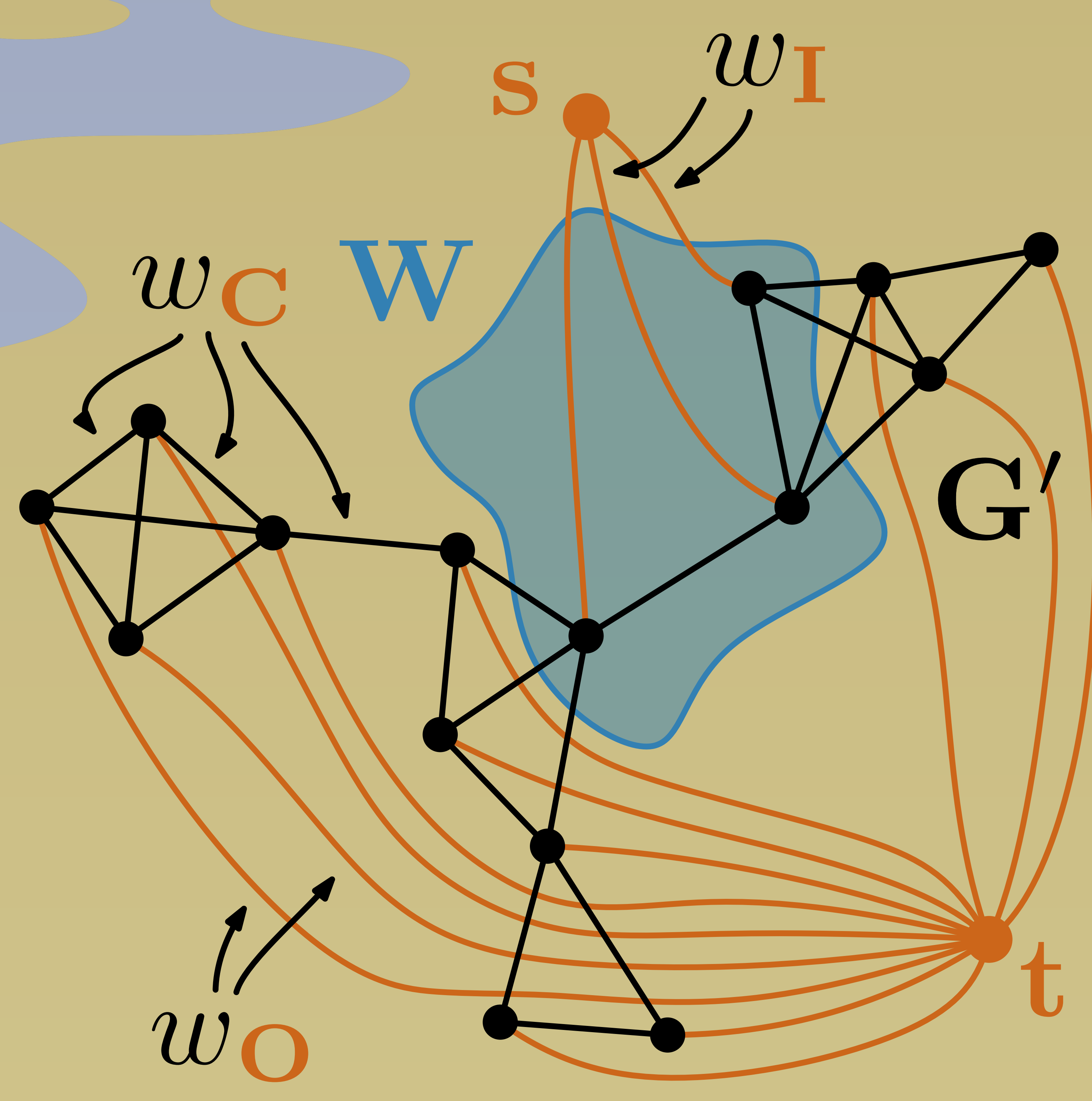


We create a new vertex **s**, attached to all vertices in **W**.

We also create a new vertex **t**, attached to all vertices outside **W**.

THE FLOW OF THE WAVE



Let $\mathbf{G}'$ be this graph, where:

- original edges of G have weight $w_{\mathbf{C}}$,
- edges attached to $\mathbf{s}$ have weight $w_{\mathbf{I}}$, and
- edges attached to $\mathbf{t}$ have weight $w_{\mathbf{O}}$.

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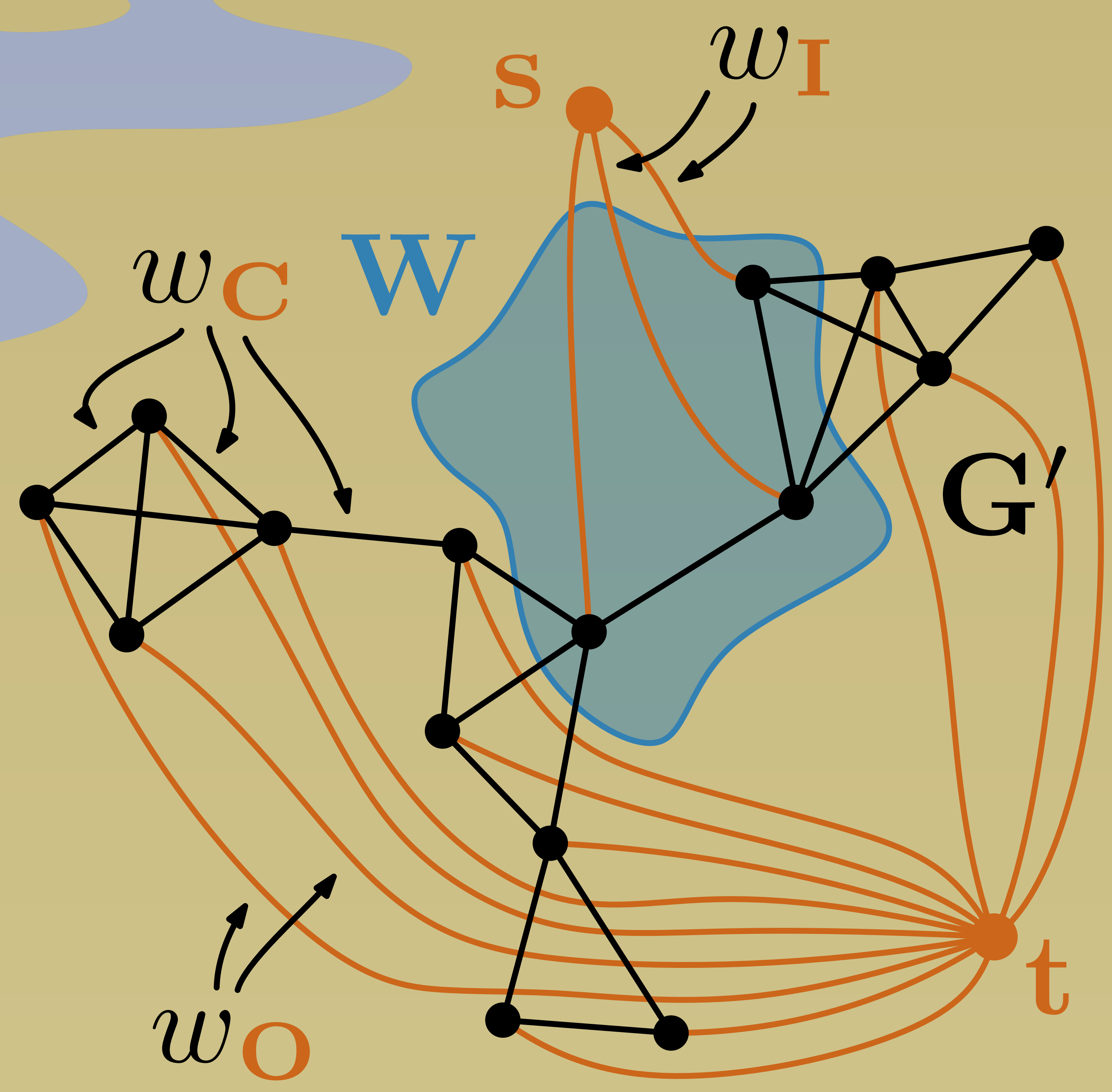

LEMMA
A minimum cut in $\mathbf{G}'$ that separates $\mathbf{s}$ from $\mathbf{t}$ corresponds with a set $\mathbf{M}$ that maximises $\mathbf{F}(\mathbf{M})$.

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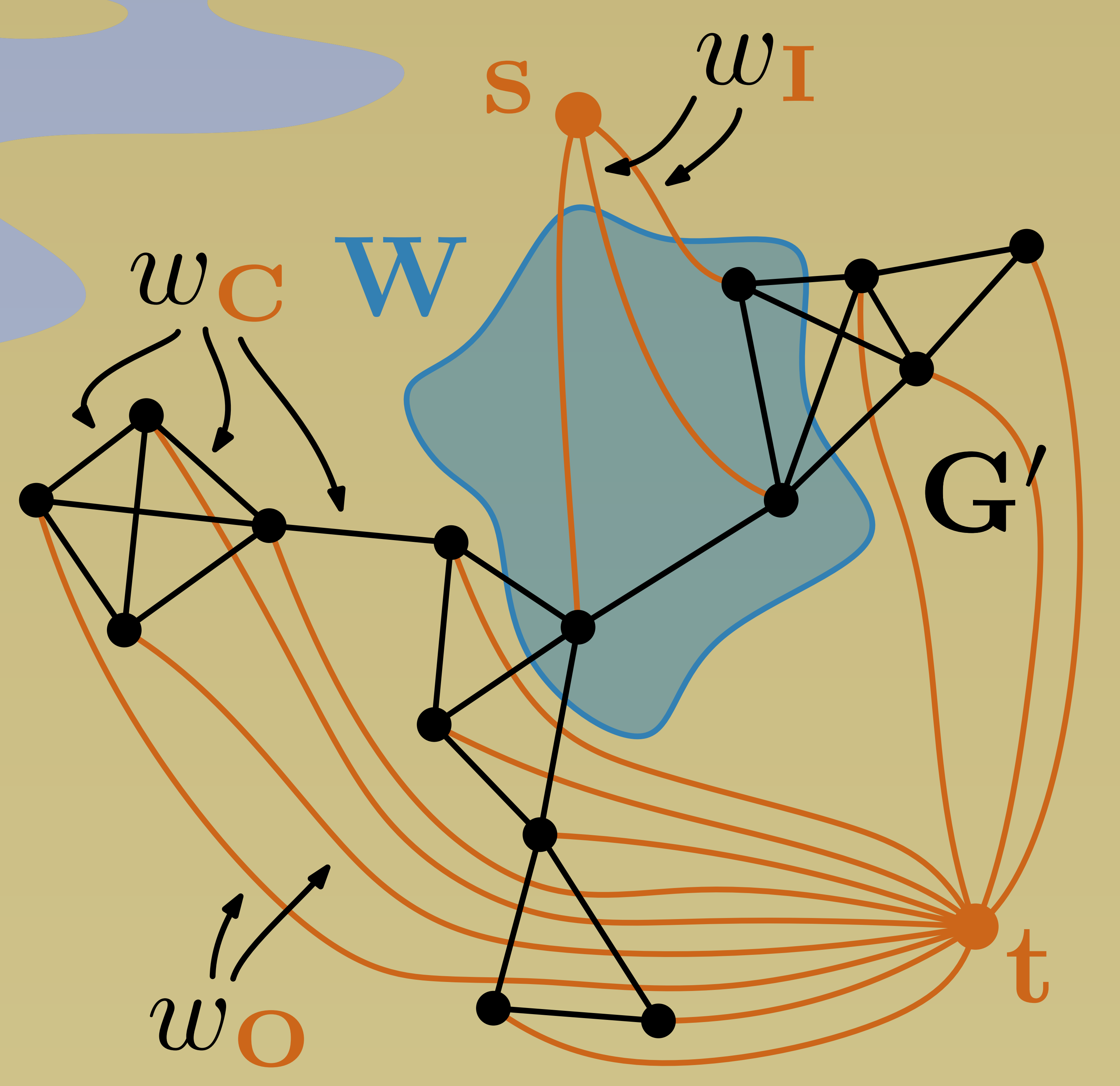

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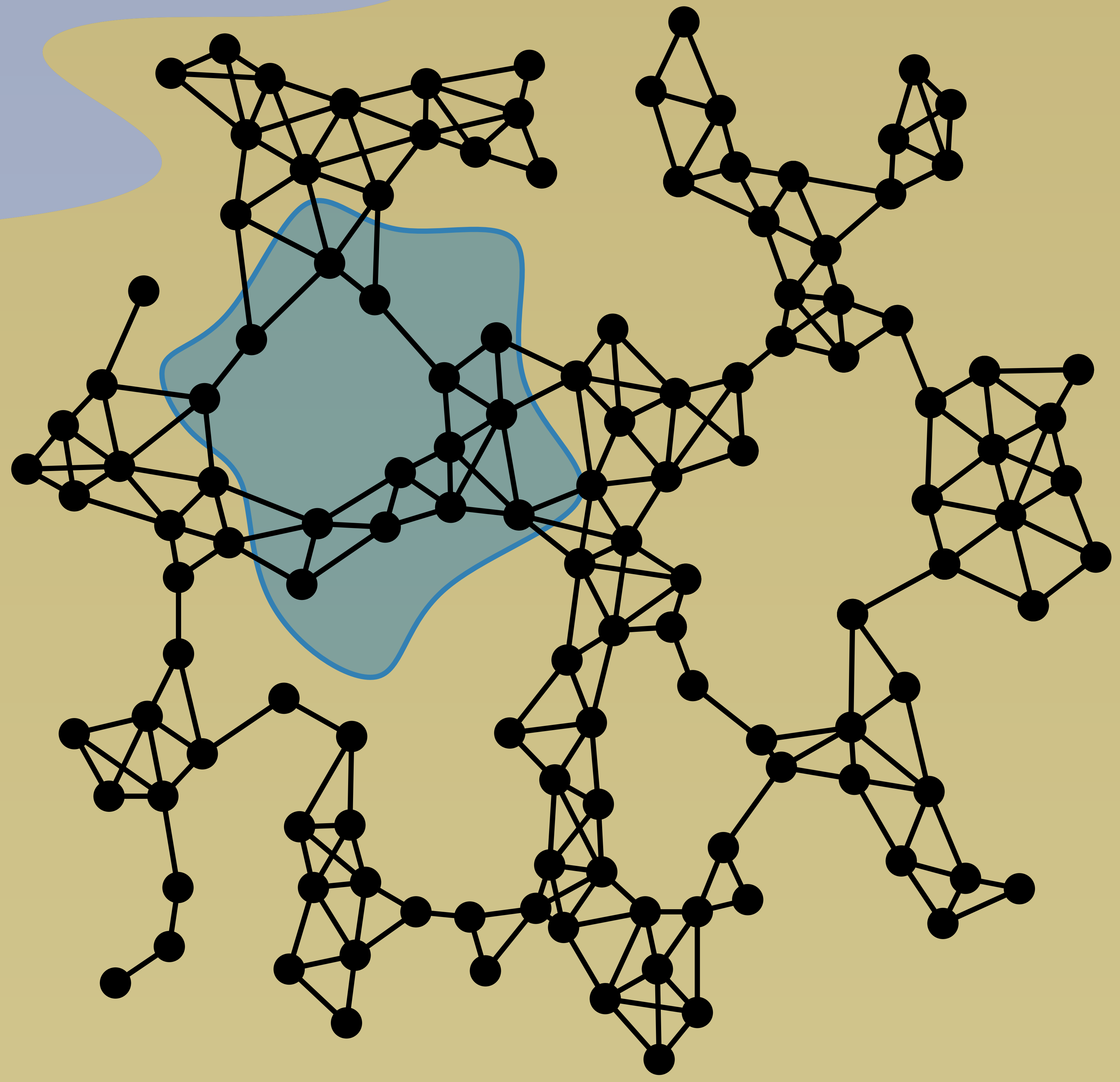
THE FLOW OF THE WAVE

THEOREM
Given a set of n mussels in $\mathbb{R}^2$, and a wave impact zone $\mathbf{W}$, we can compute the set of mussels $\mathbf{M}$ that maximises $\mathbf{F}(\mathbf{M})$ in $O(\phi^2 n^2)$ time.

EXPLOITING THE GEOMETRY



In large graphs, intuitively, mussels that are far away from $\mathbf{W}$ should not influence the computation time.

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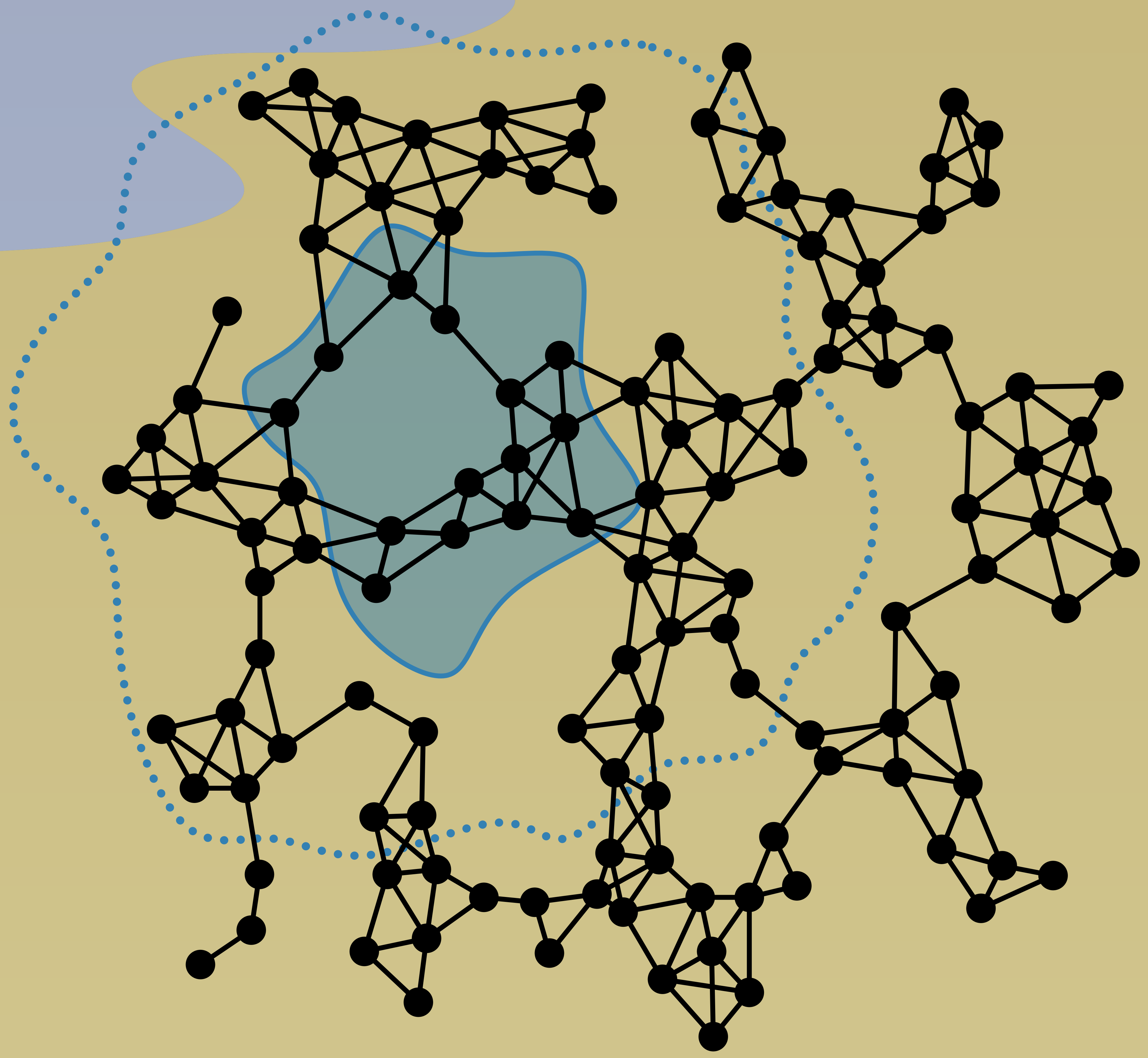

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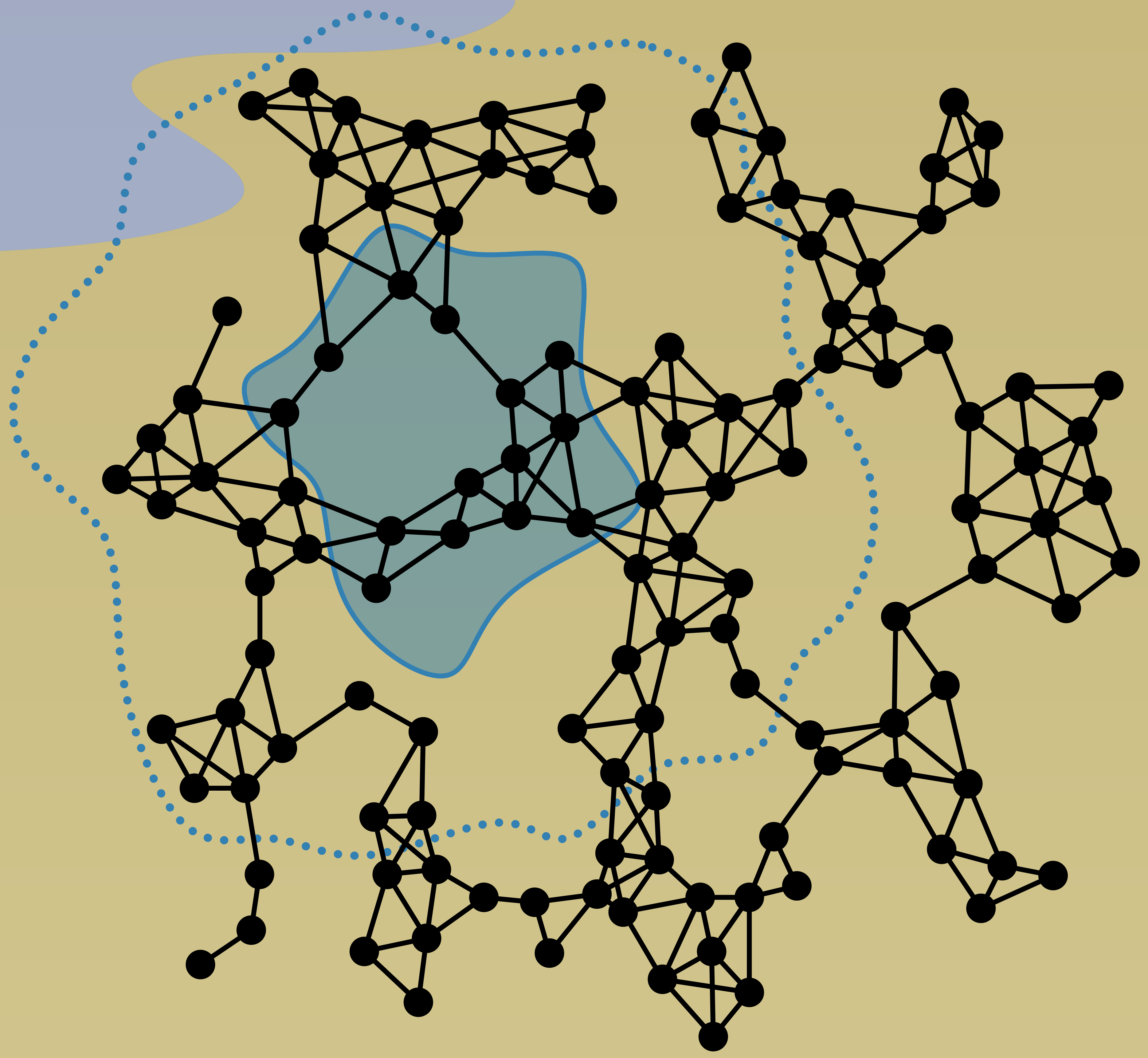

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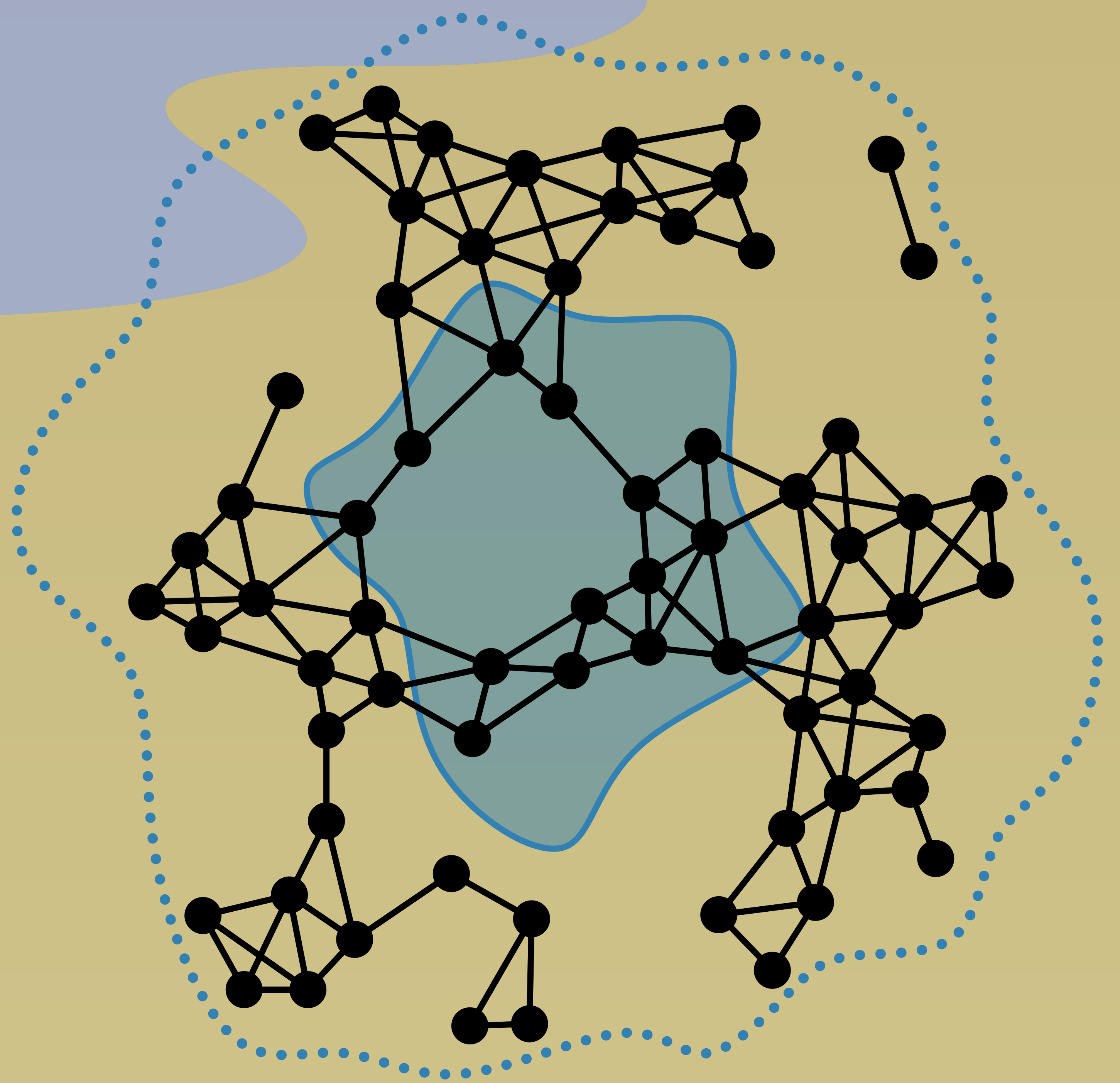

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EXPLOITING THE GEOMETRY

THEOREM
Given a set of n mussels in $\mathbb{R}^2$, and a wave impact zone $\mathbf{W}$ or radius r, we can compute the set of mussels $\mathbf{M}$ that maximises $\mathbf{F}(\mathbf{M})$ in $O(n \log n + n\phi^2 + (\frac{w_\mathbf{C}}{w_\mathbf{O}})^2 r^4 \phi^{10})$ time.


THANK YOU SO MUCH, AND ARE THERE ANY QUESTIONS?
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