GENERAL RELATIVITY

Homework problem set 8, due at 20.01.2017.

■ PROBLEM 16 Komar integrals. *(9 points)*

Using Komar integrals, calculate the following quantities:

(a) *(3 points)* Mass of the Reissner-Nordström black hole.

(b) *(3 points)* Electric charge of the Reissner-Nordström black hole. (You have calculated the electromagnetic field strength tensor in the previous homework.)

(c) *(3 points)* Angular momentum of the Kerr black hole.

■ PROBLEM 17 Perturbations around Minkowski space-time. *(9 points)*

Consider an infinitesimal coordinate transformation of the type

$$x^\mu \to \widetilde{x}^\mu = x^\mu + \xi^\mu(x) \ , \qquad |\xi| \ll 1 \ . \tag{17.1}$$

(a) *(3 points)* Show that, to linear order in ξ, the metric transforms as

$$g_{\mu\nu}(x) \to \widetilde{g}_{\mu\nu}(\widetilde{x}) = g_{\mu\nu}(\widetilde{x}) - \left(\widetilde{\nabla}_\mu \xi_\nu(\widetilde{x}) + \widetilde{\nabla}_\nu \xi_\mu(\widetilde{x})\right) \ . \tag{17.2}$$

Now assume that the metric can be written as

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) \ , \qquad |h_{\mu\nu}| \ll 1 \ . \tag{17.3}$$

It means that we are dealing with small perturbations around Minkowski space-time, $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$.

(b) *(2 points)* How does $h_{\mu\nu}$ transform under the coordinate transformation (17.1) to linear order in ξ and $h_{\mu\nu}$?

(c) *(2 points)* Show that, given the form of the metric (17.3), the form of the Riemann tensor to first order in $h_{\mu\nu}$ is

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\Big(\partial_\rho\partial_\nu h_{\mu\sigma} + \partial_\sigma\partial_\mu h_{\nu\rho} - \partial_\rho\partial_\mu h_{\nu\sigma} - \partial_\sigma\partial_\nu h_{\mu\rho}\Big) \ . \tag{17.4}$$

(d) *(2 points)* Show that (17.4) is invariant under the infinitesimal coordinate transformations (17.1).