GENERAL RELATIVITY

Homework problem set 6, due at 09.12.2016.

■ **PROBLEM 11** A careful explorer. *(4 points)*

An observer decides to explore the geometry outside a Schwarzschild black hole of mass M by starting with an initial velocity at infinity and then falling freely on an orbit that will come close to the black hole and then move out to infinity again. What is the closest that the observer can come to the black hole on an orbit of this kind? How can the observer arrange to maximize the time to study the geometry between crossing the radius $r = 4MG_N$ and crossing it again?

■ **PROBLEM 12** A careless explorer. *(4 points)*

Consider an observer (not necessarily on a geodesic) that has fallen inside the event horizon, $r < 2G_N M$. Use the ordinary Schwarzschild coordinates $(t, r, \vartheta, \varphi)$. Show that the radial coordinate must decrease at a minimum rate given by

$$\left|\frac{dr}{d\tau}\right| \geq \sqrt{\frac{2MG_N}{r} - 1}\ . \tag{12.1}$$

Calculate the maximum proper time of this observer along a trajectory from $r = 2MG_N$ to $r = 0$.

■ **PROBLEM 13** Light around a black hole. *(4 points)*

Find all closed orbits for light in Schwarzschild space-time. Determine whether these orbits are stable or unstable.