# Topologie en meetkunde – Syllabus

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## Practical information

**Lectures:**
Tuesdays from 10:00 to 11:45 at Ruppert 111
Thursdays from 13:15 to 15:00 at HFG 611AB

**Exercises:**
Tuesdays from 12:00 to 12:45 at Ruppert 111
Thursdays from 15:15 to 18:00 at BBG 161

**Warning:** This schedule is only valid for the first four weeks of class. After this, there is a scheduling conflict with a master course I have to give on Thursday afternoon. Then we will switch to the following schedule:
**Lectures:**
Tuesdays from 9:45 to 12:45 at Ruppert 111
Thursdays from 13:00 to 13:45 at HFG 611AB
**Exercises:**
Thursdays from 14:00 to 15:00 at HFG 611 AB
Thursdays from 15:15 to 18:00 at BBG 161

**Homework:** There will be two-weekly hand-in homework, which will count for 20% of the total grade. Additionally there will be lots of exercises for the werkcolleges.

**Tests and exams:** On Thursday, March 15, we will have a midterm, which will count for further 30% of the grade. The final will count for 50%.

If you do the re-take exam, the original grades of midterm and final will be discarded and the re-take counts 80%.

## Content

We will start with a discussion, what topology is about. Can we classify spaces up to homeomorphism? Can we apply topology in analysis or algebra? This will quickly lead to the notion of a homotopy, a central concept for our course.

Using this notion, we will define the fundamental group of a space. Our first aim is to calculate it for $S^1$. This will have several direct applications, like the Brouwer fixed point theorem.

We will describe a way to calculate the fundamental group of cell complexes, i.e. spaces build from spheres and disks (which includes most manifolds). The central tool is the van Kampen theorem that allows us to compute the fundamental group of a union of two spaces.

In the middle of the course, we will prove our first classification results. More precisely, we will classify closed one-dimensional manifolds (which is easy) and closed two-dimensional manifolds.

After this, we will introduce the theory of covering spaces, which is deeply intertwined with the theory of fundamental groups.

In the last weeks we will introduce the concept of homology. This can be used to solve higher-dimensional problems the fundamental group cannot deal with. Moreover it provides a way to show the topological invariance of the Euler characteristic, a number that first arose in Eulers's theorem on polyhedra. This opens the road to further applications.

## Background material

I will not follow any book directly and thus I really recommend to come to the lectures. The following are some recommended references, especially the first two:


- Lee, John M., *Introduction to Topological Manifolds*, Springer, GTM 202, 2nd ed. 2011 edition
- Hatcher, Allen, *Algebraic Topology*.
- Massey, W. *A Basic Course in Algebraic Topology*, Springer GTM 127, 1991.
- Spanier, Edwin, *Algebraic Topology*, Springer, Corr. 3rd edition, 1994
- Zeeman, *The Classification theorem for Surfaces*