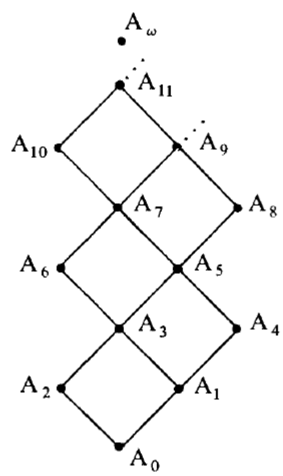
1 Exercisesheet 1

Exercise 1. We will look at the one-variable formulas in IPC, generated from a single propositional letter P by means of the logical operators $\to$, $\wedge$, $\vee$ and $\bot$. Assume that, modulo provable equivalence, all such formulas occur in the sequence $\langle A_n(P)\rangle_{n\in\mathbb{N}\cup\{\omega\}}$ given by:

$$A_0(P) :=\bot, \quad A_1(P) := P, \quad A_2(P) := \neg P,$$

$$A_{2n+1}(P) := A_{2n-1} \vee A_{2n}(P),$$
$$A_{2n+2}(P) := A_{2n}(P) \to A_{2n-1}(P),$$
$$A_\omega(P) := P \to P,$$

which can be arranged in the following poset:



Show that $A_i \leq A_j \implies A_i \to A_j$.

Exercise 2. For each of the following statements, show by means of a Kripke model that they are not Kripke valid.
i) $\neg\neg A \to A$,
ii) $((A \to B) \to A) \to A$,
iii) $\neg\neg\exists x A(x) \to \exists x \neg\neg A(x)$.