Competition between electron-solid and electron-liquid phases in quantum Hall effect systems

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Outline:

• Integral quantum Hall effect (IQHE): a single-particle picture

• Model of electrons restricted to a single level:
  (a) electron-solid phases
  (b) quantum-liquid phases

• Competition between solid and liquid phases - reentrant IQHE

• Phase transition and experiments
Quantum Hall Effect
Experimental data at $T = 100mK$

- From classical calculations, we expect:

$$\rho_{xy} \propto B$$

- One measures

$$\rho_{xy} = \frac{1}{\nu} \frac{h}{e^2} \quad \nu = \frac{n_e}{n_\phi}$$

**Integer Quantum Hall Effect:** $\nu$ integer

**Fractional Quantum Hall Effect:** $\nu = \frac{k}{(2pk \pm 1)}$, $p, k \in \mathbb{Z}$
1. Introduction

Quantum Hall system = 2D electron gas in a perpendicular magnetic field

\[ R_{xx} = \frac{V_{xx}}{I} \]  
(longitudinal resistance)

\[ R_{xy} = \frac{V_{xy}}{I} \]  
(Hall resistance)

Experimental data at \( T = 100 \text{mK} \)
Theoretical Description

\[ \hat{H} = \sum_i \left[ \frac{\left( \vec{p}_i + e\vec{A}_i \right)^2}{2m} + V_{\text{ext}}(\vec{r}_i) \right] + \hat{H}_{\text{el-el}} \]

**1-particle Hamiltonian**

*Coulomb interactions*

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**Single-particle Hamiltonian (no spin):**

\[ \hat{H}_0 = \frac{(\vec{p} + e\vec{A})^2}{2m} + V_{\text{ext}}(\vec{r}) \]

**Classical solution:**
*cyclotron motion*

**Quantum solution:**
*harmonic oscillator*

\[ \varepsilon_n = \hbar \omega_c \]

\[ n=0 \]

\[ n=1 \]

\[ n=2 \]

(Landau levels)

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- Density of states per LL: \( n_B = 1/2\pi l_B^2 \)
- Filling factor: \( \nu = n_{el}/n_B \)
- Magnetic length: \( l_B^2 = \hbar/eB \)
2. When Coulomb becomes essential: 
\[ (V_C \ll \hbar e B/m) \]

\[ \nu = n \quad \nu < 1 \quad \bar{\nu} < 1 \]

- Hamiltonian in the \( n \)th LL


\[ \hat{H}_n = \frac{1}{2} \sum_{\vec{q}} v_n(q) \tilde{\rho}(-\vec{q}) \tilde{\rho}(\vec{q}) \]

projected density: \( \langle \rho(\vec{q}) \rangle_n = F_n(q) \tilde{\rho}(\vec{q}) \)
effective potential in the \( n \)th LL:

\[ v_n(q) = \frac{2\pi e^2}{q \varepsilon_n(q)} [F_n(q)]^2, \quad v_n(r) = \frac{\tilde{v}(r/R_c)}{\sqrt{2n + 1}} \]

- Interactions lift degeneracy

(\( d \): average separation between particles)

\( (i) \ d \gg R_c, \quad (ii) \ d \ll R_c, \quad (iii) \ d \sim R_c \)
A. Wigner Crystal (WC): $d \gg R_c$

- System is quasi-classical:

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\begin{align*}
R_c \\
d
\end{align*}
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Overlap vanishes (of order $R_c / d$):
- Exchange corrections
- Correlation corrections
- To classical result

Classical result: *electron crystallization to minimize Coulomb interaction*

B. Charge Density Waves (CDW): $d \ll R_c$

- Limit does not exist in lowest LL, $R_c = l_B$ is the smallest length scale

- Clustering leads to CDW formation:

1) $d$ $d$ $\text{énergie} \approx 2u$
2) $d'$ $2d - d' > 2R_c$ $\text{énergie} \approx u$
C. Quantum Limit \((d \sim R_c), \text{ FQHE}\)

- Exchange and correlation effects not negligible
  
  mean-field approximation fails

- Experiments suggest reminiscence of the system to IQHE at \(\nu = p/(2ps \pm 1)\):
  
  existence of a gap, localization of some “quasi-particle”

- Composite fermions(CF):
  
  2\(s\) vortices attached to each electron

Reduced magnetic field: \(B^* = B - 2s\Phi_0n\)

Weakly coupled CFs: \(V_C = e^2/(\epsilon l_B^*) \propto \sqrt{B^*}\)

Composite fermion filling factor:

\[
\nu^* = \frac{n_{el}}{|B^*|/\phi_0}
\]
Physical properties of the Laughlin liquid

- Composite fermions: electrons bind to partner
  \( \text{partner} = \text{“pseudo-vortex”} \)

- \( \text{CF’s with charge } e^* = (1 - c^2)e = e/(2ps + 1) \text{ fill LL’s} \)

  CF charge leads to renormalized magnetic length/filling factor:
  \[
  l_B^* = \hbar / e^* B, \quad \nu^* = 2\pi l_B^2 n_{el}
  \]
  \( \nu^* = p \): \( p \) completely filled LL’s

- \( \text{FQHE of electrons} = \text{IQHE of composite fermions} \)

At \( \bar{\nu} \neq 1/(2s + 1) \): excitations of CFs/holes at \( p = 1 \)

\( \Rightarrow \) energy of quantum liquid:

\[
E^{\text{liq}}(n; s, \bar{\nu}) = E^L(n; s) + [\bar{\nu}(2s + 1) - 1]\Delta^n(s),
\]

\( \Delta^n(s) \): energy of quasi-particle/-hole

analytical calculation: Hamiltonian theory of Murthy/Shankar
4. Results


4.a) Results for $n = 1$

**Effect of impurities:**

(dashed lines)
- Weak/strong pinning of the bubble phase

\[
\delta E^{\text{weak}}(M, \nu) = - \frac{(2\pi)^{3/2} V_0^2/\xi^2}{\sqrt{M_0^3/2} e^2/\epsilon l_B},
\]

\[
\delta E^{\text{strong}}(M = 1) = -V_0,
\]

$V_0$ : energy of impurity potential (Gaussian),
$\xi$ : correlation length of impurity potential
- no effect over quantum liquids incompressible

single-particle localization:
not important for reentrant phenomenon
4.b) Results for $n = 2$

5.) Phase Transitions


5.a) Solid-Liquid Phase Transition

- **first order phase transition**
  (discontinuity in the first derivative of the energy)

- Possible experimental verification:

  *hysteresis* in the Hall resistance at the transition, due to supercooling at the transition point (first order)
5.b) **Solid-Solid Phase Transition** $M \leftrightarrow (M + 1)$

- **first order phase transition**
- **convex envelope**
  \[ \Rightarrow \text{mixed phase} \]

**Microwave irradiation experiments:**

- **resonance in the longitudinal conductivity due to pinning mode of the crystal** (figure: WC around $\nu = 1$)

  \[ \frac{\omega_p}{\omega_C} \sim M^{-1/4} \nu^{-3/4} \]

  (weak field),

  \[ \frac{\omega_p}{\omega_C} \sim M^{-1/2} \nu^{-3/2} \]

  (strong field)

- **expected response within the mixed phase:**
  **double resonance in the longitudinal conductivity**
  (response of **bubbles** with $M$ and $M + 1$ electrons)
Conclusions

**QHE:**

*ground state is separated from excited states by an energy gap*

+ 

*insulating behavior of elementary excitations*

- **IQHE:** *insulating behavior of electrons (holes) due to*
  (a) *localization of electrons (holes) at small $\tilde{\nu}$*
  *single-particle effect*

  (b) *formation of a triangular CDW pinned by impurities*
  *(bubble phase, electron-solid)*
  
  $N$ *particles correlation effects*

- **FQHE (quantum liquid):** *insulating behavior of quasi-particles/holes (CFs)*
  
  $N$ *particles correlation effects*

- **For $n \geq 1$:** *alternation of electron-solid and - liquid phases as a function of $\tilde{\nu}$ (first order phase transitions )*
  
  $\rightarrow$ *reentrant IQHE*
**The Quantum Hall Effect**

**Brief historical:**
1980 : discovery of the **IQHE**
(K. v. Klitzing, N. P. 1985)
1983 : discovery of the **FQHE**
(H. Störmer, D. Tsui, N. P. 1998); theory : trial wavefunction
(R. Laughlin, N. P. 1998)
1989 : composite fermions (J. Jain)
2002 : new class of FQHE (4/11, W. Pan, et al.)
2002 : reentrant **IQHE**
(J. Eisenstein, et al.)