**GENERAL RELATIVITY**

Homework problem set 5, due at 25.11.2016.

## ■ PROBLEM 9 Redshift. *(5 points)*

Consider the expanding universe metric,

$$ds^2 = -c^2dt^2 + a^2(t)\delta_{ij}dx^i dx^j \ . \tag{9.1}$$

It describes the homogeneous, isotropic universe in the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological model.

**(a)** *(2 points)* Find the geodesic equations for a particle in this space-time. (Either by explicitly constructing the Christoffel symbols, or by varying the particle action.)

**(b)** *(3 points)* Using the geodesic equations and the expression for $ds^2$, show that for a photon moving along the $x$-axis, the coordinate time as a function of the parameter $\lambda$, which parametrizes the path, is given by

$$\frac{dt}{d\lambda} = \frac{\omega_0}{a(t)}, \tag{9.2}$$

where $\omega_0$ is a constant.

For a photon-like particle, we define $p^\mu = dx^\mu/d\lambda$. The energy of the photon is defined as $E = -p_\mu u^\mu$, where $u^\mu$ is the velocity of a comoving observer (here $u^\mu = (1,0,0,0)$). It turns out that the energy of the photon-like particle for a comoving observer is $E = \omega_0/a(t)$. The energy decreses as $a$ (the scale factor) increases. This is known as the *cosmological redshift*.

## ■ PROBLEM 10 Energy-momentum tensor for electromagnetic field. *(6 points)*

The action for electromagnetism in curved space-time is

$$S_{EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - g^{\mu\nu}A_\mu J_\nu\right) \ , \tag{10.1}$$

where $J^\mu$ is the conserved charge current.

**(a)** *(2 points)* Derive the energy-momentum tensor by functional differentiation with respect to the metric.

**(b)** *(4 points)* Consider adding a new term to the action,

$$S' = \int d^4x\sqrt{-g}\,\beta R^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \ . \tag{10.2}$$

How are Maxwell's equations altered in the presence of this term? How about Einstein's equations? Is the current $J^\mu$ still conserved?