Seminar on Set Theory

Hand-in exercise 2

September 25, 2015

1 Boolean Algebras and Propositional Logic

In the lecture, it was shown that if we are given a consistent theory T in a classical propositional language $\mathcal{L}$, then there is an associated Boolean algebra $B(T)$ known as the Lindenbaum algebra of T.

Prove that for any Boolean algebra B, there is a classical propositional theory T, such that B and $B(T)$ are isomorphic. You do **not** have to prove that your theory T is consistent. Hints:

1. Formulate the theory T in a classical propositional language $\mathcal{L}$ with propositional variables P_x for each $x \in B$.
2. Use Proposition 1.3 on the hand-out.
3. Use induction to show that your homomorphism is surjective.

2 Cantor's Theorem

As we have seen in the lecture, Zermelo disposes of Russel's paradox by proving the theorem that every nonempty set M has a subset M_0 that is not an element of M. Cantor's paradox in naive set theory originates from considering the (naive) set of all sets $U = \{x : x = x\}$. We dispose of this paradox by proving Cantor's Theorem: *For any set X the power set $\mathcal{P}(X)$ has a strictly greater cardinality than X itself.*

a) Prove Cantor's theorem using Zermelo's third axiom. (Hint: Follow Zermelo's proof of the theorem disposing of Russel's paradox, discussed in the lecture.)

b) Conclude that the set of all sets does not exist.