

The impact of rotation on the flow through inclined rectangular channels

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Abstract :

Flows in nature can turn unstable and generate waves that, depending on circumstances, retard or accelerate flows. The importance of rotation on this process is studied by pumping fluid through a rectangular container on a rotating platform, and by measuring the cross channel pressure difference as well as flow rate, given an applied pump rate. As the flow passes the (rigid-lid) container, instabilities develop, leading to inertial waves. Depending on a lateral tilt of this container, these waves may or may not be focused onto a wave attractor, which may impact the through-flow.

Résumé :

When possible, write the abstract of the paper in French (150 words maximum)

Key-words : inertial waves; wave attractor; fluid experiments

1 Introduction

When a fluid is pumped through a container with a given pressure head, is rotation going to aid the through-flow, or obstruct it? One naively expects that rotation leads to additional drag on the fluid motion. Not only there is energy needed to support the secondary circulation set-up by Ekman fluxes in horizontal boundary layers, but also does the sheared through-flow turn unstable on the pressure side of the channel, the instabilities sucking up energy. With increasing rotation rate, all of this should lead to an increased pressure drop in the down-channel direction, which, in other words, is sensed as increased friction (Mårtensson et al 2002). Remarkably though, there are cases when the pressure drop does not seem to grow with increasing rotation rate (Dobner 1959). This suggests that the answer to our question is more subtle, and that, in fact, rotation may perhaps facilitate an *increasing* flow rate. The reason for this might lie in the fate of instabilities. In a three-dimensional, non-rotating fluid, instability will bring energy to the smallest scale through a process of nonlinear interactions, where energy is degraded into heat. Rotation, however, endows a fluid with elastic properties, supporting inertial waves that quickly propagate throughout the fluid, up to the largest scale available, that of the container to which the waves are confined. Thus, when instability (turbulence) manifests itself in the form of inertial waves this may completely change the cascading of energy; it is no longer necessarily through nonlinear interaction but might as well proceed through the organization brought about by multiply-reflecting inertial waves. It thus becomes of interest to consider the fate of these inertial waves. Do they spread ergodically through the container, thus eventually again losing their energy due to internal friction and upon reflection at boundaries? Or, do the waves organize themselves, e.g. in forming eigenmodes - spatially standing waves? It appears, perhaps somewhat surprisingly, that the shape of the boundary may play an important role here, as it turns out to be important whether the container shape is breaking the symmetry imposed by rotation, or not. When it does (when one or more container side is inclined relative to the rotation axis or its perpendicular plane) wave energy appears to collect on a limit cycle, called

wave attractor (see review in Maas 2005). This is a well-defined orbit, that is determined by geometry and the ratio of wave frequency, ω , to inertial frequency, 2Ω , where Ω is the frame's rotation rate (see Fig. 1). It is on approach of this wave attractor that the inertial waves mix fluid that is stratified in angular momentum, which, as a result sets up a (cyclonic) mean flow (Maas 2001) that might actually amplify also the through flow.

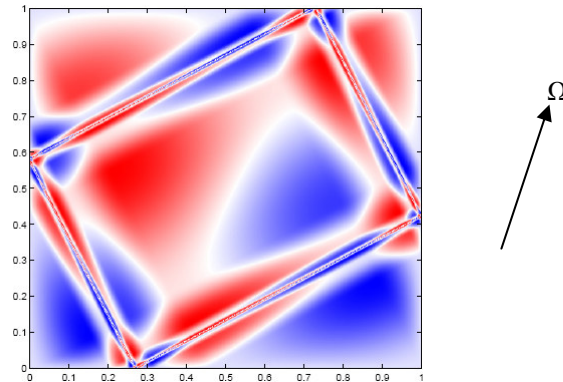


Fig 1. Example of a wave attractor, the rectangular shape visible in the streamfunction field, of focused inertial waves that occurs in a tilted rectangular channel. The tilted rotation axis indicates we are here looking at in the tilted reference frame.

The aim of this contribution is to present a number of fluid experiments that were performed to check this hypothesis. Preliminary results will be reported here. A more comprehensive discussion awaits further analysis.

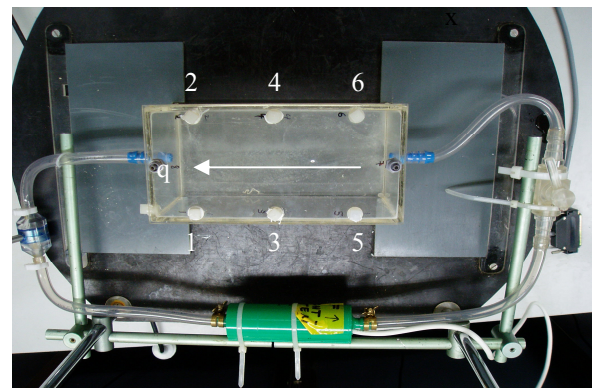
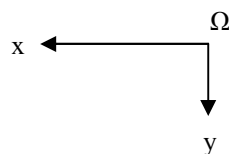


Fig. 2 : topview of container, pump (green) and propellor vane, all on turn table

2 Experiments

2.1 Experimental set-up

Fig. 2 shows the (fully enclosed) rectangular container ($10 \times 10 \times 20 \text{ cm}^3$) as viewed from the top. It is given in its horizontal position. It is filled with homogeneous water. Via connecting tubes of 0.5 cm width, this is pumped through the tank by the green pump, in the direction

indicated by the arrow, at a rate q [l/min]. Here, $q = .4572 V_q + .0639$, where V_q (varying from 0.25-7.0V) is the applied pump voltage. Pressure differences are measured between pressure holes, indicated by numbers 1-6. Flow rate is measured by a propeller vane (Höntzsch, Fa 40/10), the device indicated on the left.

The platform is set into rotation by a KMF WD251 electromotor (Electro ABI) at an angular velocity $\pm\Omega$ [rad/s]. Here, $\Omega = 1.004V_\Omega - 0.2942$, where V_Ω (varying from 0.25-7.0V) is the voltage applied to rotate the turntable.

Differential pressure, e.g. between pressure holes 5 and 6, is measured with the LPM5480 sensor (Druck), which can measure up to ± 200 Pa, with a precision of ± 0.08 Pa.

Automatic control of applied pump and rotation rate over prescribed measurement and adjustment periods, T_m and T_a respectively, as well as measurement (at 4 Hz rate) of differential pressure and flow rate, is made by means of software package LabView. Typically a scan is made over the full indicated pump and rotation voltage ranges. We usually chose $T_a = 50$ s and $T_m = 150$ s, meaning that at fixed pump rate, the rotation rate is kept fixed over 200s, of which the final 150 s are used to measure differential pressure and flow speed. Then, the rotation rate is increased by an increment (0.2 or 0.25V) and this is repeated until maximum rotation rate is obtained. Subsequently, the flow rate is increased (in similar increments), and the rotation rate is incrementally decreased until the lowest rotation rate is reached again, where the flow rate is increased again, and this whole procedure is repeated a number of times.

2.2 Results

When the container is horizontal, we measure a time-average pressure difference as given in figure 3. The table is rotating anticlockwise. We thus expect that the Coriolis force deflects the through flow to the right, so that (see figure 2) the pressure at hole 6 should be increased relative to that at hole 5. As the pressure at hole 5 minus that at 6 is presented in figure 3, this is indeed observed in the triangular area to the right of the straight solid line; that is, for relatively strong rotation. In this region, we see that the pressure difference is nearly constant along hyperbola (curved solid lines), betraying the geostrophic equilibrium which the lateral pressure gradient apparently satisfies: $dp/dy = 2 \Omega \rho u$. Here ρ is the density of the fluid. The straight line itself is close to a line for which the Rossby number, $Ro = u/2 \Omega L$, is constant, where L is a length scale to be discussed below. The part above the straight line consists of a positive 'ridge' (red), and an anomalous negative pressure difference, perhaps a brief return to geostrophy. The interruption of the latter region at $u = 4$ mm/s is not an artefact. It is found repeatedly in this region and betrays the presence of multiple equilibria, which are reached depending on whether the rotation rate is incrementally increased or decreased. Physically, it is caused by the presence of an eddy. After spin-up to a new rotation rate, the container is usually filled with two vortices: one, big cyclonic one, that fills almost the entire container, and a smaller anticyclonic vortex, which sits near the entrance of the container, on the low-pressure side (in this case, below hole 5, on the left of the through-flow), which seems to cause the anomaly.

The aforementioned length scale L is usually taken as a scale associated with the geometry. The frequency spectrum of the differential pressure signal, however, suggest this scale to be much smaller though. The spectrum shows significant variance usually only below 2Ω , indicative of inertial waves. For low pump voltages this is present in particular near $\omega = \Omega$, probably due to a slight misalignment of the table's rotation axis with gravity. For higher pump rates, this spreads out. However, below the straight line, the energetic peaks all stay below the

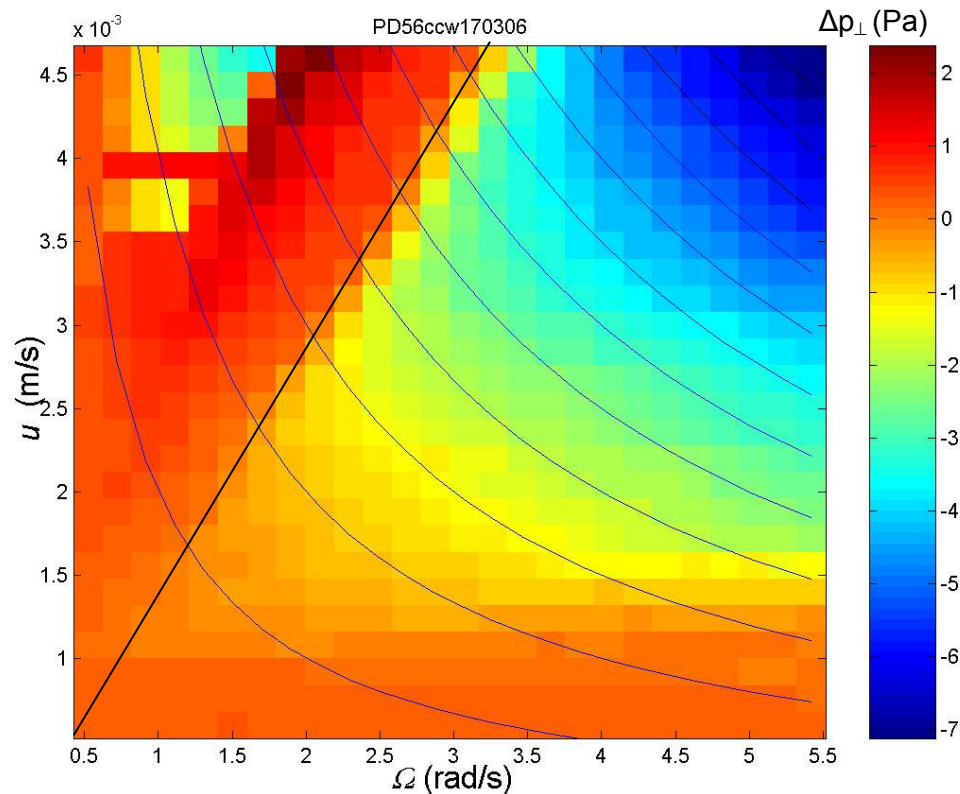


Fig. 3 Observed time-averaged differential pressure between holes 5 and 6 [Pa], measured by varying rotation rate Ω and pump rate. The latter is here expressed as the through flow $u=q/A$, obtained from pump rate q divided by cross-sectional area A , that would result in the absence of rotation, and which is linearly related to the applied pump voltage.

inertial frequency, 2Ω . Only above the straight line, the energy no longer seems to be confined to the inertial wave frequency band and energy is also found in bands above 2Ω . This strongly suggests that physically the straight line should be interpreted as $Ro=1$, which separates a strongly-rotating 'hyperbolic' area ($Ro<1$), where the variance ('turbulence') is taking shape in the form of a 'sea of inertial waves' (Tritton 1978), from a weakly-rotating 'elliptic' area ($Ro>1$), where variance appears as a spectrally broad-band phenomenon, as turbulence in non-rotating fluids. However, this interpretation suggests the length scale L present in Ro , to be set by the diameter of the feeding tube or perhaps by the Ekman boundary layer thickness.

Concerning the property of most concern here, the through flow, let us look at observed flows obtained from (1) the horizontal rotating tank compared to the non-rotating tank (Fig. 4a), and (2) the tilted rotating tank compared to the horizontal (non-tilted) tank (Fig. 4b,c). In the latter case the tank is tilted 10 and 20 degrees respectively.

Surprisingly, even in the non-tilted case (Fig. 4a), rotation seems to enhance the through-flow, particularly in the rapidly rotating triangular ($Ro<1$) region of the parameter plane spanned by the rotation rate (x-axis) and pump rate (y-axis), both expressed in terms of Volt.

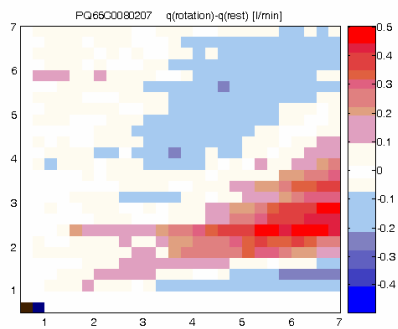
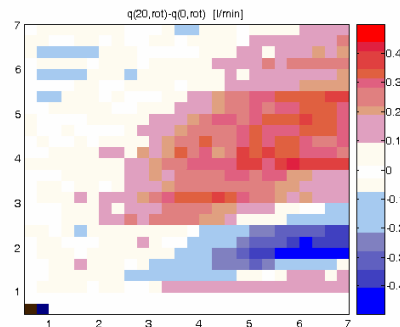
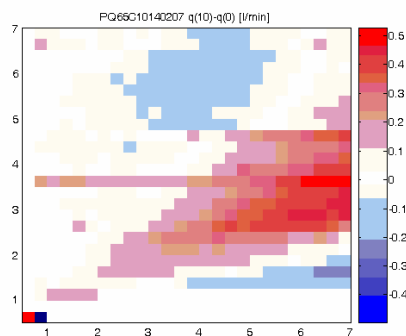


Fig. 4 (a) $q(\text{rot})-q(\text{rest})$: measured flow rate [l/min] in rotating system, relative to flow rate without rotation for horizontal (flat) tank. (b) $q(10,\text{rot})-q(0,\text{rot})$: measured flow rate in tilted, rotating frame (10 degrees tilt with horizontal) compared to that in flat, rotating frame. (c) Same as b, but for a 20 degrees tilt.

The colored spot near the origin of the graphs (0.5,0.5) should be ignored.



For $Ro < 1$, a further enhancement is found over that already present when the tank is tilted (Figs. 4b,c), with the exception of the region where the pump speed is small for which the through flow is instead obstructed.

3 Conclusions

The results presented raise a number of surprising issues. The spectral distribution of pressure perturbations tells us that the length scale that apparently sets the Rossby number, separating the 3D turbulent regime from the inertial wave regime, is much smaller than the length scale of the container. In the inertial wave regime, the average cross-channel pressure difference is found to be in near-geostrophic balance (for $Ro \ll 1$). The perturbations in this area take the form of inertial waves. For low pump rates, these are concentrated around certain spectral peaks. For higher pump rates, the spectrum is broader (albeit always staying well below the inertial frequency cut-off 2Ω). While the naïve expectation, that rotation will inhibit the through flow, is in general found to be true for $Ro > 1$, this is surprisingly not true in the inertial wave regime ($Ro < 1$). Apart from the enhancement already found when the container is still in its flat position (which is not understood well), there is a further enhancement observed when the tank is tilted, especially when the pump rate is relatively large (and the spectrum rich). We speculate that the latter enhancement is brought about by the organization of inertial waves. For some of these frequencies they will be focused onto a simply-shaped wave attractor, where, as was found in an earlier study (Maas 2001), this mixes background angular momentum leading to the generation of a cyclonic mean flow. It is speculated that for the larger flow rates, the broad-band spectrum encompasses those frequencies for which a wave attractor can be obtained and thus a mean and through flow is driven, while for the low pump rates the few frequencies observed are likely not inside an attractor window, so that they only lead to extra damping, reducing the through-flow. Further analysis is needed to substantiate these speculations.

4 Acknowledgements

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