Seminar on Set Theory
Hand-in Exercise 10

Exercise 1

In this exercise you are asked to verify some minor details in the proof of the independence of the axiom of choice. All constant names are as defined on page 83 of Bell.

(a) [1 point] Show by induction on the complexity of formulas that for every formula $\phi(v_1, \ldots, v_n)$ and $x_1, \ldots, x_n \in V^\Gamma$ it holds that

$$g \cdot [\![u \in v]\!]^\Gamma = [\![\phi(gx_1, \ldots, gx_n]\!]^\Gamma.$$

(b) [1 point] Show that $\langle g, b\rangle \mapsto gb$ is indeed an action of G on B.

(c) [1 point] Show that Γ is indeed normal.

(d) [2 points] Let $n, n' \in \omega$ such that $n \neq n'$. Show that $V^{(\Gamma)} \models u_n \neq u_{n'}$.

Exercise 2

[2 points] Give a proof of Theorem 4.1, that is, show that for any formula $\phi(v_1, ..., v_n)$ and any $x_1, ..., x_n \in M^{(B)}$ we have that $M^{(B)}/U \models \phi[x_1^U, ..., x_n^U]$ precisely when $[\![\phi(x_1, ..., x_n)]\!] \in U$. Here M is a transitive $\in$-model of ZFC, $B \in M$ is a complete Boolean algebra in M, and $U \subset B$ is an ultrafilter.

Exercise 3

[3 points] Let $\{a_i : i \in I\}$ be a partition of unity in B, and let $\{x_i : i \in I\} \subseteq V$ be such that $x_i \neq x_j$ whenever $i \neq j$. Show there is a $x \in V^{(B)}$ such that $[\![x = \hat{x}_i]\!] = a_i$ for all $i \in I$. *Hint: does this remind you of a certain Lemma?*