SEMINAR ON CAMPANA POINTS

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This is a seminar on Campana points. We start by introducing the various different definitions present in the literature.

Talk 1 (Campana points on curves). Introduce the orbifold base of a fibration [Abr09, Definitions 2.1.3-2.1.4] and [Cam05, §1.2]. Introduce Campana points on curves [Abr09, §2.1.7] and [Cam05, §4.1]. Prove [Abr09, Proposition 2.1.8]. Give an overview of the orbifold Mordell conjecture[Cam05, §4.2], [Cam05, §3.4-3.5], [KPS22, Theorem 1.7]. Read also [Poo06] for inspiration.

Talk 2 (Campana points PSTVA). Introduce Campana orbifolds and Campana points. Cover [PSTVA21, §3.1, 3.2, 3.2.1] and [BBK⁺23, Example 3.8].

Talk 3 (Campana points on singular varieties). Compare the definitions of orbifold pairs and of Campana points from the papers [Str22, §2.1], [NS20, §2.1] and [MNS22, §2] (cf. [BBK⁺23, Example 3.8]). Does [Abr09, Proposition 2.1.8] hold in this general setting?

Talk 4 (Campana points AVA). Introduce Campana points as in [AVA18] and give an overview of the paper [AVA18].

Talk 5 (20 min) finish the proof of [AVA18]

Talk 6 (30 min) Proof of [Abr09, Proposition 2.1.8] and its generalization to singular varieties.

Talk 7 (Abramovich's constellations) Cover [Abr09, §2.2] in detail.

Talk 8 (Abramovich's firmaments) Cover [Abr09, §2.4] in detail.

Talk 9 (Darmon points and stacks, M-points). Introduce Darmon points as in [Poo06] and [MNS22, §2], clarify their relation to root stacks. If there is time, introduce M-points, and interpret the notions in [Poo06] in terms of M-points.

Talk 10 (Weak approximation for the Hilbert property) Based on [NS20].

Talk 11 (M-points and M-approximation on toric varieties) Based on upcoming work of Boaz Moerman.

Talk 12 (Campana points of bounded height) Based on [PS20, BBK⁺23, PSTVA21, Poo06].

Date: April 5, 2024.

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References

- [Abr09] D. Abramovich. Birational geometry for number theorists. In Arithmetic geometry, volume 8 of Clay Math. Proc., pages 335–373. Amer. Math. Soc., Providence, RI, 2009.
- [AVA18] D. Abramovich and A. Várilly-Alvarado. Campana points, Vojta's conjecture, and level structures on semistable abelian varieties. J. Théor. Nombres Bordeaux, 30(2):525–532, 2018.
- [BBK⁺23] F. Balestrieri, J. Brandes, M. Kaesberg, J. Ortmann, M. Pieropan, and R. Winter. Campana points on diagonal hypersurfaces. arXiv:2302.08164, 2023.
- [BVV12] T. D. Browning and K. Van Valckenborgh. Sums of three squareful numbers. Exp. Math., 21(2):204–211, 2012.
- [BY21] Tim Browning and Shuntaro Yamagishi. Arithmetic of higher-dimensional orbifolds and a mixed Waring problem. *Math. Z.*, 299(1-2):1071–1101, 2021.
- [Cam05] F. Campana. Fibres multiples sur les surfaces: aspects geométriques, hyperboliques et arithmétiques. Manuscripta Math., 117(4):429–461, 2005.
- [KPS22] S. Kebekus, J. V. Pereira, and A. Smeets. Failure of the Brauer-Manin principle for a simply connected fourfold over a global function field, via orbifold Mordell. *Duke Math. J.*, 171(17):3515–3591, 2022.
- [MNS22] V. Mitankin, M. Nakahara, and S. Streeter. Semi-integral brauer-manin obstruction and quadric orbifold pairs. arXiv:2209.15582, 2022.
- [NS20] M. Nakahara and S. Streeter. Weak approximation and the Hilbert property for Campana points. arXiv:2010.12555, 2020.
- [Poo06] B. Poonen. The projective line minus three fractional points. https://math.mit.edu/ ~poonen/slides/campana_s.pdf, 2006.
- [PS20] M. Pieropan and D. Schindler. Hyperbola method on toric varieties. arXiv:2001.09815, 2020.
- [PSTVA21] M. Pieropan, A. Smeets, S. Tanimoto, and A. Várilly-Alvarado. Campana points of bounded height on vector group compactifications. Proc. Lond. Math. Soc. (3), 123(1):57–101, 2021.
- [RTW21] E. Rousseau, A. Turchet, and J. T.-Y. Wang. Nonspecial varieties and generalised Lang-Vojta conjectures. Forum Math. Sigma, 9:Paper No. e11, 29, 2021.
- [Shu21a] A. Shute. Sums of four squareful numbers. arXiv:2104.06966, 2021.
- [Shu21b] A. Shute. On the leading constant in the Manin-type conjecture for Campana points. arXiv:2104.14946, 2021.
- [Str22] S. Streeter. Campana points and powerful values of norm forms. Math. Z., 301(1):627– 664, 2022.
- [Tan21] S. Tanimoto. Campana points, Height zeta functions, and log Manin's conjecture. arXiv:2102.12689, 2021.
- [VV12] K. Van Valckenborgh. Squareful numbers in hyperplanes. Algebra Number Theory, 6(5):1019–1041, 2012.

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