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On the nature of internal wave spectra near a continental slope

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[1] Amongst the most energetic motions in the deep ocean are internal waves supported by stable vertical stratification in density. Previously, these waves were considered freely propagating as described by a smooth continuum internal wave band (IWB) frequency spectrum. We studied details of the IWB using yearlong current observations from the Bay of Biscay. Instead of observing continuum IWB spectra near a continental slope, we show that (at least) the first half-decade of the IWB is dominated by motions at localized frequencies determined by strong non-linear interactions between waves at the fundamental semidiurnal tidal and atmospherically induced inertial frequencies. *INDEX TERMS:* 4544 Oceanography: Physical: Internal and inertial waves; 4512 Oceanography: Physical: Currents; 4560 Oceanography: Physical: Surface waves and tides (1255)

1. Introduction

[2] *Munk and Wunsch* [1998] suggested that enhanced *tidal* turbulent mixing near bottom topography contributed about half of the mixing required to maintain the large-scale meridional overturning circulation in the deep ocean. This inevitably leads to the question 'Are motions close to the '*inertial*' frequency (f) also responsible for this mixing?' After all, these motions are amongst the most energetic in the ocean and turbulence-generating vertical current shear is largest at f. They also dominate gravity wave energy in the ocean interior [*Munk*, 1981]. However, the precise relationship between such internal waves and mixing, and the remarkable similarity [*Garrett and Munk*, 1972; *GM*] of IWB spectra in different ocean areas, are still widely discussed [*GM*; *Thorpe*, 1975, 1999; *Wunsch*, 1976]. Here we investigate an important role for strong non-linear interactions, following theoretical suggestions [*Phillips*, 1977] and in contrast with continuous smooth spectra [*GM*] describing a symmetric and isotropic saturated linear wave field following weak resonant interaction models [*Müller et al.*, 1986].

[3] In theory, internal gravity waves can exist in the frequency (σ) band between $f(\varphi) < \sigma < N(z)$, where the depth ($-z$) dependent buoyancy frequency $N(z) = (-g d\ln\rho/dz)^{1/2}$, g is the acceleration of gravity, $f(\varphi) = 2\Omega\sin\varphi$ is twice the local vertical component of the Earth's rotation vector Ω at latitude φ. Enhanced energy near f is attributed to local generation [*Fu*, 1981], interference of reflected waves at their 'turning latitude' [*Munk and Phillips*, 1968], or bottom trapping at this latitude [*Maas*, 2001]. Generally, it is assumed that near-inertial motions are generated near the surface by widely varying atmospheric disturbances [*Gill*, 1984; *Garrett*, 2001].

[4] Recent attention focuses on a suggestion that near-inertial motions are important for transfer of energy inside the IWB through non-linear interaction with semidiurnal tidal motions [*Mihaly et al.*, 1998; *van Haren et al.*, 1999; *Müller and Briscoe*, 1999]. In this paper we study enhanced energy at sequences of interaction frequencies suggestive of non-linear advection to the point of waves breaking. Such non-linearly forced waves are distinguished from free internal waves, the latter satisfying a dispersion relation.

2. Observations

[5] Currents were measured at seven moorings deployed between water depths H = 2000–4810 m down the continental slope into the abyssal plain of the Bay of Biscay during 11 months. The rough continental slope generated internal (tidal) waves and possibly focused them by reflection. Our deepest site was on the abyssal plain, ~100 km away from the slope. From CTD density profiles, obtained near the moorings, we estimated $N(z) = (1 \pm 0.5)(20 + 0.0034z)$ cpd, $-4480 < z < -2740$ m (frequency was calculated in cycles per day, 1 cpd = $2\pi/86400\ \mathrm{s}^{-1}$). Over the indicated depth range this profile differed by less than 25% from the [*GM*] $N(z) = N_0\exp(z/b)$, $N_0 = 72$ cpd, b = 1300 m. Our depth dependence changed abruptly to stronger linear dependence above $z > -2740$ m. At $z = -1500$ m, $N = 30 \pm 15$ cpd.

[6] In our spectral analysis we applied variable smoothing, $\nu \approx$ 3–1100 degrees of freedom (df). We used only a cosine-tapered time series without further smoothing ($\nu \approx 3$ df yielded a bandwidth of ~0.006 cpd for an 11 months record) regarding some (e.g. tidal) motions deterministic rather than a particular realization of a stochastic process. Heavy smoothing ($\nu > 1000$ df) created such large bandwidths that enhanced energy in tidal and inertial bands merged with the spectral environment, yielding smooth spectra as in [*GM*].

[7] Observed kinetic energy spectra $P(\sigma)$ revealed larger energy at shallower depth (larger N), *except at f*. At each depth, we found most energy at localized frequencies within the inertial-semidiurnal tidal band (indicated as f, M_2 in Figure 1). In the latter band about 60% of the variance was supported by highly deterministic narrowband large scale signals at tidal constituent frequencies, being either barotropic of coherent baroclinic. The enhanced energy of the inertial and semidiurnal tidal bands was superposed on a continuum $P_c(\sigma)$ showing no gaps and containing most energy at lower frequencies. Its spectral fall-off rate varied with frequency, as a first continuum $P_c(\sigma) \sim \sigma^{-1}$ for $\sigma <$ ~7–10 cpd ($\equiv \sigma_t$, a transition frequency) merged into a second $P_c(\sigma) \sim \sigma^{-2}$ for $\sigma_t < \sigma < N$. The latter resembled free wave model [*GM*] $P(\sigma) \sim N(z)\sigma^{-q}$, $1.5 < q < 2.5$. At $z < -3000$ m, $N = \sigma_t$ and $P_c(\sigma) \sim \sigma^{-3}$ for $\sigma_t < \sigma <$ ~3N. If N > 10 cpd we found $\sigma_t < N$, indicating a change in 'wave' properties.

[8] For the entire band $f < \sigma < N$ we found larger energy exceeding the second continuum $P_c(\sigma) \sim \sigma^{-2}$ at higher tidal harmonics frequencies (M_4, M_6,...). Larger energy was also found at frequencies such as $M_2 + f$, $M_4 + f$..., but only when $\sigma_t = N$. In contrast, energy at frequencies 2f, 3f (large in previous observations [*Pinkel*, 1983]), and $M_2 + 2f$, $M_2 + 3f$..., was not exceeding the σ^{-1}-continuum. We noted that this also applied for weak diurnal tidal constituents, considering odd higher tidal harmonics $M_3 \approx 2f$ at our site, so that $M_2 + 2f \approx M_5$. This difference in energy enhancement at different sequences of interaction frequencies was partially understood because fundamental tidal harmonics were more energetic than inertial motions. Clearly a strong coupling existed between near-inertial and tidal motions, but we observed differences for different stratification.

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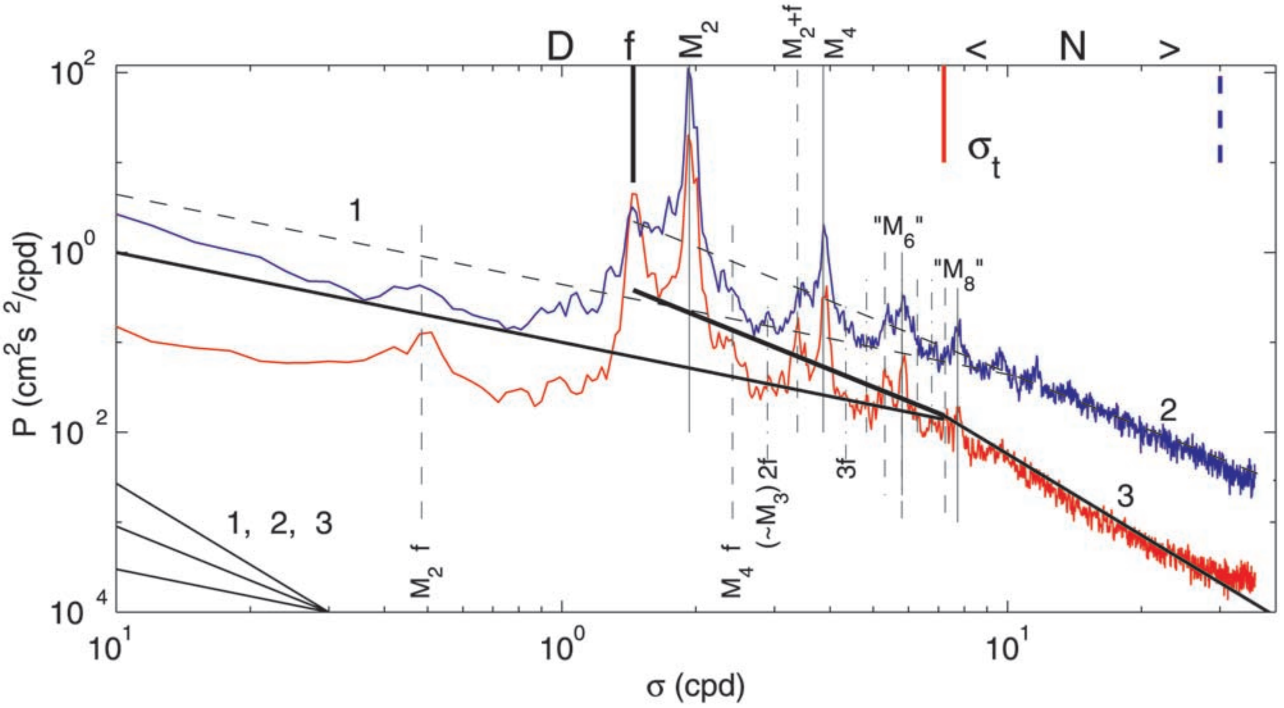

Figure 1. Kinetic energy spectra from 11 months of Aanderaa RCM-8 current meter observations at 1000 meters above the bottom in H = 4810 m water depth at 45°48′ N, 06°50′ W (red spectrum) and in H = 2450 m, 46°39′ N, 05°29′ W (blue). Spectra were moderately smoothed ($v \approx 30$ df) and not offset vertically. The difference in energy levels between the spectra corresponded to the difference in N(z), which variation was indicated between the vertical bars in the top-right corner. This happened to be the vertical distance between the sloping lines at fall-off rates σ^{-1} and σ^{-2} (solid and dashed corresponding to red and blue spectra, respectively). "D" indicated the diurnal band. The "M_6"-group contained frequencies like M_2 + 2f ($\approx M_5$; dash-dotted line), M_4 + f (dashed), M_6 (solid). "M_8" contained frequencies like M_2 + 3f (dashed), M_4 + 2f ($\approx M_7$; dash-dotted line), M_6 + f (dashed) and M_8 (solid). In the lower left corner constant slopes were indicated "−1, −2, −3" representing σ^{-1}, σ^{-2}, σ^{-3}, respectively.

[9] As in previous models [*GM*] our IWB spectra scaled with N to within a factor of 2 (Figure 2). At $\sigma < f$ and at large-scale (barotropic) tidal frequencies inside the IWB our spectra scaled with H^2, being ~N-scaling, by chance. However, new in our observations was the impact of non-linear constituents on the spectral slope. We inferred this from departures from N-scaling, such as at f and M_2 + f, to a lesser extent at M_4 − f, but not at M_4 + f and M_6 + f (Figure 1). This difference in departures was due to the downslope increase of energy at f, with most energy at the deepest mooring even without scaling, possibly due to bottom trapping [*Maas*, 2001]. We found that when energy was large at f (for small N), energy at f-interaction frequencies (e.g. M_2 + f) scaled like $\sim\sigma^{-3}$, which was typical for higher tidal harmonics at our sites. When energy at f was reduced (for large N), energy at M_n + f, n = 2, 4,. . ., scaled like $\sim\sigma^{-2}$. When smoothed strongly (Figure 2), the latter records showed overall spectral fall-off rates close to $\sim\sigma^{-2}$ for $f < \sigma < N$. For the former (relatively large inertial and tidal harmonics) we found $\sim\sigma^{-3}$, as observed for $\sigma > N$. This spectral continuation beyond the IWB reflected that local variations in buoyancy frequency (up to ~3N) could be important in broadening the IWB instead of sharp roll-off at N towards instrumental noise levels.

[10] The transition between these regimes of different smooth IWB slopes was quite abrupt (Figure 2). Apparently, removal of energy at f (for large N) resulted in a σ^{-2} spectral slope. Noting that our observed spectral slopes all had a *limited* extent in frequency, we compared them with noise models [*Schroeder*, 1991], describing *continuous* spectral fall-off rates σ^{-n}, n = 1, 2. Variance preserving σ^{-1}-noise resembled deterministic chaotic systems, on which we will not elaborate here. A white-gradient

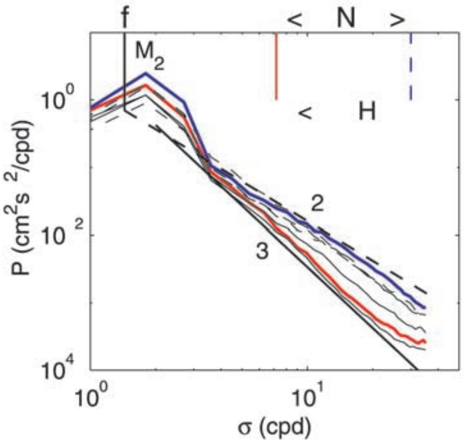

Figure 2. Strongly smoothed ($v \approx 1100$ df) IWB spectra from 500 m above the bottom in H = 2000 m, and 1000 m above the bottom on six moorings between H = 2450–4810 m. All spectra were multiplied by N(−3800 m)/N(z). The range of N(z) was indicated in the upper right corner with the change in H. The heavy red and blue solid spectra corresponded to those in Figure 1, which were typical for two extremes $P(\sigma) \sim \sigma^{-2}$ for $z > -2500$ m and $\sim\sigma^{-3}$ for $z < -3000$ m, with a transition over the small depth range in between. The straight sloping lines ("−2", "−3") represented σ^{-2} (dashed) and σ^{-3} (solid), respectively.

σ^{-2}-noise spectrum apparently described free internal waves. Previously, no model existed for the spectral shape of non-linear higher harmonics ($\sim\sigma^{-3}$), although there were studies about relative importance of such constituents in tidal context, mainly for shallow seas [*Dronkers*, 1964; *Pingree and Maddock*, 1978; *Parker*, 1991].

3. Discussion

[11] Future modeling on non-linear advection is required to explain our observations. Such modeling may be along the lines describing highly non-linear shock waves [*Platzman*, 1964], or more general and more complex non-linear flows [*Shrira*, 1981]. Time series from which our spectra were constructed showed sudden changes (shocks) in amplitude and phase of oscillatory motions, passing our moored instruments irregularly in time. Hence, future modeling should also account for internal waves propagating in groups and non-linearity may be imperative to prevent dispersion of the waves [*Thorpe*, 1999].

[12] Non-linear flows have a ratio of particle displacement speed (u) over phase speed (c) of order unity. Such flows are also characterized by a gradient Richardson number $\mathrm{Ri} \approx 1$, implying marginal dynamical stability or a subtle balance between destabilizing vertical current shear and statically stable stratification [*Phillips*, 1977; *van Haren et al.*, 1999; *D'Asaro and Lien*, 2000]. Similar Ri-value was used to propose saturation of IWB spectra [*Munk*, 1981].

[13] Assuming $u/c \sim O(1)$ we inferred horizontal wavelengths of the fundamental constituents of O(1–10 km) using $u \sim 0.05$ m s^{-1} as observed, for both inertial-tidal and tidal higher harmonics. This was the typical scale for near-inertial internal waves [*Ozmidov*, 1965; *Kunze*, 1985]. This unique length scale suggested a distorted wave shape. The observed higher inertial harmonics barely exceeding the background continuum suggested tidal-inertial harmonics to be dominated by tidal length-scales (horizontal inertial length-scales being larger).

[14] Details on the nature of the IWB were inferred from spectral differences related to changing conditions N, H, f (Figure 1). The more or less constant transition frequency (σ_t), indicating the change of continuum slope in the IWB, for entirely different N suggested a dependence of the IWB on stratification across the entire fluid domain, besides small scale variations in buoyancy frequency. Spectra also varied at a single location during shorter periods of time (not shown). We observed energy at f to vary strongly over time (by more than one decade within a week) with similar changes in energy levels at interaction frequencies. Similarly, varying width of tidal and inertial spectral bands was due to the effects of different (unresolved) spectral constituents, as was observed from nearly raw spectra (not shown). This indicated a confined size limit to internal wave groups associated with ocean mixing [*Alford and Pinkel*, 2000].

[15] We attributed the universality of strongly smoothed spectra to finestructure of free fundamental harmonic tidal and inertial waves and non-linearly forced higher tidal harmonics. In addition to the latter and about equally important we found free (small energy at f) or non-linearly forced (large energy at f) wave motions at tidal-inertial interaction frequencies. Future clarification of a relationship between such forced motions and deep-ocean mixing is needed using accepted non-linear internal wave models. Interpreting our observations as suggested previously [*Munk and Wunsch*, 1998], we conclude that tidal and inertial motions are equally important factors in large scale overturning circulation, because their interaction and shear dominate the IWB and, thereby likely, IWB-mixing. We suggest future climate change studies monitor internal wave variability using more detailed and long-term observations.

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