Exercise 1

Let $\mathcal{P}$ be a set of projections.

- ***(i)*** Let $P, Q \in \mathcal{P}$. Show that if P and Q commute, then they are compatible.
  (This part is worth 2 points)
- ***(ii)*** Let $\{P\} \cup \{Q_i \; i \in I\} \subseteq \mathcal{P}$. Show that if P is compatible with each Q_i for $i \in I$, the P is compatible with both $\bigvee_{i\in I} Q_i$ and $\bigwedge_{i\in I} Q_i$.
  Hint: First prove that P and Q are compatible, iff there exist $P_1, P_2 \in \mathcal{P}$ such that $P = P_1 \vee P_2, P_1 \leq Q$ and $P_2 \leq Q^c$
  (This part is worth 4 points)

Exercise 2

Let $u, v \in \mathbb{R}^{(\mathcal{B})}$ with A, B corresponding self-adjoint operators in $\bar{\mathcal{B}}$ and E_λ, E'_λ corresponding spectral families.
Show that $[\![u \leq v]\!] = 1$ if and only if $\forall\lambda \in \mathbb{R}(E'_\lambda \leq E_\lambda)$.
This property is equivalent to $A \leq B$ (not part of exercise). *Hint:* $[\![\hat{\mathbb{Q}} = \mathbb{Q}]\!] = 1$ *because they are both similarly defined from ω.*
(This part is worth 4 points)