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Baryogenesis

$$\eta = 6.1 \pm 0.3 \times 10^{-10}$$

Author:
Jan Weenink

Supervisor:
Dr. Tomislav Prokopec

Institute for Theoretical Physics, Utrecht University

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1 Introduction

The matter-antimatter asymmetry, usually called the baryon asymmetry, is one of the great mysteries of the universe. In everyday life we encounter only matter and no antimatter, luckily for us. Particle accelerators are the only places on earth where 'large' quantities of antimatter are produced, although it only exists for a tiny moment before it annihilates with matter. On larger scales there is also evidence that there is more matter than antimatter, and we have good reason to believe that the entire universe is made of matter. The abundance of matter over antimatter is usually expressed by the baryon-to-entropy density ratio $\eta = 6.1 \pm 0.3 \times 10^{-10}$, and was calculated from Big Bang Nucleosynthesis (BBN) and only recently independently from the Cosmic Microwave Background radiation (CMB). In section 2, I will explain more about the observational evidence for the baryon asymmetry of the universe.

From particle physics and the Standard Model, it is not obvious that there should be more matter than antimatter. The SM is nearly symmetric with respect to particles and antiparticles and thus predicts a nearly baryon symmetric universe. The only observed asymmetry in the SM is a tiny C and CP-violation in the K^0-$\bar{K}^0$-system, which is much too small to explain the observed baryon asymmetry. The big question for physicists is therefore: how can the baryon asymmetry be created from a baryon symmetric universe?

Baryogenesis is the dynamical creation of a baryon asymmetry from an initially baryon symmetric universe. In 1967, Sakharov already realised the need for baryogenesis, and gave the three ingredients necessary for successful baryogenesis. These three ingredients are baryon number violation, C and CP violation and out-of-equilibrium conditions, and they are usually called Sakharov's conditions. In section 3, I will explain more about Sakharov's conditions and why they are necessary for baryogenesis.

In the late 1970's Grand Unification Theories (GUTs) were invented, and it did not take long for people to realize that these GUTs satisfied all three of Sakharov's conditions, and could explain the baryon asymmetry. Section 4 contains a short introduction about GUTs, the application to Sakharov's conditions and a simple calculation of the baryon asymmetry. Although there are reasons to believe that GUT baryogenesis is not the origin of the baryon asymmetry (as will be explained), it still provides a nice examples of baryogenesis.

For many years people thought that the Standard Model did not satisfy Sakharov's conditions, but in the 1980's it was shown that baryogenesis is possible in the SM itself. In section 5 I will explain more about this so-called Electroweak Baryogenesis scenario. For good reviews on baryogenesis, see for example [1], [2], [3], [4], [5] and [6].

2 Observational evidence for the baryon asymmetry

As mentioned in the introduction, the earth is made entirely of matter. But we also know that other planets and objects in our solar system are made of matter, since, for example, it has not occurred that an explorer annihilated with some source of antimatter. The sun itself is also made of matter, which we know from solar winds and solar cosmic rays that contain mostly protons. Extrasolar cosmic rays are highly energetic particles coming from another source than the sun, such as black holes, supernovae and quasars. These cosmic rays consist mostly of protons. The ratio of antiprotons to protons in these rays is $\sim 10^{-4}$ and the existence of antiprotons is usually explained from high energy collision reactions of cosmic rays with matter, such as $p + p \to 3p + \bar{p}$. Thus we can also say that our own galaxy, and other galaxies are composed of matter. The possibility of entire galaxies or clusters made of either matter or antimatter is not very likely, and we would expect to see large fluxes of gamma-particles coming from the annihilation of hydrogen and antihydrogen particles in the region separating these galaxies. All in all, we have good reasons to believe that the universe is entirely made of matter.
The baryon asymmetry of the universe is characterized by the baryon-to-entropy density ratio, which currently has the value[7]

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = \frac{n_B}{n_\gamma} = 6.1^{+0.3}_{-0.2} \times 10^{-10} \qquad \text{(WMAP 2008)}. \tag{1}$$

Since at the present there is no antimatter ($n_{\bar{b}} = 0$), this ratio is actually $\eta = n_b/n_\gamma$. The baryon-to-entropy density ratio was first calculated from Big Bang Nucleosynthesis (BBN), which is the epoch where deuterium (D), helium (^{3}He and ^{4}He) and lithium (^{7}Li) were created. The abundances of these light particles are sensitive to the baryon to entropy ratio. Only recently the baryon-to-entropy density ratio has also been determined independently from the Cosmic Microwave Background radiation (CMB), see Fig. 1. The second acoustic peak is sensitive to this ratio and since the launch of NASA's WMAP this has given us the most accurate result. The results from BBN are consistent with this number.

The baryon-minus-antibaryon density $n_B = n_b - n_{\bar{b}}$ per comoving volume is conserved, i.e. $\frac{d}{dt}(a^3 n_B) = 0$, so we are basically saying that the number of baryons minus the number of antibaryons is conserved (i.e. quark conservation). Since every baryon either decays to another baryon, or annihilates with an antibaryon this statement remains true. The photon density n_γ however is only conserved at late times, since at early times heavy particles annihilated to produce more photons but not baryons. A better ratio to consider is therefore the baryon-to-entropy density ratio, n_B/s, since the entropy density per comoving volume $S = a^3 s$ is conserved, and therefore n_B/s is conserved. The photon density is calculated from the particle number density for relativistic particles

$$N = g_* \frac{\zeta(3)}{\pi^2} \left(\frac{k_B T}{\hbar c} \right)^3 , \tag{2}$$

where for photons the relativistic degrees of freedom $g_* = g_{*\gamma} = 2$ and $T = 2.73\text{K}$ (for fermions g_* gets an additional factor 3/4). For the entropy density,

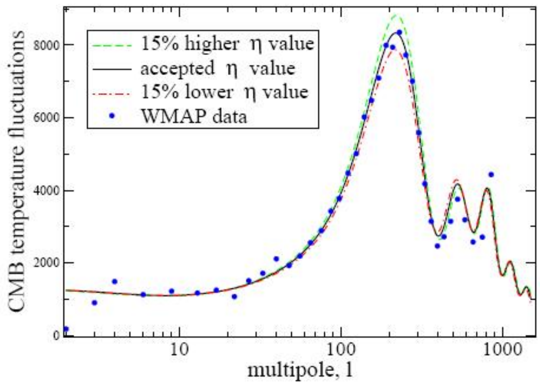

Figure 1: The dependence of the acoustic peaks in the power spectrum of the CMB temperature anisotropy on the baryon-to-photon density ratio η. The accepted value for $\eta = 6.1^{+0.3}_{-0.2} \times 10^{-10}$, measured by NASA's WMAP mission.

this is

$$s = g_{*S} \frac{2\pi^4}{45} \left(\frac{k_B T}{\hbar c} \right)^3 , \tag{3}$$

where g_{*S} counts the effective relativistic degrees of freedom,

$$g_{*S} = \sum_{a=bosons} g_a (T_a/T)^3 + \frac{7}{8} \sum_{a=fermions} g_a (T_a/T)^3 . \tag{4}$$

In this equation the g_a are the degrees of freedo for each species of bosons or fermions. So for the relation between entropy density and photon density we get

$$\frac{s}{n_\gamma} = \frac{g_{*S}}{g_{*\gamma}} \frac{2\pi^2}{45\zeta(3)} . \tag{5}$$

Currently, the only relativistic particles are photons ($g_\gamma = 2$) and neutrinos ($g_\nu = 6$, $T_\nu = 1.95$K), such that $g_{*S} = 3.91$. This finally gives $s = 7.04 n_\gamma$, such that in the present universe

$$\frac{n_B}{s} = \frac{1}{7.04} \frac{n_B}{n_\gamma} = \frac{1}{7.04} \eta = 8.7 \pm 0.3 \times 10^{-11} . \tag{6}$$

This ratio remained constants during the expansion of the universe and was therefore the same in the early, hot universe. When the universe was very hot, all particles were relativistic, and $g_{*S} \simeq 10^2$. Therefore $\eta \simeq g_{*S} n_B / s \simeq 10^{-8}$, and assuming there were about as many baryons as antibaryons as photons ($n_b \sim n_{\bar{b}} \sim n_\gamma$), this gives $(n_b - n_{\bar{b}})/n_b \simeq 10^{-8}$. Now assume that all the baryons were quarks, i.e. no nucleons had been formed yet, and we can conclude that for every 100 million antiquarks there were roughly 100 million and 1 quark. A tiny asymmetry but with important consequences!

3 Sakharov's conditions

The baryon asymmetry was first considered to be an initial condition, something that just existed after the Big Bang but cannot be explained because of the Big Bang singularity. However, inflation[8] excluded this possibility. Any pre-exisiting baryon asymmetry would be diluted to a negligible value after inflation. During reheating the temperature of the universe increases dramatically, and therefore there is an enormous production of entropy and η is effectively zero. Thus the possibility of an initially baryon asymmetric universe is excluded.

In 1967 Andrei Dimitriev Sakharov already realized the need for a dynamical generation of the baryon asymmetry from an initially baryon symmetric universe[9]. For this so-called baryogenesis scenario he listed three ingredients (although he did not name them explicitly), that are now widely considered to be both very generic and necessary for successful baryogenesis.

Sakharov's conditions are the following: 1. Baryon number violation, 2. C and CP violation, 3. Departure from thermal equilibrium. In the following part I will give a detailed explanation of these three conditions.

3.1 Baryon number violation

Baryon number is a nearly conserved quantum number of a system in particle physics. It is defined as

$$B = \frac{N_q - N_{\bar{q}}}{3}, \tag{7}$$

where N_q is the number of quarks and $N_{\bar{q}}$ is the number of antiquarks. Historically, before the discovery of quarks, the baryon number of a system was defined as the number of baryons minus the number of antibaryons, i.e. $B = N_b - N_{\bar{b}}$ and all elementary particles have a baryon number of +1, 0 or -1. For examples protons, neutrons have a baryon number of 1 and are therefore called baryons. Similarly antibaryons (antiprotons etc.) have a baryon number -1. Leptons such as electrons have a baryon number 0. Only later it was discovered that every baryon is actually composed of three quarks, which therefore changed the definition to Eq. (7). Quarks have by definition a baryon number of 1/3 and for antiquarks $B = -1/3$.

In a baryon symmetric universe there are as many baryons as antibaryons, and therefore $B = 0$. To create a universe where $B \neq 0$, baryon number must be violated. However, all Feynman diagrams in the Standard Model of particles conserve baryon number. Thus one might think that baryogenesis is not possible within the SM, but baryon number is actually violated through a non-perturbative effect. In section 5 I will come back to this.

Let us for simplicity assume that there are reactions that violate baryon number. A very heavy particle X has two possible decay channels. For one decay a baryon number B_1 is created and has a branching ratio (probability) r, whereas the other has a baryon number B_2 and a branching ratio $1-r$. As we will see in section 4, in certain grand unified models, there are heavy X gauge bosons that can decay to either two quarks, creating a baryon number of 2/3, or a lepton and an antiquark, with lepton number 1 and baryon number -1/3. In all these models $B - L$ is conserved.

Also there is an equal (we are considering an initially baryon symmetric universe) amount of heavy antiparticles $\bar{X}$ that has two decay channels which create baryon numbers $-B_1$ and $-B_2$ and has branching ratios $\bar{r}$ and $1-\bar{r}$. The average baryon number violation produced in the decay of one X particle and its corresponding antiparticle is then

$$\Delta B = rB_1 + (1-r)B_2 - \bar{r}B_1 - (1-\bar{r})B_2 = (r-\bar{r})(B_1 - B_2). \tag{8}$$

Two things can be noticed from this equation. First of all, there need to be two decay channels that create a different baryon number, i.e. $B_1 \neq B_2$, in order to violate baryon number. The reason for this is that if the same baryon number is created in both decay channels, then we might as well have assigned that baryon number to the unknown heavy particle X and there would be no B violation. Secondly, the branching ratios for the decay of the particle X have to be different from the branching ratio of its antiparticle $\bar{X}$, i.e. $r \neq \bar{r}$. If they would be the same, just as many baryons as antibaryons would be created and there would not be an asymmetry. $r \neq \bar{r}$ only when both **C** and **CP** are violated.

3.2 C and CP violation

Consider again a heavy X particle that can decay to, for example, two quarks with a branching ratio r. The branching ratio is defined as

$$r = \frac{\Gamma(X \to q + q)}{\Gamma_X}, \tag{9}$$

where Γ_X is the total decay rate (remember that for B-violation we need at least one other decay channel). For the branching ratio of the antiparticle $\bar{X}$ we have a similar definition

$$\bar{r} = \frac{\Gamma(\bar{X} \to \bar{q} + \bar{q})}{\Gamma_{\bar{X}}}. \tag{10}$$

Conservation of **CPT** tells us that the total decay rate of a particle and its antiparticle is equal, $\Gamma_X = \Gamma_{\bar{X}}$. Therefore we can write

$$r - \bar{r} = \frac{\Gamma(X \to q + q) - \Gamma(\bar{X} \to \bar{q} + \bar{q})}{\Gamma_X}. \tag{11}$$

Under **C**, all particles are replaced by its antiparticles and vice versa. When **C** is conserved,

$$\Gamma(X \to q + q) = \Gamma(\bar{X} \to \bar{q} + \bar{q}), \tag{12}$$

and therefore $r - \bar{r} = 0$ and baryon number is not violated. Thus we need **C** violation. However, we also need **CP** violation. To see this, we write

$$\Gamma(X \to q + q) = \Gamma(X \to q_L + q_L) + \Gamma(X \to q_R + q_R), \tag{13}$$

and similarly for the antiparticle. This gives

$$r-\bar{r} = \frac{\Gamma(X \to q_L + q_L) + \Gamma(X \to q_R + q_R) - \Gamma(\bar{X} \to \bar{q}_L + \bar{q}_L) - \Gamma(\bar{X} \to \bar{q}_R + \bar{q}_R)}{\Gamma_X}. \tag{14}$$

Under **CP**, $q_L \to \bar{q}_R$ and $q_R \to \bar{q}_L$, and when **CP** is conserved,

$$\Gamma(X \to q_L + q_L) = \Gamma(\bar{X} \to \bar{q}_R + \bar{q}_R) \quad (15)$$
$$\Gamma(X \to q_R + q_R) = \Gamma(\bar{X} \to \bar{q}_L + \bar{q}_L). \quad (16)$$

Again, $r - \bar{r} = 0$ and baryon number is not violated. Thus, we need both **C** and **CP** violation in order create a baryon asymmetry.

3.3 Departure from Thermal Equilibrium

The last of Sakharov's conditions is quite tricky. In thermal equilibrium particles and antiparticles will have identical distribution functions. To see this let's consider the phase space distribution functions

$$f(p) = \frac{1}{\exp[(\mu + E)/(k_B T)] \pm 1}, \quad (17)$$

where the $+$-sign is for fermions and the $-$ for bosons, μ is the chemical potential and $E = \sqrt{p^2c^2 + m^2c^4}$ the energy of the particles. The particle number density for a certain species is then

$$N = g \int \frac{d^3p}{(2\pi\hbar)^3} f(p), \quad (18)$$

where g is the number of degrees of freedom for the particle species. For relativistic particles, $k_B T \gg m, \mu$, Eq. (2) is obtained. For non-relativistic particles $m \gg k_B T, \mu$, this is evaluated to give

$$N = g \left(\frac{m k_B T}{2\pi\hbar^2 c^2} \right)^{3/2} e^{-(\mu+m)/(k_B T)}. \quad (19)$$

For relativistic particles and antiparticles it is easy to see (from Eq. (2)) that they have the same particle number densities, since g is the same for a particle and its antiparticle. Therefore, for relativistic particles and antiparticles in thermal equilibrium there is no baryon asymmetry. For non-relativistic particles it is a bit harder to see. The **CPT** theorem ensures that particle and antiparticle masses are equal, $m_b = m_{\bar{b}}$. Also, in chemical equilibrium the chemical potentials must be conserved, i.e for a reaction $a + b \to c + d$ the relation $\mu_a + \mu_b = \mu_c + \mu_d$ must hold. Since we have reactions like $b + \bar{b} \to 2\gamma$, we can see that $\mu_b + \mu_{\bar{b}} = 2\mu_\gamma = 0$, thus $\mu_b = -\mu_{\bar{b}}$. But also, since we have baryon number non-conserving reactions, all chemical potentials must vanish. Therefore, in thermal equilibrium the particle distribution functions and the particle number densities of particles and antiparticles are equal, $n_b \equiv n_{\bar{b}}$. So in order to create a baryon asymmetry, a departure from thermal equilibrium is necessary.

Now we will see how the universe can depart from thermal equilibrium. Suppose we have heavy particles X and antiparticles $\bar{X}$ with masses $M_X = M_{\bar{X}}$ that exist in equal numbers, i.e. an initially baryon symmetric universe. At very high temperatures, $k_B T \gg M_X$, the decay rate of the particles Γ is equal to the inverse decay rate Γ_{ID}, i.e. particles are decaying as fast as they are formed and no net baryon number is produced. As the temperature drops below the mass of the particles, the X particles become non-relativistic and they want to decay

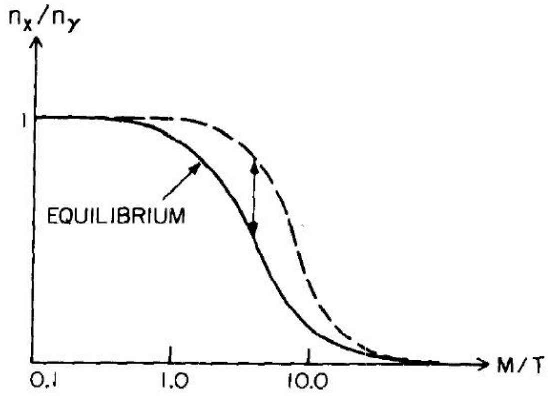

Figure 2: The X particle density-to-photon density ratio as a function of M/T, where M is the mass of the particle. For $T > M$, the X particles are as abundant as photons and they do not decay. As $T \sim M$, the X particle wants to decay and as long as $\Gamma_D > H$ the equilibrium abundance is maintained. For the dotted line Γ_D is smaller than H when the temperature drops below M. The particle cannot decay and it remains as abundant as photons. When H falls below Γ_D, the X particle finally decays, but its abundance its much greater than its equilibrium abundance, shown by the vertical arrow. This is the departure from thermal equilibrium.

in a B violating reaction. Their thermal equilibrium abundance with respect to (relativistic) photons can be obtained from Eqs. (2) and (19), and forgetting about numerical prefactors this gives,

$$\left(\frac{n_X}{n_\gamma}\right)_{EQ} \sim \left(\frac{m_X}{k_B T}\right)^{3/2} e^{-m_X/(k_B T)}, \tag{20}$$

and similarly for $\bar{X}$. The X and $\bar{X}$ are decaying in a B non-conserving way, but remember that since we are in thermal equilibrium, the number densities of particles and antiparticles are equal and no baryon number is created. However, the equilibrium abundance as in Eq. (20) is only maintained as long as the decay rate is greater than the expansion of the universe, $\Gamma_D \gg H$. Or because H^{-1} is also the characteristic timescale at which the temperature T is changing, we can also say that the reactions that drive the universe to thermal equilibrium have to occur on a timescale shorter than the timescale at which the temperature is changing.

Now suppose that however $\Gamma_D < H$. As long as the temperature is high, $k_B T \gg m_X$, the X and $\bar{X}$ do not want to decay and exist in equal numbers, comparable to the number density of photons $n_X = n_{\bar{X}} \simeq n_\gamma$. But due to the expansion of the universe the temperature drops until at some point $k_B T \simeq m_X$, and the particles want to decay. However, the particles cannot decay because the expansion rate of the universe is too big compared to the decay rate Γ_D. Thus the X and $\bar{X}$ remain as abundant as photons and the decay rate stays the same, whereas in thermal equilibrium their abundance would look like Eq. (20). This overabundance with respect to the equilibrium abundance is the departure from equilibrium, which is shown in Fig. 2

Let's make our discussion about the departure from thermal equilibrium more quantitative, where Fig. 3 illustrates the Hubble rate and various rates for

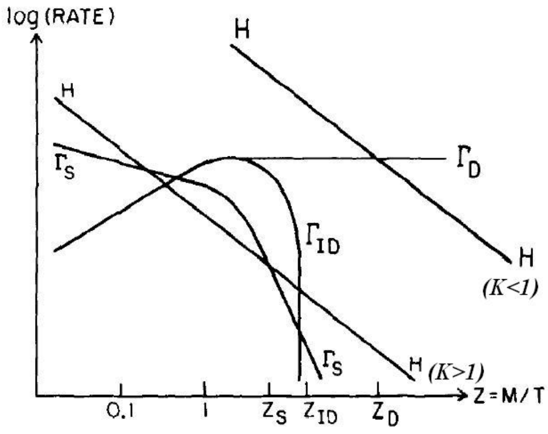

Figure 3: The log of important rates as a function of $Z = M/T$. The scattering rate Γ_S is not important in our simple model. The decay rate Γ_D scales as $1/T$ for $T \gg M$, but when $T \sim M$ it reaches a constant value. The inverse decay rate Γ_{ID} is at high temperatures equal to the decay rate, but is Boltzmann suppressed when the temperature drops below the mass scale. For the Hubble rate H two lines are shown that both scale as T^2. For the first line $K = (\Gamma_D/H)_{T=M} > 1$, i.e. as the temperature drops below the mass scale the decay rate is greater than the Hubble rate H. In that case thermal equilibrium is maintained and no baryon asymmetry is created. For the second line $K < 1$, so $\Gamma_D < H$ when the temperature drops below the mass scale, and there will be a departure from thermal equilibrium.

particle interactions, including (inverse) decays and scattering. First we take a closer look at the Hubble rate H,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\hbar\rho}{3cM_P^2}, \tag{21}$$

where $M_p^2 = \hbar c/8\pi G_N = 2.4 \times 10^{18}$GeV is the reduced Planck mass and ρ is the energy density, which for an ideal relativistic fluid is given by

$$\rho = g_* \frac{\pi^2}{30} \frac{(k_B T)^4}{(\hbar c)^3}, , \tag{22}$$

where g_* counts the effective relativistic degrees of freedom of all species in the relativistic fluid, where for each fermionic degree of freedom we need to include a factor 7/8. Combining Eqs. (21) and (22) gives us the following expression for H,

$$H = g_*^{1/2} \frac{\pi}{\sqrt{90}} \frac{(k_B T)^2}{\hbar M_p c^2} = 0.33 g_*^{1/2} \frac{(k_B T)^2}{\hbar M_p c^2}. \tag{23}$$

As the universe expands, the temperature drops and therefore the Hubble rate decreases. Thus at some point the Hubble rate again falls below the decay rate of the X particles, and the X particles will decay. At really high temperatures the decay rate is suppressed by a factor $1/T$ due to a relativistic Lorentz γ factor. When the temperature becomes comparable to the mass scale of the X particles the decay rate reaches a constant value

$$\Gamma_D = \alpha_X \frac{m_X c^2}{\hbar}. \tag{24}$$

The decay rate is proportional to the mass of the particles and a dimensionless parameter α_X that depends on the strength of the decay interactions and the number of decay channels. The particle will decay when $\Gamma_D \simeq H$, i.e. at a temperature

$$k_B T_X \simeq \sqrt{3\alpha_X m_X c^2 m_P c^2 / g_*^{1/2}}. \tag{25}$$

This temperature still needs to be below the mass scale of the particle, $m_X c^2 > k_B T$, otherwise the particle will not decay at all. This means that

$$m_X \geq 3\alpha_X M_p / g_*^{1/2}. \tag{26}$$

As an example, if the particles would decay through the weak interactions, then $\alpha_X \simeq 10^{-2}$, and taking g_* to be of the order 10^2 (in the SM $g_* = 106.75$), this means that $m_X \simeq 10^{16}$GeV. In Grand Unified Theories the unification scale is about 10^{16}GeV, so this value seems reasonable. The next section contains more information about GUTs and its application to baryogenesis.

Once the departure from equilibrium is established and the Hubble rate falls below the decay rate, the particle freely decays since the inverse decay is blocked by a Boltzmann factor $\exp[-m_X/(k_B T)]$. Every decay of a particle/antiparticle pair creates on the average a baryon number $\Delta B = (r - \bar{r})(B_1 - B_2)$. Suppose now that the number density of particles is of the same order as the photon density n_γ. Then the total baryon number created is $n_B = n_b - n_{\bar{b}} \simeq \Delta B n_\gamma$. Eq. (5) gives us the relation between n_γ and s, and omitting numerical factors this relation is $s \simeq g_* n_\gamma$, with g_* of the order of 10^2. So this means that $n_B/s = \Delta B n_\gamma / g_* n_\gamma = \Delta B / g_*$. For $n_B/s \simeq 10^{-10}$ we only need a tiny **C**/**CP** violation of 10^{-8}, which seems reasonable.

4 GUT Baryogenesis

4.1 Short introduction to grand unification

In the previous section a mechanism was sketched that could create the baryon asymmetry. The ingredients we need are a very heavy particle X that decays into baryons and antibaryons in a B-nonconserving way while also violating **C** and **CP** at the same time. In the Standard Model baryon number is pertubatively conserved, i.e. all Feynman diagrams conserve baryon ("quark") number. As we will see in section 5, baryon number is violated in the Standard Model through a non-pertubative effect. However, for a long time it was believed that baryogenesis could not occur within the SM.
In the late 1970's Grand Unification Theories (GUTs) were developed, that contained reactions that explicitly violate baryon number. Also, the energy scale of grand unification was of the order of 10^{16} GeV, roughly the mass we need for a heavy particle X in order to depart from thermal equilibrium and create the baryon asymmetry. As a consequence, the baryogenesis scenario was investigated from only the late 1970's, whereas Sakharov already proposed baryogenesis in 1967!
First a short introduction on GUTs. The renormalized couplings of the strong, weak and electromagnetic interactions are not constant but depend on the energy scale, which is known as the running of the couplings. The couplings seem to meet at the same point (once you include supersymmetry), at an energy scale of $E_{GUT} = 2 \times 10^{16}$ GeV. Thus, the strengths of all the forces were equal at some point in the early universe.
This led physicist to develop Grand Unification, the idea that the strong, weak and electromagnetic forces can be unified in a field theory with a single coupling constant. Below the GUT energy scale the theory breaks down to the familiar Standard Model (through spontaneous symmetry breaking). We already have seen this happen in nature: the electroweak theory unifies the weak and electromagnetic forces into a single force, the weak force at a temperature above ~ 100 GeV. Why not also unify the strong and electroweak force?
The Standard Model can be described by the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group, where $SU(3)$ represents the strong interaction through eight massless gluons, $SU(2)_L$ the weak interaction through the three (massless) W-bosons, and $U(1)_Y$ represents the hypercharge field B. The SM gauge group is spontaneously broken down to $SU(3)_c \times U(1)_{EM}$ at the electroweak phase transition, where three of the four gauge bosons in $SU(2)_L \times U(1)_Y$ acquire a mass (the $W^{\pm}$ and Z-bosons), and one remains massless (the photon), representing the $U(1)_{EM}$ symmetry of electromagnetism.
Suppose now we have a grand unification gauge group G that spontaneously breaks down to the SM gauge group at some high energy scale, or perhaps in multiple steps, i.e.

$$G \to ... \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{EM}. \tag{27}$$

We want to find a gauge group G that contains all of the observed particles. For the fermions we have three families, where each family has 15 members, namely two quarks (eg. u and d) that each come in three colors and can be either left- or righthanded ($2 \times 3 \times 2 = 12$), two left-handed leptons (eg. e_L^- and $(\nu_e)_L$), and one righthanded lepton (e_R^-). The observation of neutrino oscillations allows for

the extension with a righthanded neutrino.
The simplest gauge group that can accommodate all 15 particles in its representations is $SU(5)$, the group of 5×5 unitary matrices. The 15 particles cannot be accomodated in a single irreducible representation, but rather two, a 5- and 10-dimensional representation. In our discussion on GUT baryogenesis, this group will be used as an example, because it is very simple and can illustrate the principles. However, this group is more or less ruled out as GUT, because it predicts a proton lifetime of $\tau_p \sim 10^{30}$ years, whereas the experimental bound is $\tau_p \geq 10^{35}$ years [10]. An attractive option is $SO(10)$, which is not yet ruled out by proton decay. It contains a 16-dimensional irreducible representation and has $SU(5)$ as a subgroup. With the extra singlet another particle can be incorporated in the group, namely the right-handed neutrino!
GUTs have many predictions and therefore both many successes and problems. As for successes, GUTs predict charge quantization, the correct value of $\sin^2 \theta_w$ (θ_w being the Weinberg mixing angle) and small neutrino masses appear naturally through the see-saw mechanism. It also predicts certain relations between fermion masses, of which some are correct, but most are not. More importantly for our discussion on baryogenesis, it predicts baryon number violating interactions. One consequence is proton decay, a generic feature of GUT models, that has not been observed yet. Grand unification theories also predict magnetic monopoles, superheavy and extremely long-lived particles that are created at high temperatures and that we should still see today (but do not). Inflation solves the monopole problem, but it complicates the mechanism for GUT baryogenesis (more on this in the last part of this chapter). Even though the $SU(5)$ GUT model has practically been ruled out, I will still illustrate the principles of GUT baryogenesis by looking at this model, because it is a nice way to show how baryogenesis works. For more information on Grand Unification Theories and GUT baryogenesis see, for example [11] and [12].

4.2 $SU(5)$ GUT model

As mentioned before, In $SU(5)$ the 15 fermions can be accommodated in two irreducible representations. The 5-dimensional fundamental representation of one family of quarks and leptons takes the form

$$\Psi = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e^- \\ \nu \end{pmatrix}, \tag{28}$$

where the d_i represent the colored down quarks and the e^- and ν the leptons. Here it is convenient to consider all the quarks and leptons as being left-handed, whereas the antiquarks are right-handed. The Lagrangian contains gauge invariant terms

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu D_\mu \Psi + ... = i\bar{\Psi}\gamma^\mu(\partial_\mu + ig\sum_{i=1}^{24}\frac{1}{2}\lambda_i A^i_\mu)\Psi + ... \tag{29}$$

where the A^i_μ are the $5^2 - 1 = 24$ gauge bosons. These include the 8 gluons that couple the quarks, and the 3 W bosons and the B hypercharge field that couple

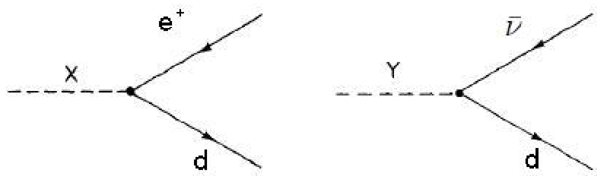

Figure 4: Examples of Feynman diagrams in $SU(5)$ involving the gauge bosons X and Y originating from the term $i\bar{\Psi}\gamma_\mu D_\mu \Psi$

the leptons. The other 12 bosons we denote as superheavy X and Y bosons (and their antipartners $\bar{X}$ and $\bar{Y}$). We can write a general 5×5-matrix for the gauge bosons

$$A \equiv \sum_{i=1}^{24} \frac{1}{2}\lambda_i A_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} & & & \bar{X}_1 & \bar{Y}_1 \\ & g & & \bar{X}_2 & \bar{Y}_2 \\ & & & \bar{X}_3 & \bar{Y}_3 \\ X_1 & X_2 & X_3 & W & W \\ Y_1 & Y_2 & Y_3 & W & W \end{pmatrix} + \frac{1}{2}\lambda_{24} B. \quad (30)$$

When using this general form in Eq. (29), we see that the X and Y bosons interact with (anti)quarks and (anti)leptons, and are therefore usually called leptoquarks. For example (see Fig. 4), a e^+ can decay to a d through an X boson, and similarly a $\bar{\nu}$ to a d through a Y boson, making it is also easy to see that X bosons have a charge $Q = \frac{4}{3}$ and Y bosons a charge $Q = \frac{1}{3}$.

Apart from the 5-dimensional fundamental irreducible representation, we have a 10-dimensional adjoint irreducible representation that contains the remaining fields. It transforms as the antisymmetric product of two 5-dimensional representations and for one family it takes the form

$$\chi_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_2 & -\bar{u}_1 & u_1 & d_1 \\ -\bar{u}_2 & 0 & \bar{u}_3 & u_2 & d_2 \\ \bar{u}_1 & -\bar{u}_3 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}. \quad (31)$$

The part of the Lagrangian containing this term takes the form

$$\begin{aligned} \mathcal{L} &= i\mathrm{Tr}(\bar{\chi}\gamma^\mu D_\mu \chi) = i\bar{\chi}_{ac}\gamma^\mu (D_\mu \chi)_{ac} = \\ &= i\bar{\chi}_{ac}\gamma^\mu [\partial_\mu + ig(\frac{1}{2}\lambda \cdot A_\mu)_{ad}\chi_{dc} + ig(\frac{1}{2}\lambda \cdot A_\mu)_{cd}\chi_{ad}] \\ &= i\mathrm{Tr}(\bar{\chi}\gamma^\mu \partial_\mu \chi) - 2g\bar{\chi}_{ac}\gamma^\mu (\frac{1}{2}\lambda \cdot A_\mu)_{ab}\chi_{bc}. \quad (32) \end{aligned}$$

Using the general form of A from Eq. (30), a number of new interactions arise, see Fig. 5. For example, a u can decay to a $\bar{u}$ through an X boson, or a d to a $\bar{u}$ through a Y boson. These processes, together with the interactions from Eq. (29), allow the decay of a the proton (uud) to a positron (e^+) and a neutral pion ($\bar{u}u$) through the exchange of an X or Y boson. We see that in such processes B is explicitly violated, since two quarks ($B = \frac{1}{3}, L = 0$) have a baryon number

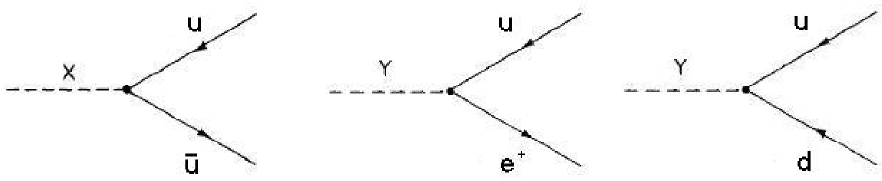

Figure 5: Examples of Feynman diagrams in $SU(5)$ involving the gauge bosons X and Y originating from the term $i\mathrm{Tr}(\bar{\chi}\gamma^{\mu}D_{\mu}\chi)$

$\frac{2}{3}$, but an antiquark ($B = -\frac{1}{3}, L = 0$)and an antilepton ($B = 0, L = -1$) have a baryon number $B = -\frac{1}{3}$. However, $B - L$ *is conserved in all the interactions of* $SU(5)$. This will be important when we discuss electroweak baryogenesis in the next chapter.

Apart from the heavy X and Y bosons, we need additional Higgs fields in order to spontaneously break the symmetry at a very high energy scale $\sim M_X$ to give a mass to the heavy X bosons. This introduces more Higgs fields of which twelve are eaten by the gauge bosons X. The Higgs particles also couple to the fermions through Yukawa couplings but for our discussion on baryogenesis it is not necessary to go into detail about this. We instead consider a simplified model and show how GUT baryogenesis works.

4.3 Simplified $SU(5)$ GUT Model

Suppose we have the following interaction Lagrangian

$$\mathcal{L}_{int} = g_1 X \bar{i}_2 i_1 + g_2 X \bar{i}_4 i_3 + g_3 Y \bar{i}_1 i_3 + g_4 Y \bar{i}_2 i_4 + \text{h.c.}.$$

X and Y are two superheavy bosons, $i_{1\to 4}$ are light fermions and $\bar{i}_a \equiv i_a^{\dagger}\gamma^0$. This interaction Lagrangian gives rise to the following decays of the X and Y boson

$$\begin{aligned} X &\to \bar{i}_1 i_2 \quad X \to \bar{i}_3 i_4 \\ Y &\to \bar{i}_3 i_1 \quad Y \to \bar{i}_4 i_2, \end{aligned} \tag{33}$$

and similarly for the antiparticles $\bar{X}$ and $\bar{Y}$. In view of the previous section, we could identify i_1 and i_2 with u, i_3 with e^- and i_4 with d, and of course X and Y with the heavy gauge bosons. The corresponding Feyman diagrams for the decay at tree level are shown in Fig. 6. The goal is to calculate the average baryon number violation produced in a decay of an X and a Y particle. Recall section 3.1 and specifically Eq. (8), and note that we can write the average baryon number produced in the decay of an X and $\bar{X}$ boson as

$$\begin{aligned} \Delta B \;=\; & \frac{1}{\Gamma_X}[(B_{i_2} - B_{i_1})(\Gamma(X \to \bar{i}_1 i_2) - \Gamma(\bar{X} \to i_1 \bar{i}_2)) \\ & + (B_{i_4} - B_{i_3})(\Gamma(X \to \bar{i}_3 i_4) - \Gamma(\bar{X} \to i_3 \bar{i}_4))], \end{aligned} \tag{34}$$

where Γ_X is the total decay rate of the X particle, which is equal to the total decay rate of the antiparticle $\Gamma_{\bar{X}}$ because of **CPT** conservation. $B_{i_{1\to 4}}$ are the

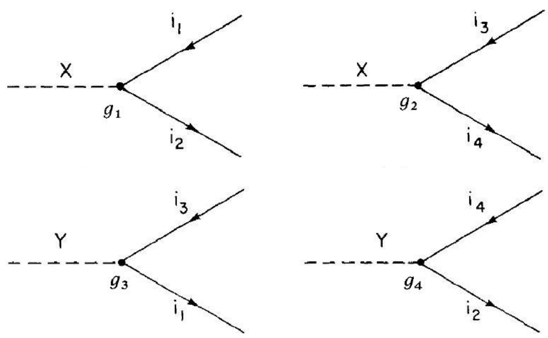

Figure 6: Tree-level Feynman diagrams for the decay of an X and Y boson in a simplified $SU(5)$ GUT model.

baryon numbers of the light particles. We now want to calculate the actual decay rates, and we will start with the tree diagrams. For the decay $X \to \bar{i}_1 i_2$ the amplitude is $\mathcal{M}_X = g_1$, and similarly for $\bar{X} \to i_1 \bar{i}_2$ the amplitude is $\mathcal{M}_{\bar{X}} = g_1^*$. The corresponding decay rates are

$$\begin{aligned} \Gamma(X \to \bar{i}_1 i_2) &= \int |\mathcal{M}_{\mathcal{X}}|^2 dPS = |g_1|^2 I_X \\ \Gamma(\bar{X} \to i_1 \bar{i}_2) &= \int |\mathcal{M}_{\bar{\mathcal{X}}}|^2 dPS = |g_1^*|^2 I_{\bar{X}} \end{aligned} \tag{35}$$

where $I_X = \int dPS$ is a Lorentz invariant phase space measure. Since the masses of the X particle and its antipartner $\bar{X}$ are equal, $I_X = I_{\bar{X}}$. This means that at tree level, $\Gamma(X \to \bar{i}_1 i_2) = \Gamma(\bar{X} \to i_1 \bar{i}_2)$ (and similarly $\Gamma(X \to \bar{i}_3 i_4) = \Gamma(\bar{X} \to i_3 \bar{i}_4)$), thus according to Eq. (34) no net baryon number is created.
Now let us look at quantum corrections to the decay process in the one-loop case. The corresponding Feynman diagrams for the X and Y boson are shown in Fig. 7. Now we can write the amplitude for the decay $X \to \bar{i}_1 i_2$ as the sum of the tree-level diagram and the one-loop diagram, i.e. $\mathcal{M}_{\mathcal{X}} = g_1 + g_2 g_3^* g_4 F_Y$. F_Y is an integral over the internal momenta,

$$\begin{aligned} F_Y &= \int \frac{d^4 q}{(2\pi)^4} \\ &\mathrm{Tr}\Big(\bar{u}(p_1) \frac{i(\not{p}_1 + \not{q}) + m_3}{(p_1+q)^2 + m_1^2 + i\epsilon} \gamma^\mu \frac{1}{q^2 + M_Y^2} \gamma^\nu \frac{i(\not{p}_2 - \not{q}) + m_4}{(p_2-q)^2 + m_2^2 + i\epsilon} v(p_2) \epsilon^\alpha(p_X)\Big), \end{aligned} \tag{36}$$

where $p_{1,2}$ and $m_{1,2}$ are the momenta and masses of the outgoing fermions and q is the internal momentum of the Y-boson and M_Y its mass. Furthermore, $\not{p} = \gamma^\mu p_\mu$, with γ^μ matrices that satisfy the Dirac algebra, i.e. $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. Also we have added a row spinor $\bar{u}(p_1)$ for the outgoing anti-fermion, a column spinor for the outgoing fermion $v(p_2)$ and a polarization vector for the incoming vector boson $\epsilon^\alpha(p_X)$. Eq. (36) can have an imaginary part, which we will see is important to create the baryon asymmetry. This imaginary part will only exist when the X, Y bosons are heavier than $m_1 + m_2$ and $m_3 + m_4$, see also [13].

Now we find for the decay rate

$$
\begin{aligned}
\Gamma(X \to \bar{i}_1 i_2) &= \int \mathcal{M}_X^* \mathcal{M}_X dPS \\
&= \int (g_1^* + g_2^* g_3 g_4^* F_Y^*)(g_1 + g_2 g_3^* g_4 F_Y) dPS \\
&= \int \Big(|g_1|^2 |F_X|^2 + |g_2|^2 |g_3|^2 |g_4|^2 |F_Y|^2 \qquad (37) \\
&\quad + g_1 g_2^* g_3 g_4^* F_Y + (g_1 g_2^* g_3 g_4^* F_Y)^* \Big) dPS \\
&= |g_1|^2 I_X + |g_2|^2 |g_3|^2 |g_4|^2 I_Y + g_1 g_2^* g_3 g_4^* I_{XY} + (g_1 g_2^* g_3 g_4^* I_{XY})^*,
\end{aligned}
$$

where $I_{XY} = \int F(Y) dPS$. Similarly we find for the decay $\bar{X} \to i_1 \bar{i}_2$

$$
\Gamma(\bar{X} \to i_1 \bar{i}_2) = |g_1^*|^2 I_X + |g_2^*|^2 |g_3^*|^2 |g_4^*|^2 I_Y + g_1^* g_2 g_3^* g_4 I_{XY} + (g_1^* g_2 g_3^* g_4 I_{XY})^*. \qquad (38)
$$

Therefore we find for the difference in decay rates

$$
\begin{aligned}
\Gamma(X \to \bar{i}_1 i_2) - \Gamma(\bar{X} \to i_1 \bar{i}_2) &= g_1 g_2^* g_3 g_4^* I_{XY} + (g_1 g_2^* g_3 g_4^* I_{XY})^* \\
&\quad - g_1^* g_2 g_3^* g_4 I_{XY} - (g_1^* g_2 g_3^* g_4 I_{XY})^* \\
&= 2 I_{XY} \mathrm{Im}(g_1 g_2^* g_3 g_4^*) - 2 I_{XY}^* \mathrm{Im}(g_1 g_2^* g_3 g_4^*) \\
&= 4 \mathrm{Im} I_{XY} \mathrm{Im}(g_1 g_2^* g_3 g_4^*). \qquad (39)
\end{aligned}
$$

By following the same steps in the calculation for $X \to \bar{i}_3 i_4$, we find that

$$
\begin{aligned}
\Gamma(X \to \bar{i}_3 i_4) - \Gamma(\bar{X} \to i_3 \bar{i}_4) &= -g_1 g_2^* g_3 g_4^* I_{XY} - (g_1 g_2^* g_3 g_4^* I_{XY})^* \\
&\quad + g_1^* g_2 g_3^* g_4 I_{XY} + (g_1^* g_2 g_3^* g_4 I_{XY})^* \\
&= -2 I_{XY} \mathrm{Im}(g_1 g_2^* g_3 g_4^*) + 2 I_{XY}^* \mathrm{Im}(g_1 g_2^* g_3 g_4^*) \\
&= -4 \mathrm{Im} I_{XY} \mathrm{Im}(g_1 g_2^* g_3 g_4^*) \\
&= -[\Gamma(X \to \bar{i}_1 i_2) - \Gamma(\bar{X} \to i_1 \bar{i}_2)]. \qquad (40)
\end{aligned}
$$

Finally, we find for the average baryon number produced in the decay of an X boson and its antipartner $\bar{X}$

$$
\Delta B_X = \frac{4 \mathrm{Im} I_{XY} \mathrm{Im}(g_1 g_2^* g_3 g_4^*)}{\Gamma_X} [(B_{i_1} - B_{i_2}) - (B_{i_3} - B_{i_4})]. \qquad (41)
$$

The alert reader might object that there is another one loop contribution for the decay of X, namely with the X boson itself in the loop. However, it can easily be verified that we would get contributions $I_{XY} g_1 g_1^* g_2 g_2^*$, thus real and we would not have any **C** violation. We could do exactly the same calculation for the decay of the Y boson, and we would find

$$
\Delta B_Y = -\frac{4 \mathrm{Im} I_{YX} \mathrm{Im}(g_1 g_2^* g_3 g_4^*)}{\Gamma_Y} [(B_{i_1} - B_{i_2}) - (B_{i_3} - B_{i_4})]. \qquad (42)
$$

Combining Eqs. (41) and (42), we find

$$
\begin{aligned}
\Delta B &= \Delta B_X + \Delta B_Y \qquad (43) \\
&= 4 \left\{ \frac{\mathrm{Im} I_{XY}}{\Gamma_X} - \frac{\mathrm{Im} I_{YX}}{\Gamma_Y} \right\} \mathrm{Im}(g_1 g_2^* g_3 g_4^*) [(B_{i_1} - B_{i_2}) - (B_{i_3} - B_{i_4})].
\end{aligned}
$$

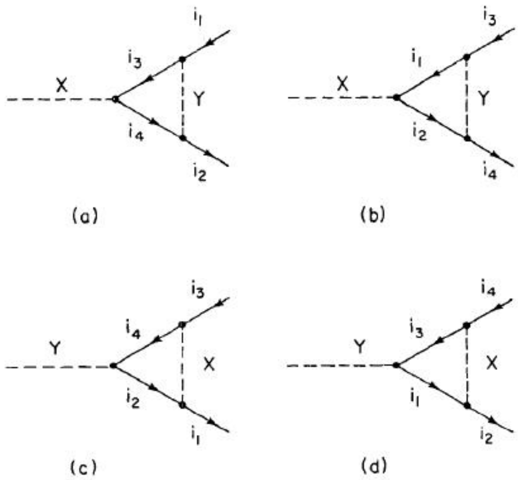

Figure 7: One-loop Feynman diagrams for the decay of an $X(Y)$ boson with a $Y(X)$ boson in the loop in a simplified $SU(5)$ GUT model.

From this equation we can see a few conditions that are necessary for baryogenesis. First of all, (some of) the couplings have to be complex. In the Standard Model it is known from the CKM-matrix that some Yukawa couplings are complex from the discovery of **C** and **CP** in the kaon system. There is therefore no reason to believe this could not happen in a theory at a much higher temperature scale. Secondly, the baryon number produced in the final states must be different for the different decay channels. If both decays (of X for example) would produce the same baryon number, we could have assigned precisely that baryon number to the X particle and there would be no B violation. As mentioned before, the masses of the X and Y bosons must be larger than the combined masses of the particles they decay to (eg. $m_1 + m_2$ and $m_3 + m_4$), because only then the phase space integrals I_{XY} and I_{YX} have an imaginary part. Finally, the X and Y bosons must not be degenerate in mass, otherwise $\Gamma_X = \Gamma_Y$ and $\text{Im}(I_{XY}) = \text{Im}(I_{YX})$ and the Y bosons would destroy as much baryon number as the X bosons create.
Coming back to GUT baryogenesis in $SU(5)$, we see that all these requirements can be met. We have already seen that the X and Y bosons decay to final states with different baryon number, so baryon number is explicitly violated. As for the complex coupling constants, there is no reason to believe that they are not complex. The X and Y bosons are estimated to have a mass of 10^{16} GeV, much larger than the quark and lepton masses. And finally, just like the W bosons, the X and Y bosons are likely to have different masses.
All in all, one might say that GUT baryogenesis might have created the baryon asymmetry of the universe. However, as already mentioned in the introduction to this section, $SU(5)$ predicts a lifetime of the proton of $\tau_p < 10^{30}$ year, whereas the experimental bound is $\tau_p > 10^{35}$ year. Other GUT models can still cope with this high lower bound on the proton lifetime, but then also inflation complicates GUT baryogenesis. If there were an initial baryon asymmetry, inflation would have diluted this number to a negligible value. As mentioned in section

3.3, for a departure from equilibrium and successful baryogenesis extremely high temperatures of $T = 10^{16}$ GeV are needed. In the 1990s it was believed that the reheating temperature never reached values higher than 10^9 GeV, because at higher temperatures magnetic monopoles can be created. This is more or less a necessity for inflationary models, since the magnetic monopole problem (the absence of magnetic monopoles) was one of the original motivations for inflation. However it was shown in [14] that very heavy particles that violate B can still be created at low temperatures through a non-perturbative effect. GUT baryogenesis is therefore still a possibility. For more on GUT bayogenesis after inflation see for example [15]. This concludes our discussion of GUT baryogenesis. In the next section we will see how baryogenesis is possible within the Standard Model itself, and that this electroweak baryogenesis puts more restrictions on GUT baryogenesis.

5 Electroweak Baryogenesis

5.1 Baryon number violation in the Standard Model

At first it was thought that the Standard Model did not satisfy all three of Sakharov's conditions. Two of the three conditions could easily be seen: **C** and **CP** violation have been discovered in the K^0-$\bar{\mathrm{K}}^0$-system in 1957 and 1964 respectively. Furthermore, the universe is expected to have often undergone phase transitions, creating a departure from thermal equilibrium and the possibility of baryogenesis. The most obvious condition for creating the baryon asymmetry from a baryon symmetric, baryon number violation, is however not so easily seen to be satisfied.
At the classical level baryon number is conserved in the Standard Model. A different way to say this is that all Feynman diagrams at tree level conserve baryon number, i.e. the number of quarks minus antiquarks stays the same. When one calculates the baryonic and leptonic current (the quark and lepton parts of the vector current $J^\mu_V = \bar{\Psi}\gamma^\mu\Psi$, defined as

$$\begin{aligned} J^\mu_B &= \bar{q}\gamma^\mu q \\ J^\mu_L &= \bar{l}\gamma^\mu l, \end{aligned} \tag{44}$$

one sees that these are conserved at the classical level, i.e. $\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = 0$. This means that there are time independent conserved charges, given by

$$\begin{aligned} \hat{B} &= \int \mathrm{d}^3\mathrm{x} J^0_B \\ \hat{L} &= \int \mathrm{d}^3\mathrm{x} J^0_L, \end{aligned} \tag{45}$$

where $J^0_{B,L} = \rho_{B,L}$. The time independence can easily be verified by acting with a time derivative on $\hat{B}$ and $\hat{L}$, and using the current conservation condition $\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = 0$. When these currents are not conserved, the charges $\hat{B}$ and $\hat{L}$ are violated, i.e. there is baryon and lepton number violation! It turns out that the baryonic and leptonic currents are not conserved when we look at quantum corrections of the Standard Model and the so-called chiral (or triangle) anomaly, corresponding to a Feynman diagram as shown in Fig. (8). First we split the general vector current in a left- and right-handed part, i.e.

$$J^\mu_V = \bar{\Psi}\gamma^\mu\Psi = \bar{\Psi}_L\gamma^\mu\Psi_L + \bar{\Psi}_R\gamma^\mu\Psi_R \tag{46}$$

When we now calculate these currents, we would find that

$$\begin{aligned} \partial_\mu\bar{\Psi}_L\gamma^\mu\Psi_L &= -c_L\frac{g^2}{16\pi^2}\mathrm{Tr}(F^{\mu\nu}\tilde{F}_{\mu\nu}) \\ \partial_\mu\bar{\Psi}_R\gamma^\mu\Psi_R &= c_R\frac{g^2}{16\pi^2}\mathrm{Tr}(F^{\mu\nu}\tilde{F}_{\mu\nu}), \end{aligned} \tag{47}$$

where $F^{\mu\nu}_a$ is the field strength tensor, which could for example be the $SU(2)$ or $U(1)$ field strength tensor, and g is the corresponding gauge coupling. $\tilde{F}^a_{\mu\nu}$ is the dual of $F^{\mu\nu}_a$, defined by $\tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}_a$. The constants c_L and c_R depend on the specific gauge group. For example the gluons couple equally to the left- and

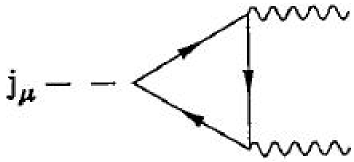

Figure 8: Feynmann diagram that is responsible for the chiral (triangle) anomaly.

right-handed baryonic current, thus $c_L = c_R$ and therefore the vector current is conserved for $SU(3)$ and there is no baryon and lepton number violation in QCD. However, in $SU(2)_L$ the W-bosons do not couple to the right-handed quarks and leptons (i.e. $c_R = 0$), and in $U(1)_Y$ the gauge boson B couples differently to left- and righthanded quarks and leptons (i.e. $c_L \neq c_R$). The final result for the baryonic and leptonic current is

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = -N_f \frac{g_w^2}{16\pi^2} \mathrm{Tr}(W^{\mu\nu}\tilde{W}_{\mu\nu}) + N_f \frac{g_1^2}{32\pi^2} B^{\mu\nu}\tilde{B}_{\mu\nu}, \tag{48}$$

where $W^{\mu\nu}$ is actually the sum of the nonabelian $SU(2)_L$ field strength tensor with the weak coupling g_w and $B^{\mu\nu}$ is the abelian $U(1)_Y$ field tensor with coupling g_1. $N_f = 3$ is the number of flavors. An important thing to notice is that although both the baryon and lepton current are not conserved, the combination $J^\mu_B - J^\mu_L$ is and therefore $B - L$ is a conserved quantity in the Standard Model. One might now think that because the right-hand side of Eq. (48) is nonzero, that baryon and lepton number are badly violated in the SM. The problem is a bit more subtle than this. The term on the right-hand side can be written as a derivative itself! Thus we can write

$$\begin{aligned}
\partial_\mu J^\mu_B &= \frac{N_f}{16\pi^2}\left[-g_w^2 \partial_\mu K^\mu + g_1^2 \partial_\mu k^\mu\right] \\
K^\mu &= \epsilon^{\mu\nu\rho\sigma} W^a_\nu \left(\partial_\rho W^a_\sigma + \frac{g_w}{3}\epsilon^{abc} W^b_\rho W^c_\sigma\right) \\
k^\mu &= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} B_\nu \partial_\rho B_\sigma,
\end{aligned} \tag{49}$$

where W_μ and B_μ are the $SU(2)_L$ and $U(1)_Y$ fields. It seems that we can simply shift the baryonic current and define a new current $\tilde{J}^\mu_B = J^\mu_B - \frac{N_f}{16\pi^2}\left[-g_w^2 K^\mu + g_1^2 k^\mu\right]$ that is conserved. For the abelian fields in k^μ this is fine, because the fields fall off quickly at infinity and its integral is zero. However, it turns out that their are physical consequences for the shift in the current for the nonabelian fields, because

$$\frac{g_w^2}{16\pi^2}\int d^3x dt \partial_\mu K^\mu = \frac{g_w^2}{16\pi^2}\left[\int d^3x K^0\right]_{t=-\infty}^{t=+\infty} = N_{CS}(\infty) - N_{CS}(-\infty) \equiv \Delta N_{CS} \tag{50}$$

where N_{CS} is the Chern-Simons number. This means that baryon number is violated by $\Delta B = N_F \Delta N_{CS} = 3\Delta N_{CS}$. The point is that the fields W^a_μ can be gauged away locally, but it cannot be gauged away throughout all space by a gauge transformation. As long as the quantum fluctuations around the local vacuum $W^a_\mu = 0$ are small, then $\Delta N_{CS} = 0$, but there exist large fields that

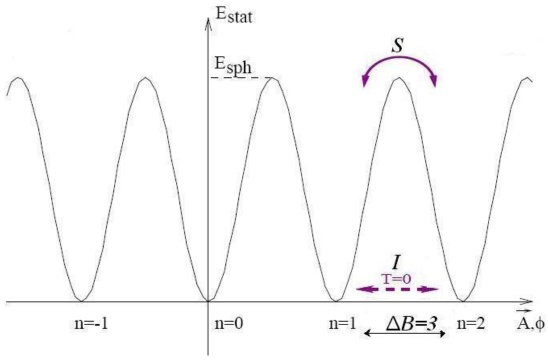

Figure 9: Yang-Mills vacuum structere for non-abelian gauge theory. There are an infinite number of topologically different vacua that differ by a Chern-Simons number. At $T = 0$ tunneling can occur from one vacuum to another via instantons, violating baryon and lepton number by 3. At finite T transitions can occur that hop over the barrier, so called sphalerons transitions, where the sphaleron energy $E_{sph} \simeq 10$ TeV.

can violate baryon number in steps of 3. We can make a picture of the so-called vacuum structure of the gauge fields, see Fig. (9). There are an infinite number of vacua, separated by some potential barrier. These barriers exist because two vacua with different N_{CS} cannot continuously be deformed into each other without generating non-vacuum gauge fields. In 1976 't Hooft[16] showed that large field configurations can tunnel through the barrier from a state $|\Psi, B>$ with a baryon number B to a state $|\Psi, B \pm 3>$with a baryon number $B \pm 3$ with a probability per unit volume

$$< \Psi, B \pm 3 \mid \Psi, B > \propto e^{-S_{inst}} = e^{-4\pi/\alpha_W} \sim 10^{-164}, \tag{51}$$

where S_{inst} is the action of these special field configurations, "instantons", and $\alpha_W = \frac{g_w^2}{4\pi} \simeq 1/29$, the weak coupling constant. This result is valid at zero temperature, and it explains why we have never seen baryon number violation in experiments, because the probability is so small that it has never happened in the lifetime of the universe. An easy way to verify this is to look at the Hubble volume $V = (c/H^3 \simeq 10^{78}$, and thus the probability that such a baryon violating process would happen in the entire universe is only $\sim 10^{-86}$. When we now look at the lifetime of the universe $\simeq 10^17$ s, it is evident that such a process has never happened.
Kuzmin, Rubakov and Shaposhnikov[17] showed in 1985 that the story changes at finite temperature. Then there are field configurations that could in principle not tunnel through the barrier, but "jump" over the barrier from one vacuum to another. These field configurations are static saddle point solutions of the classical field equations and sit on top of the barrier. They are called sphalerons, which means "ready to fall" in Greek, named by Klinkhamer and Manton[18]. To find the rate of these sphaleron processes, i.e finite T transitions between the topological different vacua which violate B and L by 3, you need to calculate fluctuations of the sphaleron around the saddle point. This rate per unit volume was calculated by Arnold and McLerran and is [19]

$$\frac{\Gamma_{sph}}{V} = c \left(\frac{E_{sph}}{k_B T}\right)^3 \left(\frac{m_W(T)}{k_B T}\right)^4 (k_B T)^4 e^{-\frac{E_{sph}}{k_B T}}, \tag{52}$$

where $m_W = g_W v/2$ is the mass of the W-boson and c is a constant. The sphaleron energy can be calculated by looking at the saddle point solution, and is

$$E_{sph} = \frac{2m_W T}{\alpha_W} B\left(\frac{m_H}{m_W}\right), \tag{53}$$

where m_H is the mass of the Higgs boson and the function B takes values between 1.56 and 2.72. So the energy of the sphaleron is approximately 10 TeV, and the rate is therefore really small when we are at finite T but below the electroweak phase transition (EWPT) at $\sim T = 100$ GeV. Above the EWPT, the Higgs boson is still in its symmetric vacuum, no symmetry breaking has occurred and the mass of the W-boson is zero. Therefore, above the EWPT, the energy of the sphaleron $E_{sph} = 0$, so the energy barriers between different vacua disappear and the rate becomes unsuppressed. From dimensional analysis it was argued that Γ_{sph}/V scales as T^4, and lattice calculations showed that it is[20]

$$\frac{\Gamma_{sph}}{V} = (25.4 \pm 2.0)\alpha_W^5 \frac{(k_B T)^4}{c^3\hbar^4} = (1.06 \pm 0.08) \times 10^{-6} \frac{(k_B T)^4}{c^3\hbar^4}. \tag{54}$$

When we now take the thermal volume $V = \frac{(\hbar c)^3}{(k_B T)^3}$ we find that

$$\Gamma_{sph} \simeq 10^{-6} \frac{k_B T}{\hbar}. \tag{55}$$

Now we want to compare this to the Hubble rate and find out when the sphalerons are in or out of thermal equilibrium. The Hubble rate H was already given in Eq. (23), and equating this to Eq. (55), we find a temperature of

$$k_B T \simeq 10^{-6} g_*^{-1/2} M_p c^2 \simeq 10^{12} \text{GeV}. \tag{56}$$

Thus, sphalerons are out of thermal equilibrium at a temperature above 10^{13} GeV, and are in a state of equilibrium below this temperature. At the EWPT, the sphaleron rate is exponentially suppressed as in Eq. (52), and quickly drops below the Hubble rate again. This situation is also depicted in Fig. (10). A number of observations can be made. First of all, any net baryon and lepton number (created by for example GUT baryogenesis) at a temperature above 10^{13} GeV is washed away by sphaleron processes when these are in thermal equilibrium. However, when we have some net $B - L$ at a high temperature, since sphaleron processes only affect B and L but not $B - L$, the $B - L$ will remain the same. This is crucial for leptogenesis[21], where a lepton number can be converted in a baryon number, and this has been discussed in another seminar[22].

5.2 Electroweak Phase Transition and CP violation in the Standard Model

As outlined above, for theories that conserve $B-L$, any baryon number created at temperatures above the EWPT, when sphalerons are in thermal equilibrium, is washed out. Therefore we now consider the situation where the baryon asymmetry is created during the EWPT (see for an early review for the principles of

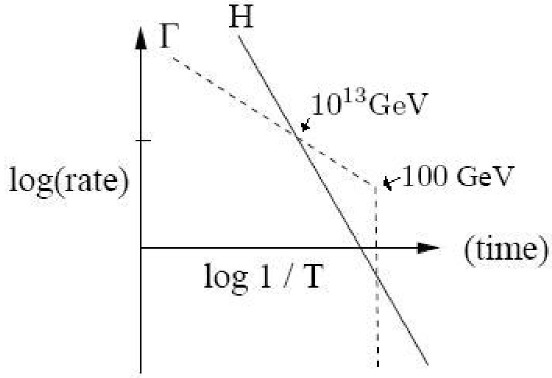

Figure 10: The log of the sphaleron rate Γ_{sph} and the Hubble rate H as a function of $1/T$. For $T > 10^{13}$ GeV sphalerons are out of thermal equilibrium, and they reach a state of thermal equilibrium at lower temperature. At the EWPT the sphaleron is exponentially suppressed and drops quickly to a negligible value.

electroweak baryogenesis[23]). An important aspect is that the baryon asymmetry created at the transition is not washed away after the transition has taken place, i.e. the sphaleron rate has to be negligible after the phase transition.
It is essential to have a strong departure from thermal equilibrium at the EWPT. This can happen when we have a sufficiently strong first order phase transition. In a first order phase transition bubbles are formed bubbles that grow and eventually fill all space. Inside the bubbles is the broken phase where the Higgs expectation value is nonzero and where we have massive W^+, W^- and Z-bosons, whereas outside the bubbles the universe still is in the symmetric phase with massless W-bosons.
For a sufficiently large Higgs vacuum expectation v the sphalerons can be out of equilibrium inside the bubbles, while they are still in thermal equilibrium outside the bubbles. In the bubble wall quantum mechanical reflection of particles takes place, and due to **CP** violating interactions there will be an asymmetry in the reflection between particles and antiparticles, thus creating the baryon asymmetry. Important is that inside the bubble wall there are no sphaleron interactions, i.e. the sphaleron rate is negligible. This happens when E_{sph} is large, and since this depends on $m_W = g_W v/2$ (see Eq. (53)), v has to be large. The quantum mechanical reflection is however very poorly understood and the calculation made is incomplete, because for example the effects of the plasma have been completely ignored.
Kainulainen et al. [24] have performed a gradient expansion to first order in $\hbar$ of the kinetic equations relevant for electroweak baryogenesis. They have derived a constraint equation for the particle density

$$(k^2 - |m|^2 + \hbar \frac{s}{\tilde{k}_0} |m|^2 \theta') g_0^s = 0, \tag{57}$$

where g_0^s corresponds to the particle density in phase space, $s = \pm 1$ is the projection of the spin of the fermions on the direction of motion of the bubble wall and the mass of the particles is $m(z) = |m| e^{i\theta(z)}$, with θ a **CP** violating phase. $k^2 = k_0^2 - \vec{k}^2$ and $\tilde{k_0} = \text{sign}[k_0] \sqrt{k_0^2 - k_\parallel^2}$, with $\vec{k}^2 = k_\parallel^2 + k_z^2$, where the z-direction is the direction of motion of the bubble wall. The constraint equation gives four solutions

$$\pm k_0 = \omega_{s\pm} = \sqrt{\vec{k}^2 + |m|^2 - \hbar \frac{s}{\tilde{k}_0} |m|^2 \theta'} \simeq \omega_0 \pm \hbar \frac{s}{2\omega_0 \tilde{\omega}_0} |m|^2 \theta', \tag{58}$$

where $\omega_0 = \sqrt{\vec{k}^2 + |m|^2}$ and $\tilde{\omega}_0 = \sqrt{\omega_0^2 - k_\parallel^2} = \sqrt{k_z^2 + |m|^2}$. Thus it is clear that particles with opposite spin have a different energy, and also particles and antiparticles have a different energy. However, particles and antiparticles with opposite spin will have the same energy. The gradient expansion to first order in $\hbar$ of the kinetic equation is

$$\partial_t f_{s\pm} + v_{s\pm} \partial_z f_{s\pm} + F_{s\pm} \partial_{k_z} f_{s\pm}, \tag{59}$$

where $f_{s\pm}$ are the distribution functions for particles (+) and antiparticles (-) with spin s and $v_{s\pm} \equiv \frac{k_z}{\omega_{s\pm}}$ is the group velocity of the particles. The **CP** violating force is then

$$F_{s\pm} = \frac{-|m|^{2\prime}}{2\omega_{s\pm}} \pm \hbar \frac{s}{2\omega_0 \tilde{\omega}_0} (|m|^2 \theta')'. \tag{60}$$

If we now draw the analogy between the separation of charged particles in an electric field by means of an opposite force acting on positively and negatively charged particles, we can see that in this case particles with spin s and antiparticles with spin $-s$ will experience the same force. On the other hand particles and antiparticles with the same spin experience an opposite force and are separated by the **CP** violating force in the bubble wall. This can create the baryon asymmetry at the EWPT. The calculation does take into account effects of the surrounding plasma, but it is only valid when the momenta of the particles are large (WKB approximation). A proper derivation of the creation of the baryon asymmetry at a first order phase transition for particles at small momenta is unfortunately still lacking.
During a second order phase transition the sphalerons also go from being in to equilibrium to out of equilibrium, but the difference is that this happens now uniformly throughout space. This means that below $T_{EWPT} \simeq 100$ GeV the sphaleron processes suddenly stop because the rate is negligible, and as a consequence the baryon and lepton numbers created before the phase transition are frozen in. But since before the EWPT the sphalerons were in thermal equilibrium (and we assume that $B - L$ is conserved), this means that any preexisting $B + L$ is washed away. Therefore, one naively expects that a second order phase transition does not create an extra baryon number, and the universe remains baryon symmetric. However, Joyce and Prokopec[25] showed that in certain non-standard cosmologies the baryon asymmetry can still be created at a second order EWPT. The baryon to entropy density calculated after freeze out is

$$\frac{n_B}{s} \sim \frac{k}{g_*} \delta_{\not{C}P} \frac{H}{T}_{\text{freeze}}, \tag{61}$$

where k is some number of order 1, $\delta_{\not{C}P}$ is a **CP**-violating parameter which is < 1 and $\frac{H}{T}_{\text{freeze}}$ is the ratio of the Hubble parameter to the temperature at the time of freeze out. In typical cosmological models this ratio is $\sim 10^{-17}$, thus

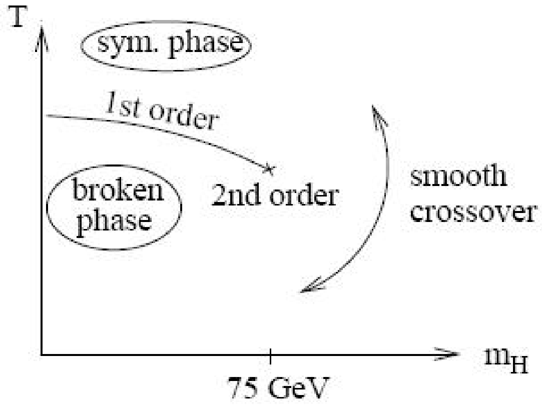

Figure 11: Phase diagram for the EWPT. For $m_H < 75$ GeV, the phase transition is first order and a baryon asymmetry can be created. For larger Higgs masses, the transition is a crossover and a baryon asymmetry is created that is too small ($\sim 10^{-20}$).

creating a baryon to entropy density ratio of $< 10^{-19}$. In certain non-standard cosmologies, such as a cosmology dominated by a kinetic term (kination era), the ratio $\frac{H}{T}_{\rm freeze}$ can be of order $\sim 10^{-7}$, thus making it still possible to get a sufficient baryon number production at a second order phase transition.
The nature of the phase transition depends on the mass of the Higgs boson. The phase diagram for the EWPT is shown in Fig. (11). For small Higgs masses $m_H < 75$ GeV the EWPT is first order, which could in principle create the baryon asymmetry. However, from experiments it is already clear that $m_H > 114$ GeV, indicating that the phase transition is a smooth crossover. This means that in typical cosmologies, the baryon asymmetry cannot be generated in the SM.
There is another problem in the electroweak baryogenesis scenario. In section 3 it was argued that we need a **C** and **CP** violation of about 10^{-8}. In the Standard Model a **CP** violating phase exists in the CKM-matrix, which originates from the Yukawa terms when one tries to diagonalize the quark masses. By field redefinitions, one can change the place where this **CP** violating phase is in the CKM-matrix. We want to find an invariant phase, and Jarlskog[26] argued that the amount of **CP** violation is of the order of 10^{-20}. This is much too small to create an $\eta \sim 10^{-10}$! However, there is still a debate going on whether or not this number is a correct estimate for the total amount of **CP** violation within the SM. For a different approach to calculate the amount of **CP** violation within the SM, see for example [27].
As a summary on electroweak baryogenesis, we have seen that baryon number violation is possible within the SM through sphaleron processes. If $B - L$ is conserved, the baryon asymmetry must be created at the electroweak phase transition, which has to be strongly first order. However, a small Higgs mass is needed for a first order transition, and experiments have already excluded this possibility. Furthermore, the amount of **CP** violation in the SM is much too small to account for the observed baryon asymmetry. In extensions of the SM, such as the MSSM (i.e. Minimal Supersymmetric Standard Model), electroweak baryogenesis is still possible. A first order phase transition can still happen for

a large Higgs mass, and new particles bring in new couplings and therefore more options for **CP** violation. Perhaps the new LHC can bring some answers by excluding or confirming supersymmetry, and hopefully we will have some results in the near future!

6 Summary and conclusions

The baryon asymmetry is one of the great mysteries of the universe. Why do we see more matter than antimatter? Initial conditions with a baryon asymmetry are excluded by the inflationary era. Therefore we need to somehow dynamically create the baryon asymmetry. Baryogenesis, proposed in 1967 by Sakharov, provides a scenario in which the asymmetry can be dynamically generated from an initially baryon symmetric universe. Only three ingredients are necessary, namely baryon number violation, **C** and **CP** violation and a departure from thermal equilibrium. There are a few scenarios in which these three conditions are satisfied.

GUT baryogenesis happens at temperatures of 10^{16} GeV, the Grand Unification scale at which the strong, weak and electromagnetic couplings are equal. Grand Unification Theories try to embed these three forces into a single theory, which naturally have baryon number violating interactions. GUTs predict proton decay, which has not been observed yet, excluding a number of these unifying theories. Furthermore, in most inflationary models the reheating temperature was never high enough to reach the GUT scale and create the baryon asymmetry. GUT baryogenesis might still be possible after inflation through non-perturbative effects, and is therefore still a possible mechanism for baryogenesis.

Baryogenesis is also possible within the Standard Model itself, because all three of Sakharov's conditions are satisfied. Baryon number violation is possible through a non-perturbative effect, so called sphaleron processes. These processes are in thermal equilibrium above the electroweak phase transition and wash out any preexisting $B + L$ asymmetry, for example an asymmetry that was created at the GUT scale. $B - L$ however is conserved through sphaleron processes. At the electroweak transition, these sphaleron processes fall out of equilibrium, and when the phase transition is strongly first order, a sufficiently large baryon asymmetry can be created. However, the Higgs mass is already too big for a first order phase transition. Another problem with electroweak baryogenesis is that the estimated total amount of **CP** violation in the SM ($\sim 10^{-20}$ is far too small to explain the baryon asymmetry $\eta \sim 10^{-10}$, although there reasons to believe that this number is not a correct estimate. Electroweak baryogenesis could still happen in extensions of the SM, such as supersymmetry, but we have to wait for results for the LHC to see what lies beyond the Standard Model.

As a final remark, there a number of interesting other mechanisms for baryogenesis, the most promising one being leptogenesis[22]. Still the mystery of the matter-antimatter asymmetry remains unsolved until now, and we can only hope for a solution in the near future.

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