Deductive and Abductive Reasoning with Causal and Evidential Information

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Abstract. In this paper, we propose the information graph (IG) formalism, which provides a precise account of the interplay between deductive and abductive inference and causal and evidential information. IGs formalise analyses performed by domain experts using the informal reasoning tools they are familiar with, such as mind maps used in crime analysis. Based on principles for reasoning with causal and evidential information given the evidence, we impose constraints on the inferences that may be performed with IGs. Moreover, we propose an argumentation formalism based on IGs that allows arguments to be formally evaluated.

Keywords. Deduction, abduction, causal and evidential information, argumentation

1. Introduction

In the legal and forensic domain, reasoning about evidence plays a central role in the rational process of proof [1]. To aid in this process, various graph-based tools exist that allow domain experts to make sense of a mass of evidence in a case, including mind maps, argument diagrams and Wigmore charts [2]. Because of their informal nature, these tools typically do not directly allow for formal evaluation using AI techniques. Hence, we wish to formalise analyses performed with such tools in a manner that allows for formal evaluation and that adheres to principles from the literature on reasoning about evidence [1,3,4] while allowing inference to be performed in a manner closely related to the way in which inference is performed using such tools.

In reasoning about evidence, inference is often performed using domain-specific *generalisations* [1], also called defaults [4], which capture knowledge about the world in conditional form. We distinguish between *causal* generalisations (e.g. fire typically causes smoke) and *evidential* generalisations (e.g. smoke is typically evidence for fire) [1,4]. Inference can be performed in a *deductive*, or forward, fashion, where from a generalisation (e.g. fire typically causes smoke) and its antecedent (fire), the consequent (smoke) is defeasibly inferred; *abduction* [3] can also be performed with causal generalisations, where by affirming the consequent (smoke) the antecedent (fire) is defeasibly inferred. Pearl [4, p. 264] argued that people generally consider it difficult to express knowledge using only causal generalisations, and in an empirical study, van den Braak and colleagues [5] found that while there are situations in which subjects consistently choose either causal or evidential modelling techniques, there are also many examples in which people use both types of generalisations in their reasoning. For instance, subjects often considered testimonies to be evidential, whereas a motive for committing an act

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is considered a cause for committing that act. This illustrates that in formal accounts of reasoning about evidence, it is important to allow for both types of generalisations [1].

When performing analyses using aforementioned tools such as mind maps, domain experts naturally mix both causal and evidential generalisations and perform both deductive and abductive inferences, where the used generalisations and the inference type (deduction, abduction) are typically left implicit. Hence, in formalising such analyses we need a precise account of the interplay between the different types of inferences and generalisations and the constraints on performing inference we need to impose. In this paper we propose the *information graph* (IG) formalism, which provides such an account. IGs are knowledge representations that formalise analyses performed by domain experts using the informal reasoning tools they are familiar with in a manner that makes the causal and evidential generalisations used in performing inference explicit. Based on principles for reasoning with causal and evidential generalisations, we then define how deduction and abduction can be performed with IGs given a set of propositions labelled evidence. Most existing formalisms that allow both inference types with causal and evidential information are logic-based (e.g. [1,6]); instead, we propose a graph-based formalism to remain closely related to analyses performed using aforementioned graph-based tools.

Our argumentation formalism generates an abstract argumentation framework as in Dung [7], that is, a set of arguments with a binary attack relation, which thus allows arguments to be formally evaluated according to Dung's classical semantics. Moreover, our argumentation formalism adheres to the constraints imposed by Pearl's C-E system [4], which say that, in performing inference, care should be taken that no cause for an effect is inferred in case an alternative cause for this effect was already previously inferred.

The paper is structured as follows. In Section 2, we provide principles for reasoning about evidence. In Section 3, we present an example of an analysis performed using a mind mapping tool, which illustrates that both deduction and abduction is performed by domain experts, using both causal and evidential generalisations. Based on this example, in Section 4 we motivate and define our IG-formalism. In Section 5, we then propose an argumentation formalism based on our IG-formalism. In Section 6, we discuss related work. In Section 7, we discuss future work and conclude.

2. Reasoning about Evidence

In this section, we provide principles for and review the terminology used to describe reasoning about evidence. Inference is the process of inferring claims from the observed evidence using *generalisations* [1]. We distinguish between causal and evidential generalisations [1,4]. Causal generalisations are of the form '$c_1, \ldots, c_n$ *usually/normally/typically causes* e', whereas evidential generalisations are of the form '$e_1, \ldots, e_n$ *is usually/normally/typically evidence for* c'. We denote generalisations as *fire* $\rightarrow$ *smoke*, where *fire* is the generalisation's *antecedent* and *smoke* its *consequent*. A generalisation may have multiple antecedents, in which case the generalisation expresses that only the antecedents together allow us to infer the consequent. The notation $\rightarrow_c$ and $\rightarrow_e$ is used for causal and evidential generalisations, respectively.

Deductive Inference Inference can be performed in a deductive fashion, where from a causal or evidential generalisation and by affirming the antecedents, the consequent is inferred by modus ponens on the generalisation. Note that while deduction is typically equated with strict inference (cf. [8]) in which the consequent universally holds given the antecedents, we use the term 'deduction' for defeasible 'forward' inference in which the consequent tentatively holds given the antecedents (cf. [9]). Hence, deduction is not necessarily a stronger or more reliable form of inference than abduction.

Abductive Inference Abduction [3] can also be performed: from a causal generalisation and by affirming the consequent, the antecedents are inferred, since if the antecedents are true it would allow us to deductively infer the consequent modus-ponens-style. In case causes $c_1,\ldots,c_n$ and $c'_1,\ldots,c'_m$ are abductively inferred from common effect e using causal generalisations $c_1,\ldots,c_n \rightarrow_C e$ and $c'_1,\ldots,c'_m \rightarrow_C e$, then c_i and c'_j for $i \in \{1,\ldots,n\}$, $j \in \{1,\ldots,m\}$ are considered to be *competing alternative explanations* for e. We assume that causes c_i (and c'_j) are not in competition among themselves. For instance, consider the causal generalisations *fire* $\rightarrow_C$ *smoke* and *smoke_machine* $\rightarrow_C$ *smoke*. By affirming the common consequent (*smoke*), *fire* and *smoke_machine* are inferred, which are then competing causes for *smoke*.

Mixed and Ambiguous Inference Deduction and abduction can be iteratively performed, where *mixed* abductive-deductive inference is also possible. Suppose that from the causal generalisation *fire* $\rightarrow_C$ *smoke* and by affirming the consequent (*smoke*), the antecedent (*fire*) is inferred. Now, if the additional causal generalisation *fire* $\rightarrow_C$ *heat* is provided, then consequent *heat* can be deductively inferred (or predicted [9]) as antecedent *fire* has been previously abductively inferred.

Mixed deduction, using both causal and evidential generalisations, can also be performed [6], but as noted by Pearl [4] deductively chaining a causal and an evidential generalisation can lead to undesirable results. Consider the example in which a causal generalisation *smoke_machine* $\rightarrow_C$ *smoke* and an evidential generalisation *smoke* $\rightarrow_E$ *fire* are provided. Deductively chaining these generalisations would make us infer there is a fire when seeing a smoke machine, which is clearly undesirable. Similarly, in performing mixed deductive-abductive inference, care should be taken that no cause for an effect is inferred if an alternative cause for this effect was already previously inferred. Consider the above example, where instead of an evidential generalisation *smoke* $\rightarrow_E$ *fire* a causal generalisation *fire* $\rightarrow_C$ *smoke* is provided. Upon seeing a smoke machine, this would make us infer there is a fire in case deduction and abduction are iteratively performed, which is again undesirable. Accordingly, we wish to prohibit these types of inference patterns, and refer to the constraint that no cause for an effect should be inferred if an alternative cause for this effect was already previously inferred as *Pearl's constraint* [4].

Finally, situations may arise in practice in which both deduction and abduction can be performed with the same causal generalisation. For instance, consider the causal generalisation *fire* $\rightarrow_C$ *smoke* and assume that both *fire* and *smoke* are affirmed but not observed, then both deduction and abduction can be performed to either infer *smoke* from *fire* or *fire* from *smoke*, respectively. The inference type is, therefore, *ambiguous*.

3. Example of an Analysis Performed Using a Mind Mapping Tool

In this section, we present an example of an analysis performed using a mind mapping tool [2], which is an example of a tool typically used by domain experts, for instance in crime analysis. Based on this example, we motivate and illustrate the design choices for our IG-formalism in Section 4. A mind map usually takes the shape of a diagram in which hypotheses and claims are represented by boxes and underlined text, and undirected edges symbolise relations between these hypotheses and claims. The mind map represents various scenario-elements and the crime analyst uses evidence to support or oppose these elements, indicated by plus and minus symbols, respectively.

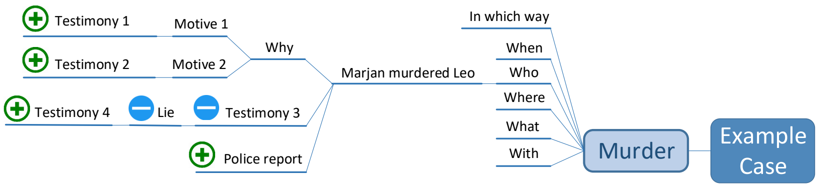

Figure 1. Example of a partially filled out mind map.

Example 1 *An example of a partially filled out mind map is depicted in Figure 1, which also serves as our running example. In this example, adapted from [1], the high-level hypothesis 'Murder' is considered. The case concerns the murder of Leo de Jager. Leo's body was found on the property of Marjan van der E.; we are interested in her involvement in the murder. As a police report (*Police report*) indicates that Leo's body was found on Marjan's property, claim* Marjan murdered Leo *is added as an answer to the 'Who' question. By means of a plus symbol and an undirected edge connecting the evidence to the claim, it is indicated that the police report supports the claim that Marjan murdered Leo. Possible motives (*Motive 1 *and* Motive 2*) are provided as to why Marjan may have wanted to murder Leo, which are connected to the 'Why' question via undirected edges.* Testimony 1 *and* Testimony 2 *support these two motives, indicated by the plus symbols connected to these claims. In her testimony (*Testimony 3*), Marjan denied any involvement in the murder of Leo, which is indicated by a minus symbol. This opposes the claim that Marjan murdered Leo. Further testimony (*Testimony 4*) indicates that Marjan had reason to lie when giving her testimony (*Lie*). By means of a minus symbol and an undirected edge connecting* Lie *to* Testimony 3*, it is indicated that this claim weakens the inference from her testimony to the claim that she did not murder Leo.* □

As the edges in a mind map are undirected, it is unclear from this graphical representation alone which types of generalisations and inferences were used in constructing this map. Establishing this with certainty would require directly consulting the domain experts involved in constructing the chart. We note, however, that the reasoning performed in constructing this mind map can be interpreted in at least two possible ways. One interpretation is that the domain expert first (preliminarily) inferred that Marjan murdered Leo from the police report via deduction using the evidential generalisation *Police report* $\rightarrow_e$ *Marjan murdered Leo*, and then abductively inferred the two possible motives using the causal generalisations g_i: *Motive i* $\rightarrow_c$ *Marjan murdered Leo*; $i = 1,2$. These two causes are then competing alternative explanations as to why Marjan murdered Leo and are subsequently grounded in evidence, namely via deduction from the testimonial evidence using evidential generalisations g'_j: *Testimony j* $\rightarrow_e$ *Motive j*; $j = 1,2$. An alternative interpretation is that the mind map was constructed iteratively from the observed evidence, where from testimonial evidence the motives are inferred via deduction using evidential generalisations g'_1 and g'_2. The claim that Marjan murdered Leo is then inferred modus-ponens style: from causal generalisations g_1 and g_2 and the previously inferred antecedents, the consequent is deductively inferred. In this way, the two motives are not in competition for the common effect that Marjan murdered Leo.

Lastly, note that in mind maps the exact manner in which claims and links conflict is not precisely specified: a minus symbol can either indicate support for the opposing claim (e.g. *Testimony 3* supports the negation of *Marjan murdered Leo*) or indicate an exception to the performed inference step (e.g. *Lie* opposes the inference step from *Testimony 3* to the negation of *Marjan murdered Leo*).

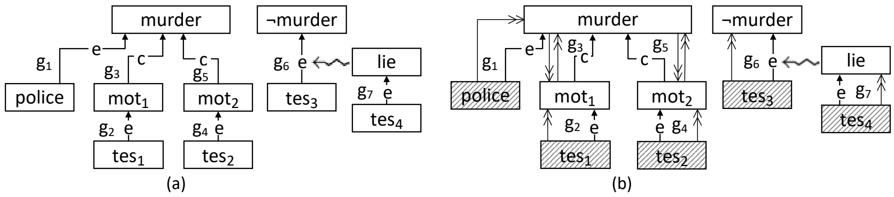

Figure 2. An IG corresponding to a possible interpretation of the mind map of Figure 1 (a); the same IG, where evidence set **E** (shaded) and resulting inference steps ($\twoheadrightarrow$) are also indicated (b).

4. The Information Graph Formalism

The example from Section 3 makes it plausible that both deduction and abduction is performed by domain experts when performing analyses using reasoning tools they are familiar with. In performing such analyses, the used generalisation, as well as the inference type (deduction, abduction), are left implicit. Furthermore, the assumptions of domain experts underlying their analyses are typically not explicitly stated, making these analyses ambiguous to interpret. For our current purposes of providing a precise account of the interplay between the different types of inferences and generalisations, we wish to formalise and disambiguate these analyses in a manner that makes the used generalisations explicit. *Information graphs* (IGs), which we define in Section 4.1, are knowledge representations that explicitly describe causal and evidential generalisations in the graph. In Section 4.2, we define how deductive and abductive inferences can be read from IGs, based on the principles for reasoning about evidence discussed in Section 2.

4.1. Information Graphs

IGs are defined as follows.

Definition 1 (Information graph) *An* information graph *(IG) is a directed graph $G = (\mathbf{P}, \mathbf{A})$, where* $\mathbf{P}$ *is a set of nodes representing propositions from a propositional literal language with ordinary negation symbol $\neg$.* $\mathbf{A} = \mathbf{G} \cup \mathbf{X}$ *is a set of directed (hyper)arcs with $\mathbf{G} \cap \mathbf{X} = \emptyset$, where* $\mathbf{G}$ *and* $\mathbf{X}$ *are sets of generalisation arcs and exception arcs, defined in Definitions 2 and 3, respectively.*

We write $p = -q$ in case $p = \neg q$ or $q = \neg p$. Note that an IG G does not have to be a connected graph (see Figure 2a). In the remainder of this paper, let $G = (\mathbf{P}, \mathbf{A})$ be an IG.

Definition 2 (Generalisation arc) *A* generalisation arc $g \in \mathbf{G} \subseteq \mathbf{A}$ *is a directed (hyper)arc $g\colon \{p_1, \ldots, p_n\} \to p$, indicating a generalisation with antecedents $\mathbf{P_1} = \{p_1, \ldots, p_n\} \subseteq \mathbf{P}$ and consequent $p \in \mathbf{P} \setminus \mathbf{P_1}$. Here, propositions in $\mathbf{P_1}$ are called the* tails *of g, denoted by $\mathbf{Tails}(g)$, and p is called the* head *of g, denoted by $Head(g)$.* $\mathbf{G}$ *divides into two disjoint subsets $\mathbf{G}^{\mathrm{c}}$ and $\mathbf{G}^{\mathrm{e}}$ of causal and evidential generalisation arcs, respectively.*

Curly brackets are omitted in case $|\mathbf{Tails}(g)| = 1$. In figures in this paper, generalisation arcs are denoted by solid (hyper)arcs, which are labelled 'c' for $g \in \mathbf{G}^{\mathrm{c}}$ and 'e' for $g \in \mathbf{G}^{\mathrm{e}}$.

Example 2 *In Figure 2a, an IG is depicted for a possible interpretation of the running example. First, we consider the undirected edges connected to the testimonies and the police report in the mind map of Figure 1. As noted earlier, testimonies are often considered to be evidential [5], where generalisations are of the form* 'Testimony to fact x is normally evidence for x'. *Police reports can similarly be considered evidential. The IG therefore includes generalisation arcs $g_1, g_2, g_4, g_7 \in \mathbf{G}^{\mathrm{e}}$ to denote these generalisations.*

As tes$_3$ *concerns Marjan's testimony to denying any involvement in the murder,* $\neg$murder *is included in* $\mathbf{P}$ *and* $g_6\colon$ tes$_3 \rightarrow \neg$murder *in* $\mathbf{G}^{\mathrm{e}}$*. A motive for committing an act can be considered a cause for committing that act [5]. The IG therefore includes arcs* $g_3\colon$ mot$_1 \rightarrow$ murder *and* $g_5\colon$ mot$_2 \rightarrow$ murder *in* $\mathbf{G}^{\mathrm{c}}$ *to denote these generalisations.* $\square$

As generalisations hardly ever hold universally, exceptional circumstances can be provided under which a generalisation may not hold; hence, we allow exceptions to generalisations to be specified in IGs by means of exception arcs.

Definition 3 (Exception arc) *An* exception arc $x \in \mathbf{X} \subseteq \mathbf{A}$ *is a hyperarc* $x\colon\ p \rightsquigarrow g$*, where* $p \in \mathbf{P}$ *is called an* exception *to generalisation* $g \in \mathbf{G}$.

An exception arc directed from p to g indicates that p provides exceptional circumstances under which g may not hold.

Example 3 *Proposition* lie*, which states that Marjan had reason to lie when giving her testimony, provides an exception to evidential generalisation* $g_6\colon$ tes$_3 \rightarrow \neg$murder *in* $\mathbf{G}^{\mathrm{e}}$*. In Figure 2a, this is indicated by a curved hyperarc* $x\colon$ lie $\rightsquigarrow g_6$ *in* $\mathbf{X}$. $\square$

4.2. Reading Inferences from Information Graphs

We now define how deductive and abductive inferences can be read from IGs. By itself, a generalisation arc only expresses that the tails together allow us to infer the head in case this generalisation is used in deductive inference, or that the tails together can be inferred from the head in case of abductive inference. Only when considering the available evidence can directionality of inference actually be read from the graph.

Definition 4 (Evidence set) *An* evidence set *is a subset* $\mathbf{E} \subseteq \mathbf{P}$ *for which it holds that for every* $p \in \mathbf{E}$*,* $\neg p \notin \mathbf{E}$.

In the remainder of this paper, let $\mathbf{E}$ be an evidence set. The restriction that for every $p \in \mathbf{E}$ it holds that $\neg p \notin \mathbf{E}$ ensures that not both a proposition and its negation are observed. In figures in this paper, nodes in G corresponding to elements of $\mathbf{E}$ are shaded and all shaded nodes correspond to elements of $\mathbf{E}$. We emphasise that various sets $\mathbf{E}$ can be used to establish inferences from the same IG.

Example 4 *In the running example, the evidence consists of the testimonies and the police report. In Figure 2b, the IG of Figure 2a is again depicted, with nodes in* $\mathbf{E} = \{$tes$_1$, tes$_2$, tes$_3$, tes$_4$, police$\}$ *shaded.* $\square$

4.2.1. Deductive Inference

First, we specify under which conditions we consider a configuration of generalisation arcs and evidence to express deductive inference.

Definition 5 (Deductive inference) *Let* $p_1, \ldots, p_n, q \in \mathbf{P}$*, with* $q \notin \mathbf{E}$*. Then given* $\mathbf{E}$*,* q *is* deductively inferred from propositions $p_1, \ldots, p_n$ using a generalisation $g\colon\ \{p_1, \ldots, p_n\} \rightarrow q$ in $\mathbf{G}$*, denoted* $p_1, \ldots, p_n \twoheadrightarrow_g q$*, iff* $\forall p_i$*,* $i = 1, \ldots, n$*:*

1. $p_i \in \mathbf{E}$*, or;*
2. p_i *is deductively inferred from propositions* $r_1, \ldots, r_m \in \mathbf{P}$ *using a generalisation* $g'\colon\ \{r_1, \ldots, r_m\} \rightarrow p_i$*, where* $g' \in \mathbf{G}^{\mathrm{e}}$ *if* $g \in \mathbf{G}^{\mathrm{e}}$*, or;*
3. p_i *is abductively inferred from a proposition* $r \in \mathbf{P}$ *using a generalisation* $g'\colon\ \{p_i, r_1, \ldots, r_m\} \rightarrow r$ *in* $\mathbf{G}^{\mathrm{c}}$*,* $g \neq g'$*,* $r_1, \ldots, r_m \in \mathbf{P}$ *(see Definition 6).*

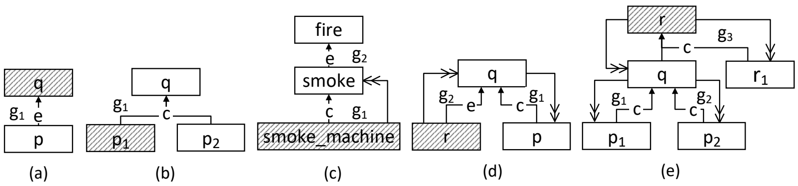

Figure 3. Examples of IGs illustrating the restrictions put on performing deduction within our IG-formalism (a-c); examples of IGs illustrating abductive inference (d-e).

In accordance with our assumptions stated in Section 2, deduction can be performed using generalisations in both $\mathbf{G}^{\mathrm{c}}$ and $\mathbf{G}^{\mathrm{e}}$. The condition $q \notin \mathbf{E}$ ensures that deduction cannot be performed to infer propositions that are already observed. Deduction can only be performed using a $g \in \mathbf{G}$ to infer $Head(g)$ from $\mathbf{Tails}(g)$ in case every tail $p_i \in \mathbf{Tails}(g)$ has been affirmed in that either $p_i \in \mathbf{E}$, p_i is itself deductively inferred, or p_i is abductively inferred. In correspondence with Pearl's constraint (see Section 2), we assume in condition 2 that a proposition $q \in \mathbf{P}$ cannot be deductively inferred from $p_1, \ldots, p_n \in \mathbf{P}$ using a $g \in \mathbf{G}^{\mathrm{e}}$ if at least one of $p_1, \ldots, p_n$ is deductively inferred using a $g' \in \mathbf{G}^{\mathrm{c}}$. Condition 3 of Definition 5 is further explained in Section 4.2.3, after abduction is defined.

Example 5 *Consider the running example. In Figure 2b,* mot_1 *and* mot_2 *are deductively inferred from* tes_1 *and* tes_2 *using generalisations g_2 and g_4, respectively, as* $\mathrm{tes}_1, \mathrm{tes}_2 \in \mathbf{E}$ *(condition* 1 *of Definition 5). Similarly,* murder, $\neg$murder *and* lie *are deductively inferred from* police, tes_3 *and* tes_4 *using generalisations g_1, g_6 and g_7, respectively, as* police, tes_3, $\mathrm{tes}_4 \in \mathbf{E}$. *Proposition* murder *is also deductively inferred from* mot_1 *and* mot_2 *using causal generalisations g_3 and g_5, as* mot_1 *and* mot_2 *are deductively inferred (condition* 2 *of Definition 5). This illustrates mixed deduction using both types of generalisations.* □

We now illustrate the restrictions put on performing deduction within our IG-formalism.

Example 6 *Figure 3a depicts an example of an IG in which q cannot be deductively inferred from p using g_1, as $Head(g_1) = q \in \mathbf{E}$. In Figure 3b, q cannot be deductively inferred from p_1 and p_2 using g_1, as $p_2 \notin \mathbf{E}$ and p_2 is neither deductively nor abductively inferred. In Figure 3c, the example of Section 2 illustrating Pearl's constraint for deduction is modelled. As* smoke_machine $\in \mathbf{E}$, smoke *is deductively inferred from* smoke_machine *using g_1 by condition* 1 *of Definition 5.* fire *cannot in turn be inferred from* smoke *using g_2, as $g_2 \in \mathbf{G}^{\mathrm{e}}$ and* smoke *is deductively inferred using $g_1 \in \mathbf{G}^{\mathrm{c}}$.* □

4.2.2. Abductive Inference

Next, we specify under which conditions we consider a configuration of generalisation arcs and evidence to express abductive inference.

Definition 6 (Abductive inference) *Let $p_1, \ldots, p_n, q \in \mathbf{P}$, with $\{p_1, \ldots, p_n\} \cap \mathbf{E} = \emptyset$. Then given* $\mathbf{E}$, propositions $p_1, \ldots, p_n$ are abductively inferred from q using a generalisation $g\colon \{p_1, \ldots, p_n\} \to q$ in $\mathbf{G}^{\mathrm{c}}$, *denoted $q \twoheadrightarrow_g p_1; \ldots; q \twoheadrightarrow_g p_n$, iff:*

1. *$q \in \mathbf{E}$, or;*
2. *q is deductively inferred from propositions $r_1, \ldots, r_m \in \mathbf{P}$ using a generalisation $g'\colon r_1, \ldots, r_m \to q$ in $\mathbf{G}$, $g \neq g'$ (see Definition 5), where $g' \in \mathbf{G} \setminus \mathbf{G}^{\mathrm{c}}$, or;*
3. *q is abductively inferred from a proposition $r \in \mathbf{P}$ using a generalisation $g'\colon \{q, r_1, \ldots, r_m\} \to r$ in $\mathbf{G}^{\mathrm{c}}$, $r_1, \ldots, r_m \in \mathbf{P}$.*

In accordance with our assumptions stated in Section 2, abduction is modelled using only causal generalisations. The condition $\{p_1,\ldots,p_n\} \cap \mathbf{E} = \emptyset$ ensures that abduction cannot be performed to infer propositions that are already observed. Furthermore, abduction can only be performed using a $g \in \mathbf{G}^c$ to infer $\mathbf{Tails}(g)$ from $Head(g)$ in case $Head(g)$ has been affirmed in that either $Head(g) \in \mathbf{E}$, $Head(g)$ is deductively inferred, or $Head(g)$ is itself abductively inferred. In correspondence with Pearl's constraint (see Section 2), we assume in condition 2 that propositions $p_1,\ldots,p_n \in \mathbf{P}$ cannot be abductively inferred from a proposition $q \in \mathbf{P}$ using a $g \in \mathbf{G}^c$ if q is deductively inferred using a $g' \neq g \in \mathbf{G}^c$.

Example 7 *In Figure 3d, p is abductively inferred from q using generalisation $g_1 \in \mathbf{G}^c$ by condition 2 of Definition 6, as q has been deductively inferred from r using generalisation $g_2 \in \mathbf{G}^e$. In Figure 3e, q and r_1 are abductively inferred from r using generalisation $g_3\colon \{q,r_1\} \to r$ in $\mathbf{G}^c$ by condition 1 of Definition 6, as $r \in \mathbf{E}$. Then by condition 3 of Definition 6, p_1 and p_2 are abductively inferred from q using generalisations g_1 and g_2, respectively. Consider Figure 4b, which illustrates that Pearl's constraint for mixed deductive-abductive inference is adhered to (see Section 2). As* smoke_machine $\in \mathbf{E}$*,* smoke *is deductively inferred from* smoke_machine *using $g_1 \in \mathbf{G}^c$.* fire *cannot be inferred from* smoke*, as $g_2 \in \mathbf{G}^c$ (condition 2 of Definition 6).* □

4.2.3. Mixed Abductive-Deductive and Ambiguous Inference

As apparent from Definitions 5 and 6, mixed abductive-deductive inference can be performed within our IG-formalism.

Example 8 *In Figure 4a, the example of Section 2 illustrating mixed abduction-deduction is modelled. From* smoke $\in \mathbf{E}$*,* fire *is abductively inferred using g_1. Then* heat *is deductively inferred (or predicted) from* fire *using g_2 (Definition 5, condition 3).* □

The conditions under which we consider a configuration of generalisation arcs and evidence to express deduction and abduction according to Definitions 5 and 6 are not mutually exclusive. Under specific conditions, both inference types can be established from the same $g \in \mathbf{G}^c$ in an IG given the provided evidence; the inference type is, therefore, ambiguous (see Section 2). Examples of such inferences are provided in Figure 2b.

5. An Argumentation Formalism Based on Information Graphs

Based on our IG-formalism, we now propose an argumentation formalism that allows for both deductive and abductive argumentation. Our approach generates an abstract argumentation framework as in Dung [7], that is, a set of arguments with a binary attack relation, which thus allows arguments to be formally evaluated according to Dung's classical semantics. In Section 5.1, we define arguments on the basis of a provided G and $\mathbf{E}$, which capture sequences of inference steps as defined in Definitions 5 and 6 starting with elements from $\mathbf{E}$. We then formally prove that arguments constructed on the basis of IGs conform to Pearl's constraint. In Section 5.2, we define several types of attacks between arguments on the basis of IGs and instantiate Dung's abstract approach.

5.1. Arguments

In defining arguments on the basis of a G and $\mathbf{E}$, we take inspiration from the definition of an argument as given in [8]. In what follows, for a given argument A, the function CONC returns its conclusion, SUB returns its sub-arguments (including itself), IMMSUB returns its immediate sub-arguments, GEN returns all the generalisations used in constructing A, and TOPGEN returns the last generalisation used in constructing A.

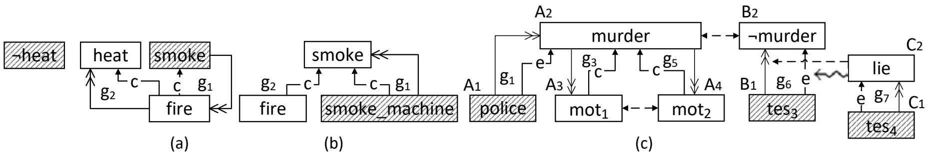

Figure 4. IGs illustrating mixed abduction-deduction (a) and Pearl's constraint for mixed deduction-abduction (b); adjustment to the IG of Figure 2b, where arguments and attacks ($\dashrightarrow$) are also indicated (c).

Definition 7 (Argument) *An* argument A on the basis of G and $\mathbf{E}$ *is any structure obtainable by applying one or more of the following steps finitely many times:*

1. *p if $p \in \mathbf{E}$, where:* $\textsc{Conc}(A) = p$*;* $\textsc{Sub}(A) = \{A\}$*;* $\textsc{ImmSub}(A) = \emptyset$*;* $\textsc{Gen}(A) = \emptyset$*;* $\textsc{TopGen}(A) =$ *undefined.*
2. *$A_1, \ldots, A_n \twoheadrightarrow_g p$ if $A_1, \ldots, A_n$ are arguments such that p is deductively inferred from* $\textsc{Conc}(A_1), \ldots, \textsc{Conc}(A_n)$ *using a generalisation $g \in \mathbf{G} \setminus (\textsc{Gen}(A_1) \cup \ldots \cup \textsc{Gen}(A_n)), g\colon \{\textsc{Conc}(A_1), \ldots, \textsc{Conc}(A_n)\} \to p$ according to Definition 5, where:* $\textsc{Conc}(A) = p$*;* $\textsc{Sub}(A) = \textsc{Sub}(A_1) \cup \ldots \cup \textsc{Sub}(A_n) \cup \{A\}$*;* $\textsc{ImmSub}(A) = \{A_1, \ldots, A_n\}$*;* $\textsc{Gen}(A) = \textsc{Gen}(A_1) \cup \ldots \cup \textsc{Gen}(A_n) \cup \{g\}$*;* $\textsc{TopGen}(A) = g$*.*
3. *$A' \twoheadrightarrow_g p$ if A' is an argument such that p is abductively inferred from* $\textsc{Conc}(A')$ *using a generalisation $g \in \mathbf{G} \setminus \textsc{Gen}(A'), g\colon \{p, p_1, \ldots, p_n\} \to \textsc{Conc}(A')$ for some propositions $p_1, \ldots, p_n \in \mathbf{P}$ according to Definition 6, where:* $\textsc{Conc}(A) = p$*;* $\textsc{Sub}(A) = \textsc{Sub}(A') \cup \{A\}$*;* $\textsc{ImmSub}(A) = \{A'\}$*;* $\textsc{Gen}(A) = \textsc{Gen}(A') \cup \{g\}$*;* $\textsc{TopGen}(A) = g$*.*

In the remainder of this paper, let the set of all arguments on the basis of G and $\mathbf{E}$ be denoted by $\mathcal{A}$. An argument $A \in \mathcal{A}$ is called a *premise argument* if only step 1 of Definition 7 is applied, *deductive* if only steps 1 and 2 are applied, *abductive* if only steps 1 and 3 are applied, and *mixed* otherwise. The restrictions in steps 2 and 3 that $g \notin (\textsc{Gen}(A_1) \cup \ldots \cup \textsc{Gen}(A_n))$ and $g \notin \textsc{Gen}(A')$, respectively, ensure that cycles in which two propositions are iteratively deductively and abductively inferred from each other using the same g are avoided in argument construction.

Example 9 *Consider the adjustment to the IG of Figure 2b depicted in Figure 4c, in which arguments on the basis of this IG and* $\mathbf{E} = \{\text{police}, \text{tes}_3, \text{tes}_4\}$ *are also indicated. According to step* 1 *of Definition 7, $A_1\colon$* police *is a premise argument. Based on A_1, deductive argument $A_2\colon A_1 \twoheadrightarrow_{g_1}$* murder *is constructed by step* 2 *of Definition 7, as* murder *is deductively inferred from* police *using $g_1\colon$* police $\to$ murder. *Then $A_3\colon A_2 \twoheadrightarrow_{g_3}$* mot_1 *is a mixed argument by step* 3 *of Definition 7, as* mot_1 *is abductively inferred from* murder *using $g_3\colon$* $\text{mot}_1 \to$ murder. *Consider Figure 3e, which illustrates step* 3 *in more detail. On the basis of this IG and $\mathbf{E} = \{r\}$, $A'_1\colon r$ is a premise argument. From A'_1, arguments $A'_2\colon A'_1 \twoheadrightarrow_{g_3} r_1$ and $A'_3\colon A'_1 \twoheadrightarrow_{g_3} q$ are constructed by step* 3 *of Definition 7, as q and r_1 are abductively inferred from* $\textsc{Conc}(A'_1)$ *using $g_3\colon \{q, r_1\} \to r$. Again by step* 3*, $A'_4\colon A'_3 \twoheadrightarrow_{g_1} p_1$ and $A'_5\colon A'_3 \twoheadrightarrow_{g_2} p_2$ are constructed using g_1 and g_2, respectively.* $\square$

In performing inference, care should be taken that no cause for an effect is inferred in case an alternative cause for this effect was already previously inferred (Pearl's constraint, see Section 2). In the context of IGs, for $g \in \mathbf{G}^{\mathrm{c}}$, propositions in $\mathbf{Tails}(g)$ express causes for the common effect expressed by $Head(g)$, and for $g \in \mathbf{G}^{\mathrm{e}}$, $Head(g)$ expresses a cause for propositions in $\mathbf{Tails}(g)$. Hence, in defining how inferences can be read from IGs, restrictions are put in Definitions 5 and 6 such that Pearl's constraint is adhered to. We now formally prove that Pearl's constraint is indeed never violated when constructing arguments on the basis of an IG G and an evidence set $\mathbf{E}$.

Proposition 1 (Adherence to Pearl's constraint) *Let $c_1, c_2 \in \mathbf{P}$ be alternative causes of $e \in \mathbf{P}$ in that either:*

1. $\exists g \in \mathbf{G}^{\mathrm{e}}$, $e \in \mathbf{Tails}(g)$, $Head(g) = c_1$, *and either:*
   1a) $\exists g' \neq g \in \mathbf{G}^{\mathrm{e}}$, $e \in \mathbf{Tails}(g')$, $Head(g') = c_2$, *or;*
   1b) $\exists g' \in \mathbf{G}^{\mathrm{c}}$, $c_2 \in \mathbf{Tails}(g')$, $Head(g') = e$.
2. $\exists g \in \mathbf{G}^{\mathrm{c}}$, $c_1 \in \mathbf{Tails}(g)$, $Head(g) = e$, *and either:*
   2a) $\exists g' \neq g \in \mathbf{G}^{\mathrm{c}}$, $c_2 \in \mathbf{Tails}(g')$, $Head(g') = e$, *or;*
   2b) $\exists g' \in \mathbf{G}^{\mathrm{e}}$, $e \in \mathbf{Tails}(g')$, $Head(g') = c_2$.

Assume arguments A and B exist in $\mathcal{A}$ with $\textsc{Conc}(B) = e$*,* $A \in \textsc{ImmSub}(B)$*, and* $\textsc{Conc}(A) = c_1$*. Then no argument C exists in $\mathcal{A}$ with* $B \in \textsc{ImmSub}(C)$, $\textsc{Conc}(C) = c_2$.

Proof. In constructing B from A, a generalisation $g \in \mathbf{G}^{\mathrm{e}}$, $e \in \mathbf{Tails}(g)$, $Head(g) = c_1$ could not have been used (case 1), as this would be an instance of abduction while per the restrictions of Definition 6 abduction can only be performed using generalisations $g \in \mathbf{G}^{\mathrm{c}}$. Thus, we only need to consider case 2, which is a deductive inference. First, consider case 2a. Then by Definition 6 (condition 2), no argument C with $\textsc{Conc}(C) = c_2$ can be constructed from B using g'. Next, consider case 2b. Then by Definition 5 (condition 2), no argument C with $\textsc{Conc}(C) = c_2$ can be constructed from B using g'. ■

5.2. Attack

In this section, several types of attacks between arguments on the basis of IGs are defined. In argumentation, two types of attacks are typically distinguished, namely rebuttal and undercutting attack [8]. We also distinguish a third type of attack, namely alternative attack, inspired by [6]. In our argumentation formalism, these three types of attacks directly follow from the constructed arguments and the specified exception arcs in an IG.

Definition 8 (Attack) *Let $A, B \in \mathcal{A}$. Then A* attacks *B iff A rebuts B, A undercuts B, or A alternative attacks B, as defined in Definitions 9, 10 and 11, respectively.*

First, rebuttal attack is considered, which informally is an attack on a $p \notin \mathbf{E}$.

Definition 9 (Rebuttal attack) *Let $A, B, B' \in \mathcal{A}$ with $B' \in$* $\textsc{Sub}(B)$*. Then A rebuts B (on B') iff* $\textsc{Conc}(B') \notin \mathbf{E}$ *and* $\textsc{Conc}(A) = -\textsc{Conc}(B')$.

Example 10 *Consider the IG of Figure 4c. Let A_1, A_2 be the arguments introduced in Example 9. Let B_1 :* tes$_3$ *and let B_2 : $B_1 \twoheadrightarrow_{g_6}$* ¬murder*. Then A_2 rebuts B_2 (on B_2) and B_2 rebuts A_2 (on A_2), as* $\textsc{Conc}(A_2) =$ murder, $\textsc{Conc}(B_2) =$ ¬murder*, and both* murder, ¬murder $\notin \mathbf{E}$*. This symmetric rebuttal is indicated in Figure 4c by means of a bidirectional dashed arc between these propositions. Consider again Example 8 and Figure 4a, in which* heat *is predicted from* fire*. Assume that contrary to this prediction we observe that there is no heat (*¬heat $\in \mathbf{E}$*). Let A''_1 :* smoke*; A''_2 : $A''_1 \twoheadrightarrow_{g_1}$* fire*; A''_3 : $A''_2 \twoheadrightarrow_{g_2}$* heat*; B''_1 :* ¬heat*. Then B''_1 rebuts A''_2 (on A''_2), but A''_2 does not rebut B''_1 as* $\textsc{Conc}(B''_1) \in \mathbf{E}$. □

Next, undercutting attack is considered. Informally, an undercutter attacks an inference by providing exceptional circumstances under which the inference may not be applicable. Undercutting attacks between arguments follow from the specified exception arcs in G. Specifically, as an exception arc directed from $p \in \mathbf{P}$ to $g \in \mathbf{G}$ specifies an exception to g, an argument $A \in \mathcal{A}$ with $\textsc{Conc}(A) = p$ undercuts an argument $B \in \mathcal{A}$ with $g \in \textsc{Gen}(B)$.

Definition 10 (Undercutting attack) *Let $A, B, B' \in \mathcal{A}$ with $B' \in$* $\textsc{Sub}(B)$*. Then A* undercuts *B (on B') iff there exists an $x \in \mathbf{X}$ such that x :* $\textsc{Conc}(A) \rightsquigarrow g$ *and* $\textsc{TopGen}(B') = g$.

Example 11 *Consider the IG of Figure 4c. Let B_1, B_2 be the arguments introduced in Example 10. Let C_1:* tes$_4$*; C_2: $C_1 \twoheadrightarrow_{g_7}$* lie*. Then C_2 undercuts B_2 (on B_2), as x:* lie *$\rightsquigarrow g_6$ in* $\mathbf{X}$ *and* $\text{TOPGEN}(B_2) = g_6$*. This attack is indicated in Figure 4c by means of a dashed arc directed from* lie *to inference* tes$_3 \twoheadrightarrow_{g_6}$ ¬murder. □

Lastly, alternative attack is defined. Arguments are involved in alternative attack iff their abductively inferred conclusions are in competition for a common effect (see Section 2).

Definition 11 (Alternative attack) *Let $A, B, B' \in \mathcal{A}$ with $B' \in \text{SUB}(B)$. Then A alternative attacks B (on B') iff there exists an argument $C \in \text{IMMSUB}(A) \cap \text{IMMSUB}(B')$ such that* $\text{CONC}(A)$ *and* $\text{CONC}(B')$ *are abductively inferred from* $\text{CONC}(\text{C})$ *using generalisations g and g' in* $\mathbf{G}^{\text{C}}$*, $g \neq g'$, respectively.*

Under the conditions set out in Definition 11, arguments $A_i\colon C \twoheadrightarrow_g p_i$ for $p_i \in \textbf{Tails}(g)$ constructed from C via abduction are involved in alternative attack with $A'_j\colon C \twoheadrightarrow_{g'} p'_j$ for $p'_j \in \textbf{Tails}(g')$ constructed from C via abduction. Arguments A_i (and A'_j) are not involved in alternative attack *among themselves*, in accordance with our assumption that the antecedents of causal generalisations are not in competition (see Section 2).

Example 12 *Consider the IG of Figure 4c. Let A_1, A_2, A_3 be the arguments introduced in Example 9, and let $A_4\colon A_2 \twoheadrightarrow_{g_5}$* mot$_2$*, where* mot$_2$ *is abductively inferred from* murder. *Then A_3 and A_4 are involved in alternative attack, as indicated in Figure 4c by means of a bidirectional dashed arc between their conclusions.* □

Finally, we instantiate [7]'s abstract approach with arguments and attacks based on IGs.

Definition 12 (Argumentation framework) *An* argumentation framework defined by G and $\mathbf{E}$ *is a pair $(\mathcal{A}, \mathcal{C})$, where $(A, B) \in \mathcal{C}$ iff $A \in \mathcal{A}$ attacks $B \in \mathcal{A}$ (see Definition 8).*

Given an argumentation framework, we can use any semantics for argumentation frameworks as defined by [7] for determining the acceptability status of arguments (cf. [8]).

6. Related Work

In this paper, we have introduced the graph-based IG-formalism for deductive and abductive inference with causal and evidential information. Most related formalisms for inference with this type of information are logic-based. In the hybrid theory proposed by Bex [1], deduction and abduction are used in constructing evidential arguments and causal stories, which are completely separate entities with their own definitions related to conflict and evaluation. In comparison, our argumentation formalism based on IGs allows for the construction of both deductive and abductive arguments. Building on the hybrid theory, Bex proposed the integrated theory of causal and evidential arguments [6]. In the integrated theory, the roles of generalisation and inference are not separated; instead, causal and evidential inferences are defined and arguments are constructed by chaining such inferences. Actual abduction is thus not performed by constructing arguments.

Graph-based formalisms for reasoning with causality information have also been proposed, notably Pearl's causal diagrams [10]. Compared to IGs, causal diagrams do not allow for capturing asymmetric conflicts such as exceptions in the graph.

7. Conclusion and Future Work

In this paper, we have introduced the IG-formalism, which provides a principled way for representing and reasoning with causal and evidential information. Based on our IG-formalism, we have proposed an argumentation formalism that generates an abstract argumentation framework as in Dung [7], that is, a set of arguments with a binary attack relation, which thus allows arguments to be formally evaluated according to Dung's classical semantics. Moreover, our argumentation formalism adheres to the constraints imposed by Pearl's C-E system [4]. The added value of our argumentation formalism is that it allows both deductive and abductive argumentation, the latter of which has received relatively little attention in argumentation. In defining our argumentation formalism, we were inspired by the ASPIC$^+$ argumentation framework [8]. Our argumentation formalism can be regarded as an adaptation of a special case of ASPIC$^+$, which would among other things require introducing a new form of attack, namely alternative attack, and restricting the manner in which arguments are constructed within this framework. In future work, we intend to investigate the relations between our argumentation formalism and ASPIC$^+$ and whether Caminada and Amgoud's rationality postulates [11] are satisfied.

IGs formalise analyses performed by domain experts using the informal reasoning tools they are familiar with, such as mind maps. In interpreting a performed analysis as an IG, an additional knowledge elicitation step may be required, as the generalisations used in performing inference are typically left implicit in tools domain experts use. IGs may also be directly constructed by domain experts in case work. In our future work, we intend to investigate possible applications of our IG-formalism as intermediate formalism between informal tools and formalisms that allow for formal evaluation other than those for argumentation, for instance by extending on our previous work on facilitating Bayesian network (BN) construction from a preliminary form of IGs [12].

In our future work, we also intend to increase the expressivity of our IG-formalism by allowing generalisations that are neither causal nor evidential. For instance, definitions, or abstractions [13], allow for reasoning at different levels of abstraction, such as stating that guns can generally be considered deadly weapons.

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