Homework 10

Student Seminar on Hilbert's Tenth Problem
Due December 2
Saskia van den Hoeven

1. Let $n, m \in \mathbb{Z}_+$. Prove that $n|m$ if and only if $p^n - 1|p^m - 1$.

2. Let $s, r \in \mathbb{Z}_+$ and let $s \geq 1$. Prove that $(p^{sr} - 1)/(p^s - 1) \equiv r \bmod p^s - 1$.

3. Prove that the relation $m = nk$ is Diophantine over $\mathbb{N}$ in the language $L_0 = \{0, 1, +, /_p, P, t\}$.

4. We have proven that the existential problem for $F[[t]]$ in the language $L = \{0, 1, +, \cdot, P, t\}$ is undecidable. Prove that for a ring R such that $F[t] \subset R \subset K((t))$, the existential problem for R in the language L is undecidable too.