Defending the Hierarchical Result Models of Precedential Constraint

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Abstract. In recent years, hierarchical case-based-reasoning models of precedential constraint have been proposed. Trevor Bench-Capon criticised these models, among other things, on the grounds that they would not account for the possibility that intermediate factors are established with different strengths by different base-level factors. In this paper we respond to these criticisms for van Woerkom's result-based hierarchical models. We argue that in some examples Bench-Capon seems to interpret intermediate factors as dimensions, and that applying van Woerkom's dimension-based version of the hierarchical result model to these examples avoids Bench-Capon's criticisms.

Keywords. case-based reasoning, precedential constraint, intermediate factors

1. Introduction

In [7] a hierarchical factor-based model of precedential constraint was proposed, extending the 'flat' factor-based result and reason models of precedential constraint of [5]. Very briefly, the flat model represents *cases* as a binary decision plus a *fact situation*, which consists of two sets of factors pro and con the decision. A decision in a new fact situation is *forced* if a precedent for that decision exists that cannot be distinguished from the new fact situation, i.e., if the latter contains at least all pro-decision factors of the precedent and at most all con-decision factors of the precedent. In [7] this is refined in terms of a factor hierarchy [1], in which *basic* factors are pro or con more *abstract* factors, which ultimately are pro or con the final binary decision. This allows to make the forcedness of the final decision relative to intermediate decisions instead of to basic factors.

In [2,3], Trevor Bench-Capon criticised the resulting model, among other things, on the ground that it does not account for the possibility that intermediate factors are established with different strengths by different base-level factors. In this paper we argue that in some examples Bench-Capon seems to interpret intermediate factors as dimensions, and that applying [6]'s dimension-based version of the hierarchical result model to these examples avoids Bench-Capon's criticisms.

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2. Examples and Bench-Capon's criticisms

Bench-Capon discusses two examples relevant for us about a family with parents Jack and Jo with two children Emma and Max, where the parents want to consistently decide about whether their children can have ice cream. Bench-Capon assumes the factor hierarchy depicted in Figure 1, adapted from [4].

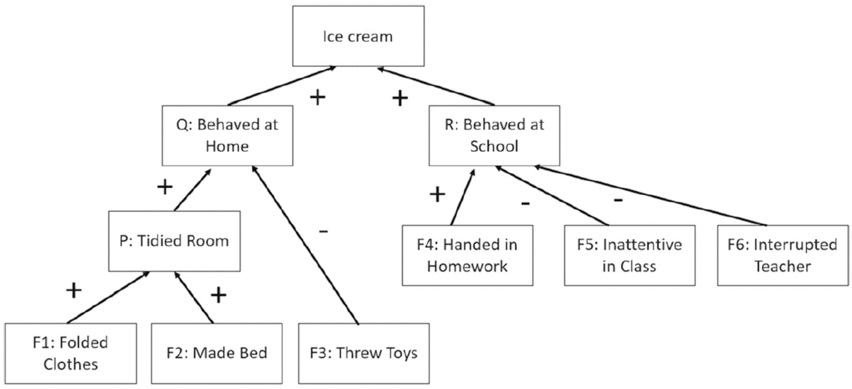

Figure 1. A factor hierarchy (from [3], as adapted from [4])

In Bench-Capon's first example, the case of **MaxMonday**, F_1 (Max had folded his clothes) and F_5 (Max had been inattentive in class) were present and parent Jo decided that Max could have his ice cream. Bench-Capon says that in a hierarchical model of precedential constraint there are two possible interpretations of this case.

- MMH: $Q > R$ (behaving at home is more important than behaving at school)
- MMF: $F_1 > F_5$ (folding one's clothes is more important than being inattentive in class)

Note that MMH and MMF correspond to the hierarchical and flat constraint model interpretations, respectively. According to Bench-Capon [3, p. 831], the choice between these two interpretations is not obvious. It is worthwhile quoting him in full:

> Jo may give clues as to her interpretation. If she says *ok, you tidied your room, which meant you behaved at home, so you can have ice cream even though you misbehaved at school*, she is clearly thinking in hierarchical terms. If, on the other hand she says *ok, you folded your clothes so you can have ice cream even though you were inattentive in class*, she is clearly thinking in terms of MMF.

In our opinion, which outcome is intuitively correct is relative to what is given. If the factor hierarchy is given in a way in which all base-level factors establish intermediate factors with the same strength, then there is nothing wrong with MMH. Bench-Capon's intuition can then be explained by noting that intermediate factors are not always established with the same strength. This becomes clear from his discussion of another example.

In Bench-Capon's second example, the case of **EmmaMonday**, F_2 (Emma had made her bed) and F_6 (Emma had interrupted her teacher) were present. According to Bench-Capon, Emma would appeal to MMH and argue that Jo's decision about Max constrains Jack's decision about her to the effect that she should also have her ice cream. Bench-Capon argues that Jack could instead decide that Emma will not have her ice cream on the grounds of $F_6 > F_2$, since this preference is consistent with $F_1 > F_5$. We argue that the latter decision in fact amounts to believing that intermediate factors can in different cases be established with different strengths. Accepting both $F_1 > F_5$ and $F_6 > F_2$ in fact says that Q is more strongly established by F_1 than by F_2 and that R is less strongly established by F_5 than by F_6. In the next sections we will argue that the dimension-based hierarchical result model of precedential constraint proposed by [6] is suitable for modelling this approach and allows a modelling of the two examples that satisfies Bench-Capon's intuitions.

3. The dimension-based result model

We next summarise the dimension-based flat and hierarchical result models of precedential constraint (abbreviated as DRM and DHRM) as presented in [6]. A *dimension* is a nonempty set. Given a finite set D of dimensions, a (dimension-based) *fact situation* is a partial choice function on D, i.e., a partial function $X : D \rightharpoonup \bigcup D$ such that $X(d) \in d$ for every $d \in D$ on which X is defined. A fact situation is *complete* if it is defined on all of D. A *case* is a pair (Y, s) where $s \in \{\pi, \delta\}$ and Y is a complete dimension-based fact situation. Finally, each dimension d is equipped with a partial order $\preceq$ on d, where $v \preceq v'$ intuitively means that v' is at least as good for π as v and v is at least as good for δ as v'.

Then 'flat' precedential constraint is defined for the DRM as follows.

Definition 3.1. Let *CB* be a case base for a set D of dimensions. We say that *CB* *forces* the decision of a fact situation X for π, written $CB, X \models \pi$, iff there exists a case $(Y, \pi) \in CB$ such with $Y(d) \preceq X(d)$ for all $d \in D$. Likewise, X is forced for δ iff there exists a case $(Y, \pi) \in CB$ such with $X(d) \preceq Y(d)$ for all $d \in D$.

The DHRM gives the set of dimensions a hierarchical structure.

Definition 3.2. A *dimension hierarchy* is a pair (D, H) with D a finite set of dimensions and H a relation on D satisfying

1. the transitive closure of H is irreflexive;
2. D contains exactly one H-maximal element.

A dimension is *base-level* if it is H-minimal, and *abstract* otherwise. In contrast to factors, dimensions are not assumed to have an inherent polarity.[2] As such, we use the notation $H(d) = \{e \in D \mid H(e, d\}$ instead of the notations $\texttt{Pro}(d)$ and $\texttt{Con}(d)$ to refer to the direct subordinates of d in the hierarchy. The case outcome

[2]We make this deviation from the definitions of [6] for the sake of simplicity.

is now considered part of the hierarchy, as phrased by point 2 of Definition 3.2. As such, a case is now simply defined as a complete fact situation.

Hierarchical constraint is now defined for the DHRM as follows.

Definition 3.3. Given a case base CB and a value v in some dimension d, a fact situation X is *lower bound by* v and CB, written $CB \models v \preceq X(d)$ iff:

- $v \preceq X(d)$; or
- $d \in A$ and there is $Y \in CB$ satisfying $v \preceq Y(d)$ such that $CB \models Y(e) \preceq X(e)$ holds for all $e \in H(d)$.

The *upper bound by* v, written $CB \models X(d) \preceq v$, is defined analogously.

Together, the lower and upper bound of a dimension d for X define the range of values that d can have in X given the case base. As stated by [6], the idea of the recursive clause is that there is a precedent Y which forces X to take a value v which is at least $Y(d)$, and therefore $v \preceq X(d)$ follows by transitivity.

4. Applying the dimension-based hierarchical result model to the examples

We now formalise our intuitions about the family example that intermediate factors can be established with varying strengths. For ease of explanation we use the natural numbers as values but our analysis applies in exactly the same way to any partially ordered set of values. For present purposes, all we need to represent is that R is satisfied more strongly in *MaxMonday* than in *EmmaMonday*.

Let 'the child can (not) have ice cream' be denoted by π (δ). The factors are now dimensions but all dimensions except P, Q and R are kept two-valued as follows: we assume two values 0 and 1 where $0 \preceq 1$ for dimensions that correspond to pro-π factors (in this case F_1, F_2, F_4) and $1 \preceq 0$ for dimensions that correspond to pro-δ factors (in this case F_3, F_5, F_6). Moreover, P, Q and R can have any natural number as value, where $v \preceq v'$ iff $v \leq v'$. Their hierarchical structure remains as in Figure 1.

We now model the intuition that making one's bed (F_2) makes the room more tidied (P) than folding one's clothes (F_1), and that interrupting the teacher (F_6) is worse behaviour at school than being inattentive in class (F_5). Moreover, the more the room is tidied, the better the behaviour at home (Q). This is modelled by assuming fact situations M and E which assign 0 to all dimensions except:

$$
\begin{array}{ll}
M\text{:} & F_1 = 1, F_5 = 1, P = 2, Q = 2, R = 3, \pi = 1 \\
E\text{:} & F_2 = 1, F_6 = 1, P = 3, Q = 3, R = ?, \pi = ?
\end{array}
$$

Note that we leave E undefined on R and π. The question is now whether the case M forces a decision to grant ice cream to Emma according to the DHRM. Formally, this boils down to whether there is a lower bound $M \models 1 \preceq E(\pi)$. We show that this is not the case. Intuitively, the point is that although Emma behaved better at home than Max, she behaved worse at school than Max and the latter is a relevant difference that allows the parents to deny Emma her ice cream.

Let us see how this follows formally, by unfolding Definition 3.3 with respect to the statement $M \models 1 \preceq E(\pi)$.

To have $M \models 1 \preceq E(\pi)$ (i.e., E is forced to be decided for π) we must either have $1 \preceq E(\pi)$, which is not the case because E is undefined on π, or else both (1) $M \models 2 \preceq E(Q)$ and (2) $M \models 3 \preceq E(R)$. The first we indeed have, since $2 \preceq 3 = E(Q)$, so let us consider (2). Successively applying Definition 3.3 and simplifying, we get:

$$
\begin{aligned}
& M \models 3 \preceq E(R) && (1)\\
\text{iff} \quad & 3 \leq E(R), \text{ or} && (2)\\
& M \models M(F_4) \preceq E(F_4), \text{ and} && (3)\\
& M \models M(F_5) \preceq E(F_5), \text{ and} && (4)\\
& M \models M(F_6) \preceq E(F_6) && (5)\\
\text{iff} \quad & 0 \preceq 1 \text{ (as values in } F_4), \text{ and} && (6)\\
& 1 \preceq 0 \text{ (as values in } F_5), \text{ and} && (7)\\
& 0 \preceq 1 \text{ (as values in } F_6). && (8)
\end{aligned}
$$

In the first step, we must either have $3 \leq E(R)$, which we do not have since E is undefined on R, or instantiate Y in the second bullet of Definition 3.3 by M, resulting in (3)–(5). As F_4, F_5, and F_6 are base-level dimensions, on all of which E is defined, these statements respectively simplify to (6)–(8). The last of these conjuncts, (8), does not hold: $1 \preceq 0$ in F_6 by definition, because interrupting the teacher is a con-π factor. In other words, E can be distinguished on F_6 with respect to M, and so M does not constrain E to take a value of at least 3 on R. We have thus verified that $M \not\models 1 \preceq E(\pi)$ according to the logic of the DHRM, so Emma can be denied her ice cream without violating precedential constraint.

We now vary the example in a way that makes Emma's case lower bounded by the decision that she is allowed to have ice cream. Consider a fact situation E' which is as E except that we replace F_6 by F_5, so $E'(F_5) = 1$ and $E'(F_6) = 0$. Because M and E' have the same values on the subordinate dimensions F_4, F_5, F_6 of R, we now do have the lower bound $M \models 3 \preceq E'(R)$. Moreover, we also have $M \models M(Q) \preceq E'(Q)$ because $M(Q) = 2 \leq E'(Q) = 3$. Putting these together, we now do have that $M \models 1 \preceq E'(\pi)$; so Emma should be allowed to have ice cream.

5. Conclusion

In this paper we have responded to Trevor Bench-Capon's criticism in [2,3] of hierarchical result models of precedential constraint. We have argued that his point that the models do not account for the possibility that intermediate factors are established with different strengths by different base-level factors does not apply to [6]'s dimension-based version. We have shown that in the analysis of some examples Bench-Capon seems to interpret some intermediate factors as dimensions, and that applying [6]'s hierarchical result model to these examples avoids Bench-Capon's criticisms.

References

[1] V. Aleven. *Teaching Case-Based Argumentation Through a Model and Examples.* PhD Dissertation University of Pittsburgh, 1997.

[2] T.J.M. Bench-Capon. A note on hierarchical constraint. In G. Sileno, J. Spanakis, and G. van Dijck, editors, *Legal Knowledge and Information Systems. JURIX 2023: The Thirty-Sixth Annual Conference*, pages 3–12. IOS Press, Amsterdam etc., 2023.

[3] T.J.M. Bench-Capon. Intermediate factors and precedential constraint. *Artificial Intelligence and Law*, 33:827–846, 2025.

[4] I. Canavotto and J. Horty. Reasoning with hierarchies of open-textured predicates. In *Proceedings of the 19th International Conference on Artificial Intelligence and Law*, pages 52–61, New York, 2023. ACM Press.

[5] J. Horty. Rules and reasons in the theory of precedent. *Legal Theory*, 17:1–33, 2011.

[6] W. van Woerkom, D. Grossi, H. Prakken, and B. Verheij. Hierarchical a fortiori reasoning with dimensions. In G. Sileno, J. Spanakis, and G. van Dijck, editors, *Legal Knowledge and Information Systems. JURIX 2023: The Thirty-Sixth Annual Conference*, pages 43–52. IOS Press, Amsterdam etc., 2023.

[7] W. van Woerkom, D. Grossi, H. Prakken, and B. Verheij. Hierarchical precedential constraint. In *Proceedings of the 19th International Conference on Artificial Intelligence and Law*, pages 333–342, New York, 2023. ACM Press.