

## 1 Register machine A - 2 points

We create a machine with 5 states and a stopping state and number the normal states 1 to 5. In every state, we decrease the value of register  $r_1$  and go the next state (after state 5 we will go to state 1). If  $r_1$  is equal to zero, we stay in the same state unless we are in state 3, then we go to the stopping state.

In this way, starting at state 1, we simply keep subtracting 5 and halt when we end at 2 in this way, giving us exactly what we want.

## 2 Register machine B - 1 point

The machine moves the value from  $r_1$  to  $r_2$ . It decreases  $r_1$  until it is zero and for every decrement, we increase  $r_2$ .

## 3 Register machine C - 4 points

First note that  $\binom{x}{2}$  is equal to the  $x$ -th triangle number, so what we are going to do is add the value of  $r_1$  to another register and then decrease  $r_1$  after which we repeat the process.

One state will decrease the value in  $r_1$ , going to a special state  $s_{end}$  when it is zero and going to a copy state  $s_{copy}$  otherwise. The state  $s_{copy}$  will then copy the value of  $r_1$  twice (while emptying  $r_1$ ), once to a register  $r_2$  and once to  $r_3$ . One of the copies will then be returned to register  $r_1$ , emptying one of the copy-registers. From then on, we will add the second copy register value to the answer-register  $r_4$  after which we return to the starting state that controls  $r_1$ .

In this way, we keep track of the value of  $r_1$ , which we want to be decreased in every ‘round’, while still maintaining that value and adding it to  $r_4$ . Now state  $s_{end}$  will make sure (together with some other states) that registers  $r_1, r_2, r_3$  are emptied.

## 4 Register machine D - 3 points

It computes  $2^x$ . State  $s_a$  puts a 1 in register  $r_2$  to be doubled, state  $s_0$  decreases  $r_1$  for every round of doubling  $r_2$ , the doubling happens in states  $s_1, s_2, s_3$  and the states  $s_{c1}, s_{c2}$  move the doubled value to  $r_2$  again.

## 5 How points are awarded

In every exercise, 1 point is awarded for a correct answer or a description of a machine that does the most important part of the computation. The other points are awarded for a good explanation of the answer and/or a detailed formal (or as precise as a formal) description.