Topological defects in cosmology: Cosmic Strings

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Abstract

This paper will discuss cosmic strings. Two toy models show how cosmic strings are created and a more topological viewpoint is also presented. A scaling solution will be derived and their position as possible source of galaxy formation is discussed. The connections with fundamental string theory and SUSY GUTs are also presented. We end with a discussion of their gravitational effects and possible observations.

Contents

1 Introduction

The high point of interest in cosmic strings was in the late eighties and most of the nineties as people thought they might offer an alternative to inflation as a means of generating primordial density perturbations from which clusters and galaxies grew. As I will show GUT-scale strings will give rise to geometric perturbations of the right magnitude. However in the late nineties the cosmic microwave background radiation measurements (COBE, BOOMERanG and later WMAP) contradicted the predictions made by cosmic string theory and made them very improbable as the source of primordial density fluctuations. As I will discuss later on, they might have still contributed a little, but are certainly not the main source. This, of course, caused a drop in interest in the field and the number of papers on cosmic strings dropped from 67 in 1997 to a mere 27 in 2001. This is also shown in figure 1[1]. However the interest in cosmic strings

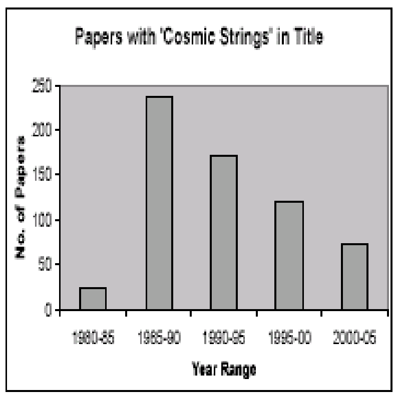

Figure 1: Number of papers with the words cosmic string in the title [1].

then started to rekindle and the number of papers on cosmic strings began to rise again up to 46 in 2003. For this there are four important reasons, two of them are theoretical, the others are observational.

The thirst theoretical reason finds its base in fundamental string theory. Originally people thought that there was no connection between cosmic strings and fundamental strings. This was mainly because the energy scales were very different, the GUT scale or less for cosmic strings and the Planck scale for fundamental strings. But this no longer holds. It turns out that the string scale may be substantially less and moreover string theory or M-theory predicts, the existence of macroscopic defects of which cosmic strings are an example [2]. The second theoretical reason is that supersymmetric GUTs seem to demand cosmic strings. An important paper on this subject was written in 2004 by Rocher et al. and I will discuss this later.

The observational reasons on the other side are not, unfortunately, made by an unambiguous discovery, but there have been tantalizing hints. These lie in

two observations, whose most natural explanation seemed to be cosmic strings. The first one constitutes an observation of possible lensing by a straight cosmic string in the system SLC-1 made in 2003 by Sazhin et al. Although closer inspection by the Hubble space telescope in 2006 showed that this was not a case of lensing, this was partly responsible for the revival of interest in cosmic strings and therefore I will also discuss this observation in detail. The second observation is maybe even more tantalizing. This consists of the observation of anomalous fluctuations in the system Q0957+561 made by Schild et al. in 2004. They show that the most obvious explanation for this would be a cosmic string loop but also state that much more work is still needed.
In this paper I will only discuss cosmic strings, other topological defects such as domain walls or monopoles I will not discuss. In the first section I will describe how cosmic strings are formed by introducing two toy models. Then I will derive how they evolve in the universe and show that cosmic strings have a scaling solution. Next I will talk about why they were thought to generate primordial density perturbations and which evidence there is against this. Then I will continue with a discussion of the connection between fundamental and cosmic strings and how they appear in supersymmetric GUT and then proceed with a discussion about there gravitational effects mainly focussed on the observations of cosmic strings. Here I will also mention what observational bounds have been derived for cosmic strings. I will finish with a discussion about the possible observations I mentioned above. Lastly it is important to mention that throughout the entire paper I will work in units where $c = 1$.

2 Formation of cosmic strings

To see how cosmic strings are formed I will start with introducing two toy models, the first one leading to global and the second one to local strings and I will end this section with a topological viewpoint on cosmic strings.

2.1 Global strings

We start with a complex scalar field $\phi(x)$ and with the well known Mexican hat potential given by the formula

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi), \qquad V = \frac{1}{2}\lambda(|\phi|^2 - \frac{1}{2}\eta^2)^2, \tag{1}$$

where λ and η are constants and $\mathcal{L}$ denotes the Lagrangian density. We see that this potential has a global U(1) symmetry under transformations of the form $\phi \to \phi \mathrm{e}^{i\alpha}$. From this potential we can easily derive that the Euler-Lagrange equations become

$$[\partial^2 + \lambda(|\phi|^2 - \frac{1}{2}\eta^2)]\phi = 0. \tag{2}$$

Solving this equation, we find that a ground state given by $\phi = (\eta/\sqrt{2})e^{i\alpha_0}$ breaks the U(1) symmetry. Note that this ground state is degenerate. As we know this will give a mass to the scalar particle equal to $m_s^2 = \lambda\eta^2$. Furthermore there is a massless Nambu-Goldstone boson that is associated with the broken symmetry. However equation 2 has also a more interesting solution. This is a static solution with non-zero energy density. To find this solution, called a vortex, we start with a cylindrical symmetrical ansatz for the field ϕ

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s\rho)\mathrm{e}^{in\psi}, \tag{3}$$

where ρ, ψ, z are the usual cylindrical coordinates and n is an integer known as the winding number. The winding number denotes the total number of times the field ϕ goes around the C_1-shaped ground state if we take its values along a closed loop. The winding number is positive for loops clockwise and negative for counterclockwise and we will see strings if it is non-zero. This will become more clear when we discuss their formation from a topological viewpoint later on. The ansatz made in equation 3 is logical to make as we want to see how this Lagrangian allows for the formation of cosmic strings and we know that they have the same symmetry. If we insert this into equation 2 we find that this reduces to a non-linear ordinary differential equation for f

$$f'' + \frac{1}{\xi}f' - \frac{n^2}{\xi^2}f - \frac{1}{2}(f^2 - 1)f = 0, \tag{4}$$

where I have introduced $\xi = m_s\rho$ for notational convenience. From this equations we can derive the behavior of f for small and large ξ. First we see that continuity of ϕ requires that $f \to 0$ as $\xi \to 0$. We need continuity (and even differentiability) of f because we need the same conditions on ϕ in order for the Lagrangian density given by equation 1 to be well defined. For large ξ we see that we need $f \to 1$, so that the field returns to its ground state. This is necessary as it would otherwise contain excess energy and this would mean the total energy blows up. Therefore for large ξ it makes sense to write $f = 1 - \delta f$ and if we insert this into equation 4 we see that $\delta f \sim n^2/\xi^2$. We know that the energy density is given by the hamiltonian density and therefore becomes

$$\mathcal{E} = |\dot{\phi}|^2 + |\nabla\phi|^2 + V(\phi). \tag{5}$$

And here we encounter a problem because, although the energy density is well localized near the origin, we see that it goes as ξ^{-2} for large ξ as a result of the second term in equation 5. From this we see that the energy per unit length becomes infinite which poses a problem and therefore these solutions, known as global strings or vortices, are usually not considered to be topologically stable. We call these solutions global strings as they come from the breaking of a global symmetry. In the next part we will see what happens when the symmetry is local.

2.2 Local strings

In principal the idea for local strings is the same as for global strings, but now we only change the U(1) symmetry from a global to a local symmetry. We know that the Lagrangian density then becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi), \tag{6}$$

where $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Again we start by looking at the field equations and now they become

$$\begin{aligned} [D^2 + \lambda(|\phi|^2 - \frac{1}{2}\eta^2)]\phi &= 0, & (7)\\ \partial_\nu F^{\mu\nu} + ie(\phi^* D^\mu\phi - D^\mu\phi^*\phi) &= 0. & (8) \end{aligned}$$

We see that we still have a ground state with $\phi = (\eta/\sqrt{2})e^{i\alpha_0}$ which brakes the symmetry. Therefore we still have the Higgs with mass $m_s = \sqrt{\lambda}\eta$, but this time the Nambu-Goldstone boson is incorporated into the vector field which gains a mass $m_v = e\eta$. We can now again try to derive the vortex solution. To do this we work in the radial gauge which means that $A_\rho = 0$ and make a cylindrical symmetrical ansatz for the fields as

$$\phi = \frac{\eta}{\sqrt{2}}f(m_s\rho)\mathrm{e}^{in\psi}, \qquad A^i = \frac{n}{e\rho}\hat{\psi}^i a(m_v\rho), \tag{9}$$

where all symbols are the same as before. If we insert this in equations 7 and 8 we find a system of coupled ODEs which do not have a known solution. However we can again look at there behavior for large ξ and we find that

$$f \sim 1 - f_1\xi^{-1/2}\exp(-\beta\xi) \qquad a \sim 1 - a_1\xi^{1/2}\exp(-\xi), \tag{10}$$

where $\beta = m_s/m_v$. We see directly that δf decreases much faster than before for large ξ. Therefore we see that the energy per unit length now stays finite and becomes

$$\mu = \int \rho\mathrm{d}\rho\mathrm{d}\phi\mathcal{E}(\rho) = \pi\eta^2\epsilon(\beta), \tag{11}$$

where $\epsilon(\beta)$ is an unknown function but it has been shown analytically that $\epsilon(1) = 1$ [5]. As these vortices have a finite energy per unit length, they are more interesting from a cosmological point of view and are called local cosmic strings. From now on, when I talk about cosmic strings I will implicitly assume them to be local. There are also cases when the broken symmetry is a mixture of a local and a global symmetry but I will not go into this here.

2.3 Cosmic strings from a topological point of view

In the previous subsection we have seen how cosmic strings are formed when a U(1) symmetry is broken. However cosmic strings can also appear when other, more complicated symmetries are broken. Assume that we have a Lagrangian invariant under some Lie group G. We know denote the vacuum manifold (in the U(1) example this was just the circle, S^1) by $\mathcal{M}$. We define H as the little group of any element $\phi_0 \in \mathcal{M}$. The little group is a Lie group that consists of those elements $h \in G$ for which $h\phi_0 = \phi_0$, so we see that this corresponds to the sub-symmetry of G that is still conserved on the vacuum manifold and we know from group theory that this is independent of the choice of ϕ_0. We know that we now get that

$$\mathcal{M} = G/H. \tag{12}$$

It turns out that cosmic strings are formed if and only if $\pi_1(\mathcal{M})$ is non-trivial. From group theory and equation 12 we see that this corresponds to $\pi_0(H)$ being non-trivial. Here π_0 is the zeroth homotopy group and this is non-trivial when the manifold H is not path connected. Similarly π_1 is the first homotopy group and this is non-trivial when there are closed paths on the manifold that can not be continuously deformed into a point. The easiest example of a manifold with non-trivial π_1 is S^1, the circle. For this manifold I will first derive the homotopy group and then give a topological argument why cosmic strings have to be formed when $\mathcal{M} = S^1$. It is easy to see that a path on the circle can only be continuously deformed into another path on the circle with the same winding number n. Here the winding number is the total number of times the path goes around the circle. Usually one direction is identified with the positive values of n and the other direction with the negative values. We see that n can therefore assume all values in the integers and we find $\pi_1(S^1) = \mathbf{Z}$. To see why this would lead to cosmic string formation, first assume we are in only two spacial dimensions. During the phase transition that breaks the symmetry, every point in the universe has to select a point on the vacuum manifold $\mathcal{M} = S^1$ and as this happens simultaneously throughout the universe it is logical to assume that all values of $\mathcal{M}$ appear. Now assume we have a closed path such that along this path ϕ assumes all those values. We now look at the values of ϕ inside this path and we try to fill the entire region by continuously deforming ϕ. However when we do this, we see that there is at least one point inside the path where the value of ϕ will have to leave the vacuum manifold in order to contain continuity. It is not hard to see that the value of ϕ will have to go to 0 in this point. This point therefore represents trapped energy as its field value is not minimal and corresponds to the vortex solution of the previous subsection. If we know look in 3 dimensions we see that this point becomes a string of points, a cosmic string. From the topology it is also apparent that cosmic strings cannot just end, as this would create similar topological problems. Therefore cosmic strings either form loops or go on forever. There are some theories in which cosmic strings can end in magnetic monopoles but I will not consider those here.

3 Deriving a scaling solution

As all topological defects, if they were ever created, were created in the early universe, we have to examine how they evolve in our expanding universe. For this we will mainly look at the energy density ratio Ω_{TD} of the topological defect, which is defined as the total energy contained in cosmic strings divided by the

total energy of the universe. Obviously this can either increase, decrease or stay constant over time. If Ω_{TD} increases it would have dominated the energy density at this time and the defect would already have been observed. If it however decreases over time all the topological defects would have been spread out so far that we can neither hope to observe them any time in the near future nor can they be used in any theory. Therefore we need topological defects that have a constant energy density over time. We call this a scaling solution. In this section I will derive a scaling solution for cosmic strings.
The important thing with cosmic strings is that they can interact when they meet. The way of this interaction is very complicated and depends on the exact form of the vacuum manifold. The case for $\pi_1(M) = \mathbf{Z}$ is very well studied and it turns out that the strings almost always inter-commute. This means that the two strings exchange partners as is shown in figure 2[6]. We see that the two strings form two new strings, one consisting of the top parts of both strings and one consisting of the bottom parts of both strings. During this inter-commuting

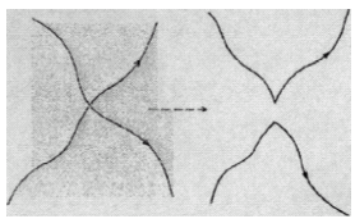

Figure 2: The process of intercommuting for strings with $\pi_1(M) = \mathbf{Z}$ [6].

a kink is formed on both strings, but this is gradually smoothed out when the universe expands. If $\pi_1(M) \neq \mathbf{Z}$, especially if it is not Abelian, the situation can be much more complicated and for instance a third string connecting the other two can be formed. This is shown in figure 3 [6]. However in order to derive the

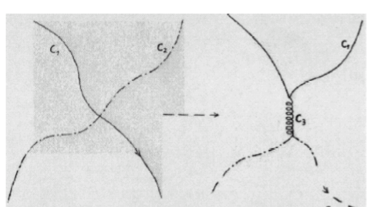

Figure 3: The process of intercommuting for strings with $\pi_1(M) \neq \mathbf{Z}$ [6].

scaling solution I will only look at the simplest case, so $\pi_1(M) = \mathbf{Z}$. The process of intercommuting is very important as it is the main energy loss mechanism of the cosmic strings, because if a string intercommutes with itself a small loop will be created, and unless this loop reconnects with a string (which is unlikely

for a small loop), it will disappear by sending out gravitational radiation.
We see that the string tension and the mass per unit length of the string are equal because of Lorentz invariance under boosts along the direction of the string as shown in [3]. This tension μ is of order $\mu \sim \eta^2$ as we have seen before. Now we can follow the calculation by Durrer [6]. First of all a number of numerical simulations ([7], [8] and [9]) has shown that about 90% of the string network consists of infinite strings, the rest are loops which decay. Therefore we focus on infinite strings. We take $v(t)$ to be the typical number of segments of infinite string inside one horizon volume at time t. Note that this volume is then equal to t^3 and that from here on I assume that $H \sim 1/t$. This then leads to the energy density of infinite strings

$$\rho_\infty \sim \frac{v(t)\mu t}{t^3}. \tag{13}$$

As strings move at relativistic speeds they experience an average $v - 1$ intersections per time t. The total number of intersections per time t per volume t^3 then becomes $\sim v(v-1)/t^4$. From this we can calculate the number of loops chopped off per time t and per volume t^3 and this becomes

$$\frac{dn}{dt} \sim pv(v-1)/t^4, \tag{14}$$

where p denotes the probability of loop creation per crossing. We estimate that the chopped off loop is of order t. This we can use to calculate the energy loss from the system per unit time

$$\frac{d}{dt}(\rho_\infty a^3) = -\mu t \frac{dn}{dt} a^3 \sim a^3 \mu p v(v-1)/t^3. \tag{15}$$

Here a is the scale factor and for the rest of the calculation I will assume we are in radiation era, so $a \propto t^{1/2}$. However the calculation goes similarly in matter era and the final result only differs in a factor. The factor a^3 is added in equation 15 to keep the calculation as clear as possible. If we now insert equation 13 into equation 15, we get for the left hand side

$$\frac{d}{dt}(\rho_\infty a^3) = \frac{d}{dt}\left(\frac{v(t)\mu a^3}{t^2}\right) = \frac{\mu a^3}{t^2}\left(\dot{v} + \frac{3}{2t}v - \frac{2}{t}v\right), \tag{16}$$

and if we insert this back in 15 we get

$$\dot{v} - \frac{1}{2t}v = -\frac{p}{t}v(v-1). \tag{17}$$

We can further rewrite this into

$$\dot{v} = \frac{v}{2}(1/2 - p(v-1)). \tag{18}$$

We see that we now have a differential equation for v. It has a two equilibria $v = 0$ and $v = 1 + 1/(2p)$. To see how strings behave we have to determine

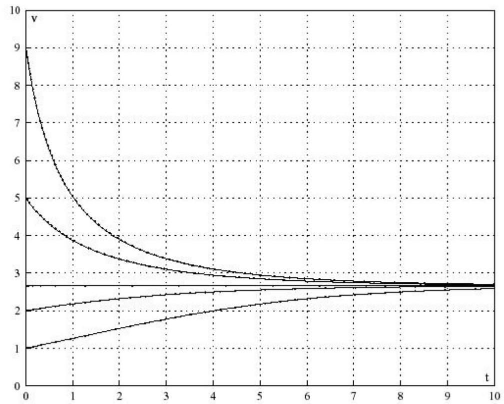

Figure 4: Solutions to the differential equation 18 made in the program CONTENT. As this is a general inquiry into the behavior of the solutions, note that we do not have units on the axis and this is not needed either, as the behavior is independent of the units or the scale.

which of these two equilibria is stable and which is unstable. We first look at the case for $p = 0.3$. The situation is shown in figure 4 and we see that the top equilibrium, $v = 1 + 1/(2p)$, is the stable one. From bifurcation theory we know that the stability of the equilibria of this kind of equations can only change when the two equilibria meet. We see that this will only happen at $p = -1/2$ and therefore for p between 0 and 1 we see that the top equilibrium is stable. So we see that v will always move towards $v = 1 + 1/(2p)$ and therefore the energy density in cosmic strings will move towards

$$\Omega_\infty = \frac{\rho_\infty}{\rho} = \frac{8\pi G}{3H^2}\rho_\infty \sim G\mu, \tag{19}$$

where we have used that $1/H \sim t$. From this we see that strings constitute a constant faction of the energy distribution in the universe, although the numerical coefficient in the final formula does change when we pass from radiation to matter era. Therefore ones strings are formed they do not die out nor come to dominate the universe. So they could be created at the end of inflation and still be observable today.

4 Cosmic strings as source of geometric perturbations

As said in the introduction, cosmic strings were once considered to be a possible source of geometric perturbations from which eventually galaxies and clusters formed. First I will explain why they were considered and next I will discuss

the evidence against this point of view.
As strings represent a lot of trapped energy it is not hard to see that cosmic strings moving around in the early universe would perturb the matter distribution. From the previous calculations we can see that the size of these perturbations are

$$\frac{\delta\rho}{\rho} \sim \Omega_\infty \sim G\mu, \tag{20}$$

where ρ is the matter density in the early universe. If we assume the cosmic strings were created around the GUT scale then these perturbations are of order 10^{-6} or 10^{-7} and this is just in the right order of magnitude to seed galaxy formation. This is the reason they were once considered to be able to seed galaxy formation and cosmic strings were so popular during the nineties. During this period accurate predictions were made, using a lot of numerical work, predicting the large-scale density perturbations explored by galactic distribution surveys and the temperature inhomogeneities in the cosmic microwave background. However, although the predictions were in the right ballpark, it turned out to be very difficult to get them to fit precisely, especially to both the large scale structure and the CMB simultaneously. The particular problem lay in explaining the anisotropies in the CMB observed first by COBE and later by WMAP. The cosmic string scenario turned out to have no explanation for the peaks in the angular power spectrum, the so-called acoustic peaks, as can be clearly seen from the numerical simulations in figure 5. On the other hand the inflation theory, which rivalled with cosmic strings to explain the density perturbations and in which the origin of the perturbations can be traced back to quantum fluctuations during an inflationary period, managed to predict these peaks exactly. However this does not mean that cosmic strings do not exist. It only means that they do not play an important role in seeding the creation of galaxies. But as we can see in figure 5 [10] cosmic strings can be fitted to the CMB data as long as they do not contribute more than approximately 10% of the density perturbations. As a result many people lost there interest in the the theory and the number of papers on the topic dropped as we saw in figure 1 and a lot of people thought that the theory was doomed. However it has made a remarkable comeback and now I will discuss why this has happend.

5 Connection with fundamental string theory

5.1 Fundamental superstring theory

As I said in the introduction, the revival of the interest in cosmic strings was partly due to a theoretical advance in the field of fundamental string theory. I will briefly describe how cosmic strings fit in superstring theory or its modern incarnation M-theory. For this I will mainly use the review of Davis and Kibble [4].
As we know one of the reasons for developing fundamental string theory came

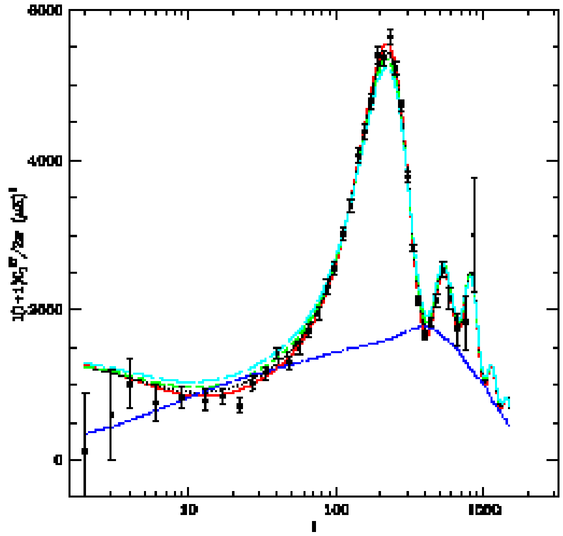

Figure 5: Angular power spectrum of CMB from WMAP. We take B to be the fraction of the power due to cosmic strings. Then the solid red line corresponds to $B = 0$, the dotted black line to $B = 0.05$, the short-dash green line to $B = 0.1$, the long-dash light blue line $B = 0.15$ and the dot-dash dark blue line to $B = 1$ [10].

from an attempt to unify gravity with the other interactions. Although this has long been considered to be the holy grail of theoretical physics, it turned out that major obstacles were formed by the appearance of infinities. In contradiction with other quantum field theories, in this case the infinities can not be renormalized and because of this we can not extract finite answers to physical questions. It turned out that no quantum version of the theory of general relativity is renormalizable. The cause of these infinities lies in the fact that we are dealing with point particles. Even in classical electromagnetic theory the self-energy of a charged particle contains infinities: the potential energy stored in a spherical distribution of charge approaches infinity as the radius tends to zero. This then led to the proposal of string theory: the particles of our theory should not be viewed as point particles, but as extended objects. The idea was that all elementary particles, like electrons, quarks, photons, etc., could be regarded as different oscillation states of a fundamental string. However we were still not able to eliminate all infinities, for this we would need to add some special features to the theory. The first of these is supersymmetry (SUSY). This remarkable feature relates bosons to fermions and in a perfect SUSY world every boson would have a fermion partner of equal mass and vice versa. However as these supersymmetric partners have never been observed, the idea is that also this symmetry must be broken. The question is then how does adding supersymmetry to our string theory help us get rid of the infinities. This is because in

a lot of cases bosons and fermions make equal but opposite contributions. This then means that if they have exact partners they would cancel and eliminate some of the infinities we are dealing with. However we were not quite there yet. We were now able to construct some superstring theories that were free of the infinities that plagued us before, but this would only happen in 10 dimensions. This led to the idea that our universe consists of 10 dimensions (9 space and 1 time), but 6 of those are curled up very small. On a macroscopic scale it would then look like our usual four-dimensional space.
Originally people did not see any connections between fundamental and cosmic strings and they were considered to be very different for multiple reasons. The first and major reason for this was the difference in energy scales, to which I already referred in the introduction. Gravity would become as strong as the other interactions around the Planck scale of around 10^{19}GeV, which is much larger than the commonly used GUT scale of $10^{15}-10^{16}$GeV. In the units I used before this would mean that for a fundamental string $G\mu \sim 1$. A second reason why people thought they very different was the length scale, it was thought that, if you would expand a fundamental string to the same size as a cosmic string, it would break into several smaller pieces.
However there have been important changes in the theory over the last years. This is mainly due to the introduction of the brane-world idea. This encompasses a method of reducing the dimension from 10 to 4 in which strings are not the only localized objects, but there also exist p-dimensional objects, known as p-branes. In this theory a 0-brane would be a particle, a 1-brane would be a string and so on. This then led to D-branes where the D denotes that we are working with Dirichlet boundary conditions and this in essence means that in addition to the closed loops that were already used in fundamental string theory, there may also be open strings whose ends are tied to D-branes. This should then be interpreted in the following way: the closed string loops still give rise to gravity, but the open strings give rise to the matter fields. It then follows that matter can be trapped on a D-brane whereas gravity can feel all the extra dimensions. In a more general way of speaking this means that matter is confined to a hypersurface (called a brane) which is embedded in a higher-dimensional space (called the bulk). This would then lead to weaker constraints on the extra dimensions, as the known particles only propagate in the usual three space dimensions and the other dimensions are only used by gravity and possibly some exotic particles, which have not yet been observed. From brain theory also the notion of cosmic superstring has arisen, which would be a comparable object to regular cosmic string. In the next part I will describe how they arise in brane theory.

5.2 Cosmic superstrings

One of the ideas in the brane-world scenario was the introduction of "warped" space-time. This would mean that the usual metric as we know it from general relativity $ds^2 = dt^2 - d\mathbf{x}^2$ no longer holds, but in stead this space-time is warped to encompass the extra dimensions. It then becomes

$$ds^2 = \mathrm{e}^{-A(\mathbf{y})}(dt^2 - d\mathbf{x}^2) - d\mathbf{y}^2, \tag{21}$$

where $\mathbf{y}$ denotes the coordinates in the extra dimensions and $A(\mathbf{y})$ is called the warp factor and should be a known, positive function. In essence this encompasses a red-shift in the compact dimensions. This then allows the creation of a hierarchy of scales in some brane theories, such that gravity, which also propagates in the bulk, can be on the Planck scale, whilst the physics that is confined to the brane can have a much lower energy scale. Now a lot depends on the behavior of the warp factor. There are some theories in which the compact dimensions form a simple space like a sphere or a torus and which have an (almost) constant warp factor. However there are also solutions in which the warp factor can vary strongly when we move through the compact dimensions, possibly with special regions known as throats where its value decreases rapidly. This is shown in figure 6 [4]. In the central region the warp factor would be

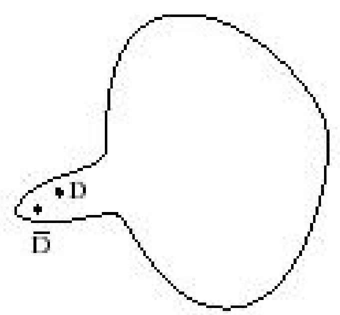

Figure 6: Space with a throat. In the middle we have compactified space with the throat on the left, where the brane, anti-brane pair lies [4].

around 1, but in the throat it would be much less than 1. There is another effect of the warped space and this is that for a four-dimensional observer like us the fundamental string mass per unit length would appear to be much less:

$$\mu = \mathrm{e}^{-A(y)}\mu_0, \tag{22}$$

where μ_0 is the ten-dimensional scale. So fundamental strings might not even have such high energies as we originally expected, even if μ_0 is at the Planck scale. So from this we see that the main objection against the similarity of cosmic and fundamental string theory no longer holds, as the difference in energy scales is much smaller than was originally thought.
As we have seen before the idea of inflation has been widely accepted. As string theory is meant to be a theory of everything, it should also describe the early universe and therefore inflation. This has then recently led to the development of brane inflation. This goes as follows: the universe contains

an extra brane and anti-brane pair (with opposite charge), which attract each other in the same way as normal charged particles. This pair is separated in the compact dimensions and the distance between them plays the role of the inflaton. Inflation is then driven by the potential energy stored in the pair. Once they meet, the pair annihilates into lower dimensional branes and it is shown that D-strings are generically formed [4]. Multiple models of this exist, but the most fully developed consists of a D3-brane, anti-brane pair placed at the bottom of the throat. These branes wrap the usual 3 dimensional space so they appear as points in the throat of the extra dimensions as shown in figure 6. The whole process of inflation and subsequent annihilation followed by the creation of lower dimensional D-branes takes place in the throat. It is proven that D1-branes (or D-strings) are formed. However in addition also fundamental strings, called F-strings can be formed. Note that although they are created in the same process D- and F-strings are very different. D-strings are similar to the cosmic strings discussed before, but F-strings are quantum mechanical objects. As both these strings are created in the throat and we know that the space there is heavily warped the energy scale of these strings can be much less than the Planck scale. The estimates for this depend on the details of the theory but vary between $10^{-11} < G\mu < 10^{-6}$. Both the D- and F-strings discussed here are known as cosmic superstrings. This idea is not new (it was first proposed in 1985 by E. Witten) but they were always dismissed because they were at a too high energy scale and unstable. However as we have seen the problem of the energy scale is solved by introducing a warped space and furthermore the throat might provide a stabilizing potential. Therefore it is now believed that cosmic superstrings might be the best way of observing actual string theory.

5.3 Cosmology of D- and F-strings

D- and F-strings behave differently from the strings described before. Especially the intercommuting is very different. When two D- or F-strings meet the probability of intercommuting is not 1 as by normal cosmic strings but is a lot less. For D-strings this is because they can miss each other in the compact dimension and F-strings are quantum mechanical so their scattering should be calculated quantum mechanically. It has been estimated that the probability for intercommuting lies between $10^{-3} < P < 1$ for F-strings and $10^{-1} < P < 1$ for D-strings. This would obviously also effect the chance of self-intersecting and therefore the string network might look quite differently. It is suggested that such a network would be denser and slower and it is likely that this would lead to an increase in the number of string loops. Furthermore normal cosmic strings can only emit the standard particles, but D- and F-strings can also emit exotic particles as a result of the underlying superstring theory.

There might be another problem arising. As D- and F-strings are different strings, they might not be able to intercommute but instead form a three-string junction with a composite DF-string, as shown in figure 7 [4]. This might be a very serious problem as this would hinder loops to form effectively. As we have seen loops play an important role in the derivation of the scaling solution

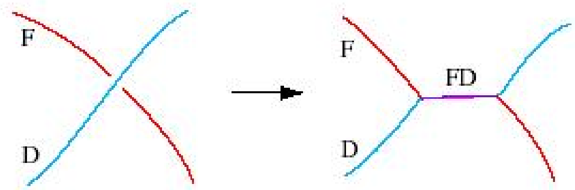

Figure 7: Possibility of intercommuting for D- and F-strings [4].

and the absence of them might therefore pose serious difficulties. There is an other possibility, namely it might be possible to have bound states of p F-strings and q D-strings. Such a network would evolve very differently and might stay frozen in time, which means that it just stretches with the expansion of the universe. As we see there are many interesting possibilities and therefore much more research is needed in this area.

6 Genericity of cosmic strings in SUSY GUT

In the previous section we saw the advances in fundamental string theory which let to a rekindle of interest in cosmic string theory. As I said in the introduction there was another theoretical reason for the renewed interest. This is found in advances made in the field of Supersymmetric Grand Unification Theories (SUSY GUTs). As we know this theory consists of a larger symmetry group which is broken down to the symmetry group of the standard model (G_{SM}) either directly or in multiple steps somewhere around the GUT scale of about 10^{16}GeV. I will not go to much into detail about all possible GUTs that exist, but here I mainly want to discuss an interesting paper by Rocher et al. from 2004 [11]. In this paper they made a number of assumptions on the way Grand Unification works and then examined all the possible groups. The first assumption they made was that there were no monopoles formed after inflation, which seems to be a good assumption as this prevents monopoles from overclosing the universe (the monopole problem). The second assumption they made however is a lot weaker and could very well be wrong. They assume that the inflaton field, that drives inflation, is included in the model as a pair of Higgs fields. However there are a lot of other possibilities for how inflation happens, for instance the brane inflation model we have seen before. Their third assumption is that baryogenesis occurs through leptogenesis, which means that the $U(1)_{B-L}$ symmetry has to be broken at the end of inflation. Although this is one of the leading theories to explain baryogenesis, there are others and therefore this assumption has to be treated carefully as well. There final assumption is that the R-parity is either contained in $U(1)_{B-L}$ or the standard model group gets expanded to $G_{SM} \times Z_2$. As this assumption is again quite standard, it should be okay as it covers the two most promising possibilities. Next they considered all groups with ranks between 4 and 8 (4 being the rank of G_{SM}) and looked at all possible symmetry breaking schemes. The considered groups included the groups $SU(5)$, $SO(10)$ and E_6, which are often mentioned as a possible Grand Unification group. The number of possible schemes considered was very large (E_6 all ready had over 1200 different schemes), but what they found was very interesting. It turned out that every symmetry breaking scheme satisfying their 4 assumptions will create cosmic strings at the end of inflation. Therefore this could be a very good indication that cosmic strings might exist, although this, as said before, will come down to the correctness of especially the second and the third assumption. Therefore more work in this field is needed to see if the assumptions made are correct. On the other hand, if we could increase the observational bounds, that I will give later, to rule out that cosmic strings are created at the GUT scale this might tell us something about how either baryogenesis or inflation works.

7 Gravitational effects of cosmic strings

In the previous section we have seen two theoretical reasons for the revival of interest in cosmic strings. In this section I will describe the gravitational effects of cosmic strings and from this we can deduce methods of detecting them. Then in the next section I will discuss recent claims of observations of cosmic strings based on these methods, which are the observational reasons for the revival of interest.

We will first look at the space-time around the string. Here we will use that the energy per unit length and the string tension are equal as follows from Lorentz invariance along the string. As a result the ordinary gravitational attraction towards the string will vanish as we will see shortly. I will do the derivation in Minkowski space as this is the only space in which we can find the solution. We start with a straight string along the z-axis and find for the string stress tensor

$$T^{\mu\nu} = \mu \text{diag}(1, 0, 0, -1)\delta(x)\delta(y). \tag{23}$$

If we introduce $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ we find that the Linearized Einstein equations become

$$\partial^2 h_{\mu\nu} = -16\pi G(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T) \tag{24}$$

and we see that this has the solution

$$h_{\mu\nu} = 8G\mu \ln(\rho/\rho_0)\text{diag}(0, 1, 1, 0), \tag{25}$$

where $\rho = \sqrt{x^2 + y^2}$ and ρ_0 is an arbitrary radial length scale. We see that this solution blows up for large ρ so it cannot be a viable solution. However if we introduce the coordinate

$$[1 - 8\pi G\mu \ln(\rho/\rho_0)]\rho^2 = (1 - 4G\mu)^2 R^2, \tag{26}$$

we see that solution transforms into a solution of the Einstein equations (which can be easily verified by plugging it in). This leads to the metric

$$ds^2 = dt^2 - dz^2 - dR^2 - (1 - 4G\mu)^2 R^2 d\psi^2 \tag{27}$$

where we have assumed that $G\mu \ll 1$ and worked up to second order in $G\mu$, which seems a reasonable assumption based on the bounds I will give later. We can show that this space is locally flat, but globally curved by doing a coordinate transformation to a new angle coordinate $\bar{\psi} = (1 - 4G\mu)\psi$. In this coordinate the metric becomes

$$ds^2 = dt^2 - dz^2 - dR^2 - R^2 d\bar{\psi}^2 \tag{28}$$

again assuming that $G\mu \ll 1$, but now only working to first order in $(G\mu)$. We see that this looks like normal flat space, but note that the angle $\bar{\psi}$ does not run from 0 to 2π but from 0 to $(2\pi - \delta)$ where the defect angle δ is given by

$$\delta = 8\pi G\mu \approx 5.2\text{''} \left(\frac{G\mu}{10^{-6}} \right), \tag{29}$$

so for a string created in a phase transition around the GUT scale this angle is a few arc seconds. What we see here is called a cone-shaped metric.
From this we see that a straight gravitational string acts as a cylindrical gravitational lens with a very characteristic pattern of lensed images. We expect to see two images of a source behind the string separated by an angle of order δ. We can calculate this angle more precisely and this becomes (see also figure 8 [4])

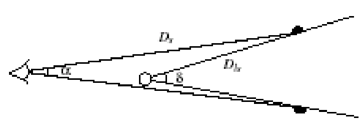

Figure 8: Lensing by a cosmic string [4].

$$\alpha = \frac{D_{ls}}{D_s} \delta \sin\theta, \tag{30}$$

where D_s is the angular diameter distance from the source to us, D_{ls} is that of the source to the lens and θ is the angle between the line of sight and the tangent to the string.
However this picture is complicated by two things. First of all, in the discussion above we have assumed that the string was standing still, which is not a correct assumption. In general, strings are moving and often with quite large velocities. In particular the velocity component perpendicular to the line of sight is important. Assume now that the string is moving with transverse velocity v, then

this will cause the two images to have slightly different red-shifts: the image behind the string will be blue-shifted relative to that ahead of it by a frequency difference $\delta\omega$ of order

$$\frac{\delta\omega}{\omega} \sim v\delta. \tag{31}$$

The same thing also applies to the CMB. If a string moves transversely this would induce a discontinuity in the temperature of the CMB and would be a unique signal of a cosmic string if it were observed. The size of this discontinuity for a string moving with transverse velocity $\mathbf{v}$ is of the magnitude

$$\delta T/T = 8\pi G\mu\gamma v_\perp, \tag{32}$$

where $\gamma = (1-\mathbf{v}^2)^{-1/2}$ (recall that we use units in which c=1, so this makes sense) and $v_\perp$ is the component of the string velocity normal to the plane containing the string and the line of sight. Using this we can derive an upper limit on the value of the parameter $G\mu$ from the WMAP observations. Pogosian et al [10] quote a limit

$$G\mu \leq 1.3 \times 10^{-6}\sqrt{\frac{B\lambda}{0.1}}, \tag{33}$$

where λ is the probability of strings inter-commuting when two strings meet (usually assumed to be 1) and B is the fraction of the CMB power spectrum attributable to cosmic strings, which satisfies $B < 0.1$ as we saw before. Another recent study by Jeong and Smoot [2], where they searched for evidence of a cosmic string in the WMAP data, yielded the tighter bound

$$G\mu \leq 3.3 \times 10^{-7}. \tag{34}$$

This result is however somewhat dependent on assumptions about string evolution.
A second consequence of a moving string is a change in the observed deficit angle α from its form in 30. However I will first discuss the second reason, why the calculation above is not completely correct, before giving the correct version of 30.
This reason is that strings can intercommute as discussed before and that as a result strings can become rather kinky. Therefore if we view a string on a larger scale, the effective energy per unit length U and the string tension T are no longer equal. If we would as an example consider a string which is a zigzag of straight elements each making an angle ψ with a median line then it can be derived [2] that

$$U = \mu \sec\psi, \tag{35}$$
$$T = \mu \cos\psi, \tag{36}$$

where $\sec\psi = 1/\cos\psi$ denotes the secant. Furthermore we note that

$$U > \mu > T \tag{37}$$

and that

$$UT = \mu^2. \tag{38}$$

In fact it can be proven that these last two results also hold for a more general kinky string [2]. As a result of 37 the ordinary gravitational attraction towards the string no longer vanishes. It is given by

$$g = \frac{2G(U-T)}{r}, \tag{39}$$

where r denotes the distance to the string [2]. As this changes the metric it also changes the defect angle and we see that this becomes

$$\delta = 4\pi G(U+T). \tag{40}$$

We can now again calculate the observed deficit angle α and we now also include the effects of the movement of the string. If we assume that the string is moving with velocity v perpendicular to its direction, then we find

$$\alpha = \frac{8\pi GU}{\gamma(1-v_r)} \frac{D_{ls}}{D_s} \sin\theta, \tag{41}$$

where v_r is the radial component of $\mathbf{v}$ [4].
So far we have seen two ways of observing cosmic strings, either by lensing or by a discontinuity in the CMB temperature. Another way of observing them would be through the gravitational radiation they emit. As the gravitational radiation is created from friction when the string moves through space the highest emission will take place on those points of the string where it is moving extremely fast. We know that loops of string undergo periodic oscillations with a period related to the size of the loop. From their dynamics it follows that during each oscillation there will be a few points at which the string instantaneously forms a cusp. In the neighborhood of these cusps the string velocity then approaches the speed of light [4]. This would generate an intense pulse of gravitational radiation and would be beamed into the direction of motion of the cusp. If strings were formed at a GUT phase transition, these pulses should be detected among the most prominent signals seen by the gravitational-wave detectors now in operation or planned, especially LIGO and LISA. From this very same effect a limit, although indirect, on $G\mu$ has already been derived. This is done by looking at the timing of millisecond pulsars. We know that gravitational waves traveling between us and the pulsar would distort space-time and therefore cause random fluctuations in the pulsar timing. However as these have not been observed (in fact pulsar timing is extremely regular), this then puts an upper limit on the amount of gravitational radiation and therefor on $G\mu$. The upper limit on the fraction of the critical density in gravitational waves with periods of up to 10 years becomes [2]

$$G_{gw} \leq 4 \times 10^{-9}. \tag{42}$$

From this we can retrieve an upper bound on $G\mu$ by making an estimate of the gravitational radiation emitted by strings. This suggests a bound

$$G\mu \leq 10^{-7}, \tag{43}$$

however we should be cautious here as this estimate is subject to considerable uncertainties, because it depends on assumptions about the evolution of small-scale structure.

8 Possible observations of cosmic strings

In the previous section, I discussed three ways of observing cosmic strings. In this section I will illustrate two of these ways with a possible observation. First I will discuss possible lensing through a cosmic string, for which I will look at the system CSL-1. Although research by the Hubble telescope in 2006 showed that this is not a case of lensing by a cosmic string, I still want to discuss it, cause it shows how lensing by a cosmic string should look and what we need to watch out for in the future. In the second part of this section I will discuss a cosmic string loop as a source of strange oscillations observed in the data from the well known system Q0957+561. The claim that these oscillations are caused by a cosmic string loop dates back to 2004, but since then this has neither been confirmed nor falsified.

8.1 Observation of cosmic string lensing

In 2003 in an Italian-Russian collaboration Sazhin et al. report the observation of a cosmic string lensing candidate, named CSL-1 (Capodimonte-Sternberg Lens Candidate no. 1)[4]. This candidate constitutes a pair of galaxy images found in the OACDF survey (Osservatorio Astronomico di Capodimonte - Deep Field). The reasons that the two galaxy images are considered to be possibly a sign of cosmic string lensing, are that they are close together (separated by 2", or approximately 20 kpc), they have identical red-shift of $z = 0.46 \pm 0.008$ and their magnitudes in three different frequency bands are equal within errors[4]:

	B	V	R
m_A	22.73 ± 0.15	20.95 ± 0.13	19.67 ± 0.20
m_B	22.57 ± 0.15	21.05 ± 0.13	19.66 ± 0.20

Here m_A denotes the results for image A (the left image in figure 10) and m_B denotes the results for image B. There are multiple explanations for these observations to be considered. First of all this might be an image of a single large galaxy from which the central part of the image is obscured by a dust lane. However it would be a remarkable coincidence that the two remaining visible parts of the galaxy are so similar. Furthermore this is ruled out by examining the spectral profile.
A second possibility which is harder to rule out is that what we see are two

different, almost identical galaxies who just happen to be seen close together. However this also would be quite remarkable.
Furthermore the lensing might be due to a more conventional object. However in that case it is unlikely to get two similar images and if it is lensing by a galaxy it would have to be very heavy and near to us, so it should be easy to detect. But no such galaxy is seen. Therefore Sazhin et al conclude that the most likely explanation is lensing by a cosmic string. If so we can estimate how large GU has to be in order to get the separation that is observed. This limit becomes

$$GU \geq 4 \times 10^{-7}, \tag{44}$$

where we have assumed that the other factors in (41) are of the order of unity. If they are much smaller GU has to be even larger. We see that if this is not to be in conflict with the limits given in (34) and (43) U has to be considerably larger than μ, so the string has to be quite kinky.
If what we see is really lensing by a cosmic string then there should be more lensed objects in the neighborhood. Therefore they looked at images of galaxies in a 4000 x 4000-pixel section of the field (16' x 16') centered on CSL-1 to see if they could find more. From other surveys they estimated that there were approximately 2200 galaxies within a magnitude range of 20 to 24 in the R band. The question is how many of these should be lensed by the cosmic string. Roughly speaking you can say that all galaxies within a strip of width 2δ centered on the string should produce double images. The amount then depends on the length of the string, which of course depends on the amount of kinks on the string. However it is estimated that the number of lensed pairs should lie between 9 (for a straight string) and around 200 (for a random walk). This is in strong contrast with an expectation of no more then two lensed pairs due to conventional lensing objects. In the survey they found 11 likely candidates, so this adds extra weight to the cosmic string lensing scenario. Furthermore as the number of lensing candidates lies close to the number expected for a straight string, we expect the lensed pairs to lie around a straight line. In figure 9 [4] we see the location of the six brightest pairs. They do not show a sharp concentration around a line but neither do they seem to be placed randomly. We see that the pairs 2,3,5 and 6 could be very well fitted to a smooth line and the others might be added with a limited number of kinks.
It is obvious that this observation created much excitement in the community when these results came out in 2003. They were partially responsible for the revived interest in cosmic strings. However a lot of research has been done on this system since then and this ruled out lensing by a cosmic string. The death blow came by a 2006 survey by the Hubble telescope. Here I will present this evidence against the cosmic string scenario.
For this I use a paper by Agol et al from April 2006[12]. They imaged CSL-1 with the Wide-Field Channel (WFC) on the Advanced Camera for Surveys (ACS) on the Hubble Space Telescope (HST). This data presented strong evidence against the cosmic string lensing scenario. In figure 10 [12] we see a direct examination of the image pair and it shows that, although they are similar elliptical galaxies, there principal axes are significantly misaligned. This effect can not be explained

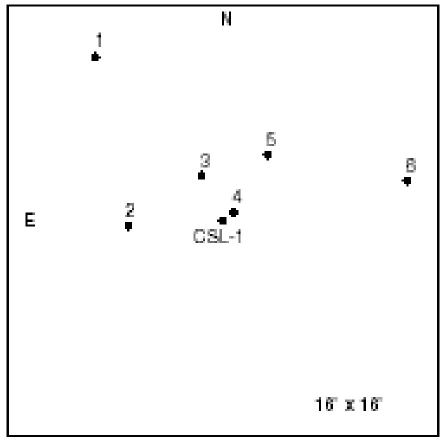

Figure 9: Position of the lensing candidates [4].

by any kind of gravitational lensing. As we already saw that the two galaxies have similar red-shifts this suggests that they are in the early stages of merging. Agol et al have done more tests to show beyond any doubt that the cosmic string scenario is wrong, but I will not quote these results here.
We have seen what cosmic string lensing candidates should look like in the example of CSL-1. Although under closer examination this system turned out to be something else we know have a good idea what to look for. Furthermore by Sazhin et al three other cosmic string lensing candidates (CSL-2 to CSl-4) have been identified but still need to be analyzed. Possibly there could be a better candidate between those.

8.2 Observation of cosmic string loop

8.2.1 Observation of anomalous fluctuations in the system Q0957+561

Here I will discuss a claim by Schild et al. that anomalous fluctuations in the observations of the brightness of the two quasar images in the system Q0957+561 are caused by a oscillating cosmic string loop. In 2004 Schild et al. found anomalous fluctuations of the brightness of the two quasars in this system[13]. This system is a known gravitational lens system and consists of two quasar images separated by approximately 6". It is known that both images are from the same quasar. It was originally discovered in 1979 by Walsh et al. and it was directly understood that it was very important for astrophysics. This was because measurements of the time delay between fluctuations in the two images would allow determination of the Hubble constant from a simple theory, independent of uncertainties that were present in other measurements. So from this time on measurements of the time delay were conducted by amongst others Schild and

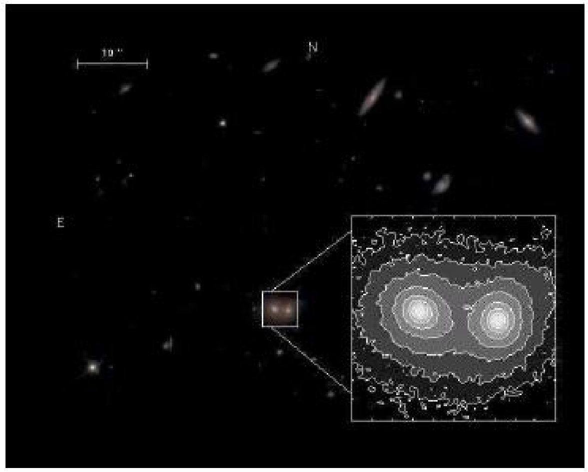

Figure 10: Data from the Hubble Space Telescope on the system CSL-1 [12].

Cholfin (1986). It was then quickly recognized that there were still differences between the time delay corrected brightness curves, but a clear pattern still emerged. The differences were then attributed to microlensing by individual massive objects in the lens galaxy. As this microlensing might allow the detection of any missing baryonic mass in the galaxy, this then justified extensive monitoring campaigns and in the 24 years since its discovery the system has been observed on more than 1500 nights. From this monitoring two principal components of fluctuations in the quasar's brightness have been revealed. There is a component due to intrinsic quasar brightness fluctuations, first seen in image A and then seen 417.1 days later in image B, and a microlensing component arising in only one image component due to individual stars along the line of sight. Schild et al. then produced evidence for a possible third component of quasar brightness fluctuations, seen in the combination of figures 11 and 12[13].

These figures plot the measured brightness of the quasar images as obtained from measurements between 1994 and 1996. In the first figure no corrections have been made for time delay and the plotted symbols are about the size of the error bars previously established (0.006 and 0.007 mag for the images of A nd B respectively). We see here the unexpected result that for approximately 400 days an oscillation of 4% amplitude and a periodicity of approximately 100 days was observed. We see that the amplitude of these fluctuations is well above the error in the measurements. However we see that the correlation between the two images for 0 lag is not perfect, as there are multiple processes simultaneously causing brightness fluctuations. It is known that microlensing can impose a randem pattern of fluctuations with durations ranging from 1 day to decades. However it is unlikely that microlensing could explain the observed fluctuations entirely, especially as it would be unexpected for them to be apparently in phase.

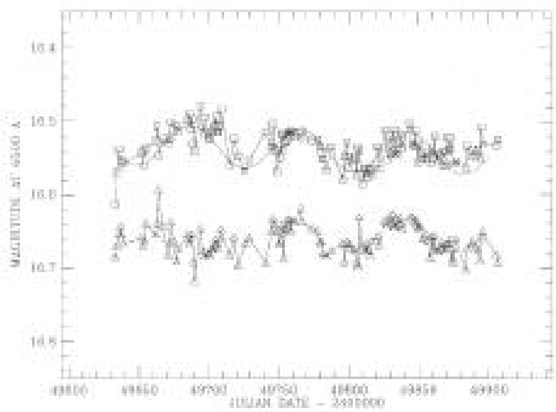

Figure 11: Brightness of the two quasar images displayed with no time delay. The upper picture shows the brightness of image A fitted to a sign curve, the bottom picture shows the brightness of image B. The axis show the magnitude at AT 6500 on the vertical axis and the Julian Date on the horizontal axis. [13].

However there is not yet a statistical test for the significance of this simultaneity. If on the other hand the fluctuations would be intrinsic to the quasar, they would also have to appear in the observations of image B 417 days earlier (left part of figure 12) and of image A 417 day later (right part of figure 12). However in these figures we see that the observed pattern is probably not present there, so it is not probable that the oscillations are intrinsic to the quasar.
The important point of these observations is that they appear simultaneously. If they were either caused by fluctuations in the quasar's intrinsic brightness or were produced in the proximity of the lens galaxy, they should be seen with a large time delay. Only if they are caused locally (i.e. between the lens galaxy and the observer, but close to the observer), then can these fluctuations be observed simultaneously. Next I will discuss two possibilities of objects that could cause these fluctuations. First I will discuss a cosmic string and secondly I will discuss the possibility of the fluctuations being caused by orbiting of binary stars. However this will turn out to be unlikely because this would require very large masses for the stars.

8.2.2 Explanation by a string loop

I start by looking at some characteristics of cosmic strings and especially of the string loops. If a loop is created at time t, the typical loop length will be $l \sim \alpha t$, i.e. about α of the horizon size t. Usually we assume that during string evolution loops constitute a fixed part of the total string network; which results

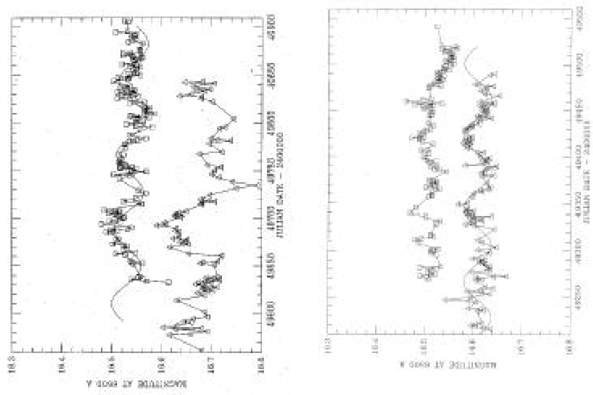

Figure 12: Brightness of the two quasar images displayed with time delay. In the left image the bottom picture shows the brightness of image B 417 days later. In the right image the top picture shows the brightness of image A 417 days earlier. The axis show the Magnitude At 6500 on the vertical axis and the Julian Date on the horizontal axis. [13].

in the loop number density

$$n_l(t) \sim \alpha^{-1} t^{-3}. \tag{45}$$

We can determine α by the gravitational back reaction and find $\alpha \sim k_g G\mu$, where $k_g \sim 50$ is a numerical constant. For a GUT string this leads to $\alpha_{GUT} \sim 10^{-4}$. We know that the loops oscillate and lose their energy and this is emitted for the largest part through gravitational radiation. Furthermore it has been determined [13] that for a loop of length l the oscillation period is $T_l = l/2$ and the lifetime is given by $\tau_l \sim l/(k_g G\mu)$. Now we can derive what parameters have to hold for a cosmic string loop to create the oscillations we saw. De Laix and Vachaspati showed in 1996 that in the simplest case of a circular loop with oscillations reduced to variations in the radius, this does not effect the image brightness of a point source if the loop does not overlap the source [13]. Therefore Schild et al take an asymmetric loop to explain the observed oscillations. They consider a maximally asymmetrical string configuration in the form of a rotating double line segment of length $2R$ transverse to the line of sight. They choose the representation

$$\begin{aligned} x_1 &= R\cos(t/R)\sin\sigma \\ x_2 &= R\sin(t/R)\sin\sigma \\ x_3 &= 0 \end{aligned} \tag{46}$$

which forms a particular solution from the family of known solutions to the string equations. In here x_1 and x_2 are the coordinates in the lens plane, x_3

is directed towards the observer and σ is the position on the string varying between 0 and 2π. Now we can obtain the lens equations from a general result of de Laix and Vachaspati and after some calculations Schild et al. derive for the solution given in 46

$$\begin{aligned}\frac{D_l}{D_s}\tilde{y}_1 &= \tilde{x}_1 - q_s R^2 \mathrm{sgn}(\tilde{x}_1) \times \\ &\times \sqrt{\frac{\sqrt{(R^2+\tilde{x}_1^2+\tilde{x}_2^2)^2 - 4R^2\tilde{x}_1^2} - R^2 - \tilde{x}_2^2 + \tilde{x}_1^2}{2((R^2+\tilde{x}_1^2+\tilde{x}_2^2)^2 - 4R^2\tilde{x}_1^2)}}, \qquad (47)\\ \frac{D_l}{D_s}\tilde{y}_2 &= \tilde{x}_2 - q_s R^2 \mathrm{sgn}(\tilde{x}_2) \times \\ &\times \sqrt{\frac{\sqrt{(R^2+\tilde{x}_1^2+\tilde{x}_2^2)^2 - 4R^2\tilde{x}_1^2} + R^2 + \tilde{x}_2^2 - \tilde{x}_1^2}{2((R^2+\tilde{x}_1^2+\tilde{x}_2^2)^2 - 4R^2\tilde{x}_1^2)}}, \qquad (48)\end{aligned}$$

where $q_s = 8\pi g\mu\frac{D_{ls}D_l}{D_s R}$, D_s, D_l and D_{ls} denote the distances from us to the source plane, to the lens plane and from the source to the lens plane respectively, $\tilde{x}_1 = x_1\cos(t/R) + x_2\sin(t/R)$, $\tilde{x}_2 = x_2\cos(t/R) - x_1\sin(t/R)$, $\tilde{y}_1 = y_1\cos(t/R) + y_2\sin(t/R)$, $\tilde{2}_2 = y_2\cos(t/R) - y_1\sin(t/R)$ and y_1, y_2 are the coordinates in the source plane [13]. As it can be shown that all the terms under the root signs are non-negative I will not elaborate on this and go on. It is known [13] that the magnification of a point-like source by such a string is

$$m = |1 - q_s^2R^4(x_1^2+x_2^2)/((R^2+x_1^2+x_2^2)^2 - 4R^2(x_1\cos(\frac{t}{R}) + x_2\sin(\frac{t}{R}))^2)^{3/2}|^{-1}. \quad (49)$$

Schild et al. now assume $q_s \sim 1$ and $R_y = RD_s/D_l \ll \rho = (y_1^2+y_2^2)^{1/2}$ and then approximate the solution of the lens equations (47,48) to obtain the magnification

$$m = 1 + \frac{q_s^2R_y^4}{\rho^4} - \frac{4q_s^3R_y^6}{\rho^6} + \frac{3q_s^2R_y^6}{\rho^8}((y_1^2-y_2^2)\cos(\frac{2t}{R}) + 2y_1y_2\sin(\frac{2t}{R})), \quad (50)$$

and from this they derive the amplitude of source brightness fluctuations to be

$$\Delta m \approx \frac{6q_s^2R_y^6}{\rho^6} \approx \frac{384\pi^2G^2\mu^2\theta_R^4}{\theta_I^6}, \quad (51)$$

where $\theta_I = \rho/D_s$ is the angular impact distance of the line-of-sight with respect to the center of the loop and $\theta_R = R/D_l$ is half of the visible angular size of the loop. As only a few oscillations were observed it is logical to assume that the loop is moving, however the number of oscillations also restricts the transverse velocity. Therefore Schild et al. restrict the transverse velocity of the loop to the values v_1, $v_2 \leq 0.1$, but leave a considerable freedom for the velocity component v_3 along the line of sight. However the only change this brings to the result above is a change in the parameter q_s by $q_s^* = q_s(1+v_3)^{-1}$.
Next we need to see what parameter values we need to reproduce the observed

data. We know that $T \approx 100$ days and that this is related to the string length $l = 2\pi R$ by $R = T_l/\pi$. If we take the relativistic motion of the string into account we get $T_l = T(1 - v_3^2)^{1/2}$. Using all this we can rewrite equation 51 as

$$\Delta m \approx 5.6 \left(\frac{\theta_I}{3"}\right)^{-6} \left(\frac{\theta_R}{1.5"}\right)^{4} \left(\frac{\mu}{10^{22}\mathrm{g/cm}}\right)^{2} (1 + v_3)^{-2}. \tag{52}$$

As we have seen that the amplitude of the oscillations in both images is almost equal this suggests that the loop should fly close to the mid-point and therefore we should have $\theta_I \approx 3"$. Furthermore Schild et al. put estimates on the different velocities involved. They take $v \approx 0.7$, $v_1 \approx 0.03$ and $v_2 \approx 0.11$. Next they state that in order to have quasi-sinusoidal variations, θ_R must be considerably smaller that the angular distance between the two images because otherwise sharp spikes and/or discontinuities will form in the dependence of the brightness upon time. Furthermore they do not want θ_R to be to small either to avoid the string mass having to become to large to be able to obtain the correct brightness fluctuation from equation 52. Therefore they take $\theta_R \approx 1.5"$ and then find that for $\Delta m \approx 0.04$ the value for μ to be $\mu \approx 4 \times 10^{20}$g/cm. In the units I used before this would become $G\mu \approx 3 \times 10^{-8}$ and this seems rather small for GUT strings. However Schild et al. claim that with more accurate parameter values obtained by numerical solutions from the equations 47-49 and without the assumption $\sqrt{y_1^2 + y_2^2} \gg R_y$ one finds the value $\mu \approx 8 \times 10^{21}$g/cm or equally $G\mu \approx 6 \times 10^{-7}$. This seems to be more in the range of GUT strings but there is still a large error in this figure. Schild et al. also made a plot of the predicted value of the brightness of the quasar of there model against the observed data and this is given in figure 13[13]. We see there is a quite good agreement.

8.2.3 Explanation by a binary system

Besides an explanation by a cosmic string loop, Schild et al. also consider the possibility of lensing by a binary system. I will here again follow their discussion. They consider a binary system of two equal point masses M orbiting their center of mass with period T. If we define r to be half of the distance between the masses and $\omega = \pi/T$ we then get that the magnification of a point-like source becomes

$$m = \left| \frac{1 - q_b^2 r^4((x_1^2 + x_2^2 + r^2)^2 - 4r^2(x_1 \sin(\omega t) - x_2 \cos(\omega t))^2)}{((x_1^2 + x_2^2 + r^2)^2 - 4r^2(x_1 \cos(\omega t) + x_2 \sin(\omega t))^2)^2} \right|^{-1} \tag{53}$$

where $q_b = 8GM \frac{D_{ls} D_l}{D_s r^2}$ [13] and the others are as in the previous part. Now we can proceed in a similar way as before to obtain an approximate formula for the magnification

$$m = 1 + \frac{q_b^2 r_y^4}{\rho^4} - \frac{4 q_b^3 r_y^6}{\rho^6} + \frac{6 q_b^2 r_y^6}{\rho^8}((y_1^2 - y_2^2)\cos(2\omega t) + 2y_1 y_2 \sin(2\omega t)) \tag{54}$$

where $r_y = rD_s/D_l$. If we now define $\theta_r = r/D_l$ and use $GM = 4\pi^2 r^3/T^2$ we can approximate the amplitude of fluctuations if we assume a circular motion

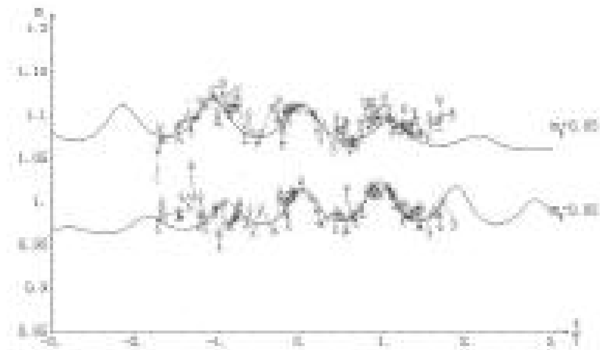

Figure 13: Brightness of the two quasar images displayed with no time delay and fitted two the expected data for lensing by a cosmic string loop (line). The upper picture shows the brightness of image A moved up by 5%, the bottom picture shows the brightness of image B moved down by 5%. The axis show the relative brightness on the vertical axis and the time on the horizontal axis. [13].

in the binary system to be

$$\Delta m \approx \frac{12 q_b^2 r_y^6}{\rho^6} \approx \frac{1.2 \times 10^6 \theta_r^8 D_l^4 D_{ls}^2}{T^4 \theta_I^6 D_s^2}. \tag{55}$$

We can now proceed from this and find in a similar way as before

$$\Delta m \approx 0.04 \left(\frac{T}{100\text{days}}\right)^{-4} \left(\frac{\theta_r}{1.5"}\right)^8 \left(\frac{\theta_I}{3"}\right)^{-6} \left(\frac{D_l}{1.2\text{pc}}\right)^4. \tag{56}$$

Now again assumptions on the parameters are necessary. Schild et al. take again $\theta_I \approx 3"$ and decide that the orbital radius should be $\theta_r D_l \approx 1.8$a.u. If we want fluctuations of the order of 4% the we find a mass $M \approx 78$ solar masses. However at such distances, note that $D_l \approx \frac{1.8\text{a.u.}}{\theta_r}$, a system this large should be easily observable. Schild et al. state that the distance to the binary system could be larger but this would also require an increase in the mass so this is not a solution to this problem.

Therefore Schild et al. conclude that the most likely explanation for the observed pattern is a cosmic string loop as it would not be logical that such a heavy binary system would not have yet been detected. However they state that much more research is necessary especially because a lot of parameters are still unknown and also because more similar events should be observed before a statistical analysis can be done on the predicted string loop size, which is much smaller then would be expected.

Furthermore I want to express caution about another point in there argument. They claim that it is unlikely for the observed fluctuations to be caused by microlensing because they are similar and in phase. However as there is not yet a test for how unlikely this is, this is a very weak assumption. Given the amount of time of observations of this system, maybe we have just noted a slightly unlikely event, just as we saw in the first part of this section were it was assumed unlikely that we had two different galaxies which were very similar and very close, but this did turn out to be the case. Therefore unless this phenomenon is observed more often or a statistical test can be devised on the likelihood of its creation by microlensing, this claim is still very uncertain.

9 Conclusion

We have seen that cosmic strings can be formed as relics of early phase transitions in the universe and could therefore tell us something about the early universe. We saw that once they were created, they do not die out nor come to overpopulate the universe. Therefore if they were created at the GUT scale phase transitions, we should still be able to observe them today. We then found that the major reason for interest in cosmic strings in the eighties and nineties, cosmic strings as a source of geometric perturbations that seeded structure formation, has been ruled out by observations of the CMB. However we have seen good reasons for the rekindle of interest. The link with fundamental strings has tightened and cosmic superstrings could be our only way of observing string theory. Furthermore there are good indications that cosmic strings appear in supersymmetric grand unification theories as we have seen from the research by Rocher et al. On the observational side we have seen that the current bounds do not yet rule out GUT-scale cosmic strings and one possible direct observation of a cosmic string is still open. Therefore the field of cosmic strings has regained its position as a field of interest and much more work is still needed.

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