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THE CHARACTERISATION OF STRUCTURE: DEFINITION VERSUS AXIOMATISATION

ABSTRACT

Crucial to structural realism is the Central Claim that *entity* B *is or has structure* $\mathfrak{S}$. We argue that neither the set-theoretical nor the category-theoretical conceptions of structure clarify the Claim in a way that serves the needs of structural realism. One of these needs is to have a viable account of reference, which almost any variety of realism needs. There is also a view of structure that can adopt *both* set-theoretical and category-theoretical conceptions of structure; this is the view that adopts B.C. van Fraassen's extension of Nelson Goodman's concept of *representation-as* from art to science. Yet the ensuing fountain of perspectives is a move away from realism, structural realism included. We then suggest that a new theory of structure is needed, one that takes the word 'structure' to express a primitive fundamental concept; the concept of structure should be axiomatised rather than defined in terms of *other* concepts. We sketch how such a theory can clarify the Central Claim in a manner that serves a descriptivist account of reference, and thereby structural realism.

1. PREAMBLE

After having discarded a number of characterisations of *scientific realism*, i.e. realism in the philosophy of science, in his classic *The Scientific Image*, Van Fraassen provided the following minimal characterisation:

> *Science aims to give us, in its theories, a literally true story of what the world is like; and acceptance of a scientific theory involves the belief that it is true.* This is the correct statement of scientific realism.[1]

The first and foremost distinction in the variety of scientific realism called *Structural Realism* (StrR[2]) is the one between *epistemic* StrR (all that science provides is knowledge of the structure of the physical world) and *ontic* StrR (all there

1 Fraassen [1980], p. 8.

2 We shall ambiguously use abbreviation 'StrR' also for 'a structural realist'.

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is in the physical world is structure).[3] Yet when we take heed of the fact that knowledge implies truth, and take truth to imply ontological adequacy, the gap between epistemic and ontic StrR narrows quickly.[4] Whether epistemic or ontic, StrR needs the concept of structure. For StR, a literally true story of what the world is like will involve *structures* only, and therefore we need to know what *structures* are.

What makes StrR stand apart from other varieties of realism is that it is supposed to be more cautious, more modest, in its realist claims, in order not to fall prey to the pessimistic meta-induction over the history of science. The premise of this inductive argument is a sequence of past scientific theories that now have all been rejected. They are plausibly false. So if you think that our *currently* accepted scientific theories are not false but true, then what are you, stupid? But StrR must remain sufficiently substantive to provide a basis for the no-miracle argument: the only explanation for the fact that in sequences of successively accepted theories each theory is empirically and technologically at least as, and generically more, successful than its predecessor is that they latch on to the structure of the physical world better and better.

We take a closer look at the set-theoretical (Section 2) and the category-theoretical (Section 3) conceptions of structure and we find them inadequate to serve the needs of StrR, specifically the need to have a literal description of the referents to which terms in scientific theories refer. Then we explore the possibility of retaining both conceptions of structure by adopting Van Fraassen's concept of *representation-as* – as opposed to *representation-of* –, which can be marshalled to evade the objections leveled against set-theoretical and category-theoretical conceptions of structure when interpreted literally. The price to pay seems however too high for realism, because this adoption introduces a perspective-dependency that stands opposed to the very idea of realism (Section 4). Since by then all available options seem exhausted, we argue for the case that StrR needs a new theory of structure, that takes the concept of structure as fundamental, that is, as a primitive concept that ought to be axiomatised rather than defined (Section 5). Such a theory will serve the needs of StrR, or so we argue.

Throughout this paper we take 'structure' to mean *mathematical structure*, because in science these structures are used in science to model, to represent, to describe, to explain, to understand, etc. the world.

2. Set-Theoretical Characterisation

Although the rigorous set-theoretical characterisation of structure is well-known and widespread, easy to explain and easy to illustrate, its rigorous definition in the formal language $\mathcal{L}_\in$ of pure set-theory (ZFC, say) is rather commanding. Bourbaki was the first to provide a definition of structure of extreme generality, within the

3 Due to J. Ladyman [1998]. H. Lyre's [2009] review in this volume and Ladyman's encyclopedia article [2009] draw more distinctions within StrR.

4 As I have argued elsewhere; see Muller [2009], Section 1.

framework of his own set-theory, in Chapter IV of his *Theory of Sets* (1949).[5] We call the sets in the domain of discourse **V** of ZFC – which harbours pure sets and nothing but pure sets – answering to this definition *set-structures*. Bourbaki propounded the view that *mathematics is the study of structure* and set out, by means of set-structures, to create Law & Order in the exuberant proliferation and progressive splintering of 20th-century mathematics, which had turned the discipline into a Tower of Babel. Patrick Suppes famously came to promulgate the use of set-structures in the philosophy of science, notably to characterise scientific theories. Suppes' Slogan: to axiomatise a theory is to define a set-theoretical predicate.[6]

Informally, a set-structure is a polytuple of the following form:

$$\Big\langle \textit{base sets, subset families, relations, functions, operations, constants} \Big\rangle\,, \qquad (1)$$

or more precisely:

$$\Big\langle\, B_1,\, \ldots,\, B_b,\, \mathfrak{F}_1,\, \ldots,\, \mathfrak{F}_s,\, R_1,\, \ldots,\, R_r,\, F_1,\, \ldots, F_f,\, O_1,\, \ldots,\, O_o,\, C \,\Big\rangle\,, \qquad (2)$$

where $b \in \mathbb{N}^+$ (positive natural number) and $r, s, f, o \in \mathbb{N}$ (natural numbers). Set-structure (2) has: b base sets; s subset families, each one of any of the base sets; r relations between the members of base sets or those of the subset families; f functions each of whose domain and co-domain is one of the base sets or subset families, or is some Cartesian product-set of these sets; o operations each of which has as a domain in a Cartesian product-set of generally one of the base sets and the same base set as its co-domain; and a set C (of Constants) which contains members of the base sets or subset families that play a special rôle (as zero, one, top, bottom, , singularity, etc.). Set C is however often omitted for the sake of brevity.

Let us consider an example from physics, that we shall take as the leading example in this paper: a Helium atom (He) in a uniform magnetic field ($\mathbf{B}_0 : \mathbb{R}^3 \to \mathbb{R}^3$, $\langle x, y, z\rangle \mapsto \langle 0, 0, B_0\rangle$). A quantum-mechanical structure used to describe this composite physical system (which we shall henceforth abbreviate by HeB) is of the following type:

$$\mathfrak{S}(\mathsf{HeB}) \;\equiv\; \Big\langle\, L^2(\mathbb{R}^3),\; H(\mathbf{B}_0),\; \psi,\; \mathrm{Pr}_t \,\Big\rangle\,, \qquad (3)$$

where the four occupants are as follows.[7] The base set is the Hilbert-space of square-integrable complex functions of three real variables ('complex wave func-

5 Bourbaki [1968]. For an attempt at an accessible exposition of Bourbaki's definition, as well as a brief description of Bourbaki's programme, see Muller [1998], pp. 106–115; for a more smooth and accessible definition of structure, see Da Costa & Chuaqui [1988], who speak of 'Suppes-predicates', in honor of Patrick Suppes [1960], [1967].

6 See Suppes [1960], [2002]; see Da Costa & French [2000] for a review of developments in this area over the past 30 years. Lyre [2009] also employs set-structures.

7 The *occupants* of a polytuple like (3) are the four items in there: $L^2(\mathbb{R}^3)$ is the *first occupant*, etc. They are not the *members* of the set (3), but iterated members. See Muller [1998], p. 24, for a rigorous definition of occupant.

tions').[8] Linear function $H(\mathbf{B}_0) : \mathcal{D} \to L^2(\mathbb{R}^3)$ is the Hamiltonian, the operator that represents the physical magnitude of energy; its domain $\mathcal{D}$ lies dense in $L^2(\mathbb{R}^3)$. (The magnetic field $\mathbf{B}_0$ and other relevant physical magnitudes, such as the linear momentum of the He-atom, are present in $H(\mathbf{B}_0)$ but suppressed notation-wise.) Function

$$\psi : \mathbb{R} \to L^2(\mathbb{R}^3),\ t \mapsto \psi(t) \tag{4}$$

is the solution of the Schrödinger-equation; it is continuous iff no measurements are performed. Finally, function $\text{Pr}_t : \Delta \mapsto \text{Pr}_t(\Delta)$ is the Born probability measure, one for every $t \in \mathbb{R}$, that gives the probability of finding a value in $\Delta \subset \mathbb{R}$ for the energy of the He-atom when measured and when the state of the He-atom is $\psi(t)$:

$$\text{Pr}_t(\Delta) = \langle \psi(t) | P^H(\Delta) | \psi(t) \rangle , \tag{5}$$

where $P^H(\Delta)$ is the relevant member of the spectral family of projectors of $H(\mathbf{B}_0)$. Mathematically there is more going on, which we have suppressed in the deceptively simple notation (3).

First of all, Hilbert-space $L^2(\mathbb{R}^3)$ is itself a structure:

$$\left\langle L^2(\mathbb{R}^3),\ +,\ \cdot,\ \langle\cdot|\cdot\rangle,\ \|\cdot\|,\ \mathit{0} \right\rangle , \tag{6}$$

where, first,

$$+ : L^2(\mathbb{R}^3) \times L^2(\mathbb{R}^3) \to L^2(\mathbb{R}^3) \tag{7}$$

is the operation of addition on the complex wave functions, leading to an Abelian additive group; secondly,

$$\cdot : \mathbb{C} \times L^2(\mathbb{R}^3) \to L^2(\mathbb{R}^3) \tag{8}$$

is the scalar multiplication of wave functions, which interacts distributively with addition, leading to a complex vector space; thirdly, mapping

$$\langle\cdot|\cdot\rangle : L^2(\mathbb{R}^3) \times L^2(\mathbb{R}^3) \to \mathbb{C} \tag{9}$$

is the inner-product; fourthly,

$$\|\cdot\| : L^2(\mathbb{R}^3) \to \mathbb{R}^+ \tag{10}$$

is the norm, generated by the inner-product, leading both to metrical and topological structure; and sixthly, *0* is the zero-function, the neutral element of the additive group. In turn, the reals ($\mathbb{R}$) also form some algebraic structure:

$$\left\langle \mathbb{R},\ <,\ +,\ \times,\ \{0,1\} \right\rangle , \tag{11}$$

8 More rigorously one has to identify members of $L^2(\mathbb{R}^3)$ which are equal almost everywhere, thus giving rise to a set of Lebesgue-equivalence classes of complex wave functions, denoted as $\mathcal{L}^2(\mathbb{R}^3)$.

and the complex numbers ($\mathbb{C}$) too. The natural numbers ($\mathbb{N}$) are always needed and they also form a particular structure:

$$\big\langle\, \mathbb{N},\ S,\ 0 \,\big\rangle\,, \tag{12}$$

where $S : \mathbb{N} \to \mathbb{N}$ is the successor-function (all arithmetical operations can be defined inductively in terms of S).

Structure $\mathfrak{S}(\mathsf{HeB})$ (3) also harbours a Kolmogorovian probability structure:

$$\big\langle\, B(\mathbb{R}),\ [0,1],\ \mathrm{Pr}_t \,\big\rangle\,, \tag{13}$$

where the probability function $\mathrm{Pr}_t : B(\mathbb{R}) \to [0,1]$ (5) is a normed measure on the Borel sets $B(\mathbb{R})$, which in turn is also a structure, a Boolean σ-lattice:

$$\big\langle\, B(\mathbb{R}),\ \subseteq,\ \cup,\ \cap,\ \backslash,\ \varnothing \,\big\rangle\,. \tag{14}$$

Thus the wave-mechanical structure $\mathfrak{S}(\mathsf{HeB})$ is in full splendour (permuting the order of the occupants):

$$\Big\langle\, \mathbb{N},\ S,\ 0;\ \ \mathbb{R},\ <,\ \ +,\ \times,\ \{0,1\};\ \ \mathbb{C},\ +,\ \times,\ \{0,1\};$$
$$L^2(\mathbb{R}^3),\ +,\ \cdot,\ \langle\cdot|\cdot\rangle,\ \|\cdot\|,\ \mathit{0}\,;\ H(\mathbf{B}_0);\ \ \psi;\ \ B(\mathbb{R}),\ \subseteq,\ \cup,\ \cap,\ \backslash,\ \varnothing,\ [0,1],\ \mathrm{Pr} \Big\rangle \tag{15}$$

Since operations are a particular kind of functions and functions are a particular kind of relations, and relations between members of two arbitrary sets, D and R say, are subsets of their Cartesian product-set $D \times R$, and thus members of the power-set of $D \times R$, and since the Cartesian product-set $D \times R$ is a member of the 3-times iterated power-set of the union-set $D \cup R$:

$$D \times R \ \in\ \wp^3(D \cup R)\,, \tag{16}$$

one sees that starting from the infinite number sets $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$, the structure $\mathfrak{S}(\mathsf{HeB})$ (3) lives at a level in the cumulative hierarchy of sets $\mathbf{V}$ that is a considerable number of applications of the power-set operation higher than were $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ live. We call to mind Cantor's Power Theorem, according to which the power-set $\wp(D)$ is strictly larger in cardinality than set D, to see that structure $\mathfrak{S}(\mathsf{HeB})$ (15) harbours various sets much larger than the cardinality of the continuum ($\mathbb{R}$).

The standard route for set-theoreticians is to take the finite von Neumann ordinals as the natural numbers ($\mathbb{N} \equiv \omega$); then there is a unique set of natural numbers. The structuralist route (Bourbaki's) is to define a 'natural number structure' by means of a set-theoretical structure-predicate (a Suppes-predicate), as a 'Peano structure' (12), or as a 'Dedekind structure', or as a 'Frege structure'; in all these cases there is no longer a unique 'natural number structure' but an absolute infinity of such structures (as many as there are sets in the domain of discourse $\mathbf{V}$ of

ZFC).[9] The same two routes are available for the other number structures (integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$; reals, $\mathbb{R}$; complex numbers, $\mathbb{C}$): they can be constructed in **V** by set-theoretical means from $\mathbb{N} = \omega$ so as to end up with unique number structures (rationals as ordered pairs of integers, reals as Bolzano-Cauchy sequences of rationals or as Dedekind-cuts, complex numbers as ordered pairs of reals); or they can be defined by structure-predicates (see footnote 9). When one follows the first, constructive-like route, then

$$\mathbb{Z} \in \wp^4(\mathbb{N}), \ \ \mathbb{Q} \in \wp^7(\mathbb{N}), \ \ \mathbb{R} \in \wp^8(\mathbb{N}), \ \ \mathbb{C} \in \wp^{10}(\mathbb{N}) \,. \tag{17}$$

Then for the set of wave functions from $\mathfrak{S}(\mathsf{HeB})$ (3) we have

$$L(\mathbb{R}^3) \ \in \ \wp^3\big(\wp^3(\wp^8\mathbb{N} \cup \wp^9\mathbb{N}) \ \cup \ \wp^9\mathbb{N}\big) \,, \tag{18}$$

and for the Hamiltonian:

$$H(\mathbf{B}_0) \ \in \ \wp^6\big(\wp^3(\wp^8\mathbb{N} \cup \wp^9\mathbb{N}) \ \cup \ \wp^9\mathbb{N}\big) \,, \tag{19}$$

and the wave function:

$$\psi \in \ \wp^3\Big(\wp^9\mathbb{N} \ \cup \ \wp^3\big(\wp^3(\wp^8\mathbb{N} \cup \wp^9\mathbb{N}) \ \cup \ \wp^9\mathbb{N}\big)\Big) \,, \tag{20}$$

and the probability measure:

$$\mathrm{Pr}_t \ \in \ \wp^3\big(\wp^8\mathbb{N} \cup \wp^9\mathbb{N}\big) \,. \tag{21}$$

For the ordered quadruple $\mathfrak{S}(\mathsf{HeB})$ (3) we then obtain:

$$\mathfrak{S}(\mathsf{HeB}) \ \in \ \wp^3\Big(L^2(\mathbb{R}^3) \ \cup \ \wp^3\big(H \ \cup \ \wp^3(\psi \cup \mathrm{Pr}_t)\big)\Big) \,. \tag{22}$$

But properly construed, as the ordered 28-tuple (15), structure $\mathfrak{S}(\mathsf{HeB})$ is a member of a far more involved set-structure. With (18), (19), (20), (21) and (22), one can work out exactly how many iterations of the power-set we are, with $\mathfrak{S}(\mathsf{HeB})$, beyond the first infinite ordinal level (ω) in the cumulative hierarchy, which we leave as an exercise for the willing readers. Presently it will become clear why we have bothered to point this all out.

Now, what does StrR claim with regard to structure $\mathfrak{S}(\mathsf{HeB})$ (3)? When we follow Patrick Suppes[10] in considering the class of structures like $\mathfrak{S}(\mathsf{HeB})$, and similar ones (with other physical magnitudes, mixed states, etc.), to constitute the theory of quantum mechanics (QM), then it trivially follows that all that QM tells us about physical reality, actually even all that QM *can* tell us about physical reality, such as about element of physical reality HeB, is that this physical system is or has structure $\mathfrak{S}(\mathsf{HeB})$. Thus John Worrall [1989] is right when he says that all science

9 See Muller [1998], pp. 56–64, where this is all spelled out.

10 Suppes [1960], [1967], [2002].

provides us with is knowledge of the structure(s) of the world, rather than of the nature(s) of the world. Epistemic StrR seems inevitable.

When knowledge implies truth, and truth implies ontological adequacy, then *knowing that* structure $\mathfrak{S}(\mathsf{HeB})$ *is* the structure HeB implies that $\mathfrak{S}(\mathsf{HeB})$ truly *is* the structure of HeB. Ontic StrR is just around the corner! Perhaps we should limit our claims to so-called *ontological substructures* of $\mathfrak{S}(\mathsf{HeB})$, but ontic StrR remains just around the corner.[11]

We get around the corner when we assume in addition that science tells us, or eventually will tell us, everything there is to tell about the physical world in general, and about He-atoms in uniform magnetic fields in particular (scientific optimism).[12] Nothing will be left unsaid. Since the physical world is built from atoms and according to ontic StrR they are structures, the physical world is composed of structures. *Ad fundum* structures determine everything there is in the physical world.

The conclusion seems to be that Suppes' structuralist view *on scientific theories* conjoined with a realist attitude yields epistemic StrR and optimistically also ontic StrR. As Worrall [2009] has recently put it: "Structural Realism is the only game in town." End of story?

Not yet. For what does it mean exactly to say that HeB *is or has structure* $\mathfrak{S}(\mathsf{HeB})$? This is an instance of the

> **Central Claim of StrR**. *Being* B *is or has structure* $\mathfrak{S}$ *(a* being *is anything, any entity, that exists), independently of us, human beings, of our activities, attitudes and capacities, of our very existence.* (23)

This Central Claim stands in need of clarification, as will emerge below.

The 'is' obviously cannot mean the identity-relation, because $\mathfrak{S}(\mathsf{HeB})$ (22) is an *abstract mathematical entity*, to wit a complicated set-theoretical construction out of the empty set, living in the cumulative hierarchy of all and only pure sets, in the domain of discourse $\mathbf{V}$ of ZFC, while HeB is a *concrete physical entity*, 'out there' in the physical world. Certainly a He-atom in a uniform magnetic field (HeB) it is not *a set*.

Perhaps, then, 'is' means predication, as does 'has'. HeB *has* a structure, a very specific structure, namely $\mathfrak{S}(\mathsf{HeB})$, just as a tomato *has* a colour, a very specific colour, namely red. Let us see where this leads us.

We express properties in our language by means of predicates. The property red – if there are 'properties' – is expressed by the predicate 'red', and the ascription of the property red to a tomato is expressed by saying that 'This tomato is red' is true, or that this tomato falls under the predicate 'red'. The obvious candidate for the predicate that ascribes the wave-mechanical structure to HeB is the

11 The idea of considering *ontological substructures* of structures for realist claims was suggested more than ten years ago, and is closely related to M.L.G. Redhead's idea of 'surplus structure'. See in Muller [1998], p. 356 ff., and Redhead [1975], p. 88.

12 See Muller [2009], Section 1.

set-theoretical one that defines structure $\mathfrak{S}(\mathsf{HeB})$ (3), call it $\boldsymbol{\xi}(\cdot)$. The general form of this predicate is:[13]

$$\begin{aligned}\boldsymbol{\xi}\big(\mathfrak{S}(\mathsf{HeB})\big) \ \text{ iff } \ & \exists\, X_1,\ \exists\, X_2,\ \exists\, X_3,\ \exists\, X_4 : \\ & \mathfrak{S}(\mathsf{HeB}) = \big\langle X_1, X_2, X_3, X_4 \big\rangle \ \wedge \\ & X_1 = L^2(\mathbb{R}^3) \ \wedge \ X_2 = H(\mathbf{B}_0) \ \wedge \\ & X_3 = \psi \ \wedge \ X_4 = \mathrm{Pr}_t\,. \end{aligned} \tag{24}$$

This will not do either, because $\boldsymbol{\xi}(\cdot)$ (24) is an open sentence in the language $\mathfrak{L}_\in$ of ZFC and thus only applies to inhabitants of **V**. Our HeB does not inhabit **V** and therefore can never fall under $\boldsymbol{\xi}(\cdot)$: formally speaking, '$\boldsymbol{\xi}(\mathsf{HeB})$' is nonsense. Now what?

The way to go without leaving set-theory seems to enrich **V** with *physical systems*.[14] This makes $\mathfrak{L}_\in$ a two-sorted language, with set-variables and *physical-system-variables*, say $\boldsymbol{a}$ and $\boldsymbol{b}$ for the new sort. Physical systems can be collected in sets, so that '$\boldsymbol{a} \in X$' etc. become well-formed atomic sentences of the enriched language, call it $\mathfrak{L}^*_\in$. Expressions '$X \in \boldsymbol{a}$', '$X = \boldsymbol{a}$', '$\boldsymbol{a} \in \boldsymbol{b}$', '$X \notin \boldsymbol{a}$' etc. are forbidden in $\mathfrak{L}^*_\in$ because the physical systems are not supposed to be entities that can have members, *they are not sets*. The axioms of ZFC have to be reformulated in the enriched language $\mathfrak{L}^*_\in$, and one axiom has to be added declaring the existence of physical systems, but that's all. Thus one obtains ZFCU. No additional axioms are present in ZFCU to govern the physical systems. (It is possible to enrich ZFC with mereological axioms that govern the physical systems, by taking the *subsystem-relation* as a primitive dyadic predicate in the language additional to the membership-predicate; theory thus obtained is a conservative extension over ZFC and therefore consistent relative to ZFC, and therefore to ZF.[15] We shall not do this here; we have done it already somewhere else (see previous footnote).)

Our HeB will now hopefully become a value of the fresh variables, because, as we all know, to be is to be the value of a variable. The conclusion that HeB is *a set* will then have been avoided. But still, structure-predicate $\boldsymbol{\xi}(\cdot)$ (24) is such that only a particular kind of polytuple, hence *a set*, falls under it, namely a polytuple of the form $\mathfrak{S}(\mathsf{HeB})$ (3). Formally, from (24) we see immediately that '$\boldsymbol{\xi}(\boldsymbol{a})$' is nonsense because '$\boldsymbol{a} = \langle ., ., ., . \rangle$' is nonsense. Therefore we have to adjust $\boldsymbol{\xi}$ (24) of $\mathfrak{L}_\in$ to some other predicate of $\mathfrak{L}^*_\in$, say $\boldsymbol{\varphi}(\cdot)$, such that '$\boldsymbol{\varphi}(\boldsymbol{a})$' makes sense and structure $\mathfrak{S}(\mathsf{HeB})$ is somehow in there – as it must, because that is what QM provides. This can be achieved in two steps.

13 Notice that the right-hand-sides of the identity-statements in the definiens (24) are assumed to be antecedently defined singular terms in the language of ZFC; this is done for brevity, more standard is to write 'X_1 is a Hilbert-space'.

14 The technical term for *objects that are not sets* is *primordial elements*, or *Ur-elements*, from the German *Urelemente*. See Fraenkel [1973], pp. 23–25.

15 See Muller [1998], pp. 189–252, for details and proofs.

The first step is to let $\boldsymbol{a}$ occupy the structure polytuple:

$$\begin{aligned}\boldsymbol{\xi}^*\big(\boldsymbol{a},\,\mathfrak{S}^*(\mathsf{HeB})\big)\ \text{ iff }\ & \exists\, X_1,\ \exists\, X_2,\ \exists\, X_3,\ \exists\, X_4 :\\ & \mathfrak{S}^*(\mathsf{HeB}) = \big\langle \boldsymbol{a},\ X_1,\ X_2,\ X_3,\ X_4 \big\rangle\ \wedge\\ & X_1 = L^2(\mathbb{R}^3)\ \wedge\ X_2 = H(\mathbf{B}_0)\ \wedge\\ & X_3 = \psi\ \wedge\ X_4 = \mathrm{Pr}_t.\end{aligned} \tag{25}$$

The dyadic predicate $\boldsymbol{\xi}^*$ expresses a relation between structure $\mathfrak{S}^*(\mathsf{HeB})$ and physical system $\boldsymbol{a}$. Since $\boldsymbol{\xi}^*$ relates a physical system, which we hope to identify with concrete physical object HeB, to an abstract object, structure $\mathfrak{S}^*(\mathsf{HeB})$, which is at the end of the day still a set, just like $\mathfrak{S}(\mathsf{HeB})$, this is not quite what StrR needs. The second step is to turn $\boldsymbol{\xi}^*$ (25) into a monadic predicate of $\boldsymbol{a}$ by existentially quantifying $\mathfrak{S}^*(\mathsf{HeB})$ away:

$$\boldsymbol{\varphi}(\boldsymbol{a})\ \text{ iff }\ \exists\,\mathfrak{S}^*(\mathsf{HeB}) :\ \boldsymbol{\xi}^*\big(\boldsymbol{a},\,\mathfrak{S}^*(\mathsf{HeB})\big)\ . \tag{26}$$

Formally, we seem to be going in the right direction. For let us compare things again to red tomatoes. Suppose there is a tomato on the plate in front of us. The sentence 'Red(this-tomato)' is true and the expression 'this-tomato' trivially *refers to* the tomato on the plate in front of us. Similarly we want to say that '$\boldsymbol{\varphi}(\boldsymbol{a})$' is true and that '$\boldsymbol{a}$' refers to a He-atom in a uniform magnetic field. But '$\boldsymbol{a}$' is a variable and variables do not refer. What we want to say instead is that $\boldsymbol{a}$ is a He-atom in a uniform magnetic field iff $\boldsymbol{\varphi}(\boldsymbol{a})$, because $\boldsymbol{\varphi}(\cdot)$ (26) is the set-theoretical translation of the characterisation of HeB that QM provides.[16] The extension of $\boldsymbol{\varphi}(\cdot)$ then includes all and only actual (and perhaps possible) He-atoms in a uniform magnetic field. The symbol 'HeB' can then be officially inaugurated as a variable running over this extension, a so-called *Helium-atom-in-a-uniform-magnetic-field-*$\mathbf{B}_0$*-variable*. If the officially inaugurated variable 'HeB' assumes a value from this extension, we can say that He-atoms in a uniform magnetic field exist, or that the variable *plurally refers* to those physical systems; or if we can locate by laser cooling techniques a single He-atom in the laboratory, and give it a name, we can say that this name *singularly refers* to the atom, just as in the case of the red tomato on the plate in front of us.

This story has to be grounded in some account of *reference*. For such unobservable physical systems as He-atoms in magnetic fields, the only viable account of reference is a *descriptivist* one.[17] The relevant description here is the description of our HeB. Science, by means of (our set-theoretically reconstructed) QM,

16 He-atoms are usually characterised by their constitutive parts (a nucleus consisting of two protons and two neutrons, and two electrons) and their mass, charge and spin. Usually this can be read of 'read off' the Hamiltonian $H(\mathbf{B}_0)$ and therefore is included but is, unlike the uniform magnetic field $\mathbf{B}_0$, notation-wise suppressed.

17 For why the only available alternative, the Kripke-Putnam causal theory of reference, fails to provide a general account of reference for science, see Gauker [2006], pp. 130–132.

delivers this description: $\varphi(\boldsymbol{a})$ (26). Let us next take a closer look at this description.

Description $\varphi(\boldsymbol{a})$ (26) literally says: there is some particular polytuple, i.e. set-structure $\mathfrak{S}^*(\mathsf{HeB})$, that has $\boldsymbol{a}$ as its first occupant (25). One can easily prove that if $\mathfrak{S}(\mathsf{HeB})$ (3) exists in $\mathbf{V}$, then $\mathfrak{S}^*(\mathsf{HeB})$ (25) exists in $\mathbf{V}^*$, *for every* $\boldsymbol{a}$ *indiscriminately*:

$$\mathsf{ZFCU} \vdash \exists \mathfrak{S} : \boldsymbol{\xi}(\mathfrak{S}) \longrightarrow \forall \boldsymbol{a} : \varphi(\boldsymbol{a}) . \tag{27}$$

Since the antecedent can also be proved, so can the consequent:

$$\mathsf{ZFCU} \vdash \forall \boldsymbol{a} : \varphi(\boldsymbol{a}) . \tag{28}$$

Recall that the idea was to obtain a description – based on QM – such that those $\boldsymbol{a}$ falling under the description can be said to be HeB. But if $\varphi(\cdot)$ (26) is that description, then as a consequence every single physical system qualifies as a HeB (28). Which is absurd.[18] The situation is actually worse than absurd, because this all generalises. For *every* set-structure $\mathfrak{A}$ in $\mathbf{V}$, one can easily prove there are as many structures as there are physical systems in that for every physical system $\boldsymbol{b}$, there is a structure $\mathfrak{A}_{\boldsymbol{b}}$ that has $\boldsymbol{b}$ as its first occupant and that shares all its occupants with $\mathfrak{A}$. Thus every physical system is everything. The descriptions are therefore void. Not only is the putative description $\varphi(\cdot)$ (26) void, in spite of appearances to the contrary, but every other description, based on any other structure $\mathfrak{A}$, rather than $\mathfrak{S}(\mathsf{HeB})$ or $\mathfrak{S}^*(\mathsf{HeB})$, will also be void. No descriptivist account of reference can take off in the context of ZFCU.

Now we are done. Our provisional conclusion is that the set-theoretical road to physical reality for StrR seems a road to nowhere. Realism without reference, then? Hmmm. Smells like realism without reality. Before realists get *that* desperate, they should explore all other options. One option to clarify the Central Claim of StrR (23) is to replace set-theory with category-theory.

3. Category-Theoretical Characterisation

When it comes to deal with structures, in particular in abstract branches of mathematics – abstract in comparison to number theory, analysis and the geometry of figures, curves and planes –, such as algebraic topology, homology and homotopy theory, universal algebra, and what have you, a vast majority of mathematicians considers Category-Theory (CT) vastly superior to set-theory. CT also is the only rival to ZFC in providing a general theory of mathematical structure and in founding the whole of mathematics. The language of CT is two-sorted: it contains *object-variables* and *arrow-variables*. An arrow sends objects to objects; an

18 When we identify the 'objects' that Brading & Landry [2006], p. 572, take to be 'presented' by a structure as Ur-elements, then theorem (27) also makes trouble for them: everything can be 'presented' by every structure, so all them 'present' everything, or conversely, every structure can 'present' anything.

identity-arrow sends an object to itself. Simply put, *structures are categories*, and a *category* is something that has objects and arrows, such that the arrows can be composed so as to form a *composition monoid*, which means that: (i) every object has an identity-arrow, and (ii) arrow-composition is associative. The languages of CT ($\mathcal{L}_{\uparrow}$) and ZFC ($\mathcal{L}_{\in}$) are inter-translatable. In CT there is the specific category **Set**, whose objects can be identified with sets and whose arrows are maps. In ZFC one can identify objects with sets and arrows with ordered pair-sets of type $\langle f, C \rangle$, consisting of a mapping f and a co-domain C.[19]

In spite of the fact that some mathematical physicists have applied categories to physics, not a single structural realist on record has advocated replacing ZFC with CT. One of the very few critics of the use of set-theory for StrR (if not the only critic) is E. M. Landry [2007], who has argued that the set-theoretical framework does not always do the work it has been suggested to do; but even she does not openly advocate CT as the superior framework for StrR, although she does advocate it for mathematical structuralism.[20]

The objects of CT are more general than the Ur-elements one can introduce in ZFC, because whereas primordial elements are not sets, the objects of CT can be *anything*, arrows, sets, functors and categories included. Similar to ZFCU is that CT does not have axioms that somehow restrict the interpretation of 'object'. A CT-object is anything that can be sent around by an arrow, similar to the fact that a set-theoretical Ur-element is anything that can be put in a set. CT-objects obtain an 'identity', a 'nature', from the category they are in: different category, different identity. Outside categories, these objects lose whatever properties and relations they had in the category they came from and they become essentially indiscernible.

One great advantage of CT is that structures, i.e. categories, are not accompanied by all these sets that arise by iterated applications of the power-set and union-set operation, as we have seen in (6), (19), (20), (21) and (22). Nevertheless, the grim story we have been telling for StrR in the framework of ZFC, can be repeated in the framework of CT, of course with a few appropriate adjustments. Objects play the rôle that Ur-elements played even better: we end up with something very similar to (28), on top of saying that HeB definitely is not a composition monoid of objects and arrows. Since there is little point in re-telling the entire story, we leave it as an exercise for the sceptical reader. The end of the story is the same problem about reference and description we landed in with ZFCU.

Our conclusion is that the category-theoretical road to physical reality for StrR to walk on also seems a road to nowhere. Before we kiss ZFC and CT goodbye, we want to explore the possibility of retaining them *both*. This seemingly impossible possibility arises when we put the concept of *representation* center stage and see whether it can help us with clarifying the Central Claim of StrR (23).

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19 See further Muller [1998], pp. 485–496.

20 When Landry [2007] argues against Suppes, French, etc. that a set-theoretical framework is *not necessary* to make things rigorous, she takes 'necessity' in a sense that is stronger than Suppes, French, etc. have ever meant it whenever they used it or sibling phrases.

4. Representation

Recently the concept of representation has gained momentum in the philosophy of science.[21] The simplest concept of representation conceivable is expressed by the following dyadic predicate: structure $\mathfrak{S}(\mathsf{HeB})$ represents HeB. S. French [2003] defended that to represent something in science is the same as to have a model for it, where models are set-structures; then 'representation' and 'model' become synonyms and so do 'to represent' and 'to model' (considered as a verb). Nevertheless, this simplest conception was quickly thrown overboard as too simple by amongst others R. N. Giere [2004], p. 743, who replaced this dyadic predicate with a quadratic predicate to express a more involved concept of representation:

$$\text{Scientist } S \text{ uses model } \mathfrak{S} \text{ to represent being } \mathsf{B} \text{ for purpose } P, \tag{29}$$

where 'model' can here be identified with 'structure'. Another step was set by B. C. van Fraassen. As early as 1994, in his contribution to J. Hilgevoord's *Physics and our View of the World*, Van Fraassen [1994] brought Nelson Goodman's distinction between *representation-of* and *representation-as* – drawn in his seminal *Languages of Art* (1968) – to bear on science; he went on to argue that all representation in science is representation-as. We *represent* a Helium atom in a uniform magnetic field *as* a set-theoretical wave-mechanical structure $\mathfrak{S}(\mathsf{HeB})$ (3). In his new tome *Scientific Representation* [2008], Van Fraassen has moved essentially to a hexadic predicate to express the most fundamental and most involved concept of representation to date:

$$\text{Repr}(S, V, \mathfrak{S}, \mathsf{B}, F, P)\ , \tag{30}$$

which reads: subject or scientist S is V-ing artefact $\mathfrak{S}$ to represent B as an F for purpose P. Example: In the 1920ies, Heisenberg (S) constructed (V) a mathematical object ($\mathfrak{S}$) to represent a Helium atom (B) as a wave-mechanical structure (F) to calculate its electro-magnetic spectrum (P). We concentrate on the following triadic predicate, which is derived from the fundamental hexadic one (30):

$$\text{ReprAs}(\mathfrak{S}, \mathsf{B}, F) \quad \text{iff} \quad \exists\, S,\ \exists\, V,\ \exists\, P:\ \text{Repr}(S, V, A, \mathsf{B}, F, P)\ , \tag{31}$$

which reads: abstract object $\mathfrak{S}$ *represents* being B *as* an F, so that $F(\mathfrak{S})$.

Brief historical interlude. Giere, Van Fraassen and contemporaries are not the first to include manifestations of human agency in their analysis of models and representation in science. Almost half a century ago, Peter Achinstein [1965], pp. 104-105, expounded the following as a characteristic of models in science:

> A theoretical model is treated as an approximation useful for certain purposes. (...)
> The value of a given model, therefore, can be judged from different though related viewpoints: how well it serves the purposes for which it is eimployed, and the completeness and accuracy of the representation it proposes. (...)
> To propose something as a *model* of X is to suggest it as way of representing X which

21 Suárez [2003], Giere [2004], Frigg [2006], Fraassen [1994], [2008].

provides at least some approximation of the actual situation; moreover, *it is to admit the possibility of alternative representations useful for different purposes*.

One year later, M. W. Wartofsky explicitly proposed, during the Annual Meeting of the American Philosophical Association, Western Division, Philadelphia, 1966, to consider a model as a genus of representation, to take in that representation involves "relevant respects for relevant for purposes", and to consider "the modelling relation triadically in this way: $M(S, x, y)$, where S takes x as a model of y".[22] Two years later, in 1968, Wartofsky wrote in his essay 'Telos and Technique: Models as Modes of Action' the following (our emphasis):

> In this sense, models are embodiments of purpose and, at the same time, instruments for carrying out such purposes. Let me attempt to clarify this idea. No entity is a model of anything simply by virtue of looking like, or being like, that thing. Anything is like anything else in an infinite number of respects and certainly in some specifiable respect; thus, *if I like, I may take anything as a model of anything else, as long as I can specify the respect in which I take it. There is no restriction on this.* Thus an array of teacups, for example, may be take as a model for the employment of infantry battalions, and matchsticks as models of mu-mesons, there being some properties that any of these things share with the others. But *when we choose something to be a model, we choose it with some end in view*, even when that end in view is simply to aid the imagination or the understanding. In the most trivial cases, then, the model is already *normative* and *telic*. It is *normative* in that is chosen to represent abstractly only certain features of the thing we model, not everything all at once, but those features we take to be important or significant or valuable. The model is *telic* in that *significance and value can exist only with respect to some end in view or purpose that the model serves*.[23]

Further, during the 1950ies and 1960ies the role of *analogies*, besides that of models, was much discussed among philosophers of science (Hesse, Achinstein, Girill, Nagel, Braithwaite, Wartofsky). We predict that several insights buried in the ensuing literature will be re-discovered by the contemporary division of representationalists. *End of brief historical interlude.*

On the basis of the general concept of representation (30), we can echo Wartofsky by asserting that almost anything can represent everything for someone for some purpose.[24] In scientific representations, *representans* and *representandum* (to introduce another pair of Latin barbarisms) will share some features, but not all features, because to represent is neither to mirror nor to copy. Realists, a-realists and anti-realists will all agree that ReprAs($\mathfrak{S}$, B, F) is true *only if* on the basis of $F(\mathfrak{S})$ one can save all phenomena that being B gives rise to, i.e. one can calculate or accommodate all measurement results obtained from observing B or experimenting with B. Whilst for *structural empiricists* like Van Fraassen this is also sufficient, for StrR it is not. StrR will want to add that structure $\mathfrak{S}$ of type F 'is

22 Collected in Wartofsky [1979], quotation on p. 6.

23 Collected in Wartofsky [1979], p. 142.

24 Almost anything, not everything: has anyone ever taken the universe as a whole to represent something?

realised', that $\mathfrak{S}$ of type F truly *is* the structure of being B or refers to B, so that also $F(\mathsf{B})$. StrR will want to order the representations of being B that scientists have constructed during the course of history as approaching the one and only true structure of B, its structure *an sich*, the Kantian regulative ideal of StrR. But this talk of truth and reference, of beings and structures *an sich*, is in dissonance with the concept of representation-as.

Some being B *can* be represented *as* many other things and all the ensuing representations are all hunky-dory if each one serves some purpose of some subject. That is the idea of (30). When the concept of representation-as is taken as pivotal to make sense of science, then the sort of 'perspectivalism' that Giere [2004] advocates is more in consonance with the ensuing view of science than realism is. Giere [2004] attempts to hammer a weak variety of realism into his 'perspectivalism': all perspectives are perspectives on *one and the same* reality and from every perspective *something* is said that can be interpreted realistically: in certain respects the representans resembles its representandum to certain degrees. A single unified picture of the world is however not to be had. Nancy Cartwright's *dappled world* seems more near to Giere's residence of patchwork realism. A unified picture of the physical world that realists dream of is completely out of the picture here. With friends like that, realism needs no enemies.

There is *prima facie* a way, however, for realists to express themselves in terms of representation, as follows. First, fix the purpose P to be: *to describe the world as it is*. When this fixed purpose leaves a variety of representations on the table, then choose the representation that is empirically superior, that is, that performs best in terms of describing the phenomena, because the phenomena are part of the world. This can be established objectively. When this still leaves more than one representation on the table, which thus save the phenomena equally well, choose the one that best *explains* the phenomena. In this context, Van Fraassen [1994] mentions the many interpretations of QM: each one constitutes a different representation of the same beings, or of only the same observable beings (phenomena), their similarities notwithstanding. Do all these interpretations provide equally good explanations? This can be established objectively too, but every judgment here will depend on which view of explanation is employed. Suppose we are left with a single structure $\mathfrak{A}$, of type G. Then we assert that '$G(\mathsf{B})$' is true. When this 'G' predicates structure to B, we still need to know what 'structure' literally means in order to know what it is that we attribute to B, of what $\mathfrak{A}$ is that B instantiates, and, even more important, we need to know this for our descriptivist account of reference, which realists need in order to be realists. Yes, we now have arrived where we were at the end of the previous two Sections. We conclude that this way for realists, to express themselves in terms of representation (as announced at the beginning of this paragraph), is a dead end. The concept of representation is not going to help them.

We conclude that applauding for a variety of different representations-as of the beings does not serve the aim of realism, StrR included. The need for substantive accounts of truth and reference fade away as soon as one adopts a view of science that takes the concept of *representation-as* as its pivotal concept. Fundamentally different kinds of mathematical structure, set-theoretical and category-theoretical, can then easily be accommodated. They are 'only representations'. That is moving away from realism, StrR included, *dissolving* rather than *solving* the problem for StrR of clarifying its Central Claim (23) – 'dissolved', because 'is or has' is replaced with 'is represented-as'. Realism wants to know what B *is*, not only how it can be represented for someone who wants to do something for some purpose. When we take it for granted that StrR needs substantive accounts of truth and reference, more specifically a descriptivist account of reference and then an account of truth by means of reference, then a characterisation of structure as directly as possible, without committing one to a profusion of abstract objects, is mandatory. This issue we address in the next and final Section.

5. Direct Characterisation

Suppose we have a zoological theory E about Elephants. The word 'elephant' is logically speaking an 'elephant-variable'. Since elephants are concrete observable animate beings easy to recognise, it serves no scientific purpose to think of postulates for E that will characterise what an elephant is. (This would be different if we were considering a theory of ants, of which there are about $12,000$ species.) That is why one will look in vain in zoology for such postulates.

Suppose next we have a mathematical theory N about Natural Numbers. Unlike elephants, natural numbers are abstract and therefore unobservable objects, but like elephants they are easy to recognise. Children recognise elephants and natural numbers effortlessly. Since natural numbers are however crucially involved in the most rigorous intellectual praxis that human civilisation so far has produced, i.e. *mathematics*, wherein *theorems* are *proved* about natural numbers and *theorems* are *proved* about other abstract objects that employ natural numbers, it serves a mathematical purpose to think of axioms for N that will characterise what sort of abstract object a natural number is. We need to know exactly what holds for them in order to know what we can use and what we cannot use in proofs of theorems that are about them or involve them. That is why one will *not* look in vain in mathematics for such axioms. Gottlob Frege, Richard Dedekind, Giuseppe Peano and William Lawvere have provided such axioms.[25]

What StrR needs, we submit, is a theory S about Structures. Just as we can say with a clear philosophical conscience that ZFC implicitly defines the set-concept, we want to say that S implicitly defines the structure-concept.[26] Hence just as the language of ZFC, $\mathcal{L}_\in$, takes the set-concept as primitive by having set-variables,

25 See Muller [1998], pp. 56–64.

26 For how to obtain a clear philosophical conscience, see Muller [2004], [2005]. This concept of 'implicit definability' should not be confused E.W. Beth's concept that is expressed by the same expression, which by the way is better expressed by 'semantic definability'.

the language of pure structure theory, call it $\mathfrak{L}_{\mathfrak{S}}$, must have structure-variables. The concept of structure ought not to be reduced to *other* concepts, such as *sets* (Section 2) or *objects-cum-arrows* (Section 3). The project to construct $\mathfrak{L}_{\mathfrak{S}}$ and S will have to wait for another occasion. For now, let us suppose that we possess S. Will that be of any help in clarifying the Central Claim of StrR (23)? Will it provide a literal description of B that any substantive account of reference requires?

To begin with, just as an elephant-realist will take his elephant-variables in E running over at least all elephants on planet Earth, StrR will take the *structure-variables* of $\mathfrak{L}_{\mathfrak{S}}$ to range over at least all structures in physical reality. The domain of discourse of S, call it $\mathbf{S}$, will of course harbour a plenitude of structures, just as $\mathbf{V}$ of ZFC harbours a plenitude of sets. When we take ZFC to provide the foundations of mathematics that is used in science, then only sets in the lower tip of $\mathbf{V}$, say below ordinal level $\omega + \omega$, will be employed. Similarly, only *some* of the structures in $\mathbf{S}$ of theory S will be candidates of which StrR will want to say that they are 'realised', or instantiated, in physical reality. Science, physics in particular, will tell us *which* ones are those candidates. StrR will then submit that those predicates in $\mathfrak{L}_{\mathfrak{S}}$ that single out these candidates provide *literal descriptions of* those structures. Exactly here, within the confines of S, a substantive descriptivist account of reference will find its Archimedean point, and reference will lead the realist to truth.

Recall that the Central Claim of StrR, *being* B *is or has structure* $\mathfrak{S}$ (23), stood in need of clarification. The clarification we have in the offing with theory S runs, in summary fashion, as follows. First, we advise StrR to say that *being* B *is a structure* $\mathfrak{S}$ *of type* F, where 'F' is a predicate in $\mathfrak{L}_{\mathfrak{S}}$ such that $F(\mathfrak{S})$, and where '$\mathfrak{S}$' now is a structure-variable of $\mathfrak{L}_{\mathfrak{S}}$. Then StrR should simply stipulate, on the basis of $F(\mathfrak{S})$, that predicate F also supplies a structural type-description *of being* B, in other words, StrR should also assert *that* $F(\mathsf{B})$. This '$F(\mathsf{B})$' is the literal description of B that any descriptivist account of reference can take aboard.

But, wait a minute, before this paper ends: how about the inscrutability of reference?

What about it? That is a problem for everyone who wants to adopt a semantic theory that unrestrictedly aims to save all *and only* observable behaviouristic facts of ascent and dissent of language-users. As soon as more is required of a semantic theory, inscrutability arguments are blocked, which is not to deny the seriousness of the problem of what precisely these additional requirements are.[27] Since inscrutability is a problem for realists and anti-realists alike, rather than a problem for scientific realism let alone StrR in particular, the issue makes little difference in the realism debate and we have therefore ignored it. ■

27 See Williams [2008] for the strongest inscrutability result so far: a permutation argument for D.K. Lewis' Montague-based *general semantics*.

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