A Study of Accrual of Arguments, with Applications to Evidential Reasoning*

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ABSTRACT

This paper presents a logical formalisation of accrual of arguments as a form of inference. The formalisation is given within the logical framework of Dung as instantiated by Pollock, and is shown to satisfy three principles that any treatment of accrual should satisfy. The formalisation of accrual as inference is contrasted to knowledge-representation treatments of accrual. Also, the formalisation is applied to some concepts from the theory of evidential legal reasoning.

1. INTRODUCTION

A common intuition is that the more reasons one has to believe or do something, the stronger one's case is. The question is how this intuition can best be formally modelled. This question is relevant both for empirical and for practical reasoning. For example, in reasoning about evidence it often arises when accumulation of evidence is considered, in reasoning about interpretation of open-textured predicates it arises when there are multiple reasons for choosing a certain interpretation and in reasoning about whether a statutory rule should be set aside by principle of purpose it arises when there are several reasons for setting aside the rule. In the AI literature two approaches have been taken to the formalisation of accrual.

The first approach, to be called the *knowledge representation* or *KR approach*), encodes the accrual of reasons by hand, as a conditional with a conjunction of the accruing reasons in the antecedent. This is the approach taken by, for instance, Pollock [10], Prakken & Sartor [12, 13] and in Probabilistic Networks (PN's). The second approach (to be called the *inference approach*) instead regards accrual as a step in the inference process: after all relevant arguments based on individual reasons have been constructed, they are somehow aggregated and then a weighing mechanism decides the conflict between the two conflicting sets of reasons. This approach is taken by, for example, [7] in their Logic of Argumentation (LA), in Hage and Verheij's Reason-Based logic [5, 6, 16] and in Verheij's Cumula [15, 16].

Both approaches may have their pros and cons. The purpose of this paper is to enable a more complete comparison between formal accounts of accrual by formalising the inference approach within a combination of two standard argument-based logics: Dung's [4] abstract approach to argumentation instantiated with Pollock's approach to the structure of arguments (e.g. [8, 9, 10]). The proposed formalisation is especially inspired by Reason-Based Logic but adapts it to an argument-based setting. The formalism of Dung and Pollock's account of the structure of arguments are widely accepted as standard but to my knowledge no one has as yet given an inference-based account of accrual within their combination. The closest is Verheij's Cumula [15, 16] but, as will be discussed in Section 8, to deal with accrual this system modifies Pollock's tree structure of arguments in one respect. A further contribution of this paper is a set of three principles that any formal modelling of accrual, whether inference- or KR-based, should satisfy and that can be used in comparing different formalisations of accrual.

In this paper the term 'reason' will be used in an informal way, to make the discussion relevant for several formalisms. Depending on the formal setting, a reason roughly is the antecedent of a conditional or a premise of an argument. Reasons are assumed to be 'prima facie sufficient' in that when there are no reasons against a certain conclusion, any reason in favour is sufficient to warrant that conclusion.

It should be noted that in this paper accrual will be modelled as an aspect of 'quick-and-dirty' commonsense reasoning, where people reason under resource limitations and with coarse qualitative approximations to the truth. Perhaps when knowledge, situations and problems can be modelled with great precision and accuracy, a more sophisticated modelling of accrual is possible, but that does not detract from the significance of the present study for other cases.

This paper is organised as follows. After a brief discussion of logical preliminaries in Section 2, three principles of accrual will be proposed in Section 3. Then the KR approach to accrual will be described in more detail in Section 4 and three reasons for pursuing the inference approach will be discussed in Section 5. Up to this point the paper's main purpose is to systematise observations about accrual as they can be found throughout the literature. Section 6 presents the main original contribution of the paper, a formalisation

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of accrual as inference. In Section 7 the new proposal is applied to some examples of evidential reasoning. Section 8 concludes with, among other things, a discussion of related research.

2. LOGICAL PRELIMINARIES

The formal part of this paper is based on Dung's [4] abstract approach to defeasible argumentation instantiated with Pollock's [8, 9, 10] approach to the structure of arguments. Here these approaches will be sketched only semiformally; for technical details the reader is referred to the original sources or to [14]. The abstract framework of [4] assumes as input a set of arguments ordered with a binary defeat relation and defines various semantics for argument-based inference, all designating one or more conflict-free sets of arguments as argument extensions. Two often-used semantics are grounded semantics, used by e.g. [8] and [12], which always produces a unique extension, and preferred semantics, used by e.g. [10] and [2], which produces more than one extension when a conflict between arguments cannot be resolved. An argument can be called *justified* in grounded (preferred) semantics if it is a member of the unique (every) extension. In all examples of this paper grounded and preferred semantics produce the same justified arguments.

As for the structure of arguments, this paper follows Pollock's treatment, with some minor modifications regarding strength of arguments. Pollock instantiates Dung's abstract approach with a tree-structure of arguments, where arguments can be constructed by chaining two kinds of inference rules into trees, viz. *strict* rules, to be denoted by $\vdash$ and *defeasible* rules, to be denoted by $\mid\!\sim$. A special kind of inference rule is an *undercutter*, which says that in certain circumstances some defeasible inference rule does not apply. Accordingly, defeasible arguments can be defeated in two ways: they can be *rebut* with an argument for the opposite conclusion and they can be *undercut* with an argument that applies the relevant undercutting reason. Arguments can be directly defeated on their final conclusion or inference step but also indirectly on intermediate ones; cases of the latter types will be called 'subargument defeat'. Rebutting conflicts between arguments are assumed to be adjudicated with some unspecified domain-specific preference relation on arguments, possibly defined in terms of a preference relation on its premises. In fact, Pollock defines strength of arguments in terms of probabilities, but to make the present discussion more widely relevant I will abstract from the precise nature of strength. For example, Pollock's treatment of strength can be easily adapted to qualitative partial orderings of premises of arguments.

In this paper, the strict inference rules are assumed to be those of standard deductive logic. Pollock distinguishes several defeasible inference rules, such as principles of perception, memory, induction, temporal persistence and statistical reasoning. The theory of this paper will hold for any set of defeasible inference rules. In examples two defeasible rules will be used, viz. temporal persistence and a qualitative counterpart of Pollock's statistical syllogism, operating on qualitative defeasible conditionals. Such conditionals will be expressed with a two-place connective $\Rightarrow$ and are assumed to satisfy just one inference rule, viz. defeasible modus ponens:

$$\varphi, \varphi \Rightarrow \psi \mid\!\sim \psi$$

A simple way will be used to express undercutters of defeasible modus ponens. Following a standard naming convention in nonmonotonic logic, each defeasible conditional $\varphi \Rightarrow \psi$ containing free variables $x_1, \ldots, x_n$ is assumed to have a unique name $n(x_1, \ldots, x_n)$. Then formulas of the form $Exception(n(t_1, \ldots, t_n))$ (where t_i is a ground substitution of x_i) can be used to express that defeasible modus ponens for this ground instance of n is undercut.

Temporal persistence can roughly be formalised as follows (see [11] for more details):

> $\ulcorner\varphi$ is true at time $t_1\urcorner$, $t_2 > t_1 \mid\!\sim \ulcorner\varphi$ is true at time $t_2\urcorner$

Obviously, the strength of this reason decreases when the length of the time interval between t_1 and t_2 increases. Pollock also considers a 'backward' version of temporal persistence, where $t_2 < t_1$, and he adds to both principles a condition that φ is "temporally projectible". Finally, Pollock formulates the following undercutter for both reasons:

> $\ulcorner\neg\varphi$ is true at time $t_3\urcorner$, $t_1 < t_3 < t_2 \mid\!\sim \neg\ulcorner\varphi$ is true at time $t_2\urcorner$

In the backward version the second condition is $t_2 < t_3 < t_1$.

3. PRINCIPLES OF ACCRUAL

In this section three principles will be formulated that any satisfactory treatment of accrual (whether KR- or inference-based) should satisfy. As for terminology, the decision or belief which is the subject of a reason or argument will be called a *claim*, a combination of reasons or arguments for the same claim will be called an *accrual* and the elements of such a combination the *accruing* reasons or arguments.

Of course, the obvious starting point of any formalisation of accrual is that adding more reasons can make one's case stronger. The following three principles are meant to constrain formalisations of this idea.

3.1 Accruals are sometimes weaker than their elements

The first principle is that an accrual is sometimes weaker than its accruing elements. This is due to the possibility that accruing reasons are not independent. Consider by way of example two reasons not to go jogging, viz. that it is hot and that it is raining [13]. For a particular runner the combination of heat and rain may be less unpleasant than heat or rain alone so that the accrual is a weaker reason not to go running than the accruing reasons. And for another jogger the combination of heat and rain may be so pleasant that it is instead a reason to go jogging. Another example is two witnesses who make the same statement [10, pp. 101-2]. If the witnesses are from a group of people who are more likely to confirm each other's statements when these statements are false than when they are true, the accrual will be weaker than the accruing reasons.

Note that the possibility of interacting reasons implies that in general the strength of an accrual cannot be calculated from the strengths of its elements.

3.2 An accrual makes its elements inapplicable

When an accrual of reasons or arguments is applicable, that is, when there are no convincing grounds to reject the accruing elements as individual reasons or arguments, then the accrual makes its elements inapplicable, even if in the end the accrual is outweighed by the conflicting accrual. More generally, any 'larger' accrual that applies makes all its 'lesser' versions inapplicable. This is since a comparison between accruals for and against a claim is meant to consider all available information, while the individual reasons or arguments take only part of the information into account. Consider again the jogging example, assume that the accrual of rain and hot is weaker than its elements and suppose that jogger John regards the rain&hot accrual as outweighed by, say, the argument that he has not been running for two days. Then it should not be possible that the latter reason is defeated by the individual reason not to go jogging that it is raining. Note, finally, that this principle is vacuous if accruals are never weaker than their elements.

3.3 Flawed reasons or arguments may not accrue

Any treatment of accrual should capture that when an individual reason or argument turns out to be flawed, it does not take part in the accrual. One way in which a reason may be flawed is that the reason turns out not to be true. For instance, the observation that it rains may be flawed (the window was wet since mother was cooking in the kitchen). This is an example of subargument defeat. A reason may also be flawed itself. For example, suppose that one of the witnesses turns out to be mentally disabled. In that case his statement is not a reason to believe what he says is true. This is an example of undercutting defeat. In both cases the flawed reason should not be allowed to accrue with other reasons for the same claim. This principle induces a two-stage argumentation process. First all the individual reasons for a certain claim are tested to see whether they may enter the accrual, then all the reasons that pass this test are accrued and compared to all accruing reasons for the opposite claim.

4. THE KNOWLEDGE-REPRESENTATION APPROACH

One way to formalise the intuition that reasons or arguments accrue is to encode this in the representation of a certain problem. It is instructive to see how this method is in fact enforced by the formalism of Probabilistic Networks (PN's). PN's are acyclic directed graphs where the nodes stand for statistical variables and the links capture probabilistic dependencies. If these dependencies are quantified as numerical probabilities, and if also prior probabilities are assigned to the node values (assigning probability 1 to the node values that represent the available evidence), then the conditional probability concerning certain nodes of interest given a body of evidence can be calculated according to the laws of probability theory, including Bayes' rule. More precisely, each node of a PN has a Conditional probability Table (CPT) which for each of its possible values and for each combination of values of all its parent nodes specifies the conditional probability that the node has this value given this particular combination of values of its parents. Let us illustrate this with a simple PN with three boolean variables.

The CPT then contains the following conditional probabilities:

$P(p \mid q\&r)$
$P(p \mid q\&\neg r)$
$P(p \mid \neg q\&r)$
$P(p \mid \neg q\&\neg r)$
(the conditional probabilities for $\neg p$ can be computed from this information).

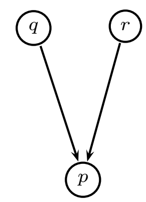


Figure 1: A simple probabilistic network

Accrual of, say, q and r as reasons for p is then modelled as follows. If both $P(p \mid q\&r) > P(p \mid q\&\neg r)$ and $P(p \mid q\&r) > P(p \mid \neg q\&r)$ then q and r may be said to genuinely accrue, otherwise we are in some exceptional situation. Note that this method obviates the need for accrual as an inference step: CPT's completely specify all possible situations by completely specifying the antecedents of the conditional probabilities: in all possible situations precisely one of these antecedents is satisfied (Vreeswijk [18] calls this the "antecedent-completeness" of CPT's). So accrual takes place by a simple look up in the CPT.

In the setting of nonmonotonic logic antecedent-completeness is not something to aim for, since such logics are meant to enable inferences in situations with incomplete information. Nevertheless, in such logics it is still possible to combine statements about the presence or absence of reasons for a claim in the antecedent of a conditional. If the same is done for reasons against the claim, then accrual boils down to defining strength relations between two conflicting rules.

For example, the jogging example can be formalised as follows:

r_1: *rain* $\wedge \neg$ *hot* $\Rightarrow \neg$ *jogging*
r_2: $\neg$ *rain* $\wedge$ *hot* $\Rightarrow \neg$ *jogging*
r_3: *rain* $\wedge$ *hot* $\Rightarrow$ *jogging*
r_4: *3daysnorun* $\Rightarrow$ *jogging*

Accrual is now captured by defining rule priorities between the rules. In this case r_3 must be given higher priority than r_1 and r_2 to reflect that the strength of the accrual is less than that of its elements. The 'normal' case of accrual is exemplified by the following slight modification of the example:

r_1: *rain* $\wedge \neg$ *hangover* $\Rightarrow \neg$ *jogging*
r_2: $\neg$ *rain* $\wedge$ *hangover* $\Rightarrow \neg$ *jogging*
r_3: *rain* $\wedge$ *hangover* $\Rightarrow \neg$ *jogging*
r_4: *3daysnorun* $\Rightarrow$ *jogging*

The normal case of accrual, where an accrual is stronger than its elements, is captured by giving r_3 higher priority than both r_1 and r_2 (but r_3 may still have lower priority than r_4).

Note that this representation does not make rules antecedent-complete, since r_1, r_2 and r_3 may apply even if nothing is known about *3daysnorun*. In a PN the propositions *3daysnorun* and its negation would be added to the antecedents of the conditional probabilities, resulting in six conditional probabilities for $\neg$ *jogging*.

One might object to this formalisation by saying that it does not capture the default nature of of rules r_1 and r_2 where, for example, r_1 means that "all other things being equal", rain is a reason not to go jogging. To capture this, the negated conditions in the rule antecedents could be replaced by default assumptions with some suitable formal mechanism. In the present formal setting the undercutter of defeasible modus ponens can be used. The formalisation is then modified as follows.

r_1: $\mathit{rain} \Rightarrow \neg\, \mathit{jogging}$
r_2: $\mathit{hot} \Rightarrow \neg\, \mathit{jogging}$
r_3: $\mathit{rain} \wedge \mathit{hot} \Rightarrow \mathit{jogging}$
r_4: $\mathit{rain} \supset \mathit{Exception}(r_2)$
r_5: $\mathit{hot} \supset \mathit{Exception}(r_1)$
r_6: $\mathit{3daysnorun} \Rightarrow \mathit{jogging}$

r_4 and r_5 express, respectively, that rain is an exceptional case of heat and heat is an exceptional case of rain. Now r_1 and r_2 are applicable if nothing is known about heat, respectively, rain. Note that the formalisation makes an individual reason inapplicable in case the accrual applies (thus giving one possible formalisation of the second principle of accrual).

Both these representation methods satisfy the three principles of accrual. They allow for accruals that are weaker than their elements (reflected in the rule preferences assigned to them), the accruing elements do not apply if the accrual applies (this holds trivially in the variant with antecedent completeness) and flawed reasons do not take part in an accrual (but see below).

5. THE INFERENCE APPROACH – WHY PURSUE IT?

If the KR approach satisfies the three principles of accrual of Section 3, why should we want to develop the inference approach? One reason that suggests itself is that in the KR approach more rules need to be formulated than in the inference approach: for each possible combination of reasons for a claim a rule must be formulated. Note that it does not suffice to formulate an extra rule just for the maximal accrual, since one of the accruing reasons may turn out to be flawed. However, this may not be such a serious disadvantage since the extra rules can be generated automatically from the single-reason rules and the KR and inference approach share the problem that the strength of each combination of reasons must be separately determined.

Another reason to pursue the inference approach is that the KR approach requires that each reason can be expressed with the same kind of conditional operator. In reality it may be that reasons of entirely different types accrue, such as applications of defeasible modus ponens and temporal persistence (see Section 7.2 below), or a teleological and a linguistic argument in a statutory interpretation problem. (This is why Pollock's suggestion in [10] to use the KR approach is not so easy to formalise within his own system, since it is based on a rich typology of defeasible inference schemes instead of on just one type of defeasible conditional).

A third and perhaps strongest reason to develop the inference approach, which to my knowledge has not been stated before in the literature, has to do with the treatment in the KR approach of the second case of flawed reasons. The following argument further develops observations in Section 6.4 of [3]. The first case, when a reason is flawed because of subargument defeat, can be naturally modelled in the KR approach. Suppose the following reasons are added to the jogging example.

r_7: $\mathit{wetwindow} \Rightarrow \mathit{rain}$
r_8: $\mathit{wetwindow} \wedge \mathit{cooking} \supset \mathit{Exception}(r_7)$

Suppose also that, except for *rain*, all antecedents of all rules are given as a fact. Then an argument can be built for *rain* with r_7 which can be extended to an accrual argument for $\neg\, \mathit{jogging}$ with r_3. However, the argument for *rain* is undercut by an argument using r_8, so that the argument using r_7 and r_3 for $\neg\, \mathit{jogging}$ is subargument-defeated. So in this case r_2 in fact expresses the accrual. However, in the second case of flawed reasons, if one of the accruing reasons is undercut, things are different. Let us represent the two-witness example as follows (leaving a naming convention of formulas implicit and letting the context disambiguate between terms and well-formed formulas).

r_1: $\mathit{testifies}(x, \varphi) \Rightarrow \varphi$
r_2: $\mathit{testifies}(x, \varphi) \wedge \mathit{testifies}(y, \varphi) \wedge x \neq y \Rightarrow \varphi$
r_3: $\mathit{testifies}(x, \varphi) \wedge \mathit{testifies}(y, \varphi) \wedge x \neq y \supset \mathit{Exception}(r_1(x, \varphi))$

The third rule expresses that if two witnesses testify to the same fact, then the rules for individual witness testimonies cannot be used. Thus r_3 formalises the second principle of accrual. However, this is not all. Suppose that two different witnesses Andy and Bob testified to the same proposition p:

f_1: $\mathit{testifies}(a, p)$
f_2: $\mathit{testifies}(b, p)$
f_3: $a \neq b$

Now r_1 is blocked for both Andy and Bob and only r_2 applies to their testimonies. But how should then the possibility that one of the witnesses is unreliable be represented? It does not suffice to formulate the following rule.

r_4: $\mathit{unreliable}(x, \varphi) \supset \mathit{Exception}(r_1(x, \varphi))$

The problem with this is that if we also have

f_4: $\mathit{unreliable}(b, p)$

then r_3 applies and r_4 has no effect so the testimony of the unreliable witness Bob still takes part in the accrual. So r_4 must be duplicated for each combination of witnesses. In other words, the dispute about the reliability of an individual witness must be formalised as a dispute about the quality of the accrual. This seems unnatural: in realistic examples people will discuss the reliability of individual witnesses and if a witness turns out to be unreliable, simply disallow this statement to enter the accrual. This is not nicely captured by saying that the witnesses' unreliability makes all many-witness conditionals in which it takes part inapplicable. What seems more natural is an approach in which first the reliability of all individual witness statements can be debated after which the witnesses that turn out to be reliable may enter the accrual and are weighed against counterevidence. More generally, it seems less natural to represent undercutters of individual reasons as undercutters of all accruals in which this reason takes part.

6. A FORMALISATION OF ACCRUAL AS INFERENCE

In this section a way will be proposed to model accrual as inference in the setting of Section 2. It is inspired by Reason-Based Logic's mechanism for 'weighing reasons' [5, 6, 16] but adapts it to an argument-based approach.

6.1 The idea

The basic idea is to label the conclusion of each individual defeasible inference step with the premises of the applied defeasible inference rule and to introduce a new defeasible inference rule that takes any set of labelled versions of a certain formula and produces the unlabelled version. All other inference rules only operate on unlabelled formulas. This leads to a reasoning process as in Reason-Based Logic, where (when regarded bottom-up) each time a defeasible conclusion is drawn, first all reasons for and against it are combined into two conflicting accruals, then the conflict between the two accruals is adjudicated, and only then the 'winning' defeasible conclusion can be used in the rest of the reasoning process. Recall further that as in Reason-Based Logic the accrual is a two-stage process, where first it is determined which reasons may enter the accrual and then the accrual is weighed against the conflicting accrual. The reason why only defeasible arguments should accrue is that accrual of deductive arguments does not make sense since they are on their own sufficient to prove their conclusion from their premises.

In formalising these ideas, the main difficulty is to reconcile the weakest-link principle that an argument is defeated if one of its subarguments is defeated with the principle of accrual that one can strengthen one's case by combining reasons that on their own may be too weak to justify their conclusion. This is a difficulty if one wants to model accrual as an inference step. Consider the following abstract example (where $A{+}B$ denotes the accrual of A and B.)

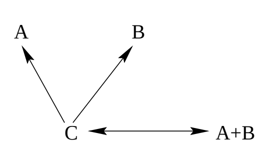
EXAMPLE 6.1.

C defeats A
C defeats B
$A{+}B$ defeats C

Intuitively we want that $A{+}B$ is justified while C is overruled. However, if A and B are regarded as subarguments of $A{+}B$ and the principle is applied that an argument is defeated if one of its subarguments is defeated, then we also have that

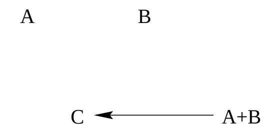
C defeats $A{+}B$

which results in the following defeat graph:



And in neither of the Dung-style semantics $A{+}B$ is justified.

The present solution is that since each conclusion based on an individual reason is labelled, it cannot be rebut by any argument for the negated conclusion, since the conclusion of that argument will have a different label. Hence in the example C does not defeat A and B any more, so it neither defeats $A{+}B$. This results in the following defeat graph:



In the new graph all semantics make A, B and $A{+}B$ justified and C overruled. More generally, in the new approach the only ways to defeat arguments based on individual reasons are to subargument-defeat or undercut them.

6.2 The formalisation

The labelling of well-formed formulas is defined as follows. Given two sets of strict and defeasible inference rules, the defeasible ones are slightly reformulated to the effect that their conclusions are labelled with the set of their premises. So, for instance, defeasible modus ponens for $\Rightarrow$ will look as follows:

$$\varphi, \varphi{\Rightarrow}\psi \mathrel{|\!\sim} \psi^{\{\varphi, \varphi\Rightarrow\psi\}}$$

In the examples below the labels will for readability often be abbreviated to $a, b, c, \ldots$ or $l_1, l_2, \ldots$.

Next the definitions of conflicts between arguments are adjusted such that for rebutting the opposite conclusions must either be both unlabelled or have the same labels and that undercutting attack requires that the attacking arguments have unlabelled conclusions.

Next a new accrual inference rule is added to the system, of the following form (in fact, the rule is a scheme for any natural number i such that $1 \leq i \leq n$):

$$\varphi^{l_1}, \ldots, \varphi^{l_n} \mathrel{|\!\sim} \varphi \text{ (Accrual)}$$

Also, arguments are now required to have subset-minimal sets of premises to infer their conclusion, otherwise many irrelevant arguments would enter an accrual.

Finally, to respect the second principle of accrual, the following undercutter scheme is formulated for any i such that $1 \leq i \leq n$.

$$\varphi^{l_1}, \ldots, \varphi^{l_n} \mathrel{|\!\sim} \neg\lceil \varphi^{l_1}, \ldots, \varphi^{l_{n-i}} \mathrel{|\!\sim} \varphi \rceil \text{ (Accrual-undercutter)}$$

The latter says that when a set of reasons accrues, no proper subset accrues.

6.3 Verifying the principles of accrual

Let us next see how the new inference rule and undercutter for accrual satisfy the three principles of Section 3. To start with, note that the new formalism does not make any assumptions on the relative strength of accruals, so that the first principle is satisfied. Also, the second principle is satisfied by the accrual undercutter. Finally, as for the third principle, note that the accrual inference rule allows the accrual of any set of arguments for a conclusion, not just of the largest such set, so that if any reason in the antecedent of the accrual undercutter is flawed, the undercutter is subargument-defeated and one of the 'lesser' accruals that it intends to undercut may be reinstated. The schematic example of Figure 2 illustrates this. On the left there is an accrual argument for p, accruing three arguments for p based on different reasons. On the right there is an

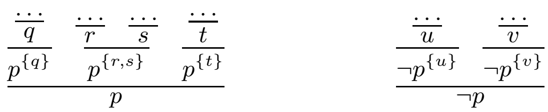


Figure 2: Accruals

accrual argument for $\neg p$, accruing two different arguments for $\neg p$. The two arguments attack each other on their last steps only, since the labelled versions of p and $\neg p$ are not contradictory. To illustrate how this deals with flawed reasons, suppose that the argument for $p^{\{q\}}$ is undercut and the argument for u is rebut. Then both accruals are flawed, which is formalised by the fact that they are subargument-defeated. So their narrowings of Figure 3 must be considered. Note that, although these two accruals are defeated

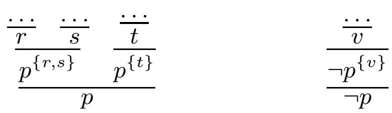


Figure 3: Lesser accruals

by their larger versions in Figure 2, they are reinstated by the fact that their larger versions are subargument-defeated.

6.4 A refinement

There are cases in which something is a reason on its own for a certain conclusion but can also be strengthened by something else that it not on its own a reason for that conclusion. For example, suppose that a witness says that p and that the witness proved to be reliable in the past. The latter fact is not on its own a reason for p but it strengthens his current testimony as a reason for p. Let us formalise this as follows.

r_1: $\mathit{testifies}(x, \varphi) \Rightarrow \varphi$
r_2: $\mathit{testifies}(x, \varphi) \wedge \mathit{reliable\text{-}before}(x) \Rightarrow \varphi$
f_1: $\mathit{testifies}(j, p)$
f_2: $\mathit{reliable\text{-}before}(j)$

We do not want that the arguments constructed with r_1 and with r_2 accrue, since r_2 already takes the witness testimony into account and does so in a more fully specified situation. Therefore, the applicability of r_2 should block the application of r_1 so that only r_2 enters the accrual for p. This can be achieved with a second undercutter for defeasible modus ponens, which is analogous to Pollock's [10] subproperty defeater for his statistical syllogism:

$$\varphi \wedge \chi, \varphi \wedge \chi \Rightarrow \psi \mid\!\sim \neg\ulcorner \varphi, \varphi \Rightarrow \psi \mid\!\sim \psi \urcorner$$

This type of case is related to another issue. It may be argued that certain types of defeasible reasons do not accrue. For instance, [17] argue that statutory rules usually do not enter into accruals with other reasons since such rules are often meant to express the resolution of a weighing process between competing sets of reasons. They give representation techniques within Reason-Based Logic to block unwanted accruals. Similar techniques could be used in the present method. Suppose r_1 is a rule for p that should not accrue while r_2 and r_3 express reasons for q and $\neg q$ that may accrue. The latter two reasons can be blocked as follows.

r_1: $p \Rightarrow q$
r_2: $r \Rightarrow q$
r_3: $s \Rightarrow \neg q$
r_4: $p \Rightarrow \mathit{Exception}(r_2) \wedge \mathit{Exception}(r_3)$
r_5: $\mathit{Exception}(r_1) \supset \mathit{Exception}(r_4)$

Thus r_2 and r_3 are blocked when r_1 applies, unless the application of r_1 is undercut; in the latter case r_4 is also undercut because of r_5.

7. APPLYING THE NEW INFERENCE RULES

In this section the new inference model of accural will be applied to some typical examples of evidential reasoning.

7.1 Examples with witness testimonies

Let us illustrate the new rules with some examples involving witness testimonies, assuming the following rules.

r_1: $\mathit{testifies}(x, \varphi) \Rightarrow \varphi$
r_2: $\mathit{unreliable}(x, \varphi) \supset \mathit{Exception}(r_1(x, \varphi))$

EXAMPLE 7.1. The first example (visualised in Figure 4) is with four witnesses Andy, Bob, Carl and Derek who all testify that p and two witnesses Emily and Fanny who both testify that $\neg p$. It turns out that Andy and Derek conferred while Fanny has lied on earlier occasions where she had to testify. Let us add the following rules on reliability.

r_3: $\mathit{conferred}(x, y, \varphi) \Rightarrow \mathit{unreliable}(x, \varphi)$
$r_{3'}$: $\mathit{conferred}(x, y, \varphi) \Rightarrow \mathit{unreliable}(y, \varphi)$
r_4: $\mathit{lied\text{-}before}(x) \Rightarrow \mathit{unreliable}(x, \varphi)$
f_1: $\mathit{conferred}(a, d, p)$
f_2: $\mathit{lied\text{-}before}(f, \neg p)$

Applying r_1 to the facts we defeasibly obtain four labelled versions of p, viz. $p^{\{a,r_1\}}, p^{\{b,r_1\}}, p^{\{c,r_1\}}$ and $p^{\{d,r_1\}}$ and two labelled versions of $\neg p$, viz. $\neg p^{\{e,r_1\}}$ and $\neg p^{\{f,r_1\}}$. Applying the accrual rule to the first four formulas results in a defeasible argument A_4 for p and applying the same rule to the last two formulas results in a defeasible argument B_2 for $\neg p$. Although these arguments rebut each other, we do not need to make a choice between them. The reason is that applying r_3 to the facts results in an argument for $\mathit{unreliable}(a, p)^{\{f_1, r_3\}}$ which, with r_2 and a trivial application of the accrual inference rule, can be extended to an argument for $\mathit{Exception}(r_1(a, p))$ and in the same way it results in an argument for $\mathit{Exception}(r_1(d, p))$. So the arguments for $p^{\{a,r_1\}}$ and $p^{\{d,r_1\}}$ are undercut. This in turn makes A_4 subargument-defeated. In an entirely similar way B_2 can be subargument-defeated by applying r_4 to the facts. (The main arguments considered thus far are displayed in the first two rows of Figure 4.) So 'lesser' accruals must be considered (see the last row of Figure 4). The only non-flawed accruals are those of p^b and p^c, accruing to an argument A_2 for p, and of $\neg p^e$, accruing to an argument B_1 for $\neg p$. Note in particular that the instances of the accrual undercutter undercutting A_2 and B_1 are themselves subargument-defeated by the above exception arguments, since these undercutters have the same subarguments as A_4 and B_2. Since both A_2 and

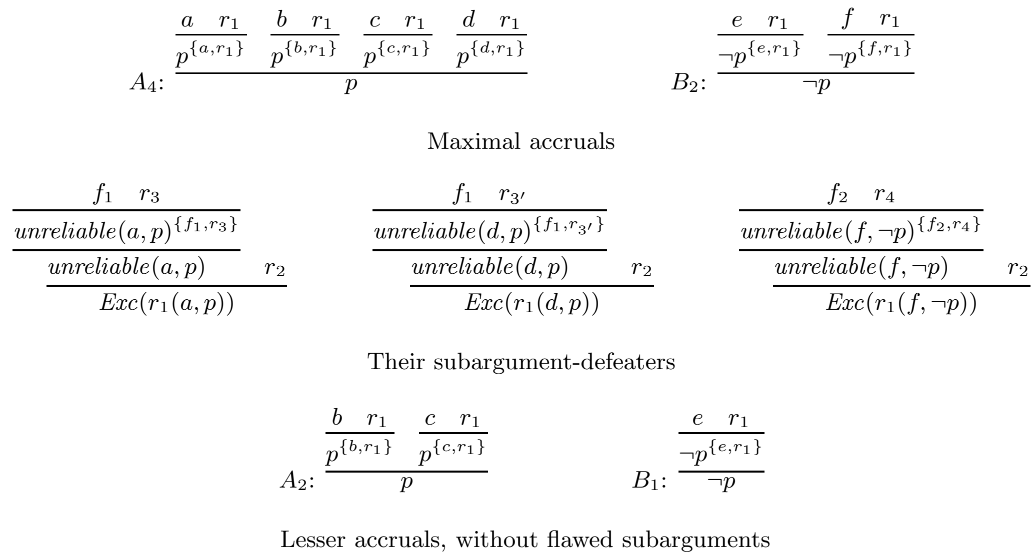


Figure 4: Some arguments in Example 7.1

B_1 have no attackers besides each other, their conflict must be decided by applying (possibly problem-specific) weighing information, expressed as argument priorities. Suppose that B_1 is preferred over A_2. Then the issue about p is settled, since the single-reason accruals of $p^{\{b,r_1\}}$ and $p^{\{c,r_1\}}$ into arguments for p are both undercut by the accrual undercutter applied to the combination of $p^{\{b,r_1\}}$ and $p^{\{c,r_1\}}$.

EXAMPLE 7.2. Example 7.2 can be extended by assuming that even though Andy and Derek conferred, their testimonies are corroborated by the testimonies of Bob and Carl, who independently declared the same as Andy and Derek. One way to model this is to add the following rules.

r_5: $\mathit{conferred}(x, y, \varphi) \wedge \mathit{confirms}(z, x, \varphi) \supset \mathit{Exception}(r_3(x, \varphi))$
r_6: $\mathit{testifies}(x, \varphi) \wedge \mathit{testifies}(y, \varphi) \wedge \mathit{independent}(x, y, \varphi) \Rightarrow \mathit{confirms}(y, x, \varphi)$
r_7: $\mathit{unreliable}(y, \varphi) \supset \mathit{Exception}(r_6(x, y, \varphi))$
r_8: $\mathit{conferred}(x, y, \varphi) \supset \neg\, \mathit{independent}(x, y, \varphi)$

With the facts of our extended example, r_6 gives rise to undefeated arguments for $\mathit{confirms}(b, a, p)$, $\mathit{confirms}(b, d, p)$, $\mathit{confirms}(c, a, p)$ and $\mathit{confirms}(c, d, p)$. Then r_5 gives rise to undefeated arguments for $\mathit{Exception}(r_3(a, p))$ and $\mathit{Exception}(r_3(d, p))$, which blocks the application of r_3 to Andy and Derek. Hence the application of r_1 to these witnesses cannot be blocked by r_2, so both Andy and John can take part in the accrual with Bob and Carl for p. Note, however, that if for some reason Bob and Carl turn out to be unreliable, the arguments that they confirm Andy and Derek are undercut, which makes all testimonies for p defeated. Note, finally, that r_8 prevents two conferring witnesses from corroborating each other's testimonies.

This notion of confirmation is not the same as accrual. When two conferring witnesses are confirmed by an independent witness, the fact that they conferred ceases to be a reason to disregard their testimonies but when they accrue with the independent witness, the fact that they conferred may still reduce the strength of the accrual.

EXAMPLE 7.3. Consider next an example in which the witnesses testify to propositions that are similar but not identical. Suppose in a certain criminal case the issue is whether the suspect was in Amsterdam on the morning of October 22nd, 2004 and consider a witness Albert who testifies he saw the suspect in the Vondelpark at 9.00am that day and a witness Bill who testifies that he saw the suspect at Central Station at 10.30am that day. When combined with some obvious background knowledge, both statements on their own deductively imply that the subject was in Amsterdam that morning. Therefore, the two witness testimonies cannot be combined in an accrual for the latter proposition, since strict inferences do not accrue. Schematically:

A: $\mathit{testifies}(a, V(s, 9.00))$, so presumably $V(s, 9.00)^a$,
so with accrual $V(s, 9.00)$,
so deductively $A(s, \mathit{morning})$

B: $\mathit{testifies}(b, S(s, 10.30))$, so presumably $S(s, 10.30)^b$,
so with accrual $S(s, 10.30)$,
so deductively $A(s, \mathit{morning})$

At first sight, the result that the witness testimonies do not accrue with respect to the proposition *morning* would seem counterintuitive. Yet the principle that accrual of strict arguments does not make sense is beyond doubt so that the intuition must be at fault. The counterintuitive feel of this example may be due to a too strong reading of the conclusion as saying that the suspect was in Amsterdam for all or the larger part of the morning. This is not what the proposition says: it says no more than that there is a time point in the morning at which the suspect was in Amsterdam.

7.2 Some concepts of evidential reasoning

In this section it will be illustrated how the present formalisation of accrual allows a formal account of some concepts of the theory of evidential reasoning. In [1, Ch. 2] three types of combinations of evidence are distinguished: *combining*, *corroborating* and *converging* evidence. These concepts are illustrated with the following example. A person Y was murdered in a house at 4:45pm (C_1). Two witnesses had seen the suspect Mr. X. enter the house at 4:30pm (T_1 and T_2) and a third witness had seen X leave the house at 5:00pm (T_3). The proposition to be proven is "It was X who murdered Y" (P). Then the statements of the first two witnesses *corroborate* to support the proposition that X entered the house at 4:30pm (C_2): each of the individual statements already supports this proposition but their combination may provide more support for the same proposition. The third witness testimony individually supports the proposition that X left the house at 5:00pm (C_3). Together, the propositions C_2 and C_3 that X entered the house at 4:30pm and left the house at 5:00pm support the proposition that X was in the house at 4:45pm (C_4). Thus the witness testimonies T_1 and T_2 on the one hand and T_3 on the other *converge* to support C_4. The difference with corroboration is that converging testimonies testify to different propositions. According to [1] the practical significance of the distinction is that corroborating evidence can only be doubted by attacking the credibility of the witnesses, while converging evidence can also be attacked on other grounds: for instance, in this example the inference of C_4 could be attacked with evidence that X left the house at 4:40pm (the reader will recognise an application of the undercutter of forward temporal persistence). The propositions C_1 that Y was murdered in the house at 4:45pm and that X was in the house at 4:45pm are then conjoined to 'Y was murdered in the house at 4:45pm and X was in the house at 4:45pm' (C_5). Thus the evidence for C_4 (the three witness testimonies) and the evidence for C_1 (unspecified) *combine* to support C_5: neither the evidence for C_4 nor the evidence for C_1 on its own can support C_5. Finally, C_5 individually supports the proposition that X had the opportunity to murder Y (C_6), which in turn individually supports the probandum P that it was X who murdered Y.

In the present formal setting, the three concepts can be analysed as follows. Let e_1 and e_2 be two pieces of evidence in support of a proposition p and let B, B_i sets of background information, such as empirical generalisations or orderings of time points. Then e_1 and e_2 *combine* to support p if there is an instance $e_1, e_2, B \mid\!\sim p$ of an inference rule but no instances of $e_1, B_1 \rightarrow p$ and $e_2, B_2 \mid\!\sim p$. And e_1 and e_2 *corroborate* to support p if all three instances exist. Finally, e_1 and e_2 *converge* to support p if they do not combine or corroborate but there are instances $e_1, B_1 \rightarrow q$, $e_2, B_2 \rightarrow r$ and $q, r, B \rightarrow p$. Combining evidence can also use a strict inference rule.

Let us illustrate this with the following formalisation of [1]'s example.

r_1: $\textit{testifies}(x, \varphi) \Rightarrow \varphi$
r_2: $C_5 \Rightarrow C_6$
r_3: $C_6 \Rightarrow P$

Premise r_1, left implicit in inferences from the individual witness testimonies, says that witnesses usually tell the truth and r_2 and r_3 are two empirical generalisations arguably left implicit in the inferences of C_6 from C_5 and P from C_6. The facts are as follows:

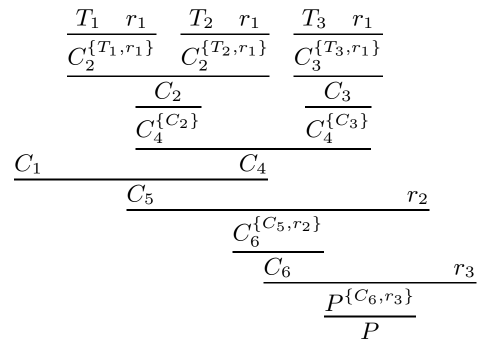


Figure 5: corroborating, converging and combining evidence

T_1: $\textit{testifies}(w_1, C_2)$
T_2: $\textit{testifies}(w_2, C_2)$
T_3: $\textit{testifies}(w_3, C_3)$
C_1: Y was murdered in his house at 4:45

Figure 5 displays the entire argument. It contains two genuine applications of the accrual inference rule: C_2 is derived by accruing two arguments from witness testimonies, and C_4 is derived by accruing a forward and backward application of temporal persistence. All other defeasible inferences in the argument trivially accrue with themselves only. The argument contains one deductive inference, viz. the derivation of C_5 from C_1 and C_4.

8. CONCLUSION

In this paper accrual of arguments has been formalised as an inference rule in a standard logical framework for defeasible argumentation. Now that such a formalisation is available, it can be compared to alternative inference-based approaches to accrual and to knowledge-representation approaches. To facilitate such a comparison, three principles were presented that any formal treatment of accrual should satisfy. The present formalisation was shown to satisfy these principles. The formalisation of accrual was illustrated with applications to evidential reasoning; as a result, formal accounts were given of some concepts of the theory of evidential reasoning. Also, some reasons were given for preferring it over KR approaches. The main reason is that the present method allows debates on the quality of individual reasons for a conclusion before these reasons enter an accrual. A possible drawback of an inference approach is that unwanted accruals must be explicitly blocked. This leads to a trade-off: with automatic accrual unwanted accruals must be explicitly blocked while with hand-made accrual wanted accruals must be explicitly expressed. The choice of method will depend on the nature of the application domain. In future research it would be interesting to investigate under which circumstances one approach leads to more compact representations or a more efficient reasoning process than the other.

As for comparisons with existing work, I know of three existing logics that have an inference-based approach to accrual, Reason-Based Logic [5, 6, 16], Verheij's *Cumula* logic [15, 16] and the Logic of Argumentation [7].

Reason-Based Logic formalises reasoning about notions related to reasons, such as the validity, application and weighing of reasons. Technically it is a logic in the style of default logic, except for its weighing mechanism. The present treatment of accrual completely respects the treatment of the weighing of reasons in Reason-Based Logic and can therefore be regarded as an argument-based reformulation of its treatment of accrual. The present proposal also adds a new element, viz. the accrual undercutter of Section 6.2, which is not available in this general form in Reason-Based Logic.

Verheij's *Cumula* logic [15, 16] logic is an argument-based logic similar to Dung's admissibility semantics, with a conception of arguments similar to Pollock's tree-based approach. Here I discuss the more mature version of [16]. As for the structure of arguments, Verheij adds one feature to a tree-based conception of arguments: different arguments for the same conclusion can be combined into a so-called *coordinated* argument, which roughly is an argument with the same conclusion but with alternative sets of premises or subarguments. The coordinating argument is called a *broadening* and its coordinated elements are called its *narrowings*. (The latter correspond to this paper's notion of 'lesser versions' of an accrual.) Following Pollock [9], Verheij defines extensions in terms of assignments of one of two statuses (*in* or *out*) to arguments in a given set. Pollock assigns *in* to an argument if, firstly, all its subarguments are assigned *in* and, secondly, all arguments defeating it are *out*. And he assigns *out* to an argument if either one of its subarguments is assigned *out* or it is defeated by an argument that is *in*. To deal with accrual, Verheij adds to this the condition that if a narrowing of an argument is *in*, the argument is *in*. Note that with contraposition this yields that if an argument is *out*, all its narrowings are *out*.

Applying these ideas to Example 6.1, it is easy to verify that they admit only one maximal status assignment , viz. the intended one in which A, B and their coordination $A + B$ are *in* and C is *out*. Cumula also respects the first and third principle of accrual of Section 3. The first principle is satisfied since it is possible to define defeat in such a way that two arguments A and B both individually defeat an argument C while their coordination $A + B$ is in turn defeated by C. The third principle is satisfied by the condition that an argument is *out* if one of its subarguments is *out*. However, the second principle of Section 3 is satisfied in a way that is too strong: since Verheij requires that if an accrual is 'out' then all its narrowings are also 'out', he cannot model situations were an accrual is 'out' because of subargument defeat so that some of its lesser versions can be 'in' (cf. Figures 2 and 3).

LA combines a logical theory of constructing and attacking tree-structured arguments with a method to assign probabilistic strengths to arguments. In LA an accrual is at least as strong as its elements, so that the first principle of accrual is trivially satisfied and there is no need to satisfy the second principle: if all accruals are at least as strong as their accruing elements, then the latter need not be made inapplicable by the accrual. On the other hand, the application of LA is thus restricted to domains where accruals are never weaker then their lesser versions. It is hard to tell whether LA satisfies the third principle, since LA's treatment of reinstatement and subargument-defeat does not follow the standard approach of Section 2.

Finally, the present account does not address the problem of valuating accrual arguments. It would be interesting to make hold by default the intuition that accrual makes a case stronger. One way to formalise this intuition was proposed in [13] in the context of our logic of [12]. In this logic, information on rule priorities can be expressed in the object language. We expressed the above intuition as the following defeasible conditional:

$$x > y \Rightarrow x^+ > y^-$$

where x^+ denotes any rule with a more specific antecedent than x and y^- denotes any rule with a less specific antecedent than y. Since this priority rule is defeasible, it can be defeated in exceptional cases.

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