Combining sceptical epistemic reasoning with credulous practical reasoning (corrected version)[1]

Henry Prakken

[a] *Department of Information and Computing Sciences, Faculty of Science, Universiteit Utrecht & Faculty of Law, University of Groningen, The Netherlands*

Abstract. This paper proposes an argument-based semantics for combined epistemic and practical reasoning, taking seriously the idea that in certain contexts epistemic reasoning is sceptical while practical reasoning is credulous. The new semantics combines grounded and preferred semantics. A dialectical proof theory is defined which is sound and complete with respect to this semantics and which combines existing argument games for sceptical reasoning in grounded semantics and credulous reasoning with preferred semantics.

Keywords. Practical vs. epistemic reasoning, argumentation, credulous and sceptical reasoning

1. Introduction

This paper is about the relation between epistemic and practical reasoning, or the relation between reasoning about beliefs and reasoning about action, where the later is understood as reasoning with motivational attitudes, such as having goals, desires or intentions. Since goals and desires often conflict or can be fulfilled in alternative ways, several researchers have proposed to formalise practical reasoning within a nonmonotonic logic. Some have used default logic (22; 6) while others have proposed argument-based accounts. Fox & Parsons (13) study the combination of medical diagnostic and treatment arguments in the argumentation logic of (14). Pollock (16) combines epistemic and practical reasoning in his OSCAR system. Atkinson and her colleagues (4; 5) give an account based on argument schemes (25). Finally, Amgoud (2) proposes a combined model of inference and decision making in a logic with tree-style defeasible arguments.

[1] This paper is a corrected version of (20). In the original version the soundness and completeness theorem does not hold. This is corrected below by distinguishing between players in an argument game (**PRO** and **CON**) and their dialectical roles (proponent and opponent). Further, on April 16, 2008 some errors in the example section were corrected (with thanks to Wietske Visser).

As is well-known, in nonmonotonic logics two different kinds of inference relations can be defined, viz. for *credulous* and for *sceptical* reasoning. They differ only when a conflict between defaults or arguments cannot be resolved. In such a case, credulous consequence notions branch into alternative sets of defeasible conclusions while sceptical consequence notions stick to one such set and leave both conclusions involved in an unresolvable conflict out of this set.

The current work on combining defeasible epistemic and practical reasoning essentially applies the same defeasible inference relation to both kinds of reasoning: either all reasoning is credulous, as in e.g. (22; 6), or all reasoning is sceptical, as in e.g. (2). (Amgoud allows for a credulous choice between alternative ways to achieve a goal, but this choice is formalised as a separate decision-making phase after all sceptically acceptable options have been computed).

However, in this paper I want to make a case for the claim that in certain contexts reasoning about beliefs should be sceptical while reasoning about action should be credulous. Consider a university lecturer John who wants to finish a paper on Friday but who has also promised to give a talk in a remote small town Faraway on the same day. John sees only two ways to travel to Faraway: by car and by bus; in both cases he will not be able to work while travelling (in the bus he always gets sick when reading or writing). So he sees no way to fulfil one desire without giving up the other and he sees no intrinsic reason to prefer one desire over the other. Then it seems rational for John to make a choice which desire he wants to fulfil. If this choice is formalised as reasoning, it must be formalised as credulous reasoning. However, let us now suppose that John's friend Bob tells him that there is a railway connection to Faraway, so that he could work while travelling and also finish his paper. Then John's other friend Mary warns him that there will be a railway strike on Friday, so that there will be no trains after all. Bob, however, says he does not believe there will be such a strike. So to form the goal of taking the train, John must first find out whether it will run on Friday. If he has to find this out on the basis of his current beliefs, his task is one of epistemic reasoning. Now suppose that John has no reason to trust one of his friends more than the other. Then it seems rational for him not to act on the credulous belief that there will be a train to Faraway on Friday.

The kind of rationality that is assumed here is that a rational agent should map out all credulously acceptable action alternatives that have sceptically acceptable epistemic support and then make a choice between them. Objections might be raised against this view of rationality; some of them will be discussed in Section 6, after the present view has been formalised.

The technical contribution of this paper is a combined formalisation for sceptical epistemic reasoning interleaved with credulous practical reasoning. More precisely, a unified formal framework will be defined for sceptical epistemic reasoning according to grounded semantics and credulous practical reasoning according to preferred semantics. The choice for sceptical grounded and credulous preferred semantics (10) has a pragmatic and a philosophical reason. The pragmatic reasoning is that sceptical grounded and credulous preferred semantics are presently the only two argument-based semantics with elegant proof-procedures in argument-game form. See e.g. (9; 21) for sceptical grounded and (24; 12; 8; 11) for credulous preferred semantics. A philosophical reason is given by Caminada (7) in his defence why sceptical reasoning should be modelled with grounded semantics and credulous reasoning with preferred semantics. Since his argument is too detailed to repeat it here, the interested reader is referred to his paper.

The rest of this paper is organised as follows. In Section 2 the framework for argument-based reasoning assumed in this paper will be introduced. Among other things, this section introduces the notions of a logical language, argument construction and defeat between arguments. These three notions will in Section 3 be refined by dividing the language into an epistemic and a practical sublanguage and by distinguishing epistemic vs. practical arguments and epistemic vs. practical ways to defeat an argument. Section 2 has also summarised grounded and preferred semantics and two argument games for these semantics. The semantics and argument games will in Section 4 be merged into a unified semantics and proof theory for combined epistemic and practical reasoning. The new formalism will be illustrated with some examples in Section 5 and some objections against its underlying account of rationality will be discussed in Section 6.

2. Logical preliminaries

The analysis of this paper is within Dung's (10) abstract approach to defeasible argumentation. First the basic notions of (10) will be summarised as far as needed here, adapting some notation of (12).

Definition 2.1 An *argument system* is a pair $\mathcal{H} = (\mathcal{A}, \mathcal{D})$, in which $\mathcal{A}$ is a set of *arguments* and $\mathcal{D} \subseteq \mathcal{A} \times \mathcal{A}$ is the *defeat* relationship for $\mathcal{H}$. When $(a, b) \in \mathcal{D}$ we say that a *defeats* b; when moreover $a \in S \subseteq \mathcal{A}$ we also say that S defeats b. For $S \subseteq \mathcal{A}$ we say that

1. $a \in \mathcal{A}$ is *acceptable with respect to* S if for every $b \in \mathcal{A}$ that defeats a there is some $c \in S$ that defeats b.
2. S is *conflict-free* if no argument in S is defeated by S.
3. S is *admissible* if S is conflict-free and every argument in S is acceptable with respect to S.
4. S is a *preferred extension* of $\mathcal{H}$ if it is a maximal (with respect to set inclusion) admissible subset of $\mathcal{A}$.
5. Let $F : \mathcal{A}^2 \longrightarrow \mathcal{A}^2$ be a function that for each subset S of $\mathcal{A}$ returns the set of all arguments that are acceptable with respect to S. Then S is the *grounded extension* of $\mathcal{H}$ if S is the least fixpoint of F.

As shown by (10), grounded semantics always produces a unique extension while preferred semantics may produce multiple extensions when a conflict between arguments cannot be resolved. This motivates the following well-known definitions. An argument is *justified with respect to* $\mathcal{H}$ in a semantics if it is in every extension of $\mathcal{H}$ according to that semantics. An argument is *defensible with respect to* $\mathcal{H}$ in preferred semantics if it is in some but not all preferred extensions of $\mathcal{H}$. Finally, an argument is *defensible with respect to* $\mathcal{H}$ in grounded semantics if it is not in the grounded extension of $\mathcal{H}$ but not defeated by it. Now *credulous* argument-based reasoning is interested whether an argument is defensible while *sceptical* argument-based reasoning checks whether an argument is justified.

This abstract approach will be instantiated with a familiar tree-style structure of arguments (15; 23; 3), where *strict* and *defeasible* inferences are chained into trees. The inference rules apply to a logical language $\mathcal{L}$ closed under negation. Strict inference rules, which are usually taken to be those of standard propositional or first-order logic, are written as $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ and defeasible rules as $\varphi_1, \ldots, \varphi_n \rightsquigarrow \varphi$ (where each φ and φ_i is a well-formed formula of $\mathcal{L}$).

Arguments chain inference rules into AND trees, starting with a subset of $\mathcal{L}$. For any argument A, its *premises*, written as $prem(A)$, are all leaf nodes of A, and its *conclusion*, $conc(A)$, is its root. An argument A is a *subargument* of an argument B if both have the same premises and $conc(A)$ is a node in B. An argument is *strict* if all its rules are strict, otherwise it is *defeasible*. It is defeasible inferences that make an argument subject to defeat.

The defeat relation will not be fully formalised in this paper; any full definition is assumed to satisfy the following conditions (which are satisfied by most argumentation systems in the literature).

Assumption 2.2 For all arguments A, B and C: if A defeats B and B is a subargument of C then A defeats C.

Observation 2.3 If an argumentation system satisfies Assumption 2.2 then for all arguments A and B and grounded or preferred extensions E:

- if A is a subargument of B and $A \notin E$ then $B \notin E$.

Finally, we define the notion of an argument game and provide two instantiations for grounded and preferred semantics. The following definitions use some notation of (19) and are relative to an unspecified argumentation system.

Argument games are between a *proponent* P and an *opponent* O of an argument from $\mathcal{A}$. The set M of *moves* of a game is defined as $\mathbb{N} \times \{P, O\} \times \mathcal{A} \times \mathbb{N}$, where the four elements of a move m_i are denoted by, respectively: $id(m)$, the *identifier* of the move, $pl(m)$, the *player* of the move, $a(m)$, the *argument* put forward in the move, $t(m)$, the *target* of the move.

The set of *argument games* (or games for short), denoted by M, is the set of all sequences $d = m_1, \ldots, m_i, \ldots$ from M such that

- each i^{th} element in the sequence has identifier i,
- $t(m_1) = 0$,
- $pl(m_i) = P$ if i is odd and $pl(m_i) = O$ if i is even;
- for all $i > 1$ it holds that $t(m_i) = j$ for some m_j preceding m_i in the sequence,
- if $d \neq \emptyset$ then $a(m_i)$ defeats $a(t(m_i))$.

The set of *finite games*, denoted by M^*, is the set of all finite sequences that satisfy these conditions. For any game $d = m_1, \ldots, m_n, \ldots$, the sequence $m_1, \ldots, m_i$ is denoted by d_i, where d_0 denotes the empty game. When $t(m) = id(m')$ we say that m *replies to* m' in d and also that m' is the *target of* m in d. Slightly abusing notation, $t(m)$ sometimes denotes a move instead of just its identifier.

A *protocol* on M is a set $R \subseteq M^*$ satisfying the condition that whenever d is in R, so are all initial sequences that d starts with. A partial function Pr : $M^* \longrightarrow \mathcal{P}(M)$ is derived from R as follows:

- $Pr(d) =$ undefined whenever $d \notin R$;
- $Pr(d) = \{m \mid d, m \in R\}$ otherwise.

The elements of $dom(Pr)$ (the domain of Pr) are called the *legal finite games*. The elements of $Pr(d)$ are the moves allowed after d. If d is a legal game and $Pr(d) = \emptyset$, then d is said to be a *terminated* game.

Within this framework an argument game for grounded semantics proposed by (21) can be stated as follows:

Definition 2.4 [G-games] An argument game is a *G-game* if for all moves m_i and finite legal games d it holds that $m_i \in Pr(d)$ iff:

1. If $i \neq 0$ then $t(m_i) = m_{i-1}$;
2. if $pl(m_i) = P$ then $a(m_i)$ was not moved by P in d.

Player *pl wins* a G-game d iff d is terminated and *pl* made the last move in d.

This game was by (21) proven to be sound and complete with respect to grounded semantics in the sense that proponent has a winning strategy for argument A if and only if A is a member of the grounded extension.[1]

In (24) the following game for preferred semantics was defined. In this definition, *a game line* is a sequence of the game where each non-initial move responds to the preceding move.

Definition 2.5 [P-games] An argument game is a *P-game* if for all moves m_i and finite legal games d it holds that $m_i \in Pr(d)$ iff:

1. if $pl(m_i) = P$ then:
   (a) If $i \neq 0$ then $t(m_i) = m_{i-1}$;
   (b) $a(m_i)$ was not moved by O in d.
2. if $pl(m_i) = O$ then $a(m_i)$ was not moved by O in d in the same game line.

Player *pl wins* a P-game d if d is terminated and *pl* made the last move in d. Furthermore, P wins d if d is infinite and O wins d if O repeats any argument earlier moved by P.

This game was by (24) proven to be sound and complete with respect to credulous preferred semantics in the sense that proponent has a winning strategy for argument A if and only if A is a member of some preferred extension.

3. Knowledge, arguments and defeat in epistemic and practical reasoning

In this section we adapt the notions of a logical language, argument construction and defeat between arguments to the distinction between epistemic and practical reasoning. Generally, two styles of reasoning about action can be recognised, which could be called *quasi-deductive* and *abductive*. The quasi-deductive ap-

[1]In fact (21) required that the target of a P move does not defeat it but this condition is redundant for this result.

proach, taken by e.g. (22; 6), formalises decision rules as "If I want to satisfy my appetite I must prepare a meal" and applies such rules to beliefs or desires in a forward, modus-ponens style of reasoning. Conflicting desires, as with "if someone opens the door for me I should thank him" vs. "I should not speak while eating", are then managed by a priority mechanism defined on the rules. The abductive approach, taken by e.g. (4), essentially applies the well-known practical syllogism from philosophy: if I believe that "preparing a meal can make me satisfy my appetite" and I want to satisfy my appetite, I can form the desire to prepare a meal with a 'backward', abductive reasoning step. Alternative ways to fulfil my desires, such as given by the knowledge that "going to a restaurant can satisfy my appetite", must then be regarded as conflicting arguments.

The formalism proposed in this paper is intended to apply to both forms of reasoning and therefore it must abstract from their particularities. To this end, this paper will confine itself to some partial assumptions on the language, the rules for argument construction and the nature of the defeat relation.

First it is assumed that the logical language is divided into two disjoint sublanguages $\mathcal{L}_e$ of *epistemic formulas* and $\mathcal{L}_p$ of *practical formulas*. A full definition of these two sublanguages is far from trivial but depends on the chosen particular argumentation system and cannot therefore be addressed in this paper.

Next a further condition is imposed on arguments. An instantiated inference rule r is called *epistemic* if $prem(r) \cup conc(r) \subseteq \mathcal{L}_e$ and it is called *practical* if $conc(r) \in \mathcal{L}_p$. Then all inferences in an argument must be either epistemic or practical. An argument is called an *e-argument* if all its inferences are epistemic, otherwise it is called a *p-argument*. For any argumentation system $\mathcal{H} = (\mathcal{A}, \mathcal{D})$, the set $\mathcal{A}$ is accordingly divided into two subsets $\mathcal{A}_e$ and $\mathcal{A}_p$; note that these sets are disjoint. Note also that these constraints rule out that inferences with a practical premise have an epistemic formula as a conclusion. This cannot be expressed as a condition on inference rules if the strict rules contain, for instance, full propositional or first-order logic, since if an epistemic formula φ is universally valid then any practical formula strictly implies it. The constraints in effect formalise the principle that no *Is* can be derived from a practical *Ought*. The reverse principle could also be formalised but for present purposes this will not be needed.

At first sight, the following example (due to Trevor Bench-Capon, personal communication) would seem to cast doubt on the constraint that no *Is* can be inferred from an *Ought*. Consider the default rule "John tends to achieve his goals" and assume that John has selected his goal to finish his paper to be carried out. Then it seems that these two premises, of which one is practical, give rise to an argument for the conclusion that John will in fact finish his paper. However, it should be noted that it is not John's goal that is a premise of this argument but the observer's belief that John has adopted it, and the latter is an epistemic formula. This is reminiscent of the distinction in deontic logic between logics of norms (expressing obligations and permissions) and logics of normative propositions (describing the content of normative system from an external perspective); see (1) for a clear account of this distinction.

An argumentation system is now called an *e-p-argumentation system* if its set of arguments consists of disjoint sets of e- and p-arguments. Moreover, for any two arguments A and B such that A defeats B we say that A *e-defeats* B if A is an epistemic argument and that A *p-defeats* B otherwise. We assume that if A defeats B such that A does not defeat a proper subargument of B, then B is an e-argument if A is an e-argument and B is a p-argument otherwise. Note, by the way, that if A e-defeats B then B may very well be a practical argument; in that case A defeats an epistemic subargument of B.

Observation 3.1 If an argumentation system satisfies Assumption 2.2 then for all arguments A and B and grounded or preferred extensions E:

1. if A e-defeats a subargument of B, then A e-defeats B;
2. no p-argument defeats an e-argument.

4. Combining epistemic and practical inference

In this section the new inference notion for combined epistemic and practical reasoning will be defined. At first sight, it would seem to suffice to first determine the grounded extension of all belief arguments, then add all justified beliefs as facts to the knowledge base and then construct the preferred extensions of the new theory. However, this does not work, since it does not respect that reasoning about beliefs and actions is interleaved (cf. also (16)). Often it is practical reasoning that determines which beliefs are relevant. For instance, in the example of the introduction John's beliefs about whether the train will run to Faraway on Friday is relevant only if he considers his goal of giving the talk. It does not make much sense for John to reason about this irrespective of his goals and desires. What is needed therefore is a single proof procedure for both kinds of reasoning. And this proof procedure in turn needs a semantics, which has the following form.

Definition 4.1 Let $\mathcal{H} = (\mathcal{A}, \mathcal{D})$ be an e-p-argumentation system with grounded extension $G_{\mathcal{H}}$. Let $\mathcal{H}_g = (\mathcal{A}_g, \mathcal{D}_g)$ be obtained from $\mathcal{H}$ by:

- removing from $\mathcal{A}$ all arguments that are defeated by an e-argument that is not defeated by an argument in $G_{\mathcal{H}}$; [2]
- and restricting $\mathcal{D}$ to $\mathcal{A}_g$.

Then S is an *e-p-extension* of $\mathcal{H}$ iff S is a preferred extension of $\mathcal{H}_g$.

Observation 4.2 For any e-p-argumentation system $\mathcal{H} = (\mathcal{A}, \mathcal{D})$ with grounded extension $G_{\mathcal{H}}$, no argument in $\mathcal{A}_g$ is defeated by an e-argument in $\mathcal{A}_g$.

The corresponding argument game is now defined by combining the G-game with the P-game as follows.

We first need to make a distinction between a player and its dialectical role.[3] (This idea is adapted from (18), where it was used to model shifts in the burden of proof in legal disputes.) The main idea is that when P is the first player to

[2] In (20) this condition was 'removing from $\mathcal{A}$ all e-arguments that are not in $G_{\mathcal{H}}$ plus all arguments of which they are a subargument'.

[3] This distinction was not made in (20).

move an epistemic argument and that argument attacks a practical argument of O, then P in fact attacks an epistemic subargument or assumption of **CON**'s practical argument and P must show that O's argument is not in the grounded extension (see the first bullet of Definition 4.1). So in fact the dialectical roles should switch here: P should become the opponent in the G-game about the epistemic subargument of O's attacked p-argument. Therefore, the players now are **PRO** and **CON** while at any stage of the game one has P-role while the other has O role, as follows. In the definition of an argument game (see Section 2) P is replaced by **PRO** and O is replaced by **CON**. Furthermore, each move m now also has a role $r(m)$, which is either P or O. Finally, when pl denotes a player then $\overline{pl}$ denotes the other player. Now:

Definition 4.3 [GP-games] An argument game is a *GP-game* if for all moves m_i and finite legal games d with last move m_l it holds that $m_i \in Pr(d)$ iff:

1. $r(m_1) = \textbf{PRO}$;
2. if $a(m_i) \in \mathcal{A}_e$ and $a(m_l) \in \mathcal{A}_p$ then $r(m_i) = O$; otherwise $r(m_i) \neq r(m_l)$;
3. if $pl(m_i) = \textbf{PRO}$ or $a(m_l) \in \mathcal{A}_e$ then $t(m_i) = m_l$;
4. if $r(m_l) = O$ and $a(m_l)$ repeats an argument earlier moved by $\overline{pl(m_l)}$ then $a(m_i) \in \mathcal{A}_e$;
5. if $r(m_i) = P$ then:
   (a) If $a(m_i) \in \mathcal{A}_e$ then $a(m_i)$ was not moved by $\underline{pl(m_i)}$ in d;
   (b) If $a(m_i) \in \mathcal{A}_p$ then $a(m_i)$ was not moved by $\overline{pl(m_i)}$ in d;
6. if $r(m_i) = O$ and $a(m_i) \in \mathcal{A}_p$ then $a(m_i)$ was not moved by $pl(m_i)$ in the same game line of d.

Player pl *wins* a GP-game d if d is terminated and pl made the last move in d. Furthermore, if d is infinite then **PRO** wins d if d contains no e-arguments or d contains a role switch while **CON** wins d otherwise.

In this definition, clause (2) says that the dialectical role of a player changes only if **PRO** attacks an epistemic subargument of a practical argument moved by **CON**: then **PRO** becomes opponent and **CON** becomes proponent in the thus started G-part of the game. Clause (3) captures that **PRO** may not backtrack in the P game and neither player may backtrack in the G-game. Clause (4) captures that when a player in O role repeats a p-argument of the other player, then the repeating player in fact wins the P-part of the GP-game so the game continues as a G-game. Clause (5a) captures that P may not repeat his own e-arguments in the G-part of the game while clause (5b) expresses that P may not repeat O's p-arguments in the P-part of the game. Finally, clause (6) repeats the rule of the P-game that O may only repeat her own moves in different lines of the dialogue.

The GP-game will now be proven sound and complete with respect to the semantics of Definition 4.1 in the sense that **PRO** has a winning strategy for argument A if and only if A is a member of some e-p-extension.

Lemma 4.4 let d be a GP-game.

1. d consists of a possibly empty sequence of p-arguments (denoted by d_p) followed by a possibly empty sequence of e-arguments;

2. d_p is a P-game;
3. Let d_e be the maximal subsequence of d such that its first move is the first in d that moves an e-argument, preceded by its target if that exists. Then d_e is a G-game.

Proof: (1) follows from Definition 4.3(3), which says that an e-argument must be immediately replied to and Observation 3.1, which says that such a reply always is an e-argument. (2) follows from (1) and since if a p-move satisfies Definition 4.3(2) then it satisfies Definition 2.5(1a), if it satisfies Definition 4.3(5b) then it satisfies Definition 2.5(1b) and if it satisfies Definition 4.3(6) then it satisfies Definition 2.5(2). Finally, (3) follows from (1) which implies that the first argument in d_e cannot be repeated and all other moves in d_e reply to their immediate predecessor. □

The reader might think that if the first argument of d_e is a p-argument, then the next move has defeated it by defeating one of its epistemic subarguments. However, this need not be the case: the p-argument itself may have been e-defeated by, for instance, an undercutter (15) or an assumption-attack (21).

Lemma 4.5 Let d be a GP-game won by **PRO** in which **CON** has played optimally and let A be the first argument in d_e.

1. $A \in \mathcal{A}_g$ iff A was moved by **PRO**.
2. if the first e-argument of d is moved by **PRO** then its target is not in $\mathcal{A}_g$.

Proof: By Lemma 4.4(3) it holds that d is a G-game. (1) Suppose first A is moved by **PRO**. Then there has been no role switch so **PRO** moved A in P-role. Since **PRO** has won against optimal play of **CON**, **PRO** has a winning strategy in the G-game for A. Then $A \in G_{\mathcal{H}}$ by soundness of the G-game so clearly $A \in \mathcal{A}_g$. Suppose next A is moved by **CON** and **PRO** replied to A with B. If A is an e-argument then B was moved in P-role and since **PRO** has won against optimal play of **CON**, **PRO** has a winning strategy as P in the G-game for B. Then $B \in G_{\mathcal{H}}$ by soundness of the G-game so clearly $A \notin \mathcal{A}_g$. If A is a p-argument then B was moved in O-role and since **PRO** has won d against optimal play of **CON**, **PRO** has a winning strategy as O in the G-game for B. Then by soundness of the G-game, B is not defeated by any argument in $G_{\mathcal{H}}$; but then $A \notin \mathcal{A}_g$.

(2) Follows immediately from (1). □

Theorem 4.6 For any e-p-argumentation system $\mathcal{H} = (\mathcal{A}, \mathcal{D})$ that satisfies Assumption 2.2 and any argument $A \in \mathcal{A}$, **PRO** has a winning strategy for A in a GP-game if and only if there exists an e-p-extension of $\mathcal{H}$ that includes A.

Proof: ($\Rightarrow$) For soundness, suppose **PRO** has a winning strategy for A. Then **PRO** can win any game for A. Consider any such game d won by **PRO** in which **CON** has played optimally. Let S be the set of all **PRO** arguments in d. Suppose for contradiction that S is not admissible on the basis of $\mathcal{H}_g$ and suppose first that S is not conflict-free. Then there exists B and C in S (so by assumption in $\mathcal{A}_g$) such that B defeats C, so CON can at some point in d repeat B after PRO has moved C. Since CON has played optimally, it has done so. Then by Definition 4.3(2) and the fact that PRO has won d, PRO has replied to B with an e-argument. But then by Lemma 4.5 it holds that $B \notin \mathcal{A}_g$. Contradiction. So there is a B outside S but in $\mathcal{A}_g$ that defeats some C in S and is not defeated by S. Then by a same line of reasoning PRO has attacked B in d with an e-argument so by Lemma 4.5 it holds that $B \notin \mathcal{A}_g$. Contradiction. So S is admissible on the basis of $\mathcal{H}_g$ which by a result of (10) implies that S (which contains the initial argument A) is included in a preferred extension of $\mathcal{H}_g$.

($\Leftarrow$) For completeness, suppose A is in some e-p-extension of $\mathcal{H}_g$. By completeness of the P-game PRO has a winning strategy S for A in the P-game on the basis of $\mathcal{H}_g$. Then as long as CON moves p-arguments from $\mathcal{A}_g$, PRO can reply in a PG-game by picking p-arguments from S.

If CON moves a p-argument B from outside $\mathcal{A}_g$, then by construction of $\mathcal{A}_g$ it holds that B is defeated by an e-argument C that by Definition 4.1 and Observation 4.2 is not overruled in grounded semantics on the basis of $\mathcal{H}$. Then if PRO moves B, PRO moves in O-role and PRO can win the PG-game by following the winning strategy that O has for B in the G-game.

Finally, suppose CON moves an e-argument B and consider the first such argument. If there does not exist a $C \in G_{\mathcal{H}}$ that e-defeats B, then the target A of B is not in $G_{\mathcal{H}}$; but then $A \notin \mathcal{A}_g$ by construction of $\mathcal{A}_g$, which contradicts the assumption that PRO has picked A from $\mathcal{A}_g$. So there exists a $C \in G_{\mathcal{H}}$ that e-defeats B. Then by completeness of the G-game PRO can win by following a winning strategy that P has for C in the G-game. □

5. Examples

In this section the new formalism will be illustrated with some symbolic examples.[4] Since the formalism abstracts from the full details of particular logics, the examples will have to be semiformal. Unless indicated otherwise, PRO's arguments are moved in P-role and CON's arguments are moved in O-role.

Consider first quasi-deductive, forward goal generation. Assume the following rules (where D stands for 'desire'):

$\{ p \Rightarrow \mathrm{D}q,\ r \Rightarrow \neg\mathrm{D}q,\ s \Rightarrow p,\ t \Rightarrow \neg p \}$

And let s, r and t be given as facts. Now let all formulas $(\neg)\mathrm{D}L$ where L is a propositional literal and all rules with occurrences of D be practical formulas and let the rest be epistemic formulas. Suppose an argument A defeats an argument B if A's conclusion contradicts a conclusion of a subargument of B. The following GP-game can be played:

PRO_1:	s, $s \Rightarrow p$, $p \Rightarrow \mathrm{D}q$, so $\mathrm{D}q$	CON'_1:	t, $t \Rightarrow \neg p$, so $\neg p$
PRO'_2:	s, $s \Rightarrow p$, so p	CON'_2:	repeats CON'_1

So PRO has no winning strategy for $\mathrm{D}q$. However, he has one for $\neg\mathrm{D}q$:

[4] In the conference version of this paper the first, fourth and last example contained some errors caused by typos in the knowledge bases, spotted and corrected by Wietske Visser.

PRO$'_1$: r, $r \Rightarrow \neg Dq$, so $\neg Dq$ **CON**$''_1$: s, $s \Rightarrow p$, $p \Rightarrow Dq$, so Dq
PRO$''_2$: repeats **PRO**$_1$

So the only action alternative with justified support is for $\neg Dq$. This agrees with the semantics: the e-arguments **PRO**$'_2$ and **CON**$'_1$ defeat each other so they are not in the grounded extension of $\mathcal{H}$. Then they are not in $\mathcal{A}_g$ so the p-argument **PRO**$_1$, which has **PRO**$'_2$ as a subargument, is also not in $\mathcal{A}_g$. So $\mathcal{H}_g$ has a unique e-p-extension, containing **CON**$_1$ = **PRO**$'_1$.

Suppose now that p becomes known as a matter of fact. Then **PRO** also has a winning strategy for Dq since **CON** cannot now win as in the second game above:

PRO$_1$: s, $s \Rightarrow p$, $p \Rightarrow Dq$, so Dq **CON**$'''_1$: t, $t \Rightarrow \neg p$, so $\neg p$
PRO$'''_2$: p

Assuming that purely factual arguments cannot be defeated, **PRO** wins this game. Nevertheless, still a choice must be made what to do since the trivial winning strategy for $\neg Dq$ still stands. Again this agrees with the semantics: **PRO**$_1$'s subargument **PRO**$'_2$ is now in the grounded extension of $\mathcal{H}$ so **PRO**$_1$ is in $\mathcal{A}_g$. Since **PRO**$_1$ defeats its only defeater in $\mathcal{A}_g$, which is **CON**$_1$, there are now two e-p-extensions, one containing **PRO**$_1$ and the other containing **CON**$_1$.

The following example illustrates a role switch (roles are now indicated between square brackets). Assume the following rules

$\{ p \Rightarrow Dq, r \Rightarrow \neg Dq, s \Rightarrow r, t \Rightarrow \neg r \}$

and let p, s and t and be given as facts. Then **PRO** has a winning strategy which includes a role switch:

PRO$_1[P]$: p, $p \Rightarrow Dq$, so Dq **CON**$_1[O]$: s, $s \Rightarrow r$, $r \Rightarrow \neg Dq$, so $\neg Dq$
PRO$_2[O]$: t, $t \Rightarrow \neg r$, so $\neg r$ **CON**$_2[P]$: s, $s \Rightarrow r$, so r
PRO$_3[O]$: repeats **PRO**$_2$

Since **CON** now has proponent role in the game on r, **CON** is not allowed to repeat **CON**$_2$ and loses.

Consider next some symbolic examples with abductive goal generation, in which from "doing a in circumstance s achieves g" and Dg the desire Da can be inferred, and from the same rule and Dg instead $\neg Da$ can be inferred. Consider

$\{ a_1 \wedge s \Rightarrow p, a_2 \Rightarrow p, r \Rightarrow s, a_1 \Rightarrow \neg p, a_2 \Rightarrow \neg q \}$

and suppose we have the fact r and the desires Dp and Dq. Let a_i be action descriptions while s is a circumstance beyond the agent's control. As before, formulas with D are in $\mathcal{L}_p$ while the rest is in $\mathcal{L}_e$. Defeat can now also happen by providing an alternative way to fulfil a desire. Now **PRO** has a winning strategy for an argument for Dp:

PRO$_1$: r, $r \Rightarrow s$, so s; also $a_1 \wedge s \Rightarrow p$ and Dp, so Da_1
CON$_1$: $a_2 \Rightarrow p$ and Dp, so Da_2
PRO$_2$: $a_2 \Rightarrow \neg q$ and Dq, so $\neg Da_2$ ("a_2 prevents another desire, so don't do it")
CON$_2$: $a_1 \Rightarrow \neg p$ and Dp, so $\neg Da_1$ (alternative reply to **PRO**$_1$)
PRO$_3$: repeats **PRO**$_1$ in reply to **CON**$_2$.

In a similar way there is a winning strategy for Da_2 so there are at least two e-p-extensions, one with arguments for Da_1 and $\neg Da_2$ and one with arguments for $\mathrm{D}a_2$ and $\neg\mathrm{D}a_1$. Note also that $\mathcal{H}_g = \mathcal{H}$ so these are also preferred extensions of $\mathcal{H}$. However, there is also a third e-p-extension, containing arguments for $\neg\mathrm{D}a_1$ and $\neg\mathrm{D}a_2$. The first of these conclusions can be proven as follows:

PRO$_1$: $a_1 \Rightarrow \neg p$ and $\mathrm{D}p$, so $\neg\mathrm{D}a_1$
CON$_1$: r, $r \Rightarrow s$, so s; also $a_1 \wedge s \Rightarrow p$ and $\mathrm{D}p$, so $\mathrm{D}a_1$
PRO$_2$: repeats **PRO**$_1$.

The proof of $\neg\mathrm{D}a_2$ is similar.

Assume now that we also know that $t \Rightarrow \neg s$ while t holds as a matter of fact. Then in the argument game for $\mathrm{D}a_1$, **CON** can win by attacking **PRO**$_1$ with an e-argument for $\neg s$, defeating its subargument for s (but also defeated by it). So the arguments for s and $\neg s$ are both not in $\mathcal{A}_g$, so **PRO**$_1$ is also not in $\mathcal{A}_g$, so we lose the first of the above three e-p-extensions.

6. Discussion

This paper has proposed an argument-based semantics and proof theory for combined epistemic and practical reasoning, taking seriously the idea that in certain contexts epistemic reasoning is sceptical while practical reasoning is credulous. As mentioned in the introduction, the kind of rationality assumed by the new formalism is that a rational agent should map out all defensible action alternatives that have justified epistemic support and then make a choice between them. Several objections might be raised against this view of rationality. For instance, it might be argued that in cases where an agent has only defensible support for his action alternatives, the present approach implies that he cannot rationally do anything. (This objection is due to Trevor Bench-Capon, personal communication.) Two alternative solutions to this problem could be proposed.

For the first, consider again the example of the introduction. It could be argued that what John should do is to compute the expected utility of his actions, incorporating the uncertainty about whether there will be a train to Faraway on Friday into a probability distribution over the possible outcomes of taking the train. This may be a sound approach if all uncertainty and utility can be quantified but it hides the fact that John will often have to *reason* about whether there will be a train (see also (17)). Moreover, often only partial and qualitative information about probability and preference is available and nonmonotonic logics are primarily meant for use in such cases.

A second option is to say that a logic for practical reasoning should simply present all credulously acceptable arguments to a user of the system and let the user decide upon which argument to act. This may be the best approach in some contexts but it also seems to blur a fundamental difference between epistemic and practical conflicts. In the first case, truth is at stake, so it is rational to do further investigations to resolve the conflict (John could phone the railway company). However, with conflicts on action there is no counterpart of truth and an arbitrary choice between credulously acceptable alternatives is perfectly rational.

Having said this, the choice between the best way to combine epistemic and practical reasoning may depend on the context of application. If precise knowledge about probabilities and utilities is available, decision-theoretic approaches may be the most suitable. Otherwise, a qualitative approach seems preferred, where in some contexts all reasoning can best be credulous while in other contexts epistemic reasoning can better be sceptical (it makes less sense to make all reasoning sceptical, since practical reasoning inevitably involves choice and a logic for such reasoning should reveal the alternatives). A merit of this paper is that it has provided a formalism for contexts in which the latter approach seems best. One such context is legal reasoning, where the facts of a case have to be proven but may lead to alternative normative consequences.

Acknowledgements

This work was partially supported by by the EU under IST-FP6-002307 (ASPIC). I thank Trevor Bench-Capon for useful discussions on the topic of this paper.

References

[1] C.E. Alchourrón. Logic of norms and logic of normative propositions. *Logique ét Analyse*, 12:242–268, 1969.

[2] L. Amgoud. A unified setting for inference and decision: an argumentation-based approach. In *Proceedings of the IJCAI-2005 Workshop on Computational Models of Natural Argument*, pages 40–43, 2005.

[3] L. Amgoud, M. Caminada, C. Cayrol, S. Doutre, M.-C. Lagasquie, H. Prakken, and G.A.W. Vreeswijk. Towards a consensus formal model: inference part. Deliverable D2.2: Draft formal semantics for inference and decision-making, ASPIC Project, 2004.

[4] K. Atkinson. *What Should We Do?: Computational Representation of Persuasive Argument in Practical Reasoning.* PhD Thesis, Department of Computer Science, University of Liverpool, Liverpool, UK, 2005.

[5] K. Atkinson, T.J.M. Bench-Capon, and P. McBurney. Computational representation of persuasive argument. *Synthese, special section Knowledge, Rationality and Action.* To appear.

[6] J. Broersen, M. Dastani, J. Hulstijn, and L. van der Torre. Goal generation in the BOID architecture. *Cognitive Science Quarterly Journal*, 2:428–447, 2002.

[7] M. Caminada. On the issue of reinstatement in argumentation. In *Proceedings of the 10th European Conference on Logic for Artificial Intelligence*, 2006. To appear.

[8] C. Cayrol, S. Doutre, and J. Mengin. On decision problems related to the preferred semantics for argumentation frameworks. *Journal of Logic and Computation*, 13, 2003. 377-403.

[9] P.M. Dung. Logic programming as dialog game. Unpublished paper, Division of Computer Science, Asian Institute of Technology, Bangkok, 1994.

[10] P.M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming, and n–person games. *Artificial Intelligence*, 77:321–357, 1995.

[11] P.M. Dung, R.A. Kowalski, and F. Toni. Dialectic proof procedures for assumption-based, admissible argumentation. *Artificial Intelligence*, 170:114–159, 2006.

[12] P.E. Dunne and T.J. Bench-Capon. Two party immediate response disputes: Properties and efficiency. *Artificial Intelligence*, 149:221–250, 2003.

[13] J. Fox and S. Parsons. On using arguments for reasoning about actions and values. In *Proceedings of the AAAI Spring Symposium on Qualitative Preferences in Deliberation and Practical Reasoning*, Stanford, CA, 1997.

[14] P. Krause, S. Ambler, M. Elvang-Gøransson, and J. Fox. A logic of argumentation for reasoning under uncertainty. *Computational Intelligence*, 11(1):113–131, 1995.

[15] J.L. Pollock. *Cognitive Carpentry. A Blueprint for How to Build a Person.* MIT Press, Cambridge, MA, 1995.

[16] J.L. Pollock. Planning agents. In M. Wooldridge and A. Rao, editors, *Foundations of Rational Agency.* Kluwer Academic Publishers, Dordrecht, 1999.

[17] J.L. Pollock. Rational choice and action omnipotence. *Philosophical Review*, 111:1–23, 2003.

[18] H. Prakken. Modelling defeasibility in law: logic or procedure? *Fundamenta Informaticae*, 48:253–271, 2001.

[19] H. Prakken. Coherence and flexibility in dialogue games for argumentation. *Journal of Logic and Computation*, 15:1009–1040, 2005.

[20] H. Prakken. Combining sceptical epistemic reasoning with credulous practical reasoning. In P. Dunne and T. Bench-Capon, editors, *Computational Models of Argument. Proceedings of COMMA-06*, pages 311–322, Amsterdam etc., 2006. IOS Press.

[21] H. Prakken and G. Sartor. Argument-based extended logic programming with defeasible priorities. *Journal of Applied Non-classical Logics*, 7:25–75, 1997.

[22] R.H. Thomason. Desires and defaults: a framework for planning with inferred goals. In *Proceedings of the 7th International Conference on Principles of Knowledge Representation and Reasoning*, pages 702–713, San Fransisco, CA, 2000. Morgan Kaufmann Publishers.

[23] G.A.W. Vreeswijk. Abstract argumentation systems. *Artificial Intelligence*, 90:225–279, 1997.

[24] G.A.W. Vreeswijk and H. Prakken. Credulous and sceptical argument games for preferred semantics. In *Proceedings of the 7th European Workshop on Logic for Artificial Intelligence (JELIA'2000)*, number 1919 in Springer Lecture Notes in AI, pages 239–253, Berlin, 2000. Springer Verlag.

[25] D.N. Walton. *Argumentation Schemes for Presumptive Reasoning.* Lawrence Erlbaum Associates, Mahwah, NJ, 1996.