Dark Matter in Astrophysics

Focussing on Weak Gravitational Lensing

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Outline

The implications of weak gravitational lensing on the dark matter distribution in our universe.

- Introduction to dark matter measurements
- Derivation of weak gravitational lensing
- Weak lensing observations
- Conclusions and outlook

Virial measurements

Fritz Zwicky (1933)
Virial Theorem

$$2\langle K\rangle = -\langle U\rangle$$

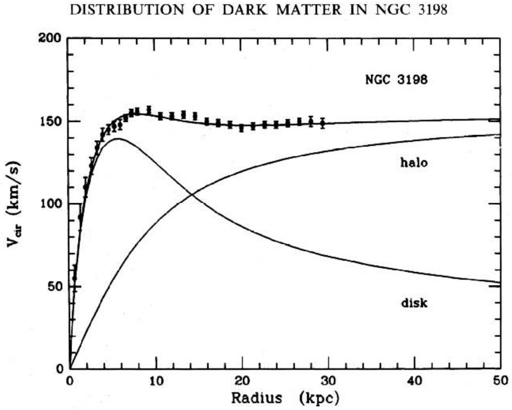
$$\left\langle v_{obs}^2\right\rangle \approx \frac{G_N M_{viral}}{5R}.$$

Virial mass 160× greater than luminous mass

⇒ Dark Matter

Coma cluster (Abell 1656)

Rotation Curves



Vera Rubin (1970)

$$\frac{G_N M}{R^2} = \frac{v_{cir}^2}{R}$$

$$\Rightarrow$$

$$v_{cir} \propto R^{-1/2}$$

Observations support halo hypothesis

Gravitational Lensing

Strong Lensing:

- Einstein rings/Arcs
- Lensed quasars

Micro Lensing:

- Blurred & indistinguisable strong lensing

Weak Lensing:

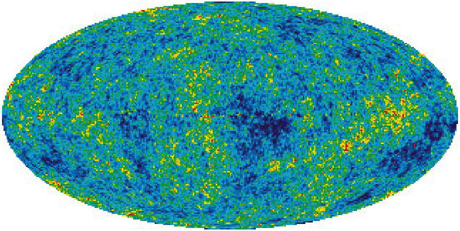
- single image

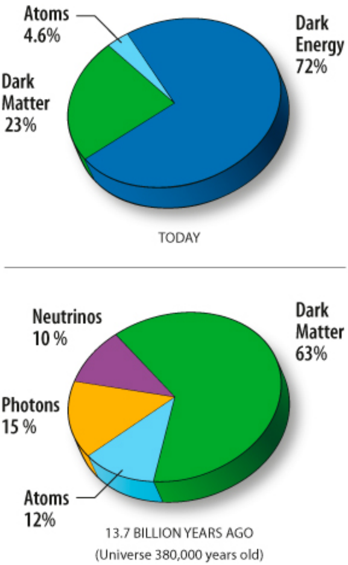
Abell(2218)

WMAP5 implications on DM(1)

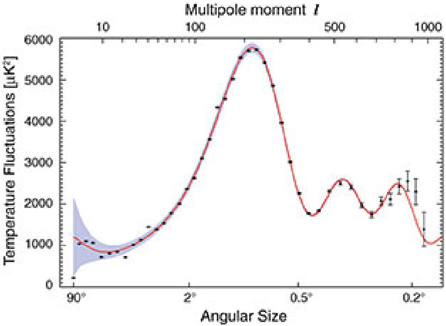
For the ΛCDM model:

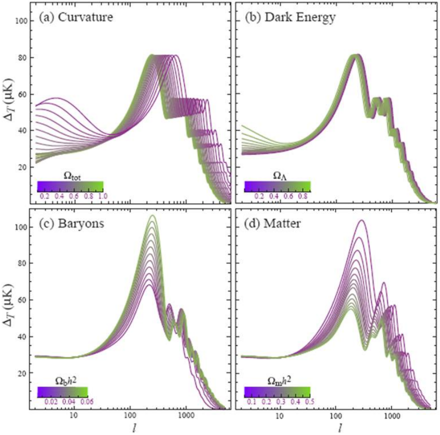
- Universe is spational flat $-.0179 < \Omega_k < 0.0081$ (95%CL)
- Super Nova measurements imply $\Omega_\Lambda \approx .72$





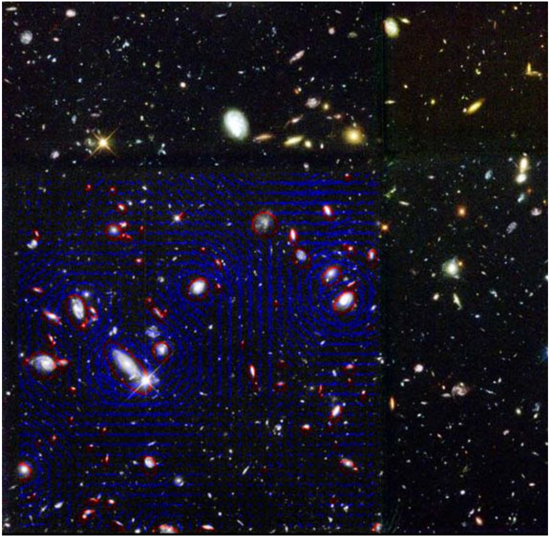
WMAP5 implications on DM(2)





Both gravitate, but only Baryonic matter undergoes sonic vibrations. Gravitation of Baryons not enough to explain third peak

See Hu& Dodelson CMB Anisotropies (2001) for details



Kubo et.al., *APJ.671:1466-1470* (2008)

Observed mass lies within 1.6σ of the virial mass.

No assumptions about virial equilibrium!

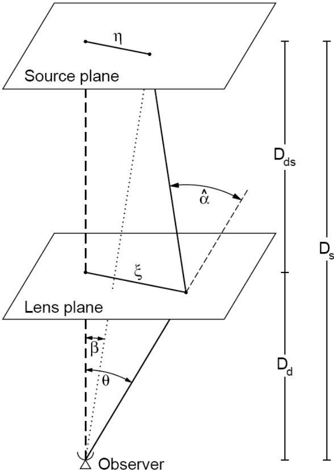
Derivation of Weak Lensing

General Relativity:
When impact parameter $\xi >> R_s = 2G_N M c^{-2}$, we have for the deflection angle $\hat{\alpha}$ of a point source

$$\hat{\alpha} = \frac{4G_N M}{c^2 r}$$

For three dimensional mass distribution $\rho(\vec{r})$

$$\hat{\alpha}(\vec{\xi}) = \frac{4G_N}{c^2} \int d^2\vec{\xi'} \int dr' \rho(\xi'_1, \xi'_2, r') \frac{\vec{\xi} - \vec{\xi'}}{\left|\vec{\xi} - \vec{\xi'}\right|^2}$$



Derivation of Weak Lensing

Introducing the surface mass density

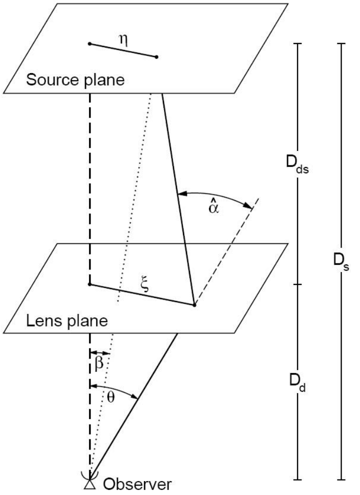
$$\Sigma(\vec{\xi}) = \int dr' \rho(\xi_1, \xi_2, r').$$

we arrive at

$$\hat{\alpha}(\vec{\xi}) = \frac{4G_N}{c^2} \int d^2\vec{\xi'} \Sigma(\vec{\xi'}) \frac{\vec{\xi} - \vec{\xi'}}{\left|\vec{\xi} - \vec{\xi'}\right|^2}$$

Valid if the mass distribution ρ does not extend from source to observer. This condition is satisfied in almost all astrophysical situations.

Derivation of Weak Lensing



Let $\vec{\eta}$ be the position of the source

$$\vec{\eta} = \frac{D_s}{D_d}\vec{\xi} - D_{ds}\hat{\alpha}(\vec{\xi})$$

Introducing $\vec{\eta} = D_s\vec{\beta}$ and $\vec{\xi} = D_d\vec{\theta}$, we can write

$$\vec{\beta} = \vec{\theta} - \frac{D_{ds}}{D_s}\hat{\alpha}(D_d\vec{\theta}) \equiv \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

- $\vec{\alpha}$ Scaled deflection angle
- $\vec{\beta}$ rescaled true position source
- $\vec{\theta}$ rescaled apparent position source

Derivation of Weak Lensing

Introducing the dimension-less surface mass density

$$k(\theta) = \frac{\Sigma(D_d\vec{\theta})}{\Sigma_{cr}} \text{ with } \Sigma_{cr} = \frac{c^2}{4\pi G_N}\frac{D_s}{D_d D_{ds}},$$

where k discriminates between weak and strong lenses.

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi}\int_{\mathbb{R}^2} d^2\theta' k(\vec{\theta'})\frac{\vec{\theta}-\vec{\theta'}}{\left|\vec{\theta}-\vec{\theta'}\right|^2}$$

Allows us to introduce the deflection potential

$$\Psi(\vec{\theta}) = \frac{1}{\pi}\int_{\mathbb{R}^2} d^2\theta' k(\vec{\theta'}) \log\left|\vec{\theta}-\vec{\theta'}\right|.$$

i.e. $\vec{\alpha} = \nabla\Psi$ and $\nabla^2\Psi(\vec{\theta}) = 2k(\vec{\theta})$.

Derivation of Weak Lensing

When source smaller than angular scale on which lens properties change the lens mapping can be locally linearised.

$$\mathcal{A}(\vec{\theta}) = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

Where we introduced the shear $\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma|\, e^{2i\phi}$

$$\gamma_1 = \frac{1}{2}(\Psi_{,11} - \Psi_{,22})\ ,\ \gamma_2 = \Psi_{,12}$$

Let $I^{(s)}(\vec{\beta})$ be the surface brightness distribution. The observed distribution is

$$I(\vec{\theta}) = I^{(s)}\left[\vec{\beta}(\vec{\theta})\right] \approx I^{(s)}\left[\vec{\beta_0} + \mathcal{A}(\vec{\theta_0}) \cdot (\vec{\theta} - \vec{\theta_0})\right]$$

Inversion problem

We want to calculate $\rho(\vec{\theta})$ from $I(\vec{\theta})$.

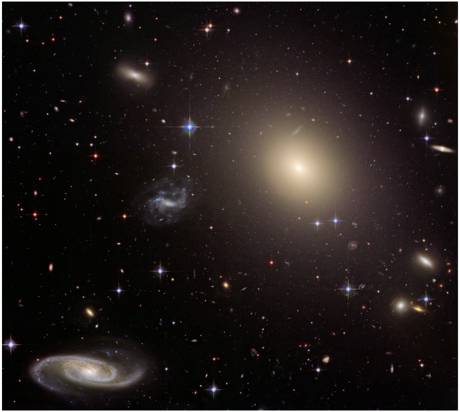
Problem:

We don't know $I^{(s)}(\vec{\theta})$

Assumptions

- Galaxies can be approximated by ellipses
- Galaxies are randomly orientated

Abell S0740



The Shape of Galaxies

For a general galaxy, we define the center of the galaxy by

$$\bar{\vec{\theta}} \equiv \frac{\int d^2\theta w_I \left[I(\vec{\theta}) \right] \vec{\theta}}{\int d^2\theta w_I \left[I(\vec{\theta}) \right]},$$

with w_I a suitable weight function.
Tensor of second brightness moments,

$$Q_{ij} = \frac{\int d^2\theta w_I \left[I(\vec{\theta}) \right] (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta w_I \left[I(\vec{\theta}) \right]}$$

If we analogously define $\bar{\vec{\beta}}$ and $Q_{ij}^{(s)}$ for the source. We find the relation

$$Q^{(s)} = \mathcal{A}(\bar{\vec{\theta}}) Q \mathcal{A}^T(\bar{\vec{\theta}})$$

The shape of Galaxies

With these definitions, we can define the size by

$$\omega = (Q_{11}Q_{22} - Q_{12}^2)^{1/2},$$

and the shape by complex ellipticity

$$\chi \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}$$

Using $Q^{(s)} = \mathcal{A}Q\mathcal{A}$ we find the relation

$$\chi^{(s)} = \frac{\chi - 2g + g^2\chi^*}{1 + |g^2| - 2\mathcal{R}(g\chi^*)},$$

where we defined the reduced shear g,

$$g(\vec{\theta}) = \frac{\gamma(\vec{\theta})}{1 - k(\vec{\theta})}.$$

The orientation of galaxies

Consider source galaxies at positions $\vec{\theta}_i$ close enough around an angle $\vec{\theta}$ such that k and γ do not change.
We expect

$$E(\chi^{(s)}) = 0.$$

Introducing the distortion

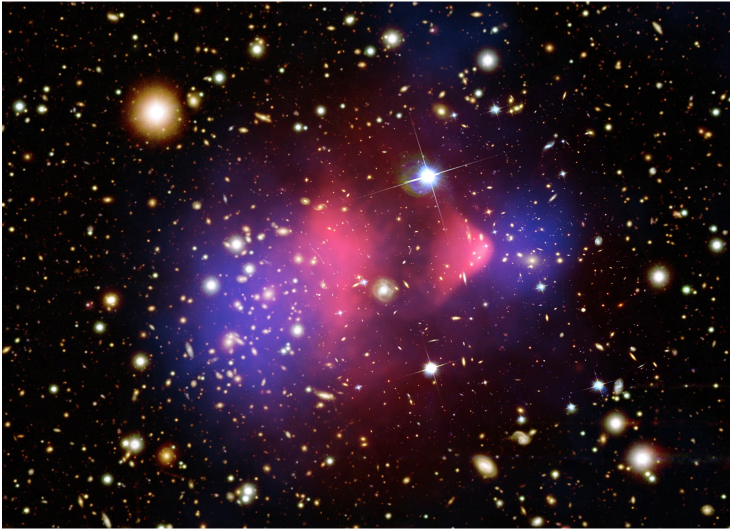
$$\delta = \frac{2g}{1 + |g|^2}$$

Schneider&Seitz(1995) showed

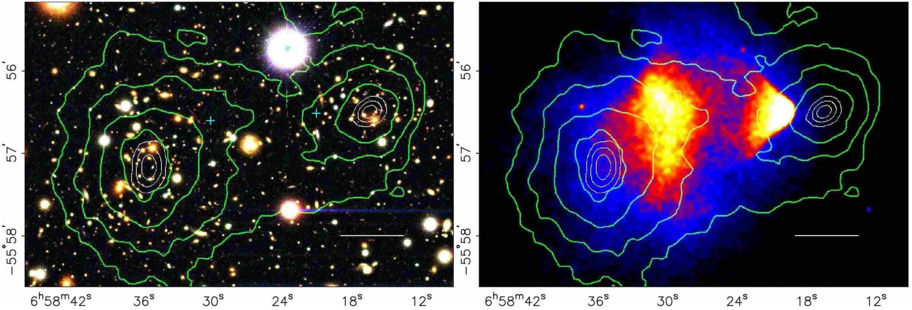
$$E(\chi^{(s)}) = 0 \Leftrightarrow \sum_i w_i \frac{\chi_i - \delta}{1 - \mathcal{R}(\delta)\chi_i^*} = 0$$

for weak lensing: $g \approx E(\chi)/2$. From the measured g we can determine k.

Bullet Cluster



Bullet Cluster



$$\nabla \log(1-k) = \frac{1}{1-g_1^2-g_2^2}\begin{pmatrix}1+g_1 & g_2\\ g_2 & 1-g_1\end{pmatrix}\begin{pmatrix}g_{1,1}+g_{2,2}\\ g_{2,1}-g_{1,2}\end{pmatrix}$$

Modified gravity cannot explain segregation.

Mass distribution proves existence of dark matter!

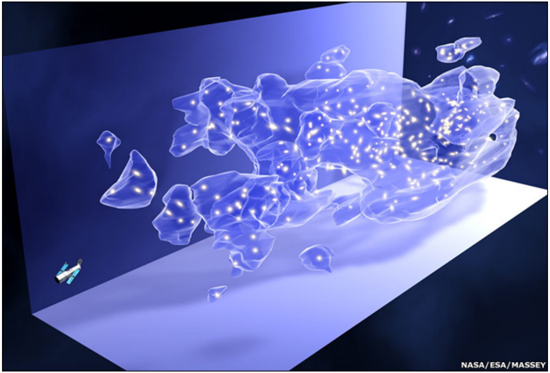
Clowe et. al., *astro-ph/0608407v1*

3D mass distribution?

Recall the definition of the dimensionless surface mass density

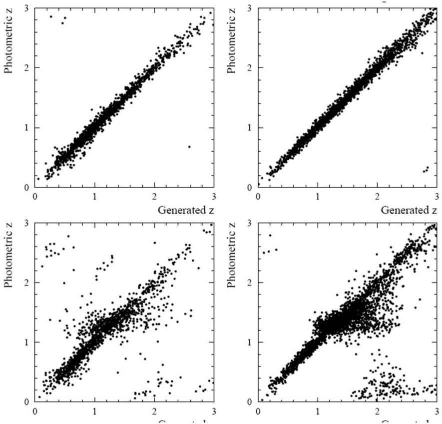
$$k(\theta) = \frac{\Sigma(D_d \vec{\theta})}{\Sigma_{cr}} \text{ with } \Sigma_{cr} = \frac{c^2}{4\pi G_N} \frac{D_s}{D_d D_{ds}}.$$

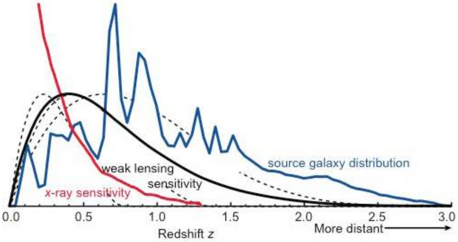
Redshift dependency allows 3D reconstruction of images!



Mapping the DM distribution in 3D

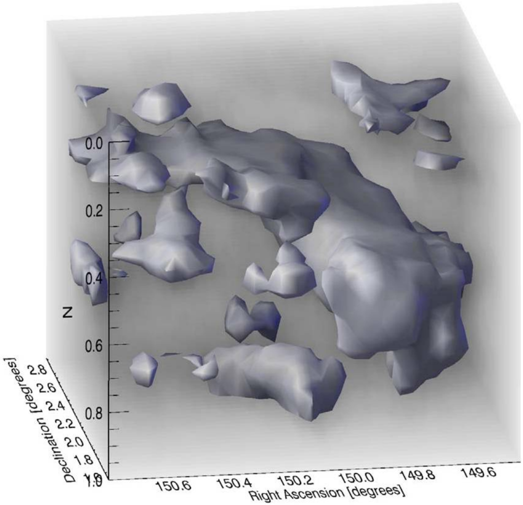
Photometric measurements provide redshift information.





Massey, Refregier, Rodes et.al. *Astron.J. 127 (2004) 3089*

3D filament structure



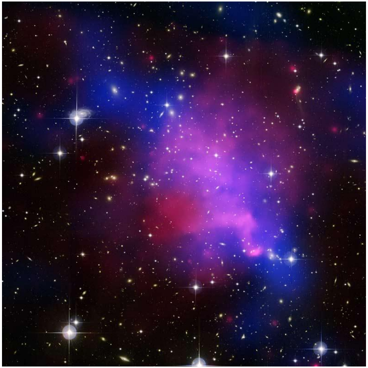
Massey, Rhodes et. al. *Nature445:286,2007*

Binsize $\Delta z = 0.05$

Structure contour
$1.6 \times 10^{12} M_{sun}$ within circle
$R = 700 kPc$

Conclusions

- Astrophysical observations support CDM universe
- Weak lensing can falsify alternative gravity theories
- Weak lensing is able to visualize 3D matter distribution in the universe



Questions?