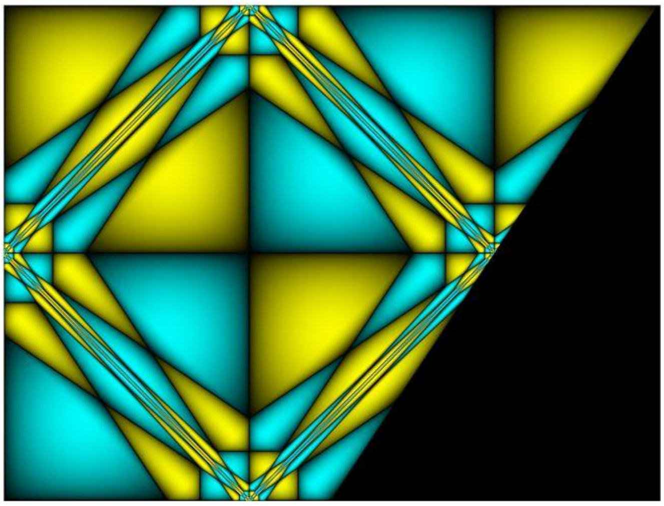
INSTITUTE OF ENGINEERING SCIENCES OF TOULON AND THE VAR

ROYAL NETHERLANDS INSTITUTE FOR SEA RESEARCH

INTERNAL GRAVITY WAVES
WAVE ATTRACTORS

Internship: 4th June – 27th July



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Acknowledgment

First, I would like to thank my supervisor Professor Leo Maas for giving me the chance to do and interesting internship at Nioz and received me very well. I would also like to thank Leo for his constant attention during my stay at Nioz. He helped me to understand internal waves' theories and to improve my knowledge. Thank him and his wife for their dinner and discussion about the Netherlands.

Then, I would express my gratitude to oceanography staff department for hosting me so nicely and especially Carola for lending me a bike. It was an amazing experience to meet all these persons.

The last but not the least, I would also like to thank my family and boy friend for supporting me during this internship.

Introduction

Depending on the nature of the ocean's stratification (function of the density field) internal gravity waves propagate differently. Indeed, for a uniformly stratified fluid these waves propagate obliquely through the ocean, and for a stratification concentrated in a thin layer the propagation is horizontally. During their propagation waves transport energy. It is speculated that these internal gravity waves may thus play an important role in maintaining the large-scale deep circulation, by providing downward mixing heat. Their mixing may also be relevant for marine ecosystems, by providing nutrients to the water column. Such processes have been observed in the deep ocean, as well as in the shallow Wadden Sea. Wave attractors are a recent discovery mainly observed in laboratory and numerical models and for the moment it has never been directly observed in the ocean because of the large scale and the 3Dimensions aspect of the ocean and also the experimental cost. For 10 years, laboratory experiments have been made to illustrate the theory. Leo Maas is specialist of this subject and has published several papers about internal wave and especially wave attractors.

I chose to do this internship to further my knowledge about physical oceanographic and to discover sea research. In a first part I will describe Royal NIOZ institute and his "Physical oceanography" Department. Then, I will speak about Internal Waves' theories and proprieties. In a second part, I would describe the material that is at my disposal to conduct my experimental studies.

My work consists in familiarize myself with the material and specific software: Digiflow (Dalziel Researh), find a frequency range where wave attractor appear in the uniformly stratified fluid and then add a surface layer with a constant density to analyze interactions. In order to exploit my recorded data, I have to create a Matlab program. The aim of my work would be **to study interactions between a uniform density gradient layer and a uniform density layer.**

1- Nioz

1.1 Presentation

Royal Netherlands Institute for Sea Research (NIOZ) is the leading research institute in the Netherlands for the marine sciences. It is located on the Wadden Island of Texel, close to the mainland.

This institute is divided in five departments: Physical Oceanography (FYS), Marine Geology (GEO), Marine Organic Biogeochemistry (BGC), Biological Oceanography (BIO), Marine Ecology (MEE) carrying out the multidisciplinary research of the institute. Approximalety 250 persons are currenlty appoint at Nioz. The mission is to gain and communicate scientific knowledge on seas and oceans for the understanding and sustainability of Earth.

NIOZ houses state of the art analytical instrumentation and operates several research vessels for coastal and ocean studies.

1.2 Physical Oceanographic Department

The Physical oceanographic department (FYS) carries out research on:

- Ocean circulation.
- Internal-wave dynamics and mixing.
- Coastal research on long-term variability and transports in the coastal zone and estuaries.

FYS aims to understand the marine ecosystem, the role of the ocean in climate and climate change, and the environment for human activities at sea with theoretical studies, hydraulic laboratory models, numerical models and observations at sea. The sea research is mainly carried out in the Dutch coastal zone and estuaries in the North Sea.

2- Internal waves

Most people think that waves are only a surface phenomenon at the interface between air and water, resulting of the wind action. But actually, it is not totally correct. One of the most important aspects to define a wave is the interface. Indeed, waves need interface to propagate. An interface is the virtual line between two layers of different density. That is why, surface waves exist : waves are supported by interface. People mainly know surface waves because they are the only waves in the ocean we can see. But if we look beneath the surface, we will discover other kind of waves: The internal waves. There are mostly hidden for eyesight. Why waves can exist into the ocean?

The ocean is decomposed into three layers: the upper mixed layer, the thermocline and the abyss. Each layer has specificities. The upper layer is mixed mainly because of wind, precipitations and evaporation. This layer is homogenous due to mixture. The abyss is the lower layer, and is composed with extremely cold water. Water motion is very slow in the area. Thermocline is located between these two layers and is stratified in density. The density field is determined by the amount of salts and heat. Stratification involves many layers of different density, i.e. numerous interfaces, so propagations of waves. Internal waves spread over the thermocline. Waves in fluids exist due to restoring forces; these forces push parcels of particle moving there out of their equilibrium position. Backing towards their position involves oscillations.

2.1 Stability

If a fluid element is perturbed vertically in a density stratified fluid, this element tends to return to its original position, due to a restoring force called: Buoyancy force. This notion plays a central role in the internal wave theory, because it is an indicator of the local stability in a stratified fluid. It is characterized by a quantity defined through:

$$N^2 = -\frac{g}{\rho_*}\frac{\partial \rho_0}{\partial z}$$

where ρ_0 is the time-averaged fluid density, ρ_* is a reference density, g is gravity pointing downward and z is the coordinate oriented vertically upward. Total density thus consists of $\rho_* + \rho_0(z) + \rho'(\underline{x}, t)$ where $\rho_* \gg |\rho_0| \gg \rho'(\underline{x}, t)$.

The water column is stable if $N^2 > 0$. This quantity N is called the Brunt Väisälä or Buoyancy frequency, its unit is radians per second.

2.2 Restoring forces

Some wave exists due to air's compression and for this case pressure is the restoring force. But for internal waves, there are two restoring forces: Buoyancy due to the ocean's stratification and Coriolis force, due to the Earth rotation. If gravity acts as the only restoring force waves are called: Internal gravity waves. (f_{wave} << $|f|$).

If the Coriolis force is only at work: gyroscopic (or inertial) wave (N=0). And if there are both waves are called: Internal intertie-gravity waves.

During my internship, I will study the internal wave in a uniformly stratified fluid. A uniform density gradient consists of a large set of infinitesimal layers whose density differs infinitesimally with the neighboring layers. Waves go along isopycnal lines (lines horizontal of same density), but because of the infinitesimal layers, wave go also vertically. When the density increases at a constant rate in the direction of gravity, the fluid – supposed incompressible- is stably stratified and characterized by a constant value of the buoyancy frequency N.

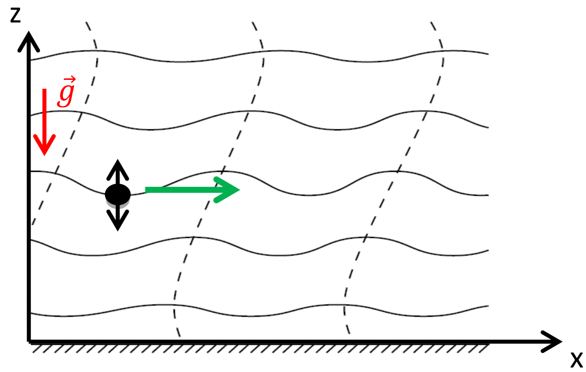


Figure 1 : Sketch of Internal waves in a stratified fluid

2.3 Origins of internal waves

There are two main mechanisms generating internal waves. The first, is due to the atmospheric forcing, mainly the wind action. Internal waves evolve from the variations involve by the wind at the surface. The second mechanism is internal tide, resulting to the surface tide. Both generate waves with low frequency between $|f|$ and N.

2.4 Equations

2.4.1 The continuity equation in a 2D plane

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial x} + \vec{u}.\nabla\rho = 0$$

Moreover, the fluid is considered incompressible. This means

$$\nabla.u = 0$$

We obtain the following continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

2.4.2 The momentum Navier Stokes equations

$$\frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla\vec{u}) = -\frac{\nabla\vec{p}}{\rho} + \vec{F}_{ext} + \mu\nabla^2\vec{u}$$

Where :

$\vec{u} = (u, v, w)$ is the velocity
$\vec{p}$ is the pressure
$\vec{F}_{ext}$ are the external forces
μ is the molecular viscosity

We ignore the viscosity, and the external force is only gravity.

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla\vec{u}) = -\frac{\nabla\vec{p}}{\rho} + \vec{g}$$

$$\begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \end{cases}$$

We consider a 2D plane (x, z), and after linearization we obtain the following reduction of Euler equation :

$$\begin{cases} \frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \\ \frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \end{cases}$$

In an external field of gravity $\vec{g}$, we consider a fluid that is initially at rest. This fluid is uniformly stratified around a constant reference density ρ_*. Thus, we have density: $\rho = \rho_* + \rho_0(z)$. The pressure field is in hydrostatic. Then, any perturbations of the initial state $p'(x, z, t)$, $\rho'(x, z, t)$ will propagate as internal waves. The complete density field is described by : $\rho = \rho_* + \rho_0(z) + \rho'(x, z, t)$

Where $\rho_* \gg |\rho_0| \gg \rho'(x, z, t)$.

2.4.3 Conservation of density

$$\frac{d\rho}{dt} = 0$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{u}.\nabla\rho = 0$$

$\rho = \rho_* + \rho_0(z) + \rho'(x, z, t)$

$$\frac{\partial \rho'}{\partial t} + w\frac{\partial \rho_0}{\partial z} + \left(u\frac{\partial \rho'}{\partial x} + w\frac{\partial \rho'}{\partial z}\right) = 0$$

The last term is neglected.

$$\frac{\partial \rho'}{\partial t} + w\frac{\partial \rho_0}{\partial z} = 0$$

We know that: $b = -g\frac{\rho'}{\rho_*}$.

$$\frac{\partial b}{\partial t} + w\left(-\frac{g}{\rho_*}\right)\frac{\partial \rho_0}{\partial z} = 0$$

Yet,

$$N^2 = -\frac{g}{\rho_*}\frac{\partial \rho_0}{\partial z} = constant\ (by\ assumption)$$

So, we obtain

$$\frac{\partial b}{\partial t} + wN^2 = 0$$

Stationary state:

$$\frac{\partial p_0}{\partial z} = -(\rho_* + \rho_0(z))g \qquad (1.1)$$

With Navier-Stokes equation we have:

$$\rho\frac{\partial w}{\partial t} = \frac{\partial p}{\partial z} - \rho g$$

$\rho = \rho_* + \rho_0(z) + \rho'(x,z,t)$
$p = p_* + p\_0(z) + p'(x,z,t)$

$$(\rho_* + \rho_0(z) + \rho'(x,z,t))\frac{\partial w}{\partial t} = \frac{\partial p_0}{\partial z} + \frac{\partial p'}{\partial z} - (\rho_* + \rho_0(z) + \rho'(x,z,t))g \qquad (1.2)$$

We subtract (1.2) with (1.1)

$$\frac{\partial w}{\partial t} = \frac{1}{\rho_*}\frac{\partial p'}{\partial z} - \frac{\rho'(x,z,t)}{\rho_*}g$$

$b = -g\frac{\rho'}{\rho_*}$. And we neglect vertical acceleration $\frac{\partial w}{\partial t}$

Hence,

$$\frac{1}{\rho_*}\frac{\partial p'}{\partial z} = b$$

We obtain the following equations :

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_*}\frac{\partial p'}{\partial x}$$

$$\frac{1}{\rho_*}\frac{\partial p'}{\partial z} = b = -g\frac{\rho'}{\rho_*}$$

$$\frac{\partial b}{\partial t} + wN^2 = 0$$

$$N^2 = -\frac{g}{\rho_*}\frac{\partial \rho_0}{\partial z}$$

2.4.4 Relation dispersion

We introduce stream functions:

$$u = -\frac{\partial \psi}{\partial z} \qquad w = \frac{\partial \psi}{\partial x}$$

With $\psi = \Psi(x,z)e^{-i\omega t}$

Hence,

$$i\omega\frac{\partial \Psi}{\partial z} = -\frac{1}{\rho_*}\frac{\partial p'}{\partial x}$$
$$-i\omega\frac{\partial \Psi}{\partial x} + \frac{1}{\rho_*}\frac{\partial p'}{\partial z} = b$$

By cross-derivative and subtraction we obtain the following single equation :

$$i\omega\frac{\partial^2 \Psi}{\partial z^2} + i\omega\frac{\partial^2 \Psi}{\partial x^2} + b = 0$$

Then we differentiate to replace b with N :

$$\omega^2\frac{\partial^2 \Psi}{\partial z^2} + \omega^2\frac{\partial^2 \Psi}{\partial x^2} - N^2\frac{\partial^2 \Psi}{\partial x^2} = 0$$

$$\frac{\partial^2 \Psi}{\partial z^2} - \frac{N^2 - \omega^2}{\omega^2}\frac{\partial^2 \Psi}{\partial x^2} = 0$$

For plane waves : $\Psi(x,z) = \widehat{\Psi}e^{i(kx+mz)}$. Hence, we have the dispersion relation :

$$\boxed{\frac{\omega^2}{N^2} = \frac{k^2}{m^2+k^2} = \cos^2\theta}$$

Where :

$$\vec{k} = k\vec{\imath} + m\vec{\jmath} = \boldsymbol{\kappa}(\cos\theta, \sin\theta)$$

2.5 Proprieties

2.5.1: Internal waves : Phase and group velocity

To illustrate one of the most important internal wave propriety, firstly we calculate group (C_g)and phase (C) velocity:

$$\overrightarrow{C_g} = \nabla_{\vec{\kappa}}\omega = \left(\frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial m}\right) \; ; \; \vec{C} = \frac{\omega}{\boldsymbol{\kappa}^2}\vec{\boldsymbol{\kappa}} = \frac{\omega}{\boldsymbol{\kappa}^2}(k,m)$$

With

$$\frac{\partial \omega}{\partial k} = \frac{N}{\boldsymbol{\kappa}^3}(\boldsymbol{\kappa}^2 - k^2) = \frac{Nm^2}{\boldsymbol{\kappa}^3} \; and \; \frac{\partial \omega}{\partial m} = -\frac{1}{2}Nk.\frac{2m}{\boldsymbol{\kappa}^3} = \frac{N(-mk)}{\boldsymbol{\kappa}^3} \; ; \; \boldsymbol{\kappa}^2 = k^2 + m^2$$

$\overrightarrow{Cg} = \frac{mN}{\boldsymbol{\kappa}^3}(m,-k) \; and \; \vec{C} = \frac{\omega}{\boldsymbol{\kappa}^2}(k,m)$

Then :

$$\boxed{\overrightarrow{Cg}.\vec{C} = \frac{\omega\, N\, m}{\boldsymbol{\kappa}^5}(mk, -km) = 0}$$

With the relation dispersion we find that group, Cg, and phase velocities, C, are perpendicular. Moreover, the horizontal components of group and phase velocities are aligned to one another, while the vertical components are in opposition.

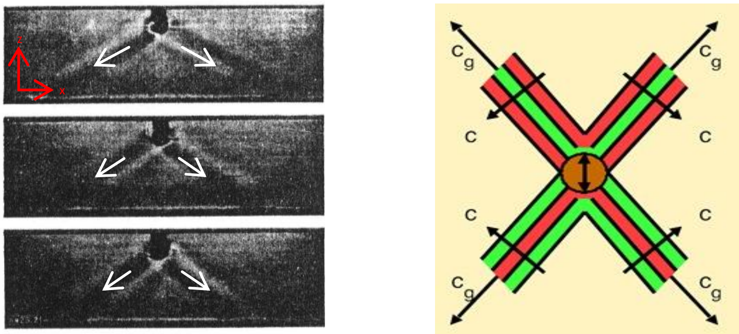


Figure 2 : Typical proprieties of these internal gravity waves were illustrated for the first time by Gortler (1943) by example from laboratory. On the lift on see internal beams emanating from the oscillating cylinder at the upper center of each image. The three images are for different frequencies, increasing from the top to the bottom. White arrows follow the wave direction. The right sketch illustrates that phase and group velocity are perpendicular.

2.5.2: Internal waves reflections

In a previous part, we have found the equations governing internal waves. Assuming a plane wave as $\propto e^{i(\boldsymbol{kx}-\omega t)}$, the horizontal momentum equation can be written :

$$u = \frac{p}{\omega} k$$

After the elimination of the buoyancy term in the vertical momentum equation, we have

$$N^2 w + \frac{\partial^2 w}{\partial t^2} = 0$$

We obtain :

$$p = -\frac{w}{m\omega}(N^2 - \omega^2)$$

The horizontal velocity can also be expressed as :

$$u = -\frac{w}{\omega^2 m}(N^2 - \omega^2)k$$

The sloping bottom is given by : $z = sx$ where bottom slope $s = \tan\alpha$ and α is the angle the bottom makes with the horizontal. A vector normal to the bottom is given by $\vec{n} = (-sin\alpha, cos\alpha)$. When there are reflections from the bottom, the wave is decomposed of an incident and reflected wave, and the vertical velocity can be written as :

$$w = A_i \exp\left[i(\boldsymbol{k_i x} - \omega_i t)\right] + A_r \exp\left[i(\boldsymbol{k_r x} - \omega_r t)\right]$$

Where suffice i and r indicate respectively incident and reflected waves. Wave has to respect boundary condition:

$$\vec{u}.\vec{n} = -u\ sin\alpha + w\ cos\alpha = 0$$

We define a wave vector slope γ which is the angle that the wave vector makes with the horizontal.

$$\gamma^2 = \tan^2\omega = \frac{m^2}{k^2} = \frac{N^2 - \omega^2}{\omega^2}$$

We insert this notation, u and w equations into boundary condition:

$$\left(\gamma_i^2 \frac{k_i}{m_i} sin\alpha + cos\alpha\right) A_i \exp[i(k_i x + m_i x tan\alpha - \omega_i t)]$$
$$+ \left(\gamma_r^2 \frac{k_r}{m_r} sin\alpha + cos\alpha\right) A_r \exp[i(k_r x + m_r x tan\alpha - \omega_r t)] = 0$$

This condition should be satisfied for each position at the bottom and for each time :

$$\omega_i = \omega_r \equiv \omega$$

$$k_i + m_i s = k_r + m_r s$$

With the dispersion relation we have :

$$\omega = N cos\theta$$

Where N is constant and θ the angle the beam makes with the vertical.
When there are reflections we have the following condition:

$$\theta_i = \theta_r \equiv \theta$$

This relation is also correct for an inconstant N. Indeed, we assume that the buoyancy frequency is the same around the reflection point.

According to the previous condition, upon reflection the angle which the beams make with respect to the direction of gravity remains fixed and as result the beams converge or diverge. Following sketches shows internal waves focus or defocus upon reflection from a sloping boundary.

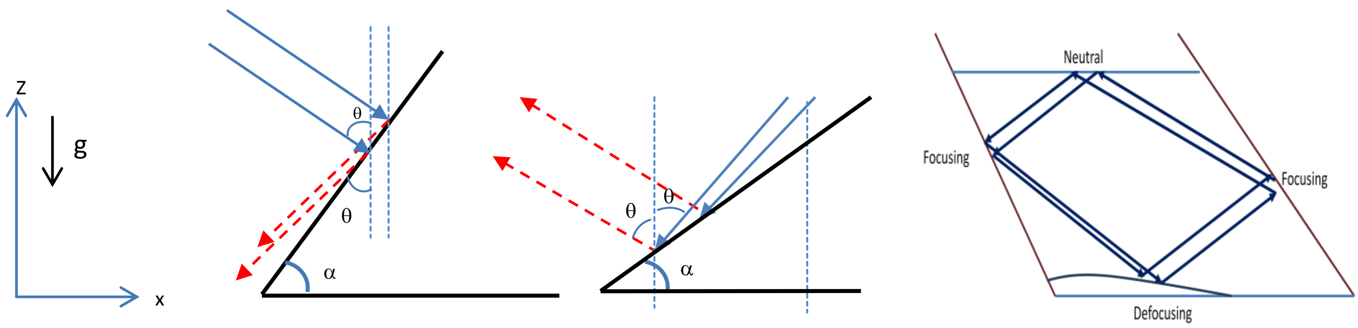


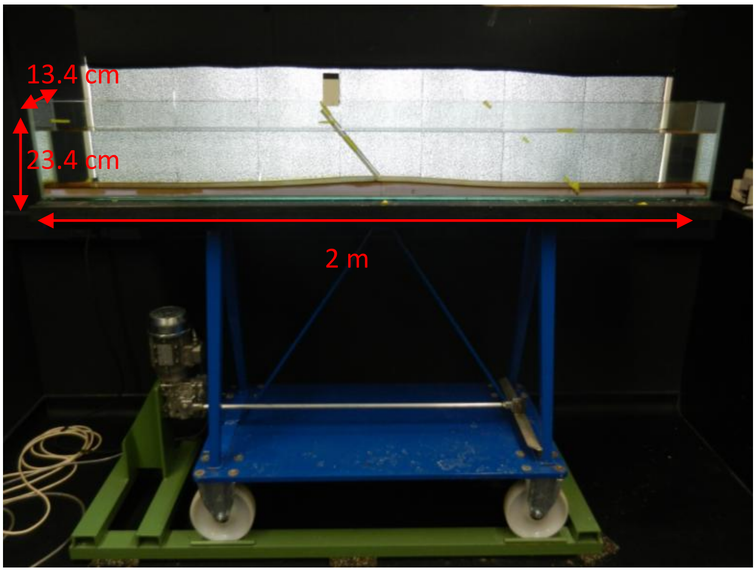
Figure 3 : Focusing and defocusing, and reflections in a closed basin

3- Laboratory experiments

During my internship, I have made several laboratory experiments. For doing these experiments, I used a tank filled with a uniformly stratified fluid, in order to study internal gravity waves. That is why uniform stratification plays a central role in my work and I have to be very care full during the creating of this stratification. Firstly, I will make experiments without an additional constant surface layer. That allow me to create a matlab program to analyze my results before doing, in a second part, others experiments.

3.1 Experimental description

I work with a narrow tank which dimensions are: 13.4 cm width, 2m long and 23.4 height.



We select our coordinate system(x, y, z) , x along the length of the tank, y across the width (the direction in which variations in the flow are negligible), and z vertically upward. We fill the tank with 15.4 cm height of uniformly stratified fluid. For the filling we use the following process.

3.1.1 Filling process

We fill the tank with 2, 3 or 4 liters of salt, mixed with warm water, to obtain 25 liters of salt water. Firstly, we have used warm water to mix the solution easier, but with this process we observe creation of a surface layer probably due to the evaporation and the slowly mixture of water. Moreover the interface of this surface layer was not flat.

We try to resolve this issue by changing the mixture process. We do not use any more warm water. Indeed, the heat diffuses more rapidly than the salt, and the mixture process continues in the tank. But now we dilute salt with fresh water in a sieve. This way seams to give good results.

We have two other 25 liters tanks filled with fresh water. Because of the density difference, when we connect tanks together the water level increase into the fresh water tanks.

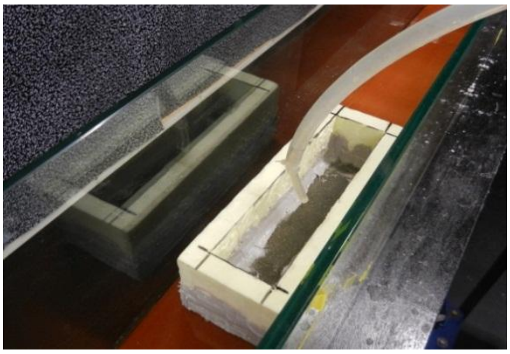


Figure 4 : On the left the installation for the filling. There are tree cylinders connected at each other and a tube goes to the sponge box to fill the tank. On the right, the sponge box into the tank where the salt water arrives.

We pump salt water and we put it to our narrow tank. When we remove salt water, fresh water enters into the salt water tank (because of the density/pressure difference) and we mix water to have a new mixture of salt water (less salt than the previous one). Thanks to this process, we create water that is subsequently less and less salt. The other important point is how we fill the tank. For this, we use a sponge box into which salt water enters very slowly. In this way we are able to fill the tank layer by layer, creating a linearly stratified fluid.

3.1.2 Salinity measurement

During the filling process, I take samples of water to measure the salinity. To this way, I check if we created a linear stratification or not. I have two different instruments to measure the salinity. A portable conductivity, salinity and temperature instrument (Ref VWR EC300) and a refractometer (Ref: AQUA MEDIC RHS-10ATC) (cf Appendix A). The portable instrument allows having very precise results but I have to take about 10cL of water. Also the instrument is limited at 70ppt. To solve this problem, I use the refractometer. Indeed, for using it, I have to put only a drop of water. Moreover, this instrument has an upper limit than the other one. So for my salinity measurements, I use the refractrometer. In a first time, I have to calibrate this instrument, for that I use the portable salinity instrument. I take eight samples of water with different salinity. I obtain the following law :

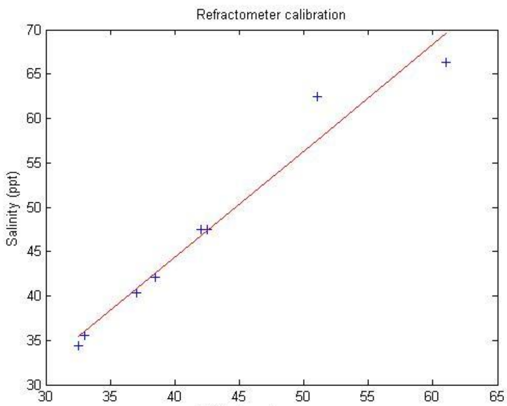


Figure 5 : Refractometer values as function of salinity (ppt)

Calibration's law :

$$Salinity = 1.1996 * Refractometer - 3.5943$$

This relation allows transforming refractometer values in salinity values. During my work, I had created different stratifications with different quantity of salts: 2 and 3 liters of salt.

The following graphs are the results of different samples I have made. There are red cross and blue cross, corresponding respectively to the right and left part of the tank. There is not a lot of different between the left and right part of the tank that means the process goes in a same way on left and on right.

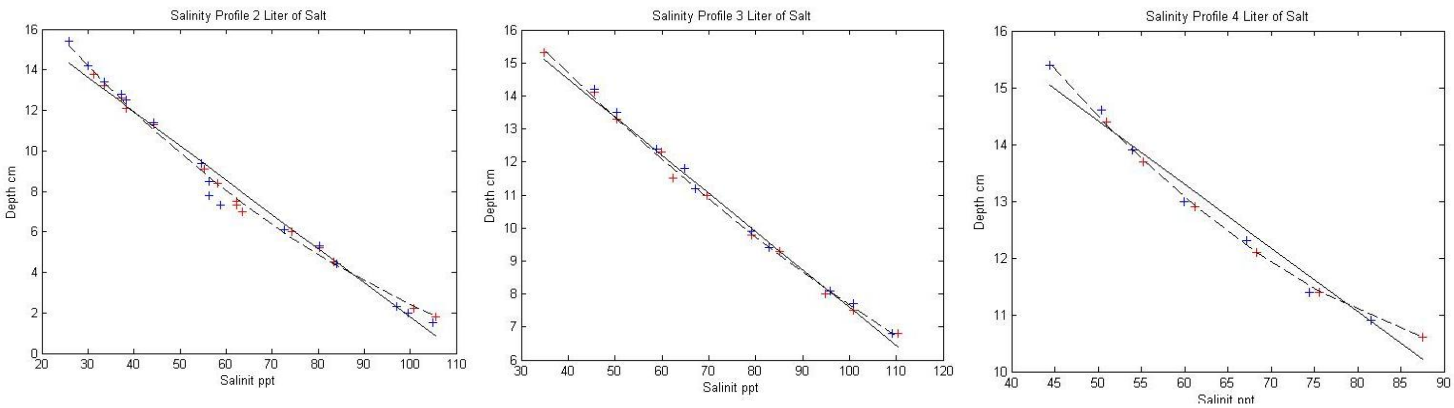


Figure 6 : Salinity (ppt) as function of depth (centimeter) for a 2liter of salt (left), 3 liter of salt (Middle) and 4 liter of salt (right). Red cross correspond to the right and blue cross to the left part of tank. solid lines are linear representations and dashed lines are curve representations

We observe that the stratification is not linear but more curved. Indeed, the black dashed line which links the cross is curved. For the next, we still consider our fluid as uniformly stratified fluid even if this curved. But, you have to keep in mind that is not exactly the case.

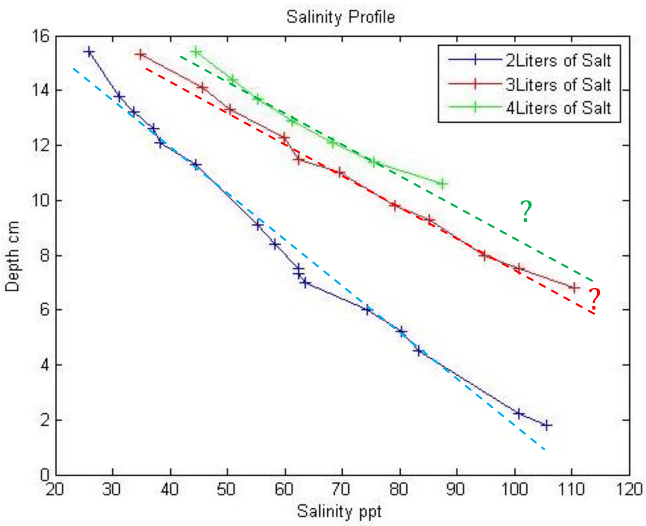


Figure 7 : Salinity (ppt) as function of depth (centimeter) for a 2liter salt (blue), a 3liter of salt (Red) and 4 liter of salt (green).

On this graphic, we can see the different results for 2, 3 and 4 liters for salt. Refractometer has a limit and for depth smaller than 6.8 cm (3 liters of salt) and 10.6cm (4 liters of salt) values are over this limit, that is why we do not have any measures before 6.8cm and 10.6cm. We notice that for 3 liters the stratification is more linear than for the 2 liters of salt. However, there are little steps on these lines, corresponding to steps of density. During the filling process lasting about 5hours, the flow rate is still not constant because of the sponge box and the tube pressure. Even if these different, we obtain a quite good linear density gradient.

When the filling is over, I insert two tilted walls in the tank, to create three different containers. The left wall make a 52 degree angle with the horizontal and the right wall a 38.3 degree angle. Closed basins with tilted boundaries focus or defocus beams.

We shake the tank periodically with a frequency f_m we choose. This frequency is actually:

$$f = \frac{f_m}{100}$$

When we have a look at tank, we cannot see anything. It is impossible to see the stratification watchful eye. To observe internal waves, there is a random pattern through which a light source behind the tank is shining. Due to density difference, light is refracted. A JAI progressive scan (Ref : CV-M4+CL) camera with a (50mm F0 95 TV) lens located approximately 3.5m from the tank is use to capture this light. Specific software called Digiflow compares these distorted images by density variations with a reference image. We use Synthetic Schlieren method to measure anomalies in real time. We see red and blue areas, corresponding respectively to the positive and negative anomaly. That is, areas where the density is higher or lower than the reference.

Because of the topography, the basin's geometry and the wave frequency, internal waves propagate at a fixed angle with the restoring force – in our case gravity, i.e. vertical axis -. Closed basins with tilted boundaries focus or defocus beams. And, after multiple reflections, rays are trapped along a specific pattern- an attractor-. The energy passes through the horizontal isopycnals (surface of constant density). When we change the frequency, we observe that the wave attractor is present inside a certain frequency range. For lower frequencies, there is no pattern and for higher frequencies the shape becomes complex.

With Digiflow we can see an attractor and we take a 60 seconds' movie with 8 images per second. After the recording, we have to analyze our movie. For doing that we use Analyze/Synthetic Schlieren/Pattern Matching. This standard Synthetic Schlieren process takes two input streams. The first is the movie and the second is the background input which is a single image. The outputs are x Gradient, y Gradient and Density images. The gradient images have the units of "per unit length", and represent respectively $\frac{1}{\rho_0}\frac{\partial \rho'}{\partial x}$ and $\frac{1}{\rho_0}\frac{\partial \rho'}{\partial y}$.

Because we have a vertical stratification, we are going to work with the y gradient movie. To exploit our results, we have to work on Matlab and for doing that we transform data from Digiflow to matrix of intensity with the command: Tools/ transform intensity.

4- Image processing

I load data on matlab. For each image (60*8=480) we have a file with three columns: X, Y and Intensity. In a first time, I recreate for each image the corresponding intensity matrix. I would like to study the evolution of the energy and phase. For doing that I study each pixel of each image. But, all the images have not the same reference because of the tank's motion. So for beginning I translate each image to the same reference, i.e tank's reference.

4.1 Translation

Each image have a pixels displacement function as the frequency, fm. According to the movie, I find the pixels' numbers which correspond to the amplitude. The tank's motion follows this expression:

$$Tank_{motion} = -\frac{Amplitude_{pixels}}{2}\cos\left(\frac{2\pi t}{T}\right) + \frac{Amplitude_{pixels}}{2}$$

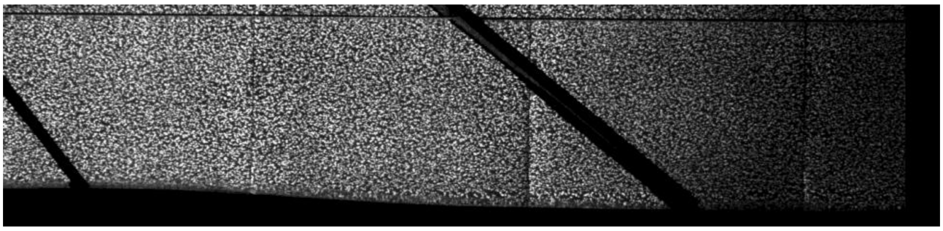
To use this model is important that each pixels displacement is an integer. So for each image, I find the nearest integer greater/ less than or equal to the pixels displacement values. Moreover, with this model is important to fix the first image as the image with the most leftward position. Indeed, when we start the recording we do not know exactly the tank's position.

I create a function called: translation_tank.m. This function translates any recording to the left. To do that, we have to input the sample frequency, the wave frequency (i. e .motor frequency) and the pixels' amplitude. For the moment it is the best option I have find. I know it is not very handy because I have to measure on Digiflow the amplitude before running the Matlab code.

Afterward, my entire matrix (images) is in the same reference. Now, I will process to remove all the pixels outside the middle basin and also the bottom.

4.2 Cutting-out

For rescale and cut my images, I have to find pixels corresponding to the bottom and outside the middle basin. For doing that, firstly I work on the real image, and find all pixels with under zero's value (corresponding to the bottom and walls). Then, I make a linear curve to represent the two walls. Hence, with a matlab function (Decoupage.m) I have created, I obtain the following result.



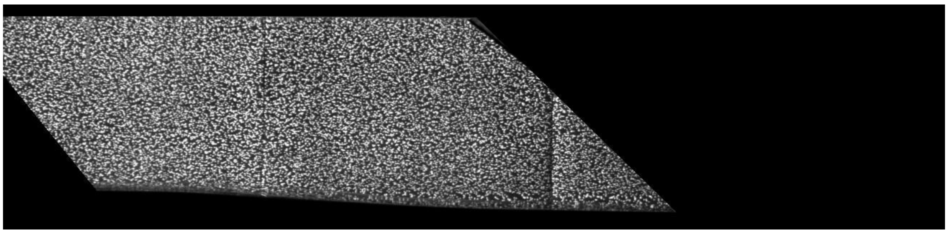


Figure 8 : Real Image before cutting out and real image after cutting out

Now, we can work with all our images, focusing in the middle basin. We are going to use least square harmonic analysis to exploit our results.

5- Least Squares Harmonic analysis

Least squares harmonic analysis is a method to approximate a solution. This method consists in minimize the error between a model we choose and the data fitting. We notice Dn the data and $\widehat{I_n}$ the model. For each pixel, we have the same process. We collect data of each pixel during a specific time scale (in general during one period).

Following, we can observe the data of a specific pixel (200,600) for a time scale equal to 8.75 second (that means 70 images).

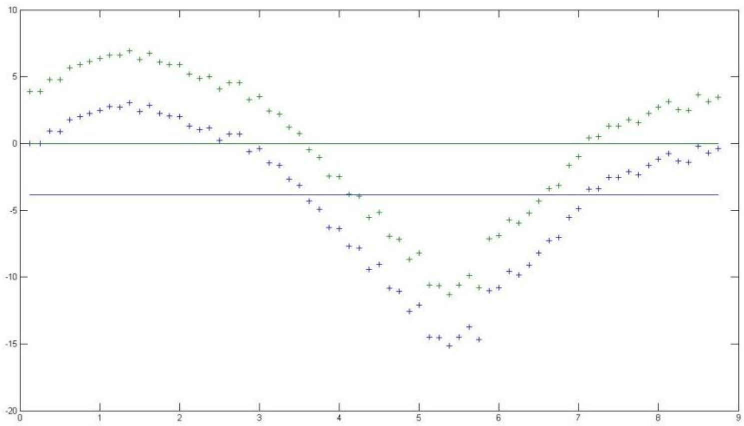


Figure 9 : Data fitting as a function of time (second) for 70 images (means 8.75 seconds) for the pixel (200,600). blue cross correspond to the observations data $\boldsymbol{D_n}$. Solid line is the time average of observations data $\overline{\boldsymbol{D_n}}$. Green cross correspond to the demeaning $\boldsymbol{I_n}$ and the solid green line is the average $\overline{\boldsymbol{I_n}}$, this value is zero by construction.

For each pixel, we calculate the demeaning:

$$\mathrm{I_n}(\mathrm{t_n}) = \mathrm{D_n} - \overline{D_n}\ , \ \ n\ is\ the\ images\ number$$

Where :

D_n represent the data of n images.
$\overline{D_n} = \frac{1}{N}\sum_{n=1}^{N} D_n$, the average at each i, j pixels.
$t_n = n\Delta t$

Least square harmonic method is considered as optimum when the sum,S of squared residuals is a minimum:

$$\mathrm{S} = \sum_{n=1}^{N} R_n^2$$

A residual is defined as the difference between the observed value and the value predicted by the model:

$$R_n = I_n - \widehat{I_n}$$

5.1 First harmonic model

We choose a first harmonic model $\hat{I}$:

$$\widehat{I_n} = \hat{I}(t_n)\ with\ \ \hat{I} = A_1 sin\omega t + B_1 cos\omega t = Ecos(\omega t - \varphi)$$

The minimum of the sum squares S is found by setting each derivative equal to zero.

$$\begin{cases} \frac{\mathrm{dS}}{\mathrm{dA_1}} = 0 \\ \frac{\mathrm{dS}}{\mathrm{dB_1}} = 0 \end{cases}$$

We have to solve these simultaneous equations in order to find A_1 et B_1 for our model.

$$\begin{cases} \sum I_n S_n - A_1 \sum S_n^2 - B_1 \sum S_n C_n = 0 \\ \sum I_n C_n - A_1 \sum C_n S_n - B_1 \sum C_n^2 = 0 \end{cases}$$

With: $S_n = \sin(\omega t_n)$
$C_n = \cos(\omega t_n)$

Hence, we have the following matrix equation :

$$\begin{pmatrix} \sum S_n^2 & \sum S_n C_n \\ \sum S_n C_n & \sum C_n^2 \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} \sum I_n S_n \\ \sum I_n C_n \end{pmatrix}$$

We obtain:

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} \sum S_n^2 & \sum S_n C_n \\ \sum S_n C_n & \sum C_n^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum I_n S_n \\ \sum I_n C_n \end{pmatrix}$$

We check if this model gives us a good approach of our data with four different pixels for 70 images (8.75 time scale, equivalent to one period) with $f_m = 12Hz$. We obtain the following results:

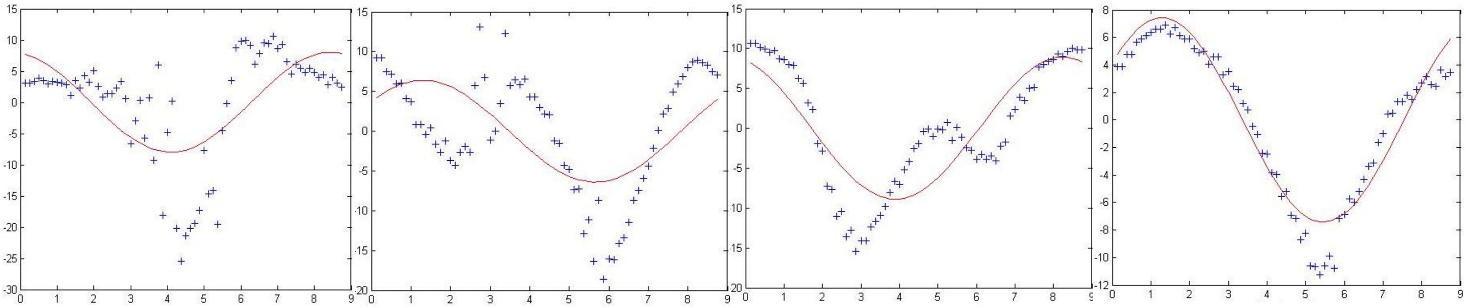


Figure 10 : Interpolation with First model harmonic for 4 different pixels (time scale 8.75 seconds)

Blue cross correspond to intensity value for a particular pixel we have chosen during 70 images (about one period 8.75 seconds). And solid red lines represent the first harmonic model. For some pixel the first harmonic model gives a quite good approach, it is the case for the last one representation. In order to quantify, the difference between the model and our data, we calculate for each pixel the residual.

We obtain for the four different pixels the percentage difference:

Pixels	N° 1	N°2	N°3	N°4
$\frac{\sum R_n^2}{\sum I_n^2}$	57.4 %	63.2 %	31.9 %	7.5 %

After residual analysis we notice a difference over 50% between the model and the data. To reduce this difference and to have a better model, we decide to work with a second harmonic model.

5.2 Second harmonic model

We add to our previous model, two additional elements which are second harmonics. Our new model is the following one:

$$\hat{I} = A_1 sin\omega t + B_1 cos\omega t + A_2 sin2\omega t + B_2 cos2\omega t$$

We process the same way to find amplitude coefficients: A_1, A_2, B_1 and B_2.

$$\begin{cases} \dfrac{\mathrm{dS}}{\mathrm{dA}_1} = 0 \\ \dfrac{\mathrm{dS}}{\mathrm{dB}_1} = 0 \\ \dfrac{\mathrm{dS}}{\mathrm{dA}_2} = 0 \\ \dfrac{\mathrm{dS}}{\mathrm{dB}_2} = 0 \end{cases}$$

Then, by combination we have this matrix equation:

$$\begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} \sum S_n^2 & \sum C_n S_n & \sum S_{2n} S_n & \sum C_{2n} S_n \\ \sum S_n C_n & \sum C_n^2 & \sum S_{2n}\, C_n & \sum C_{2n} S_n \\ \sum S_n S_{2n} & \sum C_n S_{2n} & \sum S_{2n}^2 & \sum C_{2n} S_{2n} \\ \sum S_n C_{2n} & \sum C_n C_{2n} & \sum S_{2n} C_{2n} & \sum C_{2n} \end{pmatrix}^{-1} \begin{pmatrix} \sum I_n S_n \\ \sum I_n C_n \\ \sum I_n S_{2n} \\ \sum I_n C_{2n} \end{pmatrix}$$

Thanks to that you can find the amplitude's coefficient of your model. We try to use the code on a few pixels. By the way we could see if the second harmonic elements take an important role. The following images show the evolution between the first harmonic analysis and the second harmonic analysis.

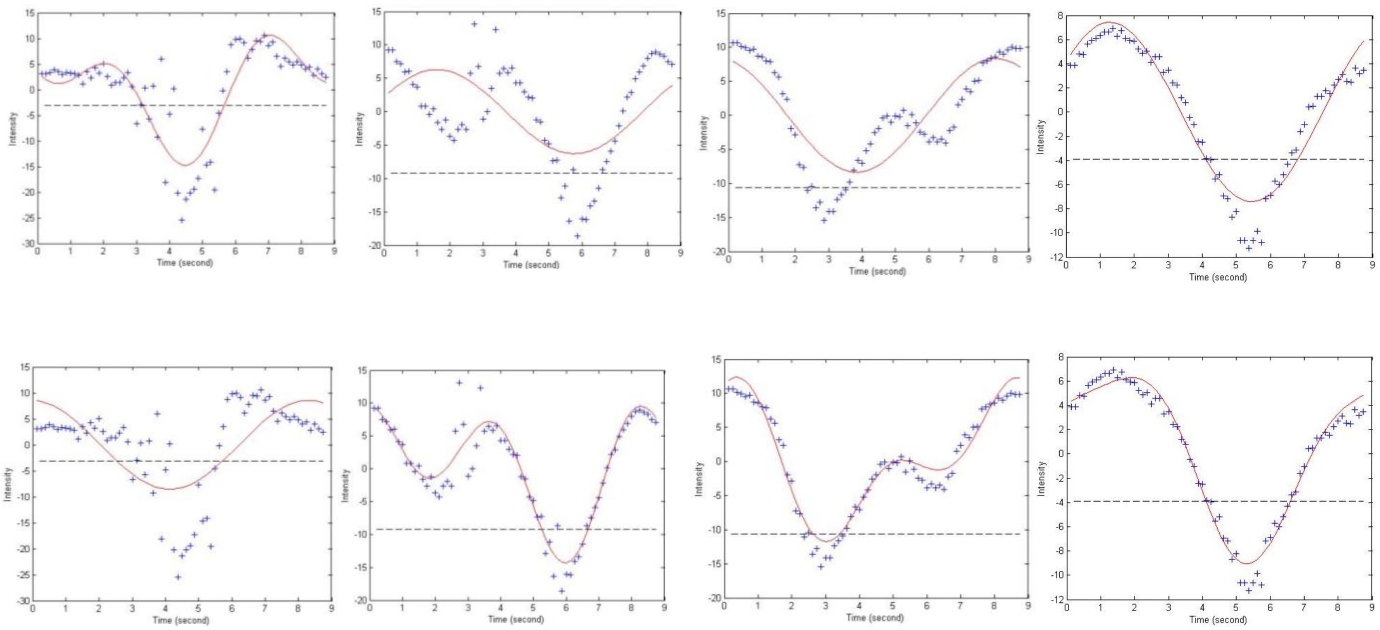


Figure 12 : Interpolation with first harmonic model for 4 different pixels (time scale 8.75 seconds)

Blue cross correspond to intensity value during 70 images, and solid red lines represent the model (first and second harmonic). The dashed black line corresponds to the time average intensity for intensity value without demeaning. We notice a difference between the two models. The second harmonic model gives us a better representation than the first harmonic. The following table illustres the percentage difference between the data and the model.

Pixels		N° 1	N°2	N°3	N°4
$\frac{\sum R_n^2}{\sum I_n^2}$	**First harmonic model**	57.4 %	63.2 %	31.9 %	7.5 %
	Second harmonic model	28.1 %	5 %	9.4 %	2.6%

The second harmonic model reduces significantly the difference between the data and the model. Indeed, the percentage difference is around 10%. We try with a third and a fourth harmonic models but representation is not quite improved. So we choose the second harmonic model for continuing our analysis.

5.3 "Energy" and Phase

Thanks to least square harmonic method, and the second harmonic model we have found amplitudes' coefficients: A_1, A_2, B_1 & B_2. These coefficients allow us to determine Phase (φ) and the "Energy" (E) with the following equations:

$$E = \sqrt{{A_1}^2 + {B_1}^2}$$

$$\varphi = \arctan\left(\frac{A_1}{B_1}\right)$$

We know that your model give us a good representation, now we will expend this analyze for each pixel, in order to have a global representation.

6- Results

6.1 Without surface layer

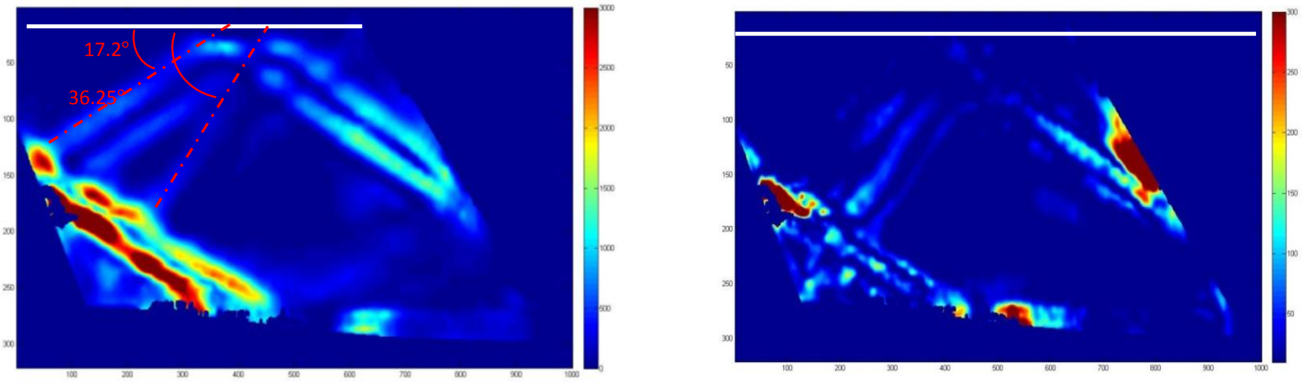


Figure 11 : "Energy" (square Amplitude) with first harmonic model and second harmonic model

Afterwards we obtain these two "Energy" representations' images. One with the first harmonic model (left) and another with the second harmonic model (right). To the left image, we observe a pattern of energy. This is the wave attractor. We observe the focus and defocus of energy around the attractor. Indeed, when the wave is defocusing, there is a loss of energy. This is the case when the beam is reflected on the bottom. The energy wave is coming from the right and is strong (red) and after the reflection the beam is defocus and light-blue. We observe the inverse phenomenon when the beam arrived on the right wall and become focused. The light blue becomes red. This experiment illustrates well the reflection theory.

Thanks to the dispersion relation we can calculate the buoyancy frequency. For doing that, with Digiflow we measure the angle formed by a beam with the horizontal. The angle is 17.2 degree. We have to transform this angle, to obtain θ which is the angle between the beam and the vertical.

$$\theta = 90 - 17.2 = 72.8°$$

<u>Dispersion relation:</u>

$$N = \frac{\omega}{cos\theta}$$

With: $\omega = 2\pi f,\quad f = \frac{f_m}{100}$ and $f_m = 12$

We obtain:

$$N = 2.55\, rad.s^{-1}$$

Moreover, we observe another beam with a different angle. This beam is also present on the left image. We suppose it is the second harmonic representation. To make sure, we calculate the second harmonic angle, with the following expression:

$$cos\theta_2 = \frac{2\omega}{N}$$

Hence,

$$\theta_2 = 53.75°$$

That mean, the angle between the beam and the horizontal is:

$$\alpha = 90 - 53.75 = 36.25°$$

This is the same value that we can measure directly with Digiflow. The second beam corresponds to the second harmonic.

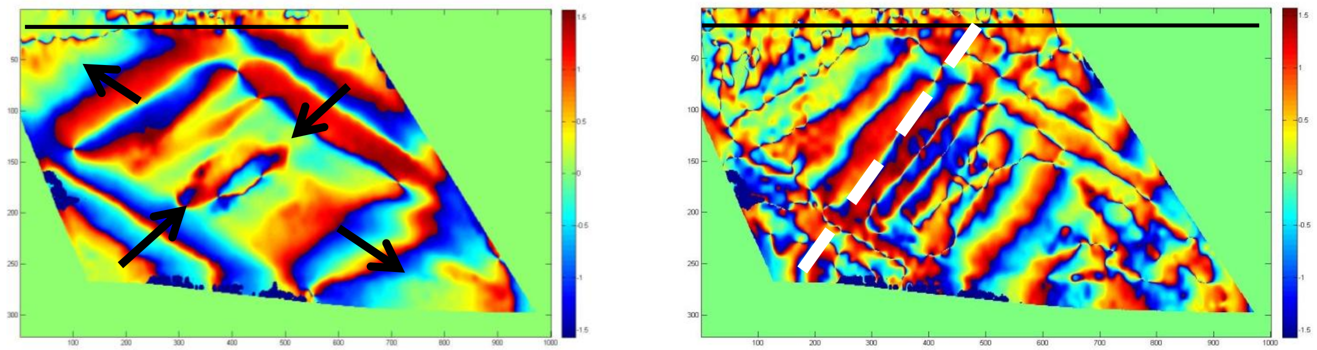


Figure 12 : Phase with first harmonic model and second harmonic model

These two figures represent phase results for the first (left) and second harmonic (right). Blue and red correspond respectively to troughs and crests. The black arrows on the first harmonic representation correspond to the phase velocity direction of propagation. Due to a fundamental propriety of internal waves (§2.5.1) we know that the group velocity (energy propagation) is perpendicular to the phase velocity. The energy propagates anti-clockwise. The white dashed line on the second harmonic result corresponds to the second harmonic branch (the same we have already observed on the energy representation).

To verify if our model still a good approximation, we plot the percentage difference between the data and the model, and we obtain the following representation.

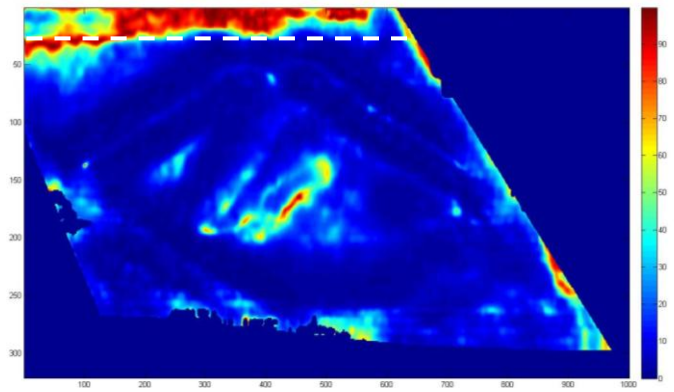


Figure 13 : representation of $\frac{\sum R_n^2}{\sum I_n^2}$ at each pixel. If the color is closed to dark blue the model is good.

This is the representation of $\frac{\sum R_n^2}{\sum I_n^2}$ at each pixel. Our model is quite good along the wave attractor structure. Outside this pattern we have a higher difference between our recording data and the model. The model is good when the Residual energy is low (ideally near 0), that means when model is close to our data.

We have found experimentally an attractor and we have created a matlab program to analysis our data. Now, we will add a constant density surface layer in order to study interactions between a uniform density gradient layer and a uniform density layer. We start with a surface layer of fresh water.

6.2-Surface layer: fresh water

We add fresh water at the surface. To do that, we place a tube at a corner of the left compartment of the tank. Firstly, the level into the left basin increases but not in the middle basin, probably because of absence of space between the wall and the tank. After a few minutes the level of the middle basin increases, but very slowly, due to the difference in pressure between the two basins.

Though the hydrostatic pressure relation, we can say that the height of the water column correspond to the water pressure.

$$\frac{\partial P}{\partial z} = -\rho g$$

Indeed, the water is higher in the left basin, so the pressure is higher than in the middle where the water is less.

Probably due to the difference of pressure, the water passes between the bottom and the slope and pushes water layers upward. The surface layer might not be composed only of fresh water, but we decided to ignore the tiny difference. After the filling, we have 1 cm of fresh water at the surface, with a salinity of 0 ‰ and a salinity step around 44 ‰.

We wait to have a nice attractor before doing measurements. The waiting time depends on the frequency. But, generally, this time fluctuates between 2 and 5minutes. We obtain the following results after Matlab running. The white dashed line corresponds to the interface between the surface layer and the stratification.

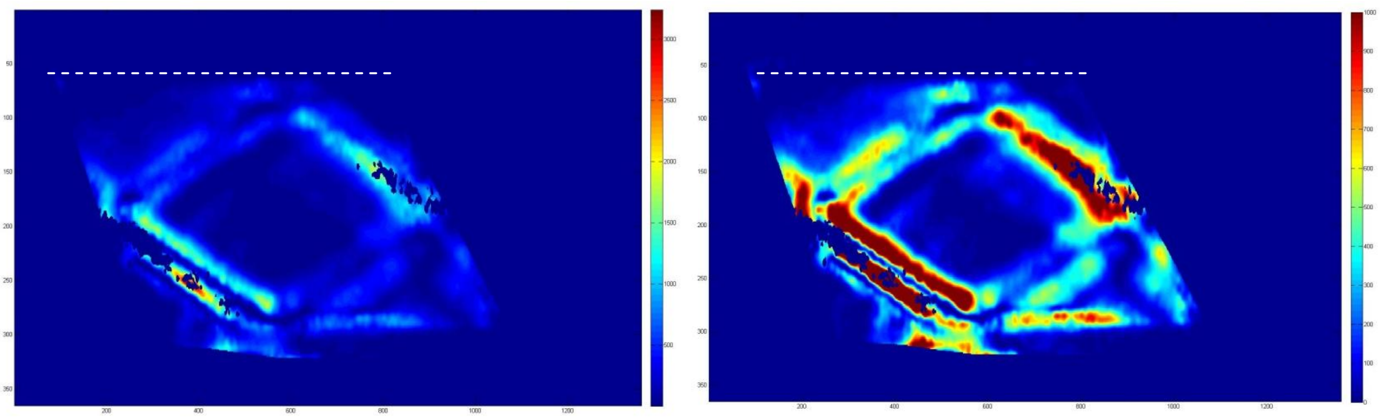


Figure 14 : Energy results for one time periode with 11.5 Hz frequency and 1cm of surface layer or fresh water. The left image corresponds to the energy results without threshold and the right image corresponds to a threshold of 1000 in order to have a better resolution. The red corresponds to high energy and blue to low energy.

We notice that no "Energy" is present in the surface layer. The interface behaves like a mirror. Indeed, the step of salinity is to important, like between water and air in the previous experiment. We will plot the phase representation to make sure nothing appear along the interface.

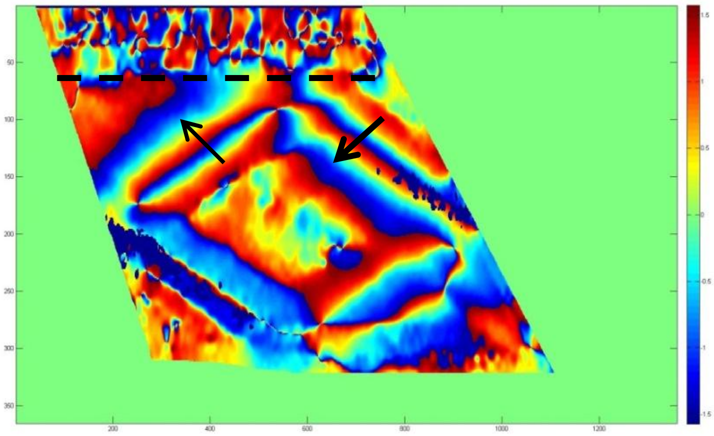


Figure 15 : Phase representation for one time periode with 11.5 Hz frequency and 1cm of surface layer of fresh water. Blue and red correspond respectively to troughs and crests.

We also observe that no interaction appears between the wave attractor and the surface layer. We have to reduce the salinity step in order to observe some interaction.

In a next time, we will make other experiment with a new kind of surface layer: salt water.

6.3- Surface layer: Salt water

After removing the surface layer, we add salt water (33.6 ‰) directly in the middle basin, because we do not want the water to pass under the wall and change the proprieties of our stratification. The salt water flows along the wall very slowly, and we are still very careful that less turbulence is created in the stratification. Indeed, there is some risk that the adding of water changes the stratification structure by mixing.

We have 0.75cm of salt water at the top of your stratification and the salinity step is around 13‰. We obtain the following results.

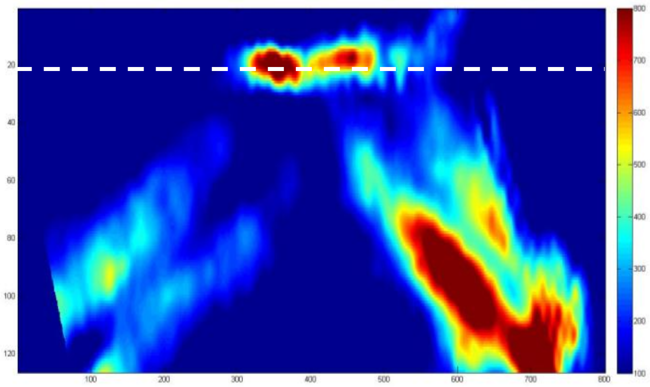


Figure 19 : "Energy" representation (Numerical Zoom)

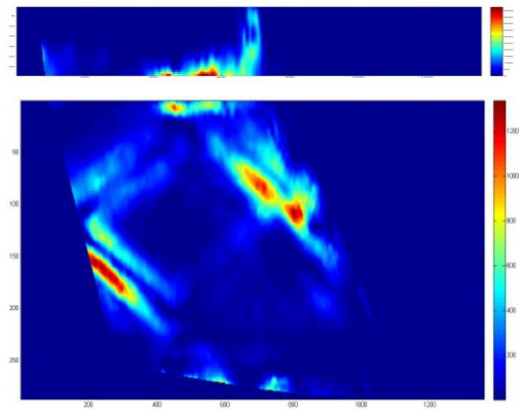


Figure 20 : Surface and Stratification "Energy"

We observe "Energy" along the interface that means a part of the global "Energy" is dissipated along the interface. We cannot observe energy propagation in the surface layer because we use the synthetic Schlieren method and this method analyses density variation. The surface layer is composed with a constant density, so it is impossible with this method to notice any motion. The only density variations we can observe are at the interface.

We will try to quantify the percentage of energy dissipates along the interface. We obtain the following table.

Energy Surface layer	Stratified fluid	Total
8.5 10^5	3.09 10^7	3.17 10^7
2.7 %	97.2 %	100 %

For less than 1 cm of surface layer with a salinity step around 13‰ we have 2.7% of the total energy located along the interface. To follow our analysis we plot the phase representation.

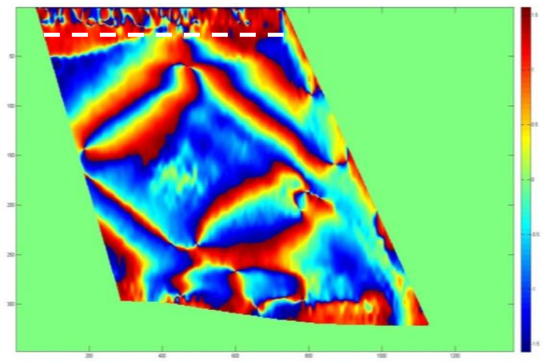


Figure 21 :Phase representation

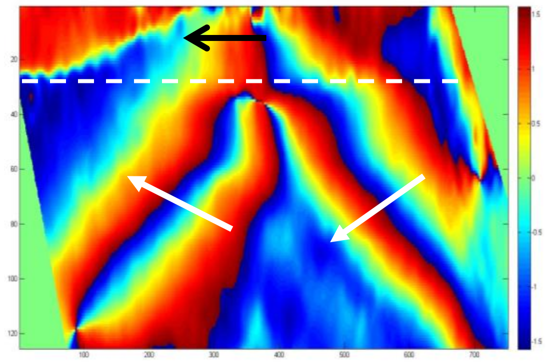


Figure 22 :Phase representation (Numerical Zoom)

These two figures are the phase representation without zoom (fig21) and with a numerical zoom (fig 22). The zoom representation allows us to see the energy propagation along the interface. Indeed, the white arrows point the phase velocity direction in the stratification layer and the black arrow points the phase velocity direction near the interface in the surface layer. We observe a propagation of energy starting to the reflection's point with the wave attractor and propagating along the interface on the left.

The "Energy" dissipates along the interface still minor (2.7%) nevertheless we observe propagation along the interface indicates that interactions exist between the stratification and the surface layer when the step of salinity is reduced.

We continue our experiments with a new stratification.

6.4. “Natural” surface layer

Previously, we have worked with a stratification composed with 4 liters of salt, and we have added water at the top to create a surface layer. This stratification was quite strong and we have easy found a well attractor. Now, we decide to empty the tank, and refill with a new stratification composed with 2 liters of salt. We choose to decrease the salinity rate in order to approach the real ocean stratification conditions. Because the density gradient is now smaller the buoyancy frequency (N) is also smaller than the previous stratification. We have to respect the following condition in order to have internal waves:

$$\omega < N$$

The good frequency for having an attractor is smaller than previously, and we find $f_m = 7.5\ Hz$. That means:

$$\omega = \frac{2\pi 7.5}{100} = 0.47\, rad.s^{-1}$$

With Digiflow I can only measure the angle with the horizontal, after transformation I find the angle θ the internal wave beam make with the gravity direction.

$$\theta = 73.1°$$

Hence, the stratification is:

$$N = \frac{\omega}{cos\theta} = 1.62\, rad.s^{-1}$$

A day after having created the new stratification, I notice the apparition of a surface layer. Samples have shown that the salinity of the surface layer is now: 31.8 ‰ instead of 27.8‰. That means the stratification has started to mix. We obtain a “natural” surface layer and we will use it to make experiments. Samples also show that the step of density between the surface layer and the top of the stratification is around 1‰. We obtain representation of “Energy” and Phase, and we zoom numerically these two representations to focus our attention around the interface area.

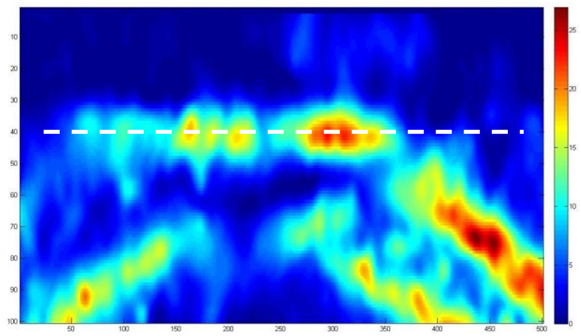


Figure 23 : ”Energy” representation (Numerical zoom)

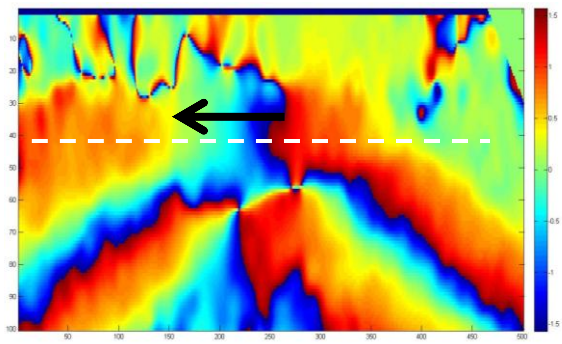


Figure 24 : Phase representation (Numerical zoom)

We observe a propagation of "Energy" along the interface involving a phase velocity to the left. The reduction of the salinity step reduces the mirror reflection effect. Indeed, the interface behaves any more like a mirror and an important quantity of "energy "is located along the interface. The following table shows the percentage of "Energy" presents in the different layers.

Energy Surface layer	Stratified fluid	Total
6.6 10^4	3.09 10^7	3.17 10^7
4.3 %	97.2 %	100 %

4.3 % of the total energy is presented in the surface layer. It is 1.5 times more important than for a salinity step of 14‰. These values illustrates well that the salinity step plays a main role in the interactions between a uniform density gradient layer and a uniform density layer.

We will study if the variation of frequency plays also a role in the interactions between stratification and a surface layer with a salinity step around 1‰. For doing that we use a new stratification and add a surface layer.

6.5. Salinity step 1‰ and variations of frequency

We create a new stratification with 3 liters of salt. In order to prevent pressure difference and mixing, we put water in each basin simultaneously. We have some problems during the filling, because it is difficult to have the same rate of flow in each sponge floater (difference of length tubes). Moreover, at the beginning two floaters were sinking because there had not been wet previously. So we have had to remove the floater, wetted it and replaced it.

We add 3 cm of mixed water and due to evaporation and the mixing process we observe 3.4cm of surface layer with a salinity step around 1‰.

We try four different frequencies: 7.5, 8, 8.5 and 9.5 Hz. We observe the variation of the pattern. Indeed, a variation of frequency leads to change the angle the wave makes with the vertical due to the dispersion relation: $\frac{\omega}{N} = cos\theta$.

We can observe this variation with following results.

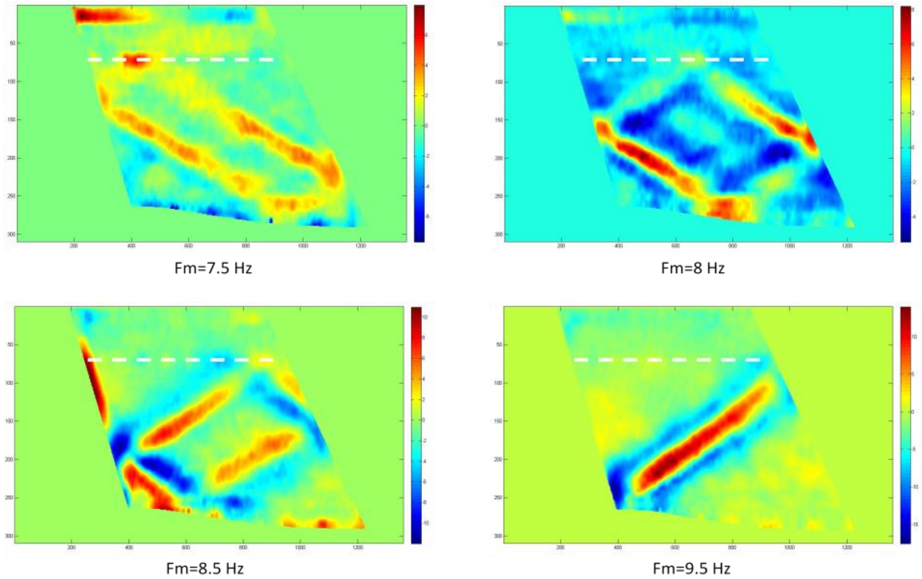


Figure 25 : Time average intensity representations for four different frequencies : 7.5, 8, 8.5 and 9.5 Hz

These representations correspond to time average intensity for each pixel and allow us to see the wave attractor quite good when the energy representations are not. We could observe the variation of the pattern. We notice that for each frequency interactions exist at the interface. Moreover for $fm = 8.5Hz$ we observe a concentration of intensity close to the left wall and at the interface. We can suppose that results from the breaking of a wave propagating along the interface.

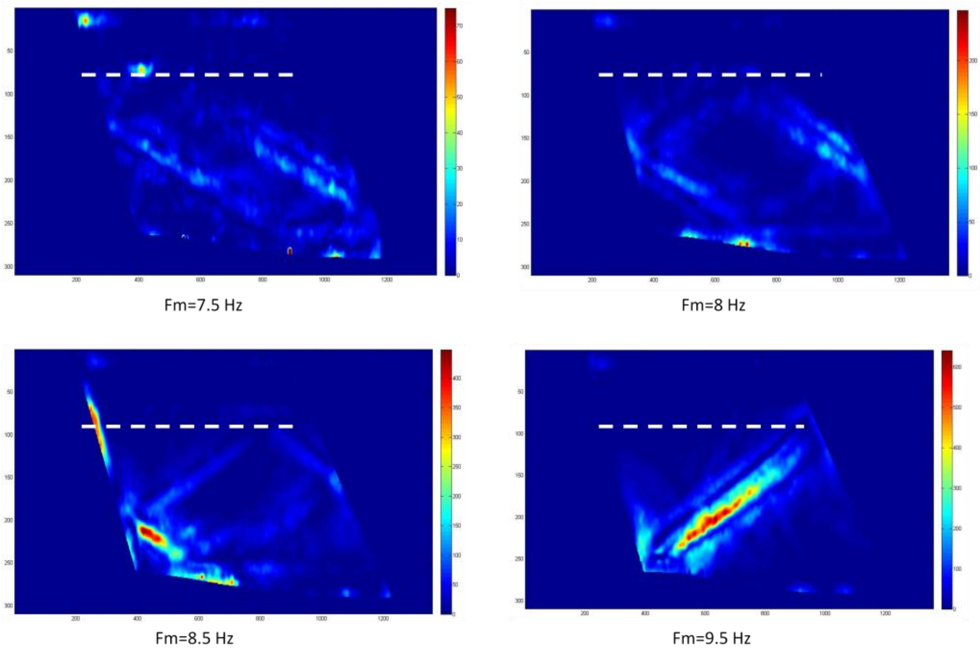


Figure 16 : "Energy" representations for four different frequencies : 7.5, 8, 8.5 and 9.5 Hz

With the "Energy" representations it is quite difficult to observe interactions along the interface. Indeed, the synthetic Schlieren method still curbs representation of motion and energy propagation in the surface layer. Moreover, peaks of energy are presented and distorted our results. We have to make thresholds but the results still difficult to analyze. Nevertheless, we observe for $fm = 8.5Hz$ the same concentration of energy near the wall and the interface. The high concentration of energy illustrates the propagation of energy along the interface and in the surface layer, even if our method does not allow us to observe something directly into the surface layer

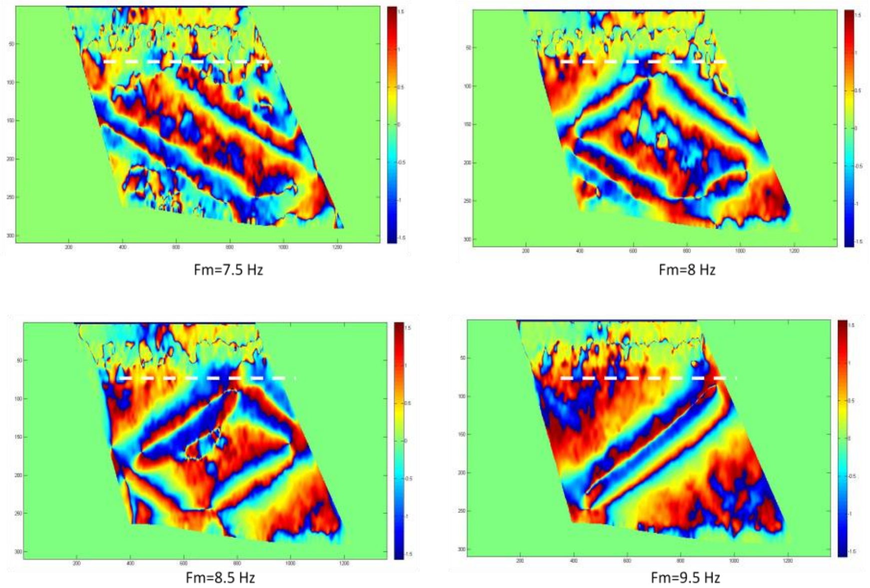


Figure 27 : Phase representations for four different frequencies : 7.5, 8, 8.5 and 9.5 Hz

We observe with the phase representations propagations of phase along the interface, mainly for $fm = 8{,}8.5\ and\ 9.5\ Hz$. Indeed, the variation of colors: yellow, red and blue at the interface are vertically presented, that means propagations occur along the interface. In a first time we decided to compare total energy for each frequency.

We have results with different frequencies, so we decided to compare total energy for each frequency.

$$Energy_{total} = \sum E(i,j)$$

We obtain the following graph:

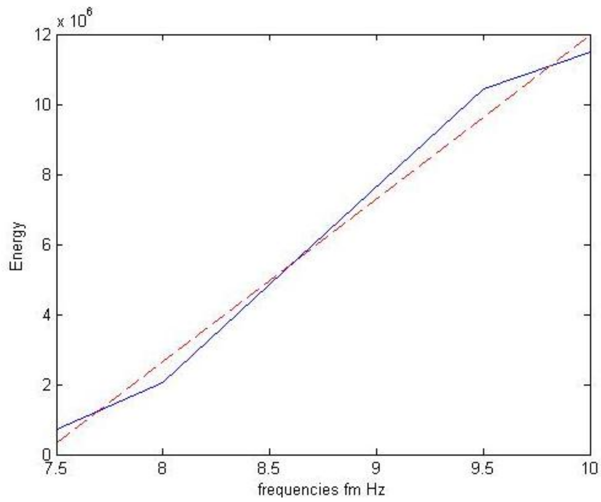


Figure 28 : Total energy functions of frequencies

We observe that the total energy increases with frequency. This increase is quite linear, and we have made a linearization. We establish a relation between the frequency and the total energy:

$$Energy_{total} = (0.4154 * Frequency - 3.0684).10^{7}$$

This relation shows us that for different frequency the global energy increases or decreases, so for different wave attractor the energy varies. We will try to see if difference of energy and also of wave attractor lead to different interactions between the surface layer and the stratification.

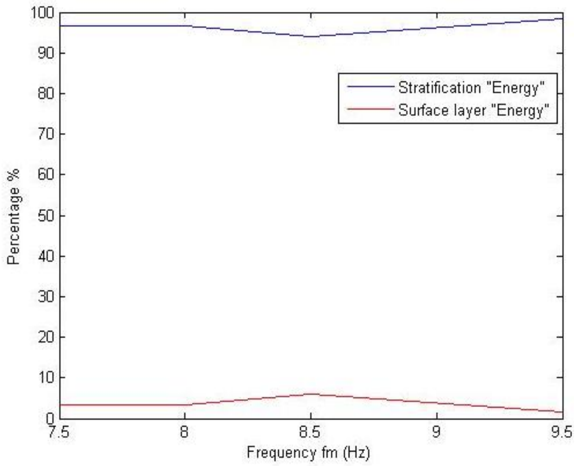


Figure 17: Percentage of energy located in the surface layer and stratification

PERCENTAGE of total "Energy"				
FREQUENCY	**9.5**	**8.5**	**8**	**7.5**
Stratified fluid	98.39 %	94,09%	96,62 %	96,61 %
Surface layer	1,61 %	4,91 %	3,38 %	3,39 %

The figure 28 shows us the percentage of energy located in the surface layer and in the stratification, recapped in the table. We observe that 4.91 % of the total energy is located in the surface layer for $fm = 8.5Hz$ and only 1.61 % with $fm = 9.5\ Hz$.

We can see a relation between our previous observations and these energy values. Indeed, we have noticed for $fm = 8.5Hz$ a concentration of energy near the left wall and the interface, and for the same frequency we find the higher value of energy located on the surface layer.

6.6. New basin

We decide to continue our study with a new basin. Indeed, we remove one wall in order to have a basin longer than the previous one. A longer basin leads to have a distance more length for the energy propagation along the interface. The following results are for a 3 liters salt stratification without additional surface layer.

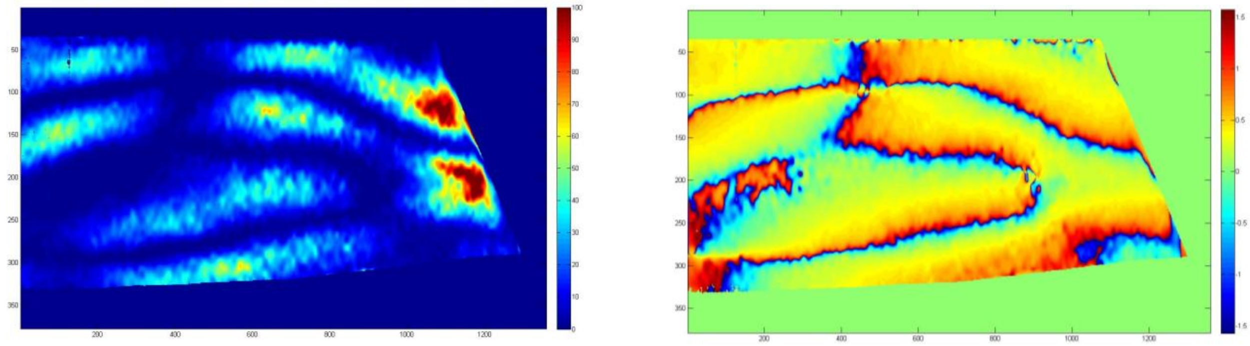


Figure 30 : Energy representation (left) and phase representation (right) for a one harmonic attractor on the left basin of the tank with $fm\ = 6\ Hz$.

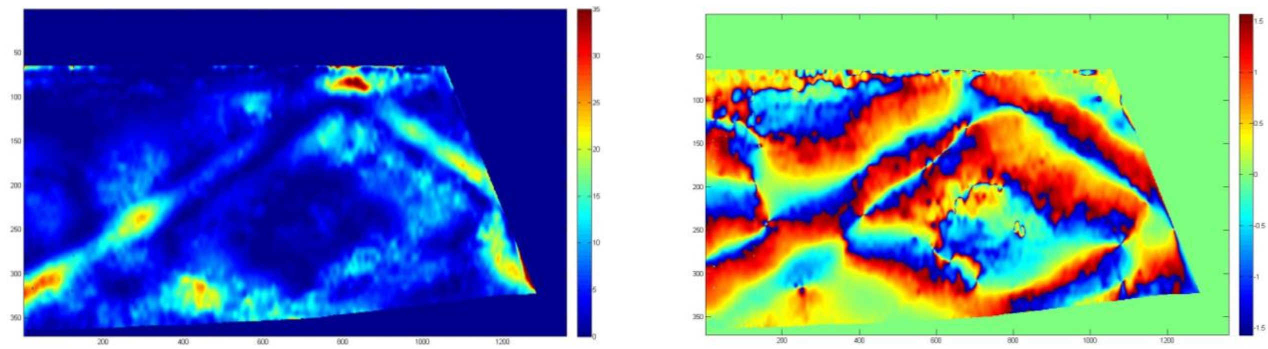


Figure 31 : Energy representation (left) and phase representation (right) for a one two wave attractor on the left basin of the thank with $fm\ = 11Hz$

Because we have a longer basin we try to find a wave attractor and also a 2,1 wave attractor. We call a 2, 1 attractor an attractor composed with two patterns. Indeed, when we increase enough the frequency we obtain a new pattern (fig 30). We find a 1,1 wave attractor for a frequency $fm = 6Hz$ and a 2,1 wave attractor for a frequency $fm = 11Hz$. We calculate the total “Energy” presents for these two kind of attractor, and we obtain the following results :

Wave attractors	1	2
“Energy”	$5.36\ \ 10^6$	$1.78\ 10^6$

It is interesting to see the loss of energy between the two attractors. The 2,1 wave attractor is more complex and less stable than the one attractor that is why we observe a diminution of Energy.

We create a surface layer of 2.7 cm with a salinity step around 1‰ and we obtain the following results :

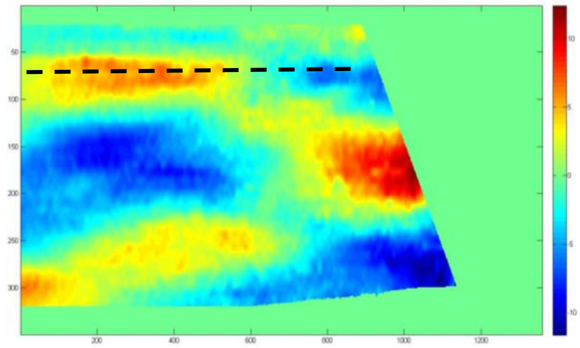


Figure 31 : Time average intensity

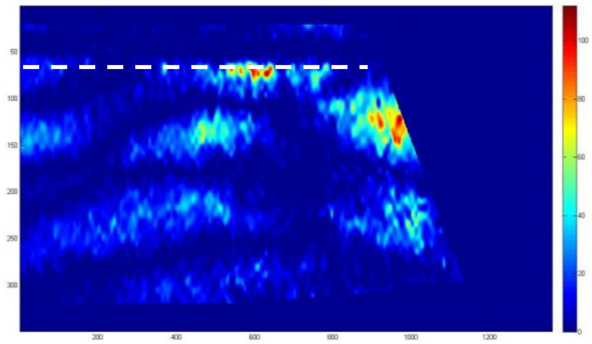


Figure 32 : "Energy" representation

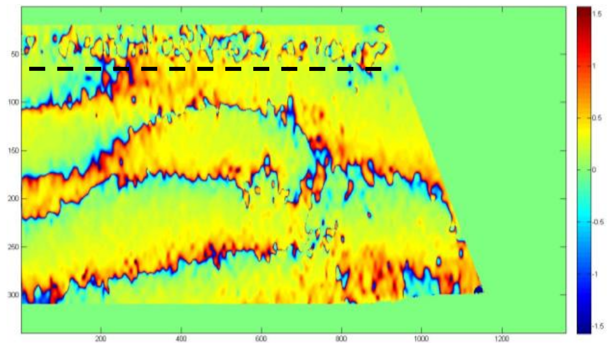


Figure 33 : Phase representation

We observe interactions between the stratification and the surface layer. The time average intensity representation shows us very well this interaction. We also notice concentration of "Energy" along the interface with the figure 32. But it is difficult to observe phase propagation along the interface between surface layer and stratification. Indeed, with a longer basin group and phase velocity are slower than for the previous basin and it is very difficult to notice something directly with the representation. We quantify the propagation of "Energy" along the interface and we obtain the following table :

"Energy" percentage %	
Stratification	97.3%
Surface layer	2.7%

A part of the total "Energy" is located in the surface layer, but the phase representation informs us that the propagation along the interface is not very present.

Concerning the 2,1 wave attractors with an additional surface layer we obtain very bad results. Indeed, this kind of pattern is not very strong, and presence of surface layer destroys completely our attractor. This observation is important, and shows us that wave attractor is very vulnerable to the external condition. Indeed, a high frequency plus a surface layer could help wave attractors to disappear.

Discussions

In this report, I have presented only principals results, but I have made other experiments. The following table summarizes my results with different height of surface layer and salinity steps. For these results, we can see the percentage of "Energy" presents in the surface layer.

Height Surface layer	Salinity Step	"Energy" surface layer
1 cm	44 ‰	0.1 %
0.75 cm	13 ‰	2.7 %
2.3 cm	1 ‰	2.7 %
3.4 cm	1 ‰	1.61 % $fm = 9.5\,Hz$ 4.91 % $fm = 8.5\,Hz$ 3.38 % $fm = 8\ \ Hz$ 3.39 % $fm = 7.5\,Hz$
3.8 cm	1 ‰	3.3 %
3.3 cm	< 1 ‰	9.5 %

We notice that about 3-4% of the total "Energy" is located along the interface for a salinity step around 1‰. But these percentages fluctuate with the salinity step, the height of surface layer and also with the frequency. Results inform us that a part of the total "Energy" is dissipated along the interface and we cannot ignore this phenomenon.

Our results are limited due to your synthetic schlieren method. Indeed, this method is useful when there are density variations, and in our surface layer, the density is the same. So this method does not allow us to observe perturbation in the surface layer, but just along the interface. It is an important point. However, it is quite difficult to create salt water with a density close to the top of the stratification density. We are able to create a 1‰ salinity step, but for one experience we have a step less than 1‰ and we observe 9.5% of the energy is located in the interface. That means if we have other materials and instruments to create and reduce the step of salinity we would be able to improve our result. Moreover with the synthetic schlieren we can observe variations in the surface layer. To solve this problem other experiments could be done with new equipments in order to follow particles in the surface layer in real time.

Conclusion

This work has been done to observe interactions between a uniform density gradient layer and a uniform density layer. After several experiments and creation of matlab programs I have found interesting results.

Indeed, around 3% of the total "Energy" is located in an additional surface layer when there is a 1 ‰ salinity step. Moreover the height of the surface layer and also the frequency play an import role. If the wave attractor is too complex it is less vulnerable to surface layer presence. It is important to have a strong pattern and a step of salinity minimal to observe propagation over 9% of total "Energy" along the interface.

This observation allows us to develop knowledge about internal waves and especially wave attractors. Although our materials limit us to study interactions in the surface layer we obtain promising results to follow research concerning this flied. It is just the begging about wave attractors' studies and other experiments in laboratory but also in the Ocean would be manage to improve knowledge about this new phenomenon. This new kind of energy transport but also of nutriments transport could play a major role in large scale deep ocean circulation and ecosystem. Our results are limited in a 2Dimension and observations in the ocean are still for the moment impossible. The international scientific community tries to find relations between their mooring line recording and wave attractors presences. Researches continue and each assumption is carefully considered to obtain the best ocean's comprehension.

This internship was a very good experience for me. Indeed, it was interesting both in my study and my education. I have worked to improve my English (particularly regarding Oceanographic terminology), and learned to work in another way than in France. I attended to conferences each week about different topics in oceanography. I could see the team-work and the good work atmosphere by attending meeting between several researchers.

I can conclude that this internship allowed me to develop further my knowledge in the field of theoretical internal waves and wave attractors. But it also allowed me to discover the word of the research and to take a small part of it. This experience will remain as amazing.

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Appendix A: Materials

Image 1 : Camera JAI progressive scan CV-M4+CL

Image 2 : A portable conductivity, salinity and temperature instrument (Ref VWR EC300)

Image 3 : Refractometer (Ref: AQUA MEDIC RHS-10ATC)

Appendix B: Matlab

```
%
%                              MAIN PROGRAM
%              Digiflow data analysis-Energy and Phase results-
%                              wave attractors
%
%                            4th June-27th July
%                              by Amandine B.
%-----------------------------------------------------------------------
%    Output :
%        - Im_E1 :"Energy" First harmonic matrix
%        - Im_E2 :"Energy" Second harmonic matrix
%        - Im_Ph1: Phase First harmonic matrix
%        - Im_Ph2: Phase Second harmonic matrix
%        - Im_R : Residual matrix
%        - Im_K : Difference between model and data (percentage) matrix
%        - Im_Dn : Time average intensity matrix
%-----------------------------------------------------------------------

close all
clear all

% Frequency
fm=12;
%
trans=translation_tank(1/8,fm,54);

% Fist image where tank is more left
im_depart=63;
% Number of images we want to run
nbr=1;

files=dir(['*.dat']);

[M,D,row,column]=Matrice(files,im_depart,nbr,trans);
[Im_E,Im_E1,Im_Ph,Im_Ph1,Im_R,Im_K,Im_Dn] =
Least_squares_harmonic_analysis(fm,row,column,D,M);



%            Translation function of the tank
%-----------------------------------------------------------
% Input :
%        - freq_sample : Sample frequency (images per second)
%        - fm : tank frequency
%        - amplitude_max : pixels number of tank amplitude
% Output :
%        - trans : Translation (pixels)



function[trans]=translation_tank(freq_sample,fm,amplitude_max)

t=[0:freq_sample:60];
f=fm/100;
T=1/f;
```

```
S=-(amplitude_max/2)*cos(2*pi*t/T)+(amplitude_max/2);

% Ceiling function : nearest integer greater than or equal
C=ceil(S);

% Floor function : nearest integer less than or equal
F=floor(S);

for i=1:length(S)
    if (S(i)<=(0.5+F(i)))
        trans(i)=F(i);
    else
        trans(i)=C(i);
    end
end

end


%              Cutting out
%-----------------------------------------------------------
% Input :
%         - Name : Matrix we want to cut
%         - row : row : Rows number
% Output :
%         - Name_cut : cutting matrix


function [Name_cut] = Decoupage (Name,row)

load('c_fresh.mat');
load('r_fresh.mat');


% 42=a1+b
% 280=a308+b

l_left=[1:row];

a_left=(280-42)/(308-1);
b_left=42-1.*a_left;

Wall_left=a_left.*l_left+b_left;

%712=a+b
%1121=332+b

l_right=[1:row];

a_right=(1121-712)/(332-1);
b_right=712-a_right;

Wall_right=a_right.*l_right+b_right;

CY_left=ceil(Wall_left);
CY_right=ceil(Wall_right);
```

```
FY_left=floor(Wall_left);
FY_right=floor(Wall_right);

for i=1:length(Wall_left)
    if (Wall_left(i)<=(0.5+FY_left(i)))
        Slope_left(i)=FY_left(i);
    else
        Slope_left(i)=CY_left(i);
    end
end

for i=1:length(Wall_right)
    if (Wall_right(i)<=(0.5+FY_right(i)))
        Slope_right(i)=FY_right(i);
    else
        Slope_right(i)=CY_right(i);
    end
end

       for i=1:row
  for j=Slope_left(i)-1:Slope_right(i)-1
     Name_cut(i,j)=Name(i,j);
  end
      end

      for i=1:length(r)
          Name_cut(r(i),c(i))=0;
      end

 end


%            Matrix creation function with Digiflow Intensity Data
%----------------------------------------------------------------------
%
% Input :
%        - files : Files with all your Intensity data (.dat)
%        - im_depart : Number of the image where the tank is to the left.
%        - nbr : Number of images we want to analyse ( Generaly one
%        periode)
%        - trans : Number of pixels each image has to be translate.
%
% Output :
%        - M : Intensity matrix
%        - D : Intensity values for each pixel during 'nbr' images
%        - row : Rows number
%        - column : columns number


function [M,D,row,column]=Matrice(files,im_depart,nbr,trans)

for ii=im_depart:im_depart+nbr


fid = fopen(files(ii).name);
C = textscan(fid, '%f32%f32%f32','TreatAsEmpty','**********');
fclose(fid);

data=cell2mat(C);
```

```
row=data(1,2);
column=data(1,1);

Lx=data(2:length(data),1);
Ly=data(2:length(data),2);
LI=data(2:length(data),3);
data=[Lx Ly LI];

M=zeros(row,column);

for i=1:row
    for j=1:column
        k=column*(i-1)+j;
        M(row-i+1,j)=LI(k);
    end
end

 for j=1:column
    n=j+trans(ii-(im_depart-1));
        if ((n>0) && (n<=column))
            M(:,j)=M(:,n);
        else
            M(:,j)=0;
        end
 end

   for i=1:row
       for j=1:column
           k=column*(i-1)+j;

           D(k,ii-(im_depart-1))=M(i,j);

           end
        end


end
end
%               Least Squares Harmonic Analysis
%----------------------------------------------------------------------
% Input :
%        - fm : Tank frequency
%        - row : rows number
%        - column : columns number
%        - D : Intensity for each pixel during an image number we have chosen
(from Matrice function)
%        - M : Intensity data matrix

% Output :
%        - Im_E1 :"Energy" First harmonic matrix
%        - Im_E2 :"Energy" Second harmonic matrix
%        - Im_Ph1: Phase First harmonic matrix
%        - Im_Ph2: Phase Second harmonic matrix
%        - Im_R : Residual matrix
%        - Im_K : Difference between model and data (percentage) matrix
%        - Im_Dn : Time average intensity matrix
```

```
function[Im_E1,Im_E2,Im_Ph1,Im_Ph2,Im_R,Im_K,Im_Dn] =
Least_squares_harmonic_analysis(fm,row,column,D,M)

f=fm*1/100;
T=1/f;
w=2*pi/T;


for i=1:length(D(:,1))

In=D(i,:)-mean(D(i,:));
tn=[1/8:1/8:(length(In))/8];



Sn=sin(w.*tn);
Cn=cos(w.*tn);
S2n=sin(2.*w.*tn);
C2n=cos(2.*w.*tn);

Mm=[sum(Sn.^2) sum(Cn.*Sn) sum(S2n.*Sn) sum(C2n.*Sn);
    sum(Cn.*Sn) sum(Cn.^2) sum(S2n.*Cn) sum(C2n.*Cn);
    sum(Sn.*S2n) sum(Cn.*S2n) sum(S2n.^2) sum(C2n.*S2n);
    sum(Sn.*C2n) sum(Cn.*C2n) sum(S2n.*C2n) sum(C2n.^2)];

F1=sum(In.*Sn);
G1=sum(In.*Cn);
F2=sum(In.*S2n);
G2=sum(In.*C2n);

R=Mm\[F1;G1;F2;G2];
A1=R(1);
B1=R(2);
A2=R(3);
B2=R(4);

% First Harmonic Model
% II=A1.*Sn+B1.*Cn;
% Second Harmonic Model
II=A1.*Sn+B1.*Cn+A2.*S2n+B2.*C2n;

for n=1:length(tn)
    R(n)=In(n)-II(n);
end

Dn(i,1)=mean(D(i,:));
SumRn(i,1)=sum(R.^2);
SumIn(i,1)=sum(In.^2);
K(i,1)=sum(R.^2)/sum(In.^2);

%"Energy" First and second harmonic
E1(i,1)=sqrt(A1^2+B1^2)^2;
E2(i,1)=sqrt(A2^2+B2^2)^2;

%Phase First and second harmonic
Ph1(i,1)=atan(A1/B1);
Ph2(i,1)=atan(A2/B2);

end
```

```
%----------------------------------------------------------------------
%                        Matrix creation
%----------------------------------------------------------------------

for i=1:row
    for j=1:column
        k=column*(i-1)+j;
        Im_E1(i,j)=E1(k);
        Im_E2(i,j)=E2(k);
        Im_Ph1(i,j)=Ph1(k);
        Im_Ph2(i,j)=Ph2(k);
        Im_R(i,j)=SumRn(k);
        Im_K(i,j)=K(k).*100;
        Im_Dn(i,j)=Dn(k);
    end
end
%                Determination of Energy
% in each part of the tank : Total, Stratification,
% surface layer. Results also in percentage of the total energy
%----------------------------------------------------------------------
% Input :
% - Matrix
% - Surface_position : Pixels number from the top to the surface layer
% - Interface_height : Pixels number of the surface layer
% Output :
% - Energy surface : Value of the surface layer energy
% - Energy strat : Value of the stratification layer energy
% - Energy_total : Total value of energy
% - Esurf_per : Percentage of surface layer energy
% - Estrat_per : Percentage of stratification layer energy
% - M_surface : Surface layer matrix
% - M_strat : Stratification layer matrix
%------------------------------------------------------------------------

function
[Energy_surface,Energy_strat,Energy_total,Esurf_per,Estrat_per,M_surface,M_strat]
=Energy(matrix_input,surface_position,interface_height)

for i=surface_position:length(matrix_input(:,1))
    for j=1:length(matrix_input(1,:))
        if isnan(matrix_input(i,j))==1
            matrix_input(i,j)=0;
        end
    M_total(i,j)=matrix_input(i,j);
    end
end

Energy_total=sum(sum(M_total));

for i=surface_position:(surface+interface_height)
    for j=1:length(matrix_input(1,:))
        M_surface(i-surface_position+1,j)=matrix_input(i,j);
    end
end

Energy_surface=sum(sum(M_surface));

for i=(surface_position+interface_height):length(matrix_input(:,1))
    for j=1:length(matrix_input(1,:))
```

```
            M_strat(i-(surface_position+interface_height)+1,j)=matrix_input(i,j);
        end
    end

    Energy_strat=sum(sum(M_strat));
    Esurf_per=Energy_surface*100/Energy_total
    Estrat_per=Energy_strat*100/Energy_total

    end
```