# Extra Practice Exercises

**Excercise 1.** Given undirected graph $G = (V, E)$, find a pair $u, v \in V$ with $u \neq v$ maximizing $N(u) \cap N(v)$ in $O(n^\omega)$ time, where $\omega$ is the matrix multiplication constant.

**Excercise 2.**

- Give an algorithm taking as input an undirected graph $G$ and a Feedback Vertex Set (FVS) $F$ of $G$, and outputs a tree decomposition of $G$ of width $|F| + O(1)$ in polynomial time.
- Suppose you have an algorithm $\texttt{fvstw}(G, (X, T))$ that given an undirected graph $G$ and a tree decomposition $(X, T)$ of $G$ of width $w$, computes a minimum size FVS in time $O^*(3^w)$. Give an algorithm that uses $\texttt{fvstw}(G, (X, T))$ as a blackbox, takes as input a graph $G$ and integer $k$, and determines whether $G$ has a FVS of size at most $k$ in $O^*(3^k)$. Hint: use iterative compression.

**Excercise 3.** A connected vertex cover (CVC) of a graph $G = (V, E)$ is a vertex cover $X \subseteq V$ such that $G[X]$ is connected. Give an algorithm that takes as input a graph $G = (V, E)$ and integer $k$ and determines in $O^*(c^k)$ time whether $G$ has a connected vertex cover of size at most $k$, for some constant $c$. You may use as blackbox an algorithm $\texttt{st}(G = (V, E), T, l)$ that solves the Steiner Tree problem in $O^*(2^{|T|})$ time, i.e. it determines whether there exists $T \subseteq Y \subseteq V$ with $|Y| \leq l$ with $G[Y]$ connected.

**Excercise 4.** Give a quadratic time algorithm that takes as input three matrices $A, B, C \in \mathbb{Z}_2^{n \times n}$ with the following properties:

- If $AB = C$, i.e. $\forall i, j : c_{ij} \equiv_2 \sum_k a_{ik} b_{kj}$, the algorithm always outputs **true** ,
- If $AB \neq C$, the algorithm outputs **false** with constant probability.

Hint: Pick $x \in \mathbb{Z}_2^n$ uniformly at random and study $Cx$ and $ABx$, use the rank-nullity theorem to bound the probability of false positives.