Planet Formation

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February 6, 2009

Abstract

The formation of planets is to this day not at all well understood. Although we believe to know what processes are in general responsible for formation, a lot of questions are still open in the theoretical model for the formation of the typical planetary system. This makes it an interesting and challenging field of research, with a lot of room for new viewpoints. In this report an overview will be given of what theories exist to explain the formation of planets. Also will be noted were these theories need to be improved, as well as what is missing. Next, the stage after planet formation will be discussed, in which the planetary systems evolve due to interactions among the planets. Then popular and less popular methods for the detection of extrasolar planets will be discussed, which are of vital importance for testing our knowledge of planets. Finally we will touch upon the question of how typical our solar system is.

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Introduction

The question of how the earth came into existence had long been a subject of debate for scientists and religious people alike. But the first attempt to set up a model to accurately describe planet formation scientifically, was made by René Decartes in 1644, who introduced the idea that planets formed from system of vortices present around the Sun. Then, later, in 1734, Emanuel Swedenborg was the one to propose the nebular hypothesis, which would be the basis for modern thoughts on planet formation. It states that the planets formed from a cloud of dust present around the Sun. In 1755 Immanuel Kant further developed this theory. However Pierre-Simon Laplace formulated a similar theory independently around 1796, in his book Exposition of a world system, and was in fact the one who first describe the process accurately. Therefore Laplace is considered by many, the founder of planetary science. Laplace had astronomy as a hobby and was intrigued by the order of the solar system, the fact that the planets have nearly circular orbits, and that these orbits are in one plane, and that the orbiting directions are the same for all planets. From this he concluded that the planets and the Sun must have formed out of one medium. This then led to the hypothesis that a large cloud (already observed in the universe at that time) contracted leading to a faster rotation. The plane of rotation then would be the plane the planets finally formed in, and the remaining gas ball in the center would be responsible for forming the Sun. A modern version of this theory is still used to this day to describe the formation of planets. The theory was however not free of problems and also other theories existed. In 1749 Georges-Louis Leclerc, Comte de Buffon proposed that the planets formed from a collision between a comet and the Sun, from which matter was expelled condensing into the planets. These theories however failed to describe the order in the solar system. The nebular hypothesis was then abandoned for a long time. mostly due to the fact that it could not explained why the planets account for 99% of the angular momentum present in the solar system.

In the 20th century scientist began to review these theories and a lot

of new theories were introduced. In 1901 Thomas Chamberlin and Forest Moulton proposed the planetesimal theory in an attempt to accurately describe how the protoplanetary disk condensed into the planets. Also much work was done on the subject by James Hopwood Jeans, Otto Schmidt, William McCrea and Micheal Woolfson. In 1978 Andrew Prentice began to revise the nebular hypothesis and then finally Victor Safronov introduced the solar nebular disk model. This is a modern version of the initial nebular hypothesis and incorporates also the work done by scientists starting from the 20th century. In Safronov's book *Evolution of the protoplanetary cloud and formation of the Earth and the planets* he published his ideas, and solved most of the problems that previous models faced.

Chapter 1

Disk formation

Since planets are known to orbit around stars, it is important to know how stars form in order to discuss planetary formation. In fact, planets are in some sense a by-product of stellar formation. Since the planets only make up a fraction of the stars mass, planets will barely influence stellar evolution, while stars are of vital importance to planet formation. This not only concerns the central star around which the planets orbit, planet formation can also be influenced by nearby stars shining very brightly. Although the process of star formation is very complicated and not yet understood too well, a global picture of star formation will be given in this chapter. This will be enough for the discussion on planet formation.

1.1 Stellar formation

Stars form from large clouds of interstellar dust. At first the density is very low, but due to the gravitational force acting among the particles of the dust, and due to their random thermal velocities, denser regions start to appear inside the cloud. Gravitational attraction then causes the cloud to fragment into dense cores. One of these cores then further collapses, eventually becoming hot enough to start shining as what is called, a class 0 object. The various stages, a star goes through in its formation are illustrated in figure 1.1. Since the innermost part of this fragment will be accelerated most, it will start radiating first as well. The outer part then is an envelope. The object radiates in far infrared wavelengths. By conservation of angular momentum, the cloud will start to rotate faster and, again by the gravitational influence, most of the dust will settle in a rotationally supported plane in which the cloud rotates. At this point the innermost part of the cloud will start to fuse hydrogen and the actual star is born. The spectrum of this class 1 object will not look like an actual star yet because it is still embedded in an envelope of gas and dust. This will show absorption lines in the spectrum. When the star continues to shine, the envelope will be dissipated due to stellar winds. Due to the dense disk, the class 2 object will still radiate in the infrared part of the spectrum. Class 3 objects consist of young stars surrounded by a transparent disk. The matter of the disk is discarded through various mechanisms such as photo-evaporation due to the central or nearby stars and accretion onto the central star. Planet formation is thought to occur during the transition from class 2 to class 3 objects.



Figure 1.1: The various stages of the formation of a stellar object are illustrated. Taken from Klahr and Brandner [8].

1.2 Disk destruction

The now present disk around the young star consists of gas and dust inherited from the interstellar medium. But since the disk is far too heavy in comparison to the final planetary system, a large part of the mass of the disk must be lost. It is estimated that the disks mass is roughly 30% that of the central star, where in the solar system, the planets make up only 1% of the mass of the Sun. Not only is disk mass loss a requirement for ending up with the right mass, but it also imposes constraints on the time scales at which planets form. For example, if all gas in the disk is lost before protoplanets even emerge, gas giant planets will never form because there is no gas left for the planets to accrete. Probably the most important mechanism for the loss of gas, especially in the outer regions of the disk, is photo-evaporation.

1.2.1 Photo-evaporation

Photo-evaporation is the process in which gas in the protoplanetary cloud is heated by radiation. This heating causes pressure gradients in the gas and results in a gas-flow in which both gas and dust can be expelled from the disk. To examine photo-evaporation, it is useful to consider two regimes of photon energies: extreme ultraviolet photons (EUV) with energy greater then 13.6 eV, and far ultraviolet photons (FUV) with energy 6 - 13.6 eV. The EUV photons can ionize hydrogen atoms, abundantly present in the gas. This heats the gas to a temperature of the order 10^4 K. FUV photons can break apart molecules, ionize carbon and heat the gas through the photoelectric effect.

Photoevaporation is especially important for O-type stars shining bright and with high flux in the EUV region. Hollenbach *et al.* [6] used, as they call it, semi analytical methods to find the mass loss rate of gaseous disks around stars. Equation (1.1) expresses the rate at which these disks are evaporated by the photons:

$$\dot{M}_{\rm EUV} \approx 4 \cdot 10^{-10} M_{\odot} {\rm yr}^{-1} \left(\frac{\phi}{10^{41} {\rm s}^{-1}}\right)^{1/2} \left(\frac{M_{\star}}{M_{\odot}}\right)^{1/2},$$
 (1.1)

where ϕ is the photon flux of the star. The 10^{41} photons per second represents a flux that the early sun could have produced. For the most massive stars, this then corresponds to a timescale of the order 10^5 years to evaporate the disk. The innermost parts of the disk will be evaporated on timescales

of 10^6 to 10^7 years, i.e. up to r_g , the gravitational radius defined by

$$r_g = \frac{GM_{\star}}{kT} \approx 100 \,\mathrm{AU} \left(\frac{T}{1000 \,\mathrm{K}}\right)^{-1} \left(\frac{M_{\star}}{M_{\odot}}\right). \tag{1.2}$$

This is far beyond the radius at which Jupiter resides. These evaporation times have also been observed [5]. The problem is that gas giant planet formation seems to be greatly suppressed if all gas is evaporated at short distances and short timescales. The problem is even more dramatic for more massive stars. As we will see the problem can be even more severe in regions with high star densities, were not only the central star is responsible for the gas evaporation but also external sources dissipate gas.

Chapter 2

Planet formation

Planets form from the accretion disk present around a young star. Rocky planets like the terrestrial planets of the solar system form mainly from the dust inside this disk while gas giants are mainly build out of gas, with a solid core. The formation occurs in various steps. The first step is for the dust in the disk to start clumping together. This happens under the influence of van der Waals forces. This step is called run away accretion because the largest bodies will grow the fastest. At some point these bodies will become large enough to gravitationally influence their environment. This is a qualitatively different phase, where a small population of large bodies, called oligarches, which are inherited from the run away phase, are allowed to become roughly equal in size. At the ice line the oligarchic growth is believed to produce large enough bodies to form gas giant planets. At smaller radii a third step occurs in which the now formed protoplanets interact to form the rocky inner planets. At and beyond the ice line protoplanets can accrete gas from the disk if this has not already dissipated. As the third step is shown to be considerably shorter in duration, formation time of the terrestrial planets and gas giant planets alike can be estimated by estimating the time it takes to form the largest protoplanets.

2.1 Planet formation timescales

To estimate formation timescales, a good starting is equation (2.1) which expresses the change of the mass of a protoplanet in the properties of the disk,

$$\frac{\mathrm{d}M}{\mathrm{d}t} \approx F \frac{\Sigma_m}{2H} \pi R_M^2 \left(1 + \frac{v_{\rm esc}^2}{v_m^2} \right) v_m, \qquad (2.1)$$

where M is the mass of a forming planet, ${\cal F}$ is the ice line enhancement factor with

$$F = \begin{cases} 1 & r < r_{\rm il} \\ 4.2 & r > r_{\rm il} \end{cases}$$
(2.2)

 Σ_m is the surface density of planetesimals in the disk, H is the disk height, R_M is the radius of the protoplanet, $v_{\rm esc}$ is the escape velocity from the surface of the protoplanet and v_m is the relative velocity between the protoplanet and the planetesimals. It can be made intuitively clear that this equation is valid. It equates the change of mass on the left hand side in the amount of mass that is available to the protoplanet. The first part expresses the density of planetesimals in the disk, the second part is the collisional cross section and the factor in brackets is the gravitational enhancement factor to the cross section. Therefore the equation expresses the fact that the protoplanet sweeps up planetesimals available.

By making the estimations that the relative velocity v_m and the disk height H are related to the inclination i_m and the eccentricity e_m of the planetesimal orbits,

$$i_m \approx e_m/2,$$

 $v_m \approx e_m r \Omega,$
 $H \approx i_m \approx e_m r/2,$ (2.3)

and using $v_{\rm esc} = (2GM/R_M)^{1/2}$ and $v_m \ll v_{\rm esc}$, we can derive a scaling law:

$$\frac{\mathrm{d}M}{\mathrm{d}t} \propto \frac{\Sigma_m M^{4/3}}{e_m^2 r^{1/2}}.\tag{2.4}$$

Because for oligarchic growth the protoplanets gravitationally stir their feeding zone of planetesimals, e_m depends on M. Therefore for oligarchic growth the scaling law becomes $\dot{M} \propto M^{2/3}$, while for runaway growth $\dot{M} \propto M^{4/3}$. This exhibits the qualitative difference between the two regimes, because for $M_1 > M_2$, the mass ratio changes as

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{M_1}{M_2}\right) \propto \begin{cases} (M_1/M_2)(M_1^{1/3} - M_2^{1/3}) > 0 & \text{runaway} \\ (M_1/M_2)(M_1^{-1/3} - M_2^{-1/3}) < 0 & \text{oligarchic} \end{cases}$$
(2.5)

So, in runaway growth, the time derivative of the mass ratio grows. Therefore the mass ratio will grow, meaning that the relatively largest mass will, will be relatively larger still, some time later. In oligarchic growth, the contrary is true. Here the time derivative of the mass ratio is smaller then zero, so the mass ratio will shrink, implying that the difference in the masses becomes smaller.

Next the eccentricities can be estimated in equilibrium by equating the gravitational viscous stirring timescale of the protoplanet and the random velocity damping due to gas drag,

$$e_m \propto M^{1/3} \rho_{\rm gas}^{-1/5}$$
. (2.6)

The result is

$$\frac{\mathrm{d}M}{\mathrm{d}t} \simeq A \Sigma_m M^{2/3},\tag{2.7}$$

where

$$A = 3.9 \frac{b^{2/5} C_D^{2/5} G^{1/2} M_{\star}^{1/6} \rho_{\rm gas}^{2/5}}{\rho_m^{4/15} \rho_M^{1/3} r^{1/10} m^{2/15}},$$

 C_D is a dimensionless drag coefficient, and b determines the distance between the protoplanets in the disk. Assuming that planetesimals undergo no radial migration, the surface density of planetesimals Σ_m can be expressed as a function of the mass M of the protoplanet,

$$\Sigma_m(M) = \Sigma_m(0) - \frac{M}{2\pi r \Delta r} = \Sigma_m(0) - \frac{3^{1/3} M_\star^{1/3} M^{2/3}}{2\pi b r^2}.$$
 (2.8)

Using this together with equation (2.7), we find

$$\frac{\mathrm{d}M}{\mathrm{d}t} \approx A M^{2/3} (\Sigma_m(0) - B M^{2/3}),$$
 (2.9)

where

$$B = \frac{3^{1/3} m_{\star}^{1/3}}{2\pi b r^2}$$

The result of this differential equation is

$$M \approx \left(\frac{\Sigma_m(0)}{B}\right)^{3/2} \tanh^3 \left[\frac{AB^{1/2}\Sigma_m(0)^{1/2}}{3}t + \tanh^{-1}\left(\frac{B^{1/2}M(0)^{1/3}}{\Sigma_m(0)^{1/2}}\right)\right].$$
(2.10)

The solution is a rising function which stops increasing at a certain time. This happens at a timescale

$$t_{\rm iso} = \frac{3}{AB^{1/2}\Sigma_m(0)^{1/2}},$$

which is the time to reach the isolation mass

$$M_{\rm iso} = \left(\frac{\Sigma_m(0)}{B}\right)^{3/2}.$$



Figure 2.1: Protoplanet sizes plotted as a function of distance to the central star. Plotted for $5 \cdot 10^5$, 10^6 , $5 \cdot 10^6$ and 10^7 years. Plots taken from Klahr and Brandner [8]

The results are plotted in figure 4.2. The behavior is characterized by an outward expanding front of growth. With the expression for $M_{\rm iso}$ and $t_{\rm iso}$ we can estimate formation times at different radii and with different disk densities. If we take $M_{\star} = M_{\odot}$ and the surface density is taken to be 1, 5 and 10 times that of the minimum mass model, which takes the density to be just large enough to form all planets, we find that just beyond the ice line large planets form, with masses of the order of 1-100 M_{\oplus} which is about

correct for giant planets, since giant planet cores are believed to be of order $10 \ M_{\oplus}$. Also the timescale on which this happens seems correct since after 10^6 years there is still a significant amount of gas present to facilitate gas giant formation. At 1 AU planets form with a mass of order $1 \ M_{\oplus}$ which is about correct to form terrestrial planets. However the timescale is even shorter then that for the giant planets whereas the formation time of the earth is estimated to be order 10^8 years. Finally far beyond the ice line at roughly 20 AU, protoplanets fail to form even within 10^7 years, and while Uranus and Neptune both have significant amounts of H and He in their atmosphere, their formation cannot be explained by this model.

2.2 Problems and improvements

We see that the theoretical model given, although it can explain giant planet formation, it does not accurately describe planet formation. Great improvements can be made by taking into account different effects, and although this makes the calculations much more difficult, numerical methods can be used to exhibit the characteristics of these more complicated models.

2.2.1 Migration due to gas drag

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One of the most logical improvements is to consider radial migration of planetesimals due to gas drag. This can be done by allowing the surface density the change not only by planetesimal sweep up, but also by migration:

$$\frac{\mathrm{d}\Sigma_m}{\mathrm{d}t} = \frac{\mathrm{d}\Sigma_m}{\mathrm{d}t}\Big|_{\mathrm{accr}} + \left.\frac{\mathrm{d}\Sigma_m}{\mathrm{d}t}\right|_{\mathrm{migr}},$$

where

$$\left. \frac{\mathrm{d}\Sigma_m}{\mathrm{d}t} \right|_{\mathrm{migr}} = -\frac{1}{r} \frac{\partial}{\partial r} (r \Sigma_m \dot{r}_m). \tag{2.11}$$

The result is a pair of coupled differential equations, which can be solved numerically. This calculation had been performed by Thommes *et al.* [12] and they found that gas drag acts as a two edged sword. Firstly gas drag damps the random thermal velocities of planetesimals in the disk, speeding up the accretion by lowering the relative velocity. But secondly gas drag also increases the rate at which Σ_m is depleted, thereby reducing $M_{\rm iso}$. So growth is quicker but results in less heavy objects. They first did the calculations within the minimum mass model. The largest protoplanets that formed had a mass of less then 1 M_{\oplus} after 10 Myears. As a result no gas giant planets can form within this model at any timescale, and the Solar system was formed from a protoplanetary disk much heavier then the minimum mass model. With a ten times minimum mass model, they found protoplanets of $\sim 10 M_{\oplus}$ in the region just beyond the ice line. Within the ice line earth mass bodies were found, but far beyond the ice line, in what corresponds to the Uranus and Neptune region, bodies of just about one tenth of an earth mass formed. They also point out that to form sufficiently large protoplanets at these radii, at least 80 times the minimum mass model is required to facilitate large enough bodies within 10 Myrs, far beyond the typical range of observationally inferred disk masses. Nevertheless this model does properly explain the formation of giant planets.

2.2.2 The Keplerian shear regime

Another effect caused by gas in the protoplanetary disk is that it has the effect of slowing down planetesimals orbiting in the disk. If this slowing down is big enough accretion can happen in the shear dominated regime, in which the planetesimals follow closely the orbit of the gas. This is opposed to the dispersion dominated regime as discussed previously, where planetesimal motion is due to random thermal motion and scattering. Because in the former regime the relative velocity is small the accretion rate is greatly enhanced. Because the planetesimals follow the gas so closely, their inclination will be relatively small. In the most extreme case this leads to the fact that the disk height is roughly equal to the protoplanet cross section resulting in a very efficient accretion of planetesimals. The accretion rate then can be seen as a 2-dimensional process. Although only a small part of planetesimals is likely to be in the shear dominated regime, Rafikov [10] showed that only 1% of the total mass needs to be in this region for the protoplanet mass change to be described by this regime.

Chapter 3

Planetary system evolution

Now that we know how a planetary system evolves from a cloud of dust and gas towards a star with a number of planets orbiting around it in nearly circular orbits, it is a good question to ask whether these systems are stable or not. From observations it is known that a large proportion of planets have large eccentricities. These eccentricities can not be properly explained using the formation model given in the last chapters. Therefore it is natural to assume that after this first phase of formation a second phase exists in which the planets interact among each other. These interactions are of the gravitational kind when the orbits of the planets happen to be close enough. Since the theoretical models are not yet elaborate enough to predict what the relative distances of the planets are after the formation stage, it is useful to look at a model in which the planets inherited from the formation phase are present in a disk in circular orbits, with random spacings between them. A way of dealing with this stage, is by simply assuming that planets have already formed, and starting from a system of nearly circular orbiting planets, and see how this system evolves. Jurić and Tremaine did simulations fur such systems [7]. They started with systems of three, ten and fifty planets, and considered only gravitational force between the planets. Collisions were fully elastic so that planets could merge with each other or with the central star, without fragmentation. In figure 3.1 the average number of planets is plotted as a function of time. As can be seen, the systems are highly unstable, since after 10^6 years most systems consist of only a few planets. Also the final number of planets seems to be insensitive to the initial number. It is important to note that similar values were obtained by Adams and Laughlin [3] and Papaloizou and Terquem [9], despite the fact that they used different initial conditions. The actual number of planets is estimated to be too low, because tidal forces by the central star are neglected. Tidal forces act as to circularize orbits of planets. Therefore, at closer orbits, less planets will collide or be expelled. Then they also plotted the distribution function they found as a function of the eccentricities of the planets, averaged over the systems for the various system sizes (figure 3.2). The planetary systems are highly eccentric, with almost no circular orbits.

To compare, in figure 4.5 observed eccentricities are plotted. Although these planets are subject to strong selection effects, this should not influence the picture since the simulations point out that eccentricity scales with neither mass nor distance. As can be seen, most planets have an eccentricity smaller then 0.2. From the simulations it is clear though that most planets should have an eccentricity of $0.2 \le e \le 0.4$. Jurić and Tremaine give two explanations for this [7]. Firstly, the observed planets are found in systems of various lifetimes, suggesting that a part of these systems has not yet had time to develop instabilities. This will show an excess in circular or nearly circular orbits. Secondly, as pointed out before, the planets in the simulations at smaller orbits are not subject to tidal forces present in real planetary systems. This will also account for a larger proportion of planets with small eccentricities in the observations.



Figure 3.1: Average number of planets as function of time. Figure taken from Jurić and Tremaine [7].



Figure 3.2: Cumulative distribution functions of planet eccentricities. Figure taken from Jurić and Tremaine [7].

Chapter 4

Planet detection

This chapter will treat some techniques to detect extrasolar planets. This is obviously of great importance to the study of planets since models should be verified by what is observed. Although today's measurement are somewhat biassed due to technical limitations, there is still a lot to be concluded from them, and combining several different detection techniques may reveal different properties.

4.1 Astrometry

Astrometry is an indirect detection technique that was the first one to be applied in the search for planets. If a planet orbits a star, planet and star will actually be orbiting a common center point, called the barycenter. Since the stars mass is much bigger than the planets mass, the distance of the planet to the barycenter will be much bigger than the stars distance, but nevertheless non-zero, and if it is big enough, the star can be directly observed in motion around it. Although only the velocity of the star is known, the distance of the planet to the star can be calculated as well as the mass of the planet. This is simple mechanics and can be done by using Kepler's third law $P^2/r^3 = \operatorname{cst.}$, and the law of gravitation $\frac{GmM_{\star}}{r^2} = \frac{mv^2}{r}$, resulting in:

$$r = \sqrt[3]{\frac{GM_{\star}}{4\pi^2 P^2}},\tag{4.1}$$

and

$$m = \frac{M_{\star}v_{\star}}{v} = \sqrt{\frac{M_{\star}r}{G}}v_{\star} \tag{4.2}$$

Depending on the planet orbiting the star, astrometry can take quite a long time to detect a planet, since the period of the star around the barycenter is the same as the planets period, and considering the fact that astrometry is more likely to find planets with large orbits, the period can easily be ten years or more. This fact together with the fact that the stars displacement is hard to spot, makes astrometry a little used technique for the detection of planets.

4.2 Doppler spectroscopy

Doppler spectroscopy or radial velocity measurement is a detection technique that detects the same motion of a star as is measured with astrometry but does this by looking at a doppler shift of the stellar light, as the star is moving towards the earth and away from it. One disadvantage as compared to astrometry is that the line of sight must make an angle with the planetary plane in order the detect the doppler shift. However, since it is far easier to spot the doppler shift then it is to spot the displacement directly, doppler spectroscopy is far more appealing then astrometry is. As an example, with astrometry, differences in the speed of a star can be measured down to 10 m/s, where with doppler spectroscopy differences of 1 m/s can be measured.

As already explained, the planetary disk has to be in the line of sight in order to be able to detect a planet at all, however is the planetary disk makes an angle with the line of sight, there will only be a partial effect, resulting in an underestimation of the planets mass. Other techniques are then used to determine the tilt of the plane. Since the velocity of the star is measured, as with astrometry, the distance of the planet to star and the mass of the planet can be calculated both.

4.3 Transits

Another useful method for planet detection is the method of transits. With this method, a planet is spotted when it passes in front of its parent star, taking away a fraction of the stellar light. The reduction of light emitted then depends on the radii of planet and star.

One obvious disadvantage, is the fact that the sphere of the planet's orbit radius is very large compared to the stellar disk, making the chance that a planet passes in front of a star very small. However, planets with a smaller orbit are easier to detect, making this method suitable for finding earth like planets. To calculate the probability that a given planetary disk had the right inclination for a transit to be observable, consider figure 4.1, where i is the



Figure 4.1: Calculating the transit probability. Taken from Sackett [11].

inclination with respect to the plane in the line of sight and d(t) is the projected distance of the planet to the star. Now a transit can only be observed when $r \cos i \leq R_{\star} + R_p$, with R_{\star} the radius of the star and R_p the radius of the planet. Now $\cos i$ takes values between 0 and 1, and since it is equally likely to take any value in between for a random orbit we can calculate the desired probability by

$$P_{\text{transit}} = \frac{\int_0^{(R_\star + R_p)/r} \mathrm{d}\cos i}{\int_0^1 \mathrm{d}\cos i} = \frac{R_\star + R_p}{r}.$$
(4.3)

In all but the most extreme cases, we have $R_{\star} \gg R_p$ so that equation 4.3 can be written as $P_{\text{transit}} = R_{\star}/r$. For a planet at 1 AU this means that the transit probability is $4.6 \cdot 10^{-3}$, so that for measurements on which reliable statistics can be done thousands of stars have to be scanned. Therefore this method will not tell wether there is a planet around one particular star.

Another disadvantage is that the method is subject to a high rate of false detections. The light emitted by stars is constantly fluctuating a tiny bit, therefore it is hard to distinguish a fluctuation and a small orbiting planet. This can however be overcome partly by doing measurements over very long periods of time.

Why then are transits useful? Were the velocity measurements discussed in the previous sections determine the mass and the distance to the star of the target planet, the transit method provides information about the size of the planet. If the stellar size is known, the size of the planet can be deduced by looking at the amount of light hidden by the planet. Furthermore by combining the transit method with spectroscopy, information about the atmosphere of the planet is won. Stellar light passing through the atmosphere will show absorption lines of the components in the atmosphere. This then is a good indication of whether conditions on the planet could resemble conditions on earth.

Currently there is a number of projects using the transit method to detect planets. The Spitzer space telescope is used for this. In 2006 the COnvection ROtation and planetary Transits (COROT) program was launched. To date COROT has detected 7 exoplanets. The most recent discovery, was announced on 3 February 2009, very recent at the time of writing, and is the smallest exoplanet detected to this date, with a radius less then twice the earth radius [1]. Although the precise environment is unknown at the time, it is known that this planet is a rocky planet, just like the terrestrial planets.

Another program which is currently still under development is the Kepler mission. Kepler's main goal is to spot transits.

4.4 Gravitational microlensing

Planets can also be detected by gravitational microlensing. This happens when a star (in the case of planet detection) moves close to the line of sight of an observer to a background star. The light rays of the background source get bended by the gravitational lens, such that multiple images of the source will be observed (see figure 4.2). From general relativity we know that the deflection angle α can be expressed in terms of the Schwarzschild radius R_S of the lens and r, the closest point of approach of the light rays to the source, if $r \gg R_S$,

$$\alpha = \frac{4GM}{c^2 r} = \frac{2R_S}{r}.\tag{4.4}$$

Using this and some geometry considerations

$$\theta_S D_S = r \frac{D_S}{D_L} - (D_S - D_L)\alpha. \tag{4.5}$$



Figure 4.2: Left picture: The light rays of a background source S are shown, as they are bended by a lens L. Images I_1 and I_2 will be observed. α is the angle between the two images. **Right picture:** The images are shown as seen by an observer. The circle at an angle θ_s gets distorted into two flattened and bended circles. θ_E is the Einstein radius, is the characteristic distance. Figure taken from Sackett [11].

which can be written as

$$\theta_S = \theta - \frac{D_{SL}}{D_S} \alpha, \tag{4.6}$$

which is known as the lens equation. It equates the angular position of the images θ , to the position of the source and the deflection angle. Using $r = D_L \theta$ and

$$\theta_E = \sqrt{\frac{2R_S D_{LS}}{D_S D_L}}.$$
(4.7)

This can be written as

$$\theta^2 - \theta_S \theta - \theta_E^2 = 0. \tag{4.8}$$

This equation then yields two solutions: the angular positions of the two images.

To calculate the magnification of the images, it is important to note that the brightness of each image is unchanged. This means that only the ratio in area is important to the magnification: the magnification is just the ratio of image area to source area. It can be shown that the magnification then can be written as

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}},\tag{4.9}$$

with $u = \theta_s / \theta_E$.



Figure 4.3: An additional peak in the lightcurve is seen due to lensing by an exoplanet. The time is in days. Figure taken from Sackett [11].

What is interesting in the case of planet detection, is microlensing by binary systems. Since there are now two gravitational sources, the image patterns will be more complicated. For the case of single lens, there are four parameters: the time t_E it takes to cross the Einstein-ring, the impact parameter u_{\min} , the time of maximum amplification t_0 and the flux of the source F_0 . In the cases of a binary system there are however three additional parameters needed to describe lensing: the separation b of the lenses, the ratio of their masses q and the angle ϕ of the source trajectory with the binary axis. Although there are much free parameters it is actually possible to predict the images in real time. The lightcurve of a detected planet is shown in figure 4.3. Since the lightcurve can be predicted real time, the mass of the planet and the distance to the star can be measured using microlensing.

A huge disadvantage is that lensing experiments cannot be repeated. The chance that a star crosses a background source is small, and the star will not return to the same spot. On the other hand, it is true that microlensing can be used on far larger distances then any other detection method, however this can also be seen as a disadvantage, because planets found by microlensing are not liable to examining any further.

Since it is hard to predict when a star will pass a background star, usually large robotic networks of telescopes are used to scan the sky for lensing events. The Optical Gravitational Lensing Experiment (OGLE) is a Polish program designed to detect dark matter using microlensing. Aside from that they also found a number of exoplanets. Then there is the Probing Lensing Anomalies NETwork (PLANET), that has several telescopes around the world onn the southern hemisphere. PLANET is even able to detect earth mass planets.

4.5 Results obtained by planet detection

The first confirmed discovery of an exoplanet was made in 1988 by B. Cambell *et al* [4]. It was very difficult to verify at the time due to technical limitations. At first the number grew slowly but now that telescopes have improved significantly, exoplanets are more readily detected. To this date 339 exoplanets have been detected [2]. Most of these planets are detected around different stars, although also multiplanet systems are known. The reason is a bias, smaller, for now undetectable, planets are likely to be present in these systems.



Figure 4.4: Mass distribution for 101 observed planets. The data is obtained by the Lick, Keck, and AAT Doppler survey. A powerlaw is fitted to the data. Picture taken from Klahr and Brandner [8].

In figure 4.4 a distribution is plotted for observed planetary masses as obtained by a doppler survey of 1330 target stars. A powerlaw is fitted to the data. In figure 4.5 an eccentricity distribution is plotted for the same



Figure 4.5: Eccentricity distribution for 101 observed planets. The data is obtained by the Lick, Keck, and AAT Doppler survey. Taken from Klahr and Brandner [8].

planets. As can be seen, the planets are highly eccentric, especially for larger orbits. For smaller orbits, the eccentricities are smaller, probably due to tidal circularization of the central star. These eccentricities are not explained by the theoretical models of planet formation, however they can be explained by assuming that the planetary systems are unstable, so that these eccentricities occur only after the planets have already formed. This is also explained in chapter 3. A property was also exhibited by plotting the metallicity of stars versus the occurrence of planets. As clearly seen from figure 4.6, there is a strong correlation between the amount of iron present in the star, and the number of planets. The physical reason for this is called *nature of nurture*, and is the effect that systems with high metallicity are subject to increased small particle condensates. The iron acts as the dust in the protoplanetary cloud. Hence there is an increased formation of planetesimals and therefore also an increased formation of planets. This is strong support for the accretion model of planet formation.



Figure 4.6: Eccentricity distribution for 101 observed planets. The data is obtained by the Lick, Keck, and AAT Doppler survey. Taken from Klahr and Brandner [8].

Chapter 5

How typical is the solar system?

As a final part of this report, it is nice to discuss the question: How typical is the solar system?. To start with the most striking difference with observed data, extra solar planets are seen to be highly eccentric. The most likely explanation for this is that the solar system is simply not old enough for the planets within it, to have interacted and obtained more eccentric orbits. However, the age of the solar system is thought to be of the order 10^9 years whereas simulations show that instabilities in planetary systems take in the order 10^6 years to develop. The most logical explanation is that the solar system is unusually stable compared to other planetary systems. It must be noted though, that these simulations are not yet that sophisticated and leave much room for improvement, most notably in the initial conditions. According to theory however the solar system is typical for young stellar systems, but then again these theories were formed to fit the solar model, so not much can be said on that account.

Of the 339 detected planets only 36 systems are known to contain multiple planets. This is mostly due to technical limitations though. A lot of exoplanets are however *hot jupiters*. When a hot jupiter is present, it is unlikely that any terrestrial planets are present in that system, because these planets close to their host star, push any other planet inwards, by migration, leaving little or no room for smaller planets at close orbits. These systems then are quite unlike the solar system. It is not known what percentage of planets is a hot jupiter. This is hard to estimate due to biasses in measurements, and limited sample size.

Although it will be possible in the future to determine exoplanets atmo-

spheres by examining transits, to this day no such measurements have been done. There are some promising projects for this, and in the future more will be known about possible earth-like conditions on exoplanets.

The bottom line is that it is to preliminary to draw any conclusions from measurements and theories alike. Great developments have taken place though and in the near future much more will be known on the subject.

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