Comparing Alternative Factor- and Precedent-based Accounts of Precedential Constraint

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Abstract. In this paper several existing dimension-based models of precedential constraint are compared and an alternative is proposed, which unlike existing models does not require that for each value assignment to a dimension it is specified whether it is for or against the case's outcome. This arguably makes the model easier to apply in practice. In addition, it is shown how several factor- and dimension-based models of precedential constraint can be embedded in a Dung-style argumentation-based form, so that general tools from the formal study of argumentation become applicable.

Keywords., case-based reasoning, precedential constraint, factors, dimensions

1. Introduction

In the formal study of legal case-based reasoning dimensions (relevant aspects of a case that can have multiple values) have received increasing attention [4,10,7]. Much of this work concerns the idea of *precedential constraint*, that is, the question under which conditions a decision in a new case is determined by a set of precedents. One aim of this paper is to compare and assess existing dimension-based models of precedential constraint and to propose an alternative. The alternative is motivated by the observation that the requirement of existing models to specify for each value assignment to a dimension in a case whether it is for or against the case's outcome is often hard to apply in practice. Instead, I will propose a model in which all that needs to be specified is which change in value favours one outcome more and the other outcome less.

A second aim of this paper is to show how both factor- and dimension-based models of precedential constraint can be embedded in a Dung-style [5] argumentation-based form, so that general tools from the formal study of argumentation become applicable. Earlier similar attempts were [9,2], which formulated argument schemes for case-based reasoning with factors or dimensions in the context of the $ASPIC^+$ framework Unlike this work, I will model case-based reasoning 'stand-alone', without embedding in a more general theory of the structure of arguments and the nature of their relations. This will allow me to focus clearly on the essence and to remain close to relevant work of others.

Below I will, after presenting the formal preliminaries in Section 2, first reconstruct Horty's [6] factor-based result and reason models of precedential constraint as Dung-style argumentation. The key idea is to define a similarity relation on precedents given a case to-be decided, and to use this relation to resolve attacks between arguments in an abstract argumentation framework. Then I will in Section 4 first adapt this embedding for Horty's [7] dimension-based result model, and then do the same for a dimension-based reason model inspired by Rigoni's [10] critique of Horty's model and for an alternative model addressing the pragmatic concerns with the Rigoni-style approach.

2. Formal preliminaries

I first summarise the formal frameworks used in this paper. An *abstract argument framework* [5] is a pair $AF = \langle \mathcal{A}, attack \rangle$, where $\mathcal{A}$ is a set of arguments and *attack* a binary relation on $\mathcal{A}$. A subset $\mathcal{B}$ of $\mathcal{A}$ is *conflict-free* if no argument in $\mathcal{B}$ attacks an argument in $\mathcal{B}$ and it is *admissible* if it is both conflict-free and also defends itself against any attack, i.e., if an argument A_1 is in $\mathcal{B}$ and some argument A_2 not in $\mathcal{B}$ attacks A_1, then some argument in $\mathcal{B}$ attacks A_2. The theory of AFs identifies sets of arguments (called *extensions*) which are all admissible but may differ on other properties. For present purposes their differences do not matter much. What suffices is that the so-called *grounded extension* is always unique and thus captures a notion of 'justified arguments', i.e., those arguments that either directly or indirectly survive all attacks. Moreover, membership can be tested with an argument game between a proponent and an opponent of a given argument. The game starts with the proponent moving the argument to be tested and the players take turns after each argument: the opponent must attack the proponent's last argument while the proponent must one-way attack the opponent's last argument (in that the attacked argument does not in turn attack the attacker). A player *wins an argument game* iff the other player cannot move. An argument is *justified* (i.e., in the grounded extension) iff the proponent has a winning strategy in a game about the argument, i.e., if the proponent can make the opponent run out of moves in whatever way the opponent plays.

I next recall some notions concerning factors and cases often used in AI & law (e.g. in [6,10,7]), although with some differences in notation. Let o and o' be two outcomes and *Pro* and *Con* be two disjoint sets of atomic propositions called, respectively, the *pro-* and *con* factors, i.e., the factors favouring, respectively, outcome o and o'. The variable s (for 'side') ranges over $\{o, o'\}$ and $\bar{s}$ denotes o' if $s = o$ while it denotes o if $s = o'$. We say that a set $F \subseteq Pro \cup Con$ favours side s (or F is pro s) if $s = o$ and $F \subseteq Pro$ or $s = o'$ and $F \subseteq Con$. For any set F of factors the set $F^s \subseteq F$ consists of all factors in F that favour side s. A *fact situation* is any subset of $Pro \cup Con$.

The notion of a case can be defined in two ways. If all factors of a case c are supposed to be relevant to its outcome (as in Horty's [6] *result model* of precedential constraint), then it can be represented as a triple $(pro(c), con(c), outcome(c))$ where $outcome(c') \in \{o, o'\}$. Moreover, if $outcome(c) = o$ then $pro(c) \subseteq Pro$ and $con(c) \subseteq Con$ and if $outcome(c) = o'$ then $pro(c) \subseteq Con$ and $con(c) \subseteq Pro$. If, by contrast, a subset of the set of factors favouring a case's outcome can be sufficient for its outcome (as in Horty's [6] *reason model* of precedential constraint), then a case can be represented as a triple $(ppro(c) \cup con(c), pro(c), outcome(c))$, where $pro(c) \subseteq ppro(c)$ and where the above constraints on $pro(c)$ also hold for $ppro(c)$ (the factors 'potentially pro' c's outcome) and the other conventions and constraints are as above. Horty calls $pro(c)$ the 'rule' of the case. It consists of those pro-decision factors that according to the decision maker are jointly sufficient to outweigh all the con-decision factors in the case.

Given all this, a case base CB is a set of cases. Below I assume it clear from the context whether cases are represented for the result model or for the reason model.

I next summarise Horty's [6] result model of precedential constraint.

Definition 2.1 [Preference relation on fact situations.] Let X and Y be two fact situations. Then $X \leq_s Y$ iff $X^s \subseteq Y^s$ and $Y^{\overline{s}} \subseteq X^{\overline{s}}$.

$X <_s Y$ is defined as usual as $X \leq Y$ and $Y \nleq X$. This definition says that Y is at least as good for s as X iff Y contains at least all pro-s factors that X contains and Y contains no more pro-$\overline{s}$ factors than X contains.

Definition 2.2 [Precedential constraint with factors: result model] Let CS be a case base and F a fact situation. Then, given CB, deciding F for s is *forced* iff there exists a case $c = (X,Y,s)$ in CB such that $X \cup Y \leq_s F$.

I finally summarise Horty's [6] reason model of precedential constraint. The following definition says that a case decision expresses a preference for any pro-decision set containing at least the pro-decision factors of the case over any con-decision set containing at most the con-decision factors of the case. This allows *a fortiori* reasoning from a precedent adding pro-decision factors an/or deleting con-decision factors.

Definition 2.3 [Preferences from cases.] Let $(ppro(c) \cup con(c), pro(c), s)$ be a case, CB a case base and X and Y sets favouring $\overline{s}$ and s, respectively. Then

1. $Y <_c X$ iff $Y \subseteq con(c)$ and $X \supseteq pro(c)$;
2. $Y <_{CB} X$ iff $Y <_c X$ for some $c \in CB$.

Definition 2.4 [(In)consistent case bases.] Let C be a case base with $<_{CB}$ the derived preference relation. Then CB is *inconsistent* if and only if there are factor sets X and Y such that $X <_{CB} Y$ and $Y <_{CB} X$. And CB is *consistent* if and only if it is not inconsistent.

The final definition says that deciding a case for a particular outcome is forced if that is the only way to keep the updated case base consistent.

Definition 2.5 [Precedential constraint with factors: reason model.] Let CB be a consistent case base and (F,R,s) a case that is not in CB. Then, given CB, c is *allowed* iff $CB \cup \{c\}$ is consistent. Moreover, deciding F for s is *forced* iff for all cases $c = (F,R,outcome(c))$ it holds that $CB \cup \{c\}$ is consistent iff $outcome(c) = s$.

Horty [6] proves that his result and reason model are equivalent on the assumption that $pro(c) = ppro(c)$ for all cases c.

3. An argumentation-based model of precedential constraint with factors

In this section I define a similarity definition on the set of cases given a focus case (a case to be decided), to be used to resolve attacks between arguments in an argumentation framework. I then prove a relation between this similarity definition and Horty's factor-based models of precedential constraint. It suffices for this purpose to look at the relevant differences between a precedent and the focus case, which are those differences that are a reason not to decide the focus case as the precedent. These are the situations in which a precedent can be distinguished in a HYPO/CATO-style approach [1], namely, when the new case lacks some features of the pro its outcome that are in the precedent or has new features con its outcome that are not in the precedent. Here it is relevant whether the two cases have the same outcome or different outcomes.

Definition 3.1 [Differences between cases with factors.] Let c and f be two cases. The set $D(c,f)$ of differences between c and f is defined as follows.

1. If $outcome(c) = outcome(f)$ then $D(c,f) = pro(c) \setminus pro(f) \cup con(f) \setminus con(c)$.
2. If $outcome(c) \neq outcome(f)$ then $D(c,f) = pro(f) \setminus con(c) \cup pro(c) \setminus con(f)$.

Intuitively, the fewer (with respect to set inclusion) the relevant differences between a case in the case base and the focus case are, the better it is. Below I formalise this by using the subset relation on sets of relevant differences with the focus case as a preference relation in an abstract argumentation framework in which arguments are cases.

Definition 3.2 [Case-based argumentation frameworks.] Given a case base CB and a focus case $f \notin CB$, an abstract argumentation framework $AF_{CB,f}$ is a pair $\langle \mathcal{A}, attack \rangle$ where:

- $\mathcal{A} = CB$;
- c attacks c' iff $outcome(c) \neq outcome(c')$ and $D(c',f) \not\subset D(c,f)$.

The idea is that a given fact situation F must be decided for s just in case there exists a justified argument for outcome S on the basis of the $AF_{CB,f}$ where $f = (F,s)$. So moving an argument in the grounded game is elliptic for 'the fact situation of the focus case must be decided as in this precedent since they are sufficiently similar'.

I next establish a formal relation between Horty's reason model of precedential constraint and the above argumentation-based reconstruction. Since Horty's result model is a special case of his reason model, this result also holds for the result model.

Proposition 3.3 Let $AF_{CB,f} = \langle \mathcal{A}, attack \rangle$ be an abstract argumentation framework defined by a consistent case base CB and a focus case f with fact situation F. Then deciding F for s is forced given CB iff there exists a case c with outcome s in CB such that $D(c,f) = \emptyset$.

Proof: Assume first that f is forced. Let $f = (F^s \cup F^{\bar{s}}, R, s)$. Then every case $f' = (F^{\bar{s}} \cup F^s, R', s)$ is inconsistent with the case base. Let $R' = F^{\bar{s}}$. Then since CB is consistent, by Observation 1 of [7] there exists a case $f'' = (X \cup Y, R'', s) \in CB$ such that $R'' <_{f'} F^{\bar{s}}$ and $F^{\bar{s}} <_{f''} R''$. The former priority entails that $R'' \subseteq Fs$. But then $pro(f'') \subseteq pro(f)$, so (1) $pro(f'') \setminus pro(f) = \emptyset$. The latter priority entails that $F^{\bar{s}} \subseteq Y$. But then (2) $con(f) \subseteq con(f'')$ so (2) $con(f) \setminus con(f'') = \emptyset$. Then observe that (1) and (2) together entail that $D(f'',f) = \emptyset$.

Assume next that there exists a $c \in CB$ with outcome s and such that $D(c,f) = \emptyset$. Then we have $con(c) <_c pro(c)$ and we have $pro(c) \subseteq pro(f)$ and $con(f) \subseteq con(c)$. But then we also have $con(f) <_c pro(f)$, so for every $R \subseteq con(f)$ we have $R <_c pro(f)$ and so $R <_c ppro(f)$. Any rule for deciding the facts of f for $\bar{s}$ requires adding a case $c' = (F,R,\bar{s})$ to CB but then $ppro(f) <_c R$ can be derived from CB, so CB is inconsistent. Moreover, this immediately implies that any case $f = (F,R,s)$ is consistent with CB. □

This proposition yields a simple syntactic criterion for determining whether a decision is forced. More generally it embeds Horty's models of precedential constraint in the formal theory of abstract argumentation. At present this embedding is still somewhat trivial, since an immediate consequence of Proposition 3.3 is that (assuming CB is consistent) deciding the fact situation of a focus case f for its outcome is forced iff there is a case in C for the same outcome that has no attackers in $AF_{CB,f}$. So dialogues for this case in the grounded game are trivial in that they stop after the proponent moves this case. However, there are ways to extend the present setup to yield more interesting dialogues, which can be explored in future research. One extension is with preferences between factors, so that cases with relevant differences could also be forced. Even more interesting is if these factor preferences can be argued for or if factors can be derived with further arguments.

4. Adapting the approach to dimensions

In this section I discuss various ways in which the above approach can be adapted to dimensions. I first show how Horty's [7] dimension-based result model can be embedded in an argumentation framework. I will not do the same for his dimension-based reason model, for two reasons. First, as Horty shows, his dimension-based reason model collapses into his result model, which arguably fails to capture the distinction between *ratio decidendi* and *obiter dicta* from common-law jurisdictions. Second, I agree with Rigoni [10] that Horty's model sometimes yields counterintuitive outcomes. For these reasons I will first formulate a reason model adapting ideas of Rigoni [10] and then present an alternative reason model motivated by some pragmatic concerns about Rigoni's approach.

4.1. Horty's dimension-based result model as argumentation

I adopt from [7] the following technical ideas (again with some notational differences). A *dimension* is a tuple $d = (V, \leq_o, \leq_{o'})$ where V is a set (of values) and $\leq_o$ and $\leq_{o'}$ two partial orders on V such that $v \leq_o v'$ iff $v' \leq_{o'} v$. A *value assignment* is a pair (d, v). The functional notation $v(d)$ denotes the value of dimension d. Then a *case* is a pair $c = (F, outcome(c))$ such that D is a set of dimensions, F is a set of value assignments to all dimensions in D and $outcome(c) \in \{o, o'\}$. Then a case base is as before a set of cases, but now explicitly assumed to be relative to a set D of dimensions in that all cases assign values to a dimension d iff $d \in D$ (an assumption also made by Horty). Likewise, a fact situation is now an assignment of values to all dimensions in D. As for notation, $v(d, c)$ denotes the value of dimension d in case c. Finally, $v \geq_s v'$ is the same as $v' \leq_s v$.

From now on I will use as a running example the fiscal-domicile example introduced in [8] and also used by [4,10,7]. The issue is whether the fiscal domicile of a person who moved abroad for some time has changed. Let us consider two dimensions d_1, the duration of the stay abroad in months and d_2 the percentage of the tax-payer's income that was earned abroad during the stay. For both values, increasingly higher values increasingly favour the outcome *change* and decreasingly favour the outcome *no change*. So, for instance, $(d_1, 12m) <_{change} (d_1, 24m)$ and so $(d_1, 24m) <_{no\ change} (d_1, 12m)$. An example of a fact situation is $F = \{v(d_1) = 30m, v(d_2) = 60\%\}$ and an example of a case is $c = (F', change)$ where $F' = \{v(d_1) = 12m, v(d_2) = 60\%\}$.

In Horty's result model a decision in a fact situation is forced iff there exists a precedent c for that decision such that on each dimension the fact situation is at least as favourable for that decision as the precedent. He formalises this idea with the help of the following preference relation between sets of value assignments.

Definition 4.1 [Preference relation on dimensional fact situations.] Let F and F' be two fact situations with the same set of dimensions. Then $F \leq_s F'$ iff for all $(d,v) \in F$ and all $(d,v') \in F'$ it holds that $v(d) \leq_s v'(d)$.

In our running example we have that $F' <_{change} F$ since F and F' are equal on d_2 while F is better for s on d_1.

Then adapting Horty's factor-based result model to dimensions is straightforward.

Definition 4.2 [Precedential constraint with dimensions: result model.] Let *CS* be a case base and and F a fact situation given a set D of dimensions. Then, given *CB*, deciding F for s is *forced* iff there exists a case $c = (F', s)$ in *CB* such that $F' \leq_s F$.

So in our running example deciding F for *change* is forced.

I next embed Horty's dimension-based result model in an argumentation framework in a similar way as I did above for his factor-based result and reason model. First Definition 3.1 of differences between cases has to be adapted to dimensions. Note that unlike in the case of factors, there is no need to indicate whether a value assignment favours a particular side in the case since the $\leq_s$ ordering suffices for this purpose.

Definition 4.3 [Differences between cases with dimensions.] Let $c = (F(c), outcome(c))$ and $f = (F(f), outcome(f))$ be two cases. The set $D(c,f)$ of differences between c and f is defined as follows.

1. If $outcome(c) = outcome(f) = s$ then $D(c,f) = \{(d,v) \in F(c) \mid v(d,c) \not\leq_s v(d,f)$.
2. If $outcome(c) \neq outcome(f)$ where $outcome(c) = s$ then $D(c,f) = \{(d,v) \in F(c) \mid v(d,c) \not\geq_{\bar{s}} v(d,f)$.

Let c be a precedent and f a focus case. Then clause (1) says that if the outcomes of the precedent and the focus case are the same, then any value assignment in the focus case that is not at least as favourable for the outcome as in the precedent is a relative difference. Clause (2) says that if the outcomes are different, then any value assignment in the focus case that is not at most as favourable for the outcome of the focus case as in the precedent is a relative difference.

In our running example, let $f = (F, change)$. Then $D(c,f) = \emptyset$. If $v(d_2,F)$ is changed from 60% to 50% then $D(c,f) = \{(d_2, 60\%)\}$ by clause 1. Next, let $g = (G, nochange)$ where $G = \{v(d_1) = 24m, v(d_2) = 60\%\}$. Then $D(c,g) = \{(d_1, 12)\}$ by clause 2.

With these definitions, Definition 3.2 of an abstract argumentation framework given a case base still applies to the setting with dimensions. This allows the following counterpart of Proposition 3.3.

Proposition 4.4 Let, given a set D of dimensions, $AF_{CB,f} = \langle \mathcal{A}, attack \rangle$ be an abstract argumentation framework defined by a case base *CB* and a focus case f with a fact situation F. Then deciding F for s is forced given *CB* according to Definition 4.2 iff there exists a case in *CB* with outcome s such that $D(c,f) = \emptyset$.

Proof: Consider first any $c = (F(c), s)$ in *CB* such that $D(c,f) = \emptyset$. Then for all $(d,v) \in F(c)$ and all $(d,v') \in F(f)$ it holds that $v(d) \leq_s v'(d)$, so $F(c) \leq_s F(f)$.

Suppose next f is forced. Then the proof is the same the other way around. □

4.2. A dimension-based reason model with complete rules

I next discuss how Horty's dimension-based result model can be turned into a dimension-based reason model. There are two features on which this can be done: by 'relaxing' an individual value assignment or by leaving some assignments out from a set of value assignments. In both ways a case is a triple $(c = (F(c), R(c), outcome(c))$, where $F(c)$ is as in the result model a value assignment to a given set D of dimensions and where $R(c)$, the rule of the case, is a set of value assignments that is in some way constrained by $F(c)$. In the first way, rule $R(c)$ consists of value assignments to each dimension in D such that for each element (d, v) in $R(c)$ and each element (d, v') in $F(c)$ it holds that $v(d) \leq_s v'(d)$. In other words, in this approach a rule of a case assigns to each of the case's dimensions a value that is at most as favourable for the case's outcome as its value in the case. Below I will call such a rule a *complete rule*. This idea is taken from Rigoni in [10], except that he also applies it to incomplete rules.

Definition 4.5 [Precedential constraint with dimensions: a reason model with complete rules.] Let, given a set D of dimensions, CS be a case base in which all cases have a complete rule and F a fact situation. Then deciding F for s is *forced* iff there exists a case $c = (F', R, s)$ in CB such that $R \leq_s F$.

This model does not collapse into the above result model. Suppose in the tax example that c has a fact situation $(\{v(d_1) = 30m, v(d_2) = 60\%\}$ and outcome *change* and consider again fact situation $F = \{v(d_1) = 24m, v(d_2) = 75\%\}$. Suppose the court in c ruled that with a percentage earned abroad of 60% a stay abroad of at least 12 months suffices for change of fiscal domicile. The rule of c then is $\{(d_1, 12m), (d_2, 60\%)\}$. Then in the reason model deciding F for *change* is forced, even though the stay abroad in F is shorter than in c, since it is still longer than its value in c's rule. By contrast, in the result model this difference suffices to make c distinguishable and deciding F for *no change* not forced.

The model also avoids an arguably counterintuitive feature of Horty's [7] model. In our example, if the rule of c is $\{(d_1, 12m)\}$ then in a new case in which the stay abroad is 24 months and the percentage of income earned abroad is 75% deciding for *change* is in Horty's model not forced by the precedent, since it is weaker for *change* than the precedent in that the stay abroad is not 30 but 24 months. However, as also argued by Rigoni in [10], this seems counterintuitive given that the court in the precedent ruled that 12 months abroad suffice for *Change* and given that the new case is stronger for this outcome in its only other dimension. With Definition 4.5 deciding for *change* is instead forced by c, since $\{(d_1, 12m), (d_2, 60\%)\} \leq_{change} \{(d_1, 24m), (d_2, 75\%)\}$.

One issue remains: Horty's factor-based reason model requires that courts select a rule in the new case that leaves the case base consistent when the case is added to it. In Horty's (and also Rigoni's [10]) model consistency is defined in terms of a preference relation between sets of reasons pro and con a decision (cf. Definition 2.3 above). However, the present model does not distinguish between pro and con value assignments, while still a notion of consistency is needed. Consider again the tax example with the two dimensions d_1 and d_2 and consider two precedents c_1 with rule $R_1 = \{(d_1, 12m), (d_2, 60\%)\}$ and outcome *change* and c_2 with rule $R_2 = \{(d_1, 8m), (d_2, 60\%)\}$ and outcome *no change*.

Consider next a fact situation F with $d_1 = 15$ and $d_2 = 60\%$. Then deciding F for *change* is forced. Suppose the court does so but formulates the rule $R_3 = \{(d_1, 10m), (d_2, 60\%)\}$. Then in a new fact situation equal to rule R_2 both deciding *change* and deciding *no change* would be forced, so adding $f = (F, R_3, change)$ would make it inconsistent in that for the same fact situation two opposite outcomes are forced. So a constraint on rule selection should be that it should leave a consistent case base consistent in this sense.

4.3. *An alternative dimension-based reason model*

The second way in which the result model can be refined into a reason model is by allowing that the rule R of a case assigns a value to a subset of its fact situation, while still adhering to the constraint that the rule's values of dimensions are at most a favourable to the case's decision as their actual values in the case. Here I would like to follow a Rigoni-style approach, in order to avoid the counterintuitive consequences of Horty's approach. However, there is a pragmatic problem here, since Rigoni requires that for each value assignment it is indicated which side it favours. The problem is that, unlike with factors, this may be hard in practice, since often this will be context-dependent (likewise [3]). In our tax example, if a case with fact situation $(\{v(d_1) = 30m, v(d_2) = 60\%\}$ has outcome *change*, are both value assignments pro this outcome, or is one pro and the other con *change*? And if the latter, then which is pro and which is con? This is not easy to say in general. On the other hand, what is uncontroversial is that increasingly higher values for these dimensions increasingly support *change* and decreasingly support *no change*. For this reason I will instead explore an approach in which all that is needed is general knowledge about which side is favoured more and which side less if a value of a dimension changes, as captured by the two partial orders $\leq_s$ and $\leq'_s$ on a dimension's values.

Below for any two sets X and Y of value assignments, $Y^{|X}$ is the subset of Y that consists of value assignments to any dimension that is also assigned a value in X.

Definition 4.6 [Precedential constraint with dimensions: an alternative reason model with possibly incomplete rules.] Let, given a set D of dimensions, CS be a case base in which all cases have a possibly incomplete rule and F a fact situation. Then deciding F for s is *forced* iff there exists a case $c = (F', R, s)$ in CB such that $R \leq_s F^{|R}$.

So deciding F for s is forced iff there is a precedent for s such that F is at least of favourable for s on all dimensions in the precedent's rule.

Moreover, like with the reason model with complete rules, the constraint on rule selection is needed that adding a new case to a consistent case base should leave the case base consistent in that for no fact situation two opposite outcomes are forced.

To see how this definition works, consider again the tax example with dimensions d_1 and d_2 and consider precedent c with fact situation $v(d_1) = 30m, v(d_2) = 60\%$, with rule $R = \{(d_1, 12m)\}$ and with outcome *change*. Consider next a fact situation F with $v(d_1) = 24m, v(d_2) = 50\%$. Then deciding F for change is forced since $F^{|R} = \{(d_1, 24m)\}$ and we have that $R = \{(d_1, 12m)\} <_{change} \{(d_1, 24m)\}$. Note that deciding F for *change* is forced by c even though F is in one dimension weaker for *change* than c, namely in d_2. The point is that d_2 is not in c's rule.

Since a rule that assigns a value to all dimensions in D is a special case, the above example that shows that Definition 4.5 does not collapse into the dimension-based result model also holds for this definition. Moreover, a counterpart of Proposition 4.4 can be obtained for this reason model by redefining the relevant differences between a precedent and a focus case as follows.

Definition 4.7 [Differences between cases with dimensions and possibly incomplete rules.] Let $c = (F(c), R(c), outcome(c))$ and $f = (F(f), R(f), outcome(f))$ be two cases. The set $D(c, f)$ of differences between c and f is defined as follows.

1. If $outcome(c) = outcome(f) = s$ then $D(c, f) = \{(d, v) \in F(f)^{|R(c)} \mid v(d, c) \not\leq_s v(d, f)$.
2. If $outcome(c) \neq outcome(f)$ where $outcome(c) = s$ then $D(c, f) = \{(d, v) \in F(f)^{|R(c)} \mid v(d, c) \not\geq_{\bar{s}} v(d, f)$.

Clause (1) says that if the outcomes of the precedent and the focus case are the same, then any value assignment in the focus case to a dimension in the precedent's rule that is not at least as favourable for the outcome as in the precedent is a relative difference. Clause (2) says that if the outcomes are different, then any value assignment in the focus case to a dimension in the precedent's rule that is not at most as favourable for the outcome of the focus case as in the precedent is a relative difference.

Proposition 4.8 given a set D of dimensions, $AF_{CB,f} = \langle \mathcal{A}, attack \rangle$ be an abstract argumentation framework defined by a case base CB in which all cases have a complete rule and let F be a fact situation. Then deciding F for s is forced given CB according to Definition 4.6 iff there exists a case $c = (F(c), R(c), outcome(c))$ in CB with the same outcome as f such that for any case $f = (F, R(f), s)$ it holds that $D(c, f) = \emptyset$.

Proof: As for Proposition 4.4 with $F(c)$ replaced by $R(c)$ and $F(f)$ replaced by $F(f)^{|R(c)}$.

On the other hand, this approach also has limitations. Consider again the last example. We saw that deciding fact situation F for *change* was forced by precedent c even though F is in one dimension weaker for *change* than c, since this is not in c's rule. This prevents that a decision maker can regard the fact that the percentage of income earned abroad was less in the new situation F than in the precedent an exception to the precedent's rule. In more general terms, in the factor-based reason model the idea of a rule has a clear intuition, namely, that the pro-decision factors in the rule are sufficient to outweigh the con-decision factors in the case. However, with dimensions this intuition does not apply, since the value assignments outside the rule do not necessarily favour the opposite outcome. All that can said is that by stating the rule the court has decided that, given the rule, the case's value assignments to the other dimensions are irrelevant. The question then is whether such a ruling is defeasible. If it is not, then every new case in which the dimensions in the precedent's rule have values that are at least as favourable to the decision as in the rule is constrained by the precedent regardless of possible differences on the other dimensions. If that is regarded as too rigid, then there are two options. The first is that value assignments to dimensions not in a precedent's rule can be a reason for distinguishing just in case in the new fact situation they are less favourable for the precedent's outcome than in the precedent. But then the model collapses into the reason model with complete rules. The second option is that *every* value assignment to a dimension that is not in the rule of the case can override the case's outcome. But then the problem with Horty's reason model reappears: in our last example any income percentage, even a percentage higher than 60%, would suffice to distinguish c. It can be concluded that a Rigoni-style approach in which value assignments are always pro a particular outcome leads to finer-grained distinctions between forced and not-forced decisions than the present approach but is arguably harder to apply in practice.

5. Conclusion

In this paper I have shown how several factor-and dimension-based models of precedential constraint can be embedded in a Dung-style setting with abstract argumentation frameworks. Thus general tools from the formal study of argumentation become available for analysing and extending these models. In addition, I have critically analysed (variants of) some existing dimension-based models of precedential constraint. I argued that a pragmatic limitation of some of them is that they require the specification of information that may be hard to obtain in practical applications and I proposed an alternative without this limitation, although also with lesser ability to distinguish between situations in which a decision is or is not forced by a body of precedents..

In future research the dropping of some limited assumptions can be investigated, such as the assumption that every case assigns a value to every dimension of a given set of dimensions. Dropping this assumption allows the introduction of new dimensions in a case but may run into the same limitations as the above alternative reason-based model. Another issue for future research is the modelling of trade-offs between dimensions with preferences and/or values, as suggested by [4]. Arguably this paper's results on the embedding in a Dung-style setting are of value here.

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