Exercise set 4 Algorithms and Complexity 2019 **Due 11 Nov 2019**

You may collaborate and submit answers in groups of at most three. Good solutions are complete and concise. Please mail to j.nederlof@tue.nl on or before the day of the deadline.

8. The knight's graph[1] is the graph on 64 vertices with a vertex for every square of a chess-board and two vertices being adjacent if they are a knight's move away from each other (formally, we have vertices $v_{i,j}$ for $1 \leq i, j \leq 8$ and an edge $(v_{i,j}, v_{i',j'})$ if either $|i-i'| = 1$ and $|j-j'| = 2$ or $|i-i'| = 2$ and $|j-j'| = 1$). Show that the knight's graph has treewidth at most 16.
If you choose to argue pictorially, you may use pen drawings (if it safes you time).

9. The MAX-SCHEDULE problem is as follows: given are m machines, n jobs, and for every $1 \leq i \leq m$ and $1 \leq j \leq n$ an integer $p_{i,j} \in \mathbb{N}_{\geq 0}$ (given in binary representation) indicating the processing time used by machine i to process job j. Additionally given is a deadline D and an integer k. The question is whether one can allocate at least k jobs to the machines such that no machine uses more than D processing time.[2]

   (a) Show how to solve this problem in polynomial time if $m = 1$.

   (b) Give an algorithm for MAX-SCHEDULE that runs in time $O^*(m^k)$. Your algorithm may have constant one-sided error probability in the following sense:

   - if the instance is a NO-instance, your algorithm should return NO,
   - otherwise, your algorithm returns YES with probability at least 1/10.

[1] See e.g. the page `https://en.wikipedia.org/wiki/Knight%27s_graph` on wikipedia.

[2] More formally stated, if we denote $M_i \subseteq \{1, \ldots, n\}$ for the set of jobs assigned to machine i in such an allocation, it is asked whether there exist disjoint subsets $M_1, \ldots, M_m \subseteq \{1, \ldots, n\}$ such that $\sum_{i=1}^{m} |M_i| \geq k$ and $\sum_{j \in M_i} p_{i,j} \leq D$ for every i.