On support relations in abstract argumentation as abstractions of inferential relations

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Abstract. Arguably the significance of an abstract model of argumentation depends on the range of realistic instantiations it allows. This paper therefore investigates for three frameworks for abstract argumentation with support relations whether they can be instantiated with the *ASPIC*$^+$ framework for structured argumentation. Both evidential argumentation systems and a simple extension of Dung's abstract frameworks with support relations proposed by Dung & Thang (2014) are shown to allow such an instantiation. However, for bipolar argumentation frameworks a positive result is only obtained for variants with only direct and secondary attacks; counterexamples are provided for variants with supported attacks, even for the special case of deductive support.

1 Introduction

There have been several recent proposals to extend [8]'s well-known abstract argumentation frameworks (AFs) with support relations. Among the best known are [5, 6]'s Bipolar Argumentation Frameworks (BAFs) and [18]'s Evidential Argumentation Systems (EASs). Arguably the significance of any abstract model of argumentation depends on the range of realistic instantiations it allows. Dung's AFs score very well in this respect, since in [8] various systems of nonmonotonic logic, argumentation and logic programming are reconstructed as AFs, namely, [24]'s default logic, [19]'s argumentation system and two semantics for logic programming. Moreover, much later work on structured argumentation was formulated to generate AFs. For example, both [23]'s system and the *ASPIC*$(^+)$ framework [4, 21, 15, 16] were explicitly defined to generate Dung-style AFs, while assumption-based argumentation (ABA) as defined in [3] was in [9] proven to instantiate AFs.

All these instantiations of AFs define relations of inferential support between (sets of) formulas in terms of definitions of structured arguments. This raises the question whether BAFs and EASs can be seen as abstractions of the inferential support relations modelled in these approaches. The present paper aims to answer this question. It should be noted that the papers on BAFs and EASs do not address this question, while [5] state that BAFs are intended for different applications, namely, for argumentation in debate contexts instead of from a given knowledge base. However, regardless of the intentions of the proponents of BAFs and EASs, the question addressed in the present paper is still legitimate, to study the significance of these frameworks as a contribution to the formal study of argumentation.

This question will be answered by investigating whether the *ASPIC*$^+$ framework of [21, 15] can be reformulated as an instantiations of BAFs or EASs. The choice of *ASPIC*$^+$ for these purposes is justified by its generality and the fact that it captures various other approaches as special cases, such as ABA as studied in [9], various forms of classical argumentation as studied in [11], and various instantiations with Tarskian abstract logics as studied by [1]. Therefore, results in terms of *ASPIC*$^+$ are representative for a large class of argumentation systems.

Below first an alternative way of adding support relations to AFs called SuppAFs will be proposed, based on an idea of [10]. Then *ASPIC*$^+$ will be shown to instantiate SuppAFs, after which a variant of BAFs with only direct and secondary attacks will turn out to be equivalent to SuppAFs and so also suitable as an abstraction of *ASPIC*$^+$. The same will be shown for an abstract version of [20]'s recursive argument labellings. However, versions of BAFs with so-called supported attacks will be shown to be inadequate as abstractions of *ASPIC*$^+$, even for the special case of deductive argumentation. Finally, *ASPIC*$^+$ will (for preferred semantics) be shown to be translatable as a special case of EASs. The question then arises of what EASs add to SuppAFs as proposed in the present paper.

2 Formal preliminaries

We first review the formal frameworks investigated in this paper.

2.1 Abstract argumentation frameworks

An **abstract argumentation framework** (AF) is a pair $(\mathcal{A}, \mathcal{D})$, where $\mathcal{A}$ is a set of *arguments* and $\mathcal{D} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary relation of *defeat*.[2] We say that A *strictly defeats* B if A defeats B while B does not defeat A. A semantics for AFs returns sets of arguments called *extensions*, which are internally coherent and defend themselves against attack.

Definition 1 Let $(\mathcal{A}, \mathcal{D})$ be an AF. For any $X \in \mathcal{A}$, X is *acceptable* w.r.t. some $S \subseteq \mathcal{A}$ iff $\forall Y$ s.t. $(Y, X) \in \mathcal{D}$ implies $\exists Z \in S$ s.t. $(Z, Y) \in \mathcal{D}$. Let $S \subseteq \mathcal{A}$ be *conflict free*, i.e., there are no A, B in S such that $(A, B) \in \mathcal{D}$. Then S is: an *admissible* set iff $X \in S$ implies X is acceptable w.r.t. S; a *complete* extension iff $X \in S$ whenever X is acceptable w.r.t. S; a *preferred* extension iff it is a set inclusion maximal complete extension; the *grounded* extension iff it is the set inclusion minimal complete extension; a *stable* extension iff it is preferred and $\forall Y \notin S, \exists X \in S$ s.t. $(X, Y) \in \mathcal{D}$.
For $T \in \{$complete, preferred, grounded, stable$\}$, X is *sceptically* or *credulously* justified under the T semantics if X belongs to all, respectively at least one, T extension.

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[2] [8] calls defeat "attack" but in this paper "defeat" is used to be compatible with the terminology in *ASPIC*$^+$.

2.2 Bipolar argumentation frameworks

Bipolar frameworks add a binary support relation $\mathcal{S}$ to AFs. Thus BAFs are a triple $(\mathcal{A}, \mathcal{D}, \mathcal{S})$[3]. In [5] but not in [6] $\mathcal{D}$ and $\mathcal{S}$ are assumed to be disjoint. In [5] a *sequence of supports* for argument B by argument A is a sequence $A\mathcal{S}B_1, \ldots \mathcal{S}B_n\mathcal{S}B$ (it is said that A *supports* B). A *supported attack* for argument B by argument A is a sequence (A, X, B) of arguments such that A supports X and X attacks B. A set $S \subseteq \mathcal{A}$ is said to *set-attack* an argument $A \in \mathcal{A}$ iff there exists a supported or direct attack on A from an element of S. Finally, a set $S \subseteq \mathcal{A}$ is $+$*conflict-free* iff there are no A and B in S such that $\{A\}$ set-attacks B. While thus BAFs as defined in [5] have a new notion of conflict freeness, they adopt Dung's original notion of acceptability of an argument with respect to a set of arguments. Then one semantics for BAFs defined by [5] is the following:

Definition 2 Given a BAF = $(\mathcal{A}, \mathcal{D}, \mathcal{S})$ a set $S \subset \mathcal{A}$ is *d-admissible* iff S is $+$conflict-free and all its elements are acceptable w.r.t. S. And S is a *d-preferred extension* of BAF iff S is maximal for $\subseteq$ among the d-admissible subsets of $\mathcal{A}$.

In [5] two further semantics for BAFs are defined. They both imply that every extension is $+$conflict-free and this will suffice for present purposes, so that they do not have to be presented here. Finally, in [6] a further notion of attack called *secondary attack* is defined as: if A supports B and C attacks A, then C (secondary-) attacks B.

2.3 Evidential argumentation systems

[18]'s **evidential argumentation systems** generalise BAFs in that both attack[4] and support is from *sets* of arguments to arguments. Several differences prevent EASs from generalising BAFs in a formal sense. Formally, an EAS is a triple $(\mathcal{A}, \mathcal{R}_a, \mathcal{R}_e)$, where $\mathcal{R}_a \subseteq 2^{\mathcal{A}} \setminus \emptyset \times A$ and $\mathcal{R}_e \subseteq 2^{\mathcal{A}} \cup \{\eta\} \times \mathcal{A}$. Here η is a special argument not in $\mathcal{A}$ that intuitively provides support from the environment. In EASs, the attack and support relation cannot intersect: there exists no $S \in 2^{\mathcal{A}}$ and $A \in \mathcal{A}$ such that both $S\mathcal{R}_aA$ and $S\mathcal{R}_eA$. Then:

Definition 3 [**Evidential support**] An argument A is *e-supported* by a set $S \subseteq \mathcal{A}$ iff

1. $\{\eta\}R_eA$; or
2. $\exists T \subset S$ such that TR_eA and $\forall X \in T$, X is e-supported by $S \setminus \{X\}$.

S is a *minimum e-support* for A if there is no $T \subset S$ such that A is e-supported by T.

Definition 4 [**Evidence-supported attack**] A set $S \subseteq \mathcal{A}$ carries out an *evidence-supported attack* on argument A iff

1. $S'R_aA$ for some $S' \subseteq S$; and
2. All elements of S' are e-supported by S.

A supported attack by S on A is *minimal* if there is no $T \subset S$ such that T carries out an evidence-supported attack on A.

Definition 5 [**Acceptability**] An argument A is *acceptable* wrt a set of arguments S iff:

1. S e-supports A; and
2. for every minimal evidence-supported attack X against A there exists a $T \in S$ such that $T\mathcal{R}_aB$ for some $B \in X$ such that $X/\{B\}$ is no longer an evidence- supported attack on A.

Finally, a set of arguments S is *conflict-free* iff $\forall Y \in S$, $\nexists X \subseteq S$ such that $X\mathcal{R}_aY$. Then the notions of admissible sets and preferred extensions are defined as usual.

2.4 The *ASPIC*$^+$ framework

The ***ASPIC***$^+$ **framework** [21, 15, 16] gives structure to Dung's arguments and defeat relation. It defines arguments as inference trees formed by applying strict or defeasible inference rules to premises formulated in some logical language. Arguments can be attacked on their (non-axiom) premises and on their applications of defeasible inference rules. Some attacks succeed as *defeats*, which is partly determined by preferences. The acceptability status of arguments is then defined by applying any of [8] semantics for abstract argumentation frameworks to the resulting set of arguments with its defeat relation. Below the version of *ASPIC*$^+$ defined in [15] is presented, more precisely, the special case with symmetric negation.

ASPIC$^+$ is not a system but a framework for specifying systems. It defines the notion of an abstract *argumentation system* as a structure consisting of a logical language $\mathcal{L}$ closed under negation, a set $\mathcal{R}$ consisting of two subsets $\mathcal{R}_s$ and $\mathcal{R}_d$ of strict and defeasible inference rules, and a naming convention n in $\mathcal{L}$ for defeasible rules in order to talk about the applicability of defeasible rules in $\mathcal{L}$. Informally, $n(r)$ is a wff in $\mathcal{L}$ which says that rule $r \in \mathcal{R}$ is applicable.

Definition 6 [**Argumentation systems**] An *argumentation system* is a triple $AS = (\mathcal{L}, \mathcal{R}, n)$ where:

- $\mathcal{L}$ is a logical language closed under negation ($\neg$).
- $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules of the form $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ and $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ respectively (where φ_i, φ are meta-variables ranging over wff in $\mathcal{L}$), and $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$.
- $n : \mathcal{R}_d \longrightarrow \mathcal{L}$ is a naming convention for defeasible rules.

We write $\psi = -\varphi$ just in case $\psi = \neg\varphi$ or $\varphi = \neg\psi$.

Definition 7 [**Knowledge bases**] A *knowledge base* in an $AS = (\mathcal{L}, \mathcal{R}, n)$ is a set $\mathcal{K} \subseteq \mathcal{L}$ consisting of two disjoint subsets $\mathcal{K}_n$ (the *axioms*) and $\mathcal{K}_p$ (the *ordinary premises*).

Arguments can be constructed step-by-step from knowledge bases by chaining inference rules into trees. In what follows, for a given argument the function `Prem` returns all its premises, `Conc` returns its conclusion, `Sub` returns all its sub-arguments and `TopRule` returns the last inference rule applied in the argument.

Definition 8 [**Arguments**] An *argument* A on the basis of a knowledge base KB in an argumentation system $(\mathcal{L}, \mathcal{R}, n)$ is:

1. φ if $\varphi \in \mathcal{K}$ with: $\texttt{Prem}(A) = \{\varphi\}$; $\texttt{Conc}(A) = \varphi$; $\texttt{Sub}(A) = \{\varphi\}$; $\texttt{TopRule}(A)$ = undefined.
2. $A_1, \ldots A_n \rightarrow/\Rightarrow \psi$ if $A_1, \ldots, A_n$ are arguments such that there exists a strict/defeasible rule $\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \rightarrow/\Rightarrow \psi$ in $\mathcal{R}_s/\mathcal{R}_d$.
   $\texttt{Prem}(A) = \texttt{Prem}(A_1) \cup \ldots \cup \texttt{Prem}(A_n)$, $\texttt{Conc}(A) = \psi$, $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$; $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \rightarrow/\Rightarrow \psi$.

[3] [5] use $\mathcal{R}_{att}$and $\mathcal{R}_{sup}$ for the defeat and support relation and call defeat "attack". Below 'attack' in descriptions of BAFs should be read as 'defeat' whenever a BAF is generated in *ASPIC*$^+$.

[4] As above, 'attack' in descriptions of EASs should be read as 'defeat' whenever an EAS is generated in *ASPIC*$^+$.

Arguments can be attacked in three ways: on their premises (undermining attack), on their conclusion (rebutting attack) or on an inference step (undercutting attack). The latter two are only possible on applications of defeasible inference rules.

Definition 9 [**Attack**] A *attacks* B iff A *undercuts*, *rebuts* or *undermines* B, where:

• A *undercuts* argument B (on B') iff $\texttt{Conc}(A) = -n(r)$ and $B' \in \texttt{Sub}(B)$ such that B''s top rule r is defeasible.

• A *rebuts* argument B (on B') iff $\texttt{Conc}(A) = -\varphi$ for some $B' \in \texttt{Sub}(B)$ of the form $B''_1, \ldots, B''_n \Rightarrow \varphi$.

• Argument A *undermines* B (on B') iff $\texttt{Conc}(A) = -\varphi$ for some $B' = \varphi$, $\varphi \notin \mathcal{K}_n$.

Argumentation systems plus knowledge bases form argumentation theories, which induce structured argumentation frameworks.

Definition 10 [**Structured Argumentation Frameworks**] Let AT be an *argumentation theory* (AS, KB). A *structured argumentation framework (*SAF*)* defined by AT, is a triple $\langle \mathcal{A}, \mathcal{C}, \preceq \rangle$ where $\mathcal{A}$ is the set of all finite arguments constructed from KB in AS, $\preceq$ is an ordering on $\mathcal{A}$, and $(X, Y) \in \mathcal{C}$ iff X attacks Y.

The notion of *defeat* can then be defined as follows. Undercutting attacks succeed as *defeats* independently of preferences over arguments, since they express exceptions to defeasible inference rules. Rebutting and undermining attacks succeed only if the attacked argument is not stronger than the attacking argument ($A \prec B$ is defined as usual as $A \preceq B$ and $B \not\preceq A$).

Definition 11 [**Defeat**] A *defeats* B iff:A undercuts B, or; A rebuts/undermines B on B' and $A \not\prec B'$.

Abstract argumentation frameworks are then generated from argumentation theories and an argument ordering as follows:

Definition 12 [**Argumentation frameworks**] An *abstract argumentation framework (AF) corresponding to a* SAF = $\langle \mathcal{A}, \mathcal{C}, \preceq \rangle$ is a pair $(\mathcal{A}, D)$ such that D is the defeat relation on $\mathcal{A}$ determined by SAF.

3 A simple framework for abstract support relations

In this section a simple way is proposed to add support relations between arguments to AFs, and *ASPIC*$^+$ will be shown to instantiate it. The idea is taken from [10] and amounts to adding a binary support relation $\mathcal{S}$ on $\mathcal{A}$ to AFs with the sole additional constraint that if B supports C and A attacks B then A also attacks C ([10] also assume that the support relation is a partial order but for present purposes this assumption is not needed). [10] actually do not make this proposal to extend AFs with support relations but as part of a proposal to combine AFs with a Tarski-style consequence notion over a logical language for conclusions of arguments and to instantiate it with special cases of *ASPIC*$^+$ and ABA.

Definition 13 [**AFs with support**] An **abstract argumentation framework with support** (SuppAF) is a triple $(\mathcal{A}, \mathcal{D}, \mathcal{S})$, where $\mathcal{A}$ is a set of *arguments*, $\mathcal{D} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary relation of *defeat* and $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary relation of *support* such that if A supports B and C defeats A then C defeats B.

The semantics of SuppAFs is simply defined by choosing one of the Dung-style semantics for the corresponding pair $(\mathcal{A}, \mathcal{D})$. Thus the support relation $\mathcal{S}$ is only used to constrain the defeat relation $\mathcal{D}$. To show that *ASPIC*$^+$ in fact generates SuppAFs, take $\mathcal{D}$ to be *ASPIC*$^+$'s defeat relation and $\mathcal{S}$ to be *ASPIC*$^+$'s subargument relation between arguments. It is then immediate from Definitions 9 and 11 that *ASPIC*$^+$'s notion of defeat satisfies [10]'s constraint on $\mathcal{D}$ in terms of $\mathcal{S}$. This proves that SuppAFS can be instantiated with *ASPIC*$^+$.

An equivalent reformulation of SuppAFs does make use of support relations in its semantics. In [22] *ASPIC*$^+$ as presented above was reformulated in terms of [20]'s recursive labellings. Abstracting this reformulation to SuppAFs we obtain the following definitions. First, [22] defines a notion of p-defeat (for "Pollock-defeat"), which captures direct defeat between arguments:

Definition 14 [**p-Attack**] A *p-attacks* B iff A *p-undercuts*, *p-rebuts* or *p-undermines* B, where:

• A *p-undercuts* argument B iff $\texttt{Conc}(A) = -n(r)$ and B has a defeasible top rule r.

• A *p-rebuts* argument B iff $\texttt{Conc}(A) = -\texttt{Conc}(B)$ and B has a defeasible top rule.

• Argument A *p-undermines* B iff $\texttt{Conc}(A) = -\varphi$ and $B = \varphi$, $\varphi \notin \mathcal{K}_n$.

Definition 15 [**p-Defeat**] A *p-defeats* B iff:A p-undercuts B, or; A p-rebuts/p-undermines B and $A \not\prec B$.

Then [22] proves that A defeats B iff A p-defeats B or a proper subargument of B. Now if the support relation of a SuppAF is taken to be *ASPIC*$^+$'s notion of an 'immediate' subargument and the defeat relation of a SuppAF is taken to be p-defeat, then the following definition is equivalent to [8]'s semantics for AFs (and so for SuppAFs).

Definition 16 [p-labellings for SuppAFs.] Let $(\mathcal{A}, \mathcal{D}, \mathcal{S})$ be a SuppAF corresponding to a SAF = $(\mathcal{A}, \mathcal{D})$ where $\mathcal{D}$ is defined as p-defeat and where $\mathcal{S}$ is defined as $(A, B) \in \mathcal{S}$ iff B is of the form $B_1, \ldots, B_n \rightarrow / \Rightarrow \varphi$ and $A = B_i$ for some $1 \leq i \leq n$. Then (In, Out) is a *p-labelling* of SuppAF iff $In \cap Out = \emptyset$ and for all $A \in \mathcal{A}$ it holds that:

1. A is labelled *in* iff:
   (a) All arguments that p-defeat A are labelled *out*; and
   (b) All B that support A are labelled *in*.
2. A is labelled *out* iff:
   (a) A is p-defeated by an argument that is labelled *in*; or
   (b) Some B that supports A is labelled *out*.

The notions of complete, stable, preferred and grounded labellings are defined as usual: a complete labelling is any labelling, a stable labelling labels all arguments, a preferred labelling maximises the set of arguments labelled *in* while a grounded labelling minimises this set. Then the corresponding notions of complete, stable, preferred and grounded extensions are defined as a set of all arguments labelled *in* in some complete (stable, preferred, grounded) labelling.

It can be shown that the extensions defined thus for SuppAFs generated from *ASPIC*$^+$ with p-defeat are exactly the extensions of SuppAFs as generated above from *ASPIC*$^+$ with defeat. The proof is a straightforward generalisation of Theorem 2 of [22].

An alternative attempt to reconstruct *ASPIC*$^+$ as an instance of SuppAFs is to define the support relation as follows: $A \in \mathcal{A}$ supports $B \in \mathcal{A}$ iff either A is a subargument of B or the conclusion of A is a premise of B. However, this proposal does not work, since it cannot distinguish between the following two situations:

Situation 1: A has premises p and q, B has conclusion p, C has conclusion q, D undercuts C.
Situation 2: A has premise p and both B and C have conclusion p, D undercuts C.

Both situations induce the same SuppAF, in which both B and C support A while D defeats both A and C. However, this is counterintuitive, since in the second situation A should not be defeated, since its premise p is still provided by an undefeated argument, namely, B.

In fact, this problem was already noted by Pollock [20] when he defined his notion of an inference graph, in which nodes are linked by both support and attack links. As a solution, the nodes in Pollock's inference graphs do not simply stand for statements but encode the way in which they are derived from previous statements. In our situation 2 there would thus be two nodes for statement p, one as derived with argument B and another as derived with argument C. This solution is also adopted in *ASPIC*$^+$, which considers two versions of A, one as supported by B and the other as supported by C.

4 BAFs as abstraction of *ASPIC*$^+$

In this section the relation between BAFs and *ASPIC*$^+$ is investigated. In fact, BAFs can in a trivial sense be instantiated with *ASPIC*$^+$ since *ASPIC*$^+$ generates Dung-style AFs and these are the special case of BAFs with an empty support relation. However, such an instantiation is clearly not insightful; what we would like is an instantiation with a non-empty support relation that corresponds in a meaningful way to *ASPIC*$^+$'s notion of inferential support. Above we saw that a definition of support as having a premise of the supported argument as the conclusion will not work. Therefore the same definition will be used as for SuppAFs, namely as *ASPIC*$^+$'s notion of a subargument.

Next, neither [10] nor *ASPIC*$^+$ adopt [5]'s constraint that the attack and support relations are disjoint. It is easy to provide instantiations of *ASPIC*$^+$ that violate this constraint. A simple example is with $\mathcal{L}$ a propositional language, $\mathcal{K}_n = \{p\}; \mathcal{K}_p = \emptyset; \mathcal{R}_s = \emptyset; \mathcal{R}_d = \{p \Rightarrow \neg p\}$. We have the following arguments:

A_1: p
A_2: $A_1 \Rightarrow \neg p$

Here A_1 supports A_2 since A_1 is a subargument of A_2 while A_1 also attacks and defeats A_2.

[5, p. 69] motivate their exclusion of such examples by saying that "…it does not seem rational to advance an argument that simultaneously attacks and supports the same other argument.". While this makes sense, it may not always be easy to detect that one argument both supports and attacks another. For this reason *ASPIC*$^+$ takes an alternative approach, namely, to allow such examples and let the logic deal with them in a rational way. For instance, in *ASPIC*$^+$ the above example has a unique preferred (and grounded) extension containing only A_1, which seems the intuitively correct outcome.

Let us now see whether BAFs can in a non-trivial sense be instantiated with *ASPIC*$^+$. It is easy to see that if in BAFs only direct and secondary attacks are used to define conflict-freeness, then BAFS are equivalent to SuppAFs, since the notion of secondary attack is equivalent to [10]'s constraint on $\mathcal{D}$. According to [17], secondary attack (and so [10]'s constraint) is suitable if support is to mean *necessary support* in that 'A supports B' means that B cannot be accepted without A. As also remarked in [7], *ASPIC*$^+$'s subargument relation is such a relation of necessary support, since in *ASPIC*$^+$ an argument cannot be in an extension if not all its subarguments are in that extension. So we have identified a realistic instantiation of [17]'s notion of necessary support.

However, things are different for variants of BAFs with supported attacks. Consider the following well-known example from the literature on nonmonotonic logic, with a propositional language and $\mathcal{K}_n = \{q, r\}; \mathcal{K}_p = \mathcal{R}_s = \emptyset; \mathcal{R}_d = \{r_1, r_2\}$ where:

r_1: $q \Rightarrow p$
r_2: $r \Rightarrow \neg p$

Read this as: quakers are typically pacifist, republicans are typically not pacifists, Richard Nixon was both a quaker and a republican. We have the following arguments:

A_1: q	B_1: r
A_2: $A_1 \Rightarrow p$	B_2: $B_2 \Rightarrow \neg p$

In *ASPIC*$^+$ as reconstructed in a SuppAF we have that A_1 supports itself and A_2, B_1 supports itself and B_2 and (if the defeasible rules have equal strength) A_2 and B_2 defeat each other by successfully rebutting each other. This yields two preferred extensions:

$E_1 = \{A_1, B_1, A_2\}$
$E_1 = \{A_1, B_1, B_2\}$

However, in BAFs neither of these extensions is +conflict-free: in E_1 this is since $\{B_1\}$ set-attacks A_2 while in E_2 this is since $\{A_1\}$ set-attacks B_2. Instead, the preferred d-extensions in BAFs are

$E_1 = \{A_1, A_2\}$
$E_2 = \{B_1, B_2\}$
$E_3 = \{A_1, B_1\}$

Thus in the BAF treatment of this example we cannot rationally accept both that Nixon was a quaker and that he was not a pacifist. This outcome shows that BAFs are not adequate as an abstraction of *ASPIC*$^+$ (a similar observation is made by [13]). Moreover, the BAF outcome is arguable counterintuitive, since there is nothing inconsistent in saying both that Nixon was a quaker and that he was not a pacifist. The point is that the two generalisations in the example are defeasible, so that it can be perfectly rational to accept their antecedent but not their consequent.

Following [2], it is suggested in [6] that supported attacks do make sense for a notion of *deductive support*, defined as: if A supports B and A is accepted, then B must also be accepted (so if B is not accepted, then A cannot be accepted). Does this abstract notion of deductive support correspond to something meaningful in *ASPIC*$^+$? One would expect that this is the case for classical-logic instantiations of *ASPIC*$^+$ in the sense of [11], where arguments are classical subset-minimal entailments from consistent subsets of a possibly inconsistent knowledge base. As shown in [15], this can be captured in *ASPIC*$^+$ by letting $\mathcal{L}$ be a propositional language, letting $\mathcal{R}_s$ be all propositionally valid inferences and by letting $\mathcal{K}_n$ and $\mathcal{R}_d$ empty. The question then reduces to the question whether BAFs with supported attacks are suitable as abstraction of such instantiations of *ASPIC*$^+$.

It turns out that this is not the case. Consider an example with $\mathcal{K}_p = \{p, q, \neg(p \wedge q)\}$ and with all arguments of equal priority. Theorem 34 of [15] implies that each maximal consistent subset of $\mathcal{K}_p$ is contained in a stable (and so preferred) extension. So there exists a stable extension of the SuppAF induced by this argumentation theory that contains both p and $\neg(p \wedge q)$. However, this extension is not +conflict-free: we have the argument $p, q \rightarrow p \wedge q$, which undermines the premise argument $\neg(p \wedge q)$. But since the premise argument p supports A, it support-attacks $\neg(p \wedge q)$. So there cannot be any d-preferred extension in BAFs that contains both p and $\neg(p \wedge q)$. The problem with the abstract notion of deductive support as defined above is that it neglects that an argument B supported by an argument A can have multiple subarguments, so if B (here $p, q \rightarrow p \wedge q$) is not accepted, one can choose to accept A (here p) and instead reject one of $B's$ other subarguments (here q).

5 *ASPIC*$^+$ as a special case of EASs

In this section it is shown that *ASPIC*$^+$ is translatable as a special case of EASs. No formal result on EASs proved in [18] depends on the constraint that their attack and support relations are disjoint, so the translation result will below be formulated for EASs without this constraint. Actually, as with BAFs a trivial translation from *ASPIC*$^+$ to EASs is possible, since as noted by [18], Dung-style AFs can be translated into EASs by letting η support all other arguments and having no further support relations, and by preserving the attack relations (now formulated from singleton sets to arguments). Since *ASPIC*$^+$ generates AFs, *ASPIC*$^+$ could be translated into EASs in this way. However, this translation is clearly not very insightful. Below a more interesting translation is provided, which translates the subargument relation of *ASPIC*$^+$ into support relations of EASs.

Definition 17 **[from *ASPIC*$^+$ to EASs]** Let $AF = (\mathcal{A}, \mathcal{D})$ be an abstract argumentation framework corresponding to a $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ induced by argumentation theory $AT = (AS, KB)$. The evidential argumentation system corresponding to AF is defined as follows:

1. $S\mathcal{R}_a A$ iff $S = \{B\}$ and B defeats A.
2. $S\mathcal{R}_e A$ iff
   (a) $S = \{\eta\}$ and $\mathtt{Sub}(A) = \{A\}$; or else
   (b) $S = \mathtt{PrSub}(A)$ (where $\mathtt{PrSub}(A) = \mathtt{Sub}(A) \setminus \{A\}$).

Lemma 1

1. Let $S = \mathtt{Sub}(A)$ for some A. Then any $x \in S$ is e-supported by $S/\{x\}$.
   Proof: with induction on the structure of arguments. For the base case, suppose $\mathtt{Sub}(x) = \{x\}$. Then $\{\eta\}\mathcal{R}_e x$ so x is e-supported by $S/\{x\} = \emptyset$. The induction hypothesis is that for any $y \in \mathtt{PrSub}(x)$ we have that y is e-supported by $S/\{y\}$. Then choose T in clause 2 of Definition 3 to be $\mathtt{PrSub}(x)$. Since $\mathtt{PrSub}(x)\mathcal{R}_e x$ we have by the induction hypothesis that this clause is satisfied.
2. A defeats B iff $\mathtt{Sub}(A)$ carries out a minimal e-supported attack on B.
   Proof from left to right, suppose A defeats B. Then $\{A\}\mathcal{R}_a B$ and $\{A\} \subseteq \mathtt{Sub}(A)$ so the first bullet of Definition 4 is satisfied. We next prove by induction on the structure of an argument that the second bullet is satisfied. For the base case, suppose $\mathtt{Sub}(A) = \{A\}$. Then $\{\eta\}\mathcal{R}_e A$ so A is e-supported by $\mathtt{Sub}(A)$. Consider next any $A \in \mathtt{Sub}(A)$ such that all elements of $\mathtt{PrSub}(A)$ are e-supported by $\mathtt{Sub}(A)$. Note that $\mathtt{PrSub}(A) \subset \mathtt{Sub}(A)$ and $\mathtt{PrSub}(A)\mathcal{R}_e A$. Then by Lemma 1(2) the second bullet is satisfied. Finally, $\mathtt{Sub}(A)$ is a minimal e-supported attack on B since in *ASPIC*$^+$ it holds that if A defeats B and C defeats B and $A \neq C$ then $\mathtt{Sub}(A) \not\subset \mathtt{Sub}(C)$.
   From right to left holds since $\mathtt{Sub}(A)$ carries out a minimal e-supported attack on B, so all its elements are needed to create the attack. But then no subargument of A defeats B so A defeats B.

Then the following theorem can be proved (below, if terminology for *ASPIC*$^+$ or AFs is also used in EASs, the EASs notions will be preceded with an e).

Theorem 2 *Let AF be an abstract argumentation framework corresponding to a SAF such that $\mathcal{S} \cap \mathcal{D}$ in the SuppAF corresponding to* ASPIC$^+$ *is empty, and let EAS correspond to AF. Then E is a preferred extension of AF iff E is an e-preferred extension of EAS.*

Proof: From left to right, assume E is a preferred extension of AF. First, E is conflict-free, so E is e-conflict-free by definition of $\mathcal{R}_a$. Next we must prove that all elements of E are e-acceptable wrt E. Note first that since *ASPIC*$^+$ satisfies closure under subarguments (see [15]), then by Lemma 1(1) E e-supports all its members. Next, since E is admissible, all its elements are acceptable wrt E. Then for all B that defeat a member A of E, there exists a C in E that defeats B. By Lemma 1(2) there exists such a B iff $\mathtt{Sub}(B)$ carries out a minimal evidence-supported attack on A. Then, since there exists a C that defeats a subargument of B, for this C we have that $\{C\}\mathcal{R}_a X$ for some $X \in \mathtt{Sub}(B)$, while moreover, $\mathtt{Sub}(B)/\{X\}$ is no longer an e-supported attack on A since $\mathtt{Sub}(B)$ is a minimal e-supported attack on A. So all elements of E are e-acceptable with respect to E. Then by Lemma 4 of [18] E is e-admissible. To prove that E is maximally e-admissible, suppose there exists an A that is e-acceptable wrt E but not in E. Then since E is a preferred extension, there exists a B that defeats A such that there is no C in E that defeats B. But A is e-acceptable wrt E, so there exists a C in E such that $\{C\}\mathcal{R}_a B'$ for some subargument B' of B. Then C also defeats B and A is acceptable wrt E. Contradiction. So E is an e-preferred extension of EAS.

From right to left, suppose E is an e-preferred extension. Since E is e-conflict free, it is also conflict free by definition of $\mathcal{R}_a$. Next we have to prove that all members of E are acceptable wrt E. Let B defeat some A in E. Then by Lemma 1(2) $\mathtt{Sub}(B)$ carries out a minimal e-supported attack on A. Since E is e-admissible, for some C in E we have that $\{C\}\mathcal{R}_A B'$ for some subargument B' of B. Then C defeats B, so A is acceptable wrt E, so E is admissible. Suppose next for contradiction that E is not maximally admissible. Then some argument A is acceptable wrt E but not in E. Since E is an e-preferred extension, A is not e-acceptable wrt E. Note that since A is acceptable wrt E, all B that defeat A are defeated by some C in E. By Lemma 1(2) there exists such a B iff $\mathtt{Sub}(B)$ carries out an evidence-supported attack on A. Then, since there exists a C that defeats a subargument of B, for this C we have that $\{C\}\mathcal{R}_a X$ for some $X \in \mathtt{Sub}(B)$, while moreover, $\mathtt{Sub}(B)/\{X\}$ is no longer an e-supported attack on A since $\mathtt{Sub}(B)$ is a minimal e-supported attack on A. So E does not e-support A, otherwise E would not be maximally e-admissible. Then there exists at least one subargument A' of A of which all elements of $\mathtt{PrSub}(A')$ are in E but A' is not in E. Then since $\mathtt{PrSub}(A')\mathcal{R}_e A'$, by Lemma 1(1) we have that E e-supports A'. Since A' is a subargument of A, we have in fact just proven that A' can be defended against all minimal evidence-supported attacks since any such attack is also a minimal e-supported attack on A. So A' is e-acceptable wrt E, so A' is in E. Contradiction.
□

Given this result, the question remains what EASs offer as advantages over the simple framework for abstract support relations proposed in the present paper. One feature of EASs that was not needed to prove the correspondence with *ASPIC*$^+$ is the possibility to have attacks from *sets* of arguments to arguments. [18] motivate this feature with the following example.

A: The bridge should be built at point x, where soft ground exists.
B: Financial considerations mean that the bridge should be built at point y $\neq$ x.
C: Financial considerations override any other considerations.

According to [18] neither B nor C alone attacks A while together they do attack A. In my opinion they here rely on an implicit distinction between attack and defeat as formalised in *ASPIC*$^+$, where they use the term “attack” for *ASPIC*$^+$’s notion of defeat and use no term for *ASPIC*$^+$’s notion of attack. Then in terms of *ASPIC*$^+$ argument C is a preference argument that makes B strictly defeat A. In my opinion this example does not show the need for attacks from sets of arguments, since it can be better modelled as an instantiation of [12]’s extended argumentation frameworks (EAFs), in which attacks on attacks (or in *ASPIC*$^+$’s terms ‘defeats of defeats’) are allowed. See [14] for such an instantiation of EAFs with *ASPIC*$^+$.

6 Conclusion

In this paper the question was addressed whether bipolar argumentation frameworks or evidential argumentation systems can be used as an abstraction of *ASPIC*$^+$-style inferential support relations between arguments. This question was investigated since arguably the significance of an abstract model of argumentation depends on the range of realistic instantiations it allows. For BAFS the answer was positive for variants with only direct and secondary attacks but negative for variants with supported attacks, even for the special case of deductive support. Moreover, a simple alternative to BAFs based on an idea of [10] called SuppAFs turned out to be suitable in general as an abstraction of *ASPIC*$^+$-style inferential support relations. The same was proven for an abstract version of [20]’s recursive argument labellings. A question that thus remains is whether other instantiations of BAFs are possible that show their significance as a contribution to the study of argumentation.

For EASs it was shown that they can be instantiated with *ASPIC*$^+$. This was proven for preferred semantics only, since that is the semantics on which [18] concentrate. Thus there now are two formalisms for abstract argumentation with both attack and support relations that can be instantiated in general with *ASPIC*$^+$, namely, EASs and SuppAFs. Clearly, EASs are more complicated as a formalism than SuppAFs. One complication is that supports in EASs are from *sets* of arguments to arguments. While this feature was used in this paper in translating *ASPIC*$^+$ to EASs, the translatability of *ASPIC*$^+$ in SuppAFs shows that abstract support relations can also be defined between single arguments. As for the second complication in EASs, namely that attacks in EASs are also from *sets* of arguments to arguments, it was above argued that this feature is not needed if a version of *ASPIC*$^+$ instantiating [12]’s extended argumentation frameworks is used. The question then remains what EASs offer over SuppAFs. This question could be answered in two ways: by providing interesting instantiations of EASs that are impossible in SuppAFs, or by showing that a metatheory of EASs can be developed that is richer than is possible for SuppAFs and that is moreover relevant for realistic instantiations. More generally, the question can be asked what frameworks with abstract support relations offer over more concrete but still abstract frameworks with support relations defined over logical languages, such as *ASPIC*$^+$, assumption-based argumentation or [1]’s approach in terms of Tarskian abstract logics. But these questions have to be left for future research.

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