# Coherent Factor Hierarchies as Noncontroversial Bipolar Argumentation Frameworks

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**Abstract.** An important notion in AI & law models of case based reasoning is that of a factor hierarchy, which models pro and con relations between more concrete and more abstract factors relevant for a case outcome. Factor hierarchies have been used to model arguments about the relevance of differences between cases and to define recursive notions of precedential constraint. This paper studies another topic, coherence constraints for factor hierarchies. A set of constraints is proposed and then shown to be formally related to so-called non-controversial bipolar argumentation frameworks. Then the introduced new concepts are used to give definitions of several argument moves involving factor hierarchies, overcoming some drawbacks of existing definitions of these moves.

**Keywords.** case-based reasoning, factor hierarchies, bipolar argumentation frameworks

## 1. Introduction

An important topic in the formal and computational study of legal argument is legal case-based reasoning. The seminal work in AI & law is the work of Ashley and Rissland on the HYPO system for arguing about misuse of trade secrets in US common law [7]. This work introduced the idea that features of a case have a tendency pro or con a decision. In HYPO the features of a case can be boolean (e.g. 'the secret was obtained by deceiving the plaintiff'), in which case they are called *factors*, or multi-valued (e.g. the number of people to whom the plaintiff had disclosed the secret or the severity of security measures taken by the plaintiff), in which case they are called *dimensions*. Vincent Aleven [1] built on the HYPO work in his development of the CATO system. While it was restricted to factors, this work introduced the notion of a factor hierarchy, which allows arguments about the relevance of differences between cases.

Cases (precedents) are in CATO represented as a triple with a boolean decision value and two sets of pro- and con-decision factors. CATO's task is to argue for a decision of a new fact situation, which is a pair of sets of pro- and con-factors. Like

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HYPO, CATO generates case-based disputes between a plaintiff and a defendant of a legal claim concerning misuse of a trade secret. The plaintiff begins by citing a *citable precedent*, that is, a precedent won by the plaintiff and sharing at least one pro-plaintiff factor with the current fact situation. The defendant can counter a citation by a *counterexample*, that is, a citable precedent for the opposite outcome. A citation may also be countered by *distinguishing* the cited precedent from the current case. This happens by indicating either a pro-decision factor in the precedent that is missing in the current case, or a con-decision factor in the current case that is absent in the cited precedent.

Formally, a factor hierarchy is a directed acyclic graph, in which more concrete factors can be a reason pro or con more abstract factors. In CATO a factor hierarchy can be used to either emphasise or downplay a distinction. For example, if a new fact situation lacks a pro-plaintiff factor *Security measures taken* of a precedent cited by the plaintiff, then if this factor is pro the more abstract factor *Efforts to maintain secrecy*, the defendant can emphasise the distinction by arguing that by not taking security measures, the plaintiff had not made efforts

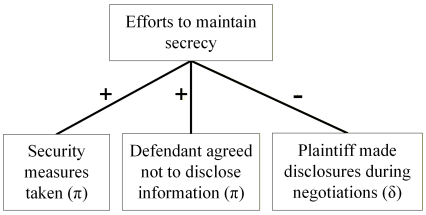



**Figure 1.** From CATO's factor hierarchy

to maintain secrecy (see Figure 1). Then if the new fact situation also contains another pro-plaintiff factor *Defendant agreed not to disclose information* that is also pro *Efforts to maintain secrecy*, the plaintiff can downplay the distinction by arguing that at the more abstract level the cases are similar since in both cases the plaintiff had made efforts to maintain secrecy, albeit in different ways. Another form of emphasing and downplaying illustrated by Figure 1 is when the defendant points at a new pro-defendant factor that was not in the precedent and is con *Efforts to maintain secrecy*, for instance, *Plaintiff made disclosures during negotiations*. The defendant can then emphasise this distinction by saying that because of the new con factor the plaintiff had not made efforts to maintain secrecy. Then if the new case again contains *Defendant agreed not to disclose information*, the plaintiff can use it to downplay the distinction by arguing that the new pro factor cancels the new con factor and so the abstract factor *Efforts to maintain secrecy* still applies in the new case.

Later work on legal case-based reasoning (starting with [6]) shifted its focus from generating realistic disputes to characterising when a particular decision is *forced* by a case base, called *precedential constraint*. The basic idea is simple: a decision in a new fact situation is forced if a precedent exists that cannot be distinguished from the new fact situation. Modelling this becomes more complicated when factor hierarchies are considered. Van Woerkom [9,10] defines a recursive account which can model precedential constraint on intermediate factors.

An until now largely neglected research question is whether the design of a factor hierarchy should be subject to coherence constraints. Consider the simple factor hierarchy in Figure 2. Given the pro path from $f_1$ to $d$ via $f_2$ it seems incoherent to regard $f_1$ as con $d$. The aim of this paper is to formulate coherence constraints on factor hierarchies that exclude such undesirable situations. It will

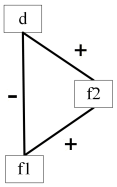



**Figure 2.** An incoherent factor hierarchy

turn out that the proposed constraints have a formal connection with a form of so-called noncontroversial bipolar argumentation frameworks, a notion introduced in [3]. A second aim of this paper is to show how the definitions used in the coherence constraints on factor hierarchies can be utilised to give precise and intuitive definitions of emphasising and downplaying moves, overcoming some drawbacks of the definitions given in [1] for CATO. Finally, in [2] constraints for factor hierarchies were proposed that only contain pro links and contain negations of all factors. I will show that their constraints are equivalent to the present ones.

This paper is organised as follows. In Section 2 van Woerkom's definition of a factor hierarchy is recalled, and four coherence constraints for factor hierarchies are proposed. In Section 3 these constraints are formally linked to the notion of a noncontroversial bipolar argumentation framework. In Section 4 the concepts introduced in Section 2 are used to give formal definitions of emphasising and downplaying moves. Section 5 concludes the paper.

## 2. Formalising Coherence Constraints on Factor Hierarchies

In this paper I use Van Woerkom's [9,10] formal definition of a factor hierarchy, with some minor notational differences. Given a set $P$ of factors, a *fact situation* $F$ is a subset of $P$. A *case* is a pair $(F, s)$ where $F$ is a fact situation and $s \in \{d, d'\}$. Here $s$ is the outcome, or decision, of the case. A case base $CB$ is a set of cases.

**Definition 2.1.** A *factor hierarchy* (FH) is a pair $(p, H)$ with $p$ a finite set of factors and $H$ a relation on $p$ satisfying

1. the transitive closure of $H$ is irreflexive;
2. $P$ contains exactly one $H$-maximal element $d$;
3. $H$ is equal to the union $\texttt{Pro} \cup \texttt{Con}$ of two disjoint relations $\texttt{Pro}$ and $\texttt{Con}$.

CATO distinguishes between weak and strong $\texttt{Pro}$ and $\texttt{Con}$ relations but for simplicity I will assume that all relations are equally strong. Because of clause (3) a factor hierarchy will alternatively be denoted as a triple $(p, \texttt{Pro}, \texttt{Con})$. A factor $p \in P$ is *basic* if there is no $q \in P$ with $H(q, p)$ and *abstract* otherwise.

Finally, $\texttt{Pro}(p) = \{q \in P \mid \texttt{Pro}(q,p)\}$ and $\texttt{Con}(p) = \{q \in P \mid \texttt{Con}(q,p)\}$. Likewise, for any fact situation $F$, $\texttt{Pro}_F(p) = \{q \in F \mid \texttt{Pro}(q,p)\}$ and $\texttt{Con}_F(p) = \{q \in F \mid \texttt{Con}(q,p)\}$.

To define when a factor hierarchy is coherent with respect to the intended meaning of its links, I now recursively define constraints on a factor hierarchy. These constraints should, for example, exclude an $FH$ with $p_1$ being pro $q$ and $d_1$ being con $p_1$ but pro $p$. The constraints are defined on the transitive closure of the untyped version of a factor hierarchy.

**Definition 2.2.** Let $f$, $f'$ and $f''$ be factors in a factor hierarchy $FH$. Then:

1. If $f \in \texttt{Pro}(f')$ then $f$ is pro $f'$.
2. If $f \in \texttt{Con}(f')$ then $f$ is con $f'$.
3. If $f$ is pro $f'$ and $f'$ is pro $f''$ then $f$ is pro $f''$.
4. If $f$ is pro $f'$ and $f'$ is con $f''$ then $f$ is con $f''$.
5. If $f$ is con $f'$ and $f'$ is pro $f''$ then $f$ is con $f''$.
6. If $f$ is con $f'$ and $f'$ is con $f''$ then $f$ is pro $f''$.
7. No other pro or con relation holds.

A factor hierarchy is *coherent* iff it satisfies these constraints and if for no pair $f, f'$ it holds that $f$ is both pro and con $f'$. Otherwise it is *incoherent*.

## 3. Relations with Noncontroversial Bipolar Argumentation Frameworks

In this section the coherence constraints on factor hierarchies will be formally linked to non-controversial bipolar argumentation frameworks. First the relevant notions of abstract and bipolar argumentation frameworks will be introduced.

### *3.1. Controversiality in Abstract and Bipolar Argumentation Frameworks*

An *abstract argumentation framework* ($AF$) is a pair $(\mathcal{A}, \mathcal{C})$, where $\mathcal{A}$ is a set of arguments and $\mathcal{C} \subseteq \mathcal{A} \times \mathcal{A}$ is a relation of attack. The theory of $AFs$ [5] defines and studies various notions of acceptable sets of arguments (called *extensions*), which are all internally conflict-free and defend themselves against attack. For present purposes these definitions are irrelevant and only the notion of controversial $AF$s will be used. It is defined in terms of two relations of indirect attack and defense between arguments (relative to an arbitrary given $AF$).

**Definition 3.1.** ([5]). Argument $a$ *indirectly attacks* argument $b$ iff there exists an odd-length path of attacks from $a$ to $b$. And $a$ *indirectly defends* $b$ iff there exists an even-length path of attacks of length $\geq 2$ from $a$ to $b$.

Then $AFs$ are in [5] defined to be (un)controversial as follows.

**Definition 3.2.** ([5]). Argument $a$ is *controversial* w.r.t. argument $b$ iff $a$ both indirectly attacks and indirectly defends $b$. An $AF$ is *controversial* iff it contains a pair $a, b \in \mathcal{A}$ such that $a$ is controversial w.r.t. $b$.

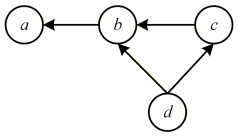



**Figure 3.** A controversial AF

Figure 3 displays an example controversial $AF$. Argument $d$ both indirectly defends and indirectly attacks argument $a$.

In [3] the notion of controversial arguments is generalised to $BAFs$, which add to Dung's $AFs$ a binary relation $\mathcal{S}$ of support on $\mathcal{A}$. Below is this definition with one addition.

**Definition 3.3.** Argument $a$ is *controversial* w.r.t. argument $b$ iff

1. $a$ supports (by a sequence of supports) an argument $x$ which indirectly attacks $b$; and
2. $a$ supports (by a sequence of supports) either $b$ or an argument $y$ which indirectly defends $b$.

A $BAF$ is controversial iff it contains a controversial argument.

The phrase 'either $b$' is added since an $FH$ with $x$ is `Pro` $a$, $a$ is `Pro` $y$ and $a$ is `Con` $y$ is incoherent but the associated $BAF$ is uncontroversial with Cayrol et al.'s original definition.

In [3] alternative argumentation-theoretic semantics are defined for controversial $BAFs$ However, such semantics are for present purposes irrelevant, since factor hierarchies are in CATO not directly used to determine outcomes of cases. Then the only relevant notion is that of controversiality and its relation with the coherence constraints on factor hierarchies proposed in Section 2.

### *3.2. Coherent Factor Hierarchies and Noncontroversial BAFs*

I now prove a relation between coherent factor hierarchies and noncontroversial $BAFs$. For one result below the following constraint on the attack relation of a $BAF$ is needed. (The formulation is adapted from [4].) Consider an arbitrary $BAF = (\mathcal{A}, \mathcal{C}, \mathcal{S})$ and any $a, b, c \in \mathcal{A}$. There is a *secondary attack* from $a$ to $b$ iff there exists an argument $c$ such that there is a sequence of supports from $c$ to $b$ and $(a, c) \in \mathcal{C}$.

**Definition 3.4.** Let $FH$ be a factor hierarchy $(P, \texttt{Pro}, \texttt{Con})$ and let $BAF = (\mathcal{A}, \mathcal{C}, \mathcal{S})$ with $\mathcal{A} = P$, $\mathcal{S} = \texttt{Pro}$ and $\mathcal{C} \subseteq \texttt{Con}$. Then $BAF$ is a bipolar argumentation framework *associated with* $FH$ iff either $\mathcal{C} = \texttt{Con}$ or $\mathcal{C} = \texttt{Con} \cup \mathcal{C}^+$ where $\mathcal{C}^+$ is the set of all secondary attacks given $BAF' = (\mathcal{A}, \mathcal{C}, \mathcal{S})$. In the latter case we say that $BAF$ is *with secondary attack*.

Note that each $FH$ has two associated $BAFs$, one with and one without secondary attack. Figure 4 displays a factor hierarchy on the left and the associated BAF with secondary attack on the right (the `Pro`, `Con`, attack and support relations go upwards). The added secondary attack from $f_2$ to $f_5$ is displayed as a dashed arrow. The factor hierarchy is incoherent since $f_1$ and $f_2$ are both pro and con $f_3$.

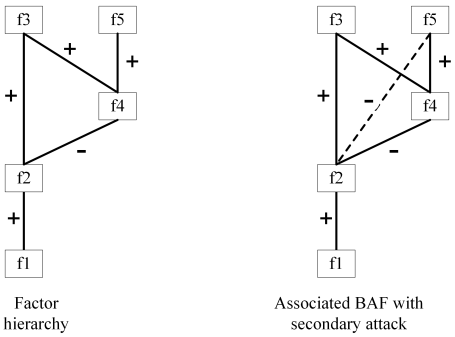



**Figure 4.** From factor hierarchy to BAF with secondary attack

**Lemma 3.5.** Let $f$ and $f'$ be factors in a factor hierarchy $FH$. Then

1. $f$ is con $f'$ iff there exists a path between $f$ and $f'$ with an odd number of `Con`-links; and
2. $f$ is pro $f'$ iff there exists a path between $f$ and $f'$ with an even number of `Con`-links.

*Proof.* (sketch) A 'con' relation can only be initiated by clause (2), (4) or (5) of Definition 2.2, which all use a single `Con`-link, and a 'con' relation can only change to 'pro' by clause (6) of Definition 2.2, which adds a single `Con`-link. □

**Lemma 3.6.** Let $BAF = (\mathcal{A}, \mathcal{C} \cup \mathcal{C}^+, \mathcal{S})$ have secondary attack. Then for any path $p$ in $BAF$ between $a$ and $b$ there is a path $p'$ in $BAF$ with only attacks from $\mathcal{C}$ and with the same number of attacks as $p$.

*Proof.* This holds since a secondary attack from $x$ to $y$ can for some $x'$ be replaced by an attack from $\mathcal{C}$ from $x$ to $x'$ and a sequence of supports from $x'$ to $y$. □

**Theorem 3.7.** For every factor hierarchy $FH$: if a $BAF$ associated with $FH$ is controversial then $FH$ is incoherent.

*Proof.* For any controversial BAF associated with $FH$ it can be shown that $FH$ is incoherent as follows. Suppose $a$ supports $x$ by a sequence of supports. Then $a$ is pro $x$. Suppose further that $x$ indirectly attacks $b$. Then by Lemma 3.5 there is an odd-length path of attacks from $\mathcal{C}$ in $BAF$ between $x$ and $b$, so $x$ is con $b$. But then $a$ is con $b$. Suppose, moreover, that $a$ supports $y$ by a sequence of supports. Then $a$ is pro $y$. Suppose finally that $y$ indirectly defends $b$. Then by Lemma 3.6 there is an even-length path of attacks from $\mathcal{C}$ in $BAF$ between $y$ and $b$, so $y$ is pro $b$. But then $a$ is pro $b$. □

**Theorem 3.8.** For every factor hierarchy $FH$: if $FH$ is incoherent and the $BAF$ associated with $FH$ is for secondary attack, then the BAF is controversial.

*Proof.* (Sketch) If $FH$ is incoherent, there exists a pair $f, f''$ such that $f$ is both pro and con $f''$. Then there are two paths $P_1$ and $P_2$ between $f$ and $f''$ such that $P_1$ contains an even number of `Con`-links while $P_2$ contains an odd number of

Con-links. Then it can be shown that the associated BAF with secondary attack is controversial as follows.

In any path $P$ in BAF from $f$ to $f''$ it holds that for every attacker $x$ of $z$ followed by a maximal sequence $S$ of supports to $y$, we have by secondary attack that $x$ attacks every argument in $S$. Then there is an attack from $x$ to $y$ in BAF. So for any $P$ there is an alternative path with from the first attacker in the sequence no supporters and the same number of attackers. But then for every $P_1$ and $P_2$ with, respectively, an odd and even number of attack links there exist pruned versions in $BAF$ starting with zero or more supports and then only attacks. These paths then satisfy, respectively, Definition 3.3(1) and Definition 3.3(2).

□

A counterexample to Theorem 3.8 without secondary attacks is an $FH$ where $x$ is $\texttt{Con}\ a_1$, $a_1$ is $\texttt{Pro}\ a_2$ which is $\texttt{Pro}\ a_3$ which is $y$ and $a_1$ is $\texttt{Con}\ a_3$. Then $FH$ is incoherent since $x$ is both pro and con $y$ but in the associated $BAF$ $x$ does not indirectly attack $y$. Adding the secondary attack from $x$ to $y$ excludes this counterexample.

### *3.3. Negated Factors*

In CATO and also in [9] factors have two values supporting different outcomes. When working with pro and con relations as above, the values of factors do not have to be made explicit. Nevertheless, this can be done, for instance, by for each factor $f$ in $FH$ adding its negation $\neg f$ and ensuring that if $p \in \texttt{Pro}(q)$ then $p \in \texttt{Con}(\neg q)$ and $\neg p \in \texttt{Con}(q)$ and that if $p \in \texttt{Con}(q)$ then $p \in \texttt{Pro}(\neg q)$ and $\neg p \in \texttt{Pro}(q)$. All definitions of this paper then still apply to the extended factor hierarchy (denoted below by $FH^{\neg}$).

Moreover, the following result holds.

**Proposition 3.9.** For any factor hierarchy $FH$ it holds that $FH$ is coherent if and only $FH^{\neg}$ is coherent.

*Proof.* (Sketch) Let $\varphi$ be a metavariable ranging over the set of all factors and their negations. First it can be shown with straightforward induction that in $FH^{\neg}$ it holds that
$\varphi$ is pro (con) $\neg q$ iff $\varphi$ is con (pro) $q$,
$\neg p$ is pro (con) $\varphi$ iff $p$ is con (pro) $\varphi$.
Then the result holds by the following observations:
$f$ is both pro and con $\neg f'$ in $FH^{\neg}$ iff $f$ is both con and pro $f'$ in $FH$.
$\neg f$ is both pro and con $f'$ in $FH^{\neg}$ iff $f$ is both pro and con $f'$ in $FH$.
$\neg f$ is both pro and con $\neg f'$ in $FH^{\neg}$ iff $f$ is both con and pro $\neg f'$ in $FH^{\neg}$ iff $f$ is both pro and con $f'$ in $FH$. □

Adding $\neg p \in \texttt{Con}(q)$ may not always be intuitive. For instance, if bribing an employee of the plaintiff is a reason pro misuse of a trade secret, then not bribing any employee is not necessarily a reason against misuse. Yet Aleven [1] assumes for CATO that every factor has a pro-plaintiff and a pro-defendant value, so for a system like CATO adding such relations seems reasonable.

As noted in the introduction, Canavotto & Horty[2] propose to replace a `Con` link between factors $p$ and $q$ by a `Pro` link between $p$ and $\neg q$. They then define the special case of Definition 2.2 considering only `Pro` links and then say that a factor hierarchy is coherent (in their terms 'factor uniform') just in case for no $p$ and $q$ it holds that $p$ is pro both $q$ and $\neg q$. The proof of Proposition 3.9 can easily be adapted to show that that $p$ is pro both $q$ and $\neg q$ in the modified hierarchy if and only if $p$ is both pro and con $q$ in the original version.

## 4. Formal Definitions of some Emphasising and Downplaying Moves

In this section I show how the formal definitions from the previous sections can be used to give precise and intuitive definitions of some emphasising and downplaying moves. One aim of this is to resolve some imperfections in Aleven's [1] description of CATO. Support is in [1] defined as for factor-value pairs but in fact the particular values do not matter: if $f$ supports one value of $g$, then it also supports the other value of $g$. Moreover, the definition of support does not consider whether a link is `Pro` or `Con`. As a consequence, the definitions allow counterintuitive forms of downplaying a distinction. Consider, for instance, the factor hierarchy in Figure 5 in which $p_1$ is `Pro`-$s$ and $d_1$ is `Con`-$s$, with these relations left implicit in the figure. Suppose a precedent has fact situation $G = \{p_1, d_1\}$ and the new fact situation $F = \{d_1\}$. Then the missing of $p_1$ in $F$ can be downplayed by arguing that $d_1$ supports $g$ in the precedent and, moreover, by saying that $d_1$ supports $g$ in $F$.

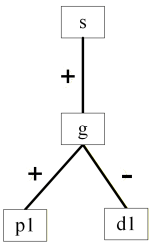



**Figure 5.** A factor hierarchy illustrating unintended downplaying

The definitions below may not be exactly what was intended in CATO but they at least illustrate how the formal definitions given above can be used to give precise and intuitive definitions of such move types. An earlier attempt to give a formal definition of CATO-style moves is Zhang et al. [11]. However, their definitions heavily rely on those of Aleven and therefore have the same drawbacks.

Recall that a case is a pair of two sets of base factors pro and con a decision plus a decision $s \in \{d, d'\}$. In AI & law applications it is assumed that every base factor of a factor hierarchy either has a `Pro` or a `Con` relation with the maximal element $d$. The same assumption holds in this section. Then a fact situation $X$ can be represented as a pair $(\texttt{Pro}_X(d), \texttt{Con}_X(d))$. The first definition below is straightforward and also applied in the work on CATO.

**Definition 4.1.** [Citable cases, distinguishing] Given a case $c = (G, s)$ and a fact situation $F$:

1. $c$ is *citable* for $s$ iff $\texttt{Pro}_G(s) \cap \texttt{Pro}_F(s) \neq \emptyset$;
2. $c$ is *distinguishable* on factor $f$ iff
   (a) $f \in \texttt{Pro}_G(s) \setminus \texttt{Pro}_F(s)$; or
   (b) $f \in \texttt{Con}_F(s) \setminus \texttt{Con}_G(s)$.

**Definition 4.2.** [Emphasising and downplaying a distinction] Consider a factor hierarchy $FH$ and a fact situation $F$.

1. Let $c$ be a citable precedent with decision $s$ which is distinguishable on $f \in \texttt{Pro}(s)$.
   (a) Then the distinction can be *emphasised* with factor $f'$ iff $f$ is pro $f'$ and $f'$ is pro $s$ (arguing that thus the more abstract factor $f'$ pro $s$ is also lacking in $F$).
   (b) The distinction can be *downplayed* with factor $f' \in \texttt{Pro}_F(s)$ iff there exists a factor $f''$ such that $f''$ is pro $s$ and both $f$ and $f'$ are pro $f''$. In this case we say that $f'$ *p-substitutes* $f$ in the new fact situation.
2. Let $c$ be a citable precedent with decision $d$ which is distinguishable on $f \in \texttt{Con}_F(d)$.
   (a) Then the distinction can be *emphasised* with factor $f'$ iff $f$ is pro $f'$ and $f'$ is con $s$, arguing that the new $\texttt{Con}(s)$ factor also makes an abstract factor con $s$ present in the new case.
   (b) This distinction can be *downplayed* with factor $g \in \texttt{Con}_c(s)$ iff $g$ is pro $f$ (arguing that the new con factor is more abstractly already in the precedent). In this case we say that $g$ *c-substitutes* $f$ in the precedent.
   (c) the distinction can also be *emphasised* with factor $f'$ if $f$ is con $f'$ and $f'$ is pro $s$, arguing that the new $\texttt{Con}(s)$ factor makes that an abstract factor pro $s$ does not apply in the new case.
   (d) This distinction can be *downplayed* with factor $g \in \texttt{Pro}_F(s)$ iff $g$ is pro $f'$ (arguing that the new pro factor cancels the new con factor as regards the presence or absence of $f'$ in the new case). In this case we say that $g$ *c-cancels* $f$ in the new case.

Further refinements are possible. For example, in CATO emphasising as in (1a) is not allowed if the distinction can be downplayed with a factor $f''$ such that $f'$ is pro $f''$ and $f''$ is pro $s$. For simplicity I do not consider such refinements but note that they can easily be added to the definitions.

Note that in coherent factor hierarchies the requirement that $f'$ is pro $s$ in (1a), that $f''$ is pro $s$ in (1b) and that $f'$ is con $s$ in (2a) are always satisfied and that the requirement that $f''$ is pro $s$ in (1b) is satisfied if the two requirements following them are satisfied. Also, in coherent factor hierarchies a distinction cannot both be emphasised and downplayed according to (1a) and (1b) involving the same abstract factor, while in incoherent hierarchies this can happen.
To illustrate the definitions, consider the factor hierarchy in Figure 6 (which leaves implicit that all basic $p$ factors are $\texttt{Pro}$ $s$ and all basic $d$ factors are $\texttt{Con}$ $s$). We have precedent $c$ with fact situation $G(c)$ and decided for $s$, and fact situation $F$:

$$\begin{array}{lcl} G(c) & = & \{p_1, d_1\} \\ F & = & \{p_2, d_2\} \end{array}$$

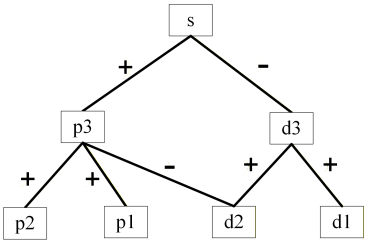



**Figure 6.** A factor hierarchy illustrating argument moves

1. Then $c$ can be distinguished on missing `Pro`-$s$ factor $p_1$.
   (a) This distinction can be emphasised with $p_3$, arguing that the intermediate pro-$s$ factor $p_3$ does not apply in $F$.
   (b) This distinction can be downplayed with $p_2$, arguing that $p_3$ still applies in $F$ since $p_2$ p-substitutes $p_1$ in $F$.
2. Moreover, $c$ can be distinguished on new `Con`-$s$ factor $d_2$.
   (a) This distinction can be emphasised with $d_3$, arguing that $d_2$ makes the more abstract `Con`-$s$ factor $d_3$ apply in $F$.
   (b) This distinction can be downplayed with $d_1$, arguing that $d_3$ already applied in $c$ since $d_1$ c-substitutes $d_2$ in $c$.
   (c) The distinction can also be emphasised with $p_3$, arguing that $d_2$, being `Con` $p_3$, makes that $p_3$ does not apply in $F$.
   (d) This distinction can be downplayed with $p_2$, arguing that $p_2$ cancels the negative effect of $d_2$ on $p_3$ in $F$.

## 5. Conclusion

Factor hierarchies are an important notion in AI & law models of case based reasoning. In this paper coherence constraints for factor hierarchies have been proposed and shown to be formally related to so-called non-controversial bipolar argumentation frameworks. The introduced new concepts have then been used to give precise and intuitive definitions of argument moves for emphasising and downplaying distinctions between cases, overcoming some drawbacks of existing definitions of these moves.

Although several notions concerning bipolar argumentation frameworks have been used in this paper, no use has been made of any semantics for such frameworks, since in CATO (the main source of inspiration for this paper) factor hierarchies are not directly used to define outcomes of cases. In future research it would be interesting to investigate whether semantics for BAFs still have uses in AI & law models of case-based reasoning, for instance, with respect to the notion of precedential constraint. Another topic for future research is whether the coherence constraints proposed in this paper can be refined to apply to dimension-based hierarchical models of case-based reasoning, such as the one from [8].

## References


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