From curves to surfaces

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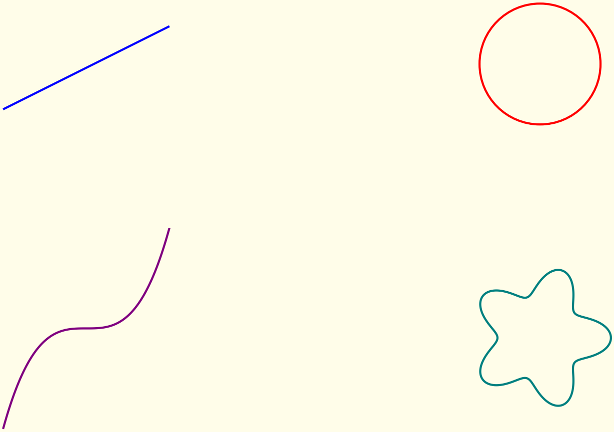
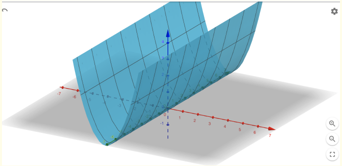
Curves

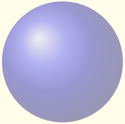
1 dimension

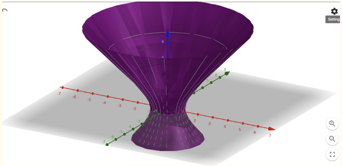


Surfaces

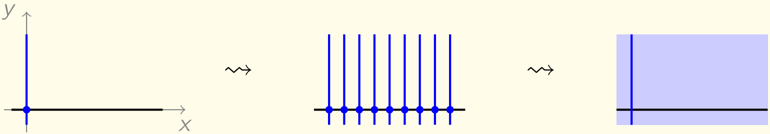
2 dimensions







Translation

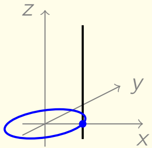


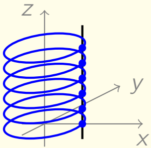
equation: $y = 0$
parameter: $(u, 0)$
equation: $x = 0$
parameter: $(0, v)$

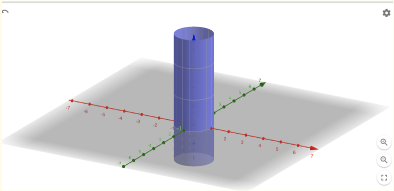
no equations,
parameters: (u, v)

Translation: sum of vectors $\quad (u, v) = (u, 0) + (0, v)$.

Translation: sum of vectors







GeoGebra: $(\cos x, \sin x, y)$

Circle: equations $x^2 + y^2 = 1$, $z = 0$

parameter $(\cos\theta, \sin\theta, 0)$

Line: equations $x = 1$, $y = 0$

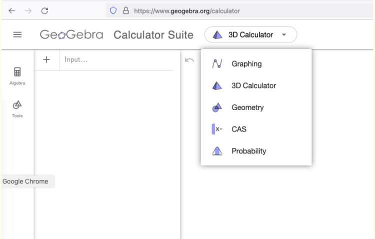
parameter $(1, 0, t)$ $\rightsquigarrow$ direction $(0, 0, t)$

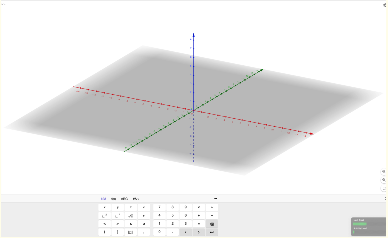
Cylinder: equation $x^2 + y^2 = 1$

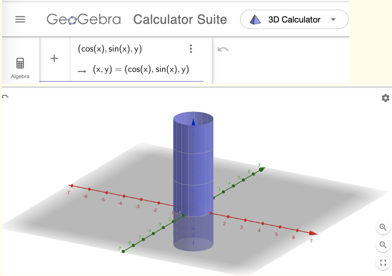
parameter $(\cos\theta, \sin\theta, t)$

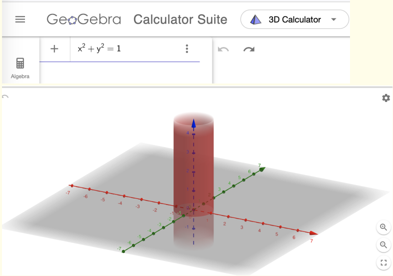
GeoGebra

https://www.geogebra.org/calculator

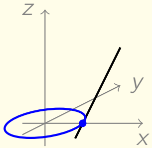


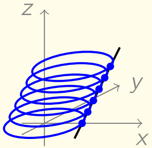


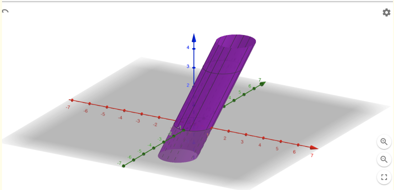




Translation: sum of vectors







GeoGebra: $(x + \cos y, \sin y, 2x)$

Circle: equations $x^2 + y^2 = 1$, $z = 0$

parameter $(\cos\theta, \sin\theta, 0)$

Line: equations $z = 2(x-1)$, $y = 0$

parameter $(t, 0, 2t-2)$

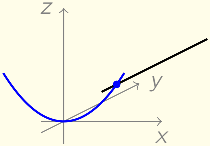
$\rightsquigarrow$ direction $(t, 0, 2t)$

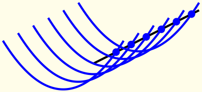
Cylinder: equation $(x - \frac{1}{2}z)^2 + y^2 = 1$

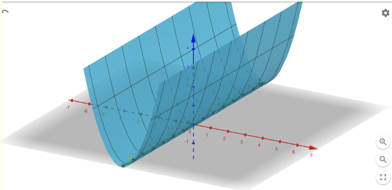
parameter $(t + \cos\theta, \sin\theta, 2t)$

Leaning Tower of Pisa

Parabolic cylinder







GeoGebra: (x, y, x^2)

Parabola: equations $z = x^2$, $y = 0$

parameter $(x, 0, x^2)$

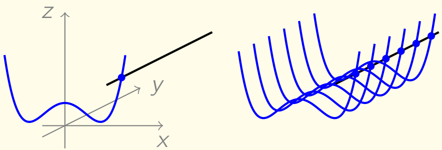
Line: equations $x = 1$, $z = 1$

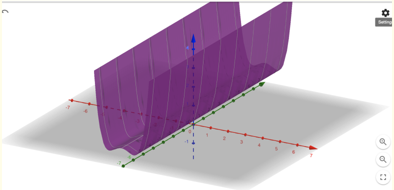
parameter $(1, t, 1)$ $\rightsquigarrow$ direction $(0, t, 0)$

Surface: equation $z = x^2$

parameter (x, t, x^2)

$z = x^4 - x^2$





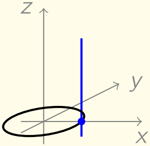
GeoGebra: $(x, t, x^4 - x^2)$

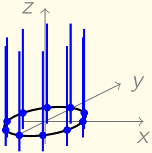
Curve: equations $z = x^4 - x^2$, $y = 0$
parameter $(x, 0, x^4 - x^2)$

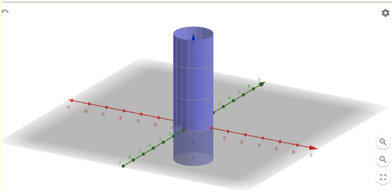
Line: equations $x = 2$, $z = 12$
parameter $(2, t, 12)$ $\rightsquigarrow$ direction $(0, t, 0)$

Surface: equation $z = x^4 - x^2$
parameter $(x, t, x^4 - x^2)$

Rotation: $(\alpha \cos\theta - \beta \sin\theta, \alpha \sin\theta + \beta \cos\theta, \gamma)$







GeoGebra: $(\cos x, \sin x, y)$

Circle: equations $x^2 + y^2 = 1$, $z = 0$

parameter $(\cos\theta, \sin\theta, 0)$

Line: equations $x = 1$, $y = 0$

parameter $(1, 0, t)$

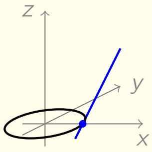
Cylinder: equation $x^2 + y^2 = 1$

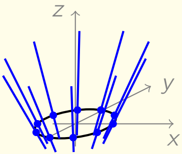
parameter $(\cos\theta, \sin\theta, t)$

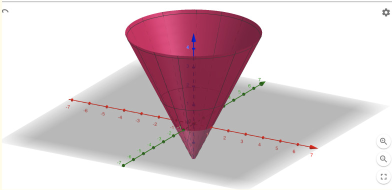
Rotation:

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\cos\theta, \sin\theta, 0), (\alpha, \beta, \gamma) \rightsquigarrow (\alpha \cos\theta - \beta \sin\theta, \alpha \sin\theta + \beta \cos\theta, \gamma)$$







GeoGebra: $(x\cos y, x\sin y, 2(x-1))$

Circle: equations $x^2+y^2=1$, $z=0$

parameter $(\cos\theta, \sin\theta, 0)$

Line: equations $z=2(x-1)$, $y=0$

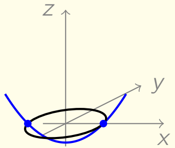
parameter $(t, 0, 2(t-1))$

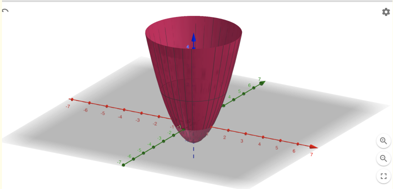
Cone: equation $x^2+y^2=(\frac{1}{2}z+1)^2$

parameter $(t\cos\theta, t\sin\theta, 2(t-1))$

$$(\cos\theta, \sin\theta, 0), (\alpha, \beta, \gamma) \rightsquigarrow (\alpha\cos\theta - \beta\sin\theta, \alpha\sin\theta + \beta\cos\theta, \gamma)$$

Paraboloid





GeoGebra: $(x \cos y, x \sin y, x^2 - 1)$

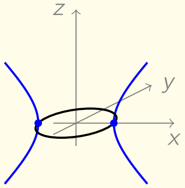
Parabola: equations $z = x^2 - 1$, $y = 0$
parameter $(x, 0, x^2 - 1)$

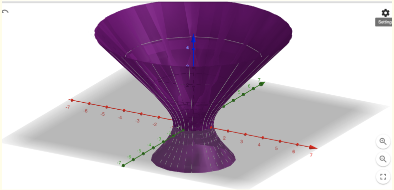
Circle: equations $x^2 + y^2 = 1$, $z = 0$
parameter $(\cos\theta, \sin\theta, 0)$

Paraboloid: equation $x^2 + y^2 = z + 1$
parameter $(t\cos\theta, t\sin\theta, t^2 - 1)$

Parabolic reflector

Hyperboloid





GeoGebra: $\left(\frac{\cos\theta}{\cos\alpha}, \frac{\sin\theta}{\cos\alpha}, \frac{\sin\alpha}{\cos\alpha}\right)$

Hyperbola: equations $x^2 - z^2 = 1$, $y = 0$

parameter $\left(\frac{1}{\cos\alpha}, 0, \frac{\sin\alpha}{\cos\alpha}\right)$

Circle: equations $x^2 + y^2 = 1$, $z = 0$

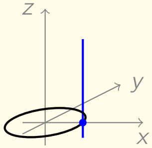
parameter $(\cos\theta, \sin\theta, 0)$

Hyperboloid: equation $x^2 + y^2 = z^2 + 1$

parameter $\left(\frac{\cos\theta}{\cos\alpha}, \frac{\sin\theta}{\cos\alpha}, \frac{\sin\alpha}{\cos\alpha}\right)$

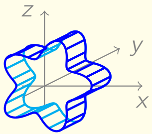
Cathedral of Brasília

Line along the circle: angle variation

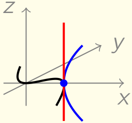


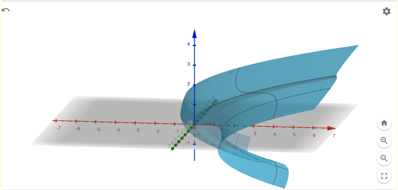
Coordinates $((z(sin(5\theta)+3)+1)cos(\theta), (z(sin(5\theta)+3)+1)sin(\theta), z)$

Not a rotation



Coordinates $((cos(5\theta)+3)cos(\theta), (cos(5\theta)+3)sin(\theta), y)$





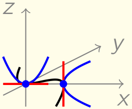
Translation $(z^2 + x, x^2 - x^3, z)$

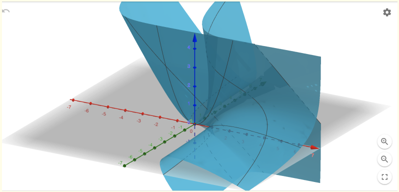
Parabola: equations $x = z^2 + 1$, $y = 0$

parameter $(z^2 + 1, 0, z)$

Cubic: equations $y = x^2 - x^3$, $z = 0$

parameter $(x, x^2 - x^3, 0)$





$(xz^2 + (1 - x)z + x, x^2 - x^3, xz + (1 - x)z^2)$

Supporting material

List of curves:
https://mathshistory.st-andrews.ac.uk/Curves/

List of surfaces:
http://www.math.rug.nl/models/

Virtual Math Museum:
https://virtualmathmuseum.org

GeoGebra:
https://www.geogebra.org/calculator