Phase Transitions in the Early Universe

Electroweak and QCD phase transitions

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Student Seminar - Cosmology

Introduction
- Timeline
- Motivation
- Toy Models

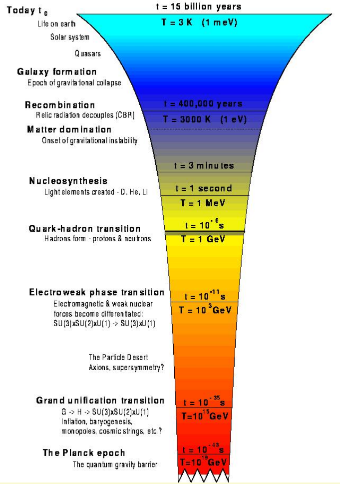
Electroweak Phase Transition
- Theory

QCD Phase Transitions
- Theory
- Experiments

Summary

Timeline

- QCD phase transition
  $t \approx 10^{-6}$s, $T \approx 100$ MeV
- Electroweak phase transition
  $t \approx 10^{-12}$s, $T \approx 100$ GeV
- Higher temperature phase transitions?



$$G \to \cdots \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{EM} \to U(1)_{EM}$$

Motivation

- Understanding the evolution of the universe
- First order phase transitions in early universe would leave a signature in present universe
- Development of theoretical models and technics (condensed matter imports to cosmology, finite temperature field theory etc.)

Toy Models - Second Order Phase Transition

$\mathcal{L}$ symmetric under $\phi \to -\phi$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4}(\phi^2 - v^2)^2$$

Extrema of the potential given by:

$$\lambda(\phi^3 - v^2\phi) = 0$$

Assuming symmetric fluctuations $\delta\phi(\mathbf{x})$ around $\langle \phi(\mathbf{x}) \rangle = \phi_0$ and averaging over fluctuations in $\frac{\partial V}{\partial \phi} = 0$:

$$\lambda(\phi_0^3 + 3\langle \delta\phi^2 \rangle \phi_0 - v^2\phi_0) = 0$$

Thermal contribution to $\langle \delta\phi^2 \rangle$:

$$\langle \delta\phi^2 \rangle = \int \frac{d^3\mathbf{p}}{(2\pi)^3\omega} \frac{1}{e^{\beta\omega} - 1} \simeq \frac{T^2}{12}$$

where $\omega = \sqrt{\mathbf{p}^2 + m^2}$, $T \gg m$.

$$\lambda\left(\phi_0^2 + \frac{T^2}{4} - v^2\right)\phi_0 = 0$$

$$\Rightarrow T_c = 2v$$

$T > T_c \Rightarrow$ symmetry restoration, $\phi_0 = 0$.

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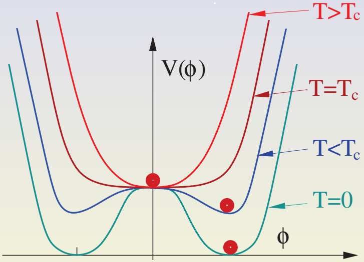
Second Order Phase Transition

- Smooth transition. Thermal equilibrium preserved at any point.
- Discontinuity in the derivative of the order parameter.
- No remnants expected.



Effective potential $V_{eff}(\phi_0)$

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with gauge field mass given by $m = g\phi_0$.

$$\Rightarrow \lambda \left[\phi_0^2 - \frac{3g^3 T \phi_0}{4\pi\lambda} + \left(\frac{1}{3} + \frac{g^2}{4\lambda} \right) T^2 - v^2 \right] \phi_0 = 0$$

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$\Rightarrow$ Theory with a first order phase transition.

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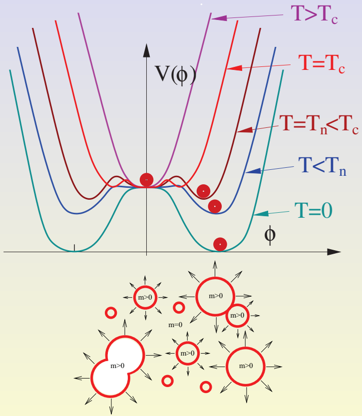
First Order Phase Transition

- Evolution between minima through bubble nucleation.
- Coexistence of phases.
- Violent in nature; can present large deviations from thermal equilibrium.



Standard Model Lagrangian

- SM Lagrangian exhibits $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry
- Massless gauge bosons, $W^{\pm}$ and Z
- Long range forces
- $\Rightarrow$ Disagreement with experiment

Solution:

Spontaneous symmetry breaking: considering a ground state which no longer obeys the symmetry of the Lagrangian

Higgs Mechanism

1. Introduce scalar doublet $H = \frac{1}{\sqrt{2}} \binom{0}{\rho} e^{i\phi^{(a)} t_{(a)}^{(F)}}$ s.t. in unitary gauge $H = \frac{1}{\sqrt{2}} \binom{0}{\rho}$
2. Add gauge invariant terms to $\mathcal{L}_{SM}$:

$$\mathcal{L}_{Higgs} = -D_\mu H^\dagger D^\mu H - \mu^2 H^\dagger H - \lambda (H^\dagger H)^2$$

3. Choose $\mu^2 < 0$ s.t. $\langle H \rangle_{T=0} = \frac{1}{\sqrt{2}} \binom{0}{v}$ and $\rho = v + h$ ($v = \sqrt{\frac{-\mu^2}{\lambda}}$, vacuum expectation value and $h(x)$, Higgs boson)
4. Through $D_\mu H^\dagger D^\mu H$ the gauge bosons which appear in the covariant derivative acquire mass.
5. The ground state remains invariant only under the action of $t_3 + Y/2 = Q$, where Q is the generator of $U(1)_{EM}$

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5. The ground state remains invariant only under the action of $t_3 + Y/2 = Q$, where Q is the generator of $U(1)_{EM}$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{EM}$$

Finite temperature

- For $T > T_c \sim m_h/g$, $v = 0$ and symmetry is restored.
- Nature of the phase transition between broken and unbroken phases given by the thermal contribution to the V_{eff}.
- Calculation of V_{eff} and the phase transition dependence on the value of Higgs mass, m_H:
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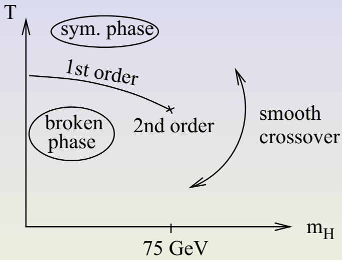
- Hope: out of equilibrium processes in bubble walls in a first order transition violate B number conservation.
- Lattice simulations: EWPT is first order for $m_H \lesssim 72$ GeV.
- Experiments excluded $m_H \lesssim 115$ GeV.



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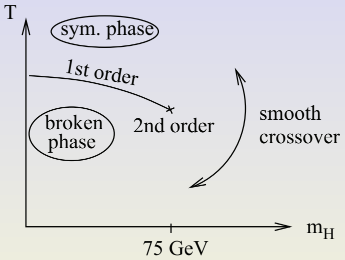

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QCDPT as (De)Confinement

- At higher energies, the coupling constant of QCD diminishes. Above $T \gtrsim 100$ MeV, asymptotic freedom can be invoked to predict deconfinement of quarks.
- Ideally, quark-gluon plasma (QGP) = perfect fluid of quarks and gluons in which perturbative theory is possible.
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Toy Model for Finding T_c

- Bag model: $E_H = \frac{4\pi}{3} R^3 B + \frac{C}{R}$, where B is vacuum energy inside the bag. (Determine B from the average hadron mass. $B^{1/4} \simeq 200$ MeV)
- Consider a phase transition hadrons $\rightarrow$ QGP s.t.:
  - only u and d quarks; m_u, $m_d \simeq 0$ (ultrarelativistic above $T \gtrsim 100 MeV$) (+ antiquarks)
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  - only lightest mesons (pions) are relevant near T_c
- Equate pressure of phases at T_c:

$$P_H = 3 \times \frac{\pi^2 T^4}{90} - B; \quad P_{QGP} = \left(2 \times 2 \times 2 \times 3 \times \frac{7}{8} + 2 \times 8\right) \frac{\pi^2 T^4}{90}$$

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QCDPT as Chiral Symmetry Breaking

- For small mass quarks at high temperature, the approximate symmetry $SU(2)_L \times SU(2)_R$ holds.
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$$\langle \overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L \rangle \neq 0$$

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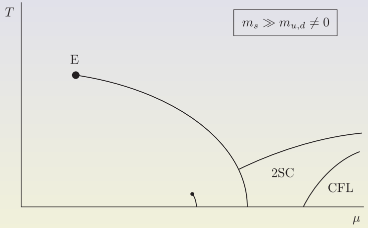
Phase Diagram

- Reliable calculation only within lattice simulations. (Still work to be done in implementing chiral fermions.)
- $T_c \simeq 170$ MeV
- Early universe: no 1st order phase transition, but a crossover.



CFL Phase

- For T much smaller compared to μ the system behaves like an almost ideal fermionic system with most of the states filled. $\mu \simeq E_F$
- CFL (color-flavor locking) phase is a theoretical model for massless u, d, s quarks in high-density regime.
- Strong interactions are the basis for formation of "Cooper pairs" $\Rightarrow$ *color superconductivity*. CFL pair:

$$\langle \psi_{iL}^{a\alpha}(\mathbf{p})\psi_{jL}^{b\beta}(-\mathbf{p})\epsilon_{ab}\rangle = -\langle \psi_{iR}^{a\alpha}(\mathbf{p})\psi_{jR}^{b\beta}(-\mathbf{p})\epsilon_{ab}\rangle = \Delta(\mathbf{p}^2)\epsilon^{\alpha\beta A}\epsilon_{ijA}$$

- $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2$
- Theoretical results:
  - Magnitude of gaps $\sim e^{const./g}$ ($\approx$ 10 - 100 MeV)
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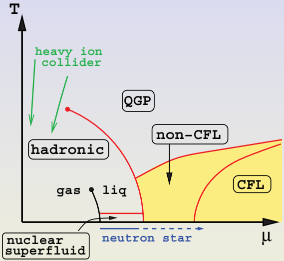
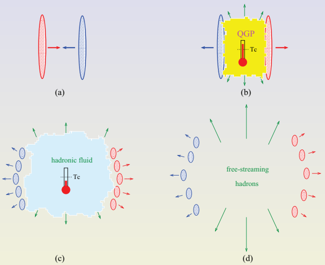
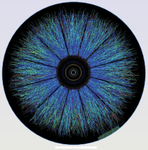
Investigating the Phase Diagram



- Is CFL phase realized inside neutron stars?
  Inert quark matter inside the neutron star (no experiments).
- QGP experiments at RHIC (SPS, AGS, ALICE).

RHIC experiment





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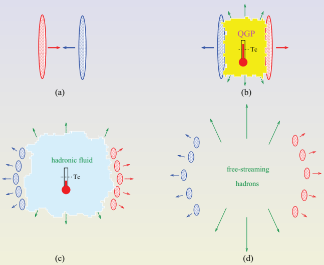

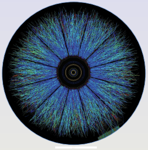

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Summary

- History of the early universe is a walk on the boulevard of broken symmetries.
- Phase transitions punctuate breaking of symmetries.
- EWPT and QCDPT cannot explain matter-antimatter asymmetry.
- Description of phases in which QCD becomes more theoretically tractable (at high T or μ)

Thank you!