Homework 2 **Graphs and Algorithms**

Planarity, Clever Enumeration, Dynamic Programming and Inclusion Exclusion

Date: 28/2/2016 Due: 11/3/2016

1. (15 pts) **Planar Separators:** Use the planar separator theorem from class to show that there is a fixed constant c such that given any $\epsilon > 0$, one can remove at most $c\epsilon n$ edges to break any planar graph on n vertices into components of size at most $1/\epsilon^2$. Prove this by writing and solving an appropriate recurrence relation.

2. (10 pts) **Faster 100-CNF-Sat:** Solve 100-CNF-Sat in $O^*((2-\epsilon)^n)$ time for some $\epsilon > 0$, where n denotes the number of variables.

3. (15 pts) **Connecting Dots with Lines:** Give an $O^*(k^{4k})$-time algorithm that takes as input n points in the plane $\mathbb{R}^2$ and an integer k, and determines whether there exist k straight lines such that every point is on some line. Hint: First, look at $k+1$ points that are on one line to find a reduction rule. Second, conclude something if n is too large when compared with k^2 and your reduction rule does not apply. Third, design an $O^*(n^{2k})$ time algorithm.

4. (10 pts) **Triangle Partition:** A triangle of a graph $G = (V, E)$ is a triple $u, v, w \in V$ such that $(u, v), (v, w), (u, w) \in E$. A triangle partition is a partition of V into triangles, e.g., a set of triangles $T_1, \ldots, T_{n/3}$ such that $T_i \cap T_j = \emptyset$ and $\cup_i T_i = V$.

   - (5pts) Give an algorithm that determines whether there is a triangle partition of a graph on n vertices in $O^*(2^n)$ time. Can you give an algorithm that uses $O^*(2^n)$ time and polynomial space?
   - (5pts) Give an algorithm that takes as input a graph G on n vertices and a vertex cover of G of size at most k, and determines whether there is a triangle partition of G using $O^*(2^k)$ time. Hint: Use that every vertex not in the vertex cover must be in a different triangle, thus a triangle partition can be formed by letting each vertex not in the vertex cover decide with which vertices it is in a triangle.